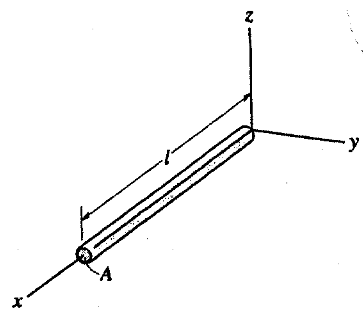
17-1. Determine the moment of inertia I_y for the slender rod. The rod's density ρ and cross-sectional area A are constant. Express the result in terms of the rod's total mass m.



$$I_y = \int_M x^2\, dm$$

$$= \int_0^l x^2\,(\rho A\, dx)$$

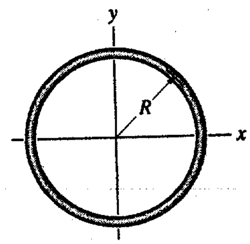
$$= \frac{1}{3}\rho A\, l^3$$

$$m = \rho A\, l$$

Thus,

$$I_y = \frac{1}{3} m\, l^2 \qquad \textbf{Ans}$$

17-2. Determine the moment of inertia of the thin ring about the z axis. The ring has a mass m.



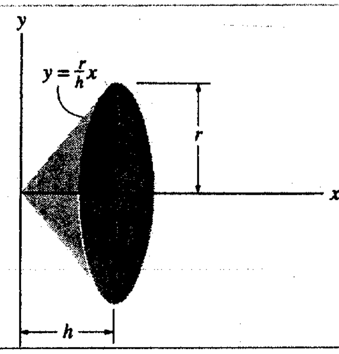
$$I_z = \int_0^{2\pi} \rho A\,(R\, d\theta)\, R^2 = 2\pi\rho A R^3$$

$$m = \int_0^{2\pi} \rho A\, R\, d\theta = 2\pi\rho A R$$

Thus,

$$I_z = m R^2 \qquad \textbf{Ans}$$

17-3. The right circular cone is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.



$$dm = \rho\, dV = \rho(\pi y^2\, dx)$$

$$m = \int_0^h \rho(\pi)\left(\frac{r^2}{h^2}\right)x^2\, dx = \rho\pi\left(\frac{r^2}{h^2}\right)\left(\frac{1}{3}\right)h^3 = \frac{1}{3}\rho\pi r^2 h$$

$$dI_x = \frac{1}{2}y^2\, dm$$

$$= \frac{1}{2}y^2\,(\rho\pi y^2\, dx)$$

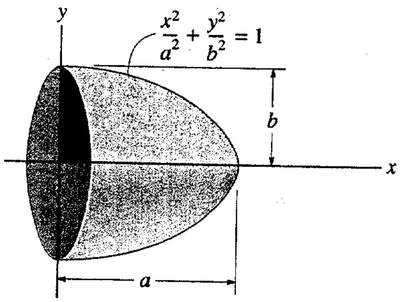
$$= \frac{1}{2}\rho(\pi)\left(\frac{r^4}{h^4}\right)x^4\, dx$$

$$I_x = \int_0^h \frac{1}{2}\rho(\pi)\left(\frac{r^4}{h^4}\right)x^4\, dx = \frac{1}{10}\rho\pi r^4 h$$

Thus

$$I_x = \frac{3}{10} m r^2 \qquad \textbf{Ans}$$

*17-4. A semiellipsoid is formed by rotating the shaded area about the x axis. Determine the moment of inertia of this solid with respect to the x axis and express the result in terms of the mass m of the solid. The material has a constant density ρ.



$$dI_x = \frac{y^2 dm}{2}$$

$$m = \int_V \rho dV$$

$$= \int_0^a \rho\pi b^2\left(1-\frac{x^2}{a^2}\right)dx$$

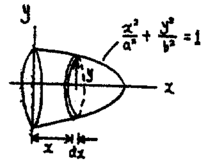
$$= \frac{2}{3}\rho\pi a b^2$$

$$I_x = \frac{1}{2}\rho\pi\int_0^a b^4\left(1-\frac{x^2}{a^2}\right)^2 dx$$

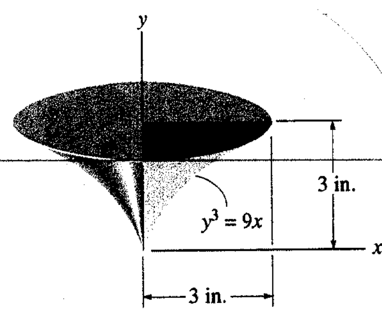
$$= \frac{4}{15}\rho\pi a b^4$$

Thus,

$$I_x = \frac{2}{5}mb^2 \qquad \text{Ans}$$



17-5. The solid is formed by revolving the shaded area around the y axis. Determine the radius of gyration k_y. The specific weight of the material is $\gamma = 380\ \text{lb/ft}^3$.



The moment of inertia of the solid : The mass of the disk element $dm = \rho\pi x^2 dy = \frac{1}{81}\rho\pi y^6 dy$.

$$dI_y = \frac{1}{2}dm x^2$$

$$= \frac{1}{2}(\rho\pi x^2 dy)x^2$$

$$= \frac{1}{2}\rho\pi x^4 dy = \frac{1}{2(9^4)}\rho\pi y^{12} dy$$

$$I_y = \int dI_y = \frac{1}{2(9^4)}\rho\pi\int_0^3 y^{12} dy$$

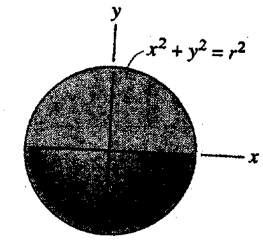
$$= 29.632\rho$$

The mass of the solid :

$$m = \int_m dm = \frac{1}{81}\rho\pi\int_0^3 y^6 dy = 12.117\rho$$

$$k_y = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{29.632\rho}{12.117\rho}} = 1.56\ \text{in.} \qquad \text{Ans}$$

17-6. The sphere is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the sphere. The material has a constant density ρ.



$$dI_x = \frac{y^2\,dm}{2}$$

$$dm = \rho\,dV = \rho\,(\pi y^2\,dx) = \rho\,\pi\,(r^2 - x^2)\,dx$$

$$dI_x = \frac{1}{2}\rho\,\pi\,(r^2 - x^2)^2\,dx$$

$$I_x = \int_{-r}^{r} \frac{1}{2}\rho\,\pi\,(r^2 - x^2)^2\,dx$$

$$= \frac{8}{15}\pi\,\rho\,r^5$$

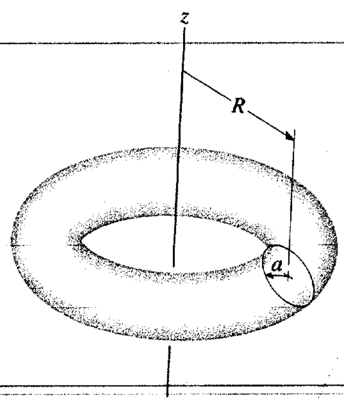
$$m = \int_{-r}^{r} \rho\,\pi\,(r^2 - x^2)\,dx$$

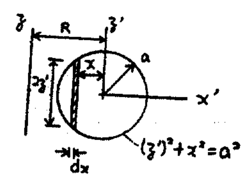
$$= \frac{4}{3}\rho\,\pi\,r^3$$

Thus,

$$I_x = \frac{2}{5} m\,r^2 \qquad \textbf{Ans}$$

17-7. Determine the moment of inertia I_z of the torus. The mass of the torus is m and the density ρ is constant. *Suggestion:* Use a shell element.





$$dm = 2\pi(R - x)(2z'\rho\,dx)$$

$$dI_z = (R - x)^2 dm$$

$$= 4\pi\rho[(R^3 - 3R^2x + 3Rx^2 - x^3)\sqrt{a^2 - x^2}\,dx]$$

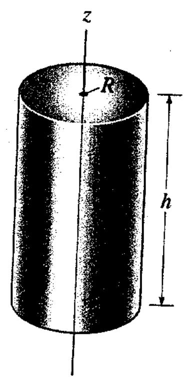
$$I_z = 4\pi\rho[R^3\int_{-a}^{a}\sqrt{a^2 - x^2}\,dx - 3R^2\int_{-a}^{a}x^3\sqrt{a^2 - x^2}\,dx + 3R\int_{-a}^{a}\sqrt{a^2 - x^2}\,dx - \int_{-a}^{a}x^3\sqrt{a^2 - x^2}$$

$$= 2\pi^2\rho R a^2(R^2 + \frac{3}{4}a^2)$$

Since $m = \rho V = 2\pi R\rho\pi a^2$

$$I_z = m(R^2 + \frac{3}{4}a^2) \qquad \textbf{Ans}$$

***17-8.** The solid cylinder has an outer radius R, height h, and is made from a material having a density that varies from its center as $\rho = k + ar^2$, where k and a are constants. Determine the mass of the cylinder and its moment of inertia about the z axis.



Consider a shell element of radius r and mass

$$dm = \rho\, dV = \rho(2\pi r\, dr)h$$

$$m = \int_0^R (k + ar^2)(2\pi r\, dr)h$$

$$m = 2\pi h\left(\frac{kR^2}{2} + \frac{aR^4}{4}\right)$$

$$m = \pi h R^2\left(k + \frac{aR^2}{2}\right) \qquad \textbf{Ans}$$

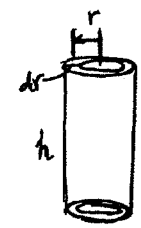
$$dI = r^2 dm = r^2(\rho)(2\pi r\, dr)h$$

$$I_z = \int_0^R r^2(k + ar^2)(2\pi r\, dr)\, h$$

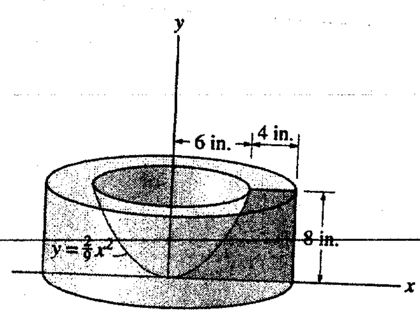
$$I_z = 2\pi h \int_0^R (k r^3 + a r^5)\, dr$$

$$I_z = 2\pi h\left[\frac{kR^4}{4} + \frac{aR^6}{6}\right]$$

$$I_z = \frac{\pi h R^4}{2}\left[k + \frac{2aR^2}{3}\right] \qquad \textbf{Ans}$$



17-9. The concrete shape is formed by rotating the shaded area about the y axis. Determine the moment of inertia I_y. The specific weight of concrete is $\gamma = 150\ \text{lb/ft}^3$.



$$dI_y = \frac{1}{2}(dm)(10)^2 - \frac{1}{2}(dm)x^2$$

$$= \frac{1}{2}\{\pi\rho\,(10)^2\,dy\}(10)^2 - \frac{1}{2}\pi\rho\, x^2\, dy\, x^2$$

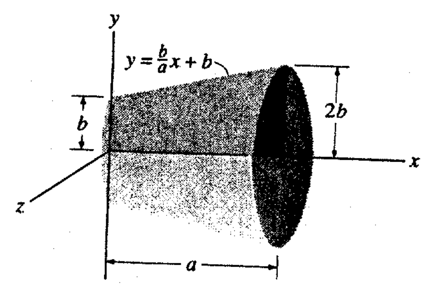
$$I_y = \frac{1}{2}\pi\rho\left[\int_0^8 (10)^4\, dy - \int_0^8 \left(\frac{9}{2}\right)^2 y^2\, dy\right]$$

$$= \frac{\frac{1}{2}\pi(150)}{32.2(12)^3}\left[(10)^4(8) - \left(\frac{9}{2}\right)^2\left(\frac{1}{3}\right)(8)^3\right]$$

$$= 324.1\ \text{slug}\cdot\text{in}^2$$

$$I_y = 2.25\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

17-10. The frustum is formed by rotating the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the frustum. The frustum has a constant density.



$$dm = \rho dV = \rho\pi y^2 dx = \rho\pi\left(\tfrac{b^2}{a^2}x^2 + \tfrac{2b^2}{a}x + b^2\right)dx$$

$$dI_x = \tfrac{1}{2}dmy^2 = \tfrac{1}{2}\rho\pi y^4 dx$$

$$dI_x = \tfrac{1}{2}\rho\pi\left(\tfrac{b^4}{a^4}x^4 + \tfrac{4b^4}{a^3}x^3 + \tfrac{6b^4}{a^2}x^2 + \tfrac{4b^4}{a}x + b^4\right)dx$$

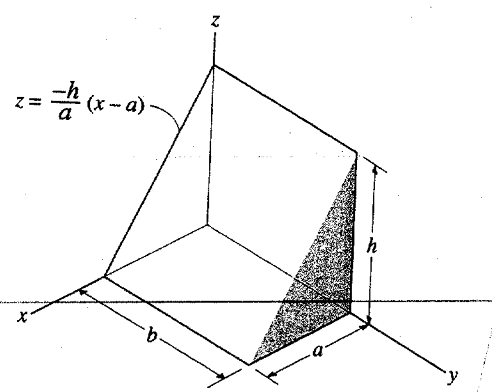
$$I_x = \int dI_x = \tfrac{1}{2}\rho\pi\int_0^a\left(\tfrac{b^4}{a^4}x^4 + \tfrac{4b^4}{a^3}x^3 + \tfrac{6b^4}{a^2}x^2 + \tfrac{4b^4}{a}x + b^4\right)dx$$

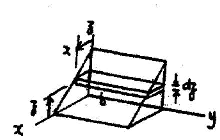
$$= \tfrac{31}{10}\rho\pi a b^4$$

$$m = \int_m dm = \rho\pi\int_0^a\left(\tfrac{b^2}{a^2}x^2 + \tfrac{2b^2}{a}x + b^2\right)dx = \tfrac{7}{3}\rho\pi a b^2$$

$I_x = \frac{93}{70}mb^2$ **Ans**

17-11. Determine the moment of inertia of the homogeneous triangular prism with respect to the y axis. Express the result in terms of the mass m of the prism. *Hint:* For integration, use thin plate elements parallel to the x–y plane and having a thickness dz.





$$dV = bx\,dz = b(a)(1 - \frac{z}{h})dz$$

$$dI_y = d\bar{I}_y + (dm)[(\frac{x}{2})^2 + z^2]$$

$$= \frac{1}{12}dm(x^2) + dm(\frac{x^2}{4}) + dmz^2$$

$$= dm(\frac{x^2}{3} + z^2)$$

$$= [b(a)(1-\frac{z}{h})dz](\rho)[\frac{a^2}{3}(1-\frac{z}{h})^2 + z^2]$$

$$I_y = ab\rho\int_0^h[\frac{a^2}{3}(\frac{h-z}{h})^3 + z^2(1-\frac{z}{h})]dz$$

$$= ab\rho[\frac{a^2}{3h^3}(h^4 - \frac{3}{2}h^4 + h^4 - \frac{1}{4}h^4) + \frac{1}{h}(\frac{1}{3}h^4 - \frac{1}{4}h^4)]$$

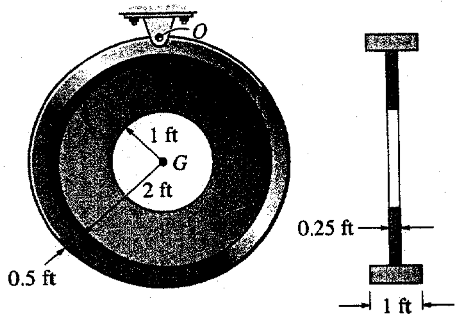
$$= \frac{1}{12}abh\rho(a^2 + h^2)$$

$$m = \rho V = \frac{1}{2}abh\rho$$

Thus,

$I_y = \frac{m}{6}(a^2+h^2)$ **Ans**

***17-12.** Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through point O. The material has a specific weight of $\gamma = 90\ \text{lb/ft}^3$.



$$I_G = \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2.5)^2(1)\right](2.5)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(1)\right](2)^2$$

$$+\frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(0.25)\right](2)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(1)^2(0.25)\right](1)^2$$

$$= 117.72\ \text{slug}\cdot\text{ft}^2$$

$$I_O = I_G + md^2$$

$$m = \left(\frac{90}{32.2}\right)\pi(2^2 - 1^2)(0.25) + \left(\frac{90}{32.2}\right)\pi(2.5^2 - 2^2)(1) = 26.343\ \text{slug}$$

$$I_O = 117.72 + 26.343(2.5)^2 = 282\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

17-13. The assembly consists of a disk having a mass of 6 kg and slender rods AB and DC which have a mass of 2 kg/m. Determine the length L of DC so that the center of mass is at the bearing O. What is the moment of inertia of the assembly about an axis perpendicular to the page and passing through O?

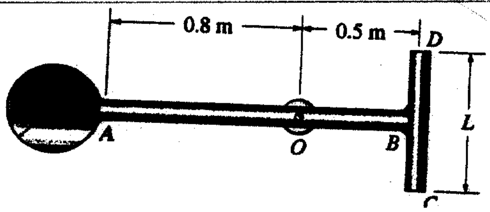
Measured from the right side,

$$\bar{y} = \frac{6(1.5) + 2(1.3)(0.65)}{6 + 1.3(2) + L(2)} = 0.5$$

$$L = 6.39\ \text{m} \qquad \textbf{Ans}$$

$$I_O = \frac{1}{2}(6)(0.2)^2 + 6(1)^2 + \frac{1}{12}(2)(1.3)(1.3)^2 + 2(1.3)(0.15)^2 + \frac{1}{12}(2)(6.39)(6.39)^2 + 2(6.39)(0.5)^2$$

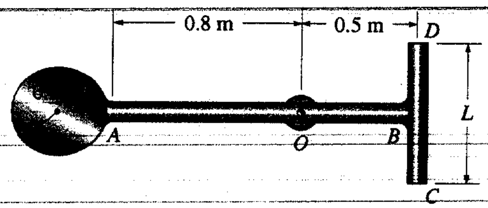
$$I_O = 53.2\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$



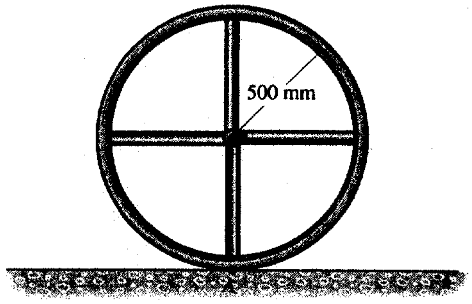
17-14. The assembly consists of a disk having a mass of 6 kg and slender rods AB and DC which have a mass of 2 kg/m. If $L = 0.75$ m, determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through O.

$$I_O = \frac{1}{2}(6)(0.2)^2 + 6(1)^2 + \frac{1}{12}(2)(1.3)(1.3)^2 + 2(1.3)(0.15)^2 + \frac{1}{12}(2)(0.75)(0.75)^2$$
$$+ 2(0.75)(0.5)^2$$

$$I_O = 6.99\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$



17-15. The wheel consists of a thin ring having a mass of 10 kg and four spokes made from slender rods and each having a mass of 2 kg. Determine the wheel's moment of inertia about an axis perpendicular to the page and passing through point A.

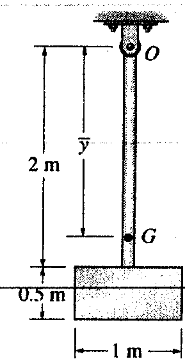


$$I_A = I_O + md^2$$

$$= \left[2\left[\frac{1}{12}(4)(1)^2\right] + 10(0.5)^2\right] + 18(0.5)^2$$

$$= 7.67 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

***17-16.** The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass G of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



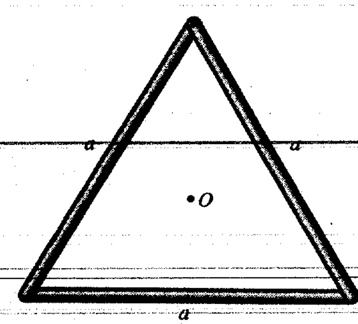
$$\bar{y} = \frac{\Sigma \tilde{y}m}{\Sigma m} = \frac{1(3) + 2.25(5)}{3+5} = 1.781 \text{ m} = 1.78 \text{ m} \qquad \textbf{Ans}$$

$$I_G = \Sigma \bar{I}_{G'} + md^2$$

$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

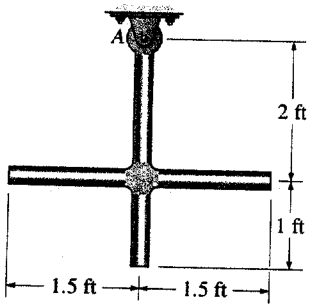
$$= 4.45 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

17-17. Each of the three rods has a mass m. Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through the center point O.



$$I_O = 3\left[\frac{1}{12}ma^2 + m\left(\frac{a\sin 60^\circ}{3}\right)^2\right] = \frac{1}{2}ma^2 \qquad \textbf{Ans}$$

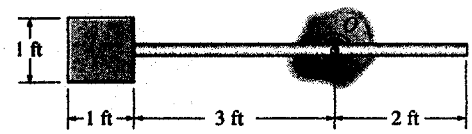
17-18. The slender rods have a weight of 3 lb/ft. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through the pin at A.



$$I_A = [\frac{1}{3}(\frac{3(3)}{32.2})(3)^2] + [\frac{1}{12}(\frac{3(3)}{32.2})(3)^2 + (\frac{3(3)}{32.2})(2)^2]$$

$$= 2.17 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

17-19. The pendulum consists of a plate having a weight of 12 lb and a slender rod having a weight of 4 lb. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.



$$I_O = \Sigma I_G + md^2$$

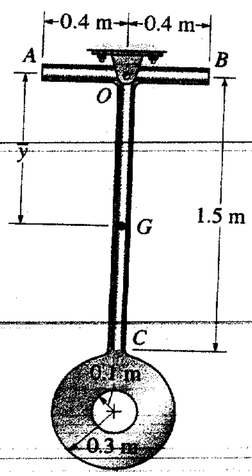
$$= \frac{1}{12}\left(\frac{4}{32.2}\right)(5)^2 + \left(\frac{4}{32.2}\right)(0.5)^2 + \frac{1}{12}\left(\frac{12}{32.2}\right)(1^2+1^2) + \left(\frac{12}{32.2}\right)(3.5)^2$$

$$= 4.917 \text{ slug}\cdot\text{ft}^2$$

$$m = \left(\frac{4}{32.2}\right) + \left(\frac{12}{32.2}\right) = 0.4969 \text{ slug}$$

$$k_O = \sqrt{\frac{I_O}{m}} = \sqrt{\frac{4.917}{0.4969}} = 3.15 \text{ ft} \qquad \textbf{Ans}$$

***17-20.** The pendulum consists of two slender rods AB and OC which have a mass of 3 kg/m. The thin plate has a mass of 12 kg/m^2. Determine the location $\bar{y}$ of the center of mass G of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



$$\bar{y} = \frac{0 + 0.75(3)(1.5) + \pi(0.3)^2(12)(1.8) - \pi(0.1)^2(12)(1.8)}{3(0.8) + 3(1.5) + \pi(0.3)^2(12) - \pi(0.1)^2(12)}$$

$$= 0.8878 = 0.888 \text{ m} \qquad \textbf{Ans}$$

$$I_O = \frac{1}{12}[3(0.8)](0.8)^2 + \frac{1}{3}[3(1.5)](1.5)^2 + \frac{1}{2}[12(\pi)(0.3)^2](0.3)^2$$

$$+ [12(\pi)(0.3)^2](1.8)^2 - \frac{1}{2}[12(\pi)(0.1)^2](0.1)^2 - [12(\pi)(0.1)^2](1.8)^2$$

$$= 13.43 \text{ kg}\cdot\text{m}^2$$

$$I_O = I_G + (9.916)(0.8878)^2$$

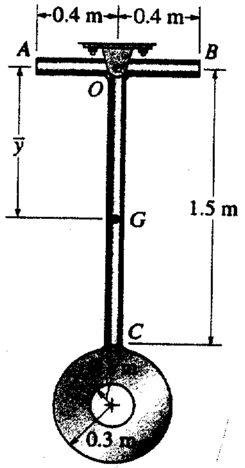
$$I_G = 5.61 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

Also:

$$I_G = \frac{1}{12}[3(0.8)](0.8)^2 + 3(0.8)(0.8878)^2 + \frac{1}{12}[3(1.5)](1.5)^2 + 3(1.5)(0.8878-0.75)^2$$
$$+ \frac{1}{2}[12(\pi)(0.3)^2](0.3)^2 + [12(\pi)(0.3)^2](1.8 - 0.8878)^2 - \frac{1}{2}[12(\pi)(0.1)^2](0.1)^2$$
$$- [12(\pi)(0.1)^2](1.8 - 0.8878)^2$$

$$= 5.61 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

17-21. The pendulum consists of two slender rods AB and OC which have a mass of 3 kg/m. The thin plate has a mass of 12 kg/m^2. Determine the moment of inertia of the pendulum about an axis perpendicular to the page and passing through the pin at O.



$$I_O = \frac{1}{12}[3(0.8)](0.8)^2 + \frac{1}{3}[3(1.5)](1.5)^2 + \frac{1}{2}[12(\pi)(0.3)^2](0.3)^2$$

$$+ [12(\pi)(0.3)^2](1.8)^2 - \frac{1}{2}[12(\pi)(0.1)^2](0.1)^2 - [12(\pi)(0.1)^2](1.8)^2$$

$$= 13.43 = 13.4 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

Also, from the solution to Prob. 17–16,

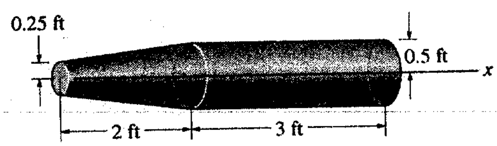
$$m = 3(0.8+1.5) + 12[\pi(0.3)^2 - \pi(0.1)^2] = 9.916 \text{ kg}$$

$$I_O = I_G + m\,d^2$$

$$= 5.61 + 9.916(0.8878)^2$$

$$= 13.4 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

17-22. Determine the moment of inertia of the solid steel assembly about the x axis. Steel has a specific weight of $\gamma_{st} = 490 \text{ lb/ft}^3$.

