45TH AUSTRALASIAN COMBINATORICS CONFERENCE
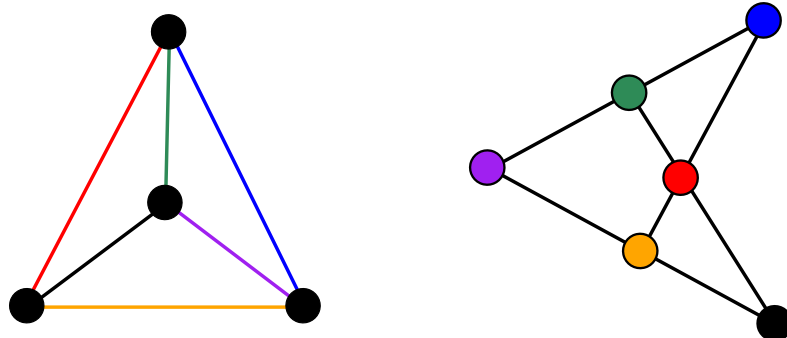
The University of Western Australia, December 11–15, 2023

Welcome!

This is the fifth time the ACC (formerly, ACCMCC) has been hosted in Perth, having previously been at UWA and/or Curtin University in the years 1984, 1992, 2001 and 2013. There are more than 65 registrants for this year, making it the second largest ACC/ACCMCC to be hosted in Western Australia. We wish you an interesting and exciting conference, and a pleasant stay in Perth.

The organisers:
John Bamberg
Alice Devillers
Michael Giudici
Luke Morgan
Cheryl Praeger
Gordon Royle

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Sunday

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1 Invited talks

The Hamilton decomposition problem

Sara Davies

The University of Queensland

Determining whether an arbitrary graph has a Hamilton cycle is a classic problem in graph theory. A *Hamilton decomposition* of a graph is a set of edge-disjoint Hamilton cycles that collectively contain all of the edges of the graph. The study of Hamilton decompositions dates back to the late 1800's and has received a lot of attention since the 1980's. In this talk, I will survey some of the progress made on this problem, especially on Hamilton decompositions of Cayley graphs, infinite graphs, line graphs and graph products.

How to design a graph with three eigenvalues

Gary Greaves

Nanyang Technological University, Singapore

(Joint work with Jose Yip)

Graphs with three distinct eigenvalues are fundamental objects of study in spectral graph theory. The most well-known examples are strongly regular graphs. In 1995, Willem Haemers posed a question at the 15th British Combinatorial Conference: "Do there exist any connected graphs having three distinct eigenvalues apart from strongly regular graphs and complete bipartite graphs?"

Muzychuk and Klin initiated the study of a graph with three distinct eigenvalues via its Weisfeiler-Leman closure (also known as the coherent closure). They classified such graphs whose Weisfeiler-Leman closure has rank at most 7. In this talk, I will provide a brief overview of the history of non-regular graphs with three distinct eigenvalues, as well as present our recent results on such graphs whose Weisfeiler-Leman closure has a small rank. Our results include the discovery of a new non-regular graph with three distinct eigenvalues obtained from a quasi-symmetric design and a new conjecturally infinite family of non-regular graphs having three distinct eigenvalues obtained by switching Latin square graphs.

Algebraic graph theory and quantum walks

Krystal Guo

Korteweg-De Vries Institute for Mathematics, University of Amsterdam and QuSoft

The interplay between the properties of graphs and the eigenvalues of their adjacency matrices is well-studied. Important graph invariants, such as diameter and chromatic number, can be understood using these eigenvalue techniques. In this talk, we bring these classical techniques in algebraic graph theory to the study of quantum walks.

A system of interacting quantum qubits can be modelled by a quantum process on an underlying graph and is, in some sense, a quantum analogue of random walk. This gives rise to a rich connection between graph theory, linear algebra and quantum computing. In this talk, I will give an overview of applications of algebraic graph theory in quantum walks, as well as various recent results on discrete-time quantum walks and strong cospectrality of vertices.

The Saturation Spectrum of odd cycles

Andre Kundgen

address

(Joint work with Ronald J. Gould and Minjung Kang)

Given a graph H, we say that a graph G is H-saturated if H is not a subgraph of G, but the addition of any new edge to G creates at least one copy of H. In this talk we will discuss all pairs (n, m) for which there is a C_5-saturated graph on n vertices and m edges. In addition, we determine all but $O(nk)$ possible sizes for n-vertex H-saturated graphs when H is an odd cycle C_{2k+1} for $k \geqslant 3$.

Quadratic forms in design theory

Padraig Ó Cathráin

Dublin City University

(Joint work with Guillermo Nuňez Ponasso, Oliver Gnilke and Oktay Olmez.)

The classification of quadratic forms over the rational numbers, due to Minkowski, Hilbert and Hasse among others, is a major achievement of mathematicians in the early twentieth century. In concrete terms, given square rational matrices A and B it yields necessary and sufficient conditions for the existence of an invertible matrix X such that $X^\top A X = B$. (In contrast, the Jordan Canonical Form gives necessary and sufficient conditions for solvability of $X^{-1}AX = B$ over an algebraically closed field, and the Frobenius Canonical Form solves the conjugacy problem over an arbitrary field.) The main tools in the classification of quadratic forms are Legendre and Hilbert symbols, which describe existence of solutions to certain quadratic equations.

Groundbreaking work of Bruck, Ryser and Chowla in the mid-twentieth century applied this theory to obtain non-existence of certain combinatorial designs. While in theory the application is straightforward, Marshall Hall described the computations as detailed and troublesome. This seems to have scared a substantial number of combinatorialists. In this talk, we aim to restore the reputation of the Bruck-Ryser-Chowla theorem by demonstrating that the algebraic manipulations are less familiar, but not more difficult, than Gaussian elimination.

I will motivate this talk by an application to a problem on symmetric designs which Darryn Bryant posed to me in 2013, while I was a postdoc at the University of Queensland.

New Ramsey multiplicity bounds and search heuristics

Tibor Szabó

Freie Universität Berlin

(Joint work with Olaf Parczyk, Sebastian Pokutta, and Christoph Spiegel.)

We study two related problems concerning the number of monochromatic cliques in two-colorings of the complete graph that go back to questions of Erdős. Most notably, we "significantly" improve the best known upper bounds on the Ramsey multiplicity of K_4 and K_5 and settle the minimum number of independent sets of size four in graphs with clique number at most four. Motivated by the elusiveness of the symmetric Ramsey multiplicity problem, we also introduce the off-diagonal variant and obtain tight results when counting monochromatic K_4 or K_5 in only one of the colors and triangles in the other. The extremal constructions turn out to be blow-ups of finite graphs and were found through search heuristics. They are complemented by lower bounds and stability results established using flag algebras, resulting in a fully computer-assisted approach. More broadly, these problems lead us to the study of the region of possible pairs of clique and independent set densities that can be realized as the limit of some sequence of graphs.

