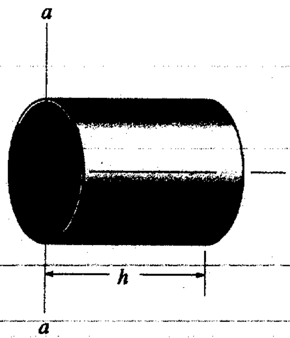
21-1. Show that the sum of the moments of inertia of a body, $I_{xx} + I_{yy} + I_{zz}$, is independent of the orientation of the x, y, z axes and thus depends only on the location of the origin.

$$I_{xx} + I_{yy} + I_{zz} = \int_m (y^2+z^2)\,dm + \int_m (x^2+z^2)\,dm + \int_m (x^2+y^2)\,dm$$
$$= 2\int_m (x^2+y^2+z^2)\,dm$$

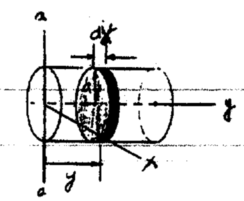
However, $x^2+y^2+z^2 = r^2$, where r is the distance from the origin O to dm. Since $|r|$ is constant, it does not depend on the orientation of the x, y, z axis. Consequently, $I_{xx} + I_{yy} + I_{zz}$ is also indepenent of the orientation of the x, y, z axis. **Q.E.D.**

21-2. Determine the moment of inertia of the cylinder with respect to the a–a axis of the cylinder. The cylinder has a mass m.



The mass of the differential element is $dm = \rho dV = \rho(\pi a^2)\,dy$.

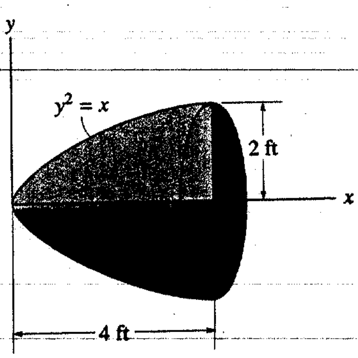
$$dI_{zz} = \frac{1}{4}dm a^2 + dm(y^2)$$
$$= \frac{1}{4}\left[\rho(\pi a^2)\,dy\right]a^2 + \left[\rho(\pi a^2)\,dy\right]y^2$$
$$= \left(\frac{1}{4}\rho\pi a^4 + \rho\pi a^2 y^2\right)dy$$
$$I_{zz} = \int dI_{zz} = \int_0^h \left(\frac{1}{4}\rho\pi a^4 + \rho\pi a^2 y^2\right)dy$$
$$= \frac{\rho\pi a^2 h}{12}\left(3a^2+4h^2\right)$$



However, $$m = \int_m dm = \int_0^h \rho(\pi a^2)\,dy = \rho\pi a^2 h$$

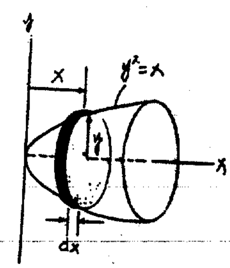
Hence, $$I_{zz} = \frac{m}{12}\left(3a^2+4h^2\right)$$ **Ans**

21-3. Determine moment of inertia I_y of the solid formed by revolving the shaded area around the x axis. The density of the material is $\rho = 12\ \text{slug/ft}^3$.

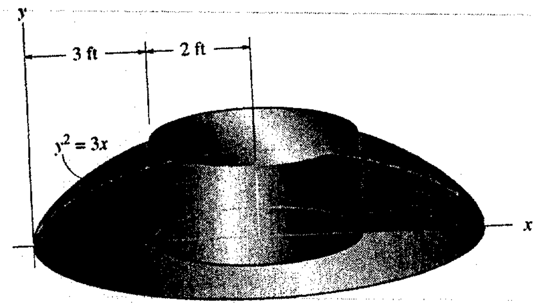


The mass of the differential element is $dm = \rho dV = \rho(\pi y^2)\,dx = \rho\pi x\,dx$.

$$dI_y = \frac{1}{4}dm y^2 + dm x^2$$
$$= \frac{1}{4}[\rho\pi x\,dx](x) + (\rho\pi x\,dx)x^2$$
$$= \rho\pi\left(\frac{1}{4}x^2 + x^3\right)dx$$
$$I_y = \int dI_y = \rho\pi\int_0^4 \left(\frac{1}{4}x^2 + x^3\right)dx = 69.33\ \pi\rho$$
$$= 69.33(\pi)(12) = 2614\ \text{slug}\cdot\text{ft}^2$$ **Ans**



***21-4.** Determine the product of inertia I_{xy} of the body formed by revolving the shaded area about the line $x = 5$ ft. Express the result in terms of the density of the material, ρ.



$$\int_0^3 dm = \rho 2\pi \int_0^3 (5-x) y \, dx = \rho 2\pi \int_0^3 (5-x)\sqrt{3x} \, dx = 38.4\rho\pi$$

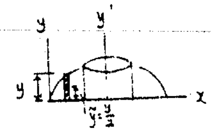
$$\int_0^3 \tilde{y} \, dm = \rho 2\pi \int_0^3 \frac{y}{2}(5-x) y \, dx$$

$$= \rho\pi \int_0^3 (5-x)(3x) \, dx$$

$$= 40.5\rho\pi$$

Thus, $\bar{y} = \dfrac{40.5\rho\pi}{38.4\rho\pi} = 1.055$ ft

The solid is symmetric about y', thus



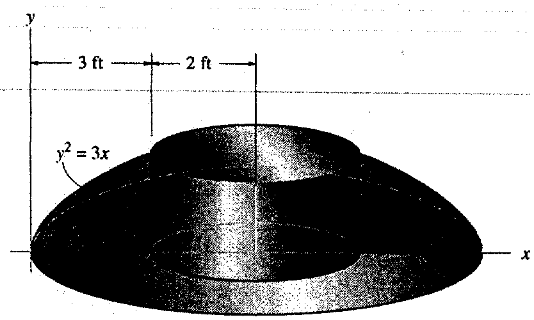
$I_{xy'} = 0$

$I_{xy} = I_{xy'} + \bar{x}\bar{y}m$

$= 0 + 5(1.055)(38.4\rho\pi)$

$I_{xy} = 636\rho$ **Ans**

21-5. Determine the moment of inertia I_y of the body formed by revolving the shaded area about the line $x = 5$ ft. Express the result in terms of the density of the material, ρ.



$$I_{y'} = \int_0^3 \frac{1}{2} dm\, r^2 - \frac{1}{2}(m')(2)^2$$

$$\int_0^3 \frac{1}{2} dm\, r^2 = \frac{1}{2}\int_0^3 \rho\, \pi\, (5-x)^4\, dy$$

$$= \frac{1}{2}\rho\, \pi \int_0^3 \left(5 - \frac{y^2}{3}\right)^4 dy$$

$$= 490.29\, \rho\, \pi$$

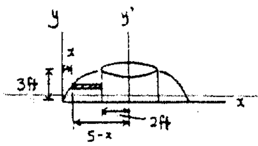
$$m' = \rho\, \pi\, (2)^2(3) = 12\, \rho\, \pi$$

$$I_{y'} = 490.29\, \rho\, \pi - \frac{1}{2}(12\, \rho\, \pi)(2)^2 = 466.29\, \rho\, \pi$$

Mass of body;

$$m = \int_0^3 \rho\, \pi\, (5-x)^2 dy - m'$$

$$= \int_0^3 \rho\, \pi\, (5 - \frac{y^2}{3})^2 dy - 12\, \rho\, \pi$$



$$= 38.4\, \rho\, \pi$$

$$I_y = 466.29\, \rho\, \pi + (38.4\, \rho\, \pi)(5)^2$$

$$= 1426.29\, \rho\, \pi$$

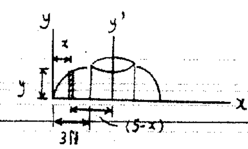
$$I_y = 4.48(10^3)\, \rho \qquad \textbf{Ans}$$

Also,

$$I_{y'} = \int_0^3 r^2 dm$$

$$= \int_0^3 (5-x)^2 \rho (2\pi)(5-x) y\, dx$$

$$= 2\, \rho\, \pi \int_0^3 (5-x)^3 (3x)^{1/2} dx$$



$$= 466.29\, \rho\, \pi$$

$$m = \int_0^3 dm$$

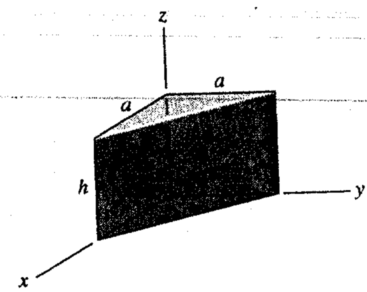
$$= 2\, \rho\, \pi \int_0^3 (5-x) y\, dx$$

$$= 2\, \rho\, \pi \int_0^3 (5-x)(3x)^{1/2} dx$$

$$= 38.4\, \rho\, \pi$$

$$I_y = 466.29\, \rho\, \pi + 38.4\, \rho\, \pi\, (5)^2 = 4.48(10^3)\rho \qquad \textbf{Ans}$$

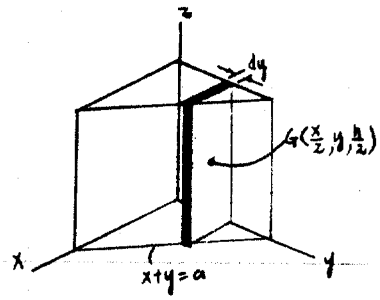
21-6. Determine by direct integration the product of inertia I_{yz} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the mass m of the prism.



The mass of the differential element is $dm = \rho dV = \rho hx dy = \rho h(a-y)dy$.

$$m = \int_m dm = \rho h \int_0^a (a-y)dy = \frac{\rho a^2 h}{2}$$

Using the parallel axis theorem :



$$dI_{yz} = (dI_{y'z'})_G + dm\, y_G z_G$$

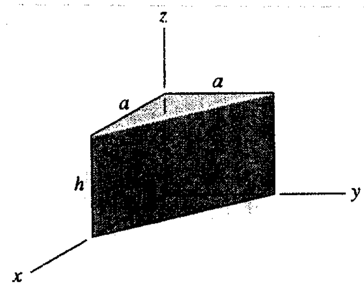
$$= 0 + (\rho hx dy)(y)\left(\frac{h}{2}\right)$$

$$= \frac{\rho h^2}{2} xy dy$$

$$= \frac{\rho h^2}{2}(ay - y^2)\, dy$$

$$I_{yz} = \frac{\rho h^2}{2}\int_0^a (ay - y^2)\, dy = \frac{\rho a^3 h^2}{12} = \frac{1}{6}\left(\frac{\rho a^2 h}{2}\right)(ah) = \frac{m}{6} ah \qquad \textbf{Ans}$$

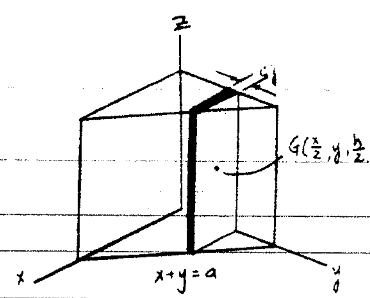
21-7. Determine by direct integration the product of inertia I_{xy} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the mass m of the prism.



The mass of the differential element is $dm = \rho dV = \rho hx dy = \rho h(a-y)dy$.

$$m = \int_m dm = \rho h \int_0^a (a-y)dy = \frac{\rho a^2 h}{2}$$

Using the parallel axis theorem :



$$dI_{xy} = (dI_{x'y'})_G + dm\, x_G y_G$$

$$= 0 + (\rho hx dy)\left(\frac{x}{2}\right)(y)$$

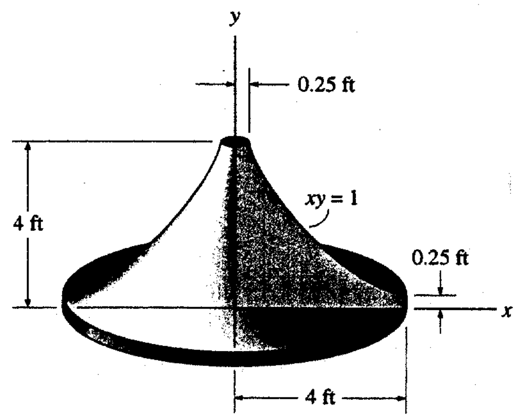
$$= \frac{\rho h}{2} x^2 y dy$$

$$= \frac{\rho h}{2}(y^3 - 2ay^2 + a^2 y)\, dy$$

$$I_{xy} = \frac{\rho h}{2}\int_0^a (y^3 - 2ay^2 + a^2 y)\, dy$$

$$= \frac{\rho a^4 h}{24} = \frac{1}{12}\left(\frac{\rho a^2 h}{2}\right) a^2 = \frac{m}{12} a^2 \qquad \textbf{Ans}$$

***21-8.** Determine the radii of gyration k_x and k_y for the solid formed by revolving the shaded area about the y axis. The density of the material is ρ.



For k_y : The mass of the differential element is $dm = \rho dV = \rho(\pi x^2)\,dy = \rho\pi\frac{dy}{y^2}$.

$$dI_y = \tfrac{1}{2}dm x^2 = \tfrac{1}{2}\left[\rho\pi\tfrac{dy}{y^2}\right]\left(\tfrac{1}{y^2}\right) = \tfrac{1}{2}\rho\pi\tfrac{dy}{y^4}$$

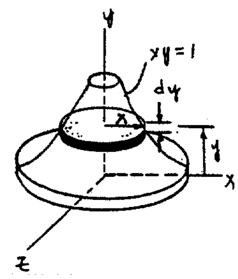
$$I_y = \int dI_y = \tfrac{1}{2}\rho\pi\int_{0.25}^{4}\frac{dy}{y^4} + \frac{1}{2}\left[\rho(\pi)(4)^2(0.25)\right](4)^2$$

$$= 134.03\rho$$

However, $$m = \int_m dm = \rho\pi\int_{0.25}^{4}\frac{dy}{y^2} + \rho\left[\pi(4)^2(0.25)\right] = 24.35\rho$$

Hence, $$k_y = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{134.03\rho}{24.35\rho}} = 2.35\text{ ft}$$ **Ans**

For k_x : $0.25\text{ ft} < y \le 4\text{ ft}$



$$dI'_x = \tfrac{1}{4}dm x^2 + dm y^2$$

$$= \tfrac{1}{4}\left[\rho\pi\tfrac{dy}{y^2}\right]\left(\tfrac{1}{y^2}\right) + \left(\rho\pi\tfrac{dy}{y^2}\right)y^2$$

$$= \rho\pi\left(\tfrac{1}{4y^4} + 1\right)dy$$

$$I'_x = \int dI'_x = \rho\pi\int_{0.25}^{4}\left(\tfrac{1}{4y^4} + 1\right)dy = 28.53\rho$$

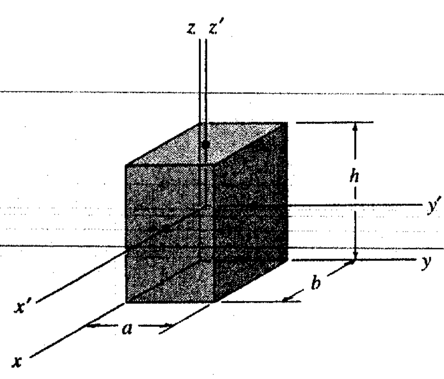
$$I''_x = \tfrac{1}{4}\left[\rho\pi(4)^2(0.25)\right](4)^2 + \left[\rho\pi(4)^2(0.25)\right](0.125)^2$$

$$= 50.46\rho$$

$$I_x = I'_x + I''_x = 28.53\rho + 50.46\rho = 78.99\rho$$

Hence, $$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{78.99\rho}{24.35\rho}} = 1.80\text{ ft}$$ **Ans**

21-9. Determine the mass moment of inertia of the homogeneous block with respect to its centroidal x' axis. The mass of the block is m.

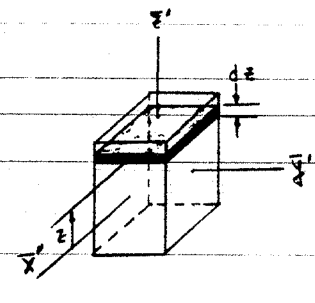


The mass of the differential element is $dm = \rho dV = \rho ab dz$.

$$dI_{x'} = \frac{1}{12}dm a^2 + dm z^2$$

$$= \frac{1}{12}(\rho ab dz)a^2 + (\rho ab dz)z^2$$

$$= \frac{\rho ab}{12}(a^2 + 12z^2)\,dz$$

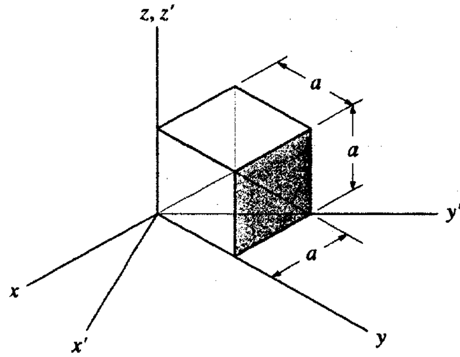


$$I_{x'} = \int dI_{x'} = \frac{\rho ab}{12}\int_{-\frac{h}{2}}^{\frac{h}{2}}(a^2 + 12z^2)\,dz = \frac{\rho abh}{12}(a^2 + h^2)$$

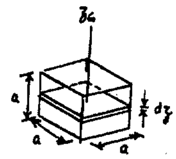
However, $$m = \int_m dm = \rho ab\int_{-\frac{h}{2}}^{\frac{h}{2}} dz = \rho abh$$

Hence, $$I_{x'} = \frac{m}{12}(a^2 + h^2)$$ **Ans**

21-10. Determine the elements of the inertia tensor for the cube with respect to the x, y, z coordinate system. The mass of the cube is m.



$$dI_{zG} = \frac{1}{12}(dm)(a^2 + a^2)$$

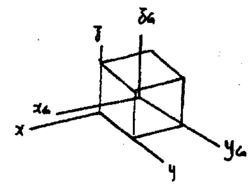


$$I_{zG} = \frac{1}{6}a^2\int_0^a \rho(a^2)\,dz$$

$$= \frac{1}{6}a^2(\rho a^3)$$

$$= \frac{1}{6}ma^2$$

$$I_x = I_y = I_z = \frac{1}{6}ma^2 + m[(\frac{a}{2})^2 + (\frac{a}{2})^2] = \frac{2}{3}ma^2$$



$$I_{xy} = 0 + m(-\frac{a}{2})(\frac{a}{2}) = -\frac{1}{4}ma^2$$

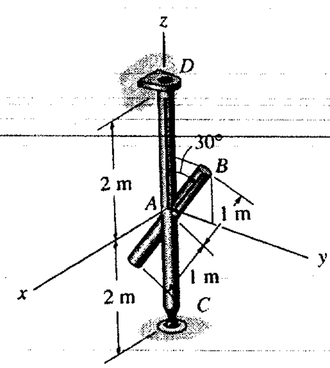
$$I_{yz} = 0 + m(\frac{a}{2})(\frac{a}{2}) = \frac{1}{4}ma^2$$

$$I_{xz} = 0 + m(-\frac{a}{2})(\frac{a}{2}) = -\frac{1}{4}ma^2$$

Changing the signs of the products of inertia as represented by the equation in the text, we have

$$\begin{pmatrix} \frac{2}{3}ma^2 & \frac{1}{4}ma^2 & \frac{1}{4}ma^2 \\ \frac{1}{4}ma^2 & \frac{2}{3}ma^2 & -\frac{1}{4}ma^2 \\ \frac{1}{4}ma^2 & -\frac{1}{4}ma^2 & \frac{2}{3}ma^2 \end{pmatrix}$$ **Ans**

21-11. Determine the moments of inertia about the x, y, z axes of the rod assembly. The rods have a mass of 0.75 kg/m.

