Lesson 15

Digital Logic

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ELSEVIER

Number Systems

- Numbers we can represent using binary representations
  - **Positive numbers**
    - Unsigned binary
  - **Negative numbers**
    - Two's complement
    - Sign/magnitude numbers

- What about **fractions**?

Numbers with Fractions

- Two common notations:
  - **Fixed-point:** binary point fixed
  - **Floating-point:** binary point floats to the right of the most significant 1

Fixed-Point Numbers

- 6.75 using 4 integer bits and 4 fraction bits:

01101100

0110.1100

$2^2 + 2^1 + 2^{-1} + 2^{-2} = 6.75$

- Binary point is implied
- The number of integer and fraction bits must be agreed upon beforehand

Fixed-Point Number Example

- Represent 7.5_{10} using 4 integer bits and 4 fraction bits.

Fixed-Point Number Example

- Represent 7.5_{10} using 4 integer bits and 4 fraction bits.

01111000 (0111.1000)

Signed Fixed-Point Numbers

- **Representations:**
  - Sign/magnitude
  - Two's complement
- **Example 1:** Represent -7.5_{10} using 4 integer and 4 fraction bits
  - **Sign/magnitude:**
  - **Two's complement:**

Signed Fixed-Point Numbers

- **Representations:**
  - Sign/magnitude
  - Two’s complement
- **Example 1:** Represent -7.5$_{10}$ using 4 integer and 4 fraction bits
  - **Sign/magnitude:**

    11111000
  - **Two’s complement:**

```
1. +7.5:          01111000
2. Invert bits:   10000111
3. Add 1 to lsb: +       1
                 ---------
                  10001000
```

Signed Fixed-Point Numbers

- **Example 2:** Two’s complement number addition

```
    0111.1000
+      10.011
-------------
```

Solution:

```
    0111.1000
+   1110.0110
-------------
   10101.1110
```

Verification: 7.5-1.625=5.875

Floating-Point Numbers*

- Binary point floats to the right of the most significant 1
- Similar to decimal scientific notation

- For example, write 273_{10} in scientific notation:

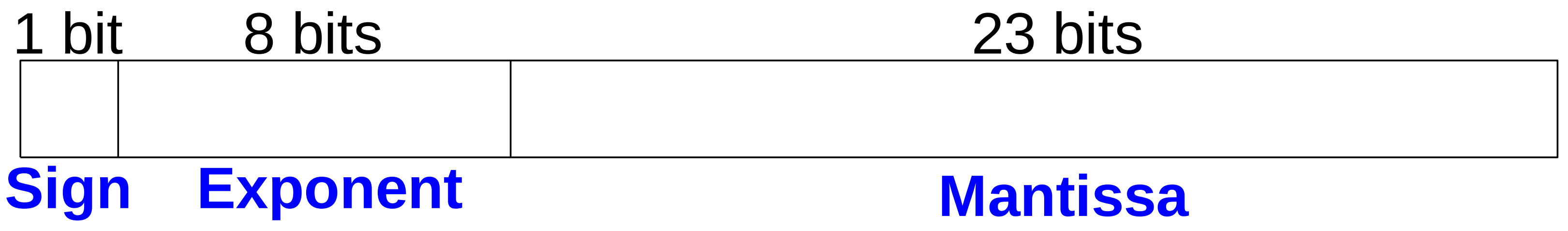
$$273 = 2.73 \times 10^2$$

- In general, a number is written in scientific notation as:

$$\pm \mathbf{M} \times \mathbf{B}^{\mathbf{E}}$$

  - **M** = mantissa
  - **B** = base
  - **E** = exponent
  - In the example, M = 2.73, B = 10, and E = 2

Floating-Point Numbers



- **Example:** represent the value 228_{10} using a 32-bit floating point representation

We only show the version called the **IEEE 754** **floating-point standard**

Floating-Point Representation

1. Convert decimal to binary (**don't reverse steps 1 & 2!**):

   $228_{10} = 11100100_2$

2. Write the number in "binary scientific notation":

   $11100100_2 = 1.11001_2 \times 2^7$

3. Fill in each field of the 32-bit floating point number:
   - The sign bit is positive (0)
   - The 8 bits are the biased exponent
   - The remaining 23 bits are the mantissa

