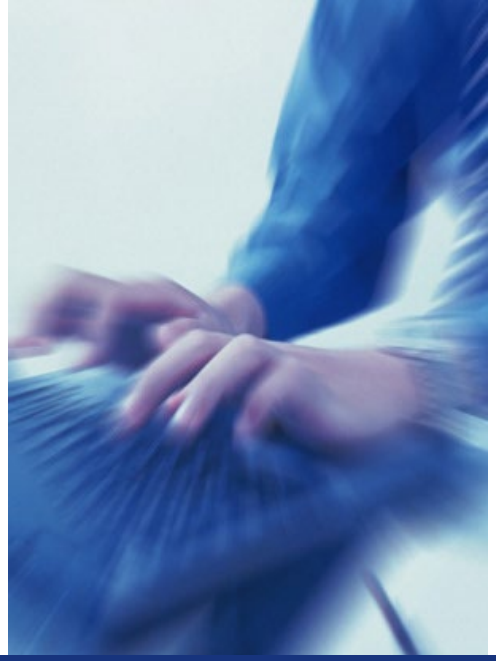
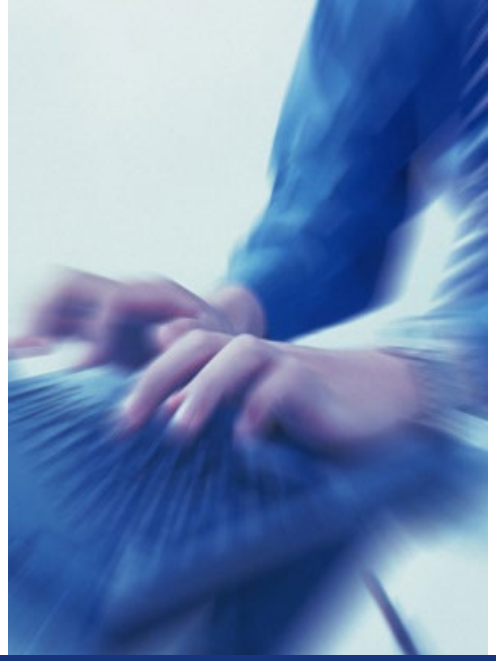


Logo

Discrete Mathematics

Dr. Han Huang

South China University of Technology



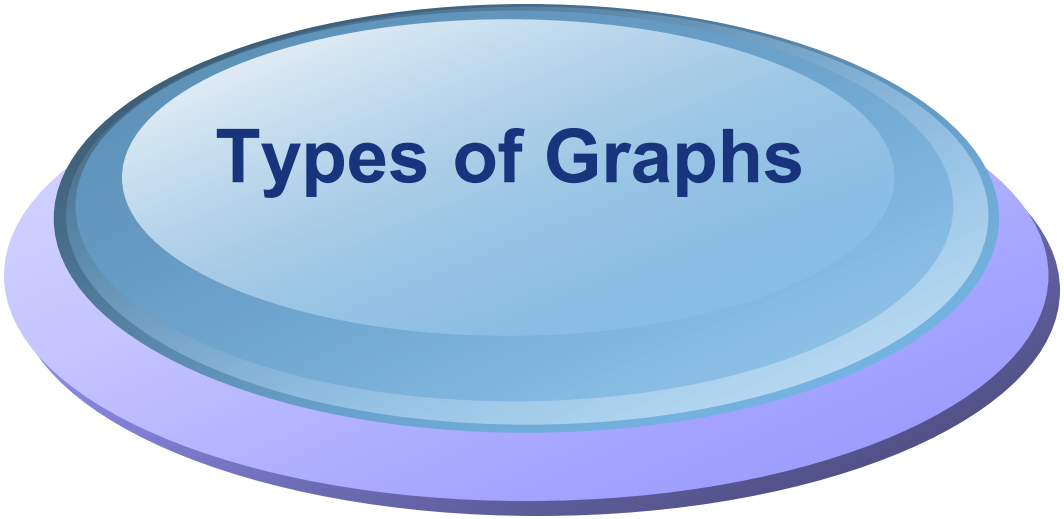
Logo

Chapter 4. Graphs

Introduction to Graphs

Section 4.1

Contents


Types of Graphs

Example

- 社交网络:图可以描述人与人之间的相识关系，对话关系，影响关系以及社交结构中的其他关系。
- 交通运输网:对于公共交通网络，停靠点是图中的顶点，各个顶点间的路线是图中的边。许多程序都使用这样的图模型，例如谷歌地图、高德地图。
- 文档链接图：链接图可以用来分析网页的相关性、最佳的信息源和较好的网站链接。
- 网络数据流量图：IP地址（Internet协议）是顶点，各个IP地址间传输的数据包构成图中的边。这类图可以用于分析网络安全。

Example

- 语义网络：顶点表示单词或概念，边表示单词或概念间的关系。语义网络已经被用于人类组织知识体系的各种模型，以及机器如何模拟这些组织行为的实践。
- 依赖图：可以用来描述项目之间的优先级以及依赖情况。依赖图通常用于大型项目中，用来规划各个组件间的依赖关系，在遵守各部分依赖的情况下降低项目的总耗时与总成本。
- 机器人计划图：顶点表示机器人可能所处的状态，边表示各个状态间可能发生的过渡。通过图的形式，可以将连续运动近似为一系列的离散步骤。例如，这种图可以用于规划自动驾驶车辆的路径。

Graphs

- Graph was introduced in 18th century by Euler, and was used to solve the Konigsberg bridge problem.
- Graph theory plays an important role in computer science.
- Graphs can be used to model discrete objects, such as networks, process scheduling, circuit layout, ..., and so on.

Konigsberg Bridge Problem

- The city of Konigsberg was divided into 4 sections by the Pregel River. The 4 sections are connected by 7 bridges.
- Is it possible to start at some location, travel across all the bridges without crossing any bridge twice, and return to the starting point?

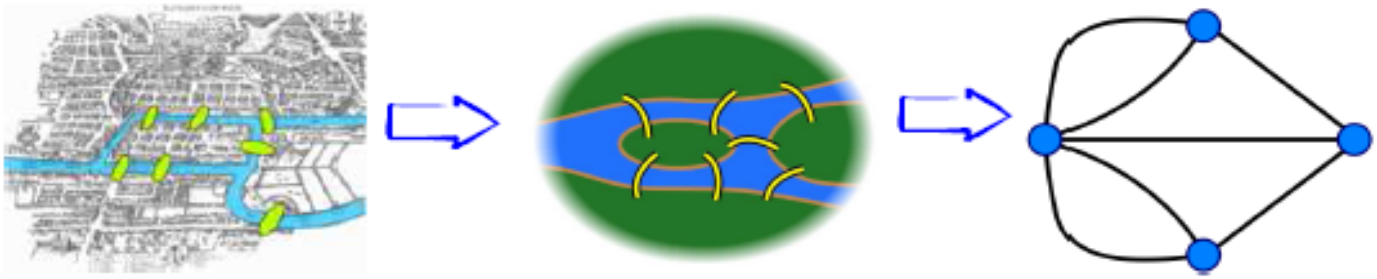
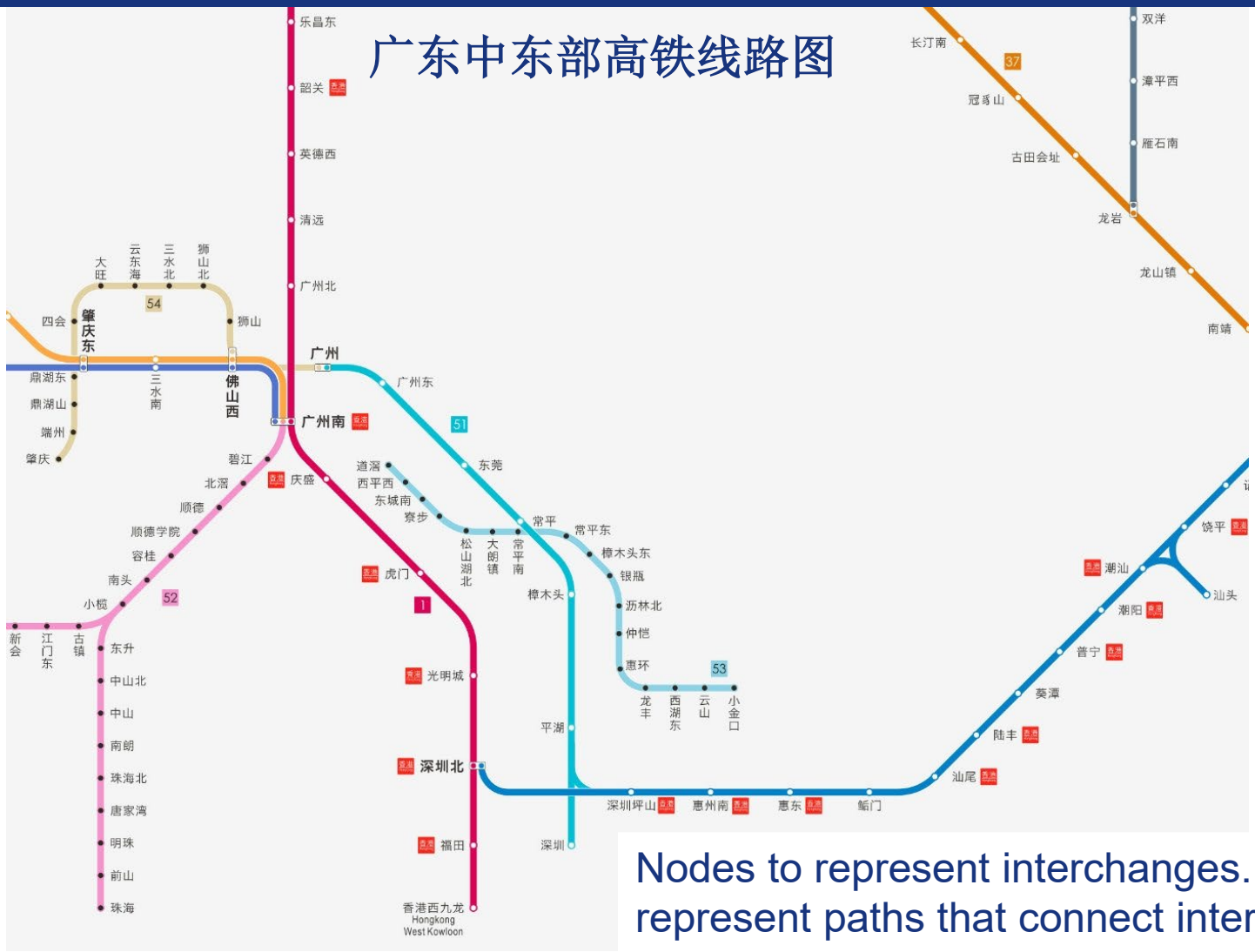


