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National Institute of Technology Goa

B.Tech. Mid Semester Examination, February-2021

Department of Humanities and Sciences

Course Name: **MATHEMATICS-I (A, B & C)** Course Code: MA100

Date: February 1, 2021 Time: 9:30 AM

Duration: 90 Min. Max. Marks: 50

ANSWER ALL QUESTIONS

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1. Let $F(x) = \dfrac{x^2 + 3x + 2}{2 - |x|}$. [2M+2M+1M]

   (a) Make tables of values of F at values of x that approach $x_0 = -2$ from above and below. Then estimate $\lim_{x \to -2} F(x)$.

   (b) Support your conclusion in part (a) by graphing F near $x_0 = -2$ and using Zoom and Trace to estimate y-values on the graph as $x \to -2$

   (c) Find $\lim_{x \to -2} F(x)$ algebraically.

2. (a) Suppose that the inequalities [2M+1M+2M]

$$\frac{1}{2} - \frac{x^2}{24} < \frac{1 - \cos x}{x^2} < \frac{1}{2}$$

   hold for values of x close to zero. Find,

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

   Give reasons for your answer.

   (b) Graph the equations $y = (1/2) - (x^2/24)$, $y = (1 - \cos x)/x^2$, and $y = 1/2$ together for $-2 \leq x \leq 2$. Comment on the behavior of the graphs as $x \to 0$.

   (c) Using ϵ-δ definition show that $\lim_{x \to 9} \sqrt{x - 5} = 2$.

3. (a) If $f'(x) = 0$ at each point x of an open interval (a, b), then prove that $f(x) = C$ for all $x \in (a, b)$, where C is constant. [2.5M+2.5M]

   (b) Does the graph of

$$f(x) = \begin{cases} x^2 \sin(1/x), & \text{if } x \neq 0. \\ 0, & \text{if } x = 0. \end{cases}$$

   have a tangent at origin? Give reasons for your answer.

4. Let $f(x) = \dfrac{(x+1)^2}{1+x^2}$. [5M]

   (a) Locate the intervals where the function is increasing and decreasing.

   (b) Locate the intervals where the function is convex and concave.

   (c) Find the points of local maximum, local minimum and point of inflection.

   (d) Find the asymptotes of f.

   (e) Sketch the graph of the function

5. (a) Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.

   (b) Consider $f(x) = \int_1^{e^x} \dfrac{2\ln t}{t} dt$. [2.5M+2.5M]

      i. Find $f(0)$.

      ii. Find df/dx.

      iii. What can you conclude about the graph of f? Give reasons for your answer.

6. (a) The region bounded by the curve $y = 4 - x^2$ and the line $y = 2 - x$ is revolved about the $x-$axis to generate a solid. Find the volume of the solid by disk method. [2.5M+2.5M]

   (b) The region bounded by the curve $y = \sqrt{x}$, the $x-$axis, and the line $y = x - 2$ is revolved about the $x-$ axis to generate a solid. Find the volume of the solid by shell method

7. (a) Find the lateral surface area of the cone generated by revolving the line segment $y = x/2, 0 \leq x \leq 4$, about the $y-$axis. Check your answer with the geometry formula [2.5M+2.5M]

$$\text{Lateral surface area} = \frac{1}{2} \times \text{base circumference} \times \text{slant height}.$$

   (b) Find the center of mass of a thin plate covering the region bounded by the parabola $x = y^2 - y$ and the line $y = x$. Assume the density of the plate at the point (x, y) is $\delta = 1$.

8. (a) Use a trigonometric substitution to evaluate $\int_0^1 \dfrac{dx}{(4+x^2)^{3/2}}$. [2.5M+2.5M]

   (b) Investigate the convergence of $\int_0^\infty \dfrac{dx}{\sqrt{x^6+1}}$

9. Prove that $\beta(m,n) = \dfrac{\gamma(m)\gamma(n)}{\gamma(m+n)}$ and hence evaluate $\int_0^{\pi/2} \sqrt{\tan x}dx$. [5M]

10. (a) Does the sequence $\left\{ \left(\dfrac{3n+1}{3n-1} \right)^n \right\}_{n=1}^{\infty}$ converge? [2.5M+2.5M]

   (b) Discuss the convergence of the infinite series $\sum_1^\infty \dfrac{10n+1}{n(n+1)(n+2)}$.

$* * *$ ALL THE BEST $* **$