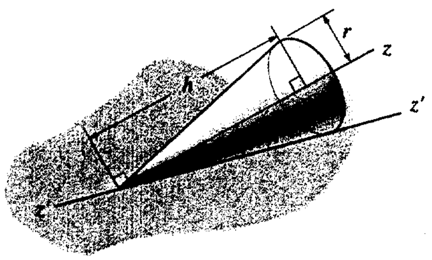


$$I_x = \frac{1}{12}[0.75(4)](4)^2 + \frac{1}{12}[0.75(2)](2)^2 = 4.50 \text{ kg}\cdot\text{m}^2$$ **Ans**

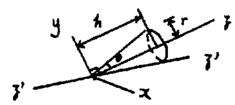
$$I_y = \frac{1}{12}[0.75(4)](4)^2 + \frac{1}{12}[0.75(2)](2\cos 30°)^2 = 4.38 \text{ kg}\cdot\text{m}^2$$ **Ans**

$$I_z = 0 + \frac{1}{12}[0.75(2)](2\sin 30°)^2 = 0.125 \text{ kg}\cdot\text{m}^2$$ **Ans**

***21-12.** Determine the moment of inertia of the cone about the z' axis. The weight of the cone is 15 lb, the *height* is h = 1.5 ft, and the radius is r = 0.5 ft.



$$\theta = \tan^{-1}\left(\frac{0.5}{1.5}\right) = 18.43^\circ$$



$$I_{xx} = I_{yy} = \left[\frac{3}{80}m\{4(0.5)^2 + (1.5)^2\}\right] + m\left[1.5-\left(\frac{1.5}{4}\right)\right]^2$$

$$I_{xx} = I_{yy} = 1.3875\,m$$

$$I_z = \frac{3}{10}m(0.5)^2 = 0.075m$$

$$I_{xy} = I_{yz} = I_{zx} = 0$$

Using Eq. 21 − 5.

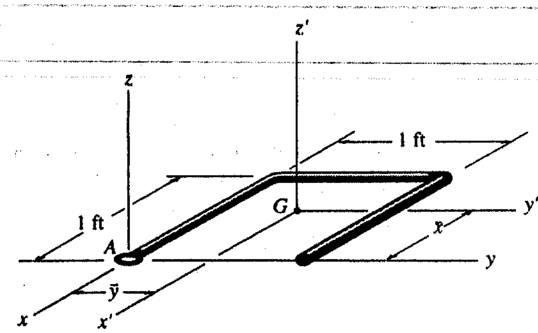
$$I_{z'z'} = u_{z'x}^2 I_{xx} + u_{z'y}^2 I_{yy} + u_{z'z}^2 I_{zz}$$

$$= 0 + [\cos(108.43^\circ)]^2(1.3875m) + [\cos(18.43^\circ)]^2(0.075m)$$

$$= 0.2062m$$

$$I_{z'z'} = 0.2062\left(\frac{15}{32.2}\right) = 0.0961 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

21-13. The bent rod has a weight of 1.5 lb/ft. Locate the center of gravity $G(\bar{x}, \bar{y})$ and determine the principal moments of inertia $I_{x'}$, $I_{y'}$, and $I_{z'}$ of the rod with respect to the x', y', z' axes.



Due to symmetry $\quad \bar{y} = 0.5$ ft $\qquad$ **Ans**

$$\bar{x} = \frac{\Sigma \tilde{x}W}{\Sigma W} = \frac{(-1)(1.5)(1) + 2[(-0.5)(1.5)(1)]}{3[1.5(1)]} = -0.667 \text{ ft} \qquad \textbf{Ans}$$

$$I_{x'} = 2\left[\left(\frac{1.5}{32.2}\right)(0.5)^2\right] + \frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2$$

$$= 0.0272 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

$$I_{y'} = 2\left[\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.667-0.5)^2\right]$$

$$+\left(\frac{1.5}{32.2}\right)(1-0.667)^2$$

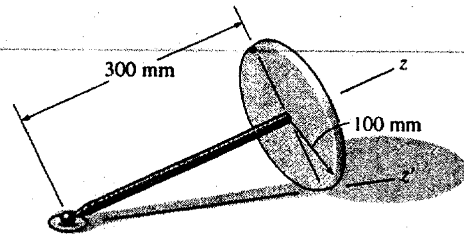
$$= 0.0155 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

$$I_{z'} = 2\left[\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.5^2 + 0.1667^2)\right]$$

$$+\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.3333)^2$$

$$= 0.0427 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

21-14. Determine the moment of inertia of both the 1.5-kg rod and 4-kg disk about the z' axis.



Due to symmetry $\quad I_{xy} = I_{yz} = I_{zx} = 0$

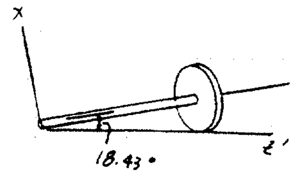
$$I_y = I_x = \left[\frac{1}{4}(4)(0.1)^2 + 4(0.3)^2\right] + \frac{1}{3}(1.5)(0.3)^2$$

$$= 0.415\ \text{kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{2}(4)(0.1)^2 = 0.02\ \text{kg}\cdot\text{m}^2$$

$$u_z = \cos(18.43°) = 0.9487, \qquad u_y = \cos 90° = 0,$$

$$u_x = \cos(90° + 18.43°) = -0.3162$$

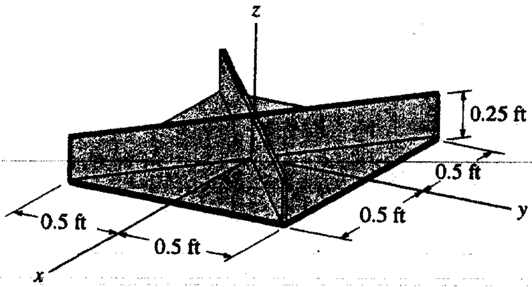


$$I_{z'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy}u_x u_y - 2I_{yz}u_y u_z - 2I_{zx}u_z u_x$$

$$= 0.415(-0.3162)^2 + 0 + 0.02(0.9487)^2 - 0 - 0 - 0$$

$$= 0.0595\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

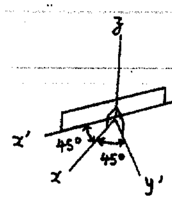
21-15. Determine the moment of inertia I_x of the composite plate assembly. The plates have a specific weight of 6 lb/ft^2.



Horizontal plate:

$$I_{xx} = \frac{1}{12}\left(\frac{6(1)(1)}{32.2}\right)(1)^2 = 0.0155$$

Vertical plates:



$$u_{xx'} = 0.707, \qquad u_{xy'} = 0.707, \qquad u_{xz'} = 0$$

$$I_{x'x'} = \frac{1}{3}\left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)\left(\frac{1}{4}\right)^2 = 0.001372$$

$$I_{y'y'} = \left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)\left(\frac{1}{12}\right)\left[\left(\frac{1}{4}\right)^2 + (1\sqrt{2})^2\right] + \left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)\left(\frac{1}{8}\right)^2$$

$$= 0.01235$$

Using Eq. 21-5,

$$I_{xx} = (0.707)^2(0.001372) + (0.707)^2(0.01235)$$

$$= 0.00686$$

Thus,

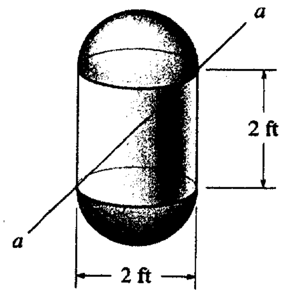
$$I_{xx} = 0.0155 + 2(0.00686) = 0.0292\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

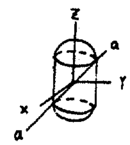
***21-16.** Determine the product of inertia I_{yz} of the composite plate assembly. The plates have a weight of 6 lb/ft^2.

Due to symmetry,

$$I_{yz} = 0 \qquad \textbf{Ans}$$

21-17. Determine the moment of inertia of the composite body about the *aa* axis. The cylinder weighs 20 lb, and each hemisphere weighs 10 lb.





$u_{az} = 0.707$

$u_{ax} = 0$

$u_{ay} = 0.707$

$I_{zz} = \frac{1}{2}(\frac{20}{32.2})(1)^2 + 2[\frac{2}{5}(\frac{10}{32.2})(1)^2]$

$= 0.5590 \text{ slug}\cdot\text{ft}^2$

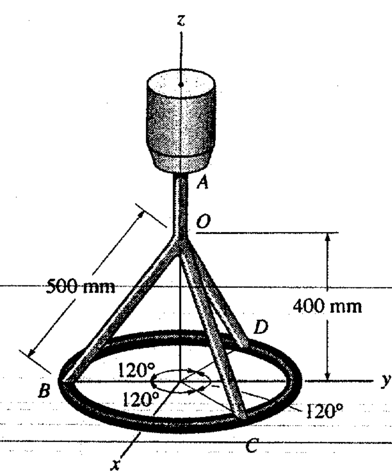
$I_{xx} = I_{yy} = \frac{1}{12}(\frac{20}{32.2})[3(1)^2 + (2)^2] + 2[0.259(\frac{10}{32.2})(1)^2 + \frac{10}{32.2}(\frac{11}{8})^2]$

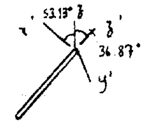
$I_{xx} = I_{yy} = 1.6975 \text{ slug}\cdot\text{ft}^2$

$I_{aa} = 0 + (0.707)^2(1.6975) + (0.707)^2(0.559)$

$I_{aa} = 1.13 \text{ slug}\cdot\text{ft}^2$ **Ans**

21-18. Determine the moment of inertia of the rod-and-thin-ring assembly about the z axis. The rods and ring have a mass of 2 kg/m.





For the rod,

$u_{x'} = 0.6, \quad u_{y'} = 0, \quad u_{z'} = 0.8$

$I_{x'} = I_{y'} = \frac{1}{3}[(0.5)(2)](0.5)^2 = 0.08333 \text{ kg}\cdot\text{m}^2$

$I_{z'} = 0$

$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$

From Eq. 21-5,

$I_z = 0.08333(0.6)^2 + 0 + 0 - 0 - 0 - 0$

$I_z = 0.03 \text{ kg}\cdot\text{m}^2$

For the ring,

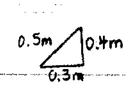
The radius is $r = 0.3$ m

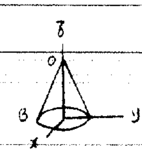
Thus,

$I_z = mR^2 = [2(2\pi)(0.3)](0.3)^2 = 0.3393 \text{ kg}\cdot\text{m}^2$

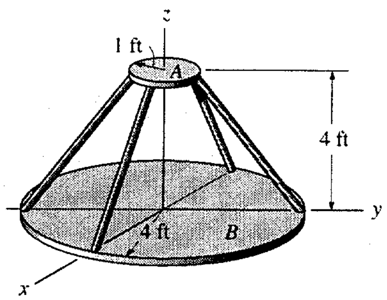
Thus the moment of inertia of the assembly is

$I_z = 3(0.03) + 0.339 = 0.429 \text{ kg}\cdot\text{m}^2$ **Ans**





21-19. The assembly consists of a 15-lb plate A, 40-lb plate B, and four 7-lb rods. Determine the moments of inertia of the assembly with respect to the principal x, y, z axes.

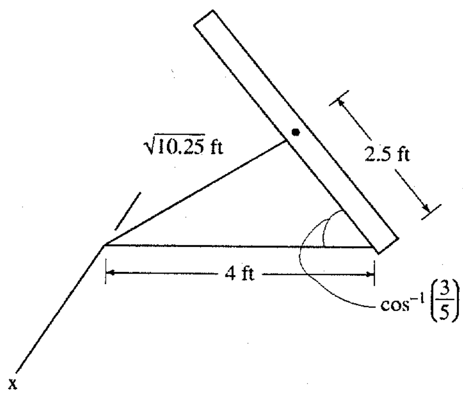


Due to symmetry

$$\bar{y} = \bar{x} = 0$$

$$\bar{z} = \frac{\sum \tilde{z} w}{\sum w} = \frac{4(15) + 0(40) + 2(28)}{15 + 40}$$

$$= 1.3976 \text{ ft}$$



$$I_z = (I_z)_{\text{upper}} + (I_z)_{\text{lower}} + (I_z)_{\text{rods}}$$

$$= \frac{1}{2}\left(\frac{15}{32.2}\right)(1)^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)(4)^2$$

$$+ 4\left[\frac{1}{12}\left(\frac{7}{32.2}\right)(3)^2 + \left(\frac{7}{32.2}\right)(2.5)^2\right]$$

$I_z = 16.3 \text{ slug} \cdot \text{ft}^2$ **Ans**

$$I_x = (I_x)_{\text{upper}} + (I_x)_{\text{lower}} + (I_x)_{\text{rods-1}} + (I_x)_{\text{rods-2}}$$

$$= \left[\frac{1}{4}\left(\frac{15}{32.2}\right)(1)^2 + \left(\frac{15}{32.2}\right)(4)^2\right] + \frac{1}{4}\left(\frac{40}{32.2}\right)(4)^2$$

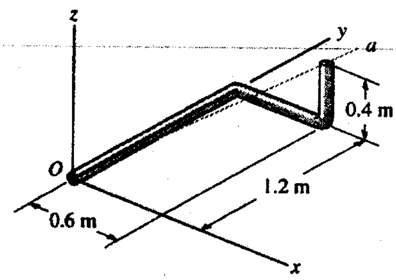
$$+ 2\left[\frac{1}{3}\left(\frac{7}{32.2}\right)(4)^2\right] + 2\left[\frac{1}{12}\left(\frac{7}{32.2}\right)(5)^2 + \left(\frac{7}{32.2}\right)(10.25)\right]$$

$I_x = 20.2 \text{ slug} \cdot \text{ft}^2$ **Ans**

And by symmetry,

$I_y = 20.2 \text{ slug} \cdot \text{ft}^2$ **Ans**

***21-20.** The bent rod has a mass of 4 kg/m. Determine the moment of inertia of the rod about the *Oa* axis.



$$I_{xy} = [4(1.2)](0)(0.6) + [4(0.6)](0.3)(1.2) + [4(0.4)](0.6)(1.2) = 2.016\ \text{kg}\cdot\text{m}^2$$

$$I_{yz} = [4(1.2)](0.6)(0) + [4(0.6)](1.2)(0) + [4(0.4)](1.2)(0.2) = 0.384\ \text{kg}\cdot\text{m}^2$$

$$I_{zx} = [4(1.2)](0)(0) + [4(0.6)](0)(0.3) + [4(0.4)](0.2)(0.6) = 0.192\ \text{kg}\cdot\text{m}^2$$

$$I_x = \frac{1}{3}[4(1.2)](1.2)^2 + [4(0.6)](1.2)^2 + \left[\frac{1}{12}[4(0.4)](0.4)^2 + [4(0.4)](1.2^2 + 0.2^2)\right]$$

$$= 8.1493\ \text{kg}\cdot\text{m}^2$$

$$I_y = 0 + \frac{1}{3}[4(0.6)](0.6)^2 + \left[\frac{1}{12}[4(0.4)](0.4)^2 + [4(0.4)](0.6^2 + 0.2^2)\right]$$

$$= 0.9493\ \text{kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{3}[4(1.2)](1.2)^2 + \left[\frac{1}{12}[4(0.6)](0.6)^2 + [4(0.6)](0.3^2 + 1.2^2)\right] + [4(0.4)](1.2^2 + 0.6^2)$$

$$= 8.9280\ \text{kg}\cdot\text{m}^2$$

$$u_{Oa} = \frac{0.6\mathbf{i} + 1.2\mathbf{j} + 0.4\mathbf{k}}{\sqrt{0.6^2 + 1.2^2 + 0.4^2}} = \frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$I_{Oa} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$$

$$= 8.1493\left(\frac{3}{7}\right)^2 + 0.9493\left(\frac{6}{7}\right)^2 + 8.9280\left(\frac{2}{7}\right)^2 - 2(2.016)\left(\frac{3}{7}\right)\left(\frac{6}{7}\right)$$

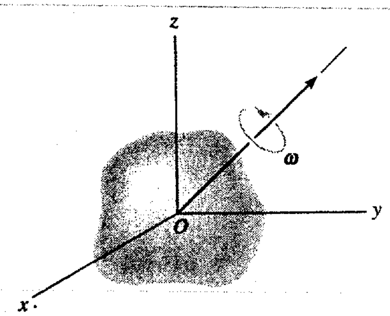
$$-2(0.384)\left(\frac{6}{7}\right)\left(\frac{2}{7}\right) - 2(0.192)\left(\frac{2}{7}\right)\left(\frac{3}{7}\right)$$

$$= 1.21\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

21-21. If a body contains *no planes of symmetry*, the principal moments of inertia can be determined mathematically. To show how this is done, consider the rigid body which is spinning with an angular velocity $\boldsymbol{\omega}$, directed along one of its principal axes of inertia. If the principal moment of inertia about this axis is I, the angular momentum can be expressed as $\mathbf{H} = I\boldsymbol{\omega} = I\omega_x\mathbf{i} + I\omega_y\mathbf{j} + I\omega_z\mathbf{k}$. The components of $\mathbf{H}$ may also be expressed by Eqs. 21–10, where the inertia tensor is assumed to be known. Equate the $\mathbf{i}, \mathbf{j}$, and $\mathbf{k}$ components of both expressions for $\mathbf{H}$ and consider ω_x, ω_y, and ω_z to be unknown. The solution of these three equations is obtained provided the determinant of the coefficients is zero. Show that this determinant, when expanded, yields the cubic equation

$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + (I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2)I - (I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I_{yz}^2 - I_{yy}I_{zx}^2 - I_{zz}I_{xy}^2) = 0$$

The three positive roots of I, obtained from the solution of this equation, represent the principal moments of inertia I_x, I_y, and I_z.



$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + (I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2)I - (I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I_{yz}^2 - I_{yy}I_{zx}^2 - I_{zz}I_{xy}^2) = 0$$

The three positive roots of I, obtained from the solution of this equation, represent the principal moments of inertia I_x, I_y, and I_z.

