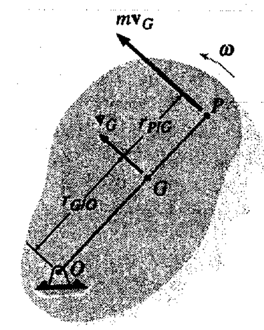
19-1. The rigid body (slab) has a mass m and is rotating with an angular velocity $\boldsymbol{\omega}$ about an axis passing through the fixed point O. Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude mv_G and acting through point P, called the *center of percussion*, which lies at a distance $r_{P/G} = k_G^2/r_{G/O}$ from the mass center G. Here k_G is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through G.

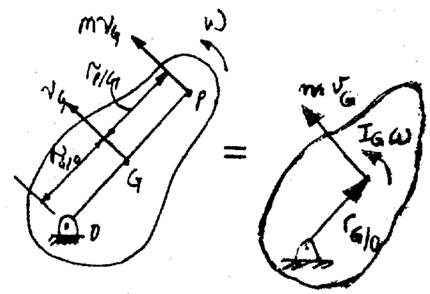


$$H_O = (r_{G/O} + r_{P/G})mv_G = r_{G/O}(mv_G) + I_G\omega, \quad \text{where } I_G = mk_G^2$$

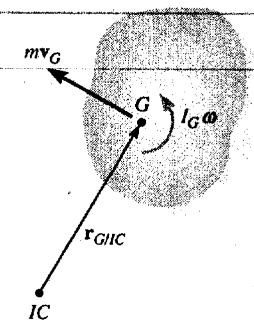
$$r_{G/O}(mv_G) + r_{P/G}(mv_G) = r_{G/O}(mv_G) + (mk_G^2)\omega$$

$$r_{P/G} = \frac{k_G^2}{v_G/\omega} \quad \text{However, } v_G = \omega r_{G/O} \text{ or } r_{G/O} = \frac{v_G}{\omega}$$

$$r_{P/G} = \frac{k_G^2}{r_{G/O}} \quad \textbf{Q.E.D.}$$



19-2. At a given instant, the body has a linear momentum $\mathbf{L} = m\mathbf{v}_G$ and an angular momentum $\mathbf{H}_G = I_G\boldsymbol{\omega}$ computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity IC can be expressed as $\mathbf{H}_{IC} = I_{IC}\boldsymbol{\omega}$, where I_{IC} represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the IC is located at a distance $r_{G/IC}$ away from the mass center G.



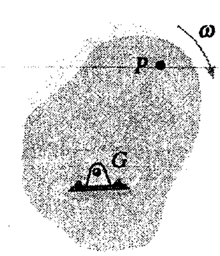
$$H_{IC} = r_{G/IC}(mv_G) + I_G\omega, \quad \text{where } v_G = \omega r_{G/IC}$$

$$= r_{G/IC}(m\omega r_{G/IC}) + I_G\omega$$

$$= (I_G + mr_{G/IC}^2)\omega$$

$$= I_{IC}\omega \quad \textbf{Q.E.D.}$$

19-3. Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center G, the angular momentum is the same when computed about any other point P on the slab.

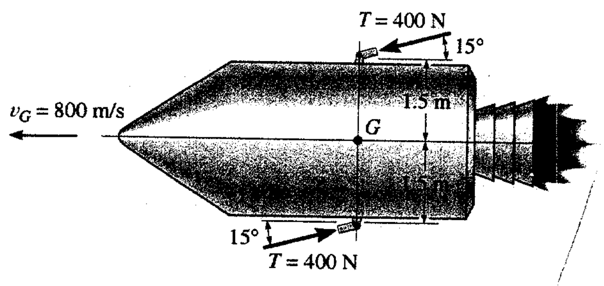


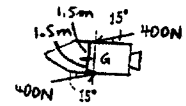
Since $v_G = 0$, the linear momentum $L = mv_G = 0$. Hence the angular momentum about any point P is

$$H_P = I_G\omega$$

Since ω is a free vector, so is $\mathbf{H}_P$. **Q.E.D.**

***19-4.** The space capsule has a mass of 1200 kg and a moment of inertia $I_G = 900\ \text{kg}\cdot\text{m}^2$ about an axis passing through G and directed perpendicular to the page. If it is traveling forward with a speed $v_G = 800$ m/s and executes a turn by means of two jets, which provide a constant thrust of 400 N for 0.3 s, determine the capsule's angular velocity just after the jets are turned off.





$(\circlearrowleft +) \quad (H_G)_1 + \Sigma\int M_G dt = (H_G)_2$

$0 + 2[400\cos 15°(0.3)(1.5)] = 900\,\omega_2$

$\omega_2 = 0.386$ rad/s **Ans**

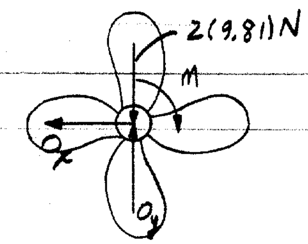
19-5. Solve Prob. 17-55 using the principle of impulse and momentum.

$(\circlearrowleft +) \quad (H_O)_1 + \Sigma\int M_O\, dt = (H_O)_2$

$0 + \int_0^4 3(1 - e^{-0.2t})dt = (0.18)\omega$

$3(t + 5e^{-0.2t})\big|_0^4 = 0.18\omega$

$\omega = 20.8$ rad/s **Ans**



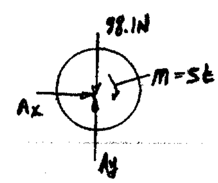
19-6. Solve Prob. 17–54 using the principle of impulse and momentum.

$(H_A)_1 + \Sigma\int M_A dt = (H_A)_2$

$0 + \int_0^3 5\,t\,dt = [10(0.2)^2]\omega$

$\frac{5}{2}(3)^2 = 0.4\omega$

$\omega = 56.2$ rad/s **Ans**



$m(v_G)_1 + \Sigma\int F dt = m(v_G)_2$

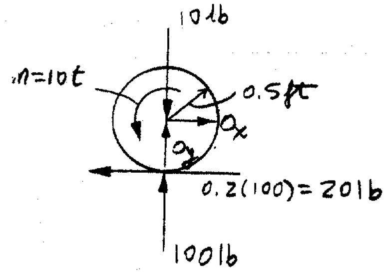
$(\overset{+}{\rightarrow}) \quad 0 + A_x(3) = 0$

$A_x = 0$ **Ans**

$(+\uparrow) \quad 0 + A_y(3) - 98.1(3) = 0$

$A_y = 98.1$ N **Ans**

19-7. Solve Prob. 17-69 using the principle of impulse and momentum.



Time to start motion :

$F = 100(0.3) = 30\text{ lb}$

$+\Sigma M_O = 0; \qquad 10t - 30(0.5) = 0$

$t = 1.5\text{ s}$

$(\circlearrowleft +) \qquad (H_O)_1 + \Sigma\int M_O\, dt = (H_O)_2$

$$0 + \int_{1.5}^{2} 10t\, dt - 20(2-1.5)(0.5) = \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2\right]\omega$$

$$5t^2\big|_{1.5}^{2} - 5 = 0.038820\omega$$

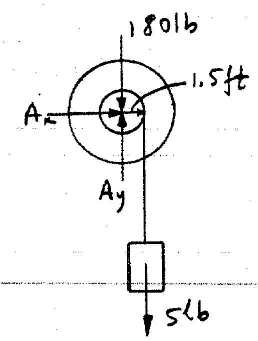
$\omega = 96.6\text{ rad/s}$ **Ans**

***19-8.** Solve Prob. 17-80 using the principle of impulse and momentum.

System :

$v_B = \omega(1.5)$

$(\circlearrowleft +) \qquad (H_A)_1 + \Sigma\int M_A\, dt = (H_A)_2$



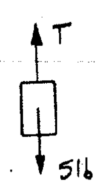
$$0 + 5(1.5)(3) = \left[\left(\frac{180}{32.2}\right)(1.25)^2\right]\omega + \left(\frac{5}{32.2}\right)[\omega(1.5)](1.5)$$

$\omega = 2.48\text{ rad/s}$ **Ans**

Block :

$v_B = \omega(1.5)$

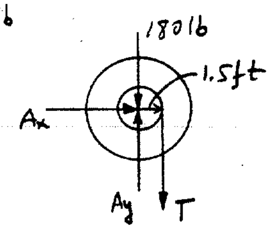
$(+\downarrow) \qquad m(v_G)_1 + \Sigma\int F_y\, dt = m(v_G)_2$



$$0 + 5(3) - T(3) = \frac{5}{32.2}[\omega(1.5)] \qquad (1)$$

Spool :

$(\circlearrowleft +) \qquad (H_A)_1 + \Sigma\int M_A\, dt = (H_A)_2$



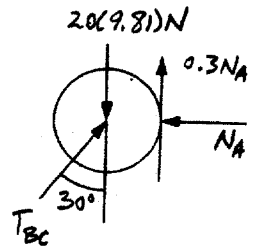
$$0 + T(1.5)(3) = \left[\left(\frac{180}{32.2}\right)(1.25)^2\right]\omega \qquad (2)$$

Solving Eqs. (1) and (2) :

$T = 4.81\text{ lb}$

$\omega = 2.48\text{ rad/s}$ **Ans**

19-9. Solve Prob. 17-73 using the principle of impulse and momentum.



$(\overset{+}{\rightarrow})$ $\quad m(v_{Gx})_1 + \Sigma\int F_x\,dt = m(v_{Gx})_2$

$$0 + T_{BC}\sin 30°(t) - N_A t = 0$$

$$0.5T_{BC} = N_A$$

$(+\uparrow)$ $\quad m(v_{Gy})_1 + \Sigma\int F_y\,dt = m(v_{Gy})_2$

$$0 + T_{BC}\cos 30° t - 20(9.81)t + 0.3N_A t = 0$$

$$0.86603T_{BC} + 0.3N_A = 196.2$$

Solving :

$T_{BC} = 193$ N **Ans**

$N_A = 96.553$ N

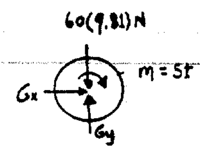
$(\circlearrowleft +)$ $\quad (H_B)_1 + \Sigma\int M_B\,dt = (H_B)_2$

$$\left[\frac{1}{2}(20)(0.15)^2\right](60) - 0.3(96.553)t(0.15) = 0$$

$t = 3.11$ s **Ans**

19-10. A flywheel has a mass of 60 kg and a radius of gyration of $k_G = 150$ mm about an axis of rotation passing through its mass center. If a motor supplies a clockwise torque having a magnitude of $M = (5t)$ N·m, where t is in seconds, determine the flywheel's angular velocity in $t = 3$ s. Initially the flywheel is rotating clockwise at $\omega_1 = 2$ rad/s.

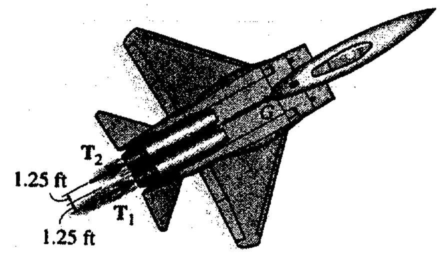
$(\circlearrowleft +)$ $\quad (H_G)_1 + \Sigma\int M\,dt = (H_G)_2$



$$60(0.15)^2(2) + \int_0^3 5t\,dt = 60(0.15)^2\omega$$

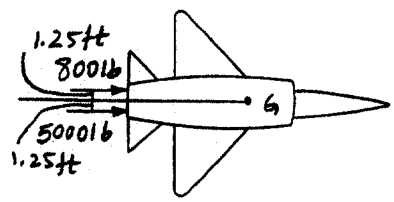
$\omega = 18.7$ rad/s **Ans**

19-11. The pilot of a crippled F-15 fighter was able to control his plane by throttling the two engines. If the plane has a weight of 17 000 lb and a radius of gyration of $k_G = 4.7$ ft about the mass center G, determine the angular velocity of the plane and the velocity of its mass center G in $t = 5$ s if the thrust in each engine is altered to $T_1 = 5000$ lb and $T_2 = 800$ lb as shown. Originally the plane is flying straight at 1200 ft/s. Neglect the effects of drag and the loss of fuel.



$$(\circlearrowleft +) \qquad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$$

$$0 + 5000(5)(1.25) - 800(5)(1.25) = \left[\left(\frac{17\,000}{32.2}\right)(4.7)^2\right]\omega$$



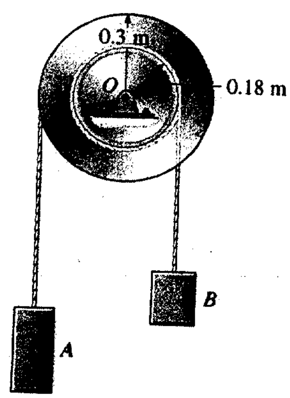
$$\omega = 2.25 \text{ rad/s} \qquad \textbf{Ans}$$

$$(\xrightarrow{+}) \qquad m(v_{Gx})_1 + \Sigma\int F_x\,dt = m(v_{Gx})_2$$

$$\left(\frac{17\,000}{32.2}\right)(1200) + 5800(5) = \left(\frac{17\,000}{32.2}\right)(v_G)_2$$

$$(v_G)_2 = 1.25(10^3) \text{ ft/s} \qquad \textbf{Ans}$$

***19-12.** The spool has a mass of 30 kg and a radius of gyration $k_O = 0.25$ m. Block A has a mass of 25 kg, and block B has a mass of 10 kg. If they are released from rest, determine the time required for block A to attain a speed of 2 m/s. Neglect the mass of the ropes.



$$v_A = 2 \text{ m/s}$$

$$\omega = \frac{2}{0.3} = 6.667 \text{ rad/s}$$

$$v_B = 6.667(0.18) = 1.20 \text{ m/s}$$

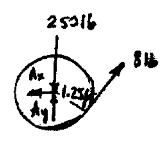
$$\circlearrowleft + \quad (H_O)_1 + \Sigma\int M\,dt = (H_O)_2$$

$$0 + 25(9.81)(0.3)\,t - 10(9.81)(0.18)(t) = 25(2)(0.3) + 30(0.25)^2(6.667) + 10(1.20)(0.18)$$

$$t = 0.530 \text{ s} \qquad \textbf{Ans}$$

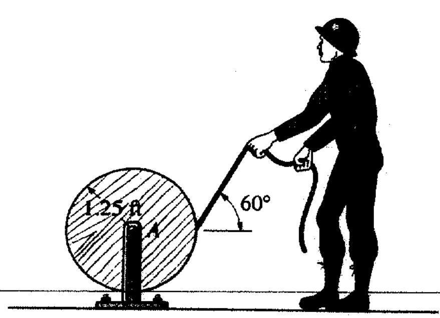
19-13. The man pulls the rope off the reel with a constant force of 8 lb in the direction shown. If the reel has a weight of 250 lb and radius of gyration $k_G = 0.8$ ft about the trunnion (pin) at A, determine the angular velocity of the reel in 3 s starting from rest. Neglect friction and the weight of rope that is removed.

$$\circlearrowleft + \quad (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$$



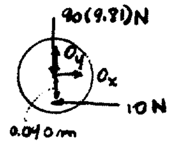
$$0 + 8(1.25)(3) = \left[\frac{250}{32.2}(0.8)^2\right]\omega$$

$$\omega = 6.04 \text{ rad/s} \qquad \textbf{Ans}$$



19-14. Angular motion is transmitted from a driver wheel A to the driven wheel B by friction between the wheels at C. If A always rotates at a constant rate of 16 rad/s, and the coefficient of kinetic friction between the wheels is $\mu_k = 0.2$, determine the time required for B to reach a constant angular velocity once the wheels make contact with a normal force of 50 N. What is the final angular velocity of wheel B? Wheel B has a mass of 90 kg and a radius of gyration about its axis of rotation of $k_G = 120$ mm.

$F = 0.2(50) = 10\text{ N}$



$\circlearrowleft+\quad (H_O)_1 + \Sigma\int M_O dt = (H_O)_2$

$0 + (10)(0.09)\,t = [90(0.120)^2]\,\omega$

$\omega_A r_A = \omega_B r_B$

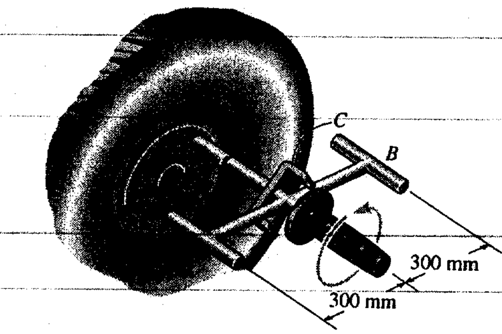
$16(20) = \omega_B(90)$

$\omega_B = 3.556\text{ rad/s} = 3.56\text{ rad/s}$ **Ans**

Thus,

$t = 5.12\text{ s}$ **Ans**

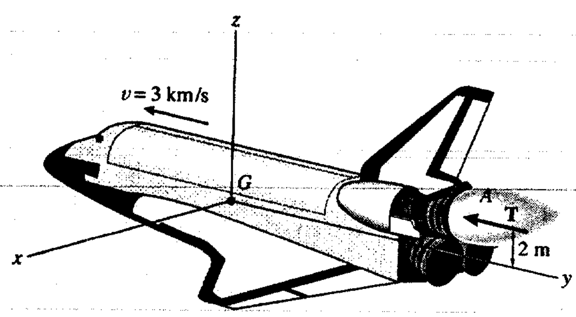
19-15. The impact wrench consists of a slender 1-kg rod AB which is 580 mm long, and cylindrical end weights at A and B that each have a diameter of 20 mm and a mass of 1 kg. This assembly is free to turn about the handle and socket, which are attached to the lug nut on the wheel of a car. If the rod AB is given an angular velocity of 4 rad/s and it strikes the bracket C on the handle without rebounding, determine the angular impulse imparted to the lug nut.



$$I_{axle} = \frac{1}{12}(1)(0.6-0.02)^2 + 2\left[\frac{1}{2}(1)(0.01)^2 + 1(0.3)^2\right] = 0.2081\text{ kg}\cdot\text{m}^2$$

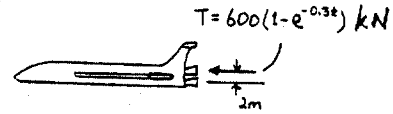
$\int M dt = I_{axle}\,\omega = 0.2081(4) = 0.833\text{ kg}\cdot\text{m}^2/\text{s}$ **Ans**

***19-16.** The space shuttle is located in "deep space," where the effects of gravity can be neglected. It has a mass of 120 Mg, a center of mass at G, and a radius of gyration $(k_G)_x = 14$ m about the x axis. It is originally traveling forward at $v = 3$ km/s when the pilot turns on the engine at A, creating a thrust $T = 600(1 - e^{-0.3t})$ kN, where t is in seconds. Determine the shuttle's angular velocity 2 s later.



$(\circlearrowleft+)\qquad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

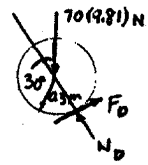
$$0 + \int_0^2 600(10^3)(1-e^{-0.3t})(2)\,dt = [120(10^3)(14)^2]\,\omega$$

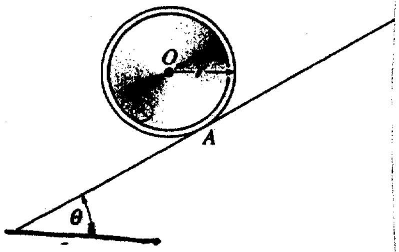


$$1200(10^3)\left[t + \frac{1}{0.3}e^{-0.3t}\right]_0^2 = 120(10^3)(14)^2\,\omega$$

$\omega = 0.0253\text{ rad/s}$ **Ans**

19-17. The drum has a mass of 70 kg, a radius of 300 mm, and radius of gyration $k_O = 125$ mm. If the coefficients of static and kinetic friction at A are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively, determine the drum's angular velocity 2 s after it is released from rest. Take $\theta = 30°$.





$$(\circlearrowleft +) \quad (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$$

$$0 + 70(9.81)(\sin 30°)(0.3)(2) = (70(0.125)^2 + 70(0.3)^2)\omega$$

$\omega = 27.863$ rad/s $= 27.9$ rad/s **Ans**

$$m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

$$(+\swarrow) \quad 0 + 70(9.81)\sin 30°(2) - F(2) = 70(27.863)(0.3)$$

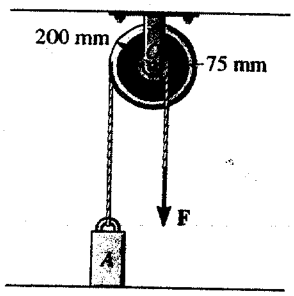
$F = 50.79$ N

$$(+\nwarrow) \quad 0 + N_D(2) - 70(9.81)\cos 30°(2) = 0$$

$N_D = 594.7$ N

$F_{max} = 0.4[(594.7] = 237.87$ N > 50.79 N **OK**

19-18. The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration $k_O = 110$ mm. If the block at A has a mass of 40 kg, determine the speed of the block in 3 s after a constant force $F = 2$ kN is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest. Neglect the mass of the rope.

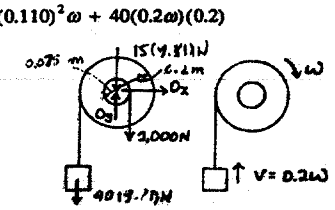


$$(\circlearrowleft +) \quad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$$

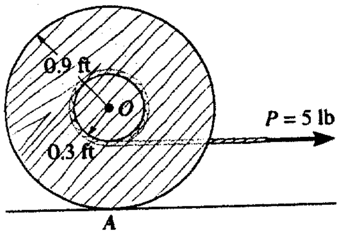
$$0 + 2000(0.075)(3) - 40(9.81)(0.2)(3) = 15(0.110)^2\omega + 40(0.2\omega)(0.2)$$

$\omega = 120.4$ rad/s

$v_A = 0.2(120.4) = 24.1$ m/s **Ans**



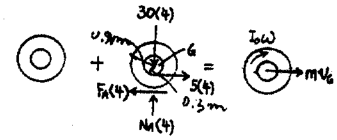
19-19. The spool has a weight of 30 lb and a radius of gyration $k_O = 0.45$ ft. A cord is wrapped around its inner hub and the end subjected to a horizontal force $P = 5$ lb. Determine the spool's angular velocity in 4 s starting from rest. Assume the spool rolls without slipping.



