15-1. A 20-lb block slides down a 30° inclined plane with an initial velocity of 2 ft/s. Determine the velocity of the block in 3 s if the coefficient of kinetic friction between the block and the plane is $\mu_k = 0.25$.

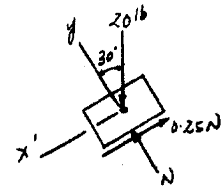
$(+\nwarrow)\qquad m(v_{y'})_1 + \Sigma\int_{t_1}^{t_2} F_{y'}\,dt = m(v_{y'})_2$

$0 + N(3) - 20\cos 30°(3) = 0 \qquad N = 17.32\text{ lb}$

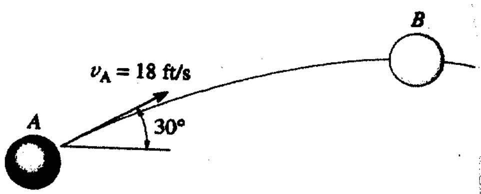
$(+\swarrow)\qquad m(v_{x'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{x'})_2$

$\frac{20}{32.2}(2) + 20\sin 30°(3) - 0.25(17.32)(3) = \frac{20}{32.2}v$

$v = 29.4\text{ ft/s}$ **Ans**



15-2. A 2-lb ball is thrown in the direction shown with an initial speed $v_A = 18$ ft/s. Determine the time needed for it to reach its highest point B and the speed at which it is traveling at B. Use the principle of impulse and momentum for the solution.



$(+\uparrow)\qquad m(v_y)_1 + \Sigma\int F\,dt = m(v_y)_2$

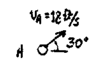
$\frac{2}{32.2}(18\sin 30°) - 2(t) = 0$

$t = 0.2795 = 0.280\text{ s}$ **Ans**

$(\overset{+}{\rightarrow})\qquad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$\frac{2}{32.2}(18\cos 30°) + 0 = \frac{2}{32.2}(v_B)$

$v_B = 15.588 = 15.6\text{ ft/s}$ **Ans**

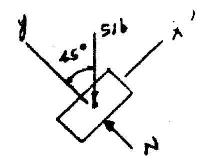


15-3. A 5-lb block is given an initial velocity of 10 ft/s up a 45° smooth slope. Determine the time it will take to travel up the slope before it stops.

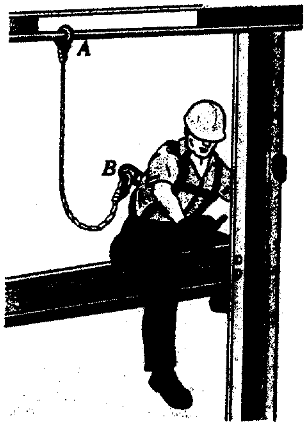
$(\overset{+}{\nearrow})\qquad m(v_{x'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{x'})_2$

$\frac{5}{32.2}(10) + (-5\sin 45°)t = 0$

$t = 0.439\text{ s}$ **Ans**



***15-4.** The 180-lb iron worker is secured by a fall-arrest system consisting of a harness and lanyard AB, which is fixed to the beam. If the lanyard has a slack of 4 ft, determine the average impulsive force developed in the lanyard if he happens to fall 4 feet. Neglect his size in the calculation and assume the impulse takes place in 0.6 seconds.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 180(4) = \frac{1}{2}\left(\frac{180}{32.2}\right)v^2$$

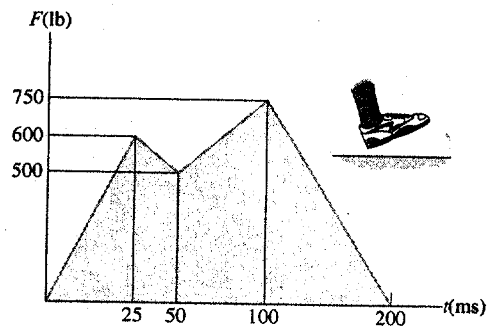
$$v = 16.05 \text{ ft/s}$$

$(+\downarrow)$ $$mv_1 + \int F\,dt = mv_2$$

$$\frac{180}{32.2}(16.05) + 180(0.6) - F(0.6) = 0$$

$$F = 329.5 \text{ lb} = 330 \text{ lb} \quad \textbf{Ans}$$

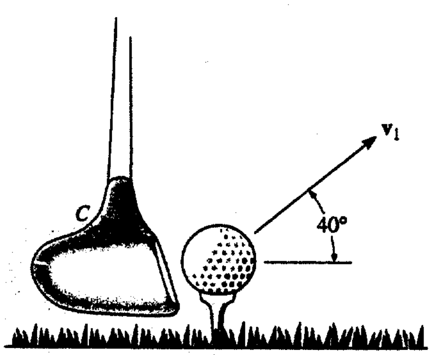
15-5. The graph shows the vertical reaction force of the shoe-ground interaction as a function of time. The first peak acts on the heel, and the second peak acts on the forefoot. Determine the total impulse acting on the shoe during the interaction.



Impulse : The total impluse acting on the shoe can be obtained by evaluating the area under the $F-t$ graph.

$$I = \frac{1}{2}(600)\left[25\left(10^{-3}\right)\right] + \frac{1}{2}(500+600)(50-25)\left(10^{-3}\right)$$
$$+\frac{1}{2}(500+750)(100-50)\left(10^{-3}\right) + \frac{1}{2}(750)\left[(200-100)\left(10^{-3}\right)\right]$$
$$= 90.0 \text{ lb}\cdot\text{s} \quad \textbf{Ans}$$

15-6. A man hits the 50-g golf ball such that it leaves the tee at an angle of 40° with the horizontal and strikes the ground at the same elevation a distance of 20 m away. Determine the impulse of the club C on the ball. Neglect the impulse caused by the ball's weight while the club is striking the ball.



$(\xrightarrow{+})$ $$s_x = (s_0)_x + (v_0)_x t$$

$$20 = 0 + v\cos 40°(t)$$

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$0 = 0 + v\sin 40°(t) - \frac{1}{2}(9.81)t^2$$

$$t = 1.85 \text{ s}$$

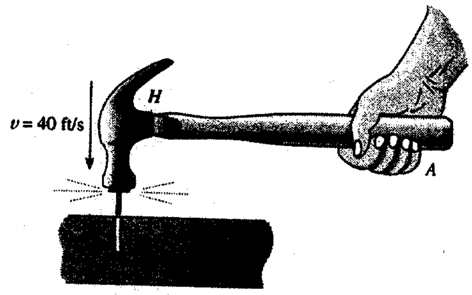
$$v = 14.115 \text{ m/s}$$

$(+\nearrow)$ $$mv_1 + \Sigma\int F\,dt = mv_2$$

$$0 + \int F\,dt = (0.05)(14.115)$$

$$\int F\,dt = 0.706 \text{ N}\cdot\text{s} \measuredangle 40° \quad \textbf{Ans}$$

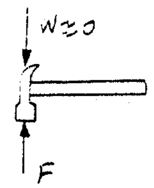
15-7. A hammer head H having a weight of 0.25 lb is moving vertically downward at 40 ft/s when it strikes the head of a nail of negligible mass and drives it into a block of wood. Find the impulse on the nail if it is assumed that the grip at A is loose, the handle has a negligible mass, and the hammer stays in contact with the nail while it comes to rest. Neglect the impulse caused by the weight of the hammer head during contact with the nail.



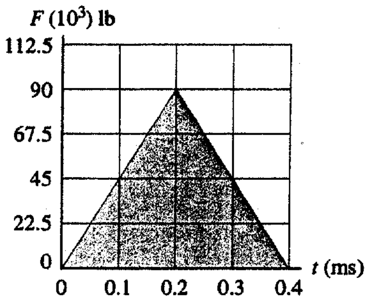
$$(+\downarrow) \qquad m(v_y)_1 + \Sigma \int F_y\, dt = m(v_y)_2$$

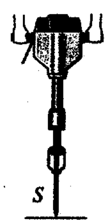
$$\left(\frac{0.25}{32.2}\right)(40) - \int F\, dt = 0$$

$$\int F\, dt = 0.311 \text{ lb}\cdot\text{s} \qquad \textbf{Ans}$$



***15-8.** During operation the breaker hammer develops on the concrete surface a force which is indicated in the graph. To achieve this the 2-lb spike S is fired from rest into the surface at 200 ft/s. Determine the speed of the spike just after rebounding.





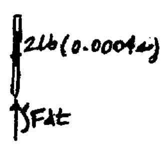
$$(+\downarrow) \qquad mv_1 + \int F dt = mv_2$$

$$\frac{2}{32.2}(200) + 2(0.0004) - Area = \frac{-2}{32.2}(v)$$

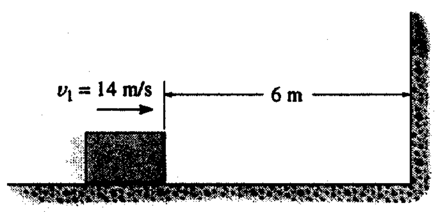
$$Area = \frac{1}{2}(90)(10^3)(0.4)(10^{-3}) = 18 \text{ lb}\cdot\text{s}$$

Thus,

$$v = 89.8 \text{ ft/s} \qquad \textbf{Ans}$$



15-9. When the 5-kg block is 6 m from the wall, it is sliding at $v_1 = 14$ m/s. If the coefficient of kinetic friction between the block and the horizontal plane is $\mu_k = 0.3$, determine the impulse of the wall on the block necessary to stop the block. Neglect the friction impulse acting on the block during the collision.



Equation of Motion : The acceleration of the block must be obtained first before one can determine the velocity of the block before it strikes the wall.

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 5(9.81) = 5(0) \qquad N = 49.05 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -0.3(49.05) = -5a \qquad a = 2.943 \text{ m/s}^2$$

Kinematics : Applying the equation $v^2 = v_0^2 + 2a_c(s - s_0)$ yields

$$(\overset{+}{\rightarrow}) \qquad v^2 = 14^2 + 2(-2.943)(6-0) \qquad v = 12.68 \text{ m/s}$$

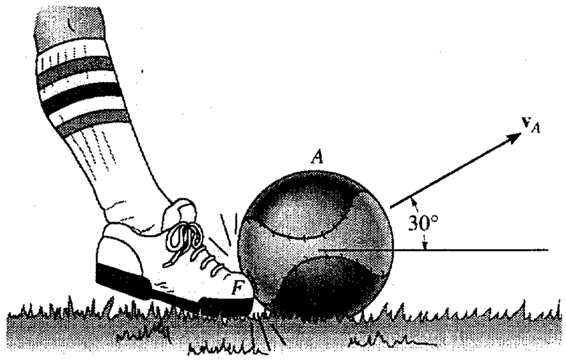
Principle of Linear Impulse and Momentum : Applying Eq. 15 – 4, we have

$$m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 5(12.68) - I = 5(0)$$

$$I = 63.4 \text{ N}\cdot\text{s} \qquad \textbf{Ans}$$

15-10. A man kicks the 200-g ball such that it leaves the ground at an angle of 30° with the horizontal and strikes the ground at the same elevation a distance of 15 m away. Determine the impulse of his foot F on the ball. Neglect the impulse caused by the ball's weight while its being kicked.



$(\overset{+}{\rightarrow})$ $\quad s_x = (s_0)_x + (v_0)_x t + \frac{1}{2}a_c t^2$

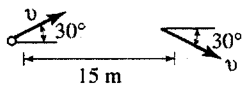
$$15 = 0 + v\cos 30° t + 0$$

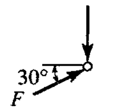
$(+\uparrow)$ $\quad v_y = (v_0)_y + a_c t$

$$-v\sin 30° = v\sin 30° - 9.81t$$

$$t = 1.329 \text{ s}$$

$$v = 13.04 \text{ m/s}$$



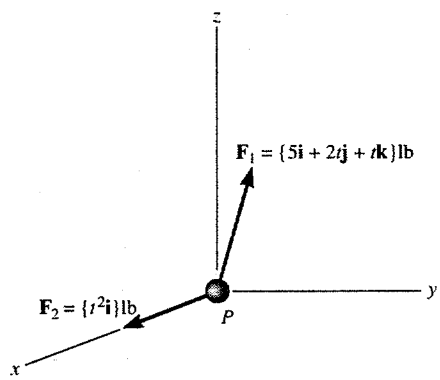


$(+\nearrow)$ $\quad mv_1 + \sum\int F\,dt = mv_2$

$$0 + \int F\,dt = 0.2(13.04)$$

$$I = \int F\,dt = 2.608 = 2.61 \text{ N}\cdot\text{s} \;\angle^{\theta}_{30°} \quad \textbf{Ans}$$

15-11. The particle P is acted upon by its weight of 3 lb and forces F_1 and F_2, where t is in seconds. If the particle originally has a velocity of $v_1 = \{3i + 1j + 6k\}$ ft/s, determine its speed after 2 s.



$$mv_1 + \sum\int_0^2 F\,dt = mv_2$$

Resolving into scalar components,

$$\frac{3}{32.2}(3) + \int_0^2 (5 + t^2)dt = \frac{3}{32.2}(v_x)$$

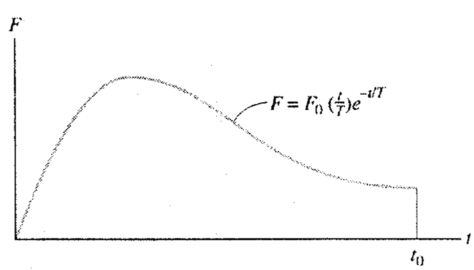
$$\frac{3}{32.2}(1) + \int_0^2 2t\,dt = \frac{3}{32.2}(v_y)$$

$$\frac{3}{32.2}(6) + \int_0^2 (t - 3)dt = \frac{3}{32.2}(v_z)$$

$v_x = 138.96$ ft/s $\quad v_y = 43.933$ ft/s $\quad v_z = -36.933$ ft/s

$$v = \sqrt{(138.96)^2 + (43.933)^2 + (-36.933)^2} = 150 \text{ ft/s} \quad \textbf{Ans}$$

***15-12.** The twitch in a muscle of the arm develops a force which can be measured as a function of time as shown in the graph. If the effective contraction of the muscle lasts for a time t_0, determine the impulse developed by the muscle.



$$I = \int F\,dt = \int_0^{t_0} F_0\left(\frac{t}{T}\right)e^{-t/T}dt$$

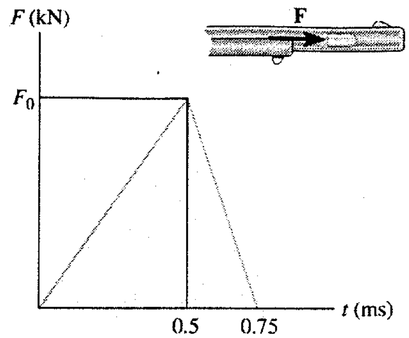
$$I = \frac{F_0}{T}\int_0^{t_0} te^{-(t/T)}dt$$

$$I = -F_0\left[Te^{-t/T}\left(\frac{t}{T} + 1\right)\right]_0^{t_0}$$

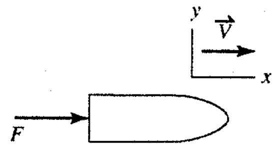
$$I = -F_0\left[Te^{-t_0/T}\left(\frac{t_0}{T} + 1\right) - T\right]$$

$$I = TF_0\left[1 - e^{-t_0/T}\left(1 + \frac{t_0}{T}\right)\right] \quad \textbf{Ans}$$

15-13. Assuming that the force acting on a 2-g bullet, as it passes horizontally through the barrel of a rifle, varies with time in the manner shown, determine the maximum net force F_0, applied to the bullet when it is fired. The muzzle velocity is 500 m/s when $t = 0.75$ ms. Neglect friction between the bullet and the rifle barrel.



Principle of Linear Impulse and Momentum: The total impluse acting on the bullet can be obtained by evaluating the area under the $F - t$ graph. Thus, $I = \sum \int_{t_1}^{t_2} F_x dt = \frac{1}{2}(F_0)[0.5(10^{-3})] + \frac{1}{2}(F_0)[(0.75 - 0.5)(10^{-3})] = 0.375(10^{-3})F_0$. Applying Eq. 15-4, we have

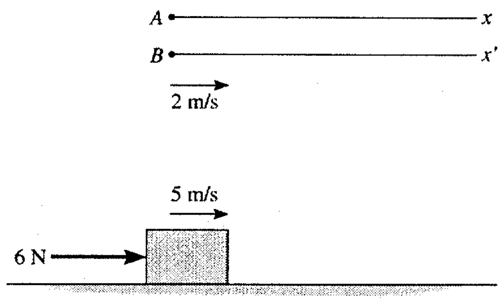


$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_k)_2$$

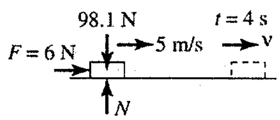
$$(\overset{+}{\rightarrow}) \qquad 0 + 0.375(10^{-3})F_0 = 2(10^{-3})(500)$$

$$F_0 = 2666.67 \text{ N} = 2.67 \text{ kN} \qquad \textbf{Ans}$$

15-14. As indicated by the derivation, the principle of impulse and momentum is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a fixed frame x, determine the final speed of the block in 4 s if it has an initial speed of 5 m/s measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis that moves at a constant velocity of 2 m/s relative to A.



Observer A:



$$(\overset{+}{\rightarrow}) \qquad mv_1 + \sum \int F dt = mv_2$$

$$10(5) + 6(4) = 10v$$

$$v = 7.40 \text{ m/s} \qquad \textbf{Ans}$$

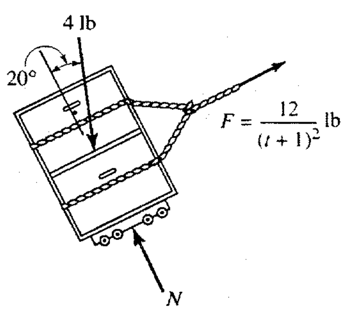
Observer B:

$$(\overset{+}{\rightarrow}) \qquad mv_1 + \sum \int F dt = mv_2$$

$$10(3) + 6(4) = 10v$$

$$v = 5.40 \text{ m/s} \qquad \textbf{Ans}$$

15-15. The 4-lb cabinet is subjected to the force $F = 12(t+1)^2$ lb where t is in seconds. If the cabinet is initially moving up the plane with a velocity of 10 ft/s, determine how long it will take before the cabinet comes to a stop. F always acts parallel to the plane. Neglect the size of the rollers.



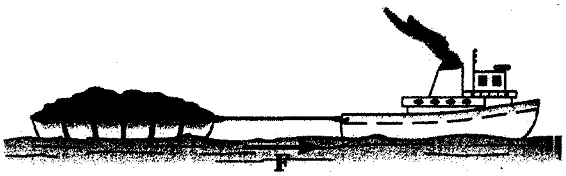
Principle of Linear Impulse and Momentum: Applying Eq. 15-4, we have

$$m(v_{x'})_1 + \sum \int_{t_1}^{t_2} F_{x'} dt = m(v_{x'})_2$$

$$\left(\frac{4}{32.2}\right)(10) + \int_0^t \frac{12}{(t+1)^2} dt - 4 \sin 20° t = \left(\frac{4}{32.2}\right)(0)$$

$$t = 8.78 \text{ s} \qquad \textbf{Ans}$$

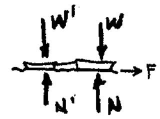
***15-16.** If it takes 35 s for the 50-Mg tugboat to increase its speed uniformly to 25 km/h, starting from rest, determine the force of the rope on the tugboat. The propeller provides the propulsion force **F** which gives the tugboat forward motion, whereas the barge moves freely. Also, determine F acting on the tugboat. The barge has a mass of 75 Mg.



$$25\left(\frac{1000}{3600}\right) = 6.944 \text{ m/s}$$

System :

$$(\overset{+}{\rightarrow}) \quad mv_1 + \Sigma\int F\,dt = mv_2$$



$$[0+0] + F(35) = (50+75)(10^3)(6.944)$$

$F = 24.8$ kN **Ans**

Barge :

$$(\overset{+}{\rightarrow}) \quad mv_1 + \Sigma\int F\,dt = mv_2$$

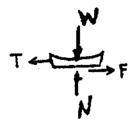
$$0 + T(35) = (75)(10^3)(6.944)$$

$T = 14.881 = 14.9$ kN **Ans**

Also, using this result for T,

Tugboat :

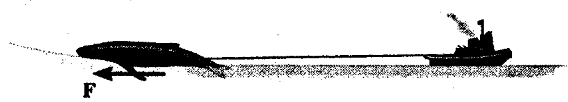
$$(\overset{+}{\rightarrow}) \quad mv_1 + \Sigma\int F\,dt = mv_2$$



$$0 + F(35) - (14.881)(35) = (50)(10^3)(6.944)$$

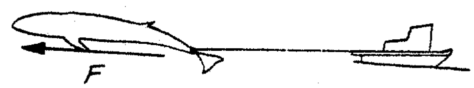
$F = 24.8$ kN **Ans**

15-17. The 5.5-Mg humpback whale is stuck on the shore due to changes in the tide. In an effort to rescue the whale, a 12-Mg tugboat is used to pull it free using an inextensible rope tied to its tail. To overcome the frictional force of the sand on the whale, the tug backs up so that the rope becomes slack and then the tug proceeds forward at 3 m/s. If the tug then turns the engines off, determine the average frictional force **F** on the whale if sliding occurs for 1.5 s before the tug stops after the rope becomes taut. Also, what is the average force on the rope during the tow?



$$(\overset{+}{\rightarrow}) \quad m_1(v_x)_1 + \Sigma\int F_x\,dt = m_2(v_x)_2$$

$$0 + 12(10^3)(3) - F(1.5) = 0 + 0$$



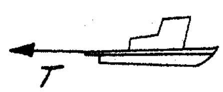
$F = 24$ kN **Ans**

Tug :

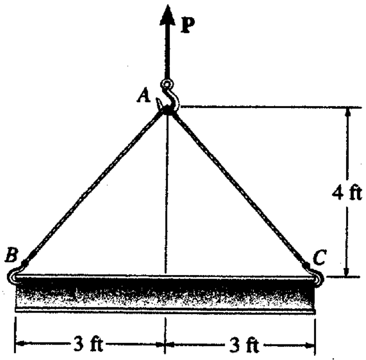
$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

$$12(10^3)(3) - T(1.5) = 0$$

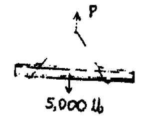
$T = 24$ kN **Ans**



15-18. The uniform beam has a weight of 5000 lb. Determine the average tension in each of the two cables AB and AC if the beam is given an upward speed of 8 ft/s in 1.5 s starting from rest. Neglect the mass of the cables.

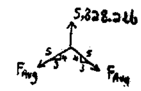


$(+\uparrow)\quad mv_1 + \Sigma\int F\,dt = mv_2$



$$0 + P_{avg}(1.5) - 5000(1.5) = \frac{5000}{32.2}(8)$$

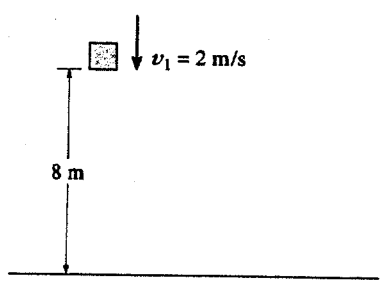
$$P_{avg} = 5828.157 = 5828.2 \text{ lb}$$



$$+\uparrow\Sigma F_y = 0;\quad 5828.157 - 2\left(\frac{4}{5}\right)F_{avg} = 0$$

$$F_{avg} = 3642.598 \text{ lb} = 3.64 \text{ kip} \qquad \textbf{Ans}$$

15-19. The 5-kg block is moving downward at $v_1 = 2$ m/s when it is 8 m from the sandy surface. Determine the impulse of the sand on the block necessary to stop its motion. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.

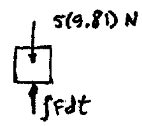


Just before impact

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(5)(2)^2 + 8(5)(9.81) = \frac{1}{2}(5)(v^2)$$

$$v = 12.687 \text{ m/s}$$

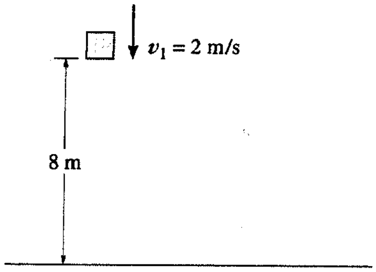


$(+\downarrow)\quad mv_1 + \Sigma\int F\,dt = mv_2$

$$5(12.687) - \int F\,dt = 0$$

$$I = \int F\,dt = 63.4 \text{ N}\cdot\text{s} \qquad \textbf{Ans}$$

***15-20.** The 5-kg block is falling downward at $v_1 = 2$ m/s when it is 8 m from the sandy surface. Determine the average impulsive force acting on the block by the sand if the motion of the block is stopped in 0.9 s once the block strikes the sand. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.

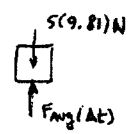


Just before impact

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(5)(2)^2 + 8(5)(9.81) = \frac{1}{2}(5)(v^2)$$

$$v = 12.69 \text{ m/s}$$

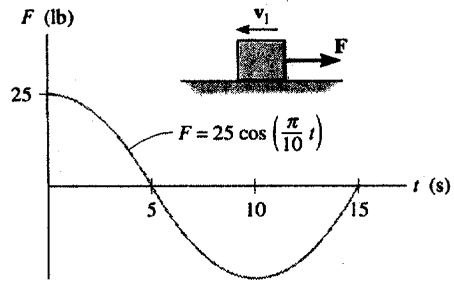


$(+\downarrow)\quad mv_1 + \Sigma\int F\,dt = mv_2$

$$5(12.69) - F_{avg}(0.9) = 0$$

$$F_{avg} = 70.5 \text{ N} \qquad \textbf{Ans}$$

15-21. A 30-lb block is initially moving along a smooth horizontal surface with a speed of $v_1 = 6$ ft/s to the left. If it is acted upon by a force **F**, which varies in the manner shown, determine the velocity of the block in 15 s.



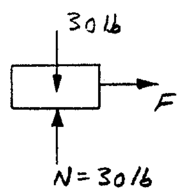
$(\xrightarrow{+})\quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$$-\left(\frac{30}{32.2}\right)(6) + \int_0^{15} 25\cos\left(\frac{\pi}{10}t\right)dt = \left(\frac{30}{32.2}\right)(v_x)_2$$

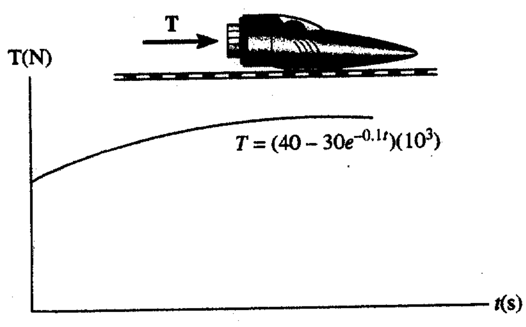
$$-5.59 + (25)\left[\sin\left(\frac{\pi}{10}t\right)\right]_0^{15}\left(\frac{10}{\pi}\right) = \left(\frac{30}{32.2}\right)(v_x)_2$$

$$-5.59 + (25)[-1]\left(\frac{10}{\pi}\right) = \left(\frac{30}{32.2}\right)(v_x)_2$$

$(v_x)_2 = -91.4 = 91.4$ ft/s $\leftarrow$ **Ans**



***15-22.** The rocket sled has a mass of 3 Mg and starts from rest when $t = 0$. If the engines provide a horizontal thrust T which varies as shown in the graph, determine the sled's velocity in $t = 4$ s. Neglect air resistance, friction, and the loss of fuel during the motion.



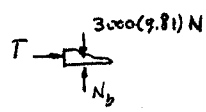
$(\xrightarrow{+})\quad mv_1 + \Sigma\int F\,dt = mv_2$

$$0 + (10^3)\int_0^4 (40 - 30e^{-0.1t})\,dt = 3(10^3)v$$

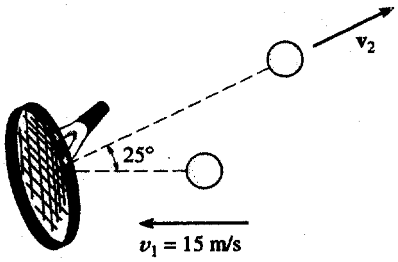
$$(10^3)[40t + 300e^{-0.1t}]_0^4 = 3(10^3)v$$

$$(10^3)[160 + 300(\frac{1}{e^{0.4}}) - 300] = 3(10^3)v$$

$v = 20.4$ m/s **Ans**



15-23. The tennis ball has a horizontal speed of 15 m/s when it is struck by the racket. If it then travels away at an angle of 25° from the horizontal and reaches a maximum altitude of 10 m, measured from the height of the racket, determine the magnitude of the net impulse of the racket on the ball. The ball has a mass of 180 g. Neglect the weight of the ball during the time the racket strikes the ball.



$(+\uparrow)\quad v_y^2 = (v_0)_y^2 + 2a_c(s_y - (s_0)_y)$

$(v_2 \sin 25°)^2 = 0 + 2(9.81)(10-0)$

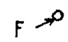
$v_2 = 33.14$ m/s

$(\overset{+}{\rightarrow})\quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$-0.180(15) + \int F_x\,dt = 0.180(33.14\cos 25°)$

$\int F_x\,dt = 8.107$ N·s

$(+\uparrow)\quad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$

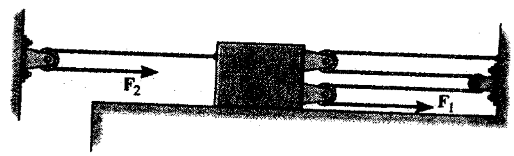


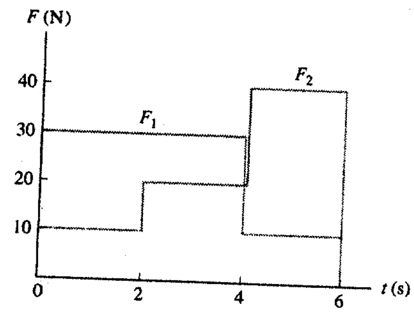
$0 + \int F_y\,dt = 0.180(33.14\sin 25°)$

$\int F_y\,dt = 2.521$ N·s

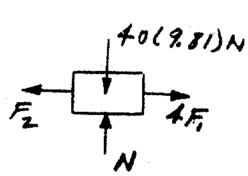
$I = \int F\,dt = \sqrt{(8.107)^2 + (2.521)^2} = 8.49$ N·s $\measuredangle^{17.3°}$ **Ans**

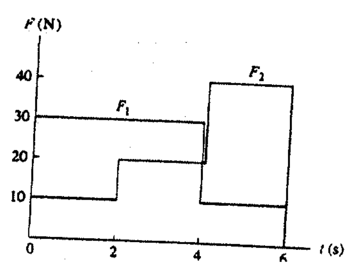
*15-24. The 40-kg slider block is moving to the right with a speed of 1.5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.





The impulses acting on the block are equal to the areas under the graph.



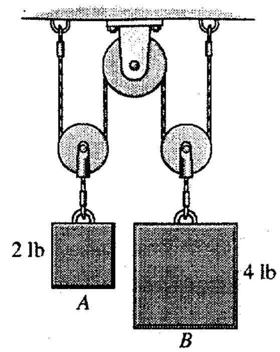


$(\overset{+}{\rightarrow})\quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$40(1.5) + 4[(30)4 + 10(6-4)] - [10(2) + 20(4-2) + 40(6-4)] = 40v_2$

$v_2 = 12.0$ m/s $(\rightarrow)$ **Ans**

15-25. Determine the velocities of blocks A and B 2 s after they are released from rest. Neglect the mass of the pulleys and cables.



$2s_A + 2s_B = l$

$v_A = -v_B$

Block A:

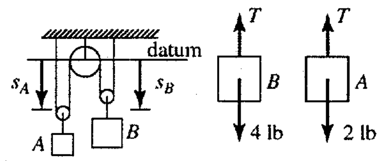
$(+\downarrow) \qquad m(v_y)_1 + \sum \int F_y dt = m(v_y)_2$

$$0 - T(2) + 2(2) = \left(\frac{2}{32.2}\right) v_A$$

Block B:

$(+\downarrow) \qquad m(v_y)_1 + \sum \int F_y dt = m(v_y)_2$

$$0 + 4(2) - T(2) = \left(\frac{4}{32.2}\right) v_B$$



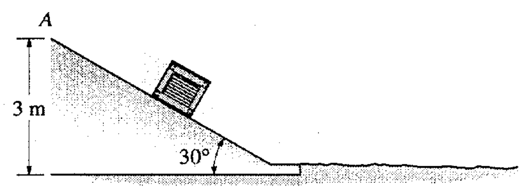
Solving,

$T = 2.67$ lb

$v_B = 21.5$ ft/s $\downarrow$ **Ans**

$v_A = -21.5$ ft/s $= 21.5$ ft/s $\uparrow$ **Ans**

15-26. The 5-kg package is released from rest at A. It slides down the smooth plane onto the rough surface having a coefficient of kinetic friction of $\mu_k = 0.2$. Determine the total time of travel before the package stops sliding. Neglect the size of the package.



Potential Energy: The datum is set at point A. When the package reaches the toe of the inclined plane, its position is 3 m *below* the datum. Its gravitational potential energy is $5(9.81)(-3) = -147.15$ N · m.

Conservation of Energy: Applying Eq. 14-21, we have

$T_1 + V_1 = T_2 + V_2$

$$0 + 0 = \frac{1}{2}(5)v^2 + (-147.15)$$

$$v = 7.672 \text{ m/s}$$

Principle of Linear Impulse and Momentum: The time taken for the package to reach the toe of the inclined plane can be obtained by applying Eq. 15-4.

$$m(v_{x'})_1 + \sum \int_{t_1}^{t_2} F_{x'} dt = m(v_{x'})_2$$

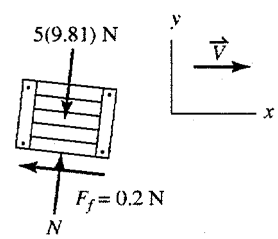
$(\searrow +) \qquad 5(0) + 5(9.81)\sin 30° t_1 = 5(7.672)$

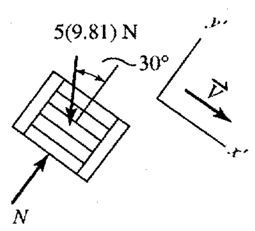
$$t_1 = 1.564 \text{ s}$$

The time taken for the package to stop when it moves along the horizontal rough surface can be obtained by applying Eq. 15-4.

