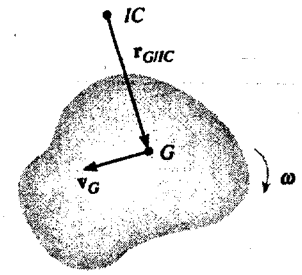
18-1. At a given instant the body of mass m has an angular velocity ω and its mass center has a velocity $\mathbf{v}_G$. Show that its kinetic energy can be represented as $T = \frac{1}{2}I_{IC}\omega^2$, where I_{IC} is the moment of inertia of the body computed about the instantaneous axis of zero velocity, located a distance $r_{G/IC}$ from the mass center as shown.



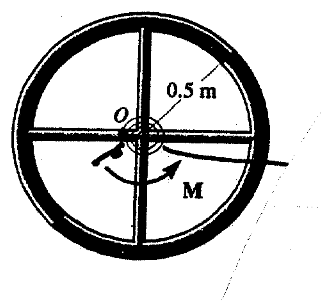
$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2 \qquad \text{where } v_G = \omega r_{G/IC}$$

$$= \frac{1}{2}m(\omega r_{G/IC})^2 + \frac{1}{2}I_G\omega^2$$

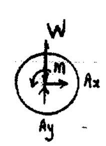
$$= \frac{1}{2}\left(mr_{G/IC}^2 + I_G\right)\omega^2 \qquad \text{However } mr_{G/IC}^2 + I_G = I_{IC}$$

$$= \frac{1}{2}I_{IC}\,\omega^2 \qquad \textbf{Q.E.D.}$$

18-2. The wheel is made from a 5-kg thin ring and two 2-kg slender rods. If the torsional spring attached to the wheel's center has a stiffness $k = 2\ \text{N}\cdot\text{m/rad}$, so that the torque on the center of the wheel is $M = (2\theta)\ \text{N}\cdot\text{m}$, where θ is in radians, determine the maximum angular velocity of the wheel if it is rotated two revolutions and then released from rest.



$$I_O = 2[\frac{1}{12}(2)(1)^2] + 5(0.5)^2 = 1.583$$



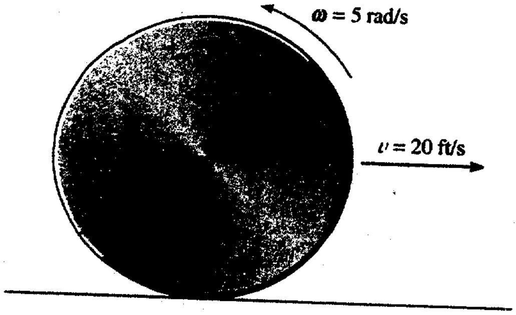
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{4\pi} 2\theta\, d\theta = \frac{1}{2}(1.583)\omega^2$$

$$(4\pi)^2 = 0.7917\omega^2$$

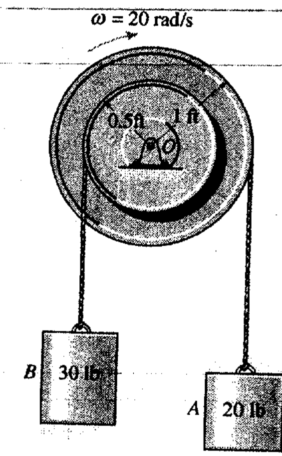
$\omega = 14.1$ rad/s **Ans**

18-3. At the instant shown, the 30-lb disk has a counterclockwise angular velocity of 5 rad/s when its center has a velocity of 20 ft/s. Determine the kinetic energy of the disk at this instant.



$$T = \frac{1}{2}\left[\frac{1}{2}\left(\frac{30}{32.2}\right)(2)^2\right](5)^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)(20)^2 = 210 \text{ ft}\cdot\text{lb} \qquad \text{Ans}$$

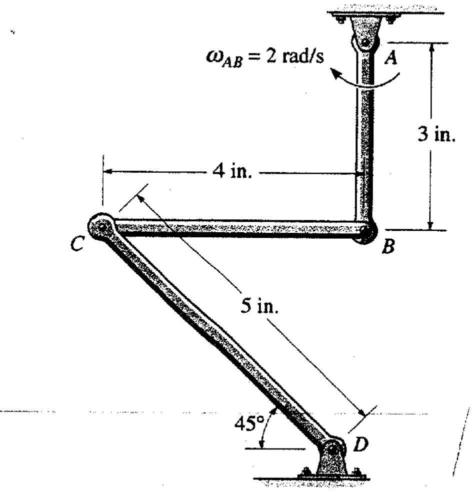
***18-4.** The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a centroidal radius of gyration of $k_O = 0.6$ ft and is turning with an angular velocity of 20 rad/s clockwise. Determine the kinetic energy of the system. Assume that neither cable slips on the pulley.



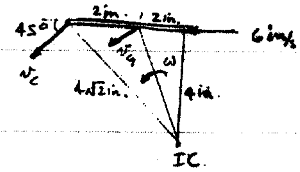
$$T = \frac{1}{2}I_O\,\omega_O^2 + \frac{1}{2}m_A\,v_A^2 + \frac{1}{2}m_B\,v_B^2$$

$$T = \frac{1}{2}\left(\frac{50}{32.2}(0.6)^2\right)(20)^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)\left[(20)(1)\right]^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)\left[(20)(0.5)\right]^2 = 283 \text{ ft}\cdot\text{lb} \qquad \text{Ans}$$

18-5. At the instant shown, link AB has an angular velocity $\omega_{AB} = 2$ rad/s. If each link is considered as a uniform slender bar with a weight of 0.5 lb/in., determine the total kinetic energy of the system.



$$\omega_{BC} = \frac{6}{4} = 1.5 \text{ rad/s}$$



$$v_C = 1.5(4\sqrt{2}) = 8.4853 \text{ in./s}$$

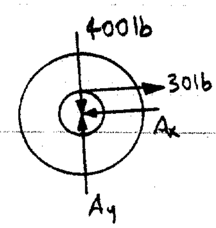
$$r_{IC-G} = \sqrt{(2)^2 + (4)^2} = 4.472$$

$$v_G = 1.5(4.472) = 6.7082 \text{ in./s}$$

$$\omega_{DC} = \frac{8.4853}{5} = 1.697 \text{ rad/s}$$

$$T = \frac{1}{2}\left[\frac{1}{3}\left(\frac{3(0.5)}{32.2}\right)\left(\frac{3}{12}\right)^2\right](2)^2 + \frac{1}{2}\left[\frac{4(0.5)}{32.2}\right]\left(\frac{6.7082}{12}\right)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{4(0.5)}{32.2}\right)\left(\frac{4}{12}\right)^2\right](1.5)^2$$
$$+ \frac{1}{2}\left[\frac{1}{3}\left(\frac{5(0.5)}{32.2}\right)\left(\frac{5}{12}\right)^2\right](1.697)^2 = 0.0188 \text{ ft}\cdot\text{lb} \qquad \textbf{Ans}$$

18-6. Solve Prob. 17-58 using the principle of work and energy.

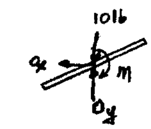


$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 30(8) = \frac{1}{2}\left[\left(\frac{400}{32.2}\right)(1.30)^2\right]\omega^2$$

$\omega = 4.78$ rad/s **Ans**

18-7. Solve Prob. 17–59 using the principle of work and energy.



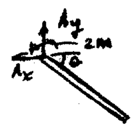
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{\pi/2} 5\theta\, d\theta = \frac{1}{2}\left[\frac{1}{12}\left(\frac{10}{32.2}\right)(2)^2\right]\omega^2$$

$$\frac{5}{2}\left(\frac{\pi}{2}\right)^2 = 0.05176\omega^2$$

$\omega = 10.9$ rad/s **Ans**

***18-8.** Solve Prob. 17–63 using the principle of work and energy.



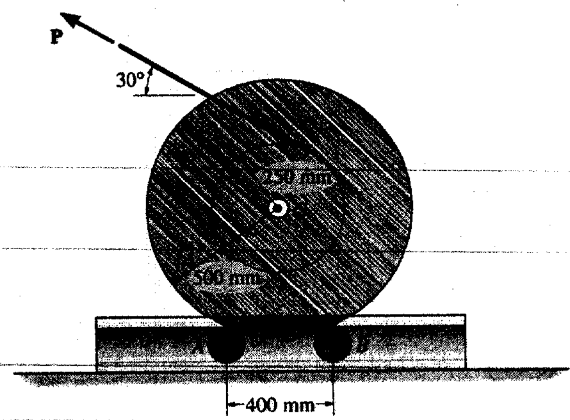
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 2\int_0^{\pi/2} 50\theta\, d\theta = \frac{1}{2}\left[\frac{1}{3}(70)(1.2)^2\right]\omega^2$$

$$100\left(\frac{\theta^2}{2}\right)\Big|_0^{\pi/2} = 16.8\,\omega^2$$

$\omega = 2.71$ rad/s **Ans**

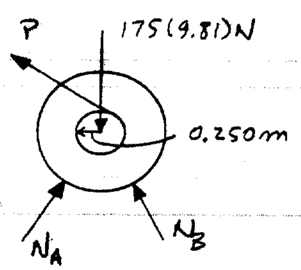
18-9. A force of $P = 20$ N is applied to the cable, which causes the 175-kg reel to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. The radius of gyration of the reel about its center axis is $k_G = 0.42$ m.



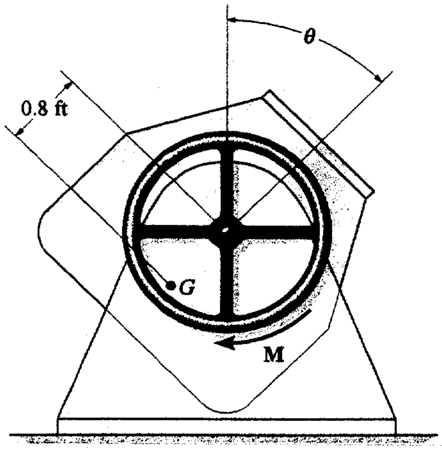
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 20(2)(2\pi)(0.250) = \frac{1}{2}\left[175(0.42)^2\right]\omega^2$$

$\omega = 2.02$ rad/s **Ans**

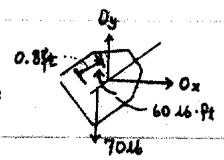


18-10. The tub of the mixer has a weight of 70 lb and a radius of gyration $k_G = 1.3$ ft about its center of gravity. If a constant torque $M = 60$ lb·ft is applied to the dumping wheel, determine the angular velocity of the tub when it has rotated $\theta = 90°$. Originally the tub is at rest when $\theta = 0°$.