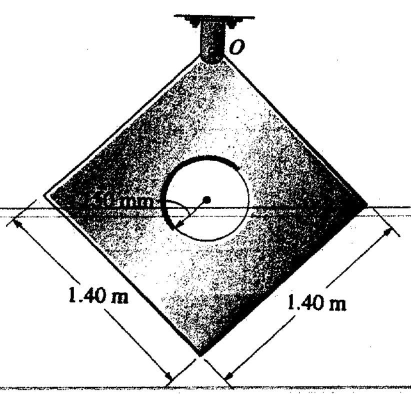


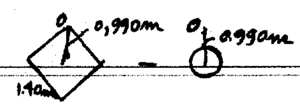
$$I_x = \frac{1}{2}m_1(0.5)^2 + \frac{3}{10}m_2(0.5)^2 - \frac{3}{10}m_3(0.25)^2$$

$$= \left[\frac{1}{2}\pi(0.5)^2(3)(0.5)^2 + \frac{3}{10}\left(\frac{1}{3}\right)\pi(0.5)^2(4)(0.5)^2 - \frac{3}{10}\left(\frac{1}{3}\right)\pi(0.25)^2(2)(0.25)^2\right]\left(\frac{490}{32.2}\right)$$

$$= 5.64 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

17-23. Determine the moment of inertia about an axis perpendicular to the page and passing through the pin at O. The thin plate has a hole in its center. Its thickness is 50 mm, and the material has a density $\rho = 50 \text{ kg/m}^3$.





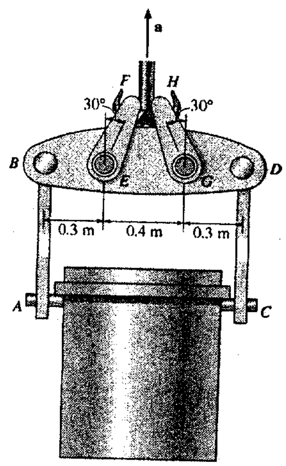
$$m_p = 50(1.40)^2(0.050) = 4.90 \text{ kg}$$

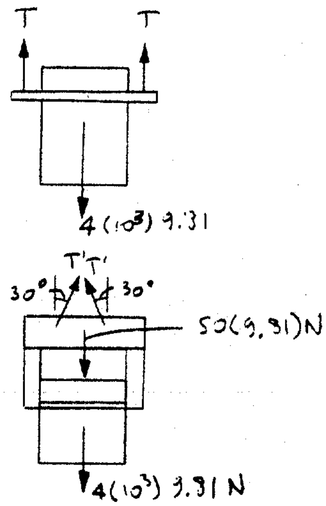
$$m_h = (50)(\pi)(0.15)^2(0.050) = 0.1767 \text{ kg}$$

$$I_O = \frac{1}{12}(4.90)[(1.40)^2 + (1.40)^2] + 4.90(0.990)^2 - [\frac{1}{2}(0.1767)(0.15)^2 + (0.1767)(0.990)^2]$$

$$I_O = 6.23 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

***17-24.** The 4-Mg canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the force in each of the links AB, CD, EF, and GH when the system is lifted with an acceleration of $a = 2\ \text{m/s}^2$ for a short period of time.





Canister :

$+\uparrow \Sigma F_y = m(a_G)_y;\quad 2T - 4(10^3)(9.81) = 4(10^3)(2)$

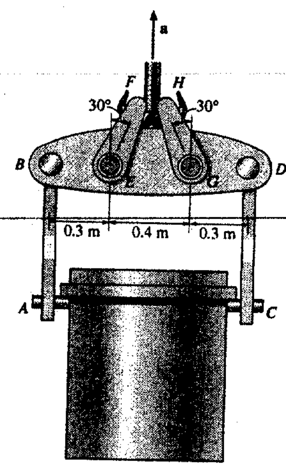
$T_{AB} = T_{CD} = T = 23.6\ \text{kN}$ **Ans**

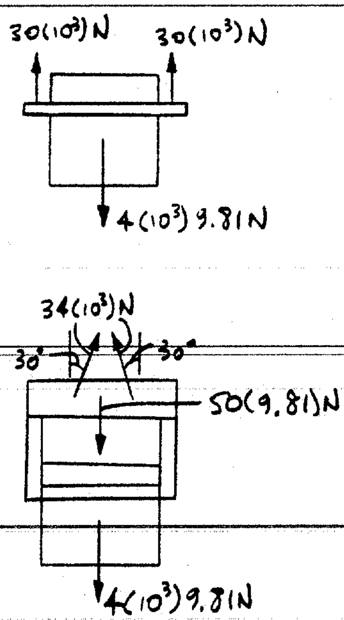
System :

$+\uparrow \Sigma F_y = m(a_G)_y;\quad 2T'\cos 30° - 4050(9.81) = 4050(2)$

$T_{EF} = T_{GH} = T' = 27.6\ \text{kN}$ **Ans**

17-25. The 4-Mg canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the largest vertical acceleration **a** of the system so that each of the links AB and CD are not subjected to a force greater than 30 kN and links EF and GH are not subjected to a force greater than 34 kN.





Canister :

$+\uparrow \Sigma F_y = m(a_G)_y;\quad 2(30)(10^3) - 4(10^3)(9.81) = 4(10^3)a$

$a = 5.19\ \text{m/s}^2$

System :

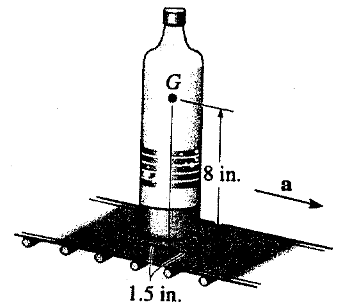
$+\uparrow \Sigma F_y = m(a_G)_y;\quad 2[34(10^3)\cos 30°] - 4050(9.81) = 4050a$

$a = 4.73\ \text{m/s}^2$

Thus,

$a_{max} = 4.73\ \text{m/s}^2$ **Ans**

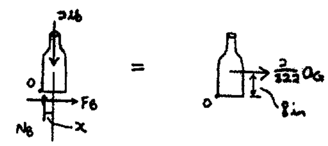
17-26. The 2-lb bottle rests on the check-out conveyor at a grocery store. If the coefficient of static friction is $\mu_s = 0.2$, determine the largest acceleration the conveyor can have without causing the bottle to slip or tip. The center of gravity is at G.



$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad F_B = \frac{2}{32.2}a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_B - 2 = 0$

$\circlearrowleft + \Sigma M_O = \Sigma(M_K)_O; \quad 2x = \frac{2}{32.2}a_G(8)$



Assume bottle is about to slip.

$N_B = 2$ lb, $\quad F_B = 0.2(2) = 0.4$ lb

$a_G = 6.44$ ft/s², $\quad x = 1.6$ in. > 1.5 in.

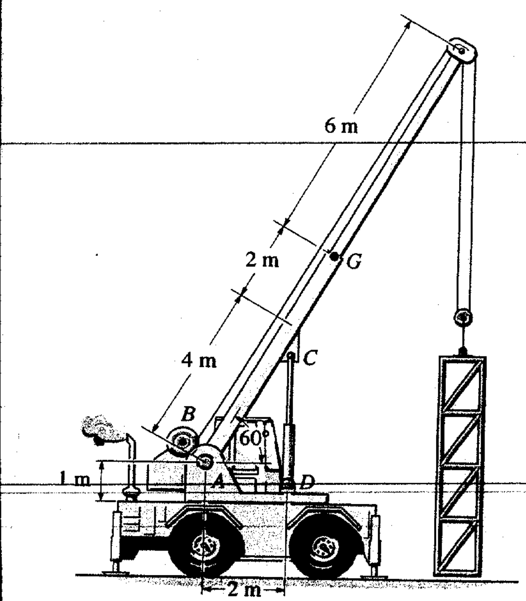
Bottle will tip before slipping.

Set $x = 1.5$ in.

$N_B = 2$ lb, $\quad a_G = 6.04$ ft/s² **Ans**

$F_B = 0.375$ lb < 0.4 lb **(O.K!)**

17-27. The assembly has a mass of 8 Mg and is hoisted using the boom and pulley system. If the winch at B draws in the cable with an acceleration of 2 m/s², determine the compressive force in the hydraulic cylinder needed to support the boom. The boom has a mass of 2 Mg and mass center at G.

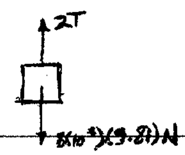


$s_B + 2s_L = l$

$a_B = -2a_L$

$2 = -2a_L$

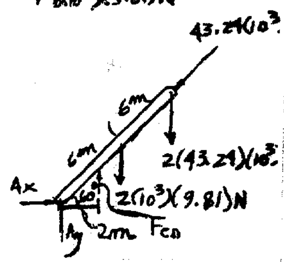
$a_L = -1$ m/s²



Assembly:

$+\uparrow \Sigma F_y = m\,a_y; \quad 2T - 8(10^3)(9.81) = 8(10^3)(1)$

$T = 43.24$ kN

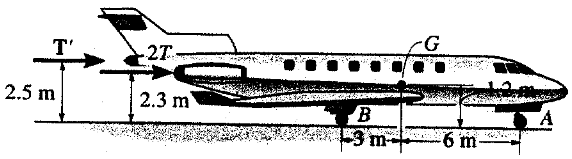


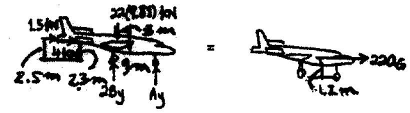
Boom:

$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(2) - 2(10^3)(9.81)(6\cos 60°) - 2(43.24)(10^3)(12\cos 60°) = 0$

$F_{CD} = 289$ kN **Ans**

***17-28.** The jet aircraft has a total mass of 22 Mg and a center of mass at G. Initially at take-off the engines provide a thrust $2T = 4$ kN and $T' = 1.5$ kN. Determine the acceleration of the plane and the normal reactions on the nose wheel and each of the *two* wing wheels located at B. Neglect the mass of the wheels and, due to low velocity, neglect any lift caused by the wings.





$\xrightarrow{+} \Sigma F_x = ma_x;$ $\quad 1.5 + 4 = 22a_G$

$+\uparrow \Sigma F_y = 0;$ $\quad 2B_y + A_y - 22(9.81) = 0$

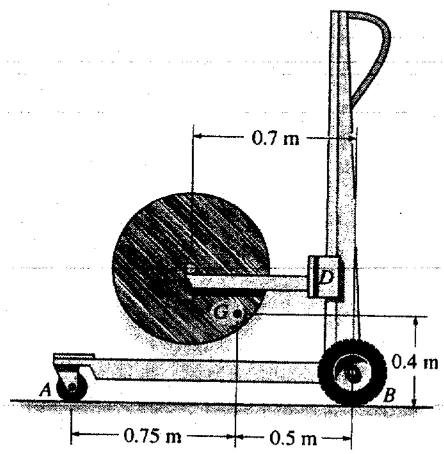
$\curvearrowleft +\Sigma M_B = \Sigma(M_K)_B;$ $\quad -4(2.3) - 1.5(2.5) - 22(9.81)(3) + A_y(9) = -22a_G(1.2)$

$A_y = 72.6$ kN **Ans**

$B_y = 71.6$ kN **Ans**

$a_G = 0.250$ m/s^2 **Ans**

17-29. The lift truck has a mass of 70 kg and mass center at G. If it lifts the 120-kg spool with an acceleration of 3 m/s^2, determine the reactions of each of the four wheels on the ground. The loading is symmetric. Neglect the mass of the movable arm CD.

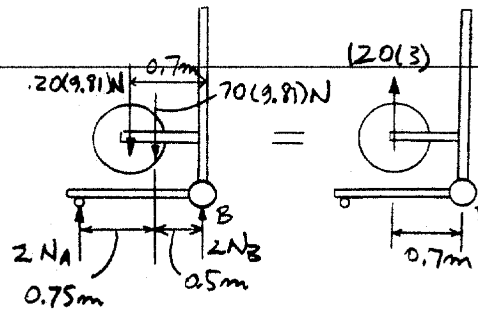


$\curvearrowleft +\Sigma M_B = \Sigma(M_k)_B;$ $\quad 70(9.81)(0.5) + 120(9.81)(0.7) - 2N_A(1.25) = -120(3)(0.7)$

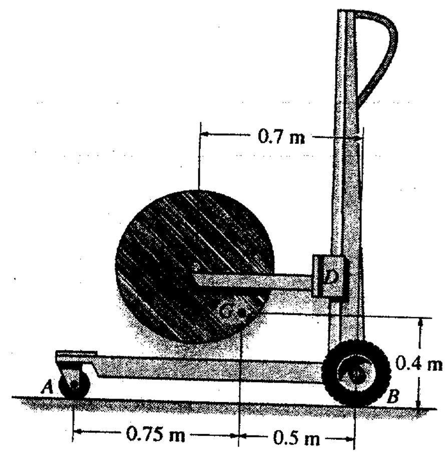
$N_A = 567.76$ N $= 568$ N **Ans**

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad 2(567.76) + 2N_B - 120(9.81) - 70(9.81) = 120(3)$

$N_B = 544$ N **Ans**



17-30. The lift truck has a mass of 70 kg and mass center at G. Determine the largest upward acceleration of the 120-kg spool so that no reaction of the wheels on the ground exceeds 600 N.



Assume $N_A = 600$ N,

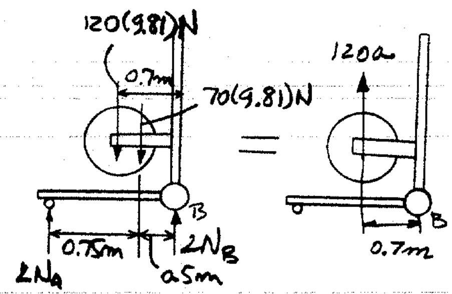
$\circlearrowleft + \Sigma M_B = \Sigma (M_k)_B$; $\quad 70(9.81)(0.5) + 120(9.81)(0.7) - 2(600)(1.25) = -120a(0.7)$

$a = 3.960 \text{ m/s}^2$

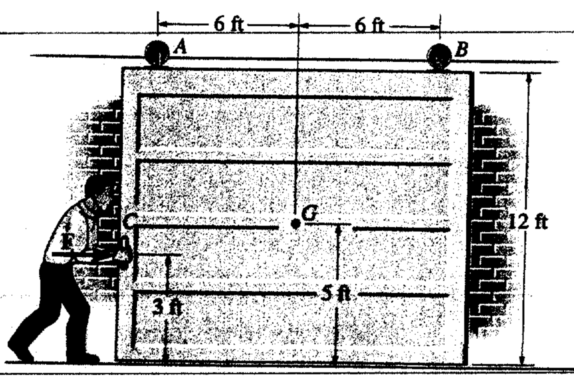
$+\uparrow \Sigma F_y = m(a_G)_y$; $\quad 2(600) + 2N_B - 120(9.81) - 70(9.81) = 120(3.960)$

$N_B = 570 \text{ N} < 600\text{N}$ OK

Thus $\quad a = 3.96 \text{ m/s}^2$ **Ans**

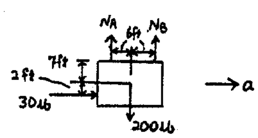


17-31. The door has a weight of 200 lb and a center of gravity at G. Determine how far the door moves in 2 s, starting from rest, if a man pushes on it at C with a horizontal force $F = 30$ lb. Also, find the vertical reactions at the rollers A and B.



$\xrightarrow{+} \Sigma F_x = m(a_G)_x$; $\quad 30 = (\frac{200}{32.2})a_G$

$a_G = 4.83 \text{ ft/s}^2$



$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A$; $\quad N_B(12) - 200(6) + 30(9) = (\frac{200}{32.2})(4.83)(7)$

$N_B = 95.0 \text{ lb}$ **Ans**

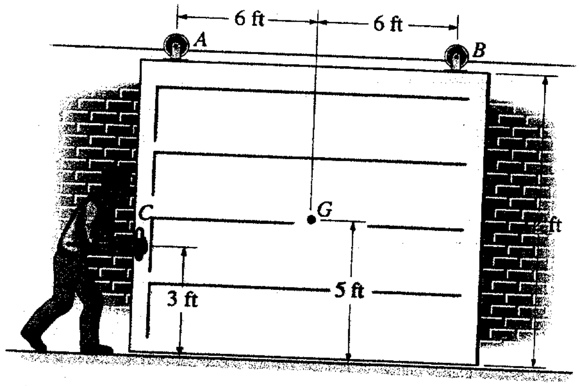
$+\uparrow \Sigma F_y = m(a_G)_y$; $\quad N_A + 95.0 - 200 = 0$

$N_A = 105 \text{ lb}$ **Ans**

$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_G t^2$

$s = 0 + 0 + \frac{1}{2}(4.83)(2)^2 = 9.66 \text{ ft}$ **Ans**

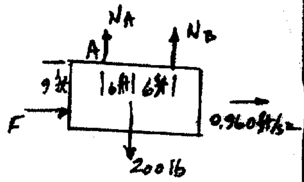
***17-32.** The door has a weight of 200 lb and a center of gravity at G. Determine the constant force F that must be applied to the door to push it open 12 ft to the right in 5 s, starting from rest. Also, find the vertical reactions at the rollers A and B.



$$(\xrightarrow{+})\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$12 = 0 + 0 + \frac{1}{2}a_c(5)^2$$

$$a_c = 0.960\ \text{ft/s}^2$$



$$\xrightarrow{+}\Sigma F_x = m(a_G)_x; \quad F = \frac{200}{32.2}(0.960)$$

$$F = 5.9627\ \text{lb} = 5.96\ \text{lb} \qquad \textbf{Ans}$$

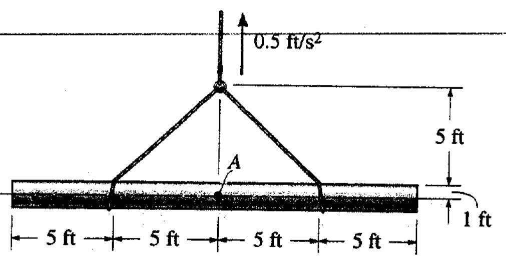
$$+\Sigma M_A = \Sigma(M_k)_A; \quad N_B(12) - 200(6) + 5.9627(9) = \frac{200}{32.2}(0.960)(7)$$

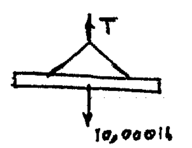
$$N_B = 99.0\ \text{lb} \qquad \textbf{Ans}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \quad N_A + 99.0 - 200 = 0$$

$$N_A = 101\ \text{lb} \qquad \textbf{Ans}$$

17-33. The uniform pipe has a weight of 500 lb/ft and diameter of 2 ft. If it is hoisted as shown with an acceleration of 0.5 ft/s², determine the internal moment at the center A of the pipe due to the lift.

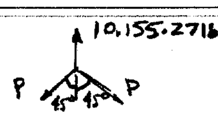




Pipe:

$$+\uparrow\Sigma F_y = m\,a_y; \quad T - 10\,000 = \frac{10\,000}{32.2}(0.5)$$

$$T = 10\,155.27\ \text{lb}$$

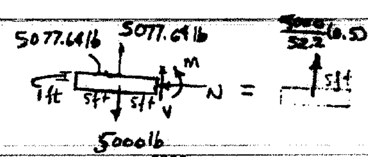


Cables:

$$+\uparrow\Sigma F_y = 0; \quad 10\,155.27 - 2P\cos 45° = 0$$

$$P = 7\,180.867\ \text{lb}$$

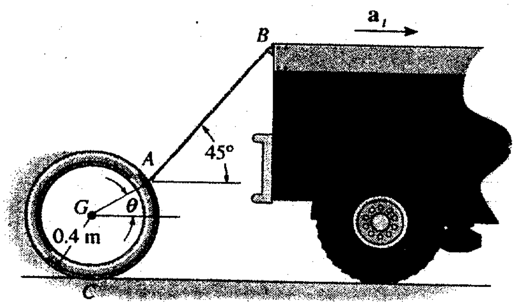
$$P_x = P_y = 7\,180.867\left(\frac{1}{\sqrt{2}}\right) = 5\,077.64\ \text{lb}$$



$$(+\Sigma M_A = \Sigma(M_k)_A; \quad M_A + 5000(5) - 5077.64(5) - 5077.64(1) = \frac{-5000}{32.2}(0.5)(5)$$

$$M_A = 5077.6\ \text{lb}\cdot\text{ft} = 5.08(10^3)\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

17-34. The pipe has a mass of 800 kg and is being towed behind the truck. If the acceleration of the truck is $a_t = 0.5\ \text{m/s}^2$, determine the angle θ and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.



$\xrightarrow{+} \Sigma F_x = ma_x;\qquad -0.1N_C + T\cos 45° = 800(0.5)$

$+\uparrow \Sigma F_y = ma_y;\qquad N_C - 800(9.81) + T\sin 45° = 0$

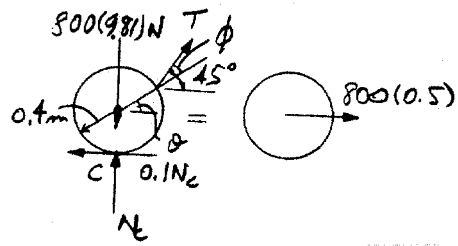
$\curvearrowleft + \Sigma M_G = 0;\qquad -0.1N_C(0.4) + T\sin\phi(0.4) = 0$

$N_C = 6770.9\ \text{N}$

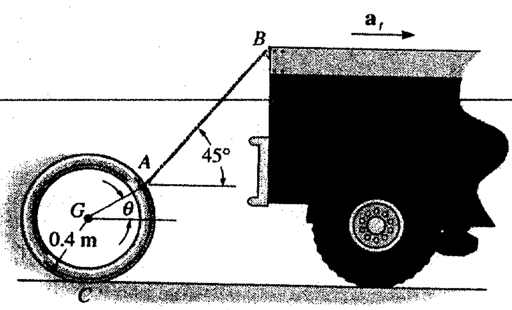
$T = 1523.24\ \text{N} = 1.52\ \text{kN}$ **Ans**

$\sin\phi = \dfrac{0.1(6770.9)}{1523.24}\qquad \phi = 26.39°$

$\theta = 45° - \phi = 18.6°$ **Ans**



17-35. The pipe has a mass of 800 kg and is being towed behind a truck. If the angle $\theta = 30°$, determine the acceleration of the truck and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.



