Logo

Discrete Mathematics

Dr. Han Huang

South China University of Technology

Logo

Chapter 1. Logic and Proof, Sets, and Function

Rules of Inference

Section 1.5

Contents

Logo

Rules of Inference

Logo

Some Terminologies

- A **theorem** is a statement that can be shown to be true.
- A **proof** is that a sequence of statements that form an argument so that the theorem is true.
- **Axioms** are the underlying assumptions about mathematical structures.
- **Postulates** are hypotheses of the theorem to be proved.

Some Terminologies

- **Fallacies** are some forms of incorrect reasoning, which will help clarity that makes a correct proof.
- A **Lemma** is a simple theorem used in the proof of other theorems.
- A **corollary** is proposition that can be established directly from a theorem that has been proved.
- A **conjecture** is a statement whose truth value is unknown.

Logo

Inference Rules - General Form

- An *Inference Rule* is
  - A pattern establishing that if we know that a set of *premisses* are all true, then we can deduce that a certain *conclusion* statement is true.
- *premiss 1*
  premiss 2 …
  ∴ *conclusion*

"∴" means "therefore"

Logo

A Simple Example

- "If it snows today, then we will go skiing"
- "it is snowing today" *hypothesis*
- If *hypothesis* is true, the conclusion that "we will go skiing" is true.

- $$\boxed{\begin{array}{l} p \\ \underline{p \rightarrow q} \\ \therefore q \end{array}}$$

Rule of *modus ponens*
(a.k.a. *law of detachment*)

Logo

Some Inference Rules for propositional logic

- $$\frac{p}{\therefore\ p \vee q}$$ Rule of Addition
- $$\frac{p \wedge q}{\therefore\ p}$$ Rule of Simplification
- $$\frac{\begin{array}{l} p \\ q \end{array}}{\therefore\ p \wedge q}$$ Rule of Conjunction

Modus Ponens & Tollens

- p
  $\underline{p \rightarrow q}$
  $\therefore q$

Rule of *modus ponens* (a.k.a. *law of detachment*)

"the mode of affirming"

- $\neg q$
  $\underline{p \rightarrow q}$
  $\therefore \neg p$

Rule of *modus tollens*

"the mode of denying"

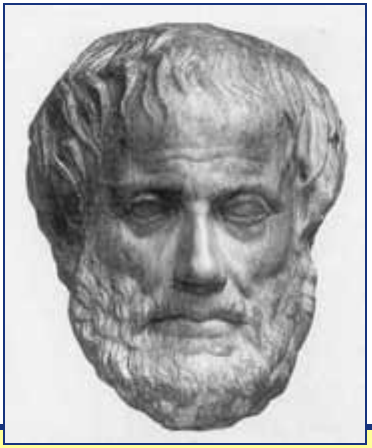
Syllogism Inference Rules

- $$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

  Rule of hypothetical syllogism [a.k.a. transitivity]

- $$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore\ q \end{array}$$

  Rule of disjunctive syllogism



Aristotle
(ca. 384-322 B.C.)

Logo

Rule of Resolution

- $$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore\ q \vee r \end{array}$$

- $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$ *Tautology*

Valid Arguments

- An argument form is called **valid** if whenever all the hypotheses are true, the conclusion is also true.

- ***$(p_1 \land p_2 \land \ldots \land p_n) \rightarrow q$ is True***
- ***$p_1, p_2, \ldots, p_n$ can lead to a correct conclusion.***
- $(p_1 \land p_2 \land \ldots \land p_n) \Rightarrow q$

(1) 附加 $p \Rightarrow (p \vee q)$

(2) 化简 $p \wedge q \Rightarrow p$

(3) 假言推理 $((p \rightarrow q) \wedge p) \Rightarrow q$

(4) 拒取式 $((p \rightarrow q) \wedge \neg q) \Rightarrow \neg p$

(5) 析取三段论 $((p \vee q) \wedge \neg p) \Rightarrow q$

(6) 假言三段论 $((p \rightarrow q) \wedge (q \rightarrow r)) \Rightarrow (p \rightarrow r)$

Logo

(7) 等价三段论

$$((A \leftrightarrow B) \wedge (B \leftrightarrow C)) \Rightarrow (A \leftrightarrow C)$$

(8) 构造性二难

$$(A \rightarrow B) \wedge (C \rightarrow D) \wedge (A \vee C) \Rightarrow (B \vee D)$$

(9) 破坏性二难

$$(A \rightarrow B) \wedge (C \rightarrow D) \wedge (\neg B \vee \neg D) \Rightarrow (\neg A \vee \neg C)$$

Valid Argument, Resolution and Fallacies

Logo

Example 6

- "It didn't rain this afternoon" p
- "it is colder than yesterday" q
- "We will go to park" r
- "We will go to the mall" s
- "We will be home before dusk" t
- *Hypotheses*
- $\neg p \wedge q \quad r \rightarrow p \quad \neg r \rightarrow s \quad s \rightarrow t$

