Logo

Discrete Mathematics

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Logo

Chapter 3. Relations

Relations and Their Properties

Section 3.1

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Contents

Introduction

Example

- 在一群学生中间，我们可以说，如果两位学生是同一个班的话，那么这两位学生是有关系的。
- 在一组计算机程序中，我们可以说，假若两个程序共享一些数据的话，那么这两个程序是有关系的。
- 在计算机科学中我们会碰到许多关系，如数据库的数据特性关系，计算机语言的字符关系，一种计算语言与这个语言的一个有效语句之间的关系，计算机程序的输入输出关系，一个程序与它所使用的一个变量之间的关系，等等。

Binary Relations

- Let A, B be any sets. A *binary relation* R from A to B, (i.e., with signature $R{:}A\times B$) can be identified with a subset of $A\times B$.
  - E.g., $<$ can be seen as $\{(n,m) \mid n < m\}$
- $(a,b)\in R$ means that a is related to b (by R)
- Also written as aRb; also $R(a,b)$
  - E.g., $a<b$ and $<(a,b)$ both mean $(a,b)\in <$
- A binary relation R corresponds to a characteristic function $P_R{:}A\times B\rightarrow\{T,F\}$

Example 1

- Let A be the set of students in your school.
- Let B be the set of courses.
- Let R be the relation that consists of the pairs (a, b) where a is a student enrolled in course b.
- (小白, 离散数学)
- (小黄, 离散数学)
- (小黄, 算法设计)

Example 2

- Let A={ 0, 1, 2 } and B={ a, b }.
- Then { (0,a),(0,b),(1,b),(2,a) } is a relation from A to B.
- $0 \ R \ a \qquad\qquad 2 \ \cancel{R} \ b$

Inverse Relations

- Any binary relation $R{:}A\times B$ has an *inverse* relation $R^{-1}{:}B\times A$, defined by

  $R^{-1} :\equiv \{(b,a) \mid (a,b)\in R\}$.

  E.g., $<^{-1} = \{(a,b) \mid a<b\}^{-1} = \{(b,a) \mid b>a\} = >$.

- *E.g.*, if R:People x Foods is defined by

  $a\ R\ b \Leftrightarrow a$ *eats* b, then:

  $b\ R^{-1}\ a \Leftrightarrow b$ *is eaten by* a. (Passive voice.)

Function and Relations on A Set

Functionality

- A relation R: $A \times B$ is *functional* iff, for every $a \in A$, there is *at most one* $b \in B$ such that $(a,b) \in R$.

- Say this in predicate logic

Functionality

- A relation R: $A \times B$ is *functional* iff, for every $a \in A$, there is *at most one* $b \in B$ such that $(a,b) \in R$.
  $\forall a \in A: \neg\exists\, b_1, b_2 \in B\ (b_1 \neq b_2 \wedge aRb_1 \wedge aRb_2)$.
- If R is functional, then R can be seen as a function or a *partial function* $R: A \rightarrow B$
  (hence one can write R(a)=b as well as aRb, R(a,b), and (a,b)$\in$ R. Each of these means the same.)
- NB A functional relation R: $A \times B$ does not have to be total (i.e., there may be $a \in A$ such that $\neg\exists b \in B\ (aRb)$).

Functionality

- Theorem: A relation R is a (*total) function* $R{:}A\rightarrow B$ iff it is functional and total (i.e., iff $\forall a\in A{:}\ \exists b{:}\ aRb$.)
- *Definition: R* is *anti-functional* iff its inverse relation R^{-1} is functional.
- (Exercise: Show that iff R is functional and anti-functional, and both it and its inverse are total, then it is a bijective function.)