Floating-Point Representation

- *Biased exponent*: bias = 127 (01111111_2)
  - Biased exponent = bias + exponent
  - Exponent of 7 is stored as:

$$127 + 7 = 134 = 10000110_2$$

- The **IEEE 754 32-bit floating-point representation** of 228_{10}

1 bit	8 bits	23 bits
0	10000110	110 0100 0000 0000 0000 0000
Sign	**Biased Exponent**	**Fraction**

in hexadecimal: **0x43640000**

Floating-Point Example

Write -58.25_{10} in floating point (IEEE 754)

Floating-Point Example

Write -58.25_{10} in floating point (IEEE 754)

1. Convert decimal to binary:

   58.25_{10} = **111010.01_2**

2. Write in binary scientific notation:

   1.1101001×2^5

3. Fill in fields:

   Sign bit: 1 (negative)
   8 exponent bits: $(127 + 5) = 132 =$ **10000100_2**
   23 fraction bits: **110 1001 0000 0000 0000 0000**

1 bit	8 bits	23 bits
1	100 0010 0	110 1001 0000 0000 0000 0000
Sign	**Exponent**	**Fraction**

in hexadecimal: **0xC2690000**

Floating-Point: Special Cases

Number	Sign	Exponent	Fraction
0	X	00000000	00000000000000000000000
∞	0	11111111	00000000000000000000000
- ∞	1	11111111	00000000000000000000000
NaN	X	11111111	non-zero

Floating-Point Precision

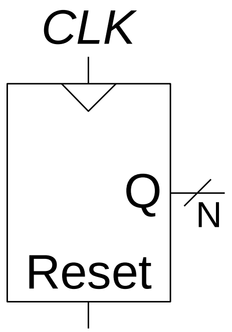
- **Single-Precision:**
  - 32-bit
  - 1 sign bit, 8 exponent bits, 23 fraction bits
  - bias = 127

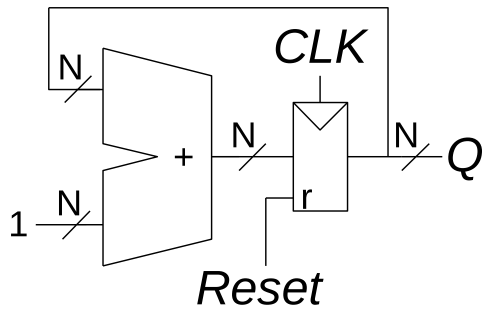
- **Double-Precision:**
  - 64-bit
  - 1 sign bit, 11 exponent bits, 52 fraction bits
  - bias = 1023

Counters

- Increments on each clock edge
- Used to cycle through numbers. For example,
  - 000, 001, 010, 011, 100, 101, 110, 111, 000, 001…
- Example uses:
  - Digital clock displays
  - Program counter: keeps track of current instruction executing

Symbol **Implementation**

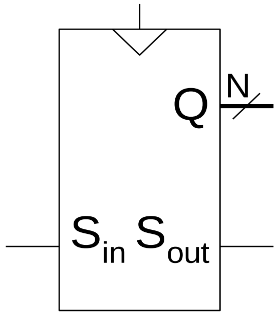




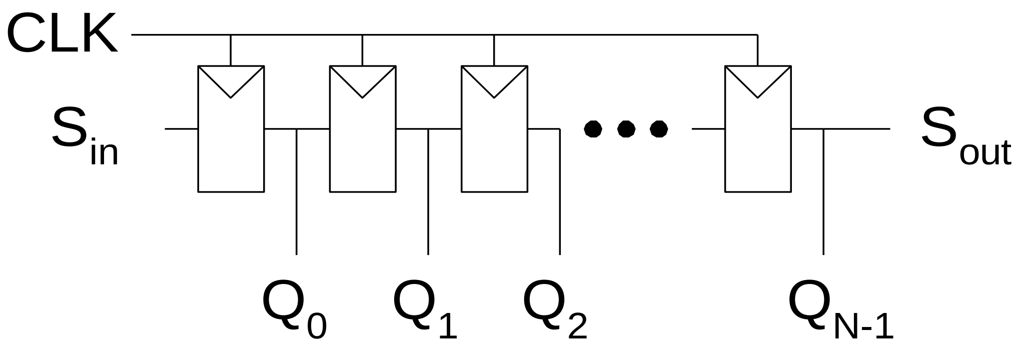
Shift Registers

- Shift a new bit in on each clock edge
- Shift a bit out on each clock edge
- *Serial-to-parallel converter*: converts serial input (S_{in}) to parallel output ($Q_{N-1:0}$)

