Logo

Discrete Mathematics

Dr. Han Huang

South China University of Technology

Logo

Chapter 2. Set model

Sets and Set Operations

Section 2.1

Contents

Introduction of Sets

Introduction to Set Theory (§ 1.6)

- A *set* is an *unordered collection of objects*.

- **Cantor's work:**
- **Paradoxes   Axioms   Naive set theory**

- **Sets are ubiquitous in computer software systems.**
- **All of mathematics can be defined in terms of the form of set theory.**

- The objects in a set are also called the *elements*, or *members*, of the set. A set is said to *contain* its elements.
- Example 1: V={a,e,I,o,u}
- Example 2: O={1,3,5,7,9}
- Example 3: {a,2,Wang,Guang,Zhou}
- Example 4:

N = {0, 1, 2, …} The Natural numbers.
Z = {…, -2, -1, 0, 1, 2, …} The integers.

- **Q**={ *p / q* | p $\in$ **Z**, *q* $\in$ **Z,** *q≠0*} is the set of rational number.

- Two sets are *equal* if and only if they have the same elements.
- {1, 3, 5} and {3, 5, 1} are equal.
- {1, 3, 3, 3, 5, 5, 5, 5} is the same as {1, 3, 5}
- No matter what objects a, b, and c denote, **{a, b, c} = {a, c, b} = {b, a, c} = …**
- Multiple listings make no difference:

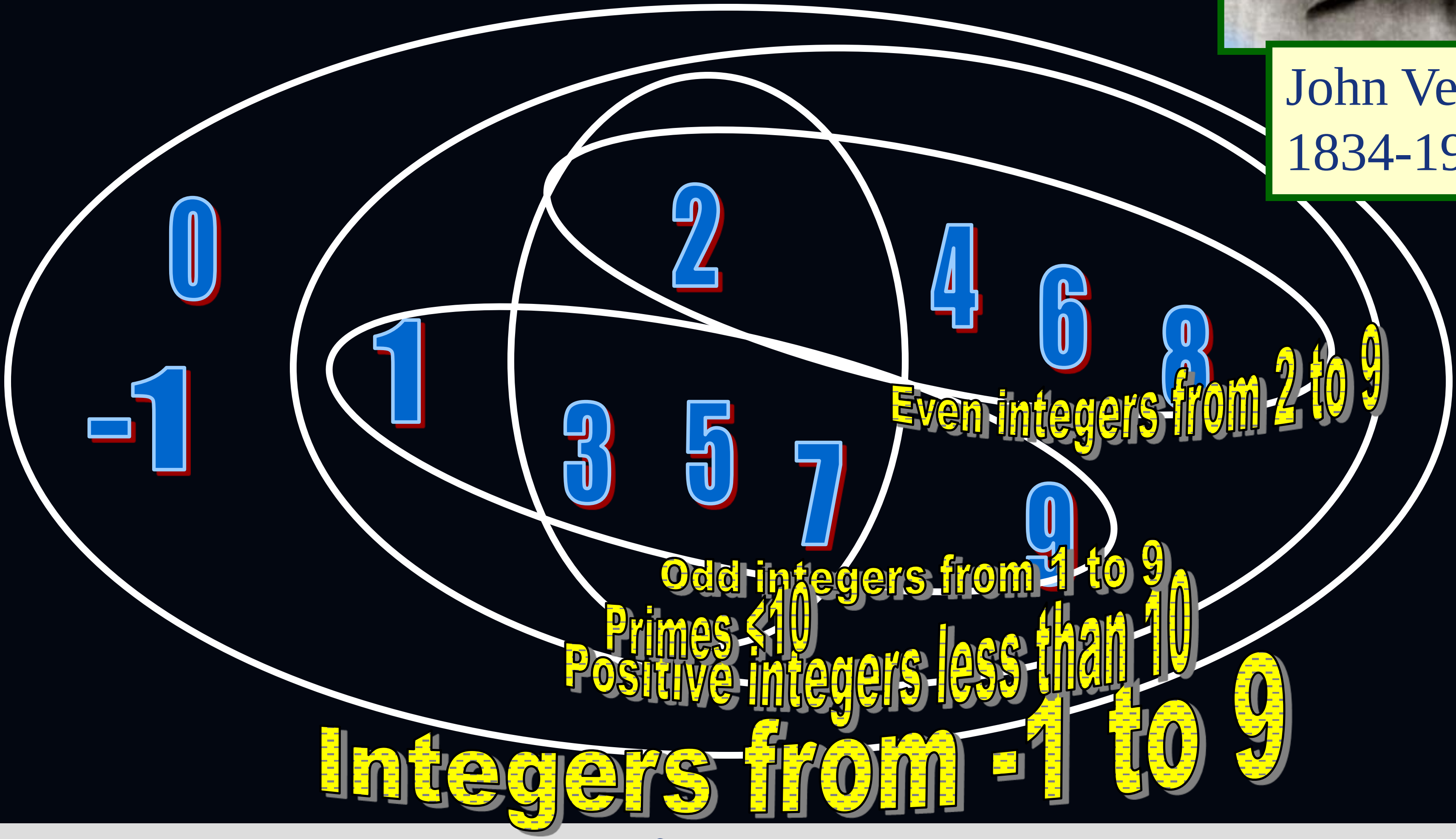
  {a, a, c, c, c, c}={a,c}.

