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Discrete Mathematics

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Chapter 3. Relations

Closures of Relations

Section 3.4

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Closures

§ 7.4: Closures of Relations

- For any property *X*, the "*X* closure" of a set *A* is defined as the "smallest" superset of *A* that has property *X*. More specifically,
- The *reflexive closure* of a relation *R* on *A* is the smallest superset of R that is reflexive.
- The *symmetric closure* of *R* is the smallest superset of R that is symmetric
- The *transitive closure* of *R* is the smallest superset of R that is transitive

Calculating closures

- The *reflexive closure* of a relation R on A is obtained by "adding" (a,a) to R for each $a \in A$. *I.e.*, it is $R \cup I_A$ (Check that this is the r.c.)

- The *symmetric closure* of R is obtained by "adding" (b,a) to R for each (a,b) in R. *I.e.*, it is $R \cup R^{-1}$ (Check that this is the s.c.)

- The *transitive closure* of R is obtained by *"repeatedly"* adding (a,c) to R for each $(a,b),(b,c)$ in R …

Calculating closures

- Adore={(a,b),(b,c),(c,c)}
- Detest={(b,d),(c,a),(c,b)}
- The *symmetric closure* of ...

  ... Adore={(a,b),(b,c),(c,c),(b,a), (c,b)}

  ... Detest={(b,d),(c,a),(c,b), (d,b), (a,c),(b,c)}

Calculating closures

- Adore={(a,b),(b,c),(c,c)}
- Detest={(b,d),(c,a),(c,b)}
- The *transitive closure* of ...

  ... Adore={(a,b),(b,c),(c,c),(a,c)}

  ... Detest={(b,d),(c,a),(c,b),(c,d)}

Example 1

- What is the reflexive closure of the relation $R = \{ (a,b) \mid a < b \}$ on the set of integers?
- Solution:

The reflexive closure of R is

$$R \cup \Delta = \{ (a,b) \mid a<b \} \cup \{ (a,a) \mid a \in Z \}$$
$$= \{(a,b) \mid a \leq b \}$$

Example 2

- What is the symmetric closure of the relation $R = \{ (a,b) \mid a>b \}$ on the set of positive integers?
- Solution:

The symmetric of closure of R is the relation

$$R \cup R^{-1} = \{ (a,b) \mid a>b \} \cup \{ (b,a) \mid b>a\}$$
$$= \{ (a,b) \mid a \neq b \}$$

Transitive Closures

Definition 2

- Let **R** be a relation on a set **A**.
- The connectivity relation **R*** consists of the pairs **(a,b)** such that there is a path of length at the least one from **a** to **b** in **R**.

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R*

How would you formally define R*?

Here's a safe bet

$$\bigcup_{i=1}^{\infty} R^i$$

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R*

How would you formally define R*?

Here's a finite variant, where n = |A| *(proof in book* that n *is large enough)*

$$\bigcup_{i=1}^{n} R^i$$

Example 4

- Let R be the relation on the set of all people in the world that contains (a,b) if a has met b, What is R^n, where n is a positive integer greater than one?

- R^n consists of those pair (a,b) such that there are people $x_1, x_2, \ldots, x_{n-1}$ such that a has met x_1, x_1 has met x_2,…, and x_{n-1} has met b.

Example 4

- Let **R** be the relation on the set of all people in the world that contains **(a,b)** if **a** has met **b**, **What is R***, where **n** is a positive integer greater than one?

- The relation **R*** contains **(a,b)** if there is a sequence of people, starting with **a** and ending with **b**, such that each person in the sequence has met the next person in the sequence.

Proof of Theorem 2

Theorem: R^* = the transitive closure of R

We need to prove that R^* is the smallest transitive superset of R.

1. Proof that R^* is transitive:

Suppose xR^*y and yR^*z.
E.g., xR^ny and yR^mz
Then $xR^{n+m}z$, hence xR^*z

Proof of Theorem 2

Theorem: R* = the transitive closure of R

2. We now know that R* is transitive. Evidently, $R \subseteq R^*$. Prove that there cannot be a smaller transitive superset S of R than R*:

- Suppose such a transitive superset of R existed. This would mean that there exists a pair (x,y) such that xR*y while $\neg$xSy.
- But xR*y means $\exists$n such that xR^ny. But since $R \subseteq S$, it would follow that xS^ny; but because S is transitive, this would imply that xSy. Contradiction.

