

Q#1 Muhammad Tanveer ID F2016266241

```
int func(int arr[], int l, int r, int x) {
```

```
while (l <= r) {
```

```
int mid = l + (r - l) / 2
```

```
if (arr[mid] == x)
```

```
return mid;
```

```
else if (arr[mid] < x)
```

```
l = mid + 1;
```

```
else
```

```
r = mid - 1;
```

```
}
```

```
return -1;
```

```
}
```

cost	Time	cost time
C_1	1	C_1
C_2	$T(N)$	$C_2 T(N)$
C_3	$T(N)$	$C_3 T(N)$
C_4	$T(N)$	$C_4 T(N)$
C_5	1	C_5
C_6	$T(N)$	$C_6 T(N)$
C_7	$T(N)$	$C_7 T(N)$
C_8	$T(N)$	$C_8 T(N)$
C_9	$T(N)$	$C_9 T(N)$
C_{10}	$T(N)$	$C_{10} T(N)$
C_{11}	1	C_{11}

Continue Q#1 Muhammad Tanveer ID F2016266241

Note: Return statements only execute once, hence (5th) For $T(N)$ consider the following

① Assume the array $[]$ contains N # of elements

② $T(N)$ = Consider that each iteration reducing the search space by $N/2$ as we are calculating "mid"

\therefore

$$T(N) = C + T(N/2)$$

$$T(N/2) = C + T(N/4) \quad \text{Put in eq above } T(N) = C + C + T(N/4)$$

$$T(N/4) = C + T(N/8) \quad \text{Put in eq above } T(N) = 2C + T(N/4)$$

$$\text{Put in eq above } T(N) = 2C + C + T(N/8)$$

$$= 3C + T(N/8)$$

$$T(N) = C + T(N/2)$$

$$= 2C + T(N/4)$$

$$= 3C + T(N/8)$$

$$T(N) = 1C + T(N/8)$$

Continue Q #1

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$$T(N) = i.c + T\left(\frac{N}{2^i}\right)$$

Now at some point, as $\frac{N}{2^i}$ decrease it will reach 1

$$\frac{N}{2^i} \xrightarrow{\text{as } i \text{ large value}} 1$$

$$\therefore T\left(\frac{N}{2^i}\right) = T(1)$$

$T(1)$ means that array has only 1 element \therefore time for execution $\propto 1$

$$\Rightarrow \frac{N}{2^i} = 1$$

take \log_2 on both side

$$N = 2^i$$

$$\log_2 N = \log_2 2^i$$

$$\log_2 N = i \log_2 2$$

$$\log_2 N = i$$

Put this back in our eq

$$T(N) = i.c + T\left(\frac{N}{2^i}\right) = c \cdot \log_2 N + T\left[\frac{N}{2^{\log_2 N}}\right]$$

$$= c \cdot \log_2 N + T\left[\frac{N}{N}\right]$$

$$= c \log_2 N + T(1)$$

$$= c \log_2 N + 1$$

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$$\begin{aligned}\therefore \text{Total cost} &\Rightarrow C_1 + C_5 + C_{11} + [C_2 + C_3 + C_4 + C_6 + C_7 + C_8 + C_9] \cdot T(N) \\ &\Rightarrow a + b[c \log_2 N + 1] \Rightarrow a + b + bc \log_2 cN) \\ &\Rightarrow k_1 + k_2 \log_2(N) \longrightarrow O[\log_2(N)]\end{aligned}$$

Worst case time complexity

$$O[\log_2(N)]$$

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 Q#2 part a

Quick sort

$\{17, 9, 6, 23, 14, 29\}$

① chose a pivot take 1st
middle and last

$\{17, 23, 29\}$
 $\downarrow \quad \downarrow \quad \downarrow$
 1st middle last

② $\underline{17, 9, 6, 14} \quad \triangle_{23} \quad \underline{29}$
 $\leq 23 \quad \quad \quad > 23$

Elements are ^{1 to 5} sorted smaller list we can
 use insertion sort

③ $\underline{6, 9, 14, 17} \quad \triangle_{23} \quad \underline{29}$
 $\leq 23 \quad \quad \quad > 23$