Basic properties of sets

- *Set builder notation*: For any proposition $P(x)$ over any universe of discourse, $\{x|P(x)\}$ is *the set of all x such that P(x).*
- **For example:**
  {1, 2, 3, 4} =
  $\{x \mid x$ is an integer where $x>0$ and $x<5\}$ =
  $\{x \mid x$ is a positive integer whose square is >0 and $<25\}$

Venn/Euler Diagrams

John Venn
1834-1923



The Empty Set

- We have seen that there exists exactly one empty set, so we can give it a name:
- $\varnothing$ ("the empty set") is the unique set that contains no elements whatsoever.
- $\varnothing = \{\} = \{x|x \neq x\} = \ldots = \{x|\textbf{False}\}$

Subset and Superset Relations

- $S \subseteq T$ ("S is a subset of T") means that every element of S is also an element of T.
- $S \subseteq T :\equiv_{\text{def}} \forall x\, (x \in S \rightarrow x \in T)$
- What do you think about these?
  - $\emptyset \subseteq S$?
  - $S \subseteq S$?

Subset and Superset Relations

- $S \subseteq T$ ("S is a subset of T") means that every element of S is also an element of T.
- $S \subseteq T :\equiv_{\text{def}} \forall x\, (x \in S \rightarrow x \in T)$
- What do you think about these?
  - **$\emptyset \subseteq S$? Yes**
  - **$S \subseteq S$? Yes**

Subset and Superset Relations

- *More notation:*
- $S \supseteq T$ ("S is a superset of T") $:\equiv_{\text{def}}$ $T \subseteq S$.

  Note $S=T \Leftrightarrow S \subseteq T \wedge S \supseteq T$.

- $S \not\subseteq T :\equiv_{\text{def}} \neg(S \subseteq T)$, *i.e.* $\exists x(x \in S \wedge x \notin T)$

$$\neg(S \subseteq T) \Leftrightarrow \neg\forall x(x \in S \rightarrow x \in T)$$

$$\Leftrightarrow \exists x \neg(\neg(x \in S) \vee (x \in T))$$

$$\Leftrightarrow \exists x(x \in S \wedge x \notin T)$$

Proper (Strict) Subsets & Supersets

- $S \subset T$ ("S is a proper subset of T") means that $S \subseteq T$ but $T \not\subseteq S$.

Example: $\{1,2\} \subset \{1,2,3\}$

We have $\{1,2,3\} \subseteq \{1,2,3\}$,

but not $\{1,2,3\} \subset \{1,2,3\}$

Sets Are Objects, Too!

- The objects that are elements of a set may *themselves* be sets.
- *E.g.* let $S=\{x \mid x \subseteq \{1,2,3\}\}$ then S = …

Sets Are Objects, Too!

- The objects that are elements of a set may *themselves* be sets.
- *E.g.* let $S=\{x \mid x \subseteq \{1,2,3\}\}$ then $S=\{\varnothing,$
  $\{1\}, \{2\}, \{3\},$
  $\{1,2\}, \{1,3\}, \{2,3\},$
  $\{1,2,3\}\}$
- Note that $1 \neq \{1\} \neq \{\{1\}\}$

Cardinality and Finiteness

- $|S|$ (read "the *cardinality* of S") is a measure of how many different elements S has.
- *E.g.*, $|\varnothing|=0$, $|\{1,2,3\}| = 3$, $|\{a,b\}| = 2$, $|\{\{1,2,3\},\{4,5\}\}| = \underline{\ 2\ }$
- If $|S| \in \mathbf{N}$, then we say S is *finite*. Otherwise, we say S is *infinite*.

Example

在一个班级的 50 个学生中，有 26 人在离散数学的考试中取得了优秀的成绩；21 人在程序设计的考试中取得了优秀的成绩。假如有 17 人在两次考试中都没有取得优秀成绩，问有多少人在两次考试中都取得了优秀成绩?

分别用 A,B 表示在离散和程序设计的考试中取得优秀成绩的学生集合， U 表示全体学生集合：则 $|A|=26$ ， $|B|=21$ ， $|A\cup B|=50-17=33$ ，则两次考试中都取得了优秀成绩的学生人数为 26+21-33=14 人。

$$|A\cup B|=|A|+|B|-|A\cap B|$$

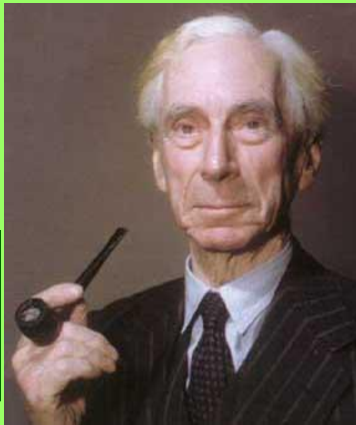
This version of Set Theory is inconsistent

Russell's paradox:

- Consider the set that corresponds with the predicate $x \notin x$:

$$S = \{x \mid x \notin x\}.$$

- Now ask: is $S \in S$?



Bertrand Russell
1872-1970

Logo

Russell's paradox

- *Let S* = {*x* | *x*$\notin$*x* }. Is *S*$\in$*S*?
- If *S*$\in$*S*, then S is one of those objects *x* for which *x*$\notin$*x*. In other words, *S*$\notin$*S*
  By Reductio, we have S$\notin$S
- If *S*$\notin$*S*, then S is not one of those objects *x* for which *x*$\notin$*x*. In other words, *S*$\in$*S*
  By Reductio, we have S$\in$S
- We conclude that both S$\in$S and S$\notin$S
- Paradox! (There's no assumption that we can blame, so we cannot Reductio again)

Power Sets

Logo

The *Power Set* Operation

- The *power set* P(S) of a set S is the set of all subsets of S. $P(S) :\equiv \{x \mid x \subseteq S\}$.
- *E.g.* P({a,b}) = {∅, {a}, {b}, {a,b}}.
- Sometimes P(S) is written $\mathbf{2}^S$, because $|P(S)| = 2^{|S|}$.
- It turns out $\forall S{:}|P(S)|>|S|$, *e.g.* $|P(\mathbf{N})| > |\mathbf{N}|$.