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$(+\uparrow) \qquad 5(0) + N(t_2) - 5(9.81)t_2 = 5(0)$





$N = 49.05$ N

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

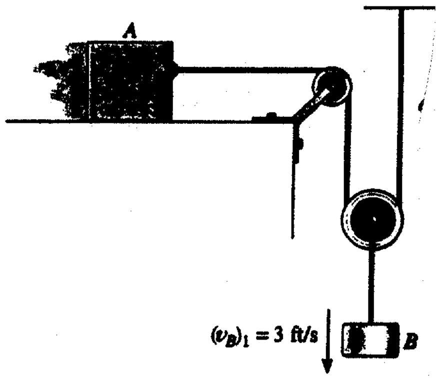
$(\overset{+}{\rightarrow}) \qquad 5(7.672) + [-0.2(49.05)t_2] = 5(0)$

$$t = 3.910 \text{ s}$$

Thus, the total package's traveling time is

$t = t_1 + t_2 = 1.564 + 3.910 = 5.47$ s **Ans**

15-27. Block A weighs 10 lb and block B weighs 3 lb. If B is moving downward with a velocity $(v_B)_1 = 3$ ft/s at $t = 0$, determine the velocity of A when $t = 1$ s.



$$s_A + 2s_B = l$$

$$v_A = -2v_B$$

$(\overset{+}{\leftarrow})$ $$mv_1 + \Sigma\int F\,dt = mv_2$$

$$-\frac{10}{32.2}(2)(3) - T(1) = \frac{10}{32.2}(v_A)_2$$

$(+\downarrow)$ $$mv_1 + \Sigma\int F\,dt = mv_2$$

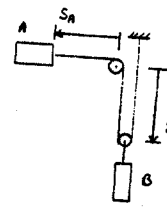
$$\frac{3}{32.2}(3) + 3(1) - 2T(1) = \frac{3}{32.2}\left(-\frac{(v_A)_2}{2}\right)$$

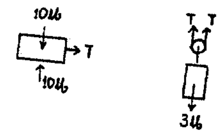
$$-32.2T - 10(v_A)_2 = 60$$

$$-64.4T + 1.5(v_A)_2 = -105.6$$

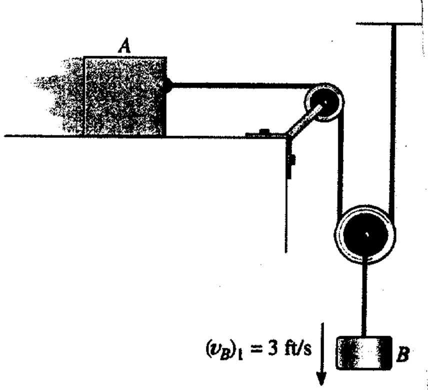
$$T = 1.40 \text{ lb}$$

$(v_A)_2 = -10.5$ ft/s $= 10.5$ ft/s $\rightarrow$ **Ans**





***15-28.** Block A weighs 10 lb and block B weighs 3 lb. If B is moving downward with a velocity $(\mu_B)_1 = 3$ ft/s at $t = 0$, determine the velocity of A when $t = 1$ s. The coefficient of kinetic friction between the horizontal plane and block A is $\mu_A = 0.15$.



$$s_A + 2s_B = l$$

$$v_A = -2v_B$$

$(\overset{+}{\leftarrow})$ $$mv_1 + \Sigma\int F\,dt = mv_2$$

$$-\frac{10}{32.2}(2)(3) - T(1) + 0.15(10) = \frac{10}{32.2}(v_A)_2$$

$(+\downarrow)$ $$mv_1 + \Sigma\int F\,dt = mv_2$$

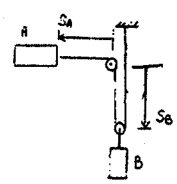
$$\frac{3}{32.2}(3) + 3(1) - 2T(1) = \frac{3}{32.2}\left(-\frac{(v_A)_2}{2}\right)$$

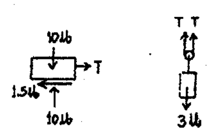
$$-32.2T - 10(v_A)_2 = 11.70$$

$$-64.4T + 1.5(v_A)_2 = -105.6$$

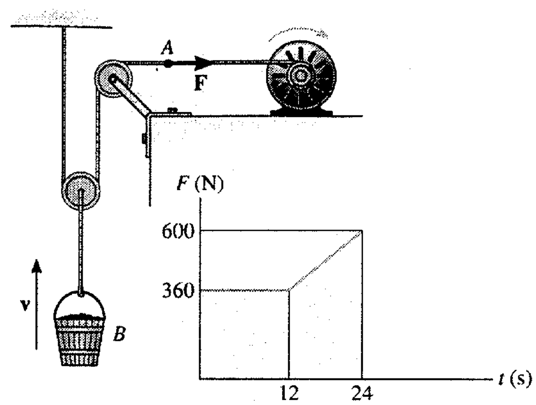
$$T = 1.50 \text{ lb}$$

$(v_A)_2 = -6.00$ ft/s $= 6.00$ ft/s $\rightarrow$ **Ans**

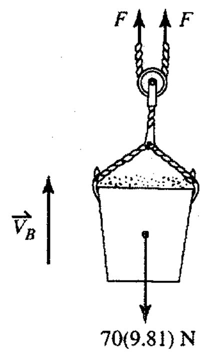




15-29. The winch delivers a horizontal towing force F to its cable at A which varies as shown in the graph. Determine the speed of the 70-kg bucket B when $t = 18$ s. Originally the bucket is moving upward at $v_1 = 3$ m/s.



Principle of Linear Impulse and Momentum: For the time period $12 \text{ s} \le t < 18 \text{ s}$, $\dfrac{F - 360}{t - 12} = \dfrac{600 - 360}{24 - 12}$, $F = (20t + 120)$ N. Applying Eq. 15-4 to bucket B, we have

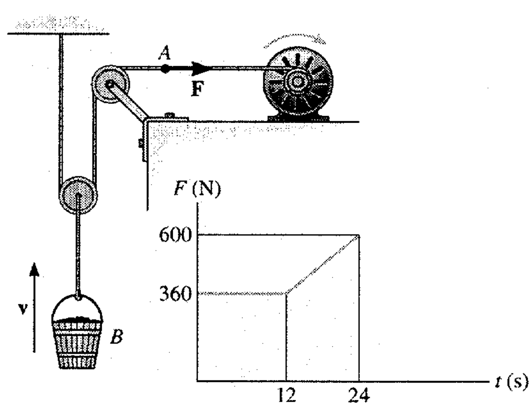


$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$(+\uparrow) \qquad 70(3) + 2\left[360(12) + \int_{12s}^{18s} (20t + 120)dt\right] - 70(9.81)(18) = 70v_2$$

$v_2 = 21.8$ m/s **Ans**

15-30. The winch delivers a horizontal towing force F to its cable at A which varies as shown in the graph. Determine the speed of the 80-kg bucket B when $t = 24$ s. Originally the bucket is moving downward at 20 m/s.

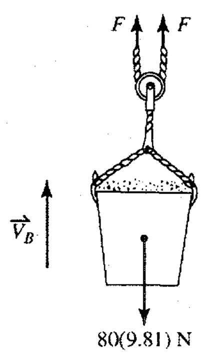


Principle of Linear Impulse and Momentum: The total impluse exerted on bucket B can be obtained by evaluating the area under the $F - t$ graph. Thus,

$$I = \sum \int_{t_1}^{t_2} F_y dt = 2\left[360(12) + \frac{1}{2}(360 + 600)(24 - 12)\right]$$

$$= 20\,160 \text{ N} \cdot \text{s}.$$

Applying Eq. 15-4 to the bucket B, we have

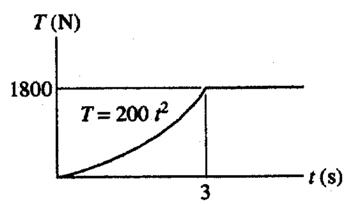


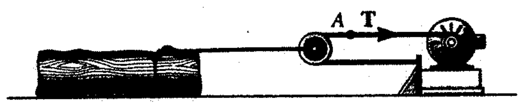
$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

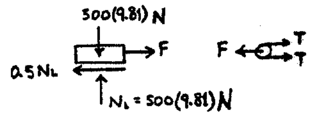
$$(+\uparrow) \qquad 80(-20) + 20\,160 - 80(9.81)(24) = 80v_2$$

$v_2 = -3.44$ m/s $= 3.44$ m/s $\downarrow$ **Ans**

15-31. The log has a mass of 500 kg and rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the log when $t = 5$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the log.







$\xrightarrow{+} \Sigma F_x = 0; \quad F - 0.5(500)(9.81) = 0$

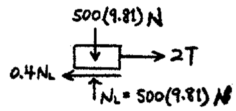
$F = 2452.5$ N

Thus,

$2T = F$

$2(200t^2) = 2452.5$

$t = 2.476$ s to start log moving



$(\xrightarrow{+}) \quad m v_1 + \Sigma \int F\,dt = m v_2$

$0 + 2\int_{2.476}^{3} 200t^2\,dt + 2(1800)(5-3) - 0.4(500)(9.81)(5-2.476) = 500v_2$

$400(\frac{t^3}{3})\Big|_{2.476}^{3} + 2247.91 = 500v_2$

$v_2 = 7.65$ m/s **Ans**

***15-32.** A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

$(\xrightarrow{+}) \quad \Sigma m v_1 = \Sigma m v_2$

$15\,000(1.5) - 12\,000(0.75) = 27\,000(v_2)$

$v_2 = 0.5$ m/s **Ans**

$T_1 = \frac{1}{2}(15\,000)(1.5)^2 + \frac{1}{2}(12\,000)(0.75)^2 = 20.25$ kJ

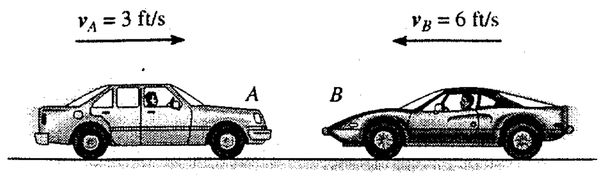
$T_2 = \frac{1}{2}(27\,000)(0.5)^2 = 3.375$ kJ

$\Delta T = T_2 - T_1$

$= 3.375 - 20.25 = -16.9$ kJ **Ans**

This energy is dissipated as noise, shock, and heat during the coupling.

15-33. The car A has a weight of 4500 lb and is traveling to the right at 3 ft/s. Meanwhile a 3000-lb car B is traveling at 6 ft/s to the left. If the cars crash head-on and become entangled, determine their common velocity just after the collision. Assume that the brakes are not applied during collision.

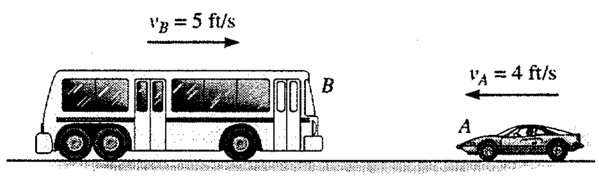


$(\overset{+}{\rightarrow})$ $\quad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$

$$\frac{4500}{32.2}(3) - \frac{3000}{32.2}(6) = \frac{7500}{32.2}v_2$$

$v_2 = -0.600$ ft/s $= 0.600$ ft/s $\leftarrow$ **Ans**

15-34. The bus B has a weight of 15 000 lb and is traveling to the right at 5 ft/s. Meanwhile a 3000-lb car A is traveling at 4 ft/s to the left. If the vehicles crash head-on and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.

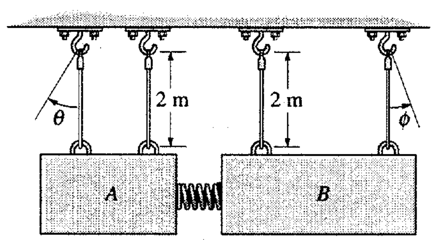


$(\overset{+}{\rightarrow})$ $\quad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v$

$$\frac{15\,000}{32.2}(5) - \frac{3000}{32.2}(4) = \frac{18\,000}{32.2}v$$

$v = 3.5$ ft/s $\rightarrow$ **Ans**

15-35. The two blocks A and B each have a mass of 5 kg and are suspended from parallel cords. A spring, having a stiffness of $k = 60$ N/m, is attached to B and is compressed 0.3 m against A and B as shown. Determine the maximum angles θ and ϕ of the cords when the blocks are released from rest and the spring becomes unstretched.



$(\overset{+}{\rightarrow})$ $\quad \sum mv_1 = \sum mv_2$

$$0 + 0 = -5v_A + 5v_B$$

$$v_A = v_B = v$$

Just before the blocks begin to rise:

$$T_1 + V_1 = T_2 + V_2$$

$$(0+0) + \frac{1}{2}(60)(0.3)^2 = \frac{1}{2}(5)(v)^2 + \frac{1}{2}(5)(v)^2 + 0$$

$v = 0.7348$ m/s

For A or B:

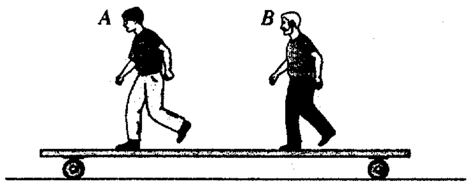
Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(5)(0.7348)^2 + 0 = 0 + 5(9.81)(2)(1 - \cos\theta)$$

$\theta = \phi = 9.52°$ **Ans**

*15-36. Two men A and B, each having a weight of 160 lb, stand on the 200-lb, cart. Each runs with a speed of 3 ft/s measured relative to the cart. Determine the final speed of the cart if (a) A runs and jumps off, then B runs and jumps off the same end, and (b) both run at the same time and jump off at the same time. Neglect the mass of the wheels and assume the jumps are made horizontally.



(a) A jumps first.

$(\overset{+}{\leftarrow})\ 0+0=m_A v_A-(m_C+m_B)v_C'$ However, $v_A=-v_C'+3$

$$0=\frac{160}{32.2}(-v_C'+3)-\frac{360}{32.2}v_C'$$

$$v_C'=0.9231 \text{ ft/s} \rightarrow$$

And then B jumps

$0+(m_C+m_B)v_C'=m_B v_B-m_C v_C$ However, $v_B=-v_C+3$

$$\frac{360}{32.2}(-0.9231)=\frac{160}{32.2}(-v_C+3)-\frac{200}{32.2}v_C$$

$v_C=2.26 \text{ ft/s} \rightarrow$ **Ans**

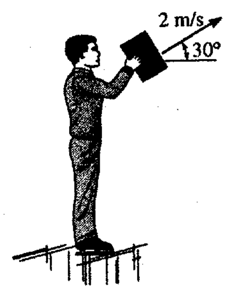
(b) Both men jump at the same time

$(\overset{+}{\leftarrow})\ 0+0=(m_A+m_B)v-m_C v_C$ However, $v=-v_C+3$

$$0=\left(\frac{160}{32.2}+\frac{160}{32.2}\right)(-v_C+3)-\frac{200}{32.2}v_C$$

$v_C=1.85 \text{ ft/s} \rightarrow$ **Ans**

15-37. A man wearing ice skates throws an 8-kg block with an initial velocity of 2 m/s, measured relative to himself, in the direction shown. If he is originally at rest and completes the throw in 1.5 s while keeping his legs rigid, determine the horizontal velocity of the man just after releasing the block. What is the vertical reaction of both his skates on the ice during the throw? The man has a mass of 70 kg. Neglect friction and the motion of his arms.



$(\xrightarrow{+})\quad 0 = -m_M\, v_M + m_B\,(v_B)_x \qquad (1)$

However, $\mathbf{v}_B = \mathbf{v}_M + \mathbf{v}_{B/M}$

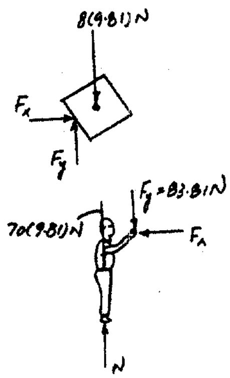
$(\xrightarrow{+})\quad (v_B)_x = -v_M + 2\cos 30° \qquad (2)$

$(+\uparrow)\quad (v_B)_y = 0 + 2\sin 30° = 1\text{ m/s}$

Substituting Eq. (2) into (1) yields :

$0 = -m_M\, v_M + m_B\,(-v_M + 2\cos 30°)$

$$v_M = \frac{2m_B\cos 30°}{m_B + m_M} = \frac{2(8)\cos 30°}{8+70} = 0.178\text{ m/s} \qquad \textbf{Ans}$$



For the block

$(+\uparrow)\quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$

$0 + F_y(1.5) - 8(9.81)(1.5) = 8(2\sin 30°) \qquad F_y = 83.81\text{ N}$

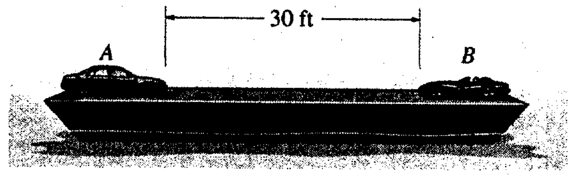
For the man

$(+\uparrow)\quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$

$0 + N(1.5) - 70(9.81)(1.5) - 83.81(1.5) = 0$

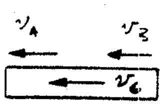
$N = 771\text{ N} \qquad$ **Ans**

15-38. The barge weighs 45 000 lb and supports two automobiles A and B, which weigh 4000 lb and 3000 lb, respectively. If the automobiles start from rest and drive towards each other, accelerating at $a_A = 4\ \text{ft/s}^2$ and $a_B = 8\ \text{ft/s}^2$ until they reach a constant speed of 6 ft/s relative to the barge, determine the speed of the barge just before the automobiles collide. How much time does this take? Originally the barge is at rest. Neglect water resistance.



$(\overset{+}{\leftarrow})$ $\quad v_A = v_C + v_{A/C} = v_C - 6$

$(\overset{+}{\leftarrow})$ $\quad v_B = v_C + v_{B/C} = v_C + 6$



$(\overset{+}{\leftarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 = m_A(v_C - 6) + m_B(v_C + 6) + m_C v_C$$

$$0 = \left(\frac{4000}{32.2}\right)(v_C - 6) + \left(\frac{3000}{32.2}\right)(v_C + 6) + \left(\frac{45\,000}{32.2}\right)v_C$$

$v_C = 0.1154\ \text{ft/s} = 0.115\ \text{ft/s}$ **Ans**

For A:

$(\overset{+}{\rightarrow})$ $\quad v = v_0 + a_c t$

$$6 = 0 + 4t_A$$

$$t_A = 1.5\ \text{s}$$

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$s = 0 + 0 + \frac{1}{2}(4)(1.5)^2 = 4.5\ \text{ft}$$

For B:

$(\overset{+}{\leftarrow})$ $\quad v = v_0 + a_c t$

$$6 = 0 + 8t_B$$

$$t_B = 0.75\ \text{s}$$

$(\overset{+}{\leftarrow})$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$s = 0 + 0 + \frac{1}{2}(8)(0.75)^2 = 2.25\ \text{ft}$$

For the remaining $(1.5 - 0.75)\text{s} = 0.75\ \text{s}$

$$s = vt = 6(0.75) = 4.5\ \text{ft}$$

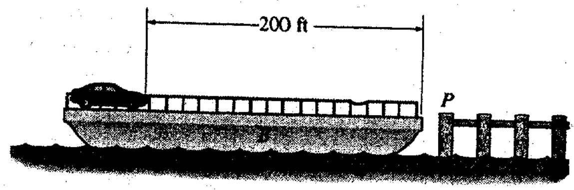
Thus,

$$s = 30 - 4.5 - 4.5 - 2.25 = 18.75\ \text{ft}$$

$$t' = \frac{s/2}{v} = \frac{18.75/2}{6} = 1.5625$$

$t = 1.5 + 1.5625 = 3.06\ \text{s}$ **Ans**

15-39. The barge B weighs 30 000 lb and supports an automobile weighing 3000 lb. If the barge is not tied to the pier P and someone drives the automobile to the other side of the barge for unloading, determine how far the barge moves away from the pier. Neglect the resistance of the water.



Relative Velocity : The relative velocity of the car with respect to the barge is $v_{c/b}$. Thus, the velocity of the car is

$$(\overset{+}{\rightarrow}) \qquad v_c = -v_b + v_{c/b} \qquad [1]$$

Conservation of Linear Momentum : If we consider the car and the barge as a system, then the impulsive force caused by the traction of the tires is *internal* to the system. Therefore, it will cancel out. As the result, the linear momentum is conserved along the x axis.

$$0 = m_c v_c + m_b v_b$$

$$(\overset{+}{\rightarrow}) \qquad 0+0 = \left(\frac{3000}{32.2}\right)v_c + \left(\frac{30\ 000}{32.2}\right)v_b \qquad [2]$$

Substituting Eq. [1] into [2] yields

$$11v_b - v_{c/b} = 0 \qquad [3]$$

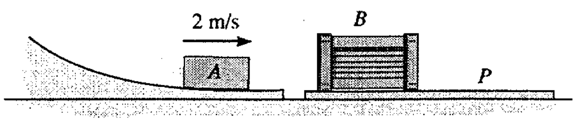
Integrating Eq. [3] becomes

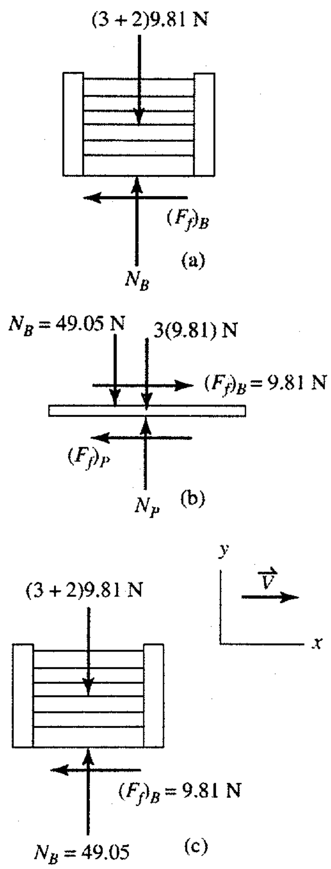
$$(\overset{+}{\rightarrow}) \qquad 11s_b - s_{c/b} = 0 \qquad [4]$$

Here, $s_{c/b} = 200$ ft. Then, from Eq. [4]

$$11s_b - 200 = 0 \qquad s_b = 18.2 \text{ ft} \qquad \textbf{Ans}$$

***15-40.** The block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu_k' = 0.4$. Also, the coefficient of static friction between the plate and the floor is $\mu_s' = 0.5$.





Equations of Equilibrium: From FBD(a),

$$+\uparrow \sum F_y = 0; \quad N_B - (3+2)(9.81) = 0 \quad N_B = 49.05 \text{ N}$$

When box B slides on top of plate P, $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b), and assuming the plate does not move.

$$+\uparrow \sum F_y = 0; \quad N_P - 49.05 - 3(9.81) = 0 \quad N_P = 78.48 \text{ N}$$

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad 9.81 - (F_f)_P = 0 \quad (F_f)_P = 9.81 \text{ N}$$

Since $(F_f)_P < [(F_f)_P]_{\max} = \mu_s' N_P = 0.5(78.48) = 39.24$ N, plate P does not move. Thus

$s_P = 0$ **Ans**

Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along the x axis.

$$m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$(\overset{+}{\rightarrow}) \quad 2(2) + 0 = (2+3)v_2$$

$$v_2 = 0.800 \text{ m/s} \rightarrow$$

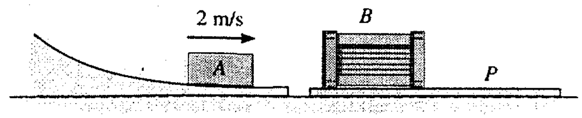
Principle of Linear Impulse and Momentum: Applying Eq. 15-4, we have

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \quad 5(0.8) + [-9.81(t)] = 5(0)$$

$t = 0.408$ s **Ans**

15-41. The block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu_k' = 0.1$. Also, the coefficient of static friction between the plate and the floor is $\mu_s' = 0.12$.



Equations of Equilibrium: From FBD(a),

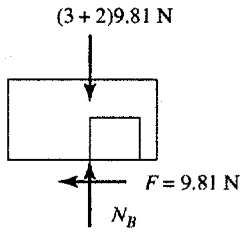
$$+\uparrow \sum F_y = 0; \quad N_B - (3+2)(9.81) = 0 \quad N_B = 49.05 \text{ N}$$

When box B slides on top of plate P, $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b), and assuming the plate does not move,

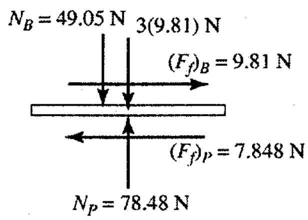
$$+\uparrow \sum F_y = 0; \qquad N_P - 49.05 - 3(9.81) = 0 \quad N_P = 78.48 \text{ N}$$

$$\overset{+}{\rightarrow} \sum F_x = 0; \qquad 9.81 - (F_f)_P = 0 \quad (F_f)_P = 9.81 \text{ N}$$

Since $(F_f)_P > [(F_f)_P]_{\max} = \mu_s' N_P = 0.12(78.48) = 9.418$ N, plate P slides. Thus, $(F_f)_P = \mu_k' N_P = 0.1(78.48) = 7.848$ N.



(a)



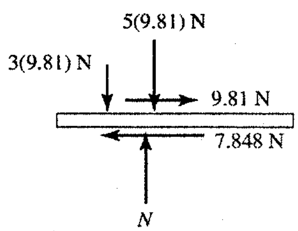
(b)

Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along x axis.

$$m_A(v_A)_1 + m_R(v_R)_1 = (m_A + m_R)v_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(2) + 0 = (2+3)v_2$$

$$v_2 = 0.800 \text{ m/s} \rightarrow$$



(c)

Principle of Linear Impulse and Momentum: In order for box B to stop sliding on plate P, both box B and plate P must have same speed v_3. Applying Eq. 15-4 to box B [FBD(a)], we have

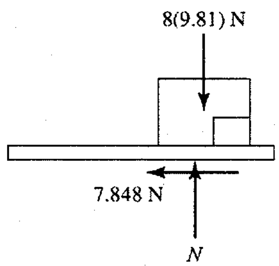
$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 5(0.8) + [-9.81(t_1)] = 5v_3 \qquad [1]$$

Applying Eq. 15-4 to plate P[FBD(d)], we have

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 3(0) + 9.81(t_1) - 7.848(t_1) = 3v_3 \quad [2]$$



(d)

Solving Eqs. [1] and [2] yields

$t_1 = 0.3058$ s $\quad v_3 = 0.200$ m/s

15-41. (Cont.)

Equation of Motion: From FBD(c), the acceleration of plate P when box B still slides on top of it is given by

$$\xrightarrow{+} \sum F_x = ma_x; \quad 9.81 - 7.848 = 3(a_P)_1 \quad (a_P)_1 = 0.654 \text{ m/s}^2$$

When box B stops sliding on top of plate, $(F_f)_B = 0$. From this instant onward plate P and box B act as a unit and slide together. From FBD(d), the acceleration of plate P and box B is given by

$$\xrightarrow{+} \sum F_x = ma_x; \quad -7.848 = 8(a_P)_2 \quad (a_P)_2 = -0.981 \text{ m/s}^2$$

Kinematics: Plate P travels a distance d_1 before box B stop sliding.

$$(\xrightarrow{+}) \quad d_1 = (v_0)_P t_1 + \frac{1}{2}(a_P)_1 t_1^2$$

$$= 0 + \frac{1}{2}(0.654)(0.3058^2) = 0.03058 \text{ m}$$

The time t_2 for plate P to stop after box B stop sliding is given by

$$(\xrightarrow{+}) \quad v_4 = v_3 + (a_P)_2 t_2$$

$$0 = 0.200 + (-0.981)t_2 \quad t_2 = 0.2039 \text{ s}$$

The distance d_2 traveled by plate P after box B stop sliding is given by

$$(\xrightarrow{+}) \quad v_4^2 = v_3^2 + 2(a_P)_2 d_2$$

$$0 = 0.200^2 + 2(-0.981)d_2 \quad d_2 = 0.02039 \text{ m}$$

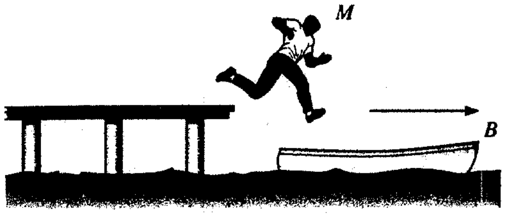
The total distance traveled by plate P is

$$d_P = d_1 + d_2 = 0.03058 + 0.02039 = 0.05097 \text{ m} = 0.0510 \text{ m} \quad \textbf{Ans}$$

The total time taken to cease all the motion is

$$t_{\text{Tot}} = t_1 + t_2 = 0.3058 + 0.2039 = 0.510 \text{ s} \quad \textbf{Ans}$$

***15-42.** The man M weighs 150 lb and jumps onto the boat B which has a weight of 200 lb. If he has a horizontal component of velocity *relative to the boat* of 3 ft/s, just before he enters the boat, and the boat is traveling $v_B = 2$ ft/s away from the pier when he makes the jump, determine the resulting velocity of the man and boat.



$(\xrightarrow{+})\quad v_M = v_B + v_{M/B}$

$v_M = 2+3$

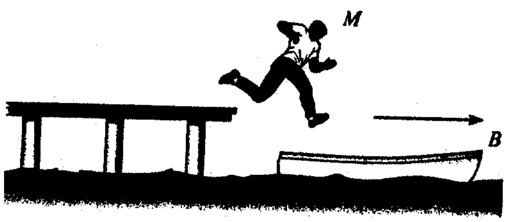
$v_M = 5 \text{ ft/s}$

$(\xrightarrow{+})\quad \Sigma m\, v_1 = \Sigma m\, v_2$

$$\frac{150}{32.2}(5) + \frac{200}{32.2}(2) = \frac{350}{32.2}(v_B)_2$$

$(v_B)_2 = 3.29 \text{ ft/s}$ **Ans**

***15-43.** The man M weighs 150 lb and jumps onto the boat B which is originally at rest. If he has a horizontal component of velocity of 3 ft/s just before he enters the boat, determine the weight of the boat if it has a velocity of 2 ft/s once the man enters it.



$(\xrightarrow{+})\quad v_M = v_B + v_{M/B}$

$v_M = 0+3$

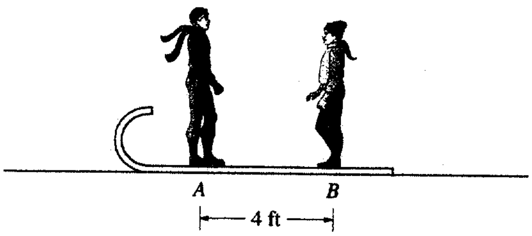
$v_M = 3 \text{ ft/s}$

$(\xrightarrow{+})\quad \Sigma m(v_1) = \Sigma m(v_2)$

$$\frac{150}{32.2}(3) + \frac{W_B}{32.2}(0) = \frac{(W_B+150)}{32.2}(2)$$

$W_B = 75 \text{ lb}$ **Ans**

*15-44. A boy A having a weight of 80 lb and a girl B having a weight of 65 lb stand motionless at the ends of the toboggan, which has a weight of 20 lb. If A walks to B and stops, and both walk back together to the original position of A, determine the final position of the toboggan just after the motion stops. Neglect friction.



A goes to B,

$(\overset{+}{\rightarrow})$ $\Sigma mv_1 = \Sigma mv_2$

$$0 = m_A v_A - (m_t + m_B)v_B$$

$$0 = m_A s_A - (m_t + m_B)s_B$$

Assume B moves x to the left, then A moves $(4-x)$ to the right

$$0 = m_A(4-x) - (m_t + m_B)x$$

$$x = \frac{4m_A}{m_A + m_B + m_t}$$

$$= \frac{4(80)}{80+65+20} = 1.939 \text{ ft} \leftarrow$$

A and B go to other end.

$(\overset{+}{\rightarrow})$ $\Sigma mv_1 = \Sigma mv_2$

$$0 = -m_B v - m_A v + m_t v_t$$

$$0 = -m_B s - m_A s + m_t s_t$$

Assume the toboggan moves x' to the right, then A and B move $(4-x')$ to the left

$$0 = -m_B(4-x') - m_A(4-x') + m_t x'$$

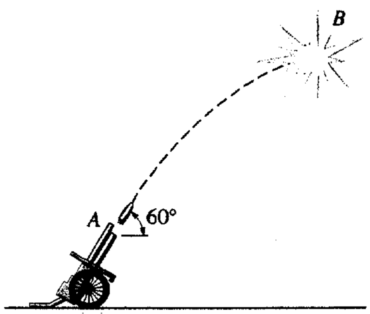
$$x' = \frac{4(m_B + m_A)}{m_A + m_B + m_t}$$

$$= \frac{4(65+80)}{80+65+20} = 3.515 \text{ ft} \rightarrow$$

Net movement of sled is

$(\overset{+}{\rightarrow})$ $x = 3.515 - 1.939 = 1.58 \text{ ft} \rightarrow$ **Ans**

15-45. The 10-lb projectile is fired from ground level with an initial velocity of $v_A = 80$ ft/s in the direction shown. When it reaches its highest point B it explodes into two 5-lb fragments. If one fragment travels vertically upward at 12 ft/s, determine the distance between the fragments after they strike the ground. Neglect the size of the gun.



$(\overset{+}{\rightarrow})\quad v_x = (v_0)_x$

$v_B = 80\cos 60° = 40$ ft/s

$(+\uparrow)\quad v_y^2 = (v_0)_y^2 + 2a_c(s_y - (s_0)_y)$

$0 = (80\sin 60°)^2 + 2(-32.2)(h - 0)$

$h = 74.53$ ft

$(\overset{+}{\rightarrow})\quad \Sigma mv_1 = \Sigma mv_2$

$\frac{10}{32.2}(80\cos 60°) = \frac{5}{32.2}(v_F)_x$

$(v_F)_x = 80$ ft/s $\rightarrow$

$(+\uparrow)\quad \Sigma mv_1 = \Sigma mv_2$

$0 = \frac{5}{32.2}(v_F)_y + \frac{5}{32.2}(12)$

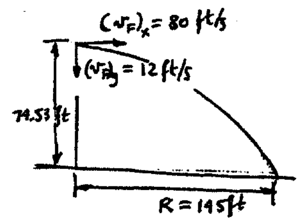
$(v_F)_y = -12$ ft/s $= 12$ ft/s $\downarrow$

$(+\downarrow)\quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}a_c t^2$

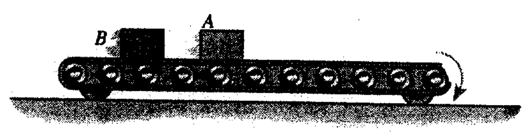
$74.53 = 0 + 12t + \frac{1}{2}(32.2)t^2$

$t = 1.81$ s

$(\overset{+}{\rightarrow})\quad R = 80(1.81) = 145$ ft **Ans**



15-46. Two boxes A and B, each having a weight of 160 lb, sit on the 500-lb conveyor which is free to roll on the ground. If the belt starts from rest and begins to run with a speed of 3 ft/s, determine the final speed of the conveyor if (a) the boxes are not stacked and A falls off then B falls off, and (b) A is stacked on top of B and both fall off together.



a) Let v_b be the velocity of A and B.

$(\overset{+}{\rightarrow})\quad \Sigma mv_1 = \Sigma mv_2$

$$0 = \left(\frac{320}{32.2}\right)(v_b) - \left(\frac{500}{32.2}\right)(v_c)$$

$(\overset{+}{\rightarrow})\quad v_b = v_c + v_{b/c}$

$v_b = -v_c + 3$

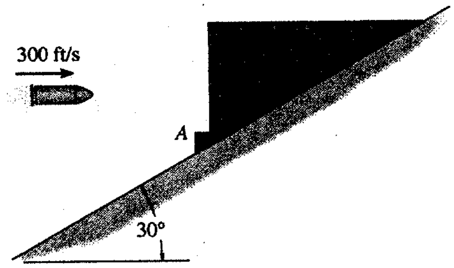
Thus, $v_b = 1.83$ ft/s $\rightarrow$ $\quad v_c = 1.17$ ft/s $\leftarrow$

When a box falls off, it exerts no impulse on the conveyor, and so does not alter the momentum of the conveyor. Thus,

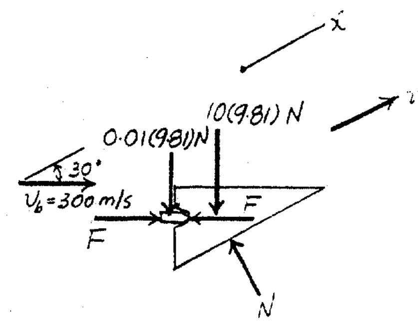
a) $v_c = 1.17$ ft/s $\leftarrow$ **Ans**

b) $v_c = 1.17$ ft/s $\leftarrow$ **Ans**

15-47. The 10-kg block is held at rest on the smooth inclined plane by the stop block at A. If the 10-g bullet is traveling at 300 m/s when it becomes embedded in the 10-kg block, determine the distance the block will slide up along the plane before momentarily stopping.