$(+\uparrow)\quad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$

$0 + N_A(4) - 30(4) = 0$

$N_A = 30$ lb



$(\xrightarrow{+})\quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$0 + 5(4) - F_A(4) = \dfrac{30}{32.2}v_G$

$(\circlearrowleft +)\quad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

$0 + F_A(4)(0.9) - 5(4)(0.3) = \dfrac{30}{32.2}(0.45)^2\,\omega$

Since no slipping occurs

Set $v_G = 0.9\omega$

$F_A = 2.33$ lb

$\omega = 12.7$ rad/s **Ans**

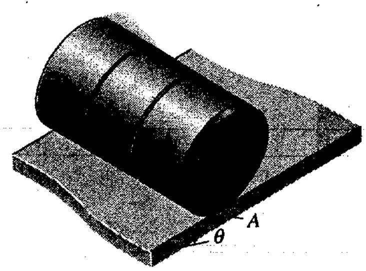
Also,

$(\circlearrowleft +)\quad (H_A)_1 + \Sigma M_A\,dt = (H_A)_2$

$0 + 5(4)(0.6) = \left[\dfrac{30}{32.2}(0.45)^2 + \dfrac{30}{32.2}(0.9)^2\right]\omega$

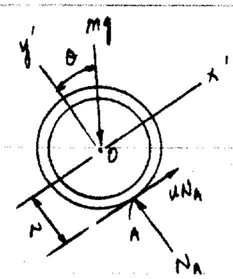
$\omega = 12.7$ rad/s **Ans**

***19-20.** The drum of mass m, radius r, and radius of gyration k_O rolls along an inclined plane for which the coefficient of static friction is μ_s. If the drum is released from rest, determine the maximum angle θ for the incline so that it rolls without slipping.



$(+\nwarrow)\quad m(v_{Oy'})_1 + \Sigma\int_{t_1}^{t_2} F_{y'}\,dt = m(v_{Oy'})_2$

$0 + N_A(t) - mg\cos\theta(t) = 0 \qquad N_A = mg\cos\theta$



$(+\swarrow)\quad m(v_{Ox'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{Ox'})_2$

$0 + mg\sin\theta(t) - \mu_s mg\cos\theta(t) = mv_O \qquad (1)$

Since no slipping occurs, $v_O = \omega r$. Hence Eq.(1) becomes

$mg\sin\theta(t) - \mu_s mg\cos\theta(t) = m\omega r$

$t = \dfrac{m\omega r}{mg(\sin\theta - \mu_s\cos\theta)} \qquad (2)$

$(\circlearrowleft +)\quad I_A\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$

$0 + mg\sin\theta(r)(t) = \left[mk_O^2 + mr^2\right]\omega$

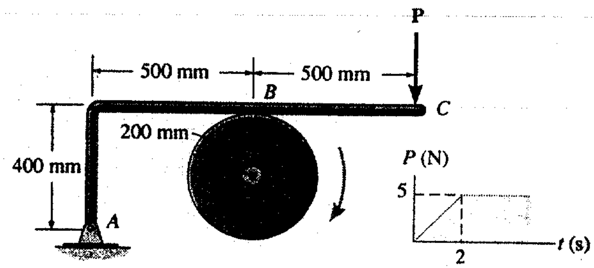
$t = \dfrac{\left[mk_O^2 + mr^2\right]\omega}{mgr\sin\theta} \qquad (3)$

Equating Eqs.(2) and (3)

$\dfrac{m\omega r}{mg(\sin\theta - \mu_s\cos\theta)} = \dfrac{\left[mk_O^2 + mr^2\right]\omega}{mgr\sin\theta}$

$\theta = \tan^{-1}\left[\dfrac{\mu_s(k_O^2 + r^2)}{k_O^2}\right]$ **Ans**

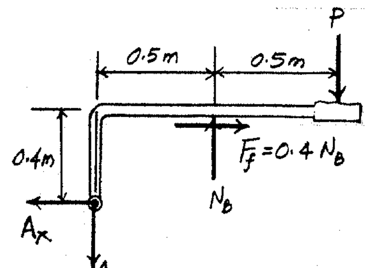
19-21. The 12-kg disk has an angular velocity of $\omega = 20$ rad/s. If the brake ABC is applied such that the magnitude of force $\mathbf{P}$ varies with time as shown, determine the time needed to stop the disk. The coefficient of kinetic friction at B is $\mu_k = 0.4$.



Equations of Equilibrium : Since slipping occurs at B, the friction $F_f = \mu_k N_B = 0.4N_B$. From FBD(a), the normal reaction N_B can be obtained directly by summing moments about point A.

$$\curvearrowleft + \Sigma M_A = 0; \qquad N_B(0.5) - 0.4N_B(0.4) - P(1) = 0$$
$$N_B = 2.941P$$

Thus, the friction $F_f = 0.4N_B = 0.4(2.941P) = 1.176P$.

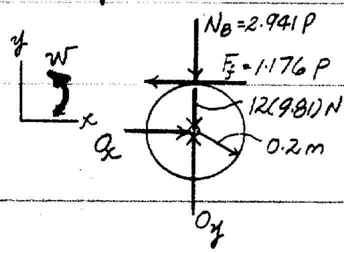


(a)

Principle of Impulse and Momentum : The mass moment inertia of the cylinder about its mass center is $I_O = \frac{1}{2}(12)(0.2^2) = 0.240\ \text{kg}\cdot\text{m}^2$. Applying Eq. 19 – 14, we have

$$I_O\omega_1 + \Sigma\int_{t_1}^{t_2} M_O dt = I_O\omega_2$$

$$(\curvearrowleft +) \qquad -0.240(20) + \left(1.176\int_0^t Pdt\right)(0.2) = 0 \qquad [1]$$

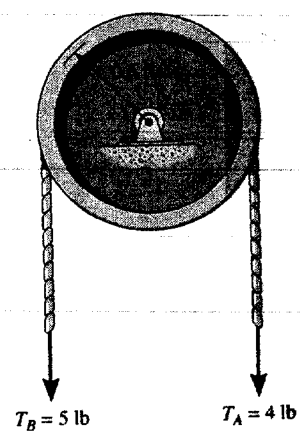


However, $\int_0^t Pdt$ is the area under the $P-t$ graph. Assuming $t > 2$ s, then $\int_0^t Pdt = \frac{1}{2}(5)(2) + 5(t-2) = (5t-5)$ N · s. Substitute into Eq. [1] yields

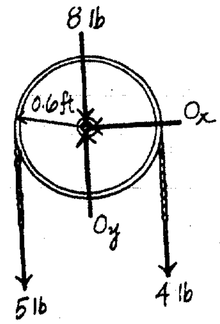
$$-0.240(20) + 1.176(5t-5)(0.2) = 0$$
$$t = 5.08 \text{ s} \qquad \textbf{Ans}$$

Since $t = 5.08$ s > 2 s, the above assumption is correct.

19-22. The pulley has a weight of 8 lb and may be treated as a thin disk. A cord wrapped over its surface is subjected to forces $T_A = 4$ lb and $T_B = 5$ lb. Determine the angular velocity of the pulley when $t = 4$ s if it starts from rest when $t = 0$. Neglect the mass of the cord.



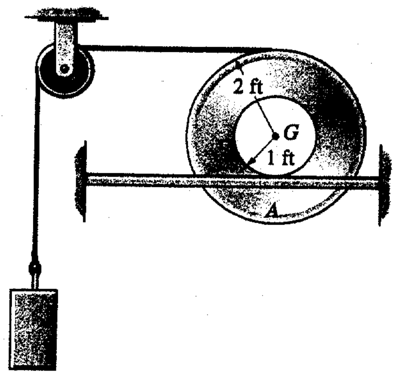
Principle of Impulse and Momentum : The mass moment inertia of the pulley about its mass center is $I_O = \frac{1}{2}\left(\frac{8}{32.2}\right)(0.6^2) = 0.04472\ \text{slug}\cdot\text{ft}^2$. Applying Eq. 19 – 14, we have

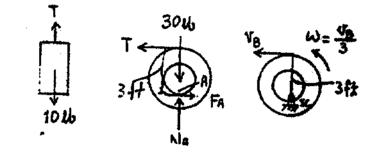


$$I_O\omega_1 + \Sigma\int_{t_1}^{t_2} M_O dt = I_O\omega_2$$

$$(\curvearrowleft +) \qquad 0 + [5(4)](0.6) - [4(4)](0.6) = 0.04472\omega_2$$
$$\omega_2 = 53.7 \text{ rad/s} \qquad \textbf{Ans}$$

19-23. The inner hub of the wheel rests on the horizontal track. If it does not slip at A, determine the speed of the 10-lb block in 2 s after the block is released from rest. The wheel has a weight of 30 lb and a radius of gyration $k_G = 1.30$ ft. Neglect the mass of the pulley and cord.





Spool,

$(\circlearrowleft +)\qquad (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$

$$0 + T(3)(2) = \left[\frac{30}{32.2}(1.3)^2 + \frac{30}{32.2}(1)^2\right]\left(\frac{v_B}{3}\right)$$

Block,

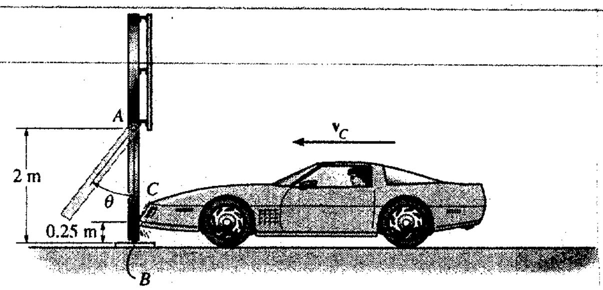
$(+\downarrow)\qquad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$

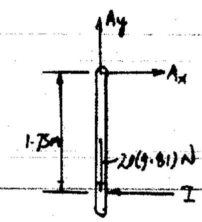
$$0 + 10(2) - T(2) = \frac{10}{32.2}v_B$$

$v_B = 34.0$ ft/s **Ans**

$T = 4.73$ lb

***19-24.** For safety reasons, the 20-kg supporting leg of a sign is designed to break away with negligible resistance at B when the leg is subjected to the impact of a car. Assuming that the leg is pin supported at A and approximates a thin rod, determine the impulse the car bumper exerts on it, if after the impact the leg appears to rotate upward to an angle of $\theta_{max} = 150°$.

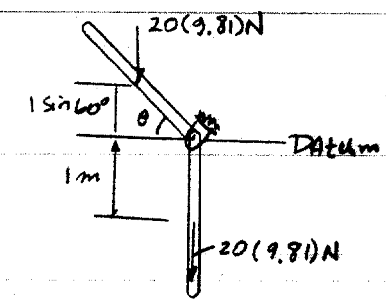




$(\circlearrowleft +)\qquad I_A\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\,\omega_2$

$$0 + I(1.75) = \left[\frac{1}{3}(20)(2)^2\right]\omega_2$$

$$\omega_2 = 0.065625I$$

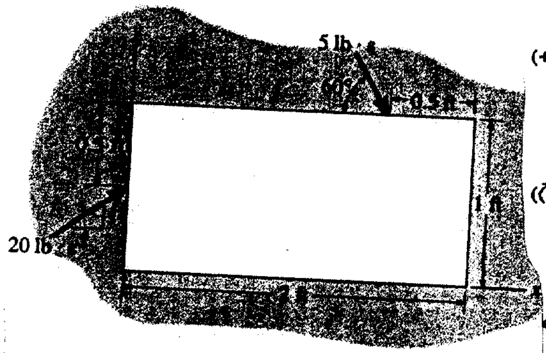


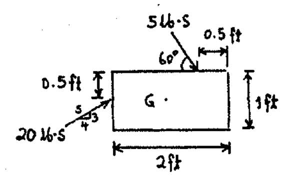
$T_2 + V_2 = T_3 + V_3$

$$\frac{1}{2}\left[\frac{1}{3}(20)(2)^2\right](0.065625I)^2 + 20(9.81)(-1) = 0 + 20(9.81)(1\sin 60°)$$

$I = 79.8$ N · s **Ans**

19-25. The 10-lb rectangular plate is at rest on a smooth *horizontal* floor. If it is given the horizontal impulses shown, determine its angular velocity and the velocity of the mass center.





$(\xrightarrow{+})\quad m(v_{Gx})_1 + \Sigma\int F_x\,dt = m(v_{Gx})_2$

$$0 + 20(\frac{4}{5}) + 5\cos 60^\circ = \frac{10}{32.2}(v_G)_x$$

$(+\uparrow)\quad m(v_{Gy})_1 + \Sigma\int F_y\,dt = m(v_{Gy})_2$

$$0 + 20(\frac{3}{5}) - 5\sin 60^\circ = \frac{10}{32.2}(v_G)_y$$

$(\circlearrowleft +)\quad I_G\omega_1 + \Sigma\int M\,dt = I_G\omega_2$

$$0 + 20(\frac{3}{5})(1) + 5\sin 60^\circ(0.5) + 5\cos 60^\circ(0.5) = \frac{1}{12}(\frac{10}{32.2})[(1)^2 + (2)^2]\omega$$

Solving,

$\omega = 119$ rad/s **Ans**

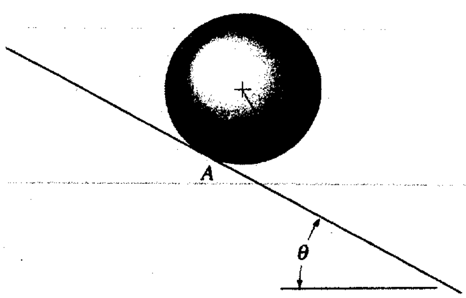
$(v_G)_x = 59.6$ ft/s

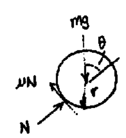
$(v_G)_y = 24.7$ ft/s

$v_G = \sqrt{(59.6)^2 + (24.7)^2} = 64.5$ ft/s **Ans**

$\theta = \tan^{-1}\frac{24.7}{59.6} = 22.5^\circ$ **Ans**

19-26. The ball of mass m and radius r rolls along an inclined plane for which the coefficient of static friction is μ. If the ball is released from rest, determine the maximum angle θ for the incline so that it rolls without slipping at A.





$+\nearrow\quad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$

$$0 + N(t) - (mg\cos\theta)t = 0$$

$\searrow +\quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$$0 + (mg\sin\theta)t - \mu Nt = m(r\omega)$$

$\circlearrowleft +\quad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

$$0 + \mu Nrt = (\frac{2}{5}mr^2)\omega$$

$$N = mg\cos\theta$$

$$mg\sin\theta\, t - \mu mg\cos\theta t = mr\omega$$

$$t = \frac{r\omega}{g(\sin\theta - \mu\cos\theta)}$$

$$\mu(mg\cos\theta)r(\frac{r\omega}{g(\sin\theta - \mu\cos\theta)}) = \frac{2}{5}mr^2\omega$$

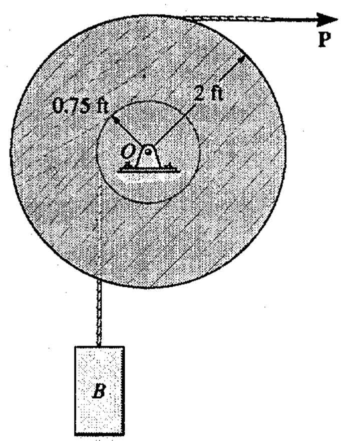
$$\mu\cos\theta = \frac{2}{5}(\sin\theta - \mu\cos\theta)$$

$$3.5\mu\cos\theta = \sin\theta$$

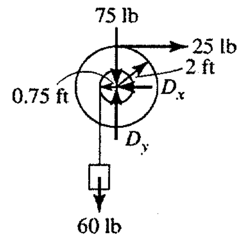
$$\tan\theta = 3.5\mu$$

$\theta = \tan^{-1}(3.5\mu)$ **Ans**

19-27. The spool has a weight of 75 lb and a radius of gyration $k_O = 1.20$ ft. If the block B weighs 60 lb, and a force $P = 25$ lb is applied to the cord, determine the speed of the block in 5 s starting from rest. Neglect the mass of the cord.



$$\circlearrowleft + \quad (H_O)_1 + \sum \int M_O dt = (H_O)_2$$



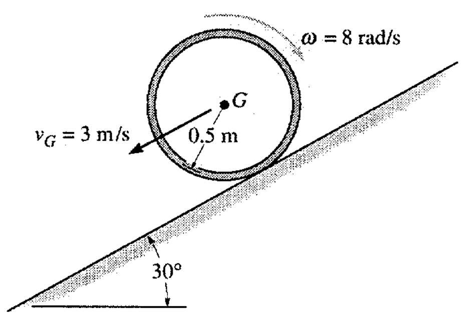
$$0 - 60(0.75)(5) + 25(2)(5) = \frac{75}{32.2}(1.20)^2\omega$$

$$+ \left[\frac{60}{32.2}(0.75\omega)\right](0.75)$$

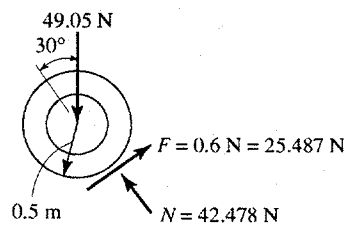
$\omega = 5.679$ rad/s

$v_B = \omega r = (5.679)(0.75) = 4.26$ ft/s **Ans**

***19-28.** The hoop (thin ring) has a mass of 5 kg and is released down the inclined plane such that it has a back-spin $\omega = 8$ rad/s and its center has a velocity $v_G = 3$ m/s as shown. If the coefficient of kinetic friction between the hoop and the plane is $\mu_k = 0.6$, determine how long the hoop rolls before it stops slipping.



$$\swarrow + \quad mv_{x1} + \sum \int F_x dt = mv_{x2}$$



$$5(3) + 49.05 \sin 30°(t) - 25.487t = 5v_G$$

$$\circlearrowleft + \quad (H_G)_1 + \sum \int M_G dt = (H_G)_2$$

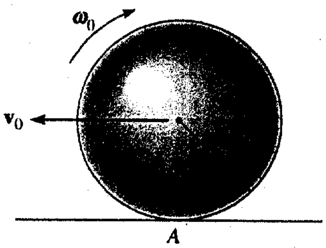
$$-5(0.5)^2(8) + 25.487(0.5)(t) = 5(0.5)^2\left(\frac{v_G}{0.5}\right)$$

Solving,

$v_G = 2.75$ m/s

$t = 1.32$ s **Ans**

19-29. If the ball has a weight W and radius r and is thrown onto a *rough surface* with a velocity $\mathbf{v}_0$ parallel to the surface, determine the amount of backspin, ω_0, it must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of friction at A for the calculation.

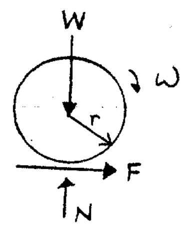


$(\overset{+}{\leftarrow})\qquad m(v_{Gx})_1 + \Sigma\int F_x\,dt = m(v_{Gx})_2$

$$\frac{W}{g}v_0 - Ft = 0 \qquad (1)$$

$(\circlearrowleft +)\qquad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

$$-\frac{2}{5}\left(\frac{W}{g}r^2\right)\omega_0 + Ft(r) = 0 \qquad (2)$$

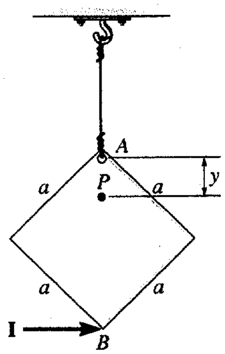


Eliminate Ft between Eqs. (1) and (2) :

$$\frac{2}{5}\left(\frac{W}{g}r^2\right)\omega_0 = \left[\frac{W}{g}\left(\frac{v_0}{t}\right)\right]t(r)$$

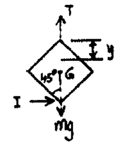
$$\omega_0 = 2.5\left(\frac{v_0}{r}\right) \qquad \textbf{Ans}$$

19-30. The square plate has a mass m and is suspended at its corner A by a cord. If it receives a horizontal impulse **I** at corner B, determine the location y of the point P about which the plate appears to rotate during the impact.



$(\circlearrowleft +)\quad (H_G)_1 + \Sigma\int M_G dt = (H_G)_2$

$$0 + I\left(\frac{a}{\sqrt{2}}\right) = \frac{m}{12}(a^2 + a^2)\omega$$

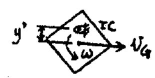


$(\overset{+}{\rightarrow})\quad m(v_{Gx})_1 + \Sigma\int F_x dt = m(v_{Gx})_2$

$$0 + I = mv_G$$

$$\omega = \frac{6I}{\sqrt{2}am}$$

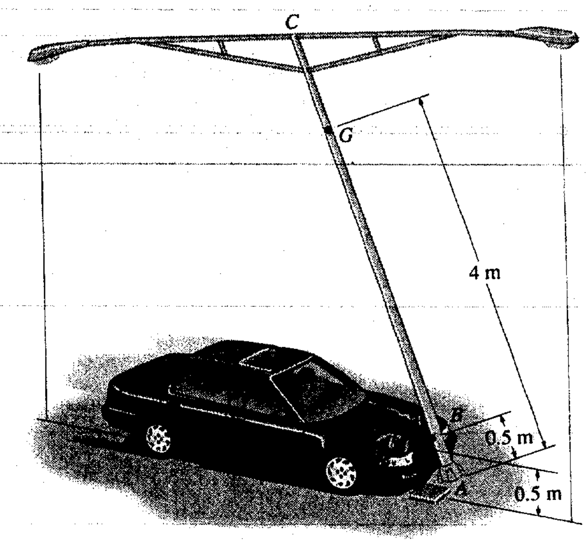
$$v_G = \frac{I}{m}$$

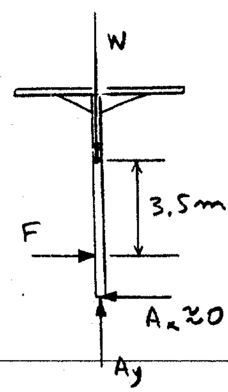


$$y' = \frac{v_G}{\omega} = \frac{\frac{I}{m}}{\frac{6I}{\sqrt{2}\,am}} = \frac{\sqrt{2}a}{6}$$

$$y = \frac{3\sqrt{2}}{6}a - \frac{\sqrt{2}}{6}a = \frac{\sqrt{2}}{3}a \qquad \textbf{Ans}$$

19-31. The car strikes the side of a light standard, which is designed to break away from its base with negligible resistance. From a video taken of the collision it is observed that the pole was given an angular velocity of 60 rad/s when AC was vertical. The pole has a mass of 175 kg, a center of mass at G, and a radius of gyration about an axis perpendicular to the plane of the pole assembly and passing through G of $k_G = 2.25$ m. Determine the horizontal impulse which the car exerts on the pole while AC is essentially vertical.



