EEEN3006J

Wireless Systems

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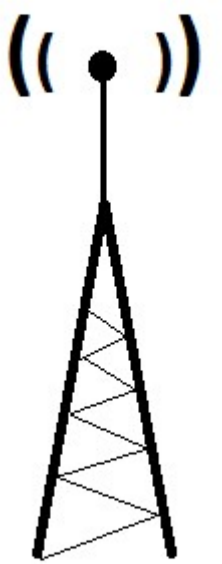
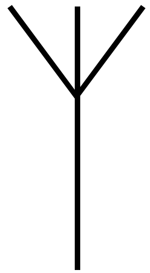
Antennas

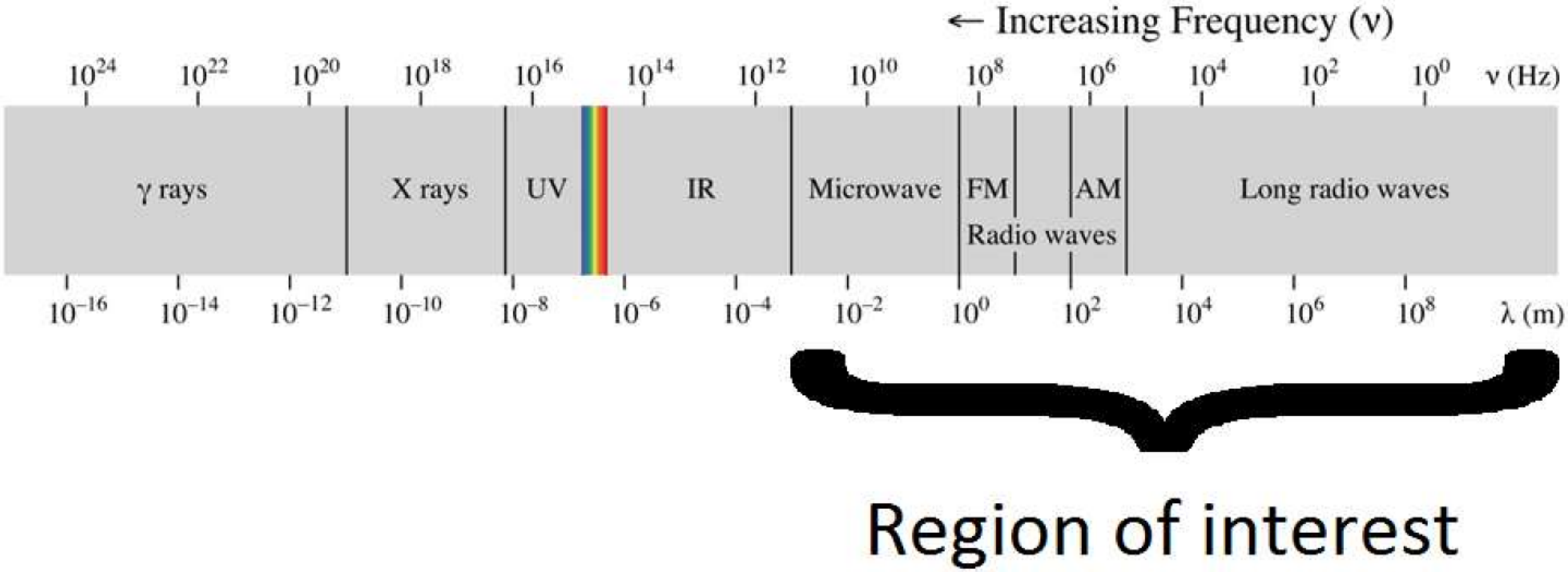
An antenna is the physical structure associated with the transition of an electromagnetic wave from being a guided wave to being a free-space wave (transmitting) or vice versa (receiving).