④ merge all

⑤ $[6, 9, 14, 17, 23, 29]$ sorted

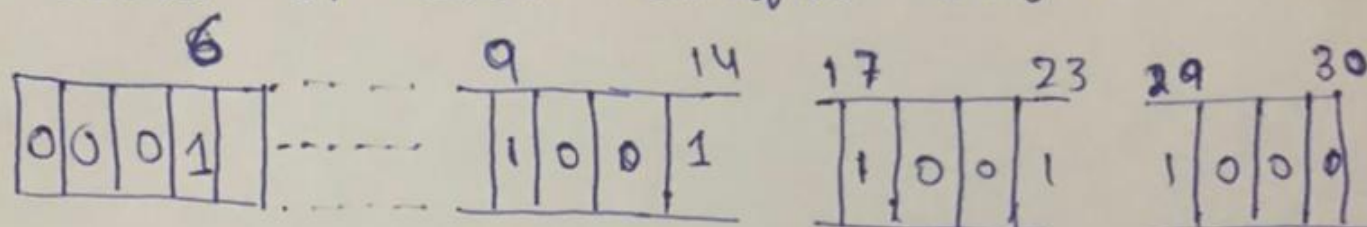
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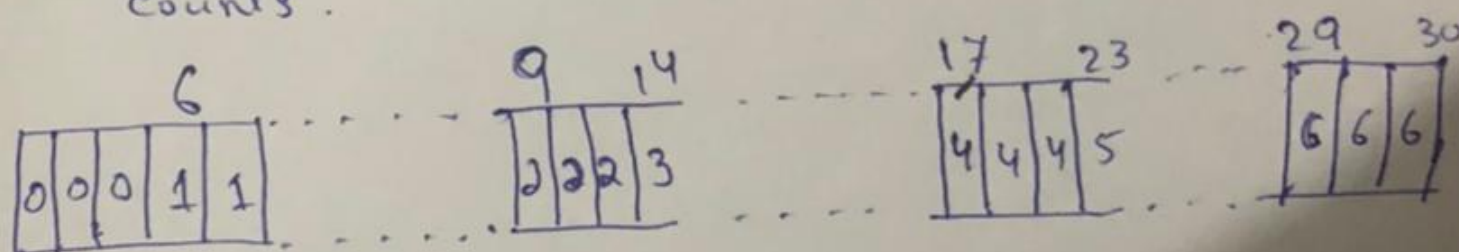
① #2 part ②

{17, 9, 6, 23, 14, 29}

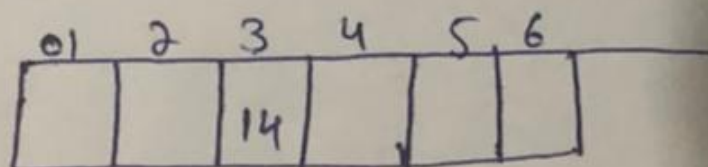
- ① simplicity consider data in range from 0 → 30. create array to store count of each unique object



- ② Modify the count array by adding the previous counts.



- ③ Corresponding values represent the places in the count array. Place the objects in the correct position and decrease the count by one.



Note:- as the total count is 6 we need only 6 places.

First element in original array = 14
 index [14] in count array = 3. ∴ 14 is placed in loc 3
 and index [14] in count array = 3 - 1 = 2

Q#2

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(4) ^{continue past 6} Next element = 6 count Array [6] = 1 \therefore place pos 1

Reduce

count Array [6] = 1 - 1 = 0

1	2	3	4	5	6
6		14			

(5) Next element = 17

count [17] = 4

 \therefore place 17 in pos 4

Reduce count [17] = 17 = 4 - 1 = 3

1	2	3	4	5	6
6		14	17		

(6) Next element = 23

count [23] = 5

place 23 in loc 5

count [23] = 5 - 1 = 4

1	2	3	4	5	6
6		14	17	23	

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Q#2 continue part
B

- ⑦ Next element = 9
count [9] = 2
Place 9 in loc 2
count [9] = 2 - 1 = 1

1	2	3	4	5	6
6	9	14	17	23	

- ⑧ Next element = 29
count [29] = 6
Place 29 in loc 6
count [29] = 6 - 1 = 5

1	2	3	4	5	6
6	9	14	17	23	29

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Sig. Director
ID 2016266241
Muhmmad Tanveer
Q # a part c
Sig. Customer

RADIX SORT

A = {17, 9, 6, 23, 14, 29}

① place element
in 1 column

17
9
6
23
14
29

② sort element
on per lower
radix

23
14
17
29
09
06

③ sort list 2nd
per 2nd
Radix

06
09
14
17
23
29

list are sorted

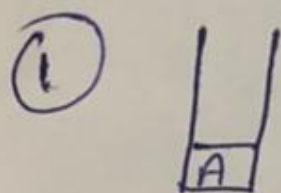
① #3 ^{Muhammad} Tanveer ID 2016266241

Breadth first search:

- ① Use a queue
- ② state are { Undiscovered, discovered & processed }
- ③ Put elements in queue only if their state is undiscovered. After putting them in queue, mark them as discovered.
- ④ Those that are discovered, make them discovered (parents) of undiscovered value.
- ⑤ To process a node removes from queue and put in queue undiscovered neighbours of the node.

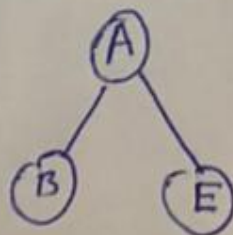
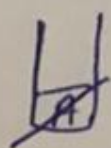
A undiscovered