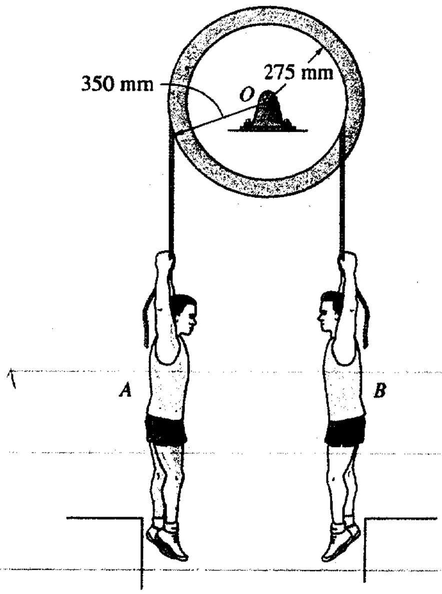


$(\circlearrowleft +)\quad (H_G)_1 + \Sigma\int M_G\, dt = (H_G)_2$

$$0 + \left[\int F\, dt\right](3.5) = 175(2.25)^2(60)$$

$$\int F\, dt = 15.2 \text{ kN}\cdot\text{s} \qquad \textbf{Ans}$$

***19-32.** The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 30 kg and a radius of gyration $k_O = 250$ mm. If two men A and B grab the suspended ropes and step off the ledges at the same time, determine their speeds in 4 s starting from rest. The men A and B have a mass of 60 kg and 70 kg, respectively. Assume they do not move relative to the rope during the motion. Neglect the mass of the rope.



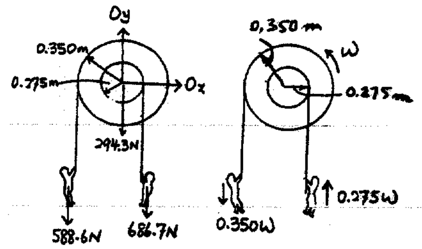
$$\circlearrowleft+ \quad (H_O)_1 + \Sigma\int M_O dt = (H_O)_2$$

$$0 + 588.6(0.350)(4) - 686.7(0.275)(4) = 30(0.25)^2\omega + 60(0.35\omega)(0.35) + 70(0.275\omega)(0.275$$

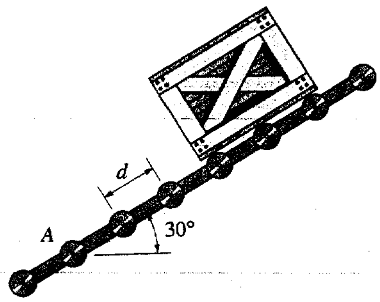
$\omega = 4.73$ rad/s

$v_A = 0.35(4.73) = 1.66$ m/s **Ans**

$v_B = 0.275(4.73) = 1.30$ m/s **Ans**



19-33. The crate has a mass m_c. Determine the constant speed v_0 it acquires as it moves down the conveyor. The rollers each have a radius of r, mass m, and are spaced d apart. Note that friction causes each roller to rotate when the crate comes in contact with it.



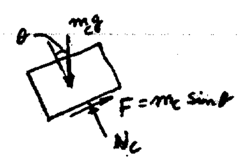
The number of rollers per unit length is $1/d$.

Thus in one second, $\frac{v_0}{d}$ rollers are contacted.

If a roller is brought to full angular speed of $\omega = \frac{v_0}{r}$ in t_0 seconds,

then the moment of inertia that is effected is

$$I' = I(\frac{v_0}{d})(t_0) = (\frac{1}{2}m r^2)(\frac{v_0}{d})t_0$$



Since the frictional impluse is

$F = m_c \sin\theta$ then

$$\circlearrowleft+ \quad (H_G)_1 + \Sigma\int M_G dt = (H_G)_2$$

$$0 + (m_c \sin\theta)\, r t_0 = \left[\left(\frac{1}{2}m r^2\right)\left(\frac{v_0}{d}\right)t_0\right]\left(\frac{v_0}{r}\right)$$

$$v_0 = \sqrt{(2g \sin\theta\, d)(\frac{m_c}{m})}$$ **Ans**

19-34. Two wheels A and B have masses m_A and m_B, and radii of gyration about their central vertical axes of k_A and k_B, respectively. If they are freely rotating in the same direction at $\boldsymbol{\omega}_A$ and $\boldsymbol{\omega}_B$ about the same vertical axis, determine their common angular velocity after they are brought into contact and slipping between them stops.

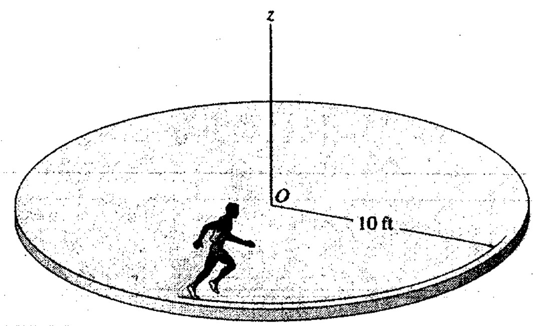
$$(\Sigma \text{Syst. Ang. Mom.})_1 = (\Sigma \text{Syst. Ang. Mom.})_2$$

$$(m_A k_A^2)\,\omega_A + (m_B k_B^2)\,\omega_B = (m_A k_A^2)\,\omega'_A + (m_B k_B^2)\,\omega'_B$$

Set $\omega'_A = \omega'_B = \omega$, then

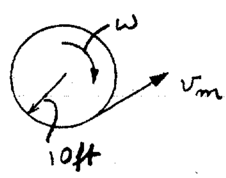
$$\omega = \frac{m_A k_A^2 \omega_A + m_B k_B^2 \omega_B}{m_A k_A^2 + m_B k_B^2}$$ **Ans**

19-35. A horizontal circular platform has a weight of 300 lb and a radius of gyration $k_z = 8$ ft about the z axis passing through its center O. The platform is free to rotate about the z axis and is initially at rest. A man having a weight of 150 lb begins to run along the edge in a circular path of radius 10 ft. If he has a speed of 4 ft/s and maintains this speed relative to the platform, determine the angular velocity of the platform. Neglect friction.



$$\mathbf{v}_m = \mathbf{v}_p + \mathbf{v}_{m/p}$$

$(\overset{+}{\rightarrow})$ $\quad v_m = -10\omega + 4$

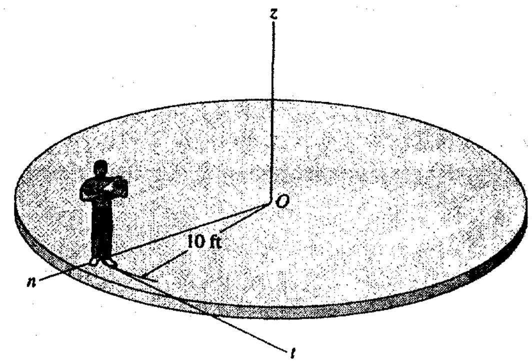


$(\circlearrowleft +)$ $\quad (H_z)_1 = (H_z)_2$

$$0 = -\left(\frac{300}{32.2}\right)(8)^2\,\omega + \left(\frac{150}{32.2}\right)(-10\omega + 4)(10)$$

$\omega = 0.175$ rad/s **Ans**

***19-36.** A horizontal circular platform has a weight of 300 lb and a radius of gyration $k_z = 8$ ft about the z axis passing through its center O. The platform is free to rotate about the z axis and is initially at rest. A man having a weight of 150 lb throws a 15-lb block off the edge of the platform with a horizontal velocity of 5 ft/s, *measured relative to the platform*. Determine the angular velocity of the platform if the block is thrown (a) tangent to the platform, along the $+t$ axis, and (b) outward along a radial line, or $+n$ axis. Neglect the size of the man.

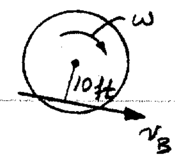


$$0+0=\left(\frac{15}{32.2}\right)(v_b)(10)-\left(\frac{300}{32.2}\right)(8)^2\,\omega-\left(\frac{150}{32.2}\right)(10\omega)(10)$$

$$v_b = 228\omega$$

$$\mathbf{v}_b = \mathbf{v}_m + \mathbf{v}_{b/m}$$

$$(\overset{+}{\rightarrow}) \qquad v_b = -10\omega + 5$$



$$228\omega = -10\omega + 5$$

$$\omega = 0.0210 \text{ rad/s} \qquad \textbf{Ans}$$

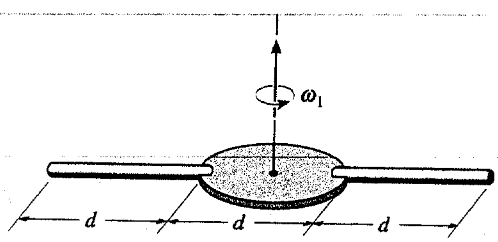
b)

$$(H_z)_1 = (H_z)_2$$

$$0+0=0-\left(\frac{300}{32.2}\right)(8)^2\,\omega-\left(\frac{150}{32.2}\right)(10\omega)(10)$$

$$\omega = 0 \qquad \textbf{Ans}$$

19-37. Each of the two slender rods and the disk have the same mass m. Also, the length of each rod is equal to the diameter d of the disk. If the assembly is rotating with an angular velocity ω_1 when the rods are directed outward, determine the angular velocity of the assembly if by internal means the rods are brought to an upright vertical position.

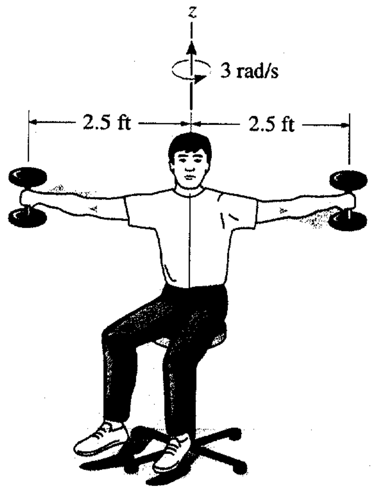


$$H_1 = H_2$$

$$\left[\frac{1}{2}m\left(\frac{d}{2}\right)^2\right]\omega_1 + 2\left(\frac{1}{12}md^2\right)\omega_1 + 2(md^2\,\omega_1) = \left[\frac{1}{2}m\left(\frac{d}{2}\right)^2\right]\omega' + 2m\left(\frac{d}{2}\right)^2\omega'$$

$$\omega' = \frac{11}{3}\omega_1 \qquad \textbf{Ans}$$

19-38. The man sits on the swivel chair holding two 5-lb weights with his arms outstretched. If he is turning at 3 rad/s in this position, determine his angular velocity when the weights are drawn in and held 0.3 ft from the axis of rotation. Assume he weighs 160 lb and has a radius of gyration $k_z = 0.55$ ft about the z axis. Neglect the mass of his arms and the size of the weights for the calculation.



Mass Moment of Inertia : The mass moment inertia of the man and the weights about z axis when the man arms is fully stretched is

$$(I_z)_1 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(2.5^2)\right] = 3.444 \text{ slug} \cdot \text{ft}^2$$

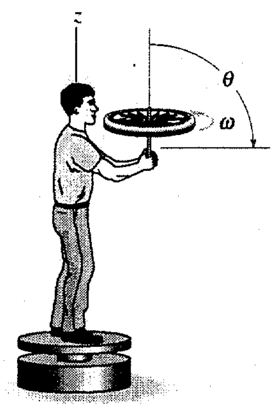
The mass moment inertia of the man and the weights about z axis when the weights are drawn in to a distance 0.3 ft from z axis

$$(I_z)_2 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(0.3^2)\right] = 1.531 \text{ slug} \cdot \text{ft}^2$$

Conservation of Angular Momentum : Applying Eq. 19 – 17, we have

$$(H_z)_1 = (H_z)_2$$
$$3.444(3) = 1.531(\omega_z)_2$$
$$(\omega_z)_2 = 6.75 \text{ rad/s} \qquad \textbf{Ans}$$

19-39. A man has a moment of inertia I_z about the z axis. He is originally at rest and standing on a small platform which can turn freely. If he is handed a wheel when it is at *rest* and he starts it spinning with an angular velocity ω, determine his angular velocity if (a) he holds the wheel upright as shown, (b) turns the wheel out, $\theta = 90°$, and (c) turns the wheel downward, $\theta = 180°$.



(a)

$$\sum(H_z)_1 = \sum(H_z)_2; \quad 0 + I\omega = I_z\omega_M + I\omega \quad \omega_M = 0 \qquad \textbf{Ans}$$

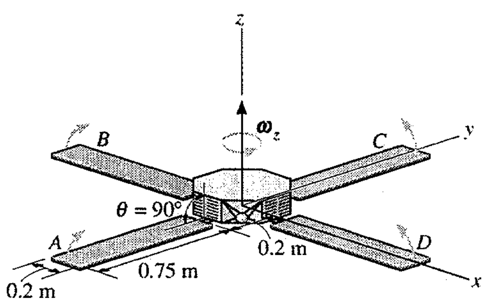
(b)

$$\sum(H_z)_1 = \sum(H_z)_2; \quad 0 + I\omega = I_z\omega_M + 0 \quad \omega_M = \frac{I}{I_z}\omega \qquad \textbf{Ans}$$

(c)

$$\sum(H_z)_1 = \sum(H_z)_2; \quad 0 + I\omega = I_z\omega_M - I\omega \quad \omega_M = \frac{2I}{I_z}\omega \qquad \textbf{Ans}$$

***19-40.** The space satellite has a mass of 125 kg and a moment of inertia $I_z = 0.940\ \text{kg}\cdot\text{m}^2$, excluding the four solar panels A, B, C, and D. Each solar panel has a mass of 20 kg and can be approximated as a thin plate. If the satellite is originally spinning about the z axis at a constant rate $\omega_z = 0.5$ rad/s when $\theta = 90°$, determine the rate of spin if all the panels are raised and reach the upward position, $\theta = 0°$, at the same instant



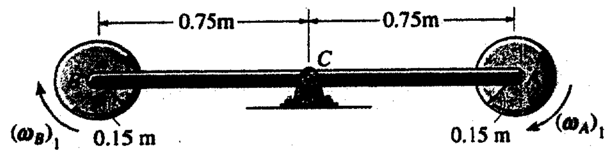
$+$ $\quad H_1 = H_2$

$$2\left[\frac{1}{2}(4)(0.15)^2\right](5) = 2\left[\frac{1}{2}(4)(0.15)^2\right]\omega$$

$$+ 2[4(0.75\omega)(0.75)] + \left[\frac{1}{12}(2)(1.50)^2\right]\omega$$

$\omega = 0.0906$ rad/s $\qquad$ **Ans**

19-41. The 2-kg rod *ACB* supports the two 4-kg disks at its ends. If both disks are given a clockwise angular velocity $(\omega_A)_1 = (\omega_B)_1 = 5$ rad/s while the rod is held stationary and then released, determine the angular velocity of the rod after both disks have stopped spinning relative to the rod due to frictional resistance at the pins *A* and *B*. Motion is in the *horizontal plane*. Neglect friction at pin *C*.

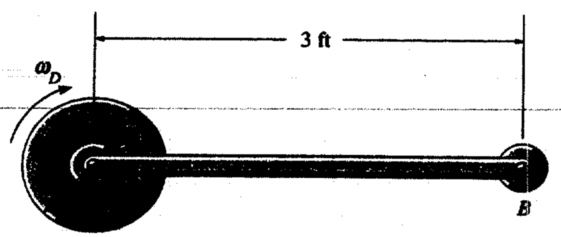


$$\circlearrowleft + \quad H_1 = H_2$$

$$2[\frac{1}{2}(4)(0.15)^2](5) = 2[\frac{1}{2}(4)(0.15)^2]\omega + 2[4(0.75\omega)(0.75)] + [\frac{1}{12}(2)(1.50)^2]\omega$$

$\omega = 0.0906$ rad/s **Ans**

19-42. The 5-lb rod *AB* supports the 3-lb disk at its end. If the disk is given an angular velocity $\omega_D = 8$ rad/s while the rod is held stationary and then released, determine the angular velocity of the rod after the disk has stopped spinning relative to the rod due to frictional resistance at the bearing *A*. Motion is in the *horizontal plane*. Neglect friction at the fixed bearing *B*.

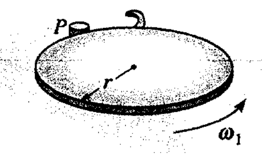


$$\Sigma(H_B)_1 = \Sigma(H_B)_2$$

$$\left[\tfrac{1}{2}\left(\tfrac{3}{32.2}\right)(0.5)^2\right](8) + 0 = \left[\tfrac{1}{3}\left(\tfrac{5}{32.2}\right)(3)^2\right]\omega + \left[\tfrac{1}{2}\left(\tfrac{3}{32.2}\right)(0.5)^2\right]\omega + \left(\tfrac{3}{32.2}\right)(3\omega)(3)$$

$\omega = 0.0708$ rad/s **Ans**

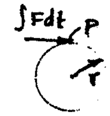
19-43. A thin disk of mass m has an angular velocity ω_r while rotating on a smooth surface. Determine its new angular velocity just after the hook at its edge strikes the peg P and the disk starts to rotate about P without rebounding.



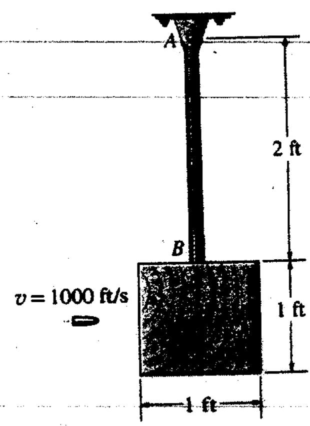
$$H_1 = H_2$$

$$\left(\frac{1}{2}mr^2\right)\omega_1 = \left[\frac{1}{2}mr^2 + mr^2\right]\omega_2$$

$$\omega_2 = \frac{1}{3}\omega_1 \qquad \textbf{Ans}$$

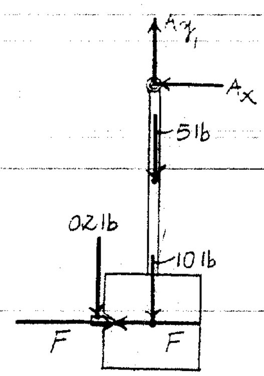


***19-44.** The pendulum consists of a 5-lb slender rod AB and a 10-lb wooden block. A projectile weighing 0.2 lb is fired into the center of the block with a velocity of 1000 ft/s. If the pendulum is initially at rest, and the projectile embeds itself into the block, determine the angular velocity of the pendulum just after the impact.



Mass Moment of Inertia **:** The mass moment inertia of the pendulum and the embeded bullet about point A is

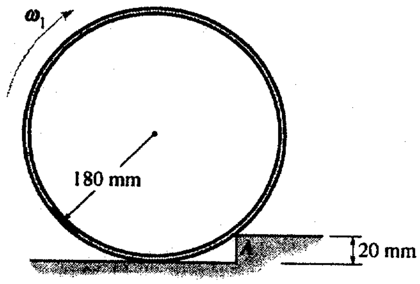
$$(I_A)_2 = \frac{1}{12}\left(\frac{5}{32.2}\right)(2^2) + \frac{5}{32.2}(1^2)$$
$$+ \frac{1}{12}\left(\frac{10}{32.2}\right)(1^2 + 1^2) + \frac{10}{32.2}(2.5^2) + \frac{0.2}{32.2}(2.5^2)$$
$$= 2.239 \text{ slug} \cdot \text{ft}^2$$



Conservation of Angular Momentum **:** Since force F due to the impact is *internal* to the system consisting of the pendulum and the bullet, it will cancel out. Thus, angular momentum is conserved about point A. Applying Eq. 19 – 17, we have

$$(H_A)_1 = (H_A)_2$$
$$(m_b v_b)(r_b) = (I_A)_2\,\omega_2$$
$$\left(\frac{0.2}{32.2}\right)(1000)(2.5) = 2.239\omega_2$$
$$\omega_2 = 6.94 \text{ rad/s} \qquad \textbf{Ans}$$

19-45. A thin ring having a mass of 15 kg strikes the 20-mm-high step. Determine the largest angular velocity ω_1 the ring can have so that it will not rebound off the step at A when it strikes it.



The weight is non – impulsive.

$(H_A)_1 = (H_A)_2$

$15(\omega_1)(0.18)(0.18-0.02)+\left[15(0.18)^2\right](\omega_1)=\left[15(0.18)^2+15(0.18)^2\right]\omega_2$

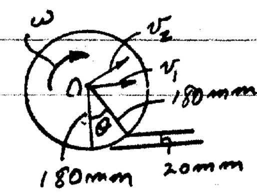
$\omega_2 = 0.9444\omega_1$

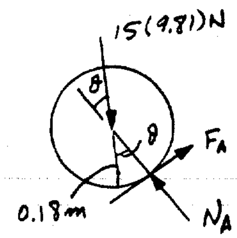
$+\searrow\Sigma F_n = m(a_G)_n;\quad (15)(9.81)\cos\theta - N_A = 15\omega_2^2(0.18)$

When hoop is about to rebound, $N_A = 0$. Also, $\cos\theta = \dfrac{160}{180}$, and so

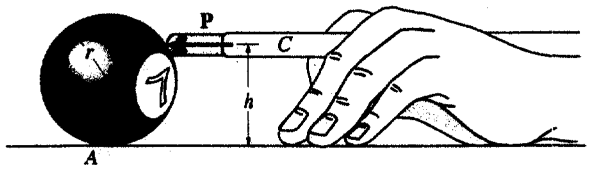
$\omega_2 = 6.9602$ rad/s

$\omega_1 = \dfrac{6.9602}{0.9444} = 7.37$ rad/s **Ans**





19-46. **Determine the height h at which a billiard ball of mass m must be struck so that no frictional force develops between it and the table at A. Assume that the cue C only exerts a horizontal force $\mathbf{P}$ on the ball.**



For the ball

$$(\overset{+}{\leftarrow})\quad mv_1 + \Sigma\int F\,dt = mv_2$$

$$0 + P(\Delta t) = mv_2 \qquad (1)$$

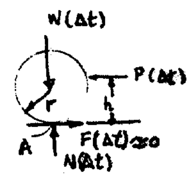
$$\circlearrowleft + \quad (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$$

$$0 + (P)\Delta t(h) = [\tfrac{2}{5}m\,r^2 + m\,r^2]\omega_2 \qquad (2)$$

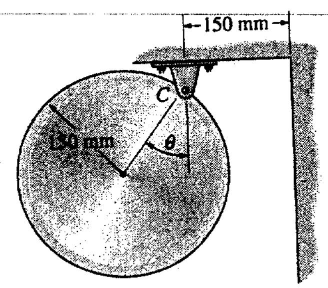
Require $v_2 = \omega_2 r$ $\qquad$ **(3)**

Solving Eqs. (1) – (3) for h yields

$$h = \frac{7}{5}r \qquad \textbf{Ans}$$



19-47. The disk has a mass of 15 kg. If it is released from rest when $\theta = 30°$, determine the maximum angle θ of rebound after it collides with the wall. The coefficient of restitution between the disk and the wall is $e = 0.6$. When $\theta = 0°$, the disk hangs such that it just touches the wall. Neglect friction at the pin C.



