COMBINATIONAL LOGIC DESIGN

Lesson 10

Digital Logic

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In-class Test 2 Answers

- 2.1
  - Recall SOP canonical form (Page 59).
  - Sum the minterms of all the rows (in the truth table) with outputs as 1's.

$$\text{(a) } Y = \bar{A}B + A\bar{B} + AB = \sum(m_1, m_2, m_3) = \sum(1,2,3)$$

$$\text{(b) } Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}$$

$$= \sum(m_1, m_2, m_3, m_4, m_6) = \sum(1,2,3,4,6)$$

- Some students simplified the equation.
- Incorrect, because the question asks for **canonical form**.

In-class Test 2 Answers

- 2.2(a)
  - You can choose to use Boolean theorems or K-maps.
  - For 2-variable equations, using Boolean algebra to simplify the equations is easy. For 3- and 4-variable equations, using K-maps is more convenient. K-maps are usually used for equations with up to 4 variables.

$$Y = \bar{A}B + A\bar{B} + AB = \bar{A}B + A\bar{B} + (AB + AB)$$

$$= (\bar{A}B + AB) + (A\bar{B} + AB) = (\bar{A} + A)B + (\bar{B} + B)A$$

$$= B + A = A + B$$

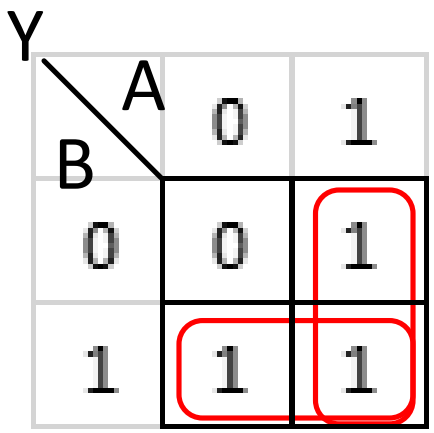
- Some students used Boolean algebra to simplify the equation but got the result as AB+A or AB+B.
- This is not the minimized form.

In-class Test 2 Answers

- 2.2(a) (continued)

  Use K-map:

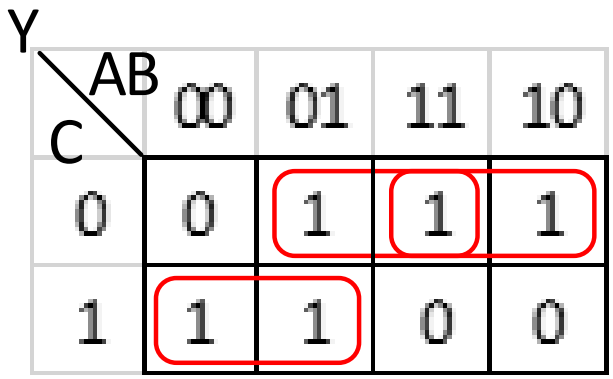
  – Variable coding method: Gray code (The adjacent codes have only one bit changed.)



$$Y = A + B$$

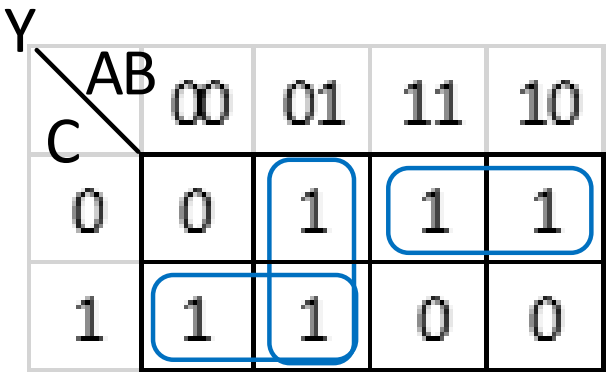
In-class Test 2 Answers

- 2.2(b)



Solution 1:

$$Y = \bar{A}C + B\bar{C} + A\bar{C}$$



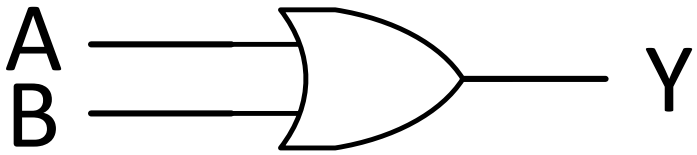
Solution 2:

$$Y = \bar{A}C + \bar{A}B + A\bar{C}$$

In-class Test 2 Answers

- 2.3(a)

