Logo

Discrete Mathematics

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Logo

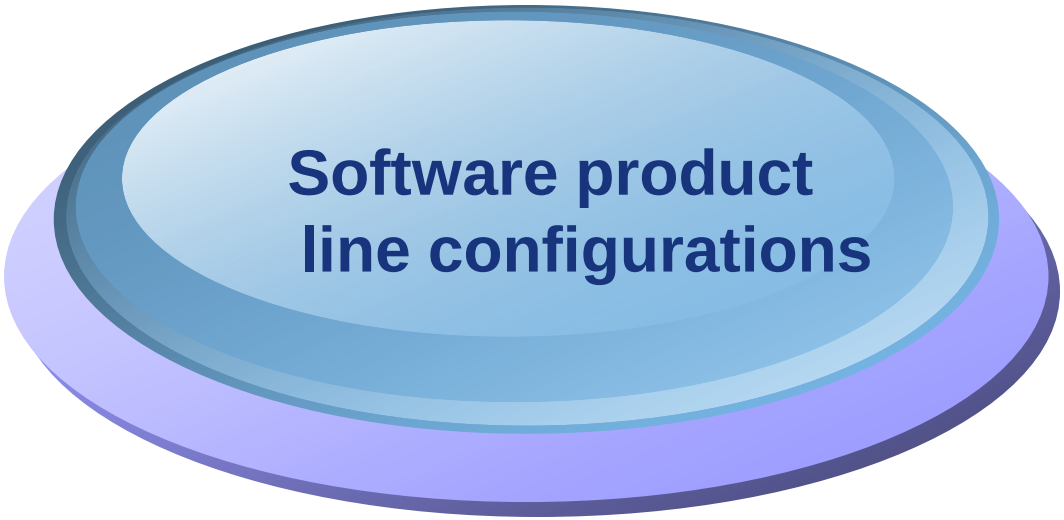
Chapter 1. Binary

Mathematical Modeling and Application

Section 1.7

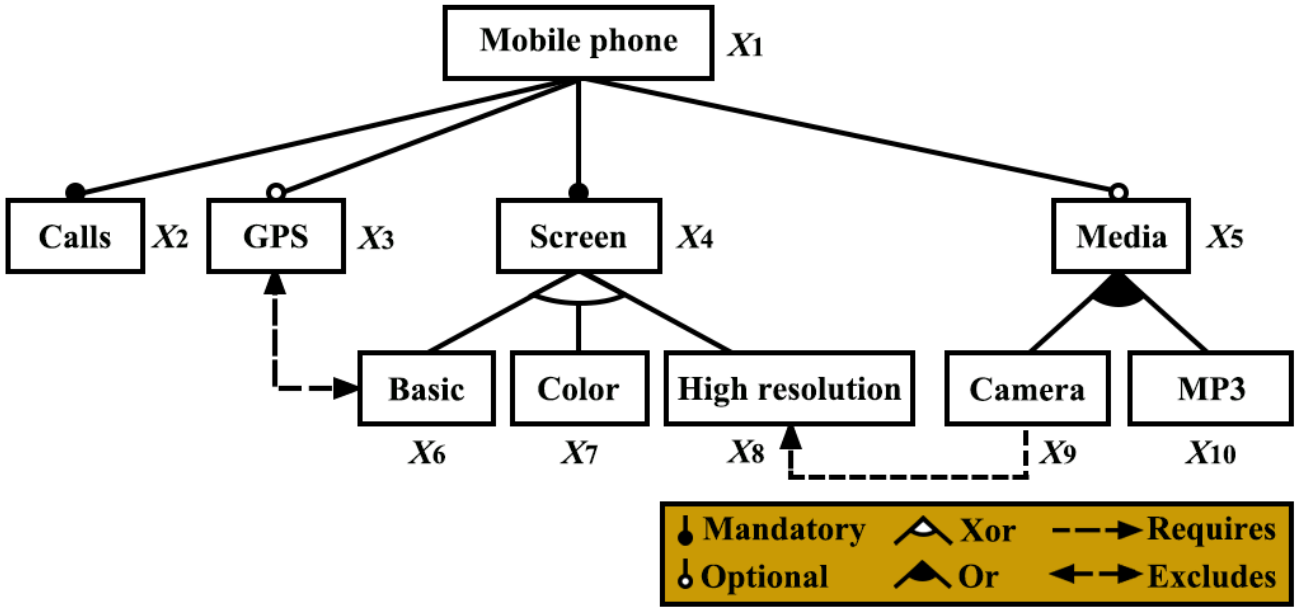

Logo

Contents

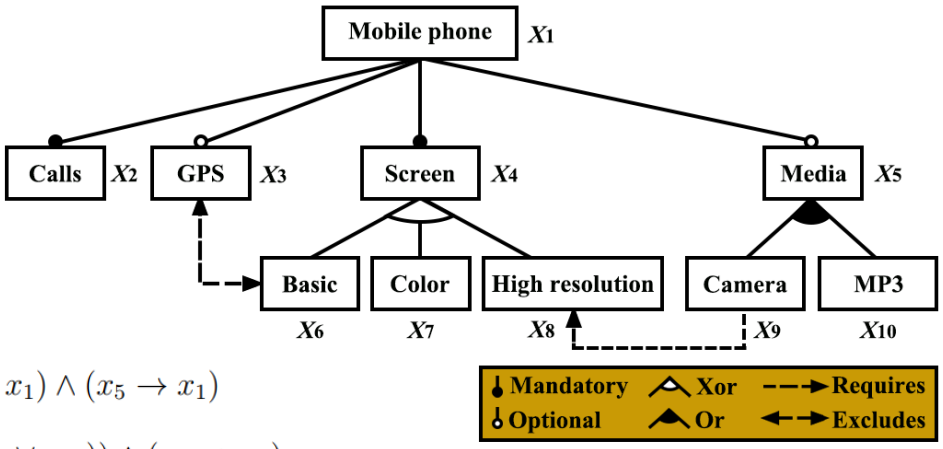

Software product
line configurations

Introduction

- The Software Product Line defines a series of software products that can be automatically assembled with modular software components.



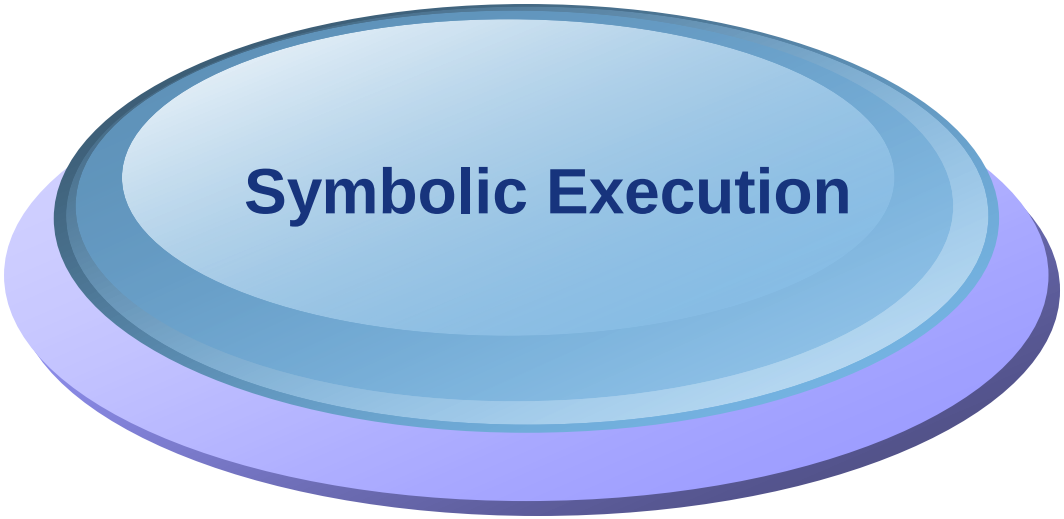
Modeling



$$FeatureModel = x_1 \wedge (x_1 \leftrightarrow x_2) \wedge (x_1 \leftrightarrow x_4) \wedge (x_3 \rightarrow x_1) \wedge (x_5 \rightarrow x_1)$$
$$\wedge (x_4 \leftrightarrow xor(x_6, x_7, x_8)) \wedge (x_5 \leftrightarrow (x_9 \vee x_{10})) \wedge (x_9 \rightarrow x_8)$$
$$\wedge \neg(x_3 \wedge x_6)$$

$$FeatureModel = x_1 \wedge (\neg x_1 \vee x_2) \wedge (x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_4) \wedge (x_1 \vee \neg x_4)$$
$$\wedge (x_1 \vee \neg x_3) \wedge (x_1 \vee \neg x_5) \wedge (x_4 \vee \neg x_6) \wedge (x_4 \vee \neg x_7) \wedge (x_4 \vee \neg x$$
$$\wedge (\neg x_4 \vee x_6 \vee x_7 \vee x_8) \wedge (\neg x_6 \vee \neg x_7) \wedge (\neg x_6 \vee \neg x_8)$$
$$\wedge (\neg x_7 \vee \neg x_8) \wedge (x_5 \vee \neg x_9) \wedge (x_5 \vee \neg x_{10}) \wedge (\neg x_5 \vee x_9 \vee x_{10})$$
$$\wedge (\neg x_9 \vee x_8) \wedge (\neg x_3 \vee \neg x_6)$$

- This approach can help improve the efficiency and quality of software product line development and enable rapid generation of customized software products


