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Project Report

Network Based Modelling for the Spread of Scientific Ideas

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1 Abstract

Using simulations, we investigated the changes in networks with nodes that held different ‘scientific ideas’ and influenced each other through a complex contagion mechanism. This was divided into two parts: by varying the network structure and observing features of the distribution of ideas in networks, and by varying the starting idea distributions of the nodes and observing features of the network structures. The update step included a rewiring probability, a complex contagion threshold, and a probability of innovation (producing a new idea). We found that both structures and idea distributions influenced each other’s features. Values of the update parameter also played a role.

2 Individual contributions

This report represents a group effort by all members.

3 Introduction and Motivation

We live in a time in which aspects of our lives and of our world become more and more connected with each other. To understand one aspect, we must understand many other aspects, which all together form a large, complex system, or network. Globalization has changed the meaning of ‘distance’ and communication, allowing seemingly unrelated and unconnected individuals to share more than they ever could before. Fields of studies are overlapping with each other, creating new interdisciplinary domains and building a diverse playground for the sharing of ideas. But how do ideas spread? This question is especially interesting with the increase of technology that allows us to record and visualize the networks that connect individuals and ideas, and in particular the ability to ‘see’ how they change. This can lead to insight about why and when ideas spread and into the complexity of the matter. Research has shown that not only does the nature of information or innovation influence the diffusion of it, but also that the structure of a network influences the diffusion dynamics. Here, we try to simulate the spread of scientific ideas in different networks. The model presented is based on two studies: one that investigated critical parameter values for complex contagion (Centola and Macy, 2007) and another that investigated critical values of a rewiring parameter (Holme and Newman, 2006).

3.1 Fundamental Questions

The main goals of this simulation study are to investigate how network structure influences the distribution of ideas, and how the distribution of ideas influences network structure. For a list of the terminology that will be used throughout the paper, please refer to Table 1. By varying three parameters - probability of rewiring, rate of innovation, and complex contagion threshold (ϕ , α , and δ respectively) - and using different network structures and idea distributions, we observed how network structure characteristics changed. Similarly, we observed how the distribution of ideas and the connections between them changed. Below we describe our questions more specifically.

3.1.1 Effects of Network Structure on Idea Distribution

Given a starting network and a random idea distribution, how do different network structures affect the distance between nodes that have the same idea (intra-idea distance)? How do they affect the neighbourhood index? How do they change the emergence of dominant ideas and their time of dominance? How do their effects depend on the values of ϕ , α and δ ?

More rigid network structures (those with less ‘randomness’, such as the caveman

Table 1: Some definitions

	Definition
Neighborhood index	The fraction of the holders of the same idea who are neighbor as well averaged over all ideas.
Intra-idea distance	The average distance between holders of the same idea in the network.
Dominant frequency	The frequency of the dominant idea in the network at each time steps of the simulation.
Average dominance time	The average number of time steps in which the dominant idea keeps its dominance.
Novelty index	The fraction of newly generated ideas.
Average shortest path	The average number of steps along the shortest paths for all possible pairs of network nodes.
Clustering coefficient	a measure of degree to which nodes in a graph tend to cluster together.
Degree of connectivity	The number of edges incident to the vertex.
Connected component	a subgraph in which any two vertices are connected to each other by paths, and which is connected to no additional vertices in the supergraph.
Diameter of network	the longest of all the calculated shortest paths in a network.

and the small world networks) may make it more difficult for ‘like-minded’ nodes (that is, nodes with the same idea) to connect and may thus have smaller neighbourhood indexes and larger intra-idea distances than the more random network structures (such as the random and scale-free networks). Their effects may be more sensitive to the values of ϕ (because this affects how likely it is for their structure to change) and to values of δ because being restricted to a more closed group of nodes makes it difficult to reach a threshold necessary to become similar to surrounding nodes. Values of α may decrease the neighbourhood indexes by creating larger diversity among neighbouring nodes.

If more rigid network structures do make it more difficult for like-minded nodes to connect, then it would be more difficult for a dominant idea to emerge in these networks. These effects may be smaller for larger values of ϕ since these values would allow for the structure to change more. For larger values of ϕ therefore one could expect that the effects of the network structures on the characteristics of the idea distribution are more similar since allowing to change the structure removes their

initial influence.

3.1.2 Effects of Idea Distribution on Network Structure

Given a starting idea distribution and a caveman network structure, how do different idea distributions affect the average path length and diameter of the network? Do they change the number of connected components in the network? Do clusters form differently, and how does the clustering coefficient change? What does the distribution of node degree looks like? How do these effects depend on the values of ϕ , α , and δ ?

If the starting idea distribution is parallel to the caveman network structure (see Table 1 for definitions), then like-minded nodes will already be connected and thus rewiring will probably not change much of the average path length, nor will it change the diameter. Similarly, the clustering coefficient will remain high just like the starting value. The distribution of the node degree will also not change (nodes will have one of two values for their degree). In other words, if the idea distribution is parallel to the network structure, the structure will not change much. Changing ϕ and δ will not change these effects, and perhaps increasing α will decrease the clustering coefficient and will increase the number of connected components because nodes will disconnect from nodes with novel ideas and will rewire to nodes with the same idea.

If the starting idea distribution is random, then the network's caves will disintegrate as nodes will rewire with other nodes outside of their caves. This will change the degree distribution by increasing its variance (nodes will have a variety of different degree values). Depending on the value of δ this disintegration may be reduced because nodes have a higher chance of forming dominant ideas within caves. Similarly, increasing ϕ will increase the disintegration of caves. Thus for this idea distribution the parameter values may play a larger role.

If the starting idea distribution is anti-parallel, nodes within each cave will initially be connected with nodes that do not hold the same idea as them. Therefore the threshold δ will not be met in order for nodes to change their ideas, and they will rewire with other nodes outside of their cave. The clustering coefficient and the number of connected components will likely decrease, and the diameter and the average path length will decrease as well since the structure will change significantly. The degree distribution will increase in variance. Increasing ϕ and α and decreasing δ will probably increase the magnitude of these effects. Thus, having an anti-parallel idea distribution will probably display the most changes in the characteristics of the network structure that are in question.

4 Description of the Model

The model used here is based on a study by Holme and Newman (2006). Each simulation begins with a specified network structure as well as a distribution of the ‘idea’ (or state) of the nodes. At each time step a node either changes its idea to that of one of its neighbours’ ideas if its frequency surpasses a defined threshold, rewires to connect with a node that has the same idea, or generates a novel idea (this is the innovation parameter).

Given the network structure and node states, three parameters are introduced: ϕ (probability of rewiring), α (probability of innovation), and δ (complex contagion threshold). As in Holme and Newman (2006), ϕ is a value from zero to one, and is the probability that one of the edges of a randomly chosen node i will be changed to connect to another node j that i is unconnected with. We decided to add one more criterion to this definition: node j is a node that has the same idea as node i . This encourages the simulations to reflect a common tendency of individuals to seek out others who think like them. If there is no such node j , then the chosen node will do nothing.

At each time step a node may ‘come up with a new idea’ with a probability of α . This value is small to reflect that novel ideas are not frequently observed.

We introduced a node threshold δ to the general model in order to investigate the behaviour of complex contagion as opposed to simple contagion. This was motivated by a study by Centola and Macy (2007). Simple contagion is well suited for modeling the spread of diseases since they may often be passed on by a single contact with an infected individual. However, as our intuition may suggest, other kinds of innovations raise questions about the legitimacy and credibility of the innovations themselves, and may thus require exposure to multiple sources of the innovation. This is called complex contagion. Models usually represent it in two ways regarding the number of connected sources that must have adopted an innovation in order to influence the agent (or individual) in question: it is either a fixed number (≥ 1) or a fraction (between zero and one, inclusive). Our simulation used the second formulation since the degree of nodes varied across network structures.

4.1 Networks

For the purposes of our simulations, we used four network structures. *******NETWORK STRUCTURE FIGURE****** illustrates them. These structures can be characterized by properties such as average shortest path lengths, clustering coefficients, and the degree of connectivity (see Table 1 for definitions). Below are short descriptions of each network structure. In order to compare between different structures, the network structure parameters were chosen such that the mean degree

of the networks were similar (approximately 30). For further details about parameter values, see Table 2.

Table 2: Table of parameters used in simulations.

Network	Parameter	Value		
	ϕ	0.1	0.3	0.5
	α	0.01	0.05	0.1
	δ	0.001	0.01	0.05
	n	1000		
	t_{end}	1000		
Caveman	m	40		
	p	40		
Random (Erdos-Renyi)	$prob$	0.025		
Scale free	m_0	24		
	m_1	12		
Small world	κ	24		
	β	0.1		

4.1.1 Random Graph

Random graphs have a short average path length. The graph is defined by the total number of nodes, and by the probability of any two nodes to be connected. Thus all connections are random. They typically have a small clustering coefficient. Here we used a variant of the Erdős-Rényi random graph model (Erdős and Rényi, 1960) as implemented by Brugger and Schwirzer (2011).

4.1.2 Caveman Graph

The caveman structure, as defined by Watts (2003), has k isolated and fully connected ‘cliques’ from which one link is changed to connect one clique to another, rendering all cliques to be connected. . Thus, relative to random graphs, they have a high clustering coefficient and a large average shortest path length.

4.1.3 Small World Graph

Small world graphs have characteristics that lie in between random graphs and highly clustered graphs (such as caveman graphs): they have a high clustering coefficient similar to the latter, but also have a small average shortest path similar to the former. Many real-world networks have been observed to have a small world structure, and

thus we included it in our simulations. Here we used the graph as defined by Watts and Strogatz (1998), and implemented by Brugger and Schwirzer (2011).

4.1.4 Scale-Free Graph

Scale-free network structures are often found where new nodes are constantly being added, and they are connected to already well-connected nodes. Such a structure displays a scale-free power-law distribution of the degree (connectivity) of nodes. Thus, there are few nodes that are highly connected, and more nodes that are moderately or mildly connected. Compared to random graphs, they have a smaller average shortest path. This graph was implemented as defined by Barabási and Albert (1999) and implemented by Brugger and Schwirzer (2011).

4.2 Ideas

After choosing a starting structure for our model, we then chose a distribution for the starting ideas (states) of nodes. Each node was randomly assigned one of these ideas, thus allowing for multiple nodes to have the same idea. For the caveman structure, however, there were two other options: to either distribute the starting ideas ‘parallel’ to the structure, i.e. such that all nodes in a cave shared the same idea, or ‘antiparallel’ such that all nodes in a cave had a different idea. This was used for the analysis of the effect of the idea distribution on the network structure. *****FIGURE OF IDEA DISTRIBUTION***** illustrates these idea distributions. Why was the caveman structure investigated? There were two reasons: firstly, it was straightforward how to define idea distributions that are in accord or disaccord with the connections of nodes in the network. Secondly, in the scientific community research teams may often be made up of closely-connected members that are only weakly connected to other research teams, and within these teams, members may or may not be interested in the same ideas for research.

As previously mentioned, each node had a small probability α of adopting a novel idea from a virtually unlimited number of new ideas.

5 Implementation

Our simulation comprises mainly two parts, the phase 1 corresponding to the study of the effect of network structure on to the opinion distribution and phase 2 where we study how the distribution of the ideas affect the topology of the network. These two phases are implemented in the same MATLAB file **mainscript.m**. Implementation of both phases is divided in three steps.

Step 1: Structure network is chosen and we generate the adjacency matrix corresponding to that structure. Also is generated the initial idea distribution.

Step 2: The updating processes is done using the chosen structure network.

Step 3: At this step a series of functions are called to getting the results.

5.1 Step 1: Generating structure networks and initial idea distribution

For the first phase one of the functions **step1_scalefree**, **step1_caveman**, **step1_smallworld** and **step1_randomgraph** is called, depending in our choice. Each function is found in the MATLAB files **step1_scalefree.m**, **step1_caveman.m**, **step1_randomgraph.m** and **step1_smallworld.m** correspondingly. Each of these functions generates the adjacency matrix corresponding to the chosen network structure. Also in this step the initial idea distribution is done, in the first phase we have a random initial distribution.

For the second phase we just generate the adjacency matrix for the caveman structure network, and then we chose among three different initial idea distributions: random, parallel or nonparallel. After choosing one of them the initial idea distribution vector is generated.

5.2 Step 2: update

This step is the same for both phases.

The updating process is implemented in the file **step2.m** and done by function **step2** which requires the next parameters:

- **t_end**: number of iterations
- **phi**: network reorganization rate
- **alpha**: innovation rate
- **mat**: initial connectivity matrix
- **vec**: initial idea vector

- **p**: initial number of opinions

And has as outputs:

- **mat**: connectivity matrix after simulation
- **vec**: idea vector after simulation
- **dominant_freq**: the vector holding the frequency of dominant idea
- **most_freq**: the vector holding the index of dominating idea in each time

The function **step2** in which the updating is executed follows the next algorithm:

Algorithm 1 Update process

```

for each of the iterations do
  choose a node  $x1$  in the network at random
  generate a random number  $a1$ 
  if  $a1 < \phi$  then                                 $\triangleright$  With probability  $\phi$  we reorganize the network
    eliminate the connection between  $x1$  and one of his neighbors with different idea
    select another node at random among the nodes with the same idea than  $x1$  and
    that are not already neighbors of  $x1$  and create a connection between them
  else change the idea of  $x1$  to one of the ideas of his neighbors which meet the threshold
  end if
  choose a node  $y$  in the random to come up with a nobel idea
  generate a random number  $a2$ 
  if  $a2 < \alpha$  then  $y$  comes up with a new idea     $\triangleright$  With probability  $\alpha$   $y$  comes up
  with a new idea
  end if
end for

```

5.3 Step 3: Getting results

In the phase 1 we want to observe the influence of a certain network structure on the distribution of ideas after the up dating process (step 2). For this is necessary to search for parameters that reflects the final distribution of ideas. In this study we observe the following parameters:

- **n_index** which is the average neighbor index of the network
- **intra_idea_distance** which is the average of the average shortest distance between agents holding the same idea

- **average_dominance_time** which is the average of the dominance time for different dominance periods

All this parameters are obtained in **step3a.m**, **step3b.m** and **step 3d.m** which are called in **main_script.m**.