Conservation of Linear Momentum : If we consider the block and the bullet as a system, then from the FBD, the *impulsive* force F caused by the impact is *internal* to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are *nonimpulsive forces*. As the result, linear momentum is conserved along the x' axis.



$$m_b(v_b)_{x'} = (m_b + m_B)\,v_x$$

$$(+\nearrow) \quad 0.01(300\cos 30°) = (0.01 + 10)\,v$$

$$v = 0.2595 \text{ m/s}$$

Conservation of Energy : The datum is set at the block's initial position. When the block and the embedded bullet is at their highest point, they are h *above* the datum. Their gravitational potential energy is $(10+0.01)(9.81)h = 98.1981h$. Applying Eq. 14–21, we have

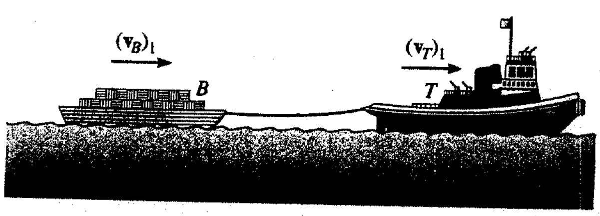
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(10+0.01)\left(0.2595^2\right) = 0 + 98.1981h$$

$$h = 0.003433 \text{ m} = 3.43 \text{ mm}$$

$$d = 3.43 / \sin 30° = 6.87 \text{ mm} \qquad \textbf{Ans}$$

***15-48.** A tugboat T having a mass of 19 Mg is tied to a barge B having a mass of 75 Mg. If the rope is "elastic" such that it has a stiffness k = 600 kN/m, determine the maximum stretch in the rope during the initial towing. Originally both the tugboat and barge are moving in the same direction with speeds $(v_T)_1$ = 15 km/h and $(v_B)_1$ = 10 km/h, respectively. Neglect the resistance of the water.



$(v_T)_1 = 15 \text{ km/h} = 4.167 \text{ m/s}$

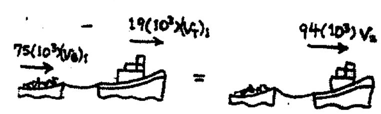
$(v_B)_1 = 10 \text{ km/h} = 2.778 \text{ m/s}$

When the rope is stretched to its maximum, both the tug and barge have a common velocity. Hence,

$$(\xrightarrow{+}) \quad \Sigma m v_1 = \Sigma m v_2$$

$$19\,000(4.167) + 75\,000(2.778) = (19\,000 + 75\,000)v_2$$

$$v_2 = 3.059 \text{ m/s}$$



$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = \frac{1}{2}(19\,000)(4.167)^2 + \frac{1}{2}(75\,000)(2.778)^2 = 454.282 \text{ kJ}$$

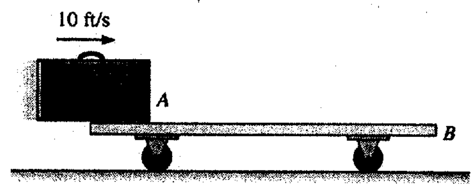
$$T_2 = \frac{1}{2}(19\,000 + 75\,000)(3.059)^2 = 439.661 \text{ kJ}$$

Hence,

$$454.282(10^3) + 0 = 439.661(10^3) + \frac{1}{2}(600)(10^3)x^2$$

$$x = 0.221 \text{ m} \qquad \textbf{Ans}$$

15-49. The 20-lb cart B is supported on rollers of negligible size. If a 10-lb suitcase A is thrown horizontally on it at 10 ft/s, determine the length of time that A slides relative to B, and the final velocity of A and B. The coefficient of kinetic friction between A and B is $\mu_k = 0.4$.

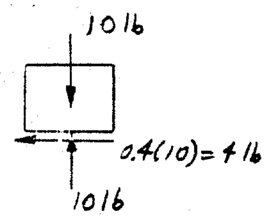


System :

$(\xrightarrow{+})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$\left(\frac{10}{32.2}\right)(10) + 0 = \left(\frac{10+20}{32.2}\right)v$$

$v = 3.33$ ft/s $\quad$ **Ans**



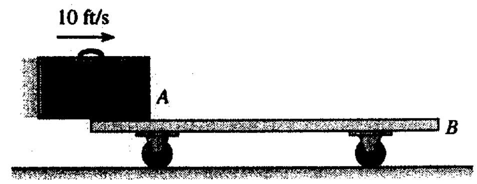
For A :

$$mv_1 + \Sigma \int F\,dt = mv_2$$

$$\left(\frac{10}{32.2}\right)(10) - 4t = \left(\frac{10}{32.2}\right)(3.33)$$

$t = 0.5176 = 0.518$ s $\quad$ **Ans**

15-50. The 20-lb cart B is supported on rollers of negligible size. If a 10-lb suitcase A is thrown horizontally on it at 10 ft/s, determine the time t and the distance B moves before A stops relative to B. The coefficient of kinetic friction between A and B is $\mu_k = 0.4$.



System :

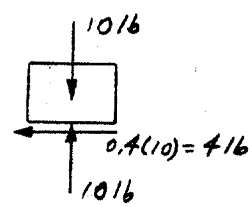
$(\overset{+}{\rightarrow})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$\left(\frac{10}{32.2}\right)(10) + 0 = \left(\frac{10+20}{32.2}\right)v$$

$v = 3.33$ ft/s

For A :

$m v_1 + \Sigma \int F\, dt = m v_2$



$$\left(\frac{10}{32.2}\right)(10) - 4t = \left(\frac{10}{32.2}\right)(3.33)$$

$t = 0.5176 = 0.518$ s **Ans**

For B :

$(\overset{+}{\rightarrow})$ $\quad v = v_0 + a_c t$

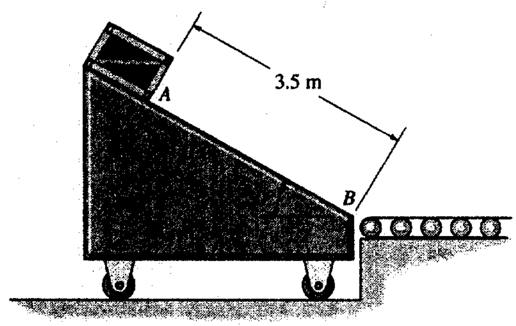
$3.33 = 0 + a_c\,(0.5176)$

$a_c = 6.440\ \text{ft/s}^2$

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s = 0 + 0 + \frac{1}{2}(6.440)(0.5176)^2 = 0.863$ ft **Ans**

15-51. The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at A and slides down 3.5 m to point B. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches B. Also, what is the velocity of the crate?

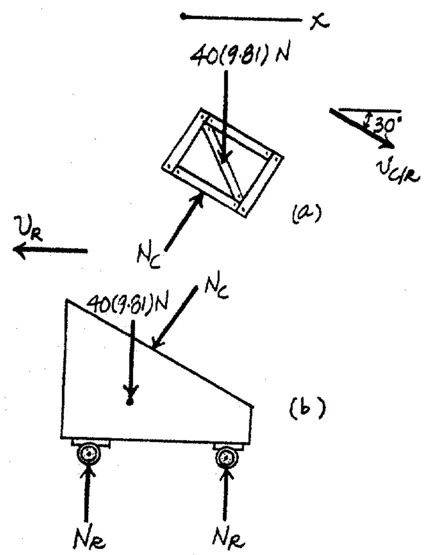


Conservation of Energy : The datum is set at lowest point B. When the crate is at point A, it is $3.5\sin 30° = 1.75$ m *above* the datum. Its gravitational potential energy is $10(9.81)(1.75) = 171.675$ N · m. Applying Eq. 14 – 21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 171.675 = \frac{1}{2}(10)\,v_C^2 + \frac{1}{2}(40)\,v_R^2$$

$$171.675 = 5v_C^2 + 20v_R^2 \qquad [1]$$



Relative Velocity : The velocity of the crate is given by

$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_R + \mathbf{v}_{C/R}\\ &= -v_R\mathbf{i} + (v_{C/R}\cos 30°\mathbf{i} - v_{C/R}\sin 30°\mathbf{j})\\ &= (0.8660v_{C/R} - v_R)\mathbf{i} - 0.5v_{C/R}\mathbf{j}\end{aligned} \qquad [2]$$

The magnitude of $\mathbf{v}_C$ is

$$\begin{aligned}v_C &= \sqrt{(0.8660v_{C/R} - v_R)^2 + (-0.5v_{C/R})^2}\\ &= \sqrt{v_{C/R}^2 + v_R^2 - 1.732v_R\,v_{C/R}}\end{aligned} \qquad [3]$$

Conservation of Linear Momentum : If we consider the crate and the ramp as a system, from the FBD, one realizes that the normal reaction N_C (*impulsive force*) is *internal* to the system and will cancel each other. As the result, the linear momentum is conserved along the x axis.

$$0 = m_C(v_C)_x + m_R v_R$$

$$(\overset{+}{\rightarrow}) \qquad 0 = 10(0.8660v_{C/R} - v_R) + 40(-v_R)$$

$$0 = 8.660v_{C/B} - 50v_B \qquad [4]$$

Solving Eqs.[1], [3] and [4] yields

$$v_R = 1.101 \text{ m/s} = 1.10 \text{ m/s} \qquad v_C = 5.43 \text{ m/s} \qquad \textbf{Ans}$$

$$v_{C/R} = 6.356 \text{ m/s}$$

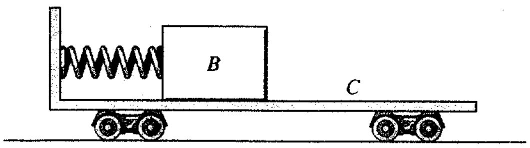
From Eq.[1]

$$\mathbf{v}_C = [0.8660(6.356) - 1.101]\mathbf{i} - 0.5(6.356)\mathbf{j} = \{4.403\mathbf{i} - 3.178\mathbf{j}\} \text{ m/s}$$

Thus, the directional angle ϕ of v_C is

$$\phi = \tan^{-1}\frac{3.178}{4.403} = 35.8° \qquad \textbf{Ans}$$

*15-52. The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring which is attached to the cart and not the block is compressed 0.2 m and the system is released from rest, determine the speed of the block after the spring becomes undeformed. Neglect the mass of the cart's wheels and the spring in the calculation. Also neglect friction. Take k = 300 N/m.



$$T_1 + V_1 = T_2 + V_2$$

$$[0+0] + \frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)(v_b)^2 + \frac{1}{2}(75)(v_c)^2$$

$$12 = 50\,v_b^2 + 75\,v_c^2$$

$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

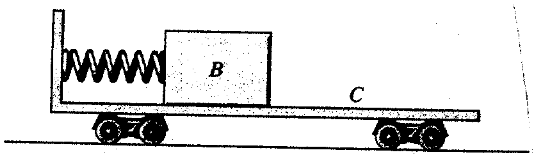
$$0 + 0 = 50\,v_b - 75\,v_c$$

$$v_b = 1.5 v_c$$

$$v_c = 0.253 \text{ m/s} \leftarrow$$

$$v_b = 0.379 \text{ m/s} \rightarrow \qquad \textbf{Ans}$$

15-53. The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring which is attached to the cart and not the block is compressed 0.2 m and the system is released from rest, determine the speed of the block with respect to the cart after the spring becomes undeformed. Neglect the mass of the wheels and the spring in the calculation. Also neglect friction. Take k = 300 N/m.



$$T_1 + V_1 = T_2 + V_2$$

$$[0+0] + \frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)(v_b)^2 + \frac{1}{2}(75)(v_c)^2$$

$$12 = 50\,v_b^2 + 75\,v_c^2$$

$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = 50\,v_b - 75\,v_c$$

$$v_b = 1.5 v_c$$

$$v_c = 0.253 \text{ m/s} \leftarrow$$

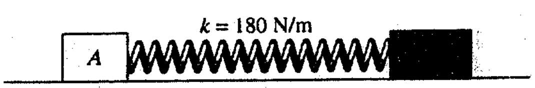
$$v_b = 0.379 \text{ m/s} \rightarrow$$

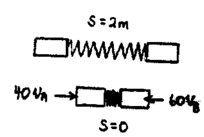
$$\mathbf{v}_b = \mathbf{v}_c + \mathbf{v}_{b/c}$$

$(\overset{+}{\rightarrow})$ $\quad 0.379 = -0.253 + v_{b/c}$

$$v_{b/c} = 0.632 \text{ m/s} \rightarrow \qquad \textbf{Ans}$$

15-54. Blocks A and B have masses of 40 kg and 60 kg, respectively. They are placed on a smooth surface and the spring connected between them is stretched 2 m. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.





$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = 40\,v_A - 60\,v_B$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(180)(2)^2 = \frac{1}{2}(40)(v_A)^2 + \frac{1}{2}(60)(v_B)^2$$

$$v_A = 3.29 \text{ m/s} \qquad \textbf{Ans}$$

$$v_B = 2.19 \text{ m/s} \qquad \textbf{Ans}$$

15-55. An ivory ball having a mass of 200 g is released from rest at a height of 400 mm above a very large fixed metal surface. If the ball rebounds to a height of 325 mm above the surface, determine the coefficient of restitution between the ball and the surface.

Before impact

$T_1 + V_1 = T_2 + V_2$

$0 + 0.2(9.81)(0.4) = \frac{1}{2}(0.2)v_1^2 + 0$

$v_1 = 2.801$ m/s

After the impact

$\frac{1}{2}(0.2)v_2^2 = 0 + 0.2(9.81)(0.325)$

$v_2 = 2.525$ m/s

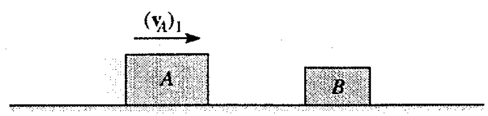
Coefficient of restitution:

$(+\downarrow) \qquad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$

$= \frac{0 - (-2.525)}{2.801 - 0}$

$= 0.901$ **Ans**

***15-56.** Block A has a mass of 3 kg and is sliding on a rough horizontal surface with a velocity $(v_A)_1 = 2$ m/s when it makes a direct collision with block B, which has a mass of 2 kg and is originally at rest. If the collision is perfectly elastic ($e = 1$), determine the velocity of each block just after collision and the distance between the blocks when they stop sliding. The coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.3$.



$(\xrightarrow{+}) \qquad \sum mv_1 = \sum mv_2$

$3(2) + 0 = 3(v_A)_2 + 2(v_B)_2$

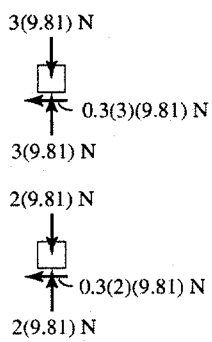
$(\xrightarrow{+}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$1 = \frac{(v_B)_2 - (v_A)_2}{2 - 0}$

Solving

$(v_A)_2 = 0.400$ m/s **Ans**

$(v_B)_2 = 2.40$ m/s **Ans**



Block A:

$T_1 + \sum U_{1-2} = T_2$

$\frac{1}{2}(3)(0.400)^2 - 3(9.81)(0.3)d_A = 0$

$d_A = 0.0272$ m

Block B:

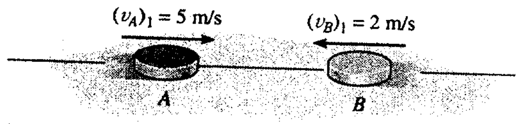
$T_1 + \sum U_{1-2} = T_2$

$\frac{1}{2}(2)(2.40)^2 - 2(9.81)(0.3)d_B = 0$

$d_B = 0.9786$ m

$d = d_B - d_A = 0.951$ m **Ans**

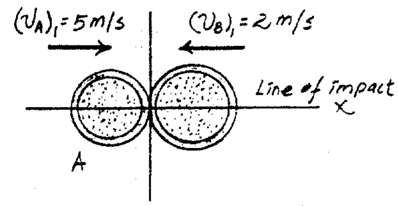
15-57. Disk A has a mass of 2 kg and is sliding forward on the *smooth* surface with a velocity $(v_A)_1 = 5$ m/s when it strikes the 4-kg disk B, which is sliding towards A at $(v_B)_1 = 2$ m/s, with direct central impact. If the coefficient of restitution between the disks is $e = 0.4$, compute the velocities of A and B just after collision.



Conservation of Momentum :

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(5) + 4(-2) = 2(v_A)_2 + 4(v_B)_2 \qquad [1]$$



Coefficient of Restitution :

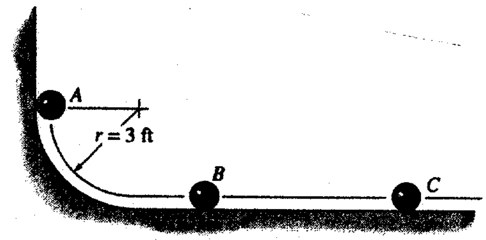
$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.4 = \frac{(v_B)_2 - (v_A)_2}{5 - (-2)} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$(v_A)_2 = -1.53$ m/s $= 1.53$ m/s $\leftarrow$ $\qquad (v_B)_2 = 1.27$ m/s $\rightarrow$ $\qquad$ **Ans**

15-58. The three balls each weigh 0.5 lb and have a coefficient of restitution of $e = 0.85$. If ball A is released from rest and strikes ball B and then ball B strikes ball C, determine the velocity of each ball after the second collision has occurred. Neglect the size of each ball.



Ball A :
Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0.5)(3) = \frac{1}{2}\left(\frac{0.5}{32.2}\right)(v_A)_1^2 + 0$$

$$(v_A)_1 = 13.90 \text{ ft/s}$$

Balls A and B :

$$(\overset{+}{\rightarrow}) \qquad \Sigma mv_1 = \Sigma mv_2$$

$$\left(\frac{0.5}{32.2}\right)(13.90) + 0 = \left(\frac{0.5}{32.2}\right)(v_A)_2 + \left(\frac{0.5}{32.2}\right)(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.85 = \frac{(v_B)_2 - (v_A)_2}{13.90 - 0}$$

Solving :

$(v_A)_2 = 1.04$ ft/s **Ans**

$(v_B)_2 = 12.86$ ft/s

Balls B and C :

$$(\overset{+}{\rightarrow}) \qquad \Sigma mv_2 = \Sigma mv_3$$

$$\left(\frac{0.5}{32.2}\right)(12.86) + 0 = \left(\frac{0.5}{32.2}\right)(v_B)_3 + \left(\frac{0.5}{32.2}\right)(v_C)_3$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_C)_3 - (v_B)_3}{(v_B)_2 - (v_C)_2}$$

$$0.85 = \frac{(v_C)_3 - (v_B)_3}{12.86 - 0}$$

Solving :

$(v_B)_3 = 0.964$ ft/s **Ans**

$(v_C)_3 = 11.9$ ft/s **Ans**

15-59. If two disks A and B have the same mass and are subjected to direct central impact such that the collision is perfectly elastic ($e = 1$), prove that the kinetic energy before collision equals the kinetic energy after collision. The surface upon which they slide is smooth.

$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2$$

$$m_A\left[(v_A)_1 - (v_A)_2\right] = m_B\left[(v_B)_2 - (v_B)_1\right] \qquad (1)$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = 1$

$$(v_B)_2 - (v_A)_2 = (v_A)_1 - (v_B)_1 \qquad (2)$$

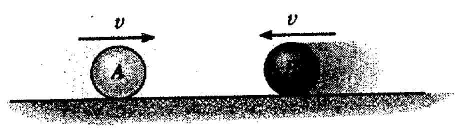
Combining Eqs. (1) and (2) :

$$m_A\left[(v_A)_1 - (v_A)_2\right]\left[(v_A)_1 + (v_A)_2\right] = m_B\left[(v_B)_2 - (v_B)_1\right]\left[(v_B)_2 + (v_B)_1\right]$$

Expand and multiply by $\frac{1}{2}$:

$$\frac{1}{2} m_A (v_A)_1^2 + \frac{1}{2} m_B (v_B)_1^2 = \frac{1}{2} m_A (v_A)_2^2 + \frac{1}{2} m_B (v_B)_2^2 \qquad \text{Q.E.D.}$$

***15-60.** Each ball has a mass m and the coefficient of restitution between the balls is e. If they are moving towards one another with a velocity v, determine their speeds after collision. Also, determine their common velocity when they reach the state of maximum deformation. Neglect the size of each ball.



$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$mv - mv = mv_A + mv_B$$

$$v_A = -v_B$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = \dfrac{v_B - v_A}{v - (-v)}$

$$2ve = 2v_B$$

$v_B = ve \rightarrow$ **Ans**

$v_A = -ve = ve \leftarrow$ **Ans**

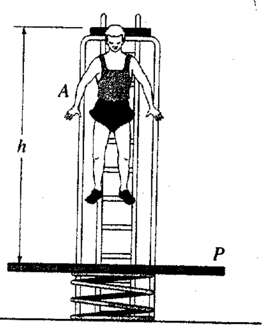
At maximum deformation $v_A = v_B = v'$.

$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$mv - mv = (2m)v'$$

$v' = 0$ **Ans**

15-61. The man A has a weight of 175 lb and jumps from rest $h = 8$ ft onto a platform P that has a weight of 60 lb. The platform is mounted on a spring, which has a stiffness $k = 200$ lb/ft. Determine (a) the velocities of A and P just after impact and (b) the maximum compression imparted to the spring by the impact. Assume the coefficient of restitution between the man and the platform is $e = 0.6$, and the man holds himself rigid during the motion.



$$T_0 + V_0 = T_1 + V_1$$

$$0 + 175(8) = \frac{1}{2}\left(\frac{175}{32.2}\right)(v_{m1})^2 + 0$$

$$v_{m1} = 22.698 \text{ ft/s}$$

$$(+\downarrow) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$\frac{175}{32.2}(22.698) + 0 = \frac{175}{32.2}(v_{m2}) + \frac{60}{32.2}(v_{P2})$$

$$(+\downarrow) \qquad e = \frac{v_{p2} - v_{m2}}{v_{m1} - v_{p1}}$$

$$0.6 = \frac{v_{p2} - v_{m2}}{22.698 - 0}$$

Solving,

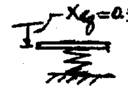
$v_{p2} = 27.04$ ft/s $= 27.0$ ft/s **Ans**

$v_{m2} = 13.4$ ft/s **Ans**

$$60 = 200(x_{eq})$$

$$x_{eq} = 0.3 \text{ ft}$$

$x_{eq} = 0.3$ ft



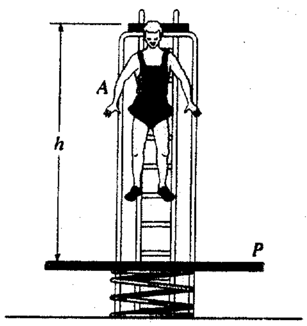
$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left(\frac{60}{32.2}\right)(27.04)^2 + \frac{1}{2}(200)(0.3)^2 + 0 = 0 + \frac{1}{2}(200)(x + 0.3)^2 - 60(x)$$

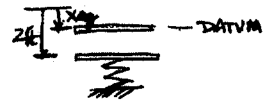
$$100x^2 - 681.206 = 0$$

$x = 2.61$ ft **Ans**

15-62. The man A has a weight of 100 lb and jumps from rest onto the platform P that has a weight of 60 lb. The platform is mounted on a spring, which has a stiffness $k = 200$ lb/ft. If the coefficient of restitution between the man and the platform is $e = 0.6$, and the man holds himself rigid during the motion, determine the required height h of the jump if the maximum compression of the spring becomes 2 ft.



For the platform after collision:



$$60 = 200(x_{eq})$$

$$x_{eq} = 0.3 \text{ ft}$$

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left(\frac{60}{32.2}\right)(v_{p2})^2 + \frac{1}{2}(200)(0.3)^2 + 0 = 0 + \frac{1}{2}(200)(2)^2 - 60(2-0.3)$$

$$v_{p2} = 17.612 \text{ ft/s}$$

$$(+\downarrow) \qquad e = \frac{v_{p2} - v_{m2}}{v_{m1} - v_{p1}}$$

$$0.6 = \frac{17.612 - v_{m2}}{v_{m1} - 0}$$

$$(+\downarrow) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$\frac{100}{32.2}v_{m1} + 0 = \frac{100}{32.2}(v_{m2}) + \frac{60}{32.2}(17.612)$$

Solving,

$$v_{m1} = 17.612 \text{ m/s}$$

$$v_{m2} = 7.045 \text{ m/s}$$

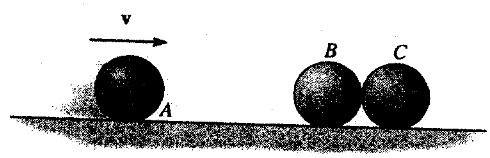
For the man just before striking the platform

$$T_0 + V_0 = T_1 + V_1$$

$$0 + 100h = \frac{1}{2}\left(\frac{100}{32.2}\right)(17.612)^2 + 0$$

$$h = 4.82 \text{ ft} \qquad \textbf{Ans}$$

15-63. The three balls each have the same mass m. If A has a speed v just before a direct collision with B, determine the speed of C after collision. The coefficient of restitution between each ball is e. Neglect the size of each ball.



Conservation of Momentum : When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad mv + 0 = m(v_A)_2 + m(v_B)_2 \qquad [1]$$

Coefficient of Restitution :

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{v - 0} \qquad [2]$$

Solving Eqs.[1] and [2] yields

$$(v_A)_2 = \frac{v(1-e)}{2} \qquad (v_B)_2 = \frac{v(1+e)}{2}$$

Conservation of Momentum : When ball B strikes ball C, we have

$$m_B(v_B)_2 + m_C(v_C)_1 = m_B(v_B)_3 + m_C(v_C)_2$$

$$(\overset{+}{\rightarrow}) \qquad m\left[\frac{v(1+e)}{2}\right] + 0 = m(v_B)_3 + m(v_C)_2 \qquad [3]$$

Coefficient of Restitution :

$$e = \frac{(v_C)_2 - (v_B)_3}{(v_B)_2 - (v_C)_1}$$

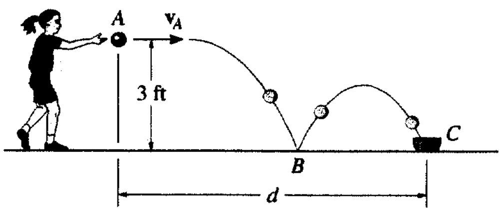
$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_C)_2 - (v_B)_3}{\frac{v(1+e)}{2} - 0} \qquad [4]$$

Solving Eqs.[3] and [4] yields

$$(v_C)_2 = \frac{v(1+e)^2}{4} \qquad \textbf{Ans}$$

$$(v_B)_3 = \frac{v(1-e^2)}{4}$$

***15-64.** If the girl throws the ball with a horizontal velocity of 8 ft/s, determine the distance d so that the ball bounces once on the smooth surface and then lands in the cup at C. Take $e = 0.8$.



$(+\downarrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$(v_1)_y^2 = 0 + 2(32.2)(3)$

$(v_1)_y = 13.90 \downarrow$

$(+\downarrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$3 = 0 + 0 + \frac{1}{2}(32.2)(t_{AB})^2$

$t_{AB} = 0.43167$ s

$(+\downarrow)\quad e = \frac{(v_2)_y}{(v_1)_y}$

$0.8 = \frac{(v_2)_y}{13.90}$

$(v_2)_y = 11.1197 \uparrow$

$(+\downarrow)\quad v = v_0 + a_c t$

$11.1197 = -11.1197 + 32.2(t_{BC})$

$t_{BC} = 0.6907$ s

Total time is $t_{AC} = 1.1224$ s

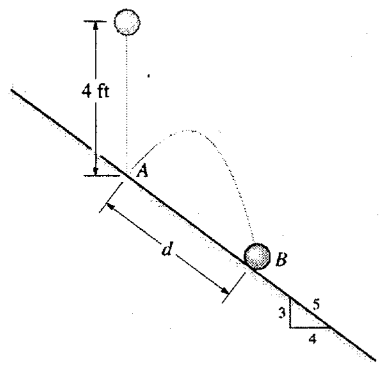
Since the x component of momentum is conserved

$d = v_A(t_{AC})$

$d = 8(1.1224)$

$d = 8.98$ ft **Ans**

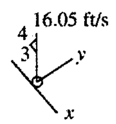
15-65. The ball is dropped from rest and falls a distance of 4 ft before striking the smooth plane at A. If $e = 0.8$, determine the distance d to where it again strikes the plane at B.



$$T_1 + V_1 = T_2 + V_2$$

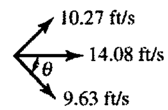
$$0 + 0 = \frac{1}{2}(m)(v_A)_1^2 - m(32.2)(4)$$

$$(v_A)_1 = \sqrt{2(32.2)(4)} = 16.05 \text{ ft/s}$$



$$\searrow + \quad (v_A)_{2x} = \frac{3}{5}(16.05) = 9.63 \text{ ft/s}$$

$$\nearrow + \quad (v_A)_{2y} = 0.8\left(\frac{4}{5}\right)(16.05) = 10.27 \text{ ft/s}$$



$$(v_A)_2 = \sqrt{(9.63)^2 + (10.27)^2} = 14.08 \text{ ft/s}$$

$$\theta = \tan^{-1}\left(\frac{10.27}{9.63}\right) = 46.85^\circ$$

$$\phi = 46.85^\circ - \tan^{-1}\left(\frac{3}{4}\right) = 9.977^\circ$$

$$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$$

$$d\left(\frac{4}{5}\right) = 0 + 14.08 \cos 9.977^\circ (t)$$

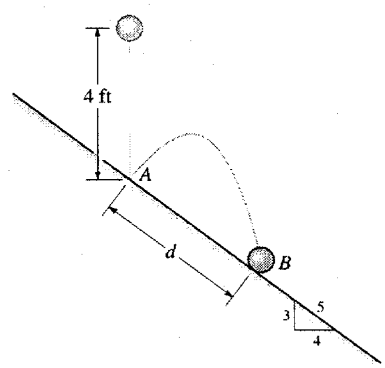
$$(+\downarrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$d\left(\frac{3}{5}\right) = 0 - 14.08 \sin 9.977^\circ (t) + \frac{1}{2}(32.2)t^2$$

$$t = 0.798 \text{ s}$$

$d = 13.8$ ft **Ans**

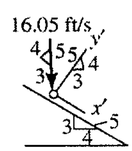
15-66. The ball is dropped from rest and falls a distance of 4 ft before striking the smooth plane at A. If it rebounds and in $t = 0.5$ s again strikes the plane at B, determine the coefficient of restitution e between the ball and the plane. Also, what is the distance d?



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(m)(v_A)_1^2 - m(32.2)(4)$$

$$(v_A)_1 = \sqrt{2(32.2)(4)} = 16.05 \text{ ft/s}$$



$$+\searrow \quad (v_A)_{2x'} = \frac{3}{5}(16.05) = 9.63 \text{ ft/s}$$

$$\nearrow + \quad (v_A)_{2y'} = e\left(\frac{4}{5}\right)(16.05) = 12.84e \text{ ft/s}$$

$$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$$

$$\frac{4}{5}(d) = 0 + v_{A2x}(0.5)$$

$$(+\downarrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$\frac{3}{5}(d) = 0 - v_{A2y}(0.5) + \frac{1}{2}(32.2)(0.5)^2$$

$$(\overset{+}{\rightarrow}) \quad 0.5\left[9.63\left(\frac{4}{5}\right) + 12.84e\left(\frac{3}{5}\right)\right] = \frac{4}{5}d$$

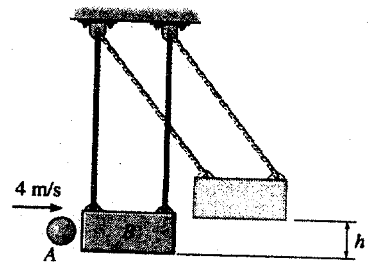
$$(+\uparrow) \quad 0.5\left[-9.63\left(\frac{3}{5}\right) + 12.84e\left(\frac{4}{5}\right)\right] = 4.025 - \frac{3}{5}d$$

Solving,

$e = 0.502$ **Ans**

$d = 7.23$ ft **Ans**

15-67. The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the coefficient of restitution between the ball and the block is $e = 0.8$, determine the maximum height h to which the block will swing before it momentarily stops.



System :

$(\overset{+}{\rightarrow})$ $\Sigma m_1 v_1 = \Sigma m_2 v_2$

$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$(\overset{+}{\rightarrow})$ $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.8 = \frac{(v_B)_2 - (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$

Solving :

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

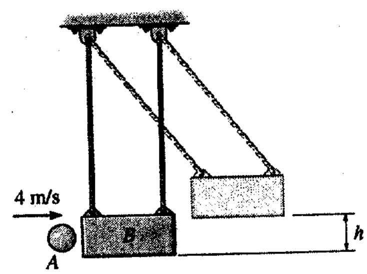
Block :
Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(20)(0.6545)^2 + 0 = 0 + 20(9.81)h$$

$h = 0.0218$ m $= 21.8$ mm **Ans**

***15-68.** The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the time of impact between the ball and the block is 0.005 s, determine the average normal force exerted on the block during this time. Take $e = 0.8$.



System :

$(\overset{+}{\rightarrow})$ $\Sigma m_1 v_1 = \Sigma m_2 v_2$

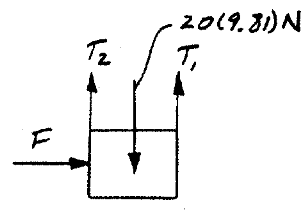
$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$(\overset{+}{\rightarrow})$ $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.8 = \frac{(v_B)_2 - (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$



Solving :

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

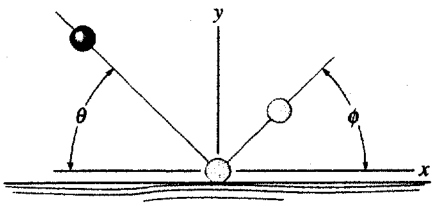
Block :

$(\overset{+}{\rightarrow})$ $m v_1 + \Sigma \int F\,dt = m v_2$

$$0 + F(0.005) = 20(0.6545)$$

$F = 2618$ N $= 2.62$ kN **Ans**

15-69. A ball is thrown onto a rough floor at an angle θ. If it rebounds at an angle ϕ and the coefficient of kinetic friction is μ, determine the coefficient of restitution e. Neglect the size of the ball. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.



$(+\downarrow)$ $$e = \frac{0-[-v_2 \sin\phi]}{v_1 \sin\theta - 0} \qquad e = \frac{v_2 \sin\phi}{v_1 \sin\theta} \qquad [1]$$

$(\overset{+}{\rightarrow})$ $$m(v_x)_1 + \int_{t_1}^{t_2} F_x\,dx = m(v_x)_2$$

$$mv_1 \cos\theta - F_x \Delta t = mv_2 \cos\phi$$

$$F_x = \frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} \qquad [2]$$

$(+\downarrow)$ $$m(v_y)_1 + \int_{t_1}^{t_2} F_y\,dx = m(v_y)_2$$

$$mv_1 \sin\theta - F_y \Delta t = -mv_2 \sin\phi$$

$$F_y = \frac{mv_1 \sin\theta + mv_2 \sin\phi}{\Delta t} \qquad [3]$$

Since $F_x = \mu F_y$, from Eqs.[2] and [3]

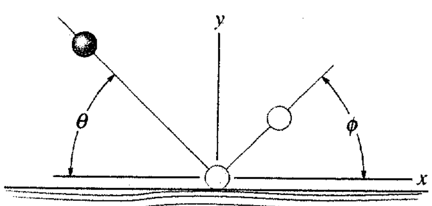
$$\frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} = \frac{\mu(mv_1 \sin\theta + mv_2 \sin\phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi} \qquad [4]$$

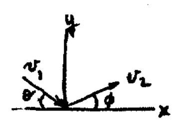
Substituting Eq.[4] into [1] yields :

$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu\sin\theta}{\mu\sin\phi + \cos\phi}\right) \qquad \textbf{Ans}$$

15-70. A ball is thrown onto a rough floor at an angle of $\theta = 45°$. If it rebounds at the same angle $\phi = 45°$, determine the coefficient of kinetic friction between the floor and the ball. The coefficient of restitution is $e = 0.6$. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.