Symbolic Execution

Introduction

- Symbolic execution simulates the execution of a program by using abstract symbols rather than concrete exact values as input parameters to the program to be tested.
- All variables in the set of variables are assigned a value of their domain that satisfies the condition that the path expression is true. Such constraint-satisfying solutions can be used as concrete inputs for testing the corresponding program paths.

```
1   int twice (int v) {
2           return 2*v;
3    }
4
5   void testme (int x, int y) {
6           z = twice(y);
7           if (z == x) {
8                   if (x >y+10)
9                       ERROR; }
10                  }
11          }
12  }
13
14  /* simple driver exercising testme() with sym inputs */
15  int main()  {
16          x =sym-input();
17          y =sym-input();
18          testme (x, y);
19          return 0;
20   }
```

Modeling

```
1   int twice (int v) {
2           return 2*v;
3    }
4
5   void testme (int x, int y) {
6           z = twice(y);
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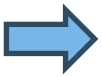
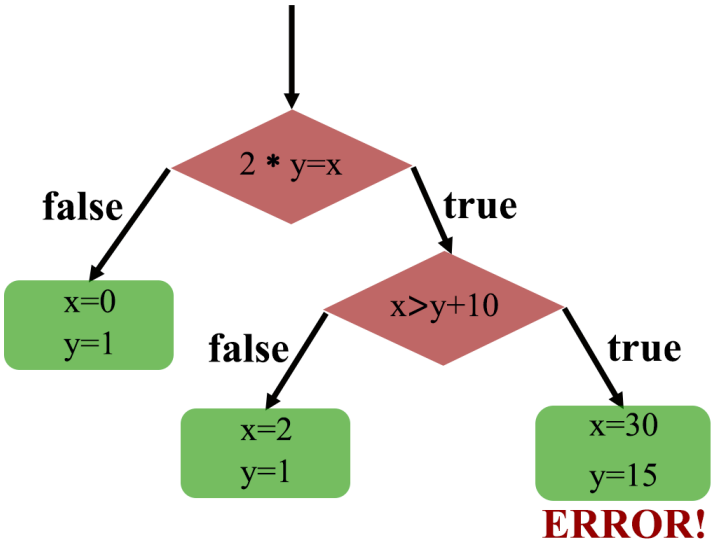




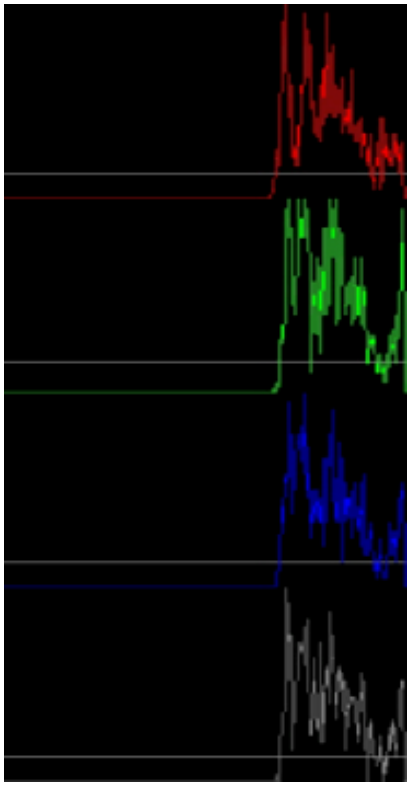
- In the symbolic execution case, we utilize composite propositions to represent a series of paths with input symbols as variables. The test of each path is realized by finding the values of the variables that make the path expression true to obtain a solution to the expression.

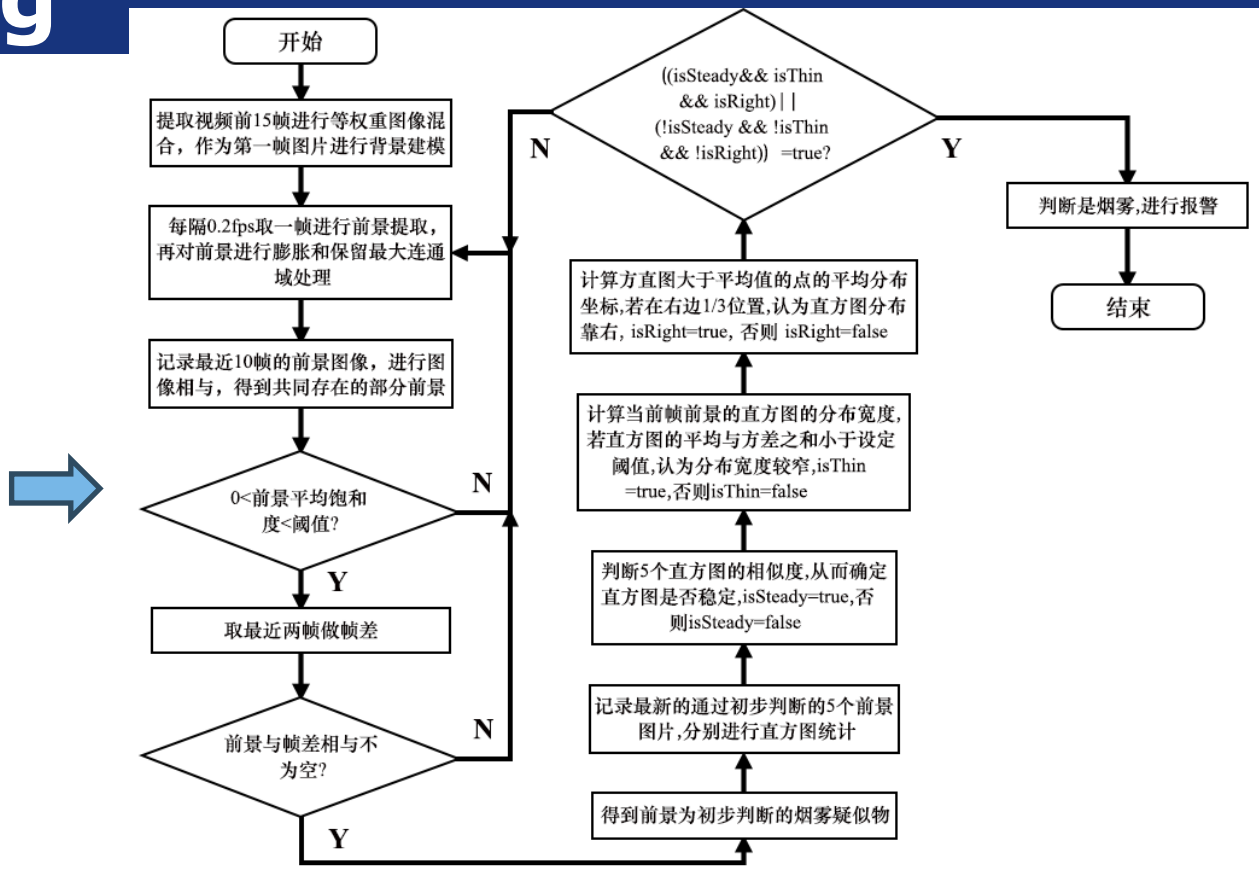

Video Smoke
Detection

Introduction