'Segre-type' theorems: combinatorial characterisations for algebraic objects

Geertrui Van de Voorde

The University of Canterbury

One of the most beautiful results within finite geometry is Segre's characterisation of conics in Desarguesian projective planes of odd order. In 1955, Segre showed that in those planes, the coordinates of a point set that has the same *combinatorial* properties as a conic, must have the same *algebraic* property of satisfying a quadratic equation. In even order planes, the situation is vastly different, and the classification of ovals remains is still an open problem.

Several 'Segre-type' questions have been studied for objects such as *quadrics*, *Hermitian varieties*, and more generally, for sets with *few intersection numbers*.

In this talk, I'll give an overview of some of the history of this subject and present new recent results.

Local actions and eigenspaces of vertex-transitive graphs

Gabriel Verret

The University of Auckland

When studying families of vertex-transitive graphs, it is often important to have control of the size of vertex-stabilisers of the automorphism groups. It turns out that the "local" action of the automorphism group plays a crucial role. I'll explain this connection, describe some known results and some more recent connection with the size of the eigenspaces of such graphs over some finite fields.

2 Contributed talks

Latin squares without proper subsquares

Jack Allsop*

Monash University

(Joint work with Ian Wanless)

A Latin square of order n is an $n \times n$ matrix of n symbols, such that each symbol occurs exactly once in each row and column. A subsquare of order k is a $k \times k$ submatrix of a Latin square that is itself a Latin square. Every Latin square of order n contains n^2 subsquares of order one, and one subsquare of order n. All other subsquares are called proper. If a Latin square contains no proper subsquares then it is called N_∞. Around 50 years ago Hilton conjectured that an N_∞ Latin square of order n exists for all sufficiently large n. Hilton's conjecture was previously known to hold for all integers n not of the form $2^a 3^b$ for integers $a \geqslant 1$ and $b \geqslant 0$. We resolve Hilton's conjecture by constructing N_∞ Latin squares for all previously unresolved orders.

Groups of Lie Type Acting on Generalised Quadrangles

Vishnuram Arumugam*

The University of Western Australia

Incidence geometry is the study of geometric structures involving a collec- tion of points and lines along with a relation (called incidence) which tells us whether a point lies on a line. A generalised polygon is a type of point-line incidence structure that was introduced by Jacques Tits in 1959 to study the groups of Lie type as the symmetries of geometric objects. Since then, these objects have been studied extensively in the areas of group theory and finite geometry. The classification of these objects started from Weiss and Tits and many results about the existence (and non-existence) of generalised polygons under various symmetry conditions (point primitivity, flag transitivity and so on) since then. I will provide a survey of the work that has been done and some recent progress in this classification.

Point-box incidences and logarithmic density of semilinear graphs

Abdul Basit

Monash University

(Joint work with Artëm Chernikov, Sergei Starchenko, Terence Tao, and Chieu-Minh Tran)

Zarankiewicz's problem in extremal graph theory asks for the maximum number of edges in a bipartite graph on n vertices which does not contain a copy of $K_{k,k}$, the complete bipartite graph with k vertices in both classes. We will consider this question for incidence graphs of geometric objects. Significantly better bounds are known in this setting, in particular when the geometric objects are defined by systems of algebraic inequalities. We show even stronger bounds under the additional constraint that the defining inequalities are linear. We will also discuss connections of these results to combinatorial geometry and model theory.

List Colouring Graphs with bounded Maximal Local Edge Connectivity

Sam Bastida*

Victoria University Wellington

(Joint work with Nick Brettell)

List colouring is a generalisation of the traditional notion of colouring where each vertex of the graph can have a different palette. A *proper colouring* of a graph G maps each vertex of G to a colour such that adjacent vertices have different colours. A *k-list assignment* L is an assignment of a list of k colours to each vertex of G. A graph is *L-colourable* if it has a proper colouring where the colour for each vertex v is in the list $L(v)$. A graph G is *k-choosable* if for every k-list assignment L, the graph G is L-colourable. This notion generalises k-colouring: a graph is *k-colourable* if it is ϕ-colourable where ϕ maps each vertex to the same list of k colours. While some results about k-colourability generalise to k-choosability, such as Brooks' Theorem, others, such as the Four Colour Theorem, do not. Brooks' Theorem states that a connected graph G with maximum degree Δ is Δ-colourable, except when G is a complete graph or odd cycle. Stiebitz and Toft (2018) generalised Brooks' Theorem, showing that a graph G is k-colourable, where k is the maximum number of edge-disjoint paths between two vertices of G, except when each block of G can be obtained from complete graphs or odd cycles using Hajós joins. We consider an extension of this result to k-choosability, specifically in the case where $k = 3$.

A comparison of graph width parameters

Nick Brettell

Victoria University of Wellington

(Joint work with Andrea Munaro, Daniel Paulusma, and Shizhou Yang.)

The classic example of a width parameter is treewidth, which, loosely speaking, gives a measure of how tree-like a graph is. Due to Courcelle's theorem, many problems are known to be polynomial-time solvable for a class of graphs with bounded treewidth. Say that a parameter p is *less restrictive* than a parameter q if there exists a function f such that $p(G) \leqslant f(q(G))$ for every graph G (it is "less restrictive" in the sense that a class may have bounded p-width but unbounded q-width). These days, there is a rich landscape of width parameters that are less restrictive than treewidth, but, like treewidth, facilitate efficient algorithms. In this talk, we'll be interested in clique-width, mim-width, sim-width, and tree-independence number. I'll give a brief introduction to each of these parameters, and touch on why they are of interest. We'll then compare them when restricted to a class of graphs with no $K_{t,t}$ subgraph, the class of line graphs, and the common generalisation of the class of graphs with no induced $K_{t,t}$ subgraph. In particular, Gurski and Wanke (2000) showed that although clique-width is less restrictive than treewidth, these parameters are equivalent for graphs with no $K_{t,t}$ subgraph. Gurski and Wanke (2007) also showed that a class of graphs has bounded treewidth if and only if the corresponding class of line graphs has bounded clique-width. We generalise these results to mim-width, sim-width, and tree-independence number.

Association schemes on triples from two-transitive groups

Dom Vito A. Briones

University of the Philippines - Diliman

Association schemes on triples (ASTs) are higher-dimensional analogues of classical association schemes where the relations and adjacency algebras are ternary instead of binary. Analogous to Schurian association schemes, ASTs arise from the actions of two-transitive groups. In this presentation, we provide the sizes and third valencies of the ASTs obtained from the two-transitive permutation groups. We also determine the intersection numbers of the ASTs obtained from the sporadic two-transitive groups and some subgroups of the affine and projective semilinear groups $A\Gamma L(k,n)$ and $P\Gamma L(k,n)$.