In phase 2 we want to find out how the distribution of ideas change the network structure, for this reason we just look at a fixed structure network, that would be the caveman structure. After step 2 we want to see how the initial structure was modified by the rewiring process, in order to see this we look for the next parameters

- **clust_coeff**: the clustering coefficient of the network
- **[dgr, frq]**: degree vector and its corresponding frequency vector
- **average_path_length**: the average path length for the graph
- **graph_diameter**: outputs the diameter of the graph

6 Simulation Results and Discussion

For the first section of the results, ideas were assigned randomly onto the nodes of four different network structures (caveman, small world, random, and scale-free). Simulations were run for each network structure for 27 different parameter combinations (three values for each of ϕ , δ and α). The effects of each network structure on the idea distribution was evaluated on five features: the average dominance time of dominant (i.e., most frequent) ideas, the novelty index, the intra-idea distance, the neighbourhood index, and the frequency of dominance of an idea (see ***TABLE OF DEFINITIONS***). We also investigated how these effects varied with the three parameters (ϕ , δ , α).

The second section of the results investigated the opposite direction of effects: how changing the idea distribution affected the resulting network structure. To do this we applied three different idea distributions (random, parallel, and antiparallel to the structure) to a caveman network (see ***TABLE OF DEFINITIONS*** with 40 caves. There were again 27 different parameter combinations for each idea distribution, as in the previous analysis. The five features of the network structure that we evaluated were: the clustering coefficient, the degree distribution of the nodes, the number of connected components, the average path length, and the network diameter.

We found that there were different behaviours for some features of the idea distribution depending on the network structure, and that there were different behaviours for the structure features depending on the idea distribution. Both of these results were at least partly influenced by the values of ϕ , δ , and α .

6.1 Effects of Network Structure on Idea Distribution

The results of the average dominance time and the novelty index were too dependent on the parameter values and were not included; the network structure does not seem to play a strong enough role over all parameter combinations in influencing these two features. It is not surprising, however, that the novelty index was highly dependent on the value of α , but it is not so intuitive why the average dominance time was not correlated with α at all: increasing the number of novel ideas decreases the possible number of ‘followers’ for already-established ideas, which should affect the frequency of dominance and therefore the dominance time. Perhaps the value of α would need to be increased to observe this behaviour.

Furthermore, we observed two different results for the intra-idea distance and the neighbourhood index: those from the caveman and small world structures, and those from the random and scale-free structures. All three parameter values affected these results.

6.1.1 Intra-Idea Distance and Neighbourhood Index

For any parameter combination, the caveman and small world structures resulted in larger intra-idea distances (respectively) than those of the random and scale-free structures, which were very similar to each other (**FIGURE 1.1**). Interestingly, a similar influence was found on the neighbourhood index: the caveman structure held the largest index regardless of parameter combination, followed by the small world, random, and scale-free structures (**FIGURE 1.2**). It seems that the caveman structure encourages nodes to be within the direct neighbourhood of like-minded nodes (nodes with the same idea) and at a farther distance from like-minded nodes that are not in their direct neighbourhood, whereas the random and scale-free structures have a tendency to keep like-minded nodes in each other's direct neighbourhood but to also keep those like-minded nodes not in their direct neighbourhood at a shorter distance. The difference in intra-idea distance between network structures could be a result of the general smaller average path distance that random and scale-free networks have as compared to the caveman and small world graphs.

Dependence on α For all network structures, increasing the level of innovation α decreased the intra-idea distance. This effect was more pronounced in the scale-free and random networks (**FIGURE 1.5-1.8**). Does innovation bring people together in scientific communities? Perhaps, and perhaps not. It is possible that these results are due to the construction of our model: because the mechanism of influence involves complex contagion, new ideas would not be able to spread (since they are held only by the originator) and therefore these ideas would technically have an intra-idea distance of zero.

Higher values of α also increased the neighbourhood index for all network structures. This correlation was again most visible for the scale-free network, followed by the random network (**FIGURE 1.9-1.12**). One possible explanation for this correlation is that the more novel ideas nodes create, the less likely it is that the contagion threshold is met for other ideas, and thus nodes will only be rewiring instead of also changing their ideas. Thus more like-minded nodes will be connected, and the index increases.

Dependence on ϕ By increasing ϕ , the intra-idea distance decreased for all network structures. This correlation is quite an intuitive result since ϕ is the probability of deleting a connection between two nodes with different ideas and the formation of a new connection between two nodes with the same idea, and thus, by increasing ϕ the distance between like-minded nodes decreases. Unlike the α parameter, the effects of ϕ are more pronounced for the caveman and small world structures rather than for the random and scale-free networks (**FIGURES 1.13-1.16**).

On the other hand, the effects of ϕ on the neighborhood index varied between networks. Increasing ϕ increased the neighbourhood indexes of the random and scale-free networks, while it decreased the neighbourhood index of the caveman network and had no correlation with changes in the small world network (**FIGURES 1.17-1.20**).

Dependence on δ Increasing values of the complex contagion threshold δ slightly decreased the intra-idea distance for the caveman and small world network structures (****FIGURE 1.21-1.24**).

No correlation was found between the neighbourhood index of the small world, scale-free, and random network structures and values of δ . However, the neighbourhood index for the caveman network increased as δ increased (****FIGURE 1.25**). This is an interesting result. On one hand, it is intuitive since requiring more like-minded nodes to be in the direct neighbourhood for a node to adopt their idea automatically increases the number of like-minded nodes in the neighbourhood (if the node adopts their idea). However, increasing δ could have the opposite effect: if the threshold is too high, nodes will not adopt their neighbours' ideas and thus the neighbourhood index would remain small. The influence of δ only on the caveman network could be due to this structure's higher degree per node: almost all nodes have the same degree, whereas the other structures have the same average amount but with greater variance.

6.1.2 Frequency of Dominance

This feature is interesting if one considers scientific society. How dominant are dominant ideas in the scientific community? Does the structure of the community influence this dominance? For all parameter combinations and for all network structures in our simulations, the frequency of dominance of ideas increased with time. This may suggest that none of these structures impede the adoption of new ideas. More surprisingly, however, the increase in dominance frequency progressed more quickly for the caveman network structure (**FIGURE 1.3**). Could it be that this structure encourages nodes to adopt dominant ideas more easily? This may not be generalizable because the results are quite sensitive to parameter values due to the stochasticity of the simulations. For example, ****FIGURE 1.4** shows a different combination of parameters, and here the faster increase in the dominance is not observed for the caveman structure.

6.2 Effects of Idea Distribution on Network Structure

We observed that the results of applying a parallel idea distribution on the features of the network structure were quite different to the results of the random and antiparallel idea distributions. It is interesting that for all parameter combinations and for each idea distribution, the network always remained fully connected. This could be because of the nature of our model: in order to disconnect with one node, there must be another node with the same idea to connect with. The nodes in a caveman network structure are, in a sense, ‘saturated’ since they are fully connected to their caves, and thus they remain connected to at least one of their original cave members. While the networks always remained fully connected (**FIGURE 2.4****), the remaining features changed. The parameter α did not change these effects for any of the idea distributions (see ***FIGURES 2.6-2.14***). Given these simulation results, perhaps a caveman-structured scientific community would also manage certain levels of innovativity without changing its fundamental structure.

6.2.1 Clustering Coefficient

The clustering coefficient of resulting networks varied with the type of idea distribution. When nodes in the same cave shared the same idea (parallel distribution), the clustering coefficient was larger (***FIGURE 2.3***). Additionally, the clustering coefficients for both the random and the anti-parallel idea distributions decreased in a similar manner regardless of the parameter combinations. Both of these results are intuitive since less rewiring would have taken place for the parallel distribution case, and the high clustering coefficient of the caveman structure would have been conserved, whereas more rewiring would have occurred for the two other distributions, thus decreasing the clustering coefficients.

Dependence on Parameters Increasing values of the rewiring parameter ϕ decreased the clustering coefficient for all starting distributions, especially for the random and antiparallel distributions (***FIGURES 2.21-2.23***). This again is intuitive since ϕ increases the chances of changing connections in a highly clustered network. Increasing values of δ , however, only slightly increased the clustering coefficient when using a parallel idea distribution (***FIGURES 2.30-2.32***).

6.2.2 Average Path Length

The average path length of resulting networks was larger when a parallel idea distribution was used (***FIGURE 2.1****). This is not surprising, since the structure of the caveman network was more preserved (because most nodes were already connected to

like-minded nodes and did not need to rewire), and its average path length is larger than that of more randomized networks, such as the ones resulting from a larger amount of rewiring.

Dependence on Parameters Increasing ϕ , regardless of the idea distribution, decreased the average path length. This effect was more pronounced for the random and anti-parallel idea distribution cases (**FIGURE 2.15-2.17**). This is another intuitive result since more rewiring naturally disturbs the rigid structure of a cave-man network, thereby decreasing its large average path length; rewiring also occurs less frequently in the parallel idea distribution case because most nodes are already connected to like-minded nodes. Changing values of δ (which requires more than one neighbouring node to have the same idea in order to influence the chosen node) did not change the effects of idea distributions on the average path length (**FIGURES 2.24-2.26**). This is natural, since nodes were either already connected to a significant number of nodes with the same idea (in the case of the parallel distribution), not connected to any other node with the same idea (the antiparallel case), or connected to nodes with random assignment of ideas. Thus the threshold δ would either already be fulfilled from the start, would definitely not be fulfilled, or would have a very small chance of being fulfilled, respectively.

6.2.3 Network Diameter

Similar to the clustering coefficient and the average path length, the network diameter was larger when the parallel idea distribution was applied (**FIGURE 2.2**). This distribution encouraged the structure of the caveman network to remain mostly unchanged, and thus the farthest distance between two nodes was larger than for the case of random or anti-parallel distributions, where more rewiring occurred.

Dependence on Parameters As ϕ increased, the network diameter decreased for all idea distributions, and slightly less for the parallel distribution (**FIGURES 2.18-2.20**). Rewiring a caveman network intuitively may decrease the network diameter by connecting more of its caves. Similar to the average path length, values of δ did not change the behaviour of the network diameter given the idea distributions (**FIGURES 2.27-2.29**).

6.2.4 Degree Distribution

Random graphs have a somewhat normally distributed degree distribution. Scale-free graphs, on the other hand, have a degree distribution that follows a scale-free power-law. We observed that the degree distribution of the networks depended on

the idea distribution (**FIGURE 2.5**). A parallel idea distribution resulted in a degree distribution similar to that of a scale-free graph, which is not surprising. Caveman graphs have two or three different degrees for their nodes, and allowing for some rewiring would 'smooth' out this discrete distribution. Similar to previous results, the random and antiparallel idea distributions behaved similarly: their degree distribution was similar to that of random graphs. It is interesting to see such a visible difference in these distributions over relatively few time steps (1000 steps), regardless of the parameter combinations.

6.3 Discussion

As previously mentioned, we observed that some features of the idea distribution varied with the type of network structure. We also observed that some features of the network structure varied with the kind of idea distribution implemented. Some of these results were intuitive and expected, while others were not.

Network structure mainly influenced two features of the idea distribution: the intra-idea distance and the neighbourhood index. The intra-idea distance was larger for the caveman and small world networks, as anticipated. However, contrary to expectations, the neighbourhood index was larger for these two structures, and the caveman structure did not decrease the frequency of dominance (and neither did any other structure). Additionally, the values of ϕ and δ did correlate with changes in the values of the intra-idea distance (by decreasing it) and the neighbourhood index (with different effects depending on the structure), but not always: δ did not vary the results of the intra-idea distance or the neighbourhood index of the random and scale-free networks, and ϕ did not vary the neighbourhood index in the small world network. Lastly, and also contrary to expectations, increasing the α values increased the neighbourhood index of networks.

Four of the five features of the network structure changed with different idea distributions. The connected component interestingly remained one, regardless of the idea distribution and parameter values. Feature values of the anti-parallel and random idea distributions differed from those of the parallel distribution. Most features changed as expected: the clustering coefficient, average path length, and network diameter were larger when the parallel idea distribution was used as compared to the other two, and their values were similar to those of a caveman network structure. The clustering coefficient of the anti-parallel idea distribution networks did decrease, as expected. The degree distribution did increase in variance (becoming more similar to a normal distribution) for the networks that were initiated with a random or anti-parallel idea distribution, but the degree distribution of the parallel idea distribution networks resembled that of a scale-free power law. Values of ϕ influenced the features of the random and antiparallel distribution networks the most. However, the structure

features of the networks with the random idea distribution were not more sensitive to parameter values as would have been expected; they behaved similar to those of the networks with the antiparallel idea distribution. Values of δ only correlated with changes in the clustering coefficient of the networks with a parallel idea distribution, and only weakly. Values of α did not correlated with any changes of feature values.