- Most of the traditional fire monitoring technologies such as ionic smoke sensors and photoelectric smoke sensors sense the temperature change of smoke particle density in the fire scenario and trigger the alarm device, but they are easily affected by the smoke concentration and the change of the surrounding airflow.
- With the development and breakthroughs in video image and surveillance technology, more and more researchers in the field of security have made great progress in utilizing the image characteristics of smoke itself for early warning of fire.

Modeling



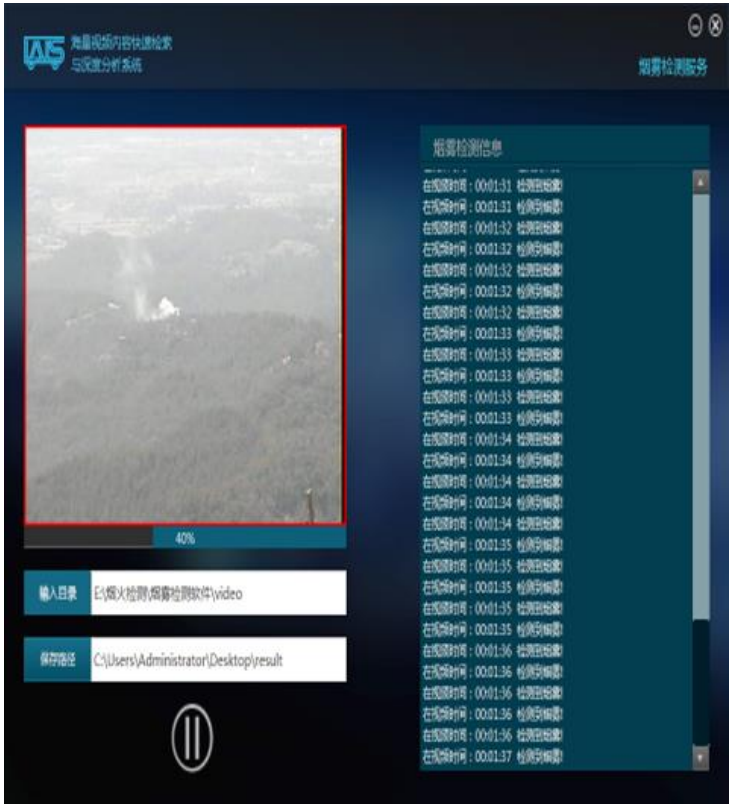


- Can accurately detect the appearance of smoke in the video and give alerts

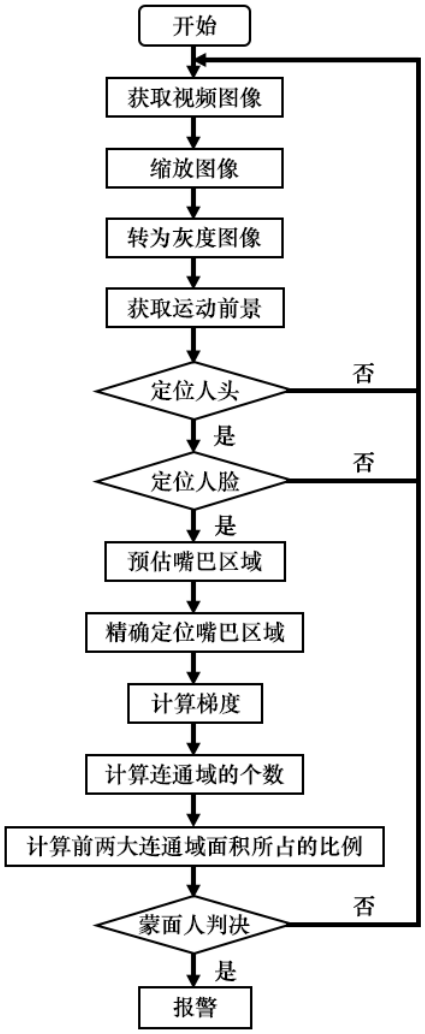

Video Masking
Detection

Introduction

- Provide a video and try to determine if a masked man appears in the video?



Modeling



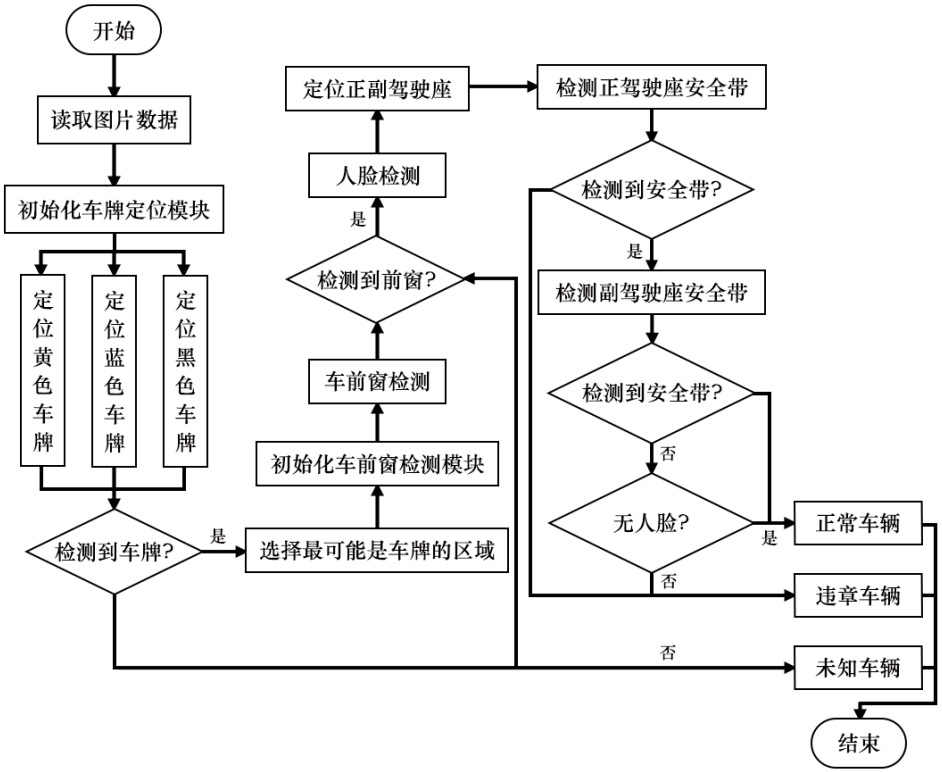
- We accurately detected the masked man appearing in the video and gave an alert.

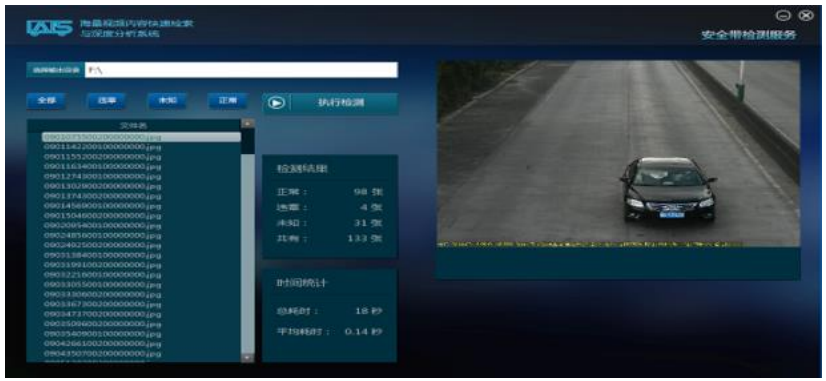

Seat Belt Detection for
Video Surveillance

Introduction

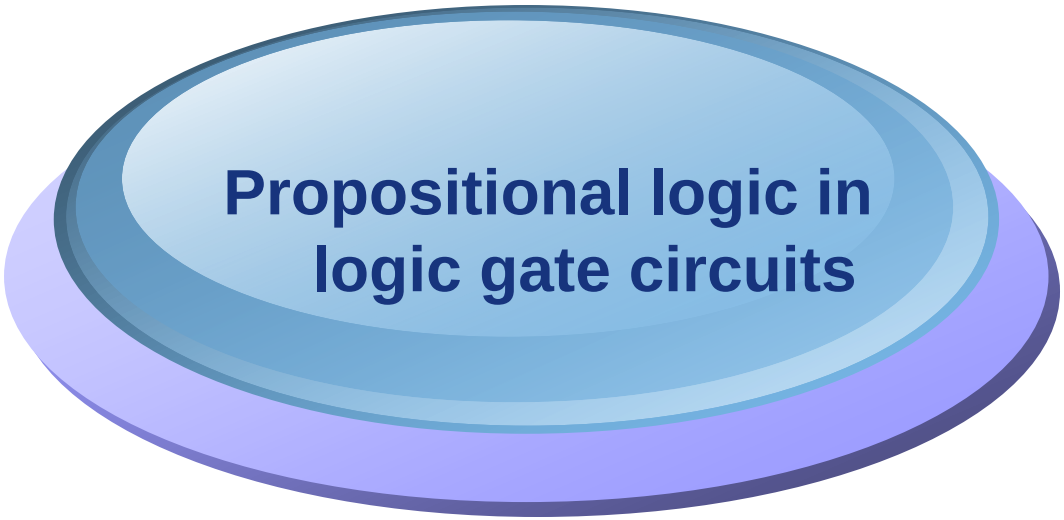
- The traditional method of manual detection is time-consuming and labor-intensive, and is not conducive to large-scale seatbelt violation detection, which has certain limitations.
- Provide a video and try to determine if the driver of the car in the video is wearing a seat belt?

Modeling





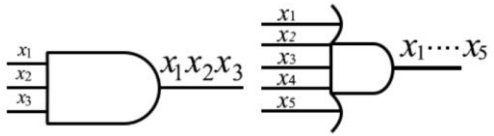
- We accurately detect the seatbelt wear of the people in each vehicle passing in the video.


Propositional logic in
logic gate circuits

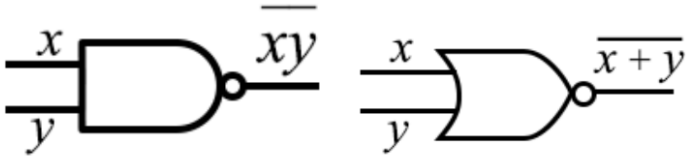
Introduction

- A computer or other electronic device is made up of thousands of electronic circuits, and electronic circuits are ultimately made up of several of the most basic types of logic circuits.

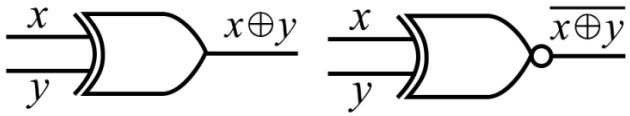
