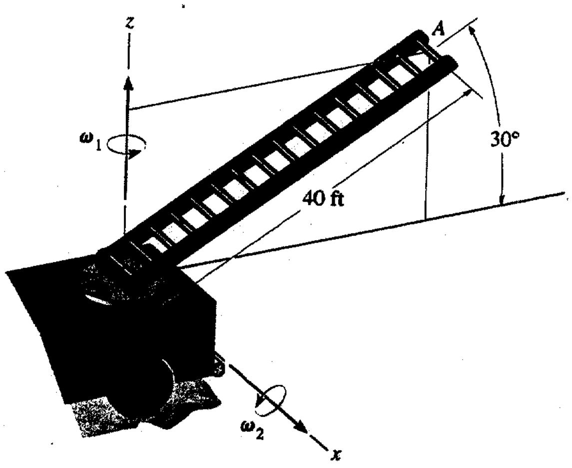
20-1. The ladder of the fire truck rotates around the z axis with an angular velocity $\omega_1 = 0.15$ rad/s, which is increasing at 0.8 rad/s^2. At the same instant it is rotating upward at a constant rate $\omega_2 = 0.6$ rad/s. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.



$\omega = \omega_1 + \omega_2 = 0.15\mathbf{k} + 0.6\mathbf{i} = \{0.6\mathbf{i} + 0.15\mathbf{k}\}$ rad/s

Angular acceleration : For ω_1, $\omega = \omega_1 = \{0.15\mathbf{k}\}$ rad/s.

$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \omega \times \omega_2$

$= 0 + (0.15\mathbf{k}) \times (0.6\mathbf{i}) = \{0.09\mathbf{j}\}$ rad/s^2

For ω_1, $\Omega = 0$.

$(\dot{\omega}_1)_{XYZ} = (\dot{\omega}_1)_{xyz} + \omega \times \omega_1 = (0.8\mathbf{k}) + 0 = \{0.8\mathbf{k}\}$ rad/s^2

$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$

$\alpha = 0.8\mathbf{k} + 0.09\mathbf{j} = \{0.09\mathbf{j} + 0.8\mathbf{k}\}$ rad/s^2

$\mathbf{r}_A = 40\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k} = \{34.641\mathbf{j} + 20\mathbf{k}\}$ ft

$\mathbf{v}_A = \omega \times \mathbf{r}_A$

$= (0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$

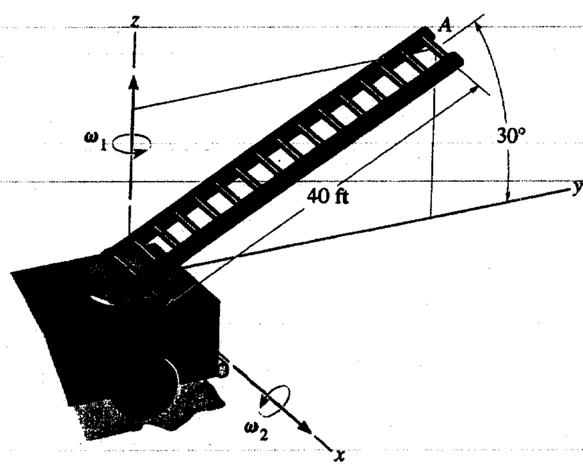
$= \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_A = \alpha \times \mathbf{r} + \omega \times \mathbf{v}_A$

$= (0.09\mathbf{j} + 0.8\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k}) + (0.6\mathbf{i} + 0.15\mathbf{k}) \times (-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k})$

$= \{-24.1\mathbf{i} - 13.3\mathbf{j} - 7.20\mathbf{k}\}$ ft/s^2 **Ans**

***20-2.** The ladder of the fire truck rotates around the z axis with an angular velocity of $\omega_1 = 0.15$ rad/s, which is increasing at 0.2 rad/s^2. At the same instant it is rotating upwards at $\omega_2 = 0.6$ rad/s while increasing at 0.4 rad/s^2. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.



$\mathbf{r}_{A/O} = 40\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k}$

$\mathbf{r}_{A/O} = \{34.641\mathbf{j} + 20\mathbf{k}\}$ ft

$\Omega = \omega_1\mathbf{k} + \omega_2\mathbf{i} = \{0.6\mathbf{i} + 0.15\mathbf{k}\}$ rad/s

$\dot{\Omega} = \dot{\omega}_1\mathbf{k} + \dot{\omega}_2\mathbf{i} + \omega_1\mathbf{k} \times \omega_2\mathbf{i}$

$\dot{\Omega} = 0.2\mathbf{k} + 0.4\mathbf{i} + 0.15\mathbf{k} \times 0.6\mathbf{i} = \{0.4\mathbf{i} + 0.09\mathbf{j} + 0.2\mathbf{k}\}$ rad/s^2

$\mathbf{v}_A = \Omega \times \mathbf{r}_{A/O} = (0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$

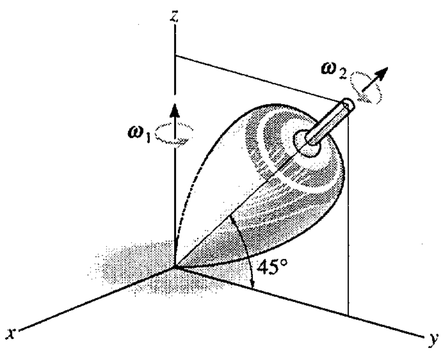
$\mathbf{v}_A = \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_A = \Omega \times (\Omega \times \mathbf{r}_{A/O}) + \dot{\Omega} \times \mathbf{r}_{A/O}$

$\mathbf{a}_A = (0.6\mathbf{i} + 0.15\mathbf{k}) \times [(0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})]$
$+ (0.4\mathbf{i} + 0.09\mathbf{j} + 0.2\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$

$\mathbf{a}_A = \{-3.33\mathbf{i} - 21.3\mathbf{j} + 6.66\mathbf{k}\}$ ft/s^2 **Ans**

20-3. The motion of the top is such that at the instant shown it is rotating about the z axis at $\omega_1 = 0.6$ rad/s, while it is spinning at $\omega_2 = 8$ rad/s. Determine the angular velocity and angular acceleration of the top at this instant. Express the result as a Cartesian vector.



$\omega = \omega_1 + \omega_2$

$\omega = 0.6\mathbf{k} + 8\cos 45°\mathbf{j} + 8\sin 45°\mathbf{k}$

$\omega = \{5.66\mathbf{j} + 6.26\mathbf{k}\}$ rad/s **Ans**

$\omega = \omega_1 + \omega_2$

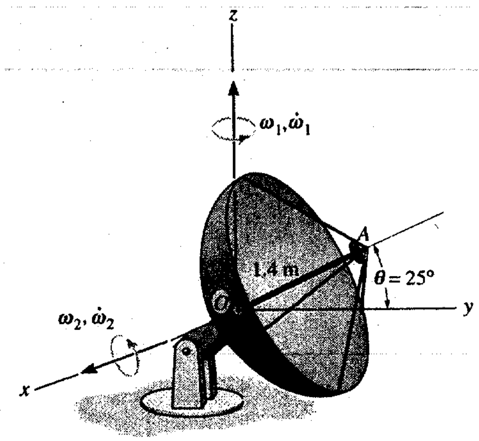
Let x, y, z axes have angular velocity of $\Omega = \omega_1$, thus

$\dot{\omega}_1 = \mathbf{0}$

$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + (\omega_1 \times \omega_2) = \mathbf{0} + (0.6\mathbf{k}) \times (8\cos 45°\mathbf{j} + 8\sin 45°\mathbf{k}) = -3.394\mathbf{i}$

$\alpha = \dot{\omega} = \{-3.39\mathbf{i}\}$ rad/s^2 **Ans**

*20-4. At a given instant, the satellite dish has an angular motion $\omega_1 = 6$ rad/s and $\dot{\omega}_1 = 3$ rad/s^2 about the z axis. At this same insant $\theta = 25°$, the angular motion about the x axis is $\omega_2 = 2$ rad/s, and $\dot{\omega}_2 = 1.5$ rad/s^2. Determine the velocity and acceleration of the signal horn A at this instant.



Angular Velocity : The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the satellite at this instant(with reference to X, Y, Z) can be expressed in terms of **i**, **j**, **k** components.