Group divisible designs with block size three and two group sizes

Yudhistira Andersen Bunjamin*

UNSW Sydney

(Joint work with Oden Petersen)

A k-GDD, or group divisible design with block size k, is a triple $(X, G, \mathcal{B})$ where X is a set of points, G is a partition of X into subsets (called groups) and $\mathcal{B}$ is a collection of k-element subsets of X (called blocks) such that any two points from distinct groups appear together in exactly one block and no two distinct points from any group appear together in any block. There are a number of known necessary conditions for the existence of a GDD. However, these conditions are not sufficient.

In this talk, we will present constructions for some 3-GDDs with two group sizes where one group size is a multiple of the other group size. The talk will have a particular focus on how some recent advancements regarding the existence of 4-GDDs with two group sizes have enabled the construction of some infinite families of 3-GDDs with two group sizes.

The distinguishing number of 2-arc-transitive bipartite graphs

Lei Chen*

The University of Western Australia

(Joint work with Alice Devillers, Luke Morgan and Friedrich Rober)

We investigate the distinguishing number of 2-arc-transitive bipartite graphs. The distinguishing number of a graph is the minimal number of colours needed to colour the vertices in such a way that the only automorphism preserving the colouring is the identity automorphism.

This work is the continuation of the paper by Devillers, Morgan and Harper in 2018, which analysed the distinguishing number of the non-bipartite graphs. In 2018 paper, it turns out that, except for finitely many examples, all of the semi-primitive non-partite graphs have distinguishing number 2.

Our research shows that for 2-arc-transitive non-partite graphs, apart from complete bipartite graphs and crown graphs, the distinguishing numbers are small.

Covering Arrays via Finite Fields

Charles Colbourn

Arizona State University

In order to construct covering arrays of strength t and index λ on q symbols, one effective and well-studied method forms a base array with "few" rows whose entries are elements of $\mathbb{F}_q^t$. Each row of the base array underlies q^t rows of the covering array. A t-tuple T of columns is covering in a row of the base array when the corresponding q^t rows of the covering array contain each of the q^t symbol tuples in T. When every t-tuple of columns is covering in at least λ rows, the base array is a covering perfect hash family (CPHF$_\lambda$). When λ is 'small' and q is 'large', CPHFs yield the best probabilistic upper bounds on sizes of covering arrays and the best current construction algorithms. In this talk we revise the conditions on CPHFs to account for the partial coverage arising from non-covering t-tuples of columns. This improves the quality of the bounds on covering array sizes, particularly when λ is 'large' or q is 'small'.

Diameter of some families of quotient-complete arc-transitive graphs

John Mel Dacaymat*

University of the Philippines Diliman

(Joint work with Carmen Amarra, Joseph Ray Clarence Damasco)

A graph is *quotient-complete* if it has at least one nontrivial normal quotient graph that is a complete graph, and if each of its nontrivial normal quotient graphs is either an empty graph or a complete graph. Quotient-complete graphs arise, via normal quotient reduction, as basic graphs in the family of vertex-transitive graphs. Quotient-complete arc-transitive graphs with diameter two are of particular interest, as they contain some important subfamilies. Among these graphs, those with at least three complete normal quotients are completely known, except for those associated with the one-dimensional affine group $\Gamma L_1(q)$. We aim to address the gap in the classification of these graphs. Specifically, we provide bounds for the diameter of quotient-complete arc-transitive graphs with at least three complete normal quotients that are associated with $\Gamma L_1(q)$. In addition, we present an example of an infinite family of quotient-complete arc-transitive graphs with exactly two complete normal quotients and find bounds on its diameter.

A strongly regular graph co-spectral and non-isomorphic to $\mathrm{NO}^+(8,2)$

Jan De Beule

Vrije Universiteit Brussel

The graph $\mathrm{NO}^+(8,2)$ is strongly regular with parameters $(120, 63, 30, 36)$. It can be constructed using a quadratic form of Witt index 4 on $\mathrm{GF}(2)^8$. Then its vertices are the set of non-singular vectors. Two vertices are adjacent if and only if they are orthogonal with relation to the quadratic form. Its automorphism group is $\mathrm{P\Gamma O}^+(8,2)$.

In their recent book – Strongly Regular Graphs – Brouwer and Van Maldeghem mention the existence of a non-isomorphic, strongly regular graph with the same parameters, admitting $\mathrm{Sym}(7)$ as automorphism group. In this talk we discuss how the adjacency relation of $\mathrm{NO}^+(8,2)$ can be modified to obtain this graph, it turns out that the unique ovoid (and spread) of the triality quadric $\mathrm{Q}^+(7,2)$ plays a central role. We also discuss further interesting properties such as that fact the cliques and co-cliques get switched by modifying the adjacency relation of $\mathrm{NO}^+(8,2)$.

Compatible groups and inverse limits

Zhaochen Ding*

University of Auckland

Two finite groups L_1 and L_2 are called compatible if there is a group G with two isomorphic normal subgroups N_1 and N_2 such that $G/N_1 \cong L_1$ and $G/N_2 \cong L_2$. In this talk, we will discuss some recent work (joint with Gabriel Verret) on compatibility of groups, including a new construction based on inverse limits.

Proper Minor-Closed Classes of Graphs have Assouad-Nagata Dimension 2

Marc Distel*

Monash University

Asymptotic dimension and Assouad-Nagata dimension are measures of the large-scale shape of a class of graphs. Bonamy et al. [J. Eur. Math. Society] showed that any proper minor-closed class has asymptotic dimension 2, dropping to 1 only if the treewidth is bounded. We improve this result by showing it also holds for the stricter Assouad-Nagata dimension. We also characterise when subdivision-closed classes of graphs have bounded Assouad-Nagata dimension.

Erdős-Ko-Rado theorems for finite general linear groups

Alena Ernst*

Paderborn University

(Joint work with Kai-Uwe Schmidt)

We call a subset Y of the finite general linear group $\mathrm{GL}(n, q)$ *t-intersecting* if $\mathrm{rk}(x - y) \leq n - t$ for all $x, y \in Y$. In this talk we give upper bounds on the size of t-intersecting sets and characterise the extremal cases that attain the bound. This is a q-analog of the corresponding result for the symmetric group, which was conjectured by Deza and Frankl in 1977 and proved by Ellis, Friedgut, and Pilpel in 2011. The results are obtained by using eigenvalue techniques and the theory of association schemes plays a crucial role.

Enumeration of the Hosoya Index in Anthracene System

Muhammad Talha Farooq*

King Mongkut's University of Technology Thonburi, Bangkok, Thailand

In chemical graph theory, topological indices play an important role and have various uses in quantitative structure–property relationship (QSPR) as well as quantitative structure-activity relationships (QSARs). The Hosoya index is one of them, performing a molecular descriptor in mathematical chemistry. Therefore, progressing with investigations requires computing the Hosoya index of distinct molecular graphs. This research presents a computational method for determining the Hosoya index of triple benzenoid systems (Anthracene) by using transfer matrix approach and the Hosoya vector.