Argument

Step	Reason
1. $\neg p \wedge q$	*Hypothesis*
2. $\neg p$	*Simplification using Step 1*
3. $r \rightarrow p$	*Hypothesis*
4. $\neg r$	*Modus tollens using 2-3*
5. $\neg r \rightarrow s$	*Hypothesis*
6. s	*Modus tollens using 4-5*
7. $s \rightarrow t$	*Hypothesis*
8. t	*Modus tollens using 6-7*

Logo

Example 7

- "You send me an e-mail message" p
- "I will finish the experiments" q
- "I will go to sleep early" r
- "I will wake up feeling refreshed" s

- $p \to q \quad \neg p \to r \quad r \to s$

Argument

- Step
- 1. $p \to q$
- 2. $\neg q \to \neg p$
- 3. $\neg p \to r$
- 4. $\neg q \to r$
- 5. $r \to s$
- 6. $\neg q \to s$

- Reason
- *Hypothesis*
- *Contrapositive of Step 1*
- *Hypothesis*
- *Hypothetical syllogim using 2-3*
- *Hypotheses*
- *Hypothetical syllogim using 4-5*

Resolution

- Computer programs have been developed to automate the task of reasoning and proving theorem.
- The *resolution* is used to for the programs as a rule of inference.
- The is *resolution* based on the tautology
- $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$

Resolution

- $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$
- $(q \vee r)$ is called the *resolvent*.
- Let $r = q$ $\qquad (p \vee q) \wedge (\neg p \vee q) \rightarrow q$
- Let $r = \mathrm{F}$ $\qquad (p \vee q) \wedge (\neg p) \rightarrow q$
- which is the tautology on which the rule of disjunctive syllogism is based.

Example 8

- **"It is snowing"** p
- **"XiaoJie is skiing"** q
- **"XiaoBai is playing basketball"** r
- ***Hypotheses*** $\neg p \vee q$ $p \vee r$
- $((\neg p \vee q) \wedge (p \vee r)) \rightarrow (r \vee q)$
- $r \vee q$ ***is the true conclusion implied by Hypotheses.***

Example 9

- $(p \wedge q) \vee r$
- $p \vee r$
- $q \vee r$

- $r \rightarrow s$
- $\neg r \vee s$

- $(p \vee r) \wedge (\neg r \vee s) \rightarrow p \vee s$ is tautology

Logo

Fallacies

- p: 你做过本书的每一道练习题　q: 你学习过离散数学
- 1、肯定结论的谬误（***fallacy*** of affirming the conclusion）
- $((p \to q) \wedge q) \to p$　表示“如果你做过本书的每一道练习题，则你就学习过离散数学。你学习过离散数学。因此，你做过本书的每一道练习题。”这种论证是无效的，称为肯定结论的谬误。
- 2、否定假设的谬误（***fallacy*** of denying the hypothesis）
- $((p \to q) \wedge \neg p) \to \neg q$　表示“如果你做过本书的每一道练习题，则你就学习过离散数学。你没做过本书的每一道习题。因此，你没学习过离散数学。”该论证为否定假设的谬误，论证无效。

Formal Proof Example

- Suppose we have the following premises:
  "It is not sunny and it is cold."
  "We will go to park only if it is sunny."
  "If we do not park, then we will go to mall."
  "If we go to mall, then we will be home early."
- Given these premises, prove the theorem **"We will be home early"** using inference rules.

Formal Proof Example

It is not sunny and it is cold.

We will go to park only if it is sunny.

If we do not park, then we will go to mall.

If we go to mall, then we will be home early

$\therefore$ **We will be home early**

sunny = **"It is sunny"**; *cold* = **"It is cold"**;

park = **"We will go to park "**;

mall = **"We will go to mall"**;

early = **"We will be home early"**.

Proof Example *cont.*

- The premises can be written as:
  (1) $\neg sunny \wedge cold$ (2) $park \rightarrow sunny$
  (3) $\neg park \rightarrow mall$ (4) $mall \rightarrow early$

- **It is not sunny and it is cold.**
- **We will go to park only if it is sunny.**
- **If we do not park, then we will go to mall.**
- **If we go to mall, then we will be home early**

Proof Example *cont*.

Step	Proved by
1. $\neg$*sunny* $\wedge$ *cold*	Premise #1.
2. $\neg$*sunny*	Simplification of 1.
3. *park*$\rightarrow$*sunny*	Premise #2.
4. $\neg$ *park*	Modus Tollens on 2,3.
5. $\neg$ *park* $\rightarrow$*mall*	Premise #3.
6. *mall*	Modus Ponens on 4,5.
7. *mall* $\rightarrow$*early*	Premise #4.
8. *early*	Modus Ponens on 6,7.