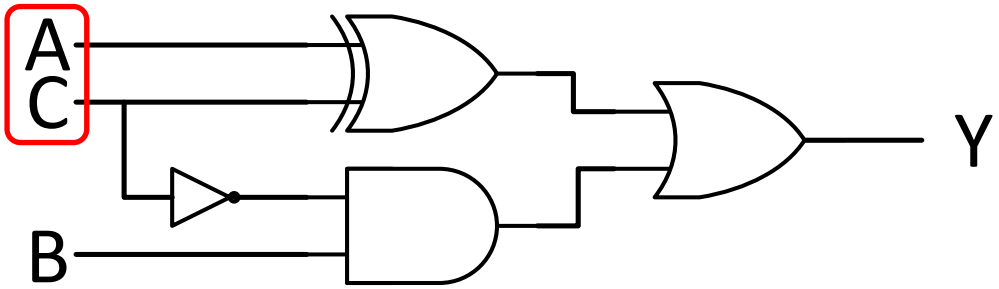
$$Y = A + B$$



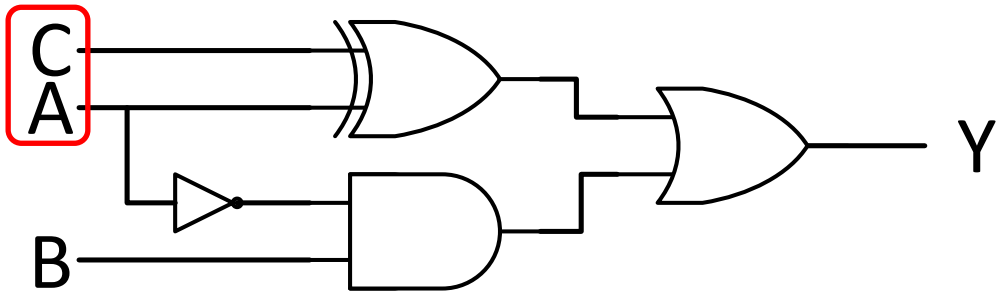
In-class Test 2 Answers

- 2.3(b)

Solution 1: $Y = \bar{A}C + B\bar{C} + A\bar{C}$



Solution 2: $Y = \bar{A}C + \bar{A}B + A\bar{C}$



In-class Test 2 Answers

- 2.4
  - Need not minimize the equation.

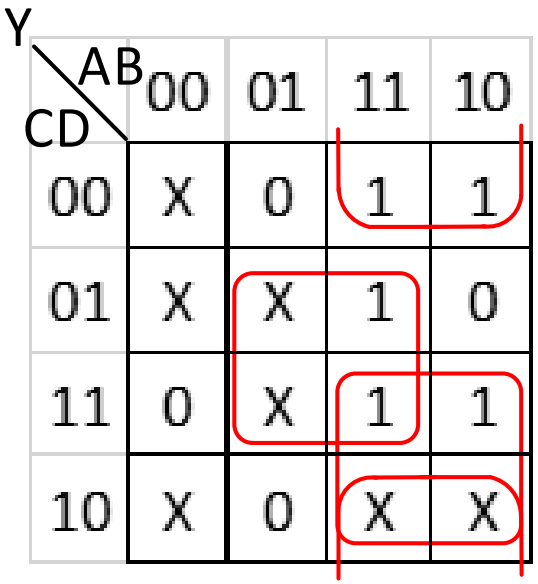
$$Y = \bar{A}D + A\bar{C}D + A\bar{B}C + ABCD$$

$$Z = BD + A\bar{C}D$$

In-class Test 2 Answers

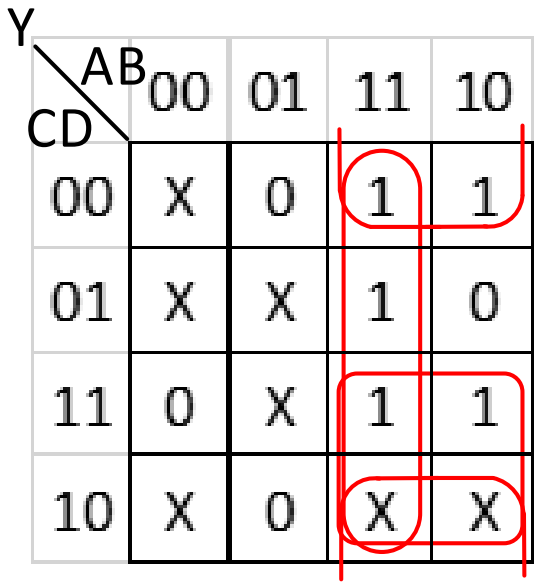
- 2.5
  - Minimal Boolean equation (several solutions)