$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 = \{2\mathbf{i} + 6\mathbf{k}\} \text{ rad/s}$$

Angular Acceleration : The angular acceleration $\boldsymbol{\alpha}$ will be determined by investigating separately the time rate of change of *each angular velocity component* with respect to the fixed XYZ frame. $\boldsymbol{\omega}_2$ is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\boldsymbol{\Omega} = \boldsymbol{\omega}_1 = \{6\mathbf{k}\}$ rad/s. Applying Eq. 20−6 with $(\dot{\boldsymbol{\omega}}_2)_{xyz} = \{1.5\mathbf{i}\}$ rad/s^2, we have

$$\dot{\boldsymbol{\omega}}_2 = (\dot{\boldsymbol{\omega}}_2)_{xyz} + \boldsymbol{\omega}_1 \times \boldsymbol{\omega}_2 = 1.5\mathbf{i} + 6\mathbf{k} \times 2\mathbf{i} = \{1.5\mathbf{i} + 12\mathbf{j}\} \text{ rad/s}^2$$

Since $\boldsymbol{\omega}_1$ is always directed along the Z axis ($\boldsymbol{\Omega} = \mathbf{0}$), then

$$\dot{\boldsymbol{\omega}}_1 = (\dot{\boldsymbol{\omega}}_1)_{xyz} + \mathbf{0} \times \boldsymbol{\omega}_1 = \{3\mathbf{k}\} \text{ rad/s}^2$$

Thus, the angular acceleration of the satellite is

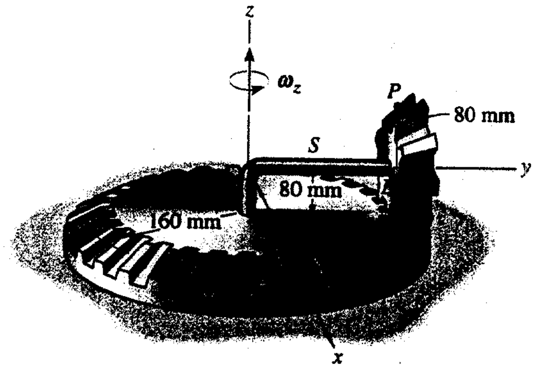
$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}_1 + \dot{\boldsymbol{\omega}}_2 = \{1.5\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}\} \text{ rad/s}^2$$

Velocity and Acceleration : Applying Eqs. 20 − 3 and 20 − 4 with the $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ obtained above and $\mathbf{r}_A = \{1.4\cos 25°\mathbf{j} + 1.4\sin 25°\mathbf{k}\}$ m $= \{1.2688\mathbf{j} + 0.5917\mathbf{k}\}$ m, we have

$$\begin{aligned}\mathbf{v}_A &= \boldsymbol{\omega} \times \mathbf{r}_A = (2\mathbf{i} + 6\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k}) \\ &= \{-7.61\mathbf{i} - 1.18\mathbf{j} + 2.54\mathbf{k}\} \text{ m/s}\end{aligned}$$ **Ans**

$$\begin{aligned}\mathbf{a}_A &= \boldsymbol{\alpha} \times \mathbf{r}_A + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_A) \\ &= (1.5\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k}) \\ &\quad + (2\mathbf{i} + 6\mathbf{k}) \times [(2\mathbf{i} + 6\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})] \\ &= \{10.4\mathbf{i} - 51.6\mathbf{j} - 0.463\mathbf{k}\} \text{ m/s}^2\end{aligned}$$ **Ans**

20-5. Gear A is fixed while gear B is free to rotate on the shaft S. If the shaft is turning about the z axis at $\omega_z = 5$ rad/s, while increasing at 2 rad/s^2, determine the velocity and acceleration of point C at the instant shown. The face of gear B lies in a vertical plane.



From Prob. 20-4.

$$\boldsymbol{\Omega} = \{5\mathbf{k} - 10\mathbf{j}\} \text{ rad/s}$$

$$\dot{\boldsymbol{\Omega}} = \{50\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{v}_C = \boldsymbol{\Omega} \times \mathbf{r}_C$$

$$\mathbf{v}_C \quad (5\mathbf{k} - 10\mathbf{j}) \times (80\mathbf{i} + 160\mathbf{j})$$

$$\mathbf{v}_C = \{-800\mathbf{i} + 400\mathbf{j} + 800\mathbf{k}\} \text{ mm/s}$$

$$= \{-0.800\mathbf{i} + 0.400\mathbf{j} + 0.800\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans}$$

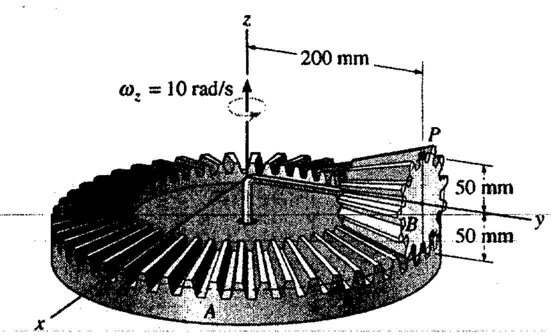
$$\mathbf{a}_C = \boldsymbol{\Omega} \times \mathbf{v}_C + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_C$$

$$\mathbf{a}_C = \{50\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\} \times (80\mathbf{i} + 160\mathbf{j}) + (-10\mathbf{j} + 5\mathbf{k}) \times (-800\mathbf{i} + 400\mathbf{j} + 800\mathbf{k})$$

$$\mathbf{a}_C = \{-10\,320\mathbf{i} - 3840\mathbf{j} + 320\mathbf{k}\} \text{ mm/s}^2$$

$$\mathbf{a}_C = \{-10.3\mathbf{i} - 3.84\mathbf{j} + 0.320\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

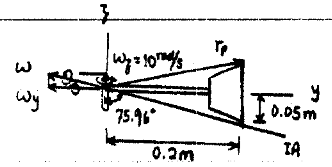
20-6. Gear B is connected to the rotating shaft, while the plate gear A is fixed. If the shaft is turning at a constant rate of $\omega_z = 10$ rad/s about the z axis, determine the magnitudes of the angular velocity and the angular acceleration of gear B. Also, determine the magnitudes of the velocity and acceleration of point P.



$\omega_z = 10$ rad/s

$\omega_y = -10\tan 75.96° = -40$ rad/s

$\omega_x = 0$



$\omega = \{-40\mathbf{j} + 10\mathbf{k}\}$ rad/s

$\omega = \sqrt{(-40)^2 + (10)^2} = 41.2$ rad/s **Ans**

$\mathbf{r}_P = \{0.2\mathbf{j} + 0.05\mathbf{k}\}$ m

$\mathbf{v}_P = \omega \times \mathbf{r}_P = (-40\mathbf{j} + 10\mathbf{k}) \times (0.2\mathbf{j} + 0.05\mathbf{k})$

$\mathbf{v}_P = \{-4\mathbf{i}\}$ m/s

$v_P = 4.00$ m/s **Ans**

Let $\Omega = \omega_z$,

$$\dot{\omega} = \left(\dot{\omega}\right)_{xyz} + \Omega \times \omega$$

$$= 0 + (10\mathbf{k}) \times (-40\mathbf{j} + 10\mathbf{k}) = \{400\mathbf{i}\} \text{ rad/s}^2$$

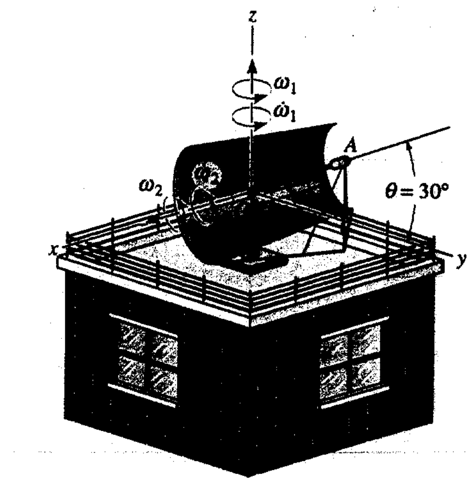
$\alpha = \dot{\omega} = 400$ rad/s^2 **Ans**

$$\mathbf{a}_P = \alpha \times \mathbf{r}_P + \omega \times \mathbf{v}_P = (400\mathbf{i}) \times (0.2\mathbf{j} + 0.05\mathbf{k}) + (-40\mathbf{j} + 10\mathbf{k}) \times (-4\mathbf{i})$$

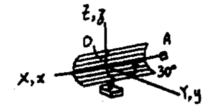
$$= \{-60\mathbf{j} - 80\mathbf{k}\} \text{ m/s}^2$$

$a_P = \sqrt{(-60)^2 + (-80)^2} = 100$ m/s^2 **Ans**

20-7. At a given instant, the antenna has an angular motion $\omega_1 = 3$ rad/s and $\dot{\omega}_1 = 2$ rad/s^2 about the z axis. At this same instant $\theta = 30°$, the angular motion about the x axis is $\omega_2 = 1.5$ rad/s, and $\dot{\omega}_2 = 4$ rad/s^2. Determine the velocity and acceleration of the signal horn A at this instant. The distance from O to A is $d = 3$ ft.