Examples

- $P(\{0, 1, 2\})$
  $=\{\varnothing,\{0\},\{1\},\{2\},\{0,1\},\{1,2\},\{0,2\},\{1,2\},\{0,1,2\}\}$
- $P(\{\varnothing\})=\{\varnothing, \{\varnothing\}\}$
- $P(\varnothing)=\{\varnothing\}$

Cartesian Products of Sets

- For sets A, B, their *Cartesian product* $A \times B :\equiv \{(a, b) \mid a \in A \wedge b \in B\}$.
- *E.g.* $\{a,b\} \times \{1,2\} = \{(a,1),(a,2),(b,1),(b,2)\}$

Example

在计算机内，一个字（或单元）有固定的 n 个有序二进制位所组成，它可以表示成有序 n 元组形式：$(a_1, a_2, \cdots, a_n)$，其中 a_i 表示字中第 i 位二进制的数字。而每个 a_i 所取之值为 0 或 1，亦即取自集合 $A=\{0,1\}$ 作为字的内容。所以，这些 n 位长的字的全体可表示为

$\underbrace{A\times A\times\cdots\times A}_{n个}=\{(a_1, a_2, \cdots, a_n) \mid a_i \in A, i=1,\cdots,n\}$ 这也可以写为 A^n。

Cartesian Products of Sets

- Note that
  - **for finite set A, B, $|A \times B| = |A| \cdot |B|$**
  - **the Cartesian product is *not* commutative: *i.e.*, $\neg\forall AB: A \times B = B \times A$.**

Using Set Notation with Quantifiers

- $\forall x \in S\ P(x)$ denotes the universal quantification
- $\exists x \in S\ P(x)$ denotes the existential quantification
- $\forall x \in R\ (x^2 \geq 0)$ and $\exists\ x \in Z\ (x^2 = 1)$

Logo

Set Operations

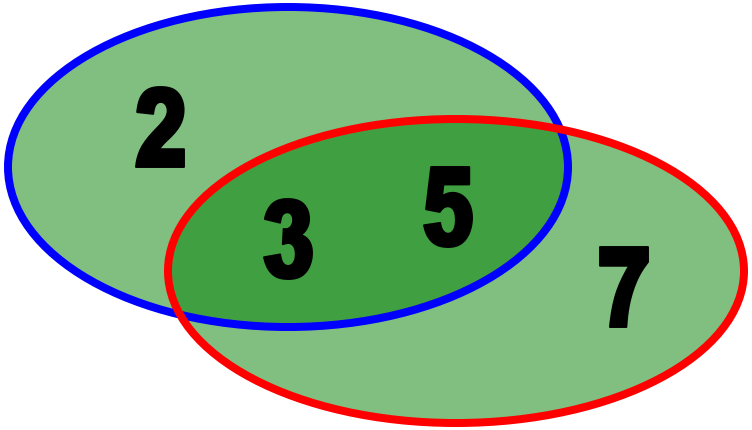
Start §2.2: The Union Operator

- For sets A, B, their *union* $A \cup B$ is the set containing all elements that are either in A, **or** ("$\vee$") in B (or, of course, in both).
- Formally, $\forall A, B: A \cup B = \{x \mid x \in A \vee x \in B\}$.
- Note that $A \cup B$ is a **superset** of both A and B (in fact, it is the smallest such superset):

$$\forall A, B: (A \cup B \supseteq A) \wedge (A \cup B \supseteq B)$$

Union Examples

- {a,b,c} ∪ {2,3} = {a,b,c,2,3}
- {2,3,5} ∪ {3,5,7} = {2,3,5,3,5,7} ={2,3,5,7}



Logo

The Intersection Operator

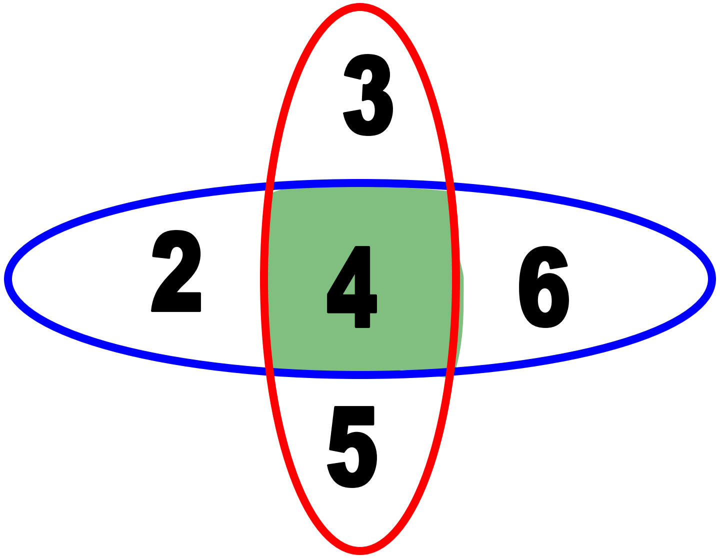
- For sets A, B, their *intersection* $A \cap B$ is the set containing all elements that are simultaneously in A **and** (“$\wedge$”) in B.
- Formally, $\forall A,B\colon A \cap B = \{x \mid x \in A \wedge x \in B\}$.
- Note that $A \cap B$ is a **subset** of both A and B (in fact it is the largest such subset):

$$\forall A, B\colon (A \cap B \subseteq A) \wedge (A \cap B \subseteq B)$$

Logo

Intersection Examples

- $\{a,b,c\} \cap \{2,3\} = \underline{\ \emptyset\ }$
- $\{2,4,6\} \cap \{3,4,5\} = \underline{\ \{4\}\ }$



Think "The **intersection** of University Ave. and W 13th St. is just that part of the road surface that lies on *both* streets."