Solution 1:



$$Y = BD + A\bar{D} + AC$$

Solution 2:

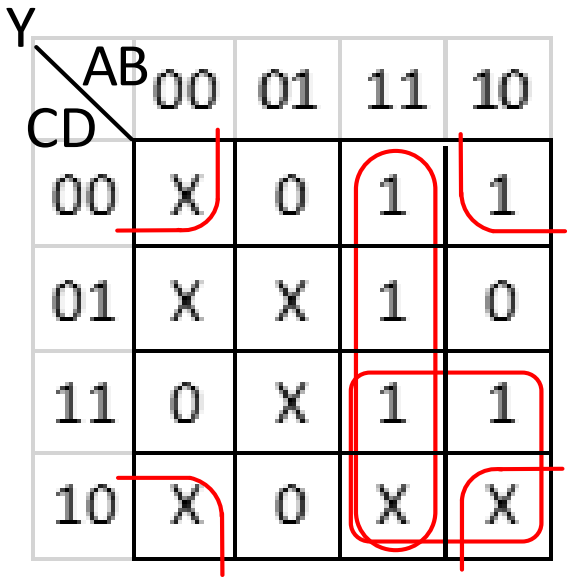


$$Y = AB + A\bar{D} + AC$$

In-class Test 2 Answers

- 2.5 (continued)

Solution 3:

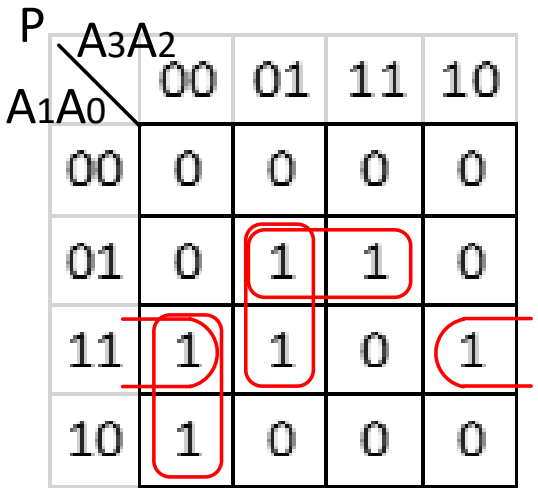


$$Y = AB + AC + \overline{B}\,\overline{D}$$

In-class Test 2 Answers

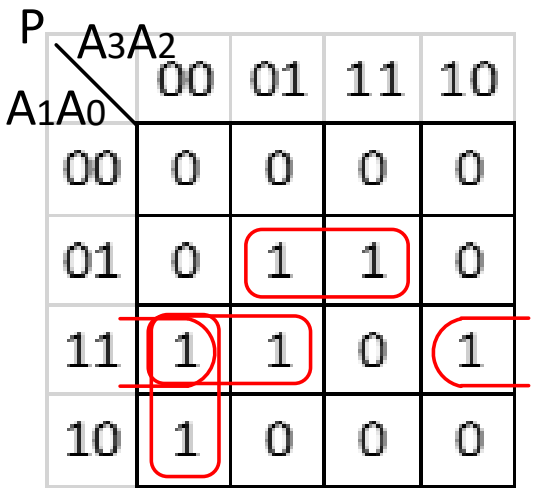
- 2.6

A3	A2	A1	A0	P	D
0	0	0	0	0	1
0	0	0	1	0	0
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	1	1	0
0	1	1	0	0	1
0	1	1	1	1	0
1	0	0	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	0	0	0
1	1	1	1	0	1



Solution 1:

$$P = A_2\overline{A_1}A_0 + \overline{A_3}A_2A_0 + \overline{A_2}A_1A_0 + \overline{A_3}\,\overline{A_2}A_1$$



Solution 2:

$$P = A_2\overline{A_1}A_0 + \overline{A_3}A_1A_0 + \overline{A_2}A_1A_0 + \overline{A_3}\,\overline{A_2}A_1$$

In-class Test 2 Answers

- 2.6 (continued)

