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National Institute of Technology Goa

B.Tech. End Semester Examination, April-2021

Department of Humanities and Sciences

Course Name: **MATHEMATICS-I (A, B & C)** Course Code: MA100

Date: April 5, 2021 Time: 9:30 AM

Duration: 3 Hours Max. Marks: 100

ANSWER ALL QUESTIONS

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1. (a) Prove that $\lim_{(x,y)\to(1,2)} (x^2 + 2y) = 5$ by using the precise definition

   (b) Consider

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & \text{otherwise} \end{cases}$$

   i. Does $f(x,y)$ continuous at $(0,0)$?

   ii. Find $f_x(0,0)$ and $f_y(0,0)$

2. (a) Show that if $w = f(u,v)$ satisfies the Laplace equation $f_{uu} + f_{vv} = 0$ and if $u = (x^2 - y^2)/2$ and $v = xy$, then w satisfies the Laplace equation $w_{xx} + w_{yy} = 0$.

   (b) Consider the function $f(x,y) = x^2 + y^2 + 2xy - x - y + 1$ over the square $0 \leq x \leq 1$ and $0 \leq y \leq 1$

   i. Show that f has an absolute minimum along the line segment $2x + 2y = 1$ in this square. What is the absolute minimum value?

   ii. Find the absolute maximum value of f over the square.

3. (a) Minimize the function $f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraints $x + 2y + 3z = 6$ and $x + 3y + 9z = 9$.

   (b) Use the Taylor's formula to find a cubic approximation of $f(x,y) = e^x \cos y$ near the origin.

4. (a) Sketch the region of integration, reverse the order of integration, and evaluate the following integral

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3}\, dy dx.$$

   (b) Evaluate $\int\int_R xy\, dA$ where R is the region bounded by the lines $y = x$, $y = 2x$, and $x + y = 2$.

   (c) Find the center of mass and moment of inertia about the $x-$axis of a thin plate bounded by the curves $x = y^2$ and $x = 2y - y^2$ if the density at the point (x,y) is $\delta = y + 1$.

5. (a) Convert the integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)dzdxdy.$$

to an equivalent integral in cylindrical coordinates and evaluate the result.

(b) Find the volume of the portion of the solid sphere $\rho \leq a$ that lies between the cones $\phi = \pi/3$ and $\phi = 2\pi/3$.

6. (a) Use the Jacobi transformation evaluate the following integral

$$\int\int_R (2x^2 - xy - y^2)dxdy$$

for the region R in the first quadrant bounded by the lines $y = -2x+4$, $y = -2x+7$, $y = x-2$ and $y = x+1$.

(b) Find the Curvature and Torsion of the following helix:

$$r(t) = 2\cos ti + 2\sin tj + 3tk.$$

7. (a) Consider **F**$= (3x^2 - 6y^2)i - (12xy - 4y)j$.

i. Prove that **F** is conservative

ii. Find a potential function for **F**

iii. Let C be the curve $x = 1 + y^3(1-y)^3$, $0 \leq y \leq 1$. Calculate $\int_C F \cdot dr$

(b) Use Green's Theorem to find the counterclockwise circulation and outward flux for the field **F** $= (y^2 - x^2)i + (x^2 + y^2)j$, where the curve C is the region by $y = 0, x = 3$ and $y = x$.

8. (a) Find the surface area of the region S: The portion of the plane $y + 2z = 2$ inside the cylinder $x^2 + y^2 = 1$.

(b) Verify the divergence theorem for $\mathbf{F} = (2x - z)i + x^2yj - xz^2k$ taken over the region bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

9. Verify Stoke's theorem for $\mathbf{F} = 3yi - xzj + yz^2k$, where S is the surface of the paraboloid $2z = x^2 + y^2$ bounded by $z = 2$ and C is its boundary.

10. Find the Fourier series of the following function.

$$f(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ \dfrac{\pi x}{4}, & 0 < x < \pi \end{cases}$$

Also deduce that $\dfrac{\pi^2}{8} = 1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \cdots$

$* * *$ ALL THE BEST $* * *$