$(+\downarrow)$ $$e = \frac{0-[-v_2 \sin\phi]}{v_1 \sin\theta - 0} \qquad e = \frac{v_2 \sin\phi}{v_1 \sin\theta} \qquad [1]$$



$(\overset{+}{\rightarrow})$ $$m(v_x)_1 + \int_{t_1}^{t_2} F_x\,dx = m(v_x)_2$$

$$mv_1 \cos\theta - F_x \Delta t = mv_2 \cos\phi$$

$$F_x = \frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} \qquad [2]$$

$(+\downarrow)$ $$m(v_y)_1 + \int_{t_1}^{t_2} F_y\,dx = m(v_y)_2$$

$$mv_1 \sin\theta - F_y \Delta t = -mv_2 \sin\phi$$

$$F_y = \frac{mv_1 \sin\theta + mv_2 \sin\phi}{\Delta t} \qquad [3]$$

Since $F_x = \mu F_y$, from Eqs.[2] and [3]

$$\frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} = \frac{\mu(mv_1 \sin\theta + mv_2 \sin\phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi} \qquad [4]$$

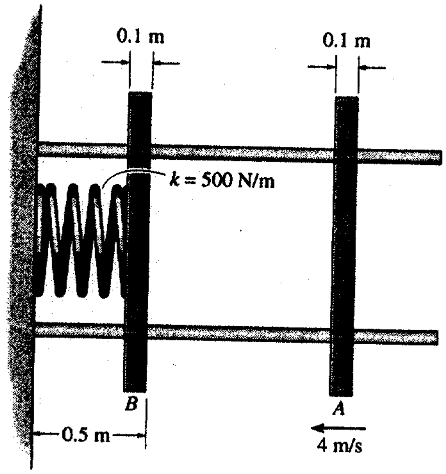
Substituting Eq.[4] into [1] yields :

$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu\sin\theta}{\mu\sin\phi + \cos\phi}\right)$$

$$0.6 = \frac{\sin 45°}{\sin 45°}\left(\frac{\cos 45° - \mu \sin 45°}{\mu \sin 45° + \cos 45°}\right)$$

$$0.6 = \frac{1-\mu}{1+\mu} \qquad \mu = 0.25 \qquad \textbf{Ans}$$

15-71. Plates A and B each have a mass of 4 kg and are restricted to move along the smooth guides. If the coefficient of restitution between the plates is $e = 0.7$, determine (a) the speed of both plates just after collision and (b) the maximum compression of the spring. Plate A has a velocity of 4 m/s just before striking B. Plate B is originally at rest and the spring is unstretched.



Conservation of Momentum :

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(\overset{+}{\leftarrow}) \qquad 4(4) + 0 = 4(v_A)_2 + 4(v_B)_2 \qquad [1]$$

Coefficient of Restitution :

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\leftarrow}) \qquad 0.7 = \frac{(v_B)_2 - (v_A)_2}{4 - 0} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = 0.600 \text{ m/s} \leftarrow \qquad (v_B)_2 = 3.40 \text{ m/s} \leftarrow \qquad \textbf{Ans}$$

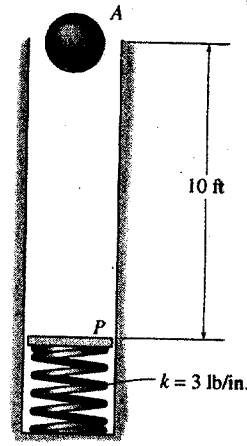
Conservation of Energy : When plate B stops momentary, the spring has been compressed to its maximum and the elastic potential energy at this instant is $\frac{1}{2}(500)s^2 = 250s^2$. Applying Eq. 14 − 21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(4)(3.40^2) + 0 = 0 + 250s^2$$

$$s = 0.3041 \text{ m} = 304 \text{ mm} \qquad \textbf{Ans}$$

*15-72. The 8-lb ball is released from rest 10 ft from the surface of a flat plate P which weighs 6 lb. Determine the maximum compression in the spring if the impact is perfectly elastic.



Datum at lowest point :

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (8)(10) = \frac{1}{2}\left(\frac{8}{32.2}\right)(v_A)_1^2 + 0$$

$$(v_A)_1 = 25.377 \text{ ft/s}$$

$$(+\downarrow) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$\frac{8}{32.2}(25.377) + 0 = \frac{8}{32.2}(v_A)_2 + \frac{6}{32.2}(v_P)_2$$

$$e = \frac{(v_P)_2 - (v_A)_2}{(v_A)_1 - (v_P)_1}$$

$$1 = \frac{(v_P)_2 - (v_A)_2}{25.377 - 0}$$

Solving,

$$(v_A)_2 = 3.625 \text{ ft/s}$$

$$(v_P)_2 = 29.00 \text{ ft/s}$$

Initially the plate is compressed

$$F_s = kx; \qquad 6 = (3)x, \qquad x = 2 \text{ in.} = \frac{1}{6} \text{ ft}$$

Datum at final plate position :

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left(\frac{6}{32.2}\right)(29.00)^2 + \frac{1}{2}(3)(12)\left(\frac{1}{6}\right)^2 + 6x = 0 + \frac{1}{2}(3)(12)\left(x + \frac{1}{6}\right)^2$$

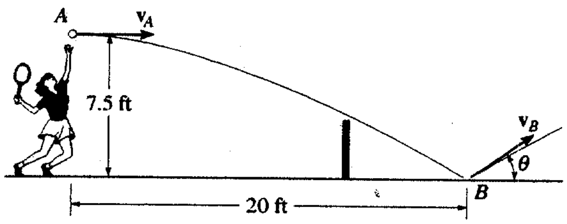
$$18x^2 - 78.367 = 0$$

$$x = 2.087 \text{ ft}$$

Maximum spring compression is

$$x_{max} = 2.087 + \frac{1}{6} = 2.25 \text{ ft} \qquad \textbf{Ans}$$

15-73. It was observed that a tennis ball when served horizontally 7.5 ft above the ground strikes the smooth ground at B 20 ft away. Determine the initial velocity $\mathbf{v}_A$ of the ball and the velocity $\mathbf{v}_B$ (and θ) of the ball just after it strikes the court at B. Take $e = 0.7$.



$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$20 = 0 + v_A t$

$(+\downarrow)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$7.5 = 0 + 0 + \frac{1}{2}(32.2)t^2$

$t = 0.682524$

$v_A = 29.303 = 29.3$ ft/s **Ans**

$v_{Bx1} = 29.303$ ft/s

$(+\downarrow)$ $v = v_0 + a_c t$

$v_{By1} = 0 + 32.2(0.68252) = 21.977$ ft/s

$(\overset{+}{\rightarrow})$ $mv_1 = mv_2$

$v_{B2x} = v_{B1x} = 29.303$ ft/s $\rightarrow$

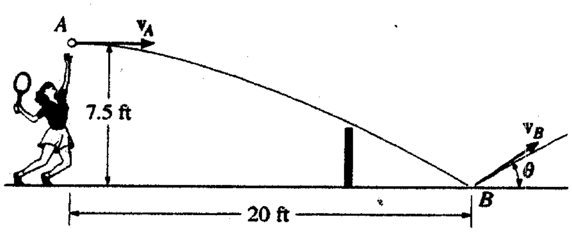
$e = \dfrac{v_{By2}}{v_{By1}}$

$0.7 = \dfrac{v_{By2}}{21.977}$, $v_{By2} = 15.384$ ft/s $\uparrow$

$v_{B2} = \sqrt{(29.303)^2 + (15.384)^2} = 33.1$ ft/s **Ans**

$\theta = \tan^{-1}\dfrac{15.384}{29.303} = 27.7°$ **Ans**

15-74. The tennis ball is struck with a horizontal velocity $\mathbf{v}_A$, strikes the smooth ground at B, and bounces upward at $\theta = 30°$. Determine the initial velocity $\mathbf{v}_A$, the final velocity $\mathbf{v}_B$, and the coefficient of restitution between the ball and the ground.



$(+\downarrow)$ $v^2 = v_0^2 + 2a_c(s - s_0)$

$(v_{By})_1^2 = 0 + 2(32.2)(7.5 - 0)$

$v_{By1} = 21.9773$ m/s

$(+\downarrow)$ $v = v_0 + a_c t$

$21.9773 = 0 + 32.2t$

$t = 0.68252$ s

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$20 = 0 + v_A(0.68252)$

$v_A = 29.303 = 29.3$ ft/s **Ans**

$(\overset{+}{\rightarrow})$ $mv_1 = mv_2$

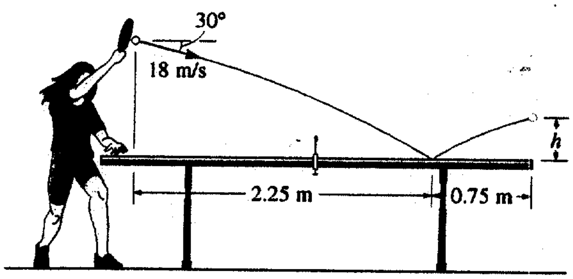
$v_{Bx2} = v_{Bx1} = v_A = 29.303$

$v_{B_2} = 29.303/\cos 30° = 33.8$ ft/s **Ans**

$v_{By2} = 29.303 \tan 30° = 16.918$ ft/s

$e = \dfrac{v_{By2}}{v_{By1}} = \dfrac{16.918}{21.9773} = 0.770$ **Ans**

15-75. The ping-pong ball has a mass of 2 g. If it is struck with the velocity shown, determine how high h it rises above the end of the smooth table after the rebound. Take $e = 0.8$.



$(\xrightarrow{+})\quad s = s_0 + v_0 t$

$2.25 = 0 + 18\cos 30° t$

$t = 0.14434$ s

$(v_x)_1 = (v_x)_2 = 18\cos 30° = 15.5885$ m/s

$(+\downarrow)\quad v = v_0 + a_c t$

$(v_y)_1 = 18\sin 30° + 9.81(0.14434)$

$(v_y)_1 = 10.4160$ m/s

$(+\uparrow)\quad e = 0.8 = \dfrac{(v_y)_2}{10.4160}$

$(v_y)_2 = 8.3328$ m/s

$(\xrightarrow{+})\quad s = s_0 + v_0 t$

$0.75 = 0 + 15.5885t$

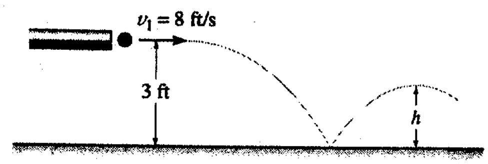
$t = 0.048112$ s

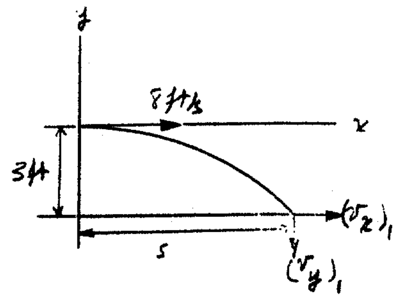
$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

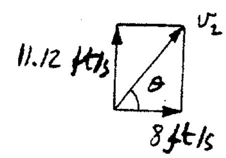
$h = 0 + 8.3328(0.048112) - \frac{1}{2}(9.81)(0.048112)^2$

$h = 0.390$ m **Ans**

***15-76.** The ball is ejected from the tube with a horizontal velocity of $v_1 = 8$ ft/s as shown. If the coefficient of restitution between the ball and the ground is $e = 0.8$, determine (a) the velocity of the ball just after it rebounds from the ground and (b) the maximum height to which the ball rises after the first bounce.







$(+\uparrow)\qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$(v_y)_1^2 = 0 + 2(-32.2)(-3-0)$

$(v_y)_1 = 13.90$ ft/s $\downarrow$

In y direction :

$(+\uparrow)\qquad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1};\qquad 0.8 = \dfrac{(v_y)_2 - 0}{0 - (-13.90)}$

$(v_y)_2 = 11.12$ ft/s $\uparrow$

$(v_x)_2 = 8$ ft/s $\rightarrow$

$v_2 = \sqrt{(8)^2 + (11.12)^2} = 13.7$ ft/s **Ans**

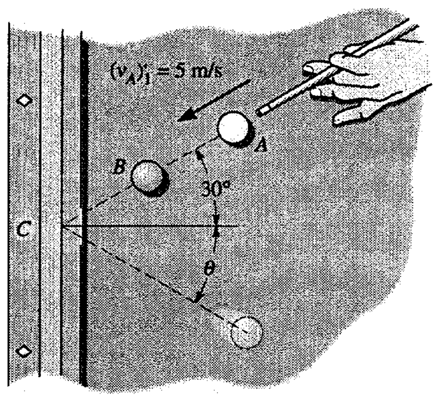
$\theta = \tan^{-1}\left(\dfrac{11.12}{8}\right) = 54.3°$ **Ans**

$(+\uparrow)\qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (11.12)^2 + 2(-32.2)(h - 0)$

$h = 1.92$ ft **Ans**

15-77. The cue ball A is given an initial velocity $(v_A)_1 = 5$ m/s. If it makes a direct collision with ball $B(e = 0.8)$, determine the velocity of B and the angle θ just after it rebounds from the cushion at $C(e' = 0.6)$. Each ball has a mass of 0.4 kg. Neglect he size of each ball.



Conservation of Momentum: When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(+) \qquad 0.4(5) + 0 = 0.4(v_A)_2 + 0.4(v_B)_2 \qquad [1]$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\leftarrow}) \quad 0.8 = \frac{(v_B)_2 - (v_A)_2}{5 - 0} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$(v_A)_2 = 0.500$ m/s $\quad (v_B)_2 = 4.50$ m/s

Conservation of "y" Momentum: When ball B strikes the cushion at C, we have

$$m_B(v_{B_y})_2 = m_B(v_{B_y})_3$$

$$(+\downarrow) \qquad 0.4(4.50 \sin 30°) = 0.4(v_B)_3 \sin\theta$$

$$(v_B)_3 \sin\theta = 2.25 \quad \textbf{Ans} \qquad [3]$$

Coefficient of Restitution (x):

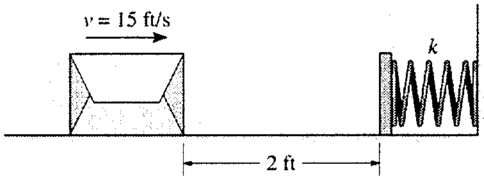
$$e = \frac{(v_C)_2 - (v_{B_x})_3}{(v_{B_x})_2 - (v_C)_1}$$

$$(\overset{+}{\leftarrow}) \qquad 0.6 = \frac{0 - [-(v_B)_3 \cos\theta]}{4.50 \cos 30° - 0} \qquad [4]$$

Solving Eqs. [1] and [2] yields

$(v_B)_3 = 3.24$ m/s $\quad \theta = 43.9°$ **Ans**

15-78. The 20-lb box slides on the surface for which $\mu_k = 0.3$. The box has a velocity $v = 15$ ft/s when it is 2 ft from the plate. If it strikes the smooth plate, which has a weight of 10 lb and is held in position by an unstretched spring of stiffness $k = 400$ lb/ft, determine the maximum compression imparted to the spring. Take $e = 0.8$ between the box and the plate.



$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{20}{32.2}\right)(15)^2 - (0.3)(20)(2) = \frac{1}{2}\left(\frac{20}{32.2}\right)(v_2)^2$$

$$v_2 = 13.65 \text{ ft/s}$$

$$(\overset{+}{\rightarrow}) \quad \sum mv_1 = \sum mv_2$$

$$\left(\frac{20}{32.2}\right)(13.65) = \left(\frac{20}{32.2}\right)v_A + \frac{10}{32.2}v_B$$

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.8 = \frac{v_P - v_A}{13.65}$$

Solving,

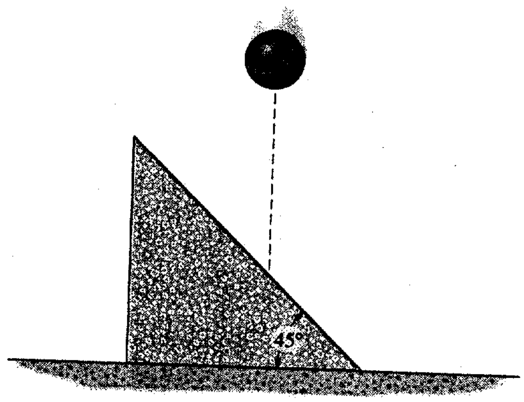
$$v_P = 16.38 \text{ ft/s}, \quad v_A = 5.46 \text{ ft/s}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(16.38)^2 + 0 = 0 + \frac{1}{2}(400)(s)^2$$

$s = 0.456$ ft **Ans**

15-79. The sphere of mass m falls and strikes the triangular block with a vertical velocity v. If the block rests on a smooth surface and has a mass $3m$, determine its velocity just after the collision. The coefficient of restitution is e.



Conservation of "x'" Momentum :

$$m(v)_1 = m(v)_2$$

$$(\;+)\qquad m(v \sin 45°) = m(v_{sx'})_2$$

$$(v_{sx'})_2 = \frac{\sqrt{2}}{2}v$$

Coefficient of Restitution (y') :

$$e = \frac{(v_b)_2 - (v_{b_{y'}})_2}{(v_{s_{y'}})_1 - (v_b)_1}$$

$$(+\;)\qquad e = \frac{v_b \cos 45° - [-(v_{sy'})_2]}{v \cos 45° - 0}$$

$$(v_{sy'})_2 = \frac{\sqrt{2}}{2}(ev - v_b)$$

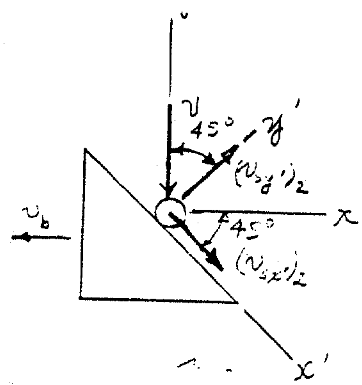
Conservation of "x" Momentum :

$$0 = m_s(v_s)_x + m_b v_b$$

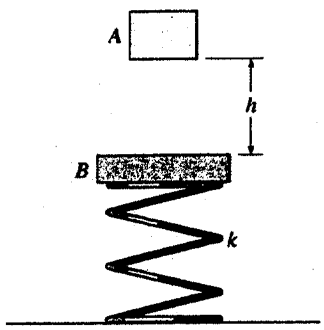
$$(\overset{+}{\leftarrow})\qquad 0 + 0 = 3mv_b - m(v_{sy'})_2 \cos 45° - m(v_{sx'})_2 \cos 45°$$

$$3v_b - \frac{\sqrt{2}}{2}(ev - v_b)\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}v\frac{\sqrt{2}}{2} = 0$$

$$v_b = \left(\frac{1+e}{7}\right)v \qquad \textbf{Ans}$$



***15-80.** Block A, having a mass m, is released from rest, falls a distance h and strikes the plate B having a mass $2m$. If the coefficient of restitution between A and B is e, determine the velocity of the plate just after collision. The spring has a stiffness k.



Just before impact, the velocity of A is

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}mv_A^2 - mgh$$

$$v_A = \sqrt{2gh}$$

$$(+\downarrow) \qquad e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gh}}$$

$$e\sqrt{2gh} = (v_B)_2 - (v_A)_2 \qquad (1)$$

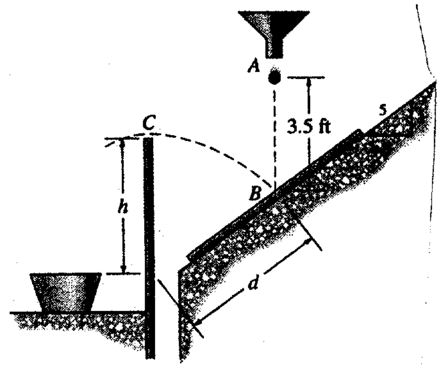
$$(+\downarrow) \qquad \Sigma mv_1 = \Sigma mv_2$$

$$m(v_A) + 0 = m(v_A)_2 + 2m(v_B)_2 \qquad (2)$$

Solving Eqs.(1) and (2) for $(v_B)_2$ yields;

$$(v_B)_2 = \frac{1}{3}\sqrt{2gh}(1+e) \qquad \textbf{Ans}$$

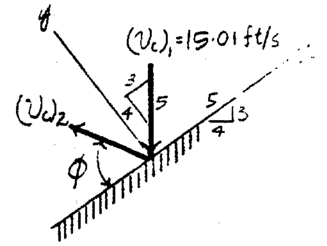
15-81. Before a cranberry can make it to your dinner plate, it must pass a bouncing test which rates its quality. If cranberries having an $e \geqslant 0.8$ are to be accepted, determine the dimensions d and h for the barrier so that when a cranberry falls from rest at A it strikes the plate at B and bounces over the barrier at C.



Conservation of Energy : The datum is set at point B. When the cranberry falls from a height of 3.5 ft *above* the datum, its initial gravitational potential energy is $W(3.5) = 3.5W$. Applying Eq. 14 − 21, we have

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 3.5W = \frac{1}{2}\left(\frac{W}{32.2}\right)(v_c)_1^2 + 0$$
$$(v_c)_1 = 15.01 \text{ ft/s}$$

Conservation of "x'" Momentum : When the cranberry strikes the plate with a speed of $(v_c)_1 = 15.01$ ft/s, it rebounds with a speed of $(v_c)_2$.



$$m_c(v_{c_{x'}})_1 = m_c(v_{c_{x'}})_2$$

$$(+\ )\qquad m_c(15.01)\left(\frac{3}{5}\right) = m_c[(v_c)_2 \cos\phi]$$

$$(v_c)_2 \cos\phi = 9.008 \qquad [1]$$

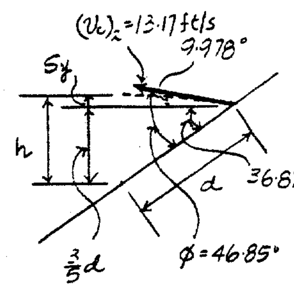
Coefficient of Restitution (y') :

$$e = \frac{(v_P)_2 - (v_{c_{y'}})_2}{(v_{c_{y'}})_1 - (v_P)_1}$$

$$(\ +)\qquad 0.8 = \frac{0 - (v_c)_2 \sin\phi}{-15.01\left(\frac{4}{5}\right) - 0} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$\phi = 46.85^\circ \qquad (v_c)_2 = 13.17 \text{ ft/s}$$



Kinematics : By considering the vertical motion of the cranberry after the impact, we have

$$(+\uparrow)\qquad v_y = (v_0)_y + a_c t$$
$$0 = 13.17 \sin 9.978^\circ + (-32.2)t \qquad t = 0.07087 \text{ s}$$

$$(+\uparrow)\qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$
$$= 0 + 13.17 \sin 9.978^\circ (0.07087) + \frac{1}{2}(-32.2)\left(0.07087^2\right)$$
$$= 0.080864 \text{ ft}$$

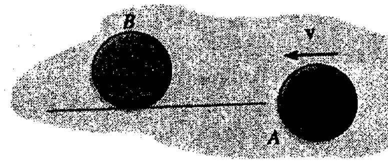
By considering the horizontal motion of the cranberry after the impact, we have

$$(\overset{+}{\leftarrow})\qquad s_x = (s_0)_x + v_x t$$
$$\frac{4}{5}d = 0 + 13.17 \cos 9.978^\circ (0.07087)$$
$$d = 1.149 \text{ ft} = 1.15 \text{ ft} \qquad \textbf{Ans}$$

Thus,

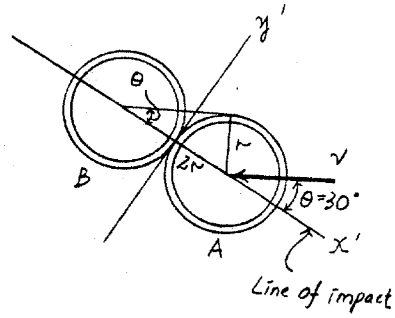
$$h = s_y + \frac{3}{5}d = 0.080864 + \frac{3}{5}(1.149) = 0.770 \text{ ft} \qquad \textbf{Ans}$$

15-82. If disk A is sliding along the tangent to disk B and strikes B with a velocity $\mathbf{v}$, determine the velocity of B after the collision and compute the loss of kinetic energy during the collision. Neglect friction. Disk B is originally at rest. The coefficient of restitution is e, and each disk has the same size and mass m.



Impact : This problem involves *oblique impact* where the *line of impact* lies along x' axis (line jointing the mass center of the two impact bodies). From the geometry $\theta = \sin^{-1}\left(\frac{r}{2r}\right) = 30°$. The x' andy' components of velocity for disk A just before impact are

$$(v_{A_{x'}})_1 = -v\cos 30° = -0.8660v \qquad (v_{A_{y'}})_1 = -v\sin 30° = -0.5v$$



Conservation of "x'" Momentum :

$$m_A(v_{A_{x'}})_1 + m_B(v_{B_{x'}})_1 = m_A(v_{A_{x'}})_2 + m_B(v_{B_{x'}})_2$$

$$(+) \qquad m(-0.8660v) + 0 = m(v_{A_{x'}})_2 + m(v_{B_{x'}})_2 \qquad [1]$$

Coefficient of Restitution (x') :

$$e = \frac{(v_{B_{x'}})_2 - (v_{A_{x'}})_2}{(v_{A_{x'}})_1 - (v_{B_{x'}})_1}$$

$$(+) \qquad e = \frac{(v_{B_{x'}})_2 - (v_{A_{x'}})_2}{-0.8660v - 0} \qquad [2]$$

Solving Eqs.[1] and [2] yields

$$(v_{B_{x'}})_2 = -\frac{\sqrt{3}}{4}(1+e)v \qquad (v_{A_{x'}})_2 = \frac{\sqrt{3}}{4}(e-1)v$$

Conservation of "y'" Momentum : The momentum is conserved along y' axis for both disks A and B.

$$(+\uparrow) \qquad m_B(v_{B_{y'}})_1 = m_B(v_{B_{y'}})_2; \qquad (v_{B_{y'}})_2 = 0$$

$$(+\uparrow) \qquad m_A(v_{A_{y'}})_1 = m_A(v_{A_{y'}})_2; \qquad (v_{A_{y'}})_2 = -0.5v$$

Thus, the magnitude the velocity for disk B just after the impact is

$$(v_B)_2 = \sqrt{(v_{B_{x'}})_2^2 + (v_{B_{y'}})_2^2}$$

$$= \sqrt{\left(-\frac{\sqrt{3}}{4}(1+e)v\right)^2 + 0} = \frac{\sqrt{3}}{4}(1+e)v \qquad \textbf{Ans}$$

nd directed toward **negative x' axis**. **Ans**

he magnitude of the velocity for disk A just after the impact is

$$(v_A)_2 = \sqrt{(v_{A_{x'}})_2^2 + (v_{A_{y'}})_2^2}$$

$$= \sqrt{\left[\frac{\sqrt{3}}{4}(e-1)v\right]^2 + (-0.5v)^2}$$

$$= \sqrt{\frac{v^2}{16}(3e^2 - 6e + 7)}$$

of Kinetic Energy : Kinetic energy of the system before the impact is

$$U_k = \frac{1}{2}mv^2$$

etic energy of the system after the impact is

$$U_k' = \frac{1}{2}m\left[\sqrt{\frac{v^2}{16}(3e^2 - 6e + 7)}\right]^2 + \frac{1}{2}m\left[\frac{\sqrt{3}}{4}(1+e)v\right]^2$$

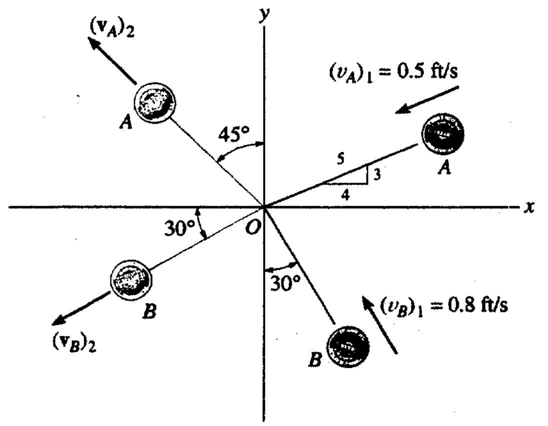
$$= \frac{mv^2}{32}(6e^2 + 10)$$

Thus, the kinetic energy loss is

$$\Delta U_k = U_k - U_k' = \frac{1}{2}mv^2 - \frac{mv^2}{32}(6e^2 + 10)$$

$$= \frac{3mv^2}{16}(1 - e^2) \qquad \textbf{Ans}$$

15-83. Two smooth coins A and B, each having the same mass, slide on a smooth surface with the motion shown. Determine the speed of each coin after collision if they move off along the dashed paths. *Hint:* Since the line of impact has not been defined, apply the conservation of momentum along the x and y axes, respectively.



$$\Sigma m v_1 = \Sigma m v_2$$

$(\overset{+}{\rightarrow})$ $$-m(0.8)\sin 30^\circ - m(0.5)\left(\frac{4}{5}\right) = -m(v_A)_2 \sin 45^\circ - m(v_B)_2 \cos 30^\circ$$

$$0.8 = 0.707(v_A)_2 + 0.866(v_B)_2$$

$(+\uparrow)$ $$m(0.8)\cos 30^\circ - m(0.5)\left(\frac{3}{5}\right) = m(v_A)_2 \cos 45^\circ - m(v_B)_2 \sin 30^\circ$$

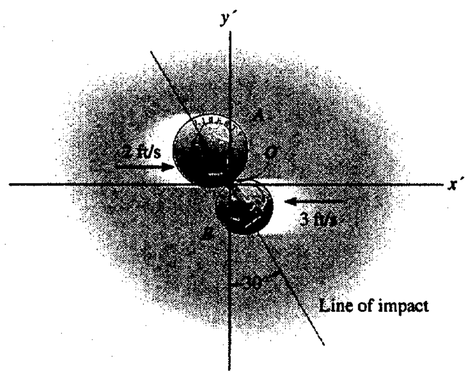
$$-0.3928 = -0.707(v_A)_2 + 0.5(v_B)_2$$

Solving,

$(v_B)_2 = 0.298$ ft/s **Ans**

$(v_A)_2 = 0.766$ ft/s **Ans**

***15-84.** Two coins A and B have the initial velocities shown just before they collide at point O. If they have weights of $W_A = 13.2(10^{-3})$ lb and $W_B = 6.60(10^{-3})$ lb and the surface upon which they slide is smooth, determine their speeds just after impact. The coefficient of restitution is $e = 0.65$.



$(+\searrow)$ $$m_A (v_{A_x})_1 + m_B (v_{B_x})_1 = m_A (v_{A_x})_2 + m_B (v_{B_x})_2$$

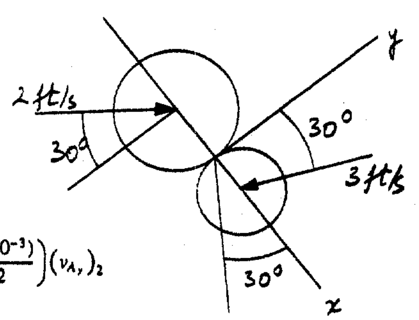
$$\left(\frac{13.2(10^{-3})}{32.2}\right)2\sin 30^\circ - \left(\frac{6.6(10^{-3})}{32.2}\right)3\sin 30^\circ = \left(\frac{13.2(10^{-3})}{32.2}\right)(v_{A_x})_2 + \left(\frac{6.6(10^{-3})}{32.2}\right)(v_{B_x})_2$$

$(+\searrow)$ $$e = \frac{(v_{B_x})_2 - (v_{A_x})_2}{(v_{A_x})_1 - (v_{B_x})_1}; \qquad 0.65 = \frac{(v_{B_x})_2 - (v_{A_x})_2}{2\sin 30^\circ - (-3\sin 30^\circ)}$$

Solving :

$(v_{A_x})_2 = -0.3750$ ft/s

$(v_{B_x})_2 = 1.250$ ft/s



$(+\nearrow)$ $$m_A (v_{A_y})_2 = m_A (v_{A_y})_2$$

$$\left(\frac{13.2(10^{-3})}{32.2}\right)2\cos 30^\circ = \left(\frac{13.2(10^{-3})}{32.2}\right)(v_{A_y})_2$$

$(v_{A_y})_2 = 1.732$ ft/s

$(+\nearrow)$ $$m_B (v_{B_y})_1 = m_B (v_{B_y})_2$$

$$\left(\frac{6.6(10^{-3})}{32.2}\right)3\cos 30^\circ = \left(\frac{6.6(10^{-3})}{32.2}\right)(v_{B_y})_2$$

$(v_{B_y})_2 = 2.598$ ft/s

Thus,

$(v_B)_2 = \sqrt{(1.250)^2 + (2.598)^2} = 2.88$ ft/s **Ans**

$(v_A)_2 = \sqrt{(-0.3750)^2 + (1.732)^2} = 1.77$ ft/s **Ans**

15-85. Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is $e = 0.75$.

$$(\overset{+}{\rightarrow}) \quad \Sigma m v_1 = \Sigma m v_2$$

$$0.5(4)\left(\frac{3}{5}\right) - 0.5(6) = 0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$$(\overset{+}{\rightarrow}) \quad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$0.75 = \frac{(v_A)_{2x} - (v_B)_{2x}}{4(\frac{3}{5}) - (-6)}$$

$$(v_A)_{2x} = 1.35 \text{ m/s} \rightarrow$$

$$(v_B)_{2x} = 4.95 \text{ m/s} \leftarrow$$

$$(+\uparrow) \quad m v_1 = m v_2$$

$$0.5\left(\frac{4}{5}\right)(4) = 0.5(v_B)_{2y}$$

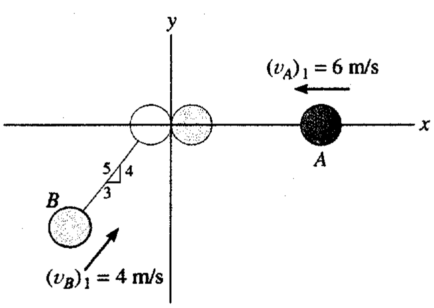
$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$

$$v_A = 1.35 \text{ m/s} \rightarrow \qquad \textbf{Ans}$$

$$v_B = \sqrt{(4.95)^2 + (3.20)^2} = 5.89 \text{ m/s} \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\frac{3.20}{4.95} = 32.9° \qquad \textbf{Ans}$$

15-86. Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision B travels along a line, 30° counterclockwise from the y axis.



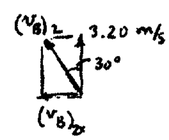
$$\Sigma m v_1 = \Sigma m v_2$$

$$(\overset{+}{\rightarrow}) \quad 0.5(4)\left(\frac{3}{5}\right) - 0.5(6) = -0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$$-3.60 = -(v_B)_{2x} + (v_A)_{2x}$$

$$(+\uparrow) \quad 0.5(4)\left(\frac{4}{5}\right) = 0.5(v_B)_{2y}$$

$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$



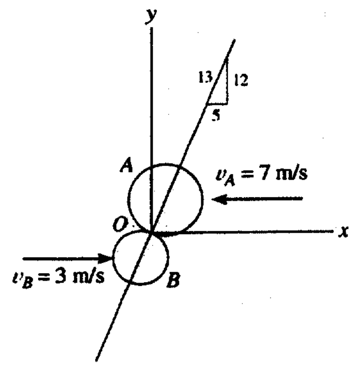
$$(v_B)_{2x} = 3.20 \tan 30° = 1.8475 \text{ m/s} \leftarrow$$

$$(v_A)_{2x} = -1.752 \text{ m/s} = 1.752 \text{ m/s} \leftarrow$$

$$(\overset{+}{\rightarrow}) \quad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$e = \frac{-1.752 - (-1.8475)}{4(\frac{3}{5}) - (-6)} = 0.0113 \qquad \textbf{Ans}$$

15-87. Two smooth disks A and B have the initial velocities shown just before they collide at O. If they have masses $m_A = 8$ kg and $m_B = 6$ kg, determine their speeds just after impact. The coefficient of restitution is $e = 0.5$.