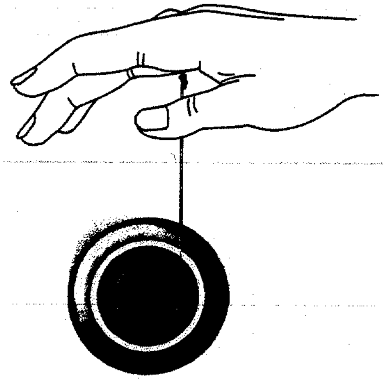
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 60\left(\frac{\pi}{2}\right) - 70(0.8) = \frac{1}{2}\left[\left(\frac{70}{32.2}\right)(1.3)^2\right](\omega)^2 + \frac{1}{2}\left[\frac{70}{32.2}\right](0.8\omega)^2$$



$\omega = 3.89$ rad/s Ans

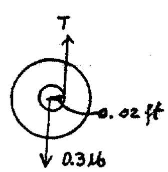
18-11. A yo-yo has a weight of 0.3 lb and a radius of gyration $k_O = 0.06$ ft. If it is released from rest, determine how far it must descend in order to attain an angular velocity $\omega = 70$ rad/s. Neglect the mass of the string and assume that the string is wound around the central peg such that the mean radius at which it unravels is $r = 0.02$ ft.



$v_G = (0.02)70 = 1.40$ ft/s

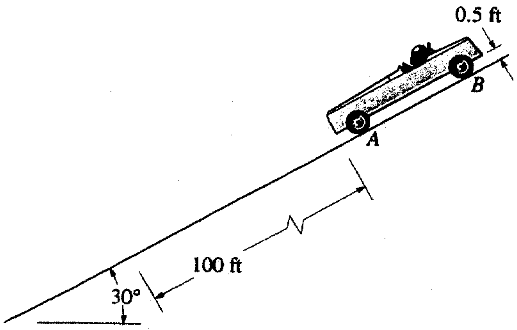
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (0.3)(s) = \frac{1}{2}\left(\frac{0.3}{32.2}\right)(1.40)^2 + \frac{1}{2}\left[(0.06)^2\left(\frac{0.3}{32.2}\right)\right](70)^2$$



$s = 0.304$ ft Ans

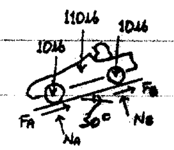
***18-12.** The soap-box car has a weight of 110 lb, including the passenger but *excluding* its four wheels. Each wheel has a weight of 5 lb, radius of 0.5 ft, and a radius of gyration $k = 0.3$ ft, computed about an axis passing through the wheel's axle. Determine the car's speed after it has traveled 100 ft starting from rest. The wheels roll without slipping. Neglect air resistance.



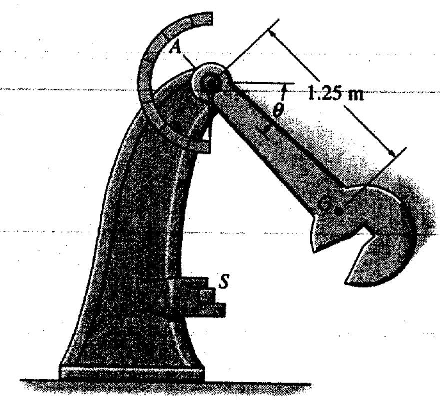
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 130(100\sin 30°) = 4\{\frac{1}{2}(\frac{5}{32.2})(v_C)^2 + \frac{1}{2}[(\frac{5}{32.2})(0.3)^2](\frac{v_C}{0.5})^2\} + \frac{1}{2}(\frac{110}{32.2})(v_C)^2$$

$v_C = 55.2$ ft/s **Ans**



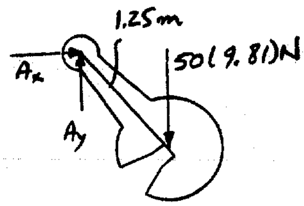
18-13. The pendulum of the Charpy impact machine has a mass of 50 kg and a radius of gyration of $k_A = 1.75$ m. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen S, $\theta = 90°$.



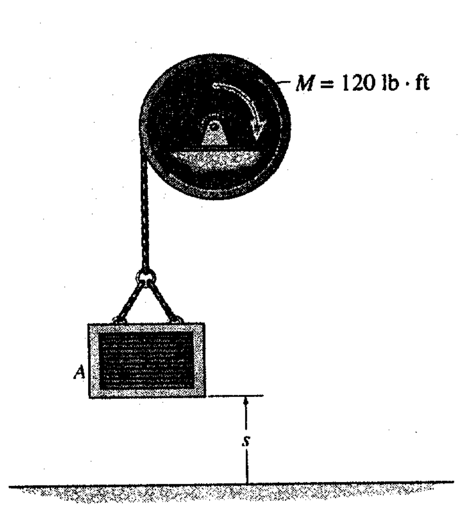
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (50)(9.81)(1.25) = \frac{1}{2}[(50)(1.75)^2]\omega_2^2$$

$\omega_2 = 2.83$ rad/s **Ans**

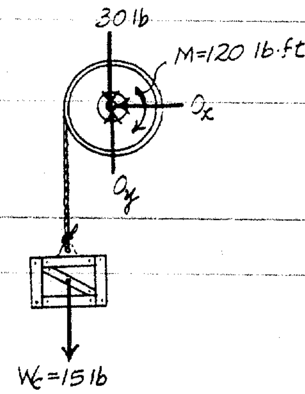


18-14. A motor supplies a constant torque or twist of $M = 120$ lb · ft to the drum. If the drum has a weight of 30 lb and a radius of gyration of $k_O = 0.8$ ft, determine the speed of the 15-lb crate A after it rises $s = 4$ ft starting from rest. Neglect the mass of the cord.



Free Body Diagram : The weight of the crate does *negative* work since it acts in the opposite direction to that of its displacement s_W. Also, the couple moment **M** does positive work as it acts in the same direction of its angular displacement θ. The reactions O_x, O_y and the weight of the drum do no work since point O does not displace.

Kinematic : Since the drum rotates about point O, the angular velocity of the drum and the speed of the crate can be related by $\omega_D = \frac{v_A}{r_D} = \frac{v_A}{1.5} = 0.6667 v_A$. When the crate rises $s = 4$ ft, the angular displacement of the drum is given by $\theta = \frac{s}{r_D} = \frac{4}{1.5} = 2.667$ rad.



Principle of Work and Energy : The mass moment of inertia of the drum about point O is $I_O = mk_O^2 = \left(\frac{30}{32.2}\right)(0.8^2) = 0.5963$ slug · ft². Applying Eq. 18 – 13, we have

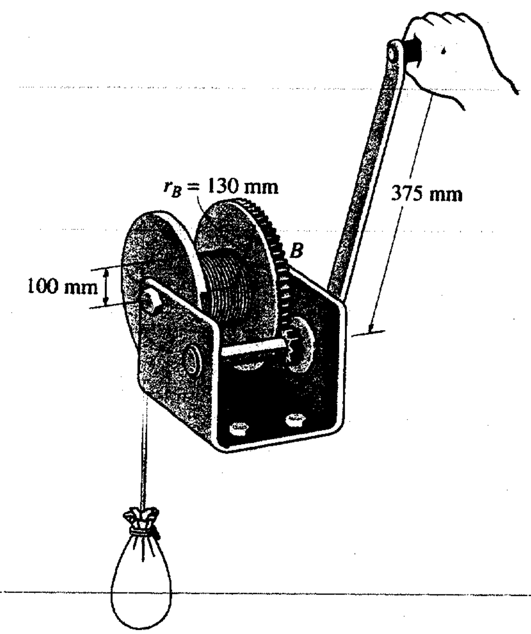
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + M\theta - W_C s_C = \frac{1}{2} I_O \omega^2 + \frac{1}{2} m_C v_C^2$$

$$0 + 120(2.667) - 15(4) = \frac{1}{2}(0.5963)(0.6667 v_A)^2 + \frac{1}{2}\left(\frac{15}{32.2}\right) v_A^2$$

$$v_A = 26.7 \text{ ft/s} \qquad \textbf{Ans}$$

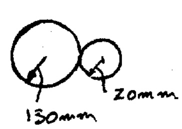
18-15. The hand winch is used to lift the 50-kg load. Determine the work required to rotate the handle five revolutions. The gear at A has a radius of 20 mm.



