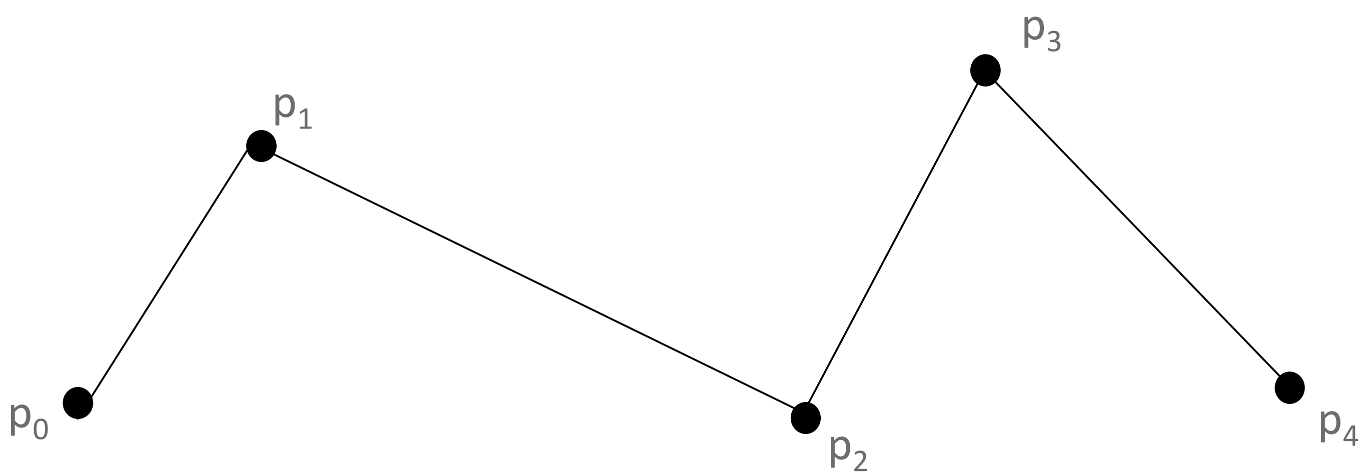
Topic:

Catmull-Romm Splines

Interpolation: in-betweening a sequence of values

Linear Interpolation



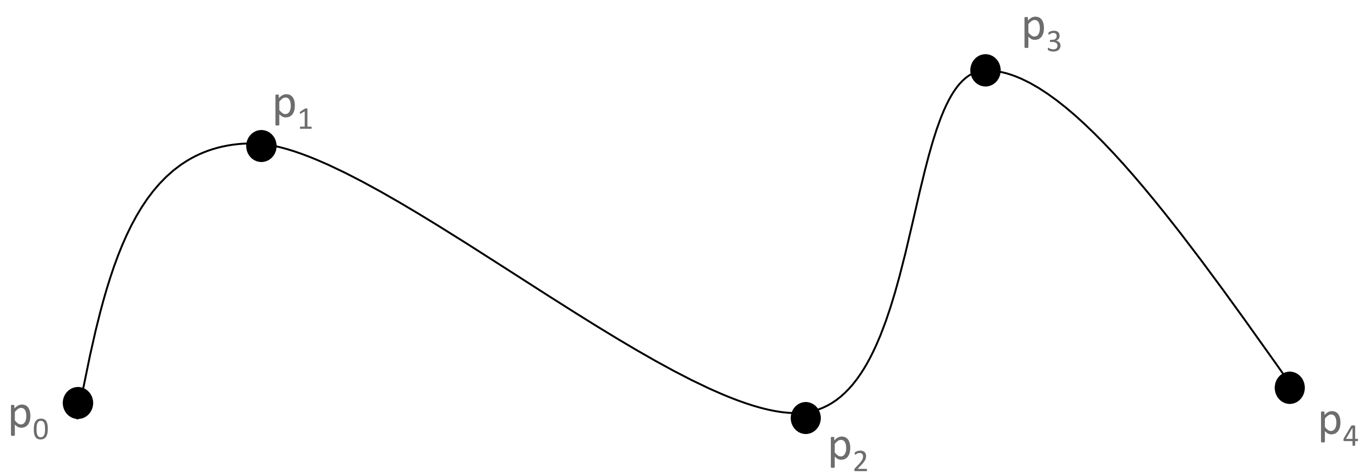
An n-degree polynomial in t has n+1 coefficients. It can be defined by n+1 constraints. A line (degree 1) thus needs two points.

$p(t)= (1-t)*p_0 + t*p_1$

$$p(t)= [t\ 1]*\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} * [p_0\ \ p_1]^T$$

Interpolation: in-between a sequence of values

Curve Interpolation



For smooth (tangent continuous) interpolation across points, we need to be able to interpolate points as well as tangents.