$\xrightarrow{+} \Sigma F_x = ma_x;\qquad T\cos 45° - 0.1N_C = 800a$

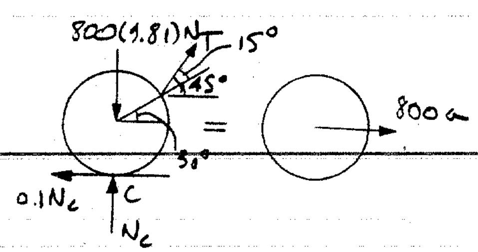
$+\uparrow \Sigma F_y = ma_y;\qquad N_C - 800(9.81) + T\sin 45° = 0$

$\curvearrowleft + \Sigma M_G = 0;\qquad T\sin 15°(0.4) - 0.1N_C(0.4) = 0$

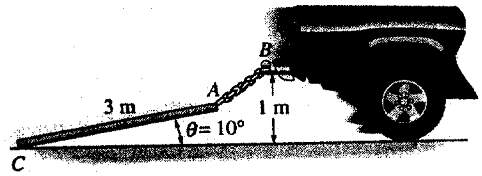
$N_C = 6164\ \text{N}$

$T = 2382\ \text{N} = 2.38\ \text{kN}$ **Ans**

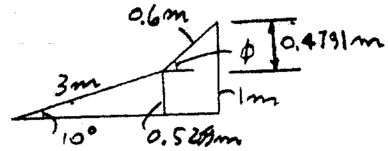
$a = 1.33\ \text{m/s}^2$ **Ans**



***17-36.** The pipe has a length of 3 m and a mass of 500 kg. It is attached to the back of the truck using a 0.6-m-long chain AB. If the coefficient of kinetic friction at C is $\mu_k = 0.4$, determine the acceleration of the truck if the angle $\theta = 10°$ with the road as shown.



$$\phi = \sin^{-1}\left(\frac{0.4791}{0.6}\right) = 52.98°$$



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad T\cos 52.98° - 0.4N_C = 500a_G$

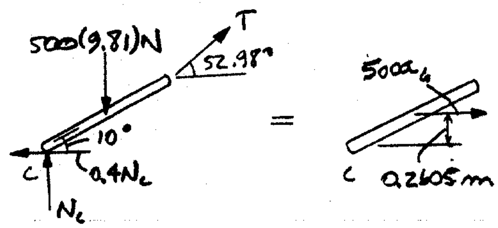
$+\uparrow \Sigma F_y = m(a_G)_y;\quad N_C - 500(9.81) + T\sin 52.98° = 0$

$\circlearrowleft + \Sigma M_C = \Sigma(M_k)_C;\quad -500(9.81)(1.5\cos 10°) + T\sin(52.98° - 10°)(3) = -500a_G(0.2605)$

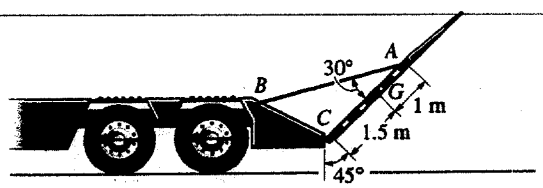
$T = 3.39$ kN

$N_C = 2.19$ kN

$a_G = 2.33\ \text{m/s}^2$ **Ans**



17-37. The drop gate at the end of the trailer has a mass of 1.25 Mg and mass center at G. If it is supported by the cable AB and hinge at C, determine the tension in the cable when the truck begins to accelerate at $5\ \text{m/s}^2$. Also, what are the horizontal and vertical components of reaction at the hinge C?



$\circlearrowleft + \Sigma M_C = \Sigma(M_k)_C;\quad T\sin 30°(2.5) - 12\,262.5(1.5\cos 45°) = 1250(5)(1.5\sin 45°)$

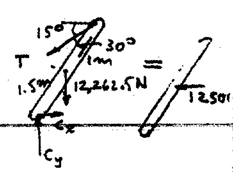
$T = 15\,708.4\ \text{N} = 15.7$ kN **Ans**

$\xleftarrow{+} \Sigma F_x = m(a_G)_x;\quad -C_x + 15\,708.4\cos 15° = 1250(5)$

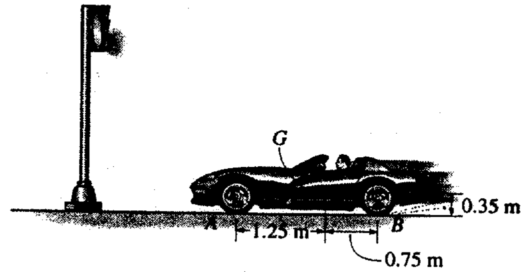
$C_x = 8.92$ kN **Ans**

$+\uparrow \Sigma F_y = m(a_G)_y;\quad C_y - 12\,262.5 - 15\,708.4\sin 15° = 0$

$C_y = 16.3$ kN **Ans**



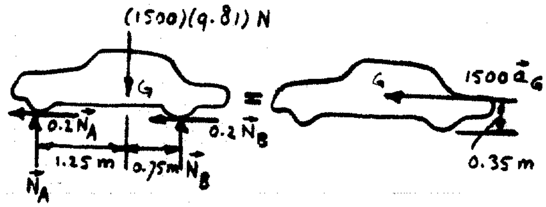
17-38. The sports car has a mass of 1.5 Mg and a center of mass at G. Determine the shortest time it takes for it to reach a speed of 80 km/h, starting from rest, if the engine only drives the rear wheels, whereas the front wheels are free rolling. The coefficient of static friction between the wheels and the road is $\mu_s = 0.2$. Neglect the mass of the wheels for the calculation. If driving power could be supplied to all four wheels, what would be the shortest time for the car to reach a speed of 80 km/h?



$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad 0.2N_A + 0.2N_B = 1500a_G \quad (1)$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 1500(9.81) = 0 \quad (2)$

$\circlearrowleft + \Sigma M_G = 0; \quad -N_A(1.25) + N_B(0.75) - (0.2N_A + 0.2N_B)(0.35) = 0 \quad (3)$



For rear-- wheel drive :

Set the friction force $0.2N_A = 0$ in Eqs. (1) and (3)

Solving yields :

$N_A = 5.18$ kN > 0 (OK); $N_B = 9.53$ kN; $a_G = 1.271$ m/s^2

Since $v = 80$ km/h $= 22.22$ m/s, then

$(\overset{+}{\rightarrow}) \quad v = v_0 + a_G t$

$22.22 = 0 + 1.271t$

$t = 17.5$ s **Ans**

For 4 - wheel drive :

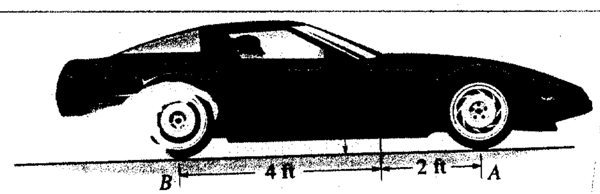
$N_A = 5.00$ kN > 0 (OK); $N_B = 9.71$ kN; $a_G = 1.962$ m/s^2

Since $v_2 = 80$ km/h $= 22.22$ m/s, then

$v_2 = v_1 + a_G t; \quad 22.22 = 0 + 1.962t$

$t = 11.3$ s **Ans**

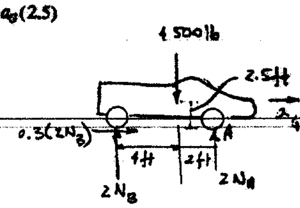
17-39. The sports car has a weight of 4500 lb and center of gravity at G. If it starts from rest it causes the rear wheels to slip as it accelerates. Determine how long it takes for it to reach a speed of 10 ft/s. Also, what are the normal reactions at *each* of the four wheels on the road? The coefficients of static and kinetic friction at the road are $\mu_s = 0.5$ and $\mu_k = 0.3$, respectively. Neglect the mass of the wheels.



$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad -2N_B(6) + 4500(2) = \dfrac{-4500}{32.2}a_G(2.5)$

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 0.3(2N_B) = \dfrac{4500}{32.2}a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad 2N_B + 2N_A - 4500 = 0$



Solving,

$N_A = 1393$ lb **Ans**

$N_B = 857$ lb **Ans**

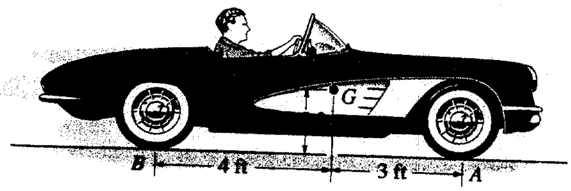
$a_G = 3.68$ ft/s^2

$(\overset{+}{\rightarrow}) \quad v = v_0 + a_c t$

$10 = 0 + 3.68\,t$

$t = 2.72$ s **Ans**

***17-40.** The car accelerates uniformly from rest to 88 ft/s in 15 seconds. If it has a weight of 3800 lb and a center of gravity at G, determine the normal reaction of *each wheel* on the pavement during the motion. Power is developed at the front wheels, whereas the rear wheels are free to roll. Neglect the mass of the wheels and take the coefficients of static and kinetic friction to be $\mu_s = 0.4$ and $\mu_k = 0.2$, respectively.

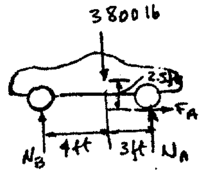


$v = v_0 + a_c t$

$88 = 0 + a_c(15)$

$a_c = 5.867 \text{ ft/s}^2$

Assume no slipping



$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x;\quad F_A = \frac{3800}{32.2}(5.867)$

$F_A = 692 \text{ lb}$

$\circlearrowleft + \Sigma M_B = \Sigma(M_k)_B;\quad 3800(4) - N_A(7) = \frac{3800}{32.2}(5.867)(2.5)$

$N_A = 1924.2 \text{ lb}$

$(F_A)_{max} = 1924.2(0.4) = 770 \text{ lb} > 692 \text{ lb}$ (O.K!)

$+\uparrow \Sigma F_y = 0;\quad N_B - 3800 + 1924.2 = 0$

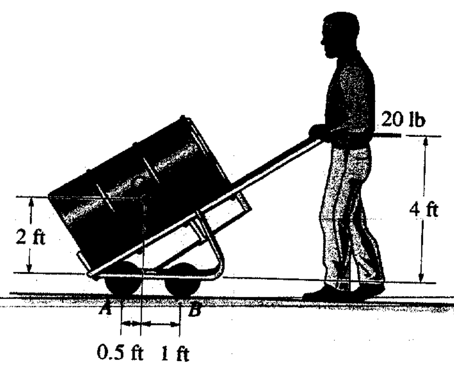
$N_B = 1875.8 \text{ lb}$

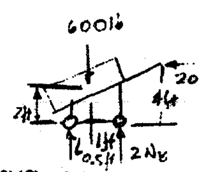
Normal reactions are

$N_A' = \frac{1924.2}{2} = 962 \text{ lb}$ **Ans**

$N_B' = \frac{1875.8}{2} = 938 \text{ lb}$ **Ans**

17-41. The drum truck supports the 600-lb drum that has a center of gravity at G. If the operator pushes it forward with a horizontal force of 20 lb, determine the acceleration of the truck and the normal reactions at each of the four wheels. Neglect the mass of the wheels.





$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x;\quad 20 = \left(\frac{600}{32.2}\right)a_G$

$a_G = 1.0733 \text{ ft/s}^2 = 1.07 \text{ ft/s}^2$ **Ans**

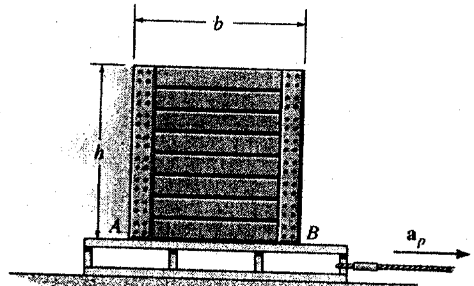
$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;\quad 20(4) - 600(0.5) + 2N_B(1.5) = \frac{600}{32.2}(1.0733)(2)$

$N_B = 86.7 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = m(a_G)_y;\quad 2N_A + 2(86.7) - 600 = 0$

$N_A = 213 \text{ lb}$ **Ans**

17-42. The uniform crate has a mass m and rests on a rough pallet for which the coefficient of static friction between the crate and pallet is μ_s. If the pallet is given an acceleration of a_p, show that the crate will tip and slip at the same time provided $\mu_s = b/h$.



$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad \mu_s N_c = m a_p$

$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \quad mg\left(\frac{b}{2}\right) = m a_p \left(\frac{h}{2}\right)$

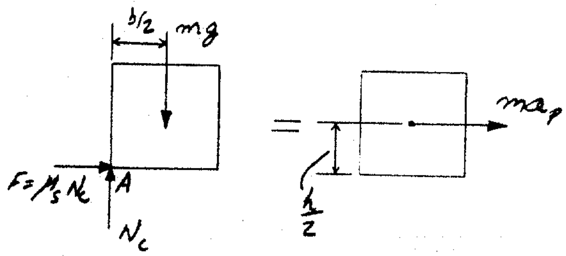
$$a_p = g\left(\frac{b}{h}\right)$$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_c - mg = 0$

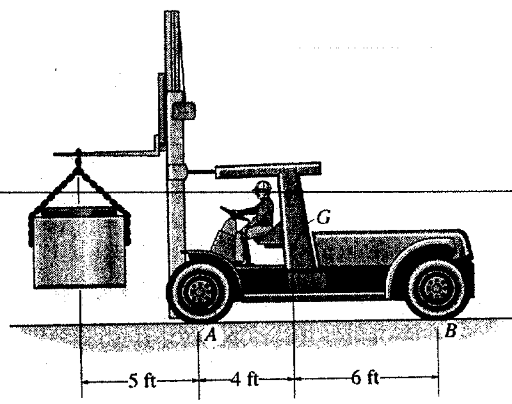
$$N_c = mg$$

$$\mu_s (mg) = m\left(g\frac{b}{h}\right)$$

$$\mu_s = \frac{b}{h} \qquad \textbf{Q.E.D.}$$



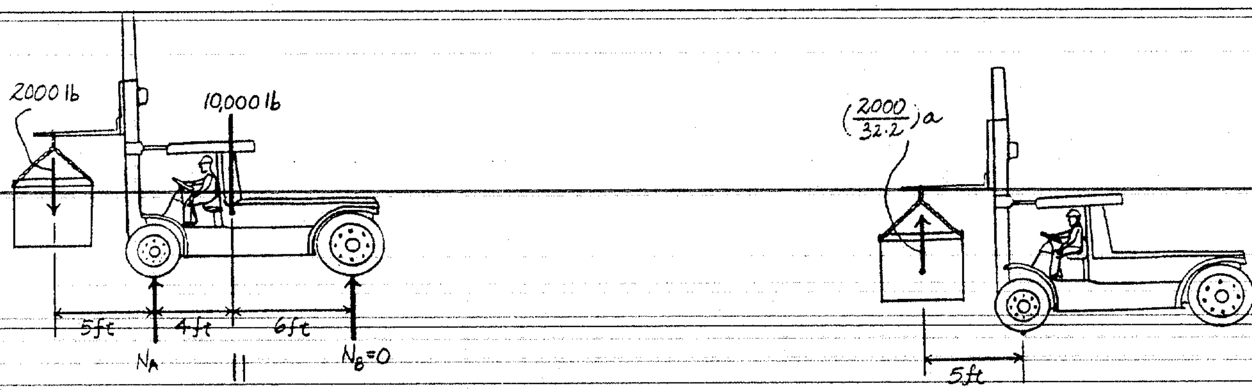
17-43. The forklift and operator have a combined weight of 10 000 lb and center of mass at G. If the forklift is used to lift the 2000-lb concrete pipe, determine the maximum vertical acceleration it can give to the pipe so that it does not tip forward on its front wheels.



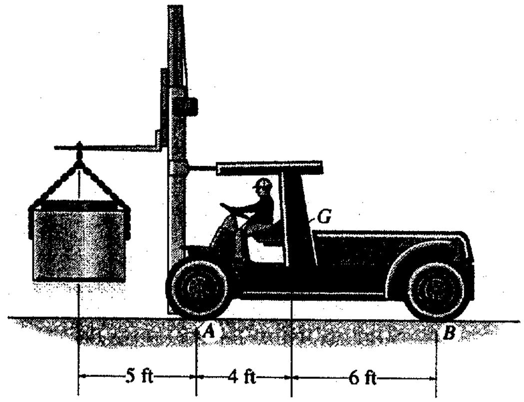
It is required that $N_B = 0$.

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 2000(5) - 10000(4) = -\left[\left(\frac{2000}{32.2}\right)a\right](5)$

$$a = 96.6 \text{ ft/s}^2 \qquad \textbf{Ans}$$



***17-44.** The forklift and operator have a combined weight of 10 000 lb and center of mass at G. If the forklift is used to lift the 2000-lb concrete pipe, determine the normal reactions on each of its four wheels if the pipe is given an upward acceleration of 4 ft/s^2.

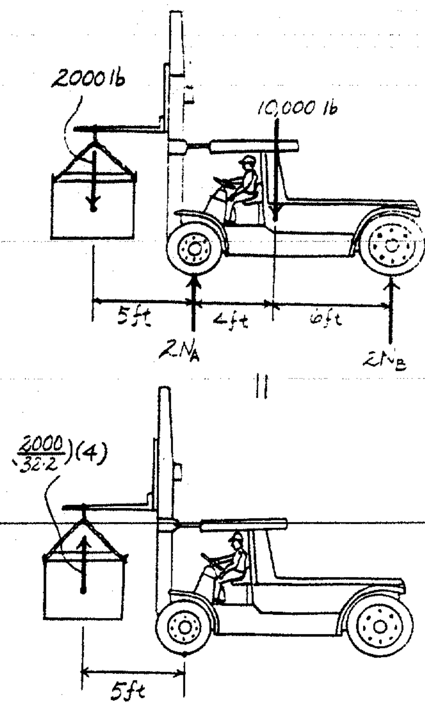


$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A;\quad 2000(5) + 2N_B(10) - 10000(4) = -\left[\left(\frac{2000}{32.2}\right)(4)\right](5)$

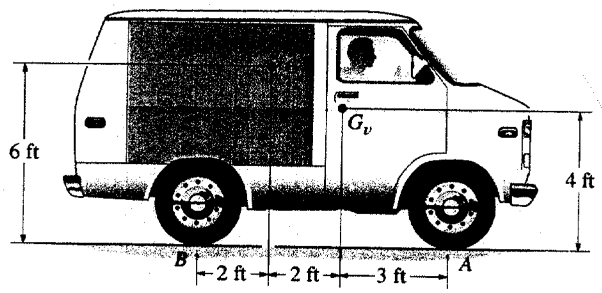
$N_B = 1437.89 \text{ lb} = 1.44 \text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = m(a_G)_y;\quad 2N_A + 2(1437.89) - 2000 - 10000 = \left(\frac{2000}{32.2}\right)(4)$

$N_A = 4686.34 \text{ lb} = 4.69 \text{ kip}$ **Ans**



17-45. The van has a weight of 4500 lb and center of gravity at G_v. It carries a fixed 800-lb load which has a center of gravity at G_l. If the van is traveling at 40 ft/s, determine the distance it skids before stopping. The brakes cause *all* the wheels to lock or skid. The coefficient of kinetic friction between the wheels and the pavement is $\mu_k = 0.3$. Compare this distance with that of the van being empty. Neglect the mass of the wheels.



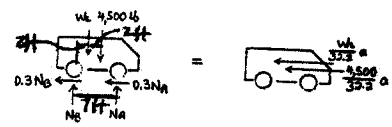
$\overset{+}{\leftarrow}\Sigma F_x = ma_x;\qquad 0.3N_B + 0.3N_A = \frac{W_L}{32.2}a + \frac{4500}{32.2}a \qquad (1)$

$+\uparrow\Sigma F_y = ma_y;\qquad N_B + N_A - W_L - 4500 = 0 \qquad (2)$

Set $W_L = 800$ lb in Eqs. (1) and (2)

$N_A + N_B = 5300$

$a = 9.66\ \text{ft/s}^2$



$(\overset{+}{\rightarrow})\qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (40)^2 + 2(-9.66)(s - 0)$

$s = 82.8$ ft **Ans**

For empty van $W_L = 0$ in Eqs. (1) and (2)

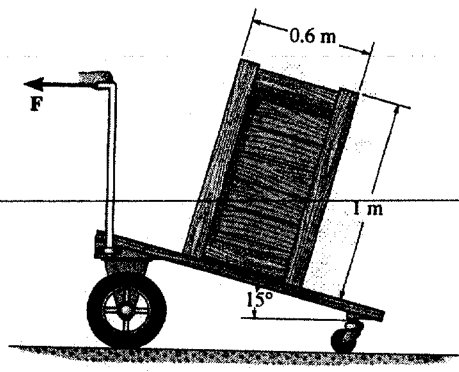
$N_A + N_B = 4500$

$a = 9.66\ \text{ft/s}^2$

Thus,

$s = 82.8$ ft as before **Ans**

17-46. The crate has a mass of 50 kg and rests on the cart having an inclined surface. Determine if the crate will tip over or slide relative to the cart when the cart is subjected to the smallest acceleration necessary to cause one of these relative motions. What is the magnitude of this acceleration? The coefficient of static friction between the crate and the cart is $\mu_s = 0.5$.

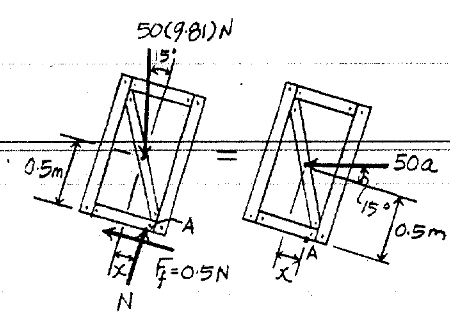


Equation of Motion: Assume that the crate slips, then $F_f = \mu_s N = 0.5N$.

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;\qquad 50(9.81)\cos 15°(x) - 50(9.81)\sin 15°(0.5) = 50a\cos 15°(0.5) + 50a\sin 15°(x) \qquad [1]$

$+\ \Sigma F_{y'} = m(a_G)_{y'};\qquad N - 50(9.81)\cos 15° = -50a\sin 15° \qquad [2]$

$+\Sigma F_{x'} = m(a_G)_{x'};\qquad 50(9.81)\sin 15° - 0.5N = -50a\cos 15° \qquad [3]$



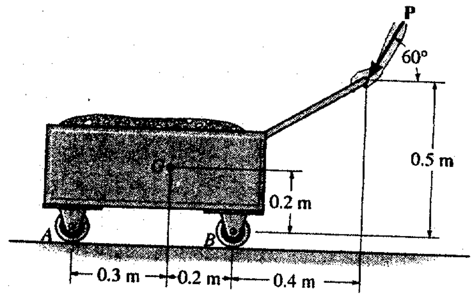
Solving Eqs. [1], [2] and [3] yields

$N = 447.81$ N $\qquad x = 0.250$ m

$a = 2.01\ \text{m/s}^2$ **Ans**

Since $x < 0.3$ m, then crate will not tip. Thus, **the crate slips.** **Ans**

17-47. The handcart has a mass of 200 kg and center of mass at G. Determine the normal reactions at each of the two wheels at A and the two wheels at B if a force of $P = 50$ N is applied to the handle. Neglect the mass of the wheels.

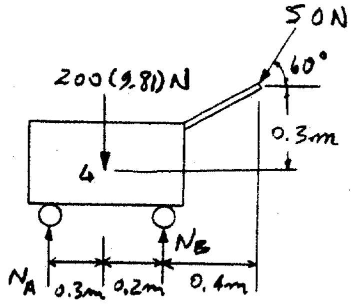


$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;$ $\quad 50 \cos 60^\circ = 200 a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N_A + N_B - 200(9.81) - 50 \sin 60^\circ = 0$

$\circlearrowleft + \Sigma M_G = 0;$ $\quad -N_A(0.3) + N_B(0.2) + 50 \cos 60^\circ(0.3) - 50 \sin 60^\circ(0.6) = 0$

$a_G = 0.125 \text{ m/s}^2;$ $\quad N_A = 765.2 \text{ N};$ $\quad N_B = 1240 \text{ N}$

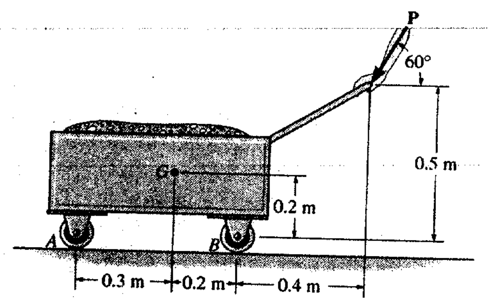


At each wheel,

$N_A' = \dfrac{N_A}{2} = 383 \text{ N}$ **Ans**

$N_B' = \dfrac{N_B}{2} = 620 \text{ N}$ **Ans**

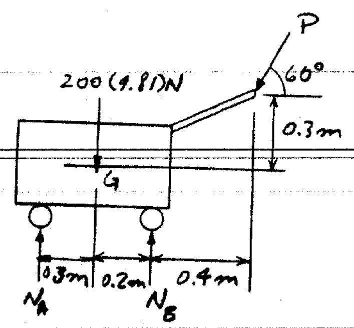
***17-48.** The handcart has a mass of 200 kg and center of mass at G. Determine the magnitude of the largest force **P** that can be applied to the handle so that the wheels at A or B continue to maintain contact with the ground. Neglect the mass of the wheels.



