Chapter 9 Arithmetic

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Computer Organization & Architecture

2-1 Integer Representation

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Integers

- An n-bit integer

  $B = b_{n-1}\ldots b_1 b_0$

  - where $b_i = 0$ or 1 for $0 \le i \le n-1$
  - represents an unsigned integer value $0 \sim 2^n-1$

  $$V(B) = b_{n-1} * 2^{n-1} + \ldots + b_1 * 2^1 + b_0 * 2^0$$

- Need to represent both positive and negative numbers
  - Signed-Magnitude representation
  - Signed One's Complement representation
  - Signed Two's Complement representation

Signed-Magnitude Representation (1)

- For an *n*-bit integer
  - The sign part is a 1-bit value that is 0 for positive numbers, and 1 for negative numbers
  - The magnitude part is an *(n-1)*-bit value that holds the absolute value of the number

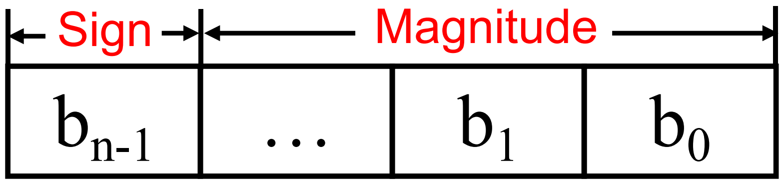
Signed-Magnitude Representation (2)

- An n-bit integer

  $B = b_{n-1} \ldots b_1 b_0$

  – $b_{n-1} = 0$, B is a positive number

  – $b_{n-1} = 1$, B is a negative number



- Example

  – $+18_{10} = \underline{0}0010010$

  – $-18_{10} = \underline{1}0010010$

  – $+0_{10} = \underline{0}0000000$

  – $-0_{10} = \underline{1}0000000$

Signed-Magnitude Representation (3)

- Representation range
  - In general, if an *n*-bit sequence of binary digits $b_{n-1}\ldots b_1 b_0$ is interpreted as an signed integer *B*, its value is

$$V(B) = \begin{cases} \sum_{i=0}^{n-2} 2^i b_i & \text{if } b_{n-1} = 0 \\ -\sum_{i=0}^{n-2} 2^i b_i & \text{if } b_{n-1} = 1 \end{cases}$$

$$-(2^{n-1}-1) \leqslant V(B) \leqslant 2^{n-1}-1$$

Signed-Magnitude Representation (4)

- Drawbacks
  - Addition and subtraction require a consideration of both the signs of the numbers and their relative magnitudes to carry out the required operation
  - There are two representations of 0
    - $+0$ = <u>0</u>0000000
    - -0 = <u>1</u>0000000

Signed One's Complement Representation (1)

- For an *n*-bit integer
  - The <u>sign part</u> is a 1-bit value that is 0 for positive numbers, and 1 for negative numbers
  - The <u>magnitude part</u> is an *(n-1)*-bit value
    - Positive numbers: equivalent to the magnitude part of a signed-magnitude integer
    - Negative numbers: represented as the bitwise complement of its absolute value

Signed One's Complement Representation (2)

- Bitwise complement
  - Take the Boolean complement of each bit of the number. That is, set each 1 to 0 and each 0 to 1
- Example
  - +18 = <u>0</u>0010010
  - - 18 = <u>1</u>1101101
  - + 0 = <u>0</u>0000000
  - - 0 = <u>1</u>1111111

Signed Two's Complement Representation (1)

- For an *n*-bit integer
  - The sign part is a 1-bit value that is 0 for positive numbers, and 1 for negative numbers
  - The magnitude part is an *(n-1)*-bit value
    - Positive numbers: equivalent to the magnitude part of a signed-magnitude integer
    - Negative numbers: represented as the bitwise complement of its absolute value +1

Signed Two's Complement Representation (2)

- Example
  - +18 = <u>0</u>0010010
  - - 18 = <u>1</u>1101110
  - + 0 = <u>0</u>0000000
  - - 0 = <u>0</u>0000000

Signed Two's Complement Representation (3)

- Representation range
  - The general case $B = b_{n-1} \ldots b_1 b_0$

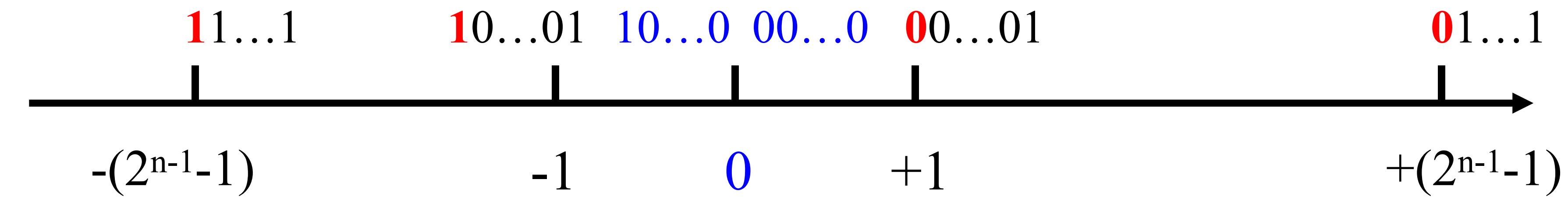
$$
V(B) = \begin{cases} \displaystyle\sum_{i=0}^{n-2} 2^i b_i & B \geqslant 0 \\ \displaystyle -2^{n-1} b_{n-1} + \sum_{i=0}^{n-2} 2^i b_i & B < 0 \end{cases}
$$