Example in Chinese

写出对应下面推理的证明。

(1) 如果今天是星期一，则要进行数据结构或离散数学考试。

如果数据结构老师有会，则不考数据结构，

今天是星期一，数据结构老师有会，

所以进行离散数学考试。

解：p：今天是星期一，q：进行数据结构考试，

r：进行离散数学考试，s：数据结构老师有会。

前提：$p \to (q \vee r), s \to \neg q, p, s$　结论：r

Example in Chinese

前提：$p \to (q \vee r), s \to \neg q, p, s$　结论：r

证明：

①	$p \to (q \vee r)$	前提引入
②	p	前提引入
③	$q \vee r$	①②假言推理
④	$s \to \neg q$	前提引入
⑤	s	前提引入
⑥	$\neg q$	④⑤假言推理
⑦	r	③⑥析取三段论

Example in Chinese

(2) 如果6是偶数，则2不能整除7；

5不是素数，或者2整除7；

5是素数。

因此，6是奇数。

解： p：6是偶数，q：2整除7，r：5是素数。

前提：$p \rightarrow \neg q, \neg r \vee q, r$

结论：$\neg p$

Example in Chinese

前提：$p \to \neg q, \neg r \vee q, r$　结论：$\neg p$

证明：① $\neg r \vee q$　前提引入

② $r \to q$　①置换规则

③ r　前提引入

④ q　②③假言推理

⑤ $p \to \neg q$　前提引入

⑥ $\neg p$　④⑤拒取式

Rules of Inference for Quantified Statement

Logo

All Inference Rules for Quantifiers

- $\dfrac{\forall x\, P(x)}{\therefore P(o)}$ **Universal instantiation** (substitute any object *o*)
- $\dfrac{P(g)}{\therefore \forall x\, P(x)}$ (for *g* an *arbitrary* element) **Universal generalization**
- $\dfrac{\exists x\, P(x)}{\therefore P(c)}$ **Existential instantiation** (substitute a *new constant c*)
- $\dfrac{P(o)}{\therefore \exists x\, P(x)}$ (for any object *o*) **Existential generalization**

Logo

Example 12

- $D(x)$ denotes "x is in this discrete mathematic class"
- $C(x)$ denotes "x has taken a course in computer science"
- The premises are $\forall x\,(D(x) \rightarrow C(x))$ and $D(\text{XiaoGang})$
- $\forall x\,(D(x) \rightarrow C(x))$ *Premise*
- $D(\text{XiaoGang}) \rightarrow C(\text{XiaoGang})$ *Universal instantiation*
- $D(\text{XiaoGang})$ *Premise*
- $C(\text{XiaoGang})$ *Modus Ponens* from #2 and #3

Example 13

- $C(x)$ denotes "x is in this class"
- $B(x)$ denotes "x has read the book"
- $P(x)$ denotes "x passed the first exam"
- *Premise* $\exists \mathbf{x}\, (C(x) \wedge \neg B(x))$
- $\forall \mathbf{x}\, (C(x) \rightarrow P(x))$

- *Conclusion* $\exists \mathbf{x}\, (P(x) \wedge \neg B(x))$?

- **1.** $\exists\, \mathbf{x}\ (C(x) \land \neg B(x))$ *Premise*
- 2. $C(a) \land \neg B(a)$ *Existential instantiation from 1*
- **3.** $C(a)$ *Simplification from 2*
- **4.** $\forall\, \mathbf{x}\ (C(x) \rightarrow P(x))$ *Premise*
- **5.** $C(a) \rightarrow P(a)$ *Universal instantiation from 4*
- **6.** $P(a)$ *Modus pones from 3 and 5*
- **7.** $\neg B(a)$ *Simplification from 2*
- **8.** $P(a) \land \neg B(a)$ *Conjunction from 6 and 7*
- **9.** $\exists\, \mathbf{x}\ (P(x) \land \neg B(x))$ *Existential generalization from 8*

Method of Proving Theorems

Other proof methods for implications

For proving implications $p \rightarrow q$, we have:

- *Direct* proof: Assume p is true, and prove q.
- *Indirect* proof: Assume $\neg q$, and prove $\neg p$.
- (*Vacuous* proof: Prove $\neg p$ by itself.
- *Trivial* proof: Prove q by itself.)
- Proof by cases:
  Show $(a \vee b) \rightarrow q$, and $(a \rightarrow q)$ and $(b \rightarrow q)$.

Direct Proof Example

- **Definition:** An integer n is called *odd* iff $n=2k+1$ for some integer k; n is *even* iff $n=2k$ for some k.
- **Theorem:** (For all numbers n) If n is an odd integer, then n^2 is an odd integer.
- **Proof:** If n is odd, then $n = 2k+1$ for some integer k. Thus, $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$. Therefore n^2 is odd.