$20(\theta_A) = \theta_B(130)$

When θ_A = 5 rev. = 10π

θ_B = 4.8332 rad

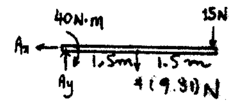


Thus load moves up

s = 4.8332(0.1 m) = 0.48332 m

U = 50(9.81)(0.48332) = 237 J **Ans**

***18-16.** The 4-kg slender rod is subjected to the force and couple moment. When it is in the position shown it has an angular velocity $\omega_1 = 6$ rad/s. Determine its angular velocity at the instant it has rotated downward 90°. The force is always applied perpendicular to the axis of the rod. Motion occurs in the vertical plane.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}[\frac{1}{3}(4)(3)^2](6)^2 + 15(\frac{\pi}{2})(3) + 4(9.81)(1.5) + 40(\frac{\pi}{2}) = \frac{1}{2}[\frac{1}{3}(4)(3)^2]\omega^2$$

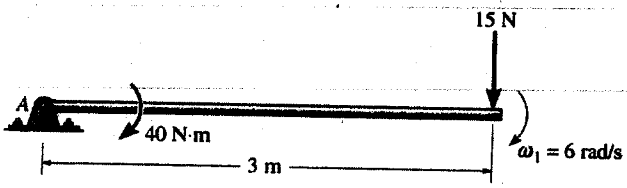
$\omega = 8.25$ rad/s **Ans**

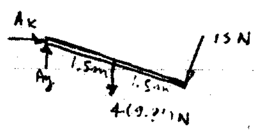
18-17. The 4-kg slender rod is subjected to the force and couple moment. When the rod is in the position shown it has a angular velocity $\omega_1 = 6$ rad/s. Determine its angular velocity at the instant it has rotated 360°. The force is always applied perpendicular to the axis of the rod and motion occurs in the vertical plane.

$$T_1 + \Sigma U_{1-2} = T_2$$

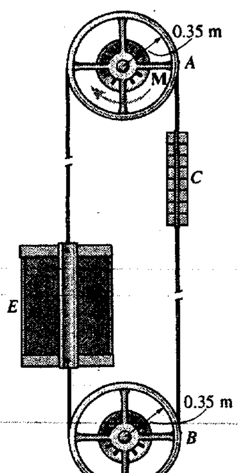
$$\frac{1}{2}[\frac{1}{3}(4)(3)^2](6)^2 + 15(2\pi)(3) + 40(2\pi) = \frac{1}{2}[\frac{1}{3}(4)(3)^2]\omega^2$$

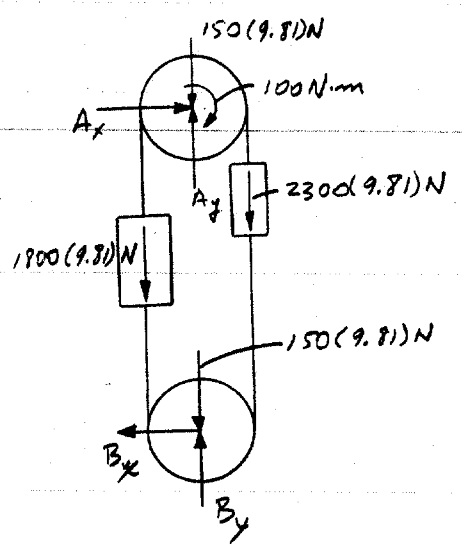
$\omega = 11.2$ rad/s **Ans**





18-18. The elevator car E has a mass of 1.80 Mg and the counterweight C has a mass of 2.30 Mg. If a motor turns the driving sheave A with a constant torque of $M = 100$ N·m, determine the speed of the elevator when it has ascended 10 m starting from rest. Each sheave A and B has a mass of 150 kg and a radius of gyration of $k = 0.2$ m about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.





$$\theta = \frac{10}{0.35} = 28.57 \text{ rad}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

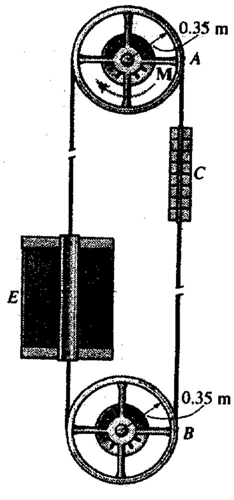
$$0 + 2300(9.81)(10) - 1800(9.81)(10) + 100(28.57)$$

$$= \frac{1}{2}(1800)(v)^2 + \frac{1}{2}(2300)(v)^2 + (2)\frac{1}{2}[150(0.2)^2]\left(\frac{v}{0.35}\right)^2$$

$$51\,907.1 = 2099v^2$$

$v = 4.97$ m/s **Ans**

18-19. The elevator car E has a mass of 1.80 Mg and the counterweight C has a mass of 2.30 Mg. If a motor turns the driving sheave A with a torque of $M = (0.06\theta^2 + 7.5)$ N·m, where θ is in radians, determine the speed of the elevator when it has ascended 12 m starting from rest. Each sheave A and B has a mass of 150 kg and a radius of gyration of $k = 0.2$ m about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.



$$\theta = \frac{12}{0.35} = 34.29 \text{ rad}$$

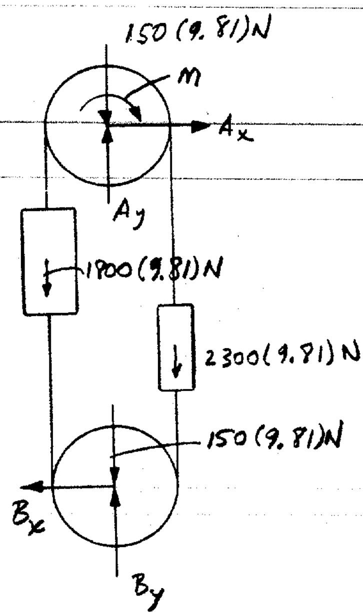
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 2300(9.81)(12) - 1800(9.81)(12) + \int_0^{34.29} (0.06\theta^2 + 7.5)\,d\theta$$

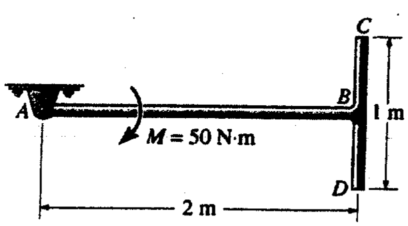
$$= \frac{1}{2}(1800)(v)^2 + \frac{1}{2}(2300)(v)^2 + (2)\frac{1}{2}[150(0.2)^2]\left(\frac{v}{0.35}\right)^2$$

$$58860 + (0.02\theta^3 + 7.5\theta)|_0^{34.29} = 2098.98v^2$$

$v = 5.34$ m/s **Ans**



***18-20.** The pendulum consists of two slender rods each having a mass of 4 kg/m. If it is acted upon by a moment $M = 50$ N·m and released from the position shown, determine its angular velocity when it has rotated (a) 90° and (b) 180°. Motion occurs in the vertical plane.

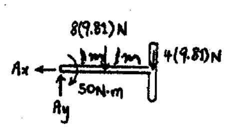


a) $T_1 + \Sigma U_{1-2} = T_2$

$$[0+0] + 8(9.81)(1) + 4(9.81)(2) + 50\left(\frac{\pi}{2}\right) = \frac{1}{2}\left[\frac{1}{3}(8)(2)^2\right]\omega^2 + \frac{1}{2}\left[\frac{1}{12}(4)(1)^2 + 4(2)^2\right]\omega^2$$

$$235.5 = 13.5\,\omega^2$$

$\omega = 4.18$ rad/s **Ans**

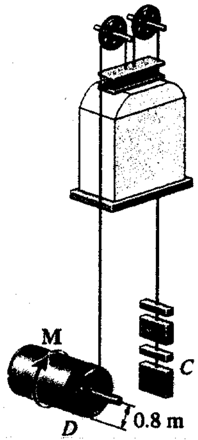


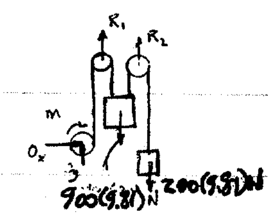
b) $T_1 + \Sigma U_{1-2} = T_2$

$$[0+0] + 50(\pi) = 13.5\omega^2$$

$\omega = 3.41$ rad/s **Ans**

18-21. A motor supplies a constant torque $M = 6$ kN·m to the winding drum that operates the elevator. If the elevator has a mass of 900 kg, the counterweight C has a mass of 200 kg, and the winding drum has a mass of 600 kg and radius of gyration about its axis of $k = 0.6$ m, determine the speed of the elevator after it rises 5 m starting from rest. Neglect the mass of the pulleys.





$v_E = v_C$

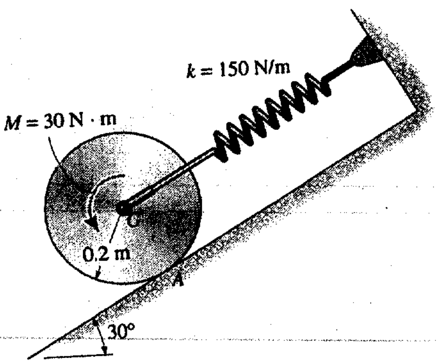
$\theta = \frac{s}{r} = \frac{5}{0.8}$

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 6000\left(\frac{5}{0.8}\right) - 900(9.81)(5) + 200(9.81)(5) = \frac{1}{2}(900)(v)^2 + \frac{1}{2}(200)(v)^2 + \frac{1}{2}[600(0.6)^2]\left(\frac{v}{0.8}\right)^2$$

$v = 2.10$ m/s **Ans**

18-22. The 20-kg disk is originally at rest, and the spring holds it in equilibrium. A couple moment of $M = 30$ N·m is then applied to the disk as shown. Determine its angular velocity at the instant its mass center G has moved 0.8 m down along the inclined plane. The disk rolls without slipping.



Initial tension in spring :

$+\Sigma M_A = 0;\quad -F_s(0.2) + 20(9.81)\sin 30°(0.2) = 0$

$F_s = 98.1$ N

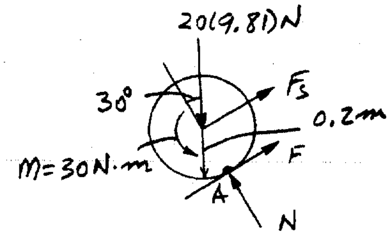
$s_1 = \frac{98.1}{150} = 0.654$ m

When $s = 0.8$ m the disk rotates $\theta = \frac{0.8}{0.2} = 4$ rad

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 20(9.81)(0.8\sin 30°) + 30(4) - \left[\frac{1}{2}(150)(0.8+0.654)^2 - \frac{1}{2}(150)(0.654)^2\right]$$

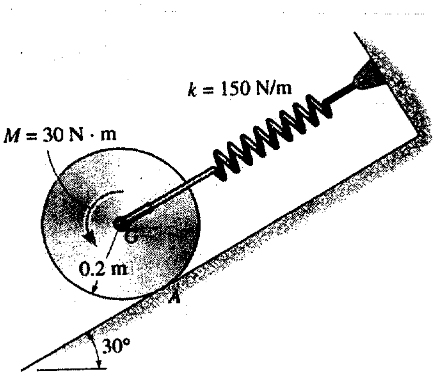
$$= \frac{1}{2}\left[\frac{1}{2}(20)(0.2)^2\right]\omega^2 + \frac{1}{2}(20)(0.2\omega)^2$$



$72 = 0.6\omega^2$

$\omega = 11.0$ rad/s **Ans**

18-23. The 20-kg disk is originally at rest, and the spring holds it in equilibrium. A couple moment of $M = 30$ N·m is then applied to the disk as shown. Determine how far the center of mass of the disk travels down along the incline, measured from the equilibrium position, before it stops. The disk rolls without slipping.