Symbol

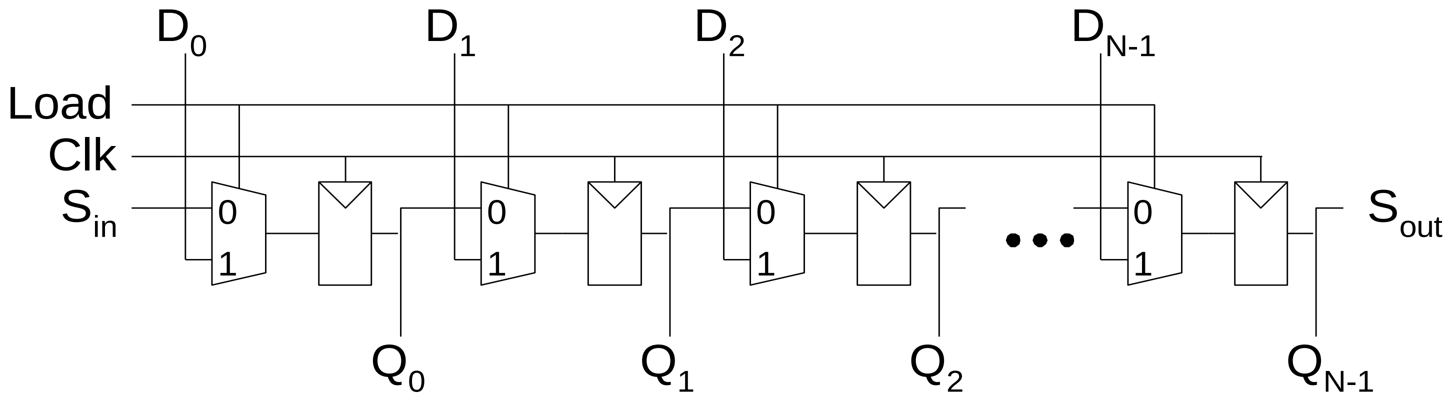


Implementation



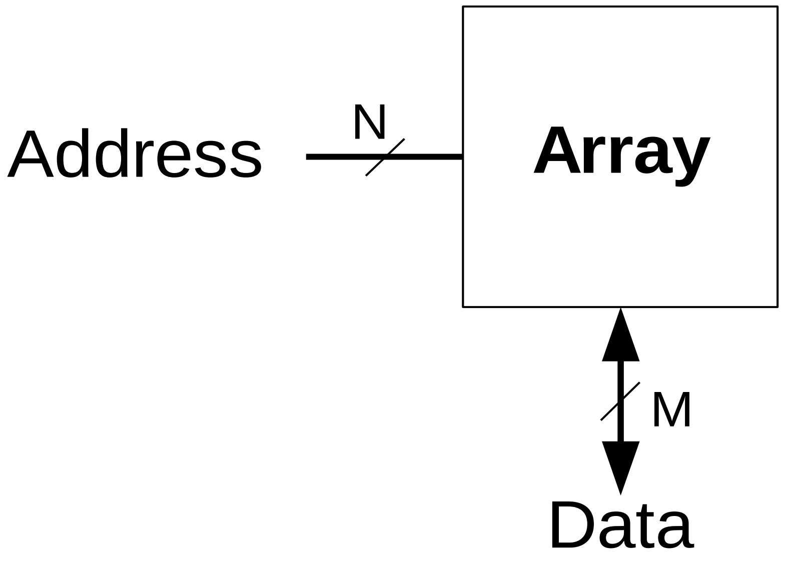
Shift Register with Parallel Load

- When *Load* = 1, acts as a normal *N*-bit register
- When *Load* = 0, acts as a shift register
- Now can act as a *serial-to-parallel converter* (S_{in} to $Q_{N-1:0}$) or a *parallel-to-serial converter* ($D_{N-1:0}$ to S_{out})