Indirect Proof Example (calculus)

- **Theorem:** (For all integers n)
  If $3n+2$ is odd, then n is odd.
- **Proof:** Suppose n is even. Then $n=2k$ for some integer k. Then $3n+2 = 3(2k)+2 = 6k+2 = 2(3k+1)$, which is even (not odd).

Vacuous and Trivial Proof

- p is false
- $p \rightarrow q$ is true according to truth value table
- **F $\rightarrow$ T or F $\rightarrow$ F** is true.
- "p is false" is called a vacuous proof.
- q is true
- $p \rightarrow q$ is true according to truth value table
- **F $\rightarrow$ T or T $\rightarrow$ T** is true.
- "q is true" is called a trivial proof.

Proof Strategy

- Example 18
- Prove that the sum of two rational numbers is rational.

- Example 19
- Prove that if n is an integer and n^2 is odd, then n is odd.

Proofs by Contradiction

- Example 20
- Show that at least four of any 22 days must fall on the same day of the week
- Example 21
- Prove that sqrt(2) is irrational by giving a proof by contradiction
- Example 22
- Given a proof by contradiction of the theorem “If 3n+2 is odd, then n is odd.”

Proof by Cases

- *To prove the implication of the form*
- $(p_1 \lor p_2 \lor \ldots \lor p_n) \to q$
- *The tautology*
- $[(p_1 \lor p_2 \lor \ldots \lor p_n) \to q]$
- $\leftrightarrow [(p_1 \to q) \land (p_2 \to q) \land \ldots \land (p_n \to q)]$
- $p_1, p_2, \ldots, p_n$ can be proved by proving each of the n implications $p_i \to q$, $i=1,\ldots,n$
- *Example 23*

Logo

Proof by Equivalence

- $(p \leftrightarrow q) \leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow p)]$
- Example 24
- The integer n is odd if and only if n^2 is odd.

- $(p \leftrightarrow q) \leftrightarrow [(p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3) \wedge \ldots \wedge (p_n \rightarrow p_1)]$
- Example 25 Show that these statements are equivalent p_1: n is an even integer.
- p_2: n-1 is an odd integer.
- p_3: n^2 is an even integer.

Logo

Others about Theorem Proof

Theorem and Quantifiers

- **Existence Proofs**
- Example 26 A Constructive Existence Proof
- Example 27 A Non-constructive Existence Proof
- **Uniqueness Proofs**
- Example 28 $p + q = 0$
- **Counterexample**
- **Example 29**
- **Example 30**

Complete the following proof

(1) Premises： $p \to r, q \to s, p \vee q$

Conclusion： $r \vee s$

解：

①	p -> r	p
②	¬p v r	①
③	p v q	p
④	r v q	②③
⑤	q -> s	p
⑥	¬q v s	⑤
⑦	r v s	④⑥

Complete the following proof

(2) Premises： $p \vee q, p \to \neg r, s \to t, \neg s \to r, \neg t$

Conclusion： q

解：			
	①	¬s -> r	p
	②	¬r -> s	①
	③	P -> ¬r	p
	④	p -> s	②③
	⑤	s -> t	p
	⑥	p -> t	④⑤
	⑦	¬t	p
	⑧	¬p	⑥⑦
	⑨	p v q	p
	⑩	q	⑧⑨

Exercise in Chinese

(3) 如果乙不参加篮球赛，那么甲就不参加；

如果乙参加篮球赛，那么甲和丙就参加。

因此，如果甲参加篮球赛，那么丙就参加。

解：p：乙参加篮球赛，q：甲参加篮球赛，

r：丙参加篮球赛。

前提：$\neg p \rightarrow \neg q, p \rightarrow (q \wedge r)$

结论：$q \rightarrow r$

Exercise in Chinese

前提：$\neg p \to \neg q,\ p \to (q \wedge r)$ 结论：$q \to r$

证明：① $\neg p \to \neg q$ 前提引入

② $q \to p$ ①置换规则

③ $p \to (q \wedge r)$ 前提引入

④ $q \to (q \wedge r)$ ②③假言三段论

⑤ $q \to r$ ④置换规则

$$\big(q \to (q \wedge r) \Leftrightarrow \neg q \vee (q \wedge r)$$
$$\Leftrightarrow (\neg q \vee q) \wedge (\neg q \vee r)$$
$$\Leftrightarrow \neg q \vee r \Leftrightarrow q \to r\ \big)$$

Logo

Exercise in Chinese

- 如果他是计算机系本科生或者是计算机系研究生，那么他一定学过DELPHI语言而且学过C++语言。只要他学过DELPHI语言或者C++语言，那么他就会编程序。因此如果他是计算机系本科生，那么他就会编程序。请用命题逻辑推理方法，证明该推理的有效结论。

Exercise in Chinese

令p：他是计算机系本科生　q：他是计算机系研究生

r：他学过DELPHI语言　　s:他学过C++语言　　t:他会编程序

前提：$(p \vee q) \rightarrow (r \wedge s), (r \vee s) \rightarrow t$

结论：$p \rightarrow t$

证：①p　　P(附加前提)

②$p \vee q$　　①

③$(p \vee q) \rightarrow (r \wedge s)$　　P(前提引入)

④$r \wedge s$　　②③

⑤r　　④

⑥$r \vee s$　　⑤

⑦$(r \vee s) \rightarrow t$　　P(前提引入)

⑧t　　⑤⑥

Exercise

L o g o

Exercises

- 1. Premises: (a) It is sunny or rainy. (b)If it is sunny, I will go to movie. (c) If I go to movie, I won’t read the book. Conclusion: If I read the book, it is rainy. Please give the process of reasoning and explain why.