A discovered

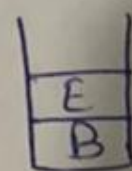


②

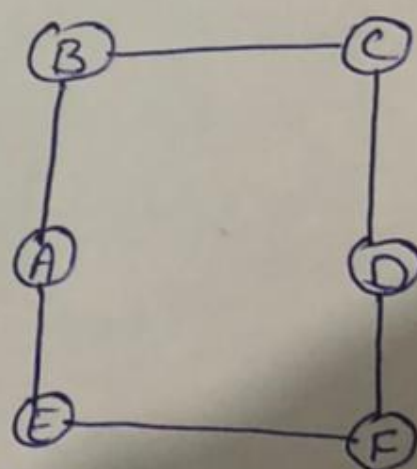
A - process



③



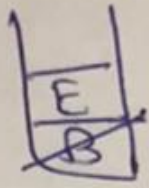
B, E = discovered



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Q3 part a continue

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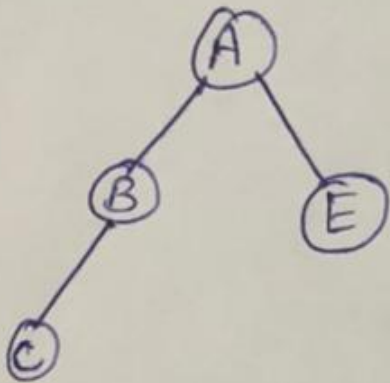
④



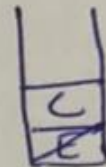
B - process

C - discovered

$\{C\} = B$



⑤

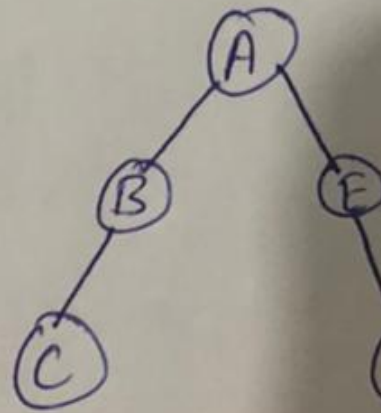
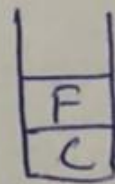


E - process

F - discovered

A already discovered
so don't ~~add~~ Added
A

⑥



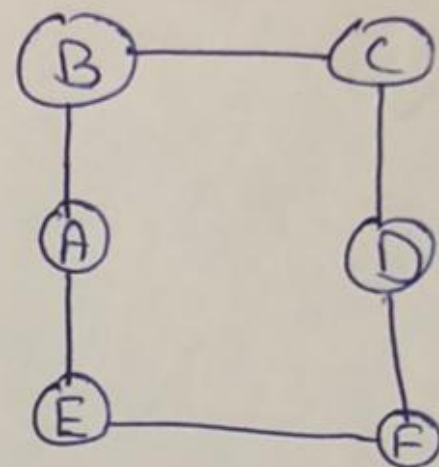
F - Process

C - Process

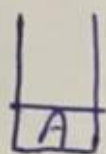
Final ~~state~~ Tree

Q#3 Part B
Muhammad Tanveer
Depth first search
Using a stack

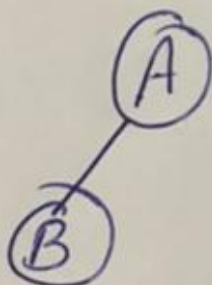
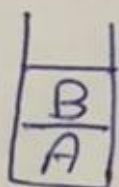
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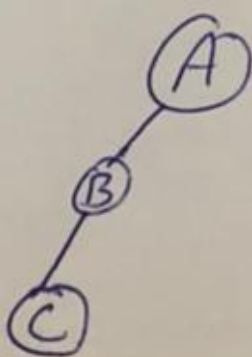
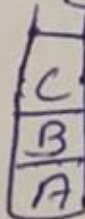
① A - Dis



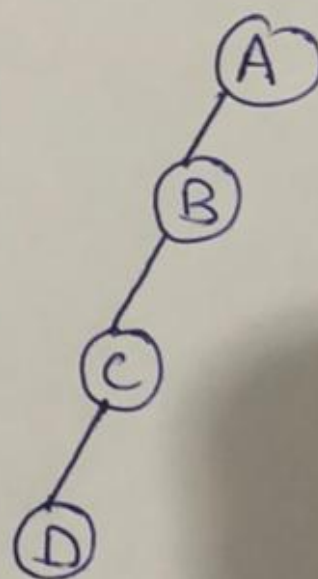
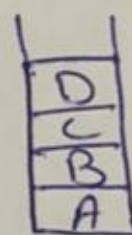
② B - Dis