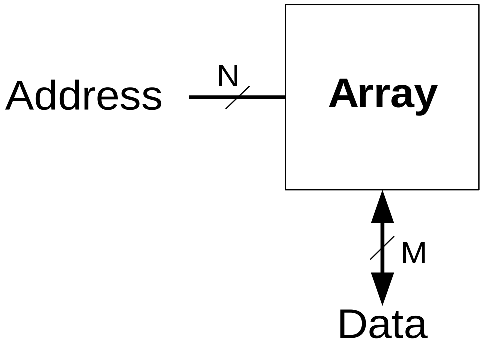
Memory Arrays

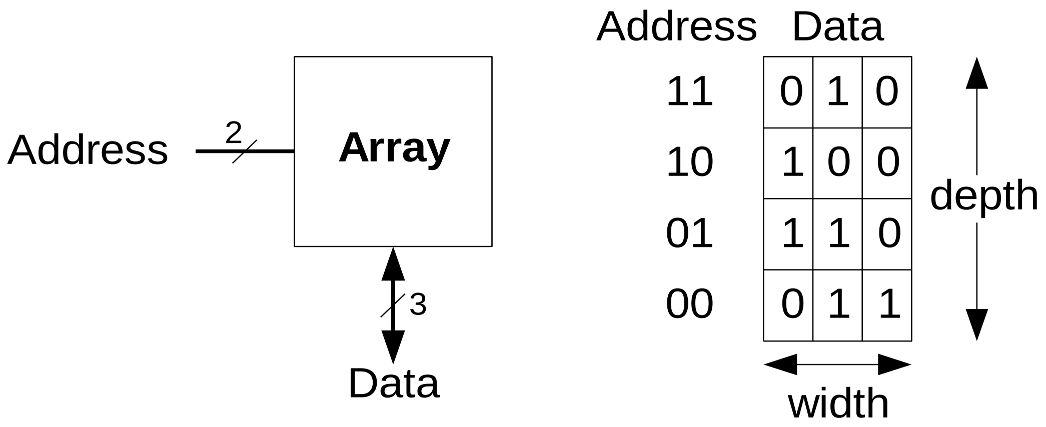
- Efficiently store large amounts of data
- 3 common types:
  - Dynamic random access memory (DRAM)
  - Static random access memory (SRAM)
  - Read only memory (ROM)
- *M*-bit data value read/ written at each unique *N*-bit address



Memory Arrays

- 2-dimensional array of bit cells
- Each bit cell stores one bit
- *N* address bits and *M* data bits:
  - 2^N rows and *M* columns
  - **Depth:** number of rows (number of words)
  - **Width:** number of columns (size of word)
  - **Array size:** depth × width = $2^N \times M$