$\mathbf{r}_A = 3\cos 30°\mathbf{j} + 3\sin 30°\mathbf{k} = \{2.598\mathbf{j} + 1.5\mathbf{k}\}$ ft



$\boldsymbol{\Omega} = \omega_1 + \omega_2 = 3\mathbf{k} + 1.5\mathbf{i}$

$\mathbf{v}_A = \boldsymbol{\Omega} \times \mathbf{r}_A$

$\mathbf{v}_A = (3\mathbf{k} + 1.5\mathbf{i}) \times (2.598\mathbf{j} + 1.5\mathbf{k})$

$= -7.794\mathbf{i} + 3.897\mathbf{k} - 2.25\mathbf{j}$

$= \{-7.79\mathbf{i} - 2.25\mathbf{j} + 3.90\mathbf{k}\}$ ft/s **Ans**

$\dot{\boldsymbol{\Omega}} = \dot{\omega}_1 + \dot{\omega}_2$

$= (2\mathbf{k} + 0) + (4\mathbf{i} + 3\mathbf{k} \times 1.5\mathbf{i})$

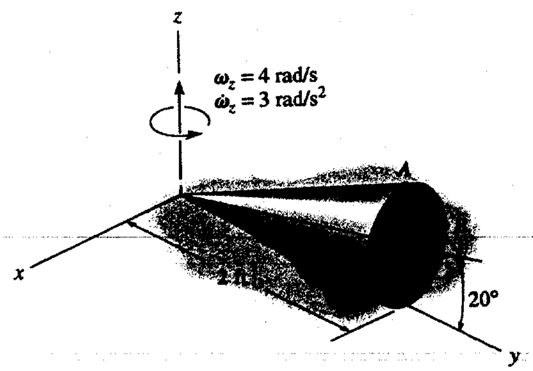
$= 4\mathbf{i} + 4.5\mathbf{j} + 2\mathbf{k}$

$\mathbf{a}_A = \dot{\boldsymbol{\Omega}} \times \mathbf{r}_A + \boldsymbol{\Omega} \times \mathbf{v}_A$

$\mathbf{a}_A = (4\mathbf{i} + 4.5\mathbf{j} + 2\mathbf{k}) \times (2.598\mathbf{j} + 1.5\mathbf{k}) + (3\mathbf{k} + 1.5\mathbf{i}) \times (-7.794\mathbf{i} - 2.25\mathbf{j} + 3.879\mathbf{k})$

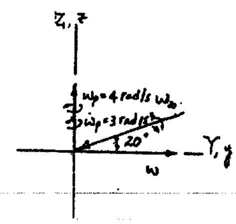
$\mathbf{a}_A = \{8.30\mathbf{i} - 35.2\mathbf{j} + 7.02\mathbf{k}\}$ ft/s^2 **Ans**

***20-8.** The cone rolls without slipping such that at the instant shown $\omega_z = 4$ rad/s and $\dot{\omega}_z = 3$ rad/s^2. Determine the velocity and acceleration of point A at this instant.



Angular velocity : The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity(y axis).

$\omega = \omega_1 + \omega_2$
$\omega\mathbf{j} = 4\mathbf{k} + (\omega_2\cos 20°\mathbf{j} + \omega_2\sin 20°\mathbf{k})$
$\omega\mathbf{j} = \omega_2\cos 20°\mathbf{j} + (4 + \omega_2\sin 20°)\mathbf{k}$



Equating $\mathbf{j}$ and $\mathbf{k}$ components

$4 + \omega_2\sin 20° = 0 \qquad \omega_2 = -11.70$ rad/s
$\omega = -11.70\cos 20° = -10.99$ rad/s

Hemce $\omega = \{-10.99\mathbf{j}\}$ rad/s
$\omega_2 = -11.70\cos 20°\mathbf{j} + (-11.70\sin 20°)\mathbf{k} = \{-10.99\mathbf{j} + 4\mathbf{k}\}$ rad/s

Angular acceleration:

$(\dot{\omega}_1)_{xyz} = \{3\mathbf{k}\}$ rad/s^2
$(\dot{\omega}_2)_{xyz} = \left(-\frac{3}{\sin 20°}\right)\cos 20°\mathbf{j} - 3\mathbf{k} = \{-8.2424\mathbf{j} - 3\mathbf{k}\}$ rad/s^2

$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$
$= \left[(\dot{\omega}_1)_{xyz} + \Omega \times \omega_1\right] + \left[(\dot{\omega}_2)_{xyz} + \Omega \times \omega_2\right]$

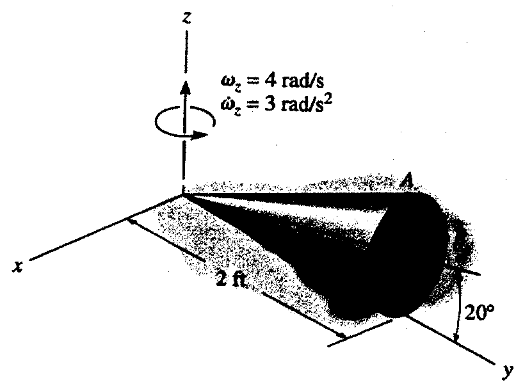
$\Omega = \omega_1 = \{4\mathbf{k}\}$ rad/s then

$\dot{\omega} = [3\mathbf{k} + 0] + [(-8.2424\mathbf{j} - 3\mathbf{k}) + 4\mathbf{k} \times (-10.99\mathbf{j} + 4\mathbf{k})]$
$= \{43.9596\mathbf{i} - 8.2424\mathbf{j}\}$ rad/s

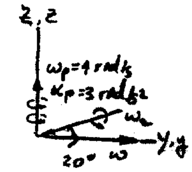
$\mathbf{r}_A = 2\cos 40°\mathbf{j} + 2\sin 40°\mathbf{k} = \{1.5321\mathbf{j} + 1.2856\mathbf{k}\}$ ft
$\mathbf{v}_A = \omega \times \mathbf{r}_A$
$= (-10.99\mathbf{j}) \times (1.5321\mathbf{j} + 1.2856\mathbf{k})$
$= \{-14.1\mathbf{i}\}$ ft/s **Ans**

$\mathbf{a}_A = \mathbf{a} \times \mathbf{r}_A + \omega \times \mathbf{v}_A$
$= (43.9596\mathbf{i} - 8.2424\mathbf{j}) \times (1.5321\mathbf{j} + 1.2856\mathbf{k}) + (-10.99\mathbf{j}) \times (-14.1\mathbf{i})$
$= \{-10.6\mathbf{i} - 56.5\mathbf{j} - 87.9\mathbf{k}\}$ ft/s^2 **Ans**

20-9. The cone rolls without slipping such that at the instant shown $\omega_z = 4$ rad/s and $\dot{\omega}_z = 3$ rad/s^2. Determine the velocity and acceleration of point B at this instant.