$\mathbf{H} = I\omega = I\omega_x\,\mathbf{i} + I\omega_y\,\mathbf{j} + I\omega_z\,\mathbf{k}$

Equating the $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components to the scalar equations (Eq. 21-10) yields

$$(I_{xx} - I)\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z = 0$$

$$-I_{yx}\,\omega_x + (I_{yy} - I)\,\omega_y - I_{yz}\,\omega_z = 0$$

$$-I_{zx}\,\omega_x - I_{zy}\,\omega_y + (I_{zz} - I)\,\omega_z = 0$$

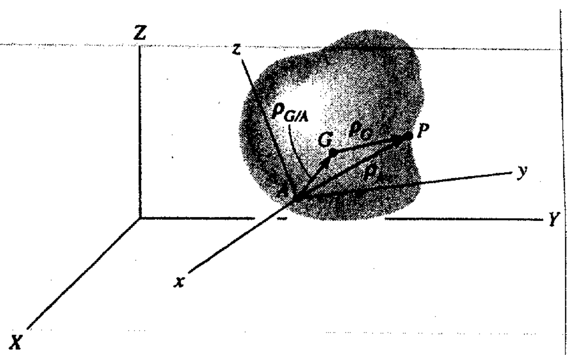
Solution for ω_x, ω_y, and ω_z requires

$$\begin{vmatrix} (I_{xx} - I) & -I_{xy} & -I_{xz} \\ -I_{yx} & (I_{yy} - I) & -I_{yz} \\ -I_{zx} & -I_{zy} & (I_{zz} - I) \end{vmatrix} = 0$$

Expanding

$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + (I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2)I$$
$$- (I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I_{yz}^2 - I_{yy}I_{zx}^2 - I_{zz}I_{xy}^2) = 0 \quad \textbf{QED}$$

21-22. Show that if the angular momentum of a body is determined with respect to an arbitrary point A, then $\mathbf{H}_A$ can be expressed by Eq. 21-9. This requires substituting $\boldsymbol{\rho}_A = \boldsymbol{\rho}_G + \boldsymbol{\rho}_{G/A}$ into Eq. 21-6 and expanding, noting that $\int \boldsymbol{\rho}_G \, dm = \mathbf{0}$ by definition of the mass center and $\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega} \times \boldsymbol{\rho}_{G/A}$.



$$\mathbf{H}_A = \left(\int_m \rho_A \, dm\right) \times \mathbf{v}_A + \int_m \rho_A \times (\omega \times \rho_A) \, dm$$

$$= \left(\int_m (\rho_G + \rho_{G/A}) \, dm\right) \times \mathbf{v}_A + \int_m (\rho_G + \rho_{G/A}) \times \left[\omega \times (\rho_G + \rho_{G/A})\right] dm$$

$$= \left(\int_m \rho_G dm\right) \times \mathbf{v}_A + (\rho_{G/A} \times \mathbf{v}_A) \int_m dm + \int_m \rho_G \times (\omega \times \rho_G) \, dm$$

$$+ \left(\int_m \rho_G dm\right) \times (\omega \times \rho_{G/A}) + \rho_{G/A} \times \left(\omega \times \int_m \rho_G dm\right) + \rho_{G/A} \times (\omega \times \rho_{G/A}) \int_m dm$$

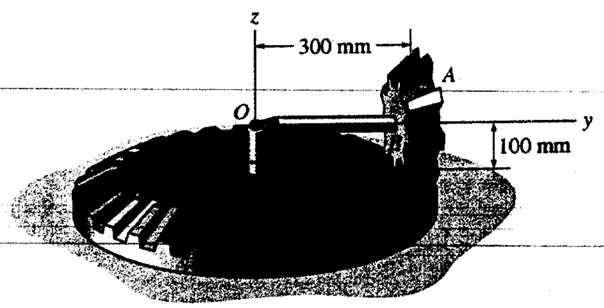
Since $\int_m \rho_G dm = 0$ and from Eq. 21-8 $\mathbf{H}_G = \int_m \rho_G \times (\omega \times \rho_G) \, dm$

$$\mathbf{H}_A = (\rho_{G/A} \times \mathbf{v}_A) m + \mathbf{H}_G + \rho_{G/A} \times (\omega \times \rho_{G/A}) m$$

$$= \rho_{G/A} \times (\mathbf{v}_A + (\omega \times \rho_{G/A})) m + \mathbf{H}_G$$

$$= (\rho_{G/A} \times m \mathbf{v}_G) + \mathbf{H}_G \qquad \text{Q.E.D.}$$

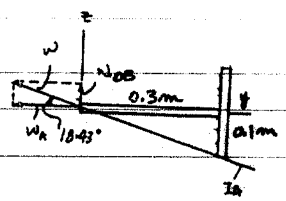
21-23. The 2-kg gear A rolls on the fixed plate gear C. Determine the angular velocity of rod OB about the z axis after it rotates one revolution about the z axis, starting from rest. The rod is acted upon by the constant moment $M = 5\ \text{N}\cdot\text{m}$. Neglect the mass of rod OB. Assume that gear A is a uniform disk having a radius of 100 mm.



$$\omega_{OB} = \omega_A \tan 18.43° = 0.3333 \omega_A \qquad [1]$$

$$I_z = \tfrac{1}{2}(2)(0.1)^2 = 0.01\ \text{kg}\cdot\text{m}^2$$

$$I_z = \tfrac{1}{4}(2)(0.1)^2 + 2(0.3)^2 = 0.185\ \text{kg}\cdot\text{m}^2$$



$$T_1 + \Sigma U_{1-2} = T_2$$

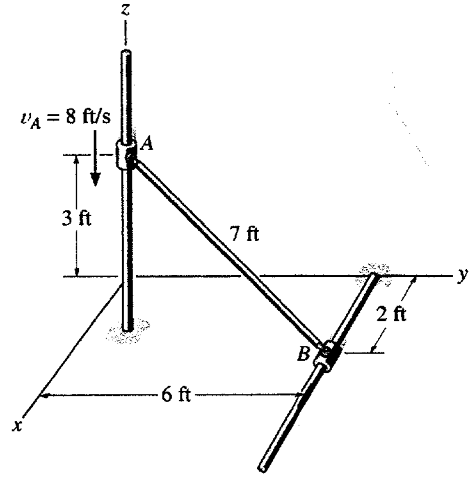
$$0 + 5(2\pi) = \tfrac{1}{2}(0.01)\omega_A^2 + \tfrac{1}{2}(0.185)\omega_{OB}^2 \qquad [2]$$

Solving Eqs.[1] and [2] yields :

$\omega_{OB} = 15.1\ \text{rad/s}$ **Ans**

$\omega_A = 45.35\ \text{rad/s}$

*21-24. Rod AB has a weight of 6 lb and is attached to two smooth collars at its end points by ball-and-socket joints. If collar A is moving downward at a speed of 8 ft/s, determine the kinetic energy of the rod at the instant shown. Assume that at this instant the angular velocity of the rod is directed perpendicular to the rod's axis.



$\mathbf{v}_A = \{-8\mathbf{k}\}$ m/s $\quad \mathbf{v}_B = v_B\mathbf{i} \quad \boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$\mathbf{r}_{B/A} = \{2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}$ m $\quad \mathbf{r}_{G/A} = \{1\mathbf{i} + 3\mathbf{j} - 1.5\mathbf{k}\}$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{i} = -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 2 & 6 & -3 \end{vmatrix}$$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ components

$-3\omega_y - 6\omega_z = v_B \qquad [1]$

$3\omega_x + 2\omega_z = 0 \qquad [2]$

$6\omega_x - 2\omega_y - 8 = 0 \qquad [3]$

Since ω_{AB}

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$$

$2\omega_x + 6\omega_y - 3\omega_z = 0 \qquad [4]$

Solving Eqs.[1] to [4] yields :

$\omega_x = 0.9796$ rad/s $\quad \omega_y = -1.0612$ rad/s $\quad \omega_z = -1.4694$ rad/s

$v_B = 12.0$ ft/s

Hence $\boldsymbol{\omega}_{AB} = \{0.9796\mathbf{i} - 1.0612\mathbf{j} - 1.4694\mathbf{k}\}$ rad/s $\quad \mathbf{v}_B = \{12.0\mathbf{i}\}$ ft/s

$$\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{G/A}$$

$$= -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9796 & -1.0612 & -1.4694 \\ 1 & 3 & -1.5 \end{vmatrix}$$

$$= \{6.0\mathbf{i} - 4.0\mathbf{k}\} \text{ ft/s}$$

$$\omega_{AB}^2 = 0.9796^2 + (-1.0612)^2 + (-1.4694)^2 = 4.2449$$

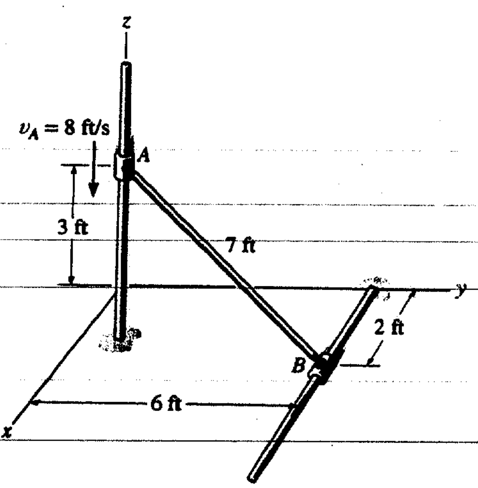
$$v_G^2 = 6^2 + 4^2 = 52$$

$$T = \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}I_G\omega_{AB}^2$$

$$= \tfrac{1}{2}\left(\tfrac{6}{32.2}\right)(52) + \tfrac{1}{2}\left[\tfrac{1}{12}\left(\tfrac{6}{32.2}\right)\left(\sqrt{2^2 + 6^2 + (-3)^2}\right)^2\right](4.2449)$$

$= 6.4596 = 6.46$ ft·lb $\qquad$ **Ans**

21-25. At the instant shown the collar at A on the 6-lb rod AB has a velocity of $v_A = 8$ ft/s. Determine the kinetic energy of the rod after the collar has descended 3 ft. Neglect friction and the thickness of the rod. Neglect the mass of the collar and the collar are attached to the rod using ball-and-socket joints.



From Prob. 21 − 32.

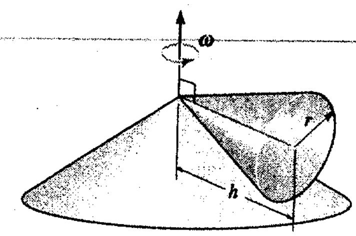
$T_1 = 6.4596$ ft·lb

$T_1 + V_1 = T_2 + V_2$

$6.4596 + 6(1.5) = T_2 + 0$

$T_2 = 15.5$ ft·lb $\qquad$ **Ans**

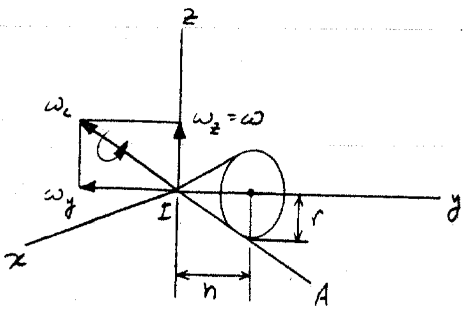
21-26. The cone has a mass m and rolls without slipping on the conical surface so that it has an angular velocity about the vertical axis of $\boldsymbol{\omega}$. Determine the kinetic energy of the cone due to this motion.



$$\frac{\omega_z}{r} = \frac{\omega_y}{h}$$

$$\omega_y = \left(\frac{h}{r}\right)\omega_z = \left(\frac{h}{r}\right)\omega$$

$$I_z = \left(\frac{3}{80}\right)m\left(4r^2+h^2\right) + m\left(\frac{3}{4}h\right)^2 = \left(\frac{3}{20}\right)mr^2 + \left(\frac{3}{5}\right)mh^2 = \left(\frac{3}{20}\right)m\left(r^2+4h^2\right)$$

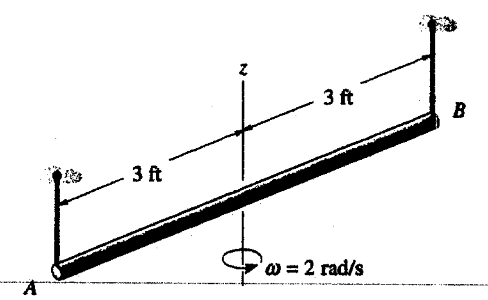


$$T = \frac{1}{2}I_x\,\omega_x^2 + \frac{1}{2}I_y\,\omega_y^2 + \frac{1}{2}I_z\,\omega_z^2$$

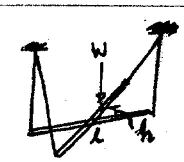
$$= 0 + \frac{1}{2}\left(\frac{3}{10}mr^2\right)\left(\frac{h}{r}\omega\right)^2 + \frac{1}{2}\left[\left(\frac{3}{20}\right)m\left(r^2+4h^2\right)\right]\omega^2 = \frac{m\omega^2}{20}\left[3h^2 + \frac{3}{2}r^2 + 6h^2\right]$$

$$T = \frac{9mh^2}{20}\left[1 + \frac{r^2}{6h^2}\right]\omega^2 \qquad \textbf{Ans}$$

21-27. The rod weighs 3 lb/ft and is suspended from parallel cords at A and B. If the rod has an angular velocity of 2 rad/s about the z axis at the instant shown, determine how high the center of the rod rises at the instant the rod momentarily stops swinging.



$$T_1 + V_1 = T_2 + V_2$$

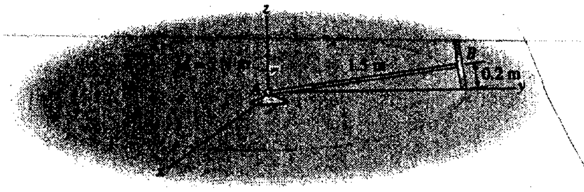


$$\frac{1}{2}\left[\frac{1}{12}\frac{W}{g}l^2\right]\omega^2 + 0 = 0 + Wh$$

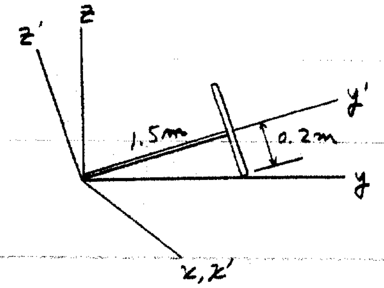
$$h = \frac{1}{24}\frac{l^2\omega^2}{g} = \frac{1}{24}\frac{(6)^2(2)^2}{(32.2)}$$

$$h = 0.1863 \text{ ft} = 2.24 \text{ in.} \qquad \textbf{Ans}$$

***21-28.** The 5-kg disk is connected to the 3-kg slender rod. If the assembly is attached to a ball-and-socket joint at A and the 5-N·m couple moment is applied, determine the angular velocity of the rod about the z axis after the assembly has made two revolutions about the z axis starting from rest. The disk rolls without slipping.



$$I_{x'} = I_{z'} = \frac{1}{4}(5)(0.2)^2 + 5(1.5)^2 + \frac{1}{3}(3)(1.5)^2 = 13.55$$



$$I_{y'} = \frac{1}{2}(5)(0.2)^2 = 0.100$$

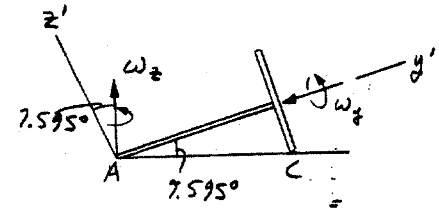
$$\omega = -\omega_{y'}\mathbf{j}' + \omega_z\mathbf{k} = -\omega_{y'}\mathbf{j}' + \omega_z\sin 7.595°\mathbf{j}' + \omega_z\cos 7.595°\mathbf{k}'$$

$$= (0.13216\omega_z - \omega_{y'})\mathbf{j}' + 0.99123\omega_z\mathbf{k}'$$

Since points A and C have zero velocity,

$$\mathbf{v}_C = \mathbf{v}_A + \omega \times \mathbf{r}_{C/A}$$

$$0 = 0 + \left[(0.13216\omega_z - \omega_{y'})\mathbf{j}' + 0.99123\omega_z\mathbf{k}'\right] \times (1.5\mathbf{j}' - 0.2\mathbf{k}')$$

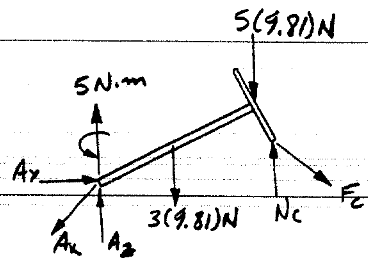


$$0 = -1.48684\omega_z - 0.026433\omega_z + 0.2\omega_{y'}$$

$$\omega_{y'} = 7.5664\omega_z$$

Thus,

$$\omega = -7.4342\omega_z\mathbf{j}' + 0.99123\omega_z\mathbf{k}'$$

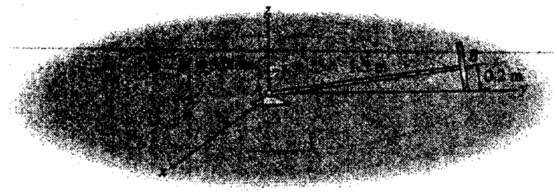


$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 5(2\pi)(2) = 0 + \frac{1}{2}(0.100)(-7.4342\omega_z)^2 + \frac{1}{2}(13.55)(0.99123\omega_z)^2$$

$\omega_z = 2.58$ rad/s **Ans**

21-29. The 5-kg disk is connected to the 3-kg slender rod. If the assembly is attached to a ball-and-socket joint at A and the 5-N·m couple moment gives it an angular velocity about the z axis of $\omega_z = 2$ rad/s, determine the magnitude of the angular momentum of the assembly about A.



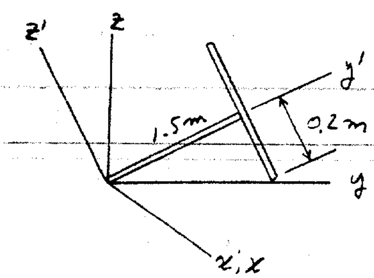
$$I_{x'} = I_{z'} = \frac{1}{4}(5)(0.2)^2 + 5(1.5)^2 + \frac{1}{3}(3)(1.5)^2 = 13.55$$

$$I_{y'} = \frac{1}{2}(5)(0.2)^2 = 0.100$$

$$\omega = -\omega_{y'}\mathbf{j}' + \omega_z\mathbf{k} = -\omega_{y'}\mathbf{j}' + \omega_z \sin 7.595°\mathbf{j}' + \omega_z \cos 7.595°\mathbf{k}'$$

$$= (0.13216\omega_z - \omega_{y'})\mathbf{j}' + 0.99123\omega_z\mathbf{k}'$$

Since points A and C have zero velocity,



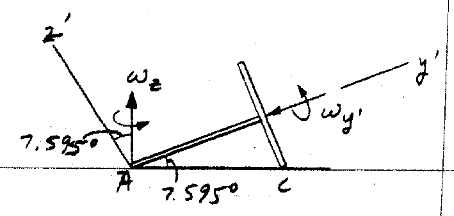
$$\mathbf{v}_C = \mathbf{v}_A + \omega \times \mathbf{r}_{C/A}$$

$$0 = 0 + \left[(0.13216\omega_z - \omega_{y'})\mathbf{j}' + 0.99123\omega_z\mathbf{k}'\right] \times (1.5\mathbf{j}' - 0.2\mathbf{k}')$$

$$0 = -1.48684\omega_z - 0.026433\omega_z + 0.2\omega_{y'}$$

$$\omega_{y'} = -7.5664\omega_z$$

Thus,



$$\omega = -7.4342\omega_z\mathbf{j}' + 0.99123\omega_z\mathbf{k}'$$

Since $\omega_z = 2$ rad/s

$$\omega = -14.868\mathbf{j}' + 1.9825\mathbf{k}'$$

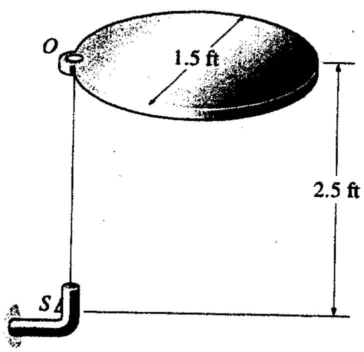
So that,

$$\mathbf{H}_A = I_{x'}\omega_{x'}\mathbf{i}' + I_{y'}\omega_{y'}\mathbf{j} + I_{z'}\omega_{z'}\mathbf{k}' = 0 + 0.100(-14.868)\mathbf{j}' + 13.55(1.9825)\mathbf{k}'$$

$$= -1.4868\mathbf{j}' + 26.862\mathbf{k}'$$

$$H_A = \sqrt{(-1.4868)^2 + (26.862)^2} = 26.9 \text{ kg} \cdot \text{m}^2/\text{s} \qquad \textbf{Ans}$$

21-30. The circular plate has a weight of 19 lb and a diameter of 1.5 ft. If it is released from rest and falls horizontally 2.5 ft onto the hook at S, which provides a permanent connection, determine the velocity of the mass center of the plate just after the connection with the hook is made.



Conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 19(2.5) = \frac{1}{2}\left(\frac{19}{32.2}\right)(v_G)_2^2 + 0$$

$$(v_G)_2 = 12.69 \text{ ft/s}$$

Conservation of momentum about point O:

$$(\mathbf{H}_O)_1 = \left[-\left(\frac{19}{32.2}\right)(12.69)(0.75)\right]\mathbf{i} = \{-5.6153\mathbf{i}\} \text{ slug}\cdot\text{ft}^2/\text{s}$$

$$I_x = \left[\frac{1}{4}\left(\frac{19}{32.2}\right)(0.75)^2 + \left(\frac{19}{32.2}\right)(0.75)^2\right] = 0.4149 \text{ slug}\cdot\text{ft}^2$$

$$I_y = \frac{1}{4}\left(\frac{19}{32.2}\right)(0.75)^2 = 0.08298 \text{ slug}\cdot\text{ft}^2$$

$$I_z = \left[\frac{1}{2}\left(\frac{19}{32.2}\right)(0.75)^2 + \left(\frac{19}{32.2}\right)(0.75)^2\right] = 0.4979 \text{ slug}\cdot\text{ft}^2$$

$$(\mathbf{H}_O)_3 = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$= 0.4149\omega_x\mathbf{i} + 0.08298\omega_y\mathbf{j} + 0.4979\omega_z\mathbf{k}$$

$$(\mathbf{H}_O)_2 = (\mathbf{H}_O)_3$$

$$-5.6153\mathbf{i} = 0.4149\omega_x\mathbf{i} + 0.08298\omega_y\mathbf{j} + 0.4979\omega_z\mathbf{k}$$

Equating **i**, **j** and **k** components

$-5.6153 = 0.4149\omega_x$ $\qquad \omega_x = -13.54$ rad/s

$0 = 0.08298\omega_y$ $\qquad \omega_y = 0$

$0 = 0.4979\omega_z$ $\qquad \omega_z = 0$

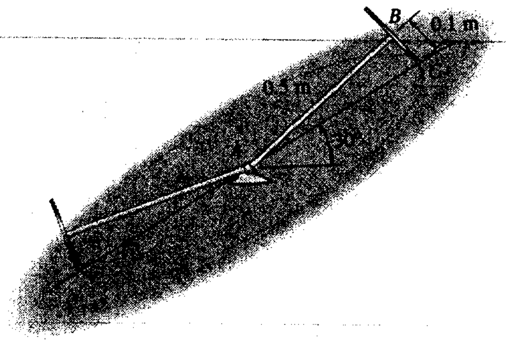
Hence $\qquad \boldsymbol{\omega} = \{-13.54\mathbf{i}\}$ rad/s

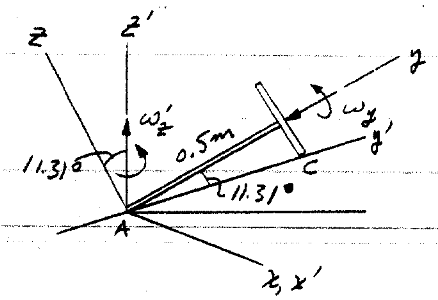
$$\mathbf{v}_G = \boldsymbol{\omega} \times \mathbf{r}_{G/O}$$

$$= (-13.54\mathbf{i}) \times (0.75\mathbf{j})$$

$$= \{-10.2\mathbf{k}\} \text{ ft/s}$$ **Ans**

21-31. The 2-kg thin disk is connected to the slender rod which is fixed to the ball-and-socket joint at A. If it is released from rest in the position shown, determine the spin of the disk about the rod when the disk reaches its lowest position. Neglect the mass of the rod. The disk rolls without slipping.



