Chapter 3

Baseband and Bandpass Signals

3.1 Baseband signal

A *baseband* signal is one whose frequency content, or spectrum, is close to zero frequency (DC). A *bandpass* signal is one whose frequency content, or spectrum, is centered on some high frequency.

Let $x(t)$ be a baseband signal, with frequency content extending only up to $f = W$ Hz, i.e. $X(f) = 0$ for $|f| > W$.

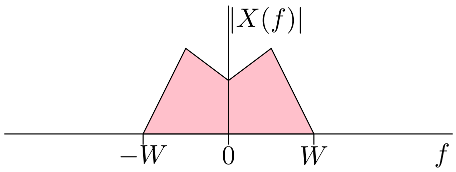

Figure 3.1.1: Spectrum of $x(t)$.

3.2 Bandpass signal

Suppose we create

$$\tilde{x}(t) = x(t)\cos(2\pi f_c t)$$

The signal is said to be frequency shifted, or *heterodyned*, up to center frequency f_c. By writing

$$\tilde{x}(t) = x(t) \cdot \frac{1}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)$$

we obtain the Fourier transform of the signal $\tilde{x}(t)$ as

$$\tilde{X}(f) = \frac{1}{2}\left[X(f - f_c) + X(f + f_c)\right]$$

Therefore, $\tilde{x}(t)$ is a bandpass signal, with frequency spectrum of bandwidth $2W$ centered on f_c Hz, as shown in Figure 3.2.1

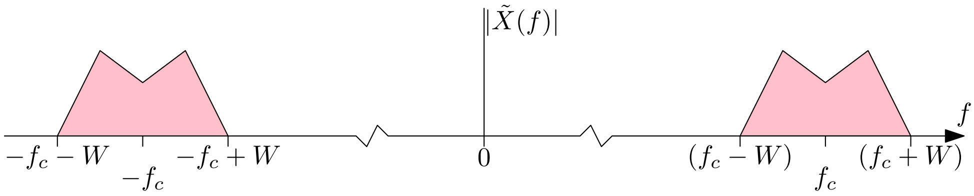

Figure 3.2.1: Spectrum of $\tilde{x}(t) = x(t)\cos(2\pi f_c t)$.

3.3 Quadrature signal

Similarly, let $y(t)$ be another baseband signal, with frequency content extending only up to $f = W$ Hz, i.e. $Y(f) = 0$ for $|f| > W$.

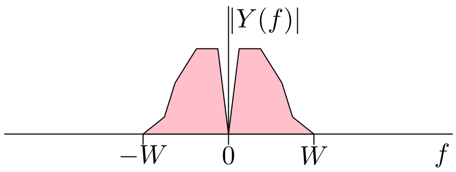

Figure 3.3.1: Spectrum of $y(t)$.

If we create

$$\tilde{y}(t) = y(t)\sin(2\pi f_c t)$$

Then, writing

$$\tilde{y}(t) = y(t) \cdot \frac{1}{2j}\left(e^{j2\pi f_c t} - e^{-j2\pi f_c t}\right)$$

we obtain the Fourier transform of $\tilde{y}(t)$ as

$$\tilde{Y}(f) = \frac{1}{2j}\left[Y(f - f_c) - Y(f + f_c)\right]$$

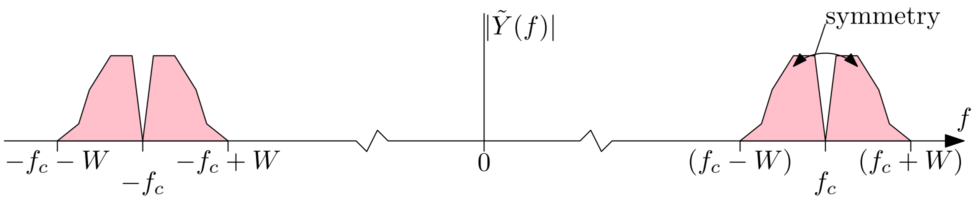

Figure 3.3.2: Spectrum of $\tilde{y}(t) = y(t)\sin(2\pi f_c t)$.

Therefore, $\tilde{y}(t)$ is also a bandpass signal, with frequency spectrum of bandwidth $2W$ centered on f_c Hz.