Spreading primitive groups of diagonal type do not exist

Saul Freedman

The University of Western Australia

(Joint work with John Bamberg and Michael Giudici)

The synchronisation hierarchy of finite permutation groups, introduced by Araújo, Cameron and Steinberg in 2017, consists of classes of groups lying between 2-transitive groups and primitive groups. This includes the classes of synchronising and separating groups, defined in terms of combinatorial properties of related graphs, and the class of spreading groups, defined in terms of sets and multisets of permuted points. Araújo et al. proved that the members of these classes are primitive of almost simple, affine or diagonal type. In addition, Bray, Cai, Cameron, Spiga and Zhang showed in 2020 that any such diagonal type group must have socle $T \times T$ for some non-abelian finite simple group T. In this talk, we prove that no spreading group of diagonal type exists, by considering transitive actions (and several character tables) of the non-abelian finite simple groups.

Levenshtein's conjecture for sequence covering arrays

Dani Gentle*

Monash University

A sequence covering array is a set of permutations of the v-element alphabet $\{0, ..., v-1\}$ such that every sequence of t distinct symbols of the alphabet appears in the specified order in at least one permutation. A key conjecture in this area attributed to Levenshtein concerns when it is possible to build such an array in which each sequence appears in exactly one permutation. In this talk, I will discuss existing results on this conjecture, and present new results for the next open case of the conjecture.

Existence of Latin Squares with Constrained Transversals

Afsane Ghafari Baghestani*

Monash University

A Latin Square is an $n \times n$ array where entries are chosen from the set $\{1, 2, \ldots, n\}$ with the property that every symbol appears exactly once in every row and column. A transversal of such a square is defined to be a selection of n entries, one from each row and each column, where we choose every symbol exactly once.

Enumerating dihypergraphs

Catherine Greenhill

UNSW Sydney

(Joint work with This is joint work with Tam a's Makai (Ludwig Maximilian University of Munich))

A dihypergraph is a directed hypergraph: that is, a set of vertices and a set of directed edges, where each edge is partitioned into a head and a tail. The head and tail of an edge must be disjoint. Directed hypergraphs arise in many applications, including modelling chemical reactions and in the study of relational databases.

I will discuss some work on finding asymptotic enumeration formulae for directed hypergraphs where the in-degrees and out-degrees of the vertices, and the head and tail sizes for the edges are all specified. If at least one of these four sequences is regular and the entries are not too large then the result follows easily from asymptotic enumeration formulae for sparse bipartite graphs. Otherwise we need a stricter assumption on the maximum degrees and maximum head/tail sizes, and the proof involves a martingale argument.

Transitive path decompositions of Cartesian products of complete graphs

Ajani De Vas Gunasekara

Monash University

An H-decomposition of a graph Γ is a partition of its edge set into subgraphs isomorphic to H. A transitive decomposition is a special kind of H-decomposition that is highly symmetrical in the sense that the subgraphs (copies of H) are preserved and transitively permuted by a group of automorphisms of Γ. This paper concerns transitive H-decompositions of the graph $K_n \square K_n$ where H is a path. When n is an odd prime, we present a construction for a transitive path decomposition where the paths in the decomposition are arbitrary large.

Perfect codes in Cayley graphs on $\mathbb{Z}_p \times \mathbb{Z}_p$ and $\mathbb{Z}_{p^k}$

Yusuf Hafidh*

University of Melbourne

(Joint work with Sanming Zhou and Binzhou Xia.)

A set of vertices C on a connected graph G is called a *perfect code* if every vertex not in C is a neighbor of exactly one vertex in C. In this talk, we will discuss the characterization of Cayley graph $Cay(G, S)$ that has a perfect code for two groups $G = \mathbb{Z}_p \times \mathbb{Z}_p$ and $G = \mathbb{Z}_{p^k}$ for prime p.

Connectivity Preserving Hamiltonian Cycles in k-Connected Dirac Graphs

Toru Hasunuma

Tokushima University

We show that for $k \geqslant 2$, there exists a function $f(k) = O(k)$ such that every k-connected graph G of order $n \geqslant f(k)$ with minimum degree at least $\frac{n}{2}$ contains a Hamiltonian cycle H such that $G - E(H)$ is k-connected. Applying Nash-Williams' result on edge-disjoint Hamiltonian cycles, we also show that for $k \geqslant 2$ and $\ell \geq 2$, there exists a function $g(k, \ell) = O(k\ell)$ such that every k-connected graph G of order $n \geqslant g(k, \ell)$ with minimum degree at least $\frac{n}{2}$ contains ℓ edge-disjoint Hamiltonian cycles $H_1, H_2, \ldots, H_\ell$ such that $G - \cup_{1 \leq i \leq \ell} E(H_i)$ is k-connected. As a corollary, we have a statement that refines the result of Nash-Williams for k-connected graphs with $k \leqslant 8$. Moreover, when the connectivity of G is exactly k, a similar result with an improved lower bound on n can be shown, which does not depend on the result of Nash-Williams.

Large sets of infinite-dimensional q-Steiner systems

Dan Hawtin

University of Rijeka

Let V be a vector space over the finite field $\mathbb{F}_q$. An $S(t,k,V)_q$, is a collection $\mathcal{B}$ of k-spaces of V such that every t-space of V is contained in a unique element of $\mathcal{B}$. An $LS(t,k,V)_q$, is a partition of the k-dimensional subspaces of V into $S(t,k,V)_q$ systems. In 1995, Cameron proved that if V has infinite dimension then an $LS(t,k,V)_q$ exists for all positive integers t,k with $t < k$. We give an explicit construction of an $LS(t,t+1,V)_q$ for all prime powers q, all positive integers t, and where V has countably infinite dimension.

Powers of planar graphs, product structure, and blocking partitions

Robert Hickingbotham*

Monash University

Graph product structure theory describes complex graphs in terms of products of simpler graphs. In this talk, I will introduce this subject and talk about a new tool called 'blocking partitions.' I'll show how this tool can be used to prove stronger product structure theorems for powers of planar graphs as well as k-planar graphs, resolving open problems of Dujmović, Morin and Wood, and Ossona de Mendez.

Spherical designs and the D_4 lattice

Masatake Hirao

Aichi Prefectural University

We study shells of the D_4 lattice with the concept of spherical design of harmonic index T (spherical T-design for short). We show that the $2m$-shell of D_4 is an antipodal spherical $\{10, 4, 2\}$-design on the 3-sphere, that the 2-shell (i.e., the D_4 root system) is a tight antipodal $\{10, 4, 2\}$-design in the terms of LP bound, and that the uniqueness of the 2-shell as an tight antipodal spherical $\{10, 4, 2\}$-design. Moreover, we report some applications of our results.

Matroid representation over finite rings

Koji Imamura

Kumamoto University

Matroids were introduced by H. Whitney to axiomatize combinatorial properties of finite sets of vectors in a vector space. Nevertheless, it is well-known that almost all matroids are non-representable as a finite set of vectors over a finite field. It is one of the most significant problems to determine whether a given matroid is representable over some field.