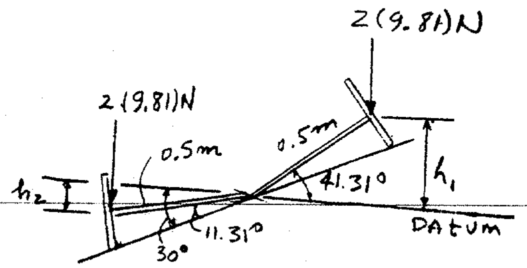


$$I_x = I_z = \frac{1}{4}(2)(0.1)^2 + 2(0.5)^2 = 0.505\ \text{kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{2}(2)(0.1)^2 = 0.01\ \text{kg}\cdot\text{m}^2$$

$$\omega = \omega_y + \omega_{z'} = -\omega_y\,\mathbf{j} + \omega_{z'}\sin 11.31^\circ\mathbf{j} + \omega_{z'}\cos 11.31^\circ\mathbf{k}$$

$$= (0.19612\omega_{z'} - \omega_y)\mathbf{j} + (0.98058\omega_{z'})\mathbf{k}$$



Since $\mathbf{v}_A = \mathbf{v}_C = \mathbf{0}$, then

$$\mathbf{v}_C = \mathbf{v}_A + \omega \times \mathbf{r}_{C/A}$$

$$\mathbf{0} = \mathbf{0} + \left[(0.19612\omega_{z'} - \omega_y)\mathbf{j} + (0.98058\omega_{z'})\mathbf{k}\right] \times (0.5\mathbf{j} - 0.1\mathbf{k})$$

$$0 = -0.019612\omega_{z'} + 0.1\omega_y - 0.49029\omega_{z'}$$

$$\omega_{z'} = 0.19612\omega_y$$

Thus,

$$\omega = -0.96154\omega_y\,\mathbf{j} + 0.19231\omega_y\,\mathbf{k}$$

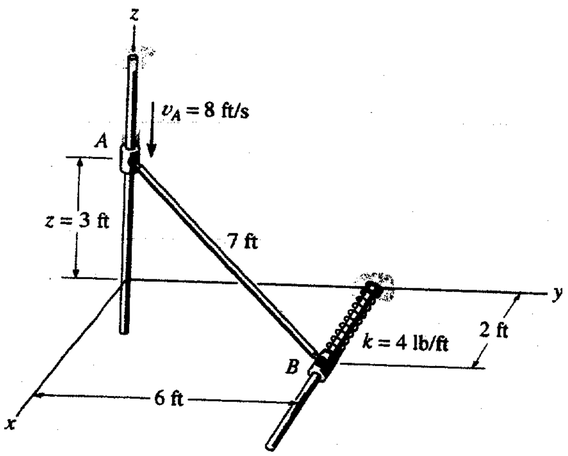
$h_1 = 0.5\sin 41.31^\circ = 0.3301$ m, $\quad h_2 = 0.5\sin 18.69^\circ = 0.1602$ m

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2(9.81)(0.3301) = \left[0 + \frac{1}{2}(0.01)(-0.96154\omega_y)^2 + \frac{1}{2}(0.505)(0.19231\omega_y)^2\right] - 2(9.81)(0.1602$$

$\omega_y = 26.2$ rad/s **Ans**

***21-32.** Rod AB has a weight of 6 lb and is attached to two smooth collars at its ends by ball-and-socket joints. If collar A is moving downward with a speed of 8 ft/s when $z = 3$ ft, determine the speed of A at the instant $z = 0$. The spring has an unstretched length of 2 ft. Neglect the mass of the collars. Assume the angular velocity of rod AB is perpendicular to its axis.



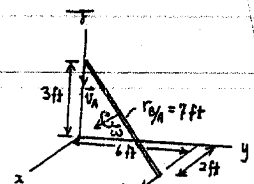
$$\mathbf{v}_A = \{-8\mathbf{k}\} \text{ ft/s}$$

$$\mathbf{v}_B = v_B\mathbf{i}$$

$$\mathbf{r}_{B/A} = \{2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\} \text{ ft}$$

$$\boldsymbol{\omega} = \{\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}\} \text{ rad/s}$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$



$$v_B\mathbf{i} = -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 2 & 6 & -3 \end{vmatrix}$$

expanding and equating components yields,

$$v_B = -3\omega_y - 6\omega_z \qquad (1)$$

$$0 = 3\omega_x + 2\omega_z \qquad (2)$$

$$0 = -8 + 6\omega_x - 2\omega_y \qquad (3)$$

Also, $\quad \boldsymbol{\omega} \cdot \mathbf{r}_{B/A} = 0$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$$

$$2\omega_x + 6\omega_y - 3\omega_z = 0 \qquad (4)$$

Solving Eqs. (1) – (4) yields;

$$\omega_x = 0.9796 \text{ rad/s}$$

$$\omega_y = -1.061 \text{ rad/s}$$

$$\omega_z = -1.469 \text{ rad/s}$$

Thus,

$$\omega = \sqrt{(0.9796)^2 + (-1.061)^2 + (-1.469)^2} = 2.06 \text{ rad/s}$$

$$\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{G/A}$$

where $\mathbf{r}_{G/A} = \frac{1}{2}\mathbf{r}_{B/A} = \frac{1}{2}(2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})$ ft

$$\mathbf{v}_G = -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9796 & -1.061 & -1.469 \\ 1 & 3 & -1.5 \end{vmatrix}$$

$$\mathbf{v}_G = \{5.9985\mathbf{i} - 4\mathbf{k}\} \text{ ft/s}$$

$$v_G = \sqrt{(5.9985)^2 + (-4)^2} = 7.2097 \text{ ft/s}$$

Hence, since ω is directed perpendicular to the axis of the rod,

$$T_1 = \frac{1}{2}I\omega^2 + \frac{1}{2}mv_G^2$$

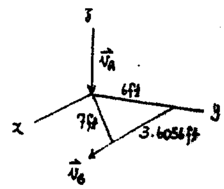
$$= \frac{1}{2}\left[\frac{1}{12}\left(\frac{6}{32.2}\right)(7)^2\right](2.06)^2 + \frac{1}{2}\left(\frac{6}{32.2}\right)(7.2097)^2$$

$$= 6.46 \text{ lb}\cdot\text{ft}$$

$$T_1 + V_1 = T_2 + V_2$$

$$6.46 + 1.5(6) = T_2 + \frac{1}{2}(4)(3.6056 - 2)^2$$

$$T_2 = 10.304 \text{ lb}\cdot\text{ft}$$



$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$-v_A\mathbf{k} = v_B\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 3.6056 & 6 & 0 \end{vmatrix}$$

$$0 = v_B - 6\omega_z$$

$$0 = \omega_z(0.36056)$$

$$-v_A = 6\omega_x - 3.6056\omega_y$$

$$\boldsymbol{\omega} \cdot \mathbf{r}_{A/B} = 0$$

$$3.6056\omega_x + 6\omega_y = 0$$

Solving,

$$\omega_z = 0$$

$$v_B = 0 \text{ (location of } IC\text{)}$$

$$v_A = 13.590\omega_y$$

$$\omega_x = -1.664\omega_y$$

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{G/B}$$

$$\mathbf{v}_G = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.664 & 1 & 0 \\ -1.803 & -3 & 0 \end{vmatrix}\omega_y = \{6.795\omega_y\mathbf{k}\} \text{ ft/s}$$

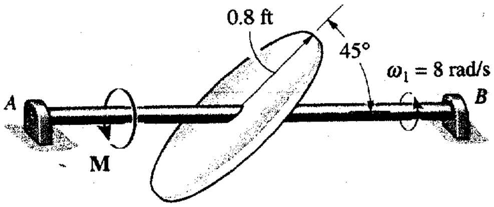
$$T_2 = \frac{1}{2}\left(\frac{6}{32.2}\right)(6.795\omega_y)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{6}{32.2}\right)(7)^2\right]\left[(-1.664\omega_y)^2 + \omega_y^2\right]$$

$$= 10.304$$

$$\omega_y = -1.34 \text{ rad/s}$$

$$v_A = 13.590(-1.34) = -18.2\mathbf{j} \text{ ft/s} \qquad \textbf{Ans}$$

21-33. The circular disk has a weight of 15 lb and is mounted on the shaft AB at an angle of 45° with the horizontal. Determine the angular velocity of the shaft when $t = 3$ s if a constant torque $M = 2$ lb · ft is applied to the shaft. The shaft is originally spinning at $\omega_1 = 8$ rad/s when the torque is applied.



Due to symmetry

$$I_{xy} = I_{yz} = I_{zx} = 0$$

$$I_y = I_z = \frac{1}{4}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.07453 \text{ slug} \cdot \text{ft}^2$$

$$I_x = \frac{1}{2}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.1491 \text{ slug} \cdot \text{ft}^2$$

For x' axis

$u_x = \cos 45° = 0.7071 \qquad u_y = \cos 45° = 0.7071$

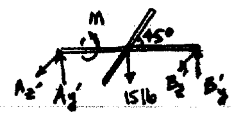
$u_z = \cos 90° = 0$

$$I_{x'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$$

$$= 0.1491(0.7071)^2 + 0.07453(0.7071)^2 + 0 - 0 - 0 - 0$$

$$= 0.1118 \text{ slug} \cdot \text{ft}^2$$

Principle of impulse and momentum:

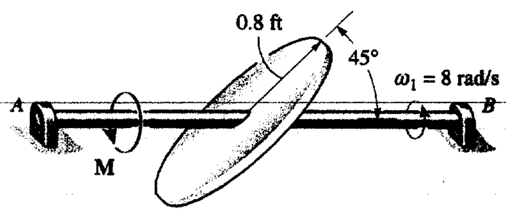


$$(H_{x'})_1 + \Sigma \int M_{x'} \, dt = (H_{x'})_2$$

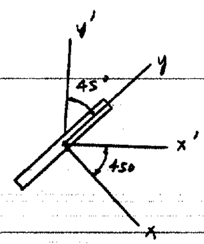
$$0.1118(8) + 2(3) = 0.1118\,\omega_2$$

$\omega_2 = 61.7$ rad/s **Ans**

21-34. The circular disk has a weight of 15 lb and is mounted on the shaft AB at an angle of 45° with the horizontal. Determine the angular velocity of the shaft when $t = 2$ s if a torque $M = (4e^{0.1t})$ lb · ft, where t is in seconds, is applied to the shaft. The shaft is originally spinning at $\omega_1 = 8$ rad/s when the torque is applied.



Due to symmetry



$$I_{xy} = I_{yz} = I_{zx} = 0$$

$$I_y = I_z = \frac{1}{4}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.07453 \text{ slug} \cdot \text{ft}^2$$

$$I_x = \frac{1}{2}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.1491 \text{ slug} \cdot \text{ft}^2$$

For x' axis

$u_x = \cos 45° = 0.7071 \qquad u_y = \cos 45° = 0.7071$

$u_z = \cos 90° = 0$

$$I_{x'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$$

$$= 0.1491(0.7071)^2 + 0.07453(0.7071)^2 + 0 - 0 - 0 - 0$$

$$= 0.1118 \text{ slug} \cdot \text{ft}^2$$

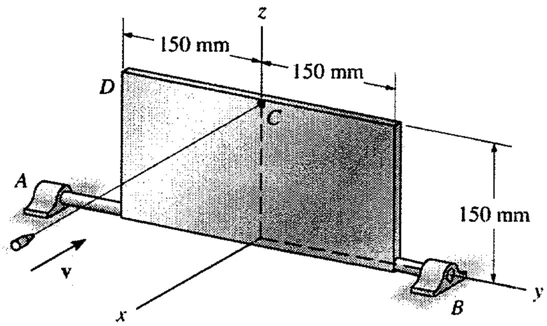
Principle of impulse and momentum:

$$(H_{x'})_1 + \Sigma \int M_{x'} \, dt = (H_{x'})_2$$

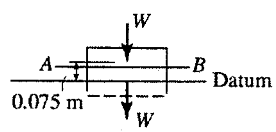
$$0.1118(8) + \int_0^2 4e^{0.1t} \, dt = 0.1118\,\omega_2$$

$\omega_2 = 87.2$ rad/s **Ans**

21-35. The 15-kg rectangular plate is free to rotate about the y axis because of the bearing supports at A and B. When the plate is balanced in the vertical plane, a 3-g bullet is fired into it, perpendicular to its surface, with a velocity $\mathbf{v} = \{-2000\mathbf{i}\}$ m/s. Compute the angular velocity of the plate at the instant it has rotated 180°. If the bullet strikes corner D with the same velocity $\mathbf{v}$, instead of at C, does the angular velocity remain the same? Why or why not?



Consider the projectile and plate as an entire system.



Angular momentum is conserved about the AB axis.

$(\mathbf{H}_{AB})_1 = -(0.003)(2000)(0.15)\mathbf{j} = \{-0.9\mathbf{j}\}$

$(\mathbf{H}_{AB})_1 = (\mathbf{H}_{AB})_2$

$-0.9\mathbf{j} = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$

Equating components,

$\omega_x = 0$

$\omega_z = 0$

$$\omega_y = \frac{-0.9}{\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right]} = -8 \text{ rad/s}$$

$T_1 + V_1 = T_2 + V_2$

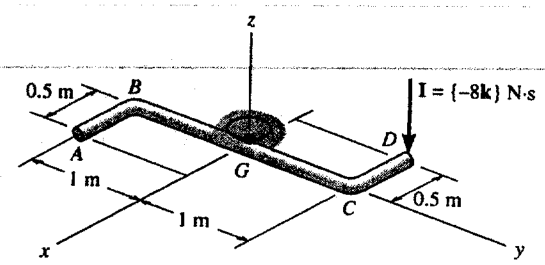
$$\frac{1}{2}\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right](8)^2 + 15(9.81)(0.15)$$

$$= \frac{1}{2}\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right]\omega_{AB}^2$$

$\omega_{AB} = 21.4$ rad/s **Ans**

If the projectile strikes the plate at D, the angular velocity is the same, only the impulsive reactions at the bearing supports A and B will be different.

*21-36. The rod assembly is supported at G by a ball-and-socket joint. Each segment has a mass of 0.5 kg/m. If the assembly is originally at rest and an impulse of $\mathbf{I} = \{-8\mathbf{k}\}$ N·s is applied at D, determine the angular velocity of the assembly just after the impact.



Moments and products of inertia :

$$I_{xx} = \frac{1}{12}\left[2(0.5)\right](2)^2 + 2\left[0.5(0.5)\right](1)^2 = 0.8333 \text{ kg}\cdot\text{m}^2$$

$$I_{yy} = \frac{1}{12}\left[1(0.5)\right](1)^2 = 0.04166 \text{ kg}\cdot\text{m}^2$$

$$I_{zz} = \frac{1}{12}\left[2(0.5)\right](2)^2 + 2\left[\frac{1}{12}\left[0.5(0.5)\right](0.5)^2 + \left[0.5(0.5)\right](1^2 + 0.25^2)\right]$$

$$= 0.875 \text{ kg}\cdot\text{m}^2$$

$$I_{xy} = \left[0.5(0.5)\right](-0.25)(1) + \left[0.5(0.5)\right](0.25)(-1) = -0.125 \text{ kg.m}^2$$

$$I_{yz} = I_{xz} = 0$$

From Eq. 21-10

$$H_x = 0.8333\omega_x + 0.125\omega_y$$

$$H_y = 0.125\omega_x + 0.04166\omega_y$$

$$H_z = 0.875\omega_z$$

$$(\mathbf{H}_G)_1 + \Sigma\int_{t_1}^{t_2}\mathbf{M}_G\,dt = (\mathbf{H}_G)_2$$

$$0 + (-0.5\mathbf{i} + 1\mathbf{j})\times(-8\mathbf{k}) = (0.8333\omega_x + 0.125\omega_y)\mathbf{i} + (0.125\omega_x + 0.04166\omega_y)\mathbf{j} + 0.875\omega_z\mathbf{k}$$

Equating **i**, **j** and **k** components

$$-8 = 0.8333\omega_x + 0.125\omega_y \qquad (1)$$

$$-4 = 0.125\omega_x + 0.04166\omega_y \qquad (2)$$

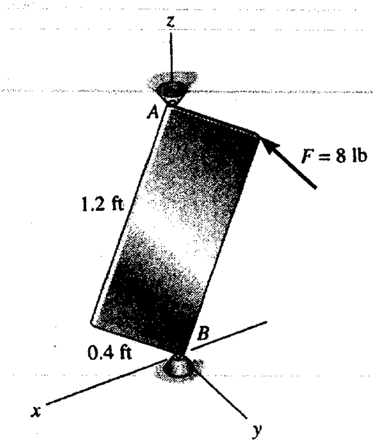
$$0 = 0.875\omega_z \qquad (3)$$

Solving Eqs.(1) to (3) yields :

$\omega_x = 8.73$ rad/s $\quad \omega_y = -122$ rad/s $\quad \omega_z = 0$

Then $\quad \boldsymbol{\omega} = \{8.73\mathbf{i} - 122\mathbf{j}\}$ rad/s $\quad$ **Ans**

21-37. The 15-lb plate is subjected to a force $F = 8$ lb which is always directed perpendicular to the face of the plate. If the plate is originally at rest, determine its angular velocity after it has rotated one revolution (360°). The plate is supported by ball-and-socket joints at A and B.