Set Difference

- For sets *A*, *B*, the *difference of A and B*, written as *A*–*B*, is the set of all elements that are in *A* but not in *B*. Formally:

$$A - B :\equiv \{x \mid x \in A \land x \notin B\}$$

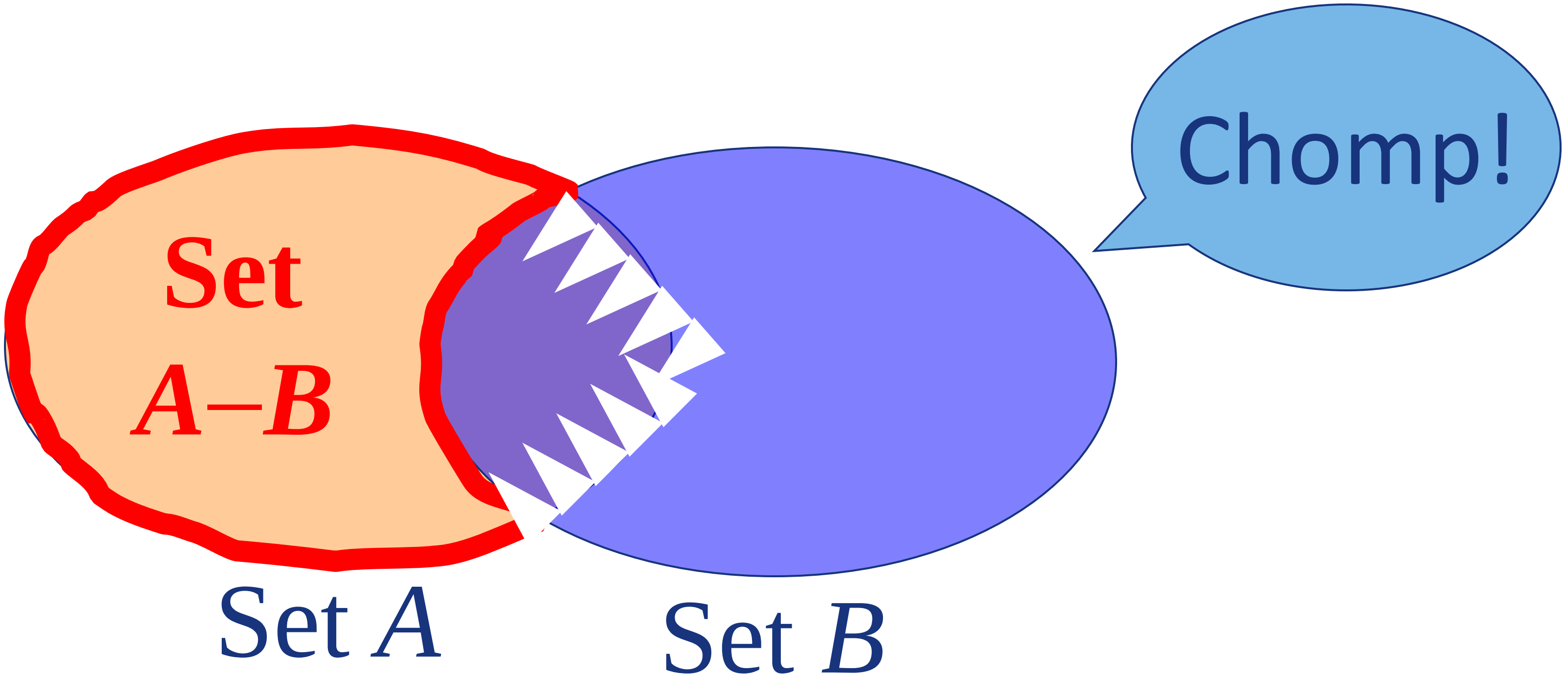
- Also called:
The *complement of B with respect to A*.

Set Difference Examples

- $\{1,\cancel{2},\cancel{3},4,\cancel{5},6\} - \{2,3,5,7,9,11\} =$
  $\underline{\qquad\{1,4,6\}\qquad}$
- $\mathbf{Z} - \mathbf{N} = \{\ldots, -1, 0, 1, 2, \ldots\} - \{0, 1, \ldots\}$
  $= \{x \mid x \text{ is an integer but not a nat. \#}\}$
  $= \{x \mid x \text{ is a negative integer}\}$
  $= \{\ldots, -3, -2, -1\}$

Set Difference - Venn Diagram

- *A*−*B* is what's left after *B* "takes a bite out of *A*"



Logo

Set Complements

- The *universe of discourse* itself can be considered a set, called *U*.
- When the context clearly defines *U*, we say that for any set $A \subseteq U$, the *complement* of *A*, written $\overline{A}$, is the complement of *A* w.r.t. *U*, *i.e.*, it is $U - A$.
- *E.g.*, If $U = \mathbf{N}$,

$$\overline{\{3,5\}} = \{0,1,2,4,6,7,\ldots\}$$

Logo

Set Identities

- $A \cup \emptyset = A = A \cap U$
- $A \cup U = U$
  $A \cap \emptyset = \emptyset$
- $A \cup A = A = A \cap A$
- $A \cup B = B \cup A$
  $A \cap B = B \cap A$
- $A \cup (B \cup C) = (A \cup B) \cup C$
- $A \cap (B \cap C) = (A \cap B) \cap C$

$$\overline{(\overline{A})} = A$$

(don't worry about their names)

- Identity: $A \cup \varnothing = A = A \cap U$
- Domination: $A \cup U = U$, $A \cap \varnothing = \varnothing$
- Idempotent: $A \cup A = A = A \cap A$
- Double complement: $\overline{(\overline{A})} = A$
- Commutative: $A \cup B = B \cup A$, $A \cap B = B \cap A$
- Associative: $A \cup (B \cup C) = (A \cup B) \cup C$,
  $A \cap (B \cap C) = (A \cap B) \cap C$

DeMorgan's Law for Sets

- Exactly analogous to (and provable from) DeMorgan's Law for propositions.

