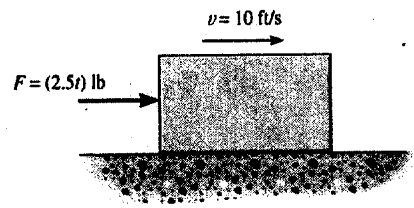
13-1. The moon has a mass of $73.5(10^{21})$ kg, and the earth has a mass of $5.98(10^{24})$ kg. If their centers are $384(10^6)$ m apart, determine the gravitational attractive force between the two bodies.

$$F = G\frac{m_1 m_2}{r^2}$$

$$F = 6.673(10^{-11})\left[\frac{73.5(10^{21})(5.98(10^{24}))}{(384(10^6))^2}\right]$$

$F = 199(10^{18})$ N **Ans**

13-2. The 10-lb block has an initial velocity of 10 ft/s on the smooth plane. If a force $F = (2.5t)$ lb, where t is in seconds, acts on the block for 3 s, determine the final velocity of the block and the distance the block travels during this time.



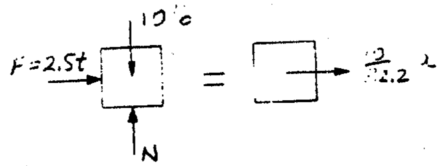
$\xrightarrow{+} \Sigma F_x = ma_x;\quad 2.5t = \left(\frac{10}{32.2}\right)a$

$a = 8.05t$

$dv = a\,dt$

$\int_{10}^{v} dv = \int_0^t 8.05t\,dt$

$v = 4.025t^2 + 10$



When $t = 3$ s,

$v = 46.2$ ft/s **Ans**

$ds = v\,dt$

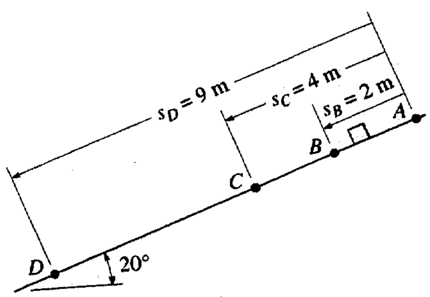
$\int_0^s ds = \int_0^t (4.025t^2 + 10)\,dt$

$s = 1.3417t^3 + 10t$

When $t = 3$ s,

$s = 66.2$ ft **Ans**

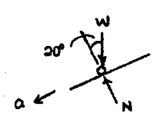
13-3. By using an inclined plane to retard the motion of a falling object, and thus make the observations more accurate, Galileo was able to determine experimentally that the distance through which an object moves in free fall is proportional to the square of the time for travel. Show that this is the case, i.e., $s \propto t^2$, by determining the time t_B, t_C, and t_D needed for a block of mass m to slide from rest at A to points B, C, and D, respectively. Neglect the effects of friction.



$W \sin 20° = \frac{W}{g}a$

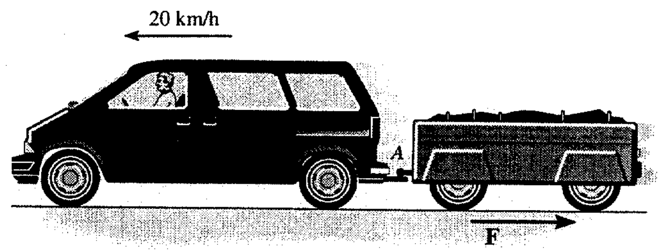
$a = 9.81(\sin 20°) = 3.355$ m/s^2

$s = \frac{1}{2}at^2$

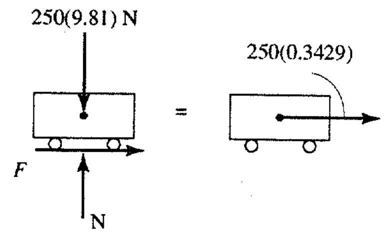


s	t	
2 m	1.09 s	**Ans**
4 m	1.54 s	**Ans**
9 m	2.32 s	**Ans**

***13-4.** The van is traveling at 20 km/h when the coupling of the trailer at A fails. If the trailer has a mass of 250 kg and coasts 45 m before coming to rest, determine the constant horizontal force F created by rolling friction which causes the trailer to stop.



$$20 \text{ km/h} = \frac{20(10^3)}{3600} = 5.556 \text{ m/s}$$



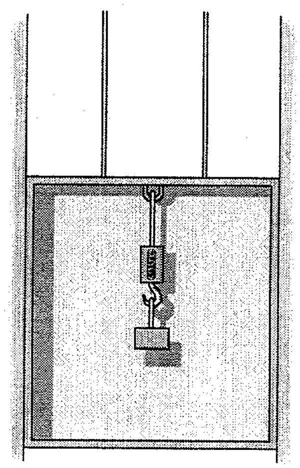
$$(\overset{+}{\leftarrow}) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = 5.556^2 + 2(a)(45 - 0)$$

$$a = -0.3429 \text{ m/s}^2 = 0.3429 \text{ m/s}^2 \rightarrow$$

$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad F = 250(0.3429) = 85.7 \text{ N} \quad \textbf{Ans}$$

13-5. A block having a mass of 2 kg is placed on a spring scale located in an elevator that is moving downward. If the scale reading, which measures the force in the spring, is 20 N, determine the acceleration of the elevator. Neglect the mass of the scale.

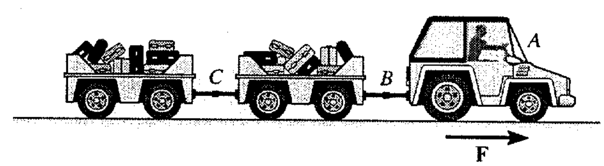


$$+\uparrow \sum F_y = ma_y; \quad 20 - 2(9.81) = 2a$$

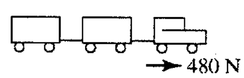
$$a = 0.19 \text{ m/s}^2 \uparrow \qquad \textbf{Ans}$$

The elevator is slowing down.

13-6. The baggage truck A has a mass of 800 kg and is used to pull the two cars, each with mass 300 kg. If the tractive force F on the truck is $F = 480$ N, determine the initial acceleration of the truck. What is the acceleration of the truck if the coupling at C suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.

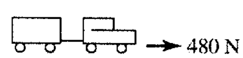


$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 480 = [800 + 2(300)]a$$



$$a = 0.3429 = 0.343 \text{ m/s}^2 \quad \textbf{Ans}$$

$$\overset{+}{\rightarrow} \sum F_x = ma_x: \quad 480 = (800 + 300)a$$

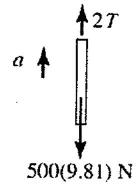


$$a = 0.436 \text{ m/s}^2 \qquad \textbf{Ans}$$

13-7. The 500-kg fuel assembly for a nuclear reactor is being lifted out from the core of the nuclear reactor using the pulley system shown. It is hoisted upward with a constant acceleration such that $s = 0$ and $v = 0$ when $t = 0$, and $s = 2.5$ m when $t = 1.5$ s. Determine the tension in the cable at A during the motion.

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$2.5 = 0 + 0 + \frac{1}{2}(a)(1.5)^2$

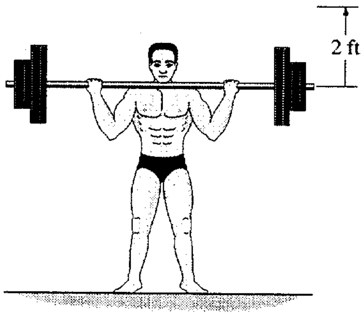


$a = 2.222\ \text{m/s}^2$

$+\uparrow \sum F_y = ma_y;\qquad 2T - 500(9.81) = 500(2.222)$

$T = 3008\ \text{N} = 3.01\ \text{kN}$ **Ans**

***13-8.** The man weighs 180 lb and supports the barbells which have a weight of 100 lb. If he lifts them 2 ft in the air in 1.5 s starting from rest, determine the reaction of *both* of his feet on the ground during the lift. Assume the motion is with uniform acceleration.

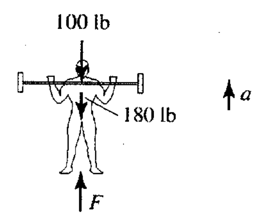


$+\uparrow \sum F_y = ma_y;\qquad F - 100 - 180 = \frac{100}{32.2}a$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$2 = 0 + 0 + \frac{1}{2}a(1.5)^2$

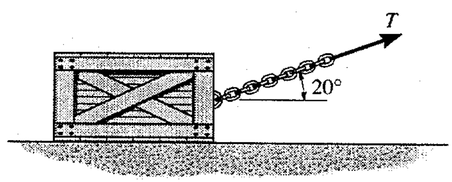
$a = 1.778\ \text{ft/s}^2$



Thus,

$F = 285.52 = 286\ \text{lb}$ **Ans**

13-9. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. If the magnitude of **T** is increased until the crate begins to slide, determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



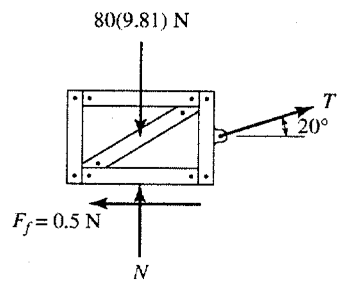
Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.5N$. From FBD(a)

$+\uparrow \sum F_y = 0;\quad N + P\sin 20° - 80(9.81) = 0\quad [1]$

$\xrightarrow{+} \sum F_x = 0;\quad T\cos 20° - 0.5N = 0\quad [2]$

Solving Eqs. [1] and [2] yields

$T = 353.29\ \text{N}\qquad N = 663.97\ \text{N}$



(a)

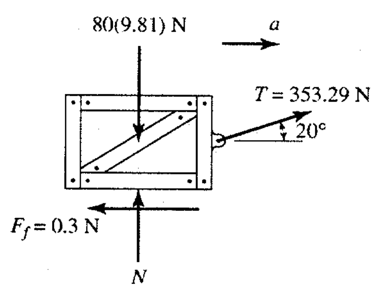
Equation of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

$+\uparrow \sum F_y = ma_y;\quad N - 80(9.81) + 353.29\sin 20° = 80(0)$

$N = 663.97\ \text{N}$

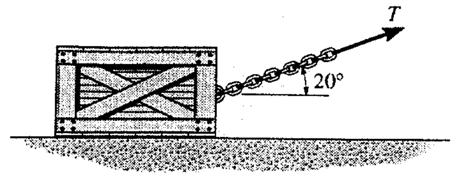
$\xrightarrow{+} \sum F_x = ma_x;\quad 353.29\cos 20° - 0.3(663.97) = 80a$

$a = 1.66\ \text{m/s}^2$ **Ans**



(b)

13-10. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. Determine the crate's acceleration in $t = 2$ s if the coefficient of static friction is $\mu_s = 0.4$ and the coefficient of kinetic friction is $\mu_k = 0.3$ and the towing force is $T = (90t^2)$ N, where t is in seconds.



Equations of Equilibrium: At $t = 2$ s, $T = 90(2^2) = 360$ N. From FBD(a)

$$+\uparrow \sum F_y = 0; \qquad N + 360\sin 20° - 80(9.81) = 0 \qquad N = 661.67 \text{ N}$$

$$\overset{+}{\rightarrow} \sum F_x = 0; \qquad 360\cos 20° - F_f = 0 \qquad F_f = 338.29 \text{ N}$$

Since $F_f > (F_f)_{max} = \mu_s N = 0.4(661.67) = 264.67$ N, the crate accelerates.

Equation of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

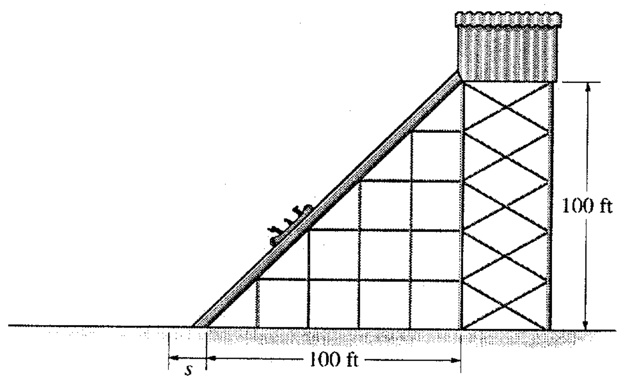
$$+\uparrow \sum F_y = ma_y; \quad N - 80(9.81) + 360\sin 20° = 80(0)$$

$$N = 661.67 \text{ N}$$

$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 360\cos 20° - 0.3(661.67) = 80a$$

$$a = 1.75 \text{ m/s}^2$$ **Ans**

13-11. The water-park ride consists of an 800-lb sled which slides from rest down the incline and then into the pool. If the frictional resistance on the incline is $F_r = 30$ lb, and in the pool for a short distance $F_r = 80$ lb, determine how fast the sled is traveling when $s = 5$ ft.



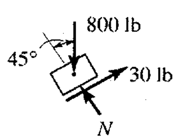
$$+\swarrow \sum F_x = ma_x; \qquad 800\sin 45° - 30 = \frac{800}{32.2}a$$

$$a = 21.561 \text{ ft/s}^2$$

$$v_1^2 = v_0^2 + 2a_c(s - s_0)$$

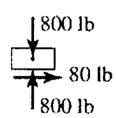
$$v_1^2 = 0 + 2(21.561)(100\sqrt{(2-0)})$$

$$v_1 = 78.093 \text{ ft/s}$$



$$\overset{+}{\leftarrow} \sum F_x = ma_x; \qquad -80 = \frac{800}{32.2}a$$

$$a = -3.22 \text{ ft/s}^2$$

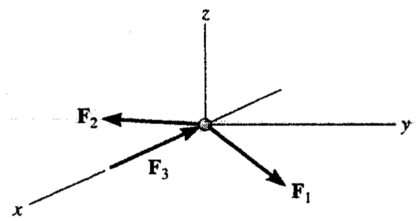


$$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$$

$$v_2^2 = (78.093)^2 + 2(-3.22)(5 - 0)$$

$$v_2 = 77.9 \text{ ft/s}$$ **Ans**

***13-12.** The 6-lb particle is subjected to the action of its weight and forces $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{-2t\mathbf{i}\}$ lb, where t is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.



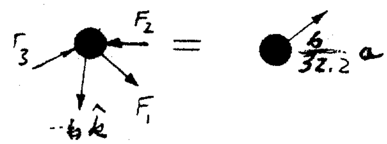
$\Sigma \mathbf{F} = m\mathbf{a};\qquad (2\mathbf{i}+6\mathbf{j}-2t\mathbf{k}) + (t^2\mathbf{i}-4t\mathbf{j}-1\mathbf{k}) - 2t\mathbf{i} - 6\mathbf{k} = \left(\frac{6}{32.2}\right)(a_x\mathbf{i}+a_y\mathbf{j}+a_z\mathbf{k})$

Equating components :

$$\left(\frac{6}{32.2}\right)a_x = t^2-2t+2 \qquad \left(\frac{6}{32.2}\right)a_y = -4t+6 \qquad \left(\frac{6}{32.2}\right)a_z = -2t-7$$

Since $dv = a\,dt$, integrating from $v=0$, $t=0$, yields

$$\left(\frac{6}{32.2}\right)v_x = \frac{t^3}{3}-t^2+2t \qquad \left(\frac{6}{32.2}\right)v_y = -2t^2+6t \qquad \left(\frac{6}{32.2}\right)v_z = -t^2-7t$$



Since $ds = v\,dt$, integrating from $s=0$, $t=0$ yields

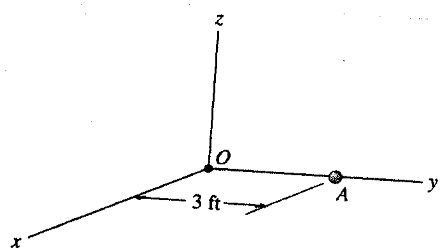
$$\left(\frac{6}{32.2}\right)s_x = \frac{t^4}{12}-\frac{t^3}{3}+t^2 \qquad \left(\frac{6}{32.2}\right)s_y = -\frac{2t^3}{3}+3t^2 \qquad \left(\frac{6}{32.2}\right)s_z = -\frac{t^3}{3}-\frac{7t^2}{2}$$

When $t = 2$ s then, $\quad s_x = 14.31$ ft, $\quad s_y = 35.78$ ft $\quad s_z = -89.44$ ft

Thus,

$s = \sqrt{(14.31)^2+(35.78)^2+(-89.44)^2} = 97.4$ ft $\qquad$ **Ans**

13-13. The 2-lb particle A is acted upon by its weight and the force system $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{3\mathbf{i} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{1\mathbf{i} - t^2\mathbf{j} - 2\mathbf{k}\}$ lb, where t is in seconds. Determine the distance the particle is from the origin 3 s after it has been released from rest.

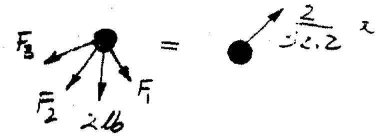


$\Sigma F_x = ma_x;\qquad 2+3+1 = \left(\frac{2}{32.2}\right)a_x$

$\Sigma F_y = ma_y;\qquad 6-t^2 = \left(\frac{2}{32.2}\right)a_y$

$\Sigma F_z = ma_z;\qquad -2-1-2-2 = \left(\frac{2}{32.2}\right)a_z$

Since $dv = a\,dt$, integrating from $v = 0$ when $t = 0$,



$$\left(\frac{2}{32.2}\right)v_x = 6t \qquad \left(\frac{2}{32.2}\right)v_y = 6t-\frac{t^3}{3} \qquad \left(\frac{2}{32.2}\right)v_z = -7t$$

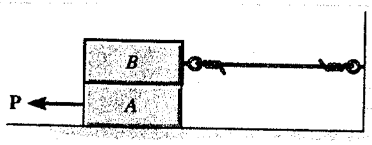
Since $ds = v\,dt$, integrating from $s_x = s_z = 0$, $s_y = 3$ ft when $t = 0$,

$$\left(\frac{2}{32.2}\right)s_x = 3t^2 \qquad \left(\frac{2}{32.2}\right)s_y = 3t^2-\frac{t^4}{12}+0.186\,34 \qquad \left(\frac{2}{32.2}\right)s_z = -\frac{7}{2}t^2$$

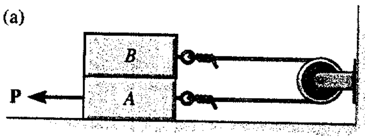
When $t = 3$ s, $\quad s_x = 434.70$ ft $\quad s_y = 329.025$ ft $\quad s_z = -507.15$ ft

$s = \sqrt{(434.70)^2+(329.25)^2+(-507.15)^2} = 745$ ft $\qquad$ **Ans**

13-14. Each of the two blocks has a mass m. The coefficient of kinetic friction at all surfaces of contact is μ. If a horizontal force **P** moves the bottom block, determine the acceleration of the bottom block in each case.



(a)

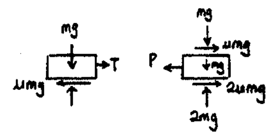


(b)

(a) **Block A :**

$\overset{+}{\leftarrow}\Sigma F_x = m\,a_x;\quad P - 3\mu mg = m\,a_A$

$a_A = \dfrac{P}{m} - 3\mu g$ **Ans**



(b) $s_B + s_A = l$

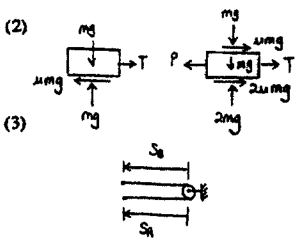
$a_A = -\,a_B$ (1)

Block A :

$\overset{+}{\leftarrow}\Sigma F_x = m\,a_x;\quad P - T - 3\mu mg = m\,a_A$ (2)

Block B :

$\overset{+}{\leftarrow}\Sigma F_x = m\,a_x;\quad \mu mg - T = m\,a_B$ (3)

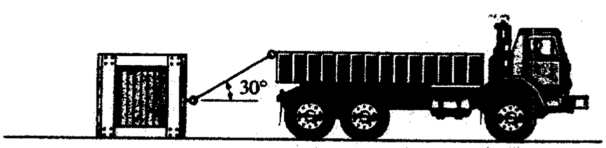


Subtract Eq.(3) from Eq.(2) :

$$P - 4\mu mg = m(a_A - a_B)$$

Use Eq.(1); $a_A = \dfrac{P}{2m} - 2\mu g$ **Ans**

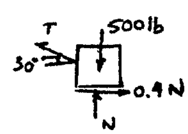
13-15. The driver attempts to tow the crate using a rope that has a tensile strength of 200 lb. If the crate is originally at rest and has a weight of 500 lb, determine the greatest acceleration it can have if the coefficient of static friction between the crate and the road is $\mu_s = 0.4$, and the coefficient of kinetic friction is $\mu_k = 0.3$.



Equilibrium : In order to slide the crate, the towing force must overcome static friction.

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad -T\cos 30° + 0.4N = 0$ [1]

$+\uparrow \Sigma F_y = 0;\quad N + T\sin 30° - 500 = 0$ [2]



Solving Eqs.[1] and [2] yields :

$T = 187.6$ lb $\quad N = 406.2$ lb

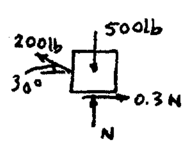
Since $T < 200$ lb, the cord will not break at the moment the crate slides.

After the crate begins to slide, the kinetic friction is used for the calculation.

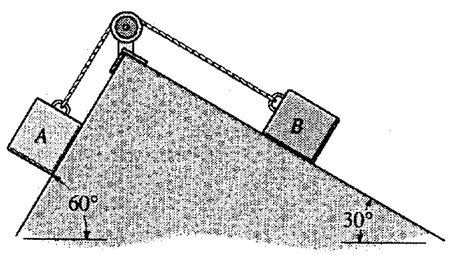
$+\uparrow \Sigma F_y = ma_y;\quad N + 200\sin 30° - 500 = 0 \quad N = 400$ lb

$\overset{+}{\rightarrow}\Sigma F_x = ma_x;\quad 200\cos 30° - 0.3(400) = \dfrac{500}{32.2}a$

$a = 3.43$ ft/s^2 **Ans**



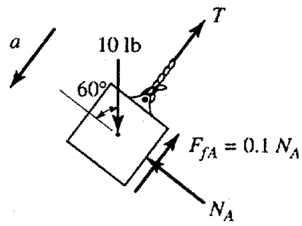
***13-16.** The double inclined plane supports two blocks A and B, each having a weight of 10 lb. If the coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.1$, determine the acceleration of each block.



Equation of Motion: Since blocks A and B are sliding along the plane, the friction forces developed between the blocks and the plane are $(F_f)_A = \mu_k N_A = 0.1 N_A$ and $(F_f)_B = \mu_k N_B = 0.1 N_B$. Here, $a_A = a_B = a$. Applying Eq. 13-7 to FBD(a), we have

$$\nwarrow + \sum F_{y'} = ma_{y'}; \quad N_A - 10\cos 60° = \left(\frac{10}{32.2}\right)(0) \quad N_A = 5.00 \text{ lb}$$

$$\nearrow + \sum F_{x'} = ma_{x'}; \quad T + 0.1(5.00) - 10\sin 60° = -\left(\frac{10}{32.2}\right)a \quad [1]$$

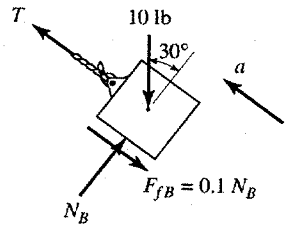


(a)

From FBD(b),

$$\nearrow + \sum F_{y'} = ma_{y'}; \quad N_B - 10\cos 30° = \left(\frac{10}{32.2}\right)(0) \quad N_B = 8.660 \text{ lb}$$

$$\nwarrow + \sum F_{x'} = ma_{x'}; \quad T - 0.1(8.660) - 10\sin 30° = \left(\frac{10}{32.2}\right)a \quad [2]$$



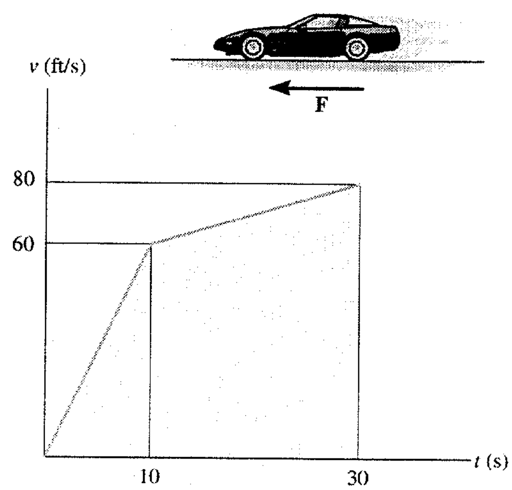
(b)

Solving Eqs. [1] and [2] yields

$a = 3.69$ ft/s^2 **Ans**

$T = 7.013$ lb

13-17. The speed of the 3500-lb sports car is plotted over the 30-s time period. Plot the variation of the traction force **F** needed to cause the motion.



Kinematics: For $\mathbf{0 \le t < 10}$ **s**, $v = \dfrac{60}{10}t = \{6t\}$ ft/s. Applying equation $a = \dfrac{dv}{dt}$, we have

$$a = \frac{dv}{dt} = 6 \text{ ft/s}^2$$

For $\mathbf{10 < t \le 30}$ **s**, $\dfrac{v - 60}{t - 10} = \dfrac{80 - 60}{30 - 10}$, $v = \{t + 50\}$ ft/s. Applying equation $a = \dfrac{dv}{dt}$, we have

$$a = \frac{dv}{dt} = 1 \text{ ft/s}^2$$

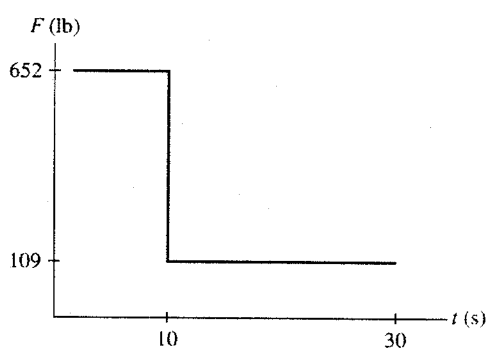
Equation of Motion:

For $\mathbf{0 \le t < 10}$ **s**

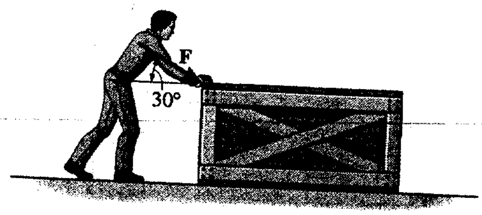
$$\overset{+}{\leftarrow} \sum F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(6) = 652 \text{ lb} \quad \textbf{Ans}$$

For $\mathbf{10 < t \le 30}$ **s**

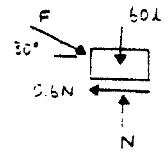
$$\overset{+}{\leftarrow} \sum F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(1) = 109 \text{ lb} \quad \textbf{Ans}$$



13-18. The man pushes on the 60-lb crate with a force **F**. The force is always directed down at 30° from the horizontal as shown, and its magnitude is increased until the crate begins to slide. Determine the crate's initial acceleration if the static coefficient of friction is $\mu_s = 0.6$ and the kinetic coefficient of friction is $\mu_k = 0.3$.