Angular velocity : The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity(y axis).



$$\omega = \omega_1 + \omega_2$$
$$\omega \mathbf{j} = 4\mathbf{k} + (\omega_2 \cos 20^\circ \mathbf{j} + \omega_2 \sin 20^\circ \mathbf{k})$$
$$\omega \mathbf{j} = \omega_2 \cos 20^\circ \mathbf{j} + (4 + \omega_2 \sin 20^\circ)\,\mathbf{k}$$

Equating j and k components

$$4 + \omega_2 \sin 20^\circ = 0 \qquad \omega_2 = -11.70 \text{ rad/s}$$
$$\omega = -11.70 \cos 20^\circ = -10.99 \text{ rad/s}$$

Hemce $\quad \omega = \{-10.99\mathbf{j}\}$ rad/s

$$\omega_2 = -11.70\cos 20^\circ \mathbf{j} + (-11.70 \sin 20^\circ)\,\mathbf{k} = \{-10.99\mathbf{j} + 4\mathbf{k}\} \text{ rad/s}$$

Angular acceleration:

$$(\dot{\omega}_1)_{xyz} = \{3\mathbf{k}\} \text{ rad/s}^2$$
$$(\dot{\omega}_2)_{xyz} = \left(-\frac{3}{\sin 20^\circ}\right)\cos 20^\circ \mathbf{j} - 3\mathbf{k} = \{-8.2424\mathbf{j} - 3\mathbf{k}\} \text{ rad/s}^2$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$
$$= \left[(\dot{\omega}_1)_{xyz} + \Omega \times \omega_1\right] + \left[(\dot{\omega}_2)_{xyz} + \Omega \times \omega_2\right]$$

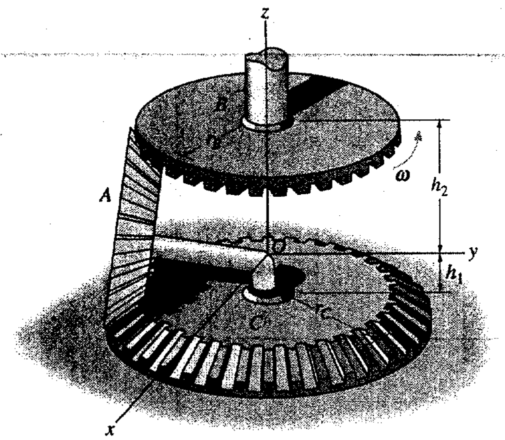
$\Omega = \omega_1 = \{4\mathbf{k}\}$ rad/s then

$$\dot{\omega} = [3\mathbf{k} + 0] + [(-8.2424\mathbf{j} - 3\mathbf{k}) + 4\mathbf{k} \times (-10.99\mathbf{j} + 4\mathbf{k})]$$
$$= \{43.9596\mathbf{i} - 8.2424\mathbf{j}\} \text{ rad/s}$$

$$\mathbf{r}_B = 2\sin 20^\circ \mathbf{i} + 2\cos 20^\circ \mathbf{j} + 2\sin 20^\circ \cos 20^\circ \mathbf{k}$$
$$= -0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k}$$
$$\mathbf{v}_B = \omega \times \mathbf{r}_B$$
$$= (-10.99\mathbf{j}) \times (-0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k})$$
$$= -7.0642\mathbf{i} - 7.5176\mathbf{k}$$
$$= \{-7.06\mathbf{i} - 7.52\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_B = \alpha \times \mathbf{r}_B + \omega \times \mathbf{v}_B$$
$$= (43.9596\mathbf{i} - 8.2424\mathbf{j}) \times (-0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k})$$
$$+ (-10.99\mathbf{j}) \times (-7.0642\mathbf{i} - 7.5176\mathbf{k})$$
$$= \{77.3\mathbf{i} - 28.3\mathbf{j} - 0.657\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans}$$

20-10. If the top gear B is rotating at a constant rate of ω, determine the angular velocity of gear A, which is free to turn about the shaft and rolls on the bottom fixed gear C.



$$\mathbf{v}_P = \omega\mathbf{k} \times (-r_B\mathbf{j}) = \omega r_B \mathbf{i}$$

Also,

$$\mathbf{v}_P = \omega_A \times (-r_B\mathbf{j} + h_2\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_{Ax} & \omega_{Ay} & \omega_{Az} \\ 0 & -r_B & h_2 \end{vmatrix}$$

$$= (\omega_{Ay} h_2 + \omega_{Az} r_B)\mathbf{i} - (\omega_{Ax} h_2)\mathbf{j} - \omega_{Ax} r_B \mathbf{k}$$

Thus,

$$\omega r_B = \omega_{Ay} h_2 + \omega_{Az} r_B \qquad (1)$$

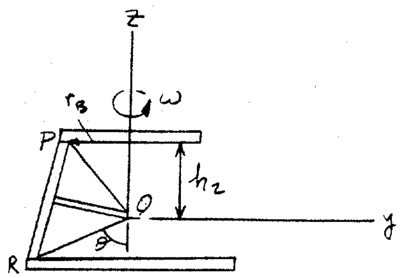
$$0 = \omega_{Ax} h_2$$

$$0 = \omega_{Ax} r_B$$

$$\omega_{Ax} = 0$$

$$\mathbf{v}_R = 0 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega_{Ay} & \omega_{Az} \\ 0 & -r_C & -h_1 \end{vmatrix} = (-\omega_{Ay} h_1 + \omega_{Az} r_C)\mathbf{i}$$

$$\omega_{Ay} = \omega_{Az}\left(\frac{r_C}{h_1}\right)$$



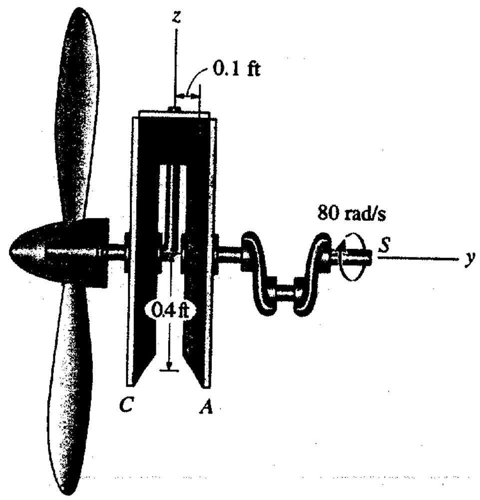
From Eq. (1)

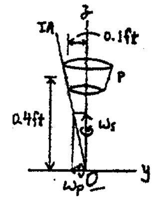
$$\omega r_B = \omega_{Az}\left[\left(\frac{r_C h_2}{h_1}\right) + r_B\right]$$

$$\omega_{Az} = \frac{r_B h_1 \omega}{r_C h_2 + r_B h_1}; \qquad \omega_{Ay} = \left(\frac{r_C}{h_1}\right)\left(\frac{r_B h_1 \omega}{r_C h_2 + r_B h_1}\right)$$

$$\omega_A = \left(\frac{r_C}{h_1}\right)\left(\frac{r_B h_1 \omega}{r_C h_2 + r_B h_1}\right)\mathbf{j} + \left(\frac{r_B h_1 \omega}{r_C h_2 + r_B h_1}\right)\mathbf{k} \qquad \textbf{Ans}$$

20-11. Gear A is fixed to the crankshaft S, while gear C is fixed and B are free to rotate. The crankshaft is turning at 80 rad/s about its axis. Determine the magnitudes of the angular velocity of the propeller and the angular acceleration of gear B.





Point P on gear B has a speed of

$$v_P = 80(0.4) = 32 \text{ ft/s}$$

The IA is located along the points of contant of B and C

$$\frac{\omega_P}{0.1} = \frac{\omega_s}{0.4}$$

$$\omega_s = 4\omega_P$$

$$\boldsymbol{\omega} = -\omega_P\mathbf{j} + \omega_s\mathbf{k}$$

$$= -\omega_P\mathbf{j} + 4\omega_P\mathbf{k}$$

$$\mathbf{r}_{P/O} = 0.1\mathbf{j} + 0.4\mathbf{k}$$

$$\mathbf{v}_P = -32\mathbf{i}$$

$$\mathbf{v}_P = \boldsymbol{\omega} \times \mathbf{r}_{P/O}$$

$$-32\mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -\omega_P & 4\omega_P \\ 0 & 0.1 & 0.4 \end{vmatrix}$$

$$-32\mathbf{i} = -0.8\omega_P\mathbf{i}$$

$$\omega_P = 40 \text{ rad/s}$$

$$\boldsymbol{\omega}_P = \{-40\mathbf{j}\} \text{ rad/s} \qquad \textbf{Ans}$$

$$\boldsymbol{\omega}_s = 4(40)\mathbf{k} = \{160\mathbf{k}\} \text{ rad/s}$$

Thus,

$$\boldsymbol{\omega} = \boldsymbol{\omega}_P + \boldsymbol{\omega}_s$$

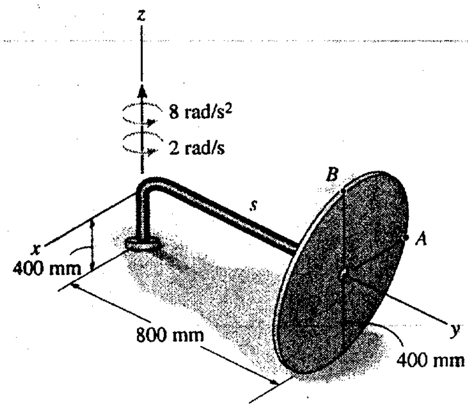
Let the x,y,z axes have an angular velocity of $\Omega \times \omega_P$, then

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}_P + \dot{\boldsymbol{\omega}}_s = 0 + \boldsymbol{\omega}_P \times (\boldsymbol{\omega}_s + \boldsymbol{\omega}_P)$$

$$\boldsymbol{\alpha} = (-40\mathbf{j}) \times (160\mathbf{k} - 40\mathbf{j})$$

$$\boldsymbol{\alpha} = \{-6400\mathbf{i}\} \text{ rad/s}^2 \qquad \textbf{Ans}$$

***20-12.** The disk B is free to rotate on the shaft S. If the shaft is turning about the z axis at $\omega_z = 2$ rad/s, while increasing at 8 rad/s^2, determine the veloicty and acceleration of point A at the instant shown.



