# Purpose of lecture

- In this lecture, we will learn about radio links.
- We will answer questions like
  - How far away can I receive the signal?
  - How much power do I need to transmit?
  - What size antenna do I require?
- These factors are related to each other. They are all part of the design of a radio system, in what are called link budget calculations.

# FRIIS FREE SPACE EQUATION

# Antenna basics

- Antenna *gain*
  - As we discussed in the last lecture.
  - We compare the antenna with an ideal *isotropic radiator*
    - This theoretical antenna transmits equally in all directions, and has no losses.
    - Does not exist: hairy ball theorem (HBT) prohibits it. $\vec{E}$ is $\perp$ to the direction of propagation, so $\parallel$ to the surface of a sphere surrounding the antenna. The HBT requires $\vec{E} = 0$ somewhere.
  - power density = power per unit area – W/m$^2$

$$gain = \frac{\text{power density in desired direction}}{\text{power density from isotropic radiator}} \text{ at distance } r$$

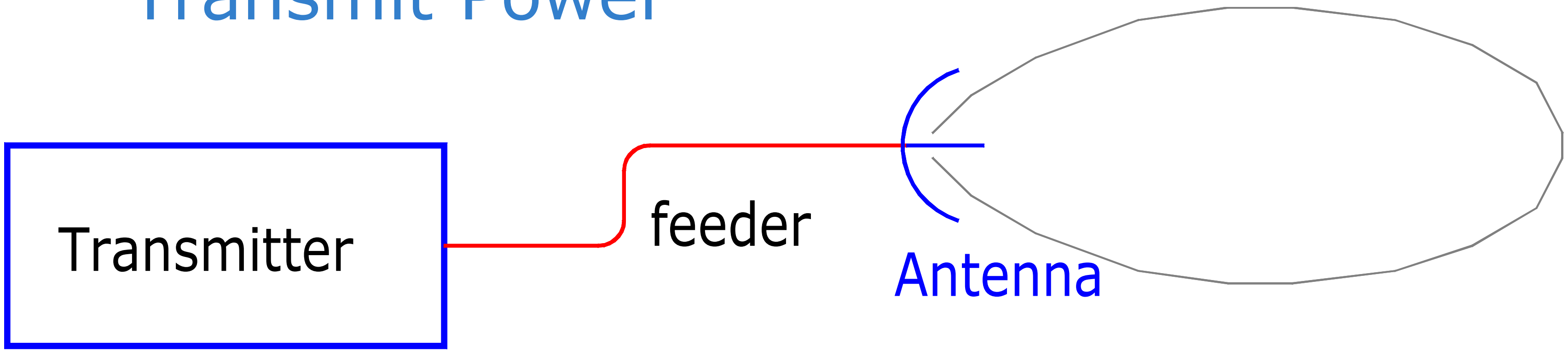
# Antenna Basics

- Receiving antenna
  - collects e-m wave and output on cable.
  - may be directional
- Effective aperture (area)
  - incoming e-m wave has power density [W/m$^2$]
  - power collected proportional to aperture
  - The gain is related to effective aperture:

$$G = \frac{4\,\pi\,A}{\lambda^2}$$

  - Note: A < actual area of dish. This accounts for losses and efficiency of the antenna.

