COMBINATIONAL LOGIC DESIGN

Lesson 3

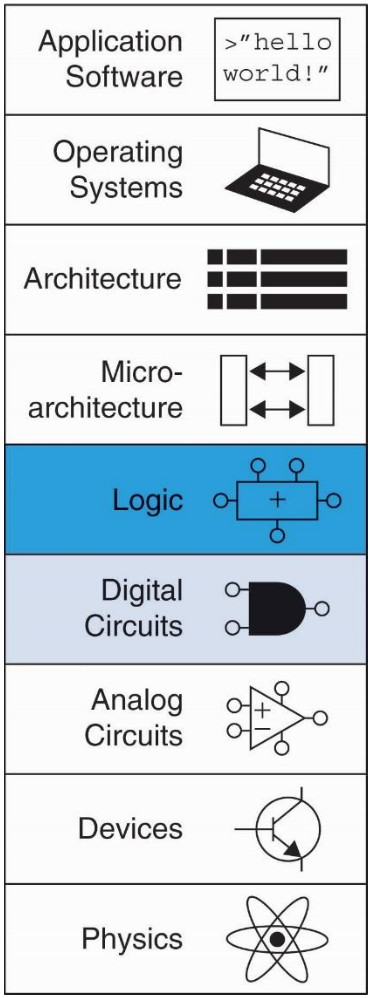
Digital Logic

Junying Chen

ELSEVIER

Chapter 2 :: Topics

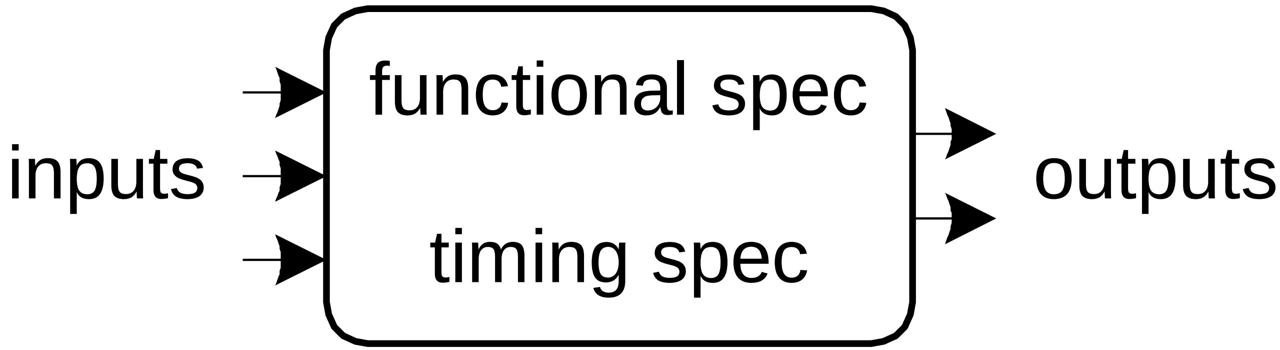
- **Introduction**
- **Boolean Equations**
- **Boolean Algebra**
- **From Logic to Gates**
- **Multilevel Combinational Logic**
- **X's and Z's**
- **Karnaugh Maps**
- **Combinational Building Blocks**
- **Timing**



Introduction

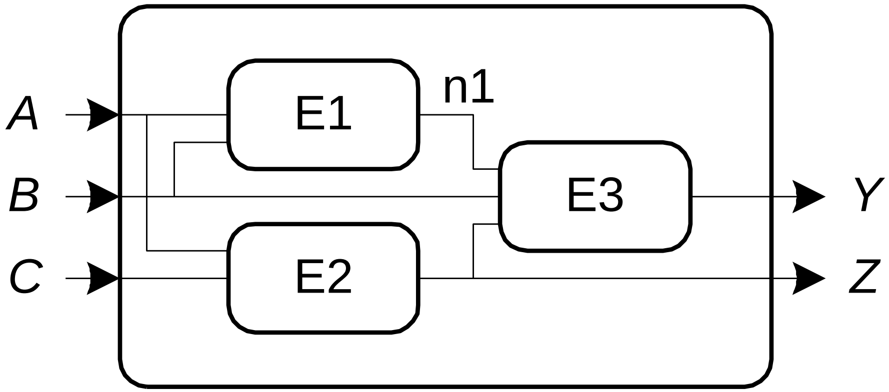
A logic circuit is composed of:

- Inputs
- Outputs
- Functional specification
- Timing specification



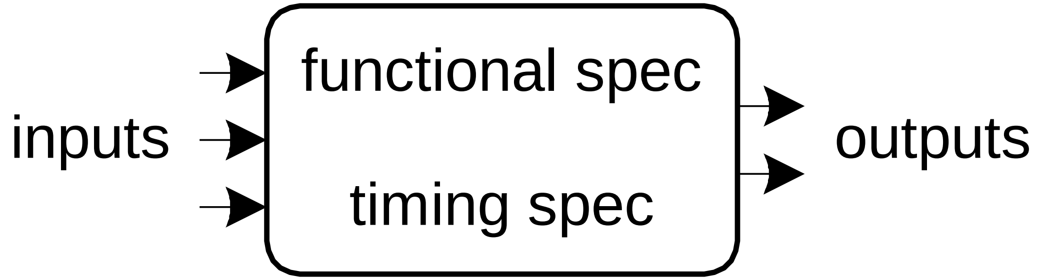
Circuits

- Nodes
  - Inputs: *A*, *B*, *C*
  - Outputs: *Y*, *Z*
  - Internal: n1
- Circuit elements
  - E1, E2, E3
  - Each a circuit