Exercises

- 1. Premises: (a) It is sunny or rainy. (b)If it is sunny, I will go to movie. (c) If I go to movie, I won't read the book. Conclusion: If I read the book, it is rainy. Please give the process of reasoning and explain why.
- Proof**:**
- M: It is sunny. Q: It is rainy. S: I will go to movie. R: I will read the book.
- The premises are: $M \vee Q$, $M \rightarrow S$, $S \rightarrow \neg R$
- The conclusion: $R \rightarrow Q$

Exercises

- 1.
- The premises are: $M \vee Q$, $M \rightarrow S$, $S \rightarrow \neg R$
- The conclusion: $R \rightarrow Q$

1. R	Premises
2. $S \rightarrow \neg R$	Premises
3. $\neg S$	T 1 and 2
4. $M \rightarrow S$	Premises
5. $\neg M$	T 3 and 4
6. $M \vee Q$	Premises
7. Q	T 5 and 6
8. $R \rightarrow Q$	CP

Logo

Exercises

- 2. Given the premises $\neg A \vee B$, $\neg C \rightarrow \neg B$, $C \rightarrow D$, how to get the conclusion $A \rightarrow D$?

Exercises

- 2. Given the premises $\neg A \vee B$, $\neg C \rightarrow \neg B$, $C \rightarrow D$, how to get the conclusion $A \rightarrow D$?
- **Proof.**
- (1) A Premise
- (2) $\neg A \vee B$ Premise
- (3) B (1)(2)
- (4) $\neg C \rightarrow \neg B$ Premise
- (5) $B \rightarrow C$ (4)
- (6) C (3)(5)
- (7) $C \rightarrow D$ Premise
- (8) D (6)(7)
- (9) $A \rightarrow D$ (1)(8)

Exercises

- 3. For the following premises, **what relevant conclusion or conclusions can be drawn?**
- “If I take the day off, it either rains or snows.”
- “I took Tuesday off or I took Thursday off.”
- “It was sunny on Tuesday.”
- “It did not snow on Thursday.”

Exercises

- 3. For the following premises, **what relevant conclusion or conclusions can be drawn?**
- “If I take the day off, it either rains or snows.”
- “I took Tuesday off or I took Thursday off.”
- “It was sunny on Tuesday.”
- “It did not snow on Thursday.”

- Valid conclusions are “I did not take Tuesday off,” “I took Thursday off,” “It rained on Thursday.”

Exercises

- 4. If a person is a university student, he is either a liberal-arts student or a science student. Some university student is an outstanding student. John is not a liberal-arts student, but he is an outstanding student. John is a university student. **Use rules of inference to show that John is a science student.**

Exercises

- 4. If a person is a university student, he is either a liberal-arts student or a science student. Some university student is an outstanding student. John is not a liberal-arts student, but he is an outstanding student. John is a university student. **Use rules of inference to show that John is a science student.**
- P(x): x is a university student.
- Q(x): x is a liberal-art student.
- S(x): x is a science student.
- T(x): x is an outstanding student.
- c: John

Exercises

- 4. $(\forall x)P(x) \rightarrow (Q(x) \vee S(x))$, $(\exists x)P(x) \wedge T(x)$, $\neg Q(c) \wedge T(c)$, $P(c)$
- Inference:

(1) $(\forall x)P(x) \rightarrow (Q(x) \vee S(x))$ Premises

(2) $P(c) \rightarrow (Q(c) \vee S(c))$ US ①

(3) $P(c)$ Premises

(4) $Q(c) \vee S(c)$ ③④

(5) $\neg Q(c) \wedge T(c)$ Premises

(6) $\neg Q(c)$ ⑤

(7) $S(c)$ ④⑥

Logo

Exercises

- 5 Given the premises, construct the conclusion.
- Premises: $((\exists x)(F(x) \land S(x)) \to (\forall y)(M(y) \to W(y)),\quad (\exists y)(M(y) \land \neg W(y))$
- Conclusion： $(\forall x)(F(x) \to \neg S(x))$