Image source – http://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg

Applications of Graphs



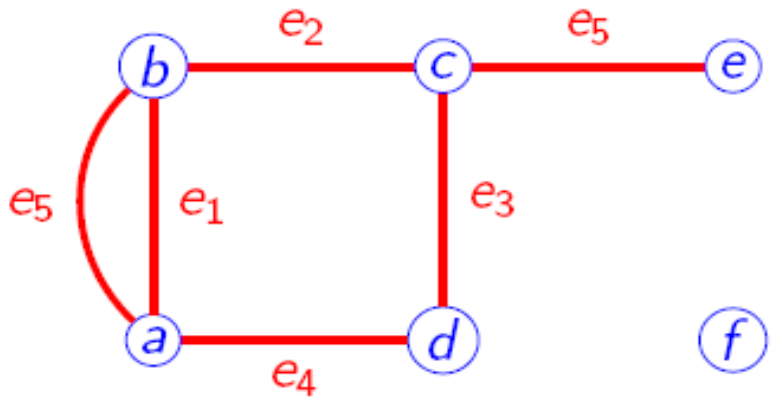
Nodes to represent interchanges. Lines to represent paths that connect interchanges

Definition

Definition 1.1 A graph (or undirected graph) $G = (V, E)$ consists of two sets as follows

- A nonempty finite set $V = \{v_1, v_2, \ldots\}$ of vertices
- A finite set $E = \{e_1, e_2, \ldots\}$ of edges such that each element e_k is identified by an unordered pair (v_i, v_j)

Note : An unordered pair means that $(v_i, v_j) = (v_j, v_i)$



$$V(G) = \{a, b, c, d, e, f\}$$

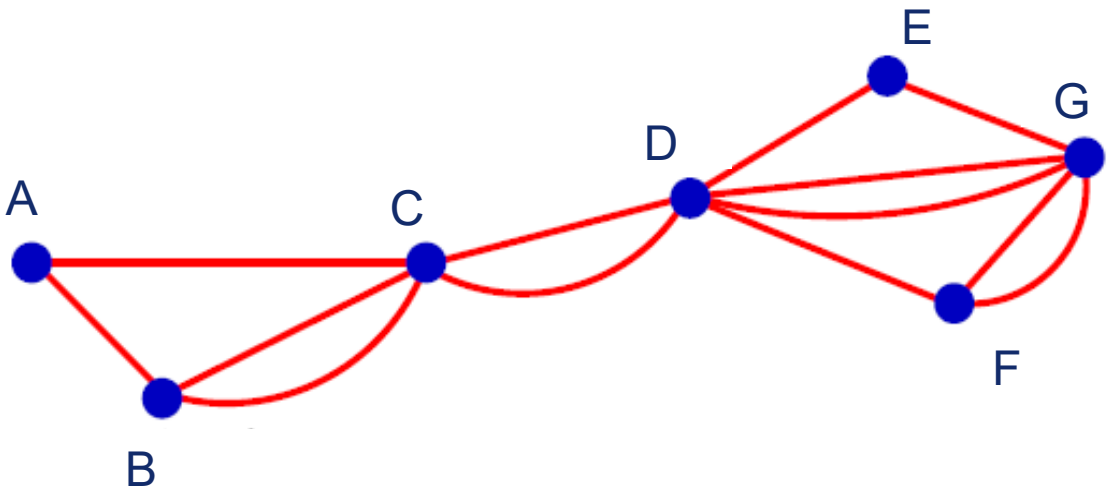
$$E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

Definition

Definition 1.2 A multigraph graph $G = (V, E)$ consists of

- a set V of vertices,
- a set E of edges, and
- a function f from E to $\{\{u, v\} \mid u, v \in V\ u \neq v\}$.

The edge e_1 and e_2 are called parallel edges if $f(e_1) = f(e_2)$

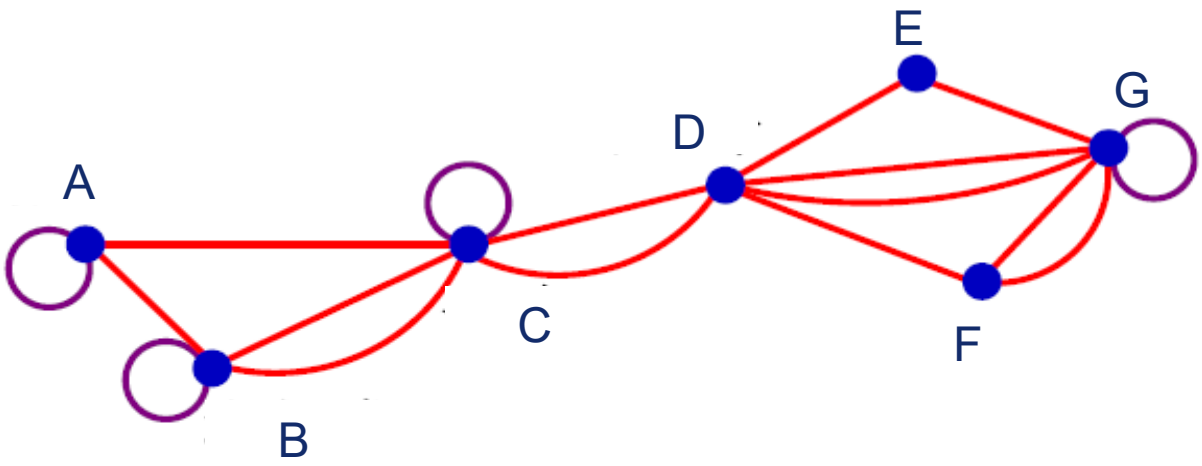


Definition

Definition 1.3 A pesudograph $G = (V, E)$ consists of

- a set V of vertices,
- a set E of edges, and
- a function f from E to $\{\{u, v\} | u, v \in V\}$.

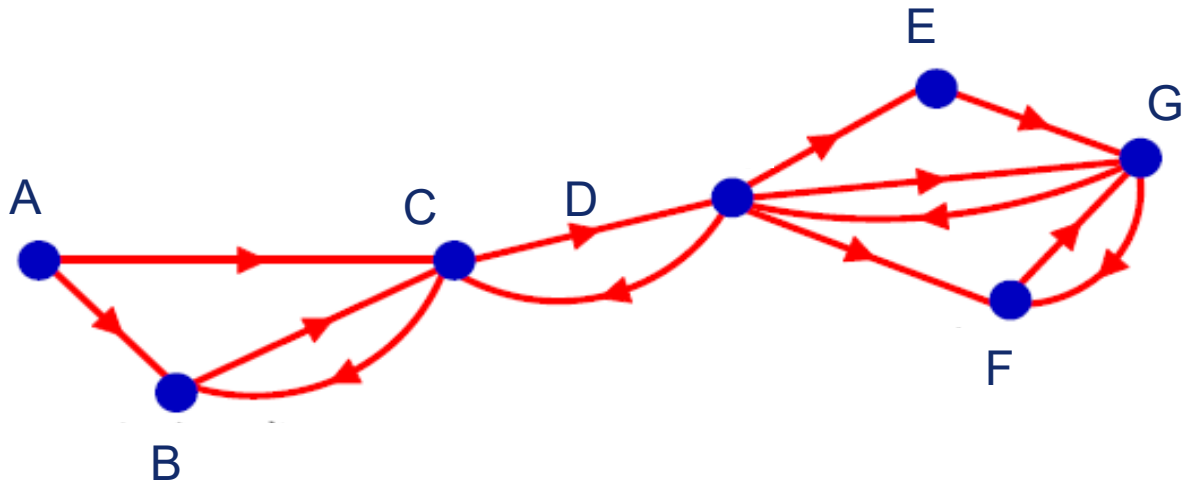
An edge is a loop if $f(e) = \{u, u\} = \{u\}$ for some $u \in V$.



Definition

Definition 1.4 A directed graph $G = (V, E)$ consists of

- a set V of vertices,
- a set E of edges that are ordered pairs of elements of V.

