Lesson 1

# Digital Logic 数字逻辑

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FROM ZERO TO ONE



ELSEVIER

# Chapter 1 :: Topics

- **Background**
- **The Game Plan**
- **The Art of Managing Complexity**
- **The Digital Abstraction**
- **Number Systems**
- **Logic Gates**

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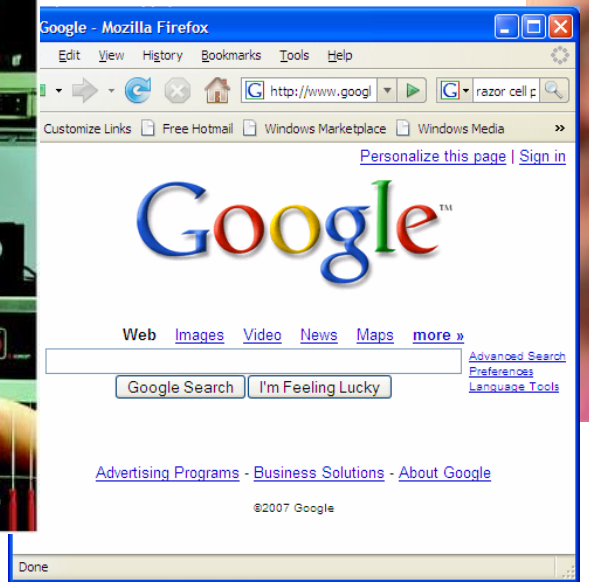
# Background

- Microprocessors have revolutionized our world
  - Cell phones, Internet, rapid advances in medicine, etc.
- The semiconductor industry has grown from $21 billion in 1985 to $306 billion in 2013




# The Game Plan

- Purpose of course:
  - Understand what's under the hood of a computer
  - **Learn the principles of digital design**
  - Learn to systematically debug increasingly complex designs

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# The Art of Managing Complexity

- Abstraction
- Discipline
- The Three –Y's
  - Hierarchy
  - Modularity
  - Regularity

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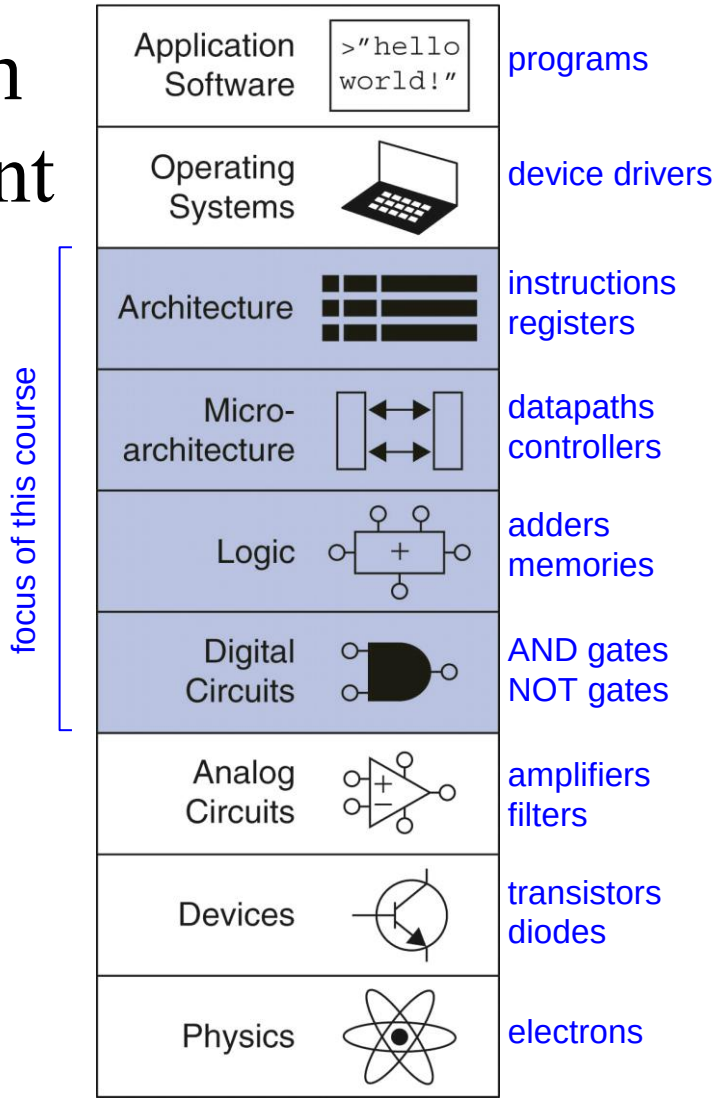
# Abstraction