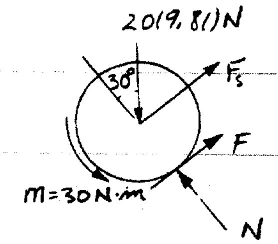


$$+\Sigma M_A = 0; \quad -F_s(0.2) + 20(9.81)\sin 30°(0.2) = 0$$

$$F_s = 98.1 \text{ N}$$

$$s_1 = \frac{98.1}{150} = 0.654 \text{ m}$$

When G moves s, the disk rotates $\theta = \dfrac{s}{0.2}$



$$T_1 + \Sigma U_{1-2} = T_2$$

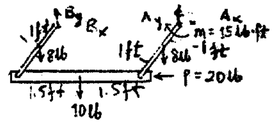
$$0 + 20(9.81)(s)\sin 30° + 30\left(\frac{s}{0.2}\right) - \left[\frac{1}{2}(150)(s+0.654)^2 - \frac{1}{2}(150)(0.654)^2\right] = 0$$

$$248.1s = 75(s^2 + 1.308s + 0.4277) - 32.08$$

$$248.1 = 75s + 98.1$$

$s = 2.00$ m **Ans**

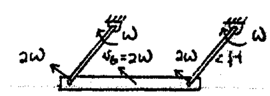
***18-24.** The linkage consists of two 8-lb rods AB and CD and a 10-lb bar AD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega_{AB} = 2$ rad/s. If rod CD is subjected to a couple moment $M = 15$ lb·ft and bar AD is subjected to a horizontal force $P = 20$ lb as shown, determine ω_{AB} at the instant $\theta = 90°$.



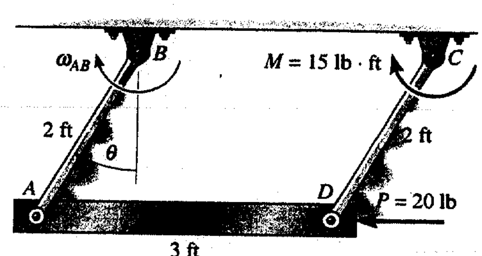
$$T_1 + \Sigma U_{1-2} = T_2$$

$$2[\tfrac{1}{2}\{\tfrac{1}{3}(\tfrac{8}{32.2})(2)^2\}(2)^2] + \tfrac{1}{2}(\tfrac{10}{32.2})(4)^2 + [20(2) + 15(\tfrac{\pi}{2}) - 2(8)(1) - 10(2)] =$$
$$2[\tfrac{1}{2}\{\tfrac{1}{3}(\tfrac{8}{32.2})(2)^2\}\omega^2] + \tfrac{1}{2}(\tfrac{10}{32.2})(2\omega)^2$$

$\omega = 5.74$ rad/s **Ans**

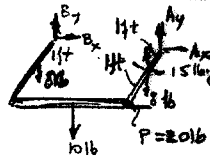


18-25. The linkage consists of two 8-lb rods AB and CD and a 10-lb bar AD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega_{AB} = 2$ rad/s. If rod CD is subjected to a couple moment $M = 15$ lb·ft and bar AD is subjected to a horizontal force $P = 20$ lb as shown, determine ω_{AB} at the instant $\theta = 45°$.



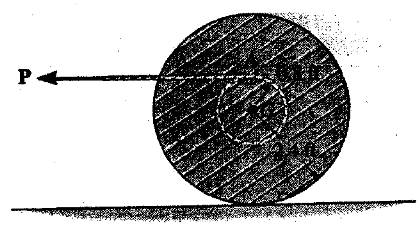
$$T_1 + \Sigma U_{1-2} = T_2$$

$$2[\tfrac{1}{2}\{\tfrac{1}{3}(\tfrac{8}{32.2})(2)^2\}(2)^2] + \tfrac{1}{2}(\tfrac{10}{32.2})(4)^2$$
$$+ [20(2\sin 45°) + 15(\tfrac{\pi}{4}) - 2(8)(1-\cos 45°) - 10(2-2\cos 45°)]$$
$$= 2[\tfrac{1}{2}\{\tfrac{1}{3}(\tfrac{8}{32.2})(2)^2\}\omega^2] + \tfrac{1}{2}(\tfrac{10}{32.2})(2\omega)^2$$



$\omega = 5.92$ rad/s **Ans**

18-26. The spool has a weight of 500 lb and a radius of gyration of $k_G = 1.75$ ft. A horizontal force of $P = 15$ lb is applied to a cable wrapped around its inner core. If the spool is originally at rest, determine its angular velocity after the mass center G has moved 6 ft to the left. The spool rolls without slipping. Neglect the mass of the cable.



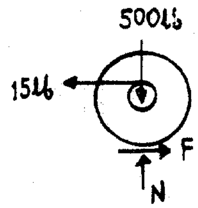
$$\frac{s_G}{2.4} = \frac{s_A}{3.2}$$

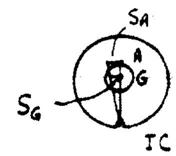
For $s_G = 6$ ft, then $s_A = 8$ ft

$$T_1 + \Sigma U_{1-2} = T_2$$

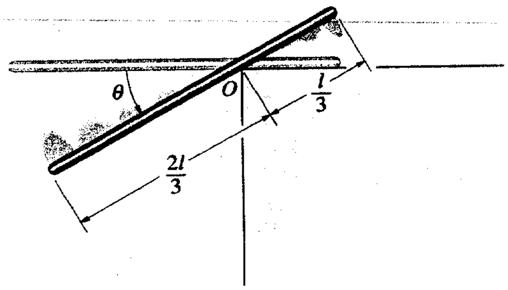
$$0 + 15(8) = \frac{1}{2}\left[\left(\frac{500}{32.2}\right)(1.75)^2\right]\omega^2 + \frac{1}{2}\left(\frac{500}{32.2}\right)(2.4\omega)^2$$

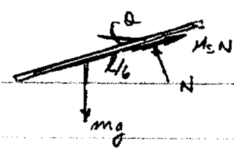
$\omega = 1.32$ rad/s **Ans**





18-27. The uniform bar has a mass m and length l. If it is released from rest when $\theta = 0°$, determine the angle θ at which it first begins to slip. The coefficient of static friction at O is $\mu_s = 0.3$.





$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + m\,g\left(\frac{l}{6}\sin\theta\right) = \frac{1}{2}\left[\frac{1}{12}m\,l^2 + m\left(\frac{l}{6}\right)^2\right]\omega^2$$

$$\omega = \sqrt{\frac{3\,g\sin\theta}{l}}$$

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \quad m\,g\cos\theta\left(\frac{l}{6}\right) = \left[\frac{1}{12}m\,l^2 + m\left(\frac{l}{6}\right)^2\right]\alpha$$

$$\alpha = \frac{3\,g\cos\theta}{2\,l}$$

$$+\Sigma F_n = m(a_G)_n; \quad \mu_s N - m\,g\sin\theta = m\left(\frac{3\,g\sin\theta}{l}\right)\left(\frac{l}{6}\right)$$

$$\mu_s N = 1.5\,m\,g\sin\theta$$

$$+\Sigma F_t = m(a_G)_t; \quad -N + m\,g\cos\theta = m\left(\frac{3\,g\cos\theta}{2\,l}\right)\left(\frac{l}{6}\right)$$

$$N = 0.75\,m\,g\cos\theta$$

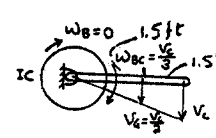
Thus,

$$\mu_s = \frac{1.5}{0.75}\tan\theta$$

$$0.3 = 2\tan\theta$$

$\theta = 8.53°$ **Ans**

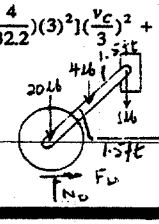
***18-28.** The system consists of a 20-lb disk A, 4-lb slender rod BC, and a 1-lb smooth collar C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e., $\theta = 0°$. The system is released from rest when $\theta = 45°$.



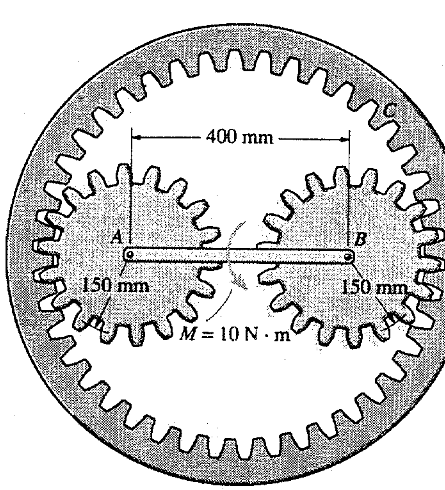
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 4(1.5\sin 45°) + 1(3\sin 45°) = \frac{1}{2}\left[\frac{1}{3}\left(\frac{4}{32.2}\right)(3)^2\right]\left(\frac{v_C}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{32.2}\right)(v_C)^2$$

$v_C = 13.3$ ft/s **Ans**



18-29. The two 2-kg gears A and B are attached to the ends of a 3-kg slender bar. The gears roll within the fixed ring gear C, which lies in the horizontal plane. If a 10-N · m torque is applied to the center of the bar as shown, determine the number of revolutions the bar must rotate starting from rest in order for it to have an angular velocity of $\omega_{AB} = 20$ rad/s. For the calculation, assume the gears can be approximated by thin disks. What is the result if the gears lie in the vertical plane?