Force to produce motion :

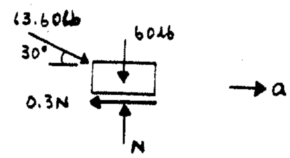


$\xrightarrow{+} \Sigma F_x = 0; \quad F\cos 30° - 0.6N = 0$

$+\uparrow \Sigma F_y = 0; \quad N - 60 - F\sin 30° = 0$

$N = 91.80 \text{ lb} \qquad F = 63.60 \text{ lb}$

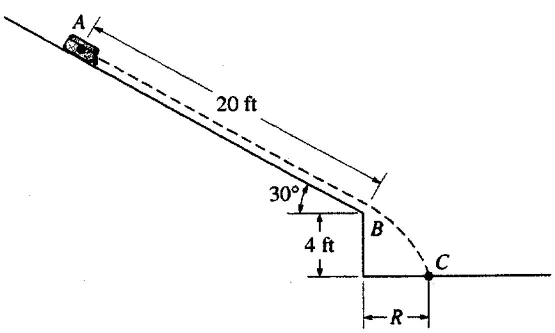
Since $N = 91.80$ lb,

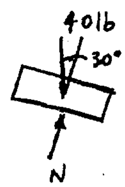


$\xrightarrow{+} \Sigma F_x = ma_x; \quad 63.60\cos 30° - 0.3(91.80) = \left(\frac{60}{32.2}\right)a$

$a = 14.8 \text{ ft/s}^2$ **Ans**

13-19. A 40-lb suitcase slides from rest 20 ft down the smooth ramp. Determine the point where it strikes the ground at C. How long does it take to go from A to C?





$+\searrow \Sigma F_x = m\,a_x; \quad 40\sin 30° = \frac{40}{32.2}a$

$a = 16.1 \text{ ft/s}^2$

$(+\searrow)\; v^2 = v_0^2 + 2a_c(s - s_0);$

$v_B^2 = 0 + 2(16.1)(20)$

$v_B = 25.38 \text{ ft/s}$

$(+\searrow)\; v = v_0 + a_c t;$

$25.38 = 0 + 16.1\, t_{AB}$

$t_{AB} = 1.576 \text{ s}$

$(\xrightarrow{+})\; s_x = (s_x)_0 + (v_x)_0\, t$

$R = 0 + 25.38\cos 30°\,(t_{BC})$

$(+\downarrow)\; s_y = (s_y)_0 + (v_y)_0\, t + \frac{1}{2}a_c t^2$

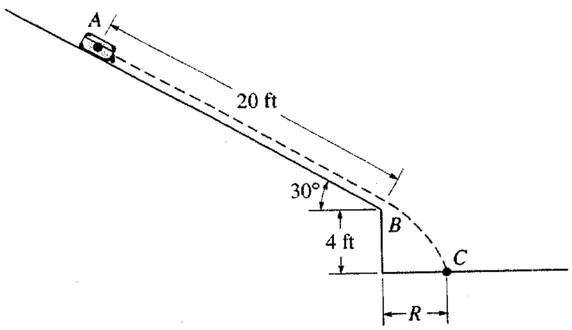
$4 = 0 + 25.38\sin 30°\, t_{BC} + \frac{1}{2}(32.2)(t_{BC})^2$

$t_{BC} = 0.2413 \text{ s}$

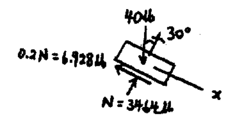
$R = 5.30 \text{ ft}$ **Ans**

Total time $= t_{AB} + t_{BC} = 1.82 \text{ s}$ **Ans**

***13-20.** Solve Prob. 13–19 if the suitcase has an initial velocity down the ramp of $v_A = 10$ ft/s and the coefficient of kinetic friction along AB is $\mu_k = 0.2$.



$+\searrow \Sigma F_x = m\,a_x;\quad 40 \sin 30^\circ - 6.928 = \dfrac{40}{32.2}a$



$a = 10.52 \text{ ft/s}^2$

$(+\searrow)\; v^2 = v_0^2 + 2a_c(s - s_0);$

$v_B^2 = (10)^2 + 2(10.52)(20)$

$v_B = 22.82$ ft/s

$(+\searrow)\; v = v_0 + a_c t;$

$22.82 = 10 + 10.52\, t_{AB}$

$t_{AB} = 1.219$ s

$(\overset{+}{\rightarrow})\; s_x = (s_x)_0 + (v_x)_0\, t$

$R = 0 + 22.82 \cos 30^\circ\, (t_{BC})$

$(+\downarrow)\; s_y = (s_y)_0 + (v_y)_0\, t + \dfrac{1}{2}a_c t^2$

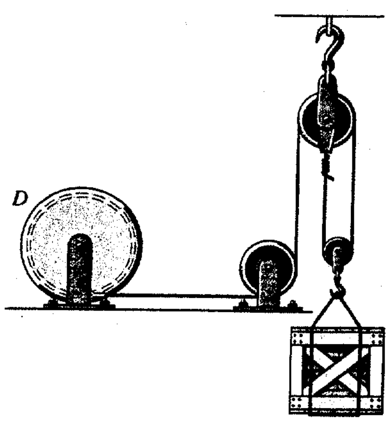
$4 = 0 + 22.82 \sin 30^\circ\, t_{BC} + \dfrac{1}{2}(32.2)(t_{BC})^2$

$t_{BC} = 0.2572$ s

$R = 5.08$ ft **Ans**

Total time $= t_{AB} + t_{BC} = 1.48$ s **Ans**

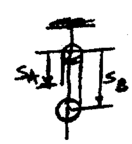
13-21. The winding drum D is drawing in the cable at an accelerated rate of 5 m/s^2. Determine the cable tension if the suspended crate has a mass of 800 kg.



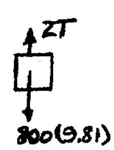
$s_A + 2s_B = l$

$a_A = -2a_B$

$5 = -2a_B$



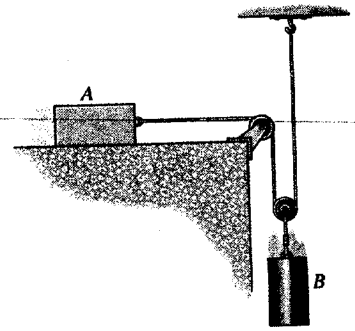
$a_B = -2.5 \text{ m/s}^2 = 2.5 \text{ m/s}^2 \uparrow$



$+\uparrow \Sigma F_y = m a_y;\quad 2T - 800(9.81) = 800(2.5)$

$T = 4924 \text{ N} = 4.92 \text{ kN}$ **Ans**

13-22. The 10-lb block A is traveling to the right at $v_A = 2$ ft/s at the instant shown. If the coefficient of kinetic friction is $\mu_k = 0.2$ between the surface and A, determine the velocity of A when it has moved 4 ft. Block B has a weight of 20 lb.

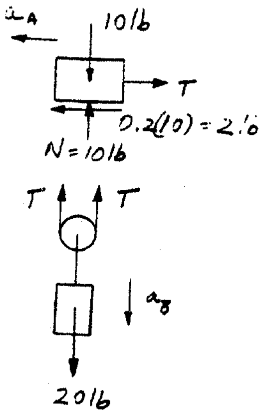


Block A :

$\overset{+}{\leftarrow} \Sigma F_x = ma_x;$ $\quad -T + 2 = \left(\dfrac{10}{32.2}\right)a_A \quad (1)$

Weight B :

$+\downarrow \Sigma F_y = ma_y;$ $\quad 20 - 2T = \left(\dfrac{20}{32.2}\right)a_B \quad (2)$



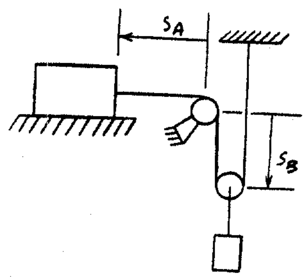
Kinematics :

$s_A + 2s_B = l$

$a_A = -2a_B \quad (3)$

Solving Eq.s (1) – (3) :

$a_A = -17.173$ ft/s^2 $\quad a_B = 8.587$ ft/s^2 $\quad T = 7.33$ lb

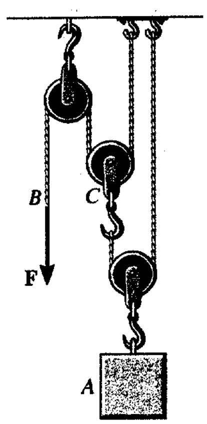


$v^2 = v_0^2 + 2a_c(s - s_0)$

$v^2 = (2)^2 + 2(17.173)(4 - 0)$

$v = 11.9$ ft/s **Ans**

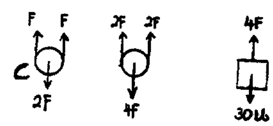
13-23. A force $F = 15$ lb is applied to the cord. Determine how high the 30-lb block A rises in 2 s starting from rest. Neglect the weight of the pulleys and cord.



$+\uparrow \Sigma F_y = ma_y;$ $\quad -30 + 4F = \dfrac{30}{32.2}a_A$

$F = 15$ lb

$a_A = 32.2$ ft/s^2

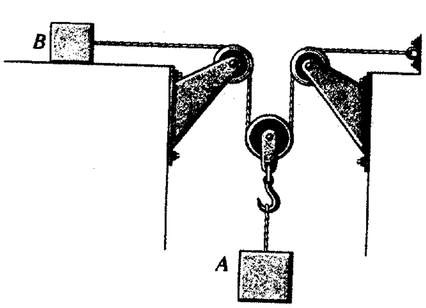


$(+\uparrow)\ s = s_0 + v_0 t + \dfrac{1}{2}a_c t^2$

$s = 0 + 0 + \dfrac{1}{2}(32.2)(2)^2$

$s = 64.4$ ft **Ans**

***13-24.** At a given instant the 10-lb block A is moving downward with a speed of 6 ft/s. Determine its speed 2 s later. Block B has a weight of 4 lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of the pulleys and cord.



Block A :

$+\downarrow \Sigma F_y = ma_y;\quad 10 - 2T = \frac{10}{32.2}a_A$

Block B :

$\overset{+}{\leftarrow} \Sigma F_x = ma_x;\quad -T + 0.2(4) = \frac{4}{32.2}a_B$

$2s_A + s_B = l$

$2a_A = -a_B$

Solving:

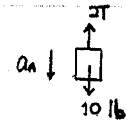
$T = 3.38$ lb

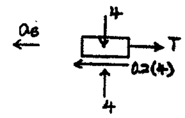
$a_A = 10.403$ ft/s^2

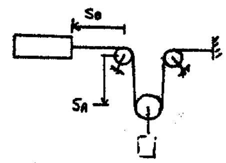
$a_B = -20.81$ ft/s^2

$(+\downarrow)\ v_A = (v_A)_0 + a_A t$

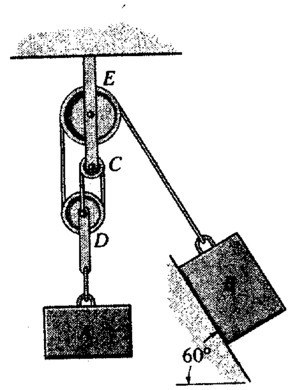
$v_A = 6 + 10.403(2) = 26.8$ ft/s **Ans**





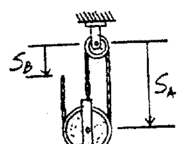


13-25. Determine the required mass of block A so that when it is released from rest it moves the 5-kg block B 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.



Kinematic : Applying equation $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$, we have

$$(\ +)\quad 0.75 = 0 + 0 + \frac{1}{2}a_B(2^2)\qquad a_B = 0.375\ \text{m/s}^2$$

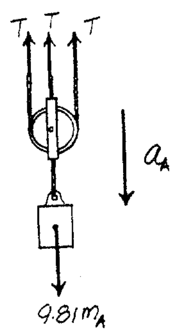


Establish the position - coordinate equation, we have

$$2s_A + (s_A - s_B) = l\qquad 3s_A - s_B = l$$

Taking time derivative twice yields

$$3a_A - a_B = 0 \qquad [1]$$



(a)

From Eq. [1],

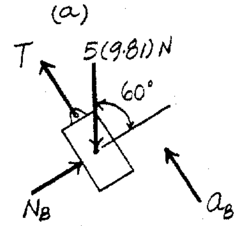
$$3a_A - 0.375 = 0\qquad a_A = 0.125\ \text{m/s}^2$$

Equation of Motion : The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(b),

$$+\Sigma F_{y'} = ma_{y'};\qquad T - 5(9.81)\sin 60° = 5(0.375)$$
$$T = 44.35\ \text{N}$$

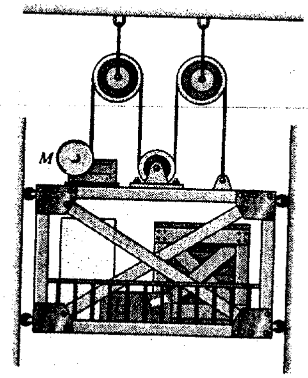
From FBD(a),

$$+\uparrow \Sigma F_y = ma_y;\qquad 3(44.35) - 9.81m_A = m_A(-0.125)$$
$$m_A = 13.7\ \text{kg}$$ **Ans**



(b)

13-26. A freight elevator, including its load, has a mass of 500 kg. It is prevented from rotating by using the track and wheels mounted along its sides. Starting from rest, in $t = 2$ s, the motor M draws in the cable with a speed of 6 m/s, *measured relative to the elevator.* Determine the constant acceleration of the elevator and the tension in the cable. Neglect the mass of the pulleys and cables.



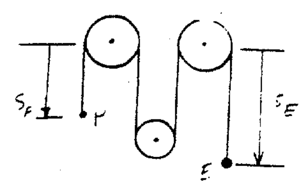
$$3s_E + s_P = l$$

$$3v_E = -v_P$$

$$(+\downarrow) \qquad v_P = v_E + v_{P/E}$$

$$-3v_E = v_E + 6$$

$$v_E = -\frac{6}{4} = -1.5 \text{ m/s} = 1.5 \text{ m/s} \uparrow$$



$$(+\uparrow) \qquad v = v_0 + a_c t$$

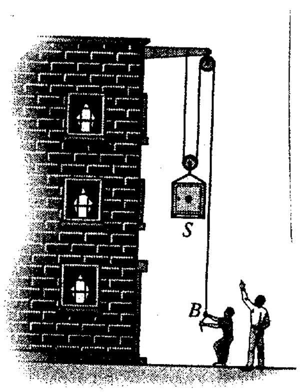
$$1.5 = 0 + a_E(2)$$

$$a_E = 0.75 \text{ m/s}^2 \uparrow \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = ma_y; \qquad 4T - 500(9.81) = 500(0.75)$$

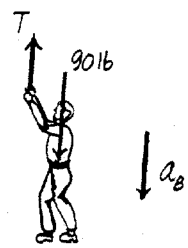
$$T = 1320 \text{ N} = 1.32 \text{ kN} \qquad \textbf{Ans}$$

13-27. The safe S has a weight of 200 lb and is supported by the rope and pulley arrangement shown. If the end of the rope is given to a boy B of weight 90 lb, determine his acceleration if in the confusion he doesn't let go of the rope. Neglect the mass of the pulleys and rope.



Equation of Motion : The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(a),

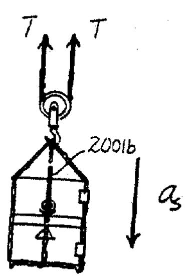
$$+\uparrow \Sigma F_y = ma_y; \qquad T - 90 = -\left(\frac{90}{32.2}\right)a_B \qquad [1]$$



(a)

From FBD(b),

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 200 = -\left(\frac{200}{32.2}\right)a_S \qquad [2]$$



(b)

Kinematic : Establish the position - coordinate equation, we have

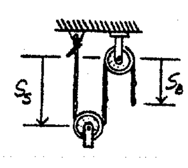
$$2s_S + s_B = l$$

Taking time derivative twice yields

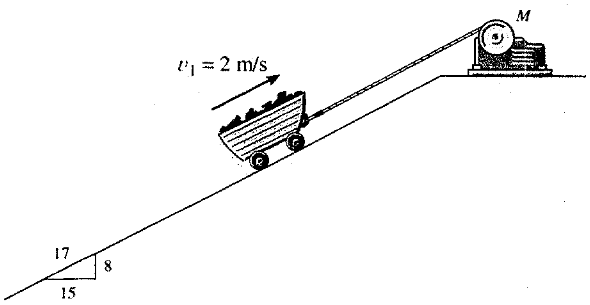
$$(+\downarrow) \qquad 2a_S + a_B = 0 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields

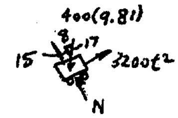
$$a_B = -2.30 \text{ ft/s}^2 = 2.30 \text{ ft/s}^2 \uparrow \qquad \textbf{Ans}$$
$$a_S = 1.15 \text{ ft/s}^2 \downarrow \qquad T = 96.43 \text{ lb}$$



***13-28.** The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s when $t = 0$, determine its velocity when $t = 2$ s.



$\nearrow + \Sigma F_{x'} = ma_{x'};\quad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$

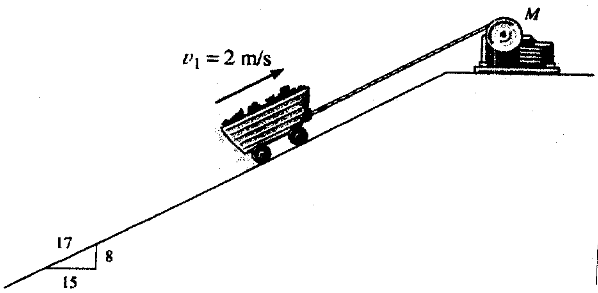


$dv = a\,dt$

$\int_2^v dv = \int_0^2 \left(8t^2 - 4.616\right) dt$

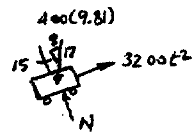
$v = 14.1$ m/s Ans

13-29. The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s at $s = 0$ and $t = 0$, determine the distance it moves up the plane when $t = 2$ s.



$\nearrow + \Sigma F_{x'} = ma_{x'};\quad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$

$dv = a\,dt$



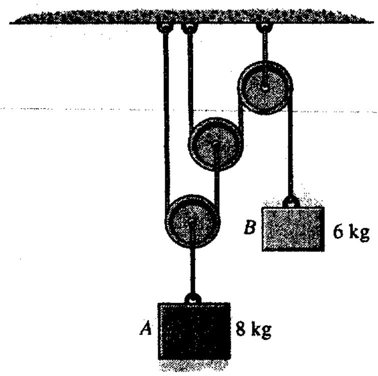
$\int_2^v dv = \int_0^t \left(8t^2 - 4.616\right) dt$

$v = \frac{ds}{dt} = 2.667t^3 - 4.616t + 2$

$\int_0^s ds = \int_0^2 \left(2.667t^3 - 4.616t + 2\right) dt$

$s = 5.43$ m Ans

13-30. Determine the tension developed in the cords attached to each block and the accelerations of the blocks. Neglect the mass of the pulleys and cords.



Equation of Motion : From FBD(a),

$+\uparrow \Sigma F_y = ma_y;$ $\qquad T_A - 8(9.81) = -8a_A$ $\qquad$ [1]

From FBD(b),

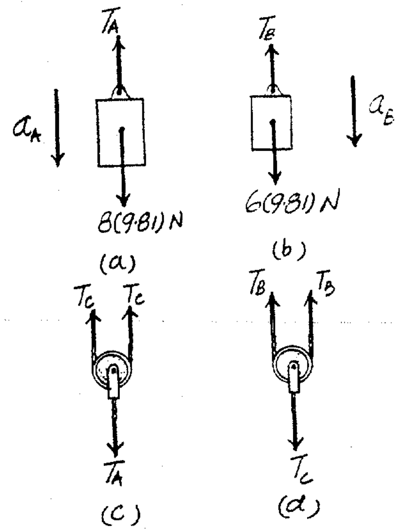
$+\uparrow \Sigma F_y = ma_y;$ $\qquad T_B - 6(9.81) = -6a_B$ $\qquad$ [2]

From FBD(c),

$+\uparrow \Sigma F_y = 0;$ $\qquad 2T_C - T_A = 0$ $\qquad$ [3]

From FBD(d),

$+\uparrow \Sigma F_y = 0;$ $\qquad 2T_B - T_C = 0$ $\qquad$ [4]

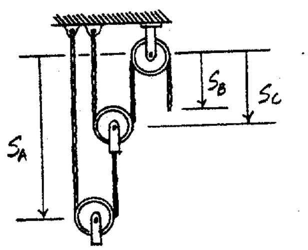


Eliminate T_C from Eqs. [3] and [4] yields

$$T_A = 4T_B \qquad [5]$$

Kinematic : Establish the position-coordinate equation, we have

$$s_A + (s_A - s_C) = l_1 \qquad [6]$$
$$2s_C + s_B = l_2 \qquad [7]$$



Eliminate s_C from Eqs. [6] and [7] yields

$$4s_A + s_B = 2l_1 + l_2 \qquad [8]$$

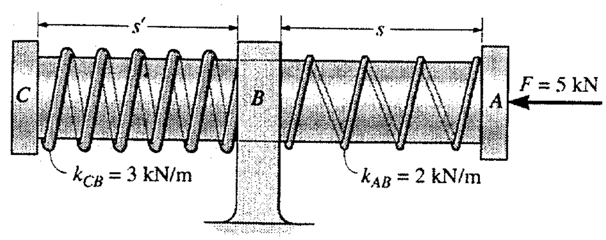
Taking time derivative twice for Eq. [8] yields

$$(+\downarrow) \qquad 4a_A + a_B = 0 \qquad [9]$$

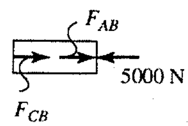
Solving Eqs. [1], [2], [5] and [9] yields

$a_A = -1.51\ \text{m/s}^2 = 1.51\ \text{m/s}^2 \uparrow$ $\qquad a_B = 6.04\ \text{m/s}^2 \downarrow$ $\qquad$ **Ans**
$T_A = 90.6\ \text{N}$ $\qquad T_B = 22.6\ \text{N}$ $\qquad$ **Ans**

13-31. The 2-kg shaft CA passes through a smooth journal bearing at B. Initially, the springs, which are coiled loosely around the shaft, are unstretched when no force is applied to the shaft. In this position $s = s' =$ 250 mm and the shaft is originally at rest. If a horizontal force of $F = 5$ kN is applied, determine the speed of the shaft at the instant $s = 50$ mm, $s' = 450$ mm. The ends of the springs are attached to the bearing at B and the caps at C and A.



$F_{CB} = k_{CB}x = 3000x \quad F_{AB} = k_{AB}x = 2000x$



$\overset{+}{\leftarrow} \sum F_x = ma_x; \qquad 5000 - 3000x - 2000x = 2a$

$2500 - 2500x = a$

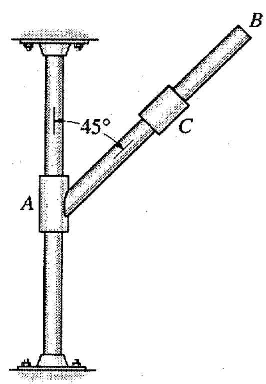
$a\,dx = v\,dv$

$$\int_0^{0.2} (2500 - 2500x)dx = \int_0^{v} v\,dv$$

$$2500(0.2) - \left(\frac{2500(0.2)^2}{2}\right) = \frac{v^2}{2}$$

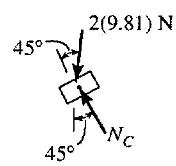
$v = 30$ m/s **Ans**

***13-32.** The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if (a) the shaft is fixed from moving, (b) collar A, which is fixed to shaft AB, moves downward at constant velocity along the vertical rod, and (c) collar A is subjected to a downward acceleration of 2 m/s². In all cases, the collar moves in the plane.



(a)

$+\swarrow \sum F_{x'} = ma_{x'}; \quad 2(9.81)\sin 45^\circ = 2a_C \quad a_C = 6.94 \text{ m/s}^2$ **Ans**

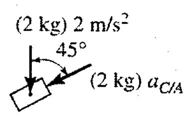


(b) From part (a) $\mathbf{a}_{C/A} = 6.94$ m/s²

$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A} \qquad$ Where $\mathbf{a}_A = 0$

$= 6.94 \text{ m/s}^2$ **Ans**

(c)



$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A}$

$= \underset{\downarrow}{2} + a_{C/A} \swarrow \quad [1]$

$+\swarrow \sum F_{x'} = ma_{x'}; \quad 2(9.81)\sin 45^\circ = 2(2\cos 45^\circ + a_{C/A})$

$a_{C/A} = 5.5225 \text{ m/s}^2 \swarrow$

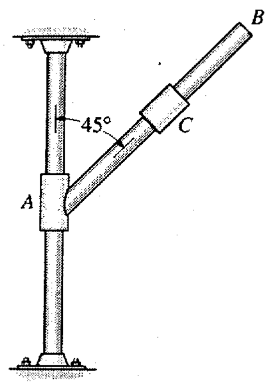
From Eq. [1]

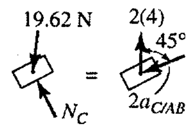
$$\mathbf{a}_C = \underset{\downarrow}{2} + \underset{\swarrow}{5.5225} = \underset{\leftarrow}{3.905} + \underset{\downarrow}{5.905}$$

$a_C = \sqrt{3.905^2 + 5.905^2} = 7.08 \text{ m/s}^2$ **Ans**

$\theta = \tan^{-1}\dfrac{5.905}{3.905} = 56.5^\circ$ **Ans**

13-33. The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if collar A is subjected to an upward acceleration of $4\ \text{m/s}^2$.