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$\begin{aligned}
\overline{A \cup B} &= \{x \mid x \in U \wedge x \notin (A \cup B)\} \\
&= \{x \mid x \notin (A \cup B)\} \\
&= \{x \mid \neg(x \in A \cup B\} \\
&= \{x \mid \neg(x \in A \vee x \in B\} \\
&= \{x \mid x \notin A \wedge x \notin B\} \\
&= \overline{A} \cap \overline{B}
\end{aligned}$$

$$\begin{aligned}
\overline{A \cap B} &= \{x \mid x \in U \wedge x \notin (A \cap B)\} \\
&= \{x \mid x \notin (A \cap B)\} \\
&= \{x \mid \neg(x \in A \cap B\} \\
&= \{x \mid \neg(x \in A \wedge x \in B\} \\
&= \{x \mid x \notin A \vee x \notin B\} \\
&= \overline{A} \cup \overline{B}
\end{aligned}$$

Proving Set Identities

To prove statements about sets, of the form $E_1 = E_2$ (where the Es are set expressions), here are three useful techniques:

1. Prove $E_1 \subseteq E_2$ and $E_2 \subseteq E_1$ separately.
2. Use set builder notation & logical equivalences.
3. Use a *membership table*.

Method 1: Mutual subsets

Example: Show $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$.

- **Part 1: Show $A\cap(B\cup C)\subseteq(A\cap B)\cup(A\cap C)$.**
- Assume $x\in A\cap(B\cup C)$, & show $x\in(A\cap B)\cup(A\cap C)$.
- We know that $x\in A$, and either $x\in B$ or $x\in C$.
- Case 1: $x\in B$. Then $x\in A\cap B$, so $x\in(A\cap B)\cup(A\cap C)$.
- Case 2: $x\in C$. Then $x\in A\cap C$, so $x\in(A\cap B)\cup(A\cap C)$.
- Therefore, $x\in(A\cap B)\cup(A\cap C)$.
- Therefore, $A\cap(B\cup C)\subseteq(A\cap B)\cup(A\cap C)$.
- **Part 2: Show $(A\cap B)\cup(A\cap C)\subseteq A\cap(B\cup C)$.** (*analogous*)

Method 2: Logic equality

- A variant of this method: translate into propositional logic, then reason within propositional logic, and finally translate back into set theory. E.g.,
- Show $A\cap(B\cup C)\subseteq(A\cap B)\cup(A\cap C)$.
  Suppose $x\in A \wedge (x\in B \vee x\in C)$.
  Prove $(x\in A \wedge x\in B) \vee (x\in A \wedge x\in C)$.

Logo

Method 3: Membership Tables

- Just like truth tables for propositional logic.
- Columns for different set expressions.
- Rows for all combinations of memberships in constituent sets.
- Use “1” to indicate membership in the derived set, “0” for non-membership.
- Prove equivalence with identical columns.

Membership Table Example

Prove $(A \cup B) - B = A - B$.

A	B	$A \cup B$	$(A \cup B) - B$	$A - B$
0	0	0	0	0
0	1	1	0	0
1	0	1	1	1
1	1	1	0	0

Membership Table Exercise

Prove $(A \cup B) - C = (A - C) \cup (B - C)$.

A	B	C	$A \cup B$	$(A \cup B) - C$	$A - C$	$B - C$	$(A - C) \cup (B - C)$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

Membership Table Exercise

Prove $(A \cup B) - C = (A - C) \cup (B - C)$.

A B C	$A \cup B$	$(A \cup B) - C$	$A - C$	$B - C$	$(A - C) \cup (B - C)$
0 0 0					
0 0 1					
0 1 0		1			
0 1 1					
1 0 0		1			
1 0 1					
1 1 0		1			
1 1 1					

Logo

Membership Table Exercise

Prove $(A \cup B) - C = (A - C) \cup (B - C)$.

A B C	$A \cup B$	$(A \cup B) - C$	$A - C$	$B - C$	$(A - C) \cup (B - C)$
0 0 0					
0 0 1					
0 1 0		1		1	
0 1 1					
1 0 0		1	1		
1 0 1					
1 1 0		1	1	1	
1 1 1					

Membership Table Exercise

Prove $(A \cup B) - C = (A - C) \cup (B - C)$.