Datum at lower position of G.

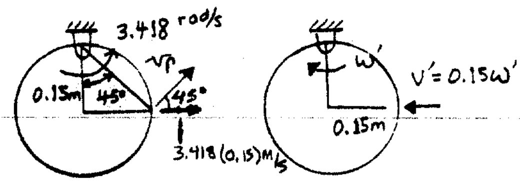
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (15)(9.81)(0.15)(1-\cos 30°) = \frac{1}{2}\left[\frac{3}{2}(15)(0.15)^2\right]\omega^2 + 0$$

$$\omega = 3.418 \text{ rad/s}$$

$$(\overset{+}{\rightarrow})\qquad e = 0.6 = \frac{0-(-0.15\omega')}{3.418(0.15)-0}$$

$$\omega' = 2.0508 \text{ rad/s}$$

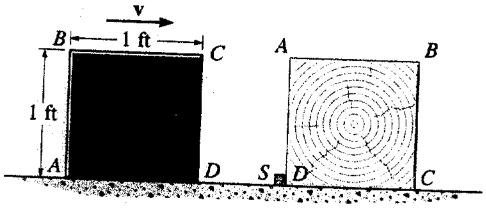


$$T_2 + V_2 = T_3 + V_3$$

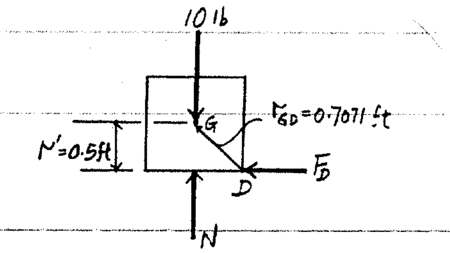
$$\frac{1}{2}\left[\frac{3}{2}(15)(0.15)^2\right](2.0508)^2 + 0 = 0 + 15(9.81)(0.15)(1-\cos\theta)$$

$$\theta = 17.9° \qquad \textbf{Ans}$$

*19-48. The 10-lb block is sliding on the smooth surface when the corner D hits a stop block S. Determine the minimum velocity $\mathbf{v}$ the block should have which would allow it to tip over on its side and land in the position shown. Neglect the size of S. *Hint:* During impact consider the weight of the block to be nonimpulsive.



Conservation of Energy : If the block tips over about point D, it must at least achieve the dash position shown. Datum is set at point D. When the block is at its initial and final position, its center of gravity is located 0.5 ft and 0.7071 ft *above* the datum. Its initial and final potential energy are $10(0.5) = 5.00$ ft · lb and $10(0.7071) = 7.071$ ft · lb. The mass moment of inertia of the block about point D is $I_D = \frac{1}{12}\left(\frac{10}{32.2}\right)(1^2+1^2)+\left(\frac{10}{32.2}\right)\left(\sqrt{0.5^2+0.5^2}\right)^2$ $= 0.2070$ slug · ft². The initial kinetic energy of the block (after the impact) is $\frac{1}{2}I_D\omega_2^2 = \frac{1}{2}(0.2070)\,\omega_2^2$. Applying Eq. 18 – 18, we have

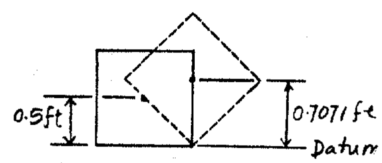


$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(0.2070)\,\omega_2^2 + 5.00 = 0 + 7.071$$

$$\omega_2 = 4.472 \text{ rad/s}$$

Conservation of Angular Momentum : Since the weight of the block and the normal reaction N are *nonimpulsive* forces, the angular momentum is conserved about point D. Applying Eq. 19 – 17, we have



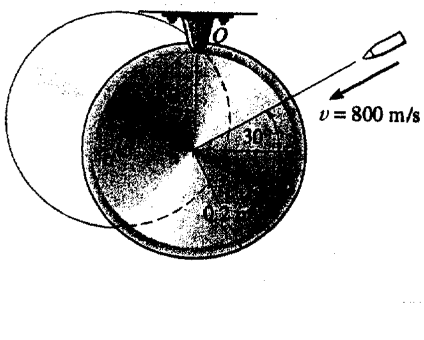
$$(H_D)_1 = (H_D)_2$$

$$(mv_G)(r') = I_D\omega_2$$

$$\left[\left(\frac{10}{32.2}\right)v\right](0.5) = 0.2070(4.472)$$

$$v = 5.96 \text{ ft/s} \qquad \textbf{Ans}$$

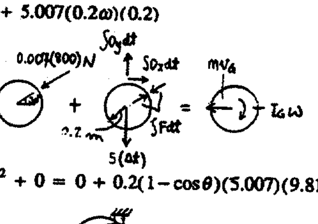
19-49. A 7-g bullet having a velocity of 800 m/s is fired into the edge of the 5-kg disk as shown. Determine the angular velocity of the disk just after the bullet becomes embedded in it. Also, calculate how far θ the disk will swing until it stops. The disk is originally at rest.



$$(\circlearrowleft +) \quad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$$

$$0.007(800)\cos 30^\circ(0.2) + 0 = \frac{1}{2}(5.007)(0.2)^2\omega + 5.007(0.2\omega)(0.2)$$

$\omega = 3.23$ rad/s **Ans**

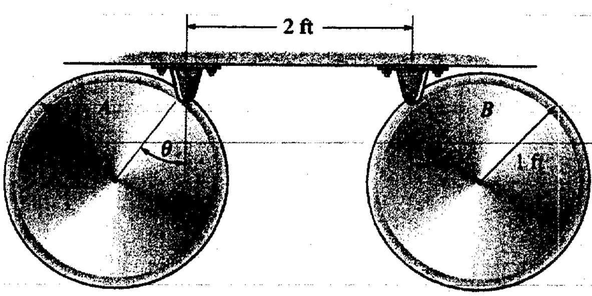


$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(5.007)[3.23(0.2)]^2 + \frac{1}{2}\left[\frac{1}{2}(5.007)(0.2)^2\right](3.23)^2 + 0 = 0 + 0.2(1-\cos\theta)(5.007)(9.81)$$

$\theta = 32.8^\circ$ **Ans**

19-50. The two disks each weigh 10 lb. If they are released from rest when $\theta = 30^\circ$, determine θ after they collide and rebound from each other. The coefficient of restitution is $e = 0.75$. When $\theta = 0^\circ$, the disks hang so that they just touch one another.



$$I_C = \frac{1}{2}\left(\frac{10}{32.2}\right)(1)^2 + \frac{10}{32.2}(1)^2 = 0.46584 \text{ slug}\cdot\text{ft}^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 10(1-\cos 30^\circ) = \frac{1}{2}(0.46584)\omega_1^2 + 0$$

$\omega_1 = 2.398$ rad/s

Coefficient of restitution:

$$e = \frac{(v_D)_{B_2} - (v_D)_{A_2}}{(v_D)_{A_1} - (v_D)_{B_1}} = 0.75 = \frac{\omega_2 - (-\omega_2)}{2.398 - (-2.398)} \qquad (1)$$

Where, v_D is the speed of point D on disk A or B. Note that $(v_D)_B = -(v_D)_A$ and $(v_D)_A = r\omega = (v_D)_B$.

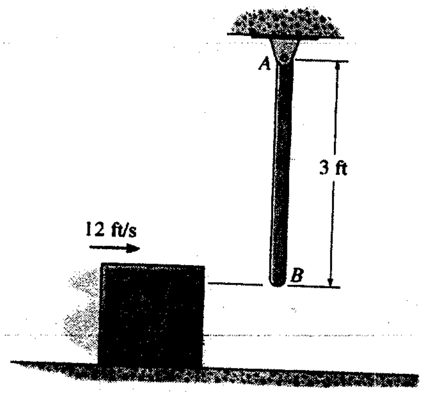
Solving Eq.(1); $\omega_2 = 1.799$ rad/s

$$T_1 + V_1 = T_2 + V_2$$

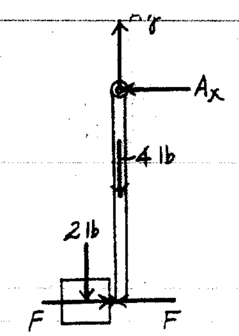
$$\frac{1}{2}(0.46584)(1.799)^2 + 0 = 0 + 10(1-\cos\theta)$$

$\theta = 22.4^\circ$ **Ans**

***19-51.** The 4-lb rod AB is hanging in the vertical position. A 2-lb block, sliding on a smooth horizontal surface with a velocity of 12 ft/s, strikes the rod at its end B. Determine the velocity of the block immediately after the collision. The coefficient of restitution between the block and the rod at B is $e = 0.8$.



Conservation of Angular Momentum : Since force F due to the impact is *internal* to the system consisting of the slender rod and the block, it will cancel out. Thus, angular momentum is conserved about point A. The mass moment of inertia of the slender rod about point A is $I_A = \frac{1}{12}\left(\frac{4}{32.2}\right)(3^2) + \frac{4}{32.2}(1.5^2) = 0.3727$ slug · ft^2. Here, $\omega_2 = \frac{(v_B)_2}{3}$. Applying Eq. 19 − 17, we have



$$(H_A)_1 = (H_A)_2$$

$$\left[m_b(v_b)_1\right](r_b) = I_A\,\omega_2 + \left[m_b(v_b)_2\right](r_b)$$

$$\left(\frac{2}{32.2}\right)(12)(3) = 0.3727\left[\frac{(v_B)_2}{3}\right] + \left(\frac{2}{32.2}\right)(v_b)_2(3) \qquad [1]$$

Coefficient of Restitution : Applying Eq. 19 − 20, we have

$$e = \frac{(v_B)_2 - (v_b)_2}{(v_b)_1 - (v_B)_1}$$

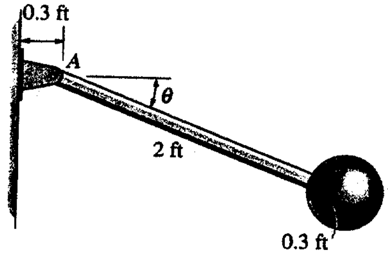
$$(\overset{+}{\rightarrow}) \qquad 0.8 = \frac{(v_B)_2 - (v_b)_2}{12 - 0} \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$(v_b)_2 = 3.36 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$

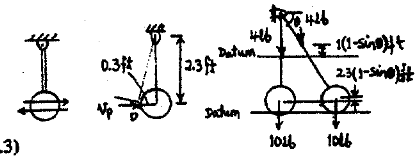
$$(v_B)_2 = 12.96 \text{ ft/s} \rightarrow$$

***19-52.** The pendulum consists of a 10-lb solid ball and 4-lb rod. If it is released from rest when $\theta_1 = 0°$, determine the angle θ_2 after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest. Take $e = 0.6$.



$$I_A = \frac{1}{3}\left(\frac{4}{32.2}\right)(2)^2 + \frac{2}{5}\left(\frac{10}{32.2}\right)(0.3)^2 + \frac{10}{32.2}(2.3)^2 = 1.8197 \text{ slug}\cdot\text{ft}^2$$

Just before impact:



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}[1.8197]\omega^2 - 4(1) - 10(2.3)$$

$$\omega = 5.4475 \text{ rad/s}$$

$$v_P = 2.3(5.4475) = 12.53 \text{ ft/s}$$

Since wall does not move

$$e = 0.6 = \frac{v_P'}{12.529}$$

$$v_P' = 7.518 \text{ ft/s}$$

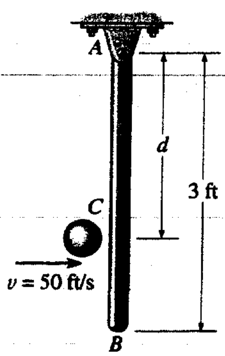
$$\omega' = \frac{7.518}{2.3} = 3.2685 \text{ rad/s}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(1.8197)(3.2685)^2 = 4(1)(1-\sin\theta) + 10(2.3)(1-\sin\theta)$$

$\theta = 39.8°$ **Ans**

19-53. The 6-lb slender rod AB is originally at rest, suspended in the vertical position. A 1-lb ball is thrown at the rod with a velocity $v = 50$ ft/s and strikes the rod at C. Determine the angular velocity of the rod just after the impact. Take $e = 0.7$ and $d = 2$ ft.



$$\circlearrowleft + (H_A)_1 = (H_A)_2$$

$$\left(\frac{1}{32.2}\right)(50)(2) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega_2 + \frac{1}{32.2}(v_{BL})(2)$$

$$e = 0.7 = \frac{v_C - v_{BL}}{50 - 0}$$

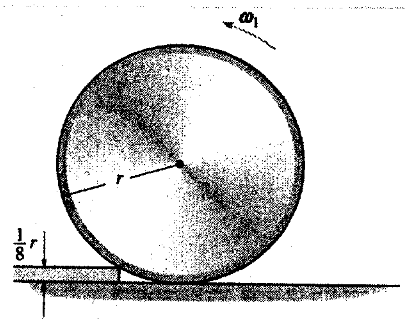
$$v_C = 2\omega_2$$

Thus,

$\omega_2 = 7.73$ rad/s **Ans**

$v_{BL} = -19.5$ ft/s

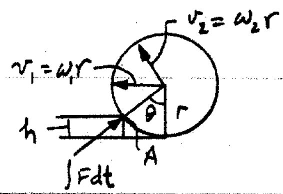
19-54. The disk has a mass m and radius r. If it strikes the rough step having a height $\frac{1}{8}r$ as shown, determine the largest angular velocity ω_1 the disk can have and not rebound off the step when it strikes it.



$$(H_A)_1 = \frac{1}{2}m\,r^2(\omega_1) + m(\omega_1 r)(r-h)$$

$$(H_A)_2 = \frac{1}{2}m\,r^2(\omega_2) + m(\omega_2 r)(r)$$

$$(H_A)_1 = (H_A)_2$$



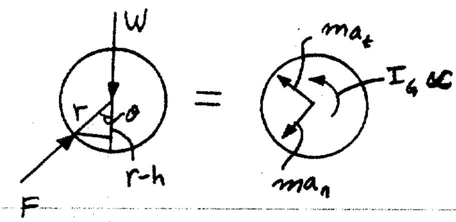
$$\left[\frac{1}{2}m\,r^2 + m\,r\,(r-h)\right]\omega_1 = \frac{3}{2}m\,r^2\omega_2$$

$$\left(\frac{3}{2}r - h\right)\omega_1 = \frac{3}{2}r\,\omega_2$$

$\swarrow + \Sigma F_n = m\,a_n$: $\quad W\cos\theta - F = m(\omega_2^2\,r)$

$$F = mg\left(\frac{r-h}{r}\right) - m(\omega_2^2 r)$$

$$F = mg\left(\frac{r-h}{r}\right) - m\,r\left(\frac{2}{3}\right)^2\left(\frac{\frac{3}{2}r - h}{r}\right)^2\omega_1^2$$

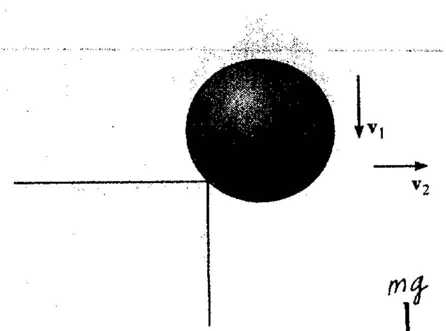


Set $h = \frac{1}{8}r$; also note that for maximum ω_1 F will approach zero. Thus

$$mg\left(\frac{r - \frac{1}{8}r}{r}\right) - mr\left(\frac{2}{3}\right)^2\left(\frac{\frac{3}{2}r - \frac{r}{8}}{r}\right)^2\omega_1^2$$

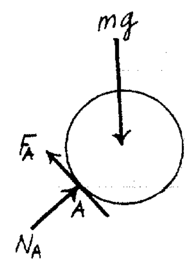
$\omega_1 = 1.02\sqrt{\frac{g}{r}}$ **Ans**

19-55. The solid ball of mass m is dropped with a velocity $\mathbf{v}_1$ onto the edge of the rough step. If it rebounds horizontally off the step with a velocity $\mathbf{v}_2$, determine the angle θ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is e.



Conservation of Angular Momentum : Since the weight of the solid ball is a *nonimpulsive force*, then angular momentum is conserved about point A. The mass moment of inertia of the solid ball about its mass center is $I_G = \frac{2}{5}mr^2$.

Here, $\omega_2 = \dfrac{v_2\cos\theta}{r}$. Applying Eq. 19 – 17, we have

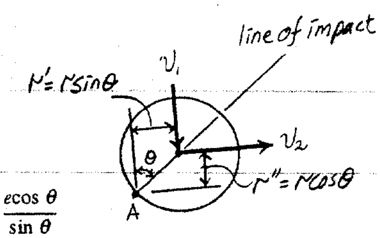


$$(H_A)_1 = (H_A)_2$$

$$[m_b(v_b)_1](r') = I_G\omega_2 + [m_b(v_b)_2](r'')$$

$$(mv_1)(r\sin\theta) = \left(\frac{2}{5}mr^2\right)\left(\frac{v_2\cos\theta}{r}\right) + (mv_2)(r\cos\theta)$$

$$\frac{v_2}{v_1} = \frac{5}{7}\tan\theta \qquad [1]$$



Coefficient of Restitution : Applying Eq. 19 – 20, we have

$$e = \frac{0-(v_b)_2}{(v_b)_1 - 0}$$

$$e = \frac{-(v_2\sin\theta)}{-v_1\cos\theta}$$

$$\frac{v_2}{v_1} = \frac{e\cos\theta}{\sin\theta} \qquad [2]$$

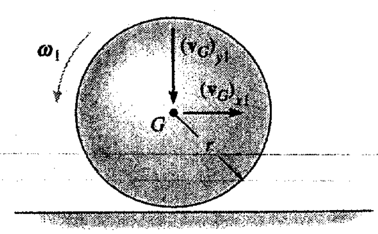
Equating Eqs. [1] and [2] yields

$$\frac{5}{7}\tan\theta = \frac{e\cos\theta}{\sin\theta}$$

$$\tan^2\theta = \frac{7}{5}e$$

$$\theta = \tan^{-1}\left(\sqrt{\frac{7}{5}e}\right) \qquad \textbf{Ans}$$

***19-56.** A solid ball with a mass m is thrown on the ground such that at the instant of contact it has an angular velocity ω_1 and velocity components $(\mathbf{v}_G)_{x1}$ and $(\mathbf{v}_G)_{y1}$ as shown. If the ground is rough so no slipping occurs, determine the components of the velocity of its mass center just after impact. The coefficient of restitution is e.



Coefficient of Restitution (y direction) :

$$(+\downarrow) \qquad e = \frac{0-(v_G)_{y2}}{(v_G)_{y1} - 0} \qquad (v_G)_{y2} = -e(v_G)_{y1} = e(v_G)_{y1}\uparrow \qquad \textbf{Ans}$$

Conservation of angular momentum about point on the ground :

$$(\circlearrowleft +) \qquad (H_A)_1 = (H_A)_2$$

$$-\frac{2}{5}mr^2\omega_1 + m(v_G)_{x1}r = \frac{2}{5}mr^2\omega_2 + m(v_G)_{x2}r$$

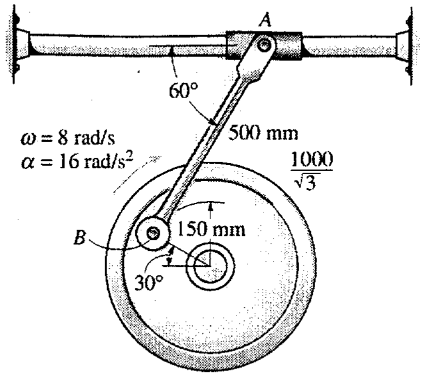
Since no slipping, $\quad (v_G)_{x2} = \omega_2 r$ then,

$$\omega_2 = \frac{5\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right)}{7r}$$

Therefore

$$(v_G)_{x2} = \frac{5}{7}\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right) \qquad \textbf{Ans}$$

R2-1. At a given instant, the wheel is rotating with the angular motions shown. Determine the acceleration of the collar at A at this instant.

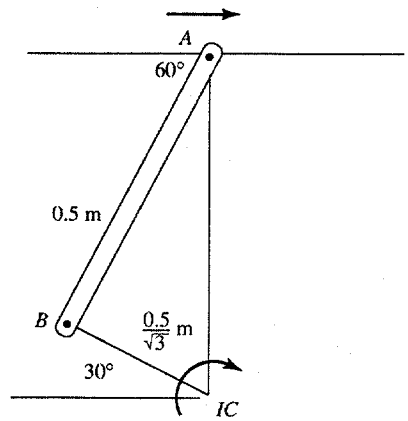


$\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B})$

$v_B = 8(0.150) = 1.20$ m/s

$$\omega_{AB} = \frac{1.20}{\left(\frac{0.500}{\sqrt{3}}\right)} = 4.1569 \text{ rad/s}$$

$$\underset{\to}{a_A} = \underset{\angle 60^\circ}{16(0.150)} + \underset{30^\circ}{(8)^2(0.150)} + \underset{30^\circ}{\alpha_{AB}(0.500)} + \underset{60^\circ}{(4.1569)^2(0.500)}$$



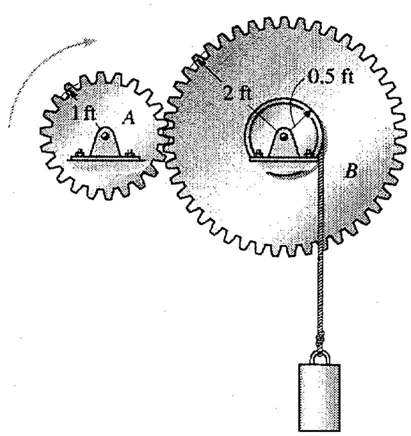
$0 = 1.2\sqrt{3} - 4.8 - 0.250\alpha_{AB} - 4.32\sqrt{3}$

$a_A = 1.2 + 4.8\sqrt{3} + (0.250\sqrt{3})\alpha_{AB} - 4.32$

$\alpha_{AB} = -40.8$ rad/s²

$a_A = -12.5$ m/s² $= 12.5$ m/s² $\leftarrow$ **Ans**

R2-2. The hoisting gear A has an initial angular velocity of 60 rad/s and a constant deceleration of 1 rad/s². Determine the velocity and deceleration of the block which is being hoisted by the hub on gear B when $t = 3$ s.