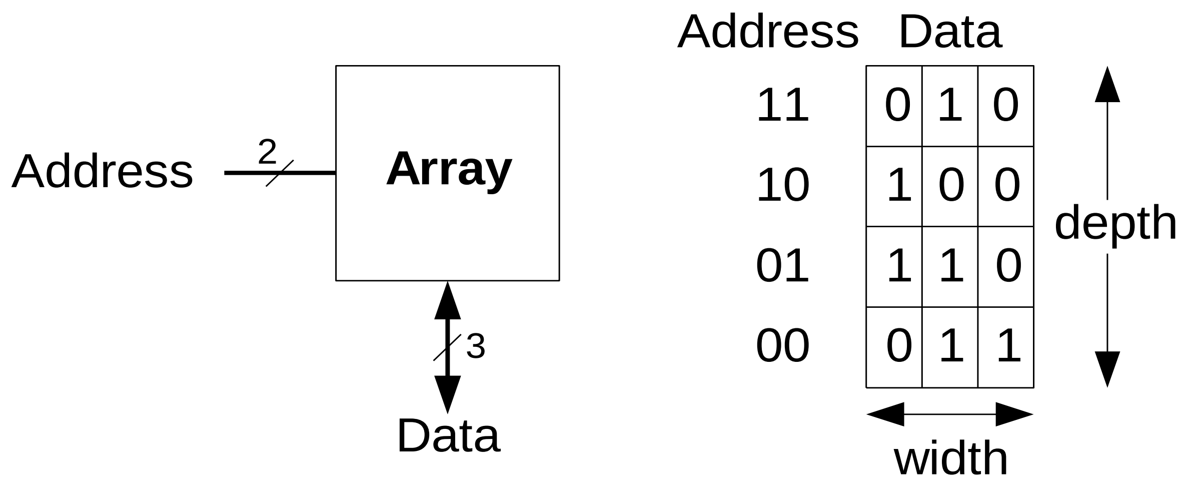




Address	Data
11	010
10	100
01	110
00	011

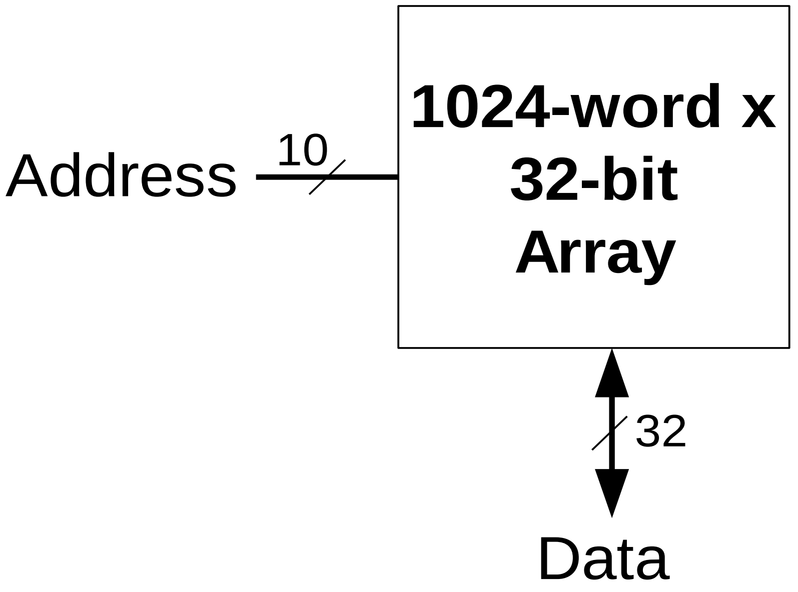
Memory Array Example

- $2^2 \times 3$-bit array
- Number of words: 4
- Word size: 3-bits
- For example, the 3-bit word stored at address 10 is 100