(a) Non-gate (b) And gate (c) Or gate



Gate circuits with multiple inputs



AND-AFR GATE and OR-AFR GATE

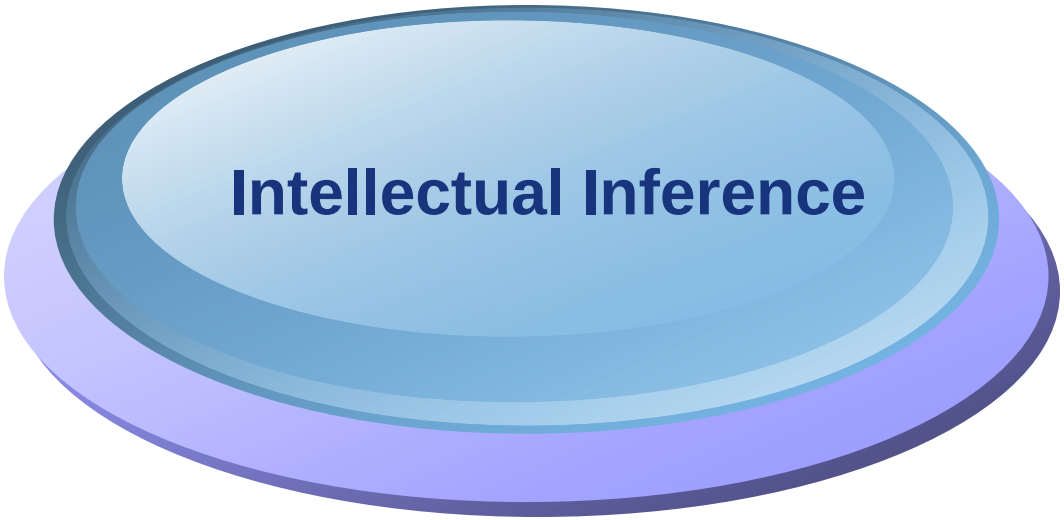


Ddifferent-or-gate and Same-or-gate

- Design an electronic circuit which is capable of adding two binary numbers and outputting the correct result.

Modeling

- For the addition of each bit, we use a different-or gate to perform a specific operation: the two corresponding bits are different-or to get the result of the addition of that bit and a signal called a "rounding" signal.
- For the rounding signal, we use an AND gate: a rounding signal is generated only if both inputs are 1.
- The rounding signal is added to the addition result of the previous bit, and the new addition result is generated again using the and gate.
- Repeat until all bits have been processed.

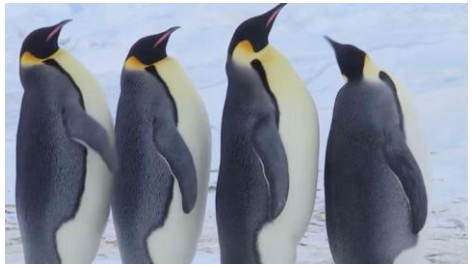

Intellectual Inference

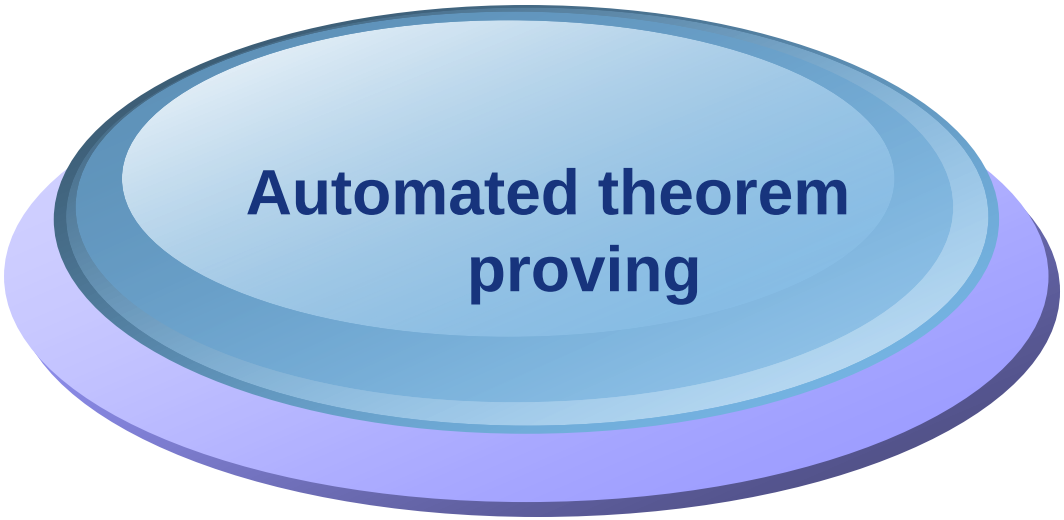
Introduction

- Early research in knowledge-based reasoning methods was mainly based on first-order predicate logic rules and Bayesian algorithms.
- Use knowledge deduction methods to answer the question: Does a penguin fly?

Modeling

- First, we need to define some related concepts and relationships in the knowledge graph, such as: "bird" as a concept with the attributes "has wings" and "can fly", and "penguin" as a special bird.
- We can also define a relation: "belongs to", which means that an entity belongs to a concept.
- Next, we use the rules of propositional logic to model our reasoning. For example, if an entity belongs to "birds", then we can infer that it has the properties "has wings" and "can fly" based on the existing property rules.