(Compare proof in book, p.500)

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Lemma 1

- **Lemma:** Let **A** be a set with **n** elements, and let **R** be a relation on **A**.
- If there is a path of length at least one in **R** from **a** to **b**, then there is such a path with length not exceeding **n**.
- Moreover, when **a ≠ b**, if there is a path of length at least one in **R** from **a** to **b**, then there is such a path with length not exceeding **n-1**.

Proof of Lemma 1

- Proof:

  Supposed there is a path from a to b in R. Let m be the length of the shortest such path.
- Supposed that x0,x1,x2,…,xm-1,xm, where x0=a and xm=b, is such a path.
- Supposed that a = b and that m > n, so that m≥n+1. By the pigeonhole principle, since there are n vertices in A, among the m vertices x0,x1,…,xm-1, at least two are equal.

- Supposed that xi=xj with 0≤i ≤j ≤m-1. Then the path contains a circuit from xi to itself.
- The circuit can be deleted from the path from a to b, leaving a path, namely, x0, x1,…,xm-1,xm, from a to b of shorter length.
- Hence, the path of shortest length must have length less than or equal to n.

- From Lemma 1, we have:
- $R^* = R^1 \cup R^2 \cup R^3 \cup \ldots \cup R^n$
- The transitive closure of R is also equal to $R^1 \cup R^2 \cup R^3 \cup \ldots \cup R^n$.

Theorem 3

- Let $M(R)$ be the zero-one matrix of the relation R on a set with n elements. Then the zero-one matrix of the transitive closure R^* is

- $M(R^*)$

  $=M(R) \vee M(R)^2 \vee M(R)^3 \vee \ldots \vee M(R)^n$

The proof is similar to Lemma 1.

Example 7

$$M(R) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \qquad M(R)^2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M(R)^3 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M(R^*) = M(R) \vee M(R)^2 \vee M(R)^3 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Algorithm of Computing TC

- Procedure transitive closure (M(R): zero-one nxn matrices)
- A:= M(R)
- B:=A
- for i:=2 to n
- begin
-     A:=A o M(R)
-     B:=B V A
- end {B is the zero-one matrix for R*}

2*(n-1)*n^3

bit operations

Warshall's Algorithm

- **Fast algorithms are available for calculating R*, especially *Warshall's algorithm***
  (also called Roy-Warshall algorithm)
- **FYI: this algorithm uses a matrix representation.**

Idea of Warshall's Algorithm

- The algorithm is based on the construction of a sequence of zero-one matrices.
- The matrices are W(0),W(1),…,W(n), where W(0) = M(R) is the zero-one matrix of this relation, and W(k) = [wij(k)].
- wij(k) = 1 if there is a path from vi to vj such that all the interior vertices of this path are in the set { v1,v2,…,vk } and is 0 otherwise.

Idea of Warshall's Algorithm

- W(n) = M(R*)
- Because the (i,j)th entry of M(R*) is 1 if and only if there is a path from vi to vj, with all interior vertices in the set

{v1, v2, …, vn}.

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Example 8

$$W(0) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad W(1) = W(2) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$W(3) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \qquad W(4) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

How to compute W(k)

- Lemma 2 give us the means to compute efficiently the matrices W(k), k=1,2,…,n.
- wij(k) = wij(k-1) V (wik(k-1) Λ wkj(k-1))

$$W(3) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \qquad W(4) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Warshall Algorithm

- Produre Warshall (M(R): n x n 0-1 Matrix)
- W := M(R)
- for k:=1 to n
- begin
-     for i:=1 to n
-     begin
-         for j:=1 to n
-             wij(k) = wij(k-1) V (wik(k-1) Λ wkj(k-1))
-     end
- end {W = [wij] is M(R*)}

2*n^3

bit operations

Exercises

Exercises

1. The transitive closures of the relations {(2, 1), (2,3), (3,1), (3,4), (4,1), (4, 3)} on {1, 2, 3, 4} is (c)

A. $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ D. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Exercises

2. Suppose S={a, b, c, d}, R={<a, b>, <b, d>, <c, c>}, then the reflexive closure of R is

{<a, b>, <b, d>, <c, c>, <a,a>,<b,b>,<d,d>}

Exercises

3. The transitive closures of the relations {(1, 2), (2, 1), (2, 3), (3, 4), (4, 1)} on {1, 2, 3, 4} is

____

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Exercises

5. Let *R* be the relation on the set {0, 1, 2, 3} containing the ordered pairs (0, 1), (1, 1), (1, 2), (2, 0), (2, 2) and (3, 0). The symmetric closure of R=________.