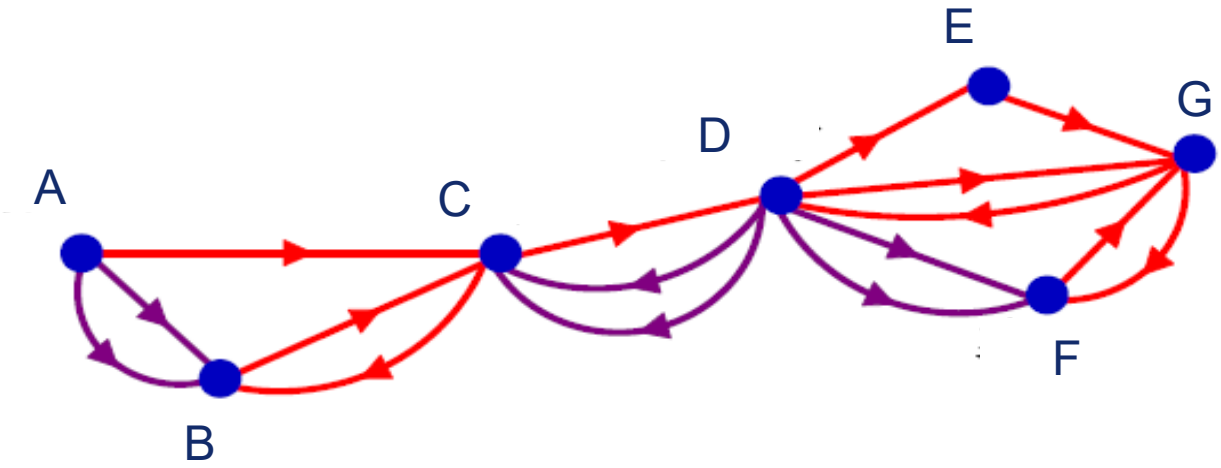


Definition

Definition 1.5 A directed multigraph $G = (V, E)$ consists of

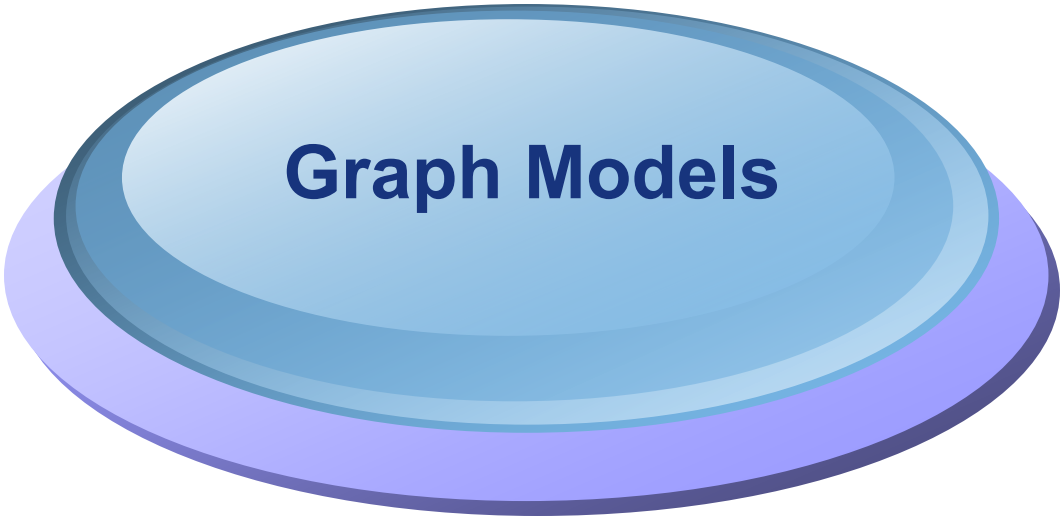
- a set V of vertices,
- a set E of edges, and
- a function f from E to $\{(u, v)|u, v \in V\}$.

The edge e_1 and e_2 are multiple edges if $f(e_1) = f(e_2)$

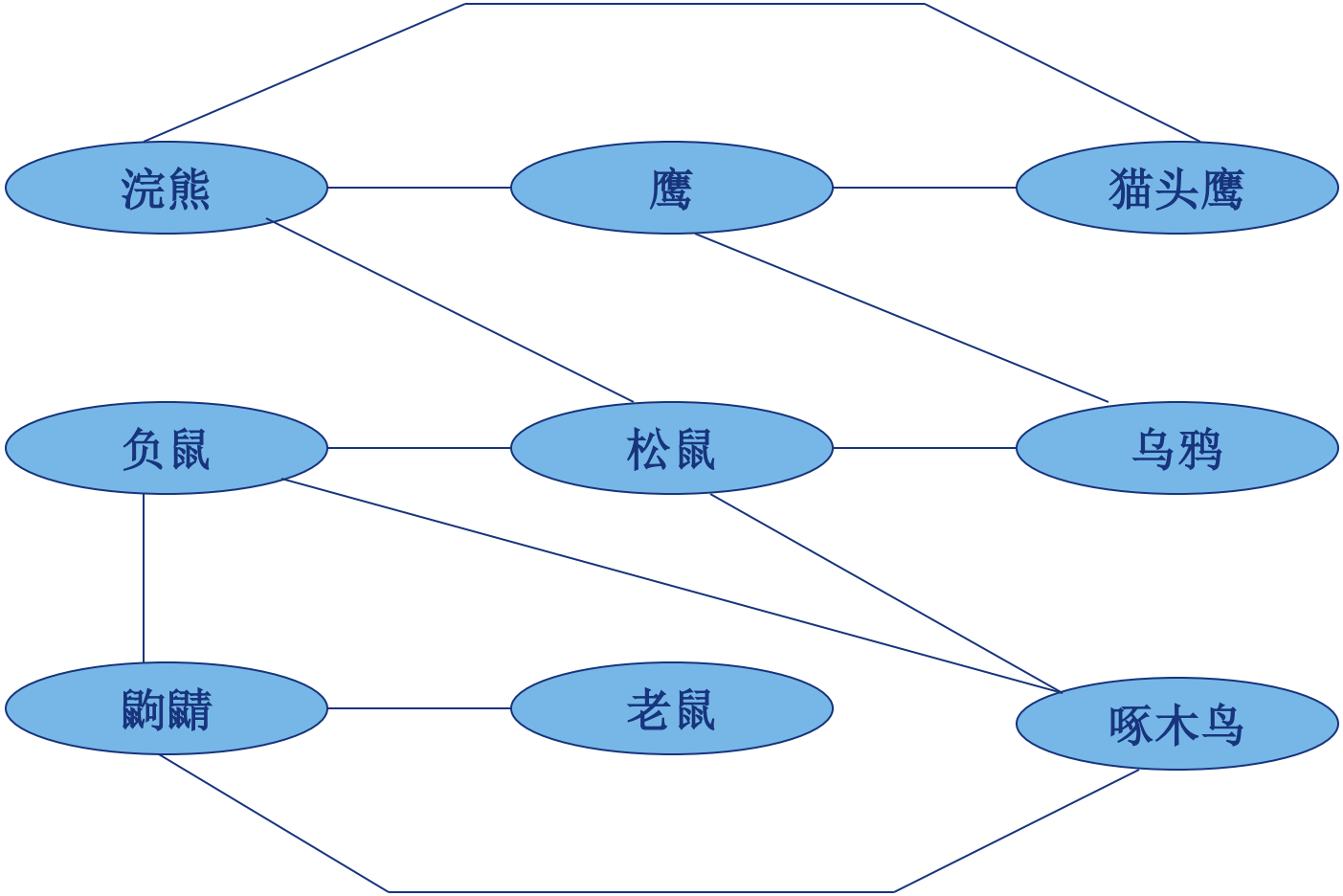


Types of Graphs

Type	Directed Edges	Multiple Edges	Loops
Simple graph	✘	✘	✘
Multigraph	✘	✔	✘
Pesudograph	✘	✔	✔
Directed graph	✔	✘	✔
Directed multigraph	✔	✔	✔


Graph Models

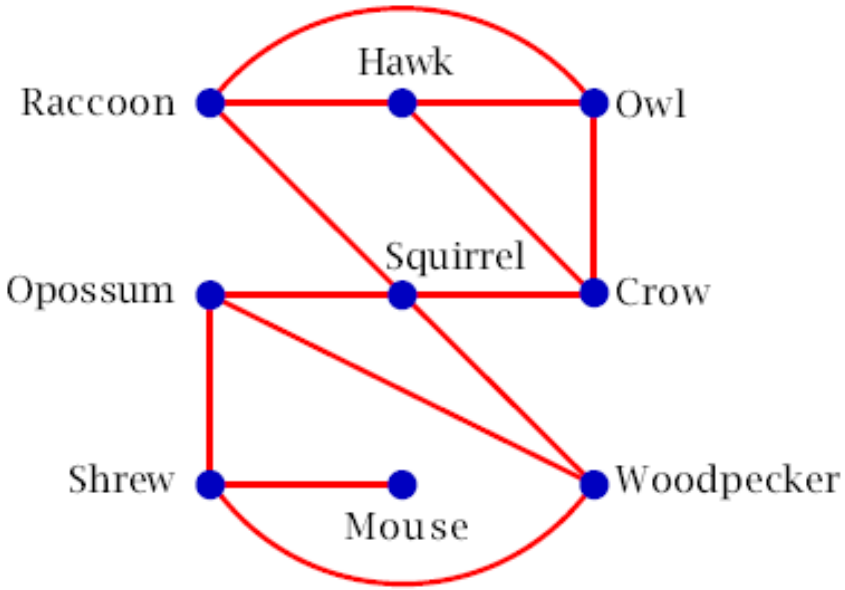
Example 1 Niche Overlap Graphs in Ecology



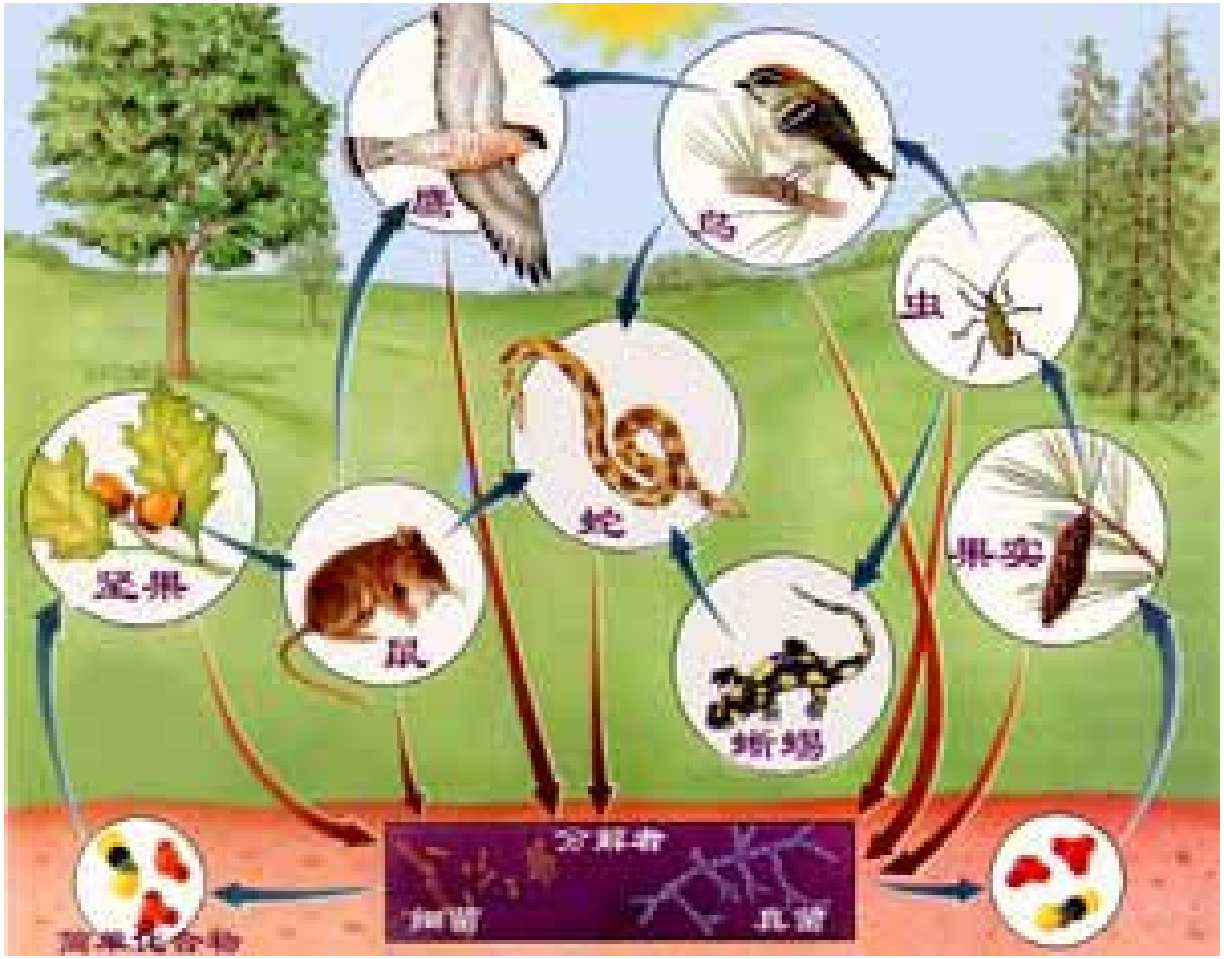
Niche Overlap graphs in Ecology

Graphs are used to model the interaction of different species of animals.