Relations on a Set

- A (binary) relation from a set *A* to itself is called a relation *on* the set *A*.
- *E.g.*, the “<” relation from earlier was defined as a relation *on* the set N of natural numbers.
- The next few slides: *relations on a set A.*

Relations on A Set

- A relation on the set *A* is a relation from *A* to *A*.
- Let *A* be the set { 1, 2, 3, 4 }.
- R = { (a, b) | a divides b }
- R = { (1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4) }

Example 5

- $R1 = \{ (a, b) \mid a \leq b \}$
- $R2 = \{ (a, b) \mid a > b \}$
- $R3 = \{ (a, b) \mid a = b \text{ or } a = -b \}$
- $R4 = \{ (a, b) \mid a = b \}$
- $R5 = \{ (a, b) \mid a = b + 1 \}$
- $R6 = \{ (a, b) \mid a + b \leq 3\}$
- (1,1) is in R1, R3, R4 and R6
- (2,1) is in R2, R5 and R6

Properties of Relations

Reflexive

- A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.
- A = { 1, 2, 3 }
- R1 = {(1,1),(1,2),(2,1),(2,2),(3,1)}
- R2 = {(1,1),(1,2),(2,1)}
- R3 = {(1,1),(1,2),(2,1),(2,2),(3,3)}
- R3 is reflexive, but others are not.

Reflexive

- **$R1 = \{ (a, b) \mid a \leq b \}$**
- **$R2 = \{ (a, b) \mid a > b \}$**
- **$R3 = \{ (a, b) \mid a = b \text{ or } a = -b \}$**
- **$R4 = \{ (a, b) \mid a = b \}$**
- **$R5 = \{ (a, b) \mid a = b + 1 \}$**
- **$R6 = \{ (a, b) \mid a + b \leq 3\}$**
- R1,R3 and R4 are reflexive.

Reflexivity and relatives

- A relation *R* on *A* is *reflexive* iff $\forall a \in A(aRa)$.
  E.g., the relation $\geq :\equiv \{(a,b) \mid a \geq b\}$ is reflexive.
  "divides" is reflexive since a|a holds.
- *R* is *irreflexive* iff $\forall a \in A(\neg aRa)$
- Note "*irreflexive*" does NOT mean "*not reflexive*", which is just $\neg\forall a \in A(aRa)$.
- E.g., if Adore={(j,m),(b,m),(m,b),(j,j)} then this relation is neither reflexive nor irreflexive

Reflexivity and relatives

- **Theorem: A relation *R* is *irreflexive* iff its *complementary* relation *R* is reflexive.**
  - **Example: < is irreflexive; ≥ is reflexive.**
  - **Proof: trivial**

- **Can you think of**
  - **Reflexive relations**
  - **Irreflexive relations**

Involving numbers, propositions or sets?

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Some examples

- **Reflexive:**

  =, 'have same cardinality', $\Leftrightarrow$

  <=, >=, $\Rightarrow$, $\subseteq$, etc.

- **Irreflexive:**

  <, >, 'have different cardinality', $\subset$

Symmetric

- A relation R on a set A is called symmetric if $(b,a) \in R$ whenever $(a,b) \in R$, for all $a,b \in A$.
- A relation R on a set A such that $(a,b) \in R$ and $(b,a) \in R$ only if $a=b$ for all $a,b \in A$, is called antisymmetric.
- "divides" is antisymmetric, for if positive integers a, b with $a|b$ and $b|a$, then $a=b$.

Symmetric

- R1 ={ (1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)}
- R2 = {(1,1),(1,2),(2,1)}
- R3 = {(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),
  (4,1),(4,4)}
- R4 = {(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)}
- R2 and R3 are symmetric.
- R4 is antisymmetric.

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Symmetric

- $R1 = \{ (a, b) \mid a \le b \}$
- $R2 = \{ (a, b) \mid a > b \}$
- $R3 = \{ (a, b) \mid a = b \text{ or } a = -b \}$
- $R4 = \{ (a, b) \mid a = b \}$
- $R5 = \{ (a, b) \mid a = b + 1 \}$
- $R6 = \{ (a, b) \mid a + b \le 3\}$
- R3, R4 and R6 are symmetric.
- R1, R2, R4 and R5 are antisymmetric.