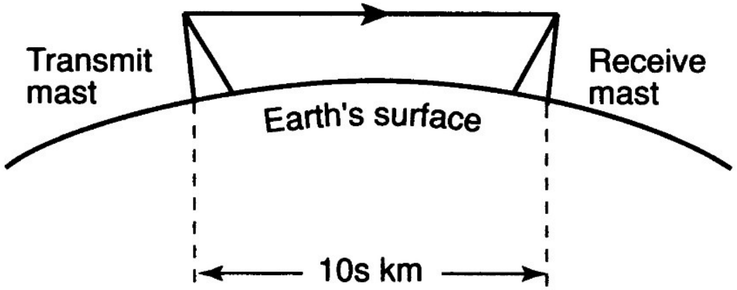
# Transmit Power

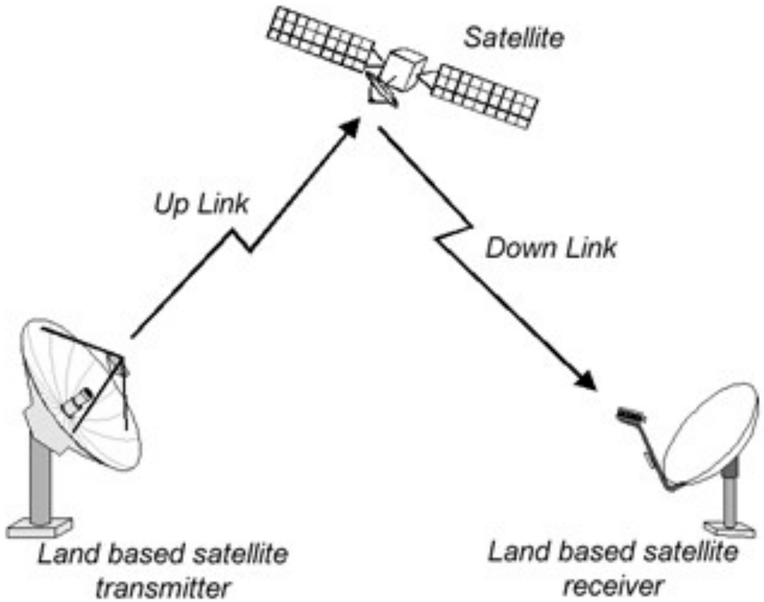



- Transmit power
  - output power of the transmitter electronics
  - reduced by loss in feeder (cable or waveguide)
  - leaves net transmit power $P_{tx}$ (W)
- Effective isotropic radiated power - EIRP
  - power needed with isotropic radiator to get same power density in desired direction
  - takes account of tx antenna gain, $G_{tx}$
  - unit W

$$EIRP = P_{tx} G_{tx}$$

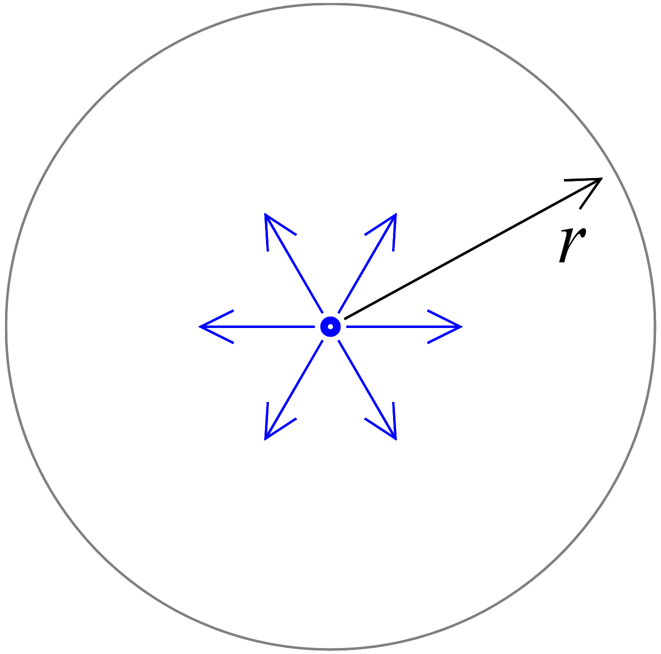
# Free Space Links




- Simplest case
  - only one path from transmitter to receiver
  - no obstructions
- Common in satellite systems
  - uplink, downlink, inter-satellite link
- Some terrestrial systems
  - directional antennas on high sites or masts
  - no obstructions or reflections

# Free Space Propagation




- Isotropic radiator
  - transmit power spread uniformly in 3 dimensions
  - imagine illuminating inside surface of sphere…
  - at distance *r*, surface area $4\pi r^2$
  - so transmit power $P_{tx}$ gives power density at distance *r* $\dfrac{P_{tx}}{4\pi r^2}$
- Real transmit antenna
  - power density in desired direction $\dfrac{P_{tx}\,G_{tx}}{4\pi r^2} = \dfrac{EIRP}{4\pi r^2}$

# Received Power

- Power density at receiver
  - falls with square of distance

$$D_{rx} = \frac{P_{tx}\,G_{tx}}{4\pi\,r^2} = \frac{EIRP}{4\pi\,r^2}$$

- Received power
  - power density at receiver × aperture of receive antenna

$$P_{rx} = D_{rx}\,A_{rx}$$

  - fundamental equation:

$$P_{rx} = \frac{P_{tx}\,G_{tx}\,A_{rx}}{4\pi\,r^2}$$

- Notes on received power:
  - independent of frequency (but $G_{tx}$ might not be)
  - falls as square of dist. (free space propagation)
  - analysis not valid close to transmit antenna