To create a quadrature signal we can combine $\tilde{x}(t)$ and $\tilde{y}(t)$ to obtain a more general bandpass signal $\tilde{s}(t) = \tilde{x}(t) - \tilde{y}(t)$, or

$$\tilde{s}(t) = x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t) \tag{3.3.1}$$

This signal $\tilde{s}(t)$ is a bandpass signal, with frequency spectrum of bandwidth $2W$ centered on f_c Hz.

$x(t)$ and $y(t)$ are called the *quadrature components* of the signal $\tilde{s}(t)$.

The two sinusoidal signals $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ have $90°$ phase difference and are therefore said to be in *phase quadrature* , i.e. one *quarter* of a whole cycle,with each other; hence the name “quadrature components”. The spectrum of the bandpass signal $s(t)$ is

$$\tilde{S}(f) = \frac{1}{2}\left[X(f - f_c) + X(f + f_c)\right] - \frac{1}{2j}\left[Y(f - f_c) - Y(f + f_c)\right] \tag{3.3.2}$$

This is illustrated in Figure 3.3.3.

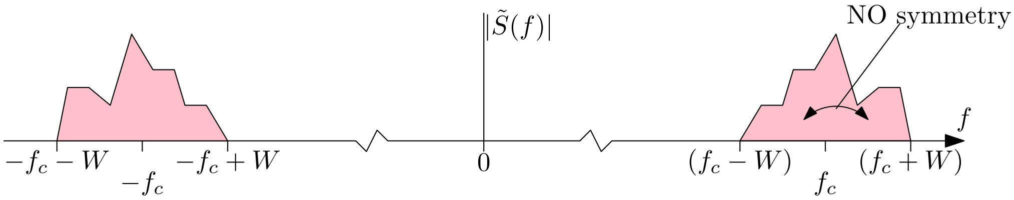

Figure 3.3.3: Spectrum of $\tilde{s}(t)$.

More generally, it may be shown that any bandpass signal with spectrum of bandwidth $2W$ centered on f_c Hz may be written in the form of equation (3.3.1) where $x(t)$ and $y(t)$ are some pair of baseband signals of bandwidth W.

3.3.1 Why?

We claim that if we receive $\tilde{s}(t)$ then a sufficiently clever receiver can compute both $x(t)$ and $y(t)$, the two quadrature components.
The proof is a not too difficult but a little long...

Exercise:

Prove that this is true...

The importance of this is that we can transmit a signal that occupies the same bandwidth as $\tilde{x}(t)$ (or $\tilde{y}(t)$) but it contains both $\tilde{x}(t)$ and $\tilde{y}(t)$ and therefore twice the amount of information. This is illustrated in Figure 3.3.4 where two audio signals are transmitted at the same time over the same channel using the same carrier frequency, but yet the receiver can separate the two signals correctly.

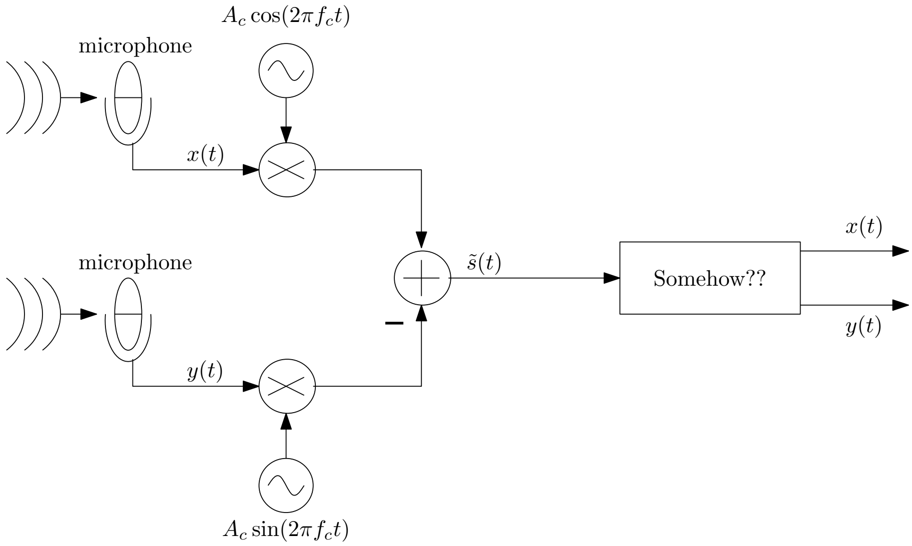

Figure 3.3.4: Two audio signals being transmitted together using the in-phase and quadrature components of a carrier.