$\overset{+}{\leftarrow} \sum F_x = ma_x;\quad N \sin 45^\circ = 2a_{C/AB} \sin 45^\circ$

$N = 2a_{C/AB}$

$+\uparrow \sum F_y = ma_y;\quad N \cos 45^\circ - 19.62 = 2(4) - 2a_{C/AB} \cos 45^\circ$

$a_{C/AB} = 9.76514$

$\mathbf{a}_C = \mathbf{a}_{AB} + \mathbf{a}_{C/AB}$

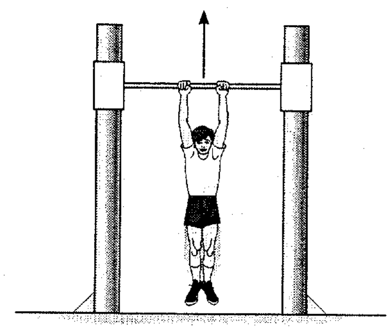
$(a_C)_x = 0 + 9.76514 \sin 45^\circ = 6.905 \leftarrow$

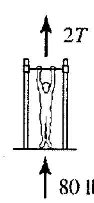
$(a_C)_y = 4 - 9.76514 \cos 45^\circ = 2.905 \downarrow$

$a_C = \sqrt{(6.905)^2 + (2.905)^2} = 7.49\ \text{m/s}^2$ **Ans**

$\theta = \tan^{-1}\left(\dfrac{2.905}{6.905}\right) = 22.8^\circ$ **Ans**

13-34. The boy having a weight of 80 lb hangs uniformly from the bar. Determine the force in each of his arms in $t = 2$ s if the bar is moving upward with (a) a constant velocity of 3 ft/s, and (b) a speed of $v = (4t^2)$ ft/s, where t is in seconds.





a) $T = 40$ lb **Ans**

b) $v = 4t^2$

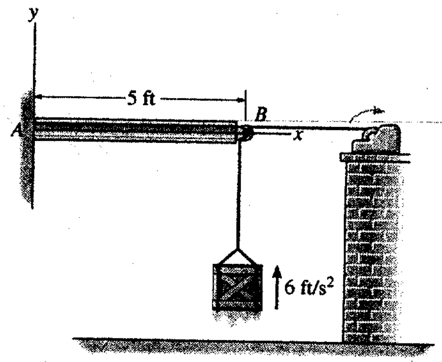
$a = 8t$

$+\uparrow \sum F_y = ma_y;\quad 2T - 80 = \dfrac{80}{32.2}(8t)$

At $t = 2$ s,

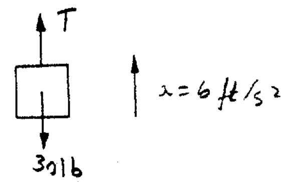
$= 59.9$ lb **Ans**

13-35. The 30-lb crate is being hoisted upward with a constant acceleration of 6 ft/s^2. If the uniform beam AB has a weight of 200 lb, determine the components of reaction at A. Neglect the size and mass of the pulley at B. *Hint:* First find the tension in the cable, then analyze the forces in the beam using statics.



Crate :

$+\uparrow \Sigma F_y = ma_y;\quad T - 30 = \left(\frac{30}{32.2}\right)(6)\qquad T = 35.59 \text{ lb}$

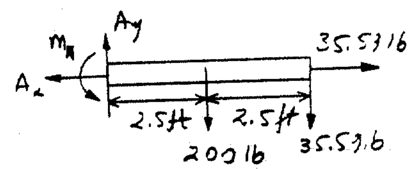


Beam :

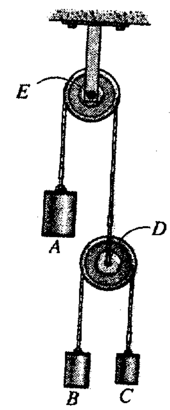
$\xrightarrow{+} \Sigma F_x = 0;\quad -A_x + 35.59 = 0\qquad A_x = 35.6 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y - 200 - 35.59 = 0\qquad A_y = 236 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_A = 0;\quad M_A - 200(2.5) - (35.59)(5) = 0\qquad M_A = 678 \text{ lb}\cdot\text{ft}$ **Ans**



***13-36.** If cylinders B and C have a mass of 15 kg and 10 kg, respectively, determine the required mass of A so that it does not move when all the cylinders are released. Neglect the mass of the pulleys and the cords.



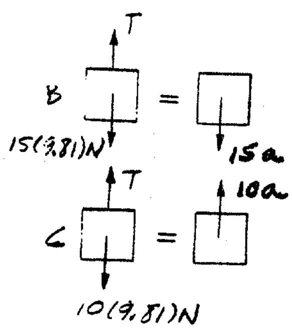
Point D does not move.

For B :

$+\downarrow \Sigma F_y = ma_y;\quad 15(9.81) - T = 15a$

For C :

$+\uparrow \Sigma F_y = ma_y;\quad -10(9.81) + T = 10a$



Solving,

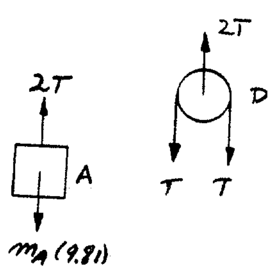
$a = 1.962 \text{ m/s}^2\qquad T = 117.72 \text{ N}$

By statics :

For A :

$+\uparrow \Sigma F_y = ma_y;\quad 2T - m_A(9.81) = 0$

$$m_A = \frac{2(117.72)}{9.81} = 24 \text{ kg}$$ **Ans**



13-37. The conveyor belt is moving at 4 m/s. If the coefficient of static friction between the conveyor and the 10-kg package B is $\mu_s = 0.2$, determine the shortest time the belt can stop so that the package does not slide on the belt.

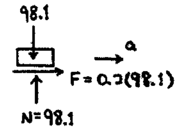
$$\xrightarrow{+} \Sigma F_x = m a_x; \quad 0.2(98.1) = 10a$$

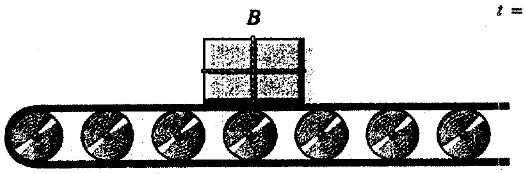
$$a = 1.962 \text{ m/s}^2$$

$$(\xrightarrow{+}) \quad v = v_0 + a_c t$$

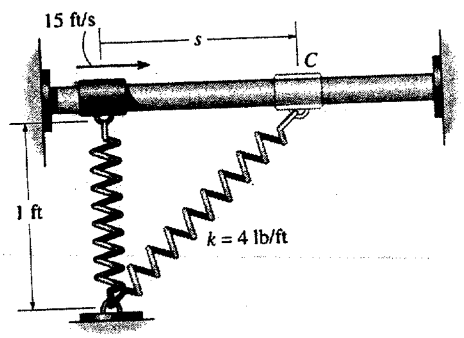
$$4 = 0 + 1.962t$$

$$t = 2.04 \text{ s} \qquad \textbf{Ans}$$





13-38. The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.



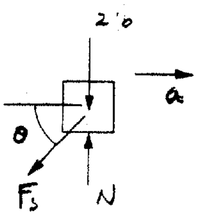
$$F_s = kx; \quad F_s = 4\left(\sqrt{1+s^2} - 1\right)$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \quad -4\left(\sqrt{1+s^2}-1\right)\left(\frac{s}{\sqrt{1+s^2}}\right) = \left(\frac{2}{32.2}\right)\left(v\frac{dv}{ds}\right)$$

$$-\int_0^1\left(4s\,ds - \frac{4s\,ds}{\sqrt{1+s^2}}\right) = \int_{15}^{v}\left(\frac{2}{32.2}\right)v\,dv$$

$$-\left[2s^2 - 4\sqrt{1+s^2}\right]_0^1 = \frac{1}{32.2}\left(v^2 - 15^2\right)$$

$$v = 14.6 \text{ ft/s} \qquad \textbf{Ans}$$



13-39. The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5$ m/s, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs. Take $\theta = 30°$.

$$\nearrow{+}\Sigma F_y = ma_y; \quad N_C - 12(9.81)\cos 30° = 0$$

$$N_C = 101.95 \text{ N}$$

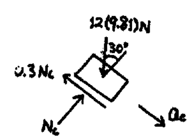
$$+\searrow \Sigma F_x = ma_x; \quad 12(9.81)\sin 30° - 0.3(101.95) = 12a_C$$

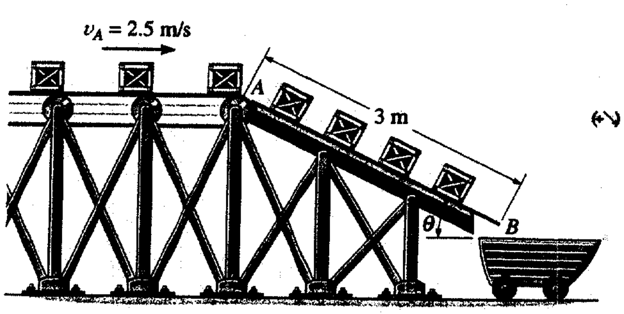
$$a_C = 2.356 \text{ m/s}^2$$

$$(+\searrow) \quad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$

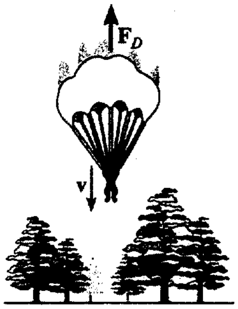
$$v_B^2 = (2.5)^2 + 2(2.356)(3-0)$$

$$v_B = 4.5152 = 4.52 \text{ m/s} \qquad \textbf{Ans}$$





*13-40. A parachutist having a mass m opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where k is a constant, determine his velocity when he has fallen for a time t. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \rightarrow \infty$.



$$+\downarrow \Sigma F_z = m\,a_z; \quad mg - kv^2 = m\frac{dv}{dt}$$

$$m\int_0^v \frac{m\,dv}{(mg-kv^2)} = \int_0^t dt$$

$$\frac{m}{k}\int_0^v \frac{dv}{\frac{mg}{k}-v^2} = t$$

$$\frac{m}{k}\left(\frac{1}{2\sqrt{\frac{mg}{k}}}\right)\ln\left[\frac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}\right]_0^v = t$$

$$\frac{k}{m}t\left(2\sqrt{\frac{mg}{k}}\right) = \ln\frac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}$$

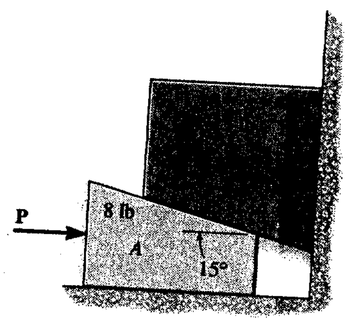
$$e^{2t\sqrt{\frac{gk}{m}}} = \frac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}$$

$$\sqrt{\frac{mg}{k}}\,e^{2t\sqrt{\frac{gk}{m}}} - v\,e^{2t\sqrt{\frac{gk}{m}}} = \sqrt{\frac{mg}{k}} + v$$

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{\frac{gk}{m}}}-1}{e^{2t\sqrt{\frac{gk}{m}}}+1}\right] \qquad \textbf{Ans}$$

When $t \rightarrow \infty \qquad v_t = \sqrt{\frac{mg}{k}}$ **Ans**

13-41. If a horizontal force $P = 12$ lb is applied to block A determine the acceleration of block B. Neglect friction.

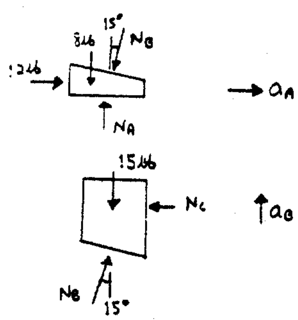


Block A:

$$\xrightarrow{+} \Sigma F_x = ma_x; \quad 12 - N_B \sin 15° = \left(\frac{8}{32.2}\right)a_A \qquad (1)$$

Block B:

$$+\uparrow \Sigma F_y = ma_y; \quad N_B \cos 15° - 15 = \left(\frac{15}{32.2}\right)a_B \qquad (2)$$

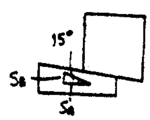


$s_B = s_A \tan 15°$

$a_B = a_A \tan 15° \qquad (3)$

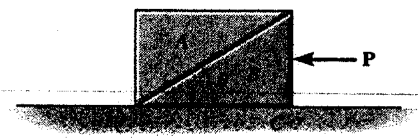
Solving Eqs. (1)−(3),

$a_A = 28.3\ \text{ft/s}^2 \qquad N_B = 19.2$ lb



$a_B = 7.59\ \text{ft/s}^2$ **Ans**

13-42. Blocks A and B each have a mass m. Determine the largest horizontal force P which can be applied to B so that A will not move relative to B. All surfaces are smooth.



Require

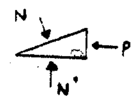
$a_A = a_B = a$

Block A :

$+\uparrow \Sigma F_y = 0;\quad N\cos\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x;\quad N\sin\theta = ma$

$a = g\tan\theta$



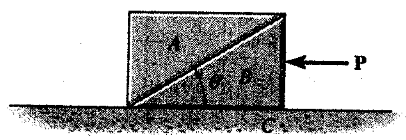
Block B :

$\overset{+}{\leftarrow} \Sigma F_x = ma_x;\quad P - N\sin\theta = ma$

$P - mg\tan\theta = mg\tan\theta$

$P = 2mg\tan\theta$ **Ans**

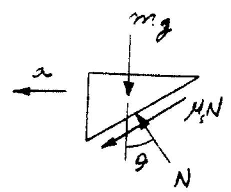
13-43. Blocks A and B each have a mass m. Determine the largest horizontal force P which can be applied to B so that A will not slip up B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.



Require

$a_A = a_B = a$

Block A :



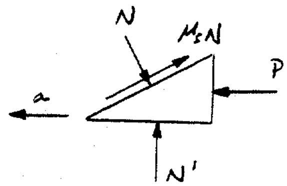
$+\uparrow \Sigma F_y = 0;\quad N\cos\theta - \mu_s N\sin\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x;\quad N\sin\theta + \mu_s N\cos\theta = ma$

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

$$a = g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

Block B :

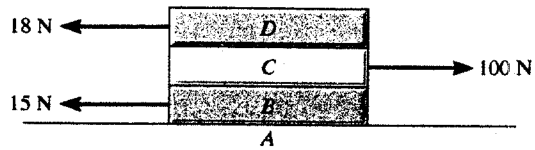


$\overset{+}{\leftarrow} \Sigma F_x = ma_x;\quad P - \mu_s N\cos\theta - N\sin\theta = ma$

$$P - mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$ **Ans**

***13-44.** Each of the three plates has a mass of 10 kg. If the coefficients of static and kinetic friction at each surface of contact are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively, determine the acceleration of each plate when the three horizontal forces are applied.



Plates B, C and D

$\xrightarrow{+} \Sigma F_x = 0; \quad 100 - 15 - 18 - F = 0$

$F = 67$ N

$F_{max} = 0.3(294.3) = 88.3 \text{ N} > 67 \text{ N}$

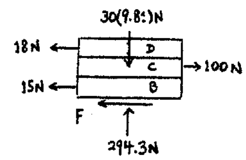


Plate B will not slip.

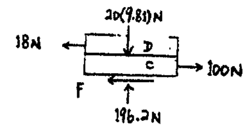
$a_B = 0$ **Ans**

Plates D and C

$\xrightarrow{+} \Sigma F_x = 0; \quad 100 - 18 - F = 0$

$F = 82$ N

$F_{max} = 0.3(196.2) = 58.86 \text{ N} < 82 \text{ N}$

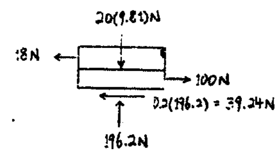


Slipping between B and C.

Assume no slipping between D and C,

$\xrightarrow{+} \Sigma F_x = m\,a_x; \quad 100 - 39.24 - 18 = 20\,a_x$

$a_x = 2.138 \text{ m/s}^2 \rightarrow$

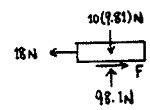


Check slipping between D and C.

$\xrightarrow{+} \Sigma F_x = m\,a_x; \quad F - 18 = 10(2.138)$

$F = 39.38$ N

$F_{max} = 0.3(98.1) = 29.43 \text{ N} < 39.38 \text{ N}$



Slipping between D and C.

Plate C:

$\xrightarrow{+} \Sigma F_x = m\,a_x; \quad 100 - 39.24 - 19.62 = 10\,a_C$

$a_C = 4.11 \text{ m/s}^2 \rightarrow$ **Ans**

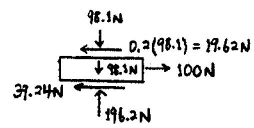
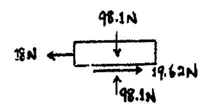


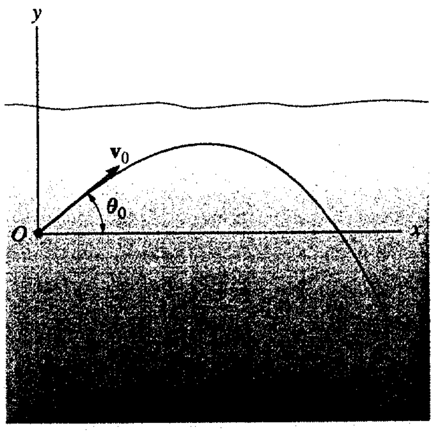
Plate D:

$\xrightarrow{+} \Sigma F_x = m\,a_x; \quad 19.62 - 18 = 10\,a_D$

$a_D = 0.162 \text{ m/s}^2 \rightarrow$ **Ans**



13-45. A projectile of mass m is fired into a liquid at an angle θ_0 with an initial velocity $\mathbf{v}_0$ as shown. If the liquid develops a frictional or drag resistance on the projectile which is proportional to its velocity, i.e., $F = kv$, where k is a constant, determine the x and y components of its position at any instant. Also, what is the maximum distance x_{max} that it travels?



$\xrightarrow{+} \Sigma F_x = m a_x; \qquad -k v \cos\theta = m a_x$

$+\uparrow \Sigma F_y = m a_y; \qquad -m g - k v \sin\theta = m a_y$

or

$$-k\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$-m g - k\frac{dy}{dt} = m\frac{d^2y}{dt^2}$$

Integrating yields

$$\ln \dot{x} = \frac{-k}{m}t + C_1$$

$$\ln\left(\dot{y} + \frac{mg}{k}\right) = \frac{k}{m}t + C_2$$

When $t = 0$, $\dot{x} = v_0\cos\theta_0$, $\quad \dot{y} = v_0\sin\theta_0$,

$$\dot{x} = v_0\cos\theta_0\, e^{-(k/m)t}$$

$$\dot{y} = -\frac{m g}{k} + \left(v_0\sin\theta_0 + \frac{m g}{k}\right)e^{-(k/m)t}$$

Integrating again,

$$x = \frac{-m v_0}{k}\cos\theta_0 e^{-(k/m)t} + C_3$$

$$y = -\frac{mg}{k}t - \left(v_0\sin\theta_0 + \frac{mg}{k}\right)\left(\frac{m}{k}\right)e^{-(k/m)t}$$

When $t = 0$, $\quad x = y = 0$, thus

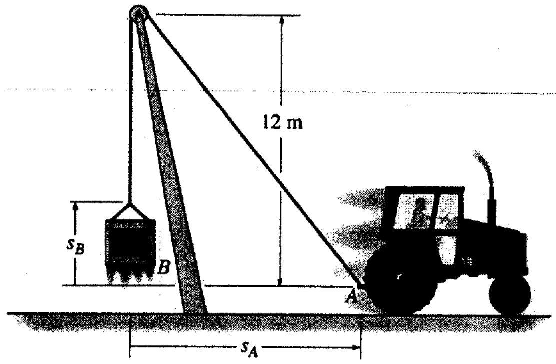
$$x = \frac{m v_0}{k}\cos\theta_0\left(1 - e^{-(k/m)t}\right) \qquad \textbf{Ans}$$

$$y = -\frac{m g t}{k} + \frac{m}{k}\left(v_0\sin\theta_0 + \frac{mg}{k}\right)\left(1 - e^{-(k/m)t}\right) \qquad \textbf{Ans}$$

As $t \to \infty$

$$x_{max} = \frac{m v_0 \cos\theta_0}{k} \qquad \textbf{Ans}$$

■13-46. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor is traveling to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.



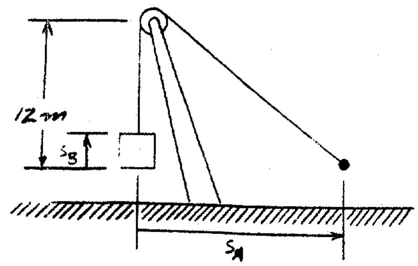
$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

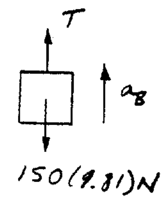
$$-\dot{s}_B + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + 0}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 1.0487 \text{ m/s}^2$$

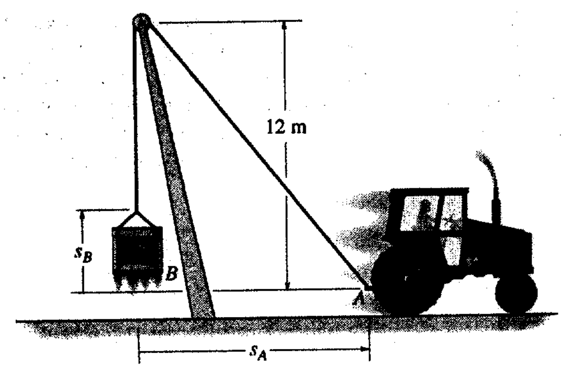




$+\uparrow \Sigma F_y = ma_y;$ $\quad T - 150(9.81) = 150(1.0487)$

$T = 1.63$ kN **Ans**

13-47. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor is traveling to the right with an acceleration of 3 m/s² and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.

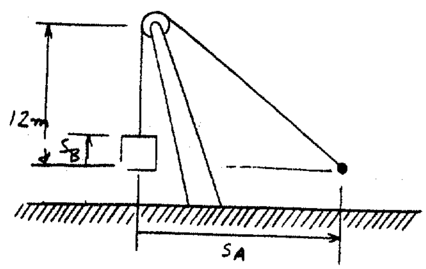


$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \frac{1}{2}\left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(2 s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

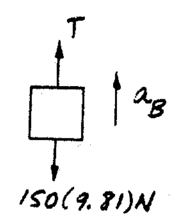
$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$



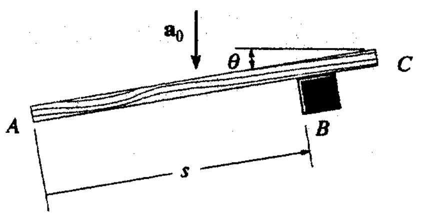
$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + (5)(3)}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 2.2025 \text{ m/s}^2$$

$+\uparrow \Sigma F_y = ma_y;$ $\quad T - 150(9.81) = 150(2.2025)$

$T = 1.80$ kN **Ans**



***13-48.** The smooth block B of negligible size has a mass m and rests on the horizontal plane. If the board AC pushes on the block at an angle θ with a constant acceleration $\mathbf{a}_0$, determine the velocity of the block along the board and the distance s the block moves along the board as a function of time t. The block starts from rest when $s = 0$, $t = 0$.



$\nearrow + \Sigma F_x = m a_x;\quad 0 = m\,a_B \sin\phi$

$\mathbf{a}_B = \mathbf{a}_{AC} + \mathbf{a}_{B/AC}$

$\mathbf{a}_B = \mathbf{a}_0 + \mathbf{a}_{B/AC}$

$\nearrow +\quad a_B \sin\phi = -a_0 \sin\theta + a_{B/AC}$

Thus,

$0 = m(-a_0 \sin\theta + a_{B/AC})$

$a_{B/AC} = a_0 \sin\theta$

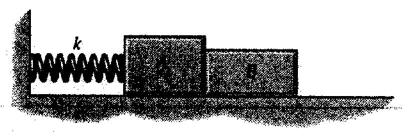
$\int_0^{v_{B/AC}} dv_{B/AC} = \int_0^t a_0 \sin\theta\, dt$

$v_{B/AC} = a_0 \sin\theta\, t$ **Ans**

$s_{B/AC} = s = \int_0^t a_0 \sin\theta\, t\, dt$

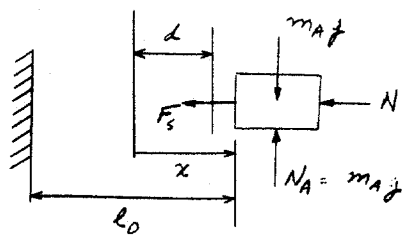
$s = \frac{1}{2} a_0 \sin\theta\, t^2$ **Ans**

13-49. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?



Block A:

$\xrightarrow{+} \Sigma F_x = m a_x;\quad -k(x-d) - N = m_A a_A$



Block B:

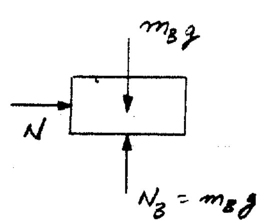
$\xrightarrow{+} \Sigma F_x = m a_x;\quad N = m_B a_B$

Since $a_A = a_B = a$,

$-k(x-d) - m_B a = m_A a$

$a = \dfrac{k(d-x)}{(m_A + m_B)}\qquad N = \dfrac{k m_B (d-x)}{(m_A + m_B)}$

$N = 0$ when $d - x = 0$, or $x = d$ **Ans**



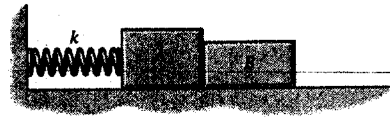
$v\,dv = a\,dx$

$\int_0^v v\,dv = \int_0^d \dfrac{k(d-x)}{(m_A + m_B)}\,dx$

$\frac{1}{2}v^2 = \dfrac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2\right]_0^d = \dfrac{1}{2}\dfrac{kd^2}{(m_A + m_B)}$

$v = \sqrt{\dfrac{kd^2}{(m_A + m_B)}}$ **Ans**

13-50. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?



Block A :

$\xrightarrow{+} \Sigma F_x = ma_x$: $-k(x-d) - N - \mu_k m_A g = m_A a_A$

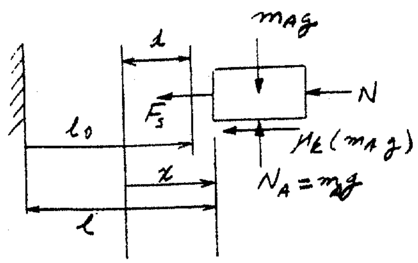
Block B :

$\xrightarrow{+} \Sigma F_x = ma_x$; $N - \mu_k m_B g = m_B a_B$

Since $a_A = a_B = a$,

$$a = \frac{k(d-x) - \mu_k g(m_A + m_B)}{(m_A + m_B)} = \frac{k(d-x)}{(m_A + m_B)} - \mu_k g$$

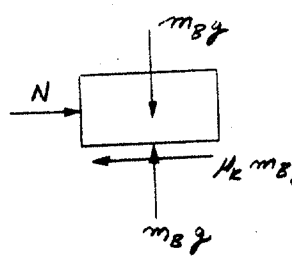
$$N = \frac{km_B(d-x)}{(m_A + m_B)}$$



$N = 0$, then $x = d$ for separation. **Ans**

At the moment of separation :

$v\,dv = a\,dx$



$$\int_0^v v\,dv = \int_0^d \left[\frac{k(d-x)}{(m_A + m_B)}\,dx - \mu_k g\right] dx$$

$$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2 - \mu_k g\,x\right]_0^d$$

$$v = \sqrt{\frac{kd^2 - 2\mu_k g(m_A + m_B)d}{(m_A + m_B)}}$$

Require $v > 0$, so that

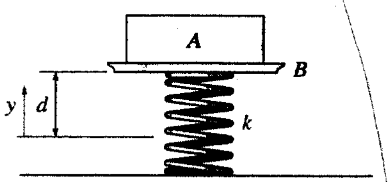
$kd^2 - 2\mu_k g(m_A + m_B)d > 0$

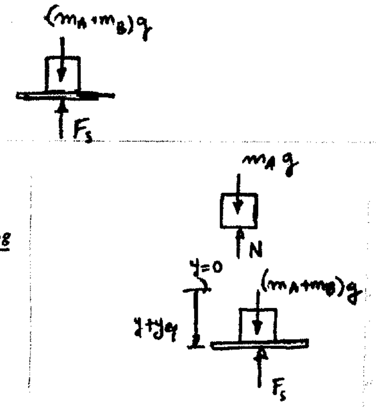
Thus,

$kd > 2\mu_k g(m_A + m_B)$

$d > \frac{2\mu_k g}{k}(m_A + m_B)$ **Q.E.D.**

13-51. The block A has a mass m_A and rests on the pan B, which has a mass m_B. Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.





