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NATIONAL INSTITUTE OF TECHNOLOGY GOA

Farmagudi, Ponda, Goa, 403401

Programme Name: B.Tech.

End Semester Examinations, December-2021

Course Name: Discrete Mathematics | Course Code: CS 203

Date: 9th December | Time: 9.30 AM

Duration: 3 Hours | Max. Marks: 100

ANSWER ALL QUESTIONS

Q1. Let Ai = {…,−2,−1, 0, 1, … , i}. Find **(5 M)**

a) $\bigcup_{i=1}^{n} A_i$. where $A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$

b) $\bigcap_{i=1}^{n} A_i$. where $A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$

Q2. Write the First Order formula for the sentence **(5 M)**

a) "Not all Engineers are intelligent", (Hint: E(x): x is an engineer, I(x): x is intelligent)

b) "Gold and Silver ornaments are precious" (Hint: G(x): x is gold, S(x): x is silver, P(x): x is precious)

Q3. Prove the equivalence using different inference rules **(10 M)**

$\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q.$

$p \to q \equiv p \leftrightarrow p \wedge q.$

Q4. Is this argument valid, "If discrete math is Good then x = 4, Discrete math is Good. Therefore x = 4 "

(10 M)

Q5. Suppose that G1 is a bipartite graph, G2 is the double of G1, G3 is the double of G2, and so forth. Use induction on n to prove that Gn is bipartite for all n>=1. **(10 M)**

Q6. Determine the properties (injective, surjective, bijective) of the functions below, and briefly explain your reasoning. **(10 M)**

i. f : R $\to$ R defined by f(x) = e^x

ii. Let S be the set of all 20-bit sequences. Let T be the set of all 10-bit sequences. Let f : S $\to$ T map each 20-bit sequence to its first 10 bits. For example: f(11110110101101010010) = 1111011010

Q7.

a) Draw the Hasse diagrams for the following relations: **(10 M)**

"Divisibility on the set of all positive divisors of 30."

b) Let S be a set of eleven 2-digit numbers. Prove that S must have two elements whose digits have the same difference (for instance in S = {10,14,19,22,26,28,49,53,70,90,93}, the digits of the numbers 28 and 93 have the same difference: 8−2 = 9−3 = 6.

Q8. **(10 M)**

a) Let **R** be a relation from a set A to a set B. The inverse relation from B to A denoted by $\mathbf{R^{-1}}$ is the set of order pairs $\{(b, a) \mid (a, b) \in R\}$. Show that the relation **R** on a set A is symmetric if and only if $\mathbf{R = R^{-1}}$.

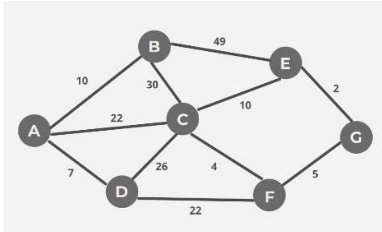
b) Show that the relation R on a set A is antisymmetric if and only if $R \cap R^{-1}$ is a subset of diagonal relation $\Delta = \{(a, a) \mid a \in A\}$.

c) Justify (Z, =) is poset.

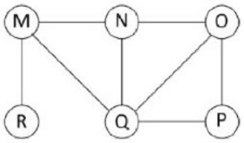
Q9.a) Let $\sigma \in S_n$ and $S_n = \{f : N \to N \mid f \text{ is one to one and onto}\}$. The σ is a K-cycle. Verify that **(10 M)**

(1456)(152) = (16)(245)

b) Show that $\mathbb{Z}^* = \mathbb{Z} \setminus \{0\}$ is group under multiplication. Justify Z is not group under multiplication

Q10. 

Use the Prim's algorithm to construct a MST starting from node A **(5 M)**



Q11. The Breadth first search (BFS) is implemented using Queue data structure. Mention the order of visiting nodes in the given graph. Mention each step of execution. **(5 M)**

Q12. A graph is called **k-regular** if degree of each vertex is "k". Let G = K6 with vertex set {1, 2, 3,4,5,6,7}. **(10 M)**

a. Given sequence of vertices [1, 3, 3, 5], is it a walk? Justify your answer.
b. What is maximum and minimum length of cycle in the given graph G?
c. How many numbers of 4-cycles in the given graph G?
d. What is the basic difference between trail and path, explain through example using the given graph G?