# Summary

$$P_{rx} = \frac{P_{tx}\, G_{tx}\, A_{rx}}{4\pi\, r^2}$$

| | |
|---|---|
| Transmitted power (W) (allowing for feeder loss) | $P_{tx}$ |
| × TX antenna gain = EIRP (W) | $EIRP = P_{tx}\, G_{tx}$ |
| ÷ surface area of sphere = power density (W/m$^2$) | $D_{rx} = \frac{EIRP}{4\pi\, r^2}$ |
| × RX antenna effective aperture = received power | $P_{rx} = D_{rx}\, A_{rx}$ |
| ÷ noise power at receiver = signal-to-noise ratio | $\frac{S}{N} = \frac{P_{rx}}{N_{rx}}$ |

# The RF world uses the decibel (dB)

$$G = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right) \quad (\text{dB})$$

$$P_{\text{dBW}} = 10\log_{10}\left(\frac{P}{1\,\text{W}}\right)$$

$$P_{\text{dBm}} = 10\log_{10}\left(\frac{P}{1\,\text{mW}}\right)$$

- decibel expresses power ratio
  - e.g. amplifier gain, antenna gain, path loss, …
- In RF systems, often also used to express power
  - extra letter added to show reference power
  - dBW = power relative to 1 W
  - dBm = power relative to 1 mW
- Example:
  - note effect of subtraction!

$$P_{in} = -3\ \text{dBm} \qquad P_{out} = +20\ \text{dBm} \qquad Gain = 23\ \text{dB}$$

# Example Calculations

Satellite downlink at 11 GHz

signal bandwidth 36 MHz

| | | |
|---|---|---|
| Transmitter output power (less feeder losses etc.) | 50 W | 17 dBW |
| TX antenna gain (0.8 m diameter @ 11 GHz, $\eta$ 59%) | 5000 | 37 dB |
| Pointing loss (receiver not on boresight) | 0.5 | -3 dB |
| EIRP (in direction of receiver) | 125 kW | 51 dBW |

# Example Calculations continued

| | | |
|---|---|---|
| Power density at receiver (40 000 km path length) | 6.2 pW/m$^2$ | |
| Receive antenna aperture (1.8 m diameter, $\eta$ 63%) | 1.6 m$^2$ | |
| RX signal power | 10 pW | -110 dBW (=-80 dBm) |
| Receiver noise temperature (see later) | 200 K | |
| Receiver noise power in 36 MHz bandwidth | 0.1 pW | -130 dBW (=-100 dBm) |
| Signal-to-noise ratio | 100 | 20 dB |

# Alternative: Path Loss Method

- Work with antenna gain only
  - related to effective aperture

$$G = \frac{4\pi A}{\lambda^2}$$

- Then calculate

$$P_{rx} = \frac{P_{tx} G_{tx} G_{rx} \lambda^2}{(4\pi)^2 r^2}$$

- Define path loss
  - looks frequency dependent
  - due to RX antenna gain

$$PL = \frac{\lambda^2}{(4\pi)^2 r^2}$$

- Simple calculation
  - can use other PL expressions for other link conditions

$$P_{rx} = P_{tx} G_{tx} G_{rx} PL$$

# Example Calculations

| | | |
|---|---|---|
| EIRP<br>(in direction of receiver) | 125 kW | 51 dBW |
| Path loss<br>(40 000 km @ 11 GHz) | $3\times10^{-21}$ | -205 dB |
| Receive antenna gain (1.8 m diameter @ 11 GHz, $\eta$ 63%) | 27000 | 44 dB |
| RX signal power | 10 pW | -110 dBW<br>-80 dBm |

EIRP obtained as before

S/N calculated as before

# Example

- Antenna on roof of UCD Engineering building
  - gain at ~500 MHz ~9 dB
  - ground height 26 m
  - building height 17 m
- Pointed at Three Rock Mountain TV transmitter
  - ground height 450 m
  - mast height 45 m
  - horizontal distance 7092 m
- What is received power on Saorview signal?