Angular Velocity : The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the disk at this instant(with reference to X, Y, Z) can be expressed in terms of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components. The velocity of the center of the disk is $v = \omega_z(0.8) = 2(0.8) = 1.60$ m/s. Since the disk rolls without slipping, its spinning angular velocity is given by $\omega_s = \dfrac{v}{r} = \dfrac{1.60}{0.4} = 4$ rad/s and is directed towards $-\mathbf{j}$. Thus, $\omega_s = \{-4\mathbf{j}\}$ rad/s.

$$\omega = \omega_z + \omega_s = \{-4\mathbf{j} + 2\mathbf{k}\} \text{ rad/s}$$

Angular Acceleration : The angular acceleration α will be determined by investigating separately the time rate of change of *each angular velocity component* with respect to the fixed XYZ frame. ω_s is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_z = \{2\mathbf{k}\}$ rad/s. The tangential acceleration of the center of the disk is $a = \dot{\omega}_z(0.8) = 8(0.8) = 6.40$ m/s^2. Since the disk rolls without slipping, its spinning anngular acceleration is given by $\dot{\omega}_s = \dfrac{a}{r} = \dfrac{6.40}{0.4} = 16$ rad/s^2 and directed towards $-\mathbf{j}$. Thus, $(\dot{\omega}_s)_{xyz} = \{-16\mathbf{j}\}$ rad/s^2. Applying Eq. 20–6, we have

$$\dot{\omega}_s = (\dot{\omega}_s)_{xyz} + \omega_z \times \omega_s = -16\mathbf{j} + 2\mathbf{k} \times (-4\mathbf{j}) = \{8\mathbf{i} - 16\mathbf{j}\} \text{ rad/s}^2$$

Since ω_z is always directed along Z axis $(\Omega = \mathbf{0})$, then

$$\dot{\omega}_z = (\dot{\omega}_z)_{xyz} + \mathbf{0} \times \omega_z = \{8\mathbf{k}\} \text{ rad/s}^2$$

Thus, the angular acceleration of the disk is

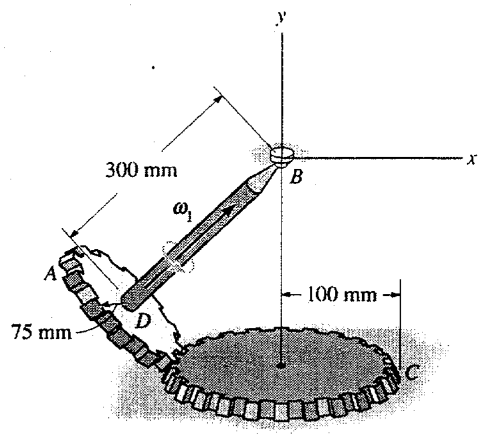
$$\alpha = \dot{\omega}_z + \dot{\omega}_s = \{8\mathbf{i} - 16\mathbf{j} + 8\mathbf{k}\} \text{ rad/s}^2$$

Velocity and Acceleration : Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{-0.4\mathbf{i} + 0.8\mathbf{j}\}$ m, we have

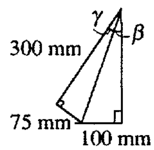
$$\begin{aligned}\mathbf{v}_A &= \omega \times \mathbf{r}_A = (-4\mathbf{j} + 2\mathbf{k}) \times (-0.4\mathbf{i} + 0.8\mathbf{j}) \\ &= \{-1.60\mathbf{i} - 0.800\mathbf{j} - 1.60\mathbf{k}\} \text{ m/s}\end{aligned}$$ **Ans**

$$\begin{aligned}\mathbf{a}_A &= \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A) \\ &= (8\mathbf{i} - 16\mathbf{j} + 8\mathbf{k}) \times (-0.4\mathbf{i} + 0.8\mathbf{j}) \\ &\quad + (-4\mathbf{j} + 2\mathbf{k}) \times [(-4\mathbf{j} + 2\mathbf{k}) \times (-0.4\mathbf{i} + 0.8\mathbf{j})] \\ &= \{1.6\mathbf{i} - 6.40\mathbf{j} - 6.40\mathbf{k}\} \text{ m/s}^2\end{aligned}$$ **Ans**

20-13. Shaft BD is connected to a ball-and-socket joint at B, and a beveled gear A is attached to its other end. The gear is in mesh with a fixed gear C. If the shaft and gear A are *spinning* with a constant angular velocity $\omega_1 = 8$ rad/s, determine the angular velocity and angular acceleration of gear A.



$$\gamma = \tan^{-1}\frac{75}{300} = 14.04^\circ \qquad \beta = \sin^{-1}\frac{100}{\sqrt{300^2 + 75^2}} = 18.87^\circ$$



The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity IA.

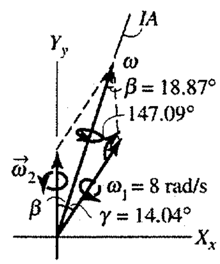
$$\frac{\omega}{\sin 147.09^\circ} = \frac{8}{\sin 18.87^\circ} \qquad \omega = 13.44 \text{ rad/s}$$

$$\omega = 13.44 \sin 18.87^\circ \mathbf{i} + 13.44 \cos 18.87^\circ \mathbf{j}$$

$$= \{4.35\mathbf{i} + 12.7\mathbf{j}\} \text{ rad/s} \qquad \textbf{Ans}$$

$$\frac{\omega_2}{\sin 14.04^\circ} = \frac{8}{\sin 18.87^\circ} \qquad \omega_2 = 6.00 \text{ rad/s}$$

$$\omega_2 = \{6\mathbf{j}\} \text{ rad/s}$$



$$\omega_1 = 8 \sin 32.91^\circ \mathbf{i} + 8 \cos 32.91^\circ \mathbf{j} = \{4.3466\mathbf{i} + 6.7162\mathbf{j}\} \text{ rad/s}$$

For ω_1, $\Omega = \omega_2 = \{6\mathbf{j}\}$ rad/s.

$$(\dot{\omega}_1)_{XYZ} = (\dot{\omega}_1)_{xyz} + \Omega \times \omega_1$$

$$= \mathbf{0} + (6\mathbf{j}) \times (4.3466\mathbf{i} + 6.7162\mathbf{j})$$

$$= \{-26.08\mathbf{k}\} \text{ rad/s}^2$$

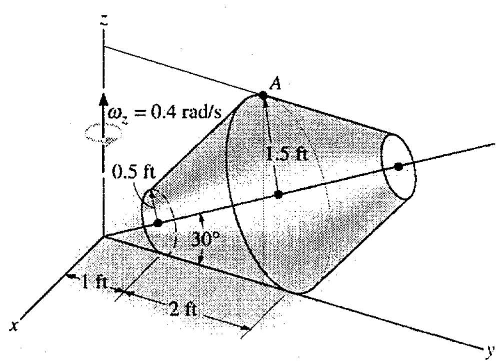
For ω_2, $\Omega = \mathbf{0}$.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \Omega \times \omega_2 = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = \mathbf{0} + (-26.08\mathbf{k}) = \{-26.1\mathbf{k}\} \text{ rad/s}^2 \qquad \textbf{Ans}$$

20-14. The truncated cone rotates about the z axis at a constant rate $\omega_z = 0.4$ rad/s without slipping on the horizontal plane. Determine the velocity and acceleration of point A on the cone.