Due to symmetry $\quad I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$

$$I_{x'} = \frac{1}{12}\left(\frac{15}{32.2}\right)(1.2)^2 = 0.05590 \text{ slug} \cdot \text{ft}^2$$

$$I_{y'} = \frac{1}{12}\left(\frac{15}{32.2}\right)(1.2^2 + 0.4^2) = 0.06211 \text{ slug} \cdot \text{ft}^2$$

$$I_{z'} = \frac{1}{12}\left(\frac{15}{32.2}\right)(0.4)^2 = 0.006211 \text{ slug} \cdot \text{ft}^2$$

For z axis

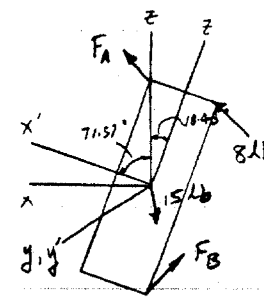
$$u_{x'} = \cos 71.57° = 0.3162 \qquad u_{y'} = \cos 90° = 0$$

$$u_{z'} = \cos 18.43° = 0.9487$$

$$I_z = I_{x'}u_{x'}^2 + I_{y'}u_{y'}^2 + I_{z'}u_{z'}^2 - 2I_{x'y'}u_{x'}u_{y'} - 2I_{y'z'}u_{y'}u_{z'} - 2I_{z'x'}u_{z'}u_{x'}$$

$$= 0.05590(0.3162)^2 + 0 + 0.006211(0.9487)^2 - 0 - 0 - 0$$

$$= 0.01118 \text{ slug} \cdot \text{ft}^2$$



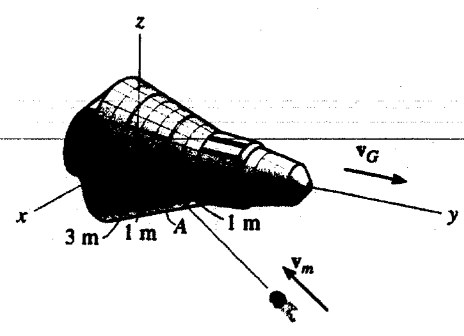
Principle of work and energy :

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 8(1.2 \sin 18.43°)(2\pi) = \frac{1}{2}(0.01118)\omega^2$$

$\omega = 58.4$ rad/s **Ans**

21-38. The space capsule has a mass of 3.5 Mg and the radii of gyration are $k_x = k_z = 0.8$ m and $k_y = 0.5$ m. If it is traveling with a velocity $\mathbf{v}_G = \{600\mathbf{j}\}$ m/s, compute its angular velocity just after it is struck by a meteoroid having a mass of 0.60 kg and a velocity $\mathbf{V}_m = \{-200\mathbf{i} - 400\mathbf{j} + 200\mathbf{k}\}$ m/s. Assume that the meteoroid embeds itself into the capsule at point A and that the capsule initially has no angular velocity.



Angular momentum for meteoriod and space capsule is conserved about G.

$$(H_G)_1 = (H_G)_2$$

$$(1\mathbf{i} + 3\mathbf{j} - 1\mathbf{k}) \times 0.6(-200\mathbf{i} - 400\mathbf{j} + 200\mathbf{k}) = 3500(0.8)^2\omega_x\mathbf{i} + 3500(0.5)^2\omega_y\mathbf{j} + 3500(0.8)^2\omega_z\mathbf{k}$$

$$120\mathbf{i} + 120\mathbf{k} = 2240\omega_x\mathbf{i} + 875\omega_y\mathbf{j} + 2240\omega_z\mathbf{k}$$

$$\omega_x = \frac{120}{2240} = 0.0536 \text{ rad/s}$$

$$\omega_y = 0$$

$$\omega_z = \frac{120}{2240} = 0.0536 \text{ rad/s}$$

$\boldsymbol{\omega} = \{0.0536\mathbf{i} + 0.0536\mathbf{k}\}$ rad/s **Ans**

21-39. Derive the scalar form of the rotational equation of motion along the x axis when $\mathbf{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *not constant* with respect to time.

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

$$= (\dot{H}_x\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\mathbf{k})_{xyz} + \Omega \times (H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

Substitute $\Omega = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left((\dot{H}_x)_{xyz} - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left((\dot{H}_y)_{xyz} - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} + \left((\dot{H}_z)_{xyz} - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Subsitute H_x , H_y and H_z using Eq. 21-10. For the $\mathbf{i}$ component

$$\Sigma M_x = \frac{d}{dt}(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z) - \Omega_z(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x) + \Omega_y(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y) \quad \textbf{Ans}$$

One can obtain y and z components in a similar manner.

***21-40.** Derive the scalar form of the rotational equation of motion along the x axis when $\mathbf{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *constant* with respect to time.

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

$$= (\dot{H}_x\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\mathbf{k})_{xyz} + \Omega \times (H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

Substitute $\Omega = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left((\dot{H}_x)_{xyz} - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left((\dot{H}_y)_{xyz} - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} + \left((\dot{H}_z)_{xyz} - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Substitute H_x , H_y and H_z using Eq. 21-10. For the $\mathbf{i}$ component

$$\Sigma M_x = \frac{d}{dt}(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z) - \Omega_z(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x) + \Omega_y(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y)$$

For constant inertia, expanding the time derivative of the above equation yields

$$\Sigma M_x = (I_x\dot{\omega}_x - I_{xy}\dot{\omega}_y - I_{xz}\dot{\omega}_z) - \Omega_z(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x) + \Omega_y(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y) \quad \textbf{Ans}$$

One can obtain y and z components in a similar manner.

21-41. Derive the Euler equations of motion for $\boldsymbol{\Omega} \neq \boldsymbol{\omega}$, i.e., Eqs. 21–26.

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

$$= \left(\dot{H}_x\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\mathbf{k}\right)_{xyz} + \boldsymbol{\Omega} \times (H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k})$$

Substitute $\boldsymbol{\Omega} = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} + \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Substitute H_x , H_y and H_z using Eq. 21-10. For the **i** component

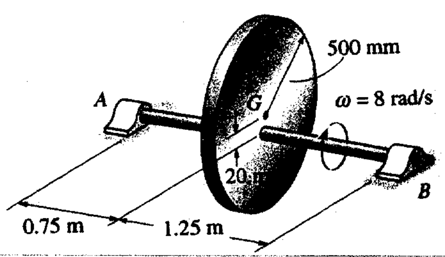
$$\Sigma M_x = \frac{d}{dt}(I_x\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z) - \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\,\omega_x) + \Omega_y\,(I_z\,\omega_z - I_{zx}\,\omega_x - I_{zy}\,\omega_y)$$

Set $I_{xy} = I_{yz} = I_{zx} = 0$ and require I_x, I_y, I_z to be constant. This yields

$$\Sigma M_x = I_x\,\dot{\omega}_x - I_y\,\Omega_z\,\omega_y + I_z\,\Omega_y\,\omega_z \qquad \textbf{Ans}$$

One can obtain y and z components in a similar manner.

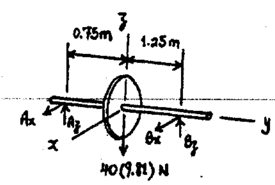
21-42. The 40-kg flywheel (disk) is mounted 20 mm off its true center at G. If the shaft is rotating at a constant speed $\omega = 8$ rad/s, determine the maximum reactions exerted on the journal bearings at A and B.



$\omega_x = 0$

$\omega_y = -8$ rad/s

$\omega_z = 0$



$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z;$

$B_z(1.25) - A_z(0.75) = 0 - 0$

$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y;$

$-B_x(1.25) + A_x(0.75) = 0 - 0$

$\Sigma F_x = ma_x; \quad A_x + B_x = 0$

$\Sigma F_z = ma_z; \quad A_z + B_z - 40(9.81) = 40(8)^2(0.020)$

Solving,

$A_x = 0$

$B_x = 0$

$A_z = 277$ N

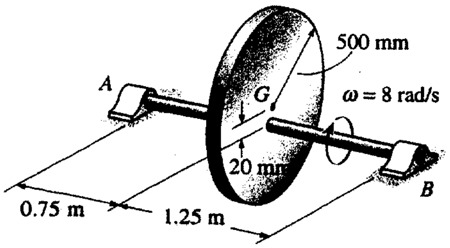
$B_z = 166$ N

Thus,

$F_A = 277$ N **Ans**

$F_B = 166$ N **Ans**

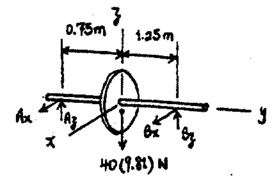
21-43. The 40-kg flywheel (disk) is mounted 20 mm off its true center at G. If the shaft is rotating at a constant speed $\omega = 8$ rad/s, determine the minimum reactions exerted on the journal bearings at A and B during the motion.



$\omega_x = 0$

$\omega_y = -8$ rad/s

$\omega_z = 0$



$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z;$

$B_z(1.25) - A_z(0.75) = 0 - 0$

$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\omega_x \omega_y;$

$-B_x(1.25) + A_x(0.75) = 0 - 0$

$\Sigma F_x = ma_x; \quad A_x + B_x = 0$

$\Sigma F_z = ma_z; \quad A_z + B_z - 40(9.81) = -40(8)^2(0.020)$

Solving,

$A_x = 0$

$B_x = 0$

$A_z = 213.25$ N

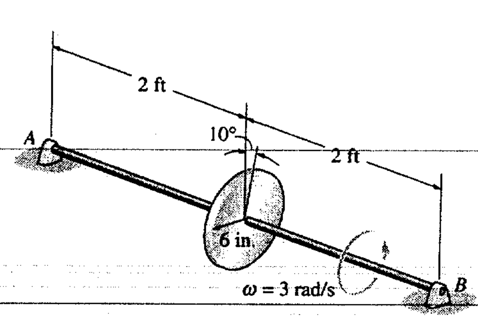
$B_z = 127.95$ N

Thus,

$F_A = 213$ N   **Ans**

$F_B = 128$ N   **Ans**

***21-44.** The 20-lb disk is mounted on the horizontal shaft AB such that its plane forms an angle of 10° with the vertical. If the shaft rotates with an angular velocity of 3 rad/s, determine the vertical reactions developed at the bearings when the disk is in the position shown.



$$I_x = \frac{1}{2}\left(\frac{20}{32.2}\right)(0.5)^2 = 0.07764 \text{ slug}\cdot\text{ft}^2$$

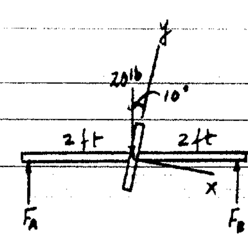
$$I_y = I_z = \frac{1}{4}\left(\frac{20}{32.2}\right)(0.5)^2 = 0.03882 \text{ slug}\cdot\text{ft}^2$$

$$\omega = 3\cos 10°\mathbf{i} + 3\sin 10°\mathbf{j} = \{2.9544\mathbf{i} + 0.5209\mathbf{j}\} \text{ rad/s}$$

Applying the third of Eqs. 21-25 with $\omega_x = 2.9544$ rad/s $\omega_y = 0.5209$ rad/s $\dot{\omega}_z = 0$

$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\omega_x \omega_y;$

$F_B(2) - F_A(2) = 0 - (0.07764 - 0.03882)(2.9544)(0.5209)$ (1)



Also,

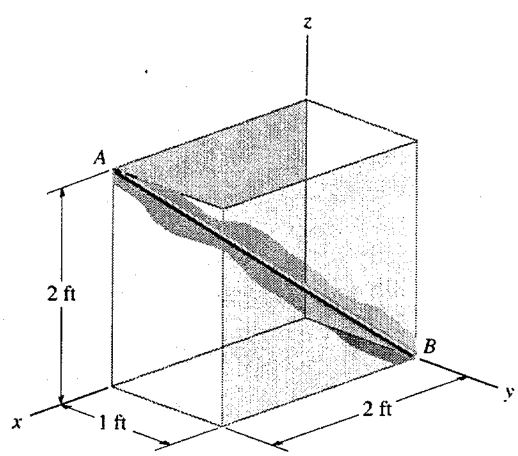
$\Sigma F_z = m(a_G)_z; \quad F_A + F_B - 20 = 0$ (2)

Solving,

$F_A = 10.0$ lb   **Ans**

$F_B = 9.99$ lb   **Ans**

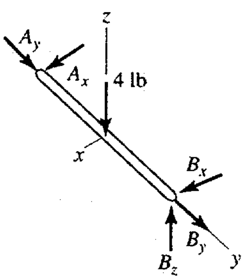
21-45. The 4-lb bar rests along the smooth corners of an open box. At the instant shown, the box has a velocity $\mathbf{v} = \{3\mathbf{j}\}$ ft/s and an acceleration $\mathbf{a} = \{-6\mathbf{j}\}$ ft/s^2. Determine the x, y, z components of force which the corners exert on the bar.



$$\sum F_x = m(a_G)_x; \quad A_x + B_x = 0 \qquad [1]$$

$$\sum F_y = m(a_G)_y; \quad A_y + B_y = \left(\frac{4}{32.2}\right)(-6) \qquad [2]$$

$$\sum F_z = m(a_G)_z; \quad B_z - 4 = 0 \quad B_z = 4 \text{ lb} \qquad \textbf{Ans}$$



Applying Eq. 21-25 with $\omega_x = \omega_y = \omega_z = 0 \; \dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$

$$\sum(M_G)_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z; \quad B_y(1) - A_y(1) + 4(0.5) = 0 \qquad [3]$$

$$\sum(M_G)_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x; \quad A_x(1) - B_x(1) + 4(1) = 0 \qquad [4]$$

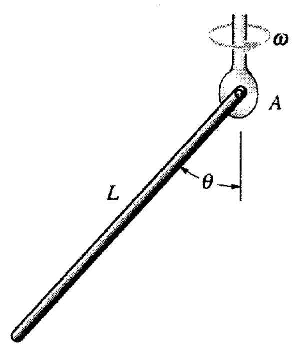
Solving Eqs. [1] to [4] yields:

$A_x = -2.00$ lb $\quad A_y = 0.627$ lb $\quad B_x = 2.00$ lb $\quad B_y = -1.37$ lb $\qquad$ **Ans**

$$\sum(M_G)_z = I_z\dot{\omega}_z - (I_x - I_y)\,\omega_x\omega_y;$$

$(-2.00)(0.5) - (2.00)(0.5) - (-1.37)(1) + (0.627)(1) = 0$ **(O.K!)**

21-46. The conical pendulum consists of a bar of mass m and length L that is supported by the pin at its end A. If the pin is subjected to a rotation ω, determine the angle θ that the bar makes with the vertical as it rotates. Also, determine the components of reaction at the pin.



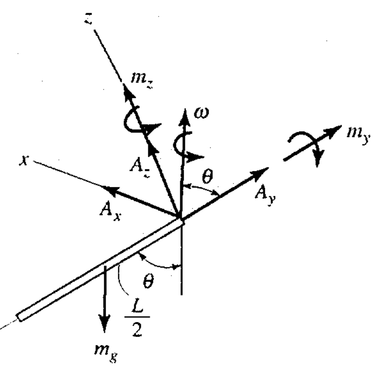
$$I_x = I_z = \frac{1}{3}mL^2, \quad I_y = 0$$

$$\omega_x = 0, \qquad \omega_y = -\omega\cos\theta, \qquad \omega_z = \omega\sin\theta$$

$$\omega_x = 0, \qquad \omega_y = 0, \qquad \omega_z = 0$$

$$\sum M_x = I_x\omega_x - (I_y - I_z)\omega_y\omega_z$$

$$-mg\left(\frac{L}{2}\sin\theta\right) = 0 - \left(0 - \frac{1}{3}mL^2\right)(-\omega\cos\theta)(\omega\sin\theta)$$

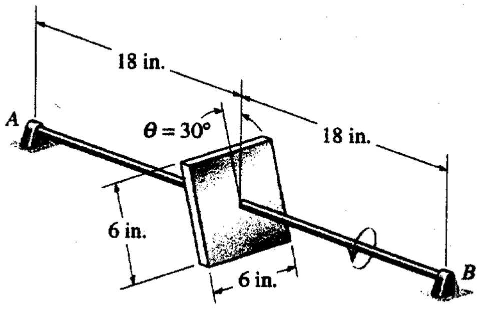


$$\frac{g}{2} = \frac{1}{3}L\omega^2\cos\theta$$

$$\cos\theta = \frac{3g}{2L\omega^2}$$

$$\theta = \cos^{-1}\left(\frac{3g}{2L\omega^2}\right) \qquad \textbf{Ans}$$

21-47. The 20-lb plate is mounted on the shaft AB so that the plane of the plate makes an angle $\theta = 30°$ with the vertical. If the shaft is turning in the direction shown with an angular velocity of 25 rad/s, determine the vertical reactions at the bearing supports A and B when the plate is in the position shown.

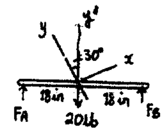


$$\omega = 25\cos 30°\mathbf{i} - 25\sin 30°\mathbf{j}$$

$$\dot{\omega} = \dot{\omega}\cos 30°\mathbf{i} - \dot{\omega}\sin 30°\mathbf{j}$$

$$I_x = \frac{20}{32.2}\left(\frac{1}{12}\right)\left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right] = 0.02588 \text{ slug}\cdot\text{ft}^2$$

$$I_y = \frac{20}{32.2}\left(\frac{1}{12}\right)\left(\frac{1}{2}\right)^2 = 0.01294 \text{ slug}\cdot\text{ft}^2$$



Using the third Eq. 21–25,

$$\Sigma M_z = F_B\left(\frac{18}{12}\right) - F_A\left(\frac{18}{12}\right)$$

$$= 0 - (0.02588 - 0.01294)(25\cos 30°)(-25\sin 30°)$$

Thus,

$$F_B - F_A = 2.3346$$

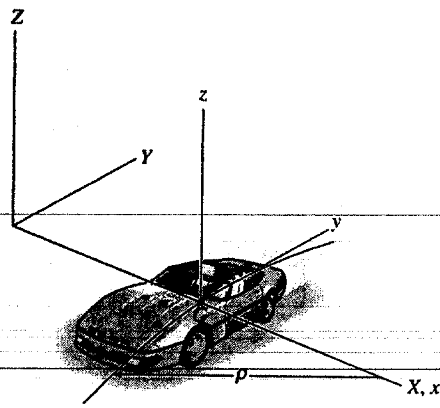
Also,

$$\Sigma F_y = m(a_G)_y; \quad F_A + F_B - 20 = 0$$

$F_A = 8.83$ lb **Ans**

$F_B = 11.2$ lb **Ans**

***21-48.** The car is traveling around the curved road of radius ρ such that its mass center has a constant speed v_G. Write the equations of rotational motion with respect to the x, y, z axes. Assume that the car's six moments and products of inertia with respect to these axes are known.



Applying Eq. 21–24 with $\omega_x = 0, \quad \omega_y = 0, \quad \omega_z = \frac{v_G}{\rho}$,

$$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$$

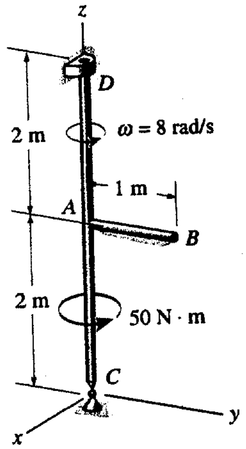
$$\Sigma M_x = -I_{yz}\left[0 - \left(\frac{v_G}{\rho}\right)^2\right] = \frac{I_{yz}}{\rho^2}v_G^2 \qquad \textbf{Ans}$$

$$\Sigma M_y = -I_{zx}\left[\left(\frac{v_G}{\rho}\right)^2 - 0\right] = -\frac{I_{zx}}{\rho^2}v_G^2 \qquad \textbf{Ans}$$

$$\Sigma M_z = 0 \qquad \textbf{Ans}$$

Note : This result indicates the normal reactions of the tires on the ground are not all necessarily equal. Instead, they depend upon the speed of the car, radius of curvature, and the products of inertia I_{yz} and I_{zx}. (See Example 13–6)

21-49. The rod assembly is supported by a ball-and-socket joint at C and a journal bearing at D, which develops only x and y force reactions. The rods have a mass of 0.75 kg/m. Determine the angular acceleration of the rods and the components of reaction at the supports at the instant $\omega = 8$ rad/s as shown.



