COMBINATIONAL LOGIC DESIGN

Lesson 4

Digital Logic

Junying Chen

Boolean Equations Example

- You are going to the cafeteria for lunch
  - You won't eat lunch ($\overline{E}$)
    - If it's not open ($\overline{O}$) or
    - If they only serve corndogs (C)
- Write a truth table for determining if you will eat lunch (E).

O	C	E
0	0	
0	1	
1	0	
1	1	

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O	*C*	*E*
0	0	0
0	1	0
1	0	1
1	1	0

SOP & POS Form

- SOP – sum-of-products

O	*C*	*E*	minterm
0	0		$\overline{O}\ \overline{C}$
0	1		$\overline{O}\ C$
1	0		$O\ \overline{C}$
1	1		$O\ C$

- POS – product-of-sums

O	*C*	*E*	maxterm
0	0		$O + C$
0	1		$O + \overline{C}$
1	0		$\overline{O} + C$
1	1		$\overline{O} + \overline{C}$

SOP & POS Form

- SOP – sum-of-products

O	C	E	minterm
0	0	0	$\overline{O}\ \overline{C}$
0	1	0	$\overline{O}\ C$
1	0	1	$O\ \overline{C}$
1	1	0	$O\ C$

$$E = O\overline{C}$$
$$= \Sigma(2)$$

- POS – product-of-sums

O	C	E	maxterm
0	0	0	$O + C$
0	1	0	$O + \overline{C}$
1	0	1	$\overline{O} + C$
1	1	0	$\overline{O} + \overline{C}$

$$E = (O + C)(O + \overline{C})(\overline{O} + \overline{C})$$
$$= \Pi(0, 1, 3)$$

Boolean Algebra

- Axioms and theorems to **simplify** Boolean equations
- Like regular algebra, but simpler: variables have only two values (1 or 0)
- **Duality** in axioms and theorems:
  - ANDs and ORs, 0's and 1's interchanged

Boolean Axioms

	Axiom		Dual	Name
A1	$B = 0 \text{ if } B \neq 1$	A1′	$B = 1 \text{ if } B \neq 0$	Binary field
A2	$\overline{0} = 1$	A2′	$\overline{1} = 0$	NOT
A3	$0 \bullet 0 = 0$	A3′	$1 + 1 = 1$	AND/OR
A4	$1 \bullet 1 = 1$	A4′	$0 + 0 = 0$	AND/OR
A5	$0 \bullet 1 = 1 \bullet 0 = 0$	A5′	$1 + 0 = 0 + 1 = 1$	AND/OR

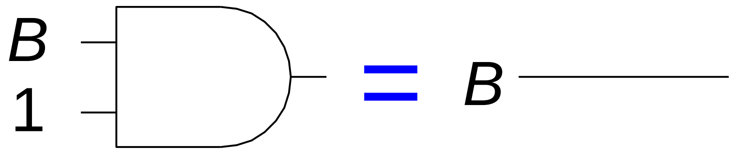
	Theorem		Dual	Name
T1	$B \bullet 1 = B$	T1′	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	T2′	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	T3′	$B + B = B$	Idempotency
T4		$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	T5′	$B + \overline{B} = 1$	Complements