Exercises

- 5. Premises: $\big((\exists x)\big(F(x) \wedge S(x)\big) \rightarrow (\forall y)\big(M(y) \rightarrow W(y)\big)\big), (\exists y)\big(M(y) \wedge \neg W(y)\big)$
- Conclusion:$(\forall x)(F(x) \rightarrow \neg S(x))$
- (1) $(\exists y)\big(M(y) \wedge \neg W(y)\big)$ P
- (2) $M(e) \wedge \neg W(e)$ ES(1)
- (3) $\neg(\neg M(e) \vee W(e))$ T(2)E
- (4) $\neg(M(e) \rightarrow W(e))$ T(3)E
- (5) $(\exists y)\neg\big(M(y) \rightarrow W(y)\big)$ EG(4)
- (6) $\neg(\forall y)\big(M(y) \rightarrow W(y)\big)$ T(5)E
- (7) $(\exists x)\big(F(x) \wedge S(x)\big) \rightarrow (\forall_y)\big(M(y) \rightarrow W(y)\big)$ P
- (8) $\neg(\exists x)\big(F(x) \wedge S(x)\big)$ T(6)(7)I
- (9)$(\forall x)\neg(F(x) \wedge S(x))$ T(8)E
- (10) $(\forall x)(F(x) \rightarrow \neg S(x))$ T(9)E

Exercises

- 6. Given the reasoning of the following predicate.

$$(\forall x)(C(x) \to W(x) \wedge R(x)) \wedge (\exists x)(C(x) \wedge Q(x)) \Rightarrow (\exists x)(Q(x) \wedge R(x))$$

Exercises

❖ 6. Given the reasoning of the following predicate.

$$(\forall x)(C(x) \to W(x) \land R(x)) \land (\exists x)(C(x) \land Q(x)) \Rightarrow (\exists x)(Q(x) \land R(x))$$

①	$(\exists x)(C(x) \land Q(X))$	P
②	$C(a) \land Q(a)$	①
③	$C(a)$	②
④	$(\forall x)(C(x) \to W(x) \land R(x))$	P
⑤	$C(a) \to W(a) \land R(a)$	④
⑥	$W(a) \land R(a)$	③⑤
⑦	$R(a)$	⑥
⑧	$Q(a)$	②
⑨	$Q(a) \land R(a)$	⑦⑧
⑩	$(\exists x)(Q(x) \land R(x))$	⑨

Logo

Exercises

- 7 For the following premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.
- "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday."
- O(x): I take the day x off. R(x): it rains on day x. S(x): it snows on day x.
- Domain: {1,2,3,4,5,6,7}
- Premise: $\forall x(O(x) \rightarrow R(x) \vee S(x))$, $O(2) \vee O(4)$, $\neg(R(2) \vee S(2))$, $\neg S(4)$

Exercises

- 7 Premise: $\forall x(O(x) \rightarrow R(x) \vee S(x))$, $O(2) \vee O(4)$, $\neg(R(2) \vee S(2))$, $\neg S(4)$
- 1 $\forall x(O(x) \rightarrow R(x) \vee S(x))$ $P(Premise)$
- 2 $O(2) \rightarrow R(2) \vee S(2)$ UI(*Universal instantiation*)
- 3 $O(4) \rightarrow R(4) \vee S(4)$ UI
- 4 $\neg\left(R(2) \vee S(2)\right)$ P
- 5 $\neg O(2)$ 2,4
- 6 $O(2) \vee O(4)$ P
- 7 $O(4)$ 5,6
- 8 $R(4) \vee S(4)$ 3,7
- 9 $\neg S(4)$ P
- 10 $R(4)$ 8,9
- Thus: $\neg O(2)$, $O(4)$, $R(4)$

Exercises

- 8. Use inference to obtain conclusion from the premises.
- All the people who like walking do not like driving.
- Every person likes driving or riding. Some people don’t like riding.
- Therefore, some people don’t like walking.

Exercises

- 8. **Symbolize the propositions.**
- W(x): x likes walking;
- D(x): x likes driving;
- R(x): x likes riding.
- Premises: $\forall x\,(W(x) \rightarrow \neg D(x))$, $\forall x\,(D(x) \vee R(x))$, $\exists x(\neg R(x))$
- Conclusion: $\exists x(\neg W(x))$

Logo

Exercises

- 8. **Premises: $\forall x\,(W(x) \rightarrow \neg D(x))$, $\forall x\,(D(x) \vee R(x))$, $\exists x(\neg R(x))$**

  Conclusion: $\exists x(\neg W(x))$
- (1) $\exists x(\neg R(x))$ (Premise)
- (2) $\neg R(c)$
- (3) $\forall x\,(D(x) \vee R(x))$ (Premise)
- (4) $D(c) \vee R(c)$
- (5) $D(c)$ (2)(4)
- (6) $\forall x\,(W(x) \rightarrow \neg D(x))$ (Premise)
- (7) $W(c) \rightarrow \neg D(c)$ (6)
- (8) $\neg W(c)$ (5)(7)
- (9) $\exists x(\neg W(x))$ (8)

Exercises

- 9. Use inference to obtain conclusion from the premises.
- If a student majors in CS or SE, then he learns Java and C. If he learns Java or C, then he can code. Therefore, if a student majors in SE, then he can code.