{(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(2,0),(2,1),(2, 2),(3,0)}

Exercises

6. Use Warshall's algorithm to find the transitive closures of the relation {(1, 2),(1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}on {1, 2, 3, 4}.

$$W_0 = W_1 = W_2 = W_3 = W_4 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Exercises

7. Suppose A = {a , b, c, d}, a relation on A is R = {<a, b>, <b, a>, <b, c>, <c, d>}. Please use the zero-one matrix to find the transitive closure of R.

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{R^2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{R^3} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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Exercises

7. Suppose A = {a , b, c, d}, a relation on A is R = {<a, b>, <b, a>, <b, c>, <c, d>}. Please use the zero-one matrix to find the transitive closure of R.

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{R^4} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{t(R)} = M_R \vee M_{R^2} \vee M_{R^3} \vee M_{R^4} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Exercises

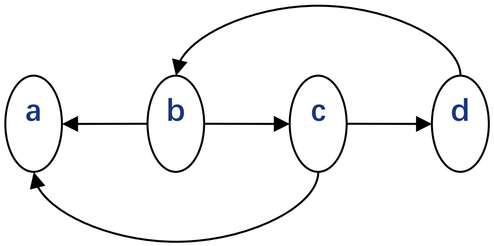
8\. Considering the relation R on A={1,2,3,4} and its adjacency matrix M_A, please apply Warshall's algorithm to calculate the transitive closure of R by completing the matrixes of W1-W4.

$$W0 = M_A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$W1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad W2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad W3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad W4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Exercises

9. Set $X = \{a, b, c, d\}$, the relationship R on X as shown:



① Write the relation matrix M_R of the relationship R

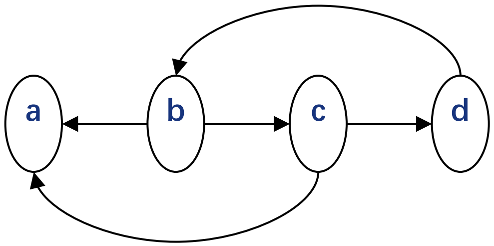
② Find the relation matrix $M_{r(R)}$ of the reflexive closure r(R) of the relationship *R*.

③ Find the relation matrix $M_{s(R)}$ of the symmetric closure s(R) of the relationship *R*.

④ Find the relation matrix $M_{t(R)}$ of the transitive closure t(*R*) of the relationship *R*.

Exercises

9. Set $X = \{a, b, c, d\}$, the relationship R on X as shown:

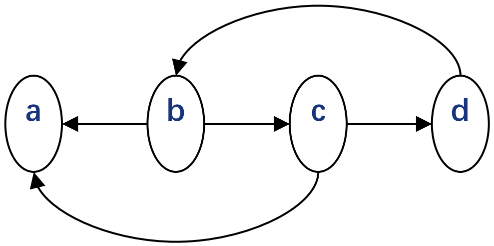


① Write the relation matrix M_R of the relationship R

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Exercises

9. Set $X=\{a,b,c,d\}$, the relationship R on X as shown:

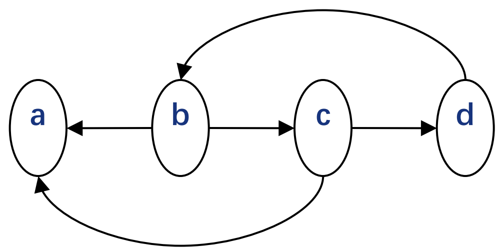


② Find the relation matrix $M_{r(R)}$ of the reflexive closure r(R) of the relationship *R*.