In this talk, we propose some representations of non-representable matroids by using matrices over finite rings. For this end, we adopted modular independence, introduced by Y.H. Park as one of the generalizations of linearly independence. It was originally defined over the ring $\mathbb{Z}_{p^e}$ of integers modulo p^e, where p is a prime and $e \in \mathbb{Z}_{>0}$, and then generalized to the case of Frobenius rings by S.T. Dougherty and H. Liu. We restrict ourselves to local rings R with the unique maximal ideal $\mathfrak{m}$, where the vectors $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_k \in R^n$ are said to be *modular independent* if $\sum a_i \boldsymbol{v}_i = \boldsymbol{0}$ implies $a_i \in \mathfrak{m}$ for all i. We will provide some conditions for a matrix over a finite ring to yield some matroid using modular independence. We also show that some well-known non-representable matroids can be represented in this way.

Cumulant expansion for counting Eulerian orientations

Mikhail Isaev

Monash University

We consider the problem of enumerating Eulerian orientations of a given graphs, that is, the orientations of its edges such that every vertex has the same in-degree and out-degree. This problem is #P-hard and corresponds to the crucial partition function in so-called "ice-type models" in statistical physics. In this work, we derive an asymptotic formula for approximating the number of Eulerian orientations of a graph with good expansion properties up to a multiplicative error $O(n^{-c})$, where c is an arbitrary fixed constant. The answer is in terms of cumulants of a multidimensional polynomial of Gaussian random variables. The proof relies on the new tail bound for the cumulant expansion series, which is of independent interest.

Safe Sets and Dominating Sets of Graphs

Pawaton Kaemawichanurat

King Mongkut's University of Technology Thonburi, Bangkok, Thailand

(Joint work with Shinya Fujita and Furuya Michitaka (Yokohama City University))

A subset S of vertices of a graph G is a safe set if, for a component H of $G - S$ and a component C of $G[S]$, we have $|V(H)| \leq |V(C)|$ whenever there is an edge joining vertices between H and C. Moreover, if the subgraph of G induced by safe set S, $G[S]$, is connected, then S is a connected safe set. The minimum cardinality of a safe set of G is called the safe number of G and is denoted by $s(G)$. Similarly, the minimum cardinality of a connected safe set of G is called the connected safe number of G and is denoted by $s_c(G)$. A subset D of vertices of a graph G is a dominating set of G if every vertex in $V(G) - D$ is adjacent to a vertex in D. Moreover, if $G[D]$ is connected, then D is called a connected dominating set of G. The minimum cardinality of a dominating set of G is called the domination number of G and is denoted by $\gamma(G)$. Similarly, the minimum cardinality of a connected dominating set of G is called the connected domination number of G and is denoted by $\gamma_c(G)$. In this paper, we prove that if G is a graph with the maximum degree Δ, then

$$f(\Delta) \leq s(G) \leq \lceil \frac{\gamma(G)(\Delta + 1)}{2} \rceil$$

where $f(\Delta) = \frac{\gamma + 6}{3}$ when $\Delta = 2$ and $f(\Delta) = \frac{\Delta^2 - 2\Delta - 3 + \sqrt{(2\Delta - \Delta^2 + 3)^2 + 4(3\Delta + \gamma(G))(\Delta - 2)}}{2(\Delta - 2)}$ when $\Delta \geq 3$. Moreover, for a connected graph G, we have

$$g(\Delta) \leq s_c(G) \leq \lceil \frac{\gamma_c(G)(\Delta - 1) + 2}{2} \rceil$$

where $g(\Delta) = \frac{\gamma_c(G) + 2}{3}$ when $\Delta = 2$ and $g(\Delta) = \frac{\Delta - 5 + \sqrt{\Delta^2 - 2\Delta + 4(\Delta - 2)\gamma_c(G) + 9}}{2(\Delta - 2)}$ when $\Delta \geq 3$. The upper bounds are shown to be sharp for some $\gamma(G), \gamma_c(G)$ and Δ. We also characterize all graphs satisfying each lower bound.

Common and Sidorenko linear patterns

Nina Kamčev

University of Zagreb

(Joint work with Anita Liebenau and Natasha Morrison)

Several classical results in Ramsey theory (including famous theorems of Schur, van der Waerden, Rado) deal with finding monochromatic linear patterns in two-colourings of the integers. Our topic will be quantitative extensions of such results. A linear system L over $\mathbb{F}_q$ is *common* if the number of monochromatic solutions to $L = 0$ in any two-colouring of $\mathbb{F}_q^n$ is asymptotically at least the expected number of monochromatic solutions in a random two-colouring of $\mathbb{F}_q^n$. Motivated by existing results for specific systems (such as Schur triples and arithmetic progressions), as well as extensive research on common and Sidorenko graphs, the systematic study of common systems of linear equations was recently initiated by Saad and Wolf. Fox, Pham and Zhao characterised common linear equations. A parallel concept of *Sidorenko* systems has also been investigated.

We will survey fundamental results on linear patterns and graphs, as well as recent progress towards a classification of common systems of two or more linear equations. For instance, any system containing a four-term arithmetic progression is uncommon.

Some Properties of q-Perfect Matroid Designs

Shinya Kawabuchi

Kumamoto University

(Joint work with Keisuke Shiromoto)

A *perfect matroid design* (PMD) was introduced in 1970 by U.S.R. Murty, P. Young and J. Edmonds. A PMD is a matroid whose flats of the same rank all have the same size. E. Byrne et al., introduced the q-analogue of PMDs (q-PMDs) and proposed a construction of a non trivial q-PMD from a q-Steiner system.

A q-matroid is a q-analogue of a matroid. We denote the collection of subspaces of a vectorspace X by $\mathcal{V}(X)$. A q-matroid $M := (E, r)$ consists of $E := \mathbb{F}_q^n$ and the so-called *rank function* $r \colon \mathcal{V}(E) \to \mathbb{Z}_{\geq 0}$ with the rank function axioms. If $r(F + x) = r(F) + 1$ for all 1-dimensional subspaces x of E not contained in F, F is called a *flat* of M. A q-PMD is a q-matroid whose flats of the same rank all have the same dimension.

A q-analogue of t-design with the parameter t-$(n, k, \lambda; q)$ is an ordered pair $(E, \mathcal{B})$ consisting of vector space $E = \mathbb{F}_q^n$ and a collection $\mathcal{B}$ of k dimensional subspaces of E satisfying that for all t-dimensional subspace X, there are only precisely λ elements of $\mathcal{B}$ include X. The element in $\mathcal{B}$ is called a *block*. If the parameter λ is equal to 1, the design is called a q-*Steiner system*.

In this talk, we show that if flats of q-PMD $M = (E, r)$ include all of the subspaces of E of dimension less than $m - 1$, the flats of the same rank are blocks of a q-analogue of a t-design. We also show how to calculate the parameter λ of the designs. Especially, in this situation, the flats of rank m is the blocks of a q-Steiner system.