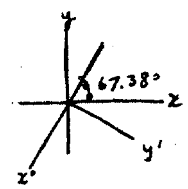


$$+\nearrow \Sigma m v_1 = \Sigma m v_2$$

$$-6(3\cos 67.38°) + 8(7\cos 67.38°) = 6(v_B)_{x'} + 8(v_A)_{x'}$$

$$(+\nearrow) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.5 = \frac{(v_B)_{x'} - (v_A)_{x'}}{7\cos 67.38° + 3\cos 67.38°}$$



Solving,

$(v_B)_{x'} = 2.14$ m/s

$(v_A)_{x'} = 0.220$ m/s

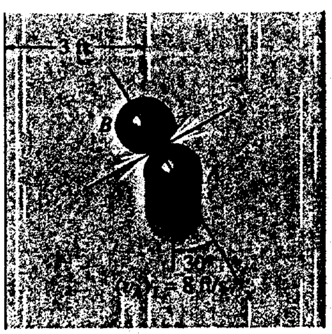
$(v_B)_{y'} = 3\sin 67.38° = 2.769$ m/s

$(v_A)_{y'} = -7\sin 67.38° = -6.462$ m/s

$v_B = \sqrt{(2.14)^2 + (2.769)^2} = 3.50$ m/s **Ans**

$v_A = \sqrt{(0.220)^2 + (6.462)^2} = 6.47$ m/s **Ans**

***15-88.** The "stone" A used in the sport of curling slides over the ice track and strikes another "stone" B as shown. If each "stone" is smooth and has a weight of 47 lb, and the coefficient of restitution between the "stones" is $e = 0.8$, determine their speeds just after collision. Initially A has a velocity of 8 ft/s and B is at rest. Neglect friction.

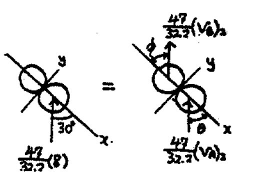


Line of impact (x-axis) :

$$\Sigma m v_1 = \Sigma m v_2$$

$$(+\searrow) \quad 0 + \frac{47}{32.2}(8)\cos 30° = \frac{47}{32.2}(v_B)_{2x} + \frac{47}{32.2}(v_A)_{2x}$$

$$(+\searrow) \quad e = 0.8 = \frac{(v_B)_{2x} - (v_A)_{2x}}{8\cos 30° - 0}$$



Solving :

$(v_A)_{2x} = 0.6928$ ft/s

$(v_B)_{2x} = 6.235$ ft/s

Plane of impact (y-axis) :

Stone A :

$$m v_1 = m v_2$$

$$(\nearrow +) \quad \frac{47}{32.2}(8)\sin 30° = \frac{47}{32.2}(v_A)_{2y}$$

$$(v_A)_{2y} = 4$$

Stone B :

$$m v_1 = m v_2$$

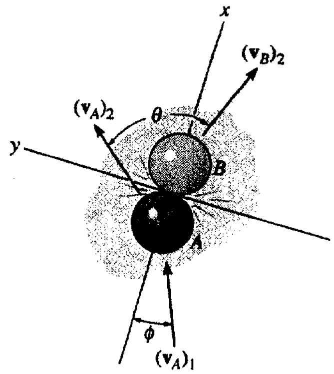
$$(\nearrow +) \quad 0 = \frac{47}{32.2}(v_B)_{2y}$$

$$(v_B)_{2y} = 0$$

$(v_A)_2 = \sqrt{(0.6928)^2 + (4)^2} = 4.06$ ft/s **Ans**

$(v_B)_2 = \sqrt{(0)^2 + (6.235)^2} = 6.235 = 6.24$ ft/s **Ans**

15-89. Ball A strikes ball B with an initial velocity of $(\mathbf{v}_A)_1$ as shown. If both balls have the same mass and the collision is perfectly elastic, determine the angle θ after collision. Ball B is originally at rest. Neglect the size of each ball.



Velocity before impact :

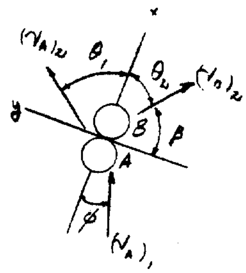
$$(v_{Ax})_1 = (v_A)_1 \cos\phi \qquad (v_{Ay})_1 = (v_A)_1 \sin\phi$$
$$(v_{Bx})_1 = 0 \qquad (v_{By})_1 = 0$$

Velocity after impact :

$$(v_{Ax})_2 = (v_A)_2 \cos\theta_1 \qquad (v_{Ay})_2 = (v_A)_2 \sin\theta_1$$
$$(v_{Bx})_2 = (v_B)_2 \cos\theta_2 \qquad (v_{By})_2 = -(v_B)_2 \sin\theta_2$$

Conservation of " y " momentum :

$$m_B (v_{By})_1 = m_B (v_{By})_2$$
$$0 = m\left[-(v_B)_2 \sin\theta_2\right] \qquad \theta_2 = 0^\circ$$



Conservation of " x " momentum :

$$m_A (v_{Ax})_1 + m_B (v_{Bx})_1 = m_A (v_{Ax})_2 + m_B (v_{Bx})_2$$
$$m(v_A)_1 \cos\phi + 0 = m(v_A)_2 \cos\theta_1 + m(v_B)_2 \cos 0^\circ$$
$$(v_A)_1 \cos\phi = (v_A)_2 \cos\theta_1 + (v_B)_2 \qquad (1)$$

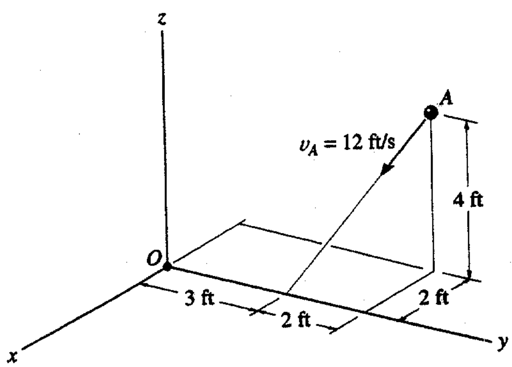
Coefficient of Restitution (x direction) :

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}\,; \qquad 1 = \frac{(v_B)_2 \cos 0^\circ - (v_A)_2 \cos\theta_1}{(v_A)_1 \cos\phi - 0}$$
$$(v_A)_1 \cos\phi = -(v_A)_2 \cos\theta_1 + (v_B)_2 \qquad (2)$$

Subtracting Eq. (1) from Eq. (2) yields :

$$2(v_A)_2 \cos\theta_1 = 0 \qquad \text{Since } 2(v_A)_2 \neq 0$$
$$\cos\theta_1 = 0 \qquad \theta_1 = 90^\circ$$
$$\theta = \theta_1 + \theta_2 = 90^\circ + 0^\circ = 90^\circ \qquad \textbf{Ans}$$

15-90. Determine the angular momentum of the 2-lb particle A about point O. Use a Cartesian vector solution.



$$m\mathbf{v}_A = \frac{2}{32.2}(12)\left(\frac{2}{\sqrt{24}}\mathbf{i} - \frac{2}{\sqrt{24}}\mathbf{j} - \frac{4}{\sqrt{24}}\mathbf{k}\right)$$

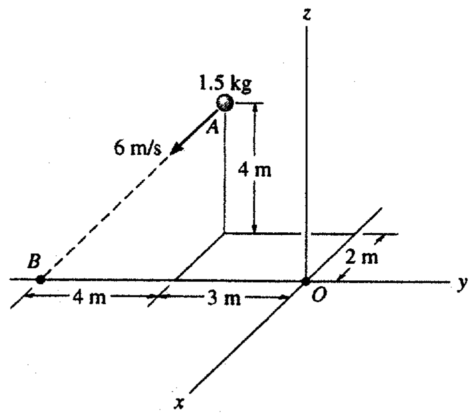
$$= \{0.3043\mathbf{i} - 0.3043\mathbf{j} - 0.6086\mathbf{k}\} \text{ slug}\cdot\text{ft/s}$$

$$(\mathbf{H}_A)_O = \mathbf{r}_A \times m\mathbf{v}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 5 & 4 \\ 0.3043 & -0.3043 & -0.6086 \end{vmatrix}$$

$$= \{-1.83\mathbf{i} - 0.913\mathbf{k}\} \text{ slug}\cdot\text{ft}^2/\text{s} \qquad \textbf{Ans}$$

15-91. Determine the angular momentum $\mathbf{H}_O$ of the particle about point O.

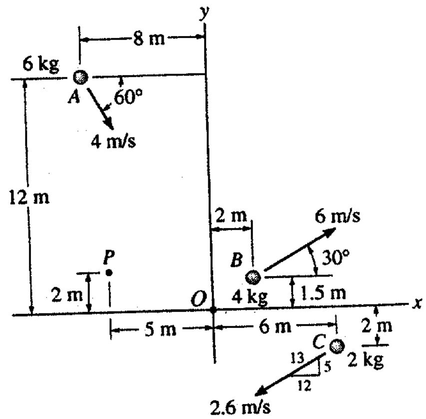


$\mathbf{r}_{OB} = \{-7\mathbf{j}\}$ m $\qquad \mathbf{v}_A = 6\left(\frac{2\mathbf{i}-4\mathbf{j}-4\mathbf{k}}{\sqrt{2^2+(-4)^2+(-4)^2}}\right) = \{2\mathbf{i}-4\mathbf{j}-4\mathbf{k}\}$ m/s

$\mathbf{H}_O = \mathbf{r}_{OB} \times m\mathbf{v}_A$

$$\mathbf{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -7 & 0 \\ 1.5(2) & 1.5(-4) & 1.5(-4) \end{vmatrix} = \{42\mathbf{i}+21\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans}$$

***15-92.** Determine the angular momentum $\mathbf{H}_O$ of each of the particles about point O.

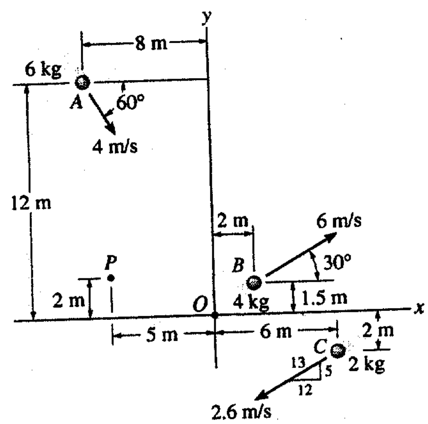


$(H_A)_O = 8(6)(4\sin 60°) - 12(6)(4\cos 60°) = 22.3\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

$(H_B)_O = -1.5(4)(6\cos 30°) + 2(4)(6\sin 30°) = -7.18\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

$(H_C)_O = -2(2)\left(\frac{12}{13}\right)(2.6) - 6(2)\left(\frac{5}{13}\right)(2.6) = -21.6\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

15-93. Determine the angular momentum $\mathbf{H}_P$ of each of the particles about point P.

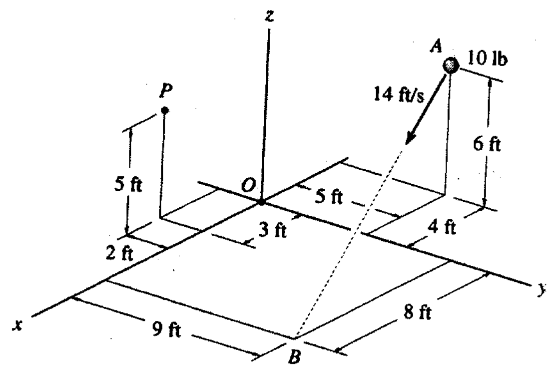


$(H_A)_P = -6(4\cos 60°)(10) + 6(4\sin 60°)(3) = -57.6\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

$(H_B)_P = 4(6\cos 30°)(0.5) + 4(6\sin 30°)(7) = 94.4\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

$(H_C)_P = -2(2.6)\left(\frac{5}{13}\right)(11) - 2(2.6)\left(\frac{12}{13}\right)(4) = -41.2\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

15-94. Determine the angular momentum $\mathbf{H}_O$ of the particle about point O.



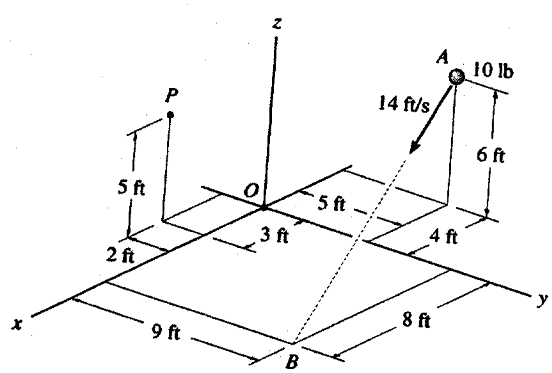
$\mathbf{r}_{OB} = \{8\mathbf{i}+9\mathbf{j}\}$ ft $\qquad \mathbf{v}_A = 14\left(\dfrac{12\mathbf{i}+4\mathbf{j}-6\mathbf{k}}{\sqrt{12^2+4^2+(-6)^2}}\right) = \{12\mathbf{i}+4\mathbf{j}-6\mathbf{k}\}$ ft/s

$$\mathbf{H}_O = \mathbf{r}_{OB}\times m\mathbf{v}_A$$

$$\mathbf{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 9 & 0 \\ \left(\frac{10}{32.2}\right)(12) & \left(\frac{10}{32.2}\right)(4) & \left(\frac{10}{32.2}\right)(-6) \end{vmatrix}$$

$$= \{-16.8\mathbf{i}+14.9\mathbf{j}-23.6\mathbf{k}\}\ \text{slug}\cdot\text{ft}^2/\text{s} \qquad \textbf{Ans}$$

15-95. Determine the angular momentum $\mathbf{H}_P$ of the particle about point P.



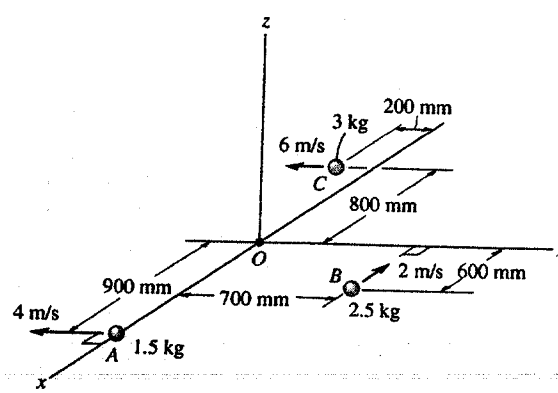
$\mathbf{r}_{PB} = \{5\mathbf{i}+11\mathbf{j}-5\mathbf{k}\}$ ft

$$\mathbf{v}_A = 14\left(\frac{12\mathbf{i}+4\mathbf{j}-6\mathbf{k}}{\sqrt{(12)^2+(4)^2+(-6)^2}}\right) = \{12\mathbf{i}+4\mathbf{j}-6\mathbf{k}\}\ \text{ft/s}$$

$$\mathbf{H}_P = \mathbf{r}_{PB}\times m\mathbf{v}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 11 & -5 \\ \left(\frac{10}{32.2}\right)(12) & \left(\frac{10}{32.2}\right)(4) & \left(\frac{10}{32.2}\right)(-6) \end{vmatrix}$$

$$= \{-14.3\mathbf{i}-9.32\mathbf{j}-34.8\mathbf{k}\}\ \text{slug}\cdot\text{ft}^2/\text{s} \qquad \textbf{Ans}$$

***15-96.** Determine the total angular momentum $\mathbf{H}_O$ for the system of three particles about point O. All the particles are moving in the x–y plane.

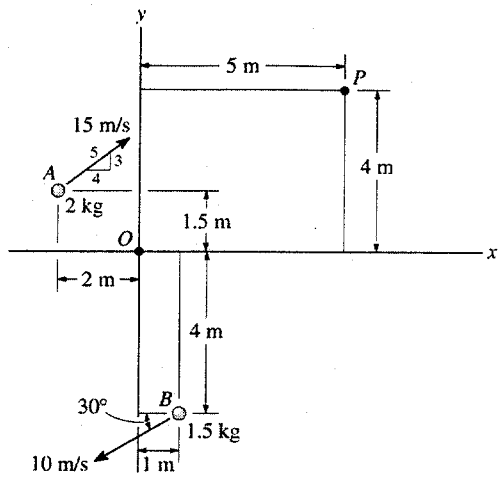


$$\mathbf{H}_O = \Sigma\,\mathbf{r}\times m\mathbf{v}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 0 \\ 0 & -1.5(4) & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 0.7 & 0 \\ -2.5(2) & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.8 & -0.2 & 0 \\ 0 & 3(-6) & 0 \end{vmatrix}$$

$$= \{12.5\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans}$$

15-97. Determine the angular momentum $\mathbf{H}_O$ of each of the two particles about point O. Use a scalar solution.



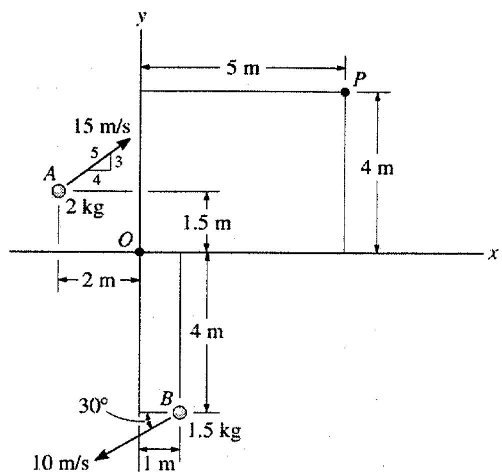
$$\circlearrowleft + (H_A)_O = -2(15)\left(\frac{4}{5}\right)(1.5) - 2(15)\left(\frac{3}{5}\right)(2)$$

$$= -72.0 \text{ kg}\cdot\text{m}^2/\text{s} = 72.0 \text{ kg}\cdot\text{m}^2/\text{s} \circlearrowright \qquad \textbf{Ans}$$

$$\circlearrowleft + (H_B)_O = -1.5(10)(\cos 30°)(4) - 1.5(10)(\sin 30°)(1)$$

$$= -59.5 \text{ kg}\cdot\text{m}^2/\text{s} = 59.5 \text{ kg}\cdot\text{m}^2/\text{s} \circlearrowright \qquad \textbf{Ans}$$

15-98. Determine the angular momentum $\mathbf{H}_P$ of each of the two particles about point P. Use a scalar solution.



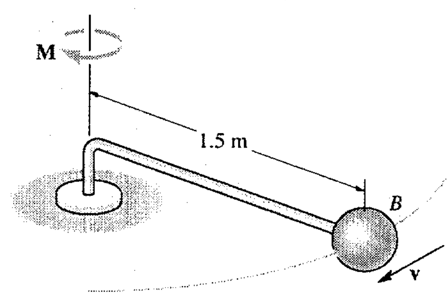
$$\circlearrowleft + (H_A)_P = 2(15)\left(\frac{4}{5}\right)(2.5) - 2(15)\left(\frac{3}{5}\right)(7)$$

$$= -66.0 \text{ kg}\cdot\text{m}^2/\text{s} = 66.0 \text{ kg}\cdot\text{m}^2/\text{s} \circlearrowright \qquad \textbf{Ans}$$

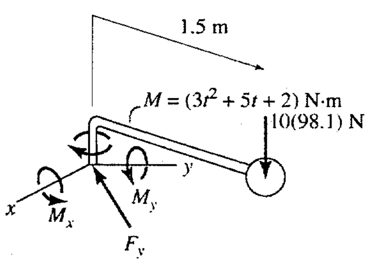
$$\circlearrowleft + (H_B)_P = -1.5(10)(\cos 30°)(8) + 1.5(10)(\sin 30°)(4)$$

$$= -73.9 \text{ kg}\cdot\text{m}^2/\text{s} = 73.9 \text{ kg}\cdot\text{m}^2/\text{s} \circlearrowright \qquad \textbf{Ans}$$

15-99. The ball B has a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = (3t^2 + 5t + 2)$ N · m, where t is in seconds, determine the speed of the ball when $t = 2$ s. The ball has a speed $v = 2$ m/s when $t = 0$.



Principle of Angular Impluse and Momentum: Applying Eq. 15-22, we have

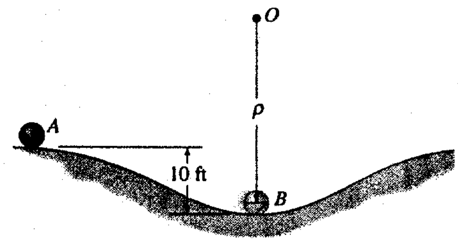


$$(H_z)_1 + \sum \int_{t_1}^{t_2} M_z dt = (H_z)_2$$

$$1.5(10)(2) + \int_0^{2s} (3t^2 + 5t + 2)dt = 1.5(10)v$$

$$v = 3.47 \text{ m/s} \qquad \textbf{Ans}$$

*15-100. The 3-lb ball located at A is released from rest and travels down the curved path. If the ball exerts a normal force of 5 lb on the path when it reaches point B, determine the angular momentum of the ball about the center of curvature, point O. *Hint:* Neglect the size of the ball. The radius of curvature at point B must be determined.

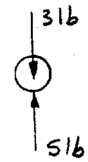


Datum at B :

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 3(10) = \frac{1}{2}\left(\frac{3}{32.2}\right)(v_B)^2 + 0$$

$$v_B = 25.38 \text{ ft/s}$$

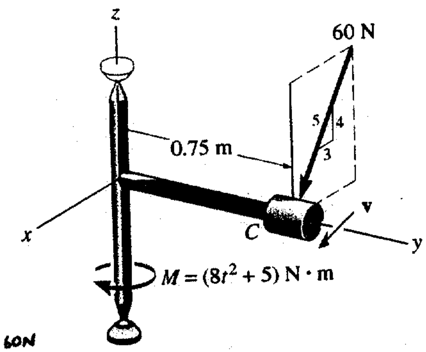


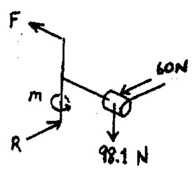
$$(+\uparrow)\Sigma F_n = ma_n; \qquad 5 - 3 = \left(\frac{3}{32.2}\right)\left(\frac{(25.38)^2}{\rho}\right)$$

$$\rho = 30 \text{ ft}$$

$$H_B = 30\left(\frac{3}{32.2}\right)(25.38) = 70.9 \text{ slug} \cdot \text{ft}^2/\text{s} \qquad \textbf{Ans}$$

15-101. The small cylinder C has a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the frame is subjected to a couple $M = (8t^2 + 5)$ N·m, where t is in seconds, and the cylinder is subjected to a force of 60 N, which is always directed as shown, determine the speed of the cylinder when $t = 2$ s. The cylinder has a speed $v_0 = 2$ m/s when $t = 0$.





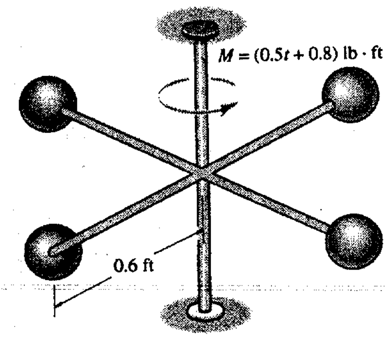
$$(H_z)_1 + \Sigma\int M_z\, dt = (H_z)_2$$

$$(10)(2)(0.75) + 60(2)\left(\frac{3}{5}\right)(0.75) + \int_0^2 (8t^2 + 5)dt = 10v(0.75)$$

$$69 + \left[\frac{8}{3}t^3 + 5t\right]_0^2 = 7.5v$$

$$v = 13.4 \text{ m/s} \qquad \textbf{Ans}$$

15-102. The four 5-lb spheres are rigidly attached to the crossbar frame having a negligible weight. If a couple moment $M = (0.5t + 0.8)$ lb·ft, where t is in seconds, is applied as shown, determine the speed of each of the spheres in 4 seconds starting from rest. Neglect the size of the spheres.



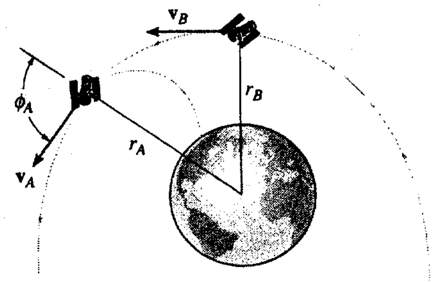
$$(H_z)_1 + \Sigma\int M_z\, dt = (H_z)_2$$

$$0 + \int_0^4 (0.5t + 0.8)\, dt = 4\left[\left(\frac{5}{32.2}\right)(0.6v_2)\right]$$

$$7.2 = 0.37267\, v_2$$

$$v_2 = 19.3 \text{ ft/s} \qquad \textbf{Ans}$$

15-103. An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of $v_A = 10$ km/s when the distance from the center of the earth is $r_A = 15$ Mm. If the launch angle at this position is $\phi_A = 70°$, determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass $M_e = 5.976(10^{24})$ kg. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = GM_e m_s/r^2$, Eq. 13–1. For part of the solution, use the conservation of energy (see Prob. 14-97).



$$(H_O)_1 = (H_O)_2$$

$$m_s(v_A \sin\phi_A)r_A = m_s(v_B)r_B$$

$$700[10(10^3)\sin 70°](15)(10^6) = 700(v_B)(r_B) \quad (1)$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m_s(v_A)^2 - \frac{GM_e m_s}{r_A} = \frac{1}{2}m_s(v_B)^2 - \frac{GM_e m_s}{r_B}$$

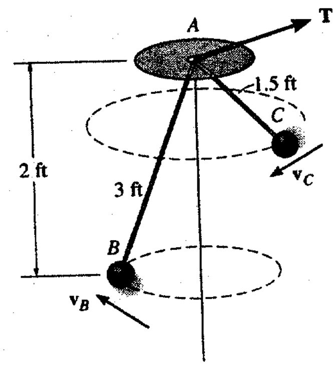
$$\frac{1}{2}(700)[10(10^3)]^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{[15(10^6)]} = \frac{1}{2}(700)(v_B)^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{r_B} \quad (2)$$

Solving,

$v_B = 10.2$ km/s **Ans**

$r_B = 13.8$ Mm **Ans**

***15-104.** The ball B has a weight of 5 lb and is originally rotating in a circle. As shown, the cord AB has a length of 3 ft and passes through the hole A, which is 2 ft above the plane of motion. If 1.5 ft of cord is pulled through the hole, determine the speed of the ball when it moves in a circular path at C.



Equation of Motion : When the ball is travelling around the first circular path, $\theta = \sin^{-1}\frac{2}{3} = 41.81°$ and $r_1 = 3\cos 41.81° = 2.236$. Applying Eq. 13 – 8, we have

$$\Sigma F_b = 0; \qquad T_1\left(\frac{2}{3}\right) - 5 = 0 \qquad T_1 = 7.50 \text{ lb}$$

$$\Sigma F_n = ma_n; \qquad 7.50\cos 41.81° = \frac{5}{32.2}\left(\frac{v_1^2}{2.236}\right)$$
$$v_1 = 8.972 \text{ ft/s}$$

When the ball is traveling around *the* second circular path, $r_2 = 1.5\cos\phi$. Applying Eq. 13 – 8, we have

$$\Sigma F_b = 0; \qquad T_2 \sin\phi - 5 = 0 \qquad [1]$$

$$\Sigma F_n = ma_n; \qquad T_2\cos\phi = \frac{5}{32.2}\left(\frac{v_2^2}{1.5\cos\phi}\right) \qquad [2]$$

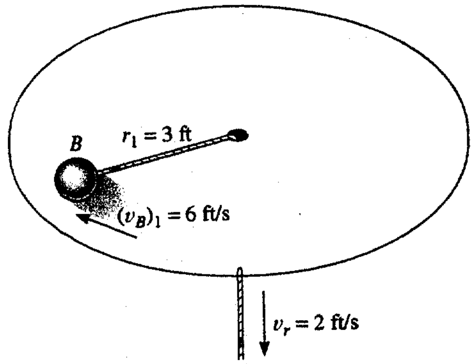
Conservation of Angular Momentum : Since no force acts on the ball along the tangent of the circular path, the angular momentum is conserved about z axis. Applying Eq. 15 – 23, we have

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$
$$r_1 m v_1 = r_2 m v_2$$
$$2.236\left(\frac{5}{32.2}\right)(8.972) = 1.5\cos\phi\left(\frac{5}{32.2}\right)v_2 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields

$$\phi = 13.8768° \qquad T_2 = 20.85 \text{ lb}$$
$$v_2 = 13.8 \text{ ft/s} \qquad \textbf{Ans}$$

15-105. A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine the ball's speed at the instant $r_2 = 2$ ft. How much work has to be done to pull down the cord? Neglect friction and the size of the ball.



$$H_1 = H_2$$

$$\frac{4}{32.2}(6)(3) = \frac{4}{32.2}v_\theta(2)$$

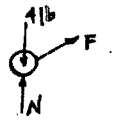
$$v_\theta = 9 \text{ ft/s}$$

$$v_2 = \sqrt{9^2 + 2^2} = 9.22 \text{ ft/s} \qquad \textbf{Ans}$$

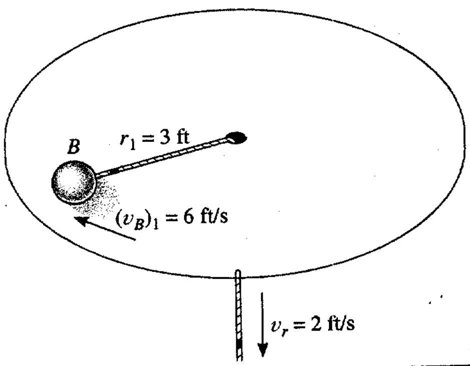
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4}{32.2}\right)(6)^2 + \Sigma U_{1-2} = \frac{1}{2}\left(\frac{4}{32.2}\right)(9.22)^2$$

$$\Sigma U_{1-2} = 3.04 \text{ ft·lb} \qquad \textbf{Ans}$$



15-106. A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine how much time is required for the ball to reach a speed of 12 ft/s. How far r_2 is the ball from the hole when this occurs? Neglect friction and the size of the ball.



$$v = \sqrt{(v_\theta)^2 + (2)^2}$$

$$12 = \sqrt{(v_\theta)^2 + (2)^2}$$

$$v_\theta = 11.832 \text{ ft/s}$$

$$H_1 = H_2$$

$$\frac{4}{32.2}(6)(3) = \frac{4}{32.2}(11.832)(r_2)$$

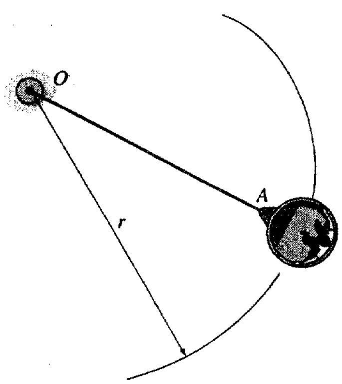
$$r_2 = 1.5213 = 1.52 \text{ ft} \qquad \textbf{Ans}$$

$$\Delta r = v_r t$$

$$(3 - 1.5213) = 2t$$

$$t = 0.739 \text{ s} \qquad \textbf{Ans}$$

15-107. An amusement park ride consists of a car which is attached to the cable OA. The car rotates in a horizontal circular path and is brought to a speed $v_1 = 4$ ft/s when $r = 12$ ft. The cable is then pulled in at the constant rate of 0.5 ft/s. Determine the speed of the car in 3 s.



Conservation of Angular Momentum : Cable OA is shorten by $\Delta r = 0.5(3) = 1.50$ ft in 3 s. Thus, at this instant $r_2 = 12 - 1.50 = 10.5$ ft. Since no force acts on the car along the tangent of the moving path, the angular momentum is conserved about point O. Applying Eq. 15 – 23, we have

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

$$r_1 m v_1 = r_2 m v'$$

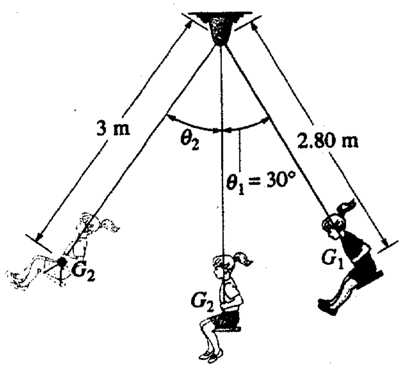
$$12(m)(4) = 10.5(m)v'$$

$$v' = 4.571 \text{ ft/s}$$

The speed of car after 3 s is

$$v_2 = \sqrt{0.5^2 + 4.571^2} = 4.60 \text{ ft/s} \qquad \textbf{Ans}$$

***15-108.** A child having a mass of 50 kg holds her legs up as shown as she swings downward from rest at $\theta_1 = 30°$. Her center of mass is located at point G_1. When she is at the bottom position $\theta = 0°$, she *suddenly* lets her legs come down, shifting her center of mass to position G_2. Determine her speed in the upswing due to this sudden movement and the angle θ_2 to which she swings before momentarily coming to rest. Treat the child's body as a particle.



First before $\theta = 0°$;

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2.80(1-\cos 30°)(50)(9.81) = \frac{1}{2}(50)(v_1)^2 + 0$$

$$v_1 = 2.713 \text{ m/s}$$

$$H_1 = H_2$$

$$50(2.713)(2.80) = 50(v_2)(3)$$

$$v_2 = 2.532 = 2.53 \text{ m/s} \qquad \textbf{Ans}$$

Just after $\theta = 0°$;

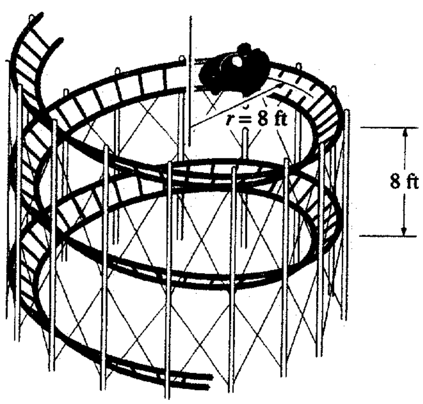
$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(50)(2.532)^2 + 0 = 0 + 50(9.81)(3)(1-\cos\theta_2)$$

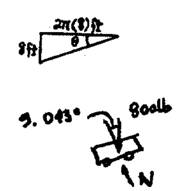
$$0.1089 = 1 - \cos\theta_2$$

$$\theta_2 = 27.0° \qquad \textbf{Ans}$$

15-109. The 800-lb roller-coaster car starts from rest on the track having the shape of a cylindrical helix. If the helix descends 8 ft for every one revolution, determine the speed of the car in $t = 4$ s. Also, how far has the car descended in this time? Neglect friction and the size of the car.



$$\theta = \tan^{-1}\left(\frac{8}{2\pi(8)}\right) = 9.043°$$



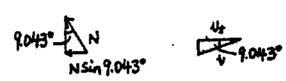
$$\Sigma F_y = 0; \qquad N - 800\cos 9.043° = 0$$

$$N = 790.1 \text{ lb}$$

$$H_1 + \int M\,dt = H_2$$

$$0 + \int_0^4 8(790.1\sin 9.043°)\,dt = \frac{800}{32.2}(8)v_t$$

$$v_t = 20.0 \text{ ft/s}$$



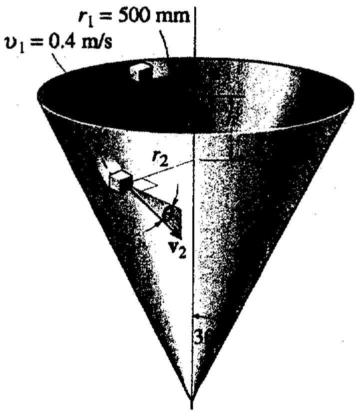
$$v = \frac{20}{\cos 9.043°} = 20.2 \text{ ft/s} \qquad \textbf{Ans}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 800h = \frac{1}{2}\left(\frac{800}{32.2}\right)(20.2)^2$$

$$h = 6.36 \text{ ft} \qquad \textbf{Ans}$$

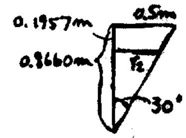
15-110. A small block having a mass of 0.1 kg is given a horizontal velocity of $v_1 = 0.4$ m/s when $r_1 = 500$ mm. It slides along the smooth conical surface. Determine the distance h it must descend for it to reach a speed of $v_2 = 2$ m/s. Also, what is the angle of descent θ, that is, the angle measured from the horizontal to the tangent of the path?