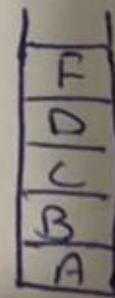
③ C - Dis



④ D - Dis

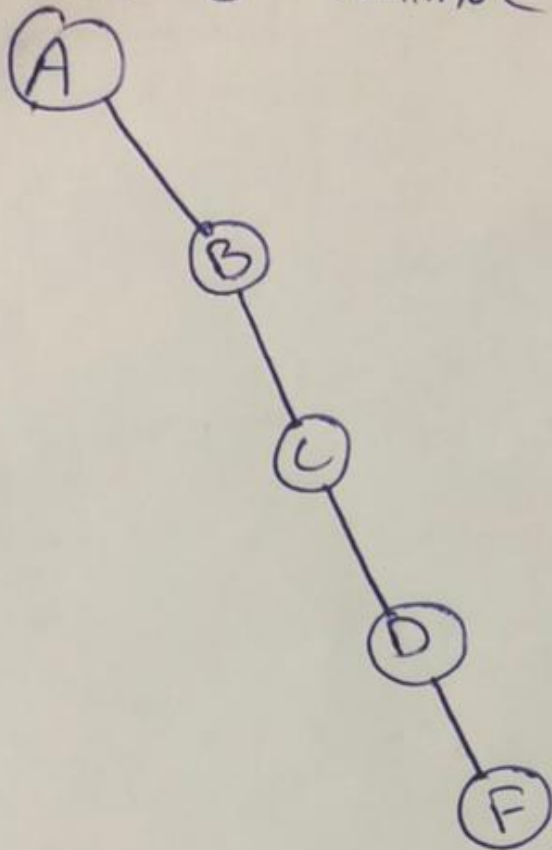
C is already
discovered

⑤ F - Dis



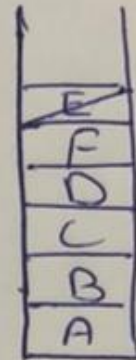
Q# 3 part b continue

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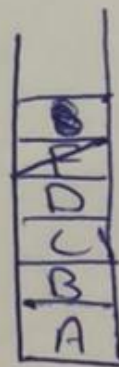
⑦ All $E = F \in A$ have been discovered

E-process

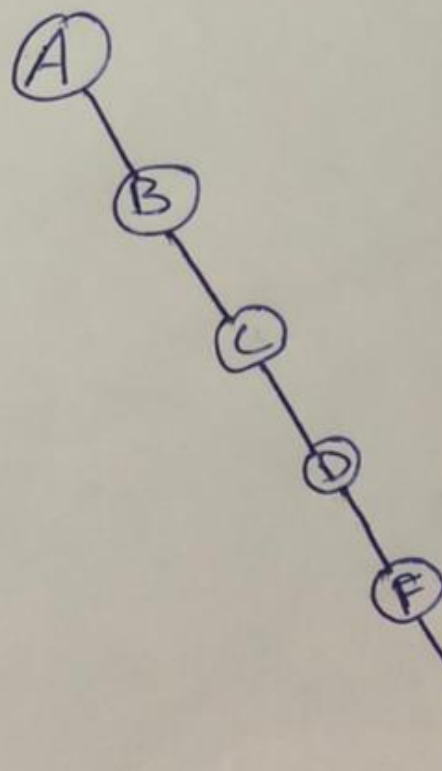
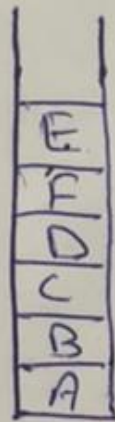