Designs in the generalised symmetric group

Lukas Klawuhn*

Paderborn University

(Joint work with Kai-Uwe Schmidt)

It is known that the notion of a transitive subgroup of a permutation group G extends naturally to the subsets of G. We study transitive subsets of the wreath product $C_r \wr S_n$ of generalised permutations acting on subsets of $\{1, \ldots, n\}$ whose elements are coloured with one of r possible colours. This includes the symmetric group for $r = 1$ and the hyperoctahedral group for $r = 2$. The group $C_r \wr S_n$ can also be interpreted as the symmetry group of a regular polytope for every r and this gives rise to an intuitively accessible definition of transitivity. We consider different notions of transitivity in $C_r \wr S_n$ and interpret these algebraically as designs in the conjugacy class association scheme of $C_r \wr S_n$ using representation theory. We also give constructions showing that there exist transitive subsets of $C_r \wr S_n$ that are small compared to the size of the group. Many of these results extend results previously known for the symmetric group S_n.

On the directed Oberwolfach problem with two tables

*Alice Lacaze-Masmonteil**

University of Ottawa

(Joint work with Daniel Horsley)

A $(\vec{C}_{m_1}, \vec{C}_{m_2})$-factor of a directed graph G is a spanning subdigraph of G comprised of two disjoint directed cycles of lengths m_1 and m_2. In this talk, we will be constructing a decomposition of the complete symmetric digraph K_n^* into $(\vec{C}_{m_1}, \vec{C}_{m_2})$-factors when $m_1 + m_2 = n$, $m_1 \in \{4, 6\}$, and $m_2 \geqslant 8$ is even. In conjunction with recent results of Kadri and Šajna (2023+), this gives rise to a complete solution to the two-table case of the directed Oberwolfach problem.

Constructing witnesses for nonspreading permutation groups

Jesse Lansdown

University of Canterbury

(Joint work with John Bamberg, Michael Giudici, and Gordon Royle.)

The class of *spreading* permutation groups lies inbetween the 2-transitive and primitive groups. Similar to a primitive group being defined by the absence of any invariant partition, a spreading group is defined by the absence of any set-multiset pair satisfying certain properties. If however a suitable set-multiset pair exists then it is called a "witness" and the group is *nonspreading*. In this talk I will consider how to construct witnesses, in particular using techniques inspired by the "AB-Lemma" used to construct hemisystems in finite geometry.

Self-avoiding walks on graphs with infinitely many ends

Florian Lehner

The University of Auckland

(Joint work with Lindorfer and Panagiotis)

The self-avoiding walk is a model from statistical physics which has been studied extensively on integer lattices. Over the last few decades, the study of self-avoiding walks on more general graphs, in particular graphs with a high degree of symmetry such as Cayley graphs of finitely generated groups, has received increasing attention.

In this talk, we focus on graphs with more than one end; intuitively these can be thought of as having some large-scale tree structure. This tree structure allows us to decompose self-avoiding walks into smaller, more manageable pieces, and answer questions for graphs with more than one end whose answers for lattices currently seem out of reach.

The talk will be aggressively non-technical. No prior knowledge of self-avoiding walks will be assumed.

The second largest eigenvalue of non-normal Cayley graphs on symmetric groups generated by cycles

Yuxuan Li*

The University of Melbourne

Aldous' Spectral Gap Conjecture states that the second largest eigenvalue of each connected Cayley graph on the symmetric group S_n with respect to a set of transpositions is attained by the standard representation of S_n. This celebrated conjecture, which was proposed in 1992 and completely proved in 2010, has inspired much interest in determining the second largest eigenvalue of Cayley graphs on S_n. For $1 \leq r < k < n$, let $C(n, k; r)$ be the set of k-cycles of S_n which move every $i \in \{1, 2, \ldots, r\}$. It is conjectured that the non-normal Cayley graph $\mathrm{Cay}(S_n, C(n, k; r))$ has the Aldous property, that is, its strictly second largest eigenvalue is achieved by the standard representations of S_n. In this talk, I will introduce the latest research developments about this conjecture, which is based on collaborative work with Binzhou Xia and Sanming Zhou.

Tensor representation of semifields and commuting polarities

Stefano Lia

University College Dublin

Finite semifields correspond to nonsingular threefold tensors and as such they admit different representation in projective spaces. In this joint work with John Sheekey, we exploit the cyclic model for threefold tensors to obtain results on a semifield invariant called BEL-rank. We show that the cyclic model allows to represent in the same space both tensors and their contraction spaces, providing a geometric interpretation of the contraction. This provides a purely geometrical proof of Dickson classification of semifields two dimensional over their center. The investigation of the nonsingularity of tensors in this model also leads to the construction of new quasi-hermitian surfaces, arising from a pair of commuting polarities related to the semifields.

Universality for graphs of bounded degeneracy

Anita Liebenau

UNSW Sydney

(Joint work with Joint with Peter Allen and Julia Böttcher.)

What is the smallest number of edges that a graph G can have if it contains all D-degenerate graphs on n vertices as subgraphs? A counting argument shows that this number is at least of order $n^{2-1/D}$, assuming n is large enough. We show that this is tight up to a polylogarithmic factor.

On generalisations of The Erdős-Ko-Rado Theorem for permutations

Adam Mammoliti

UNSW Sydney

The celebrated Erdős-Ko-Rado Theorem states that if $n \geqslant 2k$ and $\mathcal{F}$ is a family of k-subsets of $[n]$ such that $A \cap B \neq \varnothing$ for all sets $A, B \in \mathcal{F}$, then $|\mathcal{F}| \leq \binom{n-1}{k-1}$, with equality for $n > 2k$ occurring precisely when $\mathcal{F}$ is a family of all k-subsets containing a fixed element of $[n]$. Since its discovery, the Erdős-Ko-Rado Theorem has been generalised extensively and analogous results have been shown for structures other than sets. In particular, an analogue of the Erdős-Ko-Rado Theorem has been shown for families of permutations of $[n]$.

Some new results in combinatorial generation

Brendan McKay

Australian National University

The exhaustive generation of classes of combinatorial objects has been a hobby of mine since my student days. After my arrival at ANU in 1983, among my first projects were to generate cubic graphs and vertex transitive graphs with Gordon Royle. Like Gordon, I'm still addicted to the field and will discuss two recent projects. One is to compile a list of graphs extremal under not containing cycles of specified lengths, to as large an order as possible. The other is to compile a library of combinatorial 2-designs.

On the non-existence of q-ary linear codes with minimum weight $d \equiv -1 \pmod{q}$

Tatsuya Maruta

Osaka Metropolitan University

(Joint work with Hitoshi Kanda and Atsuya Kato)

A q-ary linear code is an $[n, k, d]_q$ code, which is a linear code of length n, dimension k and minimum weight d over the field of order q. When an $[n, k, d]_q$ code with d divisible by q does not exist, it is most likely that an $[n - 1, k, d - 1]_q$ code does not exist as well. We give a result on a new notion "e-locally 2-weight $\pmod{q}$" for q-ary linear codes which help us to prove the non-existence of such a code applying the well-known Extension Theorem by Hill and Lizak(1995).