- Hiding details when they aren't important



# Discipline

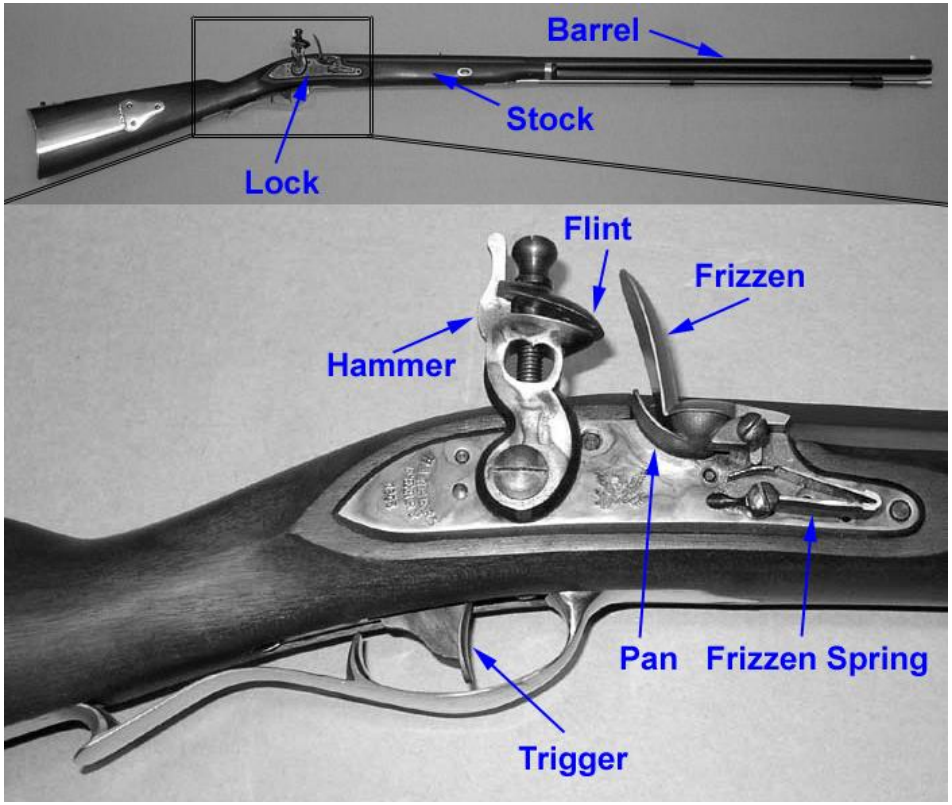
- Intentionally restrict design choices
- Example: Digital discipline
  - Discrete voltages instead of continuous
  - Simpler to design than analog circuits – can build more sophisticated systems
  - Digital systems replacing analog predecessors:
    - i.e., digital cameras, digital television, cell phones, CDs

# The Three -Y's

- **Hierarchy**
  - A system divided into modules and submodules
- **Modularity**
  - Having well-defined functions and interfaces
- **Regularity**
  - Encouraging uniformity, so modules can be easily reused