- However, for special cases such as "penguin", although it belongs to the category of "bird", it cannot fulfill the attribute of "flying" due to its special characteristics. Therefore, we need to introduce a negative proposition to describe "Penguin cannot fly".



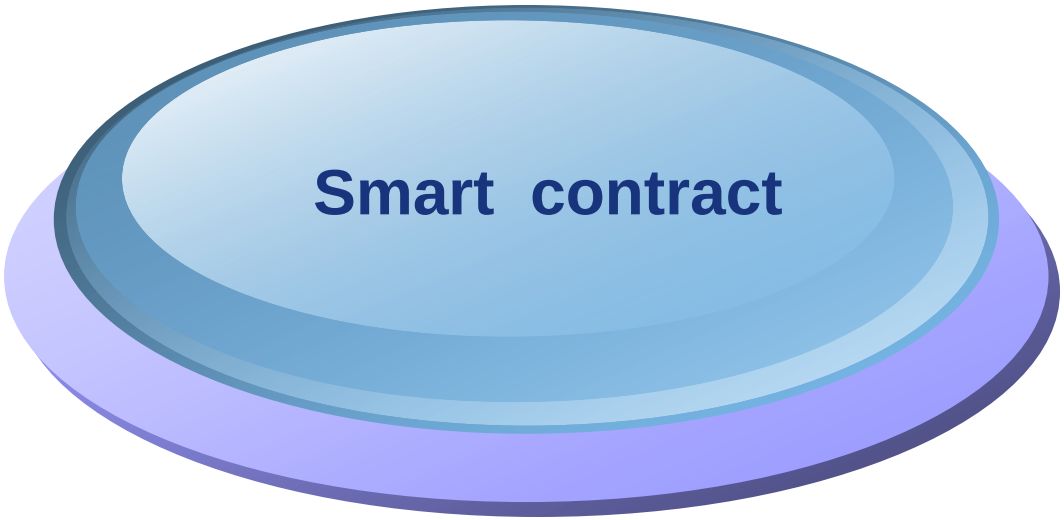

Automated theorem proving

Introduction

- The program successfully proves 38 theorems in the propositional logic (arithmetic) section of Whitehead and Russell's famous book Principia Mathematica.
- Suppose we want to prove a mathematical or logical proposition whose rule of inference is as follows: if the premise P implies the conclusion Q and the negation of the conclusion ¬Q is true, then it follows that the negation of the premise ¬P is true. We can use propositional logic to describe and reason about this proposition.

Modeling

- In order to construct a propositional logic model, we first need to define the notation and the rules of operation. We can construct logical expressions using P for premises, Q for conclusion, and the logical operators → and ¬ as set symbols.
- In propositional logic modeling, we can input given rules of deduction into an automated theorem proving system. The system performs step-by-step derivation in an automated manner based on the logical relations of the known rules and the rules of inference. In this way, the system can generate a logical proof process by applying the rules of inference in a rational way.


Smart contract

Introduction

- Since ethereum introduced smart contracts to the blockchain, the application of smart contracts has penetrated into many fields.
- On an e-commerce platform, Ming wants to purchase an item. He wants to ensure the trustworthiness and security of the transaction through a smart contract.