Equally Distributed 1-Factorisations of Graphs

Jeremy Mitchell*

The University of Queensland

The union of a pair of edge-disjoint 1-factors of a graph forms a collection of even length cycles. If t cycles formed by the union of two edge-disjoint 1-factors have lengths $a_1, a_2, \ldots, a_t$ we say the pair of 1-factors have type $(a_1, a_2, \ldots, a_t)$, if all the pairs of 1-factors of some 1-factorisation have the same type then it is a uniform 1-factorisation. Consider a 1-factorisation $\mathcal{F}$ of some graph and let $t_1, t_2, \ldots, t_m$ be all types of the pairs of 1-factors of $\mathcal{F}$. Let a_{t_i} be the number of pairs that are type t_i. If $a_{t_1} = a_{t_2} = \cdots = a_{t_m} = b$ for some integer b, then we say that $\mathcal{F}$ is an *m-equally distributed 1-factorisation* (m-ED1F) with types $(t_1, t_2, \ldots, t_m)$. We present some results on m-ED1Fs of 3- and 4-regular circulant graphs. Finally, we impose some additional conditions on m-ED1Fs and investigate when such constrained m-ED1Fs exist for complete and complete bipartite graphs.

Automorphisms of direct products of circulant graphs

Đorđe Mitrović*

The University of Auckland

For a non-bipartite graph X, the automorphisms of the direct product $X \times K_2$ play an important role in understanding the automorphism group of $X \times Y$, where Y is bipartite. A graph X is unstable if $X \times K_2$ has automorphisms that do not come from automorphisms of its factors. It is non-trivially unstable if it is unstable, connected, non-bipartite and twin-free. We provide new sufficient conditions for the instability of circulant graphs, generalising previously known results. Furthermore, we classify non-trivially unstable members of several families of circulants.

On the minimal 2-blocking sets in $\mathrm{PG}(5,2)$

Yusuke Miura

Osaka Metropolitan University

(Joint work with Koji Imamura (Kumamoto Univ.) and Tatsuya Maruta)

An n-set B in $\mathrm{PG}(r,q)$ is a *k-blocking set* if every $(r-k)$-space in $\mathrm{PG}(r,q)$ meets B in at least one point. B is called *trivial* if B contains a k-space. Bono et al.(2021) proved that there are exactly six non-trivial minimal 2-blocking sets in $\mathrm{PG}(4,2)$ up to projective equivalence. We consider the non-trivial minimal 2-blocking sets in $\mathrm{PG}(5,2)$ and their generalizations.

Routing in expanders

Rajko Nenadov

University of Auckland

We consider the problem of finding edge-disjoint paths between given pairs of vertices in a sufficiently strong d-regular expander graph G with n vertices. In particular, we describe a deterministic, polynomial time algorithm which maintains an initially empty collection of edge-disjoint paths P in G and fulfils any (infinite) series of two types of requests:

1. Given two vertices v and w such that each appears as an endpoint in $O(d)$ paths in P and, additionally, $|P| = O(nd/\log n)$, the algorithm finds a path of length at most log n connecting v and w which is edge-disjoint from all other paths in P, and adds it to P.

2. Remove a given path from P.

Importantly, each request is processed before seeing the next one. The upper bound on the length of found paths and the constraints are the best possible up to a constant factor. This establishes the first online algorithm for finding edge-disjoint paths in expanders which also allows removals, significantly strengthening a long list of previous results on the topic.

Computing with the Monster group (a public service announcement)

Tomasz Popiel

Monash University

(Joint work with Heiko Dietrich and Melissa Lee)

The Monster is the largest of the 26 sporadic finite simple groups, and is notoriously difficult to compute with, owing to a lack of sufficiently small permutation or matrix representations. As a result, various 'basic' facts about the Monster that are often needed for combinatorial applications of the Classification of the Finite Simple Groups have yet to be determined. In particular, the classification of the maximal subgroups of the Monster has remained uncompleted for some four decades. I shall report on recent joint work on this problem with Heiko Dietrich and Melissa Lee, involving software developed by Martin Seysen.

Prime labeling of some graphs with Eisenstein integers

Rovin B. Santos

Institute of Mathematics, University of the Philippines - Diliman

A graph on n vertices is said to admit a prime labeling if the vertices can be labeled with the first n natural numbers in a such a way that two adjacent vertices have relatively prime labels. In this paper, we define an order on the set of Eisenstein integers to extend the notion of prime labeling of graphs to the set of Eisenstein integers. Properties of the ordering are studied to come up with prime labeling of some families of graphs such as the flower, wheel, centipede, and double broom graphs.

Explicit $K_{3,3}$-subdivisions of Markoff mod p graphs

Shohei Satake

Kumamoto University

The Markoff mod p graph G_p, p a prime, is a graph on solutions of the Markoff equation mod p in which two solutions are adjacent if and only if one is mapped to another by a Vieta operation. This graph was introduced by Bourgain-Gamburd-Sarnak (2016), and they conjectured that G_p forms an expander family. Toward this conjecture, Courcy-Ireland (2021) proved that Gp is non-planar if $p \neq 7$, which supports the conjecture since any planar graphs cannot form an expander family. In particular he exhibited explicit K3,3-subdivisions for certain families of primes whereas there are infinitely many primes p (say, $p \equiv 3 mod 28$, for example) that no explicit K3,3-subdivisions in G_p is known.

In this talk we prove that for primes uncovered in Courcy-Ireland's work (such as $p \equiv 3 \pmod{28}$), there exist explicit $K_{3,3}$-subdivisions in G_p. We also discuss the genus of G_p as well. This talk is based on a joint work with Yoshinori Yamasaki (Ehime University).

Optimising phylogenetic diversity on phylogenetic networks

Charles Semple

University of Canterbury

(Joint work with Magnus Bordewich and Kristina Wicke)

Phylogenetic diversity (PD) is a popular measure for quantifying the biodiversity of a set of present-day taxa. This measure quantifies the extent to which the taxa spans the 'Tree of Life'. In applications, the underlying optimisation problem is to find, for a given set S of taxa and positive integer k, a subset of S of size k that maximises the phylogenetic diversity score. Historically, PD has been typically restricted to phylogenetic trees, but it extends naturally to phylogenetic networks. In this talk, we investigate such an extension.

New 2-closed groups that are not automorphism groups of digraphs

Jacob Smith*

The University of Western Australia

(Joint work with John Bamberg and Michael Giudici)

Giudici, Morgan and Zhou [1] recently published the first known examples of non-regular 2-closed groups that are not the automorphism group of any digraph. These groups all have rank 4, where the rank of a group is its number of orbital digraphs. Giudici, Bamberg and I have now generalised these groups to groups of higher rank, giving the first examples of nonregular 2-closed permutation groups of rank greater than 4 that are not the automorphism group of any digraph. We have found that certain affine groups have these properties if their field order is 3, 5, 7 or 13, but not any other prime. Proper powers of primes have yet to be investigated.