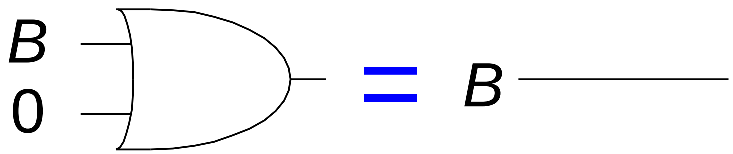
T1: Identity Theorem

- $B \cdot 1 = B$
- $B + 0 = B$

T1: Identity Theorem

- B • 1 = B
- B + 0 = B



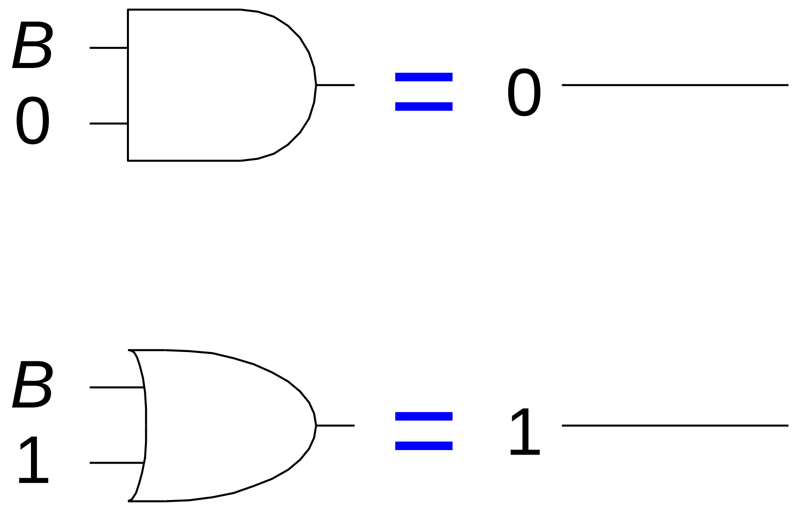


T2: Null Element Theorem

- $B \cdot 0 = 0$
- $B + 1 = 1$

T2: Null Element Theorem

- $B \bullet 0 = 0$
- $B + 1 = 1$

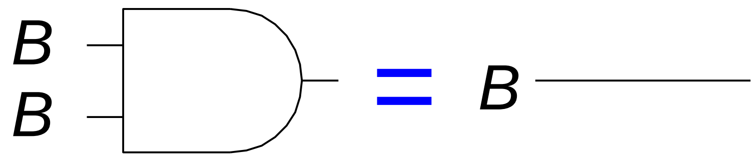


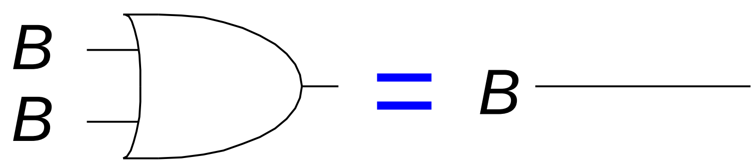
T3: Idempotency Theorem

- $B \bullet B = B$
- $B + B = B$

T3: Idempotency Theorem

- $B \cdot B = B$
- $B + B = B$





T4: Involution Theorem

- $\overline{\overline{B}} = B$

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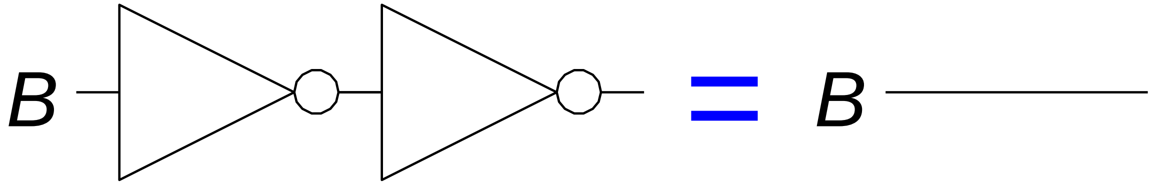

T5: Complement Theorem

- $B \bullet \overline{B} = 0$
- $B + \overline{B} = 1$

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- $B \bullet \overline{B} = 0$
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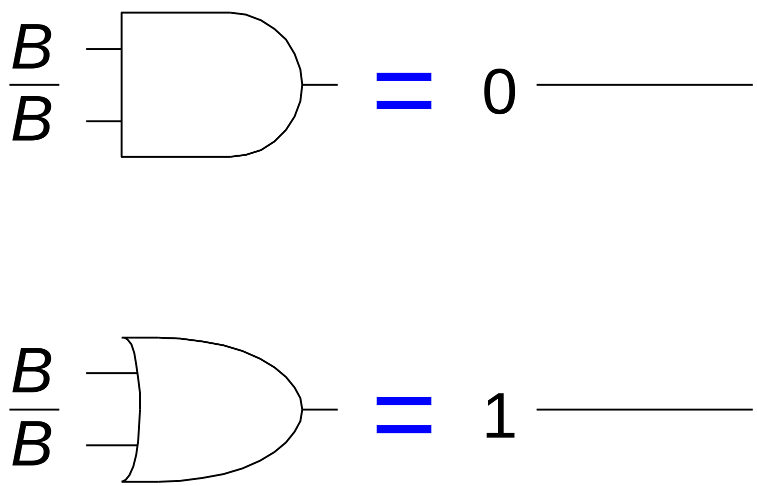

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Boolean Theorems Summary

	Theorem		Dual	Name
T1	$B \bullet 1 = B$	T1′	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	T2′	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	T3′	$B + B = B$	Idempotency
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Boolean Theorems of Several Vars

	Theorem		Dual	Name
T6	$B \bullet C = C \bullet B$	T6′	$B + C = C + B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	T7′	$(B + C) + D = B + (C + D)$	Associativity
T8	$(B \bullet C) + (B \bullet D) = B \bullet (C + D)$	T8′	$(B + C) \bullet (B + D) = B + (C \bullet D)$	Distributivity
T9	$B \bullet (B + C) = B$	T9′	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	T10′	$(B + C) \bullet (B + \overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D)$ $= B \bullet C + \overline{B} \bullet D$	T11′	$(B + C) \bullet (\overline{B} + D) \bullet (C + D)$ $= (B + C) \bullet (\overline{B} + D)$	Consensus
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \ldots}$ $= (\overline{B_0} + \overline{B_1} + \overline{B_2} \ldots)$	T12′	$\overline{B_0 + B_1 + B_2 \ldots}$ $= (\overline{B_0} \bullet \overline{B_1} \bullet \overline{B_2})$	De Morgan's Theorem

Note: T8' differs from traditional algebra: OR (+) distributes over AND (•)

Perfect induction: Use truth tables to prove theorems

Simplifying Boolean Equations

Example 1:

$$Y = AB + \overline{A}B$$

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$$Y = AB + \overline{A}B$$

$= B(A + \overline{A})$	T8
$= B(1)$	T5'
$= B$	T1

Simplifying Boolean Equations

Example 2:

$Y = A(AB + ABC)$

Simplifying Boolean Equations

Example 2:

$Y = A(AB + ABC)$

$= A(AB(1 + C))$	T8
$= A(AB(1))$	T2’
$= A(AB)$	T1
$= (AA)B$	T7
$= AB$	T3