Types of Logic Circuits

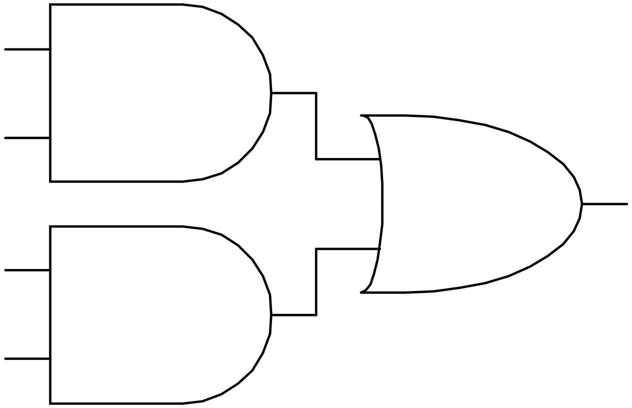
- **Combinational Logic**
  - Memoryless
  - Outputs determined by current values of inputs
- **Sequential Logic**
  - Has memory
  - Outputs determined by previous and current values of inputs



ELSEVIER

Rules of Combinational Composition

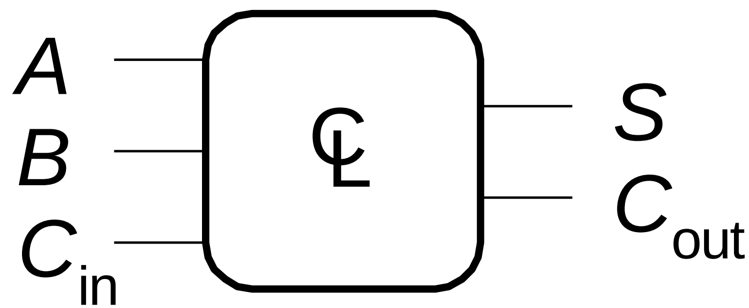
- Every element is combinational
- Every node is either an input or connects to *exactly one* output
- The circuit contains no cyclic paths
- **Example:**



Boolean Equations

- Functional specification of outputs in terms of inputs
- **Example:** $S = F(A, B, C_{in})$

  $C_{out} = F(A, B, C_{in})$



$$S = A \oplus B \oplus C_{in}$$
$$C_{out} = AB + AC_{in} + BC_{in}$$

Some Definitions

- Complement: variable with a bar over it

  $\bar{A}, \bar{B}, \bar{C}$
- Literal: variable or its complement

  $A, \bar{A}, B, \bar{B}, C, \bar{C}$
- Implicant: product of literals

  $AB\bar{C}, \bar{A}C, BC$
- Minterm: product that includes all input variables

  $AB\bar{C}, A\bar{B}\bar{C}, ABC$
- Maxterm: sum that includes all input variables

  $(A+\bar{B}+C), (\bar{A}+B+\bar{C}), (\bar{A}+\bar{B}+C)$

Sum-of-Products (SOP) Form

- All equations can be written in SOP canonical form
- Each row has a **minterm**
- A minterm is a product (AND) of literals
- Each minterm is TRUE for that row (and only that row)
- Form function by ORing minterms where the output is TRUE
- Thus, a sum (OR) of products (AND terms)

A	B	Y	minterm	minterm name
0	0	0	$\overline{A}\ \overline{B}$	m_0
0	1	1	$\overline{A}\ B$	m_1
1	0	0	$A\ \overline{B}$	m_2
1	1	1	$A\ B$	m_3

$$Y = \mathbf{F}(A, B) =$$

Sum-of-Products (SOP) Form

- All equations can be written in SOP canonical form
- Each row has a **minterm**
- A minterm is a product (AND) of literals
- Each minterm is TRUE for that row (and only that row)
- Form function by ORing minterms where the output is TRUE
- Thus, a sum (OR) of products (AND terms)

A	B	Y	minterm	minterm name
0	0	0	$\overline{A}\ \overline{B}$	m_0
0	1	1	$\overline{A}\ B$	m_1
1	0	0	$A\ \overline{B}$	m_2
1	1	1	$A\ B$	m_3

$\boldsymbol{Y = \mathrm{F}(A, B) =}$

Sum-of-Products (SOP) Form

- All equations can be written in SOP canonical form
- Each row has a **minterm**
- A minterm is a product (AND) of literals
- Each minterm is TRUE for that row (and only that row)
- Form function by ORing minterms where the output is TRUE
- Thus, a sum (OR) of products (AND terms)

A	*B*	*Y*	**minterm**	**minterm name**
0	0	0	$\overline{A}\ \overline{B}$	m_0
0	1	1	$\overline{A}\ B$	m_1
1	0	0	$A\ \overline{B}$	m_2
1	1	1	$A\ B$	m_3

$$Y = F(A, B) = \boxed{\overline{A}B} + \boxed{AB} = \Sigma(m_1, m_3) = \Sigma(1, 3)$$

Product-of-Sums (POS) Form

- All Boolean equations can be written in POS canonical form
- Each row has a **maxterm**
- A maxterm is a sum (OR) of literals
- Each maxterm is FALSE for that row (and only that row)
- Form function by ANDing the maxterms for which the output is FALSE
- Thus, a product (AND) of sums (OR terms)

A	*B*	*Y*	maxterm	maxterm name
0	0	0	$A + B$	M_0
0	1	1	$A + \overline{B}$	M_1
1	0	0	$\overline{A} + B$	M_2
1	1	1	$\overline{A} + \overline{B}$	M_3

$$Y = F(A, B) = \boxed{(A + B)}\boxed{(\overline{A} + B)} = \Pi(M_0, M_2) = \Pi(0, 2)$$