Logo

Exercises

- 9.
- p： he majors in CS q： he majors in SE
  r： he learns Java s:he learns C t:he can code
- **Premise： $(p\vee q)\to(r\wedge s),(r\vee s)\to t$**
- **Conclusion： $p\to t$**

Proof：

①	p	P(附加前提)
②	$p\vee q$	①
③	$(p\vee q)\to(r\wedge s)$	P(前提引入)
④	$r\wedge s$	②③
⑤	r	④
⑥	$r\vee s$	⑤
⑦	$(r\vee s)\to t$	P(前提引入)
⑧	t	⑤⑥

Exercises

- 10. Public security personnel review a theft case, the known facts are as follows:
- A or B stole the recorder; If A stole the recorder, the time of committing the crime cannot occur before midnight; If the testimony of B is correct, the lights in the house will not be extinguished at midnight; If the testimony of B is incorrect, the time of committing the crime occurs before midnight; The lights in the house were gone at midnight.
- Who has stolen the recorder? Symbolize the proposition and prove who stole the recorder.

Exercises

10. p: A steals the recorder;

- q: B steals the recorder;
- r: the time of the crime occurred before midnight;
- s: B's testimony is correct;
- t: the light in the house is not extinguished at midnight;
- Precondition: $p \vee q, p \rightarrow \neg r, s \rightarrow t, \neg s \rightarrow r, \neg t$

Logo

Exercises

10 Precondition: $p \vee q, p \rightarrow \neg r, s \rightarrow t, \neg s \rightarrow r, \neg t$

- ① $\neg t$ Precondition
- ②$s \rightarrow t$ Precondition
- ③$\neg s$ ①②Rejection
- ④$\neg s \rightarrow r$ Precondition
- ⑤r ③④Hypothetical reasoning
- ⑥$p \rightarrow \neg r$ Precondition
- ⑦$\neg p$ ⑤⑥Rejection
- ⑧$p \vee q$ Precondition
- ⑨q ⑦⑧Disjunctive syllogism
- So it shows that B has stolen the recorder.!

Exercises

- 11. If Wang Xiaohong studies hard, she must get good score. If Wang Xiaohong is playful or does not finish the homework on time, she will not be able to get good grade. Therefore, if Wang Xiaohong studies hard, she can finish her homework on time.

Exercises

- 11.
- ①The four simple propositions are symbolized as p, q, r, s;
- ②Symbolize the proposition, symbolize the premises and conclusion of the proposition;
- ③ Construct inferential proofs of propositions.

Exercises

- 11. (1) p: Wang Xiaohong studies hard
  q: Wang Xiaohong get good grade
  r: Wang Xiaohong is playful
  s: Finish the job on time
  (2) Precondition: $p \rightarrow q, (r \vee \neg s) \rightarrow \neg q$
  Conclusion : $p \rightarrow s$
  (3) 证明：使用附加前提条件引入证明方法
  ① p Attach precondition
  ② $p \rightarrow q$ precondition
  ③ q ①② Hypothetical reasoning
  ④ $(r \vee \neg s) \rightarrow \neg q$ precondition
  ⑤ $\neg(r \vee \neg s)$ ③④ rejection
  ⑥ $\neg r \wedge s$ ⑤ Replacement
  ⑦ s ⑥ Simplification

Exercises

- 12. Every rational number is a real number. All of the irrational number are also real numbers. Imaginary number（虚数）is not a real number. Thus, an imaginary number is neither a rational number nor an irrational number.

Exercises

- 12.**Q(x): x is a rational number. R(x): x is a real number. N(x): x is a irrational number. C(x): x is a imaginary number.**

Premise: $(\forall x)(Q(x) \to R(x))$, $(\forall z)(N(x) \to R(x))$, $(\forall x)(C(x) \to \neg R(x))$
Conclusion: $(\forall x)(C(x) \to \neg Q(x) \land \neg N(x))$

(1)$(\forall x)\big(Q(x) \to R(x)\big)$	P
(2) $Q(x) \to R(x)$	(1)
(3)$(\forall z)\big(N(x) \to R(x)\big)$	P
(4) $N(x) \to R(x)$	(3)
(5)$(\forall x)(C(x) \to \neg R(x))$	P
(6)$C(x) \to \neg R(x)$	(5)
(7)$R(x) \to \neg C(x)$	(6)
(8)$Q(x) \to \neg C(x)$	(2)(7)
(9) $N(x) \to \neg C(x)$	(4)(7)
(10) $\big(Q(x) \to \neg C(x)\big) \land \big(N(x) \to \neg C(x)\big)$	(8)(9)
(11)$C(x) \to \neg Q(x) \land \neg N(x)$	(10)
(12) $(\forall x)\big(C(x) \to \neg Q(x) \land \neg N(x)\big)$	(11)

Exercises

- 13. Machine Reading Comprehension Task (机器阅读理解任务) refers to that the machine automatically answers the user's questions based on the given context. Based on the following context and question, please use rules of inference to help the machine get the answer.
- Context: Li will go to the library tomorrow morning only if Chen goes swimming in the morning. Chen will not go swimming unless Huang agrees to go swimming too. However, Huang has decided not to go swimming.
- Question: Will Li go to the library tomorrow?