- To satisfy Ming's needs, the e-commerce platform needs to design a smart contract that can be used to ensure the following conditions:
  (1) the buyer can only pay for the item after the seller sends the item to the specified address;
  (2) after the payment is made, the buyer can apply for a refund if the seller fails to deliver the item as promised;
  (3) the buyer, upon receipt of the item and confirming that the item matches the description, can confirm that the transaction is complete and release the payment to the seller.

Modeling

We can use conditional statements, state variables, logical operators, etc. to model smart contracts.

- IF-THEN
- IF-THEN-ELSE
- AND-OR
- transaction-status

Through the application of propositional logic in smart contracts, e-commerce platforms can realize the following effects:

(1) Improve the trustworthiness and security of transactions. Through clearly defined conditions and rules, it ensures that all stages of the transaction are executed correctly and reduces potential fraud;

(2) Automated execution and verification. Smart contracts are automatically executed according to predefined conditions and verified by propositional logic, reducing the possibility of human operation and errors.


Epidemic prevention
and control

Introduction

- students who are still sick with a communicable disease at the time of registration and who are in areas with a high risk of communicable disease transmission will be deferred;
- students who have a history of living in an area with a medium or high risk of communicable disease transmission seven days prior to their return will be deferred; students who are judged to be at risk of communicable disease will be deferred; and students who are experiencing symptoms related to communicable disease will be deferred; students returning from outside of the province will be required to register on August 26;