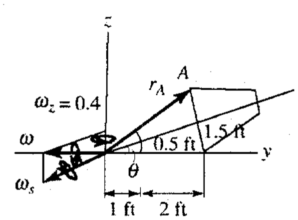
$$\theta = \sin^{-1}\left(\frac{0.5}{1}\right) = 30^\circ$$

$$\omega_s = \frac{0.4}{\sin 30^\circ} = 0.8 \text{ rad/s}$$

$$\omega = 0.8 \cos 30^\circ = 0.6928 \text{ rad/s}$$

$$\omega = \{-0.6928\mathbf{j}\} \text{ rad/s}$$

$$\Omega = 0.4\mathbf{k}$$



$$\dot{\omega} = (\dot{\omega})_{xyz} + \Omega \times \omega \qquad (1)$$

$$= 0 + (0.4\mathbf{k}) \times (-0.6928\mathbf{j})$$

$$\dot{\omega} = \{0.2771\mathbf{i}\} \text{ rad/s}^2$$

$$\mathbf{r}_A = (3 - 3\sin 30^\circ)\mathbf{j} + 3\cos 30^\circ \mathbf{k}$$

$$= (1.5\mathbf{j} + 2.598\mathbf{k}) \text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A$$

$$= (-0.6928\mathbf{j}) \times (1.5\mathbf{j} + 2.598\mathbf{k})$$

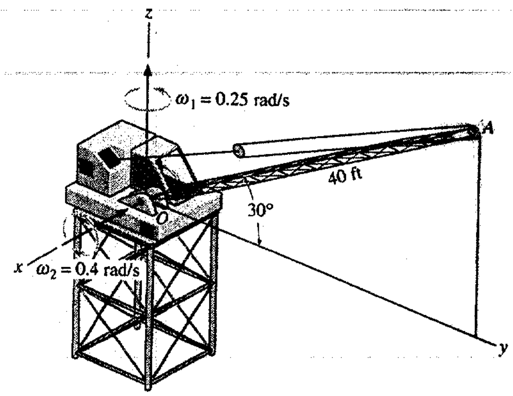
$$\mathbf{v}_A = \{-1.80\mathbf{i}\} \text{ ft/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A$$

$$= (0.2771\mathbf{i}) \times (1.5\mathbf{j} + 2.598\mathbf{k}) + (-0.6928\mathbf{j}) \times (-1.80\mathbf{i})$$

$$\mathbf{a}_A = (-0.720\mathbf{j} - 0.831\mathbf{k}) \text{ ft/s}^2 \qquad \textbf{Ans}$$

20-15. At the instant shown, the tower crane is rotating about the z axis with an angular velocity $\omega_1 = 0.25$ rad/s, which is increasing at 0.6 rad/s^2. The boom OA is rotating downward with an angular velocity $\omega_2 = 0.4$ rad/s, which is increasing at 0.8 rad/s^2. Determine the velocity and acceleration of point A located at the top of the boom at this instant.



$\omega = \omega_1 + \omega_2 = \{-0.4\mathbf{i} + 0.25\mathbf{k}\}$ rad/s

$\Omega = \{0.25\mathbf{k}\}$ rad/s

$\omega = \left(\omega\right)_{xyz} + \Omega \times \omega = (-0.8\mathbf{i} + 0.6\mathbf{k}) + (0.25\mathbf{k}) \times (-0.4\mathbf{i} + 0.25\mathbf{k})$

$= \{-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}\}$ rad/s^2

$\mathbf{r}_A = 40\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k} = \{34.64\mathbf{j} + 20\mathbf{k}\}$ ft

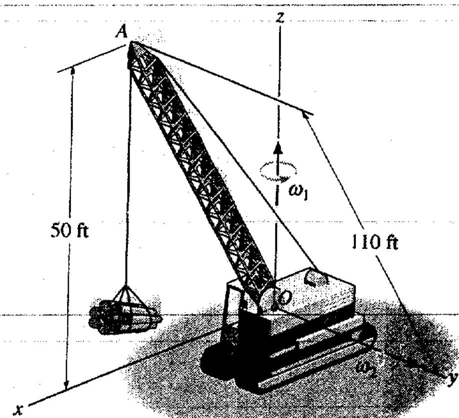
$\mathbf{v}_A = \omega \times \mathbf{r}_A = (-0.4\mathbf{i} + 0.25\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$

$\mathbf{v}_A = \{-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A = (-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k}) + (-0.4\mathbf{i} + 0.25\mathbf{k}) \times (-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k})$

$\mathbf{a}_A = \{-24.8\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}$ ft/s^2 **Ans**

***20-16.** The construction boom OA is rotating about the z axis with a constant angular velocity of $\omega_1 = 0.15$ rad/s, while it is rotating downward with a constant angular velocity of $\omega_2 = 0.2$ rad/s. Determine the velocity and acceleration of point A located at the tip of the boom at the instant shown.



$\omega = \omega_1 + \omega_2 = \{0.2\mathbf{j} + 0.15\mathbf{k}\}$ rad/s

$\omega = \omega_1 + \omega_2$

Let the x, y, z axes rotate at $\Omega = \omega_1$, then

$\alpha = \omega = \left(\omega\right)_{xyz} + \omega_1 \times \omega_2$

$\alpha = 0 + 0.15\mathbf{k} \times 0.2\mathbf{j} = \{-0.03\mathbf{i}\}$ rad/s^2

$\mathbf{r}_A = \left[\sqrt{(110)^2 - (50)^2}\right]\mathbf{i} + 50\mathbf{k} = \{97.98\mathbf{i} + 50\mathbf{k}\}$ ft

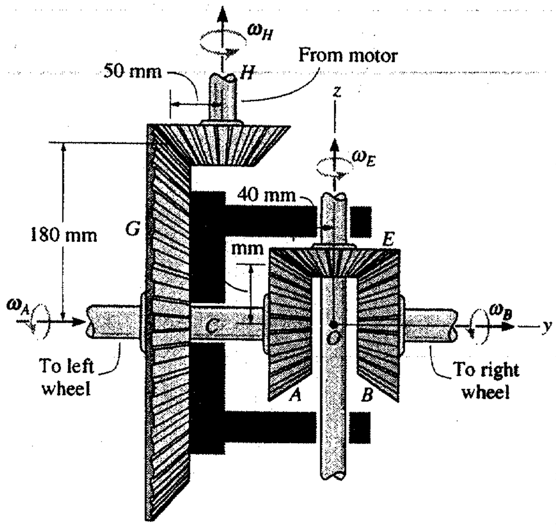
$$\mathbf{v}_A = \omega \times \mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 97.98 & 0 & 50 \end{vmatrix}$$

$\mathbf{v}_A = \{10\mathbf{i} + 14.7\mathbf{j} - 19.6\mathbf{k}\}$ ft/s **Ans**

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.03 & 0 & 0 \\ 97.98 & 0 & 50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 10 & 14.7 & -19.6 \end{vmatrix}$$

$\mathbf{a}_A = \{-6.12\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}$ ft/s^2 **Ans**

20-17. The differential of an automobile allows the two rear wheels to rotate at different speeds when the automobile travels along a curve. For operation, the rear axles are attached to the wheels at one end and have beveled gears A and B on their other ends. The differential case D is placed over the left axle but can rotate about C independent of the axle. The case supports a pinion gear E on a shaft, which meshes with gears A and B. Finally, a ring gear G is *fixed* to the differential case so that the case rotates with the ring gear when the latter is driven by the drive pinion H. This gear, like the differential case, is free to rotate about the left wheel axle. If the drive pinion is turning at $\omega_H = 100$ rad/s and the pinion gear E is spinning about its shaft at $\omega_E = 30$ rad/s, determine the angular velocity, ω_A and ω_B, of each axle.



$v_P = \omega_H r_H = 100(50) = 5000$ mm/s

$$\omega_G = \frac{5000}{180} = 27.78 \text{ rad/s}$$

Point O is a fixed point of rotation for gears A, E, and B.