$(\omega_A)_O = 60$ rad/s

$\alpha_A = -1$ rad/s²

$\omega_A = (\omega_A)_O + \alpha_A t$

$\omega_A = 60 + (-1)(3) = 57$ rad/s

$v_A = r\omega_A = (1)(57) = 57$ ft/s $= v_B$

$$\omega_B = \frac{v_B}{r} = 57/2 = 28.5 \text{ rad/s}$$

$v_W = r_C\omega_C = (0.5)(28.5) = 14.2$ ft/s **Ans**

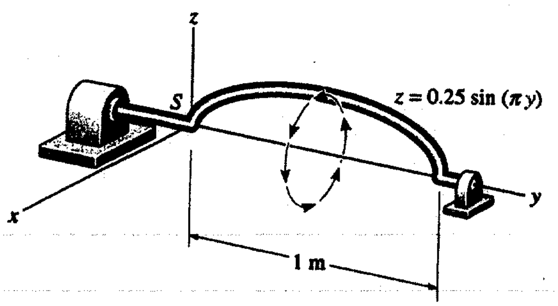
$\alpha_A = 1$

$a_{A_t} = 1(1) = 1$ ft/s²

$$\alpha_B = \frac{1}{2} = 0.5 \text{ rad/s}^2$$

$a_W = r\alpha_B = (0.5)(0.5) = 0.25$ ft/s² **Ans**

R2-3. The rod is bent into the shape of a sine curve and is forced to rotate about the y axis by connecting the spindle S to a motor. If the rod starts from rest in the position shown and a motor drives it for a short time with an angular acceleration $\alpha = (1.5e^t)$ rad/s^2, where t is in seconds, determine the magnitudes of the angular velocity and angular displacement of the rod when $t = 3$ s. Locate the point on the rod which has the greatest velocity and acceleration, and compute the magnitudes of the velocity and acceleration of this point when $t = 3$ s. The curve defining the rod is $z = 0.25\sin(\pi y)$, where the argument for the sine is given in radians when y is in meters.



$d\omega = \alpha\, dt$

$\int_0^\omega d\omega = \int_0^t 1.5\, e^t\, dt$

$\omega = 1.5\, e^t|_0^t = 1.5(e^t - 1)$

$d\theta = \omega\, dt$

$\int_0^\theta d\theta = 1.5\int_0^t (e^t - 1)\, dt$

$\theta = 1.5[e^t - t]_0^t = 1.5(e^t - t - 1)$

When $t = 3$ s,

$\omega = 1.5(e^3 - 1) = 28.6$ rad/s **Ans**

$\theta = 1.5(e^3 - 3 - 1) = 24.1$ rad **Ans**

The point having the greatest velocity and acceleration is located furthest from the axis of rotation. This is at $y = 0.5$ m, where

$z = 0.25\sin(\pi(0.5)) = 0.25$ m

Hence,

$v_P = \omega(z) = 28.6(0.25)$

$v_P = 7.16$ m/s **Ans**

and

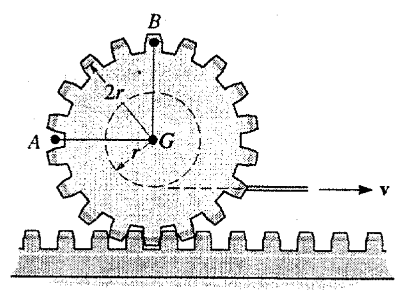
$(a_t)_P = \alpha(z) = (1.5e^3)(0.25) = 7.53$

$(a_n)_P = \omega^2(z) = (28.6)^2(0.25) = 204.90$

So that

$a_P = \sqrt{(7.53)^2 + (204.90)^2} = 205$ m/s^2 **Ans**

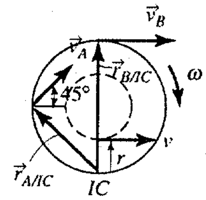
***R2-4.** A cord is wrapped around the inner spool of the gear. If it is pulled with a constant velocity v, determine the velocity and acceleration of points A and B. The gear rolls on the fixed gear rack.



$$\omega = \frac{v}{r}$$

$$v_B = \omega r_{B/IC} = \frac{v}{r}(4r) = 4v \qquad \textbf{Ans}$$

$$v_A = \omega r_{A/IC} = \frac{v}{r}\left(\sqrt{(2r)^2 + (2r)^2}\right) = 2\sqrt{2}v \qquad \textbf{Ans}$$



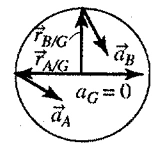
From Example 16-3, Since $\alpha_G = 0, a = 0$

$$\mathbf{r}_{B/G} = 2r\mathbf{j} \qquad \mathbf{r}_{A/G} = -2r\mathbf{i}$$

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a} \times \mathbf{r}_{B/G} - \omega^2 \mathbf{r}_{B/G}$$

$$= \mathbf{0} + \mathbf{0} - \left(\frac{v}{r}\right)^2 (2r\mathbf{j})$$

$$= \frac{2v^2}{r}$$



$$a_B = \frac{2v^2}{r} \qquad \textbf{Ans}$$

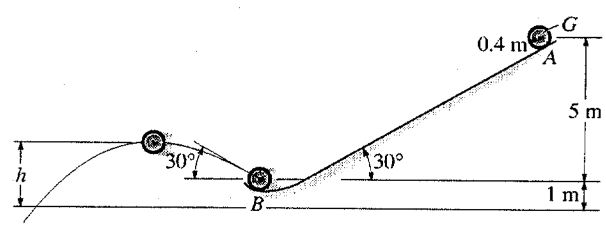
$$\mathbf{a}_A = \mathbf{a}_G + \mathbf{a} \times \mathbf{r}_{A/G} - \omega^2 \mathbf{r}_{A/G}$$

$$= \mathbf{0} + \mathbf{0} - \left(\frac{v}{r}\right)^2 (-2r\mathbf{i})$$

$$= \frac{2v^2}{r}\mathbf{i}$$

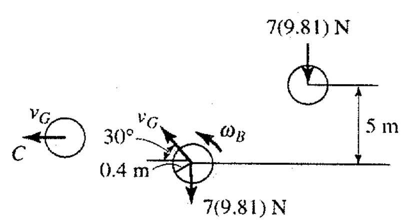
$$a_A = \frac{2v^2}{r} \qquad \textbf{Ans}$$

R2-5. A 7-kg automobile tire is released from rest at A on the incline and rolls without slipping to point B, where it then travels in free flight. Determine the maximum height h the tire attains. The radius of gyration of the tire about its mass center is $k_G = 0.3$ m.



Establish the datum at B.

Note that $v_G = 0.4\omega_B$



$$0 + 7(9.81)(5) = \frac{1}{2}[7(0.3)^2]\omega_B^2 + \frac{1}{2}(7)(0.4\omega_B)^2$$

$$\omega_B = 19.81 \text{ rad/s}$$

$$v_G = 0.4(19.81) = 7.92 \text{ m/s}$$

Let C be the high point of the tire's flight.

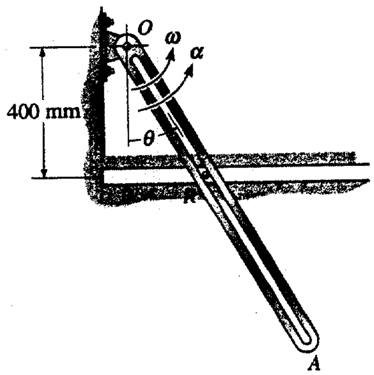
At C, $\quad (v_C)_y = 0$. Thus,

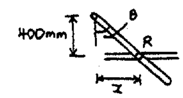
$$(+\uparrow) \qquad (v_C)_y^2 = (v_B)_y^2 + 2g((s_C)_y - (s_B)_y)$$

$$0 = (7.92 \sin 30°)^2 + 2(-9.81)(h - 1)$$

$$h = 1.80 \text{ m} \qquad \textbf{Ans}$$

R2-6. The link OA is pinned at O and rotates because of the sliding action of rod R along the horizontal groove. If R starts from rest. when $\theta = 0°$ and has a constant acceleration $a_R = 60\ \text{mm/s}^2$ to the right, determine the angular velocity and angular acceleration of OA when $t = 2$ s.





$x = 400 \tan\theta \qquad (1)$

$\dot{x} = 400 \sec^2\theta\, \dot{\theta} \qquad (2)$

$\ddot{x} = 400(2\sec^2\theta \tan\theta\, \dot{\theta})\dot{\theta} + 400 \sec^2\theta\, \ddot{\theta}$

$= 400 \sec^2\theta(2\tan\theta(\dot{\theta})^2 + \ddot{\theta}) \qquad (3)$

Since $\ddot{x} = 60\ \text{mm/s}^2$ and $\dot{x} = x = 0$ at $t = 0$;

Then in $t = 2$ s,

$v = v_0 + a_c t$

$\dot{x} = 0 + 60(2) = 120\ \text{mm/s}$

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$x = 0 + 0 + 30(2)^2 = 120\ \text{mm}$

From Eqs. (1)–(3),

$\theta = \tan^{-1}(\frac{120}{400}) = 16.7°$

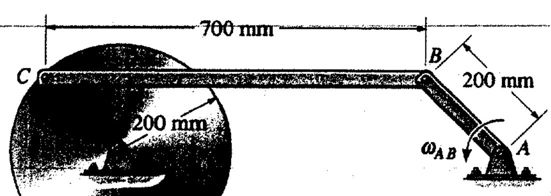
$120 = 400 \sec^2 16.7°(\dot{\theta})$

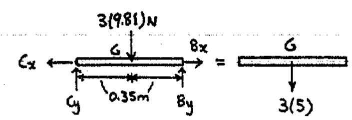
$\omega = \dot{\theta} = 0.275\ \text{rad/s} \qquad$ **Ans**

$60 = 400\sec^2 16.7°[2 \tan 16.7°(0.275)^2 + \ddot{\theta}]$

$\alpha = \ddot{\theta} = 0.0922\ \text{rad/s}^2 \qquad$ **Ans**

R2-7. The uniform connecting rod BC has a mass of 3 kg and is pin-connected at its end points. Determine the vertical forces which the pins exert on the ends B and C of the rod at the instant (a) $\theta = 0°$, and (b) $\theta = 90°$. The crank AB is turning with a constant angular velocity $\omega_{AB} = 5$ rad/s.





Rod has curvilinear motion

$\mathbf{a}_B = \mathbf{a}_G = (a_B)_n = (5)^2(0.2) = 5$

a) when $\theta = 0°$;

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad C_y + B_y - 3(9.81) = -3(5)$

$\circlearrowleft + \Sigma M_G = 0; \qquad -C_y(0.35) + B_y(0.35) = 0$

Solving,

$C_y = 7.22\ \text{N} \qquad$ **Ans**

$B_y = 7.22\ \text{N} \qquad$ **Ans**

b) When $\theta = 90°$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad C_y + B_y - 3(9.81) = 0$

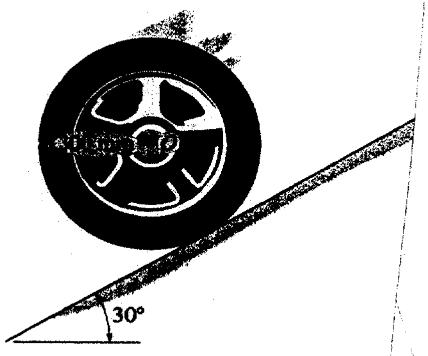
$\circlearrowleft + \Sigma M_G = 0; \qquad -C_y(0.35) + B_y(0.35) = 0$

Solving,

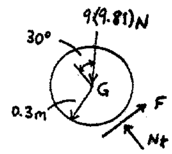
$C_y = 14.7\ \text{N} \qquad$ **Ans**

$B_y = 14.7\ \text{N} \qquad$ **Ans**

***R2-8.** The tire has a mass of 9 kg and a radius of gyration $k_O = 225$ mm. If it is released from rest and rolls down the plane without slipping, determine the speed of its center O when $t = 3$ s.



$$+\swarrow\; m(v_G)_1 + \Sigma\int F\,dt = m(v_G)_2$$



$$0 + 9(9.81)(\sin 30°)(3) - \int_0^3 F\,dt = 9(v_G)_2 \qquad (1)$$

$$\circlearrowright +\; (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$$

$$0 + \left(\int_0^3 F\,dt\right)(0.3) = [9(0.225)^2]\omega_2 \qquad (2)$$

Since $(v_G)_2 = 0.3\omega_2$,

Eliminating $\int_0^3 F\,dt$ from Eqs. (1) and (2) and solving for $(v_G)_2$ yields,

$(v_G)_2 = 9.42$ m/s **Ans**

Also,

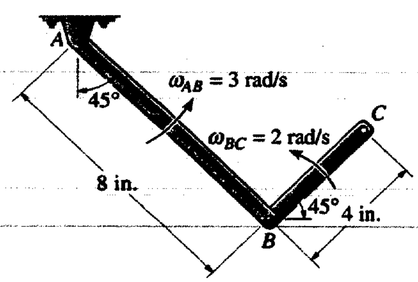
$$\circlearrowright +\; (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$$

$$0 + 9(9.81)\sin 30°(3)(0.3) = [9(0.225)^2 + 9(0.3)^2]\omega$$

$\omega = 31.39$ rad/s

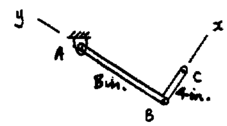
$v = 0.3(31.39) = 9.42$ m/s **Ans**

R2-9. The double pendulum consists of two rods. Rod AB has a constant angular velocity of 3 rad/s, and rod BC has a constant angular velocity of 2 rad/s. Both of these absolute motions are measured counterclockwise. Determine the velocity and acceleration of point C at the instant shown.



$\mathbf{v}_B = 3(8)\mathbf{i} = \{24\mathbf{i}\}$ in./s

$\mathbf{a}_B = (3)^2(8)\mathbf{j} = \{72\mathbf{j}\}$ in./s^2



$\mathbf{\Omega} = 3\mathbf{k}$

$\alpha = 0$

$\mathbf{r}_{C/B} = \{4\mathbf{i}\}$ in.

Note: The x, y axes rotate at 3 rad/s while BC rotates at 2 rad/s

Relative rotation of BC is therefore,

$(\omega_{BC})_{xyz} = 2\mathbf{k} - 3\mathbf{k} = -1\mathbf{k}$

$\mathbf{v}_{C/B} = (\omega_{BC})_{xyz} \times \mathbf{r}_{C/B} = -\mathbf{k} \times 4\mathbf{i} = -4\mathbf{j}$

$\mathbf{a}_{C/B} = (\omega_{BC})_{xyz} \times [(\omega_{BC}) \times \mathbf{r}_{C/B}] = -4\mathbf{i}$

$\mathbf{v}_C = \mathbf{v}_B + \mathbf{\Omega} \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$

$= 24\mathbf{i} + (3\mathbf{k} \times 4\mathbf{i}) + (-4\mathbf{j})$

$= \{24\mathbf{i} + 8\mathbf{j}\}$ in./s

$v_C = 25.3$ in./s, $\theta = 63.4°$ **Ans**

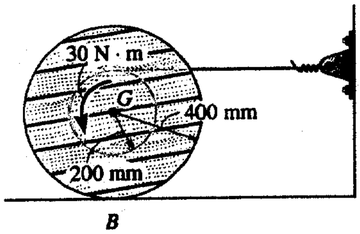
$\mathbf{a}_C = \mathbf{a}_B + \alpha \times \mathbf{r}_{C/B} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{C/B}) + 2\mathbf{\Omega} \times \mathbf{v}_{C/B} + \mathbf{a}_{C/B}$

$= 72\mathbf{j} + 0 + 3\mathbf{k} \times (3\mathbf{k} \times 4\mathbf{i}) + 2(3\mathbf{k}) \times (-4\mathbf{j}) + (-4\mathbf{i})$

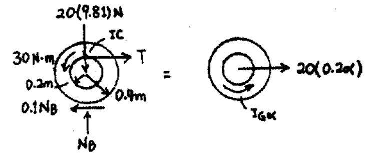
$= \{-16\mathbf{i} + 72\mathbf{j}\}$ in./s^2

$a_C = 73.8$ in./s^2, $\theta = 32.5°$ **Ans**

R2-10. The spool and wire wrapped around its core have a mass of 20 kg and a centroidal radius of gyration $k_G = 250$ mm. If the coefficient of kinetic friction at the ground is $\mu_B = 0.1$, determine the angular acceleration of the spool when the 30-N·m couple moment is applied.



$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_B - 20(9.81) = 0$



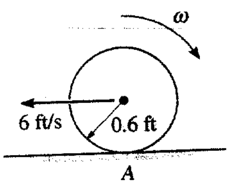
$N_B = 196.2$ N

$F_B = 0.1(196.2) = 19.62$ N

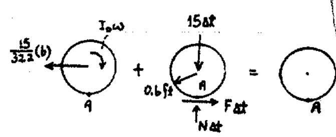
$\circlearrowleft + \Sigma M_{IC} = \Sigma(M_k)_{IC}; \quad 30 - 19.62(0.6) = 20(0.2\alpha)(0.2) + [20(0.25)^2]\alpha$

$\alpha = 8.89 \text{ rad/s}^2$ **Ans**

R2-11. If the ball has a weight of 15 lb and is thrown onto a *rough surface* so that its center has a velocity of 6 ft/s parallel to the surface, determine the amount of backspin, ω, the ball must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of kinetic friction at A for the calculation.



$\circlearrowleft + \quad (H_A)_1 + \Sigma \int M_A \, dt = (H_A)_2$



$$\frac{15}{32.2}(6)(0.6) - [\frac{2}{5}(\frac{15}{32.2})(0.6)^2]\omega + 0 = 0$$

$\omega = 25.0$ rad/s **Ans**

Also,

$\overset{+}{\leftarrow} \quad m(v_G)_1 + \Sigma \int F \, dt = m(v_G)_2$

$$\frac{15}{32.2}(6) - \int F \, dt = 0$$

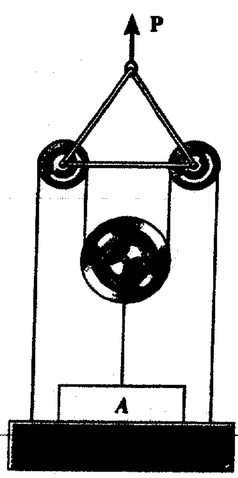
$\circlearrowright + \quad (H_G)_1 + \Sigma M_G \, dt = (H_G)_2$

$$\left[\frac{2}{5}(\frac{15}{32.2})(0.6)^2\right]\omega - (0.6)\int F \, dt = 0$$

Eliminating $\int F \, dt$ and solving for ω yields

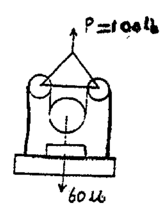
$\omega = 25.0$ rad/s **Ans**

***R2-12.** Blocks A and B weigh 50 and 10 lb, respectively. If $P = 100$ lb, determine the normal force exerted by block A on block B. Neglect friction and the weights of the pulleys, cord, and bars of the triangular frame.



System :

$+\uparrow \Sigma F_y = m a_y; \quad 100 - 60 = \frac{60}{32.2}a$



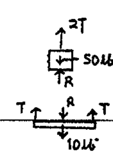
$a = 21.5 \text{ ft/s}^2$

Block A :

$+\uparrow \Sigma F_y = m a_y; \quad 2T + R - 50 = \frac{50}{32.2}(21.5)$ (1)

Block B :

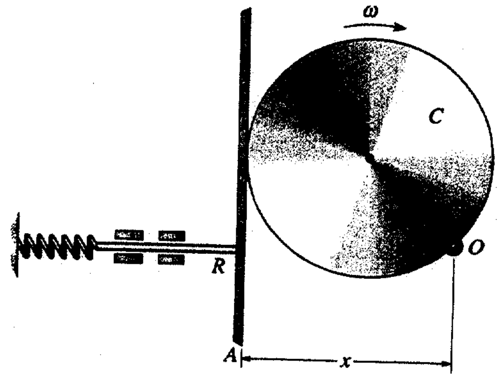
$+\uparrow \Sigma F_y = m a_y; \quad 2T - R - 10 = \frac{10}{32.2}(21.5)$ (2)



Solving Eqs. (1) and (2),

$R = 33.3$ lb **Ans**

R2-13. Determine the velocity and acceleration of rod R for any angle θ of cam C if the cam rotates with a constant angular velocity ω. The pin connection at O does not cause an interference with the motion of A on C.



$$x = r + r\cos\theta$$

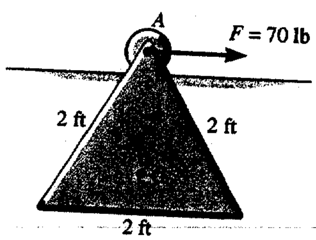
$$\dot{x} = -r\sin\theta\,\dot{\theta}$$

$$v = -r\omega\sin\theta \qquad \textbf{Ans}$$

$$\ddot{x} = -r\sin\theta\,\ddot{\theta} - r\cos\theta(\dot{\theta})^2$$

$$a = -r\omega^2\cos\theta \qquad \textbf{Ans}$$

R2-14. The uniform plate weighs 40 lb and is supported by a roller at A. If a horizontal force $F = 70$ lb is suddenly applied to the roller, determine the acceleration of the center of the roller at the instant the force is applied. The plate has a moment of inertia about its center of mass of $I_G = 0.414\ \text{slug}\cdot\text{ft}^2$. Neglect the weight of the roller.

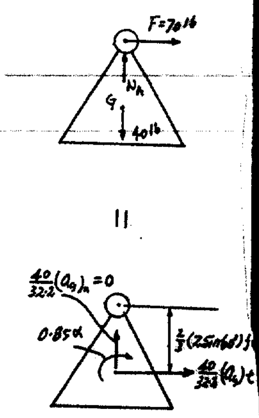


At the instant force F is applied, the normal component of acceleration of G is zero since the angular velocity is zero at this instant. As a result $a_G = (a_G)_t$.