Antisymmetry

- **Consider the relation $x \leq y$**
- **Is it symmetric? No**
- **Is it asymmetric? No**
- **Is it reflexive? Yes**
- **Is it irreflexive? No**

- **asymmetric: $\equiv$ not symmetric**

(there exist a,b $\in$A such that (a,b) $\in$R but (b,a) $\notin$R)

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Antisymmetry

- Consider the relation $x \leq y$
  - It is not symmetric. (For instance, $5 \leq 6$ but not $6 \leq 5$)
  - It is not asymmetric. (For instance, $5 \leq 5$)
  - The pattern: the only times when $(a,b) \in \leq$ and $(b,a) \in \leq$ are when $a=b$
- This is called antisymmetry

Can you say this in predicate logic?

Antisymmetry

- A binary relation *R* on *A* is *antisymmetric* iff $\forall a,b((a,b)\in R \wedge (b,a)\in R) \rightarrow a=b)$.
- Examples: $\leq$, $\geq$, $\subseteq$

- How would you define transitivity of a relation? What are its ‘relatives’?

Transitive

- **A relation R on a set A is called transitive if whenever (a,b) ∈R and (b,c) ∈R, then (a,c) ∈R, for all a,b,c ∈R.**

- **R0 = {(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)}**
- **R0 is transitive.**

- **"divides" is transitive since a|b and b|c then a|c.**

Transitive

- $R1 = \{ (a, b) \mid a \leq b \}$
- $R2 = \{ (a, b) \mid a > b \}$
- $R3 = \{ (a, b) \mid a = b \text{ or } a = -b \}$
- $R4 = \{ (a, b) \mid a = b \}$
- $R5 = \{ (a, b) \mid a = b + 1 \}$
- $R6 = \{ (a, b) \mid a + b \leq 3\}$

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Transitive

- $R1 = \{ (a, b) \mid a \leq b \}$
- $R2 = \{ (a, b) \mid a > b \}$
- $R3 = \{ (a, b) \mid a = b \text{ or } a = -b \}$
- $R4 = \{ (a, b) \mid a = b \}$
- $R5 = \{ (a, b) \mid a = b + 1 \}$
- $R6 = \{ (a, b) \mid a + b \leq 3\}$
- R1,R2,R3 and R4 are transitive.

Transitivity & relatives

- A relation R is *transitive* iff (for all a,b,c)
  $((a,b)\in R \wedge (b,c)\in R) \rightarrow (a,c)\in R$.
- A relation is non*transitive* iff it is not transitive.
- A relation R is in*transitive* iff (for all a,b,c)
  $((a,b)\in R \wedge (b,c)\in R) \rightarrow \neg(a,c)\in R$.

Transitivity & relatives

- What about these examples:
  - "x is an ancestor of y"
  - "x likes y"
  - "x is located within 1 mile of y"
  - "x +1 =y"
  - "x beat y in the tournament"
  - "x is stronger than y"

Transitivity & relatives

- What about these examples:
  - "is an ancestor of" is transitive.
  - "likes" is neither trans nor intrans.
  - "is located within 1 mile of" is neither trans nor intrans
  - "x +1 =y" is intransitive
  - "x beat y in the tournament" is neither trans nor intrans
  - "x is stronger than y" is transitive.

Transitivity & relatives

- R={ (a, b) | a比b强 } is transitive

- 前提: (小白，小黄) ∈ R,
  (小花，小白) ∈ R
- 结论: (小花，小黄) ∈ R

Application

下列关系具有哪些性质？

（1）S上的关系 $R=\{<x,y>|(x,y\in S)\wedge(x>y)\}$

（2）T={1,2,3…,10}上的关系

$$R=\{<x,y>|(x,y\in T)\wedge(x+y=10)\}$$

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Application

下列关系具有哪些性质？

（1）S上的关系 $R=\{<x,y>|(x,y\in S)\wedge(x>y)\}$

R是反对称的，反自反的，传递的。

（2）T={1,2,3…,10}上的关系

$R=\{<x,y>|(x,y\in T)\wedge(x+y=10)\}$

R是对称的。

Combining Relations

Composite Relations

- Let R:$A\times B$, and S:$B\times C$. Then the *composite* $S\circ R$ of R and S is defined as:

$$S\circ R = \{(a,c) \mid \exists b: aRb \wedge bSc\}$$

- Does this remind you of something?