A B C	$A \cup B$	$(A \cup B) - C$	$A - C$	$B - C$	$(A - C) \cup (B - C)$
0 0 0					
0 0 1					
0 1 0		1		1	1
0 1 1					
1 0 0		1	1		1
1 0 1					
1 1 0		1	1	1	1
1 1 1					

Logo

Generalized Union

- Binary union operator: $A \cup B$
- n-ary union:
  $A \cup A_2 \cup \ldots \cup A_n :\equiv ((\ldots((A_1 \cup A_2) \cup \ldots) \cup A_n)$
  (grouping & order is irrelevant)
- "Big U" notation: $\bigcup_{i=1}^{n} A_i$
- Or for infinite sets of sets: $\bigcup_{A \in X} A$

Generalized Intersection

- Binary intersection operator: $A \cap B$
- n-ary intersection:
  $A_1 \cap A_2 \cap \ldots \cap A_n \equiv ((\ldots((A_1 \cap A_2) \cap \ldots) \cap A_n)$
  (grouping & order is irrelevant)
- "Big Arch" notation: $\bigcap_{i=1}^{n} A_i$
- Or for infinite sets of sets: $\bigcap_{A \in X} A$


Application

Example 1

某个研究所有170名职工，其中120人会英语，80人会法语，60人会日语，50人会英语和法语，25人会英语和日语，30人会法语和日语，10人会英语、日语和法语。问有多少人不会这三种语言？

Example 1

某个研究所有170名职工，其中120人会英语，80人会法语，60人会日语，50人会英语和法语，25人会英语和日语，30人会法语和日语，10人会英语、日语和法语。问有多少人不会这三种语言?

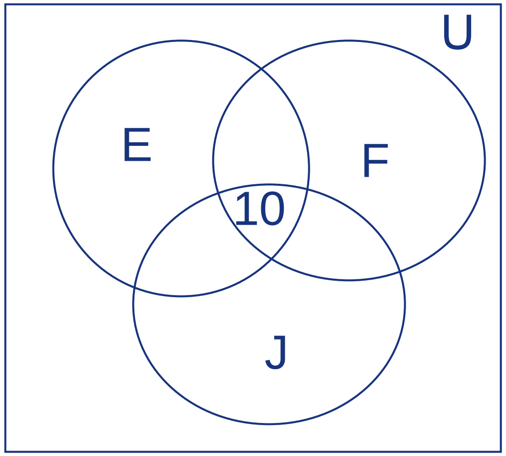
解:令U为全集，E、F、J分别为会英语、法语和日语人的集合。$|U|=170$

$|E|=120$ $|F|=80$ $|J|=60$ $|E\cap F|=50$

$|E\cap J|=25$ $|F\cap J|=30$ $|E\cap F\cap J|=10$

$|E\cup F\cup J|=|E|+|F|+|J|-|E\cap F|-|E\cap J|-|F\cap J|+|E\cap F\cap J|$

$= 120+80+60-50-25-30+10=165$



$|U-(E\cup F\cup J)|=170-165=5$ 即有5人不会这三种语言。

Example 2

75 名儿童到游乐场去玩，他们可以骑旋转木马，坐滑行铁道，乘宇宙飞船，已知其中 20 人这三种东西都玩过，其中 55 人至少乘坐过其中的两种，若每样乘坐一次的费用是 5 元，游乐场总收入 700 元，试确定有多少儿童没有乘坐其中任何一种。

Example 2

设 $A_1=\{x\mid x骑过木马\}$ ， $A_2=\{x\mid x坐过滑行铁道\}$ ， $A_3=\{x\mid x乘过宇宙飞船\}$ ，

$E=\{x\mid x为来到游乐场的儿童\}$，则 E 为全集，且 $|E|=75$ 。

设 x 为没玩过以上 3 种游乐种任何一种的人数，则 $x=|E|-|A_1\cup A_2\cup A_3|$

而 $|A_1\cup A_2\cup A_3|=(|A_1|+|A_2|+|A_3|)-(|A_1\cap A_2|+|A_1\cap A_3|+|A_2\cap A_3|)+|A_1\cap A_2\cap A_3|$

又因为 $|A_1|+|A_2|+|A_3|=700\div 5=140$， $|A_1\cap A_2\cap A_3|=20$，

$55=|(A_1\cap A_2)\cup(A_1\cap A_3)\cup(A_2\cap A_3)|$

$=(|A_1\cap A_2|+|A_1\cap A_3|+|A_2\cap A_3|)-3|A_1\cap A_2\cap A_3|+|A_1\cap A_2\cap A_3|$

$=(|A_1\cap A_2|+|A_1\cap A_3|+|A_2\cap A_3|)-2|A_1\cap A_2\cap A_3|$

$|A_1\cap A_2|+|A_1\cap A_3|+|A_2\cap A_3|=95$

所以 $|A_1\cup A_2\cup A_3|=140-95+20=65$，则 $x=75-65=10$。

Exercise

Exercises

- 1. **Which of these statements is true. ()**
- A) $0 \in \emptyset$ B) $\emptyset \in \{0\}$
- C) $\{0\} \subset \emptyset$ D) $\emptyset \subset \{0\}$

Exercises

- 1. **Which of these statements is true. (D)**
- A) $0 \in \emptyset$ B) $\emptyset \in \{0\}$
- C) $\{0\} \subset \emptyset$ D) $\emptyset \subset \{0\}$

Exercises

- **2. Which argument is true? ()**
- A) $(A \cup B) - A = B$
- B) $(A \cap B) - A = \emptyset$
- C) $(A - B) \cup B = A$
- D) $\emptyset \cup \{\emptyset\} = \emptyset$

Exercises

- 2. Which argument is true? (B)
- A) $(A \cup B) - A = B$
- B) $(A \cap B) - A = \emptyset$
- C) $(A - B) \cup B = A$
- D) $\emptyset \cup \{\emptyset\} = \emptyset$

Exercises

- **3.** Which of the following options is true? ()
- a) $|\ \emptyset\ | = 1$
- b) $|\ \{ x, x \}\ | = 2$
- c) $|\ \{x\} \cap \emptyset\ | = 0$
- d) $|\ \emptyset\ | = \emptyset$