7 Summary and Outlook

To conclude our simulation study, our results support the general idea that network structure and qualities of the ideas held in them may mutually influence each other. The more rigid the structure of a network was, the more likely that the intra-idea distance and neighbourhood index of the network was larger. These two features (the intra-idea distance and neighbourhood index) varied more with the innovation rate α for less structured networks, and varied more with the complex contagion threshold δ for the most rigid structure - the caveman network. Parameter values (ϕ , δ , and α) seemed to vary the average dominance time more than the network structures did, whereas the frequency of dominance of ideas did not decrease on average in the long run with any network structure.

A network in which the pattern of ideas held by the nodes are ‘in accord’ within the clusters of the caveman network maintained more of the structure features of a caveman network. These values varied with values of rewiring probabilities ϕ and complex contagion thresholds δ . Networks with a random pattern of ideas or with a pattern with more ‘disaccord’ within the clusters resulted in a structure that began to resemble a random graph more than a caveman network. These networks’ values varied more with the parameter ϕ of rewiring. Nodes given the chance to rewire with nodes that share the same idea had more opportunity to do so in the random and ‘disaccord’ idea patterns than in the pattern which already had much accord.

Thus, in addition to the structure of networks and the pattern of ideas in them, complex contagion thresholds, innovation rates, and the probability of creating new connections (while reflecting ‘preferences’ of being connected with like-minded others) do influence features of the network. Several extensions to the proposed model could be investigated to better understand the relationships between network structure and idea distribution.

Rewiring criteria Future simulations may compare the emerging features of idea distributions not just between network structures, but also between different rewiring criteria. It is not clear how much of the features of the idea distribution in this simulation study was a result of the network structure or of the ‘preference’ that nodes had in rewiring to like-minded nodes. Therefore, these results could be compared to (1) random rewiring or, to allow structure to play a larger role, (2) to allow random rewiring to nodes that are at most a distance of three nodes away. This is somewhat more realistic since most people connect with individuals who are somewhat in their vicinity through mutual connections.

Complex contagion and innovation Considering complex contagion, novel ideas in our model were at a disadvantage for spreading in the network. Allowing novel ideas to have a larger influence weight may be one way to grant the ability of them to spread to other nodes. Alternatively, the complex contagion threshold for novel ideas may be lowered.

Random idea adoption Future simulations may investigate the effects of network structure on idea distributions by comparing to a ‘benchmark’ model. This model would update the ideas of nodes at random, and thus the effects of connections may be better observed within network structures and then compared between network structures.

8 References

A MATLAB code

A.1 Main script

```
1 %% The first phase: Simulation to study the influence of network ...
   structure on the opinion%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2   %% The following parameters will remain constant during this study,
3   %% so we won't play with them %%
4   n=1000; %% the number of agents %%
5   m=40; %% initial number of clusters for caveman matrix%%
6   p=40; %% initial number of opinions%%
7   t_end=1000; %% number of iterations%%
8   %% for the following parameters we'll run different simulations based on
9   %% combinatorial complexity of the parameters%%
10  phi_choices=[0.1,0.3,0.5]; %% network reorganization rate%%
11  alpha_choices=[0.01,0.05,0.10]; %% innovation rate %%
12  threshold_choices=[0.001,0.01,0.05]; %% threshold for complex contagion %%
13
14
15  %% a totally random idea distribution , independent of the connectivity
16  %% matrix, so applicable for every network structure is defined for this
17  %% phase %%
18  vec1=zeros(1,n);
19  for i=1:n
20      vec1(i)=ceil(rand()*p);
21  end
22
23  for choice1=1:4
24  %% Step1: Definition of Different initial matrices ...
      %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
25      %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% For each simulation, we must choose one of the ...
          following connectivity matrices %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
26      switch choice1
27          case 1
28              %% option 1: Caveman Connectivity Matrix
29              mat1=step1_caveman(n,m);
30              s1='Caveman';
31          case 2
32              %% option 2: Random Connectivity Matrix
33              prob=0.025; %% probability of edge formation between any pairs ...
                  of edges
34              mat1=step1_randomgraph(n,prob);
35              s1='Random';
36          case 3
37              %% option 3: Scale Free Connectivity Matrix
```

```

38     m0=24; % number of initially placed nodes
39     m1=12; % number of nodes a new added node is connected to,  $1 \leq \dots$ 
        m1 < m0
40     mat1=step1.scalefree(n, m0, m1);
41     s1='Scale-free';
42     case 4
43         %% option 4: Small world Connectivity Matrix
44         ka=24; %% mean degree (assumed to be an even integer)
45         beta=0.01; %% rewiring probability
46         mat1= step1.smallworld(n, ka, beta);
47         s1='Small-world';
48     end
49
50     %% Step2: Simulation ...
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
51     for choice2=1:3
52         phi=phi.choices(choice2);
53         for choice3=1:3
54             alpha=alpha.choices(choice3);
55             for choice4=1:3
56                 threshold=threshold.choices(choice4);
57
58                 [mat2,vec2,dominant_freq,most_freq]=step2(t_end,phi,alpha,mat1,vec1,p,threshold);
                    %% obtaining the final matrix and vector after running ...
                    simulation.
59
60                 %%% We need step4c here, since it's outputs will be the input
61                 %%% for step 3b
62                 %%% step4c: number of connected components %%%%%%%%%
63                 sp_mat2=sparse(mat2);
64                 [s,c]=graphconncomp(sp_mat2); %% matlab built in function ...
                    'Bioinformatics Toolbox'
65                 %%% s: number of connected components
66                 %%% c: vector which assigns each node to a connected component
67
68                 %% Step3: Results for structure to idea ...
                    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
69
70                 %%% step3a: defining the average intra_idea ...
                    neighbourhood index %%%%%%%%%
71                 neighbor_index=step3a(mat2,vec2);
72
73                 %%% step3b: defining the average intra_idea distance ...
                    %%%%%%%%%
74                 intra_idea.distance=step3b(mat2,vec2,s,c);
75
76                 %%% step3c: frequency of dominant idea with respect to ...
                    time %%%%%%%%%
77                 %% is the third output of the step2 function ...
                    (dominant_freq)%%

```

```

78         dominant_freq;
79
80         % step3d: Fraction of novel ideas (novelity index) %
81         nov_index=(length(find(vec2>p)))/(length(vec2)); % ...
            indicates the fraction of agents holding the newly ...
            generated ideas
82
83         % step3e: defining the average dominance time (the ...
            average amount of time in which the dominating idea ...
            keeps it's dominance over differnt dominance periods)
84         average_dominance_time=step3e(most_freq);
85
86         % naming the file which saves the results
87         s2=int2str(choice2);
88         s3=int2str(choice3);
89         s4=int2str(choice4);
90         name=['phase1_',s1,'_',s2,'_',s3,'_',s4];
91         save(name);
92     end
93 end
94 end
95 end
96
97 clear;
98
99 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
100 % The second phase: Simulation to study the influence of opinion on the ...
    network structure%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
101
102 % The following three parameters will remain constant during this study,
103 % so won't play with them %
104 n=1200; % the number of agents
105 m=40; % initial number of clusters for caveman matrix
106 p=40; % initial number of opinions
107 t_end=1000; % number of iterations%
108 % for the following parameters we'll run different simulations based on
109 % combinatorial complexity of the parameters%
110 phi_choices=[0.1,0.3,0.5]; % network reorganization rate%
111 alpha_choices=[0.01,0.05,0.10]; % innovation rate %
112 threshold_choices=[0.001,0.01,0.05]; % threshold for complex contagion %
113
114
115
116 % in this phase we'll keep connectivity matrix constant, so we only use
117 % Caveman connectivity matrix %
118 mat1=step1_caveman(n,m);
119
120
121 for choice1=1:3

```

```

122 %% Step1: Definition of Different initial idea vectors ...
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
123     switch choice1
124         case 1
125             % option1: Random idea vector %
126             %% a totally random idea distribution , independent of the ...
                connectivity
127             %% matrix, so applicable for every network structure
            vec1=zeros(1,n);
128             for i=1:n
129                 vec1(i)=ceil(rand()*p);
130             end
131             s1='Random';
132         case 2
133             % option2: Parallel idea vector %
134             %% This idea vector is applicable only for caveman ...
                connectivity matrix in
135             %% which every agents inside a cluster have the same idea
            vec1=zeros(1,n);
136             for i=1:(m-1) %% for each cluster
137                 for j=1:ceil(n/m)
138                     vec1((i*ceil(n/m))+j)=i; %% all agents will hold the ...
139                         i-th idea
140                 end
141             end
142             s1='Parallel';
143         case 3
144             % option3: Antiparallel idea vector %
145             %% This idea vector is applicable only for caveman ...
                connectivity matrix in
146             %% which every agents inside a cluster have different idea
            vec1=zeros(1,n);
147             for i=1:(m-1) %% for each cluster
148                 for j=1:ceil(n/m)
149                     vec1((i*ceil(n/m))+j)=j; %% all agents will hold ...
150                         different idea
151                 end
152             end
153             s1='Antiparallel';
154     end
155 %% Step2: Simulation ...
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
156     for choice2=1:3
157         phi=phi.choices(choice2);
158     for choice3=1:3
159         alpha=alpha.choices(choice3);
160         for choice4=1:3
161             threshold=threshold.choices(choice4);
162         end
163     end
164

```

```

165     [mat2,vec2,dominant_freq,most_freq]=step2(t_end,phi,alpha,mat1,vec1,p,threshold);
        %% obtaining the final matrix and vector after running ...
        simulation.
166
167
168     %% Step3: Results for idea to structure ...
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
169
170     %%%%%%%%% step4a: clustering coefficient of the final network ...
        %%%%%%%%%
171     clust_coefficient=step4a(mat2);
172
173     %%%%%%%%% step4b: degree distribution of the final network ...
        %%%%%%%%%
174     [dgr,frq]=step4b(mat2);
175     average_degree=sum(dgr.*frq)/sum(frq);
176
177     %%%%%%%%% step4c: number of connected components %%%%%%%%%
178     sp_mat2=sparse(mat2);
179     [s,c]=graphconncomp(sp_mat2); %% matlab built in function ...
        'Bioinformatics Toolbox'
180     %% s: number of connected components
181     %% c: vector which assigns each node to a connected component
182
183     %%%%%%%%% step4d: average path length for the final network ...
        %%%%%%%%%
184     average_path_length = step4d( mat2,s,c );
185
186     %%%%%%%%% step4e: Diameter of the network %%%%%%%%%
187     diam=step4e(mat2,s,c);
188
189     %%naming and saving
190     s2=int2str(choice2);
191     s3=int2str(choice3);
192     s4=int2str(choice4);
193     name=['phase2_',s1,'_',s2,'_',s3,'_',s4];
194     save(name);
195     end
196     end
197     end
198 end
199
200 clear;

```

A.2 Step 1: generation of structure networks

randomgraph.m

```
1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2  %%% code adopted from %%%
3  %%% Modeling and Simulating Social Systems with MATLAB %%%
4  %%% http://www.soms.ethz.ch/teaching/MatlabFall2012 %%%
5  %%% Authors: Stefan Brugger and Cristoph Schwirzer, 2011 %%%
6  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
7
8  function final= step1.randomgraph(n, p)
9  % Generates an undirected random graph (without self-loops) of size n (as
10 % described in the Erdoes-Renyi model)
11 %
12 % INPUT
13 % n: [1]: number of nodes
14 % p: [1]: probability that node i and node j, i != j, are connected by ...
    an edge
15 %
16 % OUTPUT
17 % final: [n n] full symmetric adjacency matrix representing the ...
    generated graph
18
19 % Note: A generation based on sprandsym(n, p) failed (for some values of p
20 % sprandsym was far off from the expected number of n*n*p non-zeros), ...
    therefore
21 % this longish implementation instead of just doing the following:
22 %
23 % B = sprandsym(n, p);
24 % A = (B-diag(diag(B))>0);
25 %
26
27 % Idea: first generate the number of non-zero values in every row for a ...
    general
28 % 0-1-adjacency matrix. For every row this number is distributed ...
    binomially with
29 % parameters n and p.
30 %
31 % The following lines calculate "rowsize = binoinv(rand(1, n), n, p)", ...
    just in a
32 % faster way for large values of n.
33
34 % generate a vector of n values chosen u.a.r. from (0,1)
35 v = rand(1, n);
36 % Sort them and calculate the binomial cumulative distribution function with
37 % parameters n and p at values 0 to n. Afterwards match the sorted random
38 % 0-1-values to those cdf-values, i.e. associate a binomial distributed ...
    value
```

```

39 % with each value in r. Each value in v also corresponds to a value in r:
40 % permute the values in rowSize s.t. they correspond to the order given ...
    in v.
41 [r index] = sort(v); % i.e. v(index) == r holds
42 rowSize = zeros(1, n);
43 j = 0;
44 binoCDF = cumsum(binopdf(0:n, n, p));
45 for i = 1:n
46     while j<n && binoCDF(j+1)<r(i)
47         j = j + 1;
48     end
49     rowSize(i) = j;
50 end
51 rowSize(index) = rowSize;
52
53 % for every row choose the non-zero entries in it
54 nNZ = sum(rowSize);
55 I = zeros(1, nNZ);
56 J = zeros(1, nNZ);
57 j = 1;
58 for i = 1:n
59     I(j:j+rowSize(i)-1) = i;
60     J(j:j+rowSize(i)-1) = randsample(n, rowSize(i));
61     j = j + rowSize(i);
62 end
63
64 % restrict I and J to indices that correspond to entries above the main ...
    diagonal
65 % and finally construct a symmetric sparse matrix using I and J
66 upperTriu = find(I<J);
67 I = I(upperTriu);
68 J = J(upperTriu);
69 A = sparse([I;J], [J;I], ones(1, 2*size(I, 2)), n, n);
70 final=full(A);
71 end % random_graph(...)