For Equilibrium

$+\uparrow \Sigma F_y = ma_y;\quad F_s = (m_A + m_B)g$

$$y_{eq} = \frac{F_s}{k} = \frac{(m_A + m_B)g}{k}$$

Block:

$+\uparrow \Sigma F_y = ma_y;\quad -m_A g + N = m_A a$

Block and pan

$+\uparrow \Sigma F_y = ma_y;\quad -(m_A + m_B)g + k(y_{eq} + y) = (m_A + m_B)a$

Thus,

$$-(m_A + m_B)g + k\left[\left(\frac{m_A + m_B}{k}\right)g + y\right] = (m_A + m_B)\left(\frac{-m_A g + N}{m_A}\right)$$

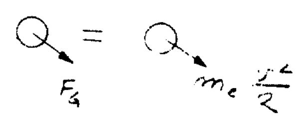
Require $y = d,\ N = 0$

$kd = -(m_A + m_B)g$

Since d is downward,

$$d = \frac{(m_A + m_B)g}{k} \qquad \textbf{Ans}$$

***13-52.** Determine the mass of the sun, knowing that the distance from the earth to the sun is $149.6(10^6)$ km. *Hint:* Use Eq. 13–1 to represent the force of gravity acting on the earth.

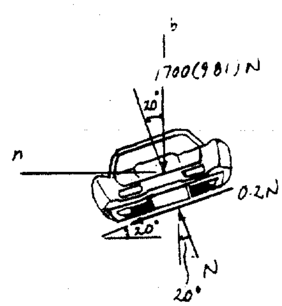


$\Sigma F_n = ma_n;\quad G\frac{M_e M_s}{R^2} = M_e\frac{v^2}{R} \qquad M_s = \frac{v^2 R}{G}$

$$v = \frac{s}{t} = \frac{2\pi(149.6)(10^9)}{365(24)(3600)} = 29.81\,(10^3)\ \text{m/s}$$

$$M_s = \frac{[(29.81)(10^3)]^2(149.6)(10^9)}{66.73(10^{-12})} = 1.99\,(10^{30})\ \text{kg} \qquad \textbf{Ans}$$

13-53. The sports car, having a mass of 1700 kg, is traveling horizontally along a 20° banked track which is circular and has a radius of curvature of $\rho = 100$ m. If the coefficient of static friction between the tires and the road is $\mu_s = 0.2$, determine the *maximum constant speed* at which the car can travel without sliding up the slope. Neglect the size of the car.



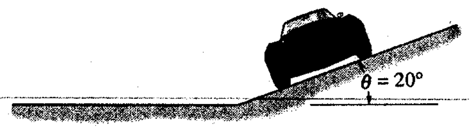
$+\uparrow \Sigma F_b = 0;\quad N\cos 20° - 0.2N\sin 20° - 1700(9.81) = 0$

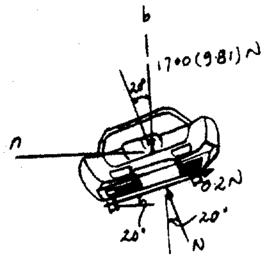
$$N = 19\,140.6\ \text{N}$$

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;\quad 19\,140.6\sin 20° + 0.2(19\,140.6)\cos 20° = 1700\left(\frac{v_{max}^2}{100}\right)$

$$v_{max} = 24.4\ \text{m/s} \qquad \textbf{Ans}$$

13-54. Using the data in Prob. 13-53, determine the *minimum speed* at which the car can travel around the track without sliding down the slope.





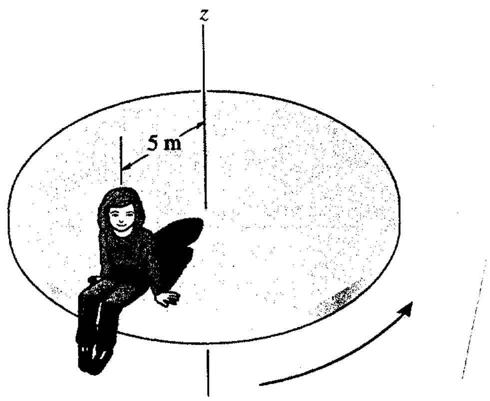
$+\uparrow \Sigma F_b = 0;\quad N\cos 20° + 0.2N\sin 20° - 1700(9.81) = 0$

$$N = 16543.1\text{ N}$$

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;\quad 16543.1\sin 20° - 0.2(16543.1)\cos 20° = 1700\left(\frac{v_{\min}^2}{100}\right)$

$$v_{\min} = 12.2\text{ m/s}$$ **Ans**

13-55. A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.

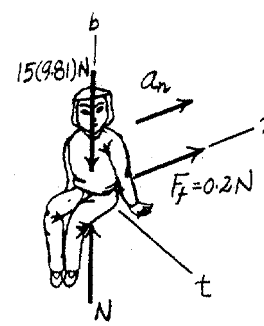


Equation of Motion : Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13 – 8, we have

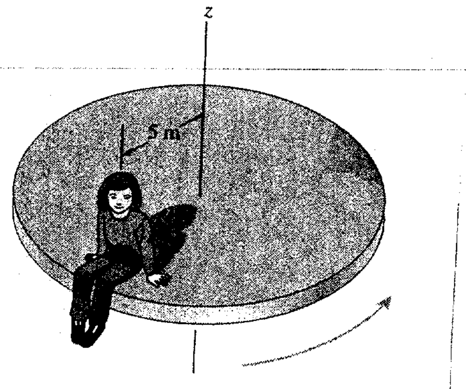
$\Sigma F_b = 0;\quad N - 15(9.81) = 0 \quad N = 147.15\text{ N}$

$\Sigma F_n = ma_n;\quad 0.2(147.15) = 15\left(\frac{v^2}{5}\right)$

$$v = 3.13\text{ m/s}$$ **Ans**



***13-56.** Solve Prob. 13–55 assuming that the platform starts rotating from rest so that the girl's speed is increased uniformly at $\dot{v} = 0.5\ \text{m/s}^2$.



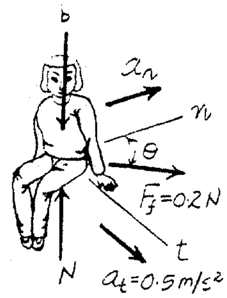
Equation of Motion : Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13 – 8, we have

$$\Sigma F_b = 0; \qquad N - 15(9.81) = 0 \qquad N = 147.15\ \text{N}$$

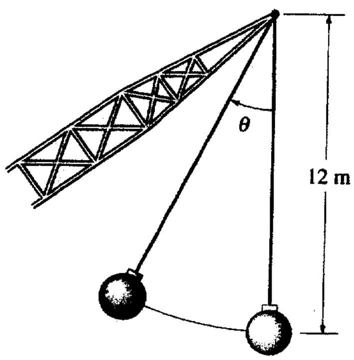
$$\Sigma F_t = ma_t; \qquad 0.2(147.15)\sin\theta = 15(0.5) \qquad \theta = 14.76°$$

$$\Sigma F_n = ma_n; \qquad 0.2(147.15)\cos 14.76° = 15\left(\frac{v^2}{5}\right)$$

$$v = 3.08\ \text{m/s} \qquad \textbf{Ans}$$



13-57. The 600-kg wrecking ball is suspended from the crane by a cable having a negligible mass. If the ball has a speed $v = 8$ m/s at the instant it is at its lowest point, $\theta = 0°$, determine the tension in the cable at this instant. Also, determine the angle θ to which the ball swings before it stops.



$$+\uparrow\Sigma F_n = m\,a_n; \qquad T - 600(9.81) = 600\left(\frac{8^2}{12}\right)$$

$$T = 9086\ \text{N} = 9.09\ \text{kN} \qquad \textbf{Ans}$$

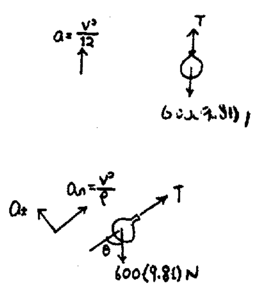
$$+\nwarrow\Sigma F_t = m\,a_t; \qquad -600(9.81)\sin\theta = 600\,a_t$$

Set $a_t(12\,d\theta) = v\,dv$

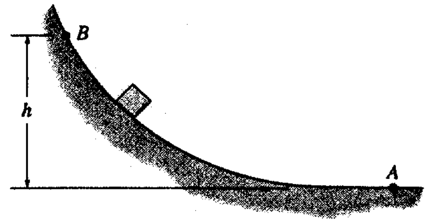
$$-9.81(12)\int_0^{\theta}\sin\theta\,d\theta = \int_8^0 v\,dv$$

$$-9.81(12)(-\cos\theta + 1) = -\frac{1}{2}(8)^2$$

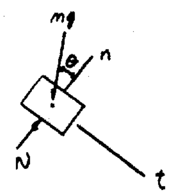
$$\theta = 43.3° \qquad \textbf{Ans}$$



13-58. Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape,* the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



$+\searrow\Sigma F_t = ma_t;\quad mg\sin\theta = ma_t \quad a_t = g\sin\theta$



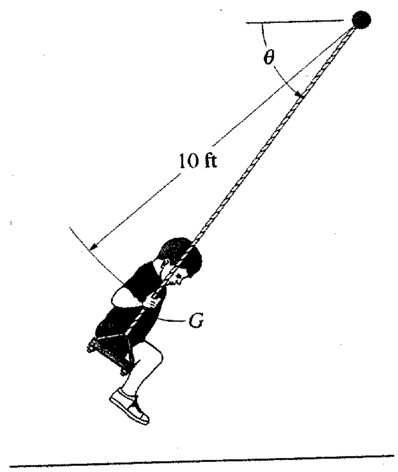
$v\,dv = a_t\,ds = g\sin\theta\,ds$ However $dy = ds\sin\theta$

$\int_0^v v\,dv = \int_0^h g\,dy$

$\frac{v^2}{2} = gh$

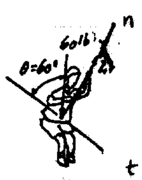
$v = \sqrt{2gh}$ **Q.E.D.**

13-59. At the instant $\theta = 60°$, the boy's center of mass G has a downward speed $v_G = 15$ ft/s. Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.

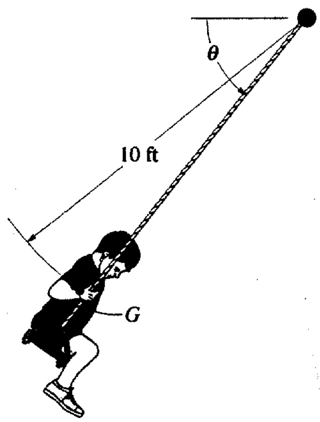


$+\searrow\Sigma F_t = ma_t;\quad 60\cos 60° = \frac{60}{32.2}a_t \quad a_t = 16.1\ \text{ft/s}^2$ **Ans**

$\nearrow+\Sigma F_n = ma_n;\quad 2T - 60\sin 60° = \frac{60}{32.2}\left(\frac{15^2}{10}\right) \quad T = 46.9\ \text{lb}$ **Ans**

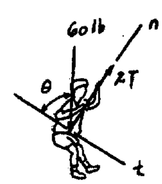


***13-60.** At the instant $\theta = 60°$, the boy's center of mass G is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = 90°$. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.



$+\searrow\Sigma F_t = ma_t;\quad 60\cos\theta = \frac{60}{32.2}a_t \quad a_t = 32.2\cos\theta$

$\nearrow+\Sigma F_n = ma_n;\quad 2T - 60\sin\theta = \frac{60}{32.2}\left(\frac{v^2}{10}\right) \quad [1]$



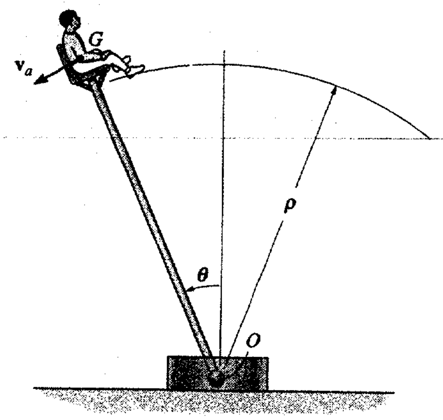
$v\,dv = a\,ds$ however $ds = 10d\theta$

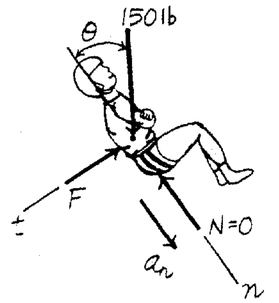
$\int_0^v v\,dv = \int_{60°}^{90°} 322\cos\theta\,d\theta$

$v = 9.289$ ft/s

From Eq.[1] $\quad 2T - 60\sin 90° = \frac{60}{32.2}\left(\frac{9.289^2}{10}\right) \quad T = 38.0\ \text{lb}$ **Ans**

13-61. An acrobat has a weight of 150 lb and is sitting on a chair which is perched on top of a pole as shown. If by a mechanical drive the pole rotates downward at a constant rate from $\theta = 0°$, such that the acrobat's center of mass G maintains a *constant speed* of $v_a = 10$ ft/s, determine the angle θ at which he begins to "fly" out of the chair. Neglect friction and assume that the distance from the pivot O to G is $\rho = 15$ ft.



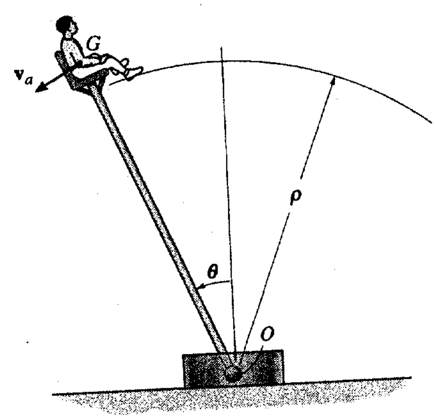


Equation of Motion : If the acrobat is about to fly off the chair, the normal reaction $N = 0$. Applying Eq. 13 − 8, we have

$$\Sigma F_n = ma_n; \qquad 150\cos\theta = \frac{150}{32.2}\left(\frac{10^2}{15}\right)$$

$$\theta = 78.1° \qquad \textbf{Ans}$$

13-62. Solve Prob. 13-61 if the speed of the acrobat's center of mass is increased from $(v_a)_0 = 10$ ft/s at $\theta = 0°$ by a constant rate of $\dot{v}_a = 0.5$ ft/s^2.



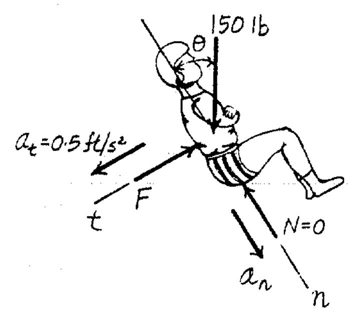
Kinematics : Applying equation $v^2 = v_0^2 + 2a_t(s - s_0)$, we have

$$v^2 = 10^2 + 2(0.5)(15\theta - 0) = 100 + 15\theta$$

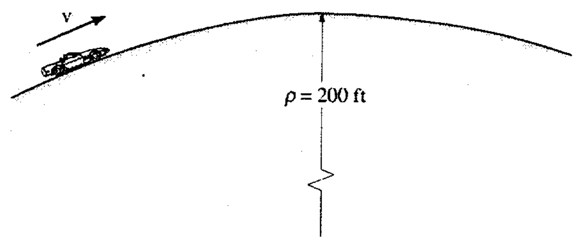
Equation of Motion : If the acrobat is about to fly off the chair, the normal reaction $N = 0$. Applying Eq. 13 − 8, we have

$$\Sigma F_n = ma_n; \qquad 150\cos\theta = \frac{150}{32.2}\left(\frac{100 + 15\theta}{15}\right)$$

Solving for θ by trial and error, $\qquad \theta = 75.6° \qquad$ **Ans**

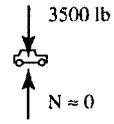


13-63. If the crest of the hill has a radius of curvature $\rho = 200$ ft, determine the maximum constant speed at which the car can travel over it without leaving the surface of the road. Neglect the size of the car in the calculation. The car has a weight of 3500 lb.

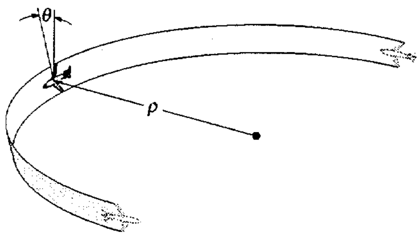


$$+\downarrow \sum F_n = ma_n; \quad 3500 = \frac{3500}{32.2}\left(\frac{v^2}{200}\right)$$

$$v = 80.2 \text{ ft/s} \qquad \textbf{Ans}$$



***13-64.** The airplane, traveling at a constant speed of 50 m/s, is executing a horizontal turn. If the plane is banked at $\theta = 15°$, when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has a mass of 70 kg?

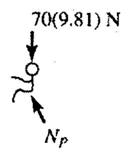


$$+\uparrow \sum F_b = ma_b; \quad N_P \sin 15° - 70(9.81) = 0$$

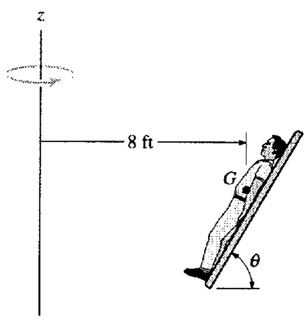
$$N_P = 2.65 \text{ kN} \qquad \textbf{Ans}$$

$$\overset{+}{\leftarrow} \sum F_n = ma_n; \quad N_P \cos 15° = 70\left(\frac{50^2}{\rho}\right)$$

$$\rho = 68.3 \text{ m} \qquad \textbf{Ans}$$



13-65. The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the z axis, he has a constant speed $v = 20$ ft/s. Neglect the size of the man. Take $\theta = 60°$.



$$+\nwarrow \sum F_y = m(a_n)_y; \quad N - 150\cos 60° = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\sin 60°$$

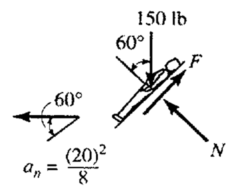
$$N = 277 \text{ lb} \qquad \textbf{Ans}$$

$$+\swarrow \sum F_x = m(a_n)_x; \quad -F + 150\sin 60° = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\cos 60°$$

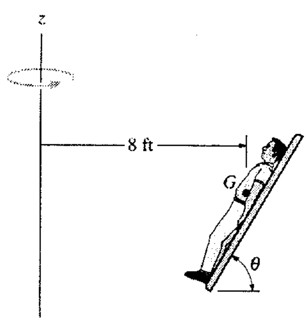
$$F = 13.4 \text{ lb} \qquad \textbf{Ans}$$

Note: No slipping occurs

Since $\mu_s N = 138.4 \text{ lb} > 13.4 \text{ lb}$



13-66. The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. If he rotates about the z axis with a constant speed $v = 30$ ft/s, determine the smallest angle θ of the cushion at which he will begin to slip off.

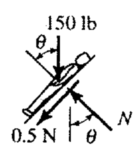


$$\overset{+}{\leftarrow} \sum F_n = ma_n; \quad 0.5N\cos\theta + N\sin\theta = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$

$$+\uparrow \sum F_b = 0; \quad -150 + N\cos\theta - 0.5N\sin\theta = 0$$

$$N = \frac{150}{\cos\theta - 0.5\sin\theta}$$

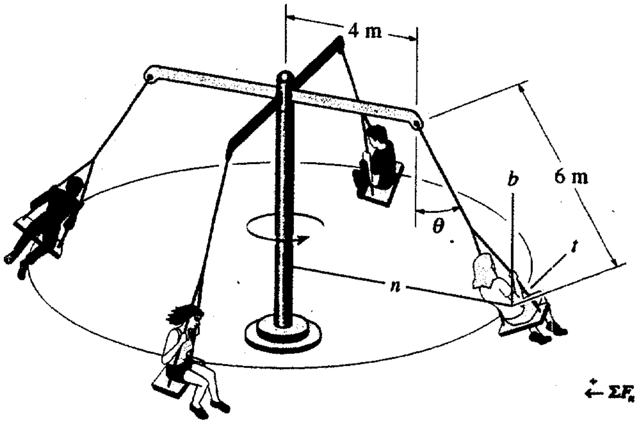
$$\frac{(0.5\cos\theta + \sin\theta)150}{(\cos\theta - 0.5\sin\theta)} = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$



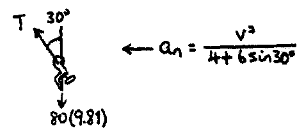
$$0.5\cos\theta + \sin\theta = 3.49378\cos\theta - 1.74689\sin\theta$$

$$\theta = 47.5° \qquad \textbf{Ans}$$

13-67. Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at $\theta = 30°$ from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the n, t, and b directions which the chair exerts on a 50-kg passenger during the motion?



$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;\quad T\sin 30° = 80\left(\dfrac{v^2}{4 + 6\sin 30°}\right)$



$+\uparrow \Sigma F_b = 0;\quad T\cos 30° - 80(9.81) = 0$

$T = 906.2$ N

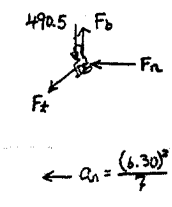
$v = 6.30$ m/s **Ans**

$\Sigma F_n = m\,a_n;\quad F_n = 50\left(\dfrac{(6.30)^2}{7}\right) = 283$ N **Ans**

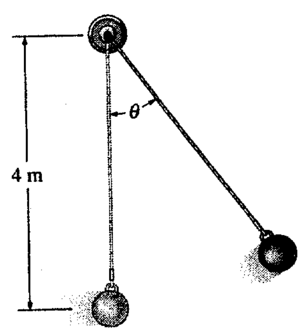
$\Sigma F_t = m\,a_t;\quad F_t = 0$ **Ans**

$\Sigma F_b = m\,a_b;\quad F_b - 490.5 = 0$

$F_b = 490$ N **Ans**



***13-68.** If the ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$, determine the tension in the cord at this instant. Also, determine the angle θ to which the ball swings at the instant it momentarily stops. Neglect the size of the ball.



$+\uparrow \Sigma F_n = ma_n;\quad T - 30(9.81) = 30\left(\dfrac{(4)^2}{4}\right)$

$T = 414$ N **Ans**

$+\swarrow \Sigma F_t = ma_t;\quad -30(9.81)\sin\theta = 30a_t$

$a_t = -9.81\sin\theta$

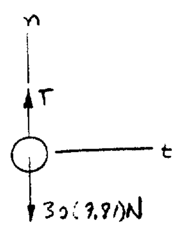
$a_t\,ds = v\,dv$ Since $ds = 4\,d\theta$, then

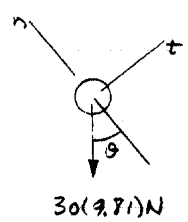
$-9.81\int_0^\theta \sin\theta\,(4d\theta) = \int_4^0 v\,dv$

$9.81(4)\cos\theta\Big]_0^\theta = -\dfrac{1}{2}(4)^2$

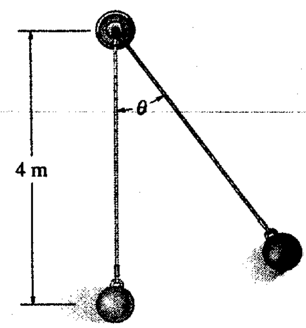
$39.24(\cos\theta - 1) = -8$

$\theta = 37.2°$ **Ans**





13-69. The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



$+\nwarrow \Sigma F_n = ma_n; \quad T - 30(9.81)\cos\theta = 30\left(\frac{v^2}{4}\right)$

$+\nearrow \Sigma F_t = ma_t; \quad -30(9.81)\sin\theta = 30a_t$

$a_t = -9.81\sin\theta$

$a_t\, ds = v\, dv$ Since $ds = 4\, d\theta$, then

$-9.81\int_0^\theta \sin\theta\,(4\, d\theta) = \int_4^v v\, dv$

$9.81(4)\cos\theta\Big|_0^\theta = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$

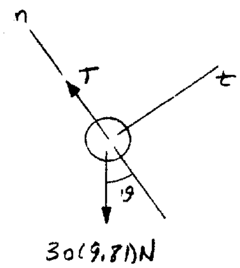
$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$

At $\theta = 20°$

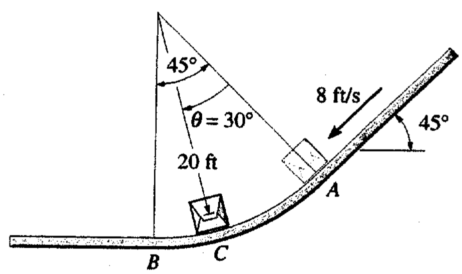
$v = 3.357$ m/s

$a_t = -3.36\ \text{m/s}^2 = 3.36\ \text{m/s}^2 \swarrow$ **Ans**

$T = 361$ N **Ans**



13-70. The package has a weight of 5 lb and slides down the chute. When it reaches the curved portion AB, it is traveling at 8 ft/s ($\theta = 0°$). If the chute is smooth, determine the speed of the package when it reaches the intermediate point C ($\theta = 30°$) and when it reaches the horizontal plane ($\theta = 45°$). Also, find the normal force on the package at C.



$+\swarrow \Sigma F_t = ma_t; \quad 5\cos\phi = \frac{5}{32.2}a_t$

$a_t = 32.2\cos\phi$

$+\nwarrow \Sigma F_n = ma_n; \quad N - 5\sin\phi = \frac{5}{32.2}\left(\frac{v^2}{20}\right)$

$v\, dv = a_t\, ds$

$\int_8^v v\, dv = \int_{45°}^{\phi} 32.2\cos\phi\,(20\, d\phi)$

$\frac{1}{2}v^2 - \frac{1}{2}(8)^2 = 644(\sin\phi - \sin 45°)$

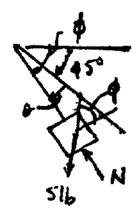
At $\phi = 45° + 30° = 75°$,

$v_C = 19.933$ ft/s $= 19.9$ ft/s **Ans**

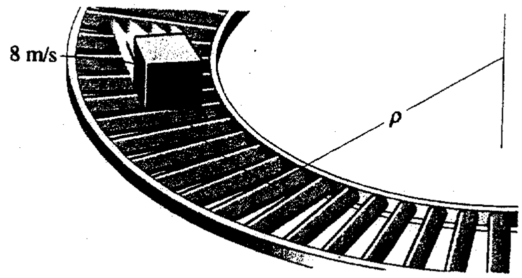
$N_C = 7.91$ lb **Ans**

At $\phi = 45° + 45° = 90°$

$v_B = 21.0$ ft/s **Ans**



13-71. Cartons having a mass of 5 kg are required to move along the assembly line at a constant speed of 8 m/s. Determine the smallest radius of curvature, ρ, for the conveyor so the cartons do not slip. The coefficients of static and kinetic friction between a carton and the conveyor are $\mu_s = 0.7$ and $\mu_k = 0.5$, respectively.