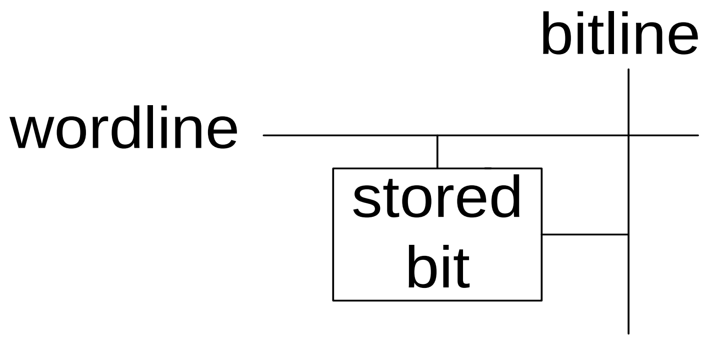


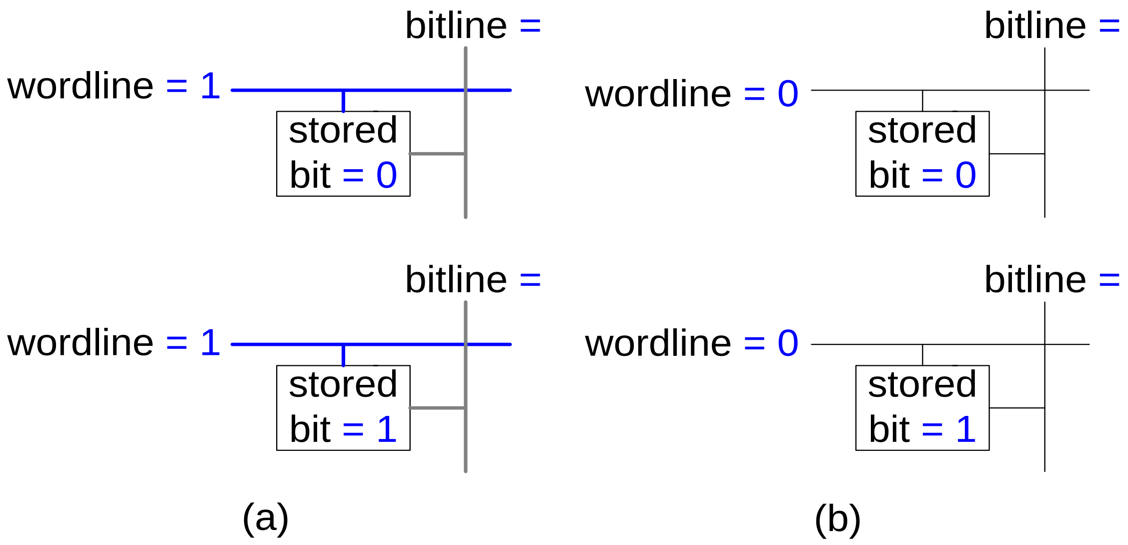
Address	Data
11	0 1 0
10	1 0 0
01	1 1 0
00	0 1 1