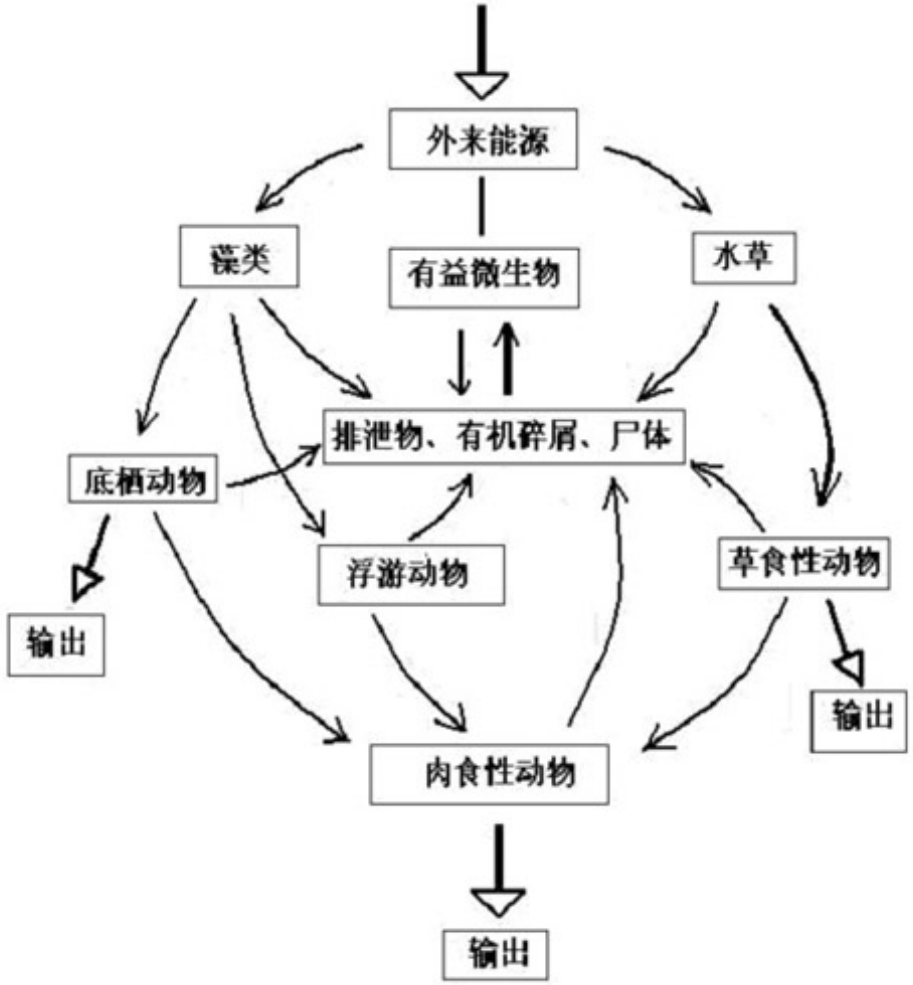
- Each species is represented by a vertex
- An undirected edge connects two species who compete. (食物鏈上有競爭關係)