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(0.1)(0.4)^2 + 0.1(9.81)(h) = \frac{1}{2}(0.1)(2)^2$$

$h = 0.1957$ m $= 196$ mm **Ans**



From similar triangles

$$r_2 = \frac{(0.8660 - 0.1957)}{0.8660}(0.5) = 0.3870 \text{ m}$$

$$(H_0)_1 = (H_0)_2$$

$$0.5(0.1)(0.4) = 0.3870(0.1)(v_2')$$

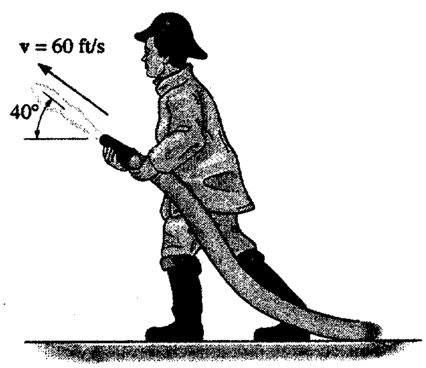
$$v_2' = 0.5168 \text{ m/s}$$

$$v_2 \cos\theta = v_2'$$

$$2 \cos\theta = 0.5168$$

$\theta = 75.0°$ **Ans**

15-111. The 150-lb fireman is holding a hose which has a nozzle diameter of 1 in. and hose diameter of 2 in. If the velocity of the water at discharge is 60 ft/s, determine the resultant normal and frictional force acting on the man's feet at the ground. Neglect the weight of the hose and the water within it. $\gamma_w = 62.4$ lb/ft^3.



Originally, the water flow is horizontal. The fireman alters the direction of flow to 40° from the horizontal.

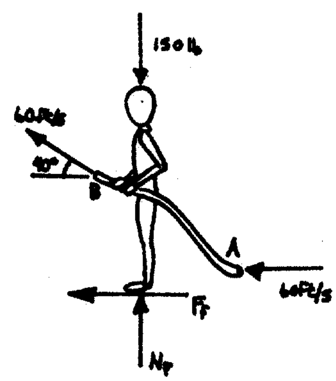
$$\frac{dm}{dt} = \rho v_B A_B = \frac{62.4}{32.2}(60)\left(\frac{\pi\left(\frac{1}{2}\right)^2}{(12)^2}\right) = 0.6342 \text{ slug/s}$$

Also, the velocity of the water through the hose is

$$\rho v_A A_A = \rho v_B A_B$$

$$\rho v_A\left(\frac{\pi(1)^2}{(12)^2}\right) = \rho(60)\left(\frac{\pi\left(\frac{1}{2}\right)^2}{(12)^2}\right)$$

$$v_A = 15 \text{ ft/s}$$



$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm}{dt}((v_B)_x - (v_A)_x)$$

$$F_f = 0.6342[60\cos 40° - 15]$$

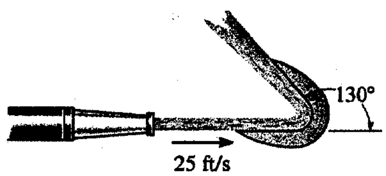
$F_f = 19.6$ lb **Ans**

$$+\uparrow \Sigma F_y = \frac{dm}{dt}((v_B)_y - (v_A)_y)$$

$$N_f - 150 = 0.6342[60\sin 40° - 0]$$

$N_f = 174$ lb Ans

*15-112. A jet of water having a cross-sectional area of 4 in^2 strikes the fixed blade with a speed of 25 ft/s. Determine the horizontal and vertical components of force which the blade exerts on the water. $\gamma_w = 62.4$ lb/ft^3.

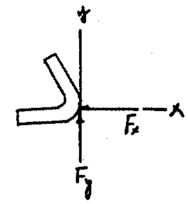


$$Q = Av = \left(\frac{4}{144}\right)(25) = 0.6944 \text{ ft}^3/\text{s}$$

$$\frac{dm}{dt} = \rho Q = \left(\frac{62.4}{32.2}\right)(0.6944) = 1.3458 \text{ slug/s}$$

$v_{Ax} = 25$ ft/s $\qquad v_{Ay} = 0$

$v_{Bx} = -25\cos 50°$ ft/s $\qquad v_{By} = 25\sin 50°$ ft/s



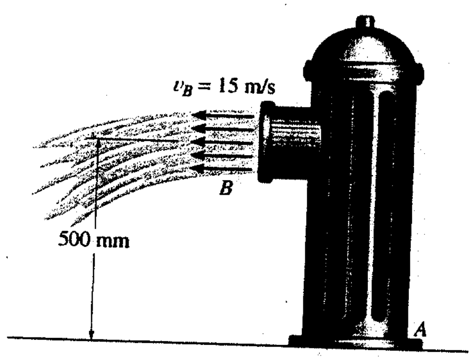
$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax});\qquad -F_x = 1.3458[-25\cos 50° - 25]$

$F_x = 55.3$ lb **Ans**

$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay});\qquad F_y = 1.3458(25\sin 50° - 0)$

$F_y = 25.8$ lb **Ans**

15-113. Water is flowing from the 150-mm-diameter fire hydrant with a velocity $v_B = 15$ m/s. Determine the horizontal and vertical components of force and the moment developed at the base joint A, if the static (gauge) pressure at A is 50 kPa. The diameter of the fire hydrant at A is 200 mm. $\rho_w = 1$ Mg/m^3.

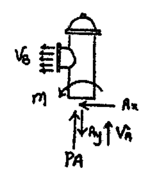


$$\frac{dm}{dt} = \rho v_A A_B = 1000(15)(\pi)(0.075)^2$$

$$\frac{dm}{dt} = 265.07 \text{ kg/s}$$

$$v_A = \left(\frac{dm}{dt}\right)\frac{1}{\rho A_A} = \frac{265.07}{1000(\pi)(0.1)^2}$$

$v_A = 8.4375$ m/s



$$\xleftarrow{+} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$$

$A_x = 265.07(15 - 0) = 3.98$ kN **Ans**

$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay})$$

$$-A_y + 50(10^3)(\pi)(0.1)^2 = 265.07(0 - 8.4375)$$

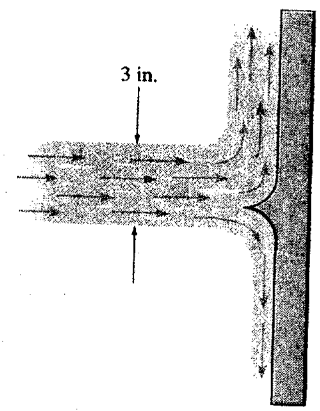
$A_y = 3.81$ kN **Ans**

$$\circlearrowleft + \Sigma M_A = \frac{dm}{dt}(d_{AB} v_B - d_{AA} v_A)$$

$M = 265.07(0.5(15) - 0)$

$M = 1.99$ kN·m **Ans**

15-114. The blade divides the jet of water having a diameter of 3 in. If one-fourth of the water flows downward while the other three-fourths flows upwards, and the total flow is $Q = 0.5\ \text{ft}^3/\text{s}$, determine the horizontal and vertical components of force exerted on the blade by the jet, $\gamma_w = 62.4\ \text{lb/ft}^3$.



Equations of Steady Flow : Here, the flow rate $Q = 0.5\ \text{ft}^3/\text{s}$. Then, $v = \dfrac{Q}{A} = \dfrac{0.5}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2} = 10.19\ \text{ft/s}$. Also, $\dfrac{dm}{dt} = \rho_w Q = \dfrac{62.4}{32.2}(0.5) = 0.9689\ \text{slug/s}$.

Applying Eq. 15 – 26, we have

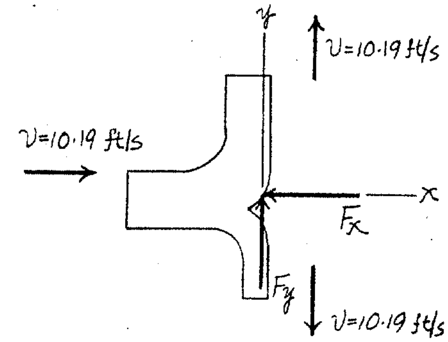
$$\Sigma F_x = \Sigma \frac{dm}{dt}\left(v_{\text{out}_x} - v_{\text{in}_x}\right);$$

$$-F_x = 0 - 0.9689(10.19) \qquad F_x = 9.87\ \text{lb} \qquad \textbf{Ans}$$

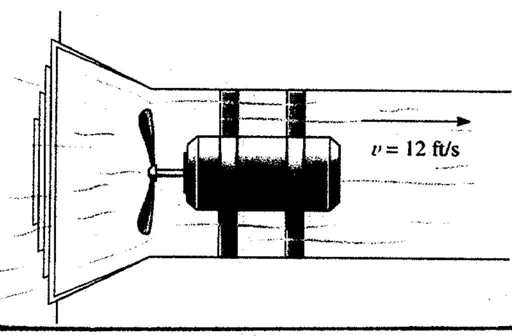
$$\Sigma F_y = \Sigma \frac{dm}{dt}\left(v_{\text{out}_y} - v_{\text{in}_y}\right);$$

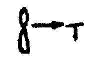
$$F_y = \frac{3}{4}(0.9689)(10.19) + \frac{1}{4}(0.9689)(-10.19)$$

$$F_y = 4.93\ \text{lb} \qquad \textbf{Ans}$$



15-115. The fan draws air through a vent with a speed of 12 ft/s. If the cross-sectional area of the vent is 2 ft², determine the horizontal thrust on the blade. The specific weight of the air is $\gamma_a = 0.076\ \text{lb/ft}^3$.





$$\frac{dm}{dt} = \rho v A$$

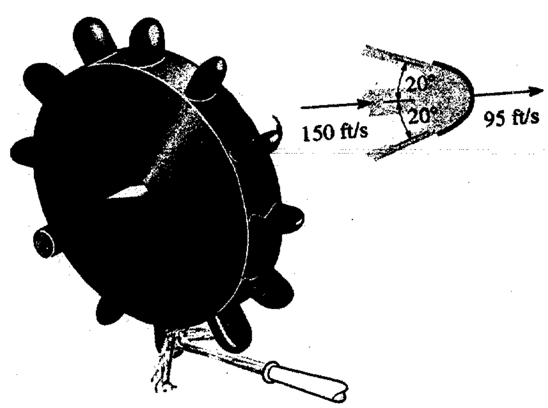
$$= \frac{0.076}{32.2}(12)(2)$$

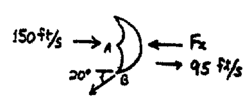
$$= 0.05665\ \text{slug/s}$$

$$\Sigma F = \frac{dm}{dt}(v_B - v_A)$$

$$T = 0.05665(12 - 0) = 0.680\ \text{lb} \qquad \textbf{Ans}$$

***15-116.** The buckets on the *Pelton wheel* are subjected to a 2-in-diameter jet of water, which has a velocity of 150 ft/s. If each bucket is traveling at 95 ft/s when the water strikes it, determine the power developed by the wheel. $\gamma_w = 62.4\ \text{lb/ft}^3$.





$$v_A = 150 - 95 = 55\ \text{ft/s} \rightarrow$$

$$(\overset{+}{\rightarrow})(v_B)_x = -55\cos 20^\circ + 95 = 43.317\ \text{ft/s} \rightarrow$$

$$\overset{+}{\leftarrow}\Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$$

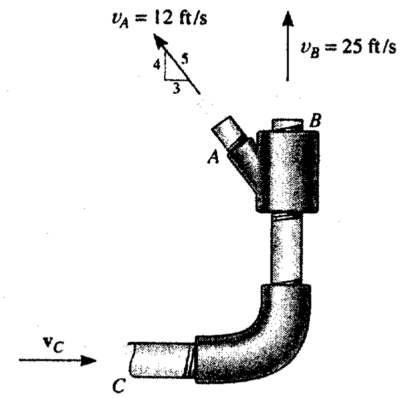
$$F_x = \left(\frac{62.4}{32.2}\right)(\pi)\left(\frac{1}{12}\right)^2(55)(-43.317 - (-55))$$

$$F_x = 27.1667\ \text{lb}$$

$$P = 27.1667(95) = 2580.835\ \text{ft}\cdot\text{lb/s}$$

$$P = 4.69\ \text{hp} \qquad \textbf{Ans}$$

15-117. The static pressure of water at C is 40 lb/in^2. If water flows out of the pipe at A and B with velocities v_A = 12 ft/s and v_B = 25 ft/s, determine the horizontal and vertical components of force exerted on the elbow at C necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 0.75 in. at C, and at A and B the diameter is 0.5 in. γ_w = 62.4 lb/ft^3.



$$\frac{dm_A}{dt} = \frac{62.4}{32.2}(12)(\pi)\left(\frac{0.25}{12}\right)^2 = 0.03171 \text{ slug/s}$$

$$\frac{dm_B}{dt} = \frac{62.4}{32.2}(25)(\pi)\left(\frac{0.25}{12}\right)^2 = 0.06606 \text{ slug/s}$$

$$\frac{dm_C}{dt} = 0.03171 + 0.06606 = 0.09777 \text{ slug/s}$$

$$v_C A_C = v_A A_A + v_B A_B$$

$$v_C(\pi)\left(\frac{0.375}{12}\right)^2 = 12(\pi)\left(\frac{0.25}{12}\right)^2 + 25(\pi)\left(\frac{0.25}{12}\right)^2$$

$$v_C = 16.44 \text{ ft/s}$$

$$\overset{+}{\rightarrow}\Sigma F_x = \frac{dm_B}{dt}v_{B_x} + \frac{dm_A}{dt}v_{A_x} - \frac{dm_C}{dt}v_{C_x}$$

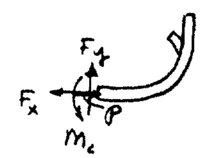
$$40(\pi)(0.375)^2 - F_x = 0 - 0.03171(12)\left(\frac{3}{5}\right) - 0.09777(16.44)$$

F_x = 19.5 lb **Ans**

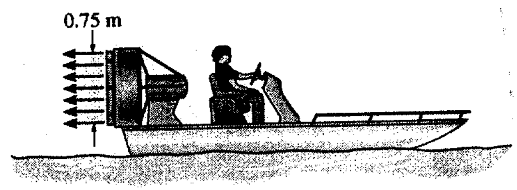
$$+\uparrow\Sigma F_y = \frac{dm_B}{dt}v_{B_y} + \frac{dm_A}{dt}v_{A_y} - \frac{dm_C}{dt}v_{C_y}$$

$$F_y = 0.06606(25) + 0.03171\left(\frac{4}{5}\right)(12) - 0$$

F_y = 1.9559 = 1.96 lb **Ans**

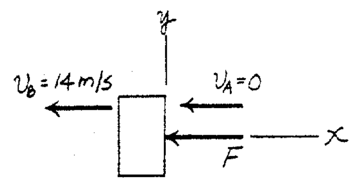


15-118. The 200-kg boat is powered by the fan which develops a slipstream having a diameter of 0.75 m. If the fan ejects air with a speed of 14 m/s, measured relative to the boat, determine the initial acceleration of the boat if it is initially at rest. Assume that air has a constant density of ρ_a = 1.22 kg/m^3 and that the entering air is essentially at rest. Neglect the drag resistance of the water.



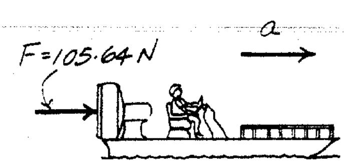
Equations of Steady Flow : Initially, the boat is at rest hence $v_B = v_{a/b}$ = 14 m/s. Then, $Q = v_B A = 14\left[\frac{\pi}{4}(0.75^2)\right] = 6.185$ m^3/s and $\frac{dm}{dt} = \rho_a Q$ = 1.22(6.185) = 7.546 kg/s. Applying Eq. 15 – 26, we have

$$\Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x}); \quad -F = 7.546(-14-0) \quad F = 105.64 \text{ N}$$

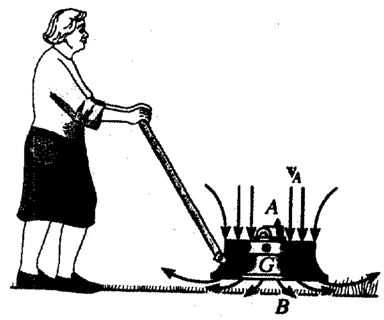


Equation of Motion :

$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \quad 105.64 = 200a \quad a = 0.528 \text{ m/s}^2$$ **Ans**



15-119. A power lawn mower hovers very close over the ground. This is done by drawing air in at a speed of 6 m/s through an intake unit A, which has a cross-sectional area of $A_A = 0.25\ \text{m}^2$, and then discharging it at the ground, B, where the cross-sectional area is $A_B = 0.35\ \text{m}^2$. If air at A is subjected only to atmospheric pressure, determine the air pressure which the lawn mower exerts on the ground when the weight of the mower is freely supported and no load is placed on the handle. The mower has a mass of 15 kg with center of mass at G. Assume that air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$.

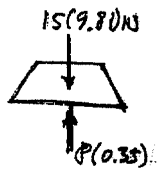


$$\frac{dm}{dt} = \rho A_A v_A = 1.22(0.25)(6) = 1.83\ \text{kg/s}$$

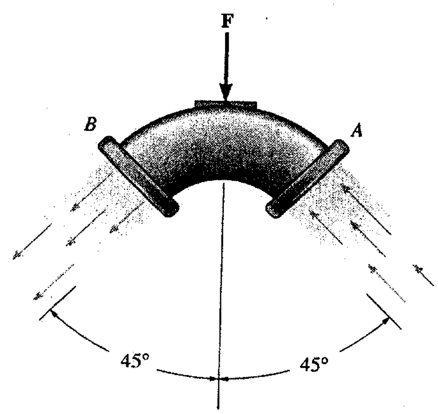
$$+\uparrow \Sigma F_y = \frac{dm}{dt}((v_B)_y - (v_A)_y)$$

$$p(0.35) - 15(9.81) = 1.83(0-(-6))$$

$p = 452$ Pa **Ans**



***15-120.** The elbow for a 5-in-diameter buried pipe is subjected to a static pressure of 10 lb/in². The speed of the water passing through it is $v = 8$ ft/s. Assuming the pipe connection at A and B do not offer any vertical force resistance on the elbow, determine the resultant vertical force **F** that the soil must then exert on the elbow in order to hold it in equilibrium. Neglect the weight of the elbow and the water within it. $\gamma_w = 62.4\ \text{lb/ft}^3$.



Equations of Steady Flow : Here, $Q = vA = 8\left[\frac{\pi}{4}\left(\frac{5}{12}\right)^2\right] = 1.091\ \text{ft}^3/\text{s}$.

Then, the mass flow rate is $\frac{dm}{dt} = \rho_w Q = \frac{62.4}{32.2}(1.091) = 2.114$ slug/s.

Also, the force induced by the water pressure at A is $F = pA = 10\left[\frac{\pi}{4}(5^2)\right] = 62.5\pi$ lb. Applying Eq. 15 – 26, we have

$$\Sigma F_y = \frac{dm}{dt}(v_{B_y} - v_{A_y});$$

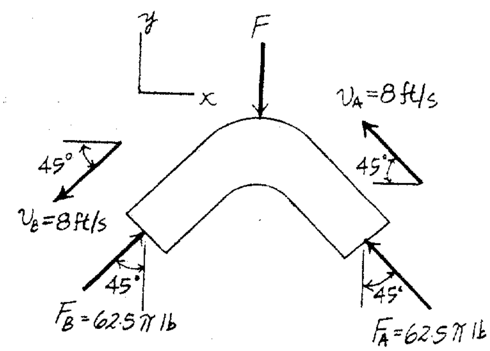
$$-F + 2(62.5\pi\cos 45°) = 2.114(-8\sin 45° - 8\sin 45°)$$

$$F = 302\ \text{lb} \qquad \textbf{Ans}$$

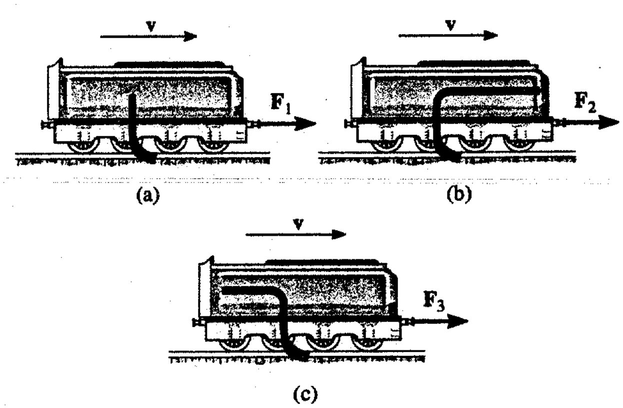
$$\Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x});$$

$$62.5\pi\sin 45° - 62.5\pi\sin 45° = 2.114[-8\cos 45° - (-8\cos 45°)]$$

$$0 = 0 \qquad (\textit{Check!})$$



15-121. The car is used to scoop up water that is lying in a trough at the tracks. Determine the force needed to pull the car forward at constant velocity **v** for each of the three cases. The scoop has a cross-sectional area A and the density of water is ρ_w.



The system consists of the car and the scoop. In all cases

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$F = 0 - v(\rho)(A)v$$

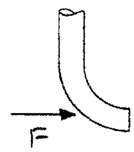
$F = v^2 \rho A$ **Ans**

15-122. A plow located on the front of a locomotive scoops up snow at the rate of 10 ft^3/s and stores it in the train. If the locomotive is traveling at a constant speed of 12 ft/s, determine the resistance to motion caused by the shoveling. The specific weight of snow is $\gamma_s = 6$ lb/ft^3.

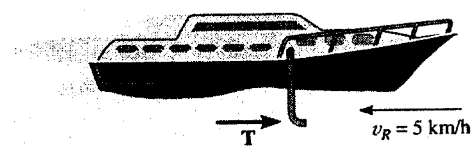
$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$F = 0 + (12 - 0)\left(\frac{10(6)}{32.2}\right)$$

$F = 22.4$ lb **Ans**



15-123. The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured *relative* to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust T on the tube that is required to overcome the resistance to the water collection. $\rho_w = 1$ Mg/m^3.



$$\frac{dm}{dt} = \frac{40}{80} = 0.5 \text{ kg/s}$$

$$v_{D/i} = (70)\left(\frac{1000}{3600}\right) = 19.444 \text{ m/s}$$

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$T = 0 + 19.444(0.5) = 9.72$ N **Ans**

***15-124.** The second stage of the two-stage rocket weighs 2000 lb (empty) and is launched from the first stage with a velocity of 3000 mi/h. The fuel in the second stage weighs 1000 lb. If it is consumed at the rate of 50 lb/s and ejected with a relative velocity of 8000 ft/s, determine the acceleration of the second stage just after the engine is fired. What is the rocket's acceleration just before all the fuel is consumed? Neglect the effect of gravitation.

Initially,

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\left(\frac{dm_e}{dt}\right)$$

$$0 = \frac{3000}{32.2}a - 8000\left(\frac{50}{32.2}\right)$$

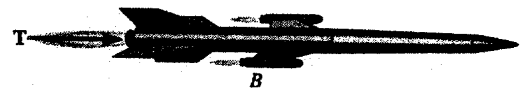
$a = 133$ ft/s^2 **Ans**

Finally,

$$0 = \frac{2000}{32.2}a - 8000\left(\frac{50}{32.2}\right)$$

$a = 200$ ft/s^2 **Ans**

15-125. The missile weighs 40 000 lb. The constant thrust provided by the turbojet engine is $T = 15\,000$ lb. Additional thrust is provided by *two* rocket boosters B. The propellant in each booster is burned at a constant rate of 150 lb/s, with a relative exhaust velocity of 3000 ft/s. If the mass of the propellant lost by the turbojet engine can be neglected, determine the velocity of the missile after the 4-s burn time of the boosters. The initial velocity of the missile is 300 mi/h.



$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

At a time t, $m = m_0 - ct$, where $c = \dfrac{dm_e}{dt}$.

$$T = (m_0 - ct)\frac{dv}{dt} - v_{D/e}c$$

$$\int_{v_0}^{v} dv = \int_0^t \left(\frac{T + cv_{D/e}}{m_0 - ct}\right)dt$$

$$v = \left(\frac{T + cv_{D/e}}{c}\right)\ln\left(\frac{m_0}{m_0 - ct}\right) + v_0 \qquad (1)$$

Here, $m_0 = \dfrac{40\,000}{32.2} = 1242.24$ slug, $c = 2\left(\dfrac{150}{32.2}\right) = 9.3168$ slug/s, $v_{D/e} = 3000$ ft/s,

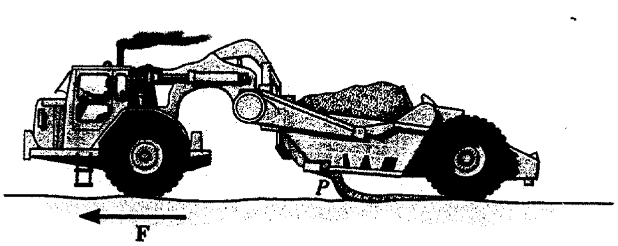
$t = 4$ s, $v_0 = \dfrac{300(5280)}{3600} = 440$ ft/s.

Substitute the numerical values into Eq.(1) :

$$v_{max} = \left(\frac{15\,000 + 9.3168(3000)}{9.3168}\right)\ln\left(\frac{1242.24}{1242.24 - 9.3168(4)}\right) + 440$$

$v_{max} = 580$ ft/s **Ans**

15-126. The earthmover initially carries 10 m^3 of sand having a density of 1520 kg/m^3. The sand is unloaded horizontally through a 2.5-m^2 dumping port P at a rate of 900 kg/s measured relative to the port. Determine the resultant tractive force **F** at its front wheels if the acceleration of the earthmover is 0.1 m/s^2 when half the sand is dumped. When empty, the earthmover has a mass of 30 Mg. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.



When half the sand remains,

$$m = 30\,000 + \frac{1}{2}(10)(1520) = 37\,600 \text{ kg}$$

$$\frac{dm}{dt} = 900 \text{ kg/s} = \rho v A$$

$$900 = 1520(v)(2.5)$$

$$v = 0.237 \text{ m/s}$$

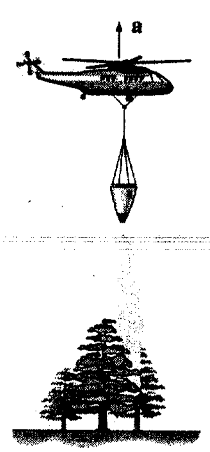
$$a = \frac{dv}{dt} = 0$$

$$\xleftarrow{+} \Sigma F_s = m\frac{dv}{dt} - \frac{dm}{dt}v$$

$$F = 37\,000(0.1) - 900(0.237)$$

$F = 3.49$ kN **Ans**

15-127. The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s, measured relative to the helicopter, determine the initial upward acceleration the helicopter experiences as the water is being released.



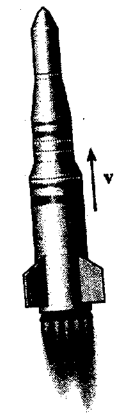
$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

Initially, the bucket is full of water, hence $m = 10(10^3) + 0.5(10^3) = 10.5(10^3)$ kg

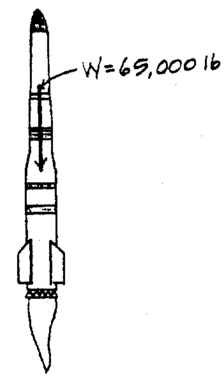
$$0 = 10.5(10^3)a - (10)(50)$$

$a = 0.0476$ m/s^2 **Ans**

***15-128.** The rocket has a mass of 65 Mg including the fuel. Determine the constant rate at which the fuel must be burned so that its thrust gives the rocket a speed of 200 ft/s in 10 s starting from rest. The fuel is expelled from the rocket at a relative speed of 3000 ft/s. Neglect the effects of air resistance and assume that g is constant.



A System That Loses Mass : Here, $W = \left(m_0 - \frac{dm_e}{dt}t\right)g$. Applying Eq. 15 – 29, we have



$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt};$$

$$-\left(m_0 - \frac{dm_e}{dt}t\right)g = \left(m_0 - \frac{dm_e}{dt}t\right)\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$\frac{dv}{dt} = \frac{v_{D/e}\frac{dm_e}{dt}}{m_0 - \frac{dm_e}{dt}t} - g$$

$$\int_0^v dv = \int_0^t \left(\frac{v_{D/e}\frac{dm_e}{dt}}{m_0 - \frac{dm_e}{dt}t} - g\right)dt$$

$$v = \left[-v_{D/e}\ln\left(m_0 - \frac{dm_e}{dt}t\right) - gt\right]\Bigg|_0^t$$

$$v = v_{D/e}\ln\left(\frac{m_0}{m_0 - \frac{dm_e}{dt}t}\right) - gt \qquad [1]$$

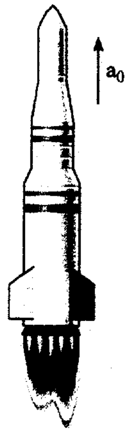
Substitute Eq. [1] with $m_0 = \frac{65\,000}{32.2} = 2018.63$ slug, $v_{D/e} = 3000$ ft/s, $v = 200$ ft/s and $t = 10$ s, we have

$$200 = 3000\ln\left[\frac{2018.63}{2018.63 - \frac{dm_e}{dt}(10)}\right] - 32.2(10)$$

$$e^{0.174} = \frac{2018.63}{2018.63 - \frac{dm_e}{dt}(10)}$$

$$\frac{dm_e}{dt} = 32.2 \text{ slug/s} \qquad \textbf{Ans}$$

15-129. The rocket has an initial mass m_0, including the fuel. For practical reasons desired for the crew, it is required that it maintain a constant upward acceleration a_0. If the fuel is expelled from the rocket at a relative speed $v_{e/r}$ determine the rate at which the fuel should be consumed to maintain the motion. Neglect air resistance, and assume that the gravitational acceleration is constant.



$$a_0 = \frac{dv}{dt}$$

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$-mg = ma_0 - v_{e/r}\frac{dm}{dt}$$

$$v_{e/r}\frac{dm}{m} = (a_0 + g)\,dt \qquad (1)$$

Since $v_{e/r}$ is constant, integrating, with $t = 0$ when $m = m_0$ yields

$$v_{e/r}\ln\left(\frac{m}{m_0}\right) = (a_0 + g)t$$

$$\frac{m}{m_0} = e^{[(a_0+g)/v_{e/r}]t}$$

The time rate of fuel consumption is determined from Eq. (1).

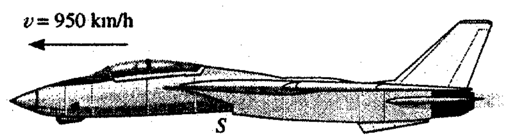
$$\frac{dm}{dt} = m\left(\frac{(a_0+g)}{v_{e/r}}\right)$$

$$\frac{dm}{dt} = m_0\left(\frac{(a_0+g)}{v_{e/r}}\right)e^{[(a_0+g)/v_{e/r}]t} \qquad \textbf{Ans}$$

Note : $v_{e/r}$ must be considered a negative quantity.

15-130. The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops S at the rate of 50 m^3/s. If the engine burns fuel at the rate of 0.4 kg/s and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m^3. *Hint:* Since mass both enters and exits the plane, Eqs. 15–29 and 15–30 must be combined to yield

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$



$$\Sigma F_s = m\frac{dv}{dt} - \frac{dm_e}{dt}(v_{D/E}) + \frac{dm_i}{dt}(v_{D/i}) \qquad (1)$$

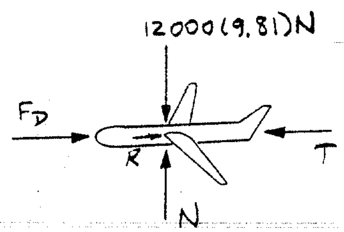
$$v = 950 \text{ km/h} = 0.2639 \text{ km/s}, \qquad \frac{dv}{dt} = 0$$

$$v_{D/E} = 0.45 \text{ km/s}$$

$$v_{D/i} = 0.2639 \text{ km/s}$$

$$\frac{dm_i}{dt} = 50(1.22) = 61.0 \text{ kg/s}$$

$$\frac{dm_e}{dt} = 0.4 + 61.0 = 61.4 \text{ kg/s}$$

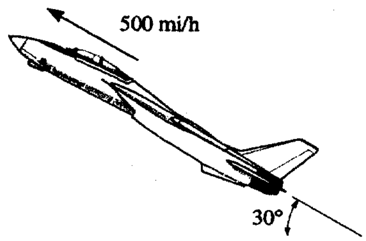


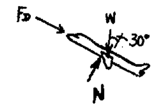
Forces T and R are incorporated into Eq. (1) as the last two terms in the equation.

$$(\overset{+}{\leftarrow})\; -F_D = 0 - (0.45)(61.4) + (0.2639)(61)$$

$$F_D = 11.5 \text{ kN} \qquad \textbf{Ans}$$

15-131. The jet is traveling at a speed of 500 mi/h, 30° with the horizontal. If the fuel is being spent at 3 lb/s, and the engine takes in air at 400 lb/s, whereas the exhaust gas (air and fuel) has a relative speed of 32 800 ft/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (0.7v^2)$ lb, where the speed is measured in ft/s. The jet has a weight of 15 000 lb. *Hint:* See Prob. 15–130.





$$\frac{dm_i}{dt} = \frac{400}{32.2} = 12.42 \text{ slug/s}$$

$$\frac{dm_e}{dt} = \frac{403}{32.2} = 12.52 \text{ slug/s}$$

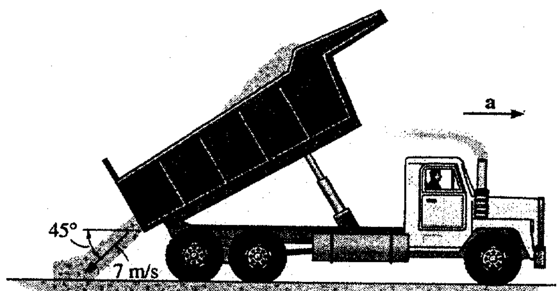
$$v = v_{D/i} = 500 \text{ mi/h} = 733.3 \text{ ft/s}$$

$$\nwarrow + \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$-(15\,000)\sin 30° - 0.7(733.3)^2 = \frac{15\,000}{32.2}\frac{dv}{dt} - 32\,800(12.52) + 733.3(12.42)$$

$$a = \frac{dv}{dt} = 37.5 \text{ ft/s}^2 \qquad \textbf{Ans}$$

***15-132.** The truck has a mass of 50 Mg when empty. When it is unloading 5 m³ of sand at a constant rate of 0.8 m³/s, the sand flows out the back at a speed of 7 m/s, measured relative to the truck, in the direction shown. If the truck is free to roll, determine its initial acceleration just as the load begins to empty. Neglect the mass of the wheels and any frictional resistance to motion. The density of sand is $\rho_s = 1520$ kg/m³.