Memory Arrays

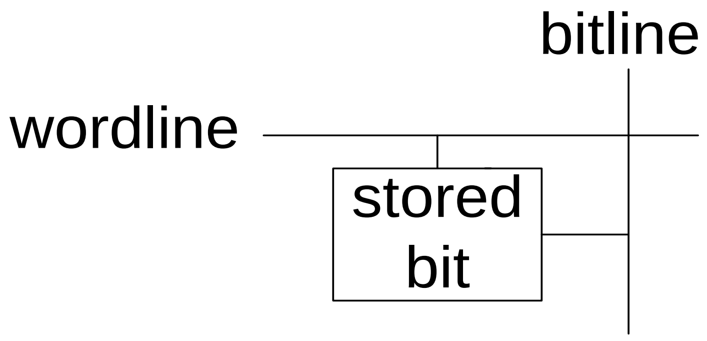


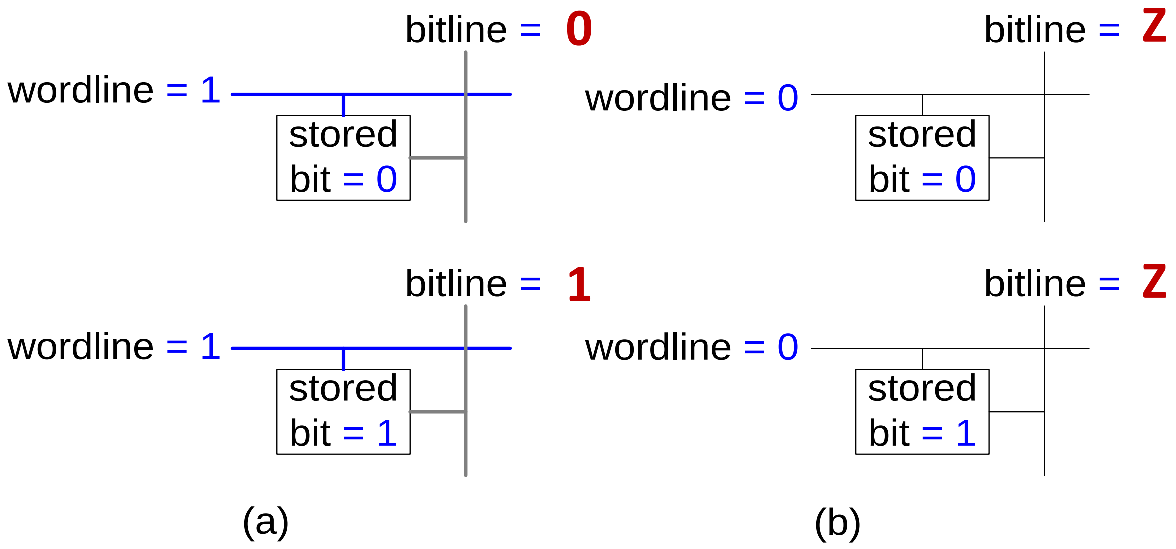
Memory Array Bit Cells





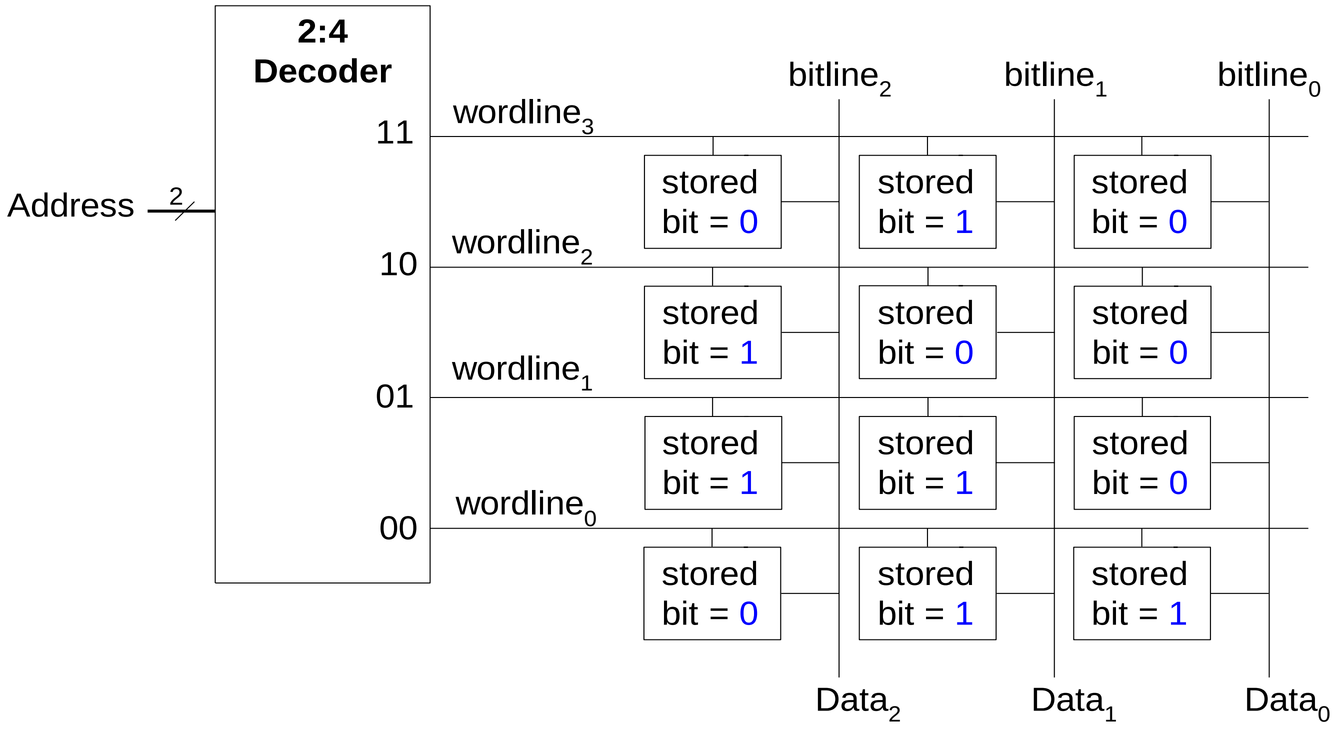
Memory Array Bit Cells





Memory Array

- **Wordline:**
  - like an enable
  - single row in memory array read/written
  - corresponds to unique address
  - only one wordline HIGH at once



Types of Memory

- Random access memory (RAM): **volatile**
- Read only memory (ROM): **nonvolatile**

RAM: Random Access Memory

- **Volatile:** loses its data when power off
- Read and written quickly
- Main memory in your computer is RAM (DRAM)

Historically called *random* access memory because any data word accessed as easily as any other (in contrast to sequential access memories such as a tape recorder)

ROM: Read Only Memory

- **Nonvolatile:** retains data when power off
- Read quickly, but writing is impossible or slow
- Flash memory in cameras, thumb drives, and digital cameras are all ROMs

Historically called *read only* memory because ROMs were written at manufacturing time or by burning fuses. Once ROM was configured, it could not be written again. This is no longer the case for Flash memory and other types of ROMs.