Exercises

- 13. A: Li will go to the library tomorrow

  B: Chen goes swimming in the morning

  C: Huang agrees to go swimming

Premises are $A \to B$ $\neg C \to \neg B$ $\neg C$

(1) $\neg C$ Premise

(2) $\neg C \to \neg B$ Premise

(3) $\neg B$ (1)(2)

(4) $A \to B$ Premise

(5) $\neg A$ (3)(4)

The conclusion is Li will not go to the library tomorrow.

Exercises

14.

Given that $\forall x(P(x) \vee Q(x))$, $\forall x(R(x) \to \neg P(x))$, $\forall x(R(x) \vee S(x))$, and $\exists x \neg Q(x)$, use rules of inference to show that $\exists x S(x)$ is true.

$\exists x S(x)$ $S(c) S(x))))$

Logo

Exercises

14.

Given that $\forall x(P(x) \vee Q(x))$, $\forall x(R(x) \to \neg P(x))$, $\forall x(R(x) \vee S(x))$, and $\exists x \neg Q(x)$, use rules of inference to show that $\exists x S(x)$ is true.

1	$\exists x \neg Q(x)$	前提
2	$\neg Q(c)$	存在量词消去
3	$\forall x(P(x) \vee Q(x))$	前提
4	$P(c) \vee Q(c)$	全称量词消去
5	$P(c)$	2，4
6	$\forall x(R(x) \to \neg P(x))$	前提
7	$R(c) \to \neg P(c)$	全称量词消去
8	$\neg R(c)$	5，7
9	$\forall x(R(x) \vee S(x))$	前提
10	$R(c) \vee S(c)$	全称量词消去
11	$S(c)$	8，10
12	$\exists x S(x)$	存在量词引入
13		

Exercises

- 15. A computer vision based program is developed for detecting masked men as Figure 4 shows. The following rules are applied to the program. If a head is detected and no face is detected, the program judges that there is a masked man; If a face is detected and a mask is detected, the program judges that there is a masked man; Otherwise, the program judges that there is no masked man. If the program judges that there is a masked man and no head is detected, <u>please use rules of inference to determine whether the program detects a mask.</u>

Exercises

15.

(1) 令“a head is detected”为 A;

令“a face is detected”为 B;

令“a mask is detected”为 C;

令“the program judges that there is a masked man”为 D;

由题意得:

$$\begin{array}{c} (A\wedge\neg B)\vee(B\wedge C)\leftrightarrow D \\ D \\ \neg A \end{array} \quad \begin{cases} A\wedge\neg B\rightarrow D \\ B\wedge C\rightarrow D \\ \neg(A\wedge\neg B)\wedge\neg(B\wedge C)\rightarrow\neg D \end{cases}$$

1. $(A\wedge\neg B)\vee(B\wedge C)\leftrightarrow D$ 前提
2. D 前提
3. $(A\wedge\neg B)\vee(B\wedge C)$ 1，2
4. $\neg A$ 前提
5. $\neg A\vee B$ 附加
6. $\neg(A\wedge\neg B)$ 德摩根律
7. $B\wedge C$ 3，6 析取三段论
8. C 7 化简

因此 a mask is detected

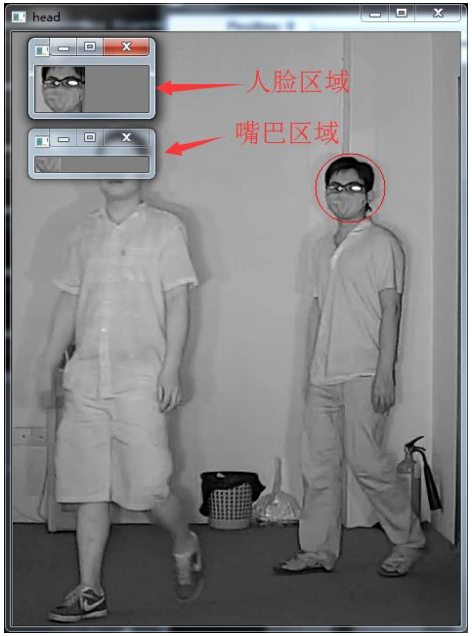


Figure 4. Graph for question

Logo

End of the Section 1.5