3.3.2 Magnitude / phase notation

We may also rewrite equation (3.3.1) in the form

$$\tilde{s}(t) = A(t)\cos(2\pi f_c t + \theta(t)) \tag{3.3.3}$$

$A(t)$ is called the *envelope* of the signal, and $\theta(t)$ is called the *phase* of the signal. Equation (3.3.3) is called the *envelope-phase* representation of the bandpass signal. The envelope and phase are related to the quadrature components via

$$A(t) = \sqrt{x^2(t) + y^2(t)}; \qquad \theta(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$$

or

$$x(t) = A(t)\cos\theta(t); \qquad y(t) = A(t)\sin\theta(t)$$

These relationships are illustrated graphically in figure 3.3.5.

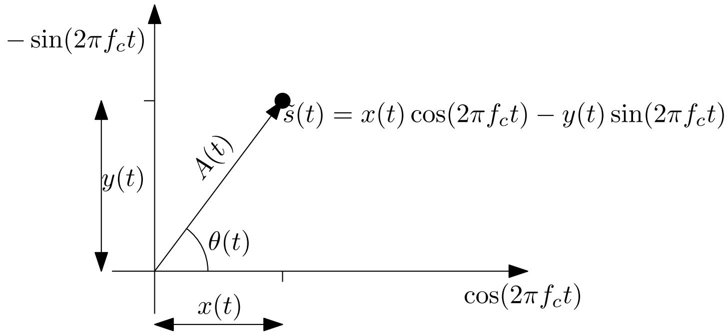

Figure 3.3.5: Illustration of the relationship between the quadrature components and the envelope and phase components of a narrowband signal.

Chapter 4

Modulation Concept

Usually the information signal $g(t)$, e.g. a voice or video signal, we wish to convey over a medium is baseband (with bandwidth W).
However many channels, most notably the wireless channel, are bandpass systems and are not directly suitable for conveying our baseband information signal (a metallic conductor is a baseband channel that would be suitable).

The engineering problem is then to transmit the base-band information signal, $g(t)$, over a bandpass channel; to achieve this we use modulation.

Modulation is the process of varying some characteristic of a sinusoidal carrier signal $c(t)$ according to the value of our information signal $g(t)$.

Consider a high-frequency sinusoid (generated by an oscillator at the transmitter), called a *carrier*, having frequency f_c that is suitable for the bandpass channel:

$$c(t) = A_c \cos(2\pi f_c t)$$

The resulting band-pass *modulated signal* is:

$$\begin{aligned}
\tilde{s}(t) &= x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t) \\
&\text{or} \\
\tilde{s}(t) &= A(t)\cos(2\pi f_c t + \theta(t))
\end{aligned}$$

has a narrow bandwidth centered on f_c, and therefore *can* pass through the channel.

For now we will focus on the second format.

The information signal $g(t)$ is used to generate $A(t)$ and / or $\theta(t)$.

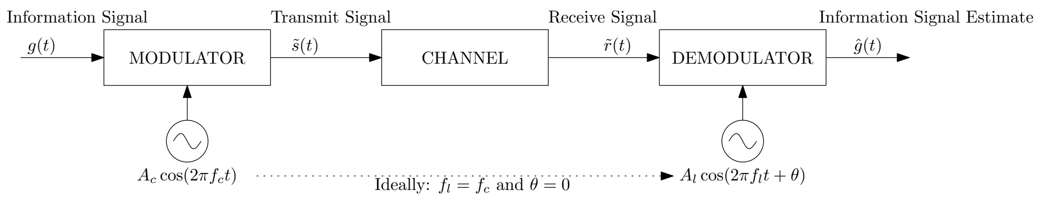

Figure 4.0.1: Bandpass Analog Communication System.