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 70 = \left(\frac{40}{32.2}\right)a_G \qquad a_G = 56.35\ \text{ft/s}^2$$

$$\Sigma M_A = \Sigma(M_k)_A; \qquad 0 = \left(\frac{40}{32.2}\right)(56.35)\left(\frac{2}{3}\right)(2\sin 60^\circ) - 0.414\alpha$$

$$\alpha = 195.2\ \text{rad/s}^2$$



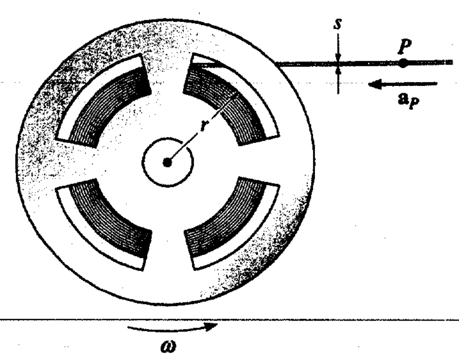
Kinematics:

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$\left[\underset{\rightarrow}{a_A}\right] = \left[\underset{\rightarrow}{56.35}\right] + \left[\underset{\rightarrow}{195.2\left(\tfrac{2}{3}\right)(2\sin 60^\circ)}\right] + [0]$$

$$(\xrightarrow{+}) \qquad a_A = 56.35 + 225.4 = 282\ \text{ft/s}^2 \qquad \textbf{Ans}$$

R2-15. A tape having a thickness s wraps around the wheel which is turning at a constant rate ω. Assuming the unwrapped portion of tape remains horizontal, determine the acceleration of point P on the tape when the radius is r. *Hint:* Since $v_p = \omega r$, take the time derivative and note that $dr/dt = \omega(s/2\pi)$.



$$v = \omega r$$

$$a = \frac{dv}{dt} = \frac{d\omega}{dt}r + \omega\frac{dr}{dt}$$

Since $\dfrac{d\omega}{dt} = 0$,

$$a = \omega\left(\frac{dr}{dt}\right)$$

In one revolution (2π rad) r increases by s, so that

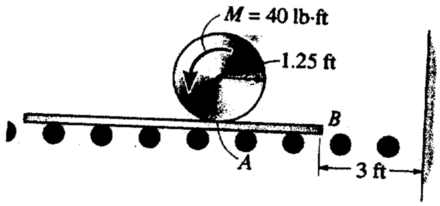
$$\frac{2\pi}{\theta} = \frac{s}{\Delta r}$$

$$\Delta r = \frac{s}{2\pi}\theta$$

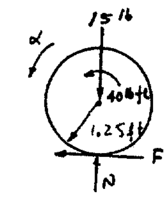
$$\frac{dr}{dt} = \frac{s}{2\pi}\omega$$

$$a = \frac{s}{2\pi}\omega^2 \qquad \textbf{Ans}$$

***R2-16.** The 15-lb cylinder is initially at rest on a 5-lb plate. If a couple moment $M = 40\ \text{lb}\cdot\text{ft}$ is applied to the cylinder, determine the angular acceleration of the cylinder and the time needed for the end B of the plate to travel 3 ft and strike the wall. Assume the cylinder does not slip on the plate, and neglect the mass of the rollers under the plate.



Cylinder:

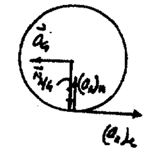


$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x;\quad F = \left(\frac{15}{32.2}\right)a_G$ [1]

$\circlearrowleft + \Sigma M_G = I_G\alpha;\quad 40 - F(1.25) = \frac{1}{2}\left(\frac{15}{32.2}\right)(1.25)^2\alpha$ [2]

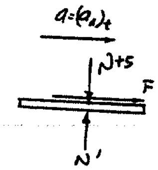
$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$

$\left[\underset{\rightarrow}{(a_A)_t}\right] + \left[\underset{\uparrow}{(a_A)_n}\right] = \left[\underset{\leftarrow}{a_G}\right] + \left[\underset{\rightarrow}{1.25\alpha}\right] + \left[\underset{\uparrow}{(a_{A/G})_n}\right]$



$(\overset{+}{\rightarrow})\ (a_A)_t = -a_G + 1.25\alpha$ [3]

Plate: Since no slipping occurs at the point of contact, the plate will move with an acceleration of $a = (a_A)_t$.



$\overset{+}{\rightarrow}\Sigma F_x = ma_x;\quad F = \left(\frac{5}{32.2}\right)(a_A)_t$ [4]

Solving Eqs.[1] to [4] yields :

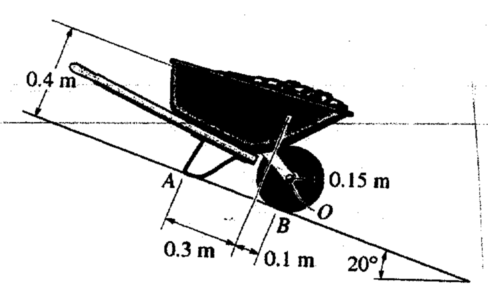
$a_G = 22.89\ \text{ft/s}^2\qquad F = 10.67\ \text{lb}\qquad (a_A)_t = 68.69\ \text{ft/s}^2$

$\alpha = 73.3\ \text{rad/s}^2$ **Ans**

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$3 = 0 + 0 + \frac{1}{2}(68.69)t^2\qquad t = 0.296\ \text{s}$ **Ans**

R2-17. The wheel barrow and its contents have a mass of 40 kg and a mass center at G, excluding the wheel. The wheel has a mass of 2 kg and a radius of gyration $k_O = 0.120$ m. If the wheelbarrow is released from rest from the position shown, determine its speed after it travels 4 m down the incline. The coefficient of kinetic friction between the incline and A is $\mu_A = 0.3$. The wheels roll without slipping at B.



$\nearrow + \Sigma F_y = m(a_G)_y;\quad N_A + N_B - 44(9.81)\cos 20° = 0$ (1)

$+\searrow \Sigma F_x = ma_x;\quad 44(9.81)\sin 20° - 0.3N_A = 40a_G + 4a_O$ (2)

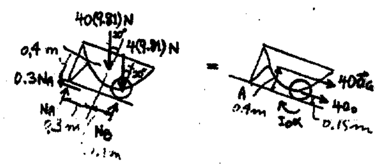
$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;\quad 40(9.81)\sin 20°(0.4) + 40(9.81)\cos 20°(0.3) + 4(9.81)\sin 20°(0.15) + 4(9.81)\cos 20°(0.4) - N_B(0.4) = 40a_G(0.4) + 4a_O(0.15) + [4(0.12)^2]\alpha$ (3)

Kinematics

$a_O = a_G$ (since the cart translates)

$a_O = a_G = 0.15\alpha$ (since the wheels roll without slipping)

Eqs.(1) – (3) reduce to



$N_A + N_B = 405.61$

$147.63 - 0.3N_A = 6.60\alpha$

$181.067 - 0.4N_B = 2.548\alpha$

Solving,

$\alpha = 19.0\ \text{rad/s}^2$

$a_G = 2.85\ \text{m/s}^2$

$N_A = 74.0\ \text{N}$

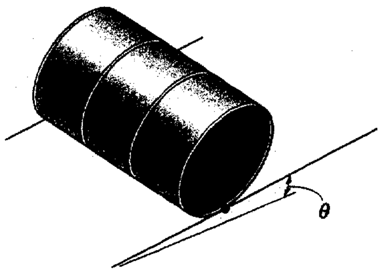
$N_B = 331.6\ \text{N}$

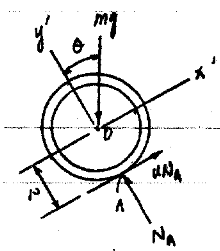
$v^2 = (v_0)^2 + 2a_G(s - s_0)$

$v^2 = 0 + 2(2.85)(4 - 0)$

$v = 4.78\ \text{m/s}$ **Ans**

R2-18. The drum of mass m, radius r, and radius of gyration k_O rolls along an inclined plane for which the coefficient of static friction is μ. If the drum is released from rest, determine the maximum angle θ for the incline so that it rolls without slipping.





$$m(v_{Oy'})_1 + \Sigma\int_{t_1}^{t_2} F_{y'}\,dt = m(v_{Oy'})_2$$

$$0 + N_A(t) - mg\cos\theta(t) = 0 \qquad N_A = mg\cos\theta$$

$$m(v_{Ox'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{Ox'})_2$$

$$0 + mg\sin\theta(t) - \mu mg\cos\theta(t) = mv_O \qquad [1]$$

Since no slipping occurs, $v_O = \omega r$. Hence Eq.[1] becomes

$$mg\sin\theta(t) - \mu mg\cos\theta(t) = m\omega r$$

$$t = \frac{m\omega r}{mg(\sin\theta - \mu\cos\theta)} \qquad [2]$$

$$I_A\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$$

$$0 + mg\sin\theta(r)(t) = \left[mk_O^2 + mr^2\right]\omega$$

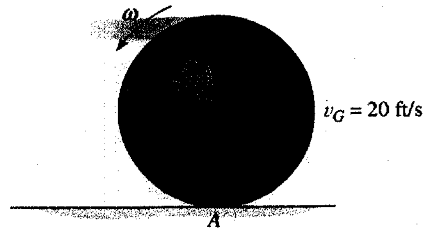
$$t = \frac{\left[mk_O^2 + mr^2\right]\omega}{mgr\sin\theta} \qquad [3]$$

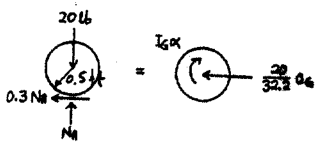
Equating Eqs.[2] and [3]

$$\frac{m\omega r}{mg(\sin\theta - \mu\cos\theta)} = \frac{\left[mk_O^2 + mr^2\right]\omega}{mgr\sin\theta}$$

$$\theta = \tan^{-1}\left[\frac{\mu(k_O^2 + r^2)}{k_O^2}\right] \qquad \text{Ans}$$

R2-19. The 20-lb solid ball is cast on the floor such that it has a backspin $\omega = 15$ rad/s and its center has an initial horizontal velocity $v_G = 20$ ft/s. If the coefficient of kinetic friction between the floor and the ball is $\mu_A = 0.3$, determine the distance it travels before it stops spinning.





$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x;\quad 0.3N_A = \frac{20}{32.2}a_G$

$+\uparrow\Sigma F_y = m(a_G)_y;\quad N_A - 20 = 0$

$\circlearrowleft+\Sigma M_G = I_G\alpha;\quad 0.3N_A(0.5) = [\frac{2}{5}(\frac{20}{32.2})(0.5)^2]\alpha$

Solving,

$N_A = 20$ lb

$a_G = 9.66$ ft/s^2

$\alpha = 48.3$ rad/s^2

$(\circlearrowleft+)\quad \omega = \omega_0 + \alpha_c t$

$0 = 15 - 48.3t$

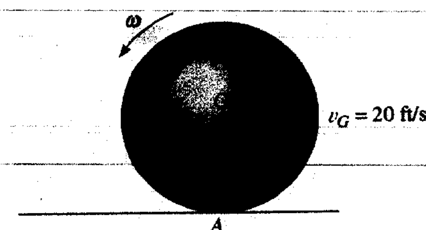
$t = 0.311$ s

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 20(0.311) - \frac{1}{2}(9.66)(0.311)^2$

$s = 5.75$ ft **Ans**

***R2-20.** Determine the backspin ω which should be given to the 20-lb ball so that when its center is given an initial horizontal velocity $v_G = 20$ ft/s it stops spinning and translating at the same instant. The coefficient of kinetic friction is $\mu_A = 0.3$.

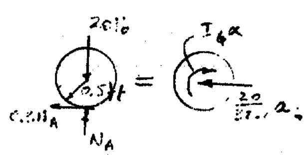


$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x;\quad 0.3N_A = \frac{20}{32.2}a_G$

$+\uparrow\Sigma F_y = m(a_G)_y;\quad N_A - 20 = 0$

$\circlearrowleft+\Sigma M_G = I_G\alpha;\quad 0.3N_A(0.5) = [\frac{2}{5}(\frac{20}{32.2})(0.5)^2]\alpha$

Solving,



$N_A = 20$ lb

$a_G = 9.66$ ft/s^2

$\alpha = 48.3$ rad/s^2

$(\circlearrowleft+)\quad \omega = \omega_0 + \alpha_c t$

$0 = \omega_1 - 48.3t$

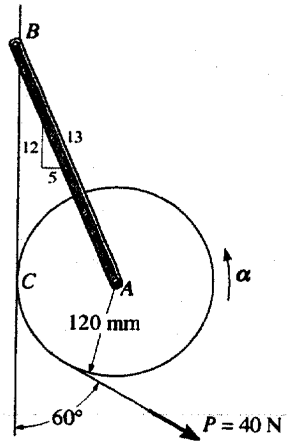
$\omega_1 = 48.3t$

$(\overset{+}{\rightarrow})\quad v = v_0 + a_c t$

$0 = 20 - 9.66(\frac{\omega}{48.3})$

$\omega = 100$ rad/s **Ans**

R2-21. A 20-kg roll of paper, originally at rest, is pin-supported at its ends to bracket AB. The roll rests against a wall for which the coefficient of kinetic friction at C is $\mu_C = 0.3$. If a force of 40 N is applied uniformly to the end of the sheet, determine the initial angular acceleration of the roll and the tension in the bracket as the paper unwraps. For the calculation, treat the roll as a cylinder.

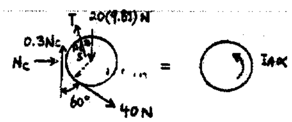


$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 40\sin 60° + N_C - (\frac{5}{13})T = 0$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad -40\cos 60° + 0.3N_C - 20(9.81) + \frac{12}{13}T = 0$

$\circlearrowleft + \Sigma M_A = I_A\alpha; \quad 40(0.120) - 0.3N_C(0.120) = [\frac{1}{2}(20)(0.120)^2]\alpha$

Solving,

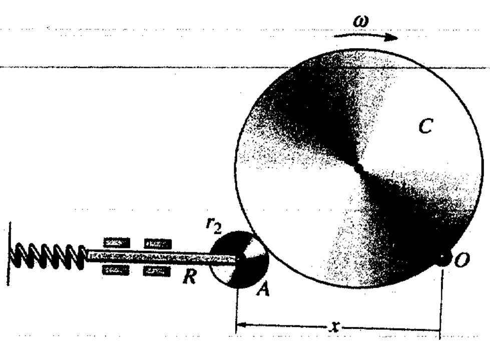


$T = 218$ N **Ans**

$N_C = 49.28$ N

$\alpha = 21.0$ rad/s^2 **Ans**

R2-22. Compute the velocity of rod R for any angle θ of the cam C if the cam rotates with a constant angular velocity ω. The pin connection at O does not cause an interference with the motion of A on C.

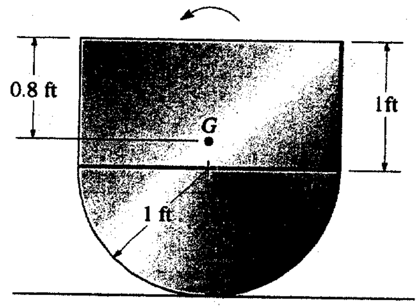


$x = r_1\cos\theta + \sqrt{(r_1+r_2)^2 - (r_1\sin\theta)^2}$

$\dot{x} = -r_1\sin\theta\dot{\theta} - \dfrac{2(r_1\sin\theta)r_1\cos\theta\dot{\theta}}{2\sqrt{(r_1+r_2)^2-(r_1\sin\theta)^2}}$

$v = -r_1\omega\sin\theta - \dfrac{r_1^2\omega\sin 2\theta}{2\sqrt{(r_1+r_2)^2-(r_1\sin\theta)^2}}$ **Ans**

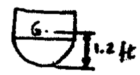
R2-23. The assembly weighs 10 lb and has a radius of gyration $k_G = 0.6$ ft about its center of mass G. The kinetic energy of the assembly is 31 ft-lb when it is in the position shown. If it is rolling counterclockwise on the surface without slipping, determine its linear momentum at this instant.



$$I_G = (0.6)^2\left(\frac{10}{32.2}\right) = 0.1118 \text{ slug}\cdot\text{ft}^2$$

$$T = \frac{1}{2}\left(\frac{10}{32.2}\right)v_G^2 + \frac{1}{2}(0.1118)\,\omega^2 = 31 \qquad (1)$$

$$v_G = 1.2\,\omega$$



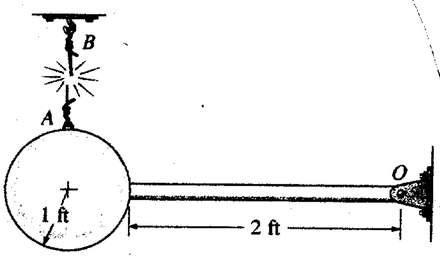
Substitute into Eq. (1),

$$\omega = 10.53 \text{ rad/s}$$

$$v_G = 10.53(1.2) = 12.64 \text{ ft/s}$$

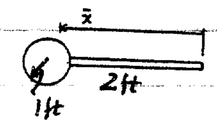
$$L = mv_G = \frac{10}{32.2}(12.64) = 3.92 \text{ slug}\cdot\text{ft/s} \qquad \textbf{Ans}$$

***R2-24.** The pendulum consists of a 30-lb sphere and a 10-lb slender rod. Compute the reaction at the pin O just after the cord AB is cut.



$$I_O = \frac{2}{5}\left(\frac{30}{32.2}\right)(1)^2 + \left(\frac{30}{32.2}\right)(3)^2 + \frac{1}{3}\left(\frac{10}{32.2}\right)(2)^2 = 9.17 \text{ slug}\cdot\text{ft}^2$$

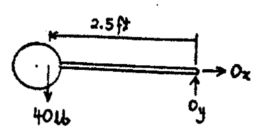
$$\bar{x} = \frac{30(3) + 10(1)}{30 + 10} = 2.5 \text{ ft}$$



$$\overset{+}{\rightarrow}\Sigma F_n = ma_n; \qquad O_x = 0$$

$$+\downarrow \Sigma F_t = ma_t; \qquad 40 - O_y = \frac{40}{32.2}a_G$$

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad 40(2.5) = 9.17\alpha$$



Kinematics,

$$a_G = 2.5\alpha$$

Solving,

$$\alpha = 10.90 \text{ rad/s}^2$$

$$a_G = 27.3 \text{ ft/s}^2$$

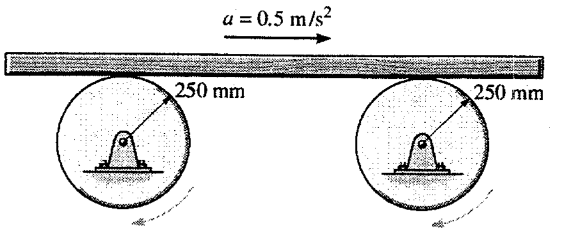
$$O_x = 0$$

$$O_y = 6.14 \text{ lb}$$

Thus;

$$F_O = 6.14 \text{ lb} \qquad \textbf{Ans}$$

R2-25. The board rests on the surface of two drums. At the instant shown, it has an acceleration of 0.5 m/s² to the right, while at the same instant points on the outer rim of each drum have an acceleration with a magnitude of 3 m/s². If the board does not slip on the drums, determine its speed due to the motion.



A point on the drum which is in contact with the board has a tangential acceleration of

$$a_t = 0.5 \text{ m/s}^2$$

$$a^2 = a_t^2 + a_n^2$$

$$(3)^2 = (0.5)^2 + a_n^2$$

$$a_n = 2.96 \text{ m/s}^2$$

$$a_n = \omega^2 r, \qquad \omega = \sqrt{\frac{2.96}{0.25}} = 3.44 \text{ rad/s}$$

$$v_B = \omega r = 3.44(0.25) = 0.860 \text{ m/s} \qquad \textbf{Ans}$$

R2-26. The center of the pulley is being lifted vertically with an acceleration of 4 m/s² at the instant it has a velocity of 2 m/s. If the cable does not slip on the pulley's surface, determine the accelerations of the cylinder B and point C on the pulley.

$$\omega = \frac{2}{0.08} = 25 \text{ rad/s}$$

$$\alpha = \frac{4}{0.08} = 50 \text{ rad/s}^2$$

$$\mathbf{a}_C = \mathbf{a}_A + (\mathbf{a}_{C/A})_n + (\mathbf{a}_{C/A})_t$$

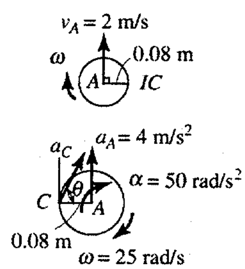
$$\mathbf{a}_C = 4\mathbf{j} + (25)^2(0.08)\mathbf{i} + 50(0.08)\mathbf{j}$$

$$\overset{+}{\rightarrow} \qquad a_C \cos\theta = 0 + 50$$

$$+\uparrow \qquad a_C \sin\theta = 4 + 0 + 4$$

Solving, $\quad a_C = 50.6 \text{ m/s}^2 \qquad$ **Ans**

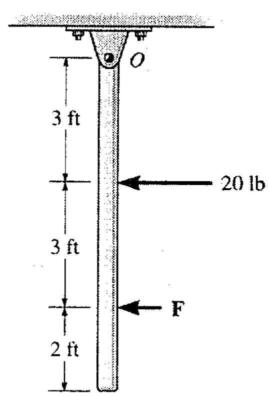
$\theta = 9.09° \qquad$ **Ans**



The block moves up with an acceleration

$$a_B = (a_C)_t = 50.6 \sin 9.09° = 8.00 \text{ m/s}^2 \uparrow \qquad \textbf{Ans}$$

R2-27. At the instant shown, two forces act on the 30-lb slender rod which is pinned at O. Determine the magnitude of force **F** and the initial angular acceleration of the rod so that the horizontal reaction which the *pin exerts on the rod* is 5 lb directed to the right.xsa



$$(a_G)_t = 4\alpha$$

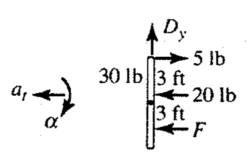
$$\overset{+}{\leftarrow} \sum F_t = m(a_G)_t; \qquad F + 20 - 5 = \frac{30}{32.2}(4\alpha)$$

$$\curvearrowleft + \sum M_O = I_O\alpha; \qquad 20(3) + F(6) = \frac{1}{3}\left(\frac{30}{32.2}\right)(8)^2\alpha$$

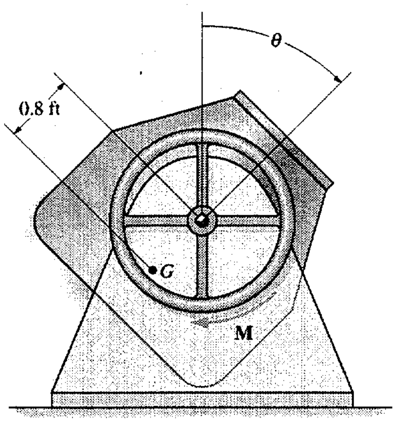
Solving,

$\alpha = 12.1 \text{ rad/s}^2 \qquad$ **Ans**

$F = 30.0 \text{ lb} \qquad$ **Ans**



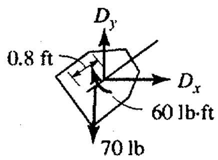
***R-28.** The tub of the mixer has a weight of 70 lb and a radius of gyration $k_G = 1.3$ ft about its center of gravity. If a constant torque $M = 60$ lb · ft is applied to the dumping wheel, determine the angular velocity of the tub when it has rotated $\theta = 90°$. Originally the tub is at rest when $\theta = 0°$.