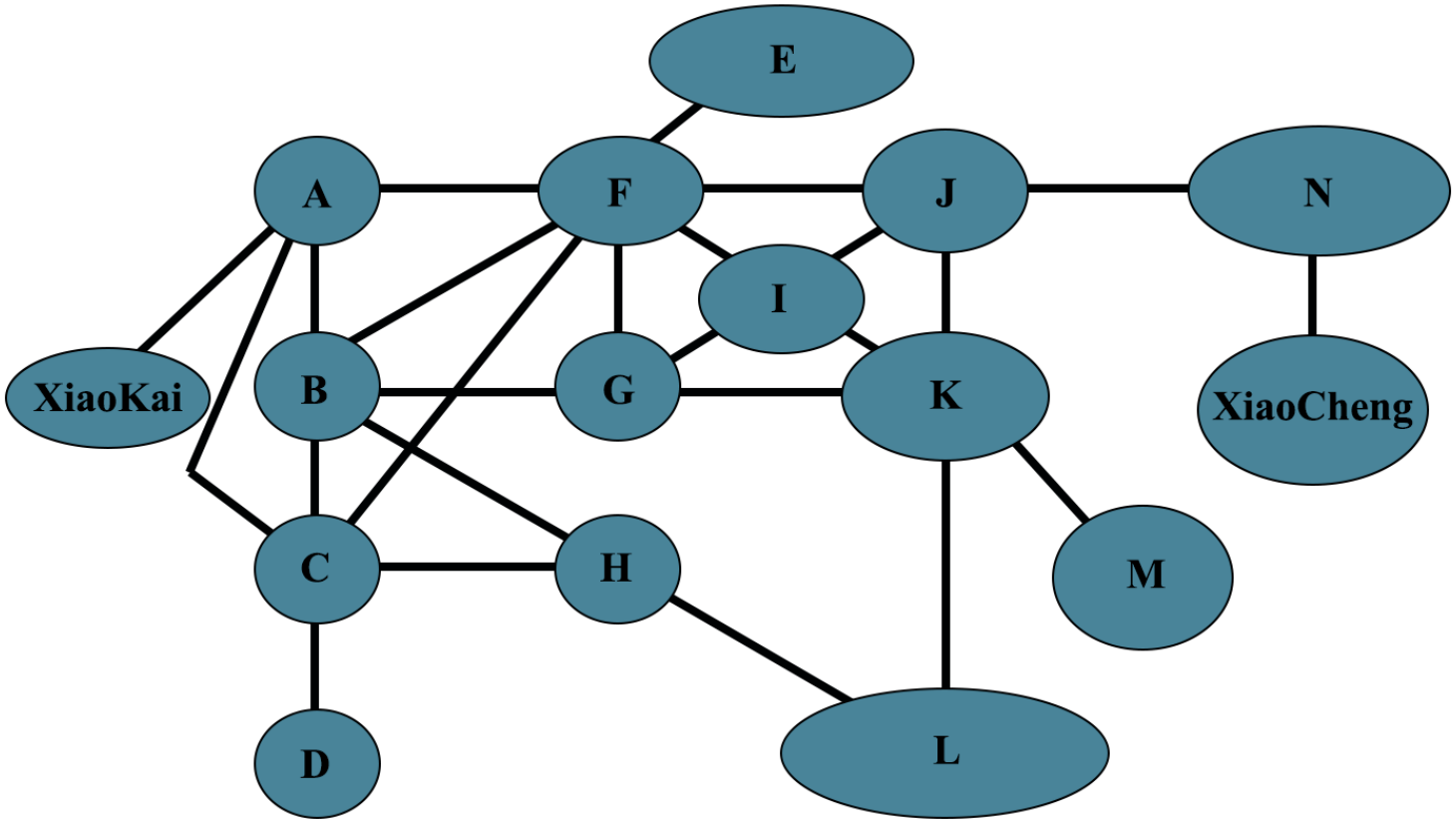
Example 1 Niche Resource Chain 1



Example 1 Niche Resource Chain 2



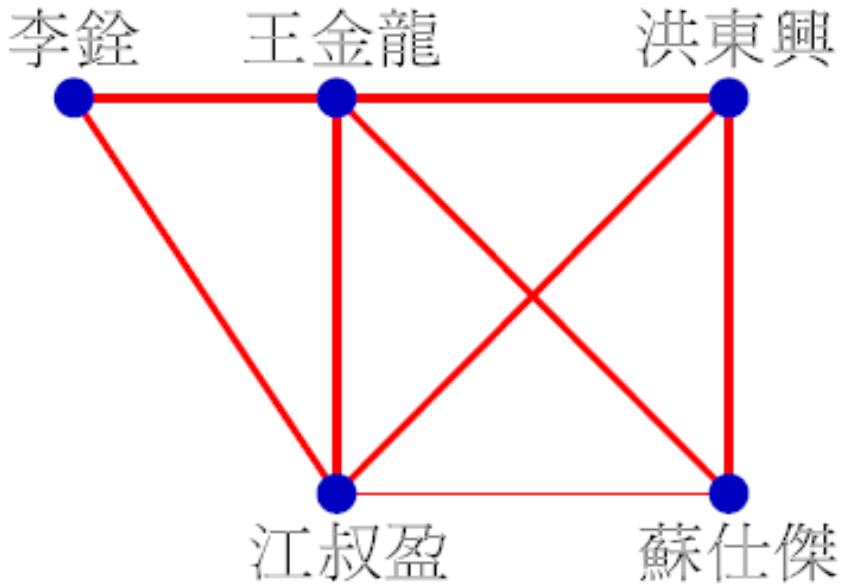
Example 2 Acquaintanceship Graphs



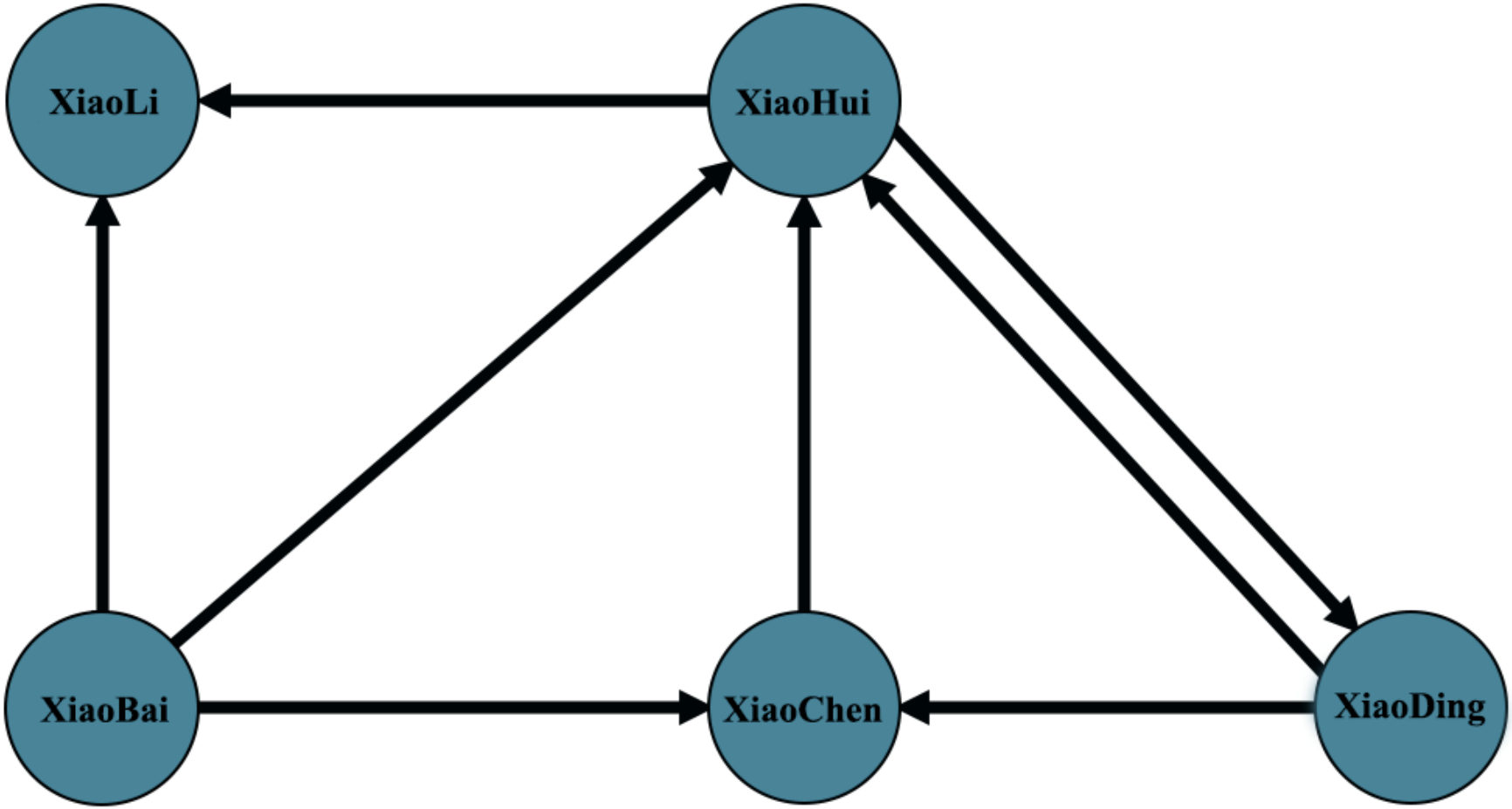
Acquaintanceship Graphs

The acquaintanceship graphs are used to model the various relationships between people.

- Each person is represented by a vertex
- An undirected graph connect two people who know each other.



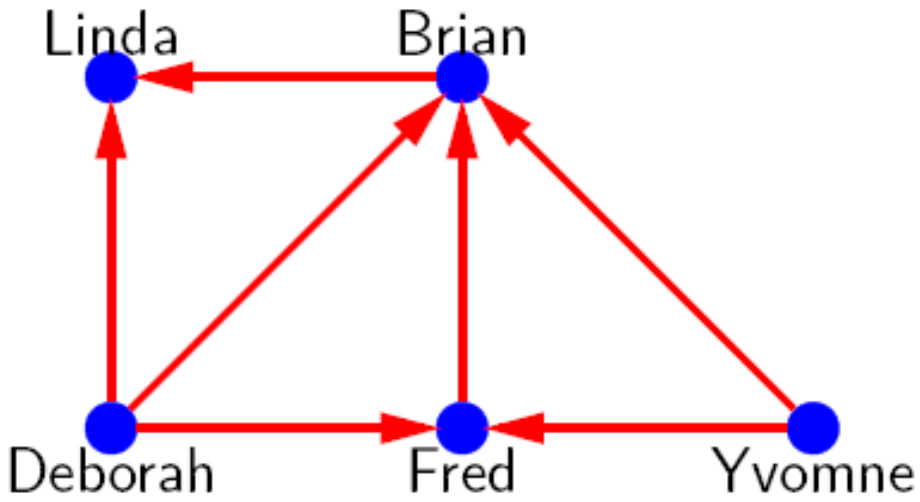
Example 3 Influence Graphs



Influence Graphs

Certain people can influence the thinking of others.

- Each person is represented by a vertex
- Directed edge : person a (initial vertex a) influences person b (terminal vertex b)



Example 4 The Hollywood Graph

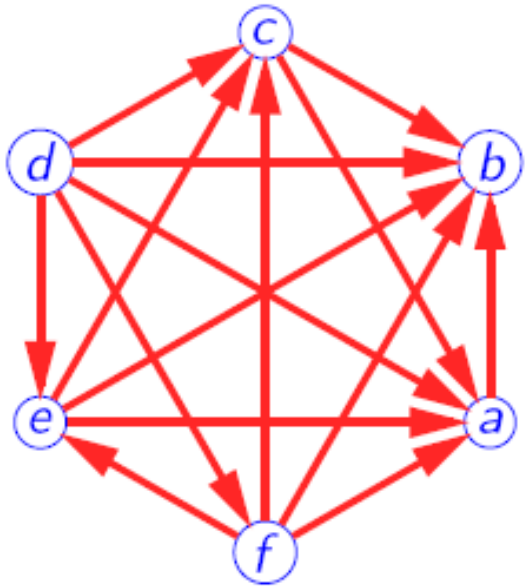
The Hollywood graph is used to model whether the actors have worked together on a movie.

- The vertices of the Hollywood graph are actors.
- An undirected edge connects two actors who have worked together on a movie.

Example 5 Round-Robin Touraments

A round-robin tournament is that every team plays every other team exactly once. The directed can model the result of a round-robin tournament.

- Each team is represented by a vertex.
- There is a directed edge from the vertex a to the vertex b if team a (initial vertex a) beats team b



Example 6 Collaboration Graphs

The collaboration graph is used to model the joint authorship of academic papers.

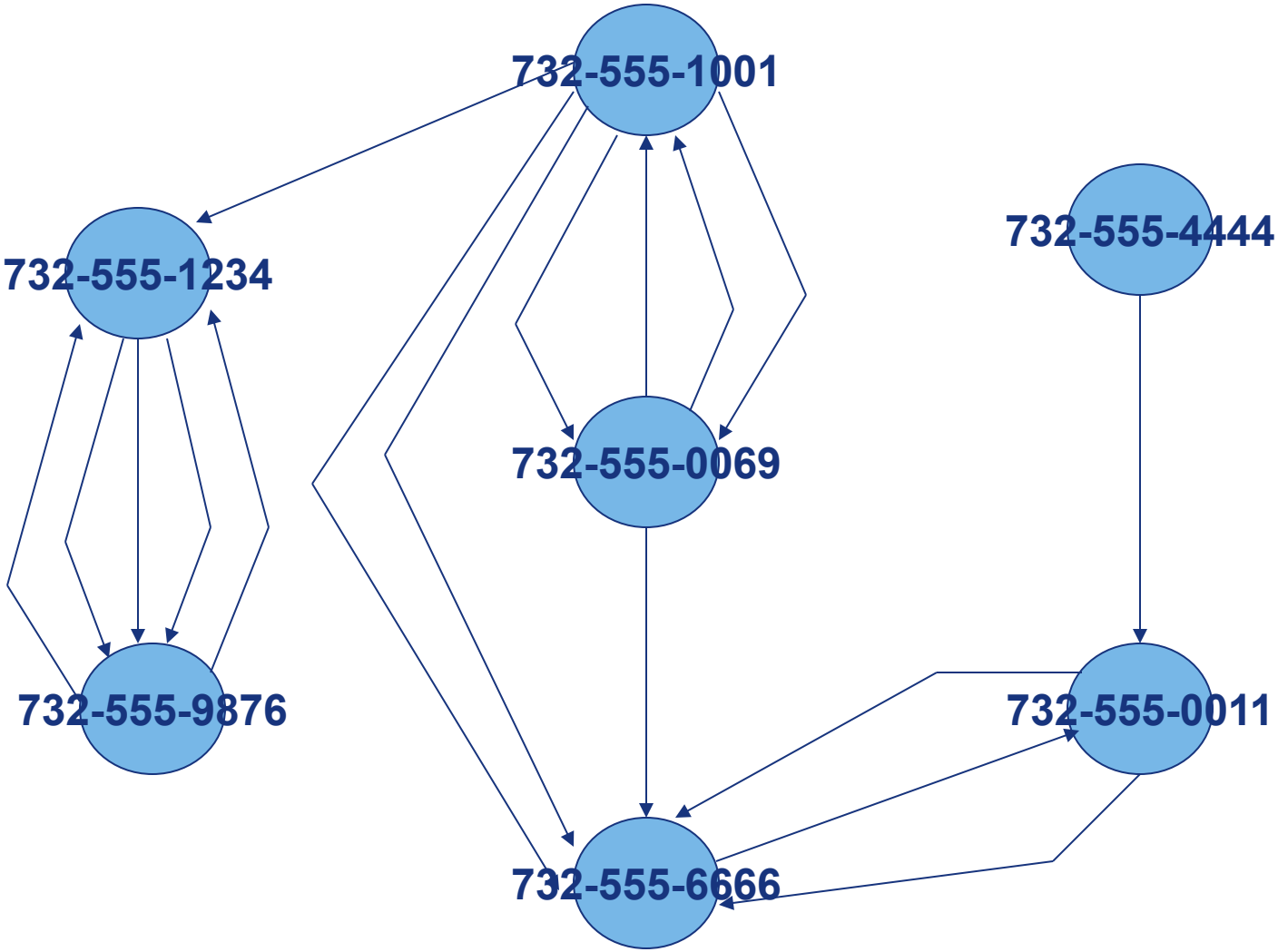
- The vertices of this graph are authors of academic papers.
- An undirected edge links two people if they have jointly written a paper.