$+\uparrow\Sigma F_b = m a_b;\quad N - W = 0$

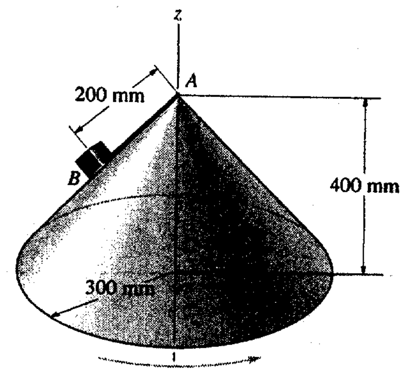
$N = W$

$F_s = 0.7W$

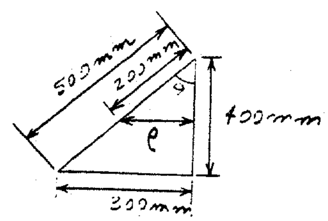
$\overset{+}{\leftarrow}\Sigma F_n = m a_n;\quad 0.7W = \frac{W}{9.81}\left(\frac{8^2}{\rho}\right)$

$\rho = 9.32$ m **Ans**

***13-72.** The smooth block B, having a mass of 0.2 kg, is attached to the vertex A of the right circular cone using a light cord. The cone is rotating at a constant angular rate about the z axis such that the block attains a speed of 0.5 m/s. At this speed, determine the tension in the cord and the reaction which the cone exerts on the block. Neglect the size of the block.



$\frac{\rho}{200} = \frac{300}{500};\quad \rho = 120 \text{ mm} = 0.120 \text{ m}$



$+\nearrow\Sigma F_y = ma_y;\quad T - 0.2(9.81)\left(\frac{4}{5}\right) = \left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{3}{5}\right)$

$T = 1.82$ N **Ans**

$+\nwarrow\Sigma F_x = ma_x;\quad N_B - 0.2(9.81)\left(\frac{3}{5}\right) = -\left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{4}{5}\right)$

$N_B = 0.844$ N **Ans**

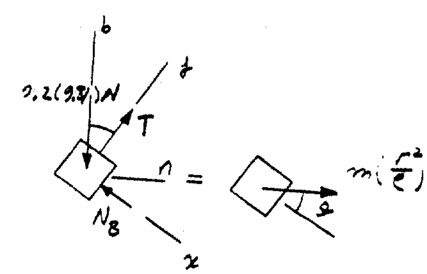
Also,

$\overset{+}{\rightarrow}\ \Sigma F_n = ma_n;\quad T\left(\frac{3}{5}\right) - N_B\left(\frac{4}{5}\right) = 0.2\left(\frac{(0.5)^2}{0.120}\right)$

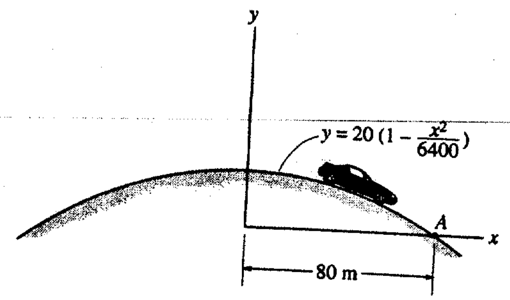
$+\uparrow\ \Sigma F_b = 0;\quad T\left(\frac{4}{5}\right) + N_B\left(\frac{3}{5}\right) - 0.2(9.81) = 0$

$T = 1.82$ N **Ans**

$N_B = 0.844$ N **Ans**



13-73. The 0.8-Mg car is traveling over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.

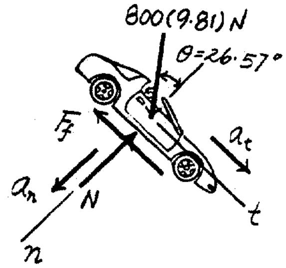


Geometry : Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57°$$

and the radius of curvature at point A is

$$\rho = \frac{\left[1+(dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1+(-0.00625x)^2\right]^{3/2}}{|-0.00625|}\right|_{x=8\text{ m}} = 223.61\text{ m}$$



Equation of Motion : Here, $a_t = 0$. Applying Eq. 13 – 8 with $\theta = 26.57°$ and $\rho = 223.61$ m, we have

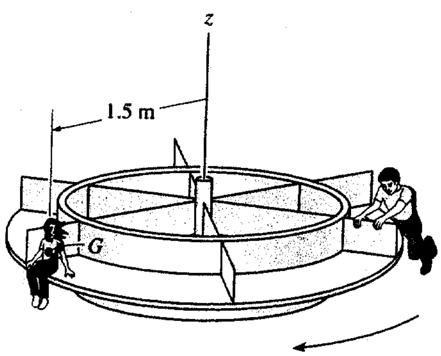
$\Sigma F_t = ma_t;$ $\qquad 800(9.81)\sin 26.57° - F_f = 800(0)$

$F_f = 3509.73\text{ N} = 3.51\text{ kN}$ **Ans**

$\Sigma F_n = ma_n;$ $\qquad 800(9.81)\cos 26.57° - N = 800\left(\frac{9^2}{223.61}\right)$

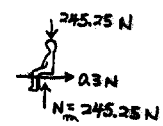
$N = 6729.67\text{ N} = 6.73\text{ kN}$ **Ans**

13-74. A girl having a mass of 25 kg sits at the edge of the merry-go-round so her center of mass G is at a distance of 1.5 m from the axis of rotation. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which she can have before she begins to slip off the merry-go-round. The coefficient of static friction between the girl and the merry-go-round is $\mu_s = 0.3$.

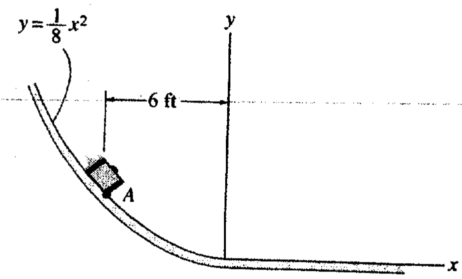


$\xrightarrow{+} \Sigma F_n = m a_n;$ $\qquad 0.3(245.25) = 25\left(\frac{v^2}{1.5}\right)$

$v = 2.10$ m/s **Ans**



13-75. The 10-lb suitcase slides down the curved ramp for which the coefficient of kinetic friction is $\mu_k = 0.2$. If at the instant it reaches point A it has a speed of 5 ft/s, determine the normal force on the suitcase and the rate of increase of its speed.

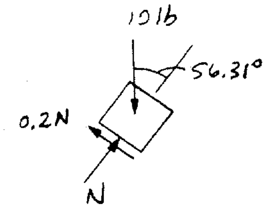


$$y = \frac{1}{8}x^2$$

$$\frac{dy}{dx} = \tan\theta = \frac{1}{4}x\Big|_{x=-6} = -1.5 \qquad \theta = -56.31°$$

$$\frac{d^2y}{dx^2} = \frac{1}{4}$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(-1.5)^2\right]^{\frac{3}{2}}}{\left|\frac{1}{4}\right|} = 23.436 \text{ ft}$$



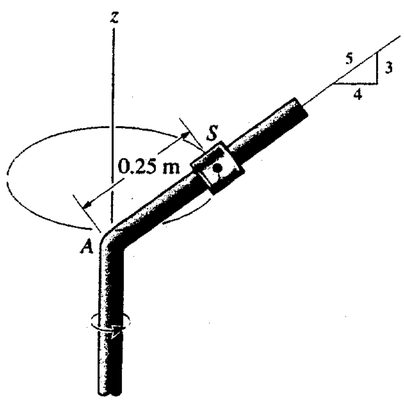
$$+\nearrow \Sigma F_n = ma_n; \qquad N - 10\cos 56.31° = \left(\frac{10}{32.2}\right)\left(\frac{(5)^2}{23.436}\right)$$

$N = 5.8783 = 5.88$ lb **Ans**

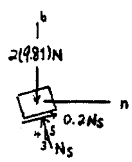
$$+\searrow \Sigma F_t = ma_t; \qquad -0.2(5.8783) + 10\sin 56.31° = \left(\frac{10}{32.2}\right)a_t$$

$a_t = 23.0$ ft/s^2 **Ans**

***13-76.** The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.



$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$



$$\overset{+}{\leftarrow} \Sigma F_n = m a_n; \qquad N_s\left(\frac{3}{5}\right) - 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

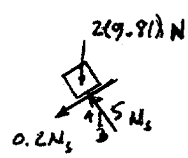
$$+\uparrow \Sigma F_b = m a_b; \qquad N_s\left(\frac{4}{5}\right) + 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

$N_s = 21.3$ N

$v = 0.969$ m/s **Ans**

13-77. The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.

$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$



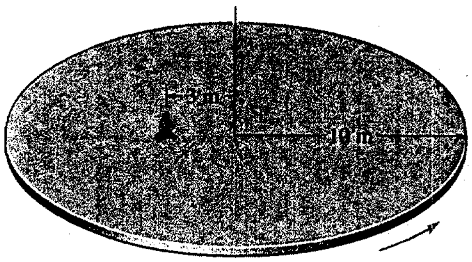
$$\overset{+}{\leftarrow} \Sigma F_n = m a_n; \qquad N_s\left(\frac{3}{5}\right) + 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

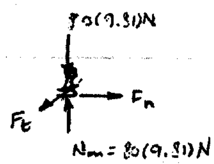
$$+\uparrow \Sigma F_b = m a_b; \qquad N_s\left(\frac{4}{5}\right) - 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

$N_s = 28.85$ N

$v = 1.48$ m/s **Ans**

13-78. The man has a mass of 80 kg and sits 3 m from the center of the rotating platform. Due to the rotation his speed is increased from rest by $\dot{v} = 0.4\ \text{m/s}^2$. If the coefficient of static friction between his clothes and the platform is $\mu_s = 0.3$, determine the time required to cause him to slip.





$\Sigma F_t = m\,a_t;\quad F_t = 80(0.4)$

$F_t = 32\ \text{N}$

$\Sigma F_n = m\,a_n;\quad F_n = (80)\dfrac{v^2}{3}$

$F = \mu_s N_m = \sqrt{(F_t)^2 + (F_n)^2}$

$0.3(80)(9.81) = \sqrt{(32)^2 + ((80)\dfrac{v^2}{3})^2}$

$55\,432 = 1024 + (6400)(\dfrac{v^4}{9})$

$v = 2.9575\ \text{m/s}$

$a_t = \dfrac{dv}{dt} = 0.4$

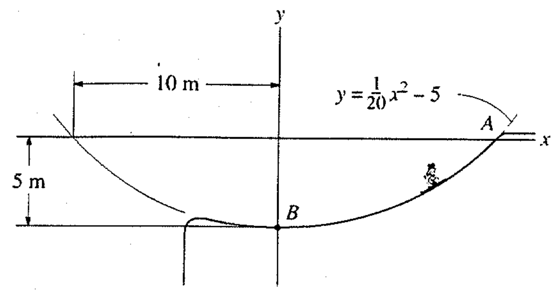
$\int_0^v dv = \int_0^t 0.4\,dt$

$v = 0.4\,t$

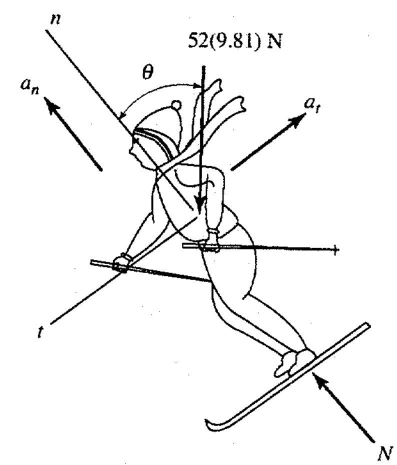
$2.9575 = 0.4\,t$

$t = 7.39\ \text{s}$ **Ans**

13-79. The skier starts from rest at $A(10\text{ m}, 0)$ and descends the smooth slope, which may be approximated by a parabola. If she has a mass of 52 kg, determine the normal force she exerts on the ground at the instant she arrives at point B. Neglect the size of the skier. *Hint:* Use the result of Prob. 13–58.



Geometry: Here, $\frac{dy}{dx} = \frac{1}{10}x$ and $\frac{d^2y}{dx^2} = \frac{1}{10}$. The slope angle θ at point B is given by



$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=0\text{ m}} = 0 \quad \theta = 0°$$

and the radius of curvature at point B is

$$\rho = \frac{[1+(dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1+\left(\frac{1}{10}x\right)^2\right]^{3/2}}{[1/10]}\right|_{x=0\text{ m}} = 10.0\text{ m}$$

Equation of Motion:

$$\sum F_t = ma_t;\quad 52(9.81)\sin\theta = -52a_t \quad a_t = -9.81\sin\theta$$

$$\sum F_n = ma_n;\quad N - 52(9.81)\cos\theta = m\left(\frac{v^2}{\rho}\right) \qquad [1]$$

Kinematics: The speed of the skier can be determined using $v\,dv = a_t\,ds$. Here, a_t must be in the direction of positive ds. Also, $ds = \sqrt{1+(dy/dx)^2}\,dx = \sqrt{1+\frac{1}{100}x^2}\,dx$. Here, $\tan\theta = \frac{1}{10}x$. Then, $\sin\theta = \frac{x}{10\sqrt{1+\frac{1}{100}x^2}}$.

$$(+)\quad \int_0^v v\,dv = -9.81\int_{10\text{ m}}^{0}\left(\frac{x}{10\sqrt{1+\frac{1}{100}x^2}}\right)\left(\sqrt{1+\frac{1}{100}x^2}\,dx\right)$$

$$v^2 = 98.1\text{ m}^2/\text{s}^2$$

Also, one can obtain v^2 by using the result of Prob. 13–58.

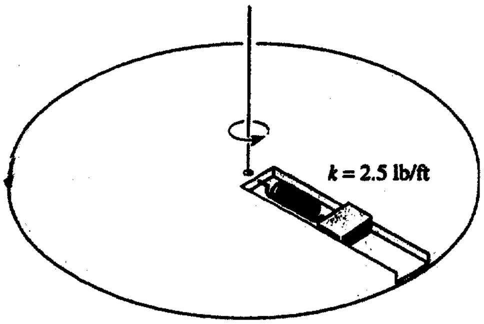
$$v^2 = 2gh = 2(9.81)(5) = 98.1\text{ m}^2/\text{s}^2$$

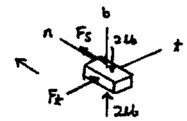
Substitute $v^2 = 98.1\text{ m}^2/\text{s}^2$, $\theta = 0°$ and $\rho = 10.0$ m into Eq. [1] yields

$$N - 52(9.81)\cos 0° = 52\left(\frac{98.1}{10.0}\right)$$

$N = 1020.24\text{ N} = 1.02\text{ kN}$ **Ans**

***13-80.** The block has a weight of 2 lb and it is free to move along the smooth slot in the rotating disk. The spring has a stiffness of 2.5 lb/ft and an unstretched length of 1.25 ft. Determine the force of the spring on the block and the tangential component of force which the slot exerts on the side of the block, when the block is at rest with respect to the disk and is traveling with a constant speed of 12 ft/s.





$\Sigma F_n = ma_n;\quad F_s = \frac{2}{32.2}\left(\frac{12^2}{\rho}\right)$

$\Sigma F_t = ma_t;\quad F_t = 0$ **Ans**

$F_s = ks;\quad F_s = 2.5(\rho - 1.25)$

$2.5(32.2)(\rho^2 - 1.25\rho) = 288$

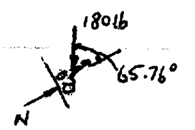
$\rho^2 - 1.25\rho - 3.58 = 0$

Choosing the positive root,

$\rho = 2.62$ ft

$F_s = 2.5(2.62 - 1.25) = 3.42$ lb **Ans**

13-81. If the bicycle and rider have a total weight of 180 lb, determine the resultant normal force acting on the bicycle when it is at point A while it is freely coasting at $v_A = 6$ ft/s. Also, compute the increase in the bicyclist's speed at this point. Neglect the resistance due to the wind and the size of the bicycle and rider.



$y = 20\cos\left(\frac{\pi}{20}x\right)$

$\frac{dy}{dx} = -\pi\sin\left(\frac{\pi}{20}x\right)\Big|_{x=5\text{ ft}} = -2.221$

$\theta = \tan^{-1}(-2.221) = -65.76°$

$\frac{d^2y}{dx^2} = -\frac{\pi^2}{20}\cos\left(\frac{\pi}{20}x\right)\Big|_{x=5\text{ ft}} = -0.3489$

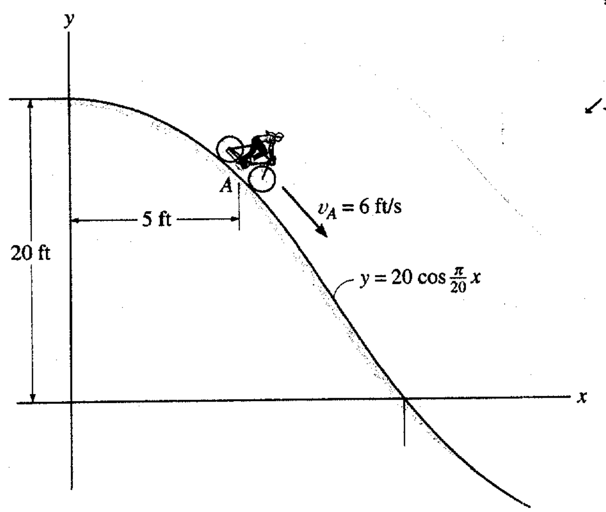
$\rho = \left|\frac{[1+(-2.221)^2]^{3/2}}{-0.3489}\right| = 41.43$ ft

$\searrow + \Sigma F_t = m\,a_t;\quad 180\sin 65.76° = \frac{180}{32.2}a_t$

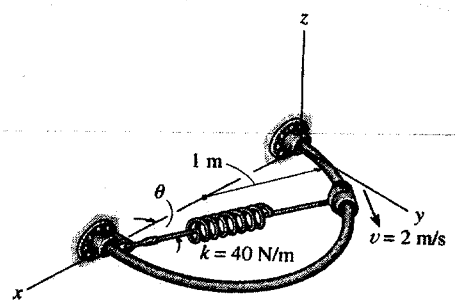
$a_t = 29.4$ ft/s^2 **Ans**

$\swarrow + \Sigma F_n = m\,a_n;\quad 180\cos 65.76° - N = \frac{180}{32.2}\left(\frac{6^2}{41.43}\right)$

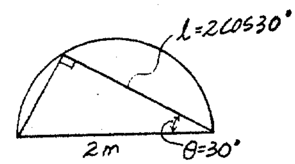
$N = 69.0$ lb **Ans**



13-82. The collar has a mass of 5 kg and is confined to move along the smooth circular rod which lies in the horizontal plane. The attached spring has an unstretched length of 200 mm. If, at the instant $\theta = 30°$, the collar has a speed $v = 2$m/s, determine the magnitude of normal force of the rod on the collar and the collar's acceleration.



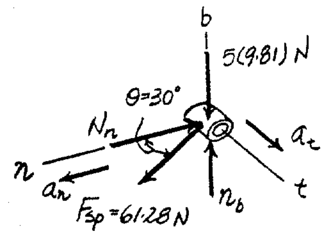
Equation of Motion : The spring force is given by $F_{sp} = k(l - l_0) = 40(2\cos 30° - 0.2) = 61.28$ N. The normal component of acceleration is $a_n = \frac{v^2}{\rho} = \frac{2^2}{1} = 4\ \text{m/s}^2$. Applying Eq. 13–8, we have



$\Sigma F_b = 0;\qquad N_b - 5(9.81) = 0 \qquad N_b = 49.05\ \text{N}$

$\Sigma F_t = ma_t;\qquad 61.28\sin 30° = 5a_t \qquad a_t = 6.128\ \text{m/s}^2$

$\Sigma F_n = ma_n;\qquad 61.28\cos 30° - N_n = 5(4) \qquad N_n = 33.07\ \text{N}$



Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{6.128^2 + 4^2} = 7.32\ \text{m/s}^2 \qquad \textbf{Ans}$$

and the magnitude of normal force is

$$N = \sqrt{N_b^2 + N_n^2} = \sqrt{49.05^2 + 33.07^2} = 59.2\ \text{N} \qquad \textbf{Ans}$$

13-83. A particle, having a mass of 1.5 kg, moves along a path defined by the equations $r = (4 + 3t)$ m, $\theta = (t^2 + 2)$ rad, and $z = (6 - t^3)$ m, where t is in seconds. Determine the r, θ, and z components of force which the path exerts on the particle when $t = 2$ s.

$r = 4 + 3t|_{t=2\,s} = 10$ m $\qquad \dot{r} = 3$ m/s $\qquad \ddot{r} = 0$

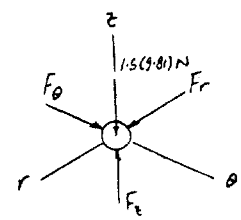
$\theta = t^2 + 2 \qquad \dot{\theta} = 2t|_{t=2\,s} = 4$ rad/s $\qquad \ddot{\theta} = 2$ rad/s^2

$z = 6 - t^3 \qquad \dot{z} = -3t^2 \qquad \ddot{z} = -6t|_{t=2\,s} = -12$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 10(4)^2 = -160\ \text{m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 10(2) + 2(3)(4) = 44\ \text{m/s}^2$

$a_z = \ddot{z} = -12\ \text{m/s}^2$

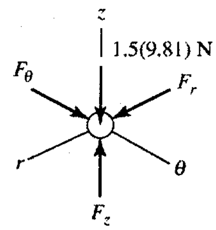


$\Sigma F_r = ma_r;\qquad F_r = 1.5(-160) = -240\ \text{N} \qquad$ **Ans**

$\Sigma F_\theta = ma_\theta;\qquad F_\theta = 1.5(44) = 66\ \text{N} \qquad$ **Ans**

$\Sigma F_z = ma_z;\qquad F_z - 1.5(9.81) = 1.5(-12) \qquad F_z = -3.28\ \text{N} \qquad$ **Ans**

***13-84.** The path of motion of a 5-lb particle in the horizontal plane is described in terms of polar coordinates as $r = (2t + 1)$ ft and $\theta = (0.5t^2 - t)$ rad, where t is in seconds. Determine the magnitude of the unbalanced force acting on the particle when $t = 2$ s.



$r = 2t + 1|_{t=2s} = 5$ ft $\quad \dot{r} = 2$ ft/s $\quad \ddot{r} = 0$

$\theta = 0.5t^2 - t|_{t=2s} = 0$ rad $\quad \dot{\theta} = t - 1|_{t=2s} = 1$ rad/s $\quad \ddot{\theta} = 1$ rad/s^2

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1)^2 = -5 \text{ ft/s}^2$$

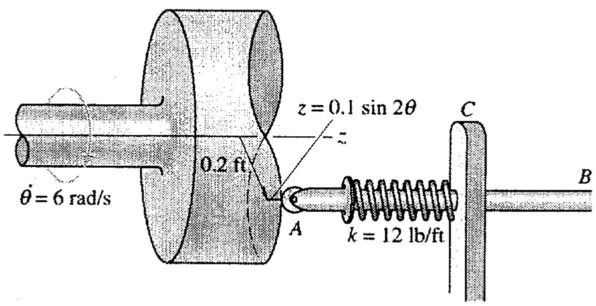
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(1) + 2(2)(1) = 9 \text{ ft/s}^2$$

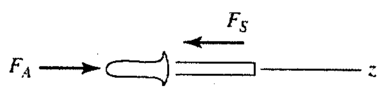
$$\sum F_r = ma_r; \quad F_r = \frac{5}{32.2}(-5) = -0.7764 \text{ lb}$$

$$\sum F_\theta = ma_\theta; \quad F_\theta = \frac{5}{32.2}(9) = 1.398 \text{ lb}$$

$$F = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(-0.7764)^2 + (1.398)^2} = 1.60 \text{ lb}$$ **Ans**

13-85. The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.2$ ft and $z = (0.1 \sin 2\theta)$ ft. If the cam is rotating at a constant rate of 6 rad/s, determine the force at the end A of the follower when $\theta = 45°$. In this position the spring is compressed 0.4 ft. Neglect friction at the bearing C.





$$z = 0.1 \sin 2\theta$$

$$\dot{z} = 0.2 \cos 2\theta\dot{\theta}$$

$$\ddot{z} = -0.4 \sin 2\theta\dot{\theta}^2 + 0.2 \cos 2\theta\ddot{\theta}$$

$$\dot{\theta} = 6 \text{ rad/s}$$

$$\ddot{\theta} = 0$$

$$\ddot{z} = -14.4 \sin 2\theta$$

$$\sum F_z = ma_z; \quad F_A - 12(z + 0.3) = m\ddot{z}$$

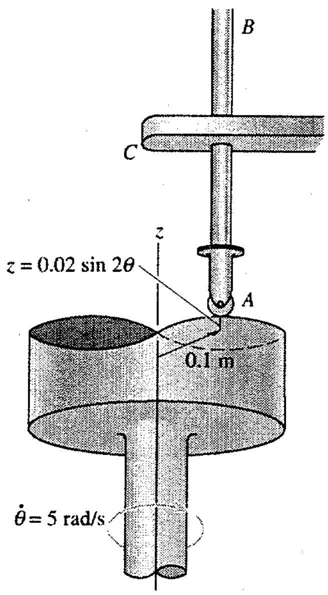
$$F_A - 12(0.1 \sin 2\theta + 0.3) = \frac{0.75}{32.2}(-14.4 \sin 2\theta)$$

For $\theta = 45°$,

$$F_A - 12(0.4) = \frac{0.75}{32.2}(-14.4)$$

$$F_A = 4.46 \text{ lb}$$ **Ans**

13-86. The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.2$ ft and $z = (0.1 \sin 2(\theta))$ ft. If the cam is rotating at a constant rate of 6 rad/s, determine the maximum and minimum force the follower exerts on the cam if the spring is compressed 0.2 ft when $\theta = 45$.