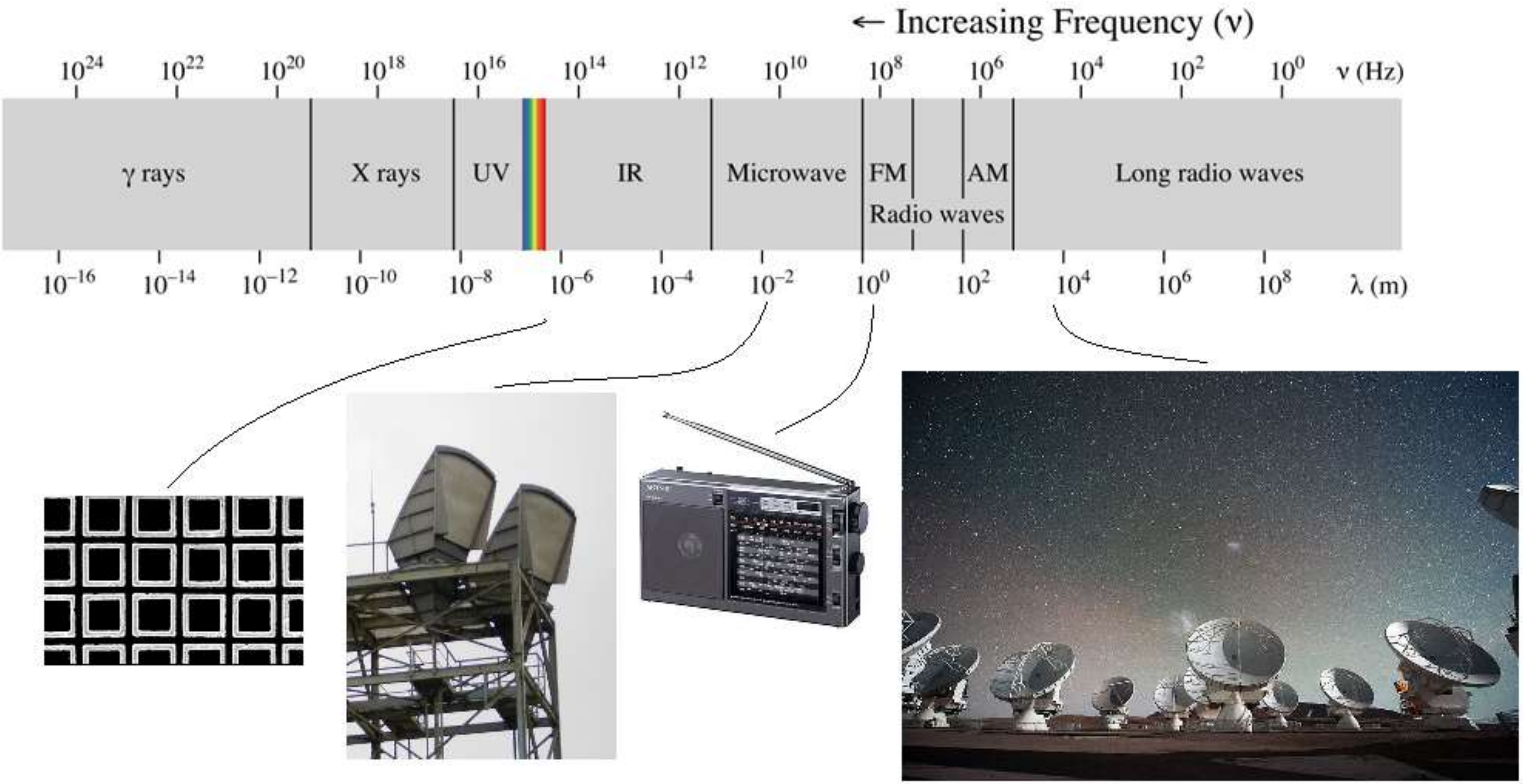


The use of antennas (some people say "antennae") and free space propagation define wireless systems. Therefore, we will make a careful study of these devices.