$$M_{r(R)}=M_R+E=\begin{bmatrix}0&0&0&0\\1&0&1&0\\1&0&0&1\\0&1&0&0\end{bmatrix}+\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0&0\\1&1&1&0\\1&0&1&1\\0&1&0&1\end{bmatrix}$$

Exercises

9. Set $X=\{a,b,c,d\}$, the relationship R on X as shown:



③ Find the relation matrix $M_{s(R)}$ of the symmetric closure s(R) of the relationship *R*.

$$M_{s(R)}=M_R+M_R^{'}=\begin{bmatrix}0&0&0&0\\1&0&1&0\\1&0&0&1\\0&1&0&0\end{bmatrix}+\begin{bmatrix}0&1&1&0\\0&0&0&1\\0&1&0&0\\0&0&1&0\end{bmatrix}=\begin{bmatrix}0&1&1&0\\1&0&1&1\\1&1&0&1\\0&1&1&0\end{bmatrix}$$

Exercises

④ Find the relation matrix $M_{t(R)}$ of the transitive closure t(*R*) of the relationship *R*.

$$M_R^2=\begin{bmatrix}0&0&0&0\\1&0&1&0\\1&0&0&1\\0&1&0&0\end{bmatrix}\begin{bmatrix}0&0&0&0\\1&0&1&0\\1&0&0&1\\0&1&0&0\end{bmatrix}=\begin{bmatrix}0&0&0&0\\1&0&0&1\\0&1&0&0\\1&0&1&0\end{bmatrix}$$

$$M_R^3=\begin{bmatrix}0&0&0&0\\1&0&0&1\\0&1&0&0\\1&0&1&0\end{bmatrix}\begin{bmatrix}0&0&0&0\\1&0&1&0\\1&0&0&1\\0&1&0&0\end{bmatrix}=\begin{bmatrix}0&0&0&0\\0&1&0&0\\1&0&1&0\\1&0&0&1\end{bmatrix}$$

$$M_R^4=\begin{bmatrix}0&0&0&0\\0&1&0&0\\1&0&1&0\\1&0&0&1\end{bmatrix}\begin{bmatrix}0&0&0&0\\1&0&1&0\\1&0&0&1\\0&1&0&0\end{bmatrix}=\begin{bmatrix}0&0&0&0\\1&0&1&0\\1&0&0&1\\0&1&0&0\end{bmatrix}$$

$$M_{t(R)}=M_R+M_R^2+M_R^3+M_R^4$$

$$=\begin{bmatrix}0&0&0&0\\1&0&1&0\\1&0&0&1\\0&1&0&0\end{bmatrix}+\begin{bmatrix}0&0&0&0\\1&0&0&1\\0&1&0&0\\1&0&1&0\end{bmatrix}+\begin{bmatrix}0&0&0&0\\0&1&0&0\\1&0&1&0\\1&0&0&1\end{bmatrix}+\begin{bmatrix}0&0&0&0\\1&0&1&0\\1&0&0&1\\0&1&0&0\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0&0\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}$$

Logo

Exercises

10. Given a relation R on A={1,2,3,4}, use Warshall's algorithm to find the adjacency matrix of the transitive closure of R.

$$R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Exercises

10.

$$W_0 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \qquad W_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Exercises

11. Given that R = {(1, 2), (2, 3), (2, 4), (3, 1)} is a relation on the set {1, 2, 3, 4}. Please find the reflexive closure _______________ and the symmetric closure ______________of R.

Exercises

11. Given that R = {(1, 2), (2, 3), (2, 4), (3, 1)} is a relation on the set {1, 2, 3, 4}. Please find the reflexive closure ______________ and the symmetric closure _____________of R.

{(1,1),(1, 2),(2,2), (2, 3), (2, 4),(3,3), (3, 1),(4,4)}
{(1,2), (2,1), (2,3), (3,2), (2,4), (4,2), (3,1), (1,3)}

Exercises

12. Suppose $S = \{a, b, c, d\}, R = \{(a, b), (b, b), (b, c), (c, a), (c, c), (d, a)\}$,then the symmetric closures of R is ________.

