Roll No								

National Institute of Technology Goa

Programme Name: **B.Tech.**

End Semester Examinations, December 2021

Course Name: **Mathematics-III**
Course Code: **MA200**
Date: **11/12/2021**
Time: **09:30 AM - 12:30 PM**
Duration : **3 Hours**
Max. Marks: **100**

ANSWER ALL QUESTIONS

1. Consider $f(z) = u(x, y) + iv(x, y)$ is an anlytic function [5M]
   (a) Can the function $v(x, y) = x^2 - y^2 + \frac{x}{x^2+y^2}$ be the Imaginary part of the function $f(z)$?
   (b) Determine all functions $u(x, y)$ such that $f(z)$ is an analytic.
   (c) Express an analytic function $f(z)$ in terms of z.

2. Expand $f(z) = \dfrac{z+4}{z^2\left(z^2+3z+2\right)}$ in a Laurent series valid for [10M]

   (a) $0 < |z| < 1$, (b) $1 < |z| < 2$, (c) $2 > |z|$, (d) $0 < |z+1| < 1$

3. Evaluate: $\oint_C \left(y^2 + 2xy\right) dx + \left(x^2 - 2xy\right) dy$ where C is the boundary of the region by $y = x^2$ and $x = y^2$. [5M]

4. Evaluate $\oint_C \dfrac{z+4}{(z^2+2z+5)} dz$ where C is the circle as follows: [10M]

   (a) $|z| = 1$, (b) $|z+1-i| = 2$, (c) $|z+1+i| = 2$.

5. Evaluate the following improper integrals [10M]

   (a) $\displaystyle\int_{-\infty}^{\infty} \frac{\cos x}{(x^4+x^2+1)} dx$, (b) $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{(x^2+2x+2)\left(x^2+1\right)^2} dx$.

6. Consider the integral [5M]

$$I = \int_0^{2\pi} \frac{\cos 3\theta}{(5 - 4\cos\theta)} dx.$$

   (a) Explain what change(s) of variable you need to make in order to transform this integral to one along the entire unit circle
   (b) Evaluate the given integral by use of residue theorem.

7. Find the bilinear transformation that maps the points $-2, -1-i, 0$ onto $-1, 0, 1$ respectively. [5M]

8. Let $P_n(x)$ be the Legendre polynomial of degree n and let [5M]

$$P_{m+1}(0) = -\frac{m}{m+1} P_{m-1}(0), m = 1, 2, \cdots .$$

   If $P_n(0) = -\frac{5}{16}$ then find $\int_{-1}^{1} P_n^2(x)dx$.

9. Consider the differential equation: [10M]

$$xy'' + y' - y = 0$$

   (a) Classify the nature of the points

   (b) Substitute $y = \sum_{n=0}^{\infty} a_n x^{m+n}$ into given ODE and express left hand side of the ODE as a power series with each term having the (common) factor x^n

   (c) Write down the indicial equation for ODE and determine its roots

   (d) Derive the recurrence formula for a_n

   (e) Determine how many independent solutions this method gives. Give the first three non vanishing term of each infinite series involved.

10. Show that the boundary value problem $y'' - y' + \lambda y = 0,\ y(0) = 0,\ y(L) = 0$ is Sturm-Liouville problem and hence find the eigenvalues and eigenfunctions. [5M]

11. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $u = u_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $u(x, t)$. [10M]

12. A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to $0°C$ and are kept at that temperature. Find the temperature function $u(x, t)$ [10M]

13. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $0 < x < \pi,\ 0 < y < \pi$, which satisfies the conditions: $u(0, y) = u(\pi, y) = u(x, \pi) = 0$ and $u(x, 0) = \sin^2 x$. [10M]

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