⑧ All neighbours are discovered

F-process



⑥ E-Dis



⑨ D-P

⑩ C-P

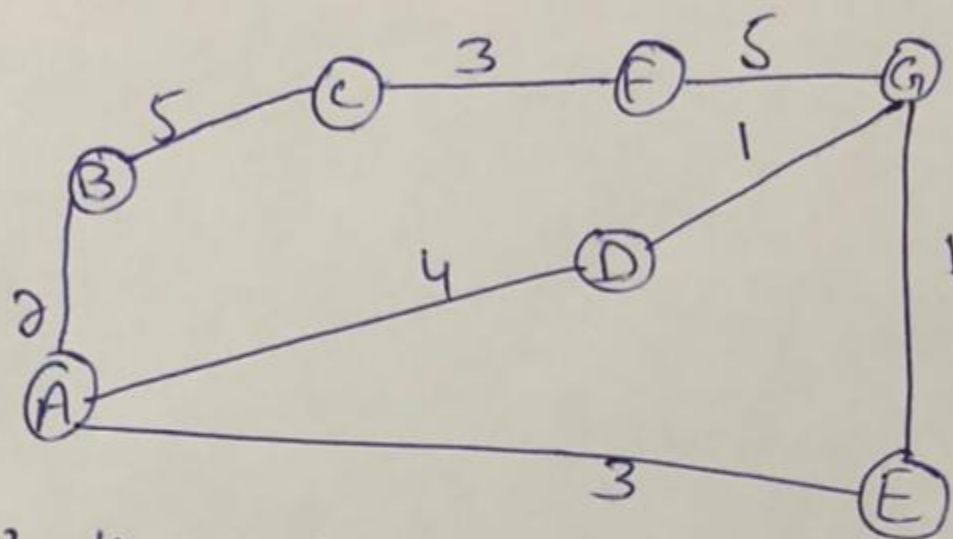
⑪ B-P

⑫ A-P

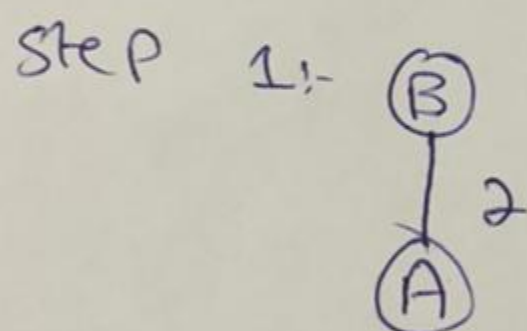
⑬ → Final Graph

M. Tanveer Khan
Question 4 Part (a) Prim's Algorithm

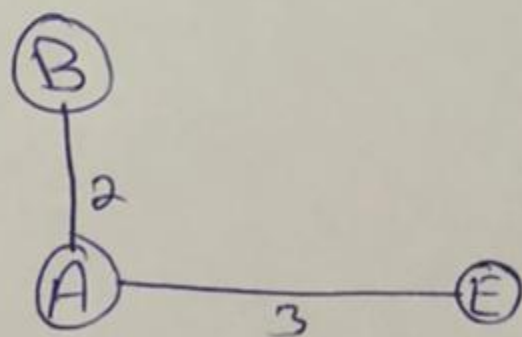
F2016266241



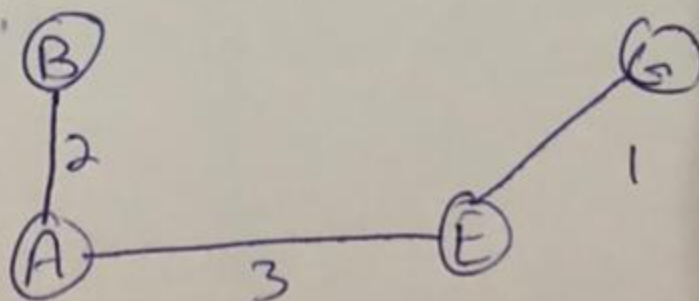
Starting From A



Step 2:-

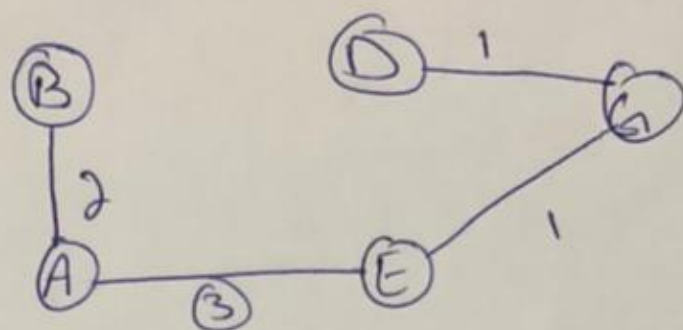


Step 3:-

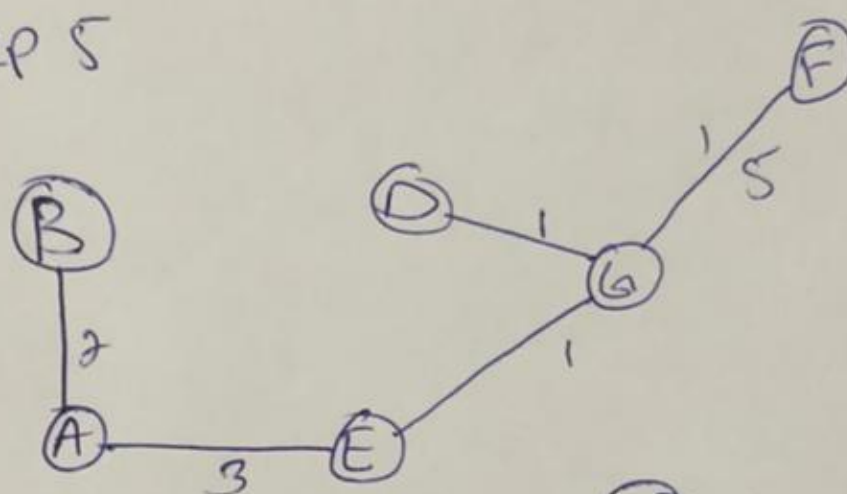


Question Part (a) continue

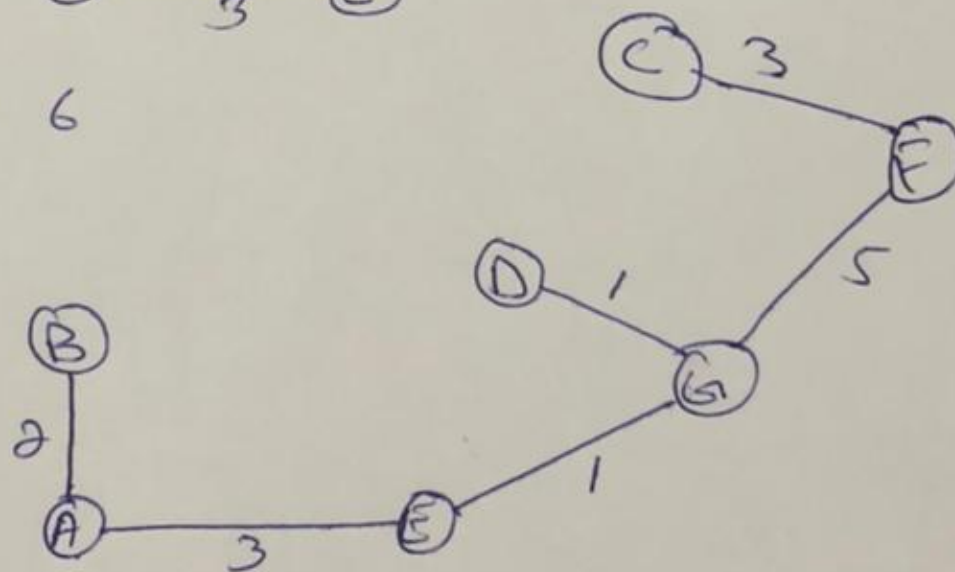
Step 4:-



Step 5



Step 6