Types of RAM

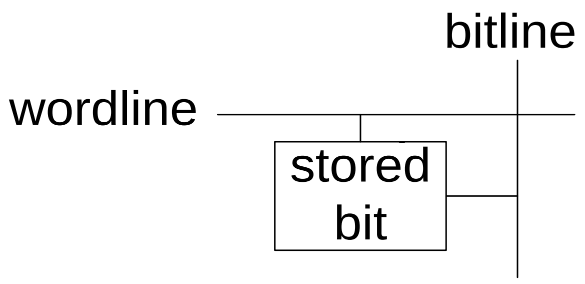
- **DRAM** (Dynamic random access memory)
- **SRAM** (Static random access memory)
- Differ in how they store data:
  - DRAM uses a capacitor
  - SRAM uses cross-coupled inverters

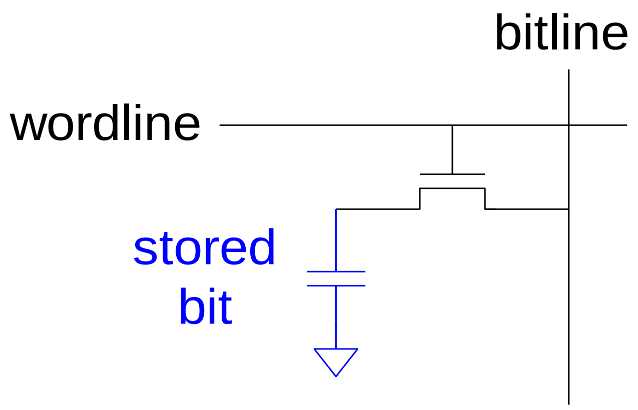
Robert Dennard, 1932 -

- Invented DRAM in 1966 at IBM
- Others were skeptical that the idea would work
- By the mid-1970's DRAM in virtually all computers

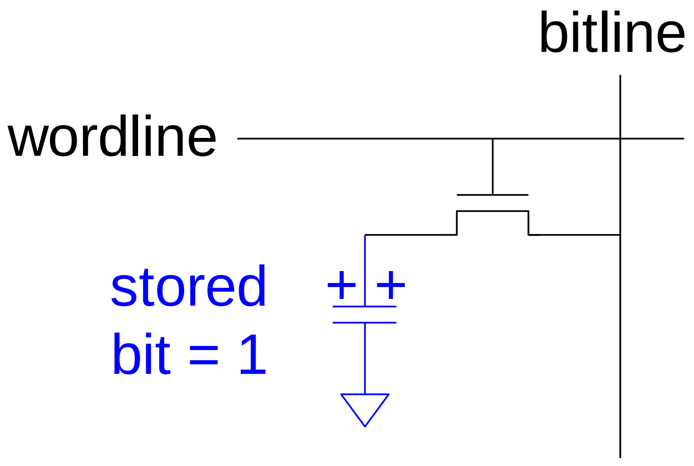
DRAM

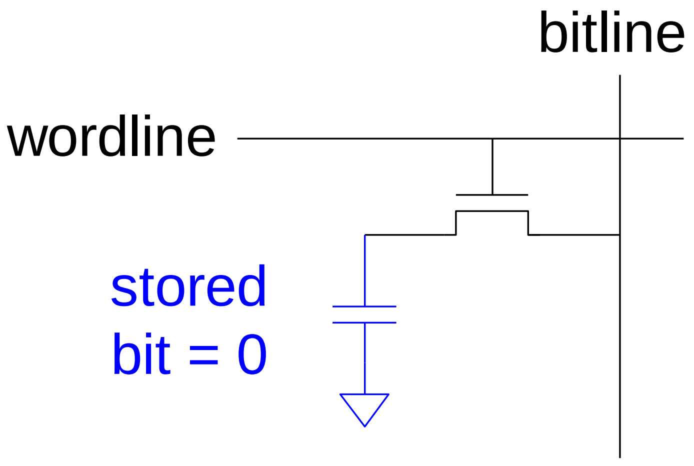
- Data bits stored on capacitor
- *Dynamic* because the value needs to be refreshed (rewritten) periodically and after read:
  - Charge leakage from the capacitor degrades the value
  - Reading destroys the stored value





DRAM


bitline
wordline
stored
bit = 1
+ +


bitline
wordline
stored
bit = 0