```

scalefree.m

```
1 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2 %%% code adopted from %%%
3 %%% Modeling and Simulating Social Systems with MATLAB %%%
4 %%% http://www.soms.ethz.ch/teaching/MatlabFall2012 %%%
5 %%% Authors: Stefan Brugger and Cristoph Schwirzer, 2011 %%%
6 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
7
8 function A = scalefree(n, m0, m)
9 % Use the Barabasi-Albert model to generate a scale free graph of size n (as
10 % described in Albert-Laszlo Barabasi & Reka Albert: "Emergence of scaling
11 % in random networks")
12 %
13 % INPUT
14 % n: [1]: number of nodes
15 % m0: [1]: number of initially placed nodes
16 % m: [1]: number of nodes a new added node is connected to,  $1 \leq m < m_0$ 
17 %
18 % OUPUT
19 % A: [n n] sparse symmetric adjacency matrix representing the generated ...
    graph
20
21 % Start with a graph of size m0 and add edges to this graph. Each of ...
    these m0
22 % nodes is connected to at least m nodes.
23 B = zeros(m0, m0);
24 for i = 1:m0
25     neighbors = randsample(m0-1, m);
26     neighbors = neighbors + (neighbors >= i);
27     B(i,neighbors) = 1;
28     B(neighbors,i) = 1;
29 end
30
31 % Create a vector of edges added so far, i.e. nodes edge(2*i) and ...
    edge(2*i-1),
32 %  $1 \leq i \leq nEdges$ , are connected by an edge.
33 [rows, columns] = find(triu(B));
34 nEdges = size(rows, 1);
35 edges = reshape([rows';columns'], 2*nEdges, 1);
36 edges = [edges; zeros(2*(n-m0)*m,1)];
37
38 % Add nodes m0+1:n, one at a time. Each node is connected to m existing ...
    nodes,
39 % where each of the existing nodes is chosen with a probability that is
40 % proportional to the number of nodes it is already connected to.
41 used = zeros(n, 1); % is a node already used in a timestep?
42 for i = m0+1:n
43     neighbors = zeros(1, m);
```



```

44     for j=1:m
45         k = edges(randi(2*nEdges));
46         while used(k)
47             k = edges(randi(2*nEdges));
48         end
49         used(k) = 1;
50         neighbors(j) = k;
51     end
52     used(neighbors) = 0;
53     edges(2*nEdges+1:2*nEdges+2*m) = reshape([repmat(i, 1, m); ...
54         neighbors], ...
55         1, 2*m);
56     nEdges = nEdges+m;
57 end
58 % finally construct a symmetric adjacency matrix using the vector of edges
59 edges = edges(1:2*nEdges);
60 first = edges(1:2:end);
61 second = edges(2:2:end);
62 A = sparse([first;second], [second;first], ones(2*nEdges, 1), n, n);
63
64 end % scale_free(...)

```

smallworld.m

```

1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2  %%% code adopted from %%%
3  %%% Modeling and Simulating Social Systems with MATLAB %%%
4  %%% http://www.soms.ethz.ch/teaching/MatlabFall2012 %%%
5  %%% Authors: Stefan Brugger and Cristoph Schwirzer, 2011 %%%
6  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
7
8  function A = smallworld(n, k, beta)
9  % Generate a small world graph using the "Watts and Strogatz model" as
10 % described in Watts, D.J.; Strogatz, S.H.: "Collective dynamics of
11 % 'small-world' networks."
12 % A graph with  $n*k/2$  edges is constructed, i.e. the nodal degree is  $n*k$  for
13 % every node.
14 %
15 % INPUT
16 % n: [1]: number of nodes of the graph to be generated
17 % k: [1]: mean degree (assumed to be an even integer)
18 % beta: [1]: rewiring probability
19 %
20 % OUTPUT
21 % A: [n n] sparse symmetric adjacency matrix representing the generated ...
    graph

```

```

22
23 % Construct a regular lattice: a graph with n nodes, each connected to k
24 % neighbors, k/2 on each side.
25 kHalf = k/2;
26 rows = reshape(repmat([1:n]', 1, k), n*k, 1);
27 columns = rows+reshape(repmat([1:kHalf] [n-kHalf:n-1]), n, 1), n*k, 1);
28 columns = mod(columns-1, n) + 1;
29 B = sparse(rows, columns, ones(n*k, 1));
30 A = sparse([], [], [], n, n);
31
32 % With probability beta rewire an edge avoiding loops and link duplication.
33 % Until step i, only the columns 1:i are generated making implicit use ...
    of A's
34 % symmetry.
35 for i = [1:n]
36     % The i-th column is stored full for fast access inside the ...
        following loop.
37     col= [full(A(i, 1:i-1))'; full(B(i:end, i))];
38     for j = i+find(col(i+1:end))'
39         if (rand()<beta)
40             col(j)=0;
41             k = randi(n);
42             while k==i || col(k)==1
43                 k = randi(n);
44             end
45             col(k) = 1;
46         end
47     end
48     A(:,i) = col;
49 end
50
51 % A is not yet symmetric: to speed things up, an edge connecting i and ...
    j, i < j
52 % implies A(i,j)==1, A(i,j) might be zero.
53 T = triu(A);
54 A = T+T';
55
56 end % small_world(...)

```

step1_caveman.m

```
1 function cave_mat = step1_caveman(n,m)
2 %%%%%%%%%%% This function outputs a Caveman Matrix %%%%%%%%%%%
3 %% n: the number of agents
4 %% m: initial number of clusters
5 %% p: initial maximum index of opinions
6 cave_mat=zeros(n,n); %Caveman Matrix
7
8 for i=1:n
9     x=ceil(i/(n/m));
10    for j=1:n
11        y=ceil(j/(n/m));
12        if x==y
13            if i≠j
14                cave_mat(i,j)=1; %definition of intracluster edges
15            end
16        end
17    end
18 end
19 %% x1 and x2 for each cluster represent the two agents who interact with
20 %% nearby clusters%%
21 x1=zeros(1,m);
22 x2=zeros(1,m);
23 for i=1:m
24     x1(i)=ceil(rand()*(n/m)+(n/m)*(i-1));
25     ind=0;
26     while(ind==0) %% This loop is used to prevent x1 and x2 to make the ...
27         same numbers
28         x2(i)=ceil(rand()*(n/m)+(n/m)*(i-1));
29         if x2(i)≠x1(i)
30             ind=1;
31         end
32     end
33 end
34 %% definition of intercluster edges
35 for i=1:(m-1)
36     cave_mat(x2(i),x1(i+1))=1;
37     cave_mat(x1(i+1),x2(i))=1;
38 end
39 cave_mat(x1(1),x2(m))=1;
40 cave_mat(x2(m),x1(1))=1;
41 end
```

step1_randomgraph.m

```
1
2
3 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
4 %%% code adopted from %%%
5 %%% Modeling and Simulating Social Systems with MATLAB %%%
6 %%% http://www.soms.ethz.ch/teaching/MatlabFall2012 %%%
7 %%% Authors: Stefan Brugger and Cristoph Schwirzer, 2011 %%%
8 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
9
10 function final= step1_randomgraph(n, p)
11 % Generates an undirected random graph (without self-loops) of size n (as
12 % described in the Erdoes-Renyi model)
13 %
14 % INPUT
15 % n: [1]: number of nodes
16 % p: [1]: probability that node i and node j, i != j, are connected by ...
    an edge
17 %
18 % OUTPUT
19 % final: [n n] full symmetric adjacency matrix representing the ...
    generated graph
20
21 % Note: A generation based on sprandsym(n, p) failed (for some values of p
22 % sprandsym was far off from the expected number of n*n*p non-zeros), ...
    therefore
23 % this longish implementation instead of just doing the following:
24 %
25 % B = sprandsym(n, p);
26 % A = (B-diag(diag(B))>0);
27 %
28
29 % Idea: first generate the number of non-zero values in every row for a ...
    general
30 % 0-1-adjacency matrix. For every row this number is distributed ...
    binomially with
31 % parameters n and p.
32 %
33 % The following lines calculate "rowsize = binoinv(rand(1, n), n, p)", ...
    just in a
34 % faster way for large values of n.
35
36 % generate a vector of n values chosen u.a.r. from (0,1)
37 v = rand(1, n);
38 % Sort them and calculate the binomial cumulative distribution function with
39 % parameters n and p at values 0 to n. Afterwards match the sorted random
40 % 0-1-values to those cdf-values, i.e. associate a binomial distributed ...
    value
```

```

41 % with each value in r. Each value in v also corresponds to a value in r:
42 % permute the values in rowSize s.t. they correspond to the order given ...
    in v.
43 [r index] = sort(v); % i.e. v(index) == r holds
44 rowSize = zeros(1, n);
45 j = 0;
46 binoCDF = cumsum(binopdf(0:n, n, p));
47 for i = 1:n
48     while j<n && binoCDF(j+1)<r(i)
49         j = j + 1;
50     end
51     rowSize(i) = j;
52 end
53 rowSize(index) = rowSize;
54
55 % for every row choose the non-zero entries in it
56 nNZ = sum(rowSize);
57 I = zeros(1, nNZ);
58 J = zeros(1, nNZ);
59 j = 1;
60 for i = 1:n
61     I(j:j+rowSize(i)-1) = i;
62     J(j:j+rowSize(i)-1) = randsample(n, rowSize(i));
63     j = j + rowSize(i);
64 end
65
66 % restrict I and J to indices that correspond to entries above the main ...
    diagonal
67 % and finally construct a symmetric sparse matrix using I and J
68 upperTriu = find(I<J);
69 I = I(upperTriu);
70 J = J(upperTriu);
71 A = sparse([I;J], [J;I], ones(1, 2*size(I, 2)), n, n);
72 final=full(A);
73 end % random_graph(...)

```

step1_scalefree.m

```
1 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2 %%% code adopted from %%%
3 %%% Modeling and Simulating Social Systems with MATLAB %%%
4 %%% http://www.soms.ethz.ch/teaching/MatlabFall2012 %%%
5 %%% Authors: Stefan Brugger and Cristoph Schwirzer, 2011 %%%
6 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
7
8 function final = step1_scalefree(n, m0, m)
9 % Use the Barabasi-Albert model to generate a scale free graph of size n (as
10 % described in Albert-Laszlo Barabasi & Reka Albert: "Emergence of scaling
11 % in random networks")
12 %
13 % INPUT
14 % n: [1]: number of nodes
15 % m0: [1]: number of initially placed nodes
16 % m: [1]: number of nodes a new added node is connected to,  $1 \leq m < m_0$ 
17 %
18 % OUTPUT
19 % final: [n n] full symmetric adjacency matrix representing the ...
    generated graph
20
21 % Start with a graph of size m0 and add edges to this graph. Each of ...
    these m0
22 % nodes is connected to at least m nodes.
23 B = zeros(m0, m0);
24 for i = 1:m0
25     neighbors = randsample(m0-1, m);
26     neighbors = neighbors + (neighbors >= i);
27     B(i,neighbors) = 1;
28     B(neighbors,i) = 1;
29 end
30
31 % Create a vector of edges added so far, i.e. nodes edge(2*i) and ...
    edge(2*i-1),
32 %  $1 \leq i \leq nEdges$ , are connected by an edge.
33 [rows, columns] = find(triu(B));
34 nEdges = size(rows, 1);
35 edges = reshape([rows';columns'], 2*nEdges, 1);
36 edges = [edges; zeros(2*(n-m0)*m,1)];
37
38 % Add nodes m0+1:n, one at a time. Each node is connected to m existing ...
    nodes,
39 % where each of the existing nodes is chosen with a probability that is
40 % proportional to the number of nodes it is already connected to.
41 used = zeros(n, 1); % is a node already used in a timestep?
42 for i = m0+1:n
43     neighbors = zeros(1, m);
```

```

44     for j=1:m
45         k = edges(randi(2*nEdges));
46         while used(k)
47             k = edges(randi(2*nEdges));
48         end
49         used(k) = 1;
50         neighbors(j) = k;
51     end
52     used(neighbors) = 0;
53     edges(2*nEdges+1:2*nEdges+2*m) = reshape([repmat(i, 1, m); ...
54         neighbors], ...
55         1, 2*m);
56     nEdges = nEdges+m;
57 end
58 % finally construct a symmetric adjacency matrix using the vector of edges
59 edges = edges(1:2*nEdges);
60 first = edges(1:2:end);
61 second = edges(2:2:end);
62 A = sparse([first;second], [second;first], ones(2*nEdges, 1), n, n);
63 final=full(A);
64 end % scale_free(...)