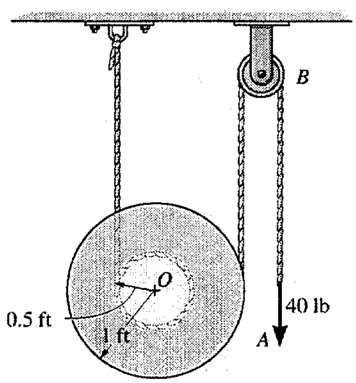
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 60\left(\frac{\pi}{2}\right) - 70(0.8) = \frac{1}{2}\left[\left(\frac{70}{32.2}\right)(1.3)^2\right](\omega)^2 + \frac{1}{2}\left[\frac{70}{32.2}\right](0.8\omega)^2$$

$\omega = 3.89$ rad/s **Ans**



R2-29. The spool has a weight of 30 lb and a radius of gyration $k_O = 0.65$ ft. If a force of 40 lb is applied to the cord at A, determine the angular velocity of the spool in $t = 3$ s starting from rest. Neglect the mass of the pulley and cord.

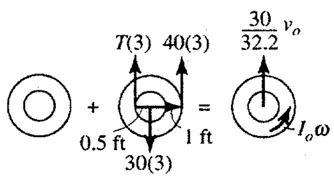


$(+\uparrow)$ $$mv_1 + \sum \int F\,dt = mv_2$$

$$0 + T(3) - 30(3) + 40(3) = \frac{30}{32.2}v_O$$

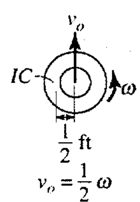
$(\circlearrowleft +)$ $$(H_O)_1 + \sum \int M_O\,dt = (H_O)_2$$

$$-T(0.5)3 + 40(1)3 = \left[\frac{30}{32.2}(0.65)^2\right]\omega$$



Kinematics,

$v_O = 0.5\omega$



Solving,

$T = 23.5$ lb

$\omega = 215$ rad/s **Ans**

$v_O = 108$ ft/s

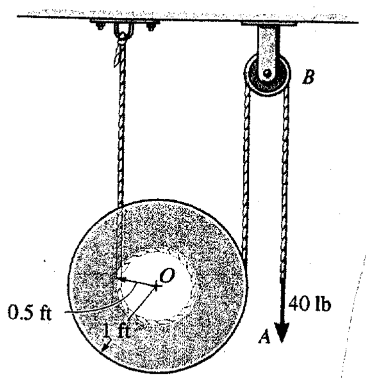
Also,

$(\circlearrowleft +)$ $$(H_{IC})_1 + \sum \int M_{IC}\,dt = (H_{IC})_2$$

$$0 - 30(0.5)(3) + 40(1.5)(3) = \left[\frac{30}{32.2}(0.65)^2 + \frac{30}{32.2}(0.5)^2\right]\omega$$

$\omega = 215$ rad/s **Ans**

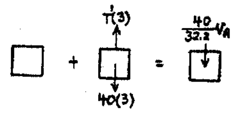
R2-30. Solve Prob. R2–29 if a 40-lb block is suspended from the cord at A, rather than applying the 40-lb force.



Block :

$(+\downarrow) \quad mv_1 + \Sigma\int F\,dt = mv_2$

$0 + 40(3) - T'(3) = \dfrac{40}{32.2}v_A \qquad (1)$



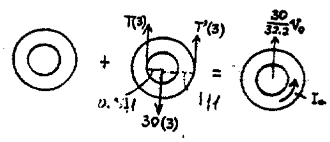
Spool :

$(+\uparrow) \quad mv_1 + \Sigma\int F\,dt = mv_2$

$0 + T(3) - 30(3) + T'(3) = \dfrac{30}{32.2}v_O$

$(\circlearrowleft +) \quad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$

$0 - T(0.5)3 + T'(1)3 = [\dfrac{30}{32.2}(0.65)^2]\omega$



Kinematics

$v_O = 0.5\omega$

$v_A = 1.5\omega$

Solving,

$T' = 15.5$ lb

$T = 20.6$ lb

$\omega = 39.5$ rad/s **Ans**

$v_O = 19.7$ ft/s

$v_A = 59.2$ ft/s

Also,

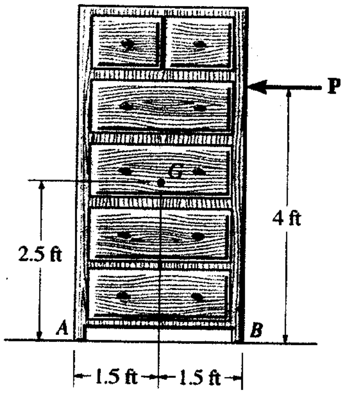
$(\circlearrowleft +) \quad (H_{IC})_1 + \Sigma\int M_{IC}\,dt = (H_{IC})_2$

$0 + T'(1.5)(3) - 30(0.5)(3) = \left(\dfrac{30}{32.2}(0.65)^2 + \dfrac{30}{32.2}(0.5)^2\right)\omega$

Since $v_A = 1.5\omega$, then with Eq. (1) we get

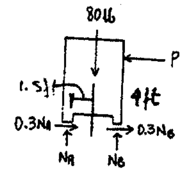
$\omega = 39.5$ rad/s **Ans**

R2-31. The dresser has a weight of 80 lb and is pushed along the floor. If the coefficient of static friction at A and B is $\mu_s = 0.3$ and the coefficient of kinetic friction is $\mu_k = 0.2$, determine the smallest horizontal force P needed to cause motion. If this force is increased slightly, determine the acceleration of the dresser. Also, what are the normal reactions at A and B when it begins to move?



For slipping:

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad -P + 0.3(N_A + N_B) = 0$



$+\uparrow\Sigma F_y = 0;\quad N_A + N_B - 80 = 0$

$P = 24$ lb **Ans**

For tipping $N_B = 0$, $N_A = 80$ lb

$\circlearrowleft +\Sigma M_A = 0;\quad P(4) - 80(1.5) = 0$

$P = 30 \text{ lb} < 24 \text{ lb}$

Therefore dresser slips.

$+\uparrow\Sigma F_y = 0;\quad N_A + N_B - 80 = 0$

$\overset{+}{\leftarrow}\Sigma F_x = ma_x;\quad 24 - 0.2N_A - 0.2N_B = \frac{80}{32.2}a_G$

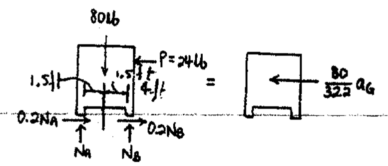
$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A;\quad 24(4) + N_B(3) - 80(1.5) = \frac{80}{32.2}a_G(2.5)$

Solving,

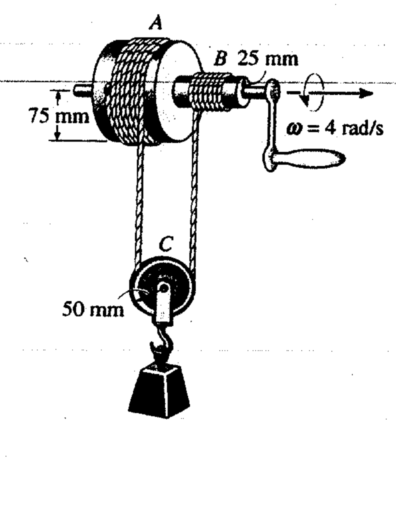
$a_G = 3.22$ ft/s^2 **Ans**

$N_B = 14.8$ lb **Ans**

$N_A = 65.3$ lb **Ans**

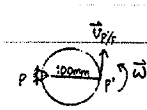


***R2-32.** When the crank on the Chinese windlass is turning, the rope on shaft A unwinds while that on shaft B winds up. Determine the speed at which the block lowers if the crank is turning with an angular velocity $\omega = 4$ rad/s. What is the angular velocity of the pulley at C? The rope segments on each side of the pulley are both parallel and vertical, and the rope does not slip on the pulley.



$v_P = \omega r_A = 4(75) = 300 \text{ mm/s} \downarrow$

$v_{P'} = \omega r_B = 4(25) = 100 \text{ mm/s} \uparrow$



$\mathbf{v}_{P'} = \mathbf{v}_P + \mathbf{v}_{P'/P}$

$100\mathbf{j} = -300\mathbf{j} + \omega(100)\mathbf{j}$

$100 = -300 + \omega(100)$

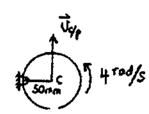
$\omega = 4$ rad/s **Ans**

$\mathbf{v}_C = \mathbf{v}_P + \mathbf{v}_{C/P}$

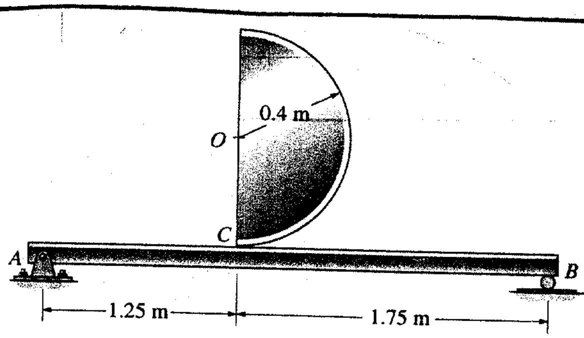
$-v_C\mathbf{j} = -300\mathbf{j} + 4(50)\mathbf{j}$

$-v_C = -300 + 200$

$v_C = 100 \text{ mm/s} \downarrow$ **Ans**



R2-33. The semicircular disk has a mass of 50 kg and is released from rest from the position shown. The coefficients of static and kinetic friction between the disk and the beam are $\mu_s = 0.5$ and $\mu_k = 0.3$, respectively. Determine the initial reactions at the pin A and roller B, used to support the beam. Neglect the mass of the beam for the calculation.



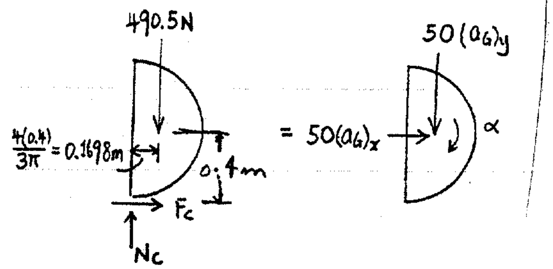
Semicircular disk :

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_C = 50(a_G)_x \qquad (1)$$

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad 490.5 - N_C = 50(a_G)_y \qquad (2)$$

$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad N_C(0.1698) - F_C(0.4) = [\frac{1}{2}(50)(0.4)^2 - 50(0.1698)^2]\alpha \qquad (3)$$

Assume no slipping at C,



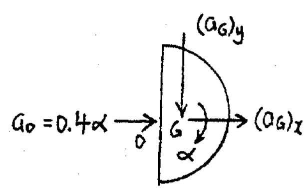
$$\mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O}$$

$$(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = 0.4\,\alpha\,\mathbf{i} - \alpha(0.1698)\mathbf{j}$$

$$(a_G)_x = 0.4\alpha$$

$$(a_G)_y = 0.1698\alpha$$

Solving,



$$\alpha = 6.94 \text{ rad/s}^2$$

$$(a_G)_x = 2.78 \text{ m/s}^2$$

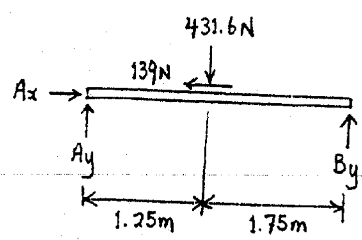
$$(a_G)_y = 1.18 \text{ m/s}^2$$

$$N_C = 431.6 \text{ N}$$

$$F_C = 139 \text{ N}$$

$$(F_C)_{max} = 0.5(431.6) = 216 \text{ N} > 139 \text{ N} \qquad \textbf{(O.K!)}$$

Beam :



$$\circlearrowleft + \Sigma M_A = 0; \qquad 431.6(1.25) - B_y(3) = 0$$

$B_y = 179.8$ N $= 180$ N **Ans**

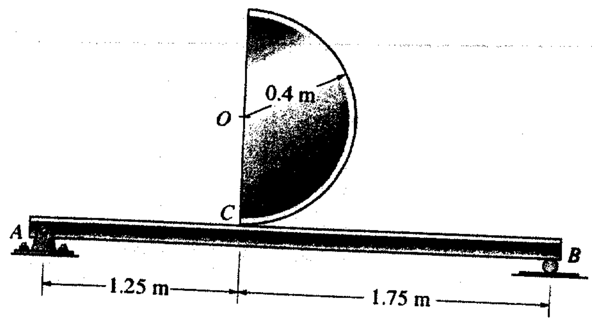
$$+\uparrow \Sigma F_y = 0; \qquad A_y + 179.8 - 431.6 = 0$$

$A_y = 252$ N **Ans**

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 139 = 0$$

$A_x = 139$ N **Ans**

R2-34. The semicircular disk has a mass of 50 kg and is released from rest from the position shown. The coefficients of static and kinetic friction between the disk and the beam are $\mu_s = 0.2$ and $\mu_k = 0.1$, respectively. Determine the initial reactions at the pin A and roller B used to support the beam. Neglect the mass of the beam for the calculation.

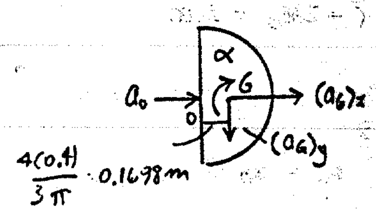


See solution to Prob. R2−33.

Here $\quad F_C = 139 \text{ N} > 0.2(432) = 86.4 \text{ N}$

Therefore disk will slip.

Thus, $\quad F_C = 0.1 N_C \qquad (4)$



$\mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O}$

$(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = a_O\mathbf{i} - \alpha(0.1698)\mathbf{j}$

$(+\downarrow) \quad (a_G)_y = \alpha(0.1698) \qquad (5)$

Solving Eqs.(1)−(3) of Prob. R2−33, and Eqs.(4)&(5).

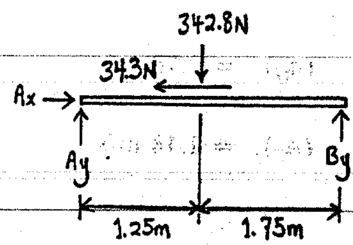
$\alpha = 17.4 \text{ rad/s}^2$

$(a_G)_x = 0.686 \text{ m/s}^2$

$(a_G)_y = 2.95 \text{ m/s}^2$

$N_C = 342.8 \text{ N}$

$F_C = 34.3 \text{ N}$



$\circlearrowleft + \Sigma M_A = 0; \quad 342.8(1.25) - B_y(3) = 0$

$B_y = 143 \text{ N} \qquad$ **Ans**

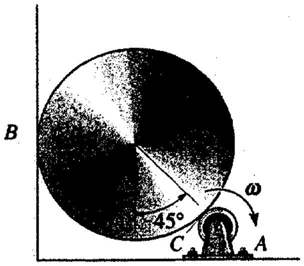
$+\uparrow \Sigma F_y = 0; \quad A_y + 143 - 342.8 = 0$

$A_y = 200 \text{ N} \qquad$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 34.3 = 0$

$A_x = 34.3 \text{ N} \qquad$ **Ans**

R2-35. The cylinder having a mass of 5 kg is initially at rest when it is placed in contact with the wall B and the rotor at A. If the rotor always maintains a constant clockwise angular velocity $\omega = 6$ rad/s, determine the initial angular acceleration of the cylinder. The coefficient of kinetic friction at the contacting surfaces B and C is $\mu_k = 0.2$.



$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A \cos 45° + 0.2N_A \sin 45° + 0.2N_B - 5(9.81) = 0$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad N_B + 0.2N_A \cos 45° - N_A \sin 45° = 0$$

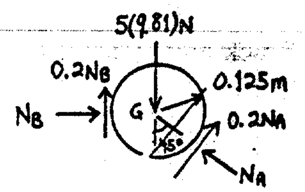
$$\circlearrowleft + \Sigma M_G = I_G \alpha; \quad 0.2N_A(0.125) - 0.2N_B(0.125) = [\frac{1}{2}(5)(0.125)^2]\alpha$$

Solving,

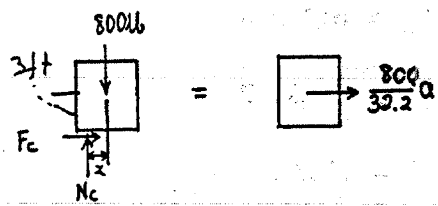
$N_A = 51.0$ N

$N_B = 28.9$ N

$\alpha = 14.2$ rad/s^2 **Ans**



***R2-36.** The truck carries the 800-lb crate which has a center of gravity at G_c. Determine the largest acceleration of the truck so that the crate will not slip or tip on the truck bed. The coefficient of static friction between the crate and the truck is $\mu_s = 0.6$.



$$\xrightarrow{+} \Sigma F_x = ma_x; \quad F_C = \frac{800}{32.2}a$$

$$+\uparrow \Sigma F_y = 0; \quad N_C - 800 = 0$$

$$\circlearrowleft + \Sigma M_{G_c} = 0; \quad -N_C(x) + F_C(3) = 0$$

Assume impending slip; $F_C = 0.6N_C$

$a = 19.32$ ft/s^2

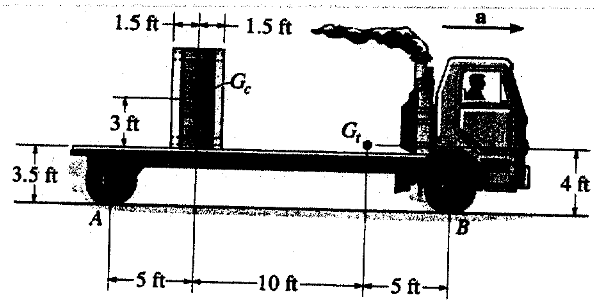
$x = 1.8$ ft $>$ 1.5 ft crate tips!

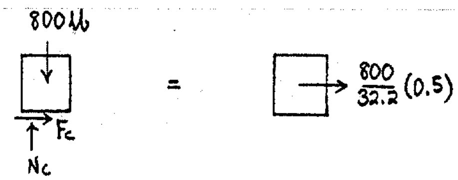
Thus, $x = 1.5$ ft

$F_C = 400$ lb

$a = 16.1$ ft/s^2 **Ans**

R2-37. The truck has a weight of 8000 lb and center of gravity at G_t. It carries the 800-lb crate, which has a center of gravity at G_c. Determine the normal reaction at *each* of its four tires if it accelerates at $a = 0.5\ \text{ft/s}^2$. Also, what is the frictional force acting between the crate and the truck, and between *each* of the rear tires and the road? Assume that power is delivered only to the rear tires. The front tires are free to roll. Neglect the mass of the tires. The crate does not slip or tip on the truck.





Crate :

$+\uparrow \Sigma F_y = 0; \qquad N_C = 800\ \text{lb}$

$\xrightarrow{+} \Sigma F_x = ma_x; \quad F_C = \dfrac{800}{32.2}(0.5) = 12.4\ \text{lb}$ **Ans**

Truck and crate :

$+\uparrow \Sigma F_y = 0; \qquad N_A + N_B - 800 - 8000 = 0$

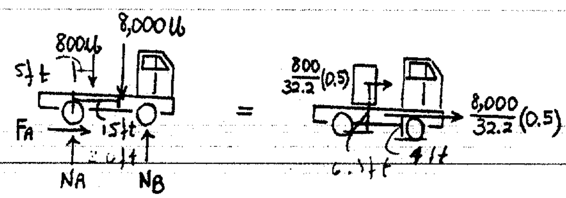
$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_A = \dfrac{800}{32.2}(0.5) + \dfrac{8000}{32.2}(0.5)$

$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad N_B(20) - 800(5) - 8000(15) = -\dfrac{800}{32.2}(0.5)(6.5) - \dfrac{8000}{32.2}(0.5)(4)$

$N_B = 6171.1\ \text{lb}$

$N_A = 2628.9\ \text{lb}$

$F_A = 136.6\ \text{lb}$



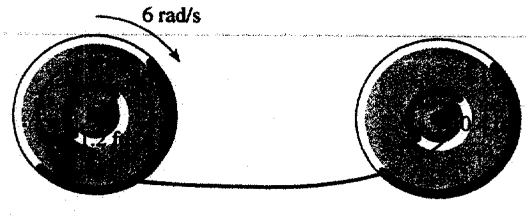
Thus,

$N_{B'} = \dfrac{N_B}{2} = 3.09\ \text{kip}$ **Ans**

$N_{A'} = \dfrac{N_A}{2} = 1.31\ \text{kip}$ **Ans**

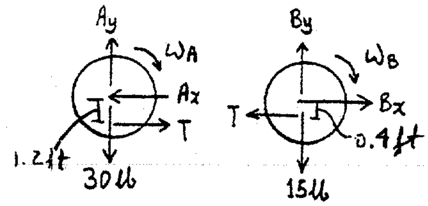
$F_{A'} = \dfrac{F_A}{2} = 68.3\ \text{lb}$ **Ans**

R2-38. Spool B is at rest and spool A is rotating at 6 rad/s when the slack in the cord connecting them is taken up. Determine the angular velocity of each spool immediately after the cord is jerked tight. The spools A and B have weights and radii of gyration $W_A = 30$ lb, $k_A = 0.8$ ft and $W_B = 15$ lb, $k_B = 0.6$ ft, respectively.



$$\circlearrowright + \quad (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$$

$$[\frac{30}{32.2}(0.8)^2](6) - \int T\,dt\,(1.2) = [\frac{30}{32.2}(0.8)^2]\,\omega_A$$



$$\circlearrowright + \quad (H_B)_1 + \Sigma\int M_B\,dt = (H_B)_2$$

$$0 + \int T\,dt\,(0.4) = [\frac{15}{32.2}(0.6)^2]\,\omega_B$$

Kinematics:

$$1.2\omega_A = 0.4\omega_B$$

$$\omega_B = 3\omega_A$$

Thus,

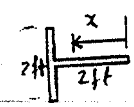
$\omega_A = 1.70$ rad/s **Ans**

$\omega_B = 5.10$ rad/s **Ans**

R2-39. The two 3-lb rods EF and HI are fixed (welded) to the link AC at E. Determine the internal axial force E_x, shear force E_y, and moment M_E, which the bar AC exerts on FE at E if at the instant $\theta = 30°$ link AB has an angular velocity $\omega = 5$ rad/s and an angular acceleration $\alpha = 8$ rad/s² as shown.