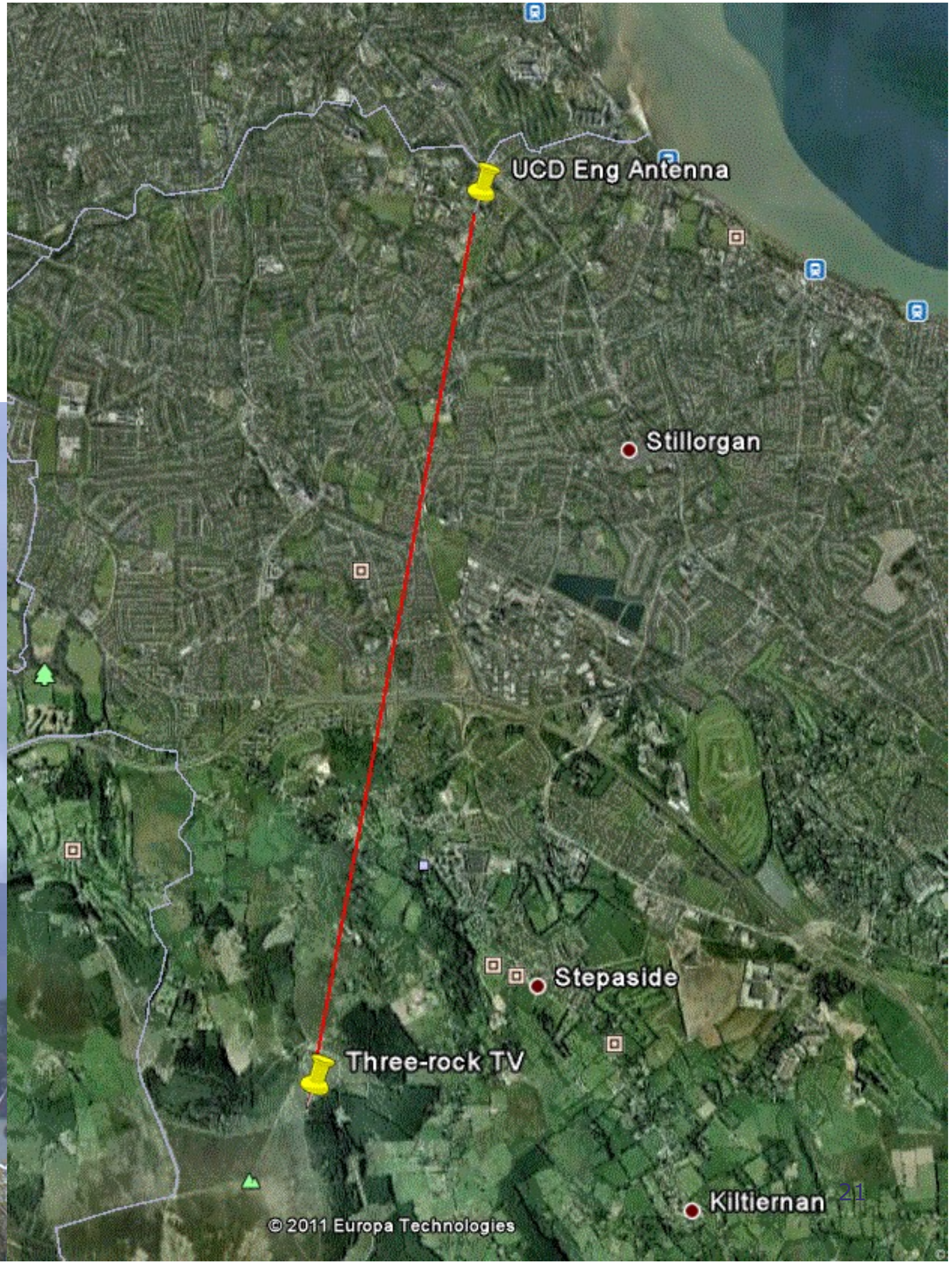
# Soarview Terrestrial – Main Transmitters

| Site | County | MUX CH. | Polarity | MAX ERP kW |
|---|---|---|---|---|
| CAIRN HILL | Longford | 47 | H | 160 |
| CLERMONT CARN | Louth | 52 | V | 160 |
| DUNGARVAN | Waterford | 55 | H | 10 |
| HOLYWELL HILL | Donegal | 30 | H | 20 |
| KIPPURE | Wicklow | 54 | H | 63 |
| MAGHERA | Clare | 48 | H | 160 |
| MOUNT LEINSTER | Wexford | 23 | H | 160 |
| MULLAGHANISH | Kerry | 21 | H | 200 |
| SPUR HILL | Cork | 45 | H | 50 |
| THREE ROCK | Dublin | 30 | H | 63 |
| TRUSKMORE | Sligo | 53 | H | 160 |
| WOODCOCK HILL | Clare | 47 | H | 10 |