$\Omega = \omega = 8\mathbf{k}$

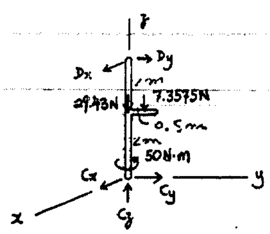
$\omega_x = \omega_y = 0, \quad \omega_z = 8$ rad/s

$\dot{\omega}_x = \dot{\omega}_y = 0, \quad \dot{\omega}_z = \dot{\omega}_z$

$I_{xz} = I_{xy} = 0$

$I_{yz} = 0.75(1)(2)(0.5) = 0.75$ kg·m^2

$I_{zz} = \frac{1}{3}(0.75)(1)(1)^2 = 0.25$ kg·m^2



Eqs. 21 – 24 become

$\Sigma M_x = I_{yz}\omega_z^2$

$\Sigma M_y = -I_{yz}\dot{\omega}_z$

$\Sigma M_z = I_{zz}\dot{\omega}_z$

Thus,

$-D_y(4) - 7.3575(0.5) = 0.75(8)^2$

$D_y = -12.9$ N $\qquad$ **Ans**

$D_x(4) = -0.75\dot{\omega}_z$

$50 = 0.25\dot{\omega}_z$

$\dot{\omega}_z = 200$ rad/s^2 $\qquad$ **Ans**

$D_x = -37.5$ N $\qquad$ **Ans**

$\Sigma F_x = m(a_G)_x; \quad C_x - 37.5 = -1(0.75)(200)(0.5)$

$C_x = -37.5$ N $\qquad$ **Ans**

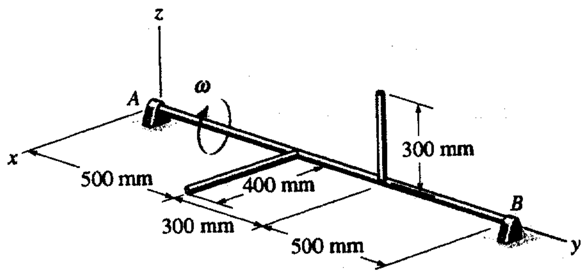
$\Sigma F_y = m(a_G)_y; \quad C_y - 12.9 = -(1)(0.75)(8)^2(0.5)$

$C_y = -11.1$ N $\qquad$ **Ans**

$\Sigma F_z = m(a_G)_z; \quad C_z - 7.3575 - 29.43 = 0$

$C_z = 36.8$ N $\qquad$ **Ans**

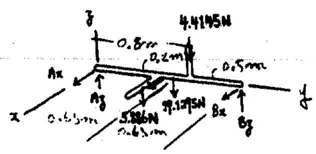
21-50. The rod assembly is supported by journal bearings at A and B, which develops only x and y force reactions on the shaft. If the shaft AB is rotating in the direction shown at $\boldsymbol{\omega} = \{-5\mathbf{j}\}$ rad/s, determine the reactions at the bearings when the assembly is in the position shown. Also, what is the shaft's angular acceleration? The mass of each rod is 1.5 kg/m.



$\omega_x = \omega_z = 0, \quad \omega_y = -5$ rad/s

$\dot{\omega}_x = \dot{\omega}_z = 0$

Eqs. 21–24 become



$$\Sigma M_x = -I_{xy}\,\dot{\omega}_y - I_{yz}\,\omega_y^2$$

$$\Sigma M_y = I_{yy}\,\dot{\omega}_y$$

$$\Sigma M_z = I_{xy}\,\omega_y^2 - I_{yz}\,\dot{\omega}_y$$

$$I_{yy} = \frac{1}{3}(0.4)(1.5)(0.4)^2 + \frac{1}{3}(0.3)(1.5)(0.3)^2 = 0.0455 \text{ kg}\cdot\text{m}^2$$

$$I_{yz} = [0 + (1.5)(0.3)(0.15)(0.8)] = 0.0540 \text{ kg}\cdot\text{m}^2$$

$$I_{xy} = [0 + (1.5)(0.4)(0.2)(0.5)] = 0.0600 \text{ kg}\cdot\text{m}^2$$

Thus,

$$-5.886(0.5) - 19.1295(0.65) - 4.4145(0.8) + B_z(1.3) = -0.0600\dot{\omega}_y - 0.0540(-5)^2$$

$$5.886(0.2) = 0.0455\dot{\omega}_y$$

$$-B_x(1.3) = 0.0600(-5)^2 - (0.0540)\dot{\omega}_y$$

$\dot{\omega}_y = 25.9 \text{ rad/s}^2$ **Ans**

$B_x = -0.0791$ N **Ans**

$B_z = 12.3$ N **Ans**

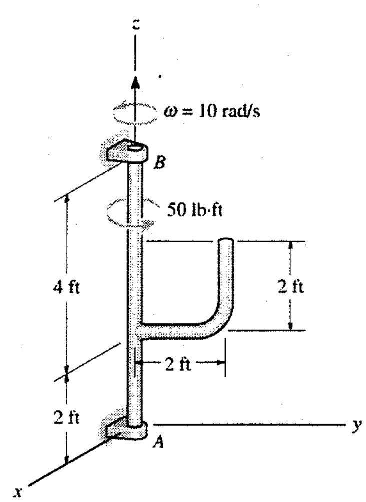
$\Sigma F_x = m(a_G)_x;\quad A_x - 0.0791 = -0.4(1.5)(5)^2(0.2) + 0.3(1.5)(25.9)(0.15)$

$A_x = -1.17$ N **Ans**

$\Sigma F_z = m(a_G)_z;\quad A_z + 12.31 - 5.886 - 19.1295 - 4.4145 = -0.4(1.5)(25.9)(0.2) - 0.3(1.5)(5)^2(0.15)$

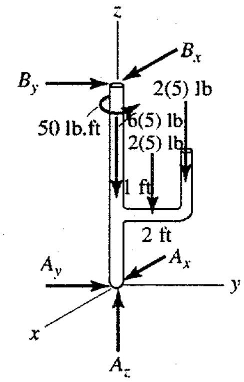
$A_z = 12.3$ N **Ans**

21-51. The rod assembly has a weight of 5 lb/ft. It is supported at B by a smooth journal bearing, which develops x and y force reactions, and at A by a smooth thrust bearing, which develops x, y, and z force reactions. If a 50-lb · ft torque is applied along rod AB, determine the components of reaction at the bearings when the assembly has an angular velocity $\omega = 10$ rad/s at the instant shown.



$$I_y = \frac{1}{3}\left[\frac{6(5)}{32.2}\right](6)^2 + \frac{1}{12}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](3)^2 + \left[\frac{2(5)}{32.2}\right](2)^2$$

$$= 15.3209 \text{ slug} \cdot \text{ft}^2$$



$$I_x = \frac{1}{3}\left[\frac{6(5)}{32.2}\right](6)^2 + \frac{1}{12}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](2^2 + 3^2)$$

$$+ \frac{1}{12}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](1^2 + 2^2)$$

$$I_x = 16.9772 \text{ slug} \cdot \text{ft}^2$$

$$I_z = \frac{1}{3}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](2)^2 = 1.6563 \text{ slug} \cdot \text{ft}^2$$

$$I_{yz} = \left[\frac{2(5)}{32.2}\right](1)(2) + \left[\frac{2(5)}{32.2}\right](2)(3) = 2.4845 \text{ slug} \cdot \text{ft}^2 \quad I_{xy} = I_{zx} = 0$$

Applying Eq. 21-24 with $\omega_x = \omega_y = 0$ $\omega_z = 10$ rad/s $\dot{\omega}_x = \dot{\omega}_y = 0$

$-B_y(6) = 0 - 0 - 0 - 2.4845(0 - 10^2) - 0 \quad B_y = -41.4$ lb **Ans**

$B_x(6) = 0 - 0 - 2.4845\dot{\omega}_z - 0 - 0 \qquad (1)$

$50 = 1.6563\dot{\omega}_z \qquad (2)$

Solving Eqs. (1) and (2) yields:

$\dot{\omega}_z = 30.19$ rad/s^2

$B_x = -12.5$ lb **Ans**

$$\sum F_x = m(a_G)_x; \quad A_x + (-12.50) = -\left[\frac{2(5)}{32.2}\right](1)(30.19) - \left[\frac{2(5)}{32.2}\right](2)(30.19)$$

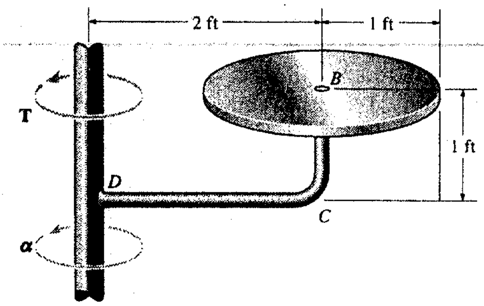
$A_x = -15.6$ lb **Ans**

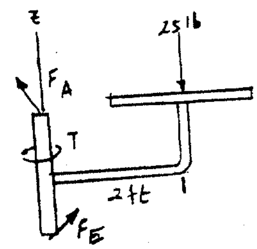
$$\sum F_y = m(a_G)_y; \quad A_y + (-41.4) = -\left[\frac{2(5)}{32.2}\right](1)(10)^2 - \left[\frac{2(5)}{32.2}\right](2)(10)^2$$

$A_y = -51.8$ lb **Ans**

$\sum F_z = m(a_G)_z; \quad A_z - 2(5) - 2(5) - 6(5) = 0 \quad A_z = 50$ lb **Ans**

***21-52.** The 25-lb disk is *fixed* to rod *BCD*, which has negligible mass. Determine the torque *T* which must be applied to the vertical shaft so that the shaft has an angular acceleration of $\alpha = 6\ \text{rad/s}^2$. The shaft is free to turn in its bearings.



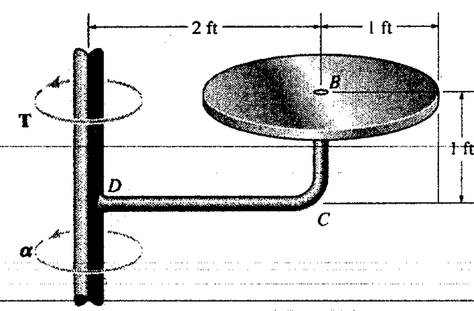


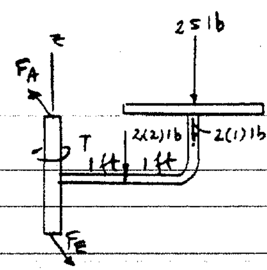
$I_z = \frac{1}{2}\left(\frac{25}{32.2}\right)(1)^2 + \left(\frac{25}{32.2}\right)(2)^2 = 3.4938\ \text{slug}\cdot\text{ft}^2$

Applying the third of Eq. 21-25 with $I_x = I_y$, $\omega_x = \omega_y = 0$, $\dot{\omega}_z = 6\ \text{rad/s}^2$

$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y;\qquad T = 3.4938(6) = 21.0\ \text{lb}\cdot\text{ft}$ **Ans**

21-53. Solve Prob. 21-52, assuming rod *BCD* has a weight of 2 lb/ft.



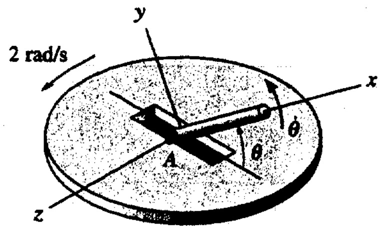


$I_z = \frac{1}{2}\left(\frac{25}{32.2}\right)(1)^2 + \left(\frac{25}{32.2}\right)(2)^2 + \frac{1}{3}\left(\frac{2(2)}{32.2}\right)(2)^2 + \left(\frac{1(2)}{32.2}\right)(2)^2 = 3.9079\ \text{slug}\cdot\text{ft}^2$

Applying the third of Eq. 21-25 with $I_x = I_y$, $\omega_x = \omega_y = 0$, $\dot{\omega}_z = 6\ \text{rad/s}^2$

$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y;\qquad T = 3.9079(6) = 23.4\ \text{lb}\cdot\text{ft}$ **Ans**

21-54. The *thin rod* has a mass of 0.8 kg and a total length of 150 mm. It is rotating about its midpoint at a constant rate $\dot{\theta} = 6$ rad/s, while the table to which its axle A is fastened is rotating at 2 rad/s. Determine the x, y, z moment components which the axle exerts on the rod when the rod is in any position θ.

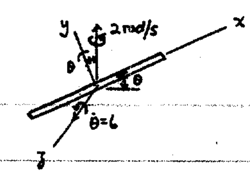


The x, y, z *kaxes* are fixed as shown.

$\omega_x = 2\sin\theta$

$\omega_y = 2\cos\theta$

$\omega_z = \dot{\theta} = 6$



$\dot{\omega}_x = 2\dot{\theta}\cos\theta = 12\cos\theta$

$\dot{\omega}_y = -2\dot{\theta}\sin\theta = -12\sin\theta$

$\dot{\omega}_z = 0$

$I_x = 0$

$I_y = I_z = \frac{1}{12}(0.8)(0.15)^2 = 1.5(10^{-3})$

Using Eqs. 21-25 :

$\Sigma M_x = 0 - 0 = 0$ **Ans**

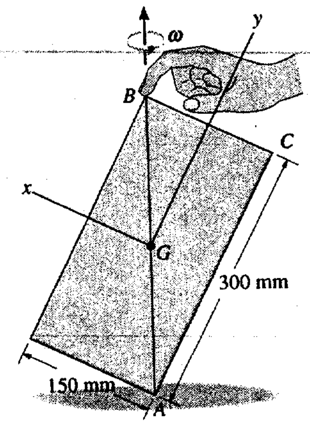
$\Sigma M_y = 1.5(10^{-3})(-12\sin\theta) - [1.5(10^{-3}) - 0](6)(2\sin\theta)$

$\Sigma M_y = (-0.036\sin\theta)$ N·m **Ans**

$\Sigma M_z = 0 - [0 - 1.5(10^{-3})](2\sin\theta)(2\cos\theta)$

$\Sigma M_z = 0.006\sin\theta\cos\theta = (0.003\sin 2\theta)$ N·m **Ans**

21-55. A thin uniform plate having a mass of 0.4 kg is spinning with a constant angular velocity $\boldsymbol{\omega}$ about its diagonal AB. If the person holding the corner of the plate at B releases his finger, the plate will fall downward on its side AC. Determine the necessary couple moment M which if applied to the plate would prevent this from happening.

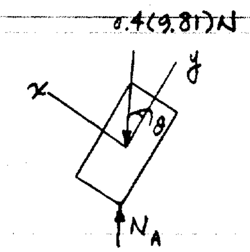


Using the principal axis shown,

$$I_x = \frac{1}{12}(0.4)(0.3)^2 = 3\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{12}(0.4)(0.15)^2 = 0.75\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{12}(0.4)\left[(0.3)^2 + (0.15)^2\right] = 3.75\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$$



$$\theta = \tan^{-1}\left(\frac{75}{150}\right) = 26.57^\circ$$

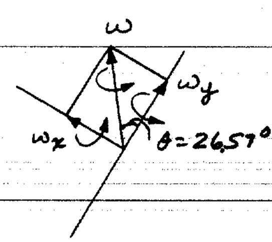
$\omega_x = \omega\sin 26.57^\circ, \qquad \dot{\omega}_x = 0$

$\omega_y = \omega\cos 26.57^\circ, \qquad \dot{\omega}_y = 0$

$\omega_z = 0, \qquad \dot{\omega}_z = 0$

$$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z$$

$$M_x = 0$$



$$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x$$

$$M_y = 0$$

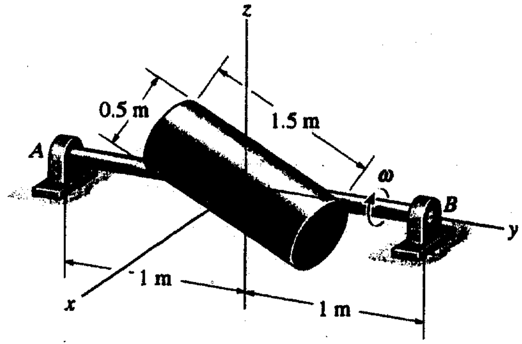
$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y$$

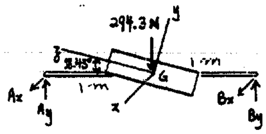
$$M_z = 0 - \left[3\left(10^{-3}\right) - 0.75\left(10^{-3}\right)\right]\omega^2\sin 26.57^\circ\cos 26.57^\circ$$

$$M_z = -0.9\left(10^{-3}\right)\omega^2\ \text{N}\cdot\text{m} = -0.9\omega^2\ \text{mN}\cdot\text{m} \qquad \textbf{Ans}$$

The couple acts outward, perpendicular to the face of the plate.

*21-56. The cylinder has a mass of 30 kg and is mounted on an axle that is supported by bearings at A and B. If the axle is turning at $\boldsymbol{\omega} = \{-40\mathbf{j}\}$ rad/s, determine the vertical components of force acting at the bearings at this instant.





$\omega_x = 0$

$\omega_y = -40\sin 18.43° = -12.65$ rad/s

$\omega_z = 40\cos 18.43° = 37.95$ rad/s

$\dot{\omega}_x = 0, \quad \dot{\omega}_y = 0, \quad \dot{\omega}_z = 0$

$(a_G)_X = (a_G)_Y = (a_G)_Z = 0$

$I_x = I_y = \frac{1}{12}(30)[3(0.25)^2 + (1.5)^2] = 6.09375 \text{ kg}\cdot\text{m}^2$

$I_z = \frac{1}{2}(30)(0.25)^2 = 0.9375 \text{ kg}\cdot\text{m}^2$

Using the first of Eqs. 21–25,

$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z$

$B_Y(1) - A_Y(1) = 0 - (6.09375 - 0.9375)(-12.65)(37.95)$

$B_Y - A_Y = 2475.348$

$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x$

$A_X(1\cos 18.43°) - B_X(1\cos 18.43°) = 0 - 0$

$A_X = B_X$

$\Sigma F_X = m(a_G)_X; \quad A_X + B_X = 0$

$\Sigma F_Y = m(a_G)_Y; \quad A_Y + B_Y = 0$

$A_X = -B_X$

~~Also, summing forces in the vertical direction~~

$\Sigma F_Y = m(a_G)_Y; \quad A_Y + B_Y - 294.3 = 0$

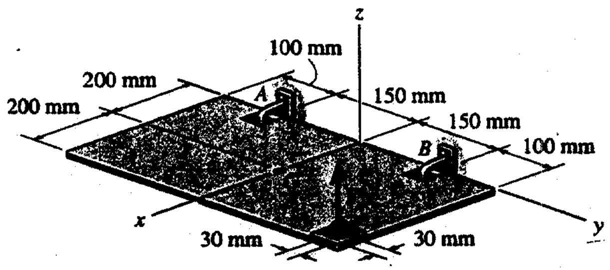
Solving,

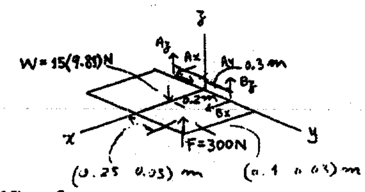
$A_X = B_X = 0$ Ans

$A_Y = -1.09$ kN Ans

$B_Y = 1.38$ kN Ans

21-57. The uniform hatch door, having a mass of 15 kg and a mass center at G, is supported in the horizontal plane by bearings at A and B. If a vertical force $F = 300$ N is applied to the door as shown, determine the components of reaction at the bearings and the angular acceleration of the door. The bearing at A will resist a component of force in the y direction, whereas the bearing at B will not. For the calculation, assume the door to be a thin plate and neglect the size of each bearing. The door is originally at rest.





$\omega_x = \omega_y = \omega_z = 0$

$\dot{\omega}_x = \dot{\omega}_z = 0$

Eqs. 21 – 25 reduce to

$\Sigma M_x = 0; \quad 300(0.25-0.03) + B_z(0.15) - A_z(0.15) = 0$

$B_z - A_z = -440 \qquad (1)$

$\Sigma M_y = I_y\,\dot{\omega}_y; \quad 15(9.81)(0.2) - (300)(0.4-0.03) = [\frac{1}{12}(15)(0.4)^2 + 15(0.2)^2]\dot{\omega}_y$

$\dot{\omega}_y = -102 \text{ rad/s}^2$ **Ans**

$\Sigma M_z = 0; \quad -B_x(0.15) + A_x(0.15) = 0$

$\Sigma F_x = m(a_G)_x; \quad -A_x + B_x = 0$

$A_x = B_x = 0$ **Ans**

$\Sigma F_y = m(a_G)_y; \quad A_y = 0$ **Ans**

$\Sigma F_z = m(a_G)_z; \quad 300 - 15(9.81) + B_z + A_z = 15(101.96)(0.2)$

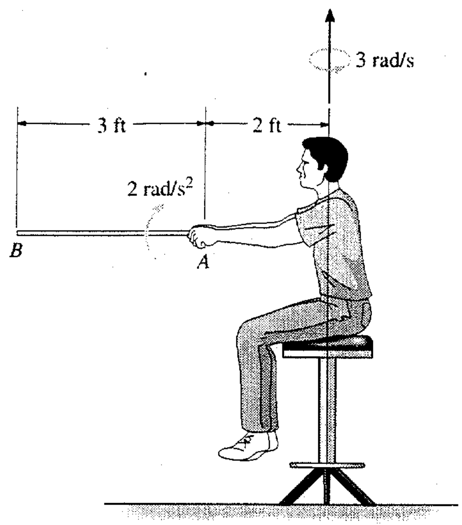
$B_z + A_z = 153.03 \qquad (2)$

Solving Eqs. (1) and (2) yields

$A_z = 297$ N **Ans**

$B_z = -143$ N **Ans**

21-58. The man sits on a swivel chair which is rotating with a constant angular velocity of 3 rad/s. He holds the uniform 5-lb rod AB horizontal. He suddenly gives it an angular acceleration of 2 rad/s^2, measured relative to him, as shown. Determine the required force and moment components at the grip, A, necessary to do this. Establish axes at the rod's center of mass G, with $+z$ upward, and $+y$ directed along the axis of the rod towards A.