```

step1_smallworld.m

```

1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2  %%% code adopted from %%%
3  %%% Modeling and Simulating Social Systems with MATLAB %%%
4  %%% http://www.soms.ethz.ch/teaching/MatlabFall2012 %%%
5  %%% Authors: Stefan Brugger and Cristoph Schwirzer, 2011 %%%
6  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
7
8  function final = step1_smallworld(n, k, beta)
9  % Generate a small world graph using the "Watts and Strogatz model" as
10 % described in Watts, D.J.; Strogatz, S.H.: "Collective dynamics of
11 % 'small-world' networks."
12 % A graph with  $n*k/2$  edges is constructed, i.e. the nodal degree is  $n*k$  for
13 % every node.
14 %
15 % INPUT
16 % n: [1]: number of nodes of the graph to be generated
17 % k: [1]: mean degree (assumed to be an even integer)
18 % beta: [1]: rewiring probability
19 %
20 % OUTPUT
21 % A: [n n] sparse symmetric adjacency matrix representing the generated ...
    graph

```

```

22
23 % Construct a regular lattice: a graph with n nodes, each connected to k
24 % neighbors, k/2 on each side.
25 kHalf = k/2;
26 rows = reshape(repmat((1:n)', 1, k), n*k, 1);
27 columns = rows+reshape(repmat([(1:kHalf) (n-kHalf:n-1)], n, 1), n*k, 1);
28 columns = mod(columns-1, n) + 1;
29 B = sparse(rows, columns, ones(n*k, 1));
30 A = sparse([], [], [], n, n);
31
32 % With probability beta rewire an edge avoiding loops and link duplication.
33 % Until step i, only the columns 1:i are generated making implicit use ...
    of A's
34 % symmetry.
35 for i = 1:n
36     % The i-th column is stored full for fast access inside the ...
        following loop.
37     col= [full(A(i, 1:i-1))'; full(B(i:end, i))];
38     for j = i+find(col(i+1:end))'
39         if (rand()<beta)
40             col(j)=0;
41             k = randi(n);
42             while k==i || col(k)==1
43                 k = randi(n);
44             end
45             col(k) = 1;
46         end
47     end
48     A(:,i) = col;
49 end
50
51 % A is not yet symmetric: to speed things up, an edge connecting i and ...
    j, i < j
52 % implies A(i,j)==1, A(i,j) might be zero.
53 T = triu(A);
54 A = T+T';
55 final=sparse(A);
56
57 end % small_world(...)

```


A.3 Step 2: Rewiring process

step2.m

```
1 function [mat, vec, dominant_freq, most_freq] = ...
   step2(t_end, phi, alpha, mat, vec, p, threshold)
2 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% step2: Runing the Simulation %%%%%%%%%%%%%
3 %%% t_end: number of iterations
4 %%% phi: network reorganization rate
5 %%% alpha: innovation rate
6 %%% mat: initial connectivity matrix
7 %%% vec: initial idea vector
8 %%% p: initial number of opinions
9 %%% then it outputs:
10 %%% mat: connectivity matrix after simulation
11 %%% vec: idea vector after simulation
12 %%% dominant_freq: the vector holding the frequency of dominant idea
13 %%% most_freq: the vector holding the index of dominating idea in each time
14
15 most_freq=zeros(1,t_end); %%% Vector for storing the index of the ...
   dominating idea in each time step.
16 dominant_freq=zeros(1,t_end); %%% Vector for storing the frequency of ...
   dominant idea in each time step.
17 a=size(mat);
18 n=a(1); %%% number of agents
19
20 for t=1:t_end
21     x1=ceil(rand()*n); %%% choosing one person randomly for network ...
       reorganization or changing idea
22     a1=rand();
23     b1=phi;
24     if a1<b1 %%% i.e with probability phi to reorganize the network
25         v00=find(mat(x1,:)==1); %%% defining neighbours of x1
26         v=find(vec(v00)~=vec(x1)); %%% define neighbours of x1 that do not ...
           have the same idea as x1
27         if ~isempty(v)
28             x2=v(ceil(rand()*length(v))); %%% choosing one neighbour with ...
               different idea randomly to remove connection with
29             mat(x1,x2)=0; %%% deletion of the edge between x1 and x2
30             mat(x2,x1)=0; %%% deletion of the edge between x2 and x1
31             similar_idea=find(vec==vec(x1)); %%% define the agents with ...
               the same idea as x1
32             non_neighbor_sim_idea=setdiff(similar_idea,v00); %%% the ...
               agents with similar idea as x1 who are not neighbor of x1
33             if ~isempty(non_neighbor_sim_idea)
34                 x3=non_neighbor_sim_idea(ceil(rand()*length(non_neighbor_sim_idea))); ...
                   %%% choose x3 randomly among the agents with the same ...
                   idea and non-neighbor with x1 as the newly connected ...
                   agent to x
```

```

35         mat(x1,x3)=1; %% formation of new edge between x1 and x3
36         mat(x3,x1)=1; %% formation of new edge between x3 and x1
37     else %% we'll forget about network reorganization in this ...
38         time by reforming the deleted edges.
39         mat(x1,x2)=1;
40         mat(x2,x1)=1;
41     end
42 end
43 else %% otherwise change the idea of x1 to that of one of it's ...
44     randomly chosen neighbours
45     v2=find(mat(x1,:)==1); %% defining the neighbours of x1
46     if ~isempty(v2)
47         vv=vec(v2); %% the corresponding ideas of the neighbours
48         vvv=unique(vv); %% vector of all the distinct ideas
49         freq=zeros(1,length(vvv)); %% vector for frequencies of the ideas
50         for i=1:length(vvv) %% to test for all distinct ideas
51             if length(find(vv==vvv(i)))>(threshold*length(v2)) ...
52                 %%whether the frequency is larger than the threshold ...
53                 [To include complex contagion definition]
54                 freq(i)=1;
55             end
56         end
57         candidates=vvv(find(freq==1)); %% The ideas meeting the threshold
58         candidates_size=length(candidates); %% The number of ideas ...
59         meeting the threshold
60         if candidates_size>0
61             chosen=ceil(rand()*candidates_size);%% Randomly choose one ...
62             of the candidates
63             vec(x1)=candidates(chosen); %% change the idea of x1 to ...
64             the chosen idea
65         end
66     end
67 end
68 y=ceil(rand()*n); %%choosing one person randomly for coming up with ...
69 a new idea
70 a2=rand();
71 b2=alpha;
72 if a2<b2 %% i.e. with probability alpha to generate a novel idea
73     bound=10^6; %% to limit the index of new ideas to 10^6 which ...
74     simulates nearly boundryless pool of ideas
75     new_idea=ceil(rand()*bound)+p; %% the index of new idea
76     vec(y)=new_idea; %% changing the idea of agent y to the novel one
77 end
78 %%%
79 most_freq(t)=mode(vec); %% the dominant idea at time t
80 dominant_freq(t)=length(find(vec==most_freq(t))); %% the frequency ...
81 of the dominant idea at time t
82 end

```

A.4 Step 3: Results

step3a.m

```

1 function n_index = step3a(mat,vec)
2 %%%%%%%%% step3a: defining the neighbourhood index between similar ideas ...
   %%%%%%%%%
3 %%% mat: adjacency matrix
4 %%% vec: idea vector
5 %%% it outputs: n_index which is the average neighbour index of the network
6 u=unique(vec); %%% collection of distinct ideas
7 w=zeros(1,length(u)); %%% initializing the vector for storing the ...
   neighbourhood index for each distinct idea
8 s=zeros(1,length(u)); %%% initializing the vector for storing the number ...
   of agents holding each distinct ideas
9 for i=1:length(u)
10     x=find(vec==u(i)); %%% defining the set of agents holding the idea u(i)
11     s(i)=length(x);
12     if s(i)>1
13         sum1=0; %%% number of agents with idea i which are in direct ...
            neighbourhood
14         for k=1:(s(i)-1) %%% These two for loops are used to test the ...
            neighborhoods of all distinct pairs
15             for j=(k+1):s(i)
16                 if mat(x(k),x(j))==1 %%% to check if they are neighbours
17                     sum1=sum1+1;
18                 end
19             end
20         end
21         max_neighbors=(s(i)*(s(i)-1)/2); %%% normalizing factor (i.e., the ...
            maximum number of pairs of distinct agents)
22         w(i)=sum1/max_neighbors; %%% neighborhood index for idea 'i'
23     else
24         w(i)=0;
25     end
26 end
27 norm1=sum(s); %%% Normalizing factor
28 count=sum(w.*s); %%% The weighted sum of neighborhood index
29 n_index=count/norm1; %%% average neighbour index of the whole network
30 end

```

steb3b.m

```

1 function intra_idea_distance = step3b(mat,vec,s,c)
2 %%%%%%%%%%% step3b: Defining the average of the average shortest ...
   distance between agents holding the same idea %%%%%%%%%%%
3 %%% mat: adjacency matrix
4 %%% vec: idea vector
5 %%% s: number of connected components
6 %%% c: vector which assigns each node to a connected component
7 %%% and it outputs the average of average intra_distance between agents
8 %%% holding the same idea %%%%%%%%%%%
9 unq=unique(vec); %%% collection of distinct ideas
10 mean_sum=zeros(1,length(unq)); %%% initializing the vector for storing ...
   the average distance between agents holding a particular idea
11 n=zeros(1,length(unq)); %%% initializing the corresponding vector for ...
   storing the number of agents holding a certain idea
12 mats=sparse(mat);
13
14 for i=1:length(unq)
15     x=find(vec==unq(i)); %%% defining the set of agents holding the ...
   idea unq(i)
16     n(i)=length(x); %%% number of agents holding the idea unq(i)
17     nn=zeros(1,s); %%% initializing the vector which will store the ...
   number of agents holding the idea unq(i) who belong to the m-th ...
   connected component
18     mean_sum0=zeros(1,s); %%% initializing the vector which stores the ...
   mean of the distances between agents holding the idea unq(i) who ...
   belong to each distinct connected component
19     for m=1:s %%% for each connected component
20         y=x(find(c(x)==m)); %%% the set of agents holding the idea ...
   unq(i) who belong to the m-th connected component
21         nn(m)=length(y); %%% number of agents holding the idea unq(i) ...
   who belong to the m-th connected component
22         sum0=0;
23         if nn(m)>1
24             for j=1:(nn(m)-1)
25                 for k=(j+1):nn(m)
26                     dis=graphshortestpath(mats,y(j),y(k)); %%% distance ...
   of each agent with similar idea from agent j ...
   inside the group [needs the bioinformatics ...
   toolbox to be installed]
27                     sum0=sum0+dis; %%% sum of the distances between ...
   agents holding the idea unq(i) who belong to m-th ...
   connected component
28                 end
29             end
30             norml0=nn(m)*(nn(m)-1)/2; %%% normalizing factor
31             mean_sum0(m)=sum0/norml0; %%% The average intra-idea distance ...
   between agents holding the idea unq(i) who belong to the ...

```

```

                                m-th connected component
32         else
33             mean_sum0(m)=0;
34         end
35     end
36
37     if n(i)>1
38         mean_sum(i)=(sum(mean_sum0.*nn))/(sum(nn)); %% The average ...
                                intra_idea distance between agents holding the idea unq(i)
39     else
40         mean_sum(i)=0;
41     end
42 end
43
44 intra_idea_distance=(sum(mean_sum.*n))/(sum(n)); %% The average ...
                                intra_idea distance for the whole network
45 end

```

step3e.m

```

1 function [ average_dominance_time ] = step3e( most_freq )
2 %%%% this function calculates the average of dominance time for the ...
                                dominating idea during the simulation
3 %%%% most_freq: is the vector obtained from simulation which holds the ...
                                index of dominating ideas for each time steps of the simulation
4 %%%% it outputs the average_time which is the average of the dominance ...
                                time for different dominance periods
5 temp=zeros(1,length(most_freq)); %% a temporary array which will hold ...
                                the dominating period for dominating idea
6 count=1; %% is the number of consecutive time steps in which an idea is ...
                                considered as dominating
7 ind=0; %% is the index of domination period
8 for i=2:length(most_freq)
9     if most_freq(i)==most_freq(i-1)
10         count=count+1; %% the number of consecutive steps is conted
11     else
12         ind=ind+1; %% as soon as another domination period gets tarted ...
                                the index of domination period adds by one
13         temp(ind)=count; %% the number of consecutive steps will be ...
                                stored in the ind-th index of tmp
14         count=0; %% and count will be reset to zero to count the ...
                                duration of the new domination period.
15     end
16 end
17
18 if ind==0 %% in case during the simulation, just one special idea ...
                                remains dominating forever

```

```

19     ind=ind+1;
20     temp(ind)=count;
21 end
22
23 %%% from ind-th element till end the temp vector will remain zero we ...
    define a new vector as follows to store the corresponding non-zero ...
    elements of temp
24 final=zeros(1,ind);
25 for j=1:ind
26     final(j)=temp(j);
27 end
28 average_dominance_time=mean(final);
29 end

```

step4a.m

```

1
2 function clust_coeff = step4a( mat )
3 % This function calculates the clustering coefficient of a network with
4 % Corresponding connectivity matrix: mat
5 % The approach used for calculation of clustering coefficient was the one ...
    which was proposed by Watts and Strogatz
6 % in which the clustering coefficient of the whole network equals to the
7 % average of the local clustering coefficient of all nodes:
8 % D. J. Watts and Steven Strogatz (June 1998). "Collective dynamics of ...
    'small-world' networks". Nature 393 (6684): 440 442.
9 a=size(mat);
10 local_clust_coeff=zeros(1,a(1)); %%% the vector to store the local ...
    clustering coefficient of each node of the network
11 for i=1:a(1) %%% for each agent in the network
12     x=find(mat(i,:)==1); %%% find all the neighbours of agent i
13     count=0;
14     l=length(x);
15     if l>1
16         for j=1:(l-1) %%% loop to investigate the neighbour relationship for ...
            all possible pairs among the neighbors of node i
17             for k=(j+1):l
18                 if mat(x(j),x(k))==1
19                     count=count+1; % to count the number of neighbour ...
                        relationships
20                 end
21             end
22         end
23         norml=1*(l-1)/2; % normalizing factor (i.e. the number of distinct ...
            pairs)
24         local_clust_coeff(i)=count/norml; % local clustering coefficient of ...
            node i