Composite Relations

- Let $R: A \times B$, and $S: B \times C$. Then the *composite* $S \circ R$ of R and S is defined as:

  $$S \circ R = \{(a,c) \mid \exists b: aRb \wedge bSc\}$$

- Does this remind you of something?
- **Function** composition ...
- ... except that $S \circ R$ accommodates the fact
  that S and R may not be functional

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Composite Relations

- Let $R:A \times B$, and $S:B \times C$. Then the *composite* $S \circ R$ of R and S is defined as:

  $$S \circ R = \{(a,c) \mid \exists b: aRb \wedge bSc\}$$

- **Function** composition is a special case of relation composition: Suppose S and R are functional. Then we have (using the definition above, then switching to function notation)

$S \circ R(a,c)$ *iff* $\exists b: aRb \wedge bSc$

iff $R(a)=b$ *and* $S(b)=c$ *iff* $S(R(a))=c$

Example

- $R = \{(1,1),(1,4),(2,3),(3,1),(3,4)\}$
- $S = \{(1,0),(2,0),(3,1),(3,2),(4,1)\}$
- $S \circ R = \{(1,0),(1,1),(2,1),(2,2),(3,0),(3,1)\}$

- $R=\{(1,1),(2,1),(3,2),(4,3)\}$
- $R^2 = R \circ R =\{(1,1),(2,1),(3,1),(4,2)\}$
- $R^3 = R \circ R \circ R =\{(1,1),(2,1),(3,1),(4,1)\}$
- $R^n = R \circ R \circ R =\{(1,1),(2,1),(3,1),(4,1)\}$
- $n \geq 3$

- Theorem 1
- The relation R on a set A is transitive if and only if $R^n \subseteq R$ for n = 1,2,3…
- Proof.

(a,b) $\in$R and (b,c) $\in$R then (a,c) $\in R^2$

and (a,c) $\in$R.

……   Use mathematical induction

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Composite Relations

- Let's see what happens when we compose R with itself

- Exercise: Prove that $R : A \times A$ is transitive iff $R \circ R = R$.

Composite relations

- **The n^{th} power R^n of a relation R on a set A**
  - *The 1^{st} power of R is R itself*
  - *The 2^{nd} power of R is $R^2 = R \circ R$*
  - *The 3^{rd} power of R is $R^3 = R \circ R \circ R$*

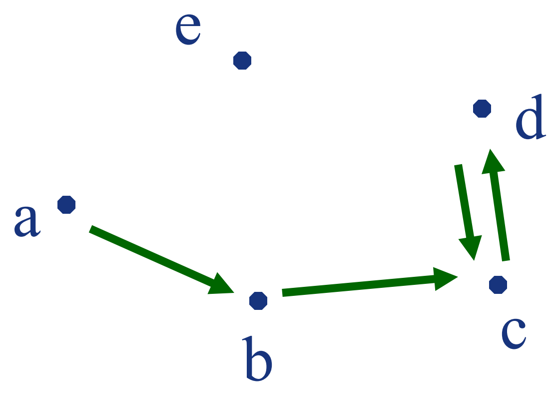
etc.

