



# National Institute of Technology Goa

Programme Name: B.Tech

Online Mid Semester Examinations, October 2022

Course Name: **Mathematics-III**

Date: 17/10/2022

Duration: 90 Minutes

Course Code: **MA200**

Time: 02:00PM - 03:30 AM

Max. Marks: 50

1. Answer All Questions.

2. No marks will be given if the explanation of your answer is missing.

3. The question paper consists of **two** pages.

1. Determine 'p' such that the function  $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \left( \frac{px}{y} \right)$  is analytic function. [4M]

2. Find whether the function  $v = x^2 - y^2 + \frac{x}{x^2 + y^2}$  can be imaginary part of an analytic function or not?. If so, find the analytic function and its real part. [6M]

If  $\tan(x + iy) = A + iB$  show that  $A^2 + B^2 + 2A \cot 2x = 1$ . [4M]

4. Determine all values and the principal value of (a)  $\ln(-4)$ , (b)  $\ln(3i)$ , (c)  $\ln(\sqrt{3} - i)$ . [6M]

5. Evaluate  $\int_C (z^2 + 3z) dz$  along:

(a) Circle  $|z| = 2$  from  $(2, 0)$  to  $(0, 2)$  in counterclockwise direction. [8M]

(b) The straight line from  $(2, 0)$  to  $(0, 2)$ .

(c) The straight lines from  $(2, 0)$  to  $(0, 2)$  and then from  $(2, 2)$  to  $(0, 2)$ .

6. If a function  $F(z)$  is defined to be

$$F(\alpha) = \oint_C \frac{5z^2 - 4z + 3}{z - \alpha} dz,$$

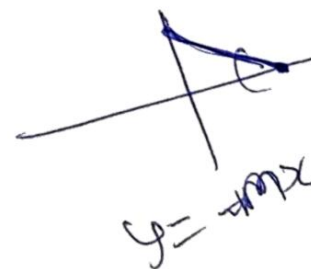
where 'C' is the ellipse  $16x^2 + 9y^2 = 144$ , then evaluate:

(a)  $F(2)$

(b)  $F(4)$

(c)  $F(-3i)$

(d)  $F'(i)$  and



(e)  $F''(-2i)$

~~7.~~ Discuss the type of singularity at singular points of  $\sin z \left( \frac{e^z+1}{e^z-1} \right)$ . [6M]

8. Evaluate  $\int_{|z|=10} \frac{e^z}{e^z-2i} dz$  using Argument principle. [4M]

9. Evaluate  $\oint_{|z|=1} e^{-\frac{1}{z}} \sin\left(\frac{1}{z}\right) dz$  using Residue theorem. [4M]

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3-5  
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912