Exercises

- **3.** Which of the following options is true? (C)
- a) $|\varnothing| = 1$
- b) $|\{x, x\}| = 2$
- c) $|\{x\} \cap \varnothing| = 0$
- d) $|\varnothing| = \varnothing$

Exercises

- **4.** Let A and B be sets. Which is not true? U is the universal set.()
- A) $A \cup \emptyset = A$
- B) $A \cap U = A$
- C) $A \cup A = A$
- D) $A - \emptyset = \emptyset$

Exercises

- **4.** Let A and B be sets. Which is not true? U is the universal set.(D)
- A) $A \cup \emptyset = A$
- B) $A \cap U = A$
- C) $A \cup A = A$
- D) $A - \emptyset = \emptyset$

Exercises

- **5.** Assume that $S = \{2, a, \{3\}, 4\}$ and $T = \{\{a\}, 3,4,1\}$, which statement is wrong? ( )
- A. $\{\{a\}, 1,3,4\} \subseteq T$
- B. $\{a\} \subseteq T$
- C. $\phi \subseteq \{\{a\}\} \subseteq T$
- D. $\phi \subseteq \{\{3\}, 4\}$

Exercises

- **5.** Assume that $S = \{2, a, \{3\}, 4\}$ and $T = \{\{a\}, 3{,}4{,}1\}$, which statement is wrong? (B)
- A. $\{\{a\}, 1{,}3{,}4\} \subseteq T$
- B. $\{a\} \subseteq T$
- C. $\phi \subseteq \{\{a\}\} \subseteq T$
- D. $\phi \subseteq \{\{3\}, 4\}$

Exercises

- **6.** Determine which of these statements is true. ()
- A) $\emptyset \in \{a, b, c\}$
- B) $\exists x(B \rightarrow A(x)) \Leftrightarrow B \rightarrow \forall x A(x)$
- C) $\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$
- D) $\{a\} \subseteq \{\{a\}, 1,2,3\}$

L o g o

Exercises

- **6.** Determine which of these statements is true. (C)
- A) $\emptyset \in \{a, b, c\}$
- B) $\exists x(B \to A(x)) \Leftrightarrow B \to \forall x A(x)$
- C) $\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$
- D) $\{a\} \subseteq \{\{a\}, 1,2,3\}$

Logo

Exercises

- **7.** Let the set A=B. Which of the following is true?(   )
- A) $A = \{\{a\}\}$, $B = \{a, \{a\}\}$
- B) $A = \{\emptyset, a, b\}$, $B = \{a, b\}$
- C) $A = \{a, b\} \cup \emptyset$, $B = \{a, b\} \cup \{\emptyset\}$
- D) $A = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$, $B = \{x \mid x$ is the subset of $\{a, b\}\}$

Exercises

- **7.** Let the set A=B. Which of the following is true?(D)
- A) $A = \{\{a\}\}$, $B = \{a, \{a\}\}$
- B) $A = \{\emptyset, a, b\}$, $B = \{a, b\}$
- C) $A = \{a, b\} \cup \emptyset$, $B = \{a, b\} \cup \{\emptyset\}$
- D) $A = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$, $B = \{x \mid x$ is the subset of $\{a, b\}\}$

Logo

Exercises

- **8.** For each of the following sets, determine whether 2 is an element of that set. ( )
- A) {2, {2}}
- B) {{2}, {{2}}}
- C) {{2}, {2,{2}}}
- D) {{{2}}}

Logo

Exercises

- **8.** For each of the following sets, determine whether 2 is an element of that set. (A)
- A) {2, {2}}
- B) {{2}, {{2}}}
- C) {{2}, {2,{2}}}
- D) {{{2}}}

Exercises

- 9. Determine which of these statements is false. ()
- A) $\{x\} \subseteq \{x\}$
- B) $\{x\} \subseteq \{x, \{x\}\}$
- C) $\{x\} \in \{x, \{x\}\}$
- D) $\{x\} \in \{x\}$

Exercises

- 9. Determine which of these statements is false. (D)
- A) $\{x\} \subseteq \{x\}$
- B) $\{x\} \subseteq \{x, \{x\}\}$
- C) $\{x\} \in \{x, \{x\}\}$
- D) $\{x\} \in \{x\}$

Exercises

- **10.** $|A \cup B \cup C| = ()$
- A. $|A|+|B \cup C|$
- B. $|A|+|B|+|C|-|A \cap B|-|C \cap B|-|A \cap C|+|A \cap B \cap C|$
- C. $|A \cup B|+|C|$
- D. $|A|+|B|+|C|+|A \cap B|+|C \cap B|+|A \cap C|-|A \cap B \cap C|$

Exercises

- **10.** $|A \cup B \cup C| =$ (B)
- A. $|A|+|B \cup C|$
- B. $|A|+|B|+|C|-|A \cap B|-|C \cap B|-|A \cap C|+|A \cap B \cap C|$
- C. $|A \cup B|+|C|$
- D. $|A|+|B|+|C|+|A \cap B|+|C \cap B|+|A \cap C|-|A \cap B \cap C|$

Logo

Exercises

- 11. Suppose that A, B and C are all sets such that $A \cap C = B \cap C$. Which answer is correct ? U is the universal set. ()
- A) $A = B$
- B) $A \neq B$
- C) if $A - C = B - C$, then $A = B$
- D) if $C = U$, then $A \neq B$

Exercises

- 11. Suppose that A, B and C are all sets such that $A \cap C = B \cap C$. Which answer is correct ? U is the universal set. (C)
- A) $A = B$
- B) $A \neq B$
- C) if $A - C = B - C$, then $A = B$
- D) if $C = U$, then $A \neq B$