- students returning from the interior will be required to register on August 27; and students returning to the interior will be required to register on August 28;
- and students returning to the interior will be required to register on August 30 Students returning from outside the province should report to school on August 26th;
- students returning from within the province should report to school on August 27th. According to the guidelines for returning to school

please use the Propositional Implications Design Rule to confirm the time for students to return to school.

Modeling

- p as having a communicable disease, high risk of communicable disease transmission area
- q as having a history of traveling to a medium or high risk of communicable disease transmission area within 7 days
- r as possibly having a communicable disease
- s as having relevant symptoms, t as returning to school in the province, m as returning to school on the 26th
- n as returning to school on the 27th
- o as holding off on returning to school.

For those who will return to school temporarily, we have $p \vee q \vee r \vee s \rightarrow o$, for those who will return on the 26th, we have $\neg o \wedge \neg t \rightarrow m$, and for those who will return on the 27th, we have $\neg o \wedge t \rightarrow n$.

第一章课堂活动

- 请各位同学参考以上案例，寻找生活中能够用命题逻辑或者推理规则表示的案例，并组队制作PPT。各组的PPT会进行评分，优秀的PPT会推荐在课堂上展示并给予奖励。
- PPT需要包括:
  1、问题描述
  2、模型分析（问题所需的本章知识点）
  3、应用效果（问题建模后的公式或推理过程）
  4、总结（知识点应用的总结）
- 各组课下完成PPT，准备时间为一周（10.8之前提交到指定链接）。

Logo

End of Section 1.7