Composite Relations

- **The n^{th} power R^n of a relation R on a set A can be defined recursively by:**

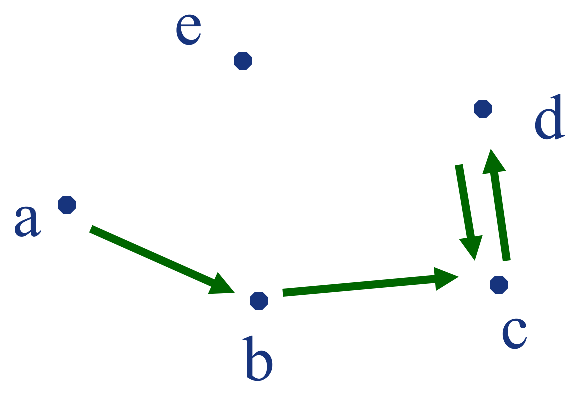
  $$R^1 :\equiv R\,; \quad R^{n+1} :\equiv R^n \circ R \quad \text{for all } n \geq 1.$$

- **E.g.,** $R^2 = R \circ R; \quad R^3 = R \circ R \circ R$



Composite Relations

- $R^2 = R \circ R = \{(a,c),(b,d),(c,c),(d,d)\}$
- a: c   d: d
  b: d   e: -
  c: c



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Application

设R和S定义在P上的二元关系，P是所有人的集合

$$R=\{<x,y>|(x,y\in P)\wedge(x\text{是}y\text{的父亲})\};$$

$$S=\{<x,y>|(x,y\in P)\wedge(x\text{是}y\text{的母亲})\};$$

（1）$R\circ R$ 表示的是什么关系。

（2）$S^{-1}\circ R$ 表示的是什么关系。

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Application

（1）$R \circ R$ 表示的是什么关系。

$$R \circ R = \{ <x,y> | (x,y \in P) \wedge (x\text{是}y\text{的祖父}) \};$$

（2）$S^{-1} \circ R$ 表示的是什么关系。

$$S^{-1} \circ R = \{ <x,y> | (x,y \in P) \wedge (x\text{和}y\text{是夫妻}) \}$$

Application

设R,S 是集合A上的关系，试证明或否定以下断言。

（1）设R,S是自反的，则 $R \circ S$ 是自反的。

（2）若R,S是传递的，则 $R \circ S$ 是传递的。

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Application

- （1）设R,S是自反的，则 $R\circ S$ 是自反的。

  正确。对任意 $x\in A$,因为R,S是自反的，所以 $<x,x>\in R,<x,x>\in S$ 。由关系映射关系则有 $<x,x>\in R\circ S$ ，所以 $R\circ S$ 是自反的。

  （2）若R,S是传递的，则 $R\circ S$ 是传递的。

  不一定。如 $R=\{<a,b>,<b,c>,<a,c>\}$,

  $S=\{<b,b>,<c,a>\}$ 都是传递的，但

  $R\circ S=\{<c,b>,<b,c>,<c,c>\}$ 不是传递的；若R和S交换, $S\circ R=\{<a,b>,<b,a>,<a,a>\}$,也是不传递的。

Exercises

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Exercises

1. How many transitive relation on the set {a， b， c}? (B)

A． 2　　B． 171　　C． 5　　D． 13

$$\{a,b,c\}\times\{a,b,c\}=\{(a,a),(b,b),(c,c),(a,b),(b,a),(a,c),(c,a),(b,c),(c,b)\}$$

由于(x,x)这种有序对不影响传递关系，因此可以分以下两种方式考虑：

（1）不考虑(a,a),(b,b),(c,c)，分析{$(a,b),(b,a),(a,c),(c,a),(b,c),(c,b)$}对应的传递关系：

①空集∅　　1

②传递关系有一个有序对：$\{(a,b)\}\ \{(b,a)\}\ \{(a,c)\}\ \{(c,a)\}\ \{(b,c)\}\ \{(c,b)\}$　　6

Exercises

③传递关系中有两个有序对：

$\{(a,b),(a,c)\}$ $\{(b,a),(c,a)\}$ $\{(b,a),(b,c)\}$ $\{(a,b),(c,b)\}$ $\{(c,a),(c,b)\}$ $\{(a,c),(b,c)\}$ 6