Energy equation (where G refers to the center of one of the two gears):

$$M\theta = T_2$$

$$10\theta = 2\left(\frac{1}{2}I_G\omega_{gear}^2\right) + 2\left(\frac{1}{2}m_{gear}\right)(0.200\omega_{AB})^2 + \frac{1}{2}I_{AB}\omega_{AB}^2$$

Using $m_{gear} = 2$ kg, $I_G = \frac{1}{2}(2)(0.150)^2 = 0.0225$ kg · m^2,

$$I_{AB} = \frac{1}{12}(3)(0.400)^2 = 0.0400 \text{ kg} \cdot \text{m}^2, \text{ and } \omega_{gear} = \frac{200}{150}\omega_{AB},$$

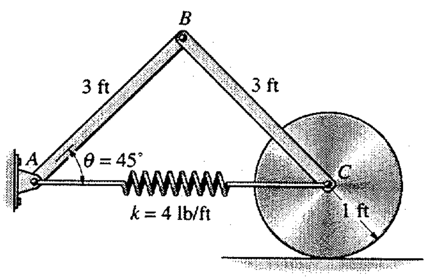
$$10\theta = 0.0225\left(\frac{200}{150}\right)^2\omega_{AB}^2 + 2(0.200)^2\omega_{AB}^2 + 0.0200\omega_{AB}^2$$

When $\omega_{AB} = 20$ rad/s,

$$\theta = 5.60 \text{ rad}$$

$= 0.891$ rev, regardless of orientation **Ans**

18-30. The assembly consists of two 15-lb slender rods and a 20-lb disk. If the spring is unstretched when $\theta = 45°$ and the assembly is released from rest at this position, determine the angular velocity of rod AB at the instant $\theta = 0°$. The disk rolls without slipping.

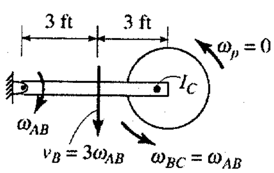


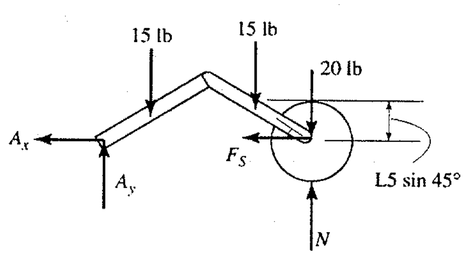
$$T_1 + \sum U_{1-2} = T_2$$

$$[0 + 0] + 2(15)(1.5)\sin 45° - \frac{1}{2}(4)[6 - 2(3)\cos 45°]^2$$

$$= 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right]$$

$\omega_{AB} = 4.28$ rad/s **Ans**

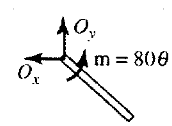




18-31. The uniform door has a mass of 20 kg and can be treated as a thin plate having the dimensions shown. If it is connected to a torsional spring at A, which has a stiffness of $k = 80$ N · m/rad, determine the required initial twist of the spring in radians so that the door has an angular velocity of 12 rad/s when it closes at $\theta = 0°$ after being opened at $\theta = 90°$ and released from rest. *Hint:* For a torsional spring $M = k\theta$, when k is the stiffness and θ is the angle of twist.

$$T_1 + \sum U_{1-2} = T_2$$

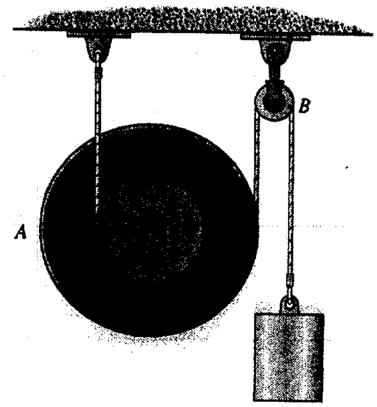
$$0 + \int_{\theta_0}^{\theta_0+\frac{\pi}{2}} 80\theta\, d\theta = \frac{1}{2}\left[\frac{1}{3}(20)(0.8)^2\right](12)^2$$



$$40\left[\left(\theta_0 + \frac{\pi}{2}\right)^2 - \theta_0^2\right] = 307.2$$

$\theta_0 = 1.66$ rad **Ans**

***18-32.** Pulley A weighs 15 lb and has a radius of gyration of $k_O = 0.8$ ft. If the system is released from rest, determine the velocity of the center O of the pulley after the 10-lb block moves downward 4 ft. Neglect the mass of the pulley at B.



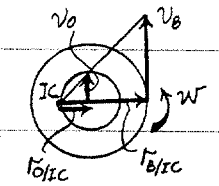
Kinematic : Since the cylinder rolls without slipping at point C, the instantaneous center of zero velocity is located at point C. Thus,

$$\frac{s_O}{0.5} = \frac{4}{1.5} \qquad s_O = 1.333 \text{ ft}$$

Also,

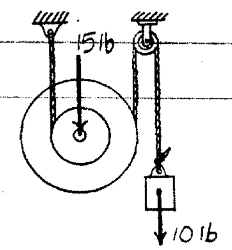
$$\omega = \frac{v_O}{r_{O/IC}} = \frac{v_B}{r_{B/IC}}$$

Thus, $\omega = \dfrac{v_O}{0.5} = 2v_O$ and $\dfrac{v_O}{0.5} = \dfrac{v_B}{1.5}$, $v_B = 3v_O$.



Free Body Diagram : The weight of the block does *positive* work since it acts in the same direction of its displacement, whereas the weight of the pulley acts in the opposite direction to that of its displacement and hence does *negative* work.

Principle of Work and Energy : The mass moment of inertia of the pulley about point O is $I_O = mk_O^2 = \left(\frac{15}{32.2}\right)(0.8^2) = 0.2981 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 18 − 13, we have



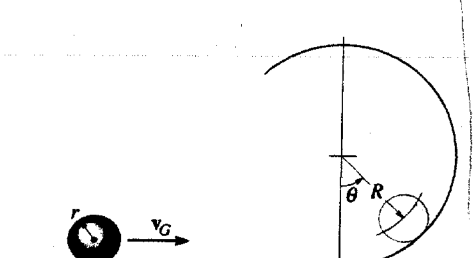
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_B s_B - W_P s_O = \frac{1}{2} m_B v_B^2 + \frac{1}{2} m_P v_O^2 + \frac{1}{2} I_O \omega^2$$

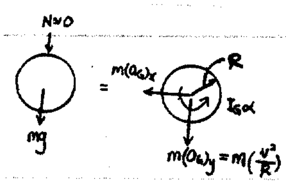
$$0 + 10(4) - 15(1.333) = \frac{1}{2}\left(\frac{10}{32.2}\right)(3v_O)^2 + \frac{1}{2}\left(\frac{15}{32.2}\right)v_O^2 + \frac{1}{2}(0.2981)(2v_O)^2$$

$$v_O = 3.00 \text{ ft/s} \qquad \textbf{Ans}$$

18-33. A ball of mass m and radius r is cast onto the horizontal surface such that it rolls without slipping. Determine the minimum speed v_G of its mass center G so that it rolls completely around the loop of radius $R + r$ without leaving the track.



$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad mg = m\left(\frac{v^2}{R}\right)$$



$$v^2 = gR$$

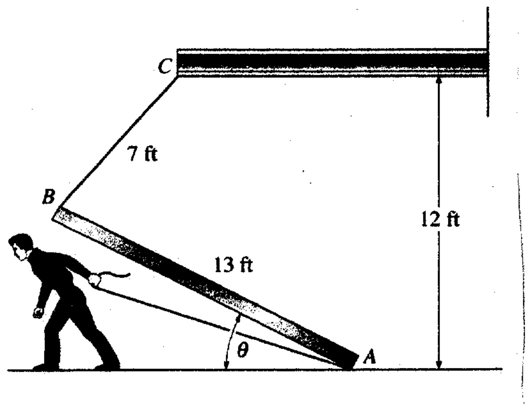
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_G^2}{r^2}\right) + \frac{1}{2}mv_G^2 - mg(2R) = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{gR}{r^2}\right) + \frac{1}{2}m(gR)$$

$$\frac{1}{5}v_G^2 + \frac{1}{2}v_G^2 = 2gR + \frac{1}{5}gR + \frac{1}{2}gR$$

$$v_G = 3\sqrt{\frac{3}{7}gR} \qquad \textbf{Ans}$$

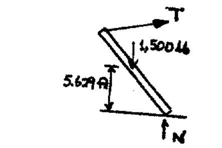
18-34. The beam has a weight of 1500 lb and is being raised to a vertical position by pulling very slowly on its bottom end A. If the cord fails when $\theta = 60°$ and the beam is essentially at rest, determine the speed of A at the instant cord BC becomes vertical. Neglect friction and the mass of the cords, and treat the beam as a slender rod.

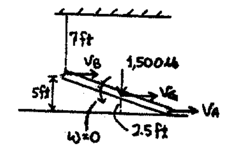


$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 1500(5.629) - 1500(2.5) = \frac{1}{2}\left(\frac{1500}{32.2}\right)(v_G)^2$$

$v_G = v_A = 14.2$ ft/s **Ans**





18-35. Solve Prob. 18-13 using the conservation of energy equation.

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 50(9.81)(1.25) = \frac{1}{2}\left[50(1.75)^2\right]\omega^2 + 0$$

$\omega = 2.83$ rad/s **Ans**

***18-36.** Solve Prob. 18–12 using the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 4\left[\frac{1}{2}\left(\frac{5}{32.2}\right)v_C^2 + \frac{1}{2}\left(\frac{5}{32.2}\right)(0.3)^2\left(\frac{v_C}{0.5}\right)^2\right] + \frac{1}{2}\left(\frac{110}{32.2}\right)v_C^2 - 130(100\sin 30°)$$

$v_C = 55.2$ ft/s **Ans**

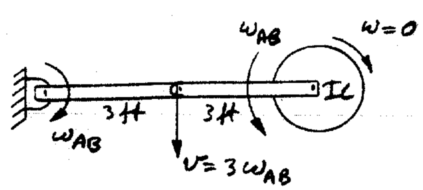
18-37. Solve Prob. 18-30 using the conservation of energy equation.

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[15(1.5\sin 45°)\right] = 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right] + \frac{1}{2}(4)\left[6 - 2(3\cos 45°)\right]^2 + 0$$

$\omega_{AB} = 4.28$ rad/s **Ans**



18-38. Solve Prob. 18–11 using the conservation of energy equation.

$$v_G = (0.02)\omega = 0.02(70) = 1.4 \text{ ft/s}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(\frac{0.3}{32.2})(1.4)^2 + \frac{1}{2}(0.06)^2(\frac{0.3}{32.2})](70)^2 - (0.3)\,s$$

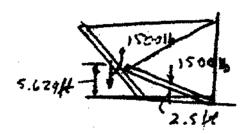
$s = 0.304$ ft **Ans**

18-39. Solve Prob. 18–34 using the conservation of energy equation.