```

```

25     else
26         local_clust_coeff(i)=0;
27     end
28 end
29 clust_coeff=sum(local_clust_coeff)/a(1); %% the average clustering ...
        coefficient of entire network
30 end

```

step4b.m

```

1 function [dgr,frq] = step4b(mat)
2 %%% this function outputs the degree vector (dgr) and its corresponding
3 %%% frequency vector (frq) from the connectivity matrix (mat). In other ...
    words,
4 %%% first the degree of all nodes in the network will be calculated and
5 %%% then the set of unique degrees will be stored in the dgr vector and the
6 %%% corresponding frequency will be stored in vector frq. For example,
7 %%% degree x will be stored in the i-th element of the dgr vector, then ...
    the number of nodes whose degree equals x
8 %%% will be calculated and will be stored in the i-th element of vector frq.
9 a=size(mat);
10 degr=zeros(1,a(1)); %% the vector which stores the degree of each node ...
    of the network
11 for i=1:a(1)
12     degr(i)=sum(mat(i,:)); %% degree for each node
13 end
14 degr=unique(degr); %% set of unique degrees of the nodes of the network
15 frq=zeros(1,length(dgr)); %% the corresponding frequency vector
16 for j=1:length(dgr)
17     frq(j)=length(find(degr==dgr(j))); %% frq(j) is the number of nodes ...
        in the network whose degree equals to dgr(j)
18 end
19
20 end

```

step4d.m

```
1 function average_path_length = step4d( mat,s,c )
2 %This function outputs the average path length for the graph
3 %mat: connectivity matrix
4 %s: number of connected components
5 %c: the connected component vector
6 av_p_l=zeros(1,s); %%% the vector which will store the average path ...
    length for each connected component of the 'mat' network
7 n=zeros(1,s); %%% the vector which will store the number of nodes ...
    belonging to each connected component.
8 sp=sparse(mat);
9
10 for i=1:s %%% for each connected component
11     x=find(c==i); %%% characterizing the nodes which belong to the i-th ...
        connected component
12     a=length(x); %%% the number of nodes in the i-th connected component
13     n(i)=a;
14     count=0;
15     if a>1
16         for k=1:(a-1)
17             for j=(k+1):a
18                 count=count+graphshortestpath(sp,x(k),x(j)); %%% sum of the path ...
                    length between all pairs of node
19             end
20         end
21         norml=a*(a-1)/2; %%% normalization factor
22         av_p_l(i)=count/norml; %%% average of path length
23     else
24         av_p_l(i)=0;
25     end
26 end
27 average_path_length=sum(av_p_l.*n)/sum(n); %%% weighted average path length
28 end
```


step4e.m

```
1 function graph.diameter = step4e( mat,s,c )
2 %This function outputs the diameter of the graph 'mat'
3 %mat: connectivity matrix
4 %s: number of connected components
5 %c: the connected component vector
6 sp=sparse(mat);
7 max_path=zeros(1,s); %% vector storing diameter for each connected ...
   components.
8 for i=1:s %% for each connected component
9     x=find(c==i); %% characterizing the nodes which belong to the i-th ...
       connected component
10    a=length(x); %% the number of nodes in the i-th connected component
11    path_length=zeros(1,(a*(a-1)/2)); %% The vector storing the ...
       shortest path between each pairs of nodes belonging to the ...
       connected component i
12    count=0;
13    if a>1
14        for k=1:(a-1)
15            for j=(k+1):a
16                count=count+1;
17                path_length(count)=graphshortestpath(sp,x(k),x(j)); %% the ...
                   vector storing the shortest path between pairs k and j
18            end
19        end
20        max_path(i)=max(path_length); %% the maximum shortest path for ...
            connected component i
21    else
22        max_path(i)=0;
23    end
24 end
25 graph.diameter=max(max_path); %% the maximum shortest path for the hole ...
   network (Diameter of the network)
26 end
```

A.5 Plote codes

plots_phase1.m

```
1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Plots of phase 1 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2  %%%
3  %%% Extracting information from the saved Mat files %%%
4  neighbor_ind=zeros(1,108);
5  int_id_dis=zeros(1,108);
6  nov_ind=zeros(1,108);
7  av_dom_tim=zeros(1,108);
8  dom_freq=zeros(108,1000);
9  alpha=[0.01,0.05,0.1];
10 phi=[0.1,0.3,0.5];
11 threshold=[0.001,0.01,0.05];
12 count=0;
13 f=figure();
14 for i=1:4
15     switch i
16         case 1
17             s1='Caveman';
18         case 2
19             s1='Random';
20         case 3
21             s1='Scale-free';
22         case 4
23             s1='Small-world';
24     end
25     for j=1:3
26         s2=int2str(j);
27         for k=1:3
28             s3=int2str(k);
29             for l=1:3
30                 count=count+1;
31                 s4=int2str(l);
32                 name=['phase1_',s1,'_',s2,'_',s3,'_',s4];
33                 a=load(name);
34                 neighbor_ind(count)=a.neighbor_index;
35                 int_id_dis(count)=a.intra_idea_distance;
36                 nov_ind(count)=a.nov_index;
37                 av_dom_tim(count)=a.average_dominance_time;
38                 dom_freq(count,:)=a.dominant_freq;
39             end
40         end
41     end
42 end
43
44 %%% phase diagrams for each parameter pairs for the results of the effects
45 %%% of network structure on idea distribution.
```

```

46
47 for i=1:4
48     switch i
49         case 1
50             s1='Caveman';
51             val=0;
52         case 2
53             s1='Random';
54             val=27;
55         case 3
56             s1='Scale free';
57             val=54;
58         case 4
59             s1='Small world';
60             val=81;
61     end
62     for j=1:4
63         switch j
64             case 1
65                 s2='neighbor index';
66                 main_vec=neighbor_ind;
67             case 2
68                 s2='intra idea distance';
69                 main_vec=int_id_dis;
70             case 3
71                 s2='nov index';
72                 main_vec=nov_ind;
73             case 4
74                 s2='average dominance time';
75                 main_vec=av_dom_tim;
76         end
77         for k=1:3
78             switch k
79                 case 1
80                     s3='(alpha:phi)';
81                     x=0:0.5:1;
82                     xl='alpha';
83                     y=0:0.5:1;
84                     yl='phi';
85                     for l=1:3
86                         M=[main_vec(val+l),main_vec(val+l+9),main_vec(val+l+18);main_vec(val+l+3),ma
87                         %
88                         [xlab,ylab]=meshgrid(x,y);
89                         hold on;
90                         view(0,90);
91                         surf(xlab,ylab,M,'EdgeColor','none');
92                         colorbar;
93                         set(gca,'FontSize',14)
94                         xlabel(xl);
95                         ylabel(yl);

```

```

96         switch l
97             case 1
98                 s5='first';
99             case 2
100                 s5='second';
101             case 3
102                 s5='third';
103         end
104         name2=['phase diagram of ',s2,' for alpha versus phi',' ...
                and','the', s5,' threshold','obtained ...
                for',s1,'network structure'];
105         title(name2);
106         name=['plotting1_', 'Pdiag_',s1,'_',s2,'_',s3,'_',s4,'AND','the', ...
                s5,' threshold'];
107         saveas(f,name);
108         hold off
109         %
110     end
111     case 2
112         s3='(alpha:threshold)';
113         x=0:0.5:1;
114         xl='alpha';
115         y=0:0.5:1;
116         yl='threshold';
117         for l=1:3
118             M=[main_vec(val+9*(l-1)+1),main_vec(val+9*(l-1)+2),main_vec(val+9*(l-1)+3);ma
119             %
120             [xlab,ylab]=meshgrid(x,y);
121             hold on;
122             view(0,90);
123             surf(xlab,ylab,M,'EdgeColor','none');
124             colorbar;
125             set(gca,'FontSize',14)
126             xlabel(xl);
127             ylabel(yl);
128             switch l
129                 case 1
130                     s5='first';
131                 case 2
132                     s5='second';
133                 case 3
134                     s5='third';
135             end
136             name2=['phase diagram of ',s2,' for alpha versus ...
                    threshold',' and','the', s5,' phi','obtained ...
                    for',s1,'network structure'];
137             title(name2);
138             name=['plotting1_', 'Pdiag_',s1,'_',s2,'_',s3,'_',s4,' ...
                    and','the', s5,' phi'];
139             saveas(f,name);

```

```

140         hold off
141         %
142     end
143
144     case 3
145     s3='(phi:threshold)';
146     x=0:0.5:1;
147     xl='phi';
148     y=0:0.5:1;
149     yl='threshold';
150     for l=1:3
151         M=[main_vec(val+3*(l-1)+1),main_vec(val+3*(l-1)+2),main_vec(val+3*(l-1)+3);m
152         %
153         [xlab,ylab]=meshgrid(x,y);
154         hold on;
155         view(0,90);
156         surf(xlab,ylab,M,'EdgeColor','none');
157         colorbar;
158         set(gca,'FontSize',14)
159         xlabel(xl);
160         ylabel(yl);
161         switch l
162         case 1
163             s5='first';
164         case 2
165             s5='second';
166         case 3
167             s5='third';
168         end
169         name2=['phase diagram of ',s2,' for phi versus ...
170             threshold',' and','the', s5,' threshold','obtained ...
171             for',s1,'network structure'];
172         title(name2);
173         name=['plotting1-', 'Pdiag-',s1,'-',s2,'-',s3,'-',s4,' ...
174             and','the', s5,' threshold'];
175         saveas(f,name);
176         hold off
177         %
178     end
179 end
180
181 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
182 %% Dependence of the property of each network to the parameters
183
184 for i=1:4
185     switch i
186     case 1

```

```

187     s1='Caveman';
188     val=0;
189     case 2
190     s1='Random';
191     val=27;
192     case 3
193     s1='Scale free';
194     val=54;
195     case 4
196     s1='Small world';
197     val=81;
198 end
199 for j=1:4
200     switch j
201     case 1
202         s2='neighborhood index';
203         main_vec=neighbor_ind;
204     case 2
205         s2='intra idea distance';
206         main_vec=int_id_dis;
207     case 3
208         s2='novelty index';
209         main_vec=nov_ind;
210     case 4
211         s2='average dominance time';
212         main_vec=av_dom_tim;
213     end
214     for k=1:3
215         switch k
216         case 1
217             s3='threshold';
218             f=figure();
219             for l=1:3
220                 line=main_vec(3*(0:8)+val+1);
221                 set(gca,'FontSize',14)
222                 xlabel('Different pairs of alpha and phi values');
223                 ylabel(s2);
224                 namek=['Dependence of',' ',s2,' ','on',' ',s3,' ...
225                        ','for',' ',s1,' ','network structure'];
226                 title(namek);
227             switch l
228             case 1
229                 plot(line,'—ks','LineWidth',2,'MarkerEdgeColor','k','MarkerF
230                 hold on
231             case 2
232                 plot(line,'—bs','LineWidth',2,'MarkerEdgeColor','b','MarkerF
233                 hold on
234             case 3
235                 plot(line,'—rs','LineWidth',2,'MarkerEdgeColor','r','MarkerF

```

```

236         end
237         legend('threshold=0.001','threshold=0.01','threshold=0.05')
238         name=['plotting1_',s1,'_',s2,'_', 'threshold_sensitivity'];
239         saveas(f,name);
240     end
241
242     case 2
243         s3='alpha';
244         f=figure();
245         for l=1:3
246             line=main_vec([1,2,3,10,11,12,19,20,21]+val+3*(l-1));
247             set(gca,'FontSize',14)
248             xlabel('Different pairs of threshold and phi ...
                values');
249             ylabel(s2);
250             namek=['Dependence of',' ',s2,' ','on',' ',s3,' ...
                ','for',' ',s1,' ','network structure'];
251             title(namek);
252
253             switch l
254                 case 1
255                     plot(line,'—ks','LineWidth',2,'MarkerEdgeColor','k','MarkerF
256                     hold on
257                 case 2
258                     plot(line,'—bs','LineWidth',2,'MarkerEdgeColor','b','MarkerF
259                     hold on
260                 case 3
261                     plot(line,'—rs','LineWidth',2,'MarkerEdgeColor','r','MarkerF
262             end
263             legend('alpha=0.01','alpha=0.05','alpha=0.10')
264             name=['plotting1_',s1,'_',s2,'_', 'alpha_sensitivity'];
265             saveas(f,name);
266         end
267
268     case 3
269         s3='phi';
270         f=figure();
271         for l=1:3
272             line=main_vec((1:9)+val+9*(l-1));
273             set(gca,'FontSize',14)
274             xlabel('Different pairs of threshold and alpha ...
                values');
275             ylabel(s2);
276             namek=['Dependence of',' ',s2,' ','on',' ',s3,' ...
                ','for',' ',s1,' ','network structure'];
277             title(namek);
278
279             switch l
280                 case 1
281                     plot(line,'—ks','LineWidth',2,'MarkerEdgeColor','k','MarkerF