Frequencies of interest



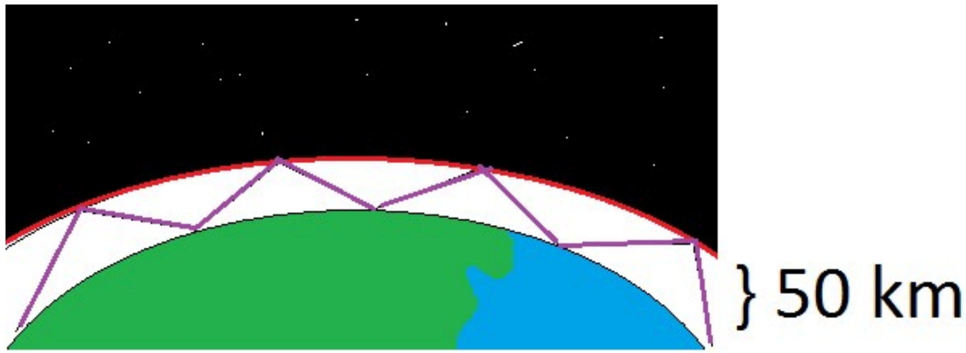
Range of antennas



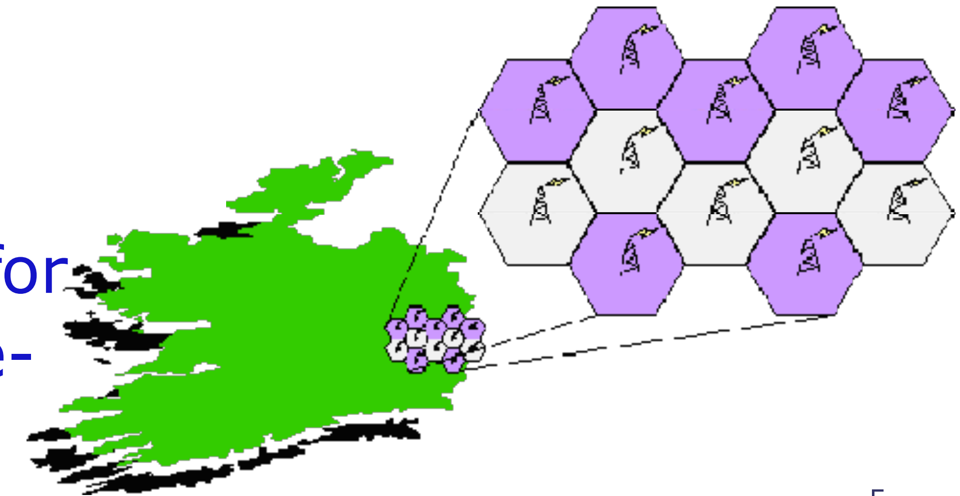
<u>Antenna length</u> is proportional to <u>wavelength</u>

Why use different frequencies?

- Low frequencies propagate longer.
- Lower absorption.
- Bounce off ionosphere; travel around world.



- Conversely, high frequencies good for local and even line-of-sight links.



Theory of Potentials – Scalar Potential

- Example: the electric potential (a.k.a. emf) at a point is the work done in bringing a unit positive charge from infinity to that point.

$$-\int_{\infty}^{r} E.\mathrm{d}l = V$$

  - E is the electric field
  - r is the location of the point.
  - V is a scalar field – that is a directionless number associated with every point in space.

- Conversely,

$$E = -\nabla V$$

Theory of Potentials – Scalar Potential

- For static fields, no sources or sinks,
$$\nabla \times E = 0$$
- Conversely, if a vector field has zero curl, then that vector field is a gradient of a scalar field, which we refer to as *the scalar field*.
- In regions of space with no current, the magnetic field is static, and there exists a magnetic field potential. However, this is of less interest in this topic.