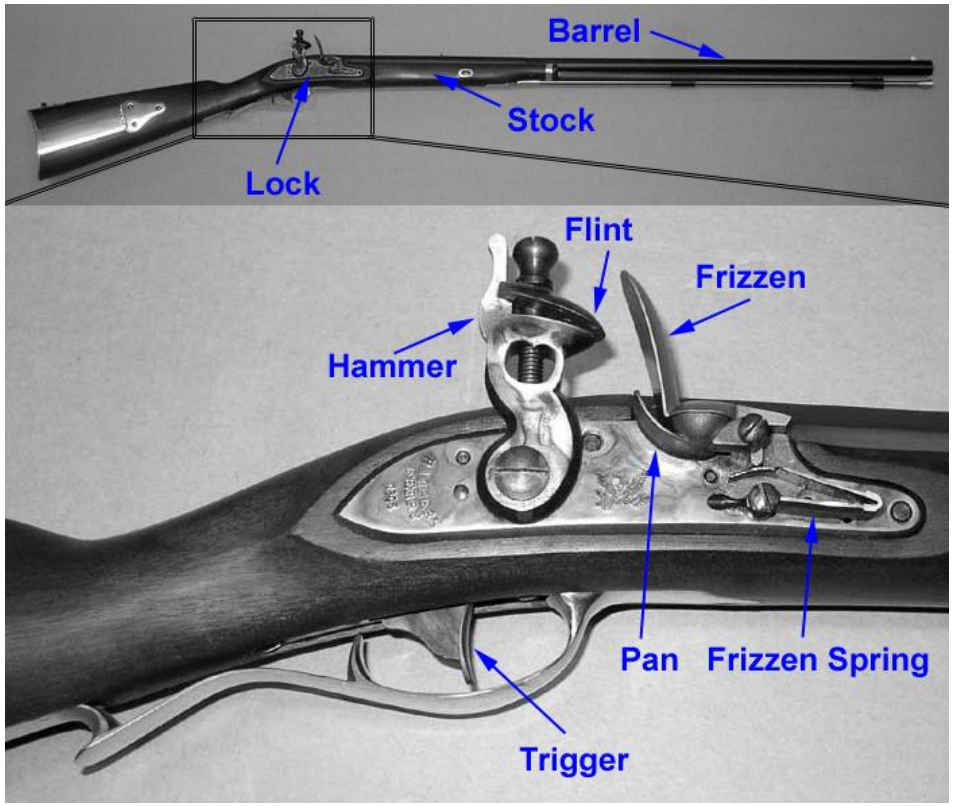
# Example: The Flintlock Rifle

- **Hierarchy**
  - **Three main modules:** lock, stock, and barrel
  - **Submodules of lock:** hammer, flint, frizzen, etc.



# Example: The Flintlock Rifle

- **Modularity**
  - **Function of stock:** mount barrel and lock
  - **Interface of stock:** length and location of mounting pins
- **Regularity**
  - Interchangeable parts

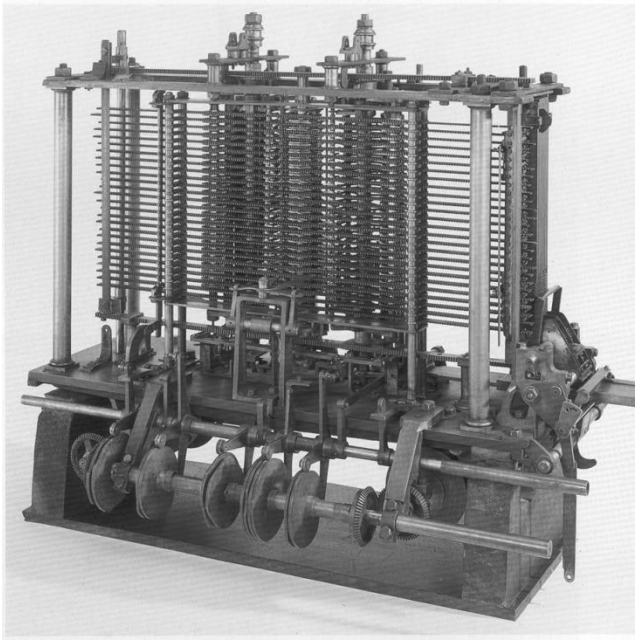


# The Digital Abstraction

- Most physical variables are **continuous**
  - Voltage on a wire
  - Frequency of an oscillation
  - Position of a mass
- Digital abstraction considers **discrete subset** of values

# The Analytical Engine

- Designed by Charles Babbage from 1834 – 1871
- Considered to be the first digital computer
- Built from mechanical gears, where each gear represented a discrete value (0-9)
- Babbage died before it was finished



# Digital Discipline: Binary Values

- **Two discrete values:**
  - 1's and 0's
  - 1, TRUE, HIGH
  - 0, FALSE, LOW
- **1 and 0:** voltage levels, rotating gears, fluid levels, etc.
- Digital circuits use **voltage** levels to represent 1 and 0
- ***Bit***: *B*inary dig*it*

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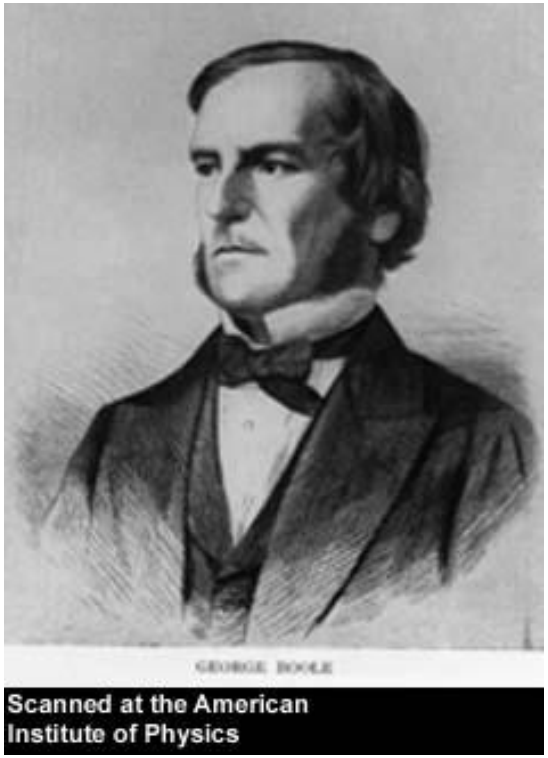
# George Boole, 1815-1864

- Born to working class parents
- Taught himself mathematics and joined the faculty of Queen's College in Ireland.
- Wrote *An Investigation of the Laws of Thought* (1854)
- Introduced binary variables
- Introduced the three fundamental logic operations: AND, OR, and NOT.