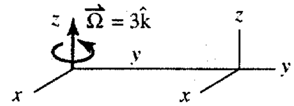
$$I_x = I_z = \frac{1}{12}\left(\frac{5}{32.2}\right)(3)^2 = 0.1165 \text{ ft}^4$$

$$I_y = 0$$

$$\Omega = \omega = 3\mathbf{k}$$

$$\omega_x = \omega_y = 0$$

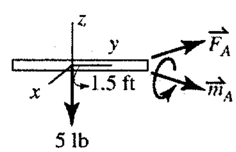
$$\omega_z = 3 \text{ rad/s}$$



$$\dot{\Omega} = (\dot{\omega}_{xyz}) + \Omega \times \omega = -2\mathbf{i} + 0$$

$$\dot{\omega}_x = -2 \text{ rad/s}^2$$

$$\dot{\omega}_y = \dot{\omega}_z = 0$$



$$(A_G)_x = 0$$

$$(A_G)_y = (3.5)(3)^2 = 31.5 \text{ ft/s}^2$$

$$(A_G)_z = 2(1.5) = 3 \text{ ft/s}^2$$

$\Sigma F_x = m(a_G)_x;\quad A_x = 0$ **Ans**

$\Sigma F_y = m(a_G)_y;\quad A_y = \dfrac{5}{32.2}(31.5) = 4.89 \text{ lb}$ **Ans**

$\Sigma F_z = m(a_G)_z;\quad -5 + A_z = \dfrac{5}{32.2}(3)$

$A_z = 5.47 \text{ lb}$ **Ans**

$$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z;$$

$$M_x + 5.47(1.5) = 0.1165(-2) - 0$$

$M_x = -8.43 \text{ lb}\cdot\text{ft}$ **Ans**

$$\Sigma M_y = I_y\dot{\omega}_x - (I_z - I_x)\omega_z\omega_x;$$

$$0 + M_y = 0 - 0$$

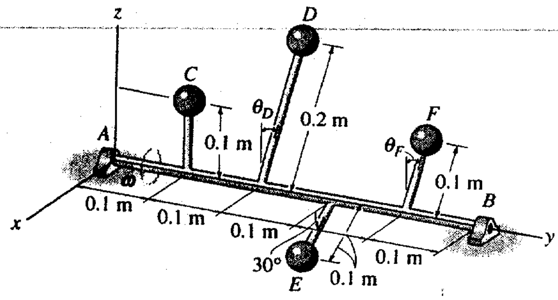
$M_y = 0$ **Ans**

$$\Sigma M_x = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y;$$

$$M_y = 0 - 0$$

$M_z = 0$ **Ans**

21-59. Four spheres are connected to shaft AB. If $m_C = 1$ kg and $m_E = 2$ kg, determine the mass of D and F and the angles of the rods, θ_D and θ_F, so that the shaft is dynamically balanced, that is, so that the bearings at A and B exert only vertical reactions on the shaft as it rotates. Neglect the mass of the rods.



For $\bar{x} = 0$; $\Sigma \bar{x}_i m_i = 0$

$(0.1\cos 30°)(2) - (0.1\sin\theta_F)m_F - (0.2\sin\theta_D)m_D = 0$ (1)

For $\bar{z} = 0$; $\Sigma \bar{z}_i m_i = 0$

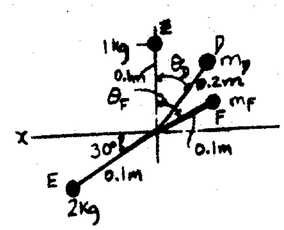
$(0.1)(1) - (0.1\sin 30°)(2) + (0.2\cos\theta_D)m_D + (0.1\cos\theta_F)m_F = 0$ (2)

For $I_{xy} = 0$; $\Sigma \bar{x}_i \bar{y}_i m_i = 0$

$-(0.2)(0.2\sin\theta_D)m_D + (0.3)(0.1\cos 30°)(2) - (0.4)(0.1\sin\theta_F)m_F = 0$ (3)

For $I_{zy} = 0$; $\Sigma \bar{z}_i \bar{y}_i m_i = 0$

$(0.1)(0.1)(1) + (0.2)(0.2\cos\theta_D)m_D - (0.3)(0.1\sin 30°)(2) + (0.1\cos\theta_F)(0.4)(m_F) = 0$ (4)



Solving,

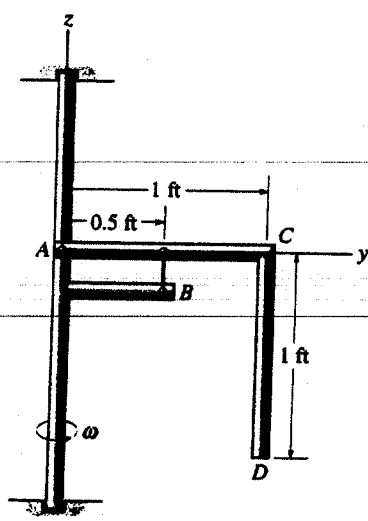
$\theta_D = 139°$ **Ans**

$m_D = 0.661$ kg **Ans**

$\theta_F = 40.9°$ **Ans**

$m_F = 1.32$ kg **Ans**

***21-60.** The bent uniform rod ACD has a weight of 5 lb/ft and is supported at A by a pin and at B by a cord. If the vertical shaft rotates with a constant angular velocity $\omega = 20$ rad/s, determine the x, y, z components of force and moment developed at A and the tension in the cord.



$\omega_x = \omega_y = 0$

$\omega_z = 20$ rad/s

$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$

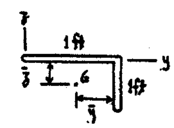
The center of mass is located at

$\bar{z} = \dfrac{5(1)(\frac{1}{2})}{5(2)} = 0.25$ ft

$\bar{y} = 0.25$ ft (symmetry)

$I_{yz} = \dfrac{5}{32.2}(-0.5)(1) = -0.0776$ slug·ft^2

$I_{xz} = 0$

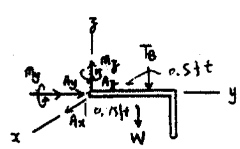


Eqs. 21 – 24 reduce to

$\Sigma M_x = I_{yz}(\omega_z)^2;$

$-T_B(0.5) - 10(0.75) = -0.0776(20)^2$

$T_B = 47.1$ lb **Ans**



$\Sigma M_y = 0;$ $M_y = 0$ **Ans**

$\Sigma M_z = 0;$ $M_z = 0$ **Ans**

$\Sigma F_x = ma_x;$ $A_x = 0$ **Ans**

$\Sigma F_y = ma_y;$ $A_y = -(\dfrac{10}{32.2})(20)^2(1-0.25)$

$A_y = -93.2$ lb **Ans**

$\Sigma F_z = ma_z;$ $A_z - 47.1 - 10 = 0$

$A_z = 57.1$ lb **Ans**

21-61. Show that the angular velocity of a body, in terms of Euler angles ϕ, θ, and ψ, may be expressed as $\omega = (\dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi)\mathbf{i} + (\dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi)\mathbf{j} + (\dot{\phi}\cos\theta + \dot{\psi})\mathbf{k}$, where **i**, **j**, and **k** are directed along the x, y, z axes as shown in Fig. 21-15*d*.

From Fig. 21-15*b*, due to rotation ϕ, the x, y, z components of $\dot{\phi}$ are simply $\dot{\phi}$ along z axis.

From Fig. 21-15*c*, due to rotation θ, the x, y, z components of $\dot{\phi}$ and $\dot{\theta}$ are $\dot{\phi}\sin\theta$ in the y direction, $\dot{\phi}\cos\theta$ in the z direction, and $\dot{\theta}$ in the x direction.

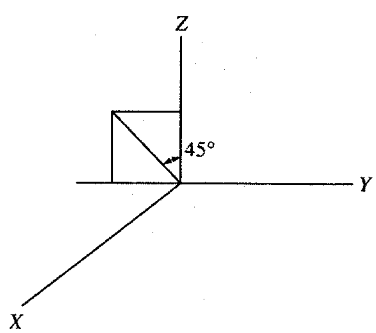
Lastly, rotation ψ, Fig. 21-15*d*, produces the final components which yields

$$\omega = (\dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi)\mathbf{i} + (\dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi)\mathbf{j}$$

$$+ (\dot{\phi}\cos\theta + \dot{\psi})\mathbf{k} \qquad \textbf{Q.E.D.}$$

21-62. A thin rod is initially coincident with the Z axis when it is given three rotations defined by the Euler angles $\phi = 30°, \theta = 45°$, and $\psi = 60°$. If these rotations are given in the order stated, determine the coordinate direction angles α, β, γ of the axis of the rod with respect to the X, Y, and Z axes. Are these directions the same for any order of the rotations? Why?

The rotations $\phi = 30°$ and $\psi = 60°$ do not change the orientation of the rod since it causes the rod to rotate about its axis only.



$$\mathbf{u} = \cos 90°\mathbf{i} + \cos 135°\mathbf{j} + \cos 45°\mathbf{k}$$

$$= -0.707\mathbf{j} + 0.707\mathbf{k}$$

Coordinate direction angles:

$\alpha = \cos^{-1}(0) = 90°$ **Ans**

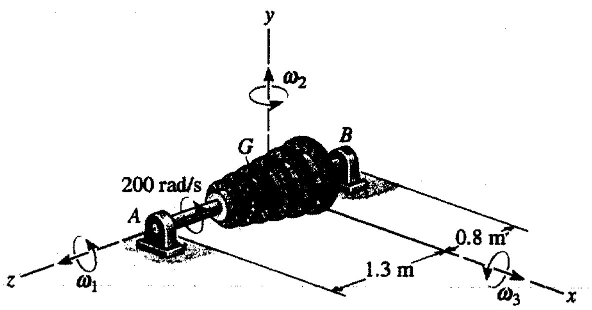
$\beta = \cos^{-1}(-0.707) = 135°$ **Ans**

$\gamma = \cos^{-1}(0.707) = 45°$ **Ans**

The orientation of the rod will not be the same for any order of rotation because finite rotations are not vectors.

No **Ans**

21-63. The turbine on a ship has a mass of 400 kg and is mounted on bearings A and B as shown. Its center of mass is at G, its radius of gyration is $k_z = 0.3$ m, and $k_x = k_y = 0.5$ m. If it is spinning at 200 rad/s, determine the vertical reactions at the bearings when the ship undergoes each of the following motions: (a) rolling, $\omega_1 = 0.2$ rad/s, (b) turning, $\omega_2 = 0.8$ rad/s, (c) pitching, $\omega_3 = 1.4$ rad/s.

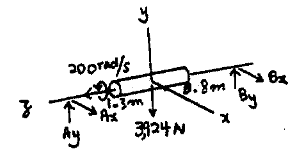


a) $\omega_z = 0.2 + 200 = 200.2$ rad/s

$\Sigma F_y = m(a_G)_y; \quad A_y + B_y - 3924 = 0$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z;$

$B_y(0.8) - A_y(1.3) = 0 - 0$



Thus,

$A_y = 1.49$ kN Ans

$B_y = 2.43$ kN Ans

b) $\Omega = 0.8\mathbf{j}$

$\omega = 0.8\mathbf{j} + 200\mathbf{k}$

$\dot{\omega} = 0 + 0.8\mathbf{j} \times (0.8\mathbf{j} + 200\mathbf{k}) = 160\mathbf{i}$

$\Sigma F_y = m(a_G)_y; \quad A_y + B_y - 3924 = 0$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z;$

$B_y(0.8) - A_y(1.3) = 400(0.5)^2(160) - [400(0.5)^2 - 400(0.3)^2](0.8)(200)$

$B_y(0.8) - A_y(1.3) = 5760$

Thus,

$A_y = -1.24$ kN Ans

$B_y = 5.17$ kN Ans

c) $\Omega = 1.4\mathbf{i}$

$\omega = 1.4\mathbf{i} + 200\mathbf{k}$

$\dot{\omega} = 1.4\mathbf{i} \times (1.4\mathbf{i} + 200\mathbf{k}) = -280\mathbf{j}$

$\Sigma F_y = m(a_G)_y; \quad A_y + B_y - 3924 = 0$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z;$

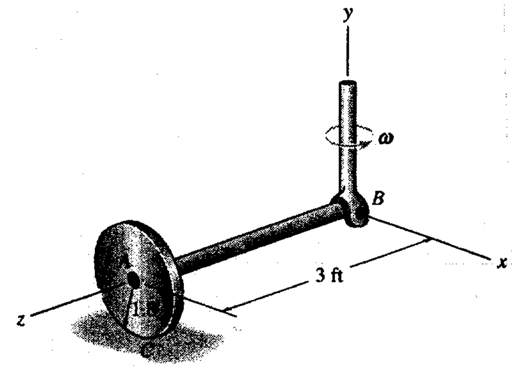
$B_y(0.8) - A_y(1.3) = 0 - 0$

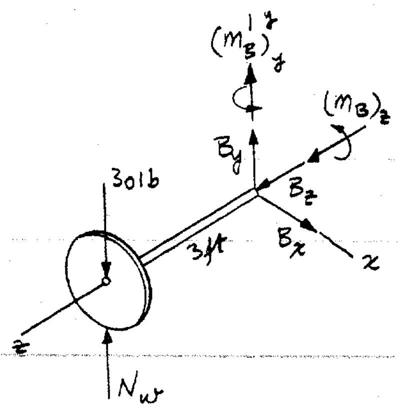
Thus,

$A_y = 1.49$ kN Ans

$B_y = 2.43$ kN Ans

*21-64. The 30-lb wheel rolls without slipping. If it has a radius of gyration $k_{AB} = 1.2$ ft about its axle AB, and the vertical drive shaft is turning at 8 rad/s, determine the normal reaction the wheel exerts on the ground at C.





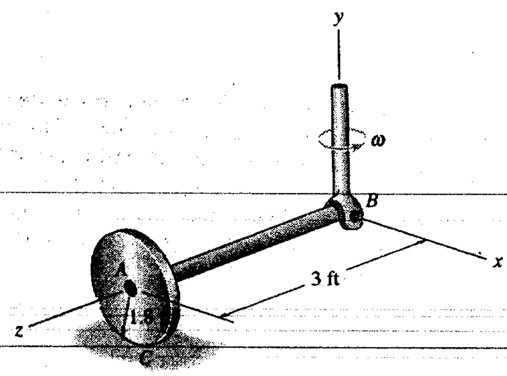
$\Omega_y = \omega = 8$ rad/s

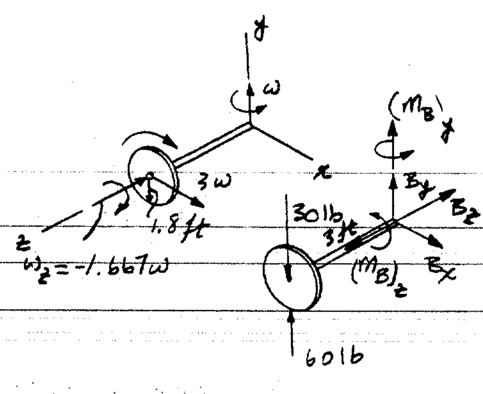
$$\omega_z = -\frac{3(8)}{1.8} = -13.33 \text{ rad/s}$$

$$\Sigma M_x = I_z \Omega_y \omega_z; \quad 30(3) - N_w(3) = \left[\left(\frac{30}{32.2}\right)(1.2)^2\right](8)(-13.33)$$

$N_w = 77.7$ lb **Ans**

21-65. The 30-lb wheel rolls without slipping. If it has a radius of gyration $k_{AB} = 1.2$ ft about its axle AB, determine its angular velocity ω so that the normal reaction at C becomes 60 lb.

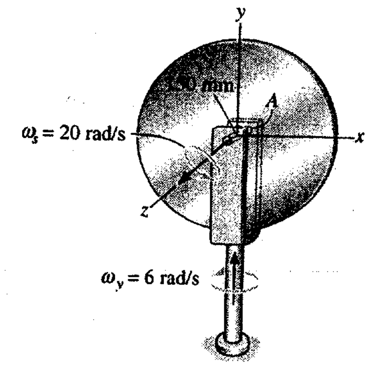


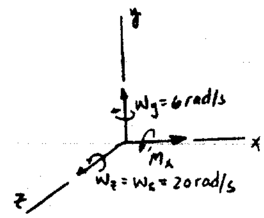


$$\Sigma M_x = I_z \Omega_y \omega_z; \quad 30(3) - 60(3) = \left[\frac{30}{32.2}(1.2)^2\right]\omega(-1.667\omega)$$

$\omega = 6.34$ rad/s **Ans**

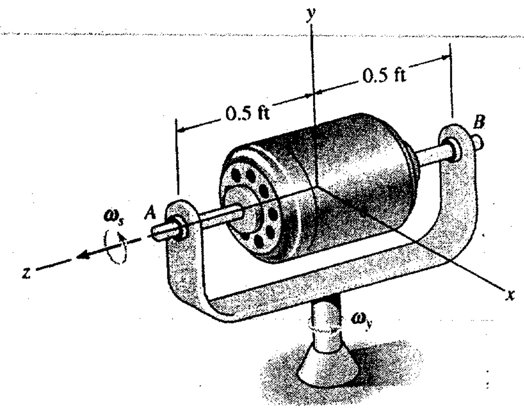
21-66. The 20-kg disk is spinning about its center at $\omega_s = 20$ rad/s while the supporting axle is rotating at $\omega_y = 6$ rad/s. Determine the gyroscopic moment caused by the force reactions which the pin A exerts on the disk due to the motion.





$\Sigma M_x = I_z \Omega_y \omega_z;$ $\quad M_x = \left[\frac{1}{2}(20)(0.15)^2\right](6)(20) = 27.0$ N · m $\quad$ **Ans**

21-67. The motor weighs 50 lb and has a radius of gyration of 0.2 ft about the z axis. The shaft of the motor is supported by bearings at A and B, and is turning at a constant rate of $\boldsymbol{\omega}_s = \{100\mathbf{k}\}$ rad/s, while the frame has an angular velocity of $\boldsymbol{\omega}_y = \{2\mathbf{j}\}$ rad/s. Determine the moment which the bearing forces at A and B exert on the shaft due to this motion.

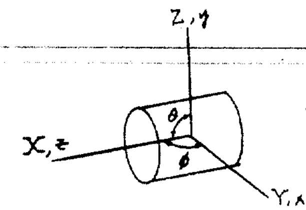


Applying Eq. 21-30 : For the coordinate system shown $\theta = 90°$ $\phi = 90°$

$\dot\theta = 0$ $\dot\phi = 2$ rad/s $\dot\psi = 100$ rad/s .

$$\Sigma M_x = -I\dot\phi^2 \sin\theta\cos\theta + I_z\dot\phi\sin\theta(\dot\phi\cos\theta + \dot\psi) \quad \text{reduces to}$$

$$\Sigma M_x = I_z\dot\phi\dot\psi; \quad M_x = \left[\left(\frac{50}{32.2}\right)(0.2)^2\right](2)(100) = 12.4 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

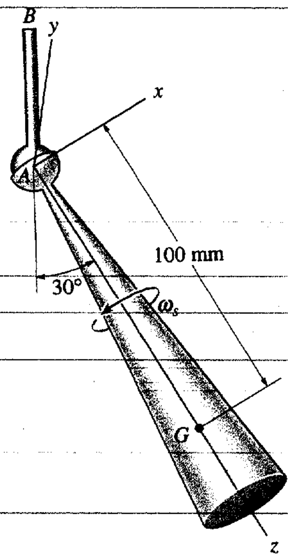


Since $\omega_x = 0$

$\Sigma M_y = 0;$ $M_y = 0$ **Ans**

$\Sigma M_z = 0;$ $M_z = 0$ **Ans**

***21-68.** The conical top has a mass of 0.8 kg, and the moments of inertia are $I_x = I_y = 3.5(10^{-3})$ kg·m² and $I_z = 0.8(10^{-3})$ kg·m². If it spins freely in the ball-and-socket joint at A with an angular velocity $\omega_s = 750$ rad/s, compute the precession of the top about the axis of the shaft AB.