$$
= -2^{n-1} b_{n-1} + \sum_{i=0}^{n-2} 2^i b_i
$$

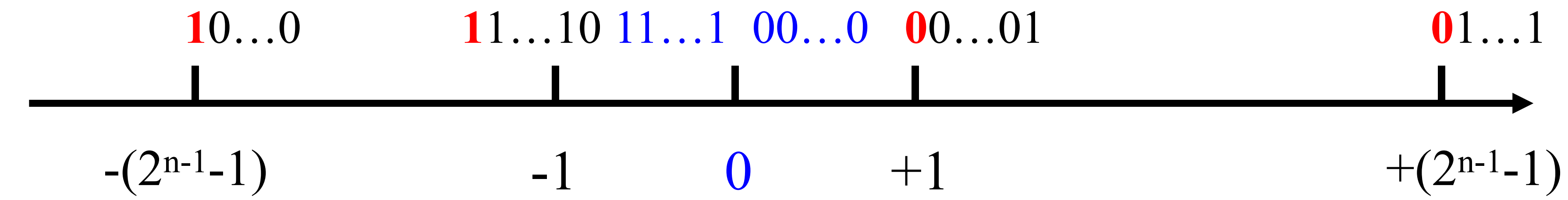
(for both positive and negative numbers)

$$
-2^{n-1} \leqslant V(B) \leqslant 2^{n-1} - 1
$$

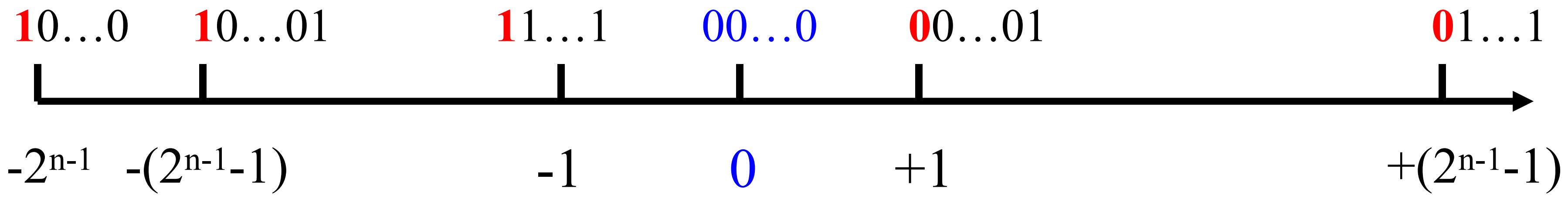
Conclusion (1)



signed-magnitude



signed one's complement



signed two's complement

Conclusion (2)

1. In all three systems, the leftmost bit is 0 for positive numbers and 1 for negative numbers.
2. Positive values have identical representations in all systems, but negative values have different representations.
3. The Two's-complement of a number is obtained by adding 1 to the One's-complement of that number.
4. The range of a *n*-bit Two's-complement integer is $-2^{n-1} \sim 2^{n-1}-1$ because of the representation of *0* is unique.

Conclusion (3)

- The Signed-Magnitude system is the simplest representation, but it is also the most awkward for addition and subtraction operations.
- The Two’s-complement system is the most efficient method for performing addition and subtraction operations.

Converting between Different Bit Lengths(1)

- Signed-Magnitude Numbers
  - Move the sign bit to the new left-most position and fill in with zeros.
  - Example
    - +18 = <u>0</u>0010010 (8 bits)
    - +18 = <u>0</u>000000000010010 (16 bits)
    - - 18 = <u>1</u>0010010 (8 bits)
    - - 18 = <u>1</u>000000000010010 (16 bits)

Converting between Different Bit Lengths(2)

- Signed Two's Complement Numbers
  - Example
    - +18 = 00010010 (8 bits)
    - +18 = 0000000000010010 (16 bits)
    - - 18 = 11101110 (8 bits)
    - - 32658 = 1000000001101110 (16 bits)

Converting between Different Bit Lengths(3)

- Signed Two's Complement Numbers
  - Sign Extension
    - Move the sign bit to the new left-most position and fill in with copies of the sign bit. For positive numbers, fill in with zeros, and for negative numbers, fill in with ones.
    - Example
      - \- 18 = 11101110 (8 bits)
      - \- 18 = 1111111111101110 (16 bits)

Quiz (1)

- ______ is the most efficient method for performing addition and subtraction operations.
  A. Signed-Magnitude Representation
  B. Signed 1's Complement Representation
  C. Signed 2's Complement Representation
  D. None of the above

- The range of an 8-bit signed 2's complement integer is _______.
  A. [-256,+256] B. [-256,+255] C. [-128,+128] D. [-128,+127]

Quiz (2)

- Given the 8-bit binary number: 10011101. What decimal number does this represent if the computer uses signed-magnitude representation?

  A. +29 B. -29 C. +99 D. -99

- Given the 8-bit binary number: 10011101. What decimal number does this represent if the computer uses signed two's complement representation?

  A. +29 B. -29 C. +99 D. -99