```

```

282             hold on
283             case 2
284                 plot(line, '--bs', 'LineWidth', 2, 'MarkerEdgeColor', 'b', 'MarkerFaceColor', 'b');
285             hold on
286             case 3
287                 plot(line, '--rs', 'LineWidth', 2, 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r');
288             end
289             legend('phi=0.1', 'phi=0.3', 'phi=0.5')
290             name=['plotting1_', s1, '-', s2, '-', 'phi_sensitivity'];
291             saveas(f, name);
292         end
293     end
294 end
295 end
296 end
297
298 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Plotting each network feature for each set of
299 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% parameters %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
300 for i=1:4
301     switch i
302     case 1
303         f=figure();
304         line1=neighbor_ind(1:27);
305         line2=neighbor_ind(28:54);
306         line3=neighbor_ind(55:81);
307         line4=neighbor_ind(82:108);
308         plot(line1, '--ks', 'LineWidth', 2, 'MarkerEdgeColor', 'k', 'MarkerFaceColor', 'k', 'MarkerSize', 10);
309         hold on
310         plot(line2, '--gs', 'LineWidth', 2, 'MarkerEdgeColor', 'g', 'MarkerFaceColor', 'g', 'MarkerSize', 10);
311         hold on
312         plot(line3, '--bs', 'LineWidth', 2, 'MarkerEdgeColor', 'b', 'MarkerFaceColor', 'b', 'MarkerSize', 10);
313         hold on
314         plot(line4, '--rs', 'LineWidth', 2, 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r', 'MarkerSize', 10);
315         set(gca, 'FontSize', 14)
316         xlabel('Different triplets of alpha ,phi and threshold values')
317         ylabel('Neighborhood index')
318         title('The influence of network structure on neighborhood ...
319             index');
320         legend('Caveman Network', 'Random Network', 'Scale Free ...
321             Network', 'Small World Network')
322         name=('Plotting1_Network Comparison_Neighborhood_index');
323         saveas(f, name);
324     case 2
325         f=figure();
326         line1=int_id_dis(1:27);
327         line2=int_id_dis(28:54);
328         line3=int_id_dis(55:81);
329         line4=int_id_dis(82:108);
330         plot(line1, '--ks', 'LineWidth', 2, 'MarkerEdgeColor', 'k', 'MarkerFaceColor', 'k', 'MarkerSize', 10);
331         hold on
332         plot(line2, '--gs', 'LineWidth', 2, 'MarkerEdgeColor', 'g', 'MarkerFaceColor', 'g', 'MarkerSize', 10);
333         hold on
334         plot(line3, '--bs', 'LineWidth', 2, 'MarkerEdgeColor', 'b', 'MarkerFaceColor', 'b', 'MarkerSize', 10);
335         hold on
336         plot(line4, '--rs', 'LineWidth', 2, 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r', 'MarkerSize', 10);
337         set(gca, 'FontSize', 14)
338         xlabel('Different triplets of alpha ,phi and threshold values')
339         ylabel('Neighborhood index')
340         title('The influence of network structure on neighborhood ...
341             index');
342         legend('Caveman Network', 'Random Network', 'Scale Free ...
343             Network', 'Small World Network')
344         name=('Plotting1_Network Comparison_Neighborhood_index');
345         saveas(f, name);
346     case 3
347         f=figure();
348         line1=int_id_dis(1:27);
349         line2=int_id_dis(28:54);
350         line3=int_id_dis(55:81);
351         line4=int_id_dis(82:108);
352         plot(line1, '--ks', 'LineWidth', 2, 'MarkerEdgeColor', 'k', 'MarkerFaceColor', 'k', 'MarkerSize', 10);
353         hold on
354         plot(line2, '--gs', 'LineWidth', 2, 'MarkerEdgeColor', 'g', 'MarkerFaceColor', 'g', 'MarkerSize', 10);
355         hold on
356         plot(line3, '--bs', 'LineWidth', 2, 'MarkerEdgeColor', 'b', 'MarkerFaceColor', 'b', 'MarkerSize', 10);
357         hold on
358         plot(line4, '--rs', 'LineWidth', 2, 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r', 'MarkerSize', 10);
359         set(gca, 'FontSize', 14)
360         xlabel('Different triplets of alpha ,phi and threshold values')
361         ylabel('Neighborhood index')
362         title('The influence of network structure on neighborhood ...
363             index');
364         legend('Caveman Network', 'Random Network', 'Scale Free ...
365             Network', 'Small World Network')
366         name=('Plotting1_Network Comparison_Neighborhood_index');
367         saveas(f, name);
368     case 4
369         f=figure();
370         line1=int_id_dis(1:27);
371         line2=int_id_dis(28:54);
372         line3=int_id_dis(55:81);
373         line4=int_id_dis(82:108);
374         plot(line1, '--ks', 'LineWidth', 2, 'MarkerEdgeColor', 'k', 'MarkerFaceColor', 'k', 'MarkerSize', 10);
375         hold on
376         plot(line2, '--gs', 'LineWidth', 2, 'MarkerEdgeColor', 'g', 'MarkerFaceColor', 'g', 'MarkerSize', 10);
377         hold on
378         plot(line3, '--bs', 'LineWidth', 2, 'MarkerEdgeColor', 'b', 'MarkerFaceColor', 'b', 'MarkerSize', 10);
379         hold on
380         plot(line4, '--rs', 'LineWidth', 2, 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r', 'MarkerSize', 10);
381         set(gca, 'FontSize', 14)
382         xlabel('Different triplets of alpha ,phi and threshold values')
383         ylabel('Neighborhood index')
384         title('The influence of network structure on neighborhood ...
385             index');
386         legend('Caveman Network', 'Random Network', 'Scale Free ...
387             Network', 'Small World Network')
388         name=('Plotting1_Network Comparison_Neighborhood_index');
389         saveas(f, name);
390     end
391 end

```



```

330     plot(line2, '--gs', 'LineWidth', 2, 'MarkerEdgeColor', 'g', 'MarkerFaceColor', 'g', 'MarkerSize', 10)
331     hold on
332     plot(line3, '--bs', 'LineWidth', 2, 'MarkerEdgeColor', 'b', 'MarkerFaceColor', 'b', 'MarkerSize', 10)
333     hold on
334     plot(line4, '--rs', 'LineWidth', 2, 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r', 'MarkerSize', 10)
335     set(gca, 'FontSize', 14)
336     xlabel('Different triplets of alpha ,phi and threshold values')
337     ylabel('Intra-idea distance')
338     title('The influence of network structure on intra-idea ...
           distance')
339     legend('Caveman Network', 'Random Network', 'Scale Free ...
           Network', 'Small World Network')
340     name=('Plotting1Network Comparison.intra_idea_distance');
341     saveas(f,name);
342 case 3
343     f=figure();
344     line1=av_dom_tim(1:27);
345     line2=av_dom_tim(28:54);
346     line3=av_dom_tim(55:81);
347     line4=av_dom_tim(82:108);
348     plot(line1, '--ks', 'LineWidth', 2, 'MarkerEdgeColor', 'k', 'MarkerFaceColor', 'k', 'MarkerSize', 10)
349     hold on
350     plot(line2, '--gs', 'LineWidth', 2, 'MarkerEdgeColor', 'g', 'MarkerFaceColor', 'g', 'MarkerSize', 10)
351     hold on
352     plot(line3, '--bs', 'LineWidth', 2, 'MarkerEdgeColor', 'b', 'MarkerFaceColor', 'b', 'MarkerSize', 10)
353     hold on
354     plot(line4, '--rs', 'LineWidth', 2, 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r', 'MarkerSize', 10)
355     set(gca, 'FontSize', 14)
356     xlabel('Different triplets of alpha ,phi and threshold values')
357     ylabel('Novelty Index')
358     title('The influence of network structure on Novelty index')
359     legend('Caveman Network', 'Random Network', 'Scale Free ...
           Network', 'Small World Network')
360     name=('Plotting1Network Comparison.intra_novelty_index');
361     saveas(f,name);
362 case 4
363     f=figure();
364     line1=av_dom_tim(1:27);
365     line2=av_dom_tim(28:54);
366     line3=av_dom_tim(55:81);
367     line4=av_dom_tim(82:108);
368     plot(line1, '--ks', 'LineWidth', 2, 'MarkerEdgeColor', 'k', 'MarkerFaceColor', 'k', 'MarkerSize', 10)
369     hold on
370     plot(line2, '--gs', 'LineWidth', 2, 'MarkerEdgeColor', 'g', 'MarkerFaceColor', 'g', 'MarkerSize', 10)
371     hold on
372     plot(line3, '--bs', 'LineWidth', 2, 'MarkerEdgeColor', 'b', 'MarkerFaceColor', 'b', 'MarkerSize', 10)
373     hold on
374     plot(line4, '--rs', 'LineWidth', 2, 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r', 'MarkerSize', 10)
375     set(gca, 'FontSize', 14)
376     xlabel('Different triplets of alpha ,phi and threshold values')

```

```

377         ylabel('Average Dominance Time')
378         title('The influence of network structure on Average ...
            Dominance Time')
379         legend('Caveman Network','Random Network','Scale Free ...
            Network','Small World Network')
380         name=('Plotting1_Network Comparison_Average Dominance Time');
381         saveas(f,name);
382     end
383 end
384
385
386 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% plotting frequency of dominant idea over time for each
387 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% parameter sets %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
388 count=0;
389 for choice1=1:3
390     switch choice1
391         case 1
392             s1='phi=0.1';
393             m1='1';
394         case 2
395             s1='phi=0.3';
396             m1='2';
397         case 3
398             s1='phi=0.5';
399             m1='3';
400     end
401
402     for choice2=1:3
403         switch choice2
404             case 1
405                 s2='alpha=0.01';
406                 m2='1';
407             case 2
408                 s2='alpha=0.05';
409                 m2='2';
410             case 3
411                 s2='alpha=0.1';
412                 m2='3';
413         end
414
415         for choice3=1:3
416             switch choice3
417                 case 1
418                     s3='threshold=0.001';
419                     m3='1';
420                 case 2
421                     s3='threshold=0.01';
422                     m3='2';
423                 case 3
424                     s3='threshold=0.05';

```

```

425         m3='3';
426     end
427     count=count+1;
428     line1=dom_freq(count,:);
429     line2=dom_freq(27+count,:);
430     line3=dom_freq(54+count,:);
431     line4=dom_freq(81+count,:);
432     f=figure();
433     plot(line1,'—ks','LineWidth',2,'MarkerEdgeColor','k','MarkerFaceColor','k','MarkerSize',10);
434     hold on
435     plot(line2,'—gs','LineWidth',2,'MarkerEdgeColor','g','MarkerFaceColor','g','MarkerSize',10);
436     hold on
437     plot(line3,'—bs','LineWidth',2,'MarkerEdgeColor','b','MarkerFaceColor','b','MarkerSize',10);
438     hold on
439     plot(line4,'—rs','LineWidth',2,'MarkerEdgeColor','r','MarkerFaceColor','r','MarkerSize',10);
440     set(gca,'FontSize',14)
441     xlabel('time')
442     ylabel('Frequency of the dominant idea')
443     title(['The influence of network structure on the frequency ...',
444           'of the dominant idea',' ',s1,' ',s2,' ',s3]);
445     legend('Caveman Network','Random Network','Scale Free ...',
446           'Network','Small World Network')
447     name=(['Plotting1_Network ...',
448           'Comparison_frequency_dominant_idea-',m1,'-',m2,'-',m3]);
449     saveas(f,name);
450 end
451 end
452 end

```

plots_phase2.m

```
1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Plots of phase 2 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2  %%
3  %% Extracting information from the saved Mat files %%%%%%%%%
4  %%%
5  clust_coefficient=zeros(1,81);
6  s=zeros(1,81);
7  average_path_length=zeros(1,81);
8  diam=zeros(1,81);
9  dgr=cell(1,81);
10 frq=cell(1,81);
11 alpha=[0.01,0.05,0.1];
12 phi=[0.1,0.3,0.5];
13 threshold=[0.001,0.01,0.05];
14
15 count=0;
16
17 for i=1:3
18     switch i
19         case 1
20             s1='Random';
21         case 2
22             s1='Parallel';
23         case 3
24             s1='Antiparallel';
25     end
26     for j=1:3
27         s2=int2str(j);
28         for k=1:3
29             s3=int2str(k);
30             for l=1:3
31                 count=count+1;
32                 s4=int2str(l);
33                 name=['phase2-',s1,'_',s2,'_',s3,'_',s4];
34                 a=load(name);
35                 clust_coefficient(count)=a.clust_coefficient;
36                 s(count)=a.s;
37                 average_path_length(count)=a.average_path_length;
38                 diam(count)=a.diam;
39                 dgr{1,count}=a.dgr;
40                 frq{1,count}=a.frq;
41             end
42         end
43     end
44 end
45
46 f=figure();
47
```

```

48  %%% phase diagrams for each parameter pairs for the results of the effects
49  %%% of idea distribution on network structure.
50
51  for i=1:3
52      switch i
53          case 1
54              s1='Random';
55              val=0;
56          case 2
57              s1='Parallel';
58              val=27;
59          case 3
60              s1='Antiparallel';
61              val=54;
62      end
63      for j=1:4
64          switch j
65              case 1
66                  s2='Clusteing Coefficient';
67                  main_vec=clust_coefficient;
68              case 2
69                  s2='Number of connected components';
70                  main_vec=s;
71              case 3
72                  s2='Average path length';
73                  main_vec=average_path_length;
74              case 4
75                  s2='Network diameter';
76                  main_vec=diam;
77          end
78          for k=1:3
79              switch k
80                  case 1
81                      s3='(alpha:phi)';
82                      x=0:0.5:1;
83                      xl='alpha';
84                      y=0:0.5:1;
85                      yl='phi';
86                      for l=1:3
87                          M=[main_vec(val+l),main_vec(val+l+9),main_vec(val+l+18);main_vec(val+l+3),ma
88                          %
89                          [xlab,ylab]=meshgrid(x,y);
90                          hold on;
91                          view(0,90);
92                          surf(xlab,ylab,M,'EdgeColor','none');
93                          colorbar;
94                          set(gca,'FontSize',14)
95                          xlabel(xl);
96                          ylabel(yl);
97                          switch l