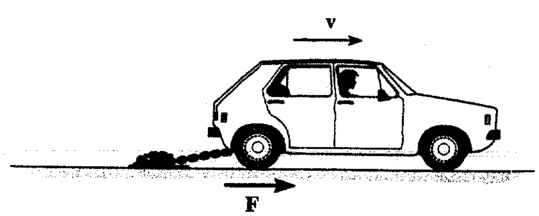
A System That Loses Mass : Initially, the total mass of the truck is $m = 50(10^3) + 5(1520) = 57.6(10^3)$ kg and $\frac{dm_e}{dt} = 0.8(1520) = 1216$ kg/s. Applying Eq. 15 – 29, we have

$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt};$$

$$0 = 57.6(10^3)\,a - (-7\cos 45°)(1216)$$

$$a = 0.104 \text{ m/s}^2 \qquad \textbf{Ans}$$

15-133. The car has a mass m_0 and is used to tow the smooth chain having a total length l and a mass per unit of length m'. If the chain is originally piled up, determine the tractive force F that must be supplied by the rear wheels of the car, necessary to maintain a constant speed v while the chain is being drawn out.



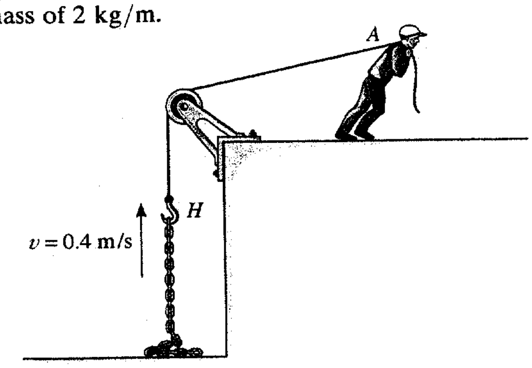
$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

At a time t, $m = m_0 + ct$, where $c = \frac{dm_i}{dt} = \frac{m'dx}{dt} = m'v$.

Here, $v_{D/i} = v$, $\frac{dv}{dt} = 0$.

$$F = (m_0 + m'vt)(0) + v(m'v) = m'v^2 \qquad \textbf{Ans}$$

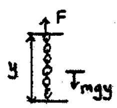
15-134. Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at A to raise the hook H with a constant speed $v = 0.4$ m/s. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass of 2 kg/m.



$$\frac{dv}{dt} = 0, \qquad y = vt$$

$$m_i = my = mvt$$

$$\frac{dm_i}{dt} = mv$$



$$+\uparrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\left(\frac{dm_i}{dt}\right)$$

$$F - mgvt = 0 + v(mv)$$

$$F = m(gvt + v^2)$$

$$= 2[9.81(0.4)t + (0.4)^2]$$

$$F = (7.85t + 0.320) \text{ N} \qquad \textbf{Ans}$$

R1-1. A sports car can accelerate at 6 m/s^2 and decelerate at 8 m/s^2. If the maximum speed it can attain is 60 m/s, determine the shortest time it takes to travel 900 m starting from rest and then stopping when s = 900 m.

Time to accelerate to 60 m/s,

$(\overset{+}{\rightarrow})\qquad v = v_0 + a_c t$

$60 = 0 + 6t$

$t = 10 \text{ s}$

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 0 + \frac{1}{2}(6)(10^2)$

$s = 300 \text{ m}$

Time to decelerate to a stop,

$(\overset{+}{\rightarrow})\qquad v = v_0 + a_c t$

$0 = 60 - 8t$

$t = 7.5 \text{ s}$

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 60(7.5) - \frac{1}{2}(8)(7.5^2)$

$s = 225 \text{ m}$

Time to travel at 60 m/s,

$900 - 300 - 225 = 375 \text{ m}$

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$

$375 = 0 + 60t$

$t = 6.25 \text{ s}$

Total time $t = 10 + 7.5 + 6.25 = 23.8 \text{ s}$ Ans

R1-2. A 2-kg particle rests on a smooth horizontal plane and is acted upon by forces $F_x = 0$ and $F_y = 3$ N. If $x = 0$, $y = 0$, $v_x = 6$ m/s, and $v_y = 2$ m/s when $t = 0$, determine the equation $y = f(x)$ which describes the path.

$$+\uparrow \Sigma F_y = ma_y; \quad 3 = 2a_y \quad a_y = 1.5 \text{ m/s}^2 \qquad [1]$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \quad 0 = 2a_x \quad a_x = 0 \qquad [2]$$

$$a_y = \frac{dv_y}{dt} = 1.5$$

$$\int_2^{v_y} dv_y = 1.5\int_0^t dt$$

$$v_y = \frac{dy}{dt} = 1.5t + 2$$

$$\int_0^y dy = \int_0^t (1.5t + 2)\,dt$$

$$y = 0.75t^2 + 2t \qquad [3]$$

$$a_x = \frac{dv_x}{dt} = 0$$

$$\int_6^{v_x} dv_x = \int_0^t 0\,dt$$

$$v_x = \frac{dx}{dt} = 6$$

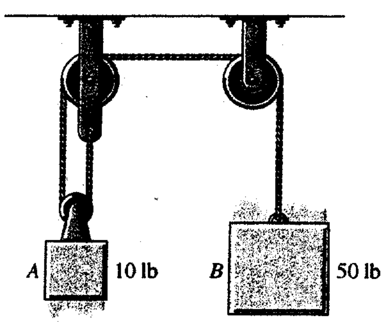
$$\int_0^x dx = \int_0^t 6\,dt$$

$$x = 6t \qquad [4]$$

Eliminating t from Eq.[3] and [4] yields :

$y = 0.0208x^2 + 0.333x$ **(Parabola)** **Ans**

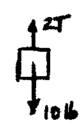
R1-3. Determine the velocity of each block 2 s after the blocks are released from rest. Neglect the mass of the pulleys and cord.



$$2s_A + s_B = l$$

$$2v_A + v_B = 0$$

$$v_A = \frac{v_B}{2}$$



$$(+\uparrow) \quad mv_1 + \Sigma\int F\,dt = mv_2$$

$$0 + 2T(2) - 10(2) = \frac{10}{32.2}\left(\frac{v_B}{2}\right)$$

$$(+\downarrow) \quad mv_1 + \Sigma\int F\,dt = mv_2$$

$$0 - T(2) + 50(2) = \frac{50}{32.2}v_B$$

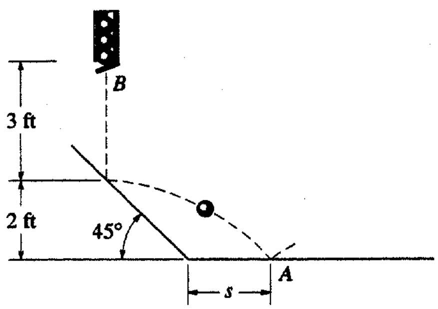
solving,

$$T = 7.14 \text{ lb}$$

$$v_B = 55.2 \text{ ft/s} \qquad \textbf{Ans}$$

Thus, $v_A = -\frac{v_B}{2} = -27.6$ ft/s $= 27.6$ ft/s **Ans**

***R1-4.** To test the manufactured properties of 2-lb steel balls, each ball is released from rest as shown and strikes a 45° inclined surface. If the coefficient of restitution is to be $e = 0.8$, determine the distance s to where the ball must strike the horizontal plane at A. At what speed does the ball strike A?

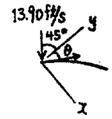


Just before impact

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2(3) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_B)_1^2 + 0$$

$$(v_B)_1 = 13.90 \text{ ft/s} \downarrow$$



Conservation of momentum in x-direction

$$\frac{2}{32.2}(13.9)\sin 45° = \frac{2}{32.2}(v_B)_2 \sin\theta$$

$$(v_B)_2 \sin\theta = 9.83 \qquad (1)$$

Coefficient of restitution

$$e = 0.8 = \frac{-(v_B)_2\cos\theta - 0}{0 - (13.9)\cos 45°}$$

$$(v_B)_2 \cos\theta = 7.86 \qquad (2)$$

Solving Eqs.(1) and (2)

$$(v_B)_2 = 12.59 \text{ ft/s}$$

$$\theta = 51.3°, \text{ so that } \triangledown\theta \;\; 6.39°$$

In y-direction:

$$(+\downarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$(v_B)_{Ay}^2 = (12.59\sin 6.34°)^2 + 2(32.2)(2-0)$$

$$(v_B)_{Ay} = 11.43 \text{ ft/s} \downarrow$$

$$(+\downarrow) \quad v = v_0 + a_c t$$

$$11.43 = 12.59\sin 6.34° + 32.2t$$

$$t = 0.312 \text{ s}$$

In x-direction:

$$(\xrightarrow{+}) \quad (v_B)_{Ax} = 12.59\cos 6.34° = 12.51 \text{ ft/s}$$

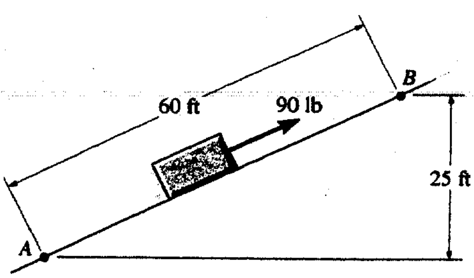
$$s = v_B t$$

$$s + 2 = 12.51(0.312)$$

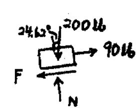
$$s = 1.90 \text{ ft} \qquad \textbf{Ans}$$

$$v_A = \sqrt{(12.51)^2 + (11.43)^2} = 16.9 \text{ ft/s} \qquad \textbf{Ans}$$

R1-5. The 90-lb force is required to drag the 200-lb block 60 ft up the *rough* inclined plane at constant velocity. If the force is removed when the block reaches point B, and the block is then released from rest, determine the block's velocity when it slides back down the plane and reaches point A.



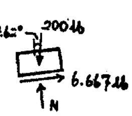
$$\theta = \sin^{-1}\left(\frac{25}{60}\right) = 24.62°$$



$$\nearrow + \Sigma F_x = 0; \quad 90 - 200\sin 24.62° - F = 0$$

$$F = 6.667 \text{ lb}$$

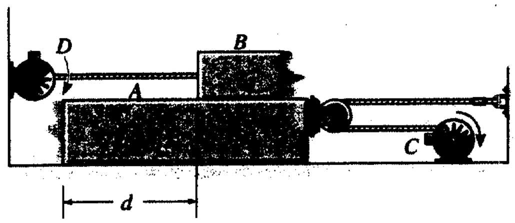
$$T_1 + \Sigma U_{1-2} = T_2$$



$$0 + 200\sin 24.62°(60) - 6.667(60) = \frac{1}{2}\left(\frac{200}{32.2}\right)v^2$$

$$v = 38.5 \text{ ft/s} \qquad \textbf{Ans}$$

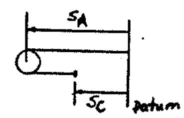
R1-6. The motor at C pulls in the cable with an acceleration $a_c = (3t^2)$ m/s^2, where t is in seconds. The motor at D draws in its cable at $a_D = 5$ m/s^2. If both motors start at the same instant from rest when $d = 3$ m, determine (a) the time needed for $d = 0$, and (b) the relative velocity of block A with respect to block B when this occurs.



$$s_A + (s_A - s_C) = l$$

$$2s_A - s_C = l$$

$$2a_A - a_C = 0 \qquad a_A = 0.5a_C = 1.5t^2$$



$$(\overset{+}{\rightarrow})\; a_{A/B} = a_A - a_B$$

$$= 1.5t^2 - (-5) = 1.5t^2 + 5$$

$$(\overset{+}{\rightarrow})\; v_{A/B} = \int_0^t (1.5t^2 + 5)\, dt = 0.5t^3 + 5t \qquad [1]$$

$$(\overset{+}{\rightarrow})\; s_{A/B} = \int_0^t (0.5t^3 + 5t)\, dt$$

$$3 = 0.125t^4 + 2.5t^2$$

$$t = 1.067 \text{ s} = 1.07 \text{ s} \qquad \textbf{Ans}$$

From Eq.[1] $\quad v_{A/B} = 0.5(1.067)^3 + 5(1.067) = 5.93 \text{ m/s} \rightarrow$ **Ans**

R1-7. A spring having a stiffness of 5 kN/m is compressed 400 mm. The stored energy in the spring is used to drive a machine which requires 80 W of power. Determine how long the spring can supply energy at the required rate.

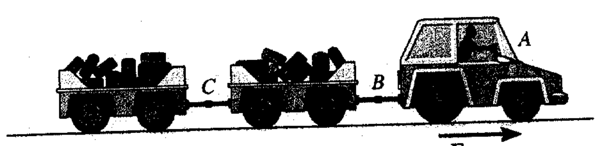
$$U = \frac{1}{2}kx^2 = \frac{1}{2}(5000)(0.4)^2 = 400 \text{ J}$$

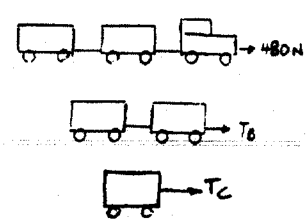
$$P = \frac{U}{t}$$

$$80 \text{ W} = \frac{400 \text{ J}}{t}$$

$t = 5$ s **Ans**

***R1-8.** The baggage truck A has a mass of 800 kg and is used to pull each of the 300-kg cars. Determine the tension in the couplings at B and C if the tractive force **F** on the truck is $F = 480$ N. What is the speed of the truck when $t = 2$ s, starting from rest? The car wheels are free to roll. Neglect the mass of the wheels.





$$\overset{+}{\rightarrow} \Sigma F_x = m a_x; \quad 480 = [800 + 2(300)]a$$

$$a = 0.3429 \text{ m/s}^2$$

$$(\overset{+}{\rightarrow}) \quad v = v_0 + a_c t$$

$$v = 0 + 0.3429(2) = 0.686 \text{ m/s} \qquad \textbf{Ans}$$

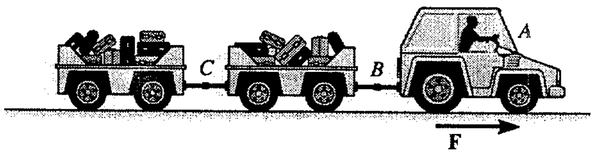
$$\overset{+}{\rightarrow} \Sigma F_x = m a_x; \quad T_B = 2(300)(0.3429)$$

$$T_B = 205.71 = 206 \text{ N} \qquad \textbf{Ans}$$

$$\overset{+}{\rightarrow} \Sigma F_x = m a_x; \quad T_C = (300)(0.3429)$$

$$T_C = 102.86 = 103 \text{ N}$$

R1-9. The baggage truck A has a mass of 800 kg and is used to pull each of the 300-kg cars. If the tractive force $\mathbf{F}$ on the truck is $F = 480$ N, determine the initial acceleration of the truck. What is the acceleration of the truck if the coupling at C suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.

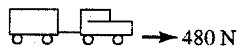


$\overset{+}{\rightarrow}\sum F_x = ma_x;$ $\quad 480 = [800 + 2(300)]a$

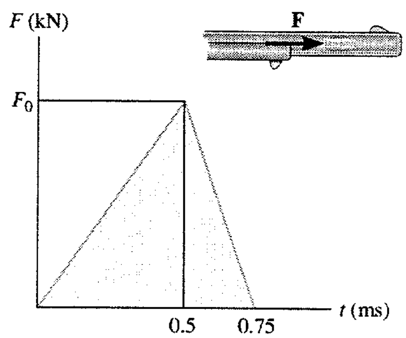
$a = 0.3429 = 0.343\ \text{m/s}^2$ **Ans**

$\overset{+}{\rightarrow}\sum F_x = ma_x;$ $\quad 480 = (800 + 300)a$

$a = 0.436\ \text{m/s}^2$ **Ans**



R1-10. Assuming that the force acting on a 2.5-g bullet, as it passes horizontally through the barrel of a rifle, varies with time in the manner shown, determine the maximum net force, F_0, applied to the bullet when it is fired. The muzzle velocity is 800 m/s when $t = 0.75$ m/s. Neglect friction between the bullet and rifle barrel.



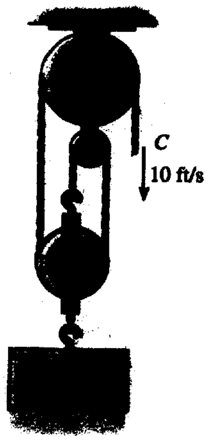
The impulse is determined from the area under the curve:

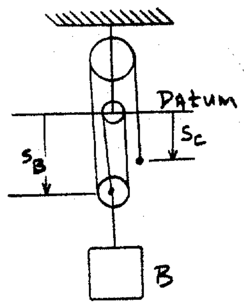
$$mv_1 + \sum \int F\,dt = mv_2$$

$$0 + \frac{1}{2}(0.75)(10^{-3})F_0 = (0.0025)(800)$$

$F_0 = 5.33$ kN **Ans**

R1-11. Determine the speed of block B if the end of the cable at C is pulled downward with a speed of 10 ft/s. What is the relative velocity of the block with respect to C?





$3s_B + s_C = l$

$3v_B = -v_C$

$3v_B = -(10)$

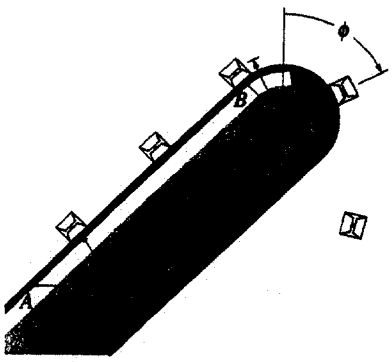
$v_B = -3.33 \text{ ft/s} = 3.33 \text{ ft/s} \uparrow$ **Ans**

$(+\downarrow)$ $v_B = v_C + v_{B/C}$

$-3.33 = 10 + v_{B/C}$

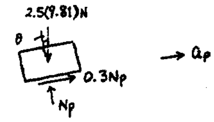
$v_{B/C} = -13.3 \text{ ft/s} = 13.3 \text{ ft/s} \uparrow$ **Ans**

***■R1-12.** Packages having a mass of 2.5 kg ride on the surface of the conveyor belt. If the belt starts from rest and with constant acceleration increases to a speed of 0.75 m/s in 2 s, determine the maximum angle of tilt, θ, so that none of the packages slip on the inclined surface AB of the belt. The coefficient of static friction between the belt and each package is $\mu_s = 0.3$. At what angle ϕ do the packages first begin to slip off the surface of the belt if the belt is moving at a constant speed of 0.75 m/s?



$\nwarrow + \Sigma F_y = ma_y; \quad N_p - 2.5(9.81)\cos\theta = 0 \quad (1)$

$\nearrow + \Sigma F_x = ma_x; \quad 0.3N_p - 2.5(9.81)\sin\theta = 2.5a_p \quad (2)$



Kinematics:

$v = v_0 + a_c t$

$0.75 = 0 + a_p(2)$

$a_p = 0.375 \text{ m/s}^2$

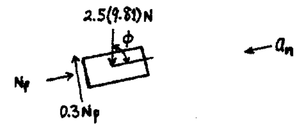
Combining Eqs.(1) and (2), using a_p,

$0.3\cos\theta - \sin\theta = 0.0382$

$\theta = 14.6°$ **Ans**

$+\Sigma F_n = ma_n; \quad 2.5(9.81)\cos\phi - N_p = 2.5[\frac{(0.75)^2}{0.350}]$

$+\Sigma F_t = ma_t; \quad 2.5(9.81)\sin\phi - 0.3N_p = 0$



Combining these equations,

$\cos\phi - 3.33\sin\phi = 0.164$

$\phi = 14.0°$

R1-13. A projectile, initially at the origin, moves along a straight-line path through a fluid medium such that its velocity is $v = 1800(1 - e^{-0.3t})$ mm/s, where t is in seconds. Determine the displacement of the projectile during the first 3 s.

$$v = \frac{ds}{dt} = 1800(1 - e^{-0.3t})$$

$$\int_0^s ds = \int_0^t 1800(1 - e^{-0.3t})\, dt$$

$$s = 1800(t + \frac{1}{0.3}e^{-0.3t}) - 6000$$

Thus, in $t = 3$ s

$$s = 1800(3 + \frac{1}{0.3}e^{-0.3(3)}) - 6000$$

$$s = 1839.4 \text{ mm} = 1.84 \text{ m}$$

R1-14. The speed of a train during the first minute of its motion has been recorded as follows:

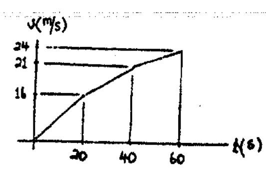
t(s)	0	20	40	60
v(m/s)	0	16	21	24

Plot the v–t graph, approximating the curve as straight line segments between the given points. Determine the total distance traveled.

s = area under curve

$$= \frac{1}{2}(20)(16) + (40-20)(\frac{21+16}{2}) + (60-40)(\frac{24+21}{2})$$

$= 980$ m **Ans**

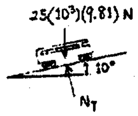


R1-15. A train car, having a mass of 25 Mg, travels up a 10° incline with a constant speed of 80 km/h. Determine the power required to overcome the force of gravity.

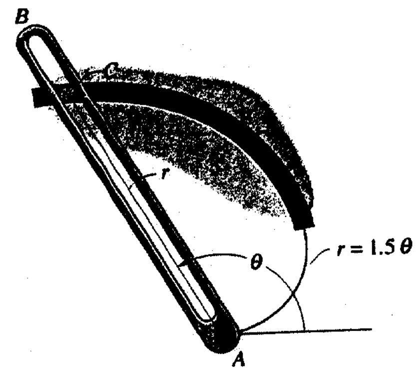
$v = 80 \text{ km/h} = 22.22 \text{ m/s}$

$P = \mathbf{F} \cdot \mathbf{v} = 25(10^3)(9.81)(22.22)(\sin 10°)$

$P = 946 \text{ kW}$ **Ans**



***R1-16.** The slotted arm AB drives the pin C through the spiral groove described by the equation $r = (1.5\theta)$ ft, where θ is in radians. If the arm starts from rest when $\theta = 60°$ and is driven at an angular rate of $\dot{\theta} = (4t)$ rad/s, where t is in seconds, determine the radial and transverse components of velocity and acceleration of the pin when $t = 1$ s.



$\dot{\theta} = 4t$

$\int_{\frac{\pi}{3}}^{\theta} d\theta = \int_0^t 4t\,dt$

$\theta - \frac{1}{3}\pi = 2t^2$

$\theta = (2t^2 + \frac{1}{3}\pi) \text{ rad}$

$\dot{\theta} = 4t$

$\ddot{\theta} = 4$

$r = 1.5\theta = 1.5(2t^2 + \frac{1}{3}\pi)$

$\dot{r} = 6t$

$\ddot{r} = 6$

When $t = 1$ s,

$\dot{\theta} = 4 \qquad r = 4.5708$

$\ddot{\theta} = 4 \qquad \dot{r} = 6$

$\ddot{r} = 6$

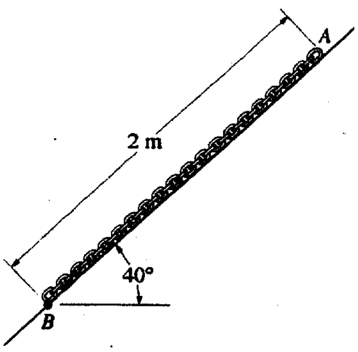
$v_r = \dot{r} = 6.00 \text{ ft/s}$ **Ans**

$v_\theta = r\dot{\theta} = 4.5708(4) = 18.3 \text{ ft/s}$ **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 4.5708(4)^2 = -67.1 \text{ ft/s}^2$ **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.5708(4) + 22(6)(4) = 66.3 \text{ ft/s}^2$ **Ans**

R1-17. The chain has a mass of 3 kg/m. If the coefficient of kinetic friction between the chain and the plane is $\mu_k = 0.2$, determine the velocity at which the end A will pass point B when the chain is released from rest.

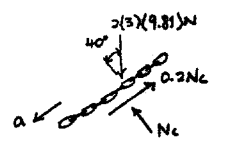


$+\nwarrow \Sigma F_y = ma_y;\quad -2(3)(9.81)\cos 40° + N_C = 0$

$N_C = 45.09\text{ N}$

$+\swarrow \Sigma F_x = ma_x;\quad 2(3)(9.81)\sin 40° - 0.2(45.09) = 2(3)\,a$

$a = 4.80\text{ m/s}^2$



$+\swarrow\quad v_2^2 = v_1^2 + 2as$

$v_2^2 = 0 + 2(4.80)(2)$

$v_2 = 4.38\text{ m/s}$ **Ans**

Also,

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 2(3)(9.81)(2\sin 40°) - 0.2(45.09)(2) = \frac{1}{2}(2)(3)(v^2)$

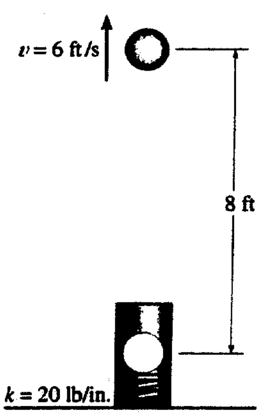
$v = 4.38\text{ m/s}$ **Ans**

R1-18. The 6-lb ball is fired from a tube by a spring having a stiffness $k = 20$ lb/in. Determine how far the spring must be compressed to fire the ball from the compressed position to a height of 8 ft, at which point it has a velocity of 6 ft/s.

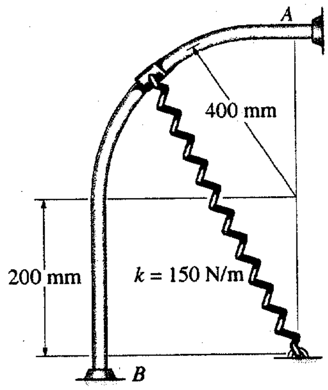
$T_1 + V_1 = T_2 + V_2$

$0 + \frac{1}{2}(20)(12)(x^2) = \frac{1}{2}\left(\frac{6}{32.2}\right)(6)^2 + 8(6)$

$x = 0.654\text{ ft} = 7.85\text{ in.}$ **Ans**



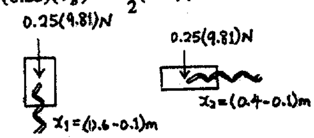
R1-19. The collar of negligible size has a mass of 0.25 kg and is attached to a spring having an unstretched length of 100 mm. If the collar is released from rest at A and travels along the smooth guide, determine its speed just before it strikes B.



$$T_A + V_A = T_B + V_B$$

$$0 + (0.25)(9.81)(0.6) + \frac{1}{2}(150)(0.6-0.1)^2 = \frac{1}{2}(0.25)(v_B)^2 + \frac{1}{2}(150)(0.4-0.1)^2$$

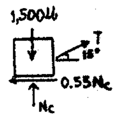
$v_B = 10.4$ m/s **Ans**



***R1-20.** A crate has a weight of 1500 lb. If it is pulled along the ground at a constant speed for a distance of 20 ft, and the towing cable makes an angle of 15° with the horizontal, determine the tension in the cable and the work done by the towing force. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.55$.

$+\uparrow \Sigma F_y = 0; \quad N_C - 1500 + T\sin 15° = 0$

$\xrightarrow{+} \Sigma F_x = 0; \quad T\cos 15° - 0.55N_C = 0$



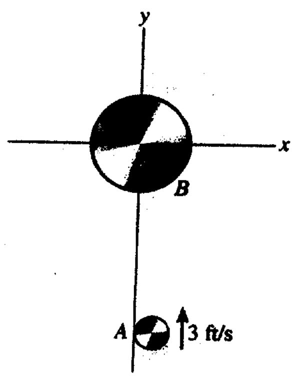
$T = 744.4 \text{ lb} = 744 \text{ lb}$ **Ans**

$N_C = 1307.3 \text{ lb}$

$U_{1-2} = (744.4\cos 15°)(20) = 14\,380.7 \text{ ft·lb}$

$U_{1-2} = 14.4 \text{ ft·kip}$ **Ans**

R1-21. Disk A weighs 2 lb and is sliding on a smooth horizontal plane with a velocity of 3 ft/s. Disk B weighs 11 lb and is initially at rest. If after the impact A has a velocity of 1 ft/s directed along the positive x axis, determine the velocity of B after impact. How much kinetic energy is lost in the collision?



Conservation of momentum for A and B in x-direction

$$0 + 0 = \frac{2}{32.2}(1) + \frac{11}{32.2}(v_B)_{2x}$$

$$(v_B)_{2x} = -0.1818 \text{ ft/s}$$

Conservation of momentum for A and B in y-direction

$$\frac{2}{32.2}(3) + 0 = 0 + \frac{11}{32.2}(v_B)_{2y}$$

$$(v_B)_{2y} = 0.54545 \text{ ft/s}$$

Thus,

$$(v_B)_2 = \sqrt{(-0.1818)^2 + (0.54545)^2} = 0.575 \text{ ft/s}$$ **Ans**

$$\theta = \tan^{-1}\left(\frac{0.54545}{0.1818}\right) = 71.5°$$ **Ans**

$$T_1 = \frac{1}{2}\left(\frac{2}{32.2}\right)(3)^2 = 0.280 \text{ ft·lb}$$

$$T_2 = \frac{1}{2}\left(\frac{2}{32.2}\right)(1)^2 + \frac{1}{2}\left(\frac{11}{32.2}\right)(0.575)^2 = 0.0875 \text{ ft·lb}$$

$$\Delta T = T_2 - T_1 = -0.192 \text{ ft·lb}$$ **Ans**

R1-22. A particle is moving along a circular path of 2-m radius such that its position as a function of time is given by $\theta = (5t^2)$ rad, where t is in seconds. Determine the magnitude of the particle's acceleration when $\theta = 30°$. The particle starts from rest when $\theta = 0°$.

$r = 2$ m $\qquad \theta = 5t^2$

$\dot{r} = 0 \qquad \dot{\theta} = 10t$

$\ddot{r} = 0 \qquad \ddot{\theta} = 10$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta$$

$$= [0 - 2(10t)^2]\mathbf{u}_r + [2(10) + 0]\mathbf{u}_\theta$$

$$= \{-200t^2\mathbf{u}_r + 20\mathbf{u}_\theta\} \text{ m/s}^2$$

When $\theta = 30° = 30(\frac{\pi}{180}) = 0.524$ rad

Then,

$0.524 = 5t^2$

$t = 0.324$ s

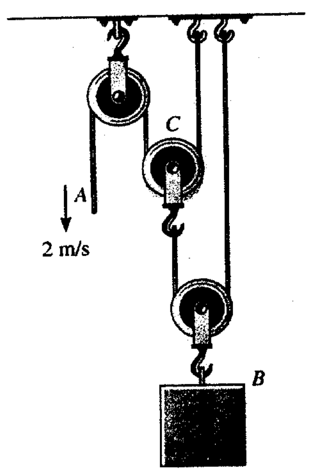
Hence,

$$\mathbf{a} = [-200(0.324)^2]\mathbf{u}_r + 20\mathbf{u}_\theta$$

$$= \{-20.9\mathbf{u}_r + 20\mathbf{u}_\theta\} \text{ m/s}^2$$

$$a = \sqrt{(-20.9)^2 + (20)^2} = 29.0 \text{ m/s}^2 \qquad \textbf{Ans}$$

R1-23. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



Two cords :

$s_A + 2s_C = l$

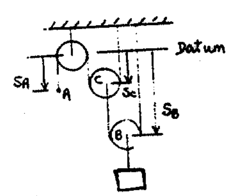
$s_B + (s_B - s_C) = l'$

Thus, $\quad v_A = -2v_C$

$2v_B = v_C$

$4v_B = -v_A$

$v_B = \frac{-2}{4} = -0.5$ m/s $= 0.5$ m/s ↑ $\qquad$ **Ans**



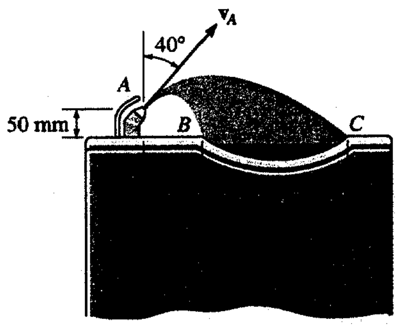
***R1-24.** A rifle has a mass of 2.5 kg. If it is loosely gripped and a 1.5-g bullet is fired from it with a horizontal muzzle velocity of 1400 m/s, determine the recoil velocity of the rifle just after firing.

$\xrightarrow{+}\quad \Sigma m v_1 = \Sigma m v_2$

$0 + 0 = 0.0015(1400) - 2.5(v_R)_2$

$(v_R)_2 = 0.840$ m/s $\qquad$ **Ans**

R1-25. The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at B and C.



$$(\overset{+}{\rightarrow})\ s_x = v_x t$$

$$R = v_A \sin 40° t \qquad t = \frac{R}{v_A \sin 40°} \qquad [1]$$

$$(+\uparrow)\ s_y = (s_y)_0 + v_y t + \tfrac{1}{2} a_c t^2$$

$$-0.05 = 0 + v_A \cos 40° t + \tfrac{1}{2}(-9.81)t^2 \qquad [2]$$

Substituting Eq.[1] into [2] yields :

$$-0.05 = v_A \cos 40°\left(\frac{R}{v_A \sin 40°}\right) + \tfrac{1}{2}(-9.81)\left(\frac{R}{v_A \sin 40°}\right)^2$$

$$v_A = \sqrt{\frac{4.905 \sin 40° R^2}{\sin^2 40°(R\cos 40° + 0.05 \sin 40°)}}$$

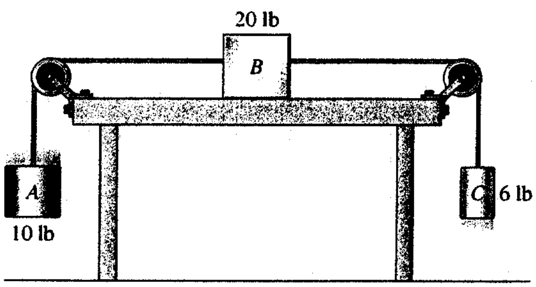
At point B, $R = 0.1$ m.

$$v_A = \sqrt{\frac{4.905 \sin 40°(0.1)^2}{\sin^2 40°(0.1\cos 40° + 0.05 \sin 40°)}} = 0.838 \text{ m/s} \qquad \textbf{Ans}$$

At point C, $R = 0.35$ m.

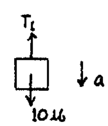
$$v_A = \sqrt{\frac{4.905 \sin 40°(0.35)^2}{\sin^2 40°(0.35\cos 40° + 0.05 \sin 40°)}} = 1.76 \text{ m/s} \qquad \textbf{Ans}$$

R1-26. The 20-lb block B rests on the surface of a table for which the coefficient of kinetic friction is $\mu_k = 0.1$. Determine the speed of the 10-lb block A after it has moved downward 2 ft from rest. Neglect the mass of the pulleys and cords.