DBLP链接: Han Huang, South China University of Technology

Example 7 Call Graphs

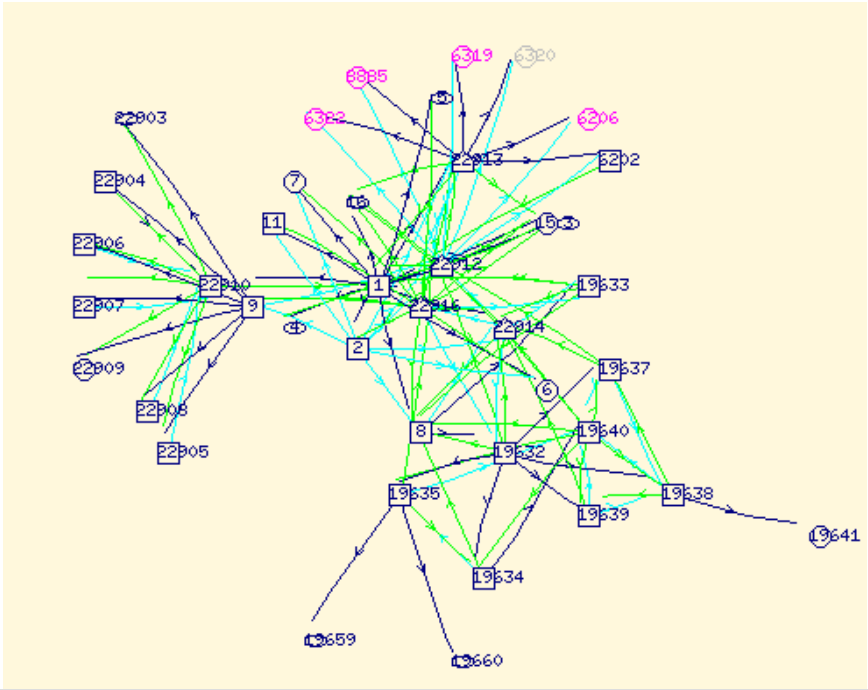
- A directed multigraph can be used to model calls where each telephone number is represented by a vertex and each telephone call is represented by directed edge.
- The edge representing a call starts at a telephone number from which the call was made and ends at the telephone number to which the call was made.

Example 7 Call Graphs

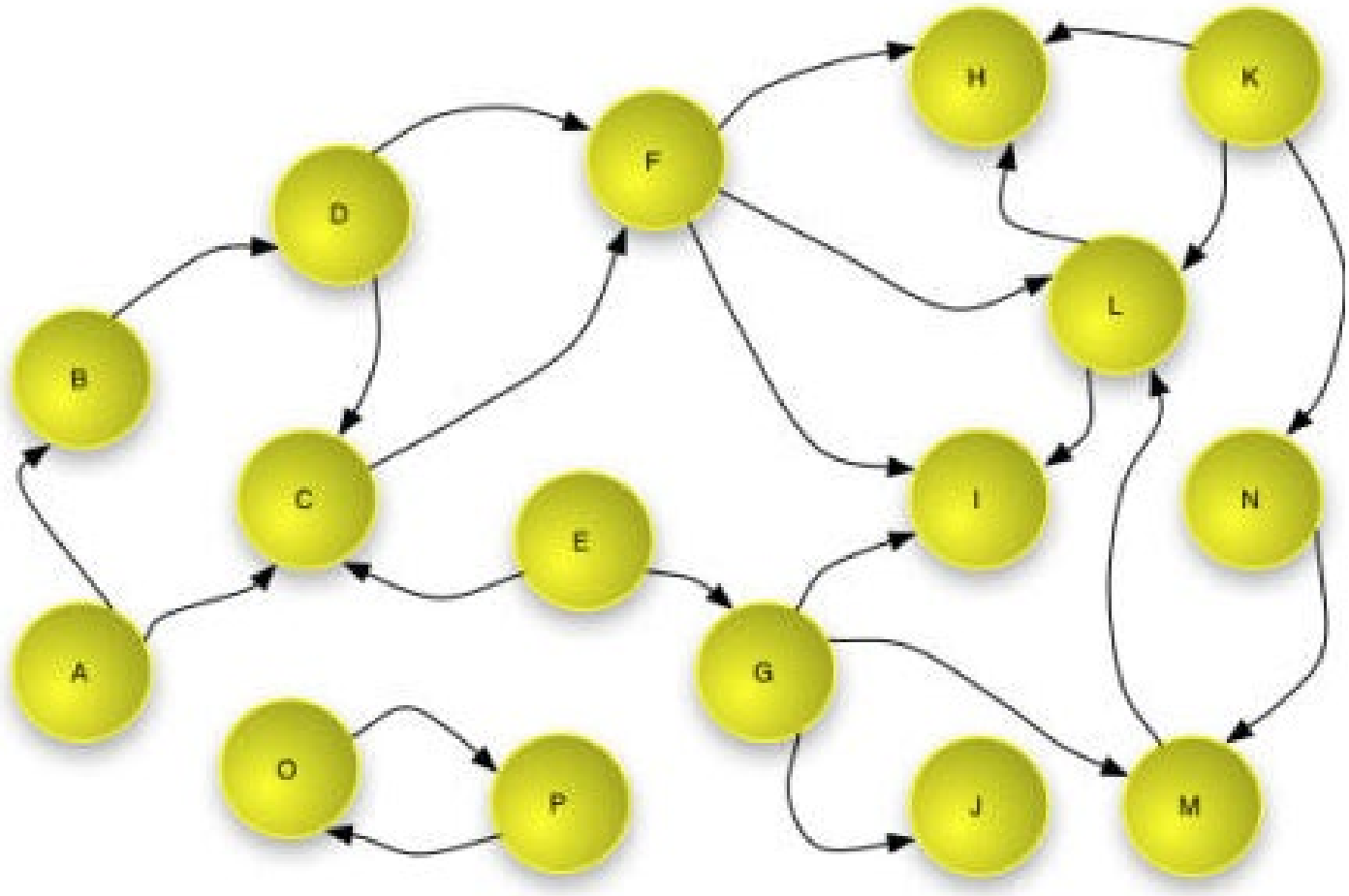


Example 8 The Web Graph

- The world wide web can be modeled as a directed graph where each Web page is represented by a vertex and where an edge starts at the Web page a and ends at the Web page b if there is a link on a pointing to b.



An example of Web Graph

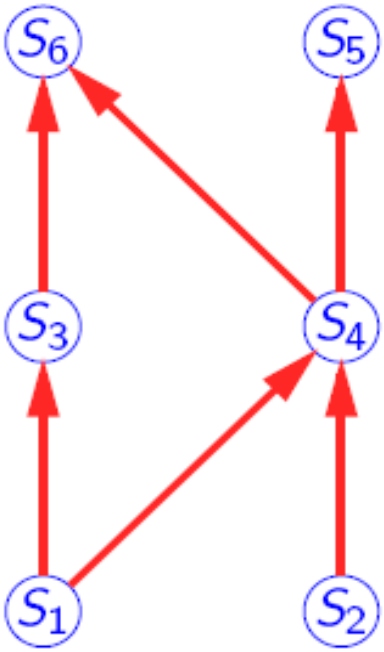


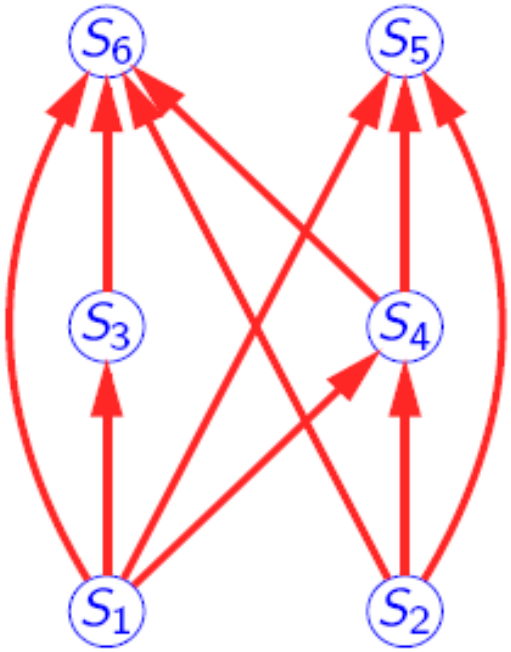
Example 9 A Precedence Graphs

The dependence of the task on previous tasks can be represented by a directed graph.

- Each task is represented by a vertex
- There is a directed edge from a task t_1 to a task t_2 if t_2 cannot be executed before t_1 has been executed

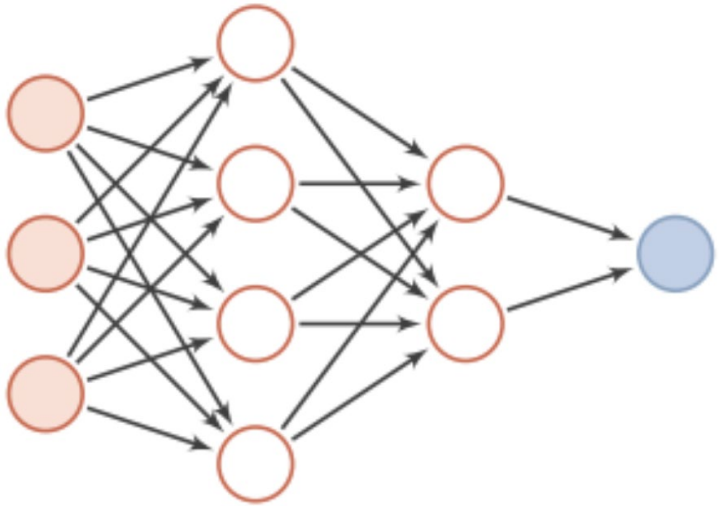
$$
\begin{aligned}
S_1\ a &= 0 \\
S_2\ b &= 1 \\
S_3\ c &= a_{s_1} + 1 \\
S_4\ d &= b_{s_2} + a_{s_1} \\
S_5\ e &= d_{s_4} + 1 \\
S_6\ e &= c_{s_3} + d_{s_4}
\end{aligned}
$$





Example 10 Artificial Neural Networks

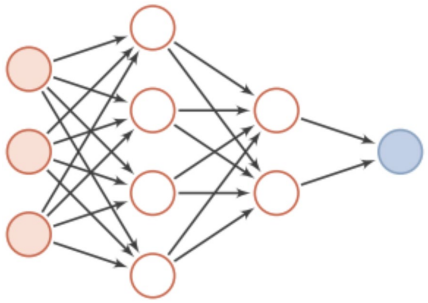
- **1. 前馈神经网络**（Feedforward Neural Network）:
- 信息从输入层开始输入，每层的神经元接收前一级输入，并输出到下一级，直至输出层。
- 整个网络信息输入传输中无反馈（循环），即任何层的输出都不会影响同级层，可用一个**有向无环图**表示.