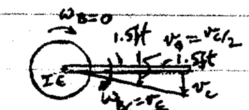
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 1500(5.629) = \frac{1}{2}(\frac{1500}{32.2})(v_G)^2 + (1500)(2.5)$$

$v_G = v_A = 14.2$ ft/s **Ans**



***18-40.** Solve Prob. 18–28 using the conservation of energy equation.

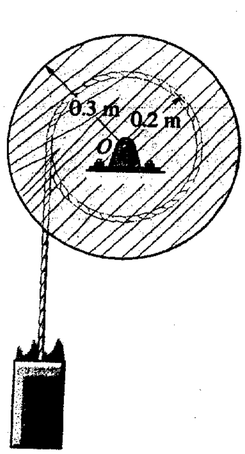


$$T_1 + V_1 = T_2 + V_2$$

$$0 + 4(1.5\sin 45°) + 1(3\sin 45°) = \frac{1}{2}[\frac{1}{3}(\frac{4}{32.2})(3)^2](\frac{v_C}{3})^2 + \frac{1}{2}(\frac{1}{32.2})(v_C)^2 + 0$$

$v_C = 13.3$ ft/s **Ans**

18-41. The spool has a mass of 50 kg and a radius of gyration $k_O = 0.280$ m. If the 20-kg block A is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 5$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.



$$v_A = 0.2\,\omega = 0.2(5) = 1 \text{ m/s}$$

System:

$$T_1 + V_1 = T_2 + V_2$$

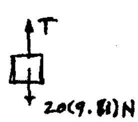
$$[0 + 0] + 0 = \frac{1}{2}(20)(1)^2 + \frac{1}{2}[50(0.280)^2](5)^2 - 20(9.81)\,s$$

$s = 0.30071$ m $= 0.301$ m **Ans**

Block:

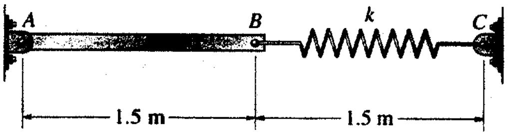
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 20(9.81)(0.30071) - T(0.30071) = \frac{1}{2}(20)(1)^2$$



$T = 163$ N **Ans**

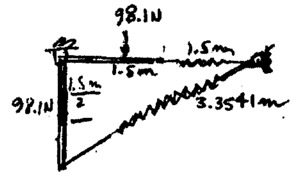
18-42. When the slender 10-kg bar AB is horizontal it is at rest and the spring is unstretched. Determine the stiffness k of the spring so that the motion of the bar is momentarily stopped when it has rotated downward 90°.



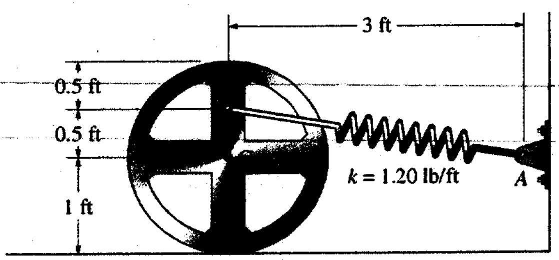
$$T_1 + V_1 = T_2 + V_2$$

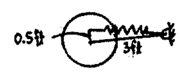
$$0 + 0 = 0 + \frac{1}{2}(k)(3.3541 - 1.5)^2 - 98.1\left(\frac{1.5}{2}\right)$$

$k = 42.8$ N/m **Ans**



18-43. The 50-lb wheel has a radius of gyration about its center of gravity G of $k_G = 0.7$ ft. If it rolls without slipping, determine its angular velocity when it has rotated clockwise 90° from the position shown. The spring AB has a stiffness $k = 1.20$ lb/ft and an unstretched length of 0.5 ft. The wheel is released from rest.

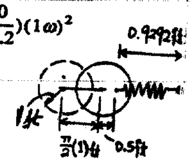




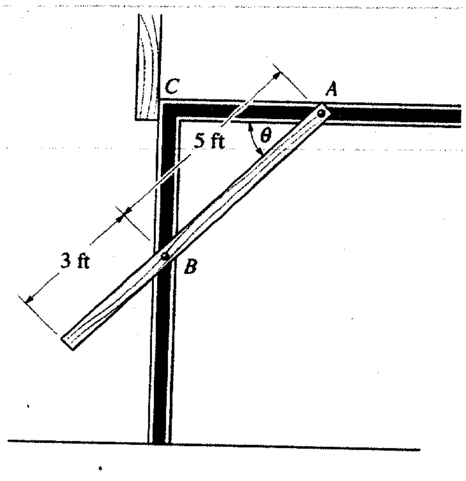
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(1.20)\left[\sqrt{(3)^2 + (0.5)^2} - 0.5\right]^2 = \frac{1}{2}\left[\frac{50}{32.2}(0.7)^2\right]\omega^2 + \frac{1}{2}\left(\frac{50}{32.2}\right)(1\omega)^2 + \frac{1}{2}(1.20)(0.9292 - 0.5)^2$$

$\omega = 1.80$ rad/s **Ans**



***18-44.** The door is made from one piece, whose ends move along the horizontal and vertical tracks. If the door is in the open position, $\theta = 0°$, and then released, determine the speed at which its end A strikes the stop at C. Assume the door is a 180-lb thin plate having a width of 10 ft.

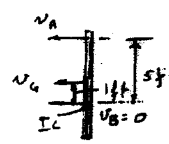


$$T_1 + V_1 = T_2 + V_2$$

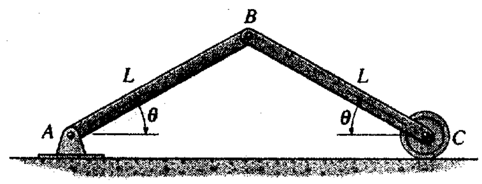
$$0 + 0 = \frac{1}{2}\left[\frac{1}{12}\left(\frac{180}{32.2}\right)(8)^2\right]\omega^2 + \frac{1}{2}\left(\frac{180}{32.2}\right)(1\omega)^2 - 180(4)$$

$\omega = 6.3776$ rad/s

$v_C = \omega(5) = 6.3776(5) = 31.9$ m/s **Ans**



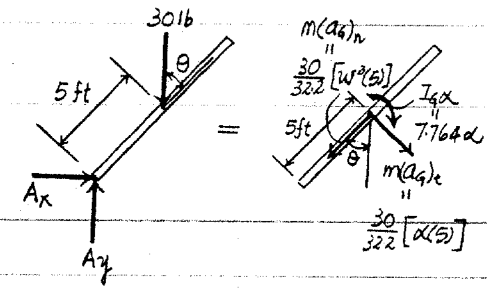
18-45. The two bars are released from rest at the position θ. Determine their angular velocities at the instant they become horizontal. Neglect the mass of the roller at C. Each bar has a mass m and length L.



Potential Energy : Datum is set at point A . When links AB and BC is at their initial position, their center of gravity is located $\frac{L}{2}\sin\theta$ *above* the datum. Their gravitational potential energy at this position is $mg\left(\frac{L}{2}\sin\theta\right)$. Thus, the initial and final potential energies are

$$V_1 = 2\left(\frac{mgL}{2}\sin\theta\right) = mgL\sin\theta \qquad V_2 = 0$$

Kinetic Energy : When links AB and BC are in the horizontal position, then $v_B = \omega_{AB}L$ which is directed vertically downward since link AB is rotating about fixed point A. Link BC is subjected to general plane motion and its instantaneous center of zero velocity is located at point C. Thus, $v_B = \omega_{BC}r_{B/IC}$ or $\omega_{AB}L = \omega_{BC}L$, hence $\omega_{AB} = \omega_{BC} = \omega$. The mass moment inertia for link AB and BC about point A and C is $(I_{AB})_A = (I_{BC})_C = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2$ $= \frac{1}{3}mL^2$. Since links AB and CD are at rest initially, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

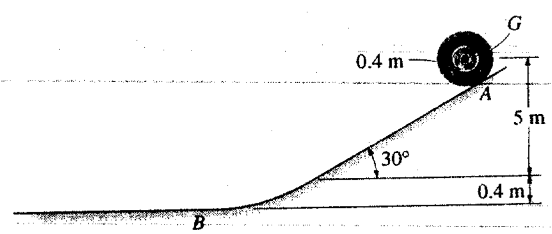


$$\begin{aligned}
T_2 &= \frac{1}{2}(I_{AB})_A\,\omega_{AB}^2 + \frac{1}{2}(I_{BC})_C\,\omega_{BC}^2 \\
&= \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 + \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 \\
&= \frac{1}{3}mL^2\omega^2
\end{aligned}$$

Conservation of Energy : Applying Eq. 18 – 18, we have

$$\begin{aligned}
T_1 + V_1 &= T_2 + V_2 \\
0 + mgL\sin\theta &= \frac{1}{3}mL^2\omega^2 + 0 \\
\omega_{AB} = \omega_{BC} = \omega &= \sqrt{\frac{3g}{L}\sin\theta}
\end{aligned}$$ **Ans**

18-46. An automobile tire has a mass of 7 kg and radius of gyration $k_G = 0.3$ m. If it is released from rest at A on the incline, determine its angular velocity when it reaches the horizontal plane. The tire rolls without slipping.



$v_G = 0.4\omega$

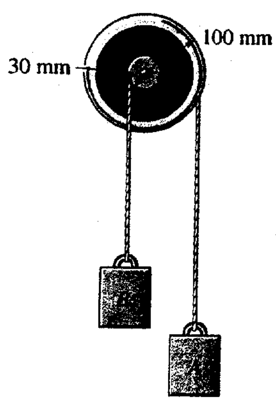
Datum at lowest point.