Two points p_0, p_1, and two tangents p'_0 , p'_1 , define a cubic (degree 3) curve.

Designing Polynomial Curves from constraints

$p(t) = TA$, where T is powers of t. for a cubic $T=[t^3\ t^2\ t^1\ 1]$.

Written with geometric constraints $p(t) = TMG$, where M is the **Basis matrix** of a design curve and G the specific design constraints.

An example of constraints for a cubic Hermite for eg. are end points and end tangents. i.e. p_0, p'_0 at t=0 and p_1, p'_1 at t=1. Plugging these constraints into $p(t) = TA$ we get.

$$
\begin{array}{lcl}
 & & \text{B} \\
p(0) = p_0 = & & [\ 0\ 0\ 0\ 1\ ]\ A_h \\
p(1) = p_1 = & & [\ 1\ 1\ 1\ 1\ ]\ A_h \\
p'(0) = p'_0 = & & [\ 0\ 0\ 1\ 0\ ]\ A_h \quad \Rightarrow \quad G=BA,\ A=MG \Rightarrow M=B^{-1} \\
p'(1) = p'_1 = & & [\ 3\ 2\ 1\ 0\ ]\ A_h
\end{array}
$$

Hermite Basis Matrix

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}^{-1}$$

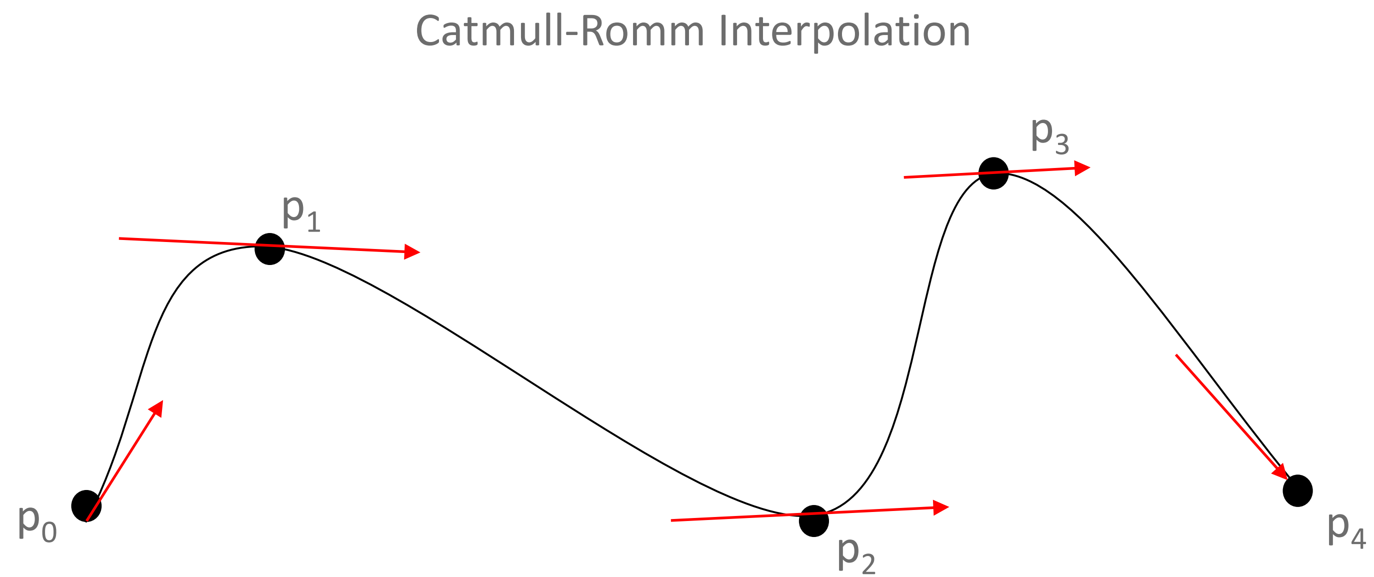
$$= M_{hermite}$$

$$\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The columns of the Basis Matrix form Basis Functions such that:

$$p(t)= (2t^3 -3t^2 +1)p_0 + (-2t^2 +3t^2)p_1 + (t^3 -2t^2 +t)p'_0 + (t^3 -t^2)p'_{1.}$$

Interpolation: Catmull-Romm Splines



Pick tangents based on a factor k (1/2 for eg.) of the vector between neighbor points.

$p'_i = k*(p_{i+1} - p_{i-1})$.

For the end-points there is only one neighbor:

$p'_0 = k*(p_1 - p_0)$.
$p'_n = k*(p_n - p_{n-1})$.