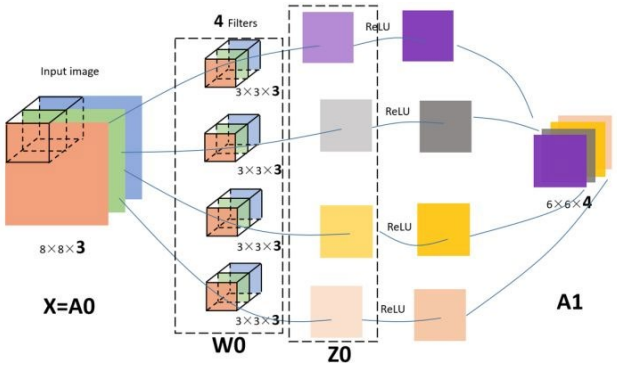


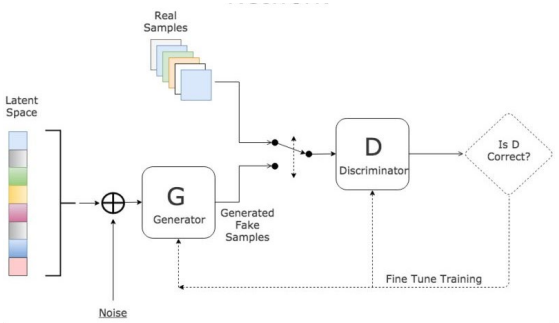
节点：神经元
边：权重
有向边：参数流向

Example 10 Artificial Neural Networks

- 常见的前馈神经网络包括：







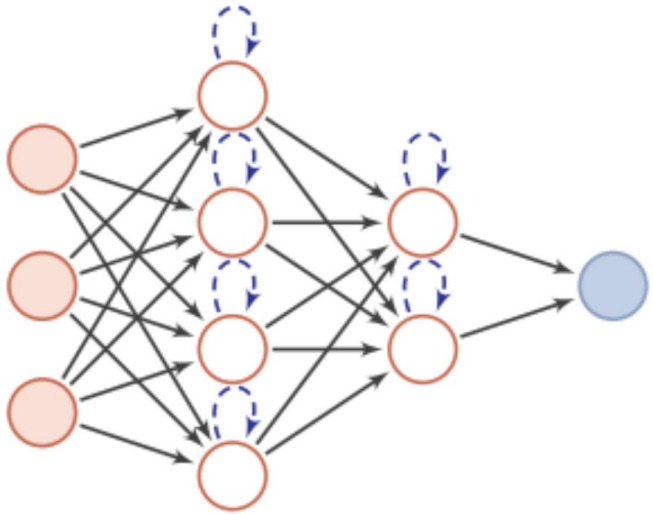
- **全连接神经网络（FCN)**
- 当前层的每个神经元都会接入前一层每个神经元的输入信号

- **卷积神经网络（CNN）**
- 卷积层可训练的，但参数明显少于标准的隐藏层，能突出并前向传播图像重要部分

- **生成对抗网络(GAN)**
- 随着时间的推移，鉴别器和生成器相互竞争

Example 10 Artificial Neural Networks

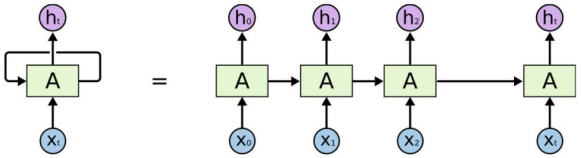
- **2.反馈神经网络（Feedback Neural Network）：**
- 神经元不但可以接收其他神经元的信号，而且可以接收自己的反馈信号。
- 反馈神经网络中的信息传播可以是单向也可以是双向传播，因此可以用一个**有向循环图**或者**无向图**来表示.



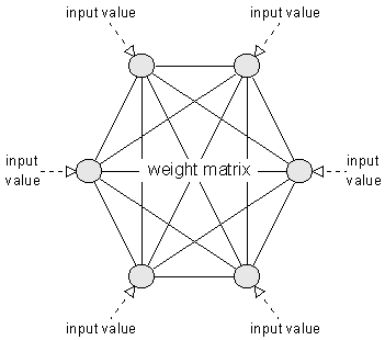
节点：神经元
边：权重
有向边：参数流向

Example 10 Artificial Neural Networks

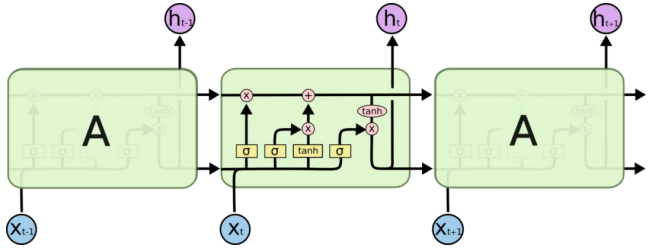
- 常见的反馈神经网络包括：



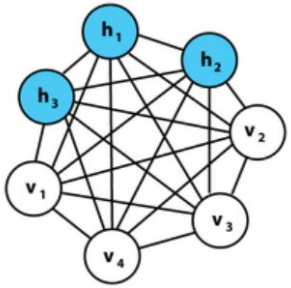
- 有向循环图
- 循环神经网络(RNN)
- 它包含环和自重复



- 完全无向图
- Hopfield网络
- 单层互相全连接的反馈型神经网络

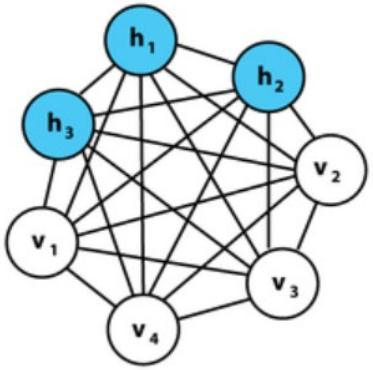


- 有向循环图
- 长短期记忆网络(LSTM)
- 每个模块包含几个循环连接的内存单元和三个门(写入、读取和重置)