Laplace's Equation

- Recall *Gauss's Law for electric fields*

$$\vec{\nabla}\circ\vec{E} = \frac{\rho}{\varepsilon}$$

- But as $E = -\nabla V$, we obtain *Poisson's Equation*

$$\vec{\nabla}\circ\vec{\nabla}\vec{V} = \nabla^2 V = \frac{\rho}{\varepsilon}$$

- In a region of space with no net charge, $\rho = 0$, and we obtain *Laplace's Equation*

$$\boxed{\nabla^2\vec{V} = 0}$$

Vector Potentials

- Consider a vector field X.
- If there exists a field *Y* such that $X = \nabla \times Y$,
  - then $\vec{\nabla} \circ \vec{X} = 0$.
- And conversely:
  - if a vector field, *X*, has zero divergence, then a vector field *Y* (called *the vector potential*) exists such that *X* is the curl of *Y*.
- We know such a field. Gauss's law for magnetic fields:

$$\vec{\nabla} \circ \vec{B} = 0$$

- We defined magnetic vector potential, A, such that B is the curl of A.

To find an expression relating A and J

- Want to relate magnetic vector potential and current density. From Ampere-Maxwell, and for a static field $\left(\frac{\partial \vec{E}}{\partial t} = 0\right)$,

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

- Hence

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{A}\right) = \mu_0 \vec{J}$$

$$\vec{\nabla}\left(\vec{\nabla} \circ \vec{A}\right) - \vec{\nabla}^2 \vec{A} = \mu_0 \vec{J}$$

- If we let $\nabla \circ A = 0$,

  Then - $\nabla^2 A = \mu J$

- So what is the solution for A that satisfies Both
- A is not unique – there are a number of solution
- One such solution is $$A = \int_L \frac{\mu I dl}{4\pi r}$$
  – The Magnetic Potential for a line current! at location r

- Or Generally ,

$$A = \frac{\mu}{4\pi}\int_{\tau} \frac{\vec{J}(x', y', z')}{r} d\tau$$

is a solution to $\vec{\nabla}^2\vec{A} = -\mu_0\vec{J}$.

- This equation is extremely useful in solving EM field problems.
- If J is in one direction only (e.g. current in a wire), then A exists only in that direction.

Retarded potential

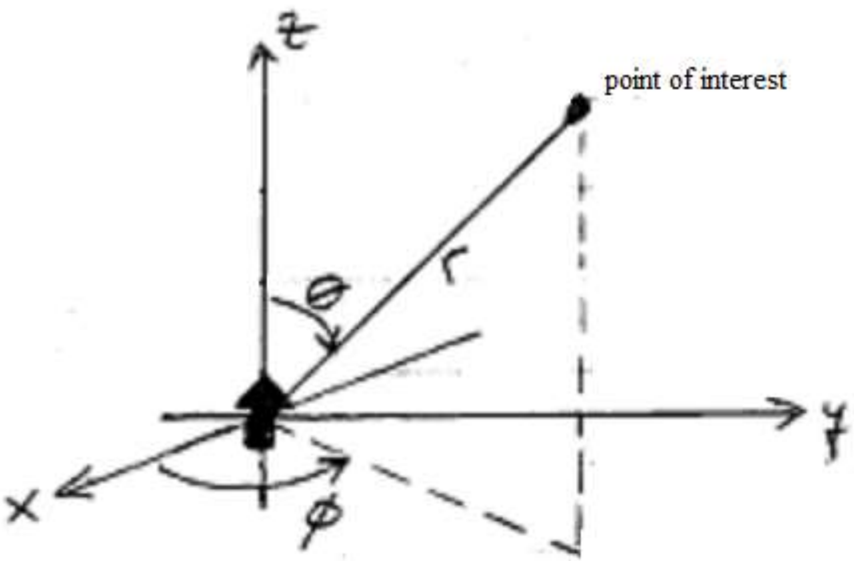
- If the current is time-varying, we must account for that as follows:

$$A = \frac{\mu}{4\pi} \int_{\tau} \frac{\vec{J}(x', y', z'; t - r/c)}{r} d\tau$$

- This proof hasn't been rigorous. We derived using the assumption of a static field. However, it produces the correct answer, and the full derivation is outside the scope of the course.