Curvilinear translation.

$$(a_G)_t = 8(3) = 24 \text{ ft/s}^2$$



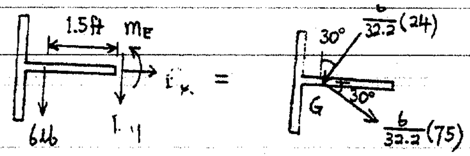
$$(a_G)_n = (5)^2(3) = 75 \text{ ft/s}^2$$

$$\bar{x} = \frac{\Sigma\tilde{x}m}{\Sigma m} = \frac{1(3) + 2(3)}{6} = 1.5 \text{ ft}$$

$$+\downarrow \Sigma F_y = m(a_G)_y; \quad E_y + 6 = \frac{6}{32.2}(24)\cos 30° + \frac{6}{32.2}(75)\sin 30°$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad E_x = \frac{6}{32.2}(75)\cos 30° - \frac{6}{32.2}(24)\sin 30°$$

$$\circlearrowleft + \Sigma M_G = 0; \quad M_E - E_y(1.5) = 0$$

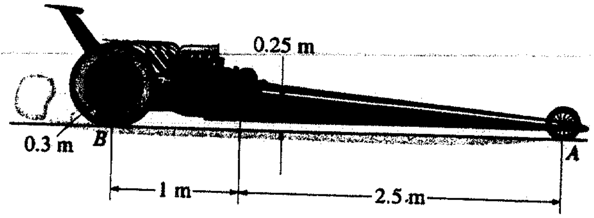


$E_x = 9.87$ lb **Ans**

$E_y = 4.86$ lb **Ans**

$M_E = 7.29$ lb·ft **Ans**

***R2-40.** The dragster has a mass of 1500 kg and a center of mass at G. If the coefficient of kinetic friction between the rear wheels and the pavement is $\mu_k = 0.6$, determine if it is possible for the driver to lift the front wheels, A, off the ground while the rear wheels are slipping. If so, what acceleration is necessary to do this? Neglect the mass of the wheels and assume that the front wheels are free to roll.



$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 0.6N_B = 1500a_G \quad (1)$$

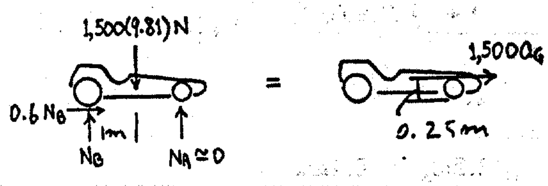
$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_B - 1500(9.81) = 0 \quad (2)$$

$$\circlearrowleft + \Sigma M_B = \Sigma(M_k)_B; \quad -1500(9.81)(1) = -1500(a_G)(0.25) \quad (3)$$

Solving Eqs.(1)&(2)

$N_B = 14715$ N

$a_G = 5.886$ m/s^2



From Eq.(3);

$a_G = 39.24$ m/s^2 $\rightarrow$

Acceleration necessary to lift front wheels is

$a_G = 39.2$ m/s^2 **Ans**

It cannot be done since 39.2 m/s^2 > 5.89 m/s^2 **Ans**

R2-41. The dragster has a mass of 1500 kg and a center of mass at G. If no slipping occurs, determine the friction force F_B which must be applied to *each* of the rear wheels B in order to develop an acceleration $a = 6$ m/s^2. What are the normal reactions of *each* wheel on the ground? Neglect the mass of the wheels and assume that the front wheels are free to roll.

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad F_B = 1500(6)$$

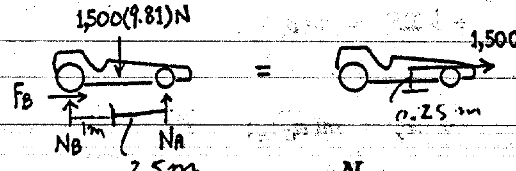
$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_B + N_A - 1500(9.81) = 0$$

$$\circlearrowleft + \Sigma M_B = \Sigma(M_k)_B; \quad -1500(9.81)(1) + N_A(3.5) = -1500(6)(0.25)$$

$F_B = 9.00$ kN

$N_A = 3.56$ kN

$N_B = 11.15$ kN

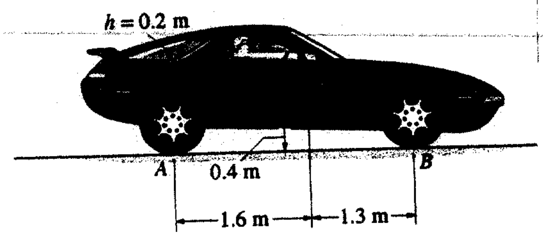


$F_{B'} = \dfrac{F_B}{2} = 4.50$ kN **Ans**

$N_{A'} = \dfrac{N_A}{2} = 1.78$ kN **Ans**

$N_{B'} = \dfrac{N_B}{2} = 5.58$ kN **Ans**

R2-42. The 1.6-Mg car shown has been "raked" by increasing the height $h = 0.2$ m of its center of mass. This was done by raising the springs on the rear axle. If the coefficient of kinetic friction between the rear wheels and the ground is $\mu_k = 0.3$, show that the car can accelerate slightly faster than its counterpart for which $h = 0$. Neglect the mass of the wheels and driver and assume the front wheels at B are free to roll while the rear wheels slip.



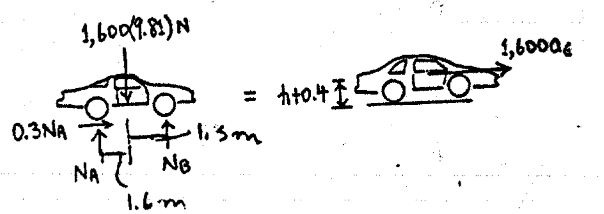
$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.3N_A = 1600a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_B + N_A - 1600(9.81) = 0$

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 1600(9.81)(1.6) - N_B(2.9) = 1600a_G(h + 0.4)$

Set $h = 0.2$ m and solving,

$a_G = 1.41 \text{ m/s}^2$ **Ans**



$N_A = 7.50$ kN

$N_B = 8.19$ kN

Set $h = 0$

$a_G = 1.38 \text{ m/s}^2$ **Ans**

$N_A = 7.34$ kN

$N_B = 8.36$ kN

R2-43. The handcart has a mass of 200 kg and center of mass at G. Determine the normal reactions at *each* of the wheels at A and B if a force $P = 50$ N is applied to the handle. Neglect the mass and rolling resistance of the wheels.

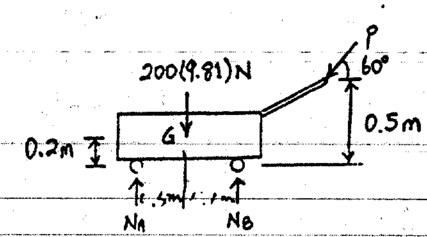
$\xleftarrow{+} \Sigma F_x = ma_x; \qquad 50\cos 60° = 200a_G \qquad (1)$

$+\uparrow \Sigma F_y = ma_y; \qquad N_A + N_B - 200(9.81) - 50\sin 60° = 0 \qquad (2)$

$\circlearrowleft + \Sigma M_G = 0; \qquad -N_A(0.3) + N_B(0.2) + 50\cos 60°(0.3) - 50\sin 60°(0.6) = 0 \qquad (3)$

Solving,

$a_G = 0.125 \text{ m/s}^2$



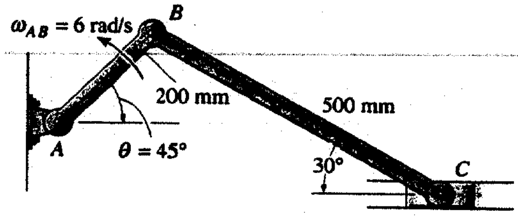
$N_A = 765.2$ N

$N_B = 1240$ N

At each wheel

$N'_A = \dfrac{N_A}{2} = 383$ N **Ans** $\qquad N'_B = \dfrac{N_B}{2} = 620$ N **Ans**

***R2-44.** If bar AB has an angular velocity $\omega_{AB} = 6$ rad/s, determine the velocity of the slider block C at the instant shown.



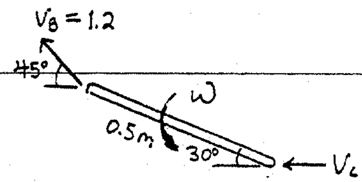
$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$$

$$v_C\mathbf{i} = (6\mathbf{k}) \times (0.2\cos 45°\mathbf{i} + 0.2\sin 45°\mathbf{j}) + (\omega\mathbf{k}) \times (0.5\cos 30°\mathbf{i} - 0.5\sin 30°\mathbf{j})$$

$$v_C = -0.8485 + \omega(0.25)$$

$$0 = 0.8485 + 0.433\,\omega$$



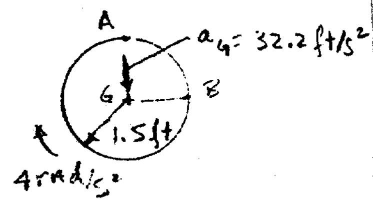
Solving

$\omega = 1.96$ rad/s **Ans**

$v_C = 1.34$ m/s **Ans**

R2-45. The disk is rotating at a constant rate $\omega = 4$ rad/s, and as it falls freely, its center has an acceleration of 32.2 ft/s^2. Determine the acceleration of points A and B on the rim of the disk at the instant shown.

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

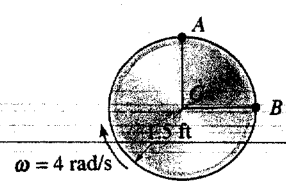


$$\left[\underset{\rightarrow}{(a_A)_x}\right] + \left[\underset{\uparrow}{(a_A)_y}\right] = \left[\underset{\downarrow}{32.2}\right] + 0 + \left[\underset{\downarrow}{(4)^2(1.5)}\right]$$

$$(\overset{+}{\rightarrow})\ (a_A)_x = 0$$

$$(+\uparrow)\ (a_A)_y = -32.2 - (4)^2(1.5) = -56.2 \text{ ft/s}^2 = 56.2 \text{ ft/s}^2 \downarrow$$

$$a_A = (a_A)_y = 56.2 \text{ ft/s}^2 \downarrow$$ **Ans**



$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$

$$\left[\underset{\rightarrow}{(a_B)_x}\right] + \left[\underset{\uparrow}{(a_B)_y}\right] = \left[\underset{\downarrow}{32.2}\right] + 0 + \left[\underset{\leftarrow}{(4)^2(1.5)}\right]$$

$$(\overset{+}{\rightarrow})\ (a_B)_x = -(4)^2(1.5) = -24 \text{ ft/s}^2 = 24 \text{ ft/s}^2 \leftarrow$$

$$(+\uparrow)\ (a_B)_y = -32.2 \text{ ft/s}^2 = 32.2 \text{ ft/s}^2 \downarrow$$

$$a_B = \sqrt{(a_B)_x^2 + (a_B)_y^2} = \sqrt{24^2 + 32.2^2} = 40.2 \text{ ft/s}^2$$ **Ans**

$$\theta = \tan^{-1}\frac{(a_B)_y}{(a_B)_x} = \tan^{-1}\frac{32.2}{24} = 53.3°$$ **Ans**

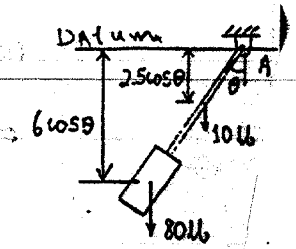
R2-46. The 80-lb cylinder is attached to the 10-lb slender rod which is pinned from point A. At the instant $\theta = 30°$ it has an angular velocity $\omega_1 = 1$ rad/s as shown. Determine the largest angle θ to which the rod swings before it momentarily stops.

$$I_A = \frac{1}{12}\left(\frac{80}{32.2}\right)[3(0.5)^2 + (2)^2] + \frac{80}{32.2}(6)^2 + \frac{1}{3}\left(\frac{10}{32.2}\right)(5)^2 = 93.01 \text{ slug}\cdot\text{ft}^2$$

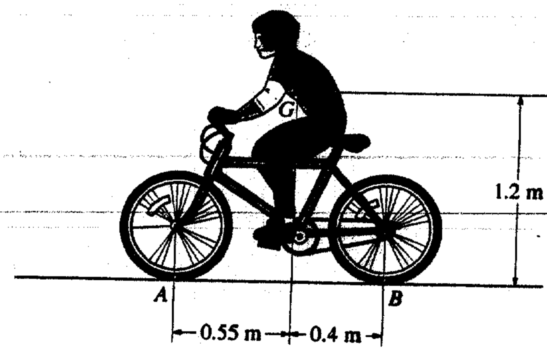
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(93.01)(1)^2 - 10(2.5\cos 30°) - 80(6\cos 30°) = [0] - 10(2.5\cos\theta) - 80(6\cos\theta)$$

$\theta = 39.3°$ **Ans**



R2-47. The bicycle and rider have a mass of 80 kg with center of mass located at G. If the coefficient of kinetic friction at the rear tire is $\mu_B = 0.8$, determine the normal reactions at the tires A and B, and the deceleration of the rider, when the rear wheel locks for braking. What is the normal reaction at the rear wheel when the bicycle is traveling at constant velocity and the brakes are not applied? Neglect the mass of the wheels.

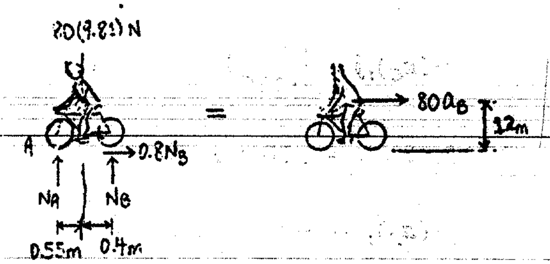


$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad 0.8N_B = 80a_B$

$+\uparrow \Sigma F_y = m(a_G)_y;\quad N_A + N_B - 80(9.81) = 0$

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;\quad -N_B(0.95) + 80(9.81)(0.55) = 80a_B(1.2)$

Solving,



$a_B = 2.26 \text{ m/s}^2$ **Ans**

$N_B = 226$ N **Ans**

$N_A = 559$ N **Ans**

For equilibrium,

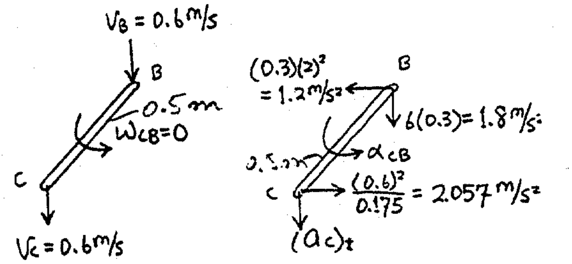
$+\uparrow \Sigma F_y = 0;\quad N_A + N_B - 80(9.81) = 0$

$\circlearrowleft + \Sigma M_A = 0;\quad -N_B(0.95) + 80(9.81)(0.55) = 0$

$N_A = 330$ N

$N_B = 454$ N **Ans**

***R2-48.** At the instant shown, link AB has an angular velocity $\omega_{AB} = 2\text{ rad/s}$ and an angular acceleration $\alpha_{AB} = 6\text{ rad/s}^2$. Determine the acceleration of the pin at C and the angular acceleration of link CB at this instant, when $\theta = 60°$.



$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$$

$$\underset{\rightarrow}{2.057} + \underset{\downarrow}{(a_C)_t} = \underset{\downarrow}{1.8} + \underset{\leftarrow}{1.2} + \underset{\angle\theta\ 30°}{\alpha_{CB}(0.5)}$$

$$(\overset{+}{\rightarrow}) \qquad 2.057 = -1.2 + \alpha_{CB}(0.5)\cos 30°$$

$$(+\downarrow) \qquad (a_C)_t = 1.8 + \alpha_{CB}(0.5)\sin 30°$$

$$\alpha_{CB} = 7.52\text{ rad/s}^2 \qquad \textbf{Ans}$$

$$(a_C)_t = 3.68\text{ m/s}^2$$

$$a_C = \sqrt{(3.68)^2 + (2.057)^2} = 4.22\text{ m/s}^2 \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{3.68}{2.057}\right) = 60.8° \ \angle\theta \qquad \textbf{Ans}$$

Also,

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{CB} \times \mathbf{r}_{C/B} - \omega^2\mathbf{r}_{C/B}$$

$$-(a_C)_t\mathbf{j} + \frac{(0.6)^2}{0.175}\mathbf{i} = -(2)^2(0.3)\mathbf{i} - 6(0.3)\mathbf{j} + (\alpha_{CB}\mathbf{k}) \times (-0.5\cos 60°\mathbf{i} - 0.5\sin 60°\mathbf{j}) - 0$$

$$2.057 = -1.20 + \alpha_{CB}(0.433)$$

$$-(a_C)_t = -1.8 - \alpha_{CB}(0.250)$$

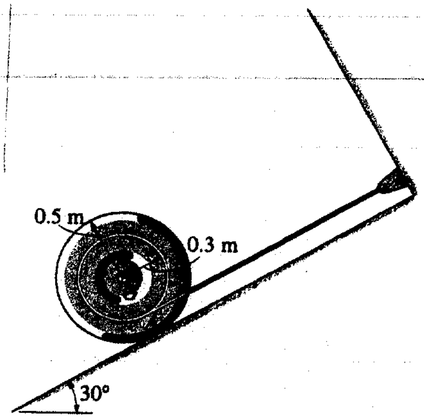
$$\alpha_{CB} = 7.52\text{ rad/s}^2 \qquad \textbf{Ans}$$

$$a_t = 3.68\text{ m/s}^2$$

$$a_C = \sqrt{(3.68)^2 + (2.057)^2} = 4.22\text{ m/s}^2 \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{3.68}{2.057}\right) = 60.8° \ \angle\theta \qquad \textbf{Ans}$$

R2-49. The spool has a mass of 60 kg and a radius of gyration $k_G = 0.3$ m. If it is released from rest, determine how far it descends down the smooth plane before it attains an angular velocity $\omega = 6$ rad/s. Neglect friction and the mass of the cord which is wound around the central core.

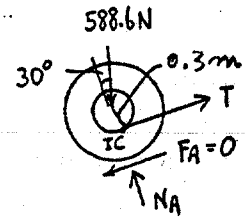


$$v_G = 0.3\omega$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 588.6\sin 30°(s) = \frac{1}{2}[60(0.3)^2](6)^2 + \frac{1}{2}(60)[0.3(6)]^2$$

$s = 0.661$ m **Ans**



R2-50. Solve Prob. R2–49 if the plane is rough, such that the coefficient of kinetic friction at A is $\mu_A = 0.2$.

$$\frac{s_G}{0.3} = \frac{s_A}{(0.5-0.3)}$$

$$s_A = 0.6667 s_G$$

$$\nwarrow + \Sigma F_y = 0; \quad N_A - 588.6\cos 30° = 0$$

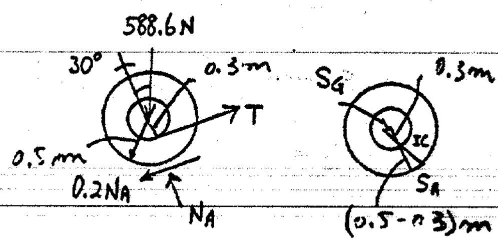
$$N_A = 509.7 \text{ N}$$

$$v_G = 0.3\omega$$

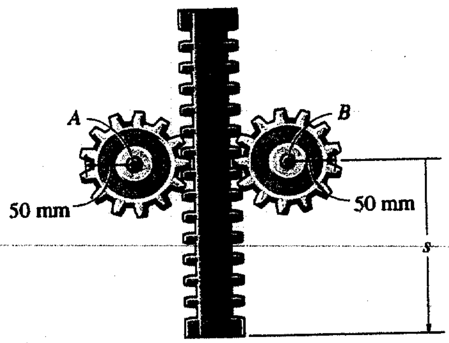
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 588.6\sin 30°(s_G) - 0.2(509.7)(0.6667 s_G) = \frac{1}{2}[60(0.3)^2](6)^2 + \frac{1}{2}(60)[(0.3)(6)]^2$$

$s_G = 0.859$ m **Ans**



R2-51. The gear rack has a mass of 6 kg, and the gears each have a mass of 4 kg and a radius of gyration $k = 30$ mm at their centers. If the rack is originally moving downward at 2 m/s, when $s = 0$, determine the speed of the rack when $s = 600$ mm. The gears are free to turn about their centers, A and B.



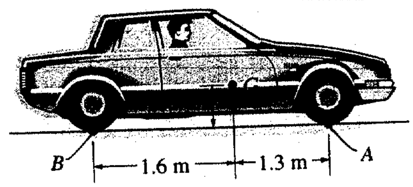
Conservation of energy : Originally, both gears are rotating with an angular velocity of $\omega_1 = \frac{2}{0.05} = 40$ rad/s. After the rack has traveled $s = 600$ mm, both gears rotate with an angular velocity of $\omega_2 = \frac{v_2}{0.05}$, where v_2 is the speed of the rack at that moment.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(6)(2)^2 + 2\{\frac{1}{2}[4(0.03)^2](40)^2\} + 0 = \frac{1}{2}(6)v_2^2 + 2\left\{\frac{1}{2}[4(0.03)^2]\left(\frac{v_2}{0.05}\right)^2\right\} - 6(9.81)(0.6)$$

$v_2 = 3.46$ m/s **Ans**

*R2-52. The car has a mass of 1.50 Mg and a mass center at G. Determine the maximum acceleration it can have if (a) power is supplied only to the rear wheels, (b) power is supplied only to the front wheels. Neglect the mass of the wheels in the calculation, and assume that the wheels that do not receive power are free to roll. Also, assume that slipping of the powered wheels occurs, where the coefficient of kinetic friction is $\mu_k = 0.3$.



(a) Rear wheel drive

Equations of motion :

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.3N_B = 1.5(10)^3 a_G \qquad [1]$$

$$\curvearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 1.5(10)^3(9.81)(1.3) - N_B(2.9) = -1.5(10)^3 a_G(0.4) \qquad [2]$$

Solving Eqs.[1] and [2] yields :

$N_B = 6881 \text{ N} = 6.88 \text{ kN}$

$a_G = 1.38 \text{ m/s}^2$ **Ans**

(b) Front wheel drive

Equations of motion :

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.3N_A = 1.5(10)^3 a_G \qquad [1]$$

$$\Sigma M_B = \Sigma(M_k)_B; \qquad -1.5(10)^3(9.81)(1.6) + N_A(2.9) = -1.5(10)^3 a_G(0.4) \qquad [2]$$

Solving Eqs.[1] and [2] yields :

$N_A = 7796 \text{ N} = 7.80 \text{ kN}$

$a_G = 1.56 \text{ m/s}^2$ **Ans**