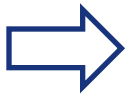
- 完全无向图
- 玻尔兹曼机
- 节点分为两类：隐藏层、可见层

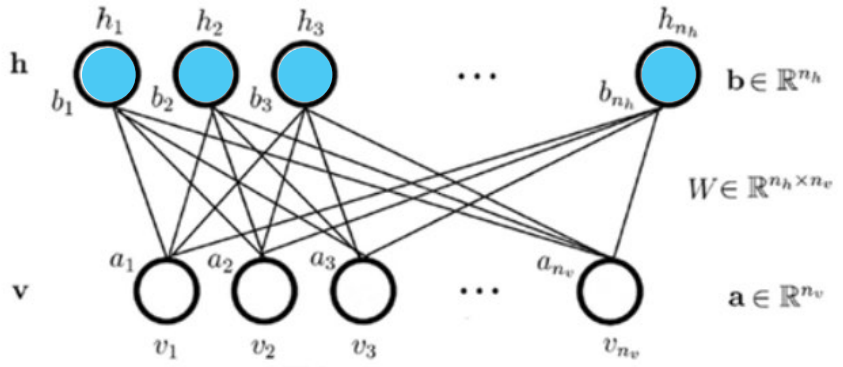
Example 10 Artificial Neural Networks



玻尔兹曼机

完全图
全连接网络
其中的节点分为两类，
蓝色：隐藏层
白色：可见层





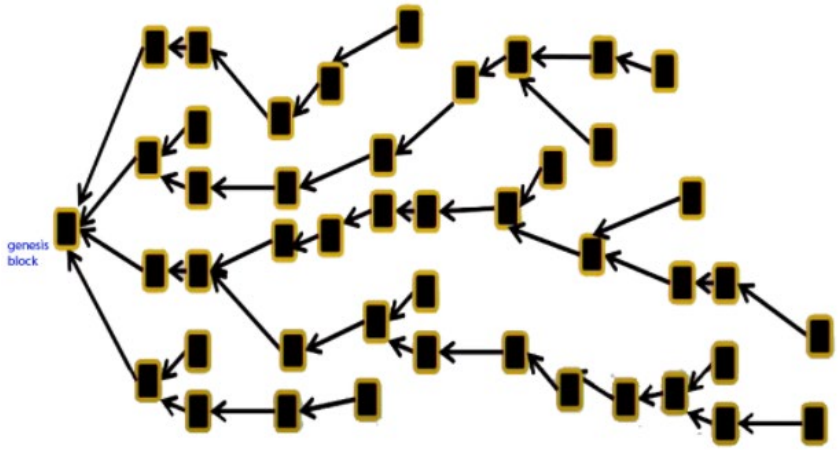
限制玻尔兹曼机

二分图
层内神经元间无连接、层间神经元全连接
各隐藏层神经元的激活是互相独立的，同理
在给定隐藏层信号后，反向传播到可见层时
，可见层神经元的激活也具有独立性

Example 11 graph structure blockchain

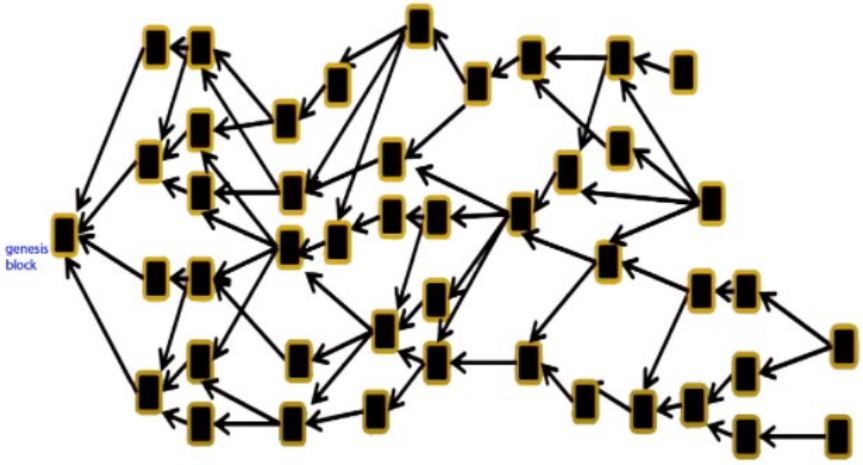
图结构区块链：

图结构利于数据的异步录入，可以解决单链区块生 成 效 率 低 下 的 问 题。



区块链结构

区块链只能指向之前的唯一的区块



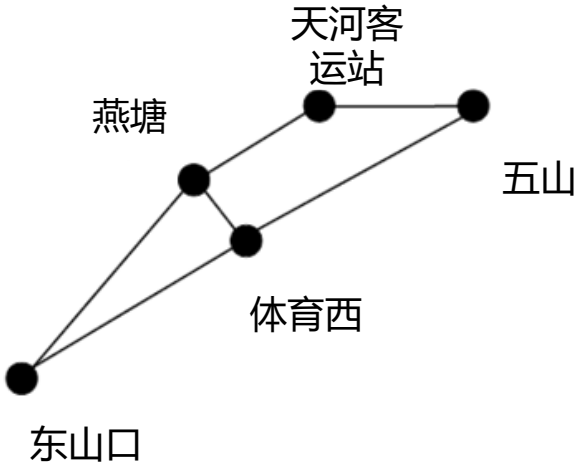
有向无环图（DAG）结构

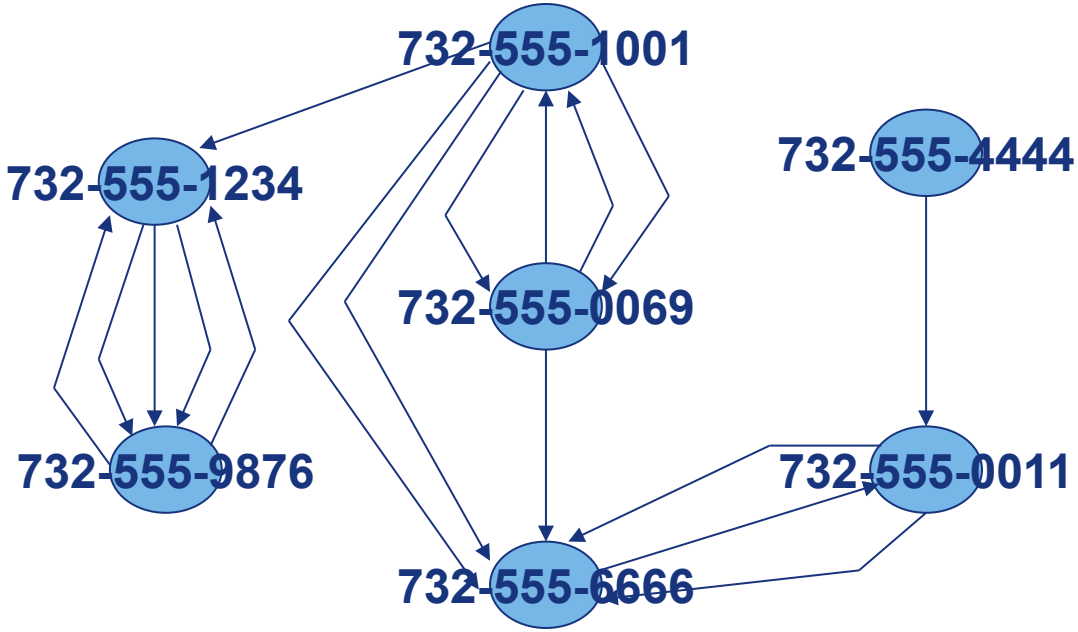
DAG可以指向之前的多个区块


Applications

Applications

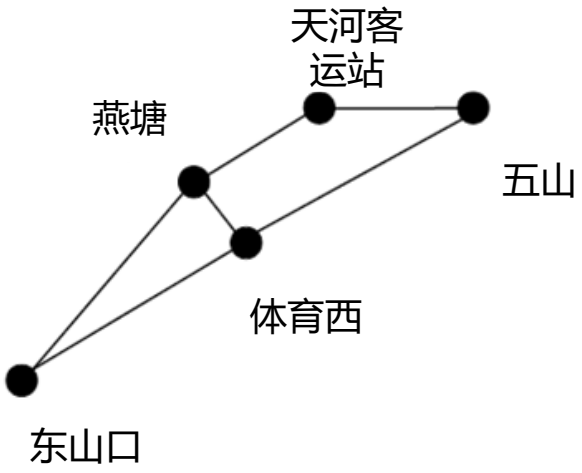
1. Tell what type of the graph is?
(simple graph, multigraph, pseudograph, directed graph or directed multigraph)



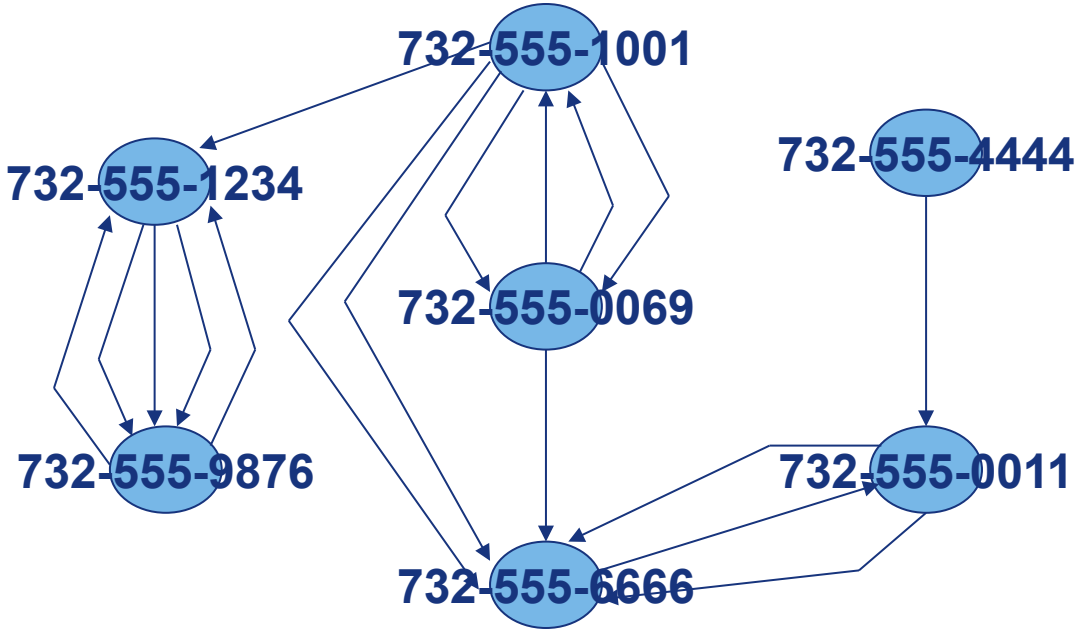


Applications

1. Tell what type of the graph is?
(simple graph, multigraph, pseudograph, directed graph or directed multigraph)



Simple graph



Directed multigraph

Applications

2. Determine the type of graphs
(simple graph, multigraph, pseudograph, directed graph or directed multigraph)

1) 新浪微博用户，与“关注”关系组成的图。
2) QQ用户，与“QQ好友”关系组成的图。 (假定自己不属于自己的好友)

Applications

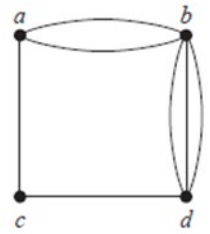
2. Determine the type of graphs.
(simple graph, multigraph, pseudograph, directed graph or directed multigraph)

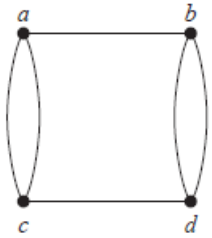
1) 新浪微博用户，与“关注”关系组成的图。（ directed graph ）
2) QQ用户，与“QQ好友”关系组成的图。(假定自己不属于自己的好友)
（ simple graph ）

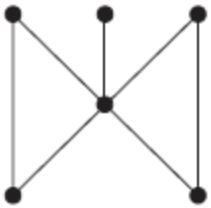

Exercises

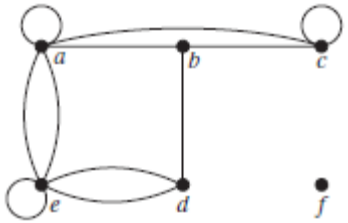
Exercises

1. Which graph is a pseudograph? (D)

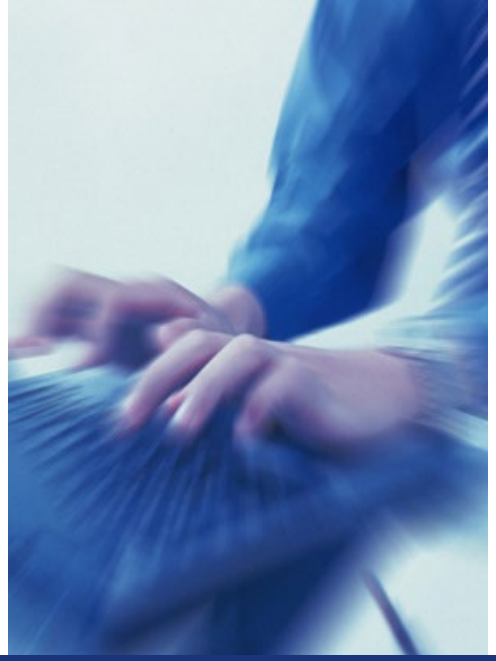
A 

B 

C 

D 

Logo



End of Section 4.1