$\omega_s = 750$ rad/s

Using Eq. 21 – 30.

$$\Sigma M_x = -I\dot\phi^2 \sin\theta\cos\theta + I_z\dot\phi\sin\theta(\dot\phi\cos\theta + \dot\phi)$$

$$0.1(0.8)(9.81)\sin 30° = -3.5(10^{-3})\dot\phi^2 \sin 30°\cos 30° + 0.8(10^{-3})\dot\phi\sin 30°(\dot\phi\cos 30° + 750)$$

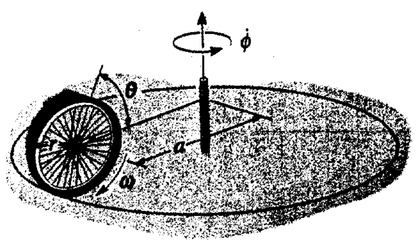
Thus,

$$1.160(10^{-3})\dot\phi^2 - 300(10^{-3})\dot\phi + 0.3924 = 0$$

$\dot\phi = 1.31$ rad/s (low precession) **Ans**

$\dot\phi = 255$ rad/s (high precession) **Ans**

21-69. A wheel of mass m and radius r rolls with constant spin ω about a circular path having a radius a. If the angle of inclination is θ, determine the rate of precession. Treat the wheel as a thin ring. No slipping occurs.

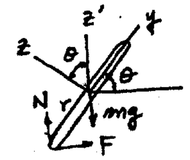


Since no slipping occurs,

$$(r)\dot{\psi} = (a + r\cos\theta)\dot{\phi}$$

or

$$\dot{\psi} = \left(\frac{a + r\cos\theta}{r}\right)\dot{\phi} \qquad (1)$$



Also,

$$\omega = \dot{\phi} + \dot{\psi}$$

$$\Sigma F_{y'} = m(a_G)_{y'}; \qquad F = m(a\dot{\phi}^2) \qquad (2)$$

$$\Sigma F_{z'} = m(a_G)_{z'}; \qquad N - mg = 0 \qquad (3)$$

$$I_x = I_y = \frac{mr^2}{2}, \qquad I_z = mr^2$$

$$\omega = \dot{\phi}\sin\theta\,\mathbf{j} + (-\dot{\psi} + \dot{\phi}\cos\theta)\mathbf{k}$$

Thus,

$$\omega_x = 0, \qquad \omega_y = \dot{\phi}\sin\theta, \qquad \omega_z = -\dot{\psi} + \dot{\phi}\cos\theta$$

$$\dot{\omega} = \dot{\phi} \times \dot{\psi} = -\dot{\phi}\dot{\psi}\sin\theta$$

$$\dot{\omega}_x = -\dot{\phi}\dot{\psi}\sin\theta, \qquad \dot{\omega}_y = \dot{\omega}_z = 0$$

Applying

$$\Sigma M_x = I_x\dot{\omega}_x + (I_z - I_y)\omega_z\omega_y$$

$$F r\sin\theta - N r\cos\theta = \frac{mr^2}{2}(-\dot{\phi}\dot{\psi}\sin\theta) + \left(mr^2 - \frac{mr^2}{2}\right)(-\dot{\psi} + \dot{\phi}\cos\theta)(\dot{\phi}\sin\theta)$$

Using Eqs. (1), (2) and (3), and eliminating $\dot{\psi}$, we have

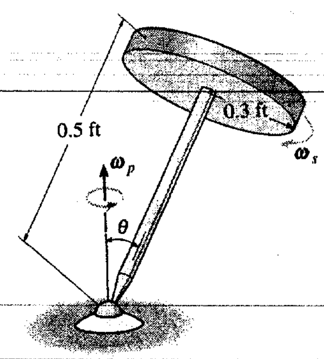
$$ma\dot{\phi}^2 r\sin\theta - mgr\cos\theta = \frac{mr^2}{2}(-\dot{\phi})\sin\theta\left(\frac{a + r\cos\theta}{r}\right)\dot{\phi} + \frac{mr^2}{2}\left(\frac{-\dot{\phi}a}{r}\right)\dot{\phi}\sin\theta$$

$$ma\dot{\phi}^2\sin\theta r - mgr\cos\theta = \frac{mr^2}{2}\left(\frac{-\dot{\phi}^2 a}{r}\right)\sin\theta - \frac{mr^2}{2}(\dot{\phi}^2\sin\theta\cos\theta)$$

$$2g\cos\theta = a\dot{\phi}^2\sin\theta + r\dot{\phi}^2\sin\theta\cos\theta$$

$$\dot{\phi} = \left(\frac{2g\,\text{ctn}\,\theta}{a + r\cos\theta}\right)^{1/2} \qquad \textbf{Ans}$$

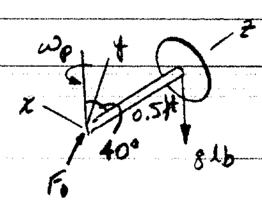
21-70. The top consists of a thin disk that has a weight of 8 lb and a radius of 0.3 ft. The rod has a negligible mass and a length of 0.5 ft. If the top is spinning with an angular velocity $\omega_s = 300$ rad/s, determine the steady-state precessional angular velocity ω_p of the rod when $\theta = 40°$.



$$\Sigma M_x = -I\dot{\phi}^2\sin\theta\cos\theta + I_z\dot{\phi}\sin\theta\left(\dot{\phi}\cos\theta + \dot{\psi}\right)$$

$$8(0.5\sin 40°) = -\left[\frac{1}{4}\left(\frac{8}{32.2}\right)(0.3)^2 + \left(\frac{8}{32.2}\right)(0.5)^2\right]\omega_p^2\sin 40°\cos 40°$$

$$+\left[\frac{1}{2}\left(\frac{8}{32.2}\right)(0.3)^2\right]\omega_p\sin 40°(\omega_p\cos 40° + 300)$$

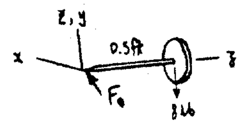


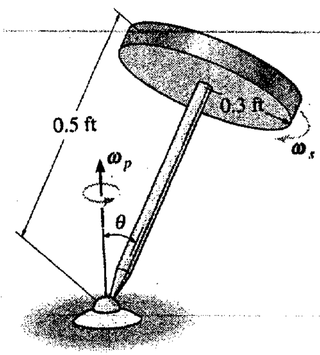
$$0.02783\omega_p^2 - 2.1559\omega_p + 2.571 = 0$$

$\omega_p = 1.21$ rad/s **Ans** (Low precession)

$\omega_p = 76.3$ rad/s **Ans** (High precession)

21-71. Solve Prob. 21-70 when $\theta = 90°$.



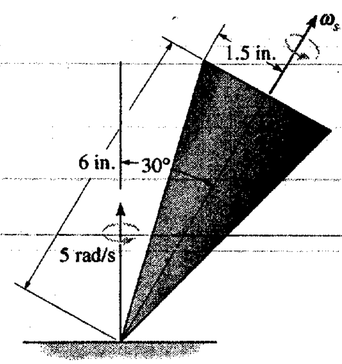


$$\Sigma M_x = I_z \Omega_y \omega_z$$

$$8(0.5) = \left[\frac{1}{2}\left(\frac{8}{32.2}\right)(0.3)^2\right]\omega_p\,(300)$$

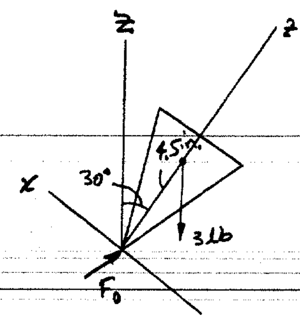
$\omega_p = 1.19$ rad/s **Ans**

***21-72.** The top has a mass of 3 lb and can be considered as a solid cone. If it is observed to precess about the vertical axis at a constant rate of 5 rad/s, determine its spin.



$$I = \frac{3}{80}\left(\frac{3}{32.2}\right)\left[4\left(\frac{1.5}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right] + \frac{3}{32.2}\left(\frac{4.5}{12}\right)^2 = 0.01419 \text{ slug}\cdot\text{ft}^2$$

$$I_z = \frac{3}{10}\left(\frac{3}{32.2}\right)\left(\frac{1.5}{12}\right)^2 = 0.43672(10^{-3}) \text{ slug}\cdot\text{ft}^2$$

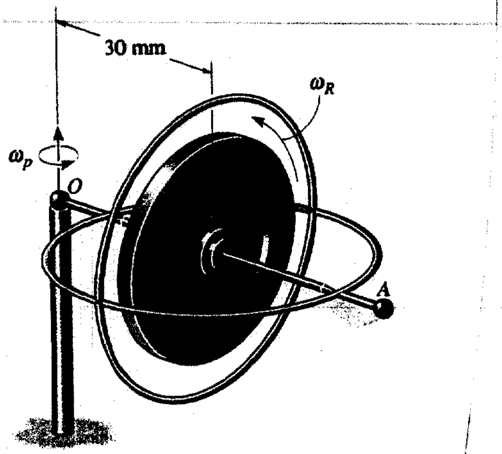


$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta\left(\dot{\phi}\cos\theta + \dot{\psi}\right)$$

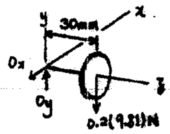
$$(3)\left(\frac{4.5}{12}\right)(\sin 30°) = -(0.01419)(5)^2 \sin 30° \cos 30° + 0.43672\left(10^{-3}\right)(5)\sin 30°\left(5\cos 30° + \dot{\psi}\right)$$

$\dot{\psi} = 652$ rad/s **Ans**

21-73. The toy gyroscope consists of a rotor R which is attached to the frame of negligible mass. If it is observed that the frame is precessing about the pivot point O at $\omega_p = 2$ rad/s, determine the angular velocity ω_R of the rotor. The stem OA moves in the horizontal plane. The rotor has a mass of 200 g and a radius of gyration $k_{OA} = 20$ mm about OA.



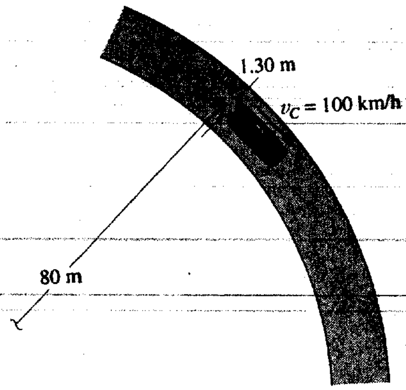
$$\Sigma M_x = I_z \Omega_y \omega_z$$



$$(0.2)(9.81)(0.03) = [0.2(0.02)^2](2)(\omega_R)$$

$\omega_R = 368$ rad/s **Ans**

21-74. The car is traveling at $v_C = 100$ km/h around the horizontal curve having a radius of 80 m. If each wheel has a mass of 16 kg, a radius of gyration $k_G = 300$ mm about its spinning axis, and a radius of 400 mm, determine the difference between the normal forces of the rear wheels, caused by the gyroscopic effect. The distance between the wheels is 1.30 m.



$$I = 2[16(0.3)^2] = 2.88 \text{ kg}\cdot\text{m}^2$$

$$\omega_s = \frac{100(1000)}{3600(0.4)} = 69.44 \text{ rad/s}$$

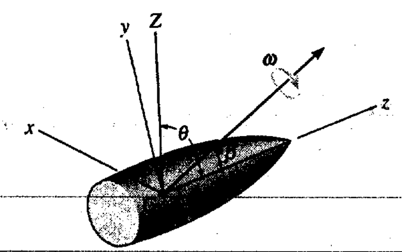
$$\omega_p = \frac{100(1000)}{80(3600)} = 0.347 \text{ rad/s}$$

$$M = I\,\omega_s\,\omega_p$$

$$\Delta F(1.30) = 2.88(69.44)(0.347)$$

$\Delta F = 53.4$ N **Ans**

21-75. The projectile shown is subjected to torque-free motion. The transverse and axial moments of inertia are I and I_z, respectively. If θ represents the angle between the precessional axis Z and the axis of symmetry z, and β is the angle between the angular velocity ω and the z axis, show that β and θ are related by the equation $\tan\theta = (I/I_z)\tan\beta$.



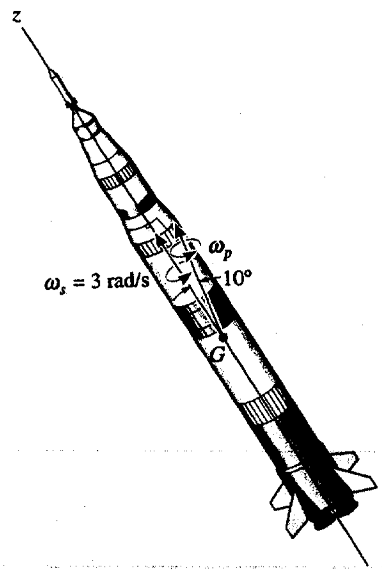
From Eq. 21-34 $\omega_y = \dfrac{H_G \sin\theta}{I}$ and $\omega_z = \dfrac{H_G\cos\theta}{I_z}$ Hence $\dfrac{\omega_y}{\omega_z} = \dfrac{I_z}{I}\tan\theta$

However, $\omega_y = \omega\sin\beta$ and $\omega_z = \omega\cos\beta$

$$\frac{\omega_y}{\omega_z} = \tan\beta = \frac{I_z}{I}\tan\theta$$

$\tan\theta = \dfrac{I}{I_z}\tan\beta$ **Q.E.D.**

***21-76.** While the rocket is in free flight, it has a spin of 3 rad/s and precesses about an axis measured 10° from the axis of spin. If the ratio of the axial to transverse moments of inertia of the rocket is 1/15, computed about axes which pass through the mass center G, determine the angle which the resultant angular velocity makes with the spin axis. Construct the body and space cones used to describe the motion. Is the precession regular or retrograde?



Determine the angle β from the result of Prob. 21-75.

$$\tan\theta = \frac{I}{I_z}\tan\beta$$

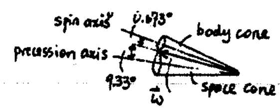
$$\tan 10° = \frac{15}{1}\tan\beta$$

$\beta = 0.673°$ **Ans**

Thus,

$$\alpha = 10° - 0.673° = 9.33°$$

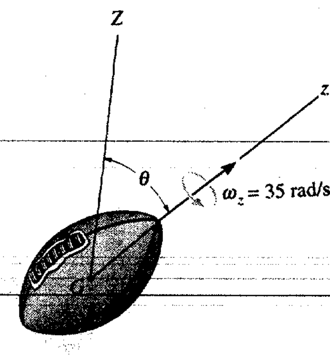
Hence,



Regular Precession **Ans**

Since $I_z < I$.

21-77. The 0.2-kg football is thrown with a spin ω_z = 35 rad/s. If the angle θ is measured as 60°, determine the precession about the Z axis



Gyroscopic Motion : Here, the spinning angular velocity $\dot{\psi} = \omega_z = 35$ rad/s. The moment inertia of the football about the z axis is $I_z = 0.2(0.05^2) = 0.5(10^{-3})$ kg · m² and the moment inertia of the satelite about its transverse axis is $I = 0.2(0.1^2) = 2(10^{-3})$ kg · m². Applying the third of Eq. 21 – 36 with $\theta = 60°$, we have

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_G \cos\theta$$

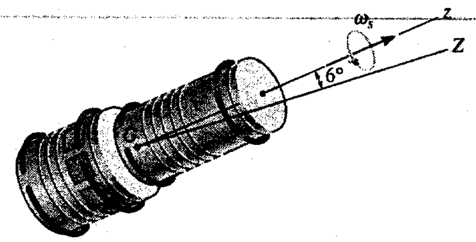
$$35 = \left[\frac{2(10^{-3}) - 0.5(10^{-3})}{2(10^{-3})0.5(10^{-3})}\right] H_G \cos 60°$$

$$H_G = 0.04667 \text{ kg} \cdot \text{m}^2/\text{s}$$

Applying the second of Eq. 21 – 36, we have

$$\dot{\phi} = \frac{H_G}{I} = \frac{004667}{2(10^{-3})} = 23.3 \text{ rad/s}$$ **Ans**

21-78. The space capsule has a mass of 3.2 Mg, and about axes passing through the mass center G the axial and transverse radii of gyration are $k_z = 0.90$ m and $k_t = 1.85$ m, respectively. If it is spinning at $\omega_s = 0.8$ rev/s, determine its angular momentum. Precession occurs about the Z axis.



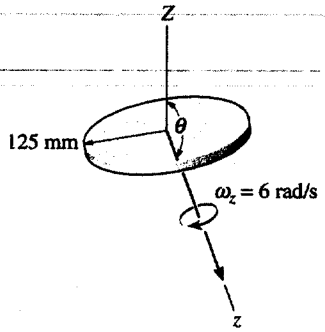
Gyroscopic Motion : Here, the spinning angular velocity $\dot{\psi} = \omega_s = 0.8(2\pi) = 1.6\pi$ rad/s. The moment of inertia of the satelite about the z axis is $I_z = 3200(0.9^2) = 2592$ kg·m^2 and the moment of inertia of the satelite about its transverse axis is $I = 3200(1.85^2) = 10\,952$ kg·m^2. Applying the third of Eq.21 – 36 with $\theta = 6°$, we have

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_G \cos\theta$$

$$1.6\pi = \left[\frac{10\,952 - 2592}{10\,952(2592)}\right] H_G \cos 6°$$

$$H_G = 17.16(10^3)\ \text{kg}\cdot\text{m}^2/\text{s} = 17.2\ \text{Mg}\cdot\text{m}^2/\text{s}$$ **Ans**

21-79. The 4-kg disk is thrown with a spin $\omega_z = 6$ rad/s. If the angle θ is measured as 160°, determine the precision about the Z axis.



$I = \frac{1}{4}(4)(0.125)^2 = 0.015625$ kg·m^2 $\quad I_z = \frac{1}{2}(4)(0.125)^2 = 0.03125$ kg·m^2

Applying Eq. 21-36 with $\theta = 160°$ $\dot{\psi} = 6$ rad/s

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_G \cos\theta$$

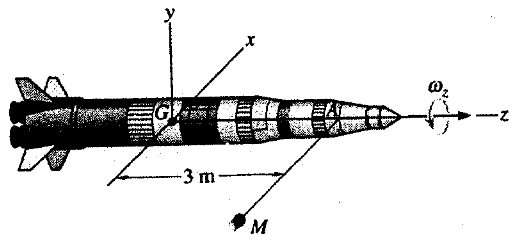
$$6 = \frac{0.015625 - 0.03125}{0.015625(0.03125)} H_G \cos 160°$$

$$H_G = 0.1995\ \text{kg.m}^2/\text{s}$$

$$\dot{\phi} = \frac{H_G}{I} = \frac{0.1995}{0.015625} = 12.8\ \text{rad/s}$$ **Ans**

Note that this is a case of retrograde precession since $I_z > I$.

***21-80.** The rocket has a mass of 4 Mg and radii of gyration $k_z = 0.85$ m and $k_y = 2.3$ m. It is initially spinning about the z axis at $\omega_z = 0.05$ rad/s when a meteoroid M strikes it at A and creates an impulse $\mathbf{I} = \{300\mathbf{i}\}$ N·s. Determine the axis of precession after the impact.



The impulse creates an angular momentum about the y axis of

$$H_y = 300(3) = 900 \text{ kg}\cdot\text{m}^2/\text{s}$$

Since

$$\omega_z = 0.05 \text{ rad/s},$$

then

$$\mathbf{H}_G = 900\mathbf{j} + \left[4000(0.85)^2\right](0.05)\mathbf{k} = 900\mathbf{j} + 144.5\mathbf{k}$$

The axis of precession is defined by $\mathbf{H}_G$.

$$\mathbf{u}_{H_G} = \frac{900\mathbf{j} + 144.5\mathbf{k}}{911.53} = 0.9874\mathbf{j} + 0.159\mathbf{k}$$

Thus,

$\alpha = \cos^{-1}(0) = 90°$ **Ans**

$\beta = \cos^{-1}(0.9874) = 9.12°$ **Ans**

$\gamma = \cos^{-1}(0.159) = 80.9°$ **Ans**