$T_1 + V_1 = T_2 + V_2$

$0 + 7(9.81)(5) = \frac{1}{2}(7)(0.4\omega)^2 + \frac{1}{2}\left[7(0.3)^2\right]\omega^2 + 0$

$\omega = 19.8$ rad/s **Ans**

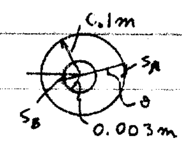
18-47. The compound disk pulley consists of a hub and attached outer rim. If it has a mass of 3 kg and a radius of gyration $k_G = 45$ mm, determine the speed of block A after A descends 0.2 m from rest. Blocks A and B each have a mass of 2 kg. Neglect the mass of the cords.



$T_1 + V_1 = T_2 + V_2$

$$[0 + 0 + 0] + [0+0] = \frac{1}{2}[3(0.045)^2]\omega^2 + \frac{1}{2}(2)(0.03\omega)^2 + \frac{1}{2}(2)(0.1\omega)^2 - 2(9.81)s_A + 2(9.81)s_B$$

$\theta = \dfrac{s_B}{0.03} = \dfrac{s_A}{0.1}$



$s_B = 0.3\, s_A$

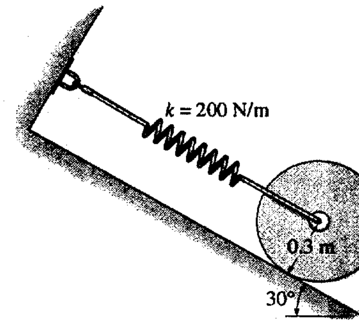
Set $s_A = 0.2$ m, $\quad s_B = 0.06$ m

Substituting and solving yields,

$\omega = 14.04$ rad/s

$v_A = 0.1(14.04) = 1.40$ m/s **Ans**

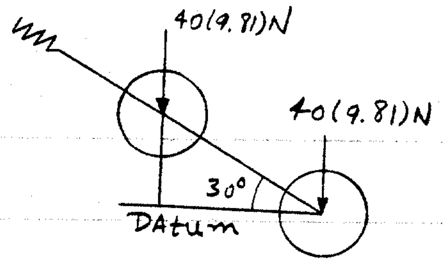
***18-48.** At the instant the spring becomes undeformed, the center of the 40-kg disk has a speed of 4 m/s. From this point determine the distance d the disk moves down the plane before momentarily stopping. The disk rolls without slipping.



Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{2}(40)(0.3)^2\right]\left(\frac{4}{0.3}\right)^2 + \frac{1}{2}(40)(4)^2 + 40(9.81)d\sin 30° = 0 + \frac{1}{2}(200)d^2$$

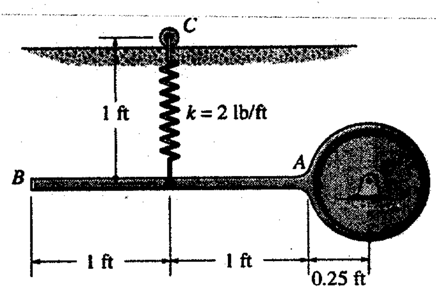


$$100d^2 - 196.2d - 480 = 0$$

Solving for the positive root

$d = 3.38$ m **Ans**

18-49. The pendulum consists of a 2-lb rod BA and a 6-lb disk. The spring is stretched 0.3 ft when the rod is horizontal as shown. If the pendulum is released from rest and rotates about point D, determine its angular velocity at the instant the rod becomes vertical. The roller at C allows the spring to remain vertical as the rod falls.



Potential Energy : Datum is set at point D. When rod AB is at vertical position, its center of gravity is located 1.25 ft *below* the datum. Its gravitational potential energy at this position is $-2(1.25)$ ft · lb. The initial and final stretch of the spring are 0.3 ft and $(1.25 + 0.3)$ ft = 1.55 ft, respectively. Hence, the initial and final elastic potential energy are $\frac{1}{2}(2)(0.3^2) = 0.09$ lb · ft and $\frac{1}{2}(2)(1.55^2)$ = 2.4025 lb · ft. Thus,

$$V_1 = 0.09 \text{ lb}\cdot\text{ft} \qquad V_2 = 2.4025 + [-2(1.25)] = -0.0975 \text{ lb}\cdot\text{ft}$$

Kinetic Energy : The mass moment inertia for rod AB and the disk about point D are $(I_{AB})_O = \frac{1}{12}\left(\frac{2}{32.2}\right)(2^2) + \left(\frac{2}{32.2}\right)(1.25^2) = 0.1178$ slug · ft² and $(I_D)_O = \frac{1}{2}\left(\frac{6}{32.2}\right)(0.25^2) = 0.005823$ slug · ft². Since rod AB and the disk are initially at rest, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

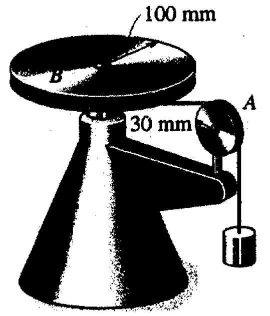
$$T_2 = \frac{1}{2}(I_{AB})_O\,\omega^2 + \frac{1}{2}(I_D)_O\,\omega^2$$
$$= \frac{1}{2}(0.1178)\,\omega^2 + \frac{1}{2}(0.005823)\,\omega^2$$
$$= 0.06179\omega^2$$

Conservation of Energy : Applying Eq. 18 – 18, we have

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 0.09 = 0.06179\omega^2 + (-0.0975)$$
$$\omega = 1.74 \text{ rad/s}$$

Ans

18-50. The assembly consists of a 3-kg pulley A and 10-kg pulley B. If a 2-kg block is suspended from the cord, determine the block's speed after it descends 0.5 m starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.



$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0 + 0] + [0] = \frac{1}{2}[\frac{1}{2}(3)(0.03)^2]\omega_A^2 + \frac{1}{2}[\frac{1}{2}(10)(0.1)^2]\omega_B^2 + \frac{1}{2}(2)(v_C)^2 - 2(9.81)(0.5)$$

$$v_C = \omega_B(0.1) = 0.03\omega_A$$

Thus,

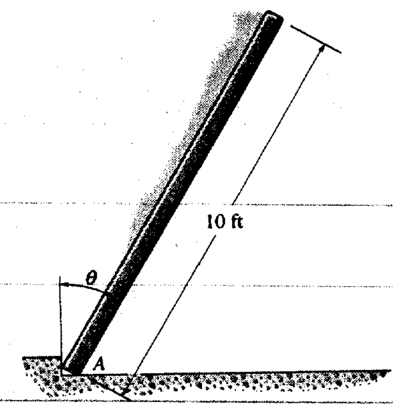
$$\omega_B = 10\,v_C$$

$$\omega_A = 33.33 v_C$$

Substituting and solving yeilds,

$v_C = 1.52$ m/s **Ans**

18-51. A uniform ladder having a weight of 30 lb is released from rest when it is in the vertical position. If it is allowed to fall freely, determine the angle θ at which the bottom end A starts to lift off the ground. For the calculation, assume the ladder to be a slender rod and neglect friction at A.



Potential Energy : Datum is set at point A. When the ladder is at its initial and final position, its center of gravity is located 5 ft and ($5\cos\theta$) ft *above* the datum. Its initial and final gravitational potential energy are $30(5) = 150$ ft · lb and $30(5\cos\theta) = 150\cos\theta$ ft · lb respectively. Thus, the initial and final potential energy are

$$V_1 = 150 \text{ ft} \cdot \text{lb} \qquad V_2 = 150\cos\theta \text{ ft} \cdot \text{lb}$$

Kinetic Energy : The mass moment inertia of the ladder about point A is $I_A = \frac{1}{12}\left(\frac{30}{32.2}\right)(10^2) + \left(\frac{30}{32.2}\right)(5^2) = 31.06$ slug · ft^2. Since the ladder is initially at rest, the initial kinectic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}I_A\,\omega^2 = \frac{1}{2}(31.06)\,\omega^2 = 15.53\omega^2$$

Conservation of Energy : Applying Eq. 18 – 18, we have

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 150 = 15.53\omega^2 + 150\cos\theta$$
$$\omega^2 = 9.66(1 - \cos\theta)$$

Equation of Motion : The mass moment inertia of the ladder about its mass center is $I_G = \frac{1}{12}\left(\frac{30}{32.2}\right)(10^2) = 7.764$ slug · ft^2. Applying Eq. 17 – 16, we have

$$+\Sigma M_A = \Sigma(M_k)_A; \quad -30\sin\theta(5) = -7.764\alpha - \left(\frac{30}{32.2}\right)[\alpha(5)](5)$$
$$\alpha = 4.83\sin\theta$$

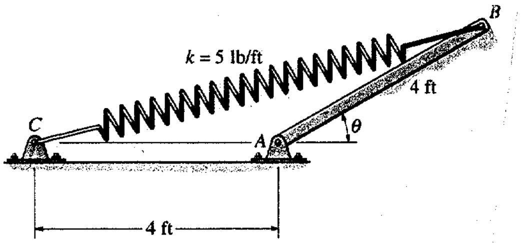
$$+\uparrow\Sigma F_y = m(a_G)_y; \quad A_y - 30 = -\frac{30}{32.2}[9.66(1-\cos\theta)(5)]\cos\theta - \frac{30}{32.2}[4.83\sin\theta(5)]\sin\theta$$

$$A_y = 30 - \frac{30}{32.2}\left(48.3\cos\theta - 48.3\cos^2\theta + 24.15\sin^2\theta\right)$$
$$= 30 - 45.0\cos\theta + 45.0\cos^2\theta - 22.5\sin^2\theta$$
$$= 30 - 45.0\cos\theta + 45.0\cos^2\theta - 22.5\left(1 - \cos^2\theta\right)$$
$$= 7.50\left(9\cos^2\theta - 6\cos\theta + 1\right)$$
$$= 7.50(3\cos\theta - 1)^2$$

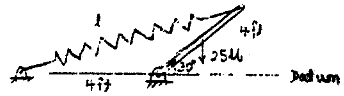
If the ladder lifts off the ground, then $A_y = 0$. Thus,

$$7.50(3\cos\theta - 1)^2 = 0$$
$$\theta = 70.5^\circ$$ **Ans**

***18-52.** The 25-lb slender rod AB is attached to a spring BC which, has an unstretched length of 4 ft. If the rod is released from rest when $\theta = 30°$, determine its angular velocity at the instant $\theta = 90°$.