$z = 0.1 \sin 2\theta$

$\dot{z} = 0.2 \cos 2\theta\dot{\theta}$

$\ddot{z} = -0.4 \sin 2\theta\dot{\theta}^2 + 0.2 \cos 2\theta\ddot{\theta}$

$\dot{\theta} = 6$ rad/s

$\ddot{\theta} = 0$

$\ddot{z} = -14.4 \sin 2\theta$

$\sum F_z = ma_z;\quad F_A - 12(z + 0.1) = m\ddot{z}$

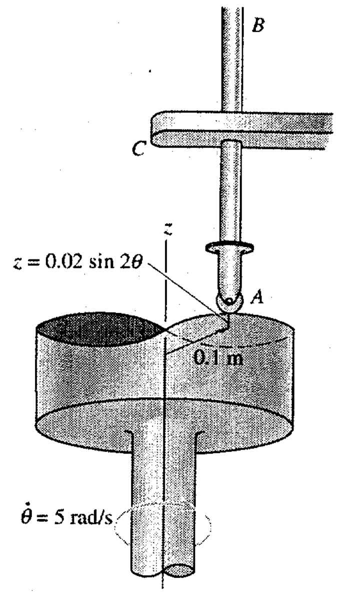
$$F_A - 12(0.1 \sin 2\theta + 0.1) = \frac{0.75}{32.2}(-14.4 \sin 2\theta)$$

$$F_A = 1.2 - 0.8646 \sin 2\theta$$

$(F_A)_{max} = 2.06$ lb **Ans**

$(F_A)_{min} = 0.335$ lb **Ans**

13-87. The 2-kg rod AB moves up and down as its end slides on the smooth contoured surface of the cam, where $r = 0.1$ m and $z = (0.02 \sin 2\theta)$ m. If the cam is rotating at a constant rate of 5 rad/s, determine the maximum and minimum force the cam exerts on the rod.



Kinematic: Taking the required time derivatives, we have

$\dot{\theta} = 5$ rad/s $\quad \ddot{\theta} = 0$

$z = 0.02 \sin 2\theta \quad \dot{z} = 0.04 \cos 2\theta\dot{\theta} \quad \ddot{z} = 0.04(\cos 2\theta\ddot{\theta} - 2\sin 2\theta\dot{\theta}^2)$

Thus,

$a_z = \ddot{z} = 0.04[\cos 2\theta(0) - 2\sin 2\theta(5^2)] = -2\sin 2\theta$

At $\theta = 45°$, $\quad a_z = -2\sin 90° = -2$ m/s^2

At $\theta = -45°$, $\quad a_z = -2\sin(-90°) = 2$ m/s^2

Equation of Motion: At $\theta = 45°$, applying Eq. 13-9, we have

$\sum F_z = ma_z; \quad (F_z)_{min} - 2(9.81) = 2(-2)$

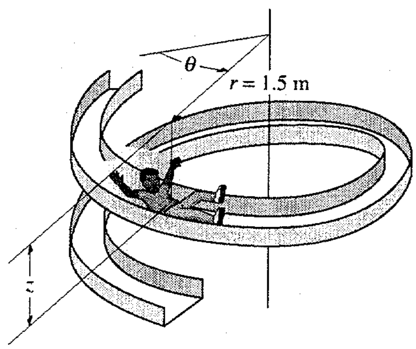
$(F_z)_{min} = 15.6$ N **Ans**

At $\theta = -45°$, we have

$\sum F_z = ma_z; \quad (F_z)_{max} - 2(9.81) = 2(2)$

$(F_z)_{max} = 23.6$ N **Ans**

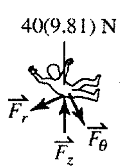
***13-88.** The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.



$r = 1.5 \quad \theta = 0.7t \quad z = -0.5t$

$\dot{r} = \ddot{r} = 0 \quad \dot{\theta} = 0.7 \quad \dot{z} = -0.5$

$\ddot{\theta} = 0 \quad \ddot{z} = 0$



$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(0.7)^2 = -0.735$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \ddot{z} = 0$

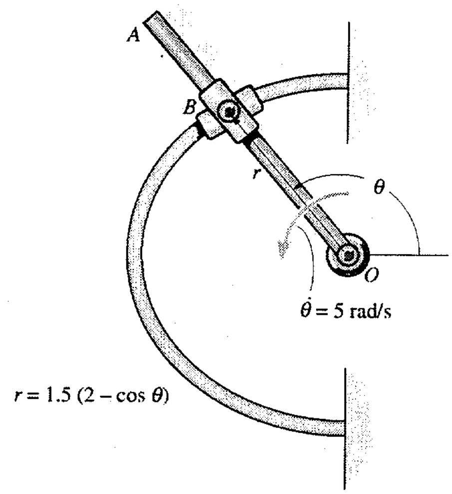
$\sum F_r = ma_r; \quad F_r = 40(-0.735) = -29.4$ N **Ans**

$\sum F_\theta = ma_\theta; \quad F_\theta = 0$ **Ans**

$\sum F_z = ma_z; \quad F_z - 40(9.81) = 0$

$F_z = 392$ N **Ans**

13-89. Rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. The double collar B is pin-connected together such that one collar slides over the rotating rod and the other slides over the *horizontal* curved rod, of which the shape is described by the equation $r = 1.5(2 - \cos\theta)$ ft. If both collars weigh 0.75 lb, determine the normal force which the curved rod exerts on one collar at the instant $\theta = 120°$. Neglect friction.



Kinematic: Here, $\dot{\theta} = 5$ rad/s and $\ddot{\theta} = 0$. Taking the required time derivatives at $\theta = 120°$, we have

$$r = 1.5(2 - \cos\theta)|_{\theta=120°} = 3.75 \text{ ft}$$

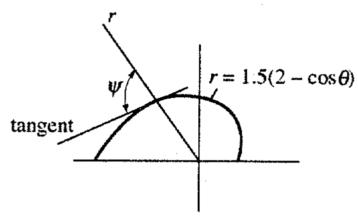
$$\dot{r} = 1.5\sin\theta\dot{\theta}|_{\theta=120°} = 6.495 \text{ ft/s}$$

$$\ddot{r} = 1.5(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)|_{\theta=120°} = -18.75 \text{ ft/s}^2$$

Applying Eqs. 12-29, we have

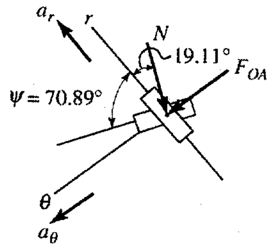
$$a_r = \ddot{r} - r\dot{\theta}^2 = -18.75 - 3.75(5^2) = -112.5 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3.75(0) + 2(6.495)(5) = 64.952 \text{ ft/s}^2$$



Equation of Motion: The angle ψ must be obtained first.

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{1.5(2-\cos\theta)}{1.5\sin\theta}\bigg|_{\theta=120°} = 2.8867 \quad \psi = 70.89°$$



Applying Eq. 13-9, we have

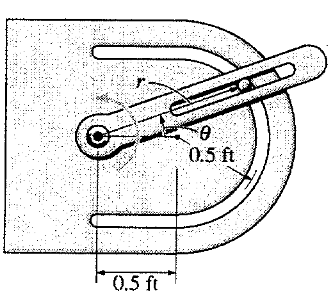
$$\sum F_r = ma_r; \quad -N\cos 19.11° = \frac{0.75}{32.2}(-112.5)$$

$$N = 2.773 \text{ lb} = 2.77 \text{ lb} \qquad \textbf{Ans}$$

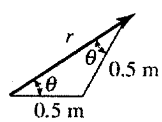
$$\sum F_\theta = ma_\theta; \quad F_{OA} + 2.773\sin 19.11° = \frac{0.75}{32.2}(64.952)$$

$$F_{OA} = 0.605 \text{ lb}$$

13-90. The 0.5-lb particle is guided along the circular path using the slotted arm guide. If the arm has an "angular velocity $\ddot{\theta} = 4$ rad/s and an angular acceleration $\dot{\theta} = 8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the guide on the particle. Motion occurs in the *horizontal plane*.



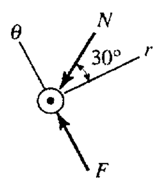
$$r = 2(0.5\cos\theta) = 1\cos\theta$$



$$\dot{r} = -\sin\theta\dot{\theta}$$

$$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$$

At $\theta = 30°$, $\dot{\theta} = 4$ rad/s and $\ddot{\theta} = 8$ rad/s^2



$$r = 1\cos 30° = 0.8660 \text{ ft}$$

$$\dot{r} = -\sin 30°(4) = -2 \text{ ft/s}$$

$$\ddot{r} = -\cos 30°(4)^2 - \sin 30°(8) = -17.856 \text{ ft/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -17.856 - 0.8660(4)^2 = -31.713 \text{ ft/s}^2$$

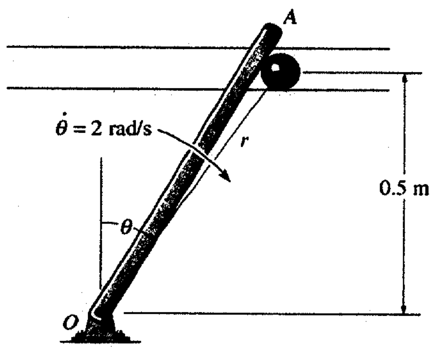
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(8) + 2(-2)(4) = -9.072 \text{ ft/s}^2$$

$$\nearrow + \sum F_r = ma_r; \quad -N\cos 30° = \frac{0.5}{32.2}(-31.713) \quad N = 0.5686 \text{ lb}$$

$$+\nwarrow \sum F_\theta = ma_\theta; \quad F - 0.5686\sin 30° = \frac{0.5}{32.2}(-9.072)$$

$$F = 0.143 \text{ lb} \qquad \textbf{Ans}$$

13-91. The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[\left(\sec\theta\tan\theta\dot{\theta}\right)\tan\theta + \sec\theta\left(\sec^2\theta\dot{\theta}\right)\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$

When $\theta = 30°$, $\dot{\theta} = 2$rad/s and $\ddot{\theta} = 0$

$$r = 0.5\sec 30° = 0.5774 \text{ m}$$

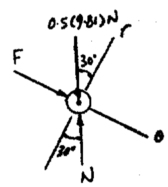
$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667 \text{ m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(0)\right]$$

$$= 3.849 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 3.849 - 0.5774(2)^2 = 1.540 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(0.6667)(2) = 2.667 \text{ m/s}^2$$



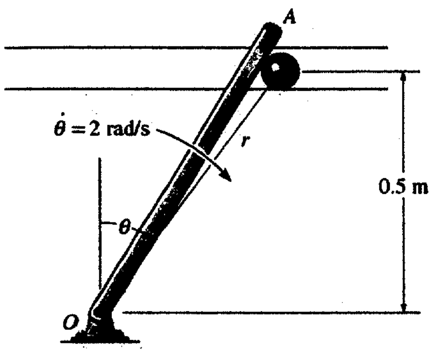
$\nearrow^{+}\Sigma F_r = ma_r;$ $\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(1.540)$

$N = 5.79$ N $\quad$ **Ans**

$+\searrow\Sigma F_\theta = ma_\theta;$ $\quad F + 0.5(9.81)\sin 30° - 5.79\sin 30° = 0.5(2.667)$

$F = 1.78$ N $\quad$ **Ans**

*13-92. Solve Problem 13–91 if the arm has an angular acceleration of $\ddot{\theta} = 3\ \text{rad/s}^2$ and $\dot{\theta} = 2\ \text{rad/s}$ at this instant. Assume the particle contacts only one side of the slot at any instant.



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[\left(\sec\theta\tan\theta\dot{\theta}\right)\tan\theta + \sec\theta\left(\sec^2\theta\dot{\theta}\right)\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$

When $\theta = 30°$, $\dot{\theta} = 2\text{rad/s}$ and $\ddot{\theta} = 3\ \text{rad/s}^2$

$$r = 0.5\sec 30° = 0.5774\ \text{m}$$

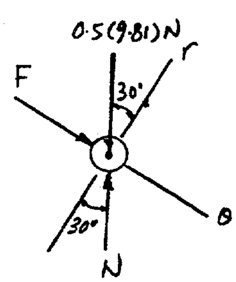
$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667\ \text{m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(3)\right]$$

$$= 4.849\ \text{m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4.849 - 0.5774(2)^2 = 2.5396\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(3) + 2(0.6667)(2) = 4.3987\ \text{m/s}^2$$



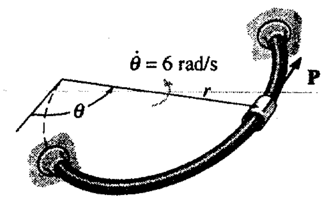
$\nearrow + \Sigma F_r = m a_r;$ $\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(2.5396)$

$N = 6.3712 = 6.37\ \text{N}$ **Ans**

$+\searrow \Sigma F_\theta = m a_\theta;$ $\quad F + 0.5(9.81)\sin 30° - 6.3712\sin 30° = 0.5(4.3987)$

$F = 2.93\ \text{N}$ **Ans**

13-93. The spool, which has a mass of 2 kg, slides along the smooth *horizontal* spiral rod, $r = (0.4\theta)$ m, where θ is in radians. If its angular rate of rotation is constant and equals $\dot{\theta} = 6$ rad/s, determine the horizontal tangential force P needed to cause the motion and the horizontal normal force component that the spool exerts on the rod at the instant $\theta = 45°$.



$r = 0.4\theta \qquad \dot{r} = 0.4\dot{\theta} \qquad \ddot{r} = 0.4\ddot{\theta}$

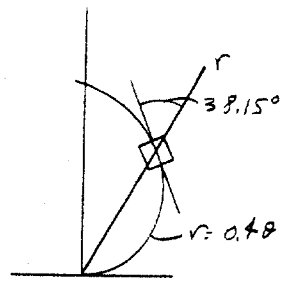
At $\theta = 45° = \dfrac{\pi}{4}$ rad, $\dot{\theta} = 6$ rad/s and $\ddot{\theta} = 0$

$r = 0.4\left(\dfrac{\pi}{4}\right) = (0.1)\,\pi$ m $\qquad \dot{r} = 0.4(6) = 2.4$ m/s $\qquad \ddot{r} = 0.4(0) = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (0.1)\pi(6)^2 = -11.310$ m/s^2

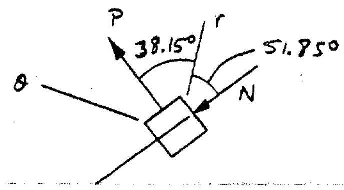
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.1)\pi(0) + 2(2.4)(6) = 28.8$ m/s^2

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{0.4\theta}{0.4} = \theta = \dfrac{\pi}{4} \qquad \psi = 38.15°$



$\nearrow\!+\,\Sigma F_r = ma_r; \qquad P\cos 38.15° - N\cos 51.85° = 2(-11.310)$

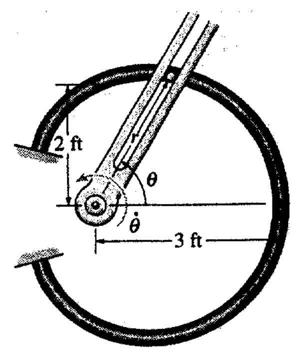
$\nwarrow\!+\,\Sigma F_\theta = ma_\theta; \qquad P\sin 38.15° + N\sin 51.85° = 2(28.8)$



Solving

$N = 59.3$ N $\qquad P = 17.8$ N $\qquad$ **Ans**

13-94. The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If at all times $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the particle at the instant $\theta = 90°$. The fork and path contact the particle on only one side.



$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 90°$, $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$

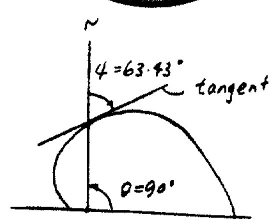
$r = 2 + \cos 90° = 2$ ft

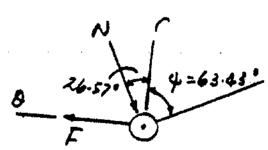
$\dot{r} = -\sin 90°(0.5) = -0.5$ ft/s

$\ddot{r} = -\cos 90°(0.5)^2 - \sin 90°(0) = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 2(0.5)^2 = -0.5$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(0) + 2(-0.5)(0.5) = -0.5$ ft/s^2





$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{2 + \cos\theta}{-\sin\theta}\Big|_{\theta = 90°} = -2 \qquad \psi = -63.43°$

$+\uparrow\Sigma F_r = ma_r; \qquad -N\cos 26.57° = \dfrac{2}{32.2}(-0.5) \qquad N = 0.03472$ lb

$\overset{+}{\leftarrow}\Sigma F_\theta = ma_\theta; \qquad F - 0.03472\sin 26.57° = \dfrac{2}{32.2}(-0.5)$

$F = -0.0155$ lb $\qquad$ **Ans**

13-95. Solve Prob. 13-94 at the instant $\theta = 60°$.

$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 60°$, $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$

$r = 2 + \cos 60° = 2.5$ ft

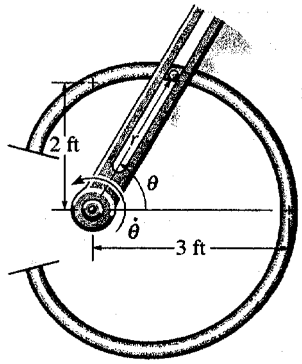
$\dot{r} = -\sin 60°(0.5) = -0.4330$ ft/s

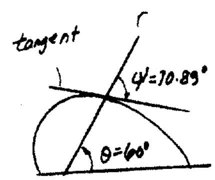
$\ddot{r} = -\cos 60°(0.5)^2 - \sin 60°(0) = -0.125$ ft/s^2

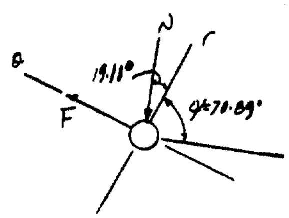
$a_r = \ddot{r} - r\dot{\theta}^2 = -0.125 - 2.5(0.5)^2 = -0.75$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.5(0) + 2(-0.4330)(0.5) = -0.4330$ ft/s^2

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{2+\cos\theta}{-\sin\theta}\bigg|_{\theta=60°} = -2.887 \qquad \psi = -70.89°$





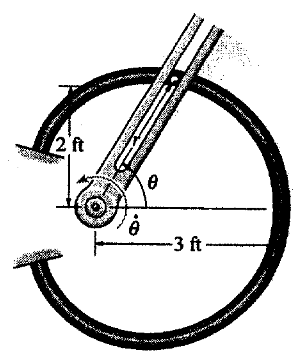


$\nearrow^{+}\Sigma F_r = ma_r; \qquad -N\cos 19.11° = \dfrac{2}{32.2}(-0.75) \qquad N = 0.04930$ lb

$\nwarrow^{+}\Sigma F_\theta = ma_\theta; \qquad F - 0.04930\sin 19.11° = \dfrac{2}{32.2}(-0.4330)$

$F = -0.0108$ lb **Ans**

***13-96.** The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.



$r = 2 + \cos\theta \qquad \theta = 0.5t^2$

$\dot{r} = -\sin\theta\dot{\theta} \qquad \dot{\theta} = t$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta} \qquad \ddot{\theta} = 1$ rad/s^2

At $t = 1$ s, $\theta = 0.5$ rad, $\dot{\theta} = 1$ rad/s and $\ddot{\theta} = 1$ rad/s^2

$r = 2 + \cos 0.5 = 2.8776$ ft

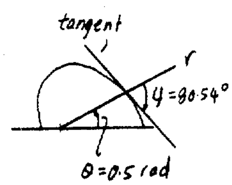
$\dot{r} = -\sin 0.5(1) = -0.4794$ ft/s

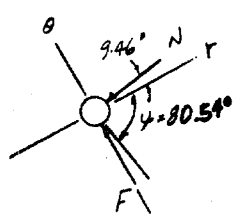
$\ddot{r} = -\cos 0.5(1)^2 - \sin 0.5(1) = -1.357$ ft/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -1.357 - 2.8776(1)^2 = -4.2346$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.8776(1) + 2(-0.4794)(1) = 1.9187$ ft/s^2

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{2+\cos\theta}{-\sin\theta}\bigg|_{\theta=0.5\text{ rad}} = -6.002 \qquad \psi = -80.54°$



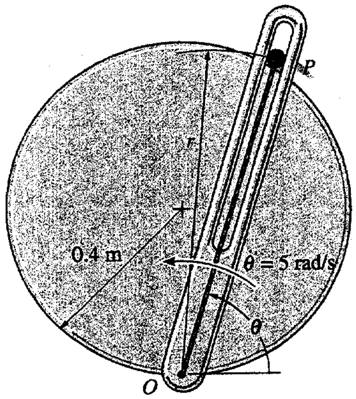


$\nearrow^{+}\Sigma F_r = ma_r; \qquad -N\cos 9.46° = \dfrac{2}{32.2}(-4.2346) \qquad N = 0.2666$ lb

$\nwarrow^{+}\Sigma F_\theta = ma_\theta; \qquad F - 0.2666\sin 9.46° = \dfrac{2}{32.2}(1.9187)$

$F = 0.163$ lb **Ans**

13-97. The smooth particle has a mass of 80 g. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = (0.8 \sin\theta)$ m. If the cord has a stiffness $k = 30$ N/m and an unstretched length of 0.25 m, determine the force of the guide on the particle when $\theta = 60°$. The guide has a constant angular velocity $\dot{\theta} = 5$ rad/s.



$r = 0.8 \sin\theta$

$\dot{r} = 0.8 \cos\theta\, \dot{\theta}$

$\ddot{r} = -0.8 \sin\theta\, (\dot{\theta})^2 + 0.8 \cos\theta\, \ddot{\theta}$

$\dot{\theta} = 5, \quad \ddot{\theta} = 0$

At $\theta = 60°$, $\quad r = 0.6928$

$\dot{r} = 2$

$\ddot{r} = -17.321$

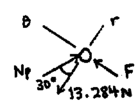
$a_r = \ddot{r} - r(\dot{\theta})^2 = -17.321 - 0.6928(5)^2 = -34.641$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(2)(5) = 20$

$F_s = ks; \quad F_s = 30(0.6928 - 0.25) = 13.284$ N

$\nearrow + \Sigma F_r = m a_r; \quad -13.284 + N_P \cos 30° = 0.08(-34.641)$

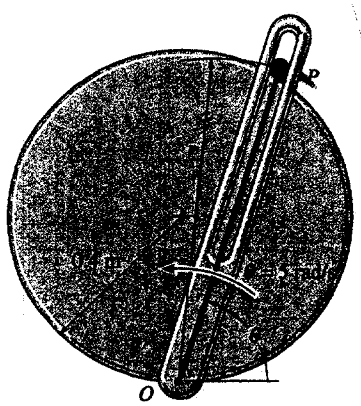
$\nwarrow + \Sigma F_\theta = m a_\theta; \quad F - N_P \sin 30° = 0.08(20)$



$F = 7.67$ N **Ans**

$N_P = 12.1$ N

13-98. Solve Prob. 13–97 if $\ddot{\theta} = 2$ rad/s^2 when $\dot{\theta} = 5$ rad/s and $\theta = 60°$.



$r = 0.8 \sin\theta$

$\dot{r} = 0.8 \cos\theta\, \dot{\theta}$

$\ddot{r} = -0.8 \sin\theta\, (\dot{\theta})^2 + 0.8 \cos\theta\, \ddot{\theta}$

$\dot{\theta} = 5, \quad \ddot{\theta} = 2$

At $\theta = 60°$, $\quad r = 0.6928$

$\dot{r} = 2$

$\ddot{r} = -16.521$

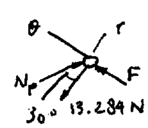
$a_r = \ddot{r} - r(\dot{\theta})^2 = -16.521 - 0.6928(5)^2 = -33.841$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6928(2) + 2(2)(5) = 21.386$

$F_s = ks; \quad F_s = 30(0.6928 - 0.25) = 13.284$ N

$\nearrow + \Sigma F_r = m a_r; \quad -13.284 + N_P \cos 30° = 0.08(-33.841)$

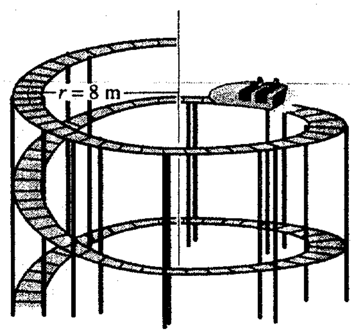
$+\nwarrow \Sigma F_\theta = m a_\theta; \quad F - N_P \sin 30° = 0.08(21.386)$



$F = 7.82$ N **Ans**

$N_P = 12.2$ N

13-99. For a short time, the 250-kg roller coaster car is traveling along the spiral track such that its position measured from the top of the track has components $r = 8$ m, $\theta = (0.1t + 0.5)$ rad, and $z = (-0.2t)$ m, where t is in seconds. Determine the magnitudes of the components of force which the track exerts on the car in the r, θ, and z directions at the instant $t = 2$ s. Neglect the size of the car.

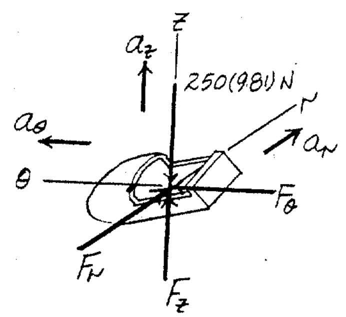


Kinematic : Here, $r = 8$ m, $\dot{r} = \ddot{r} = 0$. Taking the required time derivatives at $t = 2$ s, we have

$$\theta = 0.1t + 0.5|_{t=2s} = 0.700 \text{ rad} \qquad \dot{\theta} = 0.100 \text{ rad/s} \qquad \ddot{\theta} = 0$$

$$z = -0.2t|_{t=2s} = -0.400 \text{ m} \qquad \dot{z} = -0.200 \text{ m/s} \qquad \ddot{z} = 0$$

Applying Eqs. 12–29, we have



$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 8(0.100^2) = -0.0800 \text{ m/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 8(0) + 2(0)(0.200) = 0$$
$$a_z = \ddot{z} = 0$$

Equation of Motion :

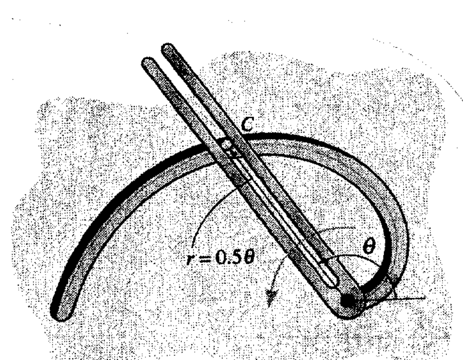
$\Sigma F_r = ma_r;$ $\quad F_r = 250(-0.0800) = -20.0$ N **Ans**

$\Sigma F_\theta = ma_\theta;$ $\quad F_\theta = 250(0) = 0$ **Ans**

$\Sigma F_z = ma_z;$ $\quad F_z - 250(9.81) = 250(0)$

$F_z = 2452.5$ N $= 2.45$ kN **Ans**

***13-100.** Using a forked rod, a smooth cylinder C having a mass of 0.5 kg is forced to move along the *vertical slotted* path $r = (0.5\theta)$ m, where θ is in radians. If the angular position of the arm is $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force of the rod on the cylinder and the normal force of the slot on the cylinder at the instant $t = 2$ s. The cylinder is in contact with only *one edge* of the rod and slot at any instant.