A3	A2	A1	A0	P	D
0	0	0	0	0	1
0	0	0	1	0	0
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	1	1	0
0	1	1	0	0	1
0	1	1	1	1	0
1	0	0	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	0	0	0
1	1	1	1	0	1

D: A1A0 \ A3A2	00	01	11	10
00	1	0	1	0
01	0	0	0	1
11	1	0	1	0
10	0	1	0	0

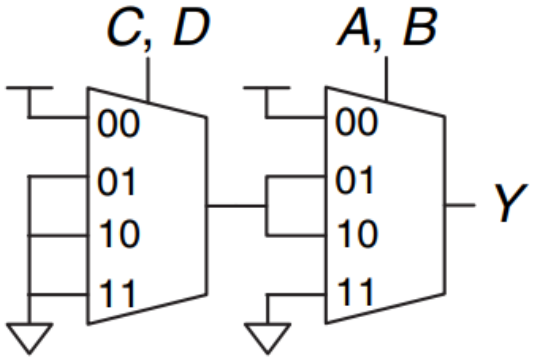
$$D = A_3 A_2 \overline{A_1} A_0 + A_3 \overline{A_2}\,\overline{A_1} A_0 + \overline{A_3}\,\overline{A_2} A_1 A_0 + A_3 A_2 A_1 A_0 + \overline{A_3} A_2 A_1 \overline{A_0} + \overline{A_3} A_2 A_1 A_0$$

- Basically, you have to write out every bit value of the input variables in the truth table.
- But you can also write the truth table with input don't cares.

In-class Test 2 Answers

- 2.7

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0



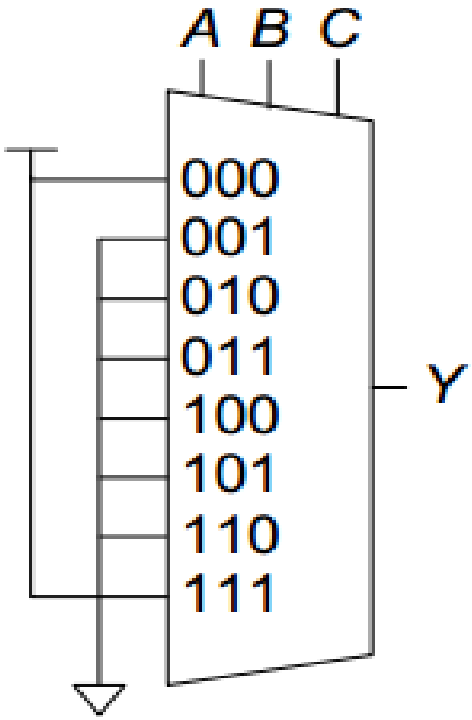
Y: AB \ CD	00	01	11	10
00	1	1	0	1
01	1	0	0	0
11	1	0	0	0
10	1	0	0	0

$$Y = \overline{A}\,\overline{B} + \overline{A}CD + B\overline{C}D$$

In-class Test 2 Answers

- 2.8(a)

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

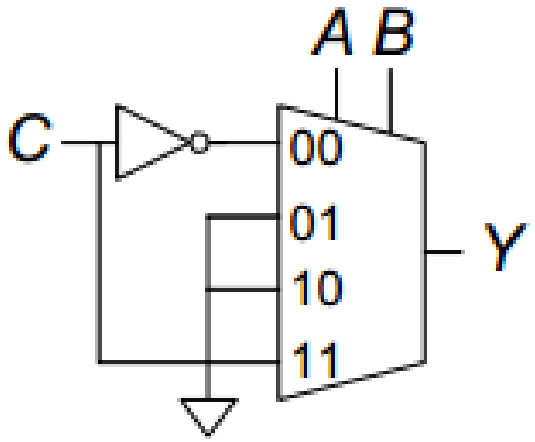


In-class Test 2 Answers

- 2.8(b)

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

A	B	Y
0	0	C'
0	1	0
1	0	0
1	1	C



In-class Test 2 Answers

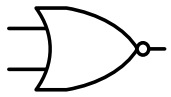
- 2.8(c)