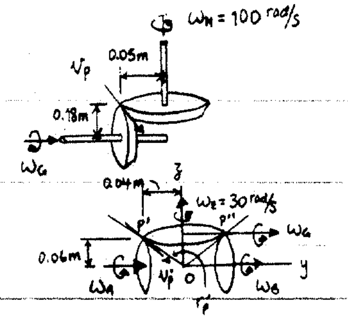
$\Omega = \omega_G + \omega_E = \{27.78\mathbf{j} + 30\mathbf{k}\}$ rad/s

$\mathbf{v}_{P'} = \Omega \times \mathbf{r}_{P'} = (27.78\mathbf{j} + 30\mathbf{k}) \times (-40\mathbf{j} + 60\mathbf{k}) = \{2866.7\mathbf{i}\}$ mm/s

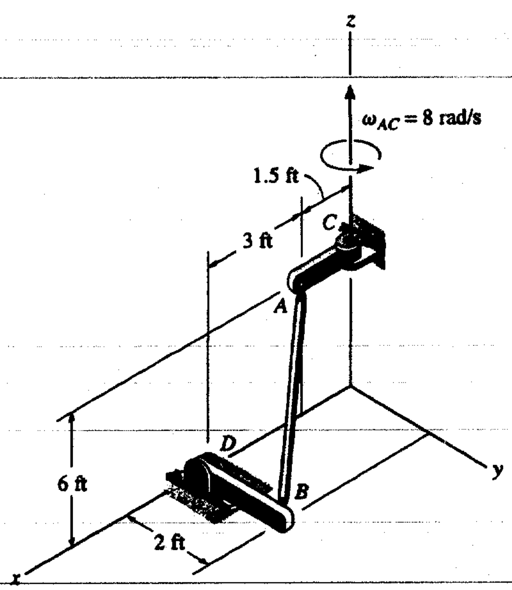
$$\omega_A = \frac{2866.7}{60} = 47.8 \text{ rad/s} \qquad \textbf{Ans}$$

$\mathbf{v}_{P''} = \Omega \times \mathbf{r}_{P''} = (27.78\mathbf{j} + 30\mathbf{k}) \times (40\mathbf{j} + 60\mathbf{k}) = \{466.7\mathbf{i}\}$ mm/s

$$\omega_B = \frac{466.2}{60} = 7.78 \text{ rad/s} \qquad \textbf{Ans}$$



20-18. Rod AB is attached to the rotating arm using ball-and-socket joints. If AC is rotating with a constant angular velocity of 8 rad/s about the pin at C, determine the angular velocity of link BD at the instant shown.



$\mathbf{v}_A = 8(1.5)\mathbf{j} = \{12\mathbf{j}\}$ ft/s

$\mathbf{v}_B = -v_B\mathbf{k}$

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$-v_B\mathbf{k} = 12\mathbf{j} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$

$0 = -6\omega_y - 2\omega_z$

$0 = 12 + 6\omega_x + 3\omega_z$

$-v_B = 2\omega_x - 3\omega_y$

Thus,

$$-v_B = 2\left(-\frac{12}{2} - \frac{3}{6}\omega_z\right) - 3\left(\frac{-\omega_z}{3}\right)$$

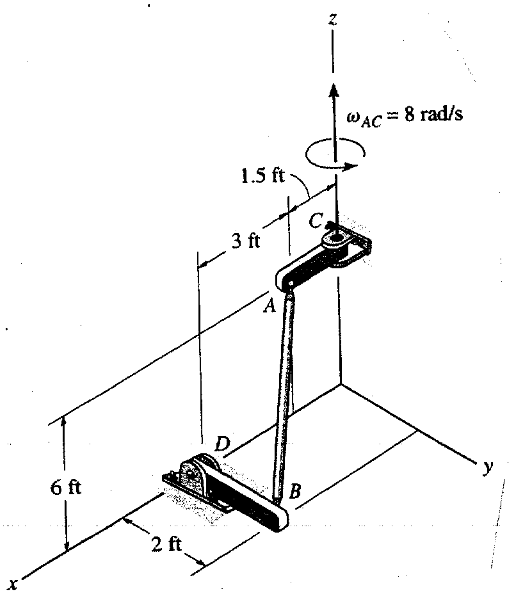
$-v_B = -4 - \omega_z + \omega_z$

$v_B = 4$ ft/s

$$\omega_{BD} = -\left(\frac{4}{2}\right)\mathbf{i}$$

$\omega_{BD} = \{-2.00\mathbf{i}\}$ rad/s $\qquad$ **Ans**

20-19. Rod AB is attached to the rotating arm using ball-and-socket joints. If AC is rotating about point C with an angular velocity of 8 rad/s and has an angular acceleration of 6 rad/s^2 at the instant shown, determine the angular velocity and angular acceleration of link BD at this instant.



See Prob. 20–18.

$\omega_{BD} = \{-2.00\mathbf{i}\}$ rad/s **Ans**

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$-4\mathbf{k} = 12\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{-12\mathbf{j} - 4\mathbf{k}\} \text{ ft/s}$$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} + \boldsymbol{\omega} \times (\mathbf{v}_{B/A})$$

$$-(2)^2(2)\mathbf{j} + (a_B)_z\mathbf{k} = -96\mathbf{i} + 9\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 3 & 2 & -6 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.633 & 0.2449 & -0.7347 \\ 0 & -12 & -4 \end{vmatrix}$$

$$0 = -96 + \alpha_y(-6) - 2\alpha_z - 9.796$$

$$-8 = 9 + 6\alpha_x + 3\alpha_z - 6.5308$$

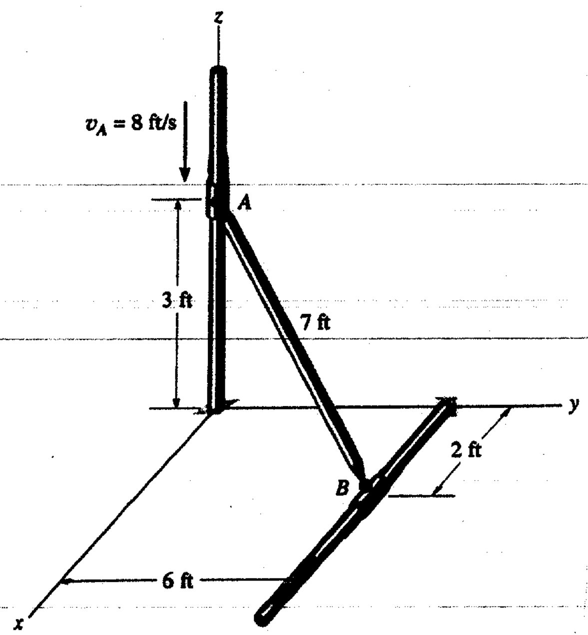
$$(a_B)_z = \alpha_x(2) - \alpha_y(3) + 19.592$$

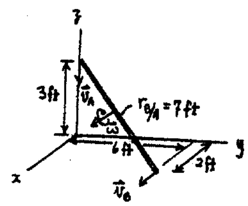
$$(a_B)_z = 69.00 \text{ ft/s}^2$$

$$\alpha_{BD} = \frac{69.00}{2} = 34.5 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_{BD} = \{34.5\mathbf{i}\}$ rad/s^2 **Ans**

***20-20. If the rod is attached with ball-and-socket joints to smooth collars A and B at its end points, determine the speed of B at the instant shown if A is moving downward at a constant speed of $v_A = 8$ ft/s. Also, determine the angular velocity of the rod if it is directed perpendicular to the axis of the rod.**





$$\mathbf{v}_A = \{-8\mathbf{k}\} \text{ ft/s}$$

$$\mathbf{v}_B = v_B\mathbf{i}$$

$$\mathbf{r}_{B/A} = \{2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\} \text{ ft}$$

$$\boldsymbol{\omega} = \{\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}\} \text{ rad/s}$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{i} = -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 2 & 6 & -3 \end{vmatrix}$$

Expanding and equating components yields :

$$v_B = -3\omega_y - 6\omega_z \qquad (1)$$

$$0 = 3\omega_x + 2\omega_z \qquad (2)$$

$$0 = -8 + 6\omega_x - 2\omega_y \qquad (3)$$

Also, $\boldsymbol{\omega} \cdot \mathbf{r}_{B/A} = 0$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$$

$$2\omega_x + 6\omega_y - 3\omega_z = 0 \qquad (4)$$

Solving Eqs. (1) – (4) yields

$$\omega_x = 0.9796 \text{ rad/s}$$

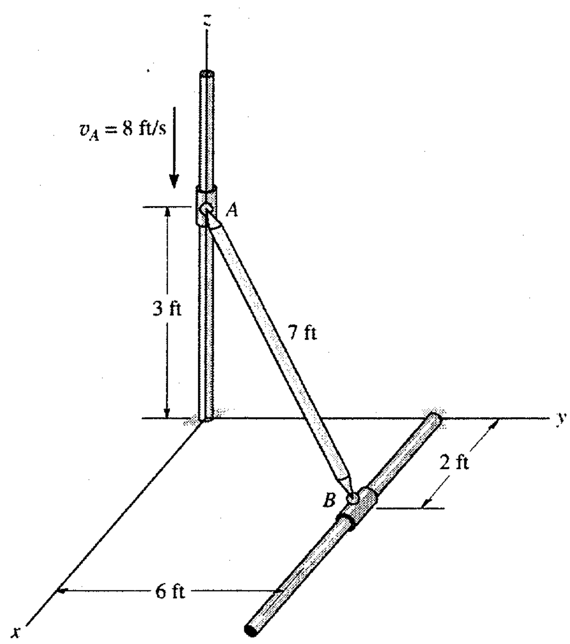
$$\omega_y = -1.061 \text{ rad/s}$$

$$\omega_z = -1.469 \text{ rad/s}$$

$\boldsymbol{\omega} = \{0.980\mathbf{i} - 1.06\mathbf{j} - 1.47\mathbf{k}\}$ rad/s **Ans**

$\mathbf{v}_B = \{12.0\mathbf{i}\}$ ft/s **Ans**

20-21. If the collar at A is moving downward with an acceleration $\mathbf{a}_A = \{-5\mathbf{k}\}$ ft/s², at the instant its speed is $v_A = 8$ ft/s, determine the acceleration of the collar at B at this instant.