$l = \sqrt{(4)^2 + (4)^2 - 2(4)(4)\cos 150°} = 7.727 \text{ ft}$

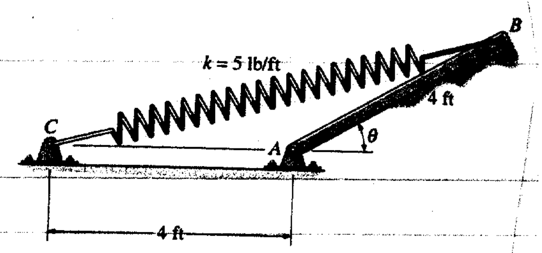


$T_1 + V_1 = T_2 + V_2$

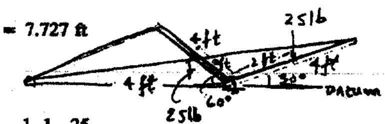
$$0 + 25(2)\sin 30° + \frac{1}{2}(5)(7.727 - 4)^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{25}{32.2}\right)(4)^2\right]\omega^2 + 25(2) + \frac{1}{2}(5)(4\sqrt{2} - 4)^2$$

$\omega = 1.18 \text{ rad/s}$ **Ans**

18-53. The 25-lb slender rod AB is attached to a spring BC which has an unstretched length of 4 ft. If the rod is released from rest when $\theta = 30°$, determine the angular velocity of the rod the instant the spring becomes unstretched.



$l = \sqrt{(4)^2 + (4)^2 - 2(4)(4)\cos 150°} = 7.727 \text{ ft}$

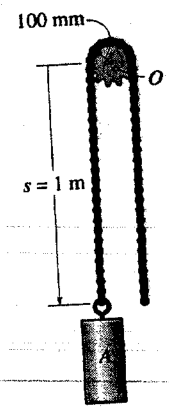


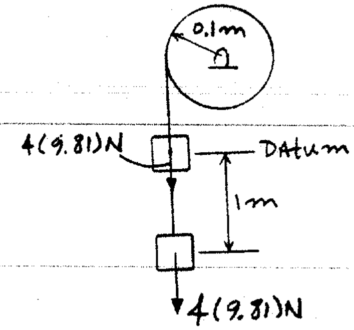
$T_1 + V_1 = T_2 + V_2$

$$0 + 25(2)\sin 30° + \frac{1}{2}(5)(7.727 - 4)^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{25}{32.2}\right)(4)^2\right]\omega^2 + 25(2)(\sin 60°) + 0$$

$\omega = 2.82 \text{ rad/s}$ **Ans**

18-54. A chain that has a negligible mass is draped over the sprocket which has a mass of 2 kg and a radius of gyration of $k_O = 50$ mm. If the 4-kg block A is released from rest in the position $s = 1$ m, determine the angular velocity of the sprocket at the instant $s = 2$ m.



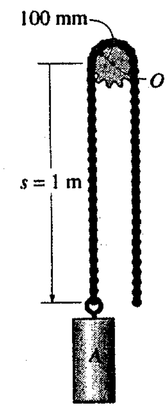


~~$T_1 + V_1 = T_2 + V_2$~~

$$0 + 0 + 0 = \frac{1}{2}(4)(0.1\omega)^2 + \frac{1}{2}\left[2(0.05)^2\right]\omega^2 - 4(9.81)(1)$$

$\omega = 41.8 \text{ rad/s}$ **Ans**

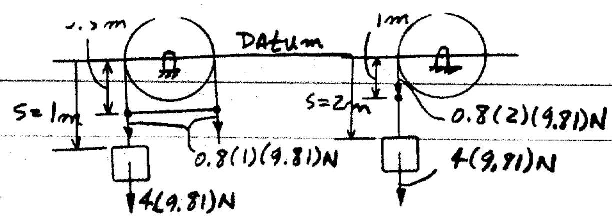
18-55. Solve Prob. 18-54 if the chain has a mass of 0.8 kg/m. For the calculation neglect the portion of the chain that wraps over the sprocket.



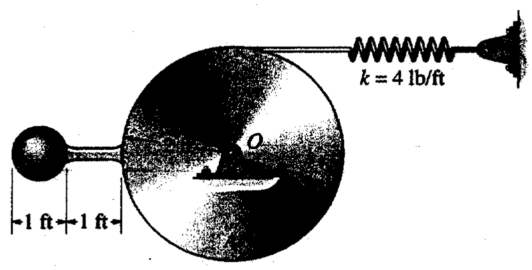
$T_1 + V_1 = T_2 + V_2$

$$0 - 4(9.81)(1) - 2\left[0.8(1)(9.81)(0.5)\right] = \frac{1}{2}(4)(0.1\omega)^2 + \frac{1}{2}\left[2(0.05)^2\right]\omega^2 + \frac{1}{2}(0.8)(2)(0.1\omega)^2$$
$$-4(9.81)(2) - 0.8(2)(9.81)(1)$$

$\omega = 39.3$ rad/s **Ans**



18-56. The disk A is pinned at O and weighs 15 lb. A 1-ft rod weighing 2 lb and a 1-ft-diameter sphere weighing 10 lb are welded to the disk, as shown. If the spring is orginally stretched 1 ft and the sphere is released from the position shown, determine the angular velocity of the disk when it has rotated 90°.



$T_1 + V_1 = T_2 + V_2$

$$[0+0+0] + \frac{1}{2}(4)(1)^2 = \frac{1}{2}[\frac{1}{2}(\frac{15}{32.2})(2)^2]\omega^2 + \frac{1}{2}[\frac{1}{12}(\frac{2}{32.2})(1)^2]\omega^2 + \frac{1}{2}(\frac{2}{32.2})(v_G)_R^2$$
$$+\frac{1}{2}[\frac{2}{5}(\frac{10}{32.2})(0.5)^2]\omega^2 + \frac{1}{2}(\frac{10}{32.2})(v_G)_S^2 - 2(2.5) - 10(3.5) + \frac{1}{2}(4)(1+2(\frac{\pi}{2}))^2$$

Since

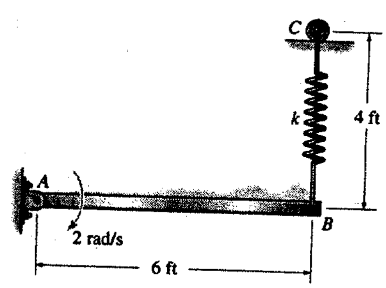
$(v_G)_S = 3.5\omega$

$(v_G)_R = 2.5\omega$

Substituting and solving, yields

$\omega = 1.73$ rad/s **Ans**

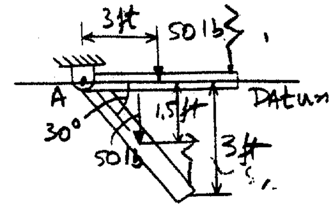
18-57. At the instant shown, the 50-lb bar is rotating downwards at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at C. If the spring has an unstretched length of 2 ft and a stiffness of $k = 6$ lb/ft, determine the angular velocity of the bar the instant it has rotated downward 30° below the horizontal.



Datum through A.

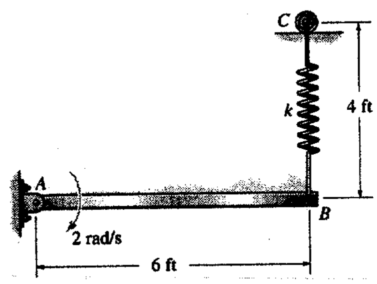
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right](2)^2 + \frac{1}{2}(6)(4-2)^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right]\omega^2 + \frac{1}{2}(6)(7-2)^2 - 50(1.5)$$



$\omega = 2.30$ rad/s **Ans**

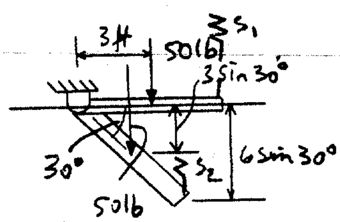
18-58. At the instant shown, the 50-lb bar is rotating downwards at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at C. If the spring has an unstretched length of 2 ft and a stiffness of $k = 12$ lb/ft, determine the angle θ, measured below the horizontal, to which the bar rotates before it momentarily stops.



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right](2)^2 + \frac{1}{2}(12)(4-2)^2 = 0 + \frac{1}{2}(12)(4 + 6\sin\theta - 2)^2 - 50(3\sin\theta)$$

$$61.2671 = 24(1 + 3\sin\theta)^2 - 150\sin\theta$$



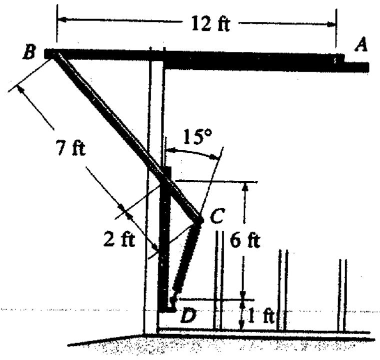
$$37.2671 = -6\sin\theta + 216\sin^2\theta$$

Set $x = \sin\theta$, and solve the quadratic equation for the positive root:

$$\sin\theta = 0.4295$$

$\theta = 25.4°$ **Ans**

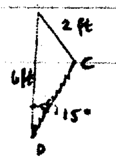
18-59. The end A of the garage door AB travels along the horizontal track, and the end of member BC is attached to a spring at C. If the spring is originally unstretched, determine the stiffness k so that when the door falls downward from rest in the position shown, it will have zero angular velocity the moment it closes, i.e., when it and BC become vertical. Neglect the mass of member BC and assume the door is a thin plate having a weight of 200 lb and a width and height of 12 ft. There is similar connection and spring on the other side of the door.



$$(2)^2 = (6)^2 + (CD)^2 - 2(6)(CD)\cos 15°$$

$$CD^2 - 11.591CD + 32 = 0$$

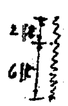
Selecting the smaller root:



$$CD = 4.5352 \text{ ft}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + 2[\frac{1}{2}(k)(8 - 4.5352)^2] - 200(6)$$



$k = 100$ lb/ft Ans