$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;$ $\quad P \cos 60^\circ = 200 a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N_A + N_B - 200(9.81) - P \sin 60^\circ = 0$

$\circlearrowleft + \Sigma M_G = 0;$ $\quad -N_A(0.3) + N_B(0.2) + P \cos 60^\circ(0.3) - P \sin 60^\circ(0.6) = 0$



For P_{max}, require

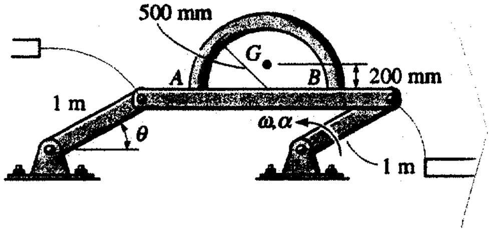
$N_A = 0$

$P = 1998 \text{ N} = 2.00 \text{ kN}$ **Ans**

$N_B = 3692 \text{ N}$

$a_G = 4.99 \text{ m/s}^2$

17-49. The arched pipe has a mass of 80 kg and rests on the surface of the platform. As it is hoisted from one level to the next, $\alpha = 0.25 \text{ rad/s}^2$ and $\omega = 0.5$ rad/s at the instant $\theta = 30°$. If it does not slip, determine the normal reactions of the arch on the platform at this instant.



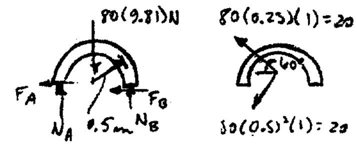
$+\uparrow\Sigma F_y = m(a_G)_y;\quad N_A + N_B - 80(9.81) = 20\sin 60° - 20\cos 60°$

$N_A + N_B = 792.12$

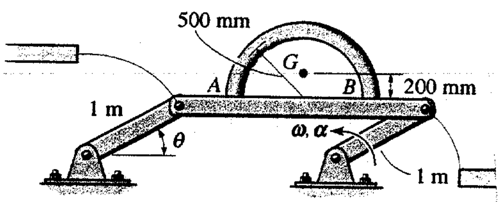
$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A;\quad N_B(1) - 80(9.81)(0.5) = 20\cos 60°(0.2) + 20\sin 60°(0.5) - 20\cos 60°(0.5) + 20\sin 60°(0.2)$

$N_B = 402$ N **Ans**

$N_A = 391$ N **Ans**



17-50. The arched pipe has a mass of 80 kg and rests on the surface of the platform for which the coefficient of static friction is $\mu_s = 0.3$. Determine the greatest angular acceleration α of the platform, starting from rest when $\theta = 45°$, without causing the pipe to slip on the platform.



$a_G = (a_G)_t = (1)(\alpha)$

$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A;\quad N_B(1) - 80(9.81)(0.5) = 80(1\alpha)(\sin 45°)(0.2) + 80(1\alpha)(\cos 45°)(0.5)$

$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x;\quad 0.3N_A + 0.3N_B = 80(1\alpha)\sin 45°$

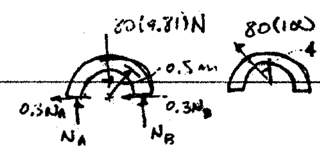
$+\uparrow\Sigma F_y = m(a_G)_y;\quad N_A + N_B - 80(9.81) = 80(1\alpha)\cos 45°$

Solving,

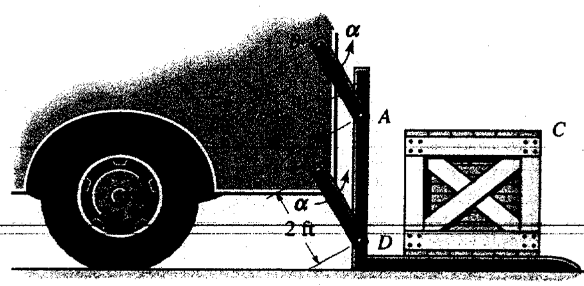
$\alpha = 5.95 \text{ rad/s}^2$ **Ans**

$N_A = 494$ N

$N_B = 628$ N



17-51. The crate C has a weight of 150 lb and rests on the truck elevator for which the coefficient of static friction is $\mu_s = 0.4$. Determine the largest initial angular acceleration α, starting from rest, which the parallel links AB and DE can have without causing the crate to slip. No tipping occurs.



$\overset{+}{\rightarrow}\Sigma F_x = ma_x;\quad 0.4N_C = \frac{150}{32.2}(a)\cos 30°$

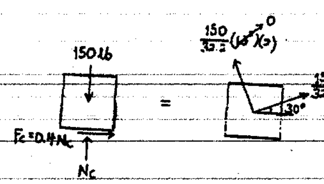
$\cancel{+\uparrow\Sigma F_y = ma_y;\quad N_C - 150 = \frac{150}{32.2}(a)\sin 30°}$

$N_C = 195.0$ lb

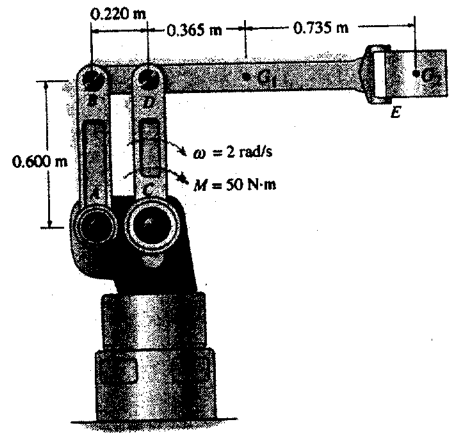
$a = 19.34 \text{ ft/s}^2$

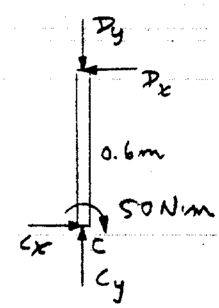
$19.34 = 2\alpha$

$\alpha = 9.67 \text{ rad/s}^2$ **Ans**



***17-52.** The arm BDE of the industrial robot manufactured by Cincinnati Milacron is activated by applying the torque of $M = 50$ N·m to link CD. Determine the reactions at the pins B and D when the links are in the position shown and have an angular velocity of 2 rad/s. The uniform arm BDE has a mass of 10 kg and a center of mass at G_1. The container held in its grip at E has a mass of 12 kg and center of mass at G_2. Neglect the mass of links AB and CD.





Curvilinear translation :

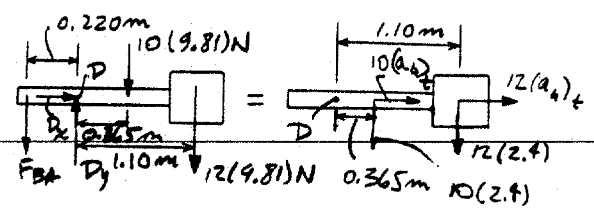
$(a_D)_n = (a_G)_n = (2)^2(0.6) = 2.4 \text{ m/s}^2$

Member DC :

$\circlearrowleft + \Sigma M_C = 0; \quad -D_x(0.6) + 50 = 0$

$D_x = 83.33 \text{ N} = 83.3 \text{ N}$ **Ans**

Member BDE :



$\circlearrowleft + \Sigma M_D = \Sigma (M_k)_D; \quad -F_{BA}(0.220) + 10(9.81)(0.365) + 12(9.81)(1.10)$

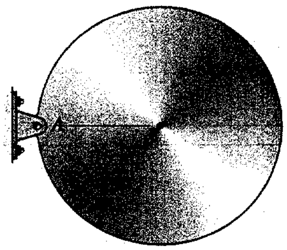
$= 10(2.4)(0.365) + 12(2.4)(1.10)$

$F_{BA} = 567.54 \text{ N} = 568 \text{ N}$ **Ans**

$+\uparrow \Sigma F_y = m(a_G)_y; \quad -567.54 + D_y - 10(9.81) - 12(9.81) = -10(2.4) - 12(2.4)$

$D_y = 731 \text{ N}$ **Ans**

17-53. The 80-kg disk is supported by a pin at A. If it is released from rest from the position shown, determine the initial horizontal and vertical components of reaction at the pin.



$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x;$ $\quad A_x = 0$ $\quad$ **Ans**

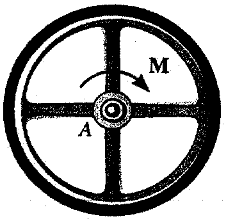
$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad A_y - 80(9.81) = -80(1.5)(\alpha)$

$\circlearrowleft + \Sigma M_A = I_A\alpha;$ $\quad 80(9.81)(1.5) = [\frac{3}{2}(80)(1.5)^2]\alpha$

$\alpha = 4.36 \text{ rad/s}^2$

$A_y = 262 \text{ N}$ $\quad$ **Ans**

17-54. The 10-kg wheel has a radius of gyration $k_A = 200$ mm. If the wheel is subjected to a moment $M = (5t)$ N·m, where t is in seconds, determine its angular velocity when $t = 3$ s starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.



$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x;$ $\quad A_x = 0$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad A_y - 10(9.81) = 0$

$\circlearrowleft + \Sigma M_A = I_A\alpha;$ $\quad 5t = 10(0.2)^2\alpha$

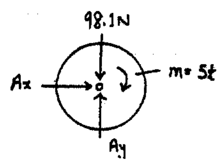
$\alpha = \frac{d\omega}{dt} = 12.5t$

$\omega = \int_0^3 12.5t\,dt = \frac{12.5}{2}(3)^2$

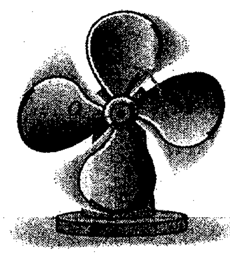
$\omega = 56.2 \text{ rad/s}$ $\quad$ **Ans**

$A_x = 0$ $\quad$ **Ans**

$A_y = 98.1 \text{ N}$ $\quad$ **Ans**



17-55. The fan blade has a mass of 2 kg and a moment of inertia $I_O = 0.18$ kg·m^2 about an axis passing through its center O. If it is subjected to a moment of $M = 3(1 - e^{-0.2t})$ N·m, where t is in seconds, determine its angular velocity when $t = 4$ s starting from rest.



$\circlearrowleft + \Sigma M_O = I_O\alpha;$ $\quad 3(1 - e^{-0.2t}) = 0.18\alpha$

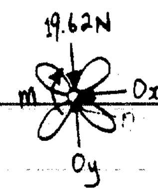
$\alpha = 16.67(1 - e^{-0.2t})$

$d\omega = \alpha\,dt$

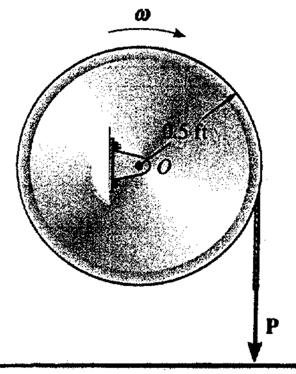
$\int_0^{\omega} d\omega = \int_0^4 16.67(1 - e^{-0.2t})\,dt$

$\omega = 16.67\left[t + \frac{1}{0.2}e^{-0.2t}\right]_0^4$

$\omega = 20.8 \text{ rad/s}$ $\quad$ **Ans**



***17-56.** The drum has a weight of 80 lb and a radius of gyration $k_O = 0.4$ ft. If the cable, which is wrapped around the drum, is subjected to a vertical force $P = 15$ lb, determine the time needed to increase the drum's angular velocity from $\omega_1 = 5$ rad/s to $\omega_2 = 25$ rad/s. Neglect the mass of the cable.



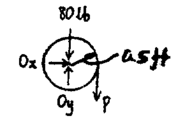
$\curvearrowright + \Sigma M_O = I_O\alpha;\qquad 15(0.5) = [\frac{80}{32.2}(0.4)^2]\alpha \qquad (1)$

$\alpha = 18.87 \text{ rad/s}^2$

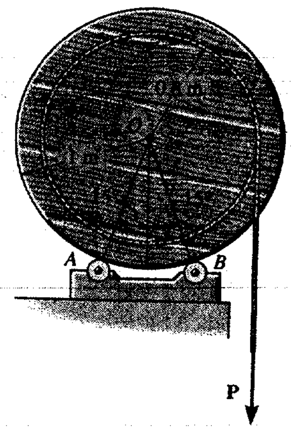
$(\curvearrowright +)\ \omega = \omega_0 + \alpha t$

$25 = 5 + 18.87\,t$

$t = 1.06$ s **Ans**



17-57. The spool is supported on small rollers at A and B. Determine the constant force P that must be applied to the cable in order to unwind 8 m of cable in 4 s starting from rest. Also calculate the normal forces at A and B during this time. The spool has a mass of 60 kg and a radius of gyration $k_O = 0.65$ m. For the calculation neglect the mass of the cable and the mass of the rollers at A and B.



$(\downarrow +)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$8 = 0 + 0 + \frac{1}{2}a_c(4)^2$

$a_c = 1 \text{ m/s}^2$

$\alpha = \frac{1}{0.8} = 1.25 \text{ rad/s}^2$

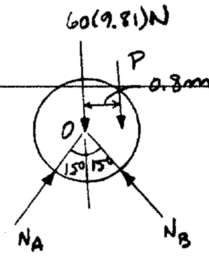
$\curvearrowleft + \Sigma M_O = I_O\alpha;\qquad P(0.8) = 60(0.65)^2(1.25)$

$P = 39.6$ N **Ans**

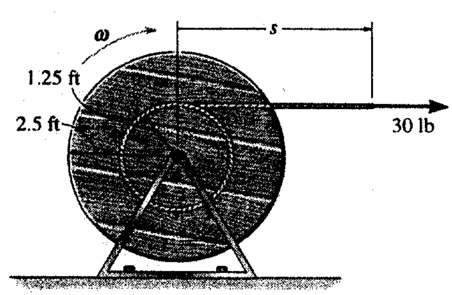
$\xrightarrow{+} \Sigma F_x = ma_x;\qquad N_A \sin 15° - N_B \sin 15° = 0$

$+\uparrow \Sigma F_y = ma_y;\qquad N_A \cos 15° + N_B \cos 15° - 39.6 - 588.6 = 0$

$N_A = N_B = 325$ N **Ans**



17-58. A cord is wrapped around the inner core of a spool. If the cord is pulled with a constant tension of 30 lb and the spool is originally at rest, determine the spool's angular velocity when $s = 8$ ft of cord has unwound. Neglect the weight of the 8-ft portion of cord. The spool and the entire cord have a total weight of 400 lb, and the radius of gyration about the axle A is $k_A = 1.30$ ft.



$I_A = mk_A^2 = \left(\frac{400}{32.2}\right)(1.30)^2 = 20.99$ slug · ft²

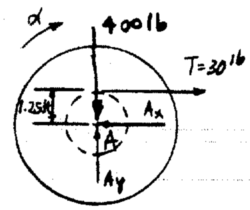
$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad 30(1.25) = 20.99(\alpha) \qquad \alpha = 1.786$ rad/s²

The angular displacement is $\theta = \frac{s}{r} = \frac{8}{1.25} = 6.4$ rad.

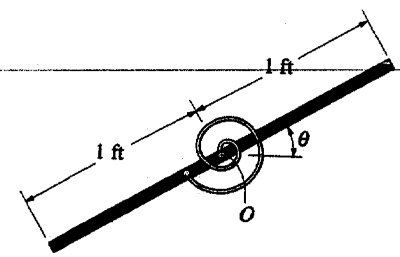
$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$\omega^2 = 0 + 2(1.786)(6.4 - 0)$

$\omega = 4.78$ rad/s **Ans**



17-59. The 10-lb bar is pinned at its center O and connected to a torsional spring. The spring has a stiffness $k = 5$ lb · ft/rad, so that the torque developed is $M = (5\theta)$ lb · ft, where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$.



$\circlearrowleft + \Sigma M_O = I_O\alpha; \quad -5\theta = [\frac{1}{12}(\frac{10}{32.2})(2)^2]\alpha$

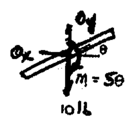
$-48.3\,\theta = \alpha$

$\alpha\, d\theta = \omega\, d\omega$

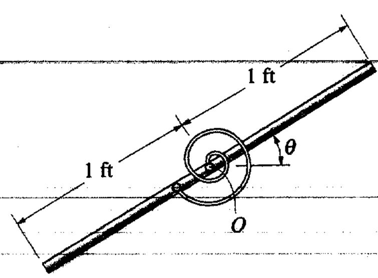
$-\int_{\frac{\pi}{2}}^{0} 48.3\,\theta\, d\theta = \int_0^{\omega} \omega\, d\omega$

$\frac{48.3}{2}(\frac{\pi}{2})^2 = \frac{1}{2}\omega^2$

$\omega = 10.9$ rad/s **Ans**



***17-60.** The 10-lb bar is pinned at its center O and connected to a torsional spring. The spring has a stiffness $k = 5$ lb · ft/rad, so that the torque developed is $M = (5\theta)$ lb · ft, where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 45°$.



$\circlearrowleft + \Sigma M_O = I_O\alpha; \quad 5\theta = [\frac{1}{12}(\frac{10}{32.2})(2)^2]\alpha$

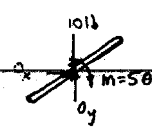
$\alpha = -48.3\theta$

$\alpha\, d\theta = \omega\, d\omega$

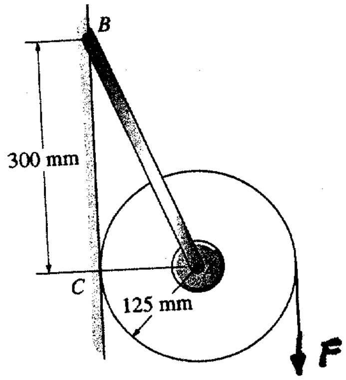
$-\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 48.3\theta\, d\theta = \int_0^{\omega} \omega\, d\omega$

$-24.15\left((\frac{\pi}{4})^2 - (\frac{\pi}{2})^2\right) = \frac{1}{2}\omega^2$

$\omega = 9.45$ rad/s **Ans**



17-61. The 20-kg roll of paper has a radius of gyration $k_A = 90$ mm about an axis passing through point A. It is pin-supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k = 0.2$ and a vertical force $F = 30$ N is applied to the end of the paper, determine the angular acceleration of the roll as the paper unrolls.



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad N_C - T_{AB}\cos 67.38° = 0$

$+\uparrow \Sigma F_y = m(a_G)_y;\quad T_{AB}\sin 67.38° - 0.2N_C - 20(9.81) - 30 = 0$

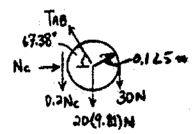
$\curvearrowleft + \Sigma M_A = I_A\,\alpha;\quad -0.2N_C(0.125) + 30(0.125) = 20(0.09)^2\alpha$

Solving;

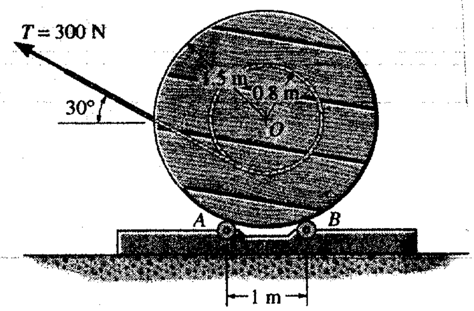
$N_C = 103$ N

$T_{AB} = 267$ N

$\alpha = 7.28\ \text{rad/s}^2$ **Ans**



17-62. Cable is unwound from a spool supported on small rollers at A and B by exerting a force of $T = 300$ N on the cable in the direction shown. Compute the time needed to unravel 5 m of cable from the spool if the spool and cable have a total mass of 600 kg and a centroidal radius of gyration of $k_O = 1.2$ m. For the calculation, neglect the mass of the cable being unwound and the mass of rollers at A and B. The rollers turn with no friction.



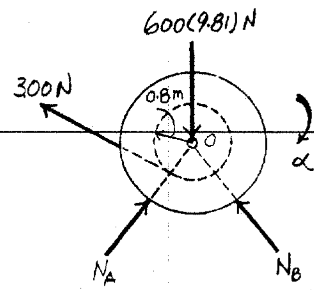
Equation of Motion : The mass moment of inertia of the spool about point O is given by $I_O = mk_O^2 = 600(1.2^2) = 864\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17 – 16, we have

$$\curvearrowleft + \Sigma M_O = I_O\,\alpha;\qquad -300(0.8) = -864\alpha \qquad \alpha = 0.2778\ \text{rad/s}^2$$

Kinematic : Here, the angular displacement $\theta = \frac{s}{r} = \frac{5}{0.8} = 6.25$ rad. Applying equation $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$, we have

$$(\curvearrowright +)\qquad 6.25 = 0 + 0 + \frac{1}{2}(0.2778)t^2$$

$$t = 6.71\ \text{s} \qquad \textbf{Ans}$$



17-63. The door will close automatically using torsional springs mounted on the hinges. Each spring has a stiffness $k = 50\ \text{N}\cdot\text{m/rad}$ so that the torque on each hinge is $M = (50\theta)\ \text{N}\cdot\text{m}$, where θ is measured in radians. If the door is released from rest when it is open at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$. For the calculation, treat the door as a thin plate having a mass of 70 kg.

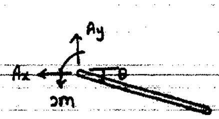
$I_{AB} = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}(70)(1.2)^2 + 70(0.6)^2 = 33.6\ \text{kg}\cdot\text{m}^2$

$\Sigma M_{AB} = I_{AB}\,\alpha;\quad 2(50\theta) = -33.6(\alpha) \qquad \alpha = -2.97620\theta$

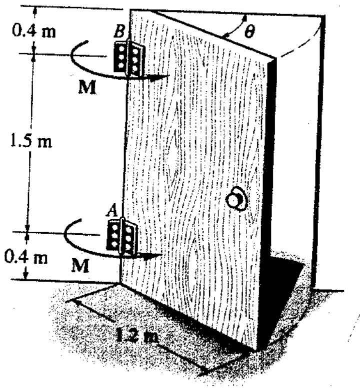
$\omega d\omega = \alpha d\theta$

$\int_0^{\omega} \omega d\omega = -\int_{\frac{\pi}{2}}^{0} 2.97620\theta d\theta$

$\omega = 2.71$ rad/s **Ans**



*17-64. The door will close automatically using torsional springs mounted on the hinges. If the torque on each hinge is $M = k\theta$, where θ is measured in radians, determine the required torsional stiffness k so that the door will close ($\theta = 0°$) with an angular velocity $\omega = 2$ rad/s when it is released from rest at $\theta = 90°$. For the calculation, treat the door as a thin plate having a mass of 70 kg.



$\Sigma M_A = I_A\alpha;\qquad 2M = -\left[\frac{1}{12}(70)(1.2)^2 + 70(0.6)^2\right](\alpha)$

$M = -16.8\alpha$

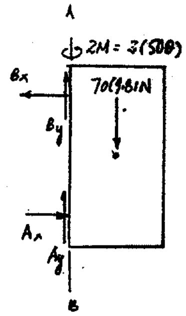
$k\theta = -16.8\alpha$

$\alpha\, d\theta = \omega\, d\omega$

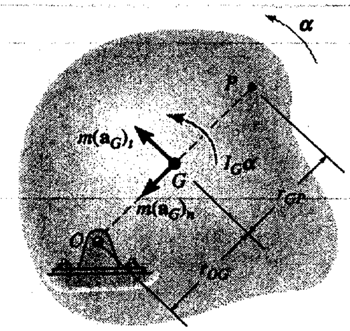
$-k\int_{\frac{\pi}{2}}^{0}\theta\, d\theta = 16.8\int_0^2 \omega\, d\omega$

$\frac{k}{2}\left(\frac{\pi}{2}\right)^2 = \frac{16.8}{2}(2)^2$

$k = 27.2$ N·m/rad **Ans**



17-65. The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis at O is shown in the figure. Show that $I_G\alpha$ may be eliminated by moving the vectors $m(\mathbf{a}_G)_t$ and $m(\mathbf{a}_G)_n$ to point P, located a distance $r_{GP} = k_G^2/r_{OG}$ from the center of mass G of the body. Here k_G represents the radius of gyration of the body about G. The point P is called the *center of percussion* of the body.