Block A :

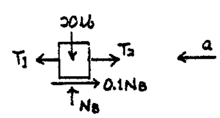
$$+\downarrow \Sigma F_y = ma_y; \qquad -T_1 + 10 = \frac{10}{32.2}a \qquad (1)$$



Block B :

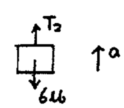
$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad -T_2 + T_1 - 0.1N_B = \frac{20}{32.2}a \qquad (2)$$

$$+\uparrow \Sigma F_y = ma_y; \qquad N_B - 20 = 0 \qquad (3)$$



Block C :

$$+\uparrow \Sigma F_y = ma_y; \qquad T_2 - 6 = \frac{6}{32.2}a \qquad (4)$$



Solving Eqs.(1) – (4) for a,

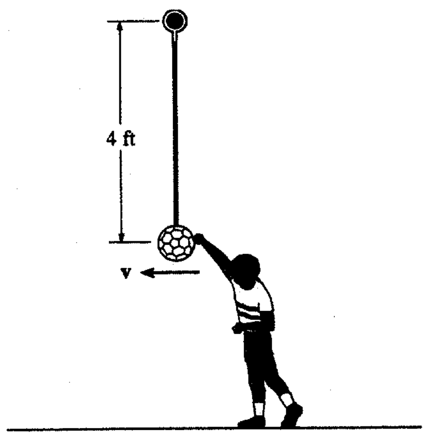
$a = 1.79$ ft/s^2

$$(+\downarrow)\ v^2 = v_0^2 + 2a_c(s - s_0)$$

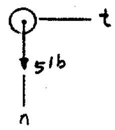
$$v^2 = 0 + 2(1.79)(2 - 0)$$

$v = 2.68$ ft/s **Ans**

R1-27. The 5-lb ball, attached to the cord, is struck by the boy. Determine the smallest speed he must impart to the ball so that it will swing around in a vertical circle, without causing the cord to become slack.



$$+\downarrow \Sigma F_n = ma_n; \quad 5 = \frac{5}{32.2}\left(\frac{v_2^2}{4}\right) \qquad v_2^2 = 128.8 \text{ ft}^2/\text{s}^2$$

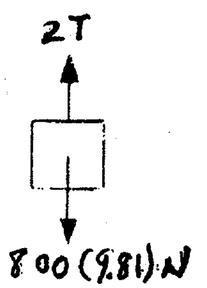


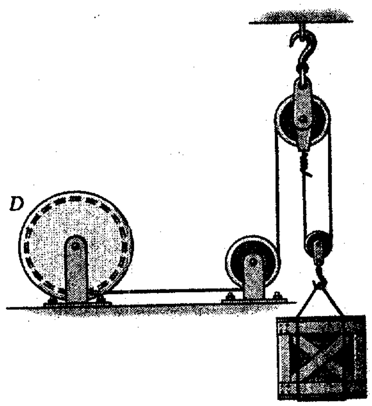
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{5}{32.2}\right)v^2 + 0 = \frac{1}{2}\left(\frac{5}{32.2}\right)(128.8) + 5(8)$$

$$v = 25.4 \text{ ft/s}$$ **Ans**

***R1-28.** The winding drum D is drawing in the cable at an accelerated rate of 5 m/s^2. Determine the cable tension if the suspended crate has a mass of 800 kg.





$$s_A + 2s_B = l$$

$$a_A = -2a_B$$

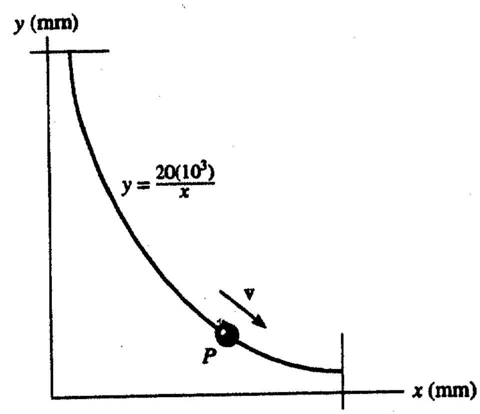
$$5 = -2a_B$$

$$a_B = -2.5 \text{ m/s}^2 = 2.5 \text{ m/s}^2 \uparrow$$

$$+\uparrow \Sigma F_y = ma_y; \quad 2T - 800(9.81) = 800(2.5)$$

$$T = 4924 \text{ N} = 4.92 \text{ kN}$$ **Ans**

R1-29. The particle P travels with a constant speed of 300 mm/s along the curve. Determine its acceleration when it is located at point (200 mm, 100 mm).



$v = 300$ mm/s

$$a_t = \frac{dv}{dt} = 0$$

$$y = \frac{2(10^4)}{x}$$

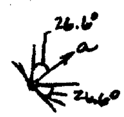
$$\left.\frac{dy}{dx}\right|_{x=200} = \frac{-2(10^4)}{x^2} = -0.5$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=200} = \frac{4(10^4)}{x^3} = 5(10^{-3})$$

$$\rho = \left|\frac{[1+(-0.5)^2]^{3/2}}{5(10^{-3})}\right| = 279.5 \text{ mm}$$

$$a_n = \frac{v^2}{\rho} = \frac{(300)^2}{279.5} = 322 \text{ mm/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = 322 \text{ mm/s}^2$$



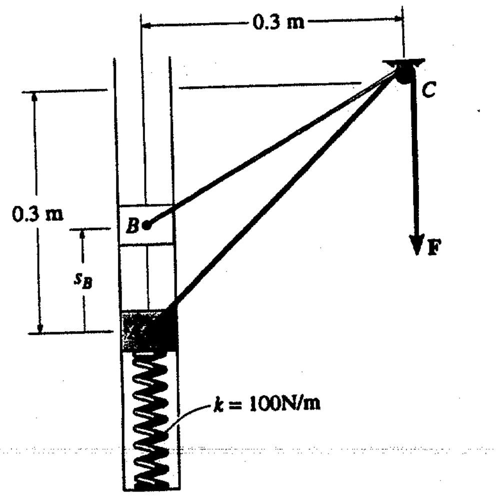
Since $\frac{dy}{dx} = -0.5$,

$$\theta = \tan^{-1}(-0.5) = -26.6°$$

Thus,

$a = 322$ m/s^2 $\measuredangle^{26.6°}$ **Ans**

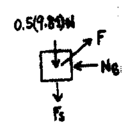
R1-30. The block has a mass of 0.5 kg and moves within the smooth vertical slot. If the block starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the mass of the cord and pulley.



The work done by F depends upon the difference in the cord length $AC - BC$.

$$T_A + \Sigma U_{A-B} = T_B$$

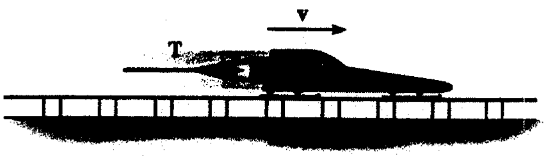
$$0 + F[\sqrt{(0.3)^2 + (0.3)^2} - \sqrt{(0.3)^2 + (0.3-0.15)^2}\,] - 0.5(9.81)(0.15)$$
$$-\frac{1}{2}(100)(0.15)^2 = \frac{1}{2}(0.5)(2.5)^2$$

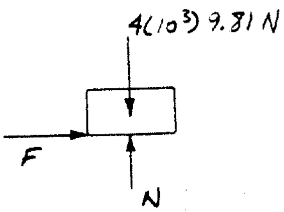


$$F(0.0889) = 3.423$$

$F = 38.5$ N **Ans**

***R1-31.** The rocket sled has a mass of 4 Mg and travels from rest along the smooth horizontal track such that it maintains a constant power output of 450 kW. Neglect the loss of fuel mass and air resistance, and determine how far it must travel to reach a speed of $v = 60$ m/s.





$$\xrightarrow{+}\Sigma F_x = m\,a_x; \qquad F = m\,a = m\left(\frac{v\,dv}{ds}\right)$$

$$P = F\,v = m\left(\frac{v^2\,dv}{ds}\right)$$

$$\int P\,ds = m\int v^2\,dv$$

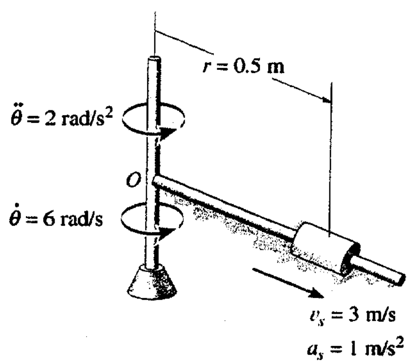
$$P\int_0^s ds = m\int_0^v v^2\,dv$$

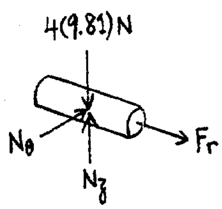
$$Ps = \frac{m\,v^3}{3}$$

$$s = \frac{m\,v^3}{3\,P}$$

$$s = \frac{4(10^3)(60)^3}{3(450)(10^3)} = 640 \text{ m} \qquad \textbf{Ans}$$

***R1-32.** The spool, which has a mass of 4 kg, slides along the rotating rod. At the instant shown, the angular rate of rotation of the rod is $\dot{\theta} = 6$ rad/s and this rotation is increasing at $\ddot{\theta} = 2$ rad/s². At this same instant, the spool has a velocity of 3 m/s and an acceleration of 1 m/s², both measured relative to the rod and directed away from the center O when $r = 0.5$ m. Determine the radial frictional force and the normal force, both exerted by the rod on the spool at this instant.





$$r = 0.5 \text{ m}$$

$$\dot{r} = 3 \text{ m/s} \qquad \dot{\theta} = 6 \text{ rad/s}$$

$$\ddot{r} = 1 \text{ m/s}^2 \qquad \ddot{\theta} = 2 \text{ rad/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 1 - 0.5(6)^2 = -17$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(2) + 2(3)(6) = 37$$

$$\Sigma F_r = m a_r; \qquad F_r = 4(-17) = -68 \text{ N}$$

$$\Sigma F_\theta = m a_\theta; \qquad N_\theta = 4(37) = 148 \text{ N}$$

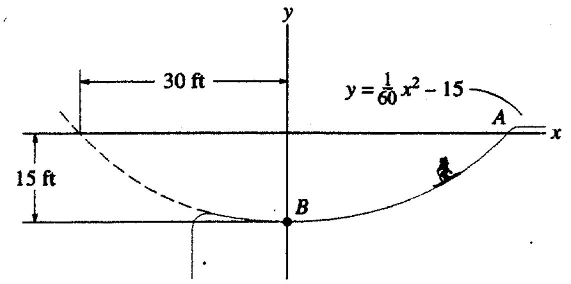
$$\Sigma F_z = m a_z; \qquad N_z - 4(9.81) = 0$$

$$N_z = 39.24 \text{ N}$$

$$F_r = -68 \text{ N} \qquad \textbf{Ans}$$

$$N_C = \sqrt{(148)^2 + (39.24)^2} = 153 \text{ N} \qquad \textbf{Ans}$$

R1-33. A skier starts from rest at A (30 ft, 0) and descends the smooth slope, which may be approximated by a parabola. If she has a weight of 120 lb, determine the normal force she exerts on the ground at the instant she arrives at point B.



$$T_A + V_A = T_B + V_B$$

$$0 + (120)(15) = \frac{1}{2}\left(\frac{120}{32.2}\right)v_B^2 + 0$$

$$v_B = 31.08 \text{ ft/s}$$

$$y = \frac{1}{60}x^2 - 15$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \left.\frac{1}{30}x\right|_{x=0} = 0$$

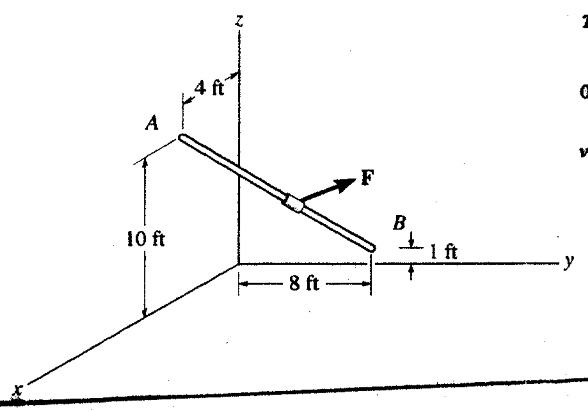
$$\frac{d^2y}{dx^2} = \frac{1}{30}$$

$$\rho = \left|\frac{[1+(\frac{dy}{dx})^2]^{3/2}}{\frac{d^2y}{dx^2}}\right| = \frac{(1+0)^{3/2}}{\frac{1}{30}} = 30 \text{ ft}$$

$$+\uparrow \Sigma F_n = ma_n; \quad N_s - 120 = \frac{120}{32.2}\left[\frac{(31.08)^2}{30}\right]$$

$$N_s = 240 \text{ lb} \qquad \textbf{Ans}$$

R1-34. The small 2-lb collar starting from rest at A slides down along the smooth rod. During the motion, the collar is acted upon by a force $\mathbf{F} = \{10\mathbf{i} + 6y\mathbf{j} + 2z\mathbf{k}\}$ lb, where x, y, z are in feet. Determine the collar's speed when it strikes the wall at B.



$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = -4\mathbf{i} + 8\mathbf{j} - 9\mathbf{k}$$

$$T_1 + \Sigma\int F ds = T_2$$

$$0 + 2(10-1) + \int_4^0 10dx + \int_0^8 6y\,dy + \int_{10}^1 2z\,dz = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

$$v_B = 47.8 \text{ ft/s} \qquad \textbf{Ans}$$

R1-35. A ball having a mass of 200 g is released from rest at a height of 400 mm above a very large fixed metal surface. If the ball rebounds to a height of 325 mm above the surface, determine the coefficient of restitution between the ball and the surface.

Just before impact

$$T_1 + T_2 = T_2 + V_2$$

$$0 + 0.2(9.81)(0.4) = \frac{1}{2}(0.2)(v_2)^2 + 0$$

$$v_2 = 2.80 \text{ m/s}$$

Just after impact

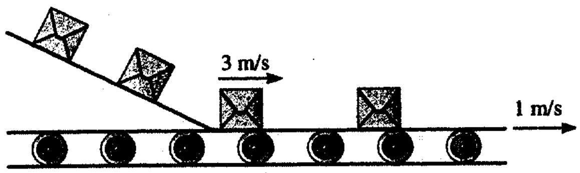
$$T_3 + V_3 = T_4 + V_4$$

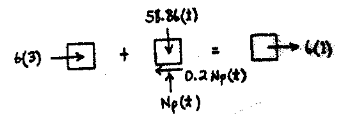
$$\frac{1}{2}(0.2)(v_3)^2 + 0 = 0 + 0.2(9.81)(0.325)$$

$$v_3 = 2.53 \text{ m/s}$$

$$e = \frac{(v_{rel})_2}{(v_{rel})_1} = \frac{2.53}{2.80} = 0.901 \qquad \textbf{Ans}$$

***R1-36.** Packages having a mass of 6 kg slide down a smooth chute and land horizontally with a speed of 3 m/s on the surface of a conveyor belt. If the coefficient of kinetic friction between the belt and a package is $\mu_k = 0.2$, determine the time needed to bring the package to rest on the belt if the belt is moving in the same direction as the package with a speed $v = 1$ m/s.





$(+\uparrow)\quad m(v_1)_y + \Sigma\int F_y\,dt = m(v_2)_y$

$0 + N_p(t) - 58.86(t) = 0$

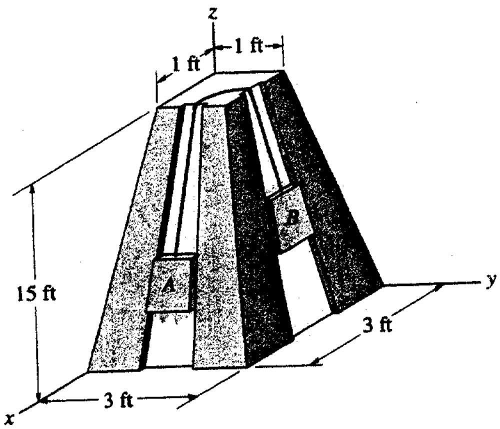
$N_p = 58.86$ N

$(\xrightarrow{+})\quad m(v_1)_x + \Sigma\int F_x\,dt = m(v_2)_x$

$6(3) - 0.2(58.86)(t) = 6(1)$

$t = 1.02$ s **Ans**

R1-37. The blocks A and B weigh 10 and 30 lb, respectively. They are connected together by a light cord and ride in the frictionless grooves. Determine the speed of each block after block A moves 6 ft up along the plane. The blocks are released from rest.



$$\frac{6}{z} = \frac{\sqrt{15^2 + 2^2}}{15}$$

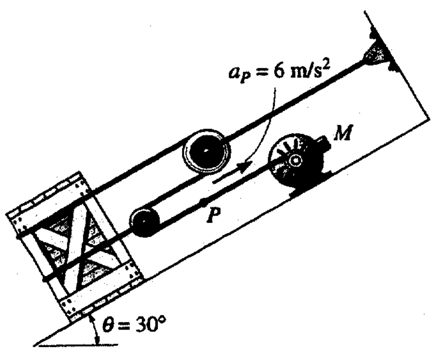
$z = 5.95$ ft

$T_1 + V_1 = T_2 + V_2$

$$0 + 0 = \frac{1}{2}\left(\frac{10}{32.2}\right)v_2^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)v_2^2 + 10(5.95) - 30(5.95)$$

$v_2 = 13.8$ ft/s **Ans**

R1-38. The motor M pulls in its attached rope with an acceleration $a_p = 6\text{ m/s}^2$. Determine the towing force exerted by M on the rope in order to move the 50-kg crate up the inclined plane. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$. Neglect the mass of the pulleys and rope.



$\nwarrow\!+\ \Sigma F_y = ma_y;\quad N_C - 50(9.81)\cos 30° = 0$

$N_C = 424.79$

$\nearrow\!+\ \Sigma F_x = ma_x;\quad 3T - 0.3(424.79) - 50(9.81)\sin 30° = 50a_C \qquad (1)$

Kinematics $\quad 2s_C + (s_C - s_P) = l$

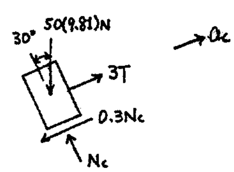
Taking two time derivatives, yields

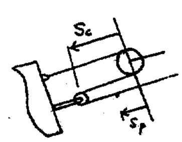
$3a_C = a_P$

Thus, $\quad a_C = \frac{6}{3} = 2$

Substituting into Eq.(1) and solving,

$T = 158$ N **Ans**





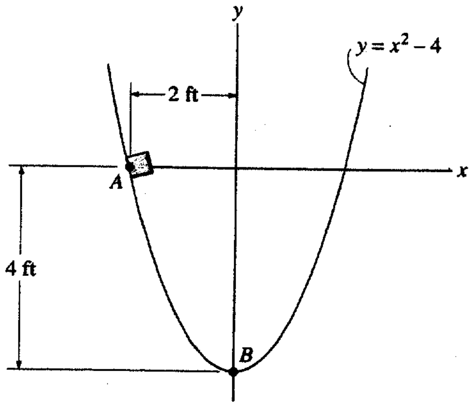
R1-39. If a particle has an initial velocity $v_0 = 12$ ft/s to the right, and a constant acceleration of 2 ft/s^2 to the left, determine the particle's displacement in 10 s. Originally $s_0 = 0$.

$(\xrightarrow{+})\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 12(10) + \frac{1}{2}(-2)(10)^2$

$s = 20.0$ ft **Ans**

***R1-40.** A 3-lb block, initially at rest at point A, slides along the smooth parabolic surface. Determine the normal force acting on the block when it reaches B. Neglect the size of the block.



$T_1 + V_1 = T_2 + V_2$

$0 + 3(4) = \frac{1}{2}(\frac{3}{32.2})v_2^2 + 0$

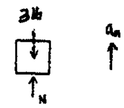
$v_2 = 16.05$ ft/s

$\rho = \frac{[1+(\frac{dy}{dx})^2]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{[1+(2x)^2]^{3/2}}{2}$

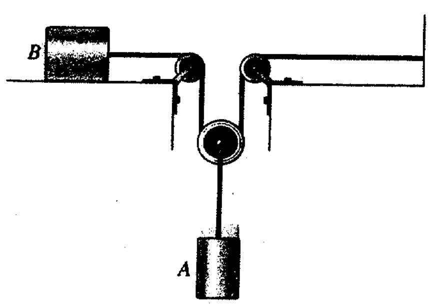
At $x = 0$, $\rho = 0.5$ ft

$+\uparrow \Sigma F_n = m a_n$; $N - 3 = \frac{3}{32.2}[\frac{(16.05)^2}{0.5}]$

$N = 51.0$ lb Ans

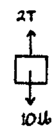


R1-41. At a given instant the 10-lb block A is moving downward with a speed of 6 ft/s. Determine its speed 2 s later. Block B has a weight of 4 lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of the pulleys and cord.



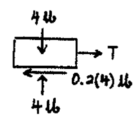
Block A:

$+\downarrow \Sigma F_y = m a_y$; $10 - 2T = \frac{10}{32.2}a_A$



Block B:

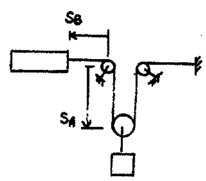
$\overset{+}{\leftarrow} \Sigma F_x = m a_x$; $-T + 0.2(4) = \frac{4}{32.2}a_B$



Kinematics:

$2s_A + s_B = l$

$2a_A = -a_B$



Solving,

$T = 3.38$ lb

$a_A = 10.40$ ft/s^2

$a_B = -20.81$ ft/s^2

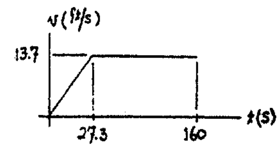
$(+\downarrow)$ $v_A = (v_A)_0 + a_A t$

$v_A = 6 + 10.40(2)$

$v_A = 26.8$ ft/s $\downarrow$ Ans

R1-42. A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2. After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v-t graph for the motion.

$0 \le t \le t'$



$a_c = 0.5$ ft/s^2

$(\overset{+}{\rightarrow})\; v = v_0 + a_c t$

$v' = 0 + 0.5t'$ (1)

$(\overset{+}{\rightarrow})\; s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$x = 0 + 0 + \frac{1}{2}(0.5)(t')^2$ (2)

$t' < t < 160$ s

$(\overset{+}{\rightarrow})\; s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$2000 = x + v'(160 - t') + 0$ (3)

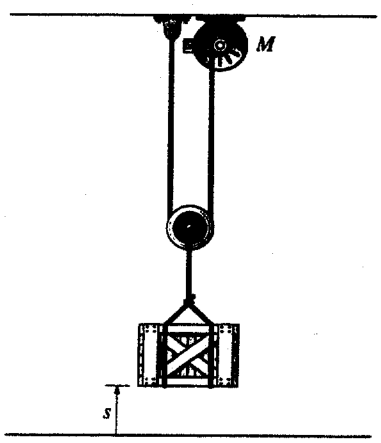
Solving Eqs. (1) – (3) for $t' < 160$ s yields

$t' = 27.3$ s **Ans**

$v' = 13.7$ ft/s

$x = 187$ ft

R1-43. The crate, having a weight of 50 lb, is hoisted by the pulley system and motor M. If the crate starts from rest and, by constant acceleration, attains a speed of 12 ft/s after rising 10 ft, determine the power that must be supplied to the motor at the instant $s = 10$ ft. The motor has an efficiency $\epsilon = 0.74$.

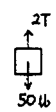


$(+\uparrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$(12)^2 = 0 + 2a_c(10 - 0)$

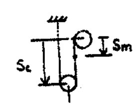
$a_c = 7.20$ ft/s^2

$+\uparrow \Sigma F_y = ma_y;\quad 2T - 50 = \frac{50}{32.2}(7.20)$



$T = 30.6$ lb

$s_C + (s_C - s_M) = l$



$v_M = 2v_C$

$v_M = 2(12) = 24$ ft/s

$P_o = \mathbf{T}\cdot\mathbf{v} = 30.6(24) = 734.2$ lb·ft/s

$P_i = \frac{734.2}{0.74} = 992.1$ lb·ft/s $= 1.80$ hp **Ans**

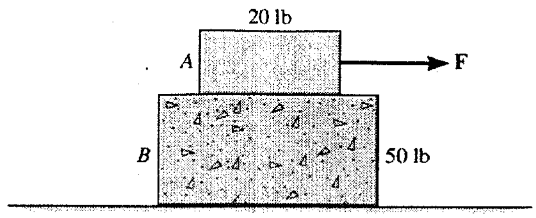
***R1-44.** An automobile is traveling with a *constant speed* along a horizontal circular curve that has a radius $\rho = 750$ ft. If the magnitude of acceleration is $a = 8$ ft/s^2, determine the speed at which the automobile is traveling.

$a = a_n = 8 = \frac{v^2}{\rho}$

$8 = \frac{v^2}{750}$

$v = 77.4$ ft/s **Ans**

R1-45. Block B rests on a smooth surface. If the coefficients of friction between A and B are $\mu_s = 0.4$ and $\mu_k = 0.3$, determine the acceleration of each block if (a) $F = 6$ lb, and (b) $F = 50$ lb.



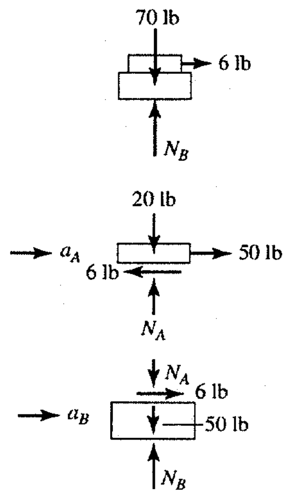
a) The maximum friction force between blocks A and B is

$$F_{max} = 0.4(20) = 8 \text{ lb} > 6 \text{ lb}$$

Thus, both blocks move together.

$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 6 = \frac{70}{32.2}a$$

$$a_B = a_A = a = 2.76 \text{ ft/s}^2 \quad \textbf{Ans}$$



b) In this case 8 lb < F = 50 lb

Block A:

$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 20(0.3) = \frac{20}{32.2}a_A$$

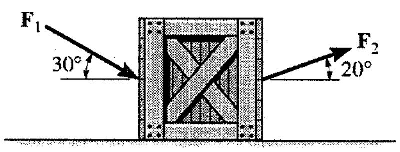
$$a_A = 70.8 \text{ ft/s}^2 \quad \textbf{Ans}$$

Block B:

$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 20(0.3) = \frac{50}{32.2}a_B$$

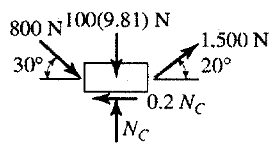
$$a_B = 3.86 \text{ ft/s}^2 \quad \textbf{Ans}$$

R1-46. The 100-kg crate is subjected to the action of two forces, $F_1 = 800$ N and $F_2 = 1.5$ kN, as shown. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



$$+\uparrow \sum F_y = 0; \quad N_C - 800 \sin 30° - 100(9.81) + 1500 \sin 20° = 0$$

$$N_C = 867.97 \text{ N}$$



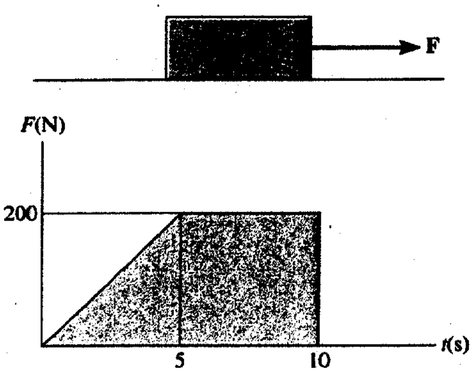
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 800\cos 30°(s) - 0.2(867.97)(s) + 1500\cos 20°(s) = \frac{1}{2}(100)(6)^2$$

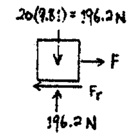
$$s(1928.7) = 1800$$

$$s = 0.933 \text{ m} \quad \textbf{Ans}$$

R1-47. A 20-kg block is originally at rest on a horizontal surface for which the coefficient of static friction is $\mu_s = 0.6$ and the coefficient of kinetic friction is $\mu_k = 0.5$. If a horizontal force F is applied such that it varies with time as shown, determine the speed of the block in 10 s. *Hint:* First determine the time needed to overcome friction and start the block moving.



The crate starts moving when



$$F = F_f = 0.6(196.2) = 117.72 \text{ N}$$

From the graph since

$$F = \frac{200}{5}t, \quad 0 \le t \le 5 \text{ s}$$

The time needed for the crate to start moving is

$$t = \frac{5}{200}(117.72) = 2.943 \text{ s}$$

Hence, the impulse due to F is equal to the area under the curve from 2.943 s $\le t \le$ 10 s

$$\xrightarrow{+} \quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

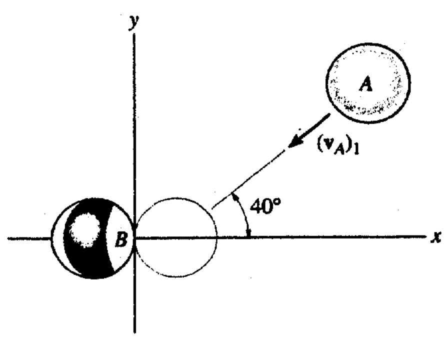
$$0 + \int_{2.943}^{5} \frac{200}{5}t\,dt + \int_{5}^{10} 200\,dt - (0.5)196.2(10-2.943) = 20v_2$$

$$40\left(\frac{1}{2}t^2\right)\Big|_{2.943}^{5} + 200(10-5) - 692.292 = 20v_2$$

$$634.483 = 20v_2$$

$$v_2 = 31.7 \text{ m/s} \qquad \textbf{Ans}$$

***R1-48.** Two smooth billiard balls A and B have an equal mass of $m = 200$ g. If A strikes B with a velocity of $(v_A)_1 = 2$ m/s as shown, determine their final velocities just after collision. Ball B is originally at rest and the coefficient of restitution is $e = 0.75$.



$$(v_A)_{x_1} = -2\cos 40° = -1.532 \text{ m/s}$$

$$(v_A)_{y_1} = -2\sin 40° = -1.285 \text{ m/s}$$

$$(\xrightarrow{+}) \quad m_A(v_A)_{x_1} + m_B(v_B)_{x_1} = m_A(v_A)_{x_2} + m_B(v_B)_{x_2}$$

$$-2(1.532) + 0 = 0.2(v_A)_{x_2} + 0.2(v_B)_{x_2} \qquad (1)$$

$$(\xrightarrow{+}) \quad e = \frac{(v_{rel})_2}{(v_{rel})_1}$$

$$0.75 = \frac{(v_A)_{x_2} - (v_B)_{x_2}}{1.532} \qquad (2)$$

Solving Eqs.(1) and (2)

$$(v_A)_{x_2} = -0.1915 \text{ m/s}$$

$$(v_B)_{x_2} = -1.3405 \text{ m/s}$$

For A :

$$(+\downarrow) \quad m_A(v_A)_{y_1} = m_A(v_A)_{y_2}$$

$$(v_A)_{y_2} = 1.285 \text{ m/s}$$

For B :

$$(+\uparrow) \quad m_B(v_B)_{y_1} = m_B(v_B)_{y_2}$$

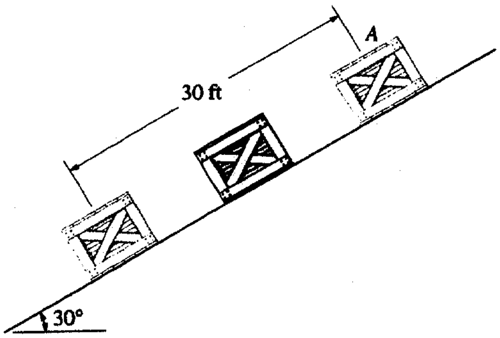
$$(v_B)_{y_2} = 0$$

Hence $(v_B)_2 = (v_B)_{x_2} = 1.34$ m/s $\leftarrow$ **Ans**

$$(v_A)_2 = \sqrt{(-0.1915)^2 + (1.285)^2} = 1.30 \text{ m/s} \qquad \textbf{Ans}$$

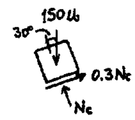
$$(\theta_A)_2 = \tan^{-1}\left(\frac{0.1915}{1.285}\right) = 8.47° \qquad \textbf{Ans}$$

R1-49. If a 150-lb crate is released from rest at A, determine its speed after it slides 30 ft down the plane. The coefficient of kinetic friction between the crate and plane is $\mu_k = 0.3$.



$+\nwarrow \Sigma F_y = 0; \quad N_C - 150\cos 30° = 0$

$N_C = 129.9 \text{ lb}$

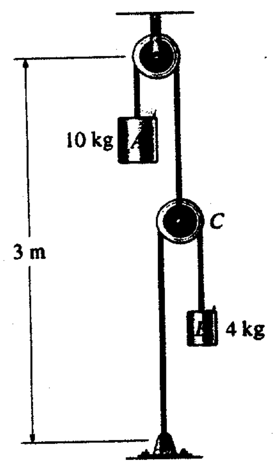


$T_1 + \Sigma U_{1-2} = T_2$

$0 + 150\sin 30°(30) - (0.3)129.9(30) = \frac{1}{2}(\frac{150}{32.2})v_2^2$

$v_2 = 21.5 \text{ ft/s}$ **Ans**

R1-50. Determine the tension developed in the two cords and the acceleration of each block. Neglect the mass of the pulleys and cords. *Hint:* Since the system consists of *two* cords, relate the motion of block A to C, and of block B to C. Then, by elimination, relate the motion of A to B.



Block A :

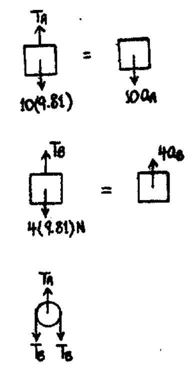
$+\downarrow \Sigma F_y = ma_y; \quad 10(9.81) - T_A = 10a_A$ (1)

Block B :

$+\uparrow \Sigma F_y = ma_y; \quad T_B - 4(9.81) = 4a_B$ (2)

Pulley C :

$+\uparrow \Sigma F_y = 0; \quad T_A - 2T_B = 0$ (3)



Kinematics:

$s_A + s_C = l$

Taking the two time derivatives :

$a_A = -a_C$

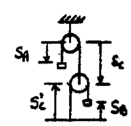
Also,

$s_C' + (s_C' - s_B) = l'$

So that $2a_C' = a_B$

Since $a_C' = -a_C$,

$a_B = 2a_A$ (4)



Solving Eqs.(1) – (4),

$a_A = 0.755 \text{ m/s}^2$ **Ans**

$a_B = 1.51 \text{ m/s}^2$ **Ans**

$T_A = 90.6 \text{ N}$ **Ans**

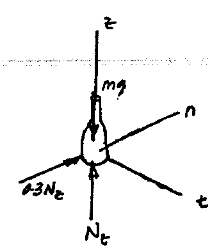
$T_B = 45.3 \text{ N}$ **Ans**

R1-51. The bottle rests at a distance of 3 ft from the center of the horizontal platform. If the coefficient of static friction between the bottle and the platform is $\mu_s = 0.3$, determine the maximum speed that the bottle can attain before slipping. Assume the angular motion of the platform is slowly increasing.

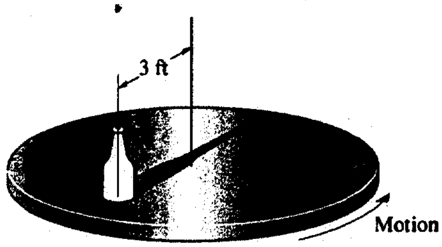
$\Sigma F_z = ma_z; \quad N_z - mg = 0 \quad N_z = mg$

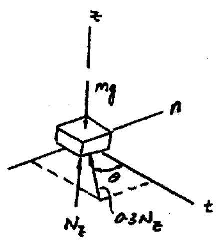
$\Sigma F_n = ma_n; \quad 0.3(mg) = m\left(\frac{v^2}{r}\right)$

$v = \sqrt{0.3gr} = \sqrt{0.3(32.2)(3)} = 5.38 \text{ ft/s}$ **Ans**



***R1-52.** Work Prob. R1–51 assuming that the platform starts rotating from rest so that the speed of the bottle is increased at 2 ft/s^2.





$\Sigma F_z = ma_b$; $N_z - mg = 0$ $N_z = mg$

$\Sigma F_t = ma_t$; $-0.3(mg)\cos\theta = -m(2)$ $\theta = 78.05°$

$\Sigma F_n = ma_n$; $0.3(mg)\sin 78.05° = m\left(\dfrac{v^2}{3}\right)$

$v = 5.32$ ft/s **Ans**