Radiation

- Consider a short piece of wire, of length L. The wire is carrying a sinusoidal current ($i(t) = \mathrm{I}\sin\omega t$) which is uniform along the length of the wire. This is called a Hertzian Dipole.



- We want to find the electric and magnetic fields around the wire.

$$A = \frac{\mu}{4\pi} \int_{\tau} \frac{\vec{J}(x', y', z'; t - r/c)}{r} d\tau$$

becomes

$$A = \frac{\mu}{4\pi} \int_{-l/2}^{l/2} \frac{\vec{I} \sin\big(\omega(t - r/v)\big)}{r} dx$$

If r>>l,

$$A = \frac{\mu \vec{I}}{4\pi r} \sin\big(\omega(t - r/v)\big) \int_{-l/2}^{l/2} dx$$

$$A = \frac{\mu \vec{I} l}{4\pi r} \sin\big(\omega(t - r/v)\big)$$

- Converting into spherical coordinates

$$A_r = \frac{\mu I l}{4\pi r} \sin\left(\omega(t - r/v)\right) \cos\theta$$

$$A_\theta = \frac{-\mu I l}{4\pi r} \sin\left(\omega(t - r/v)\right) \sin\theta$$

$$A_\phi = 0$$

- B is the curl of these equations.

Field equations

$$B_r = 0$$

$$B_\theta = 0$$

$$B_\phi = \frac{\mu I l}{4\pi r}\left(\frac{w}{v}\cos\bigl(\omega(t - r/v)\bigr)\sin\theta + \frac{1}{r}\sin\bigl(\omega(t - r/v)\bigr)\sin\theta\right)$$

- We also want the E field.

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

- But J is zero outside the wire.

$$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\mu_0 \varepsilon_0} \vec{\nabla} \times \vec{B}$$

- Substituting for B, and integrating with respect to time, we get
- $E_r = \frac{Il\cos\theta}{2\pi\varepsilon_0 r}\left\{\frac{1}{rv}\sin\omega\left(t-\frac{r}{v}\right)-\frac{1}{r^2\omega}\cos\omega\left(t-\frac{r}{v}\right)\right\}$
- $E_\theta = \frac{-Il\sin\theta}{4\pi\varepsilon_0 r}\left\{\frac{-\omega}{v^2}\cos\omega\left(t-\frac{r}{v}\right)-\frac{1}{rv}\sin\omega\left(t-\frac{r}{v}\right)+\right.$ $\left.\frac{1}{r^2\omega}\cos\omega\left(t-\frac{r}{v}\right)\right\}$
- $E_\phi = 0$
- Plus constants we can take to be zero. The physical meaning of this assumption is that the net charge on the wire is zero.

Near field

- Where r is small, r^2 and r^3 terms dominate.
- Compare Biot-Savart law.
- Terms in r^-2 are the induction field.
- Terms in r^-3 are the electrostatic field. (c.f. electric dipole.)

Far field

- Where r is large, 1/r terms dominate.
- Field equations become:

- $E_r = 0$ (1)
- $E_\theta = \frac{\eta IL \sin\theta}{2r\lambda} \cos\left(\frac{2\pi r}{\lambda} - \omega t\right)$ (2)
- $E_\phi = 0$ (3)
- $H_r = 0$ (4)
- $H_\theta = 0$ (5)
- $H_\phi = \frac{IL \sin\theta}{2r\lambda} \cos\left(\frac{2\pi r}{\lambda} - \omega t\right)$ (6)