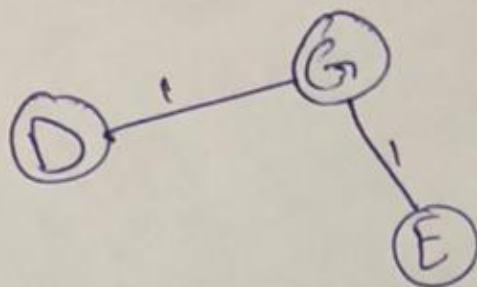


total minimum spanning tree according to
prim's is (15)

Question 2 part (b)
Kruskal Algorithm

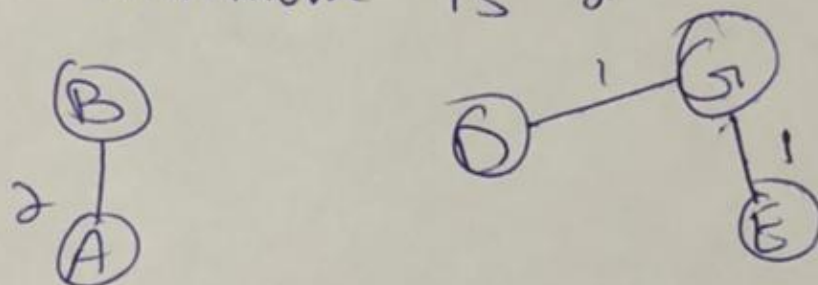
step 1:-

Minimum value is 1



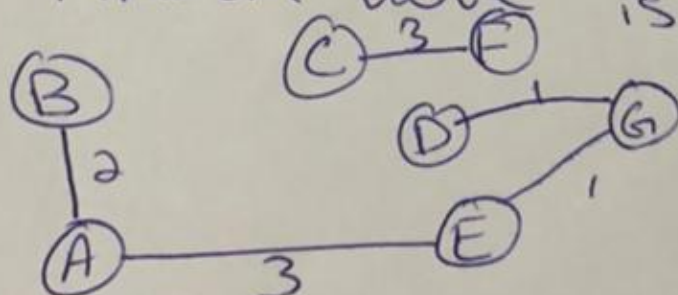
step 2:-

Minimum is 2



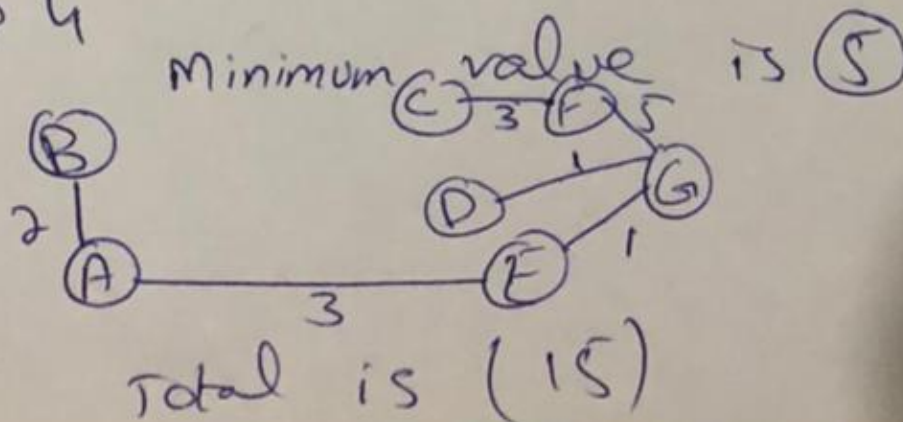
Step 3

Minimum value is 3



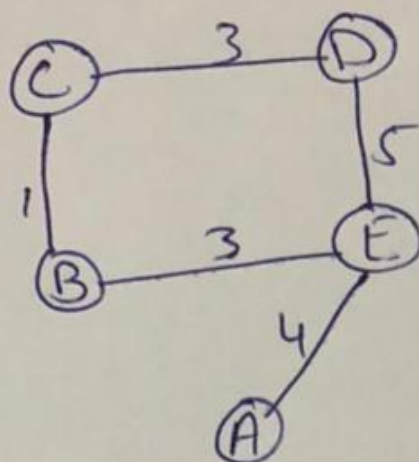
Step 4

Minimum value is 5



Q#5 Part a

(a) Starting point Node A



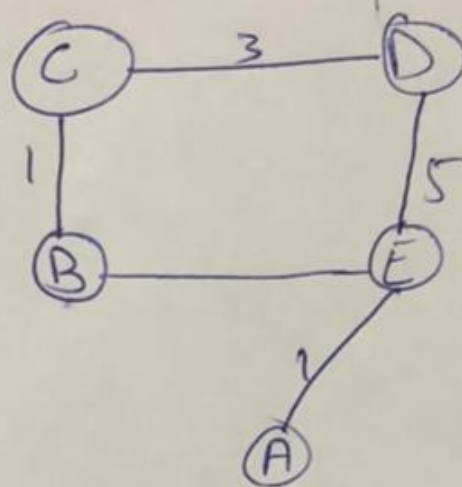
source vertex $\rightarrow A$
 considering $A = 0$