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

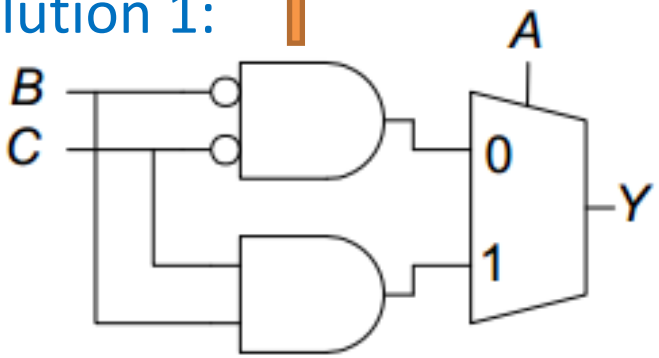
A	Y
0	B'C'
1	BC

Solution 2:

$$\overline{B}\,\overline{C} = \overline{B + C}$$



Solution 1:



In-class Test 2 Answers

- 2.8(c) (continued)

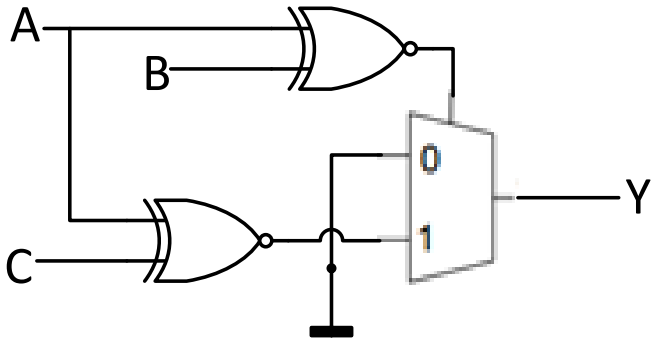
Solution 3:

– From truth table, we learn that when A, B and C are of the same value, Y outputs 1.

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

$\overline{A \oplus B}$	Y
0	0
1	$\overline{A \oplus C}$

$\overline{B \oplus C}$



A and B are interchangeable for this term.

In-class Test 2 Answers

- 2.8(c) (continued)

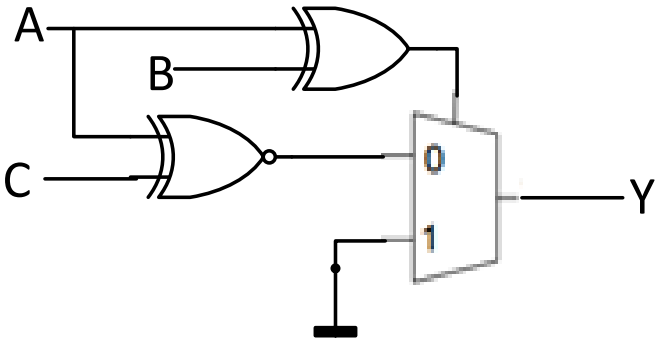
Solution 4:

– From truth table, we learn that when A, B and C are of the same value, Y outputs 1.

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

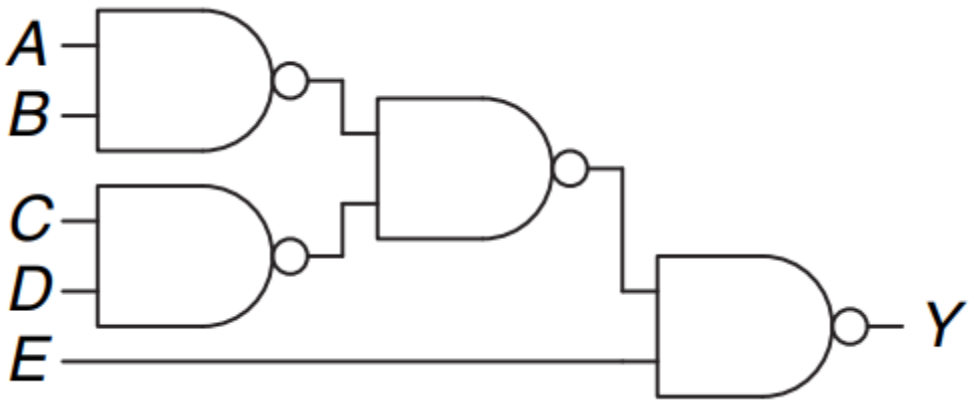
A and B are interchangeable for this term.

$A \oplus B$	$\overline{B \oplus C}$
	Y
0	$\overline{A \oplus C}$
1	0



In-class Test 2 Answers

- 2.9



  - Longest path: from A/B/C/D to Y
  - Shortest path: from E to Y

$$t_{pd} = t_{pd\_AY} = 3t_{pd\_NAND2} = 3 \times 20 = 60ps$$

$$t_{cd} = t_{cd\_EY} = t_{cd\_NAND2} = 15ps$$