④传递关系中有三个有序对： 6

$$\{(a,b),(b,c),(a,c)\}\ \{(a,c),(c,b),(a,b)\}$$
$$\{(b,a),(a,c),(b,c)\}\ \{(b,c),(c,a),(b,a)\}$$
$$\{(c,a),(a,b),(c,b)\}\ \{(c,b),(b,a),(c,a)\}$$

总共1+6+6+6=19种情况，每种情况可加入(a,a)或(b,b)或(c,c)，有2*2*2=8种选择，共19*8=152种。

（2）考虑必须包含(a,a)，(b,b)，(c,c)中的两对才能满足传递关系：

①只包含一组对称的有序对，不加入其他有序对： 3

$$\{(a,b),(b,a),(a,a),(b,b)\}$$
$$\{(a,c),(c,a),(a,a),(c,c)\}$$
$$\{(b,c),(c,b),(b,b),(c,c)\}$$

Exercises

②包含一组对称的有序对以及其他有序对：　　　　　　　　　6

$$\{(a,b),(b,a),(a,c),(b,c),(a,a),(b,b)\}$$
$$\{(a,b),(b,a),(c,a),(c,b),(a,a),(b,b)\}$$
$$\{(a,c),(c,a),(a,b),(c,b),(a,a),(c,c)\}$$
$$\{(a,c),(c,a),(b,a),(b,c),(a,a),(c,c)\}$$
$$\{(b,c),(c,b),(a,b),(a,c),(b,b),(c,c)\}$$
$$\{(b,c),(c,b),(b,a),(c,a),(b,b),(c,c)\}$$

总共3+6=9种情况，每种情况可加入(a,a),(b,b),(c,c)中的一种，有2种选择，共9*2=18种。

（2）考虑必须包含(a,a)，(b,b)，(c,c)中的三对才能满足传递关系：　　1

$$\{(a,a),(b,b),(c,c),(a,b),(b,a),(a,c),(c,a),(b,c),(c,b)\}$$

所以，总传递关系数为：152+18+1=171

Exercises

3\. The relation R, U =Z-{0}, $(x, y) \in R$ if and only if $xy \geq 1$, so R is （ D ）

A) reflexive and anti-symmetric

B) asymmetric and transitive

C) reflexive and transitive

D) reflexive, symmetric and transitive

Exercises

4. R is “ less than or equal to” relation on $Z \times Z$, then R^{-1}= $\geq$

Exercises

5. How many of the 16 different relations on {0,1} contain the pair (0,1)? （ B ）

A． 2　　B． 8　C． 171　　D． 13

$\{0,1\} \times \{0,1\} = \{(0,0), (0,1), (1,0), (1,1)\}$

当必须包含(0,1)时, (0,0), (1,0), (1,1)各有2种选择：包含或不包含，因此总组合数为$2^3 = 8$。

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Exercises

6. A={ l， m， n }， B={ a， b， c }， C={ x， y ， z }. R：A→B， S：B→C， and $R=\{<l, b>, <m, a>, <n, c>\}$, $S=\{<a, y>, <b, x>, <c, y>, <c, z>\}$, SoR=？

{<l， x>， <m， y> ， <n， y>, <n, z>}.

Exercises

7. Let $R=\{<x,y>|(x,y\in Z)\wedge(x>y)\}$ ① irreflexive ② reflexive ③ symmetric ④ antisymmetric ⑤ transitive. R has the properties of ?

①④⑤

Exercises

9. Let R be the relation R={(a, b)| a divides b} on the set of positive integers. $R^{-1}=?$

{（b , a）| b is divided by a}

Exercises

11. Determine whether the relation R, where (x,y) $\in$R if and only if x=y+1 or x=y-1, on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive.