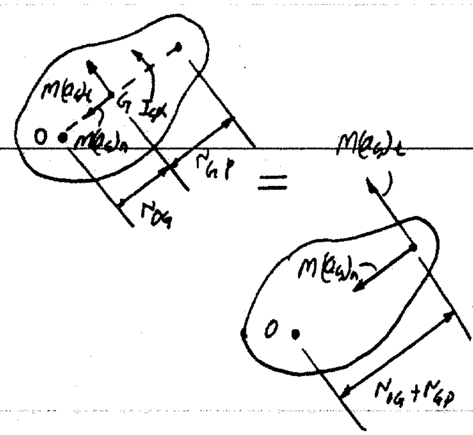


$m(a_G)_t r_{OG} + I_G\alpha = m(a_G)_t r_{OG} + (mk_G^2)\alpha$

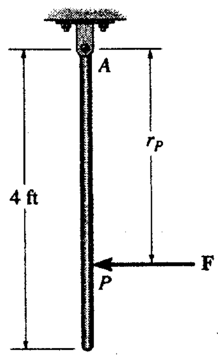
However, $\quad k_G^2 = r_{OG}r_{GP}$ and $\alpha = \frac{(a_G)_t}{r_{OG}}$

$m(a_G)_t r_{OG} + I_G\alpha = m(a_G)_t r_{OG} + (mr_{OG}r_{GP})\left[\frac{(a_G)_t}{r_{OG}}\right]$

$= m(a_G)_t(r_{OG} + r_{GP})$ **Q.E.D.**



17-66. Determine the position r_P of the center of percussion P of the 10-lb slender bar. (See Prob. 17-65.) What is the horizontal force A_x at the pin when the bar is struck at P with a force of $F = 20$ lb?

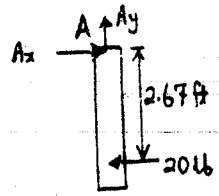


Using the result of Prob 17 - 65

$$r_{GP} = \frac{k_G^2}{r_{AG}} = \frac{\left[\sqrt{\frac{1}{12}\left(\frac{ml^2}{m}\right)}\right]^2}{\frac{l}{2}} = \frac{1}{6}l$$

Thus,

$$r_P = \frac{1}{6}l + \frac{1}{2}l = \frac{2}{3}l = \frac{2}{3}(4) = 2.67 \text{ ft} \qquad \textbf{Ans}$$



$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \quad 20(2.667) = \left[\frac{1}{3}\left(\frac{10}{32.2}\right)(4)^2\right]\alpha$$

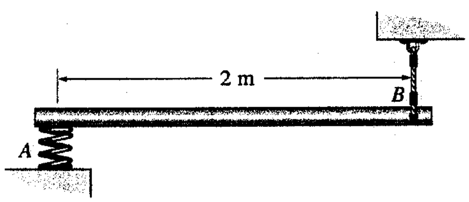
$$\alpha = 32.2 \text{ rad/s}^2$$

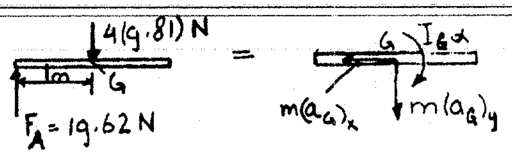
$$(a_G)_t = 2(32.2) = 64.4 \text{ ft/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_t; \quad -A_x + 20 = \left(\frac{10}{32.2}\right)(64.4)$$

$$A_x = 0 \qquad \textbf{Ans}$$

17-67. The 4-kg slender rod is supported horizontally by a spring at A and a cord at B. Determine the angular acceleration of the rod and the acceleration of the rod's mass center at the instant the cord at B is cut. *Hint:* The stiffness of the spring is not needed for the calculation.





Since the deflection of the spring is unchanged at the instant the cord is cut, the reaction at A is

$$F_A = \frac{4}{2}(9.81) = 19.62 \text{ N}$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 0 = 4(a_G)_x$$

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad 4(9.81) - 19.62 = 4(a_G)_y$$

$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad (19.62)(1) = \left[\frac{1}{12}(4)(2)^2\right]\alpha$$

Solving :

$(a_G)_x = 0$

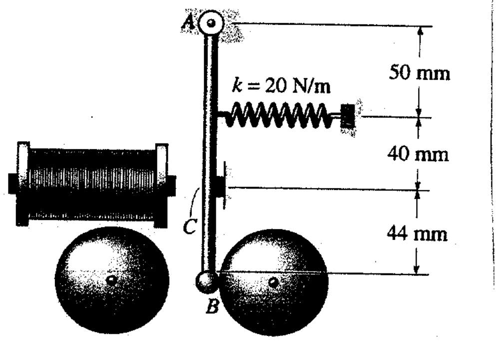
$(a_G)_y = 4.905 \text{ m/s}^2$

$\alpha = 14.7 \text{ rad/s}^2$ **Ans**

Thus,

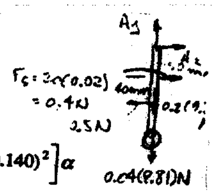
$(a_G) = 4.90 \text{ m/s}^2$ **Ans**

*17-68. The operation of the doorbell requires the use of the electromagnet, which attracts the iron clapper AB, pinned at end A and consisting of a 0.2-kg slender rod to which is attached the 0.04-kg steel ball having a radius of 6 mm. If the attractive force of the magnet at C is 0.5 N on the center of the ball when the button is pushed, determine the initial angular acceleration of the clapper. The spring is originally stretched 20 mm.



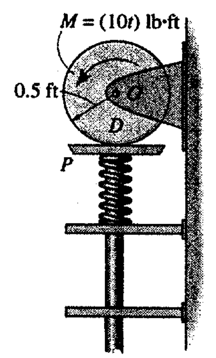
$$\circlearrowleft + \Sigma M_A = I_A \alpha; \quad 0.5(0.09) - 0.4(0.05) =$$

$$\left[\frac{1}{3}(0.2)(0.134)^2 + \frac{2}{5}(0.04)(0.006)^2 + 0.04(0.140)^2\right]\alpha$$



$\alpha = 12.6$ rad/s^2 **Ans**

17-69. The 10-lb disk D is subjected to a counterclockwise moment of $M = (10t)$ lb·ft, where t is in seconds. Determine the angular velocity of the disk 2 s after the moment is applied. Due to the spring the plate P exerts a constant force of 100 lb on the disk. The coefficients of static and kinetic friction between the disk and the plate are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. *Hint:* First find the time needed to start the disk rotating.



Determine time required to start disk in motion.

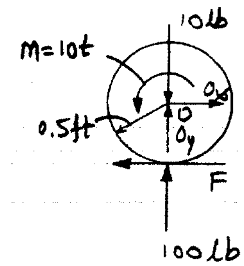
$F = 0.3(100) = 30$ lb

$\circlearrowleft + \Sigma M_O = 0; \quad 10t - 30(0.5) = 0$

$$t = 1.5 \text{ s}$$

Thus,

$F = 0.2(100) = 20$ lb



$\circlearrowleft + \Sigma M_O = I_O\alpha; \quad 10t - 20(0.5) = \left[\left(\frac{1}{2}\right)\left(\frac{10}{32.2}\right)(0.5)^2\right]\alpha$

$$\alpha = 257.6(t-1)$$

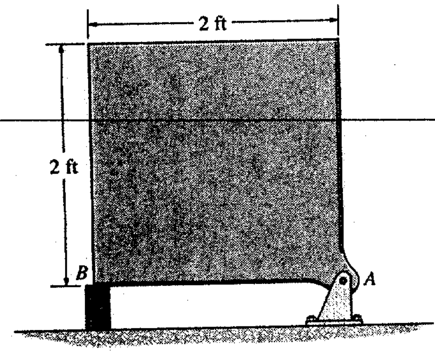
Since $\alpha = \dfrac{d\omega}{dt}$,

$$\int_0^{\omega} d\omega = \int_{1.5}^{2} 257.6(t-1)\, dt$$

$$\omega = 257.6\left(\frac{t^2}{2} - t\right)\Big|_{1.5}^{2}$$

$\omega = 96.6$ rad/s **Ans**

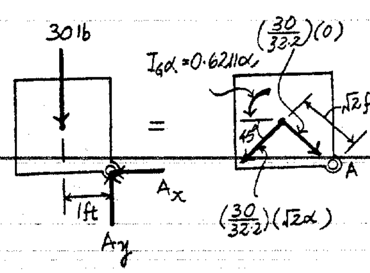
17-70. If the support at B is suddenly removed, determine the initial reactions at the pin A. The plate has a weight of 30 lb.



Equation of Motion : The mass moment inertia of the plate about its mass center is $I_G = \frac{1}{12}m(a^2+b^2) = \frac{1}{12}\left(\frac{30}{32.2}\right)(2^2+2^2) = 0.6211$ slug · ft^2. At the instant shown, the normal component of acceleration of the mass center for the plate $(a_G)_n = 0$ since the angular velocity of the plate $\omega = 0$ at that instant. The tangential component of acceleration of the mass center for the plate $(a_G)_t = \alpha r_G = \sqrt{2}\alpha$.

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 30(1) = 0.6211\alpha + \left(\frac{30}{32.2}\right)(\sqrt{2}\alpha)(\sqrt{2})$

$$\alpha = 12.075 \text{ rad/s}^2$$



$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad -A_x = -\left(\frac{30}{32.2}\right)\left[\sqrt{2}(12.075)\right]\cos 45°$

$$A_x = 11.25 \text{ lb}$$

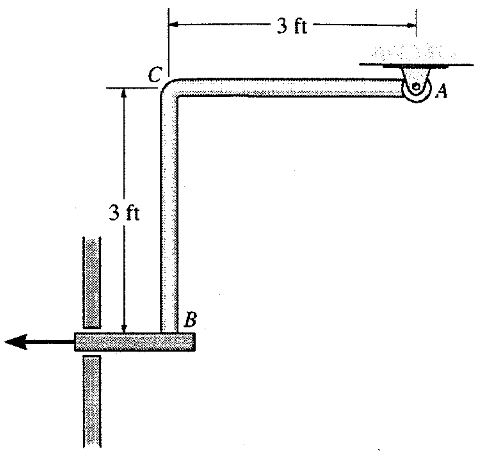
$+\uparrow \Sigma F_y = m(a_G)_y; \quad A_y - 30 = -\left(\frac{30}{32.2}\right)\left[\sqrt{2}(12.075)\right]\sin 45°$

$$A_y = 18.75 \text{ lb}$$

Thus,

$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{11.25^2 + 18.75^2} = 21.9$ lb **Ans**

17-71. If the support at B is suddenly removed, determine the initial downward acceleration of point C. Segments AC and CB each have a weight of 10 lb.

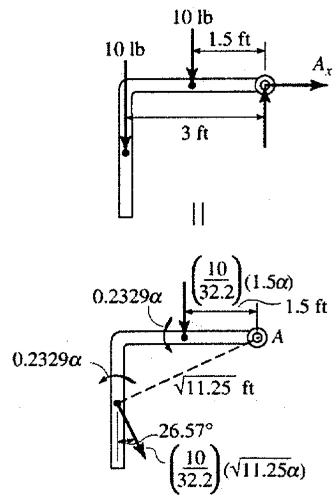


$$I_A = \frac{1}{3}\left(\frac{10}{32.2}\right)(3)^2 + \frac{1}{12}\left(\frac{10}{32.2}\right)(3)^2 + \left(\frac{10}{32.2}\right)(1.5^2 + 3^2)$$

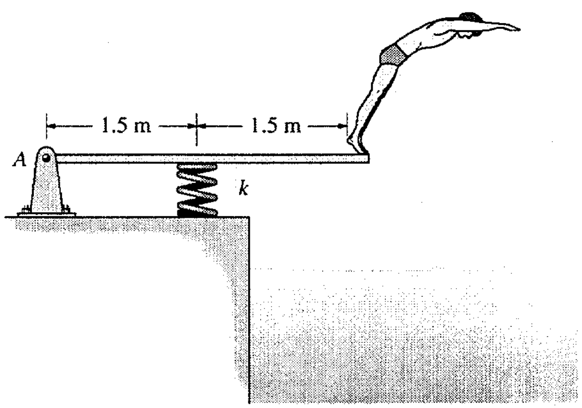
$$= 4.6584 \text{ slug} \cdot \text{ft}^2$$

$$\curvearrowleft + \sum M_A = I_A\alpha; \quad 10(3) + 10(1.5) = 4.6584(\alpha) \qquad \alpha = 9.66 \text{ rad/s}^2$$

$$a_C = \alpha r = 9.66(3) = 29.0 \text{ ft/s}^2 \quad \textbf{Ans}$$



***17-72.** Determine the angular acceleration of the 25-kg diving board and the horizontal and vertical components of reaction at the pin A the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount of 200 mm, $\omega = 0$, and the board is horizontal. Take $k = 7$ kN/m.



$$\curvearrowleft + \sum M_A = I_A\alpha; \quad 1.5(1400 - 245.25) = \left[\frac{1}{3}(25)(3)^2\right]\alpha$$

$$+\uparrow \sum F_t = m(a_G)_t; \quad 1400 - 245.25 - A_y = 25(1.5\alpha)$$

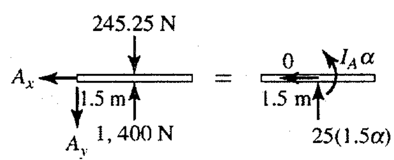
$$\overset{+}{\leftarrow} \sum F_n = m(a_G)_n; \quad A_x = 0$$

Solving,

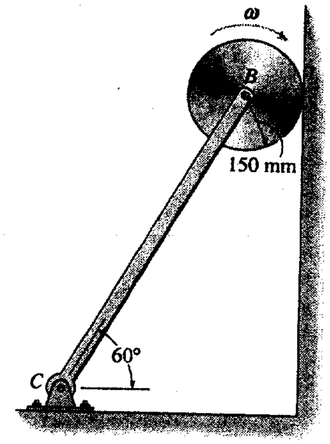
$A_x = 0$ **Ans**

$A_y = 289$ N **Ans**

$\alpha = 23.1$ rad/s^2 **Ans**



17-73. The disk has a mass of 20 kg and is originally spinning at the end of the strut with an angular velocity of $\omega = 60$ rad/s. If it is then placed against the wall, for which the coefficient of kinetic friction is $\mu_k = 0.3$, determine the time required for the motion to stop. What is the force in strut BC during this time?



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;\quad F_{CB}\sin 30^\circ - N_A = 0$

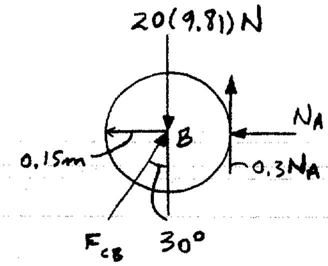
$+\uparrow \Sigma F_y = m(a_G)_y;\quad F_{CB}\cos 30^\circ - 20(9.81) + 0.3N_A = 0$

$\circlearrowleft + \Sigma M_B = I_B\,\alpha;\quad 0.3N_A\,(0.15) = \left[\frac{1}{2}(20)(0.15)^2\right]\alpha$

$N_A = 96.6$ N

$F_{CB} = 193$ N **Ans**

$\alpha = 19.3$ rad/s^2

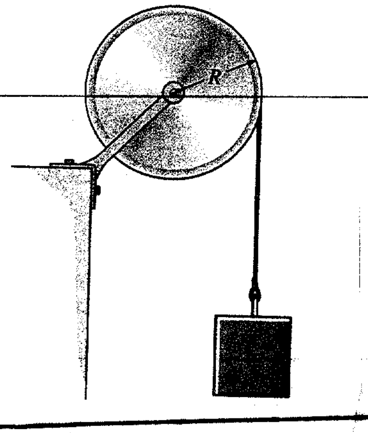


$\circlearrowleft + \omega = \omega_0 + \alpha_c t$

$0 = 60 + (-19.3)t$

$t = 3.11$ s **Ans**

17-74. The disk has a mass M and a radius R. If a block of mass m is attached to the cord, determine the angular acceleration of the disk when the block is released from rest. Also, what is the velocity of the block after it falls a distance $2R$ starting from rest?



FBD + KD

$\circlearrowleft + \Sigma M_O = \Sigma(M_k)_O;\quad mgR = \frac{1}{2}MR^2(\alpha) + m(\alpha R)R$

$\alpha = \dfrac{2mg}{R(M+2m)}$ **Ans**

$a = \alpha R$

$v^2 = v_0^2 + 2a(s - s_0)$

$v^2 = 0 + 2\left(\dfrac{2mgR}{R(M+2m)}\right)(2R - 0)$

$v = \sqrt{\dfrac{8mgR}{(M+2m)}}$ **Ans**

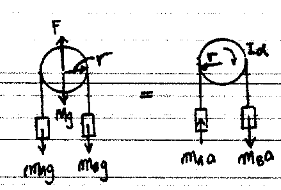
17-75. The two blocks A and B have a mass m_A and m_B, respectively, where $m_B > m_A$. If the pulley can be treated as a disk of mass M, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.

$a = \alpha r$

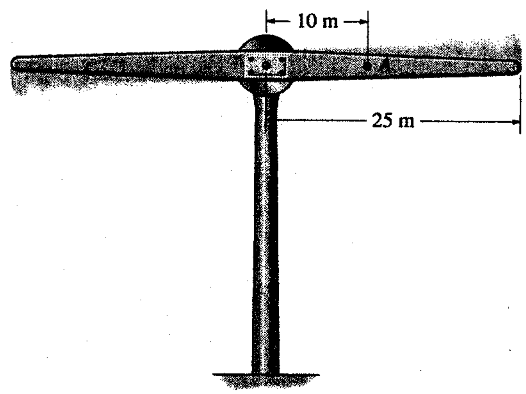
$\circlearrowleft + \Sigma M_O = \Sigma(M_O);\quad M_B g(r) - M_A g(r) = \left(\frac{1}{2}Mr^2\right)\alpha + M_B r^2\alpha + M_A r^2\alpha$

$\alpha = \dfrac{g(M_B - M_A)}{r(\frac{1}{2}M + M_B + M_A)}$

$a = \dfrac{g(M_B - M_A)}{(\frac{1}{2}M + M_B + M_A)}$ **Ans**



***17-76.** The lightweight turbine consists of a rotor which is powered from a torque applied at its center. At the instant the rotor is horizontal it has an angular velocity of 15 rad/s and an angular acceleration of 8 rad/s^2. Determine the internal normal force, shear force, and moment at a section through A. Assume the rotor is a 50-m-long slender rod, having a mass of 3 kg/m.



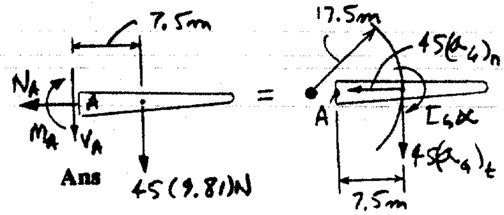
$\overset{+}{\leftarrow} \Sigma F_n = m(a_G)_n;\quad N_A = 45(15)^2(17.5) = 177 \text{ kN}$ **Ans**

$+\downarrow \Sigma F_t = m(a_G)_t;\quad V_A + 45(9.81) = 45(8)(17.5)$

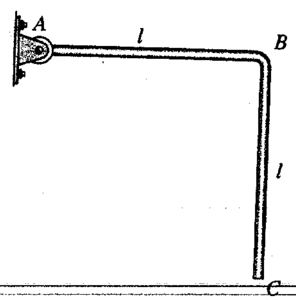
$V_A = 5.86 \text{ kN}$ **Ans**

$\curvearrowleft + \Sigma M_A = \Sigma(M_k)_A;\quad M_A + 45(9.81)(7.5) = \left[\frac{1}{12}(45)(15)^2\right](8) + [45(8)(17.5)](7.5)$

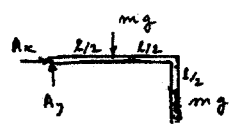
$M_A = 50.7 \text{ kN}\cdot\text{m}$ **Ans**



17-77. The two-bar assembly is released from rest in the position shown. Determine the initial bending moment at the fixed joint B. Each bar has a mass m and length l.



Assembly:



$$I_A = \frac{1}{3}ml^2 + \frac{1}{12}(m)(l)^2 + m(l^2 + (\frac{l}{2})^2)$$

$$= 1.667\, ml^2$$

$$\curvearrowleft + \Sigma M_A = I_A\,\alpha;\quad mg(\frac{l}{2}) + mg(l) = (1.667ml^2)\alpha$$

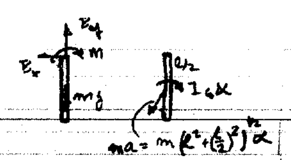
$$\alpha = \frac{0.9\,g}{l}$$

Segment BC**:**

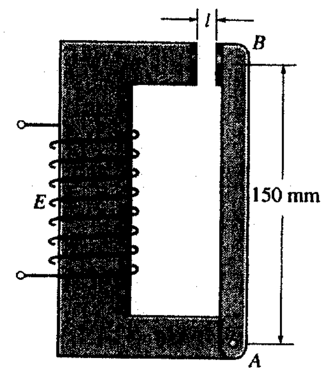
$$\curvearrowleft + \Sigma M_B = \Sigma(M_k)_B;\quad M = \left[\frac{1}{12}ml^2\right]\alpha + m(l^2 + (\frac{l}{2})^2)^{1/2}\alpha(\frac{l/2}{l^2+(\frac{l}{2})^2})(\frac{l}{2})$$

$$M = \frac{1}{3}ml^2\alpha = \frac{1}{3}ml^2(\frac{0.9g}{l})$$

$M = 0.3gml$ **Ans**



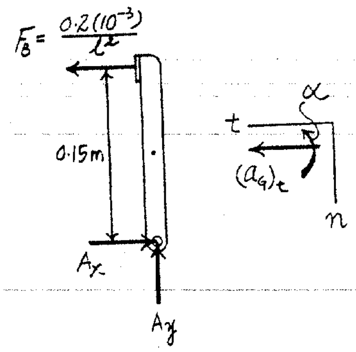
17-78. The armature (slender rod) AB has a mass of 0.2 kg and can pivot about the pin at A. Movement is controlled by the electromagnet E, which exerts a horizontal attractive force on the armature at B of $F_B = (0.2(10^{-3})l^{-2})$ N, where l in meters is the gap between the armature and the magnet at any instant. If the armature lies in the horizontal plane, and is originally at rest, determine the speed of the contact at B the instant $l = 0.01$ m. Originally $l = 0.02$ m.



Equation of Motion : The mass moment of inertia of the armature about point A is given by $I_A = I_G + mr_G^2 = \frac{1}{12}(0.2)(0.15^2) + 0.2(0.075^2) = 1.50(10^{-3})$ kg $\cdot$ m^2. Applying Eq. 17 – 16, we have

$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad \frac{0.2(10^{-3})}{l^2}(0.15) = 1.50(10^{-3})\,\alpha$$

$$\alpha = \frac{0.02}{l^2}$$



Kinematic : From the geometry, $l = 0.02 - 0.15\theta$. Then, $dl = -0.15d\theta$ or $d\theta = -\frac{dl}{0.15}$. Also, $\omega = \frac{v}{0.15}$ hence $d\omega = \frac{dv}{0.15}$. Substitute into equation $\omega d\omega = \alpha d\theta$, we have

$$\frac{v}{0.15}\left(\frac{dv}{0.15}\right) = \alpha\left(-\frac{dl}{0.15}\right)$$

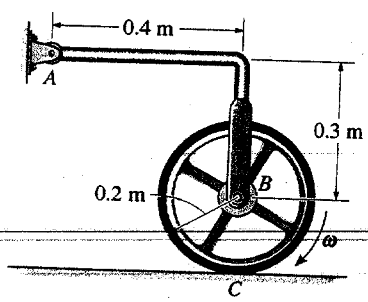
$$v dv = -0.15\alpha dl$$

$$\int_0^v v dv = \int_{0.02\text{m}}^{0.01\text{m}} -0.15\left(\frac{0.02}{l^2}\right)dl$$

$$v = 0.548 \text{ m/s}$$

Ans

17-79. The wheel has a mass of 25 kg and a radius of gyration $k_B = 0.15$ m. It is originally spinning at $\omega_1 = 40$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of reaction which the pin at A exerts on AB during this time? Neglect the mass of AB.