Substitutions

$$\lambda = \frac{v}{f} = \frac{2\pi v}{\omega}$$

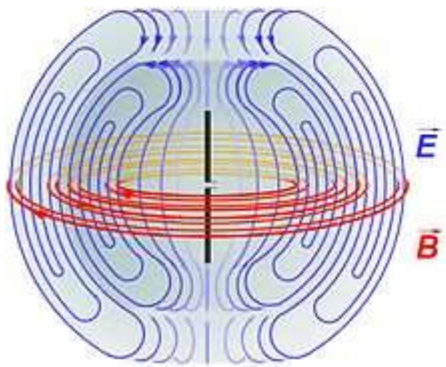
$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$B = \mu_0 H$$

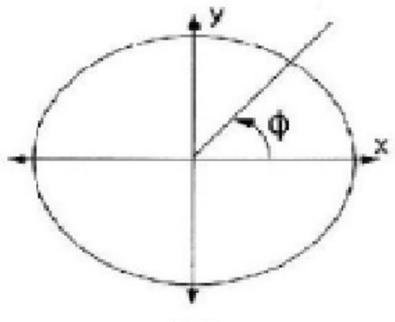
$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

- They describe the radiation field - how antennas work. (Very important.)

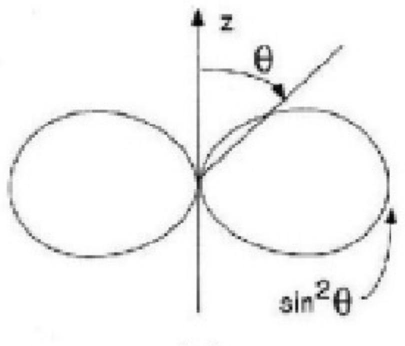
- Field due to a Hertzian dipole.
- E_θ and H_ϕ are in phase
  - Depend on $\sin\theta$
- The equation represent an em wave propagating away from the dipole. This is what we call a spherical wave.
- Field zero up or down.
- Max at equator.
- In local region ~plane wave.