Exercises

- 12 $A \subseteq B \nLeftrightarrow ()$
- A) $A \cup B = B$
- B) $A \cap B = A$
- C) $\overline{A} \supseteq \overline{B}$
- D) $(B - A) \cup A \supseteq B$

Exercises

- 12 $A \subseteq B \nLeftrightarrow$ (D)
- A) $A \cup B = B$
- B) $A \cap B = A$
- C) $\overline{A} \supseteq \overline{B}$
- D) $(B - A) \cup A \supseteq B$

Exercises

- **14** A = {∅, {∅}} and ρ(A) is the set of powers of the set A, then ρ(A)= __________

Exercises

- **14** A = {Ø, {Ø}} and ρ(A) is the set of powers of the set A, then ρ(A)= {Ø， {Ø}， {{Ø}},{Ø,{Ø}}}

Exercises

- **15** The power set $P(A) \subseteq P(B)$ holds if and only if $A \subseteq B$, True or False__________

Logo

Exercises

- **15** The power set P(A) ⊆ P(B) holds if and only if A ⊆ B, True or False ___True_____

Exercises

- **16** List the members of the set $\{x \mid x$ is a real number such that $x^2 = 1\}$

Logo

Exercises

- **16** List the members of the set {x | x is a real number such that $x^2 = 1$}

- $\{-1, 1\}$

Exercises

- **17** Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$
- $A \cap B = ?$

Exercises

- **17** Let A = {1, 2, 3, 4, 5} and B = {0, 3, 6}
- $A \cap B$ =?
- {3}

Exercises

- **18** The cardinality of set $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$ is

Exercises

- **18** The cardinality of set $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$ is

- 3

Exercises

- 19 $A=\{\phi, a, \{a\}\}$, $\rho(A\quad)$ is the power set of A. $\rho(A\quad)$=________.

Exercises

- **19** $A = \{\phi, a, \{a\}\}$, $\rho(A\quad)$ is the power set of A. $\rho(A\quad)$=________.

$\{\phi, \{\phi\}, \{a\}, \{\{a\}\}, \{\phi, a\}, \{\phi, \{a\}\}, \{a, \{a\}\}, \{\phi, a, \{a\}\}\}$

Logo

Exercises

- **20** If $A - B = \{1,5,7,8\}$, $B - A = \{2,10\}$ and $B \cap A = \{3,6,9\}$, then A=__________

Exercises

- **20** If $A - B = \{1,5,7,8\}$, $B - A = \{2,10\}$ and $B \cap A = \{3,6,9\}$, then A=__________

- {1,3,5,6,7,8,9}

Logo

Exercises

- **21** Let A = {1,2,3,4,5} and B={0,3,6}.
- Find A – B=?

Exercises

- **21** Let A = {1,2,3,4,5} and B={0,3,6}.
- Find A – B=？
- {1,2,4,5}

Logo

Exercises

- **22** $S \subseteq T :\equiv_{\mathrm{def}} \forall x\, (x \in S$ ________ $x \in T)$;
- $A - B :\equiv \{x \mid x \in A$ ________ $x \notin B\}$

Exercises

- **22.** $S \subseteq T :\equiv_{\text{def}} \forall x\,(x \in S$ ________ $x \in T)$;
- $A - B :\equiv \{x \mid x \in A$ ________ $x \notin B\}$
- $\rightarrow$
- $\wedge$

Exercises

- **23.** 150 out of 200 people speak English or German or both, if there are 85 people who speak English and 60 who speak both, how many people speak German__________

Exercises

- **23.** 150 out of 200 people speak English or German or both, if there are 85 people who speak English and 60 who speak both, how many people speak German__125______

解:令E、G分别为会英语和德语的人的集合。

$|E\cup G|=150$

$|E|=85$ $|E\cap G|=60$

因为$|E\cup G|= |E|+|G|-|E\cap G|$

所以$|G|=|E\cup G|+|E\cap G|-|E|= 150+60-85=125$

Exercises

- 24 Prove $(A-B)-C=A-(B\cup C)$ by predicate expression

Exercises

- 24 Prove $(A-B)-C = A-(B \cup C)$ by predicate expression

$$
\begin{aligned}
&(A-B)-C \\
&= \{x \mid x \in A-B \wedge x \notin C\} \\
&= \{x \mid x \in A \wedge x \notin B \wedge x \notin C\} \\
&= \{x \mid x \in A \wedge \neg(x \in B) \wedge \neg(x \in C)\} \\
&= \{x \mid x \in A \wedge \neg(x \in B \vee x \in C)\} \\
&= \{x \mid x \in A \wedge x \notin B \bigcup C\} \\
&= A-(B \bigcup C)
\end{aligned}
$$

Exercises

- **25** Let A, B and C be any sets. Prove or disprove: $A \cap (B - C) = (A \cap B) - (A \cap C)$

Logo

Exercises

- 25 Let A, B and C be any sets. Prove or disprove: $A \cap (B - C) = (A \cap B) - (A \cap C)$
- $(A \cap B) - (A \cap C)$
- $= \{x | x \in A \land x \in B \land \neg(x \in A \land x \in C)\}$
- $= \{x | x \in A \land x \in B \land (\neg x \in A \lor \neg x \in C)\}$
- $= \{x | x \in A \land x \in B \land (x \notin A \lor x \notin C)\}$
- $= \{x | x \in A \land x \in B \land x \notin C\}$
- $= \{x | x \in A \land x \in B \land \neg x \in C\}$
- $= A \cap (B - C)$

Logo

End of the Section 2.1