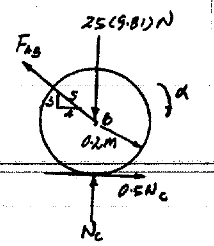


$I_B = mk_B^2 = 25(0.15)^2 = 0.5625$ kg $\cdot$ m^2

$+\uparrow \Sigma F_y = m(a_G)_y; \quad (\frac{3}{5})F_{AB} + N_C - 25(9.81) = 0 \qquad [1]$

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 0.5N_C - (\frac{4}{5})F_{AB} = 0 \qquad [2]$

$\circlearrowleft + \Sigma M_B = I_B\alpha; \quad 0.5N_C(0.2) = 0.5625(-\alpha) \qquad [3]$



Solvings Eqs.[1], [2] and [3] yields :

$F_{AB} = 111.48$ N $\quad N_C = 178.4$ N

$\alpha = -31.71$ rad/s^2

$A_x = \frac{4}{5}F_{AB} = 0.8(111.48) = 89.2$ N **Ans**

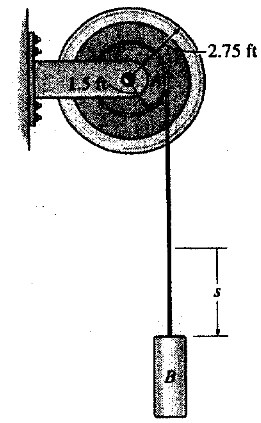
$A_y = \frac{3}{5}F_{AB} = 0.6(111.48) = 66.9$ N **Ans**

$\omega = \omega_0 + \alpha_c t$

$0 = 40 + (-31.71)t$

$t = 1.26$ s **Ans**

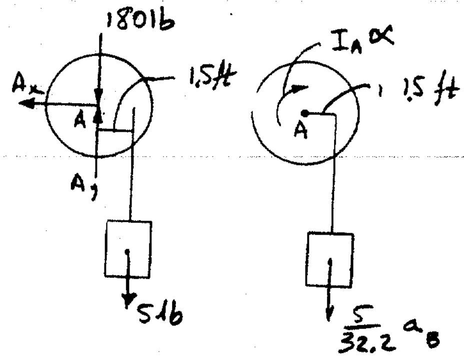
***17-80.** The cord is wrapped around the inner core of the spool. If a 5-lb block B is suspended from the cord and released from rest, determine the spool's angular velocity when $t = 3$ s. Neglect the mass of the cord. The spool has a weight of 180 lb and the radius of gyration about the axle A is $k_A = 1.25$ ft. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.



System :

$$\left(+\Sigma M_A = \Sigma (M_k)_A ; \quad 5(1.5) = \left(\frac{180}{32.2}\right)(1.25)^2\alpha + \left(\frac{5}{32.2}\right)(1.5\alpha)(1.5)\right.$$

$$\alpha = 0.8256 \text{ rad/s}^2$$



(+) $\quad \omega = \omega_0 + \alpha_c t$

$\omega = 0 + (0.8256)(3)$

$\omega = 2.48$ rad/s **Ans**

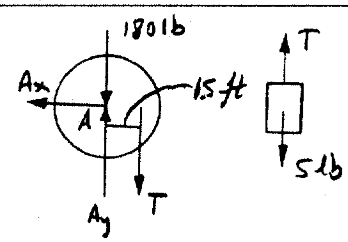
Also,

Spool :

$$\left(+\Sigma M_A = I_A\,\alpha; \quad T(1.5) = \left(\frac{180}{32.2}\right)(1.25)^2\alpha\right.$$

Weight :

$$+\downarrow \Sigma F_y = m(a_G)_y ; \quad 5 - T = \left(\frac{5}{32.2}\right)(1.5\alpha)$$



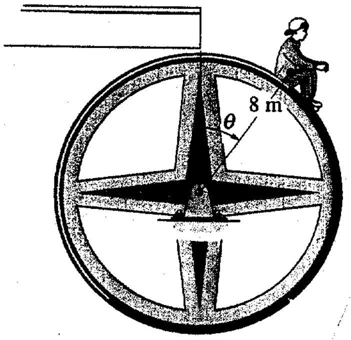
$$\alpha = 0.8256 \text{ rad/s}^2$$

(+) $\quad \omega = \omega_0 + \alpha_c t$

$\omega = 0 + (0.8256)(3)$

$\omega = 2.48$ rad/s **Ans**

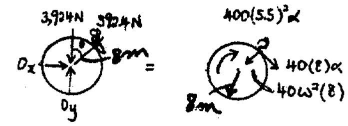
■17-81. A 40-kg boy sits on top of the large wheel which has a mass of 400 kg and a radius of gyration $k_G = 5.5$ m. If the boy essentially starts from rest at $\theta = 0°$, and the wheel begins to rotate freely, determine the angle at which the boy begins to slip. The coefficient of static friction between the wheel and the boy is $\mu_s = 0.5$. Neglect the size of the boy in the calculation.



$\zeta + \Sigma M_O = \Sigma(M_k)_O;\quad 392.4(8\sin\theta) = 400(5.5)^2\alpha + 40(8)(\alpha)(8)$

$0.2141\sin\theta = \alpha$

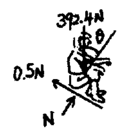
$\alpha\, d\theta = \omega\, d\omega$



$\int_0^\theta 0.2141\sin\theta\, d\theta = \int_0^\omega \omega\, d\omega$

$-0.2141\cos\theta\Big|_0^\theta = \frac{1}{2}\omega^2$

$\omega^2 = 0.4283(1-\cos\theta)$



$+\swarrow \Sigma F_{y'} = m(a_G)_{y'};\quad 392.4\cos\theta - N = 40(\omega^2)(8)$

$+\searrow \Sigma F_{x'} = m(a_G)_{x'};\quad 392.4\sin\theta - 0.5N = 40(8)(\alpha)$

$N = 392.4\cos\theta - 137.05(1-\cos\theta) = 529.45\cos\theta - 137.05$

$392.4\sin\theta - 0.5(529.45\cos\theta - 137.05) = 320(0.2141\sin\theta)$

$323.89\sin\theta - 264.73\cos\theta + 68.52 = 0$

$-\sin\theta + 0.8173\cos\theta = 0.2116$

Solve by trial and error

$\theta = 29.8°$ **Ans**

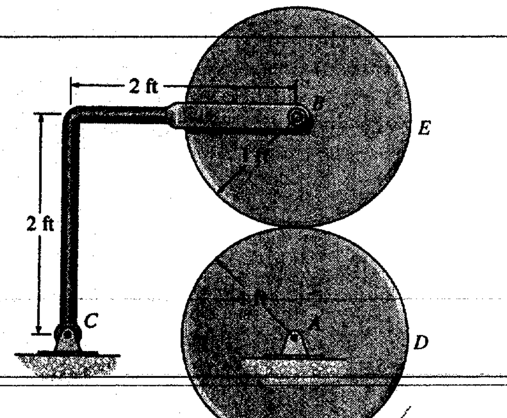
Note: The boy will loose contact with the wheel when $N = 0$, i.e.

$N = 529.45\cos\theta - 137.05 = 0$

$\theta = 75.0° > 29.8°$

Hence slipping occurs first.

17-82. Disk D turns with a constant clockwise angular velocity of 30 rad/s. Disk E has a weight of 60 lb and is initially at rest when it is brought into contact with D. Determine the time required for disk E to attain the same angular velocity as disk D. The coefficient of kinetic friction between the two disks is $\mu_k = 0.3$. Neglect the weight of bar BC.

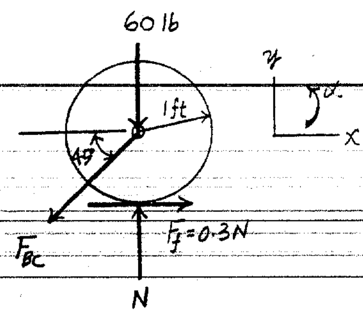


Equation of Motion: The mass moment of inertia of disk E about point B is given by $I_B = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{60}{32.2}\right)(1^2) = 0.9317$ slug · ft². Applying Eq. 17-16, we have

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad 0.3N - F_{BC}\cos 45° = 0$ [1]

$+\uparrow \Sigma F_y = m(a_G)_y;\quad N - F_{BC}\sin 45° - 60 = 0$ [2]

$\zeta + \Sigma M_O = I_O\alpha;\quad 0.3N(1) = 0.9317\alpha$ [3]



Solving Eqs. [1], [2] and [3] yields

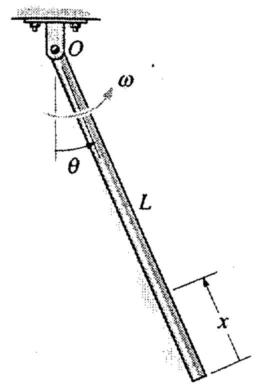
$F_{BC} = 36.37$ lb $\quad N = 85.71$ lb $\quad \alpha = 27.60$ rad/s²

Kinematic: Applying equation $\omega = \omega_0 + \alpha t$, we have

$(\zeta +)\quad 30 = 0 + 27.60t$

$t = 1.09$ s **Ans**

17-83. The bar has a weight per length of w. If it is rotating in the vertical plane at a constant rate ω about point O, determine the internal normal force, shear force, and moment as a function of x and θ.



$$a = \omega^2\left(L - \frac{x}{z}\right)$$

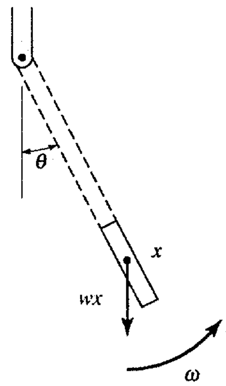
Forces:

$$\frac{wx}{g}\omega^2\left(L - \frac{x}{z}\right) = N + S + wx \downarrow \quad (1)$$

Moments:

$$I\alpha = M - S\left(\frac{x}{2}\right)$$

$$O = M - \frac{1}{2}Sx \qquad (2)$$



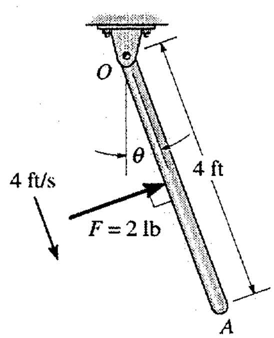
Solving (1) and (2),

$$N = wx\left[\frac{\omega^2}{g}\left(L - \frac{x}{2}\right) + \cos\theta\right]$$

$$S = wx\sin\theta$$

$$M = \frac{1}{2}wx^2\sin\theta$$

***17-84.** A force $F = 2$ lb is applied perpendicular to the axis of the 5-lb rod and moves from O to A at a constant rate of 4 ft/s. If the rod is at rest when $\theta = 0°$ and **F** is at O when $t = 0$, determine the rod's angular velocity at the instant the force is at A. Through what angle has the rod rotated when this occurs? The rod rotates in the *horizontal plane.*



$$I_O = \frac{1}{3}mR^2 = \frac{1}{3}\left(\frac{5}{32.2}\right)(4)^2 = 0.8282 \text{ slug}\cdot\text{ft}^2$$

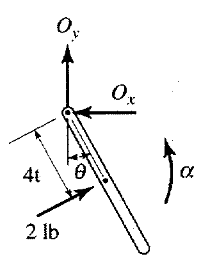
$$\circlearrowleft + \sum M_O = I_O\alpha: \quad 2(4t) = 0.8282(\alpha)$$

$$\alpha = 9.66t$$

$$d\omega = \alpha dt$$

$$\int_0^\omega d\omega = \int_0^t 9.66t\, dt$$

$$\omega = 4.83t^2$$



When $t = 1$ s,

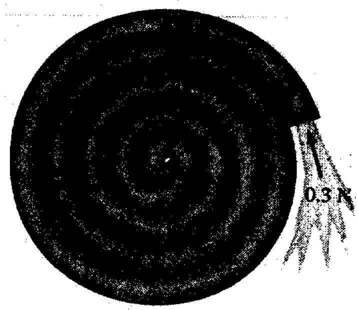
$$\omega = 4.83(1)^2 = 4.83 \text{ rad/s} \quad \textbf{Ans}$$

$$d\theta = \omega\, dt$$

$$\int_0^\theta d\theta = \int_0^t 4.83t^2 dt$$

$$\theta = 1.61 \text{ rad} = 92.2° \qquad \textbf{Ans}$$

17-85. The "Catherine wheel" is a firework that consists of a coiled tube of powder which is pinned at its center. If the powder burns at a constant rate of 20 g/s such that the exhaust gases always exert a force having a constant magnitude of 0.3 N, directed tangent to the wheel, determine the angular velocity of the wheel when 75% of the mass is burned off. Initially, the wheel is at rest and has a mass of 100 g and a radius of $r = 75$ mm. For the calculation, consider the wheel to always be a thin disk.



Mass of wheel when 75% of the powder is burned = 0.025 kg

Time to burn off 75% $= \dfrac{0.075\ \text{kg}}{0.02\ \text{kg/s}} = 3.75$ s

$m(t) = 0.1 - 0.02t$

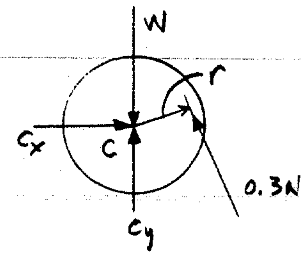
Mass of disk per unit area is

$$\rho_0 = \frac{m}{A} = \frac{0.1\ \text{kg}}{\pi(0.075\ \text{m})^2} = 5.6588\ \text{kg/m}^2$$

At any time t,

$$5.6588 = \frac{0.1 - 0.02t}{\pi r^2}$$

$$r(t) = \sqrt{\frac{0.1 - 0.02t}{\pi(5.6588)}}$$



$+\Sigma M_C = I_C\alpha;\quad 0.3r = \frac{1}{2}mr^2\alpha$

$$\alpha = \frac{0.6}{mr} = \frac{0.6}{(0.1 - 0.02t)\sqrt{\dfrac{0.1 - 0.02t}{\pi(5.6588)}}}$$

$$\alpha = 0.6\left(\sqrt{\pi(5.6588)}\right)[0.1 - 0.02t]^{-\frac{3}{2}}$$

$$\alpha = 2.530[0.1 - 0.02t]^{-\frac{3}{2}}$$

$d\omega = \alpha\, dt$

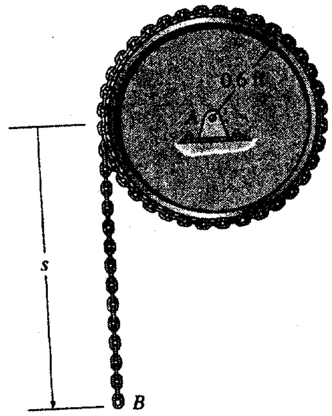
$$\int_0^{\omega} d\omega = 2.530\int_0^{t}[0.1 - 0.02t]^{-\frac{3}{2}}\, dt$$

$$\omega = 253\left[(0.1 - 0.02t)^{-\frac{1}{2}} - 3.162\right]$$

For $t = 3.75$ s,

$\omega = 800$ rad/s **Ans**

17-86. The drum has a weight of 50 lb and a radius of gyration $k_A = 0.4$ ft. A 35-ft-long chain having a weight of 2 lb/ft is wrapped around the outer surface of the drum so that a chain length of $s = 3$ ft is suspended as shown. If the drum is originally at rest, determine its angular velocity after the end B has descended $s = 13$ ft. Neglect the thickness of the chain.



$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A$; $\quad 2s(0.6) = \left(\frac{2s}{32.2}\right)[(\alpha)(0.6)](0.6) + \left[\left(\frac{50}{32.2}\right)(0.4)^2 + \frac{2(35-s)}{32.2}(0.6)^2\right]\alpha$

$1.2s = 0.02236s\alpha + (0.24845 + 0.7826 - 0.02236s)\alpha$

$1.164s = \alpha$

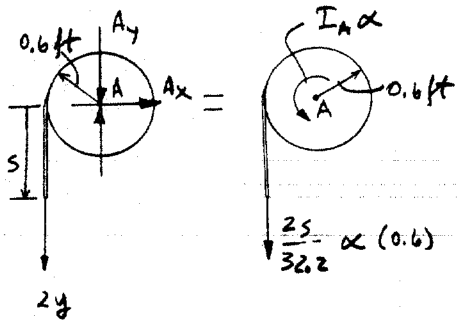
$\alpha\, d\theta = \alpha\left(\frac{ds}{0.6}\right) = \omega\, d\omega$

$1.164s\left(\frac{ds}{0.6}\right) = \omega\, d\omega$

$1.9398\int_3^{13} s\, ds = \int_0^{\omega} \omega\, d\omega$

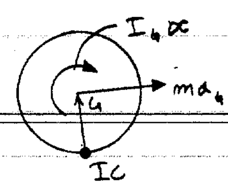
$1.9398\left[\frac{(13)^2}{2} - \frac{(3)^2}{2}\right] = \frac{1}{2}\omega^2$

$\omega = 17.6$ rad/s **Ans**



17-87. If the disk in Fig. 17-21*a* *rolls without slipping*, show that when moments are summed about the instantaneous center of zero velocity, *IC*, it is possible to use the moment equation $\Sigma M_{IC} = I_{IC}\alpha$, where I_{IC} represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.

$\circlearrowleft + \Sigma M_{IC} = \Sigma (M_k)_{IC}$; $\quad \Sigma M_{IC} = I_G\alpha + (ma_G)r$



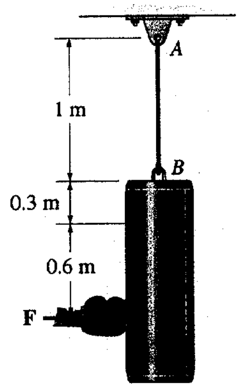
Since there is no slipping, $\quad a_G = \alpha r$

Thus, $\quad \Sigma M_{IC} = (I_G + mr^2)\alpha$

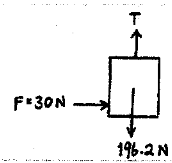
By the parallel – axis thoerem, the term in parenthesis represents I_{IC}. Thus,

$\Sigma M_{IC} = I_{IC}\alpha$ **Q.E.D.**

*17-88. The 20-kg punching bag has a radius of gyration about its center of mass G of $k_G = 0.4$ m. If it is initially at rest and is subjected to a horizontal force $F = 30$ N, determine the initial angular acceleration of the bag and the tension in the supporting cable AB.



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad 30 = 20(a_G)_x$



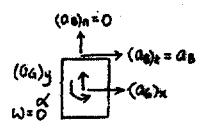
$+\uparrow \Sigma F_y = m(a_G)_y;\quad T - 196.2 = 20(a_G)_y$

$\circlearrowleft + \Sigma M_G = I_G\alpha;\quad 30(0.6) = 20(0.4)^2\alpha$

$\alpha = 5.62 \text{ rad/s}^2$ **Ans**

$(a_G)_x = 1.5 \text{ m/s}^2$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$



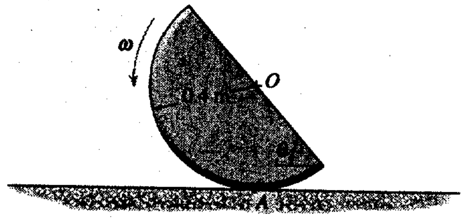
$a_B\mathbf{i} = (a_G)_y\mathbf{j} + (a_G)_x\mathbf{i} - \alpha(0.3)\mathbf{i}$

$(+\uparrow)\quad (a_G)_y = 0$

Thus,

$T = 196$ N **Ans**

17-89. The semicircular disk having a mass of 10 kg is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. If the coefficient of static friction at A is $\mu_s = 0.5$, determine if the disk slips at this instant.

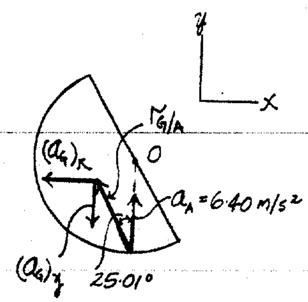


Equation of Motion : The mass moment of inertia of the semicircular disk about its center of mass is given by $I_G = \frac{1}{2}(10)(0.4^2) - 10(0.1698^2) = 0.5118\ \text{kg}\cdot\text{m}^2$. From the geometry, $r_{G/A} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)(0.4)\cos 60°} = 0.3477$ m. Also, using law of sines, $\frac{\sin\theta}{0.1698} = \frac{\sin 60°}{0.3477}$, $\theta = 25.01°$. Applying Eq. 17 – 16, we have

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 10(9.81)(0.1698\sin 60°) = 0.5118\alpha + 10(a_G)_x \cos 25.01°(0.3477) + 10(a_G)_y \sin 25.01°(0.3477) \qquad [1]$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad F_f = 10(a_G)_x \qquad [2]$$

$$+\uparrow F_y = m(a_G)_y; \quad N - 10(9.81) = -10(a_G)_y \qquad [3]$$



Kinematics : Assume that the semicircular disk does not slip at A, then $(a_A)_x = 0$. Here, $\mathbf{r}_{G/A} = \{-0.3477\sin 25.01°\mathbf{i} + 0.3477\cos 25.01°\mathbf{j}\}$ m $= \{-0.1470\mathbf{i} + 0.3151\mathbf{j}\}$ m. Applying Eq. 16 – 18, we have

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A}$$

$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = 6.40\mathbf{j} + \alpha\mathbf{k} \times (-0.1470\mathbf{i} + 0.3151\mathbf{j}) - 4^2(-0.1470\mathbf{i} + 0.3151\mathbf{j})$$

$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = (2.3523 - 0.3151\alpha)\mathbf{i} + (1.3581 - 0.1470\alpha)\mathbf{j}$$

Equating **i** and **j** components, we have

$$(a_G)_x = 0.3151\alpha - 2.3523 \qquad [4]$$

$$(a_G)_y = 0.1470\alpha - 1.3581 \qquad [5]$$

Solving Eqs. [1], [2], [3], [4] and [5] yields

$$\alpha = 13.85\ \text{rad/s}^2 \quad (a_G)_x = 2.012\ \text{m/s}^2 \quad (a_G)_y = 0.6779\ \text{m/s}^2$$

$$F_f = 20.12\ \text{N} \quad N = 91.32\ \text{N}$$

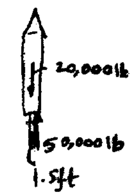
Since $F_f < (F_f)_{max} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular disk <u>does not slip.</u> **Ans**

17-90. The rocket has a weight of 20 000 lb, mass center at G, and radius of gyration about the mass center of $k_G = 21$ ft when it is fired. Each of its two engines provides a thrust $T = 50\,000$ lb. At a given instant, engine A suddenly fails to operate. Determine the angular acceleration of the rocket and the acceleration of its nose B.

$+\Sigma M_G = I_G\alpha;\qquad 50\,000(1.5) = \dfrac{20\,000}{32.2}(21)^2\alpha$

$\alpha = 0.2738\ \text{rad/s}^2 = 0.274\ \text{rad/s}^2$ **Ans**

$+\uparrow\Sigma F_y = m(a_G)_y;\qquad 50\,000 - 20\,000 = \dfrac{20\,000}{32.2}a_G$



$a_G = 48.3\ \text{ft/s}^2$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

Since $\omega = 0$

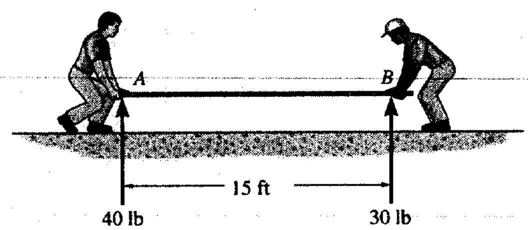
$\mathbf{a}_B = 48.3\mathbf{j} - 0.2738(30)\mathbf{i}$

$= 48.3\mathbf{j} - 8.214\mathbf{i}$

$a_B = \sqrt{(48.3)^2 + (8.214)^2} = 49.0\ \text{ft/s}^2$ **Ans**

$\theta = \tan^{-1}\dfrac{48.3}{8.214} = 80.3^\circ$

***17-91.** Two men exert constant vertical forces of 40 lb and 30 lb at the ends A and B of a uniform plank which has a weight of 50 lb. If the plank is originally at rest, determine the acceleration of its center and its angular acceleration. Assume the plank to be a slender rod.



Equation of Motion : The mass moment of inertia of the plank about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{50}{32.2}\right)(15^2) = 29.115\ \text{slug}\cdot\text{ft}^2$. Applying Eq. 17 − 14, we have

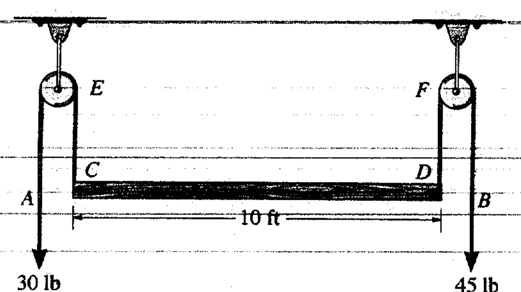
$+\uparrow\Sigma F_y = m(a_G)_y;\qquad 40 + 30 - 50 = \left(\dfrac{50}{32.2}\right)a_G$

$a_G = 12.9\ \text{ft/s}^2$ **Ans**

$\circlearrowleft + \Sigma M_G = I_G\alpha;\qquad 30(7.5) - 40(7.5) = -29.115\alpha$

$\alpha = 2.58\ \text{rad/s}^2$ **Ans**

***17-92.** The uniform 50-lb board is suspended from cords at C and D. If these cords are subjected to constant forces of 30 lb and 45 lb, respectively, determine the acceleration of the board's center and the board's angular acceleration. Assume the board is a thin plate. Neglect the mass of the pulleys at E and F.