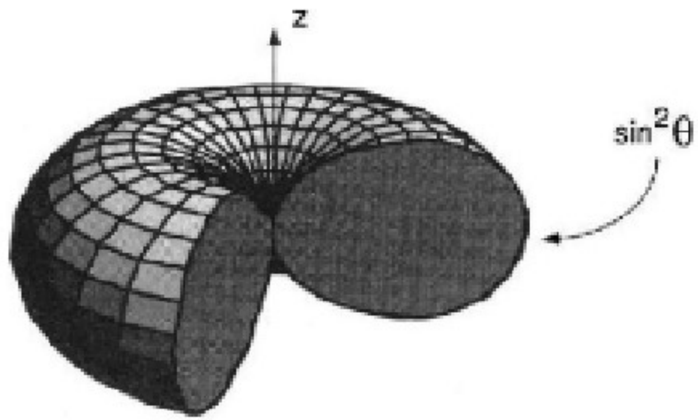
Polarisation and radiation patterns



Azimuth Pattern



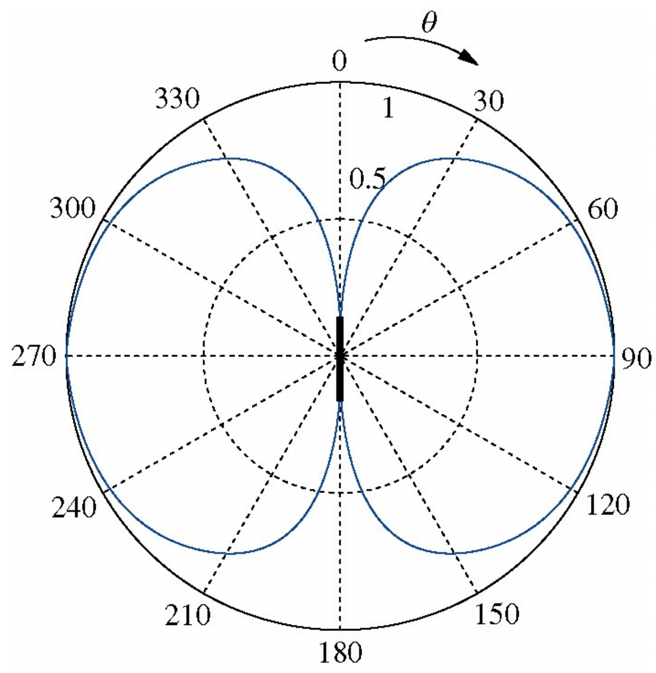
Elevation Pattern



Polar Radiation Pattern

Radiation patterns

- The angular distribution of the radiated fields is called the radiation pattern of the antenna.
- The blue contours shown are called lobes.
  - Represent the antenna's radiation pattern.
  - The lobe in the direction of maximum is called the main lobe; others are called side lobes.

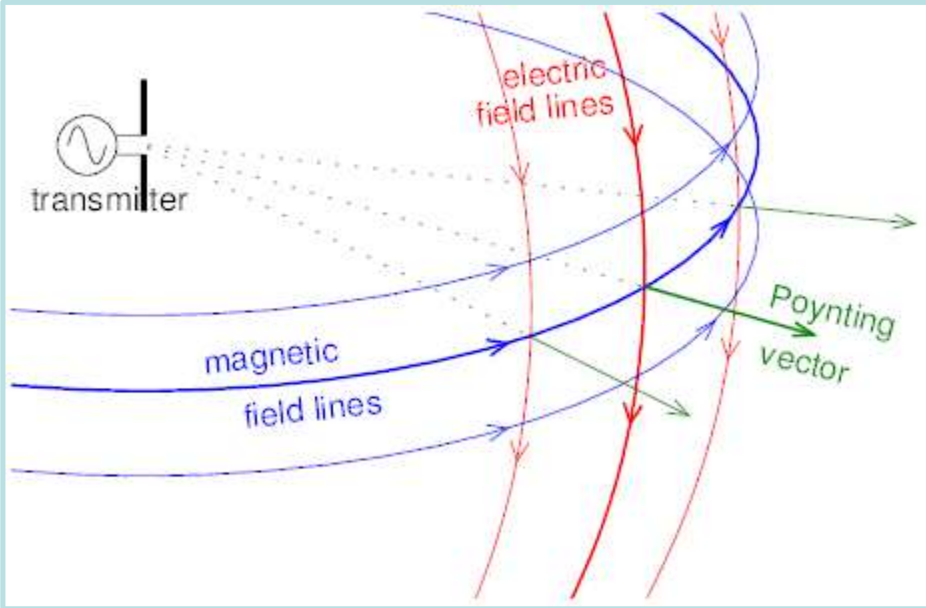


- A null is a minimum between two lobes.
- For the pattern shown, the main lobes are at 90^0 and 270^0 and nulls at 0^0 and 180^0.
- Lobes are due to constructive & destructive interference.

Far field due to Hertzian dipole

- $E_r = 0$ (1)
- $E_\theta = \frac{\eta I L \sin\theta}{2r\lambda} \cos\left(\frac{2\pi r}{\lambda} - \omega t\right)$ (2)
- $E_\phi = 0$ (3)
- $H_r = 0$ (4)
- $H_\theta = 0$ (5)
- $H_\phi = \frac{I L \sin\theta}{2r\lambda} \cos\left(\frac{2\pi r}{\lambda} - \omega t\right)$ (6)
- These equations describe the radiation field - how antennas work. (Very important.)

Poynting Vector



- The equations represent the field radiating away from the dipole.
- The $\vec{E}$ and $\vec{H}$ fields are in phase and at right angles.
- This is a spherical wave.

- *Poynting vector*: directional energy flux density of the electromagnetic field.

$$\vec{P} = \vec{E} \times \vec{H}$$