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The Screening Effectiveness of Locating Arrays

Violet Syrotiuk

Arizona State University

A (d,t)-locating array is a covering array of strength t with an additional property: Any set of d level-wise t-way interactions can be distinguished from any other such set by appearing in a distinct set of rows. Locating arrays have been proposed as experimental designs for screening experiments for complex systems due to their efficiency. In this talk, we describe how a (1,2)-locating array recovers main effects and two-way interactions from the measurements of a screening experiment. Preliminary results investigate the role of separation and d-efficiency in screening effectiveness.

On a coloring of a δ-complement graph

Wipawee Tangjai

Mahasarakham University, Thailand

A δ-complement graph was introduced in 2022. The graph is constructed in the same way as a complement graph with a restriction on taking a complement within the set of vertices with the same degree of the graph. In this work, we give several results related to a property and a chromatic number of a δ-complement graph including bounds of the chromatic number and an exact value of the chromatic number of some special classes of graphs.

Combinatorial results for certain semigroups of contraction mappings of a finite chain

Abdullahi Umar

Khalifa University, Abu Dhabi, UAE

The study of various (sub)-semigroups of transformations/mappings has made a significant contribution to semigroup theory. The most notable classes are the THREE fundamental semigroups of transformations: the full symmetric semigroup, the partial symmetric semigroup and the symmetric inverse semigroup. In this talk, we are going to discuss some combinatorial results of some classes semigroups of (partial) contraction transformations of a finite chain, which for some curious reason(s),until very recently, little is known about.

Some lower bounds on conditionally decomposable polytopes

Jie Wang*

Federation University Australia

Suppose we have two polytopes that are combinatorially equivalent, but one decomposable, the other one indecomposable. Such polytopes are called conditionally decomposable. For a conditionally decomposable polytope, we show that the minimum number of vertices is in the range $[3d - 3, 4d - 4]$; and the minimum number of facets is obtained for $d \geqslant 4$. Joint work with David Yost.

Automorphisms of quadratic quasigroups

Ian Wanless

Monash University

Let $\mathbb{F}_q$ be a finite field of odd order q. Let $\square$ denote the set of squares in $\mathbb{F}_q$. Suppose $a, b \in \mathbb{F}_q$ are such that $ab, (a-1)(b-1) \in \square \setminus \{0\}$. We define $Q_{a,b}$ to be the *quadratic quasigroup* $(\mathbb{F}_q, *)$ defined by

$$x * y = \begin{cases} x + a(y - x) \text{ if } y - x \in \square, \\ x + b(y - x) \text{ otherwise.} \end{cases}$$

We will briefly survey the combinatorial applications of quadratic quasigroups. We will then report on new work which answers the following questions.

(1) What is the automorphism group of $Q_{a,b}$?

(2) When is $Q_{a,b}$ isomorphic to $Q_{c,d}$?

(3) What are the minimal subquasigroups of $Q_{a,b}$?

(4) When is $Q_{a,b}$ isotopic to some finite group?

(5) When is $Q_{a,b}$ a Steiner quasigroup?

Joint work with Aleš Drápal, Charles University, Prague.

Proof of the Clustered Hadwiger Conjecture

David Wood

Monash University

Hadwiger's Conjecture asserts that every K_h-minor-free graph is properly $(h-1)$-colourable. We prove the following improper analogue of Hadwiger's Conjecture: for fixed h, every K_h-minor-free graph is $(h-1)$-colourable with monochromatic components of bounded size. The number of colours is best possible regardless of the size of monochromatic components. It solves an open problem of Edwards, Kang, Kim, Oum and Seymour [*SIAM J. Disc. Math.* 2015], and concludes a line of research initiated in 2007. Similarly, for fixed $t \geq s$, we show that every $K_{s,t}$-minor-free graph is $(s+1)$-colourable with monochromatic components of bounded size. The number of colours is best possible, solving an open problem of van den Heuvel and Wood [*J. London Math. Soc.* 2018]. We actually prove a single theorem from which both of the above results are immediate corollaries. For an excluded apex minor, we strengthen the result as follows: for fixed $t \geq s \geq 3$, and for any fixed apex graph X, every $K_{s,t}$-subgraph-free X-minor-free graph is $(s+1)$-colourable with monochromatic components of bounded size. The number of colours is again best possible.

This is joint work with Vida Dujmović, Louis Esperet and Pat Morin (arXiv:2306.06224).

Polytopes with minimal number of edges

David Yost

Federation University

(Joint work with Guillermo Pineda-Villavicencio and Jie Wang)

Given d, for which values of v, e does there exist a d-dimensional polytope with v vertices and e edges? This problem was solved for $d = 3$ in 1906, $d = 4$ in 1967 and $d = 5$ in 2018. An interesting feature is the gaps in the possible values. It is now known that

$$2e \notin [dv + 1, d(v + 1) - 3] \cup [d(v + 1) + 3, d(v + 2) - 7].$$

If it is not possible to determine all pairs (v, e), it is still of interest to determine the minimum value of e for fixed v, and to characterise the minimising polytopes.

On the non-existence of Griesmer linear codes

Keita Yasufuku

Osaka Metropolitan University

(Joint work with Tatsuya Maruta)

We consider the problem of determining $n_q(k, d)$, the smallest possible length n for which an $[n, k, d]_q$ code of fixed dimension k and minimum weight d over the field of order q exists. We investigate the validity of Kawabata's conjecture on the achievement of the Griesmer bound for linear codes over the field of order q, especially for $q = 5$.

On linear-algebraic notions of expansion

Chuanqi Zhang*

University of Technology Sydney

A fundamental fact about bounded-degree graph expanders is that three notions of expansion—vertex expansion, edge expansion, and spectral expansion—are all equivalent. This motivates us to study to what extent such a statement is true for linear-algebraic notions of expansion.

There are two well-studied notions of linear-algebraic expansion, namely dimension expansion [1] (defined in analogy to graph vertex expansion) and quantum expansion [2, 3] (defined in analogy to graph spectral expansion). Lubotzky and Zelmanov [4] proved that the latter implies the former. We proved that the converse is false: there are dimension expanders which are not quantum expanders.

Moreover, this asymmetry is explained by the fact that there are two distinct linear-algebraic analogues of graph edge expansion. The first of these is *quantum edge expansion*, which was introduced by Hastings [5], and which he proved to be equivalent to quantum expansion. We established a new notion, termed *dimension edge expansion*, which we proved is equivalent to dimension expansion and which is implied by quantum edge expansion. Thus, the separation above is implied by a finer one: dimension edge expansion is strictly weaker than quantum edge expansion. This new notion also led to a new and more modular proof of the Lubotzky-Zelmanov result [4] that quantum expanders are dimension expanders.

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