$r = 0.5\theta \qquad \dot{r} = 0.5\dot{\theta} \qquad \ddot{r} = 0.5\ddot{\theta}$

$\theta = 0.5t^2 \qquad \dot{\theta} = t \qquad \ddot{\theta} = 1$

At $t = 2$ s,

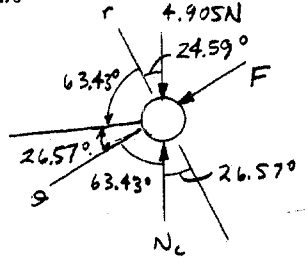
$\theta = 2 \text{ rad} = 114.59° \qquad \dot{\theta} = 2 \text{ rad/s} \qquad \ddot{\theta} = 1 \text{ rad/s}^2$

$r = 1 \text{ m} \qquad \dot{r} = 1 \text{ m/s} \qquad \ddot{r} = 0.5 \text{ m/s}^2$

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.5(2)}{0.5} \qquad \psi = 63.43°$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0.5 - 1(2)^2 = -3.5$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1(1) + 2(1)(2) = 5$$



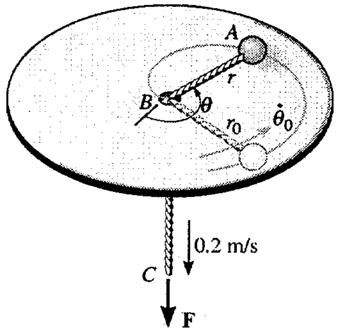
$\nwarrow + \Sigma F_r = ma_r;$ $\quad N_C \cos 26.57° - 4.905\cos 24.59° = 0.5(-3.5)$

$N_C = 3.030 = 3.03$ N **Ans**

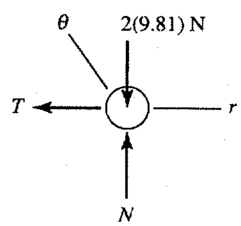
$+\swarrow \Sigma F_\theta = ma_\theta;$ $\quad F - 3.030\sin 26.57° + 4.905\sin 24.59° = 0.5(5)$

$F = 1.81$ N **Ans**

13-101. The ball has a mass of 2 kg and a negligible size. It is originally traveling around the horizontal circular path of radius $r_0 = 0.5$ m such that the angular rate of rotation is $\dot{\theta}_0 = 1$ rad/s. If the attached cord ABC is drawn down through the hole at a constant speed of 0.2 m/s, determine the tension the cord exerts on the ball at the instant $r = 0.25$ m. Also, compute the angular velocity of the ball at this instant. Neglect the effects of friction between the ball and horizontal plane. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)(d(r^2\dot{\theta})/dt) = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



$\sum F_\theta = ma_\theta; \quad 0 = m[r\ddot{\theta} + 2\dot{r}\dot{\theta}] = m\left[\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\right] = 0$



Thus,

$d(r^2\dot{\theta}) = 0$

$r^2\dot{\theta} = C$

$(0.5)^2(1) = C = (0.25)^2\dot{\theta}$

$\dot{\theta} = 4.00$ rad/s **Ans**

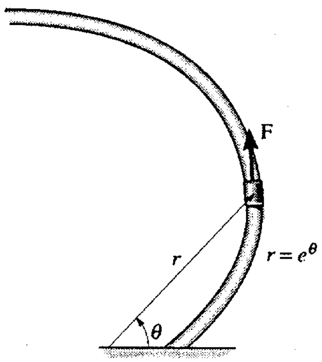
Since $r = -0.2$ m/s, $\ddot{r} = 0$

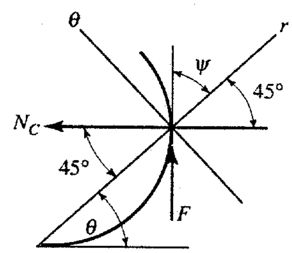
$a_r = r - r(\theta)^2 = 0 - 0.25(4.00)^2 = -4$ m/s^2

$\sum F_r = ma_r; \quad -T = 2(-4)$

$T = 8$ N **Ans**

13-102. The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 45°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.





$r = e^{\theta}$

$\dot{r} = e^{\theta}\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 45°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 2.1933$

$\dot{r} = 4.38656$

$\ddot{r} = 8.7731$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 8.7731 - 2.1933(2)^2 = 0$

$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(4.38656)(2) = 17.5462 \text{ m/s}^2$

$\tan\Psi = \dfrac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$

$\Psi = \theta = 45°$

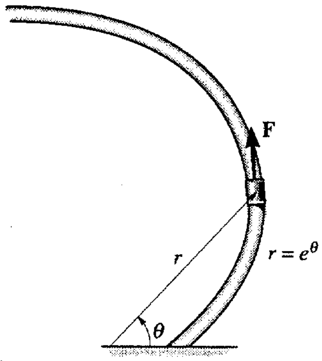
$\nearrow + \sum F_r = ma_r; \quad -N_C \cos 45° + F \cos 45° = 2(0)$

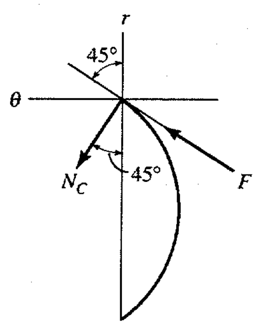
$+\nwarrow \sum F_{\theta} = ma_{\theta}; \quad F \sin 45° + N_C \sin 45° = 2(17.5462)$

$N = 24.8$ N **Ans**

$F = 24.8$ N **Ans**

13-103. The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 90°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.





$r = e^{\theta}$

$\dot{r} = e^{\theta}\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 90°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 4.8105$

$\dot{r} = 9.6210$

$\ddot{r} = 19.242$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 19.242 - 4.8105(2)^2 = 0$

$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(9.6210)(2) = 38.4838 \text{ m/s}^2$

$\tan\psi = \dfrac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$

$\psi = 45°$

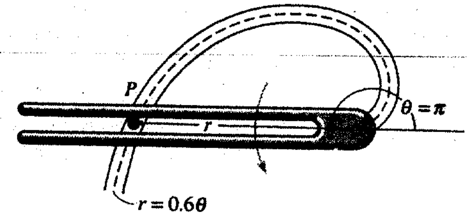
$+\uparrow \sum F_r = ma_r;\quad -N_C \cos 45° + F\cos 45° = 2(0)$

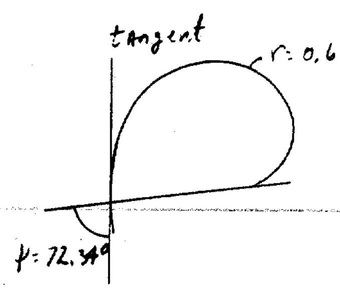
$\overset{+}{\leftarrow} \sum F_{\theta} = ma_{\theta};\quad F\sin 45° + N_C \sin 45° = 2(38.4838)$

$N_C = 54.4N$ **Ans**

$F = 54.4$ N **Ans**

***13-104.** Using a forked rod, a smooth cylinder P, having a mass of 0.4 kg, is forced to move along the *vertical slotted* path $r = (0.6\theta)$ m, where θ is in radians. If the cylinder has a constant speed of $v_C = 2$ m/s, determine the force of the rod and the normal force of the slot on the cylinder at the instant $\theta = \pi$ rad. Assume the cylinder is in contact with only *one edge* of the rod and slot at any instant. *Hint:* To obtain the time derivatives necessary to compute the cylinder's acceleration components a_r and a_θ, take the first and second time derivatives of $r = 0.6\theta$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_C = 0$, to determine $\ddot{\theta}$.





$r = 0.6\theta \qquad \dot{r} = 0.6\dot{\theta} \qquad \ddot{r} = 0.6\ddot{\theta}$

$v_r = \dot{r} = 0.6\dot{\theta} \qquad v_\theta = r\dot{\theta} = 0.6\theta\dot{\theta}$

$v^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

$2^2 = \left(0.6\dot{\theta}\right)^2 + \left(0.6\theta\dot{\theta}\right)^2 \qquad \dot{\theta} = \dfrac{2}{0.6\sqrt{1+\theta^2}}$

$0 = 0.72\dot{\theta}\ddot{\theta} + 0.36\left(2\theta\dot{\theta}^3 + 2\theta^2\dot{\theta}\ddot{\theta}\right) \qquad \ddot{\theta} = -\dfrac{\theta\dot{\theta}^2}{1+\theta^2}$

At $\theta = \pi$ rad, $\qquad \dot{\theta} = \dfrac{2}{0.6\sqrt{1+\pi^2}} = 1.011$ rad/s

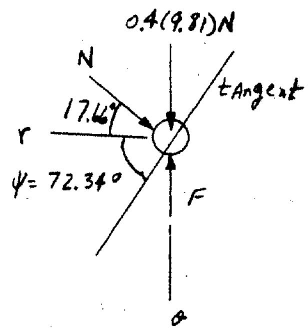
$\ddot{\theta} = -\dfrac{(\pi)(1.011)^2}{1+\pi^2} = -0.2954$ rad/s^2

$r = 0.6(\pi) = 0.6\pi$ m $\qquad \dot{r} = 0.6(1.011) = 0.6066$ m/s

$\ddot{r} = 0.6(-0.2954) = -0.1772$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1772 - 0.6\pi(1.011)^2 = -2.104$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6\pi(-0.2954) + 2(0.6066)(1.011) = 0.6698$ m/s^2



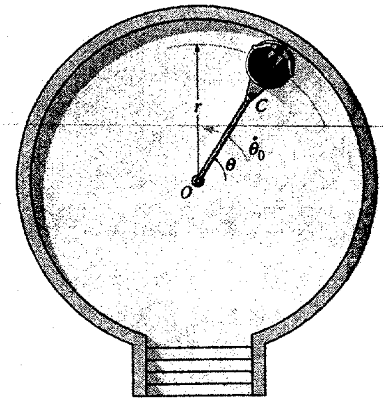
$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{0.6\theta}{0.6} = \theta = \pi \qquad \psi = 72.34°$

$\overset{+}{\leftarrow} \Sigma F_r = ma_r; \qquad -N\cos 17.66° = 0.4(-2.104) \qquad N = 0.883$ N **Ans**

$+\downarrow \Sigma F_\theta = ma_\theta; \qquad -F + 0.4(9.81) + 0.883\sin 17.66° = 0.4(0.6698)$

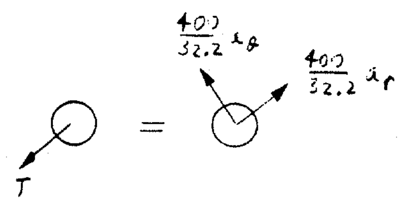
$F = 3.92$ N **Ans**

13-105. A ride in an amusement park consists of a cart which is supported by small wheels. Initially the cart is traveling in a circular path of radius $r_0 = 16$ ft such that the angular rate of rotation is $\dot{\theta}_0 = 0.2$ rad/s. If the attached cable OC is drawn inward at a constant speed of $\dot{r} = -0.5$ ft/s, determine the tension it exerts on the cart at the instant $r = 4$ ft. The cart and its passengers have a total weight of 400 lb. Neglect the effects of friction. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)d(r^2\dot{\theta})/dt = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



$+\nearrow \Sigma F_r = ma_r; \qquad -T = \left(\frac{400}{32.2}\right)\left(\ddot{r} - r\dot{\theta}^2\right) \qquad (1)$

$+\nwarrow \Sigma F_\theta = ma_\theta; \qquad 0 = \left(\frac{400}{32.2}\right)\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) \qquad (2)$



From Eq. (2), $\left(\frac{1}{r}\right)\frac{d}{dt}\left(r^2\dot{\theta}\right) = 0 \qquad r^2\dot{\theta} = c$

Since $\dot{\theta}_0 = 0.2$ rad/s when $r_0 = 16$ ft, $c = 51.2$.
Hence, when $r = 4$ ft,

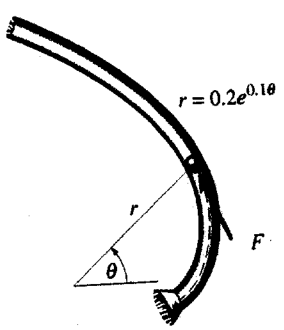
$\dot{\theta} = \left(\frac{51.2}{(4)^2}\right) = 3.2$ rad/s

Since $\dot{r} = -0.5$ ft/s, $\ddot{r} = 0$, Eq. (1) becomes

$-T = \left(\frac{400}{32.2}\right)\left(0 - (4)(3.2)^2\right)$

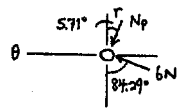
$T = 509$ lb **Ans**

13-106. Using air pressure, the 0.5-kg ball is forced to move through the tube lying in the *horizontal plane* and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to the air is 6 N, determine the rate of increase in the ball's speed at the instant $\theta = \pi/2$. What direction does it act?



$r = 0.2\,e^{0.1\theta}$

$\frac{dr}{d\theta} = 0.02\,e^{0.1\theta}$



$\tan\psi = \frac{r}{\frac{dr}{d\theta}} = \frac{0.2\,e^{0.1\theta}}{0.02\,e^{0.1\theta}} = 10$

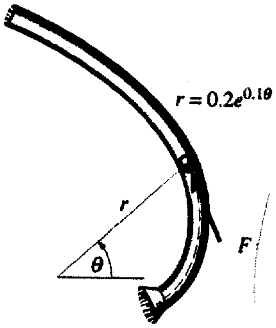
$\psi = 84.29°$

Rate of increase in speed is equivalent to the tangential component of acceleration.

$\Sigma F_t = m\,a_t; \qquad 6 = 0.5\,a_t$

$a_t = 12$ m/s² $\measuredangle$ 5.71° **Ans**

13-107. Solve Prob. 13–106 if the tube lies in a *vertical plane.*



$$r = 0.2\,e^{0.1\theta}$$

$$\frac{dr}{d\theta} = 0.02\,e^{0.1\theta}$$

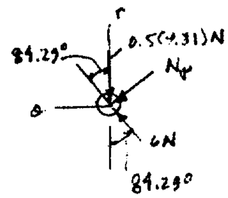
$$\tan\psi = \frac{r}{\frac{dr}{d\theta}} = \frac{0.2\,e^{0.1\theta}}{0.02\,e^{0.1\theta}} = 10$$

$$\psi = 84.29^\circ$$

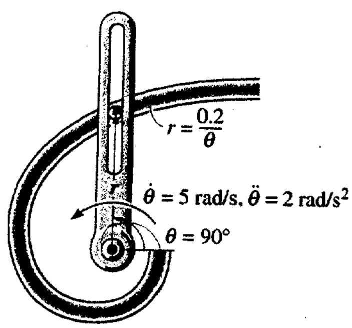
Rate of increase in speed is equivalent to the tangential component of acceleration.

$\Sigma F_t = 0.5\,a_t;\qquad 6 - 0.5(9.81)\cos 84.29^\circ = 0.5\,a_t$

$a_t = 11.0\ \text{m/s}^2$ ⦪ 5.71° **Ans**



***13-108.** The arm is rotating at a rate of $\dot{\theta} = 5$ rad/s when $\ddot{\theta} = 2$ rad/s² and $\theta = 90°$. Determine the normal force it must exert on the 0.5-kg particle if the particle is confined to move along the slotted path defined by the *horizontal* hyperbolic spiral $r\theta = 0.2$ m.



$$\theta = \frac{\pi}{2} = 90^\circ$$

$$\dot{\theta} = 5\ \text{rad/s}$$

$$\ddot{\theta} = 2\ \text{rad/s}^2$$

$$r = 0.2/\theta = 0.12732\ \text{m}$$

$$\dot{r} = -0.2\,\theta^{-2}\dot{\theta} = -0.40528\ \text{m/s}$$

$$\ddot{r} = -0.2[-2\theta^{-3}(\dot{\theta})^2 + \theta^{-2}\ddot{\theta}] = 2.41801$$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 2.41801 - 0.12732(5)^2 = -0.7651\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.12732(2) + 2(-0.40528)(5) = -3.7982\ \text{m/s}^2$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \frac{0.2/\theta}{-0.2\theta^{-2}}$$

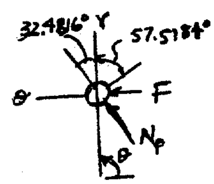
$$\psi = \tan^{-1}\left(-\frac{\pi}{2}\right) = -57.5184^\circ$$

$+\uparrow \Sigma F_r = m\,a_r;\qquad N_p\cos 32.4816^\circ = 0.5(-0.7651)$

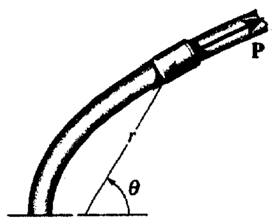
$\overset{+}{\leftarrow} \Sigma F_\theta = m\,a_\theta;\qquad F + N_p\sin 32.4816^\circ = 0.5(-3.7982)$

$N_p = -0.453$ N

$F = -1.66$ N **Ans**



13-109. The collar, which has a weight of 3 lb, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = 4/(1 - \cos\theta)$, where θ is in radians and r is in feet. If the collar's angular rate is constant and equals $\dot{\theta} = 4$ rad/s, determine the tangential retarding force P needed to cause the motion and the normal force that the collar exerts on the rod at the instant $\theta = 90°$.



$$r = \frac{4}{1-\cos\theta}$$

$$\dot{r} = \frac{-4\sin\theta\ \dot{\theta}}{(1-\cos\theta)^2}$$

$$\ddot{r} = \frac{-4\sin\theta\ \ddot{\theta}}{(1-\cos\theta)^2} + \frac{-4\cos\theta\ (\dot{\theta})^2}{(1-\cos\theta)^2} + \frac{8\sin^2\theta\ \dot{\theta}^2}{(1-\cos\theta)^3}$$

At $\theta = 90°$, $\quad \dot{\theta} = 4, \quad \ddot{\theta} = 0$

$$r = 4$$
$$\dot{r} = -16$$
$$\ddot{r} = 128$$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 128 - 4(4)^2 = 64$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-16)(4) = -128$$

$$r = \frac{4}{1-\cos\theta}$$

$$\frac{dr}{d\theta} = \frac{-4\sin\theta}{(1-\cos\theta)^2}$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \left.\frac{\frac{4}{1-\cos\theta}}{\frac{-4\sin\theta}{(1-\cos\theta)^2}}\right|_{\theta=90°} = \frac{4}{-4} = -1$$

$$\psi = -45° = 135°$$

$+\uparrow \Sigma F_r = m\,a_r; \quad P\sin 45° - N\cos 45° = \frac{3}{32.2}(64)$

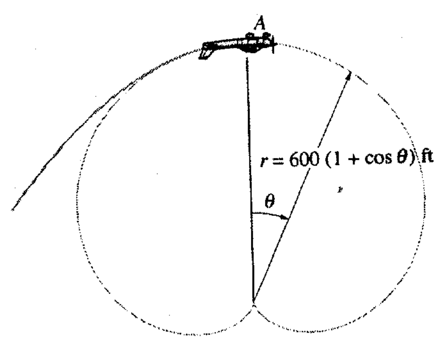
$\overset{+}{\leftarrow} \Sigma F_\theta = m a_\theta; \quad -P\cos 45° - N\sin 45° = \frac{3}{32.2}(-128)$

Solving,

$P = 12.6$ lb **Ans**

$N = 4.22$ lb **Ans**

13-110. The pilot of an airplane executes a vertical loop which in part follows the path of a cardioid, $r = 600(1 + \cos\theta)$ ft, where θ is in radians. If his speed at A ($\theta = 0°$) is a constant $v_P = 80$ ft/s, determine the vertical force the belt of his seat must exert on him to hold him to his seat when the plane is upside down at A. He weighs 150 lb. See hint related to Prob. 13–108.



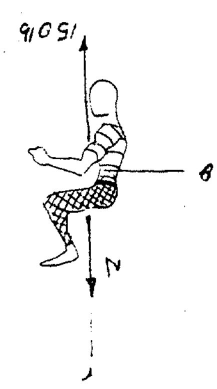
$$r = 600(1+\cos\theta)|_{\theta=0°} = 1200 \text{ ft}$$

$$\dot{r} = -600\sin\theta\dot{\theta}|_{\theta=0°} = 0$$

$$\ddot{r} = -600\sin\theta\ddot{\theta} - 600\cos\theta\dot{\theta}^2|_{\theta=0°} = -600\dot{\theta}^2$$

$$v_P^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$$

$$(80)^2 = 0 + \left(1200\dot{\theta}\right)^2 \qquad \dot{\theta} = 0.06667$$



$$2v_P\dot{v}_P = 2\dot{r}\ddot{r} + 2\left(r\dot{\theta}\right)\left(\dot{r}\dot{\theta} + r\ddot{\theta}\right)$$

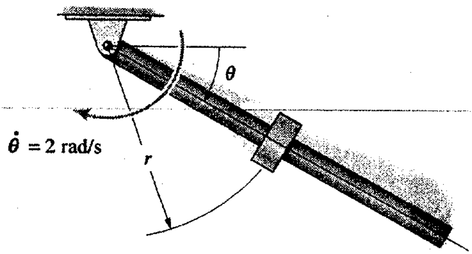
$$0 = 0 + 0 + 2r^2\dot{\theta}\ddot{\theta} \qquad \ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -600(0.06667)^2 - 1200(0.06667)^2 = -8 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$$

$+\uparrow \Sigma F_r = m a_r; \quad N - 150 = \left(\frac{150}{32.2}\right)(-8) \quad N = 113$ lb **Ans**

13-111. A 0.2-kg spool slides down along a smooth rod. If the rod has a constant angular rate of rotation $\dot{\theta} = 2$ rad/s in the vertical plane, show that the equations of motion for the spool are $\ddot{r} - 4r - 9.81 \sin\theta = 0$ and $0.8\dot{r} + N_s - 1.962\cos\theta = 0$, where N_s is the magnitude of the normal force of the rod on the spool. Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1 e^{-2t} + C_2 e^{2t} - (9.81/8)\sin 2t$. If r, $\dot{r}$, and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 determine r at the instant $\theta = \pi/4$ rad.



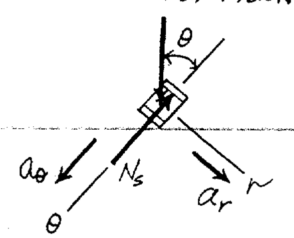
Kinematic **:** Here, $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 0$. Applying Eqs. 12 − 29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - r(2^2) = \ddot{r} - 4r$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r(0) + 2\dot{r}(2) = 4\dot{r}$$

Equation of Motion **:** Applying Eq. 13 − 9, we have

$$\Sigma F_r = ma_r; \qquad 1.962\sin\theta = 0.2(\ddot{r} - 4r)$$
$$\ddot{r} - 4r - 9.81\sin\theta = 0 \qquad (\textbf{Q.E.D.}) \qquad [1]$$

$$\Sigma F_\theta = ma_\theta; \qquad 1.962\cos\theta - N_s = 0.2(4\dot{r})$$
$$0.8\dot{r} + N_s - 1.962\cos\theta = 0 \qquad (\textbf{Q.E.D.}) \qquad [2]$$



Since $\dot{\theta} = 2$ rad/s, then, $\int_0^\theta \dot{\theta} = \int_0^t 2dt$, $\theta = 2t$. The solution of the differential equation (Eq. [1]) is given by

$$r = C_1 e^{-2t} + C_2 e^{2t} - \frac{9.81}{8}\sin 2t \qquad [3]$$

Thus,

$$\dot{r} = -2C_1 e^{-2t} + 2C_2 e^{2t} - \frac{9.81}{4}\cos 2t \qquad [4]$$

At $t = 0$, $r = 0$. From Eq. [3] $\quad 0 = C_1(1) + C_2(1) - 0 \qquad [5]$

At $t = 0$, $\dot{r} = 0$. From Eq. [4] $\quad 0 = -2C_1(1) + 2C_2(1) - \frac{9.81}{4} \qquad [6]$

Solving Eqs. [5] and [6] yields

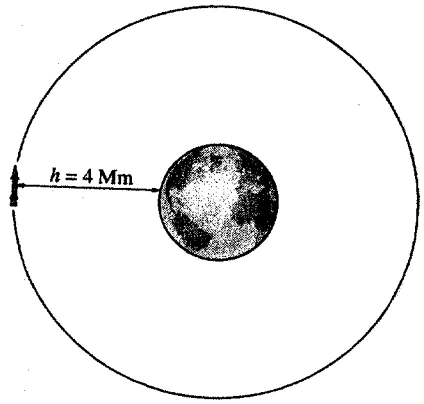
$$C_1 = -\frac{9.81}{16} \qquad C_2 = \frac{9.81}{16}$$

Thus,

$$r = -\frac{9.81}{16}e^{-2t} + \frac{9.81}{16}e^{2t} - \frac{9.81}{8}\sin 2t$$
$$= \frac{9.81}{8}\left(\frac{-e^{-2t} + e^{2t}}{2} - \sin 2t\right)$$
$$= \frac{9.81}{8}(\sinh 2t - \sin 2t)$$

At $\theta = 2t = \frac{\pi}{4}$, $\qquad r = \frac{9.81}{8}\left(\sinh\frac{\pi}{4} - \sin\frac{\pi}{4}\right) = 0.198$ m $\qquad$ **Ans**

*13-112. The rocket is in circular orbit about the earth at an altitude of $h = 4$ Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



Circular orbit :

$$v_C = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})5.976(10^{24})}{4000(10^3)+6378(10^3)}} = 6198.8 \text{ m/s}$$

Parabolic orbit :

$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})5.976(10^{24})}{4000(10^3)+6378(10^3)}} = 8766.4 \text{ m/s}$$

$\Delta v = v_e - v_C = 8766.4 - 6198.8 = 2567.6$ m/s

$\Delta v = 2.57$ km/s **Ans**

13-113. Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

From Eq. 13 – 19,

$$\frac{1}{r} = C\cos\theta + \frac{GM_s}{h^2}$$

For $\theta = 0°$ and $\theta = 180°$

$$\frac{1}{r_p} = C + \frac{GM_s}{h^2}$$

$$\frac{1}{r_a} = -C + \frac{GM_s}{h^2}$$

Eliminating C,

From Eqs. 13 – 28 and 13 – 29,

$$\frac{2a}{b^2} = \frac{2GM_s}{h^2}$$

From Eq. 13 – 31,

$$T = \frac{\pi}{h}(2a)(b)$$

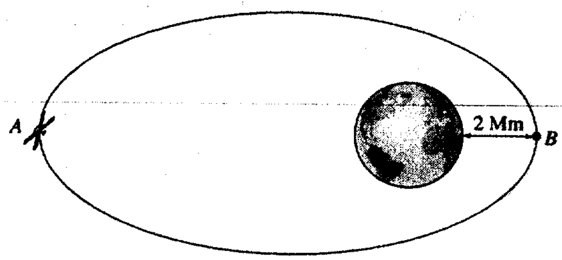
Thus,

$$b^2 = \frac{T^2h^2}{4\pi^2a^2}$$

$$\frac{4\pi^2a^3}{T^2h^2} = \frac{GM_s}{h^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3 \qquad \textbf{Q.E.D.}$$

13-114. The satellite is moving in an elliptical orbit with an eccentricity $e = 0.25$. Determine its speed when it is at its maximum distance A and minimum distance B from the earth.



$$e = \frac{Ch^2}{GM_e} \quad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

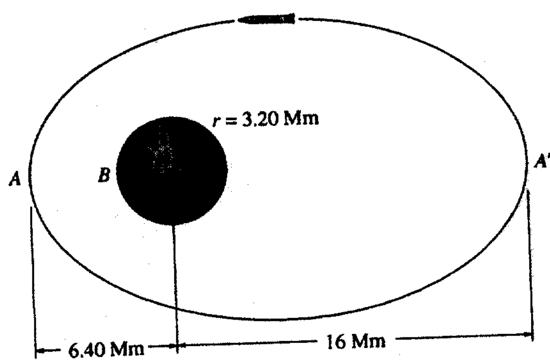
Where $r_0 = r_p = 2(10^6) + 6378(10^3) = 8.378(10^6)\,\text{m}$

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.25+1)}{8.378(10^6)}} = 7713\text{ m/s} = 7.71\text{ km/s}$$ **Ans**

$$r_a = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} = \frac{8.378(10^6)}{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)(7713)^2} - 1} = 13.96(10^6)$$

$$v_A = \frac{r_p}{r_a} v_B = \frac{8.378(10^6)}{13.96(10^6)}(7713) = 4628\text{ m/s} = 4.63\text{ km/s}$$ **Ans**

13-115. The rocket is traveling in free flight along an elliptical trajectory $A'A$. The planet has a mass 0.60 times that of the earth's. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.