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Exercises

12. Suppose $S=\{a,b,c,d\}, R=\{(a,b),(b,b),(b,c),(c,a),(c,c),(d,a)\}$,then the symmetric closures of R is ________.

$$\{(a,b),(b,a),(b,b),(b,c),(c,b),(c,a),(a,c),(c,c),(d,a),(a,d)\}$$

Exercises

13. Considering the relation R on A={1,2,3,4} and its adjacency matrix M_A, please apply Warshall's algorithm to calculate the transitive closure of R by completing the matrixes of $W_1 - W_4$.

$$W_0 = M_A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Exercises

13.

$$W_0 = M_A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad W_2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Exercises

14.Consider the relation R on A={1,2,3,4,5} given by the adjacency matrix， run Warshall's algorithm to find the adjacency matrix of the transitive closure of R.

$$M(R)=\begin{bmatrix}1&1&0&0&0\\0&1&1&0&0\\1&0&0&0&0\\0&0&0&1&1\\0&0&0&0&1\end{bmatrix}$$

Exercises

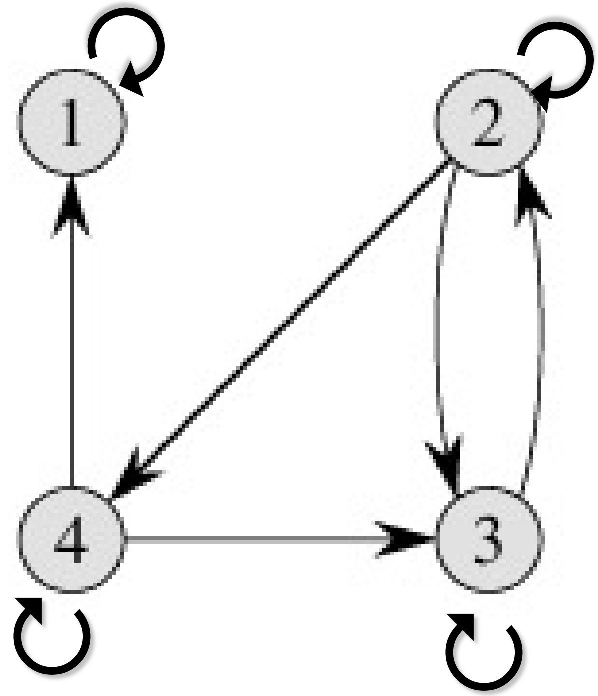
14. Solution. Consider the relation R on A={1,2,3,4,5} given by the adjacency matrix， run Warshall's algorithm to find the adjacency matrix of the transitive closure of R.

$$W_0=\begin{bmatrix}1&1&0&0&0\\0&1&1&0&0\\1&0&0&0&0\\0&0&0&1&1\\0&0&0&0&1\end{bmatrix}\quad W_1=\begin{bmatrix}1&1&0&0&0\\0&1&1&0&0\\1&1&0&0&0\\0&0&0&1&1\\0&0&0&0&1\end{bmatrix}\quad W_2=\begin{bmatrix}1&1&1&0&0\\0&1&1&0&0\\1&1&1&0&0\\0&0&0&1&1\\0&0&0&0&1\end{bmatrix}$$

$$W_3=\begin{bmatrix}1&1&1&0&0\\1&1&1&0&0\\1&1&1&0&0\\0&0&0&1&1\\0&0&0&0&1\end{bmatrix}\quad W_4=\begin{bmatrix}1&1&1&0&0\\1&1&1&0&0\\1&1&1&0&0\\0&0&0&1&1\\0&0&0&0&1\end{bmatrix}\quad W_5=\begin{bmatrix}1&1&1&0&0\\1&1&1&0&0\\1&1&1&0&0\\0&0&0&1&1\\0&0&0&0&1\end{bmatrix}$$

Exercises

15.Consider the relation R on T={1,2,3,4} shown below. Run Warshall's algorithm to find the adjacency matrix of the transitive closure of R.



Logo

Exercises

15. Solution.Consider the relation R on T={1,2,3,4} shown below. Run Warshall's algorithm to find the adjacency matrix of the transitive closure of R.

$$W_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \quad W_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \quad W_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

$$W_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad W_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

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End of Section 7.4