A di $+c(i,j)d(i)$
 the
 $d(j) = d(i) + c(i,j)$

	A	B	C	D	E
A	0	∞	∞	∞	∞
E		∞	∞	9	4
B					
C					
D					

Q# 5 part B

(B) starting point E



	A	B	C	D	E
E	4 ∞	∞	∞	∞	0
B	4	3	∞	∞	0
C			4	8	0
D				5	
A					7

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Q# 6

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Solve sudoku problem by showing
all step

①

1		2	
3		4	
			3

In sudoku every row, every column, and
every box contain 1 to 4 Numbers &
occurring only 1

②

1		2	
3		4	
		1	3

first we want 1 to 4 numbers we need
1 in box

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Q# 6 continue

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②

1		2	
		3	
3		4	
1		1	3

Now we complete column as, we need

③

1		2	
		3	
3		4	
2		1	3

we just take care of we have only
one number row and column only one
value

④

1		2	
4		3	
3		4	
2			

Now we complete 2 columns

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Q# 6 continue

(5)

1		2	
4		3	
2		4	
2	4	1	3

now we complete
2 column 1 row
in

(6)

1		2	
4		3	
3	1	4	
2	4	1	3

as we need 4 in
this row

(7)

1		2	
4		3	
3	1	4	2
2	4	1	3

in this we complete
another row

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Q# 6 continue

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8

1	3	2	
4		3	
3	1	4	2
2	4	1	3

in this we full
another box

9

1	3	2	
4	2	3	
3	1	4	2
2	4	1	3

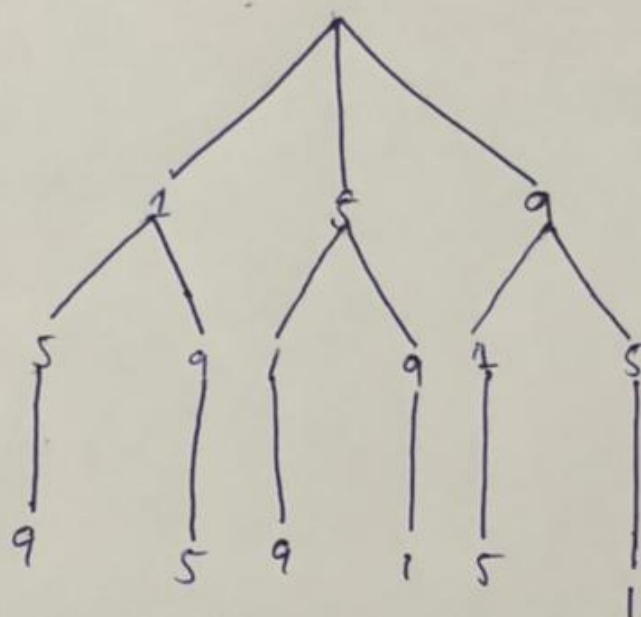
10

1	3	2	4
4	2	3	1
3	1	4	2
2	4	1	3

Q#7 Part a

(a) List of all permutation

$$S = \{1, 5, 9\}$$



$$= \{1, 5, 9\}, \{1, 9, 5\}, \{5, 1, 9\}, \{5, 9, 1\}, \\ \{9, 1, 5\}, \{9, 5, 1\}$$