- The carrier signal, $c(t) = A_c \cos(2\pi f_c t)$, ‘carries’ the information at high frequency
- An *oscillator* is required to generate the carrier having frequency f_c
- There are 3 main types of modulation:
  - Amplitude modulation (AM) - $g(t)$ varies the amplitude $A(t)$
  - Angle modulation - really there are two sub-types here
    - Phase modulation (PM) - $g(t)$ directly varies the angle $\theta(t)$
    - Frequency modulation (FM) - $g(t)$ varies the frequency $\frac{d}{dt}\theta(t)$
- *Demodulation* refers to the process of undoing the modulation, i.e. mapping the received signal $\tilde{r}(t)$ back to an estimate of the information signal, $\hat{g}(t)$. As we shall see, this process often (but not always) requires an oscillator at the same frequency (f_c) as that at the transmitter - called a *local oscillator* (LO). In systems which use a LO, it is required that the transmitter and receiver oscillators be kept at the same frequency and phase as each other – this process is called *synchronisation*. This is not a trivial task since the transmitter and receiver are usually separated by a large distance.

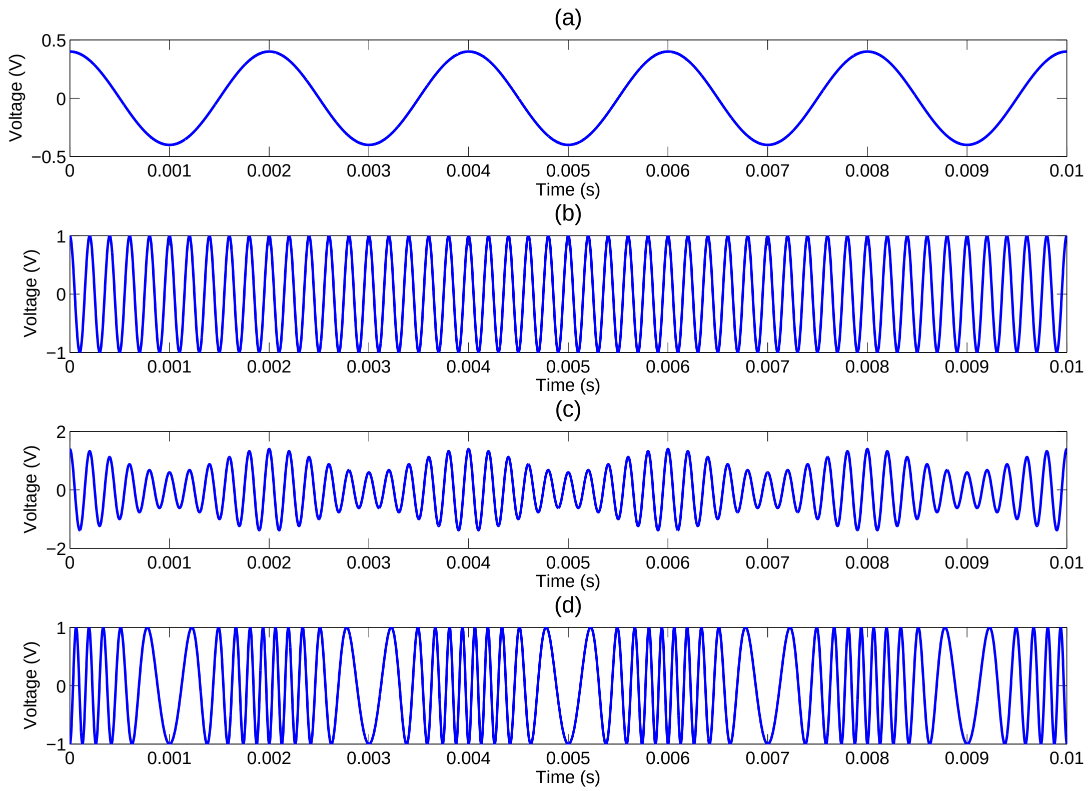

Figure 4.0.2: Illustration of amplitude and frequency modulation. (a) shows the message signal; (b) shows the carrier, a high-frequency sinusoid; (c) shows the AM signal; (d) shows the FM signal.

Modulation is performed for a number of reasons:

- To move information to a particular part of the frequency spectrum, to suit the available channel
- To put information in a better form to survive channel impairments
- To *multiplex,* i.e. to transmit many information signals on the same channel.