

## Multipliers — ALU

- Unsigned number multiplication
  - Combinational array multiplier
  - Sequential circuit binary multiplier.
- Booth method.
  - +ve and -ve multiplier
  - Spud  $\times$

## Fast Multipliers

### Bit pair recoding of multiplier

- Reduce the number of summands to  $n/2$  for  $n$ -bit operands
- Based on Booth recoding of multipliers
  - Groups pair Booth recoded multiplier bits

$$\begin{array}{rcl} M = 5 & 0 & 0 & 0 & 1 & 0 & 1 \\ Q = -11 & 1 & 1 & 0 & 1 & 0 & 1 \\ \overline{Q} & 0 & -1 & +1 & -1 & +1 & -1 \end{array}$$

$$\underline{P = -55}$$

$$M'' = 111011$$

$$0 \ 0 \ 0 \ 1 \ 0 \ 1$$

$$0 \ -1 \ +1 \ -1 \ +1 \ -1$$

$$\begin{array}{cccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & \end{array}$$

$$\begin{array}{cccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & \end{array}$$

$$\begin{array}{cccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & \end{array}$$

$$\begin{array}{cccccccccccc} 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & \end{array}$$

$$\begin{array}{cccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \end{array}$$

$$\begin{array}{cccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{array} = \underline{\underline{-55}}$$

$$\begin{array}{cccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \end{array} \rightarrow -M$$

$$\begin{array}{cccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{array} \rightarrow 2M$$

$$\begin{array}{cccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{array} = 5$$

Left shift

$$\begin{array}{cccc} 0 & 0 & 1 & 1 \\ \swarrow & \swarrow & \swarrow & \swarrow \\ 0 & 1 & 1 & 0 \\ \swarrow & \swarrow & \swarrow & \swarrow \\ 1 & 1 & 0 & 0 \end{array} = 3$$

$$= 6 = 2 \times 3$$

$$= 12 = 2 \times 6$$

Shifting left  $\rightarrow$  Multiplying by 2

Booth recoding bit pair:  $(+1 \ -1)M$

$$= 2M - M = \underline{\underline{M}}$$

Booth recoded  $\rightarrow (+1 \ -1)M$

Bit pair recoding  $\rightarrow (0, +1) \times M$

Booth recoded bit pair

Bit pair-recoding

$$(+1 \ 0)M \longrightarrow (0, +2)M$$

$$(-1, +1)M \longrightarrow (0, -1)M$$

$$(0, 0)M \longrightarrow (0, 0)M$$

$$(0, 1)M \longrightarrow (0, 1)M$$

$$(+1 \ -1)M \longrightarrow (0, +1)M$$

$$(-1, 0)M \longrightarrow (0, -2)M$$

$$(0 \ -1)M \longrightarrow (0, -1)M$$

-11  $\rightarrow$

$$\begin{array}{cccccc} 1 & 1 & 0 & 1 & 0 & 1 \\ \hline \underbrace{0 \ -1} & \underbrace{+1 \ -1} & \underbrace{+1 \ -1} & & & \\ 0 \ -1 & 0 \ +1 & 0 \ +1 & & & \end{array}$$

Bit pair  
recoding

$$\begin{array}{cccccc}
 0 & 0 & 0 & 1 & 0 & 1 & M \\
 0 & -1 & 0 & +1 & 0 & +1 & \overline{Q}
 \end{array}$$

$$\begin{array}{cccccccccccc}
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & & \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & \\
 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & & & & \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & 
 \end{array}$$

Number of summands = 3

Ex 2:

$$M = +13$$

$$0 \ 1 \ 1 \ 0 \ 1$$

$$Q = -6$$

$$1 \ 1 \ 0 \ 1 \ 0$$

$$\overline{Q}$$

$$0 \ -1 \ +1 \ -1 \ 0$$

$$0 \ 1 \ 1 \ 0 \ 1 \ M$$

$$0 \ -1 \ +1 \ -1 \ 0 \ \overline{Q}$$

$$\begin{array}{cccccccccccc}
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & & & \rightarrow ① \\
 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & & & & \rightarrow ② \\
 1 & 1 & 1 & 0 & 0 & 1 & 1 & & & & & \rightarrow ③ \\
 0 & 0 & 0 & 0 & 0 & 0 & & & & & & \\
 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & & = -78
 \end{array}$$

