Assignment-V

1 (Important) Forward elimination changes $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \boldsymbol{x} = \boldsymbol{b}$ to a triangular $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \boldsymbol{x} = \boldsymbol{c}$:

$$\begin{aligned} x + y &= \mathbf{5} \\ x + 2y &= \mathbf{7} \end{aligned} \quad \longrightarrow \quad \begin{aligned} x + y &= \mathbf{5} \\ y &= \mathbf{2} \end{aligned} \qquad \begin{bmatrix} 1 & 1 & \mathbf{5} \\ 1 & 2 & \mathbf{7} \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & \mathbf{5} \\ 0 & 1 & \mathbf{2} \end{bmatrix}$$

That step subtracted $\ell_{21} =$ _____ times row 1 from row 2. The reverse step *adds* ℓ_{21} times row 1 to row 2. The matrix for that reverse step is $L =$ _____. Multiply this L times the triangular system $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \boldsymbol{x}_1 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ to get _____ $=$ _____. In letters, L multiplies $U\boldsymbol{x} = \boldsymbol{c}$ to give _____.

11 What are L and D (the diagonal ***pivot matrix***) for this matrix A? What is U in $A = LU$ and what is the new U in $A = LDU$?

Already triangular $\qquad A = \begin{bmatrix} 2 & 4 & 8 \\ 0 & 3 & 9 \\ 0 & 0 & 7 \end{bmatrix}.$

13 (*Recommended*) Compute L and U for the symmetric matrix A:

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}.$$

Find four conditions on a, b, c, d to get $A = LU$ with four pivots.

19 *Tridiagonal matrices* have zero entries except on the main diagonal and the two adjacent diagonals. Factor these into $A = LU$ and $A = LDL^{\mathrm{T}}$:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix}.$$

23 *Easy but important.* If A has pivots $5, 9, 3$ with no row exchanges, what are the pivots for the upper left 2 by 2 submatrix A_2 (without row 3 and column 3)?