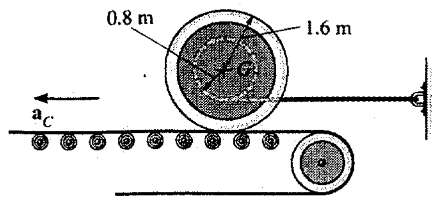
$+\uparrow\Sigma F_y = m(a_G)_y;\qquad 45 + 30 - 50 = \dfrac{50}{32.2}a_G$

$a_G = 16.1\ \text{ft/s}^2$ **Ans**

$\circlearrowleft + \Sigma M_G = I_G\alpha;\qquad -30(5) + 45(5) = \left[\dfrac{1}{12}\left(\dfrac{50}{32.2}\right)(10)^2\right]\alpha$

$\alpha = 5.80\ \text{rad/s}^2$ **Ans**

17-93. The spool has a mass of 500 kg and a radius of gyration $k_G = 1.30$ m. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.4$. If the conveyor accelerates at $a_C = 1 \text{ m/s}^2$, determine the initial tension in the wire and the angular acceleration of the spool. The spool is originally at rest.



$\xrightarrow{+} \sum F_x = m(a_G)_x; \quad -F_s + T = 500a_G$

$+\uparrow \sum F_y = m(a_G)_y; \quad N_s - 500(9.81) = 0$

$\curvearrowleft + \sum M_G = I_G\alpha; \quad F_s(1.6) - T(0.8) = 500(1.30)^2\alpha$

$\mathbf{a}_P = \mathbf{a}_G + \mathbf{a}_{P/G}$

$(a_P)_y\mathbf{j} = a_G\mathbf{i} - 0.8\alpha\mathbf{i}$

$a_G = 0.8\alpha$

$N_s = 4905$ N

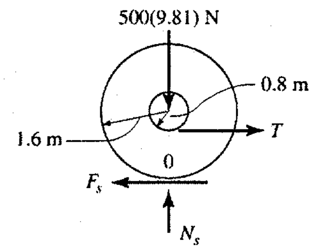
Assume no slipping

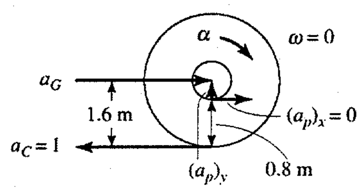
$\alpha = \dfrac{a_C}{0.8} = \dfrac{1}{0.8} = 1.25$ rad/s **Ans**

$a_G = 0.8(1.25) = 1 \text{ m/s}^2$

$T = 2.32$ kN **Ans**

$F_s = 1.82$ kN



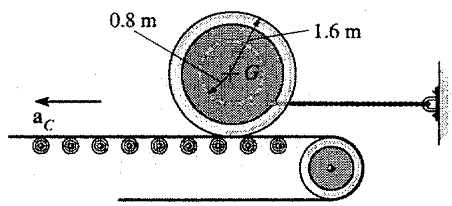


Since

$(F_s)_{max} = 0.5(4.905) = 2.45 > 1.82$

(No slipping occurs)

17-94. The spool has a mass of 500 kg and a radius of gyration $k_G = 1.30$ m. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s = 0.5$. Determine the greatest acceleration a_C of the conveyor so that the spool will not slip. Also, what are the initial tension in the wire and the angular acceleration of the spool? The spool is originally at rest.



$\xrightarrow{+} \sum F_x = m(a_G)_x; \quad T - 0.5N_s = 500a_G$

$+\uparrow \sum F_y = m(a_G)_y; \quad N_s - 500(9.81) = 0$

$\curvearrowleft + \sum M_G = I_G\alpha, \quad 0.5N_s(1.6) - T(0.8) = 500(1.30)^2\alpha$

$a_p = a_C + a_{p/G}$

$(a_p)_y\mathbf{j} = a_G\mathbf{i} - 0.8\alpha\mathbf{i}$

$a_G = 0.8\alpha$

Solving;

$N_s = 4905$ N

$T = 3.13$ kN **Ans**

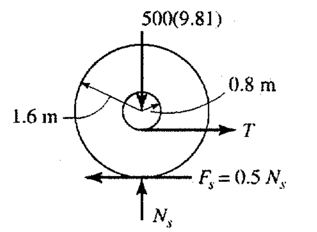
$\alpha = 1.684$ rad/s **Ans**

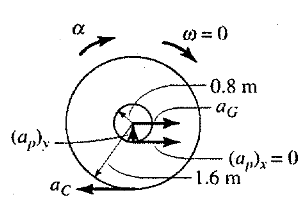
$a_G = 1.347 \text{ m/s}^2$

Since no slipping

$\mathbf{a}_C = \mathbf{a}_G + \mathbf{a}_{C/G}$

$a_C = 1.347\mathbf{i} - (1.684)(1.6)\mathbf{i}$





$a_C = 1.35 \text{ m/s}^2$ **Ans**

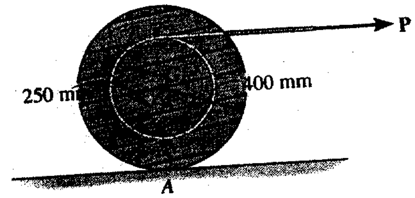
Also,

$\curvearrowleft + \sum M_{IC} = I_{IC}\alpha; \quad 0.5N_s(0.8) = [500(1.30)^2 + 500(0.8)^2]\alpha$

Since $N_s = 4905$ N

$\alpha = 1.684$ rad/s

17-95. The spool has a mass of 100 kg and a radius of gyration of $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 50$ N.



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad 50 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\quad N_A - 100(9.81) = 0$

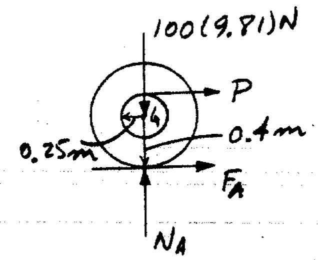
$\circlearrowleft + \Sigma M_G = I_G\alpha;\quad 50(0.25) - F_A(0.4) = \left[100(0.3)^2\right]\alpha$

Assume no slipping : $a_G = 0.4\alpha$

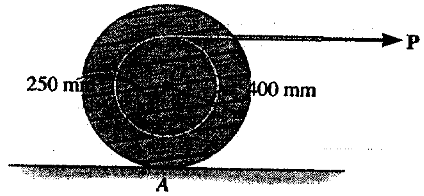
$\alpha = 1.30\ \text{rad/s}^2$ **Ans**

$a_G = 0.520\ \text{m/s}^2 \quad N_A = 981\ \text{N} \quad F_A = 2.00\ \text{N}$

Since $(F_A)_{max} = 0.2(981) = 196.2\ \text{N} > 2.00\ \text{N}$ OK



***17-96.** Solve Prob. 17-95 if the cord and force $P = 50$ N are directed vertically upwards.



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\quad N_A + 50 - 100(9.81) = 0$

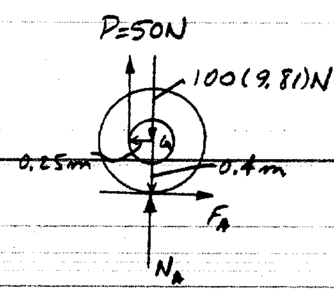
$\circlearrowleft + \Sigma M_G = I_G\alpha;\quad 50(0.25) - F_A(0.4) = \left[100(0.3)^2\right]\alpha$

Assume no slipping : $a_G = 0.4\alpha$

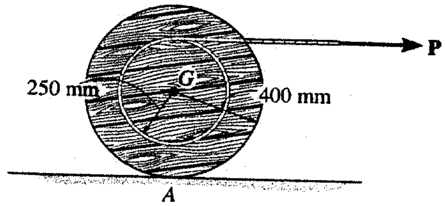
$\alpha = 0.500\ \text{rad/s}^2$ **Ans**

$a_G = 0.2\ \text{m/s}^2 \quad N_A = 931\ \text{N} \quad F_A = 20\ \text{N}$

Since $(F_A)_{max} = 0.2(931) = 186.2\ \text{N} > 20\ \text{N}$ OK



17-97. The spool has a mass of 100 kg and a radius of gyration $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 600$ N.



$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 600 + F_A = 100a_G$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 100(9.81) = 0$$

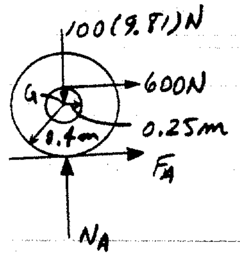
$$\curvearrowleft + \Sigma M_G = I_G\alpha; \quad 600(0.25) - F_A(0.4) = \left[100(0.3)^2\right]\alpha$$

Assume no slipping : $a_G = 0.4\alpha$

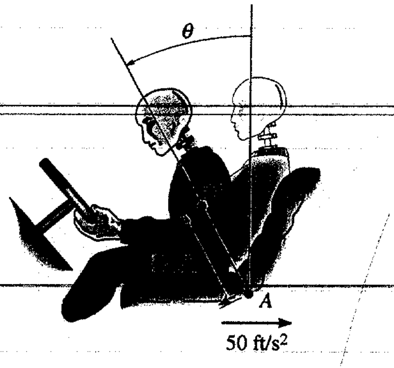
$\alpha = 15.6 \text{ rad/s}^2$ **Ans**

$a_G = 6.24 \text{ m/s}^2$ $\quad N_A = 981$ N $\quad F_A = 24.0$ N

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 24.0 \text{ N}$ OK



17-98. The upper body of the crash dummy has a mass of 75 lb, a center of gravity at G, and a radius of gyration about G of $k_G = 0.7$ ft. By means of the seat belt this body segment is assumed to be pin-connected to the seat of the car at A. If a crash causes the car to decelerate at 50 ft/s², determine the angular velocity of the body when it has rotated to $\theta = 30°$.



$$\curvearrowright + \Sigma M_A = \Sigma(M_k)_A; \quad 75(1.9\sin\theta) = \left[\left(\frac{75}{32.2}\right)(0.7)^2\right]\alpha + \left[\frac{75}{32.2}(a_G)_t\right](1.9)$$

$$\cancel{+\swarrow\ \mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_n + (\mathbf{a}_{G/A})_t}$$

$$(a_G)_t = -50\cos\theta + 0 + (\alpha)(1.9)$$

$$142.5\sin\theta = 1.1413\alpha - 221.273\cos\theta + 8.4084\alpha$$

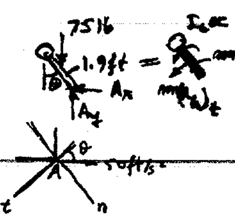
$$142.5\sin\theta + 221.273\cos\theta = 9.5497\alpha$$

$$\omega\, d\omega = \alpha\, d\theta$$

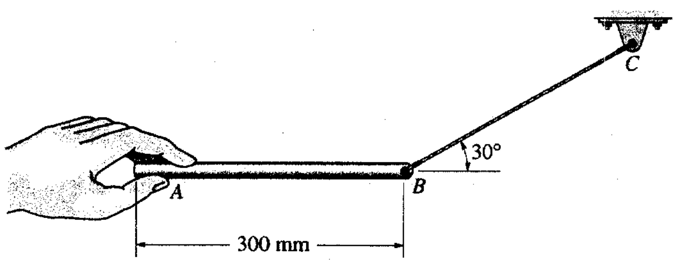
$$\int_0^{\omega} \omega\, d\omega = \int_0^{30°} (14.922\sin\theta + 23.17\cos\theta)d\theta$$

$$\frac{1}{2}\omega^2 = -14.922(\cos 30° - \cos 0°) + 23.17(\sin 30° - \sin 0°)$$

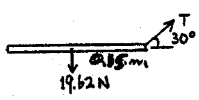
$\omega = 5.21$ rad/s **Ans**



17-99. The 2-kg slender bar is supported by cord BC and then released from rest at A. Determine the initial angular acceleration of the bar and the tension in the cord.



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;$ $\quad T\cos 30^\circ = 2(a_G)_x$



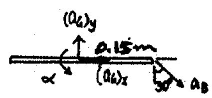
$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad T\sin 30^\circ - 19.62 = 2(a_G)_y$

$\circlearrowleft + \Sigma M_G = I_G\alpha;$ $\quad T\sin 30^\circ(0.15) = [\frac{1}{12}(2)(0.3)^2]\alpha$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$a_B \sin 30^\circ \mathbf{i} - a_B \cos 30^\circ \mathbf{j} = (a_G)_x \mathbf{i} + (a_G)_y \mathbf{j} + \alpha(0.15)\mathbf{j}$

$(\xrightarrow{+})$ $\quad (a_B)\sin 30^\circ = (a_G)_x$



$(+\uparrow)$ $\quad (a_B)\cos 30^\circ = -(a_G)_y - \alpha(0.15)$

Thus,

$1.7321(a_G)_x = -(a_G)_y - 0.15\alpha$

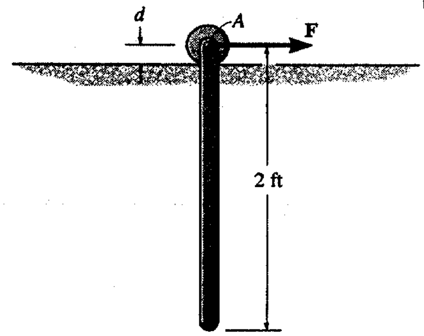
$T = 5.61 \text{ N}$ Ans

$(a_G)_x = 2.43 \text{ m/s}^2$

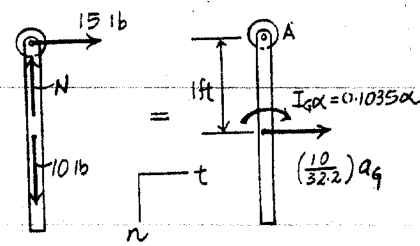
$(a_G)_y = -8.41 \text{ m/s}^2$

$\alpha = 28.0 \text{ rad/s}^2$ Ans

***17-100.** A uniform rod having a weight of 10 lb is pin-supported at A from a roller which rides on a horizontal track. If the rod is originally at rest, and a horizontal force of $F = 15$ lb is applied to the roller, determine the acceleration of the roller. Neglect the mass of the roller and its size d in the computations.



Equation of Motion : The mass moment of inertia of the rod about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035\ \text{slug}\cdot\text{ft}^2$. At the instant force $\mathbf{F}$ is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17 − 16, we have

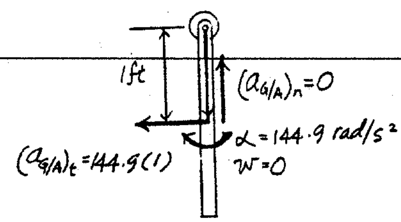


$$\Sigma F_t = m(a_G)_t; \qquad 15 = \left(\frac{10}{32.2}\right)a_G \qquad a_G = 48.3\ \text{ft/s}^2$$

$$\curvearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 0 = \left(\frac{10}{32.2}\right)(48.3)(1) - 0.1035\alpha$$

$$\alpha = 144.9\ \text{rad/s}^2$$

Kinematic : Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of roller A can be obtain by analyzing the motion of points A and G. Applying Eq. 16 − 17, we have



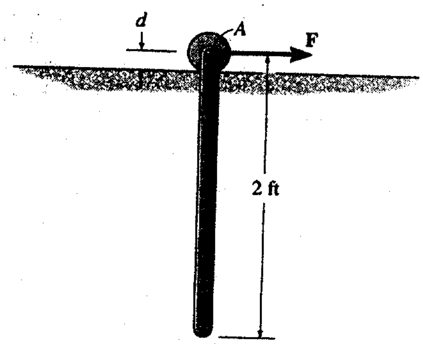
$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

$$\left[\underset{\rightarrow}{48.3}\right] = \left[\underset{\rightarrow}{a_A}\right] + \left[\underset{\leftarrow}{144.9(1)}\right] + \left[0\right]$$

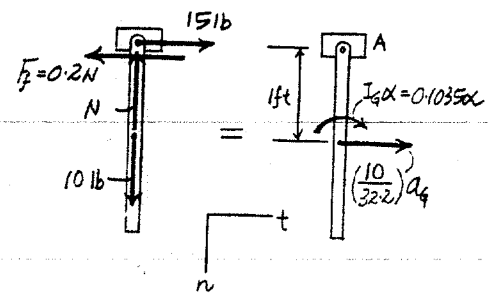
$$(\overset{+}{\rightarrow}) \qquad 48.3 = a_A - 144.9$$

$$a_A = 193\ \text{ft/s}^2 \qquad \textbf{Ans}$$

17-101. Solve Prob. 17–100 assuming that the roller at A is replaced by a slider block having a negligible mass. The coefficient of kinetic friction between the block and the track is $\mu_k = 0.2$. Neglect the dimension d and the size of the block in the computations.



Equation of Motion : The mass moment of inertia of the rod about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035\ \text{slug}\cdot\text{ft}^2$. At the instant force **F** is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17 – 16, we have



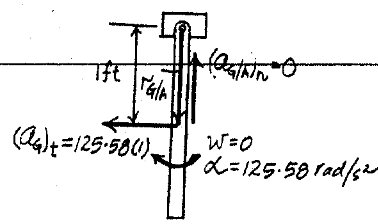
$\Sigma F_n = m(a_G)_n;\qquad 10 - N = 0\qquad N = 10.0\ \text{lb}$

$\Sigma F_t = m(a_G)_t;\qquad 15 - 0.2(10.0) = \left(\frac{10}{32.2}\right)a_G\qquad a_G = 41.86\ \text{ft/s}^2$

$\curvearrowleft + \Sigma M_A = \Sigma (M_k)_A;\qquad 0 = \left(\frac{10}{32.2}\right)(41.86)(1) - 0.1035\alpha$

$\alpha = 125.58\ \text{rad/s}^2$

Kinematic : Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of block A can be obtain by analyzing the motion of points A and G. Applying Eq. 16 – 17, we have



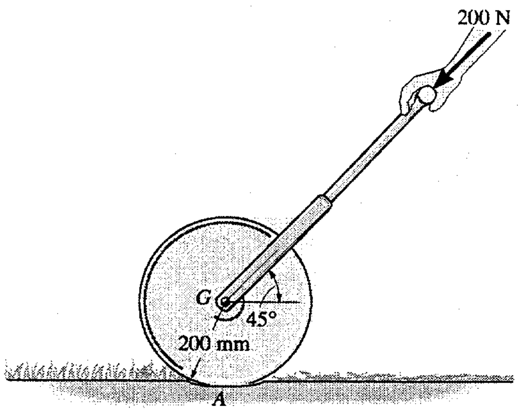
$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

$$\begin{bmatrix} 41.86 \\ \rightarrow \end{bmatrix} = \begin{bmatrix} a_A \\ \rightarrow \end{bmatrix} + \begin{bmatrix} 125.58(1) \\ \leftarrow \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$(\overset{+}{\rightarrow})\qquad 41.86 = a_A - 125.58$

$a_A = 167\ \text{ft/s}^2$ **Ans**

17-102. The lawn roller has a mass of 80 kg and a radius of gyration $k_G = 0.175$ m. If it is pushed forward with a force of 200 N when the handle is at 45°, determine its angular acceleration. The coefficients of static and kinetic friction between the ground and the roller are $\mu_s = 0.12$ and $\mu_k = 0.1$, respectively.



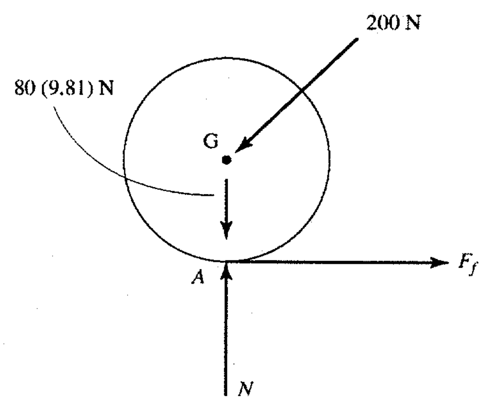
Assume no slipping.

$I_A = I_G + md^2 = 80(0.175)^2 + 80(0.200)^2 = 5.65\ \text{kg}\cdot\text{m}^2$

$M_A = 200\cos 45°(0.200) = 28.284\ \text{N}\cdot\text{m}$

$M_A = I_A\alpha$

$\alpha = \dfrac{28.284}{5.65} = 5.01\ \text{rad/s}^2$ **Ans**



Check no slippage assumption.

$(\uparrow +)\sum F_y = 0 \quad -200\sin 45° - 80(9.81) + N = 0$

$N = 926.22\ \text{N}$

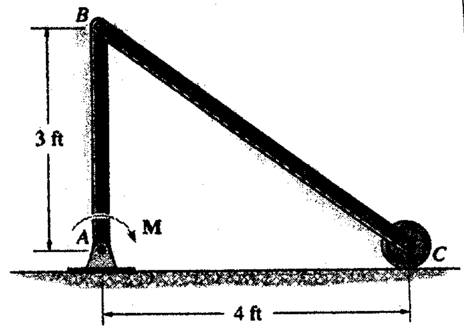
$(F_f)_{max} = 0.12(926.22) = 111.15\ \text{N}$

$I_G = 80(0.175)^2 = 2.45\ \text{kg}\cdot\text{m}^2$

$M_G = I_G\alpha = 2.45(5.01) = 12.27\ \text{N}\cdot\text{m}$

$F_f = \dfrac{12.27}{0.200} = 61.4\ \text{N} < 111.15\ \text{N}$ **O.K.**

17-103. The two pin-connected bars each have a weight of 10 lb/ft. If a moment of $M = 60$ lb·ft is applied to bar AB, determine the initial vertical reaction at C and the horizontal and vertical components of reaction at B. Neglect the size of the roller at C. The bars are initially at rest.