<u>not reflexive, symmetric, not antisymmetric, not transitive</u>

Exercises

12. For the relation {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)} on the set {1, 2, 3, 4}, decide whether it is (D)

A. Reflexive

B. symmetric

C. transitive

D. None of these properties above

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Exercises

13. For the relation $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$ on the set $\{1,2,3,4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

Reflexive. Symmetric, not antisymmetric, transitive

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Exercises

14. Determine whether the relation R on the set of all integers is transitive, where if and only if (B)

A) $x \neq y$

B) $xy \geq 1$

C) $x = y + 1$ or $x = y - 1$

D) $x = y^2$

Exercises

15. List the ordered pairs in the relation R from A = {1, 2, 3, 4} to B = {1, 2, 3}, where (a, b) $\in$ R if and only if a | b.

{(1, 1), (1, 2), (1, 3), (2, 2), (3, 3)}

Exercises

16. Supposed relation R= {(1, 2), (2, 3), (3, 4)} on the set {1, 2, 3, 4}, R is (A)

A) antisymmetric

B) symmetric

C) reflexive

D) transitive

Exercises

17. The relation R on the set of all integers. $(x,y)\in R$ if and only if $x\equiv y \pmod 7$, so R is (D) (tip: $x\equiv y \pmod 7 \Leftrightarrow (x-y) \bmod 7 = 0$)

A) reflexive and anti-symmetric

B) anti-symmetric and transitive

C) irreflexive and transitive

D) reflexive, symmetric and transitive

Logo

Exercises

18. Let R be the relation {(a,b),({a},b),({Ø},{Ø}),(Ø,{Ø})}, what are $R^{-1} \circ R^{-1}$?

$R^{-1} \circ R^{-1} = \{(\{Ø\},\{Ø\}),(\{Ø\},Ø)\}$

Exercises

19. Suppose that R and S are two relations on $A=\{1,2,3,4\}$, where $R=\{<1,1>,<2,2>,<2,3>,<4,4>\}$ and $S=\{<1,1>,<2,2>,<2,3>,<3,2>,<4,4>\}$, $(R\circ S)^{-1}=$

$$\{<1,1>,<2,2>,<2,3>,<3,2>,<3,3><4,4>\}$$

Logo

Exercises

20. Please use the propositional logic to present the transitive relation: A relation R is **transitive** iff (for all a,b,c)

$$((a,b)\in R \wedge (b,c)\in R) \rightarrow (a,c)\in R$$

Exercises

21. Set A={1, 2, 3, 4}, suppose R={(1, 2), (2, 2), (3, 1), (3, 2), (4, 4)} and S={(1, 3), (2, 3), (3, 2), (3, 3)} are relations on A, $R \circ S$=____

{<1,1>, <1,2>,<2,1>,<2,2>,<3,2>,<3,1>}

Exercises

22. Which one is **not** true? (D)

A. $f(n) = n^3$ is onto from R to R.

B. $p \leftrightarrow q$ is logically equivalent with $(p \wedge q) \vee (\neg p \wedge \neg q)$.

C. The "divides" relation on the set of all integers is antisymmetric.

D. $R_3 = \{(1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)\}$ is transitive.

Exercises

23. For the relation $R=\{(a,a),(a,b),(b,b),(b,a),(b,c),(b,d),(c,b),(d,d),(d,b)\}$ on the $S=\{a,b,c,d\}$, it is ________(**reflexive /symmetric/transitive**).

Exercises

23. For the relation $R = \{(a,a),(a,b),(b,b),(b,a),(b,c),(b,d),(c,b),(d,d),(d,b)\}$ on the $S = \{a,b,c,d\}$, it is symmetric (**reflexive /symmetric/transitive**).

Exercises

24. Let $R = \{<1,2>, <1,3>, <2,2>, <2,3>, <3,3>\}, S = \{<1,0>, <1,3>, <2,0>, <2,3>, <3,3>\}$ find $R \circ S$=________.

Exercises

24. Let $R=\{<1,2>,<1,3>,<2,2>,<2,3>,<3,3>\}, S=\{<1,0>,<1,3>,<2,0>,<2,3>,<3,3>\}$ find $R \circ S$= <u>{<1,3>,<2,3>,<3,3>}</u>.

Logo

End of Section 3.1