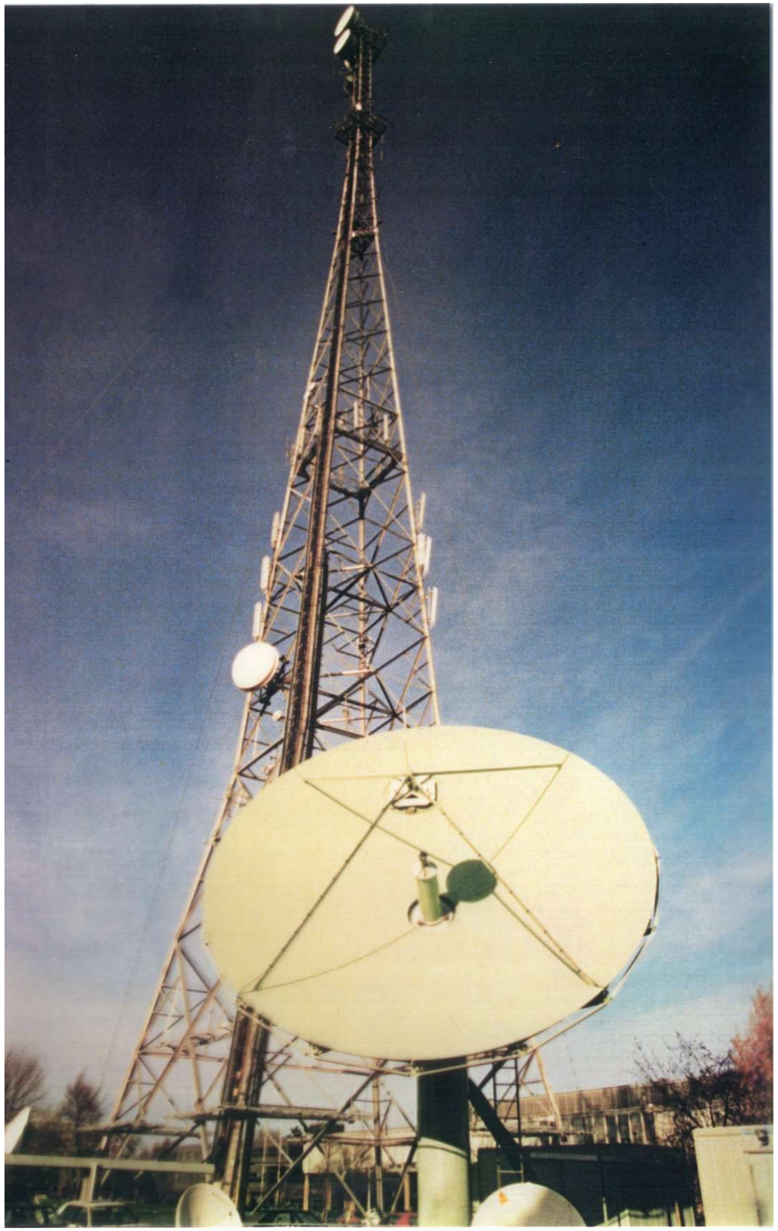
Channel 30
= 546 MHz

# Path

## Clear line of sight?



# High Site, Tall Mast



# Solution to TV Exercise

$$r_d = \sqrt{(h_{tx} - h_{rx})^2 + d^2}$$

$$P_{rx} = \frac{P_{tx}\, G_{tx}\, G_{rx}\, \lambda^2}{(4\pi)^2\, r^2}$$

- Horizontal dist. 7092 m, height diff. 452 m
  - path length 7106 m (only 14 m longer!)
- EIRP = $P_{tx}G_{tx}$ = 63 kW
- $G_{rx}$ = 9 dB – convert to ratio 7.94328
- Wavelength $\lambda = \frac{c}{f}$ = 0.54945 m
- Equation gives $P_{rx}$ = 18.95 μW or -17.2 dBm
  - using horizontal distance, get 19.02 μW
  - receiver input 75Ω, this is 37.7 mV rms
- Need to keep precision in intermediate results
  - round answer! In dB, one decimal place…

# Design Choices

E.g. suppose we want to work with smaller receive antenna…

- Higher transmit power?
  - More power on satellite or fewer transponders
- Higher gain transmit antenna?
  - reduces coverage area
  - more costly, esp. if have to unfurl after launch
- Better receiver electronics?
  - generate less noise, so need less signal
- Change modulation and/or coding
  - work with lower S/N

# In-class Exercise

Time:
8 minutes

$$P_{rx} = \frac{P_{tx}\, G_{tx}\, G_{rx}\, \lambda^2}{(4\pi)^2\, r^2}$$

- Work individually.
- A robot boat on a lake transmits at 433 MHz and has transmitter power 10 mW. The receiver is on shore with a clear line of sight. Each antenna has gain 1.6 dB.
- Find worst case receive power if the max distance ~150 m
- Find maximum range if receiver needs -105 dBm for acceptable results.
- Write your UCD student number at the top of the paper.