# Number Systems

- Decimal numbers

1's column
10's column
100's column
1000's column

$5374_{10}$ =

- Binary numbers

1's column
2's column
4's column
8's column

$1101_2$ =

# Number Systems

- Decimal numbers

1's column
10's column
100's column
1000's column

$$5374_{10} = 5 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$$

five thousands | three hundreds | seven tens | four ones

- Binary numbers

1's column
2's column
4's column
8's column

decimal equivalent

$$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 13_{10}$$

one eight | one four | no two | one one

# Powers of Two

- $2^0 =$
- $2^1 =$
- $2^2 =$
- $2^3 =$
- $2^4 =$
- $2^5 =$
- $2^6 =$
- $2^7 =$
- $2^8 =$
- $2^9 =$
- $2^{10} =$
- $2^{11} =$
- $2^{12} =$
- $2^{13} =$
- $2^{14} =$
- $2^{15} =$

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# Powers of Two

- $2^0 = 1$
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$
- $2^8 = 256$
- $2^9 = 512$
- $2^{10} = 1024$
- $2^{11} = 2048$
- $2^{12} = 4096$
- $2^{13} = 8192$
- $2^{14} = 16384$
- $2^{15} = 32768$
- Handy to memorize up to $2^9$

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# Number Conversion

- Binary to decimal conversion:
  - Convert $10011_2$ to decimal

- Decimal to binary conversion:
  - Convert $47_{10}$ to binary

# Number Conversion

- Binary to decimal conversion:
  - Convert $10011_2$ to decimal
  - $16\times1 + 8\times0 + 4\times0 + 2\times1 + 1\times1 = 19_{10}$

- Decimal to binary conversion:
  - Convert $47_{10}$ to binary
  - $32\times1 + 16\times0 + 8\times1 + 4\times1 + 2\times1 + 1\times1 = 101111_2$

# Binary Values and Range

- *N*-digit decimal number
  - How many values?
  - Range?
  - Example: 3-digit decimal number

- *N*-bit binary number
  - How many values?
  - Range?
  - Example: 3-digit binary number

# Binary Values and Range

- *N*-digit decimal number
  - How many values? $10^N$
  - Range: $[0, 10^N - 1]$
  - Example: 3-digit decimal number:
    - $10^3 = 1000$ possible values
    - Range: [0, 999]
- *N*-bit binary number
  - How many values?
  - Range?
  - Example: 3-digit binary number

# Binary Values and Range

- *N*-digit decimal number
  - How many values? $10^N$
  - Range: $[0, 10^N - 1]$
  - Example: 3-digit decimal number:
    - $10^3 = 1000$ possible values
    - Range: [0, 999]
- *N*-bit binary number
  - How many values? $2^N$
  - Range: $[0, 2^N - 1]$
  - Example: 3-digit binary number:
    - $2^3 = 8$ possible values
    - Range: $[0, 7] = [000_2$ to $111_2]$

# Hexadecimal Numbers

| Hex Digit | Decimal Equivalent | Binary Equivalent |
|---|---|---|
| 0 | 0 | |
| 1 | 1 | |
| 2 | 2 | |
| 3 | 3 | |
| 4 | 4 | |
| 5 | 5 | |
| 6 | 6 | |
| 7 | 7 | |
| 8 | 8 | |
| 9 | 9 | |
| A | 10 | |
| B | 11 | |
| C | 12 | |
| D | 13 | |
| E | 14 | |
| F | 15 | |

# Hexadecimal Numbers

| Hex Digit | Decimal Equivalent | Binary Equivalent |
|---|---|---|
| 0 | 0 | 0000 |
| 1 | 1 | 0001 |
| 2 | 2 | 0010 |
| 3 | 3 | 0011 |
| 4 | 4 | 0100 |
| 5 | 5 | 0101 |
| 6 | 6 | 0110 |
| 7 | 7 | 0111 |
| 8 | 8 | 1000 |
| 9 | 9 | 1001 |
| A | 10 | 1010 |
| B | 11 | 1011 |
| C | 12 | 1100 |
| D | 13 | 1101 |
| E | 14 | 1110 |
| F | 15 | 1111 |

Important!

# Hexadecimal Numbers

- Base 16
- Shorthand for binary

# Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - Convert $4AF_{16}$ (also written 0x4AF) to binary

- Hexadecimal to decimal conversion:
  - Convert 0x4AF to decimal

# Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - Convert $4AF_{16}$ (also written 0x4AF) to binary
  - $0100\ 1010\ 1111_2$

- Hexadecimal to decimal conversion:
  - Convert $4AF_{16}$ to decimal
  - $16^2 \times 4 + 16^1 \times 10 + 16^0 \times 15 = 1199_{10}$