$\mathbf{a}_B = a_B i, \quad \mathbf{a}_A = -5\mathbf{k}$

From Prob. 20-20.

$\omega_{AB} = 0.9796\mathbf{i} - 1.0612\mathbf{j} - 1.4694\mathbf{k}$

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$\mathbf{a}_{B/A} = \omega_{AB} \times \mathbf{v}_{B/A} + \alpha_{AB} \times \mathbf{r}_{B/A}$

$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A = 12\mathbf{i} + 8\mathbf{k}$

$$\mathbf{a}_{B/A} = \{0.9796\mathbf{i} - 1.0612\mathbf{j} - 1.4694\mathbf{k}\} \times (12\mathbf{i} + 8\mathbf{k}) + \{\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}\} \times (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})$$

$$a_B\mathbf{i} = -5\mathbf{k} + \{0.9796\mathbf{i} - 1.0612\mathbf{j} - 1.4694\mathbf{k}\} \times (12\mathbf{i} + 8\mathbf{k}) + \{(-3\alpha_y - 6\alpha_z)\mathbf{i} + (2\alpha_z + 3\alpha_x)\mathbf{j} + (6\alpha_x - 2\alpha_y)\mathbf{k}\}$$

$3\alpha_y + 6\alpha_z + a_B = -8.4897$

$-3\alpha_x - 2\alpha_z = -25.4696$

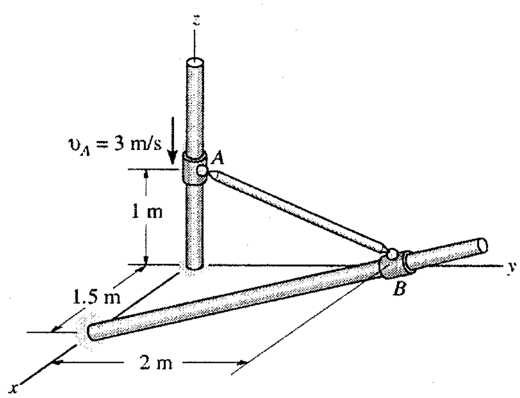
$-6\alpha_x + 2\alpha_y = 7.7344$

Solving these equations

$\mathbf{a}_B = -96.5$

$\mathbf{a}_B = \{-96.5\mathbf{i}\}$ ft/s² **Ans**

20-22. The rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a speed $v_A = 3$ m/s, determine the speed of collar B at the instant shown.



Velocity Equation: Here,

$\mathbf{r}_{B/A} = \{(2-0)\mathbf{j} + (0-1)\mathbf{k}\}$ m $= \{2\mathbf{j} - 1\mathbf{k}\}$ m,

$$\mathbf{v}_A = \{-3\mathbf{k}\} \text{ m/s}. \quad \mathbf{v}_B = v_B\left[\frac{(0-1.5)\mathbf{i} + (2-0)\mathbf{j}}{\sqrt{(0-1.5)^2 + (2-0)^2}}\right]$$

$$= -0.6v_B\mathbf{i} + 0.8v_B\mathbf{j}$$

and $\omega = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying Eq. 20-7, we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$-0.6v_B\mathbf{i} + 0.8v_B\mathbf{j} = -3\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$-0.6v_B\mathbf{i} + 0.8v_B\mathbf{j} = (-3 - \omega_y - 2\omega_z)\mathbf{i} + \omega_x\mathbf{j} + (2\omega_x - 3)\mathbf{k}$$

Equating **i**, **j** and **k** components, we have

$-0.6v_B = -\omega_y - 2\omega_z \qquad [1]$

$0.8v_B = \omega_x \qquad [2]$

$0 = 2\omega_x - 3 \qquad [3]$

If ω is specified acting *perpendicular* to the axis of the rod AB, then

$\omega \cdot \mathbf{r}_{B/A} = 0$

$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{j} - 1\mathbf{k}) = 0$

$2\omega_y - \omega_z = 0 \qquad [4]$

Solving Eqs. [1], [2], [3] and [4] yields

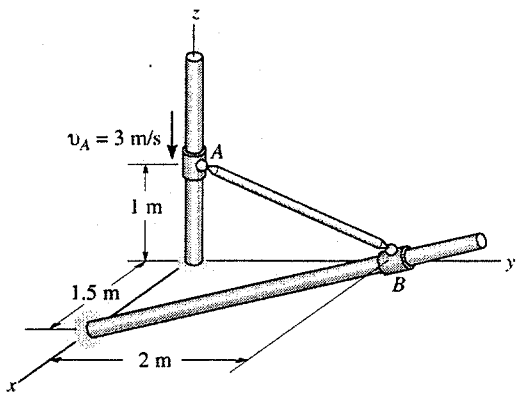
$\omega_x = 1.50$ rad/s

$\omega_y = 0.225$ rad/s

$\omega_z = 0.450$ rad/s

$v_B = 1.875$ m/s **Ans**

20-23. If the collar at A in Prob 20-22 has an acceleration of $\mathbf{a}_A = [-2\mathbf{k}]$ m/s^2 at the instant its speed is $v_A = 3$ m/s, determine the magnitude of the acceleration of the collar at B at this instant.



Velocity Equation: Here,

$$\mathbf{r}_{B/A} = \{(2-0)\mathbf{j} + (0-1)\mathbf{k}\}\ \text{m} = \{2\mathbf{j} - 1\mathbf{k}\}\ \text{m}.$$

$$\mathbf{v}_A = \{-3\mathbf{k}\}\ \text{m/s},\ \mathbf{v}_B = v_B\left[\frac{(0-1.5)\mathbf{i} + (2-0)\mathbf{j}}{\sqrt{(0-1.5)^2 + (2-0)^2}}\right]$$

$$= -0.6v_B\mathbf{i} + 0.8v_B\mathbf{j}$$

and $\omega = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying Eq. 20-7, we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$-0.6v_B\mathbf{i} + 0.8v_B\mathbf{j} = -3\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$-0.6v_B\mathbf{i} + 0.8v_B\mathbf{j} = (-3 - \omega_y - 2\omega_z)\mathbf{i} + \omega_x\mathbf{j} + (2\omega_x - 3)\mathbf{k}$$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ components, we have

$$-0.6v_B = -\omega_y - 2\omega_z \qquad [1]$$

$$0.8v_B = \omega_x \qquad [2]$$

$$0 = 2\omega_x - 3 \qquad [3]$$

If ω is specified acting *perpendicular* to the axis of the rod AB, then

$$\omega \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{j} - 1\mathbf{k}) = 0$$

$$2\omega_y - \omega_z = 0 \qquad [4]$$

Solving Eqs. [1], [2], [3] and [4] yields

$$v_B = 1.875\ \text{m/s} \quad \omega_x = 1.50\ \text{rad/s}$$

$$\omega_y = 0.225\ \text{rad/s} \quad \omega_z = 0.450\ \text{rad/s}$$

Thus,

$$\omega = \{1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}\}\ \text{rad/s}$$

Acceleration Equation: With $\alpha = \alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}$ and the result obtained above, applying Eq. 20-8, we have

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} + \omega \times (\omega \times \mathbf{r}_{B/A})$$

$$-0.6a_B\mathbf{i} + 0.8a_B\mathbf{j} = -2\mathbf