```

```

98         case 1
99             s5='first';
100        case 2
101            s5='second';
102        case 3
103            s5='third';
104    end
105    name2=['phase diagram of ',s2,' for alpha versus phi',' ...
        and','the', s5,' threshold','obtained for',s1,'idea ...
        distribution'];
106    title(name2);
107    name=['Plotting2_', 'Pdiag-',s1,'-',s2,'-',s3,'-',s4,'AND','the', ...
        s5,' threshold'];
108    saveas(f,name);
109    hold off
110    %
111    end
112    case 2
113        s3='(alpha:threshold)';
114        x=0:0.5:1;
115        xl='alpha';
116        y=0:0.5:1;
117        yl='threshold';
118        for l=1:3
119            M=[main_vec(val+9*(l-1)+1),main_vec(val+9*(l-1)+2),main_vec(val+9*(l-1)+3);ma
120            %
121            [xlab,ylab]=meshgrid(x,y);
122            hold on;
123            view(0,90);
124            surf(xlab,ylab,M,'EdgeColor','none');
125            colorbar;
126            set(gca,'FontSize',14)
127            xlabel(xl);
128            ylabel(yl);
129            switch l
130                case 1
131                    s5='first';
132                case 2
133                    s5='second';
134                case 3
135                    s5='third';
136            end
137            name2=['phase diagram of ',s2,' for alpha versus ...
                threshold',' and','the', s5,' phi','obtained ...
                for',s1,'idea distribution'];
138            title(name2);
139            name=['Plotting2_', 'Pdiag-',s1,'-',s2,'-',s3,'-',s4,' ...
                and','the', s5,' phi'];
140            saveas(f,name);
141            hold off

```

```

142         %
143     end
144
145     case 3
146         s3='(phi:threshold)';
147         x=0:0.5:1;
148         xl='phi';
149         y=0:0.5:1;
150         yl='threshold';
151         for l=1:3
152             M=[main_vec(val+3*(l-1)+1),main_vec(val+3*(l-1)+2),main_vec(val+3*(l-1)+3);ma
153             %
154             [xlab,ylab]=meshgrid(x,y);
155             hold on;
156             view(0,90);
157             surf(xlab,ylab,M,'EdgeColor','none');
158             colorbar;
159             set(gca,'FontSize',14)
160             xlabel(xl);
161             ylabel(yl);
162             switch l
163                 case 1
164                     s5='first';
165                 case 2
166                     s5='second';
167                 case 3
168                     s5='third';
169             end
170             name2=['phase diagram of ',s2,' for phi versus ...
171                 threshold',' and','the', s5,' threshold','obtained ...
172                 for',s1,'idea distribution'];
173             title(name2);
174             name=['Plotting2_', 'Pdiag-',s1,'-',s2,'-',s3,'-',s4,' ...
175                 and','the', s5,' threshold'];
176             saveas(f,name);
177             hold off
178             %
179         end
180     end
181 end
182
183 %%% Dependence of the property of each network to the parameters
184 for i=1:3
185
186     switch i
187         case 1
188             s1='Random';

```

```

189         val=0;
190         case 2
191             s1='Parallel';
192             val=27;
193         case 3
194             s1='Antiparallel';
195             val=54;
196     end
197
198     for j=1:4
199         switch j
200             case 1
201                 s2='clust_coefficient';
202                 main_vec=clust_coefficient;
203             case 2
204                 s2='s';
205                 main_vec=s;
206             case 3
207                 s2='average_path_length';
208                 main_vec=average_path_length;
209             case 4
210                 s2='diam';
211                 main_vec=diam;
212             end
213             for k=1:3
214                 switch k
215                     case 1
216                         s3='threshold';
217                         f=figure();
218                         for l=1:3
219                             line=main_vec(3*(0:8)+val+1);
220                             set(gca,'FontSize',14)
221                             xlabel('Different pairs of alpha and phi values');
222                             ylabel(s2);
223                             namek=['Dependence of',' ',s2,' ','on',' ',s3,' ...
224                                     ','for',' ',s1,' ','idea distribution'];
225                             title(namek);
226
227                             switch l
228                                 case 1
229                                     plot(line,'—ks','LineWidth',2,'MarkerEdgeColor','k','MarkerFaceColor','k');
230                                     hold on
231                                 case 2
232                                     plot(line,'—bs','LineWidth',2,'MarkerEdgeColor','b','MarkerFaceColor','b');
233                                     hold on
234                                 case 3
235                                     plot(line,'—rs','LineWidth',2,'MarkerEdgeColor','r','MarkerFaceColor','r');
236                                     hold on
237                             end
238                             legend('threshold=0.001','threshold=0.01','threshold=0.05')
239                             name=['plotting1-',s1,'-',s2,'-',s3,'-','threshold sensitivity'];

```



```

238         saveas(f,name);
239     end
240
241     case 2
242         s3='alpha';
243         f=figure();
244         for l=1:3
245             line=main_vec([1,2,3,10,11,12,19,20,21]+val+3*(l-1));
246             set(gca,'FontSize',14)
247             xlabel('Different pairs of threshold and phi ...
                values');
248             ylabel(s2);
249             namek=['Dependence of',' ',s2,' ','on',' ',s3,' ...
                ','for',' ',s1,' ','idea distribution'];
250             title(namek);
251
252             switch l
253                 case 1
254                     plot(line,'—ks','LineWidth',2,'MarkerEdgeColor','k','MarkerF
255                     hold on
256                 case 2
257                     plot(line,'—bs','LineWidth',2,'MarkerEdgeColor','b','MarkerF
258                     hold on
259                 case 3
260                     plot(line,'—rs','LineWidth',2,'MarkerEdgeColor','r','MarkerF
261             end
262             legend('alpha=0.01','alpha=0.05','alpha=0.10')
263             name=['plotting1-',s1,'-',s2,'-', 'alpha.sensitivity'];
264             saveas(f,name);
265         end
266
267     case 3
268         s3='phi';
269         f=figure();
270         for l=1:3
271             line=main_vec((1:9)+val+9*(l-1));
272             set(gca,'FontSize',14)
273             xlabel('Different pairs of threshold and alpha ...
                values');
274             ylabel(s2);
275             namek=['Dependence of',' ',s2,' ','on',' ',s3,' ...
                ','for',' ',s1,' ','idea distribution'];
276             title(namek);
277
278             switch l
279                 case 1
280                     plot(line,'—ks','LineWidth',2,'MarkerEdgeColor','k','MarkerF
281                     hold on
282                 case 2
283                     plot(line,'—bs','LineWidth',2,'MarkerEdgeColor','b','MarkerF

```

```

284         hold on
285         case 3
286             plot(line, '--rs', 'LineWidth', 2, 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r');
287         end
288         legend('phi=0.1', 'phi=0.3', 'phi=0.5')
289         name=['plotting1-', s1, '-', s2, '-', 'phi_sensitivity'];
290         saveas(f, name);
291     end
292 end
293 end
294 end
295 end
296
297
298 %%%%%%%%% generating plots for each features of network structure for each
299 %%%%%%%%% parameter sets %%%%%%%%%
300
301 for i=1:4
302     switch i
303     case 1
304         f=figure();
305         line1=clust_coefficient(1:27);
306         line2=clust_coefficient(28:54);
307         line3=clust_coefficient(55:81);
308         plot(line1, '--ks', 'LineWidth', 2, 'MarkerEdgeColor', 'k', 'MarkerFaceColor', 'k', 'MarkerSize', 10);
309         hold on
310         plot(line2, '--gs', 'LineWidth', 2, 'MarkerEdgeColor', 'g', 'MarkerFaceColor', 'g', 'MarkerSize', 10);
311         hold on
312         plot(line3, '--bs', 'LineWidth', 2, 'MarkerEdgeColor', 'b', 'MarkerFaceColor', 'b', 'MarkerSize', 10);
313         set(gca, 'FontSize', 14)
314         xlabel('Different triplets of alpha ,phi and threshold values')
315         ylabel('Clustering Coefficient')
316         title('The influence of idea distribution on clustering ...
317             coefficient');
317         legend('Random distribution', 'Parallel ...
318             distribution', 'Antiparallel distribution')
318         name=('Plotting2_idea.Comparison.clustcoeff');
319         saveas(f, name);
320     case 2
321         f=figure();
322         line1=s(1:27);
323         line2=s(28:54);
324         line3=s(55:81);
325         plot(line1, '--ks', 'LineWidth', 2, 'MarkerEdgeColor', 'k', 'MarkerFaceColor', 'k', 'MarkerSize', 10);
326         hold on
327         plot(line2, '--gs', 'LineWidth', 2, 'MarkerEdgeColor', 'g', 'MarkerFaceColor', 'g', 'MarkerSize', 10);
328         hold on
329         plot(line3, '--bs', 'LineWidth', 2, 'MarkerEdgeColor', 'b', 'MarkerFaceColor', 'b', 'MarkerSize', 10);
330         set(gca, 'FontSize', 14)
331         xlabel('Different triplets of alpha ,phi and threshold values')

```

```

332         ylabel('Number of connected components')
333         title('The influence of idea distribution on the number of ...
           connected components')
334         legend('Random distribution','Parallel ...
           distribution','Antiparallel distribution')
335         name=('Plotting2.idea.Comparison.connncmp');
336         saveas(f,name);
337     case 3
338         f=figure();
339         line1=average_path_length(1:27);
340         line2=average_path_length(28:54);
341         line3=average_path_length(55:81);
342         plot(line1,'—ks','LineWidth',2,'MarkerEdgeColor','k','MarkerFaceColor','k','MarkerSize',10)
343         hold on
344         plot(line2,'—gs','LineWidth',2,'MarkerEdgeColor','g','MarkerFaceColor','g','MarkerSize',10)
345         hold on
346         plot(line3,'—bs','LineWidth',2,'MarkerEdgeColor','b','MarkerFaceColor','b','MarkerSize',10)
347         set(gca,'FontSize',14)
348         xlabel('Different triplets of alpha ,phi and threshold values')
349         ylabel('average path length')
350         title('The influence of idea distribution on average path ...
           length')
351         legend('Random distribution','Parallel ...
           distribution','Antiparallel distribution')
352         name=('Plotting2.idea.Comparison.average_path_length');
353         saveas(f,name);
354     case 4
355         f=figure();
356         line1=diam(1:27);
357         line2=diam(28:54);
358         line3=diam(55:81);
359         plot(line1,'—ks','LineWidth',2,'MarkerEdgeColor','k','MarkerFaceColor','k','MarkerSize',10)
360         hold on
361         plot(line2,'—gs','LineWidth',2,'MarkerEdgeColor','g','MarkerFaceColor','g','MarkerSize',10)
362         hold on
363         plot(line3,'—bs','LineWidth',2,'MarkerEdgeColor','b','MarkerFaceColor','b','MarkerSize',10)
364         set(gca,'FontSize',14)
365         xlabel('Different triplets of alpha ,phi and threshold values')
366         ylabel('Network Diameter')
367         title('The influence of idea distribution on Network Diameter')
368         legend('Random distribution','Parallel ...
           distribution','Antiparallel distribution')
369         name=('Plotting2.idea.Comparison.diam');
370         saveas(f,name);
371     end
372 end
373
374 %%%%%%%%% plots for degree distribution resulting from different idea
375 %%%%%%%%% distribution
376 count=0;

```

```

377 for choice1=1:3
378     switch choice1
379         case 1
380             s1='phi=0.1';
381             m1='1';
382         case 2
383             s1='phi=0.3';
384             m1='2';
385         case 3
386             s1='phi=0.5';
387             m1='3';
388     end
389
390 for choice2=1:3
391     switch choice2
392         case 1
393             s2='alpha=0.01';
394             m2='1';
395         case 2
396             s2='alpha=0.05';
397             m2='2';
398         case 3
399             s2='alpha=0.1';
400             m2='3';
401     end
402
403 for choice3=1:3
404     switch choice3
405         case 1
406             s3='threshold=0.001';
407             m3='1';
408         case 2
409             s3='threshold=0.01';
410             m3='2';
411         case 3
412             s3='threshold=0.05';
413             m3='3';
414     end
415     count=count+1;
416     line1=dgr{count};
417     mine1=frq{count}/1000;
418     line2=dgr{27+count};
419     mine2=frq{27+count};
420     line3=dgr{54+count};
421     mine3=frq{54+count};
422     f=figure();
423     loglog(line1,mine1,'—ks','LineWidth',2,'MarkerEdgeColor','k','MarkerFaceColor','k');
424     hold on
425     plot(line2,mine2,'—rs','LineWidth',2,'MarkerEdgeColor','r','MarkerFaceColor','r');
426     hold on

```

```

427         plot(line3,mine3,'—bs','LineWidth',2,'MarkerEdgeColor','b','MarkerFaceColor','b',
428             set(gca,'FontSize',14)
429             xlabel('Log(degree)')
430             ylabel('Log(frequency)')
431             title(['The influence of idea distribution on the degree ...
                    distribution',' ',s1,' ',s2,' ',s3]);
432             legend('Random distribution','Parallel ...
                    distribution','Antiparallel distribution')
433             name=(['Plotting2_idea.comparison_degree_distribution-',m1,'-',m2,'-',m3]);
434             saveas(f,name);
435         end
436     end
437 end

```

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