(c) List of

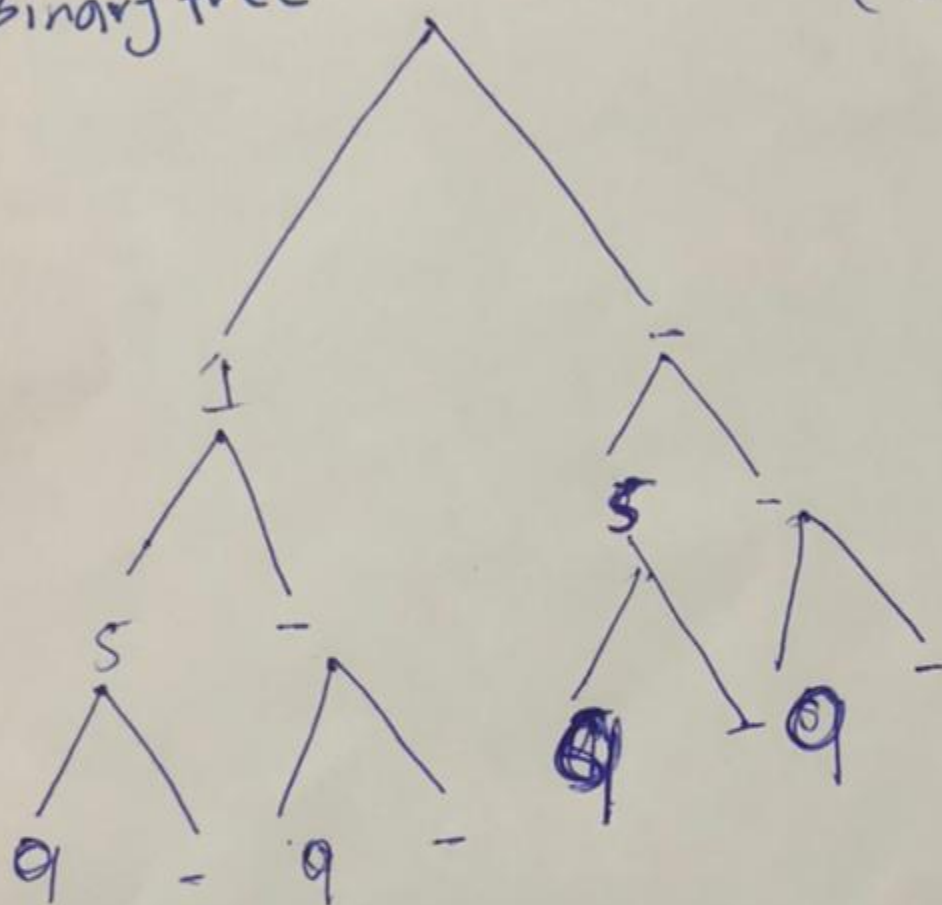
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Q#7 part b

List of subset

Binary tree

 $\{1, 5, 9\}$ $2^3 = 8$ subset

Total subset is

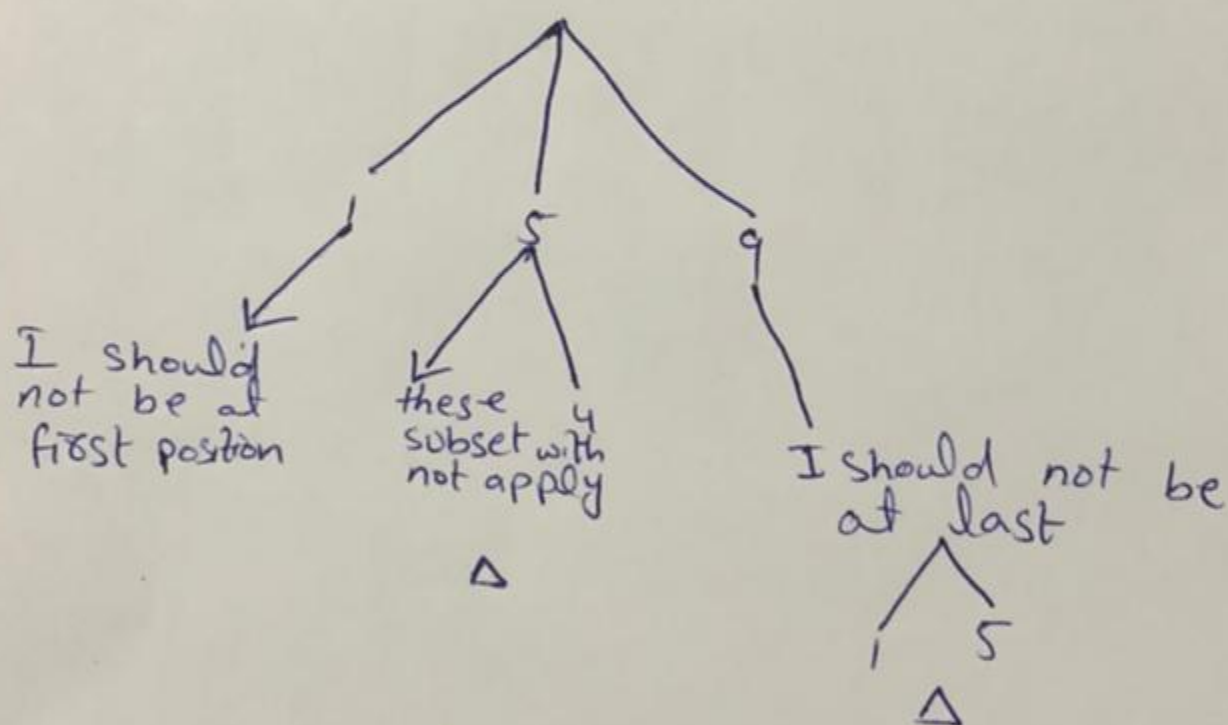
 $\{\{\}, \{1\}, \{5\}, \{9\}, \{1, 5\}, \{1, 9\}, \{5, 9\}, \{1, 5, 9\}\}$

Q #70 Part C

© List of deranged permutation
 $\{1, 5, 9\}$

means 1 should not be at
 position at 1st

9 should not be at last
 position



$$DN = N! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \dots + \frac{1}{N!} \right)$$