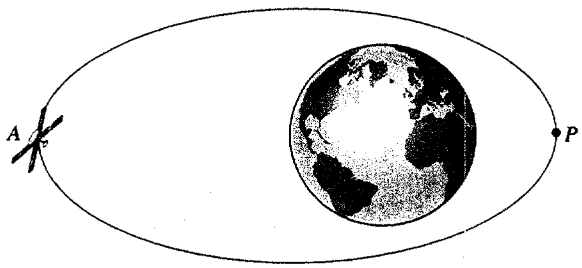


Central-Force Motion: Substitute Eq. 13–27, $r_a = \frac{r_0}{(2GM/r_0 v_0^2) - 1}$, with $r_a = 16(10^6)$ m, $r_0 = r_p = 6.40(10^6)$ m and $M = 0.60M_e$, we have

$$16(10^6) = \frac{6.40(10^6)}{\left(\frac{2(66.73)(10^{-12})(0.6)[5.976(10^{24})]}{6.40(10^6)\,v_p^2}\right) - 1}$$

$$v_p = 7308.07\text{m/s} = 7.31\text{ km/s}$$ **Ans**

***13-116.** An elliptical path of a satellite has an eccentricity $e = 0.130$. If it has a speed of 15 Mm/h when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?



$e = 0.130$

$v_P = v_0 = 15 \text{ Mm/h} = 4.167 \text{ km/s}$

$$e = \frac{Ch^2}{GM_e} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_e}\right)$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1$$

$$r_0 = \frac{(e+1)GM_e}{v_0^2}$$

$$= \frac{1.130(66.73)(10^{-12})(5.976)(10^{24})}{[4.167(10^3)]^2}$$

$= 25.96$ Mm

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1}$$

$$r_A = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e+1}\right) - 1}$$

$$r_A = \frac{r_0(e+1)}{1-e}$$

$$= \frac{25.96(10^6)(1.130)}{0.870}$$

$= 33.71(10^6)$ m $= 33.7$ Mm

$$v_A = \frac{v_0 r_0}{r_A}$$

$$= \frac{15(25.96)(10^6)}{33.71(10^6)}$$

$= 11.6$ Mm/h **Ans**

$d = 33.71(10^6) - 6.378(10^6)$

$= 27.3$ Mm **Ans**

13-117. A satellite is launched with an initial velocity $v_0 = 2500$ mi/h parallel to the surface of the earth. Determine the required altitude (or range of altitudes) above the earth's surface for launching if the free-flight trajectory is to be (a) circular, (b) parabolic, (c) elliptical, and (d) hyperbolic. Take $G = 34.4(10^{-9})$ (lb·ft²)/slug², $M_e = 409(10^{21})$ slug, the earth's radius $r_e = 3960$ mi, and 1 mi = 5280 ft.

$$v_0 = 2500 \text{ mi/h} = 3.67(10^3) \text{ ft/s}$$

(a) $$e = \frac{C^2 h}{GM_e} = 0 \qquad \text{or } C = 0$$

$$1 = \frac{GM_e}{r_0 v_0^2}$$

$$GM_e = 34.4(10^{-9})(409)(10^{21})$$

$$= 14.07(10^{15})$$

$$r_0 = \frac{GM_e}{v_0^2} = \frac{14.07(10^{15})}{[3.67(10^3)]^2} = 1.046(10^9) \text{ ft}$$

$$r = \frac{1.047(10^9)}{5280} - 3960 = 194(10^3) \text{ mi}$$ **Ans**

(b) $$e = \frac{C^2 h}{GM_e} = 1$$

$$\frac{1}{GM_e}(r_0^2 v_0^2)\left(\frac{1}{r_0}\right)\left(1 - \frac{GM_e}{r_0 v_0^2}\right) = 1$$

$$r_0 = \frac{2GM_e}{v_0^2} = \frac{2(14.07)(10^{15})}{[3.67(10^3)]^2} = 2.09(10^9) \text{ ft} = 396(10^3) \text{ mi}$$

$$r = 396(10^3) - 3960 = 392(10^3) \text{ mi}$$ **Ans**

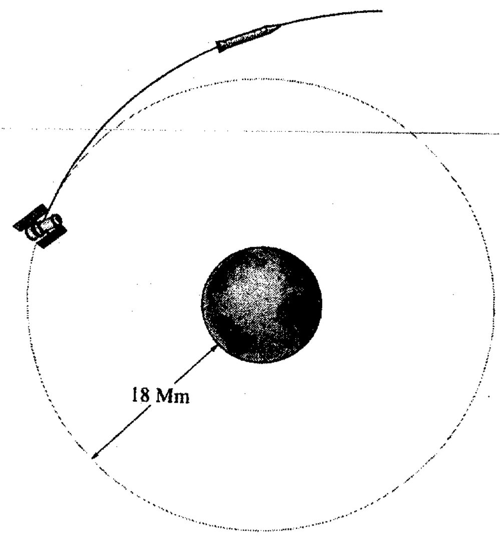
(c) $$e < 1$$

$$194(10^3) \text{ mi} < r < 392(10^3) \text{ mi}$$ **Ans**

(d) $$e > 1$$

$$r > 392(10^3) \text{ mi}$$ **Ans**

13-118. The rocket is docked next to a satellite located 18 Mm above the earth's surface. If the satellite is traveling in a circular orbit, determine the speed tangent to the earth's surface which must suddenly be given to the rocket, relative to the satellite, such that it travels in free flight away from the satellite along a parabolic trajectory as shown.



Central - Force Motion : In order for the satellite to have a circular free - flight trajectory, from Eq. 13 − 23, $e = 0$. Substitute $e = 0$ into Eq. 13 − 17, we have

$$e = \frac{Ch^2}{GM_e} = 0$$

Since $h = r_0 v_C \neq 0$, then $C = 0$. Using Eq. 13 − 21, we have

$$C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_c^2}\right) = 0 \qquad v_c = \sqrt{\frac{GM_e}{r_0}}$$

Here, $r_0 = (18 + 6.378)(10^6) = 24.378(10^6)$ m and $v_c = v_s$.

$$v_s = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{24.378(10^6)}} = 4044.52 \text{ m/s}$$

In order for the rocket to have a parabolic free - flight trajectory, from Eq. 13 − 23, $e = 1$. Here, $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13 − 21] and $h = r_0 v_0$ [Eq. 13 − 20]. Substitute these values into Eq. 13 − 17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0^2 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad [1]$$

Substitute $e = 1$ and $v_0 = v_r$ into Eq. [1], we have

$$\frac{r_0 v_r^2}{GM_e} - 1 = 1$$

$$v_r = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{24.378(10^6)}} = 5719.81 \text{ m/s}$$

Thus, the required speed of the rocket relative to the satellite is given by

$$v_{r/s} = v_r - v_s = 5719.81 - 4044.52 = 1675.29 \text{ m/s} = 1.68 \text{ km/s} \qquad \textbf{Ans}$$

13-119. Show the speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

For a circle

$$e = \frac{ch^2}{GM_e} = 0 \text{ or } ch^2 = 0$$

but $h = r_0 v_0 \neq 0$,

so that $c = 0$

Hence, $c = \frac{1}{r_0}(1 - \frac{GM_e}{r_0 v_0^2}) = 0$

$$r_0 v_0^2 = GM_e$$

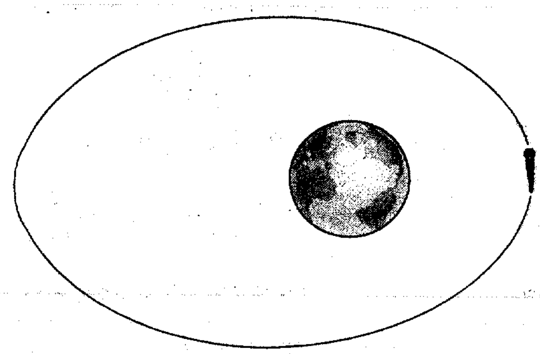
$$v_0 = \sqrt{\frac{GM_e}{r_0}} \qquad \textbf{QED}$$

For a 800 - km orbit

$$v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{(800+6378)(10^3)}}$$

$$= 7453.6 \text{ m/s} = 7.45 \text{ km/s} \qquad \textbf{Ans}$$

***13-120.** The rocket is in a free-flight elliptical orbit about the earth such that the eccentricity of its orbit is e and its perigee is r_0. Determine the minimum increment of speed it should have in order to escape the earth's gravitational field when it is at this point along its orbit.



To escape the earth's gravitation field, the rocket has to make a parabolic trajectory.

Parabolic trajectory :

$$\upsilon_e = \sqrt{\frac{2GM_e}{r_0}}$$

Elliptical orbit :

$$e = \frac{Ch^2}{GM_e} \quad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 \upsilon_0^2}\right) \text{ and } h = r_0 \upsilon_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 \upsilon_0^2}\right)(r_0 \upsilon_0)^2$$

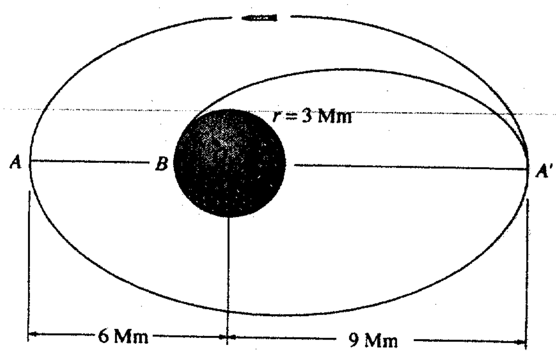
$$e = \left(\frac{r_0 \upsilon_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 \upsilon_0^2}{GM_e} = e + 1 \qquad \upsilon_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

$$\Delta\upsilon = \sqrt{\frac{2GM_e}{r_0}} - \sqrt{\frac{GM_e(e+1)}{r_0}} = \sqrt{\frac{GM_e}{r_0}}\left(\sqrt{2} - \sqrt{1+e}\right) \qquad \textbf{Ans}$$

The change in speed should occur at perigee. **Ans**

13-121. The rocket is traveling in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth's. If the rocket has an apogee and perigee as shown in the figure, determine the speed of the rocket when it is at point A.



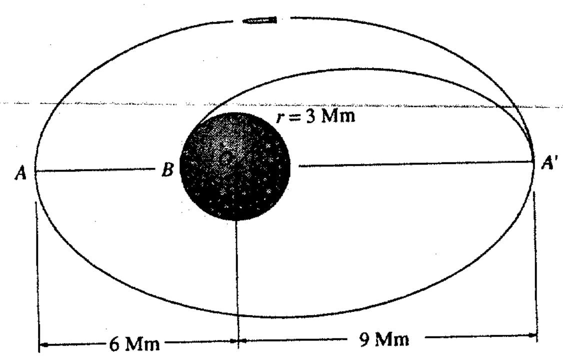
Central - Force Motion : Use $r_a = \dfrac{r_0}{(2GM/r_0 v_0^2) - 1}$,

with $r_a = 9(10^6)$ m, $r_0 = r_p = 6(10^6)$ m and $M = 0.70M_e$, we have

$$9(10^6) = \frac{6(10^6)}{\left(\dfrac{2(66.73)(10^{-12})(0.7)[5.976(10^{24})]}{6(10^6)\,v_p^2}\right) - 1}$$

$$v_p = 7471.89 \text{ m/s} = 7.47 \text{ km/s} \qquad \textbf{Ans}$$

13-122. If the rocket in Prob. 13–121 is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that it strikes the planet at B. How long does it take for the rocket to land, going from A' to B along an elliptical path?



Central - Force Motion : Use $r_a = \dfrac{r_0}{(2GM/r_0 v_0^2) - 1}$,

with $r_a = 9(10^6)$ m, $r_0 = r_p = 3(10^6)$ m and $M = 0.70M_e$, we have

$$9(10^6) = \frac{3(10^6)}{\left(\dfrac{2(66.73)(10^{-12})(0.7)[5.976(10^{24})]}{3(10^6)\,v_p^2}\right) - 1}$$

$$v_p = 11814.08 \text{ m/s}$$

Applying Eq. 13 – 20, we have

$$v_a = \left(\frac{r_p}{r_a}\right)v_p = \left[\frac{3(10^6)}{9(10^6)}\right](11814.08) = 3938.03 \text{ m/s} = 3.94 \text{ km/s} \qquad \textbf{Ans}$$

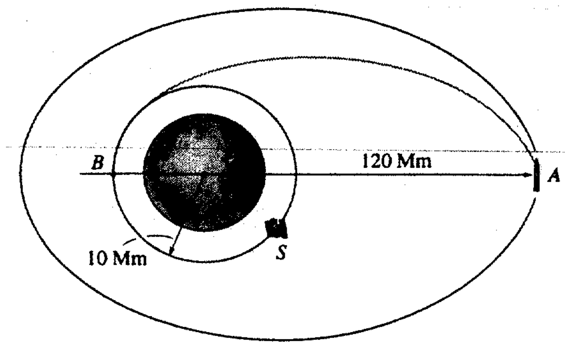
Eq. 13 – 20 gives $h = r_p v_p = 3(10^6)(11814.08) = 35.442(10^9)$ m²/s. Thus, applying Eq. 13 – 31, we have

$$\begin{aligned} T &= \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a} \\ &= \frac{\pi}{35.442(10^9)}\left[(9+3)(10^6)\right]\sqrt{3(10^6)9(10^6)} \\ &= 5527.03 \text{ s} \end{aligned}$$

The time required for the rocket to go from A' to B (half the orbit) is given by

$$t = \frac{T}{2} = 2763.51 \text{ s} = 46.1 \text{ min} \qquad \textbf{Ans}$$

13-123. A satellite S travels in a circular orbit around the earth. A rocket is located at the apogee of its elliptical orbit for which $e = 0.58$. Determine the sudden change in speed that must occur at A so that the rocket can enter the satellite's orbit while in free flight along the dashed elliptical trajectory. When it arrives at B, determine the sudden adjustment in speed that must be given to the rocket in order to maintain the circular orbit.



Central - Force Motion : Here, $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13 – 21] and $h = r_0 v_0$ [Eq. 13 – 20]. Substitute these values into Eq. 13 – 17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad [1]$$

Rearrange Eq. [1] gives

$$\frac{1}{1+e} = \frac{GM_e}{r_0 v_0^2} \qquad [2]$$

Rearrange Eq. [2] , we have

$$v_0 = \sqrt{\frac{(1+e)\,GM_e}{r_0}} \qquad [3]$$

Substitute Eq. [2] into Eq. 13 – 27, $r_a = \frac{r_0}{\left(2GM_e / r_0 v_0^2\right) - 1}$, we have

$$r_a = \frac{r_0}{2\left(\frac{1}{1+e}\right) - 1} \quad \text{or} \quad r_0 = \left(\frac{1-e}{1+e}\right) r_a \qquad [4]$$

For the first elliptical orbit $e = 0.58$, from Eq. [4]

$$(r_p)_1 = r_0 = \left(\frac{1-0.58}{1+0.58}\right)\left[120(10^6)\right] = 31.899(10^6)\ \text{m}$$

Substitute $r_0 = (r_p)_1 = 31.899(10^6)$ m into Eq. [3] yields

$$(v_p)_1 = \sqrt{\frac{(1+0.58)(66.73)(10^{-12})(5.976)(10^{24})}{31.899(10^6)}} = 4444.34\ \text{m/s}$$

Applying Eq. 13 – 20, we have

$$(v_a)_1 = \left(\frac{r_p}{r_a}\right)(v_p)_1 = \left[\frac{31.899(10^6)}{120(10^6)}\right](4444.34) = 1181.41\ \text{m/s}$$

When the rocket travels along the second elliptical orbit, from Eq. [4], we have

$$10(10^6) = \left(\frac{1-e}{1+e}\right)\left[120(10^6)\right] \qquad e = 0.8462$$

Substitute $r_0 = (r_p)_2 = 10(10^6)$ m into Eq. [3] yields

$$(v_p)_2 = \sqrt{\frac{(1+0.8462)(66.73)(10^{-12})(5.976)(10^{24})}{10(10^6)}} = 8580.25\ \text{m/s}$$

Applying Eq. 13 – 20, we have

$$(v_a)_2 = \left[\frac{(r_p)_2}{(r_a)_2}\right](v_p)_2 = \left[\frac{10(10^6)}{120(10^6)}\right](8580.25) = 715.02\ \text{m/s}$$

For the rocket to enter into orbit two from orbit one at A, its speed must be decreased by

$$\Delta v = (v_a)_1 - (v_a)_2 = 1184.41 - 715.02 = 466\ \text{m/s} \qquad \textbf{Ans}$$

If the rocket travels in a cicular free - flight trajectory, its speed is given by Eq. 13 – 25.

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{10(10^6)}} = 6314.89\ \text{m/s}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = (v_p)_2 - v_c = 8580.25 - 6314.89 = 2265.36\ \text{m/s} = 2.27\ \text{km/s} \qquad \textbf{Ans}$$

***13-124.** An asteroid is in an elliptical orbit about the sun such that its periapsis is $9.30(10^9)$ km. If the eccentricity of the orbit is $e = 0.073$, determine the apoapsis of the orbit.

$$r_p = r_0 = 9.30(10^9)\text{ km}$$

$$e = \frac{Ch^2}{GM_s} = \frac{1}{r_0}\left(1 - \frac{GM_s}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_s}\right)$$

$$e = \left(\frac{r_0 v_0^2}{GM_s} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_s} = e + 1$$

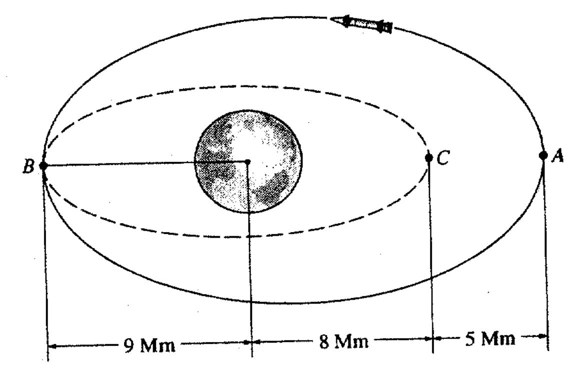
$$\frac{GM_s}{r_0 v_0^2} = \frac{1}{e+1}$$

$$r_a = \frac{r_0}{\frac{2GM_s}{r_0 v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e+1}\right) - 1}$$

$$r_a = \frac{r_0(e+1)}{1-e}$$

$$= \frac{9.30(10^9)(1.073)}{0.927} = 10.8(10^9)\text{ km} \qquad \textbf{Ans}$$

13-125. The rocket is traveling in a free-flight elliptical orbit about the earth such that $e = 0.76$ as shown. Determine its speed when it is at point A. Also determine the sudden change in speed the rocket must experience at B in order to travel in free flight along the orbit indicated by the dashed path.



$$e = \frac{Ch^2}{GM_e} \quad \text{where} \quad C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \quad \text{and} \quad h = r_0 v_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \quad \text{or} \quad \frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \qquad (1)$$

$$r_a = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} \qquad (2)$$

Substituting Eq.(1) into (2) yields :

$$r_a = \frac{r_0}{2\left(\frac{1}{e+1}\right) - 1} = \frac{r_0(e+1)}{1-e} \qquad (3)$$

$$\text{From Eq.(1)} \quad \frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.76+1)}{9(10^6)}} = 8831\text{ m/s}$$

$$v_A = \frac{r_p}{r_a} v_B = \frac{9(10^6)}{13(10^6)}(8831) = 6113\text{ m/s} = 6.11\text{ km/s} \qquad \textbf{Ans}$$

$$\text{From Eq.(3)} \quad r_a = \frac{r_0(e+1)}{1-e}$$

$$9(10)^6 = \frac{8(10^6)(e+1)}{1-e} \qquad e = 0.05882$$

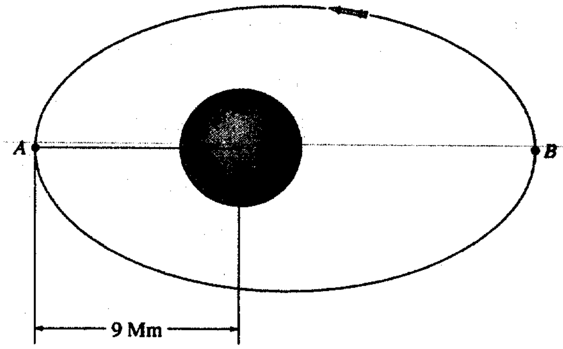
$$\text{From Eq.(1)} \quad \frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

$$v_C = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.05882+1)}{8(10^6)}} = 7265\text{ m/s}$$

$$v_B = \frac{r_p}{r_a} v_C = \frac{8(10^6)}{9(10^6)}(7265) = 6458\text{ m/s}$$

$$\Delta v_B = 6458 - 8831 = -2374\text{ m/s} = -2.37\text{ km/s} \qquad \textbf{Ans}$$

13-126. The rocket is traveling in a free-flight elliptical orbit about the earth such that $e = 0.76$ and its perigee is 9 Mm as shown. Determine its speed when it is at point B. Also determine the sudden decrease in speed the rocket must experience at A in order to travel in a circular orbit about the earth.



Central-Force Motion : Here, $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13 – 21] and $h = r_0 v_0$ [Eq. 13 – 20]. Substitute these values into Eq. 13 – 17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad [1]$$

Rearrange Eq. [1] gives

$$\frac{1}{1+e} = \frac{GM_e}{r_0 v_0^2} \qquad [2]$$

Rearrange Eq. [2], we have

$$v_0 = \sqrt{\frac{(1+e)\,GM_e}{r_0}} \qquad [3]$$

Substitute Eq. [2] into Eq. 13 – 27, $r_a = \frac{r_0}{\left(2GM_e/r_0 v_0^2\right) - 1}$, we have

$$r_a = \frac{r_0}{2\left(\frac{1}{1+e}\right) - 1} \qquad [4]$$

Rearrange Eq. [4], we have

$$r_a = \left(\frac{1+e}{1-e}\right) r_0 = \left(\frac{1+0.76}{1-0.76}\right)\left[9\left(10^6\right)\right] = 66.0\left(10^6\right)\ \text{m}$$

Substitute $r_0 = r_p = 9\left(10^6\right)$ m into Eq. [3] yields

$$v_p = \sqrt{\frac{(1+0.76)(66.73)(10^{-12})(5.976)(10^{24})}{9(10^6)}} = 8830.82\ \text{m/s}$$

Applying Eq. 13 – 20, we have

$$v_a = \left(\frac{r_p}{r_a}\right) v_p = \left[\frac{9(10^6)}{66.0(10^6)}\right](8830.82) = 1204.2\ \text{m/s} = 1.20\ \text{km/s} \qquad \textbf{Ans}$$

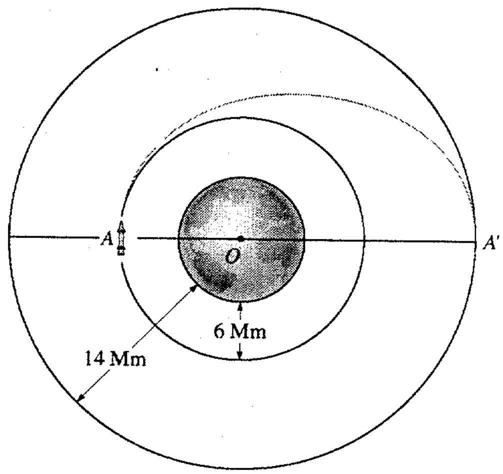
If the rocket travels in a cicular free - flight trajectory, its speed is given by Eq. 13 – 25.

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{9(10^6)}} = 6656.48\ \text{m/s}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = v_p - v_c = 8830.82 - 6656.48 = 2174.34\ \text{m/s} = 2.17\ \text{km/s} \qquad \textbf{Ans}$$

13-127. The rocket shown is originally in a circular orbit 6 Mm above the surface of the earth. It is required that it travel in another circular orbit having an altitude of 14 Mm. To do this, the rocket is given a short pulse of power at A so that it travels in free flight along the dashed elliptical path from the first orbit to the second orbit. Determine the necessary speed it must have at A just after the power pulse, and at the time required to get to the outer orbit along the path AA'. What adjustment in speed must be made at A' to maintain the second circular orbit?



Central-Force Motion: Substitute Eq. 13-27, $r_a = \dfrac{r_0}{(2GM/r_0 v_0^2) - 1}$, with $r_a = (14 + 6.378)(10^6) = 20.378(10^6)$ m and $r_0 = r_p = (6 + 6.378)(10^6) = 12.378(10^6)$ m, we have

$$20.378(10^6) = \frac{12.378(10^6)}{\left(\dfrac{2(66.73)(10^{-12})[5.976(10^{24})]}{12.378(10^6)v_p^2}\right) - 1}$$

$$v_p = 6331.27 \text{ m/s}$$

Applying Eq. 13-20, we have

$$v_a = \left(\frac{r_p}{r_a}\right) v_p = \left[\frac{12.378(10^6)}{20.378(10^6)}\right](6331.27) = 3845.74 \text{ m/s}$$

Eq. 13-20 gives $h = r_p v_p = 12.378(10^6)(6331.27) = 78.368(10^9)\ \text{m}^2/\text{s}$. Thus, applying Eq. 13-31, we have

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}$$

$$= \frac{\pi}{78.368(10^9)}[(12.378 + 20.378)(10^6)]\sqrt{12.378(20.378)}(10^6)$$

$$= 20854.54 \text{ s}$$

The time required for the rocket to go from A to A' (half the orbit) is given by

$$t = \frac{T}{2} = 10427.38 \text{ s} = 2.90 \text{ hr} \quad \textbf{Ans}$$

In order for the satellite to stay in the second circular orbit, it must achieve a speed of (Eq. 13-25)

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{20.378(10^6)}} = 4423.69 \text{ m/s} = 4.42 \text{ km/s} \quad \textbf{Ans}$$

The speed for which the rocket must be increased in order to enter the second circular orbit at A' is

$$\Delta v = v_c - v_a = 4423.69 - 3845.74 = 578 \text{ m/s} \quad \textbf{Ans}$$