Equation of Motion : The mass moment of inertia of rod AB and BC about their mass center are given by $(I_G)_{AB} = \frac{1}{12}\left(\frac{30}{32.2}\right)(3^2) = 0.6988$ slug · ft^2 and $(I_G)_{BC} = \frac{1}{12}\left(\frac{50}{32.2}\right)(5^2) = 3.2350$ slug · ft^2. At the instant moment M is applied, the angular velocity of the rod AB, $\omega_{AB} = 0$. Thus, the normal component of acceleration of the mass center for the rod AB $\left[(a_G)_{AB}\right]_n = 0$. Since rod AB is rotating about fixed point A, $(a_G)_{AB} = \left[(a_G)_{AB}\right]_t = \alpha_{AB} r_{AG} = 1.5\alpha_{AB}$. Applying Eq. 17 – 16 to FBD(a), we have

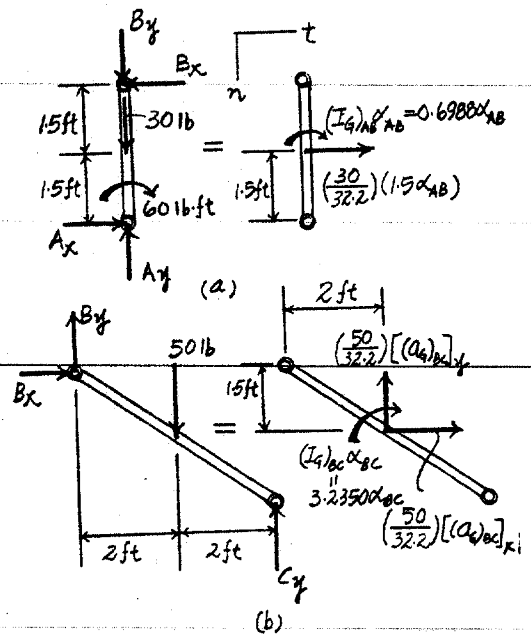
$$+\Sigma M_A = \Sigma(M_k)_A; \quad B_x(3) - 60 = -0.6988\alpha_{AB} - \left(\frac{30}{32.2}\right)(1.5\alpha_{AB})(1.5) \qquad [1]$$

Since rod AB is rotating about fixed point A and $\omega_{AB} = 0$, the normal and tangential components of acceleration of point B are given by $(a_B)_n = \omega_{AB}^2 r_{AB} = 0$ and $(a_B)_t = \alpha_{AB} r_{AB} = 3\alpha_{AB}$. Applying Eq. 17 – 16 to FBD(b), we have

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad B_x = \left(\frac{50}{32.2}\right)\left[(a_G)_{BC}\right]_x \qquad [2]$$

$$+\uparrow \Sigma F_y = m(a_G)_t; \qquad C_y + B_y - 50 = \left(\frac{50}{32.2}\right)\left[(a_G)_{BC}\right]_y \qquad [3]$$

$$\curvearrowleft + \Sigma M_B = \Sigma(M_k)_B; \quad C_y(4) - 50(2) = \left(\frac{50}{32.2}\right)\left[(a_G)_{BC}\right]_x(1.5) + \left(\frac{50}{32.2}\right)\left[(a_G)_{BC}\right]_y(2) - 3.2350\alpha_{BC} \qquad [4]$$



Kinematic : It can be proven that $\omega_{BC} = 0$. Analyzing the motion of points B and C by applying Eq. 16 – 18 with $\mathbf{r}_{C/B} = \{-4\mathbf{i} - 3\mathbf{j}\}$ ft, we have

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$
$$a_C\mathbf{i} = 3\alpha_{AB}\mathbf{i} + (-\alpha_{BC}\mathbf{k}) \times (-4\mathbf{i} - 3\mathbf{j}) - \mathbf{0}$$
$$a_C\mathbf{i} = (3\alpha_{AB} - 3\alpha_{BC})\mathbf{i} + 4\alpha_{BC}\mathbf{j}$$

Equating $\mathbf{j}$ components, we have

$$\alpha_{BC} = 0$$

Analyzing the motion of points B and G by applying Eq. 16 – 18 with $\mathbf{r}_{G/B} = \{-2\mathbf{i} - 1.5\mathbf{j}\}$ ft, we have

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{G/B} - \omega_{BC}^2 \mathbf{r}_{G/B}$$
$$\left[(a_G)_{BC}\right]_x\mathbf{i} + \left[(a_G)_{BC}\right]_y\mathbf{j} = 3\alpha_{AB}\mathbf{i} + 0\mathbf{k} \times (-2\mathbf{i} - 1.5\mathbf{j}) - \mathbf{0}$$
$$\left[(a_G)_{BC}\right]_x\mathbf{i} + \left[(a_G)_{BC}\right]_y\mathbf{j} = 3\alpha_{AB}\mathbf{i}$$

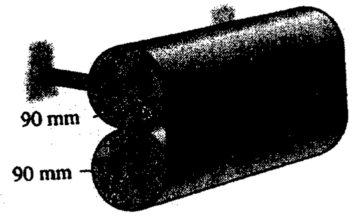
Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

$$\left[(a_G)_{BC}\right]_x = 3\alpha_{AB} \qquad \left[(a_G)_{BC}\right]_y = 0$$

Substitute the results obtained above into Eqs. [1], [2], [3] and [4] and solve yields

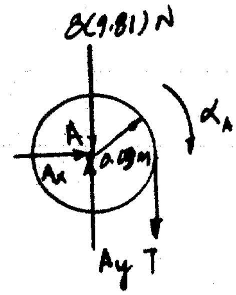
$$B_x = 16.7 \text{ lb} \qquad B_y = 18.75 \text{ lb} \qquad C_y = 31.25 \text{ lb} \qquad \textbf{Ans}$$
$$\alpha_{AB} = 3.578 \text{ rad/s}^2$$

*17-104. A long strip of paper is wrapped into two rolls, each having a mass of 8 kg. Roll A is pin supported about its center whereas roll B is not centrally supported. If B is brought into contact with A and released from rest, determine the initial tension in the paper between the rolls and the angular acceleration of each roll. For the calculation, assume the rolls to be approximated by cylinders.



For roll A.

$\circlearrowleft + \Sigma M_A = I_A \alpha; \quad T(0.09) = \frac{1}{2}(8)(0.09)^2 \alpha_A \qquad (1)$



For roll B

$\circlearrowleft + \Sigma M_O = \Sigma (M_k)_O; \quad 8(9.81)(0.09) = \frac{1}{2}(8)(0.09)^2 \alpha_B + 8a_B(0.09) \qquad (2)$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad T - 8(9.81) = -8a_B \qquad (3)$

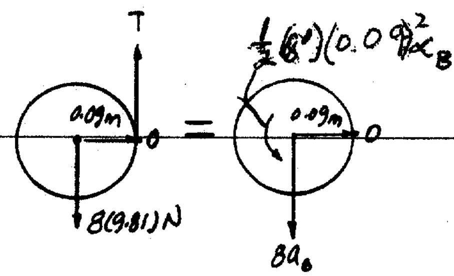
Kinematics :

$$\mathbf{a}_B = \mathbf{a}_O + (\mathbf{a}_{B/O})_t + (\mathbf{a}_{B/O})_n$$

$$\left[\underset{\downarrow}{a_B}\right] = \left[\underset{\downarrow}{a_O}\right] + \left[\underset{\downarrow}{\alpha_B(0.09)}\right] + [0]$$

$(+\downarrow) \qquad a_B = a_O + 0.09\alpha_B \qquad (4)$

also, $\quad (+\downarrow) \qquad a_O = \alpha_A(0.09) \qquad (5)$



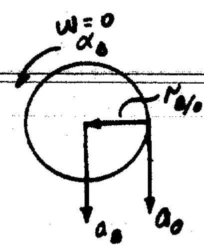
Solving Eqs. (1) – (5) yields :

$\alpha_A = 43.6 \text{ rad/s}^2$ **Ans**

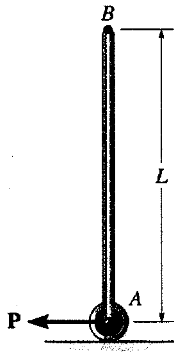
$\alpha_B = 43.6 \text{ rad/s}^2$ **Ans**

$T = 15.7 \text{ N}$ **Ans**

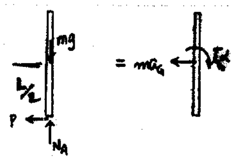
$a_B = 7.85 \text{ m/s}^2 \qquad a_O = 3.92 \text{ m/s}^2$



17-105. The uniform bar of mass m and length L is balanced in the vertical position when the horizontal force **P** is applied to the roller at A. Determine the bar's initial angular acceleration and the acceleration of its top point B.



$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x;\quad P = ma_G$



$\circlearrowleft + \Sigma M_G = I_G\alpha;\quad P(\frac{L}{2}) = (\frac{1}{12}mL^2)\alpha$

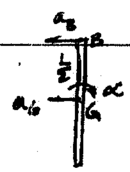
$P = \frac{1}{6}mL\alpha$

$\alpha = \frac{6P}{mL}$ Ans

$a_G = \frac{P}{m}$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$-a_B\mathbf{i} = \frac{-P}{m}\mathbf{i} + \frac{L}{2}\alpha\mathbf{i}$

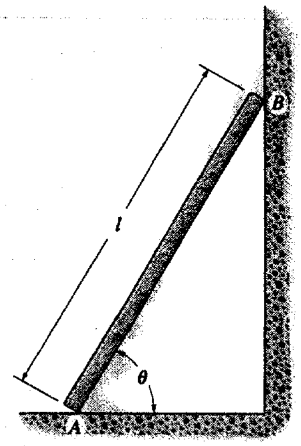


$(\overset{+}{\leftarrow})\quad a_B = \frac{P}{m} - \frac{L\alpha}{2}$

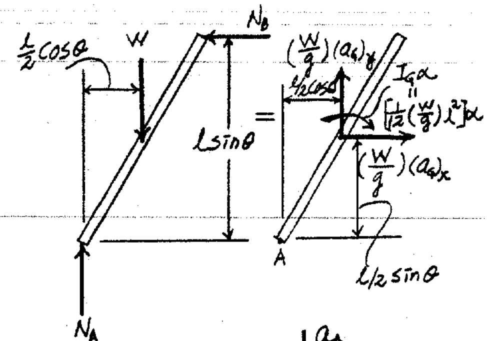
$= \frac{P}{m} - \frac{L}{2}(\frac{6P}{mL})$

$a_B = -\frac{2P}{m} = \frac{2P}{m} \rightarrow$ Ans

***17-106.** The ladder has a weight W and rests against the smooth wall and ground. Determine its angular acceleration as a function of θ when it is released and allowed to slide downward. For the calculation, treat the ladder as a slender rod.



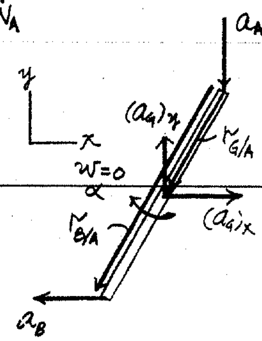
Equation of Motion : The mass moment of inertia of the ladder about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{W}{g}\right)(l^2)$. Applying Eq. 17 – 16, we have



$$\curvearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad N_B(l\sin\theta) - W\left(\frac{l}{2}\cos\theta\right) = -\left[\frac{1}{12}\left(\frac{W}{g}\right)l^2\right]\alpha + \left(\frac{W}{g}\right)(a_G)_y\left(\frac{l}{2}\cos\theta\right) - \left(\frac{W}{g}\right)(a_G)_x\left(\frac{l}{2}\sin\theta\right) \qquad [1]$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad -N_B = \left(\frac{W}{g}\right)(a_G)_x \qquad [2]$$

Kinematic : At the instant the ladder being released, the angular velocity of the ladder, $\omega = 0$. Analysing the motion of points A and B by applying Eq. 16 – 18 with $\mathbf{r}_{B/A} = -l\cos\theta\mathbf{i} - l\sin\theta\mathbf{j}$, we have



$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}\times\mathbf{r}_{B/A} - \omega^2\mathbf{r}_{B/A}$$
$$-a_B\mathbf{i} = -a_A\mathbf{j} + (-\alpha\mathbf{k})\times(-l\cos\theta\mathbf{i} - l\sin\theta\mathbf{j}) - \mathbf{0}$$
$$-a_B\mathbf{i} = -(l\sin\theta)\alpha\mathbf{i} + [(l\cos\theta)\alpha - a_A]\mathbf{j}$$

Equating $\mathbf{j}$ component, we have

$$0 = (l\cos\theta)\alpha - a_A \qquad a_A = (l\cos\theta)\alpha$$

Analysing the motion of points A and G by applying Eq. 16 – 18 with $\mathbf{r}_{G/A} = -\frac{l}{2}\cos\theta\mathbf{i} - \frac{l}{2}\sin\theta\mathbf{j}$, we have

$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha}\times\mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$
$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -(l\cos\theta)\alpha\mathbf{j} + (-\alpha\mathbf{k})\times\left(-\frac{l}{2}\cos\theta\mathbf{i} - \frac{l}{2}\sin\theta\mathbf{j}\right) - \mathbf{0}$$
$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -\left(\frac{l}{2}\sin\theta\right)\alpha\mathbf{i} - \left(\frac{l}{2}\cos\theta\right)\alpha\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ component, we have

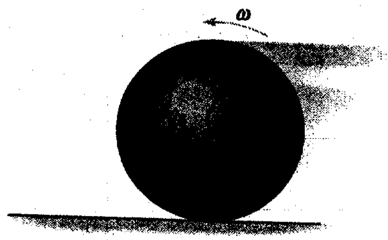
$$(a_G)_x = -\left(\frac{l}{2}\sin\theta\right)\alpha \qquad (a_G)_y = -\left(\frac{l}{2}\cos\theta\right)\alpha$$

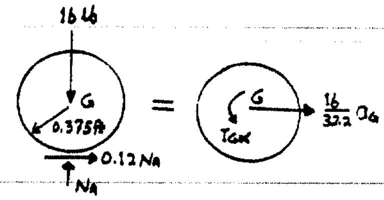
Substitute the results obtained above into Eqs. [1] and [2] and solve yields

$$N_B = \frac{3W}{8}\sin 2\theta$$

$$\alpha = \frac{3g}{2l}\cos\theta \qquad \textbf{Ans}$$

17-107. The 16-lb bowling ball is cast horizontally onto a lane such that initially $\omega = 0$ and its mass center has a velocity $v = 8$ ft/s. If the coefficient of kinetic friction between the lane and the ball is $\mu_k = 0.12$, determine the distance the ball travels before it rolls without slipping. For the calculation, neglect the finger holes in the ball and assume the ball has a uniform density.





$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 0.12N_A = \frac{16}{32.2}a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 16 = 0$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \quad 0.12N_A(0.375) = \left[\frac{2}{5}\left(\frac{16}{32.2}\right)(0.375)^2\right]\alpha$

Solving,

$N_A = 16$ lb; $\quad a_G = 3.864$ ft/s^2; $\quad \alpha = 25.76$ rad/s^2

When the ball rolls without slipping $v = \omega(0.375)$,

$(\circlearrowleft +) \quad \omega = \omega_0 + \alpha_c t$

$\frac{v}{0.375} = 0 + 25.76t$

$v = 9.660t$

$(\overset{+}{\leftarrow}) \quad v = v_0 + a_c t$

$9.660t = 8 - 3.864t$

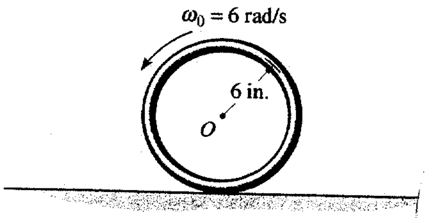
$t = 0.592$ s

$(\overset{+}{\leftarrow}) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 8(0.592) - \frac{1}{2}(3.864)(0.592)^2$

$s = 4.06$ ft **Ans**

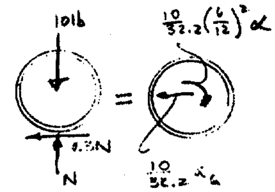
***17-108.** The 10-lb hoop or thin ring is given an initial angular velocity of 6 rad/s when it is placed on the surface. If the coefficient of kinetic friction between the hoop and the surface is $\mu_k = 0.3$, determine the distance the hoop moves before it stops slipping.



$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - 10 = 0 \qquad N = 10 \text{ lb}$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad 0.3(10) = \left(\frac{10}{32.2}\right) a_G \qquad a_G = 9.66 \text{ ft/s}^2$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \quad 0.3(10)\left(\frac{6}{12}\right) = \left(\frac{10}{32.2}\right)\left(\frac{6}{12}\right)^2 \alpha \qquad \alpha = 19.32 \text{ rad/s}^2$



When slipping ceases, $\quad v_G = \omega r = 0.5\omega \qquad$ [1]

$(\circlearrowleft +) \qquad \omega = \omega_0 + \alpha t$

$\omega = 6 + (-19.32)t \qquad$ [2]

$(\overset{+}{\leftarrow}) \qquad v_G = (v_G)_0 + a_G t$

$v_G = 0 + 9.66t \qquad$ [3]

Solving Eqs.[1] to [3] yields :

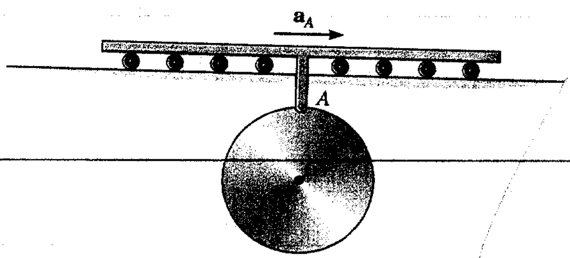
$t = 0.1553 \text{ s} \qquad v_G = 1.5 \text{ ft/s} \qquad \omega = 3 \text{ rad/s}$

$(\overset{+}{\leftarrow}) \qquad s = s_0 + (v_G)_0 t + \frac{1}{2} a_G t^2$

$= 0 + 0 + \frac{1}{2}(9.66)(0.1553)^2$

$= 0.116 \text{ ft} = 1.40 \text{ in.} \qquad$ **Ans**

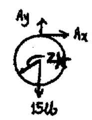
17-109. The 15-lb circular plate is suspended from a pin at A. If the pin is connected to a track which is given an acceleration $a_A = 3 \text{ ft/s}^2$, determine the horizontal and vertical components of reaction at A and the acceleration of the plate's mass center G. The plate is originally at rest.



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad A_x = \frac{15}{32.2}(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad A_y - 15 = \frac{15}{32.2}(a_G)_y$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \quad A_x(2) = \left[\frac{1}{2}\left(\frac{15}{32.2}\right)(2)^2\right]\alpha$

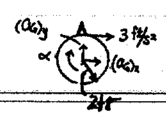


$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}$

$\mathbf{a}_G = 3\mathbf{i} - 2\alpha\,\mathbf{i}$

$(+\uparrow) \quad (a_G)_y = 0$

$(\overset{+}{\rightarrow}) \quad (a_G)_x = 3 - 2\alpha$



Thus,

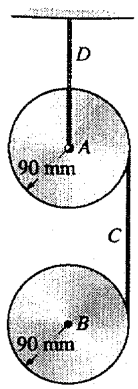
$A_y = 15.0 \text{ lb} \qquad$ **Ans**

$A_x = 0.466 \text{ lb} \qquad$ **Ans**

$\alpha = 1 \text{ rad/s}^2$

$a_G = (a_G)_x = 1.00 \text{ ft/s}^2 \qquad$ **Ans**

17-110. A cord C is wrapped around each of the two 10-kg disks. If they are released from rest, determine the tension in the fixed cord D. Neglect the mass of the cord.



For A:

$$\circlearrowleft + \sum M_A = I_A\alpha_A; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_A \qquad (1)$$

For B:

$$\circlearrowleft + \sum M_B = I_B\alpha_B; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_B \qquad (2)$$

$$+\downarrow \sum F_y = m(a_B)_y; \qquad 10(9.81) - T = 10a_B \qquad (3)$$

$$a_B = a_P + (a_{B/P})_t + (a_{B/P})_n$$

$$(+\downarrow) a_B = 0.09\alpha_A + 0.09\alpha_B + 0 \qquad (4)$$

Solving,

$a_B = 7.85\ \text{m/s}^2$

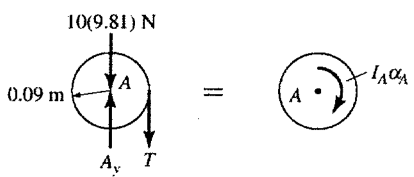
$\alpha_A = 43.6\ \text{rad/s}^2$

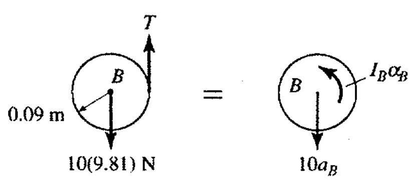
$\alpha_B = 43.6\ \text{rad/s}^2$

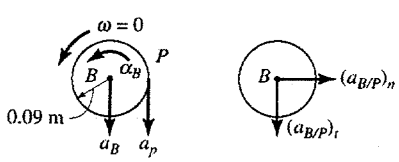
$T = 19.62\ \text{N}$

$A_y = 10(9.81) + 19.62$

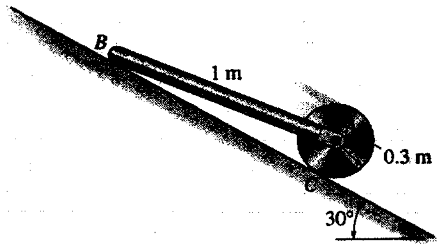
$= 118\ \text{N}$ **Ans**

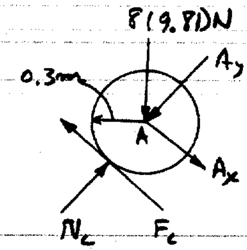






■**17-111.** The assembly consists of an 8-kg disk and a 10-kg bar which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are $\mu_s = 0.6$ and $\mu_k = 0.4$, respectively. Neglect friction at B.



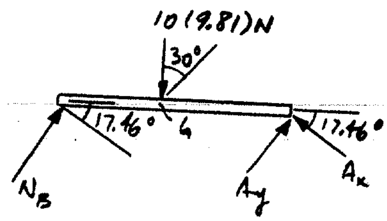


Disk :

$+\searrow \Sigma F_x = m(a_G)_x$; $A_x - F_C + 8(9.81)\sin 30° = 8a_G$ (1)

$+\nearrow \Sigma F_y = m(a_G)_y$; $N_C - A_y - 8(9.81)\cos 30° = 0$ (2)

$\circlearrowleft + \Sigma M_A = I_A\alpha$; $F_C(0.3) = \left[\frac{1}{2}(8)(0.3)^2\right]\alpha$ (3)



Bar :

$+\searrow \Sigma F_x = m(a_G)_x$; $10(9.81)\sin 30° - A_x = 10a_G$ (4)

$+\nearrow \Sigma F_y = m(a_G)_y$; $N_B + A_y - 10(9.81)\cos 30° = 0$ (5)

$\circlearrowleft + \Sigma M_G = I_G\alpha$; $-N_B(0.5\cos 17.46°) + A_x(0.5\sin 17.46°) + A_y(0.5\cos 17.46°) = 0$ (6)

Assume no slipping of the disk,

$a_G = 0.3\alpha$ (7)

Solving, Eqs. (1) – (7),

$A_x = 8.92$ N, $A_y = 41.1$ N, $N_B = 43.9$ N

$a_G = 4.01\ \text{m/s}^2$

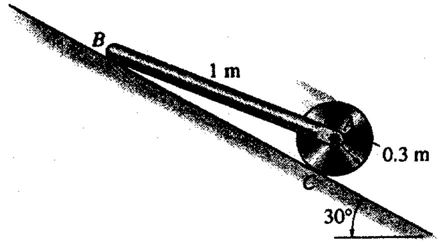
$\alpha = 13.4\ \text{rad/s}^2$ **Ans**

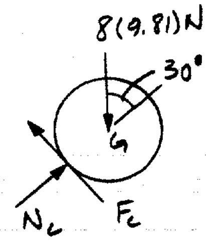
$N_C = 109$ N

$F_C = 16.1$ N

$(F_C)_{max} = 0.6(109) = 65.4\ \text{N} > 16.1\ \text{N}$ OK

*17-112. Solve Prob. 17-111 if the bar is removed. The coefficients of static and kinetic friction between the disk and inclined plane are $\mu_s = 0.15$ and $\mu_k = 0.1$, respectively.

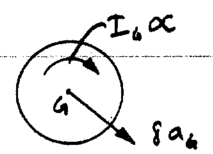




$+\searrow \Sigma F_x = m(a_G)_x$; $\quad 8(9.81)\sin 30° - F_C = 8a_G \qquad (1)$

$+\nearrow \Sigma F_y = m(a_G)_y$; $\quad -8(9.81)\cos 30° + N_C = 0 \qquad (2)$

$\circlearrowleft + \Sigma M_G = I_G\alpha$; $\quad F_C(0.3) = \left[\frac{1}{2}(8)(0.3)^2\right]\alpha \qquad (3)$



Assume no slipping : $\quad a_G = 0.3\alpha$

Solving Eqs. (1) – (3) :

$N_C = 67.97$ N

$a_G = 3.27$ m/s^2

$\alpha = 10.9$ rad/s^2

$F_C = 13.08$ N

$(F_C)_{max} = 0.15(67.97) = 10.2 \text{ N} < 13.08 \text{ N} \qquad$ NG

Slipping occurs

$F_C = 0.1N_C$

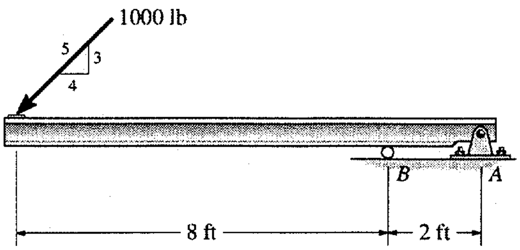
Solving Eqs. (1) – (3) :

$N_C = 67.97$ N

$\alpha = 5.66$ rad/s^2 $\qquad$ **Ans**

$a_G = 4.06$ m/s^2

17-113. The 500-lb beam is supported at A and B when it is subjected to a force of 1000 lb as shown. If the pin support at A suddenly fails, determine the beam's initial angular acceleration and the force of the roller support on the beam. For the calculation, assume that the beam is a slender rod so that its thickness can be neglected.



$\overset{+}{\leftarrow}\sum F_x = m(a_G)_x;$ $\qquad 1000\left(\frac{4}{5}\right) = \frac{500}{32.2}(a_G)_x$

$+\downarrow \sum F_y = m(a_G)_y;$ $\qquad 1000\left(\frac{3}{5}\right) + 500 - B_y = \frac{500}{32.2}(a_G)_y$

$\curvearrowleft + \sum M_B = \sum (M_k)_B;$ $\quad 500(3) + 1000\left(\frac{3}{5}\right)(8) = \frac{500}{32.2}(a_G)_y(3)$

$$+\left[\frac{1}{12}\left(\frac{500}{32.2}\right)(10)^2\right]\alpha$$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$-a_B\mathbf{i} = -(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} + \alpha(3)\mathbf{j}$

$(+\downarrow)\quad (a_G)_y = \alpha(3)$

$\alpha = 23.4 \text{ rad/s}^2$ **Ans**

$B_y = 9.62$ lb **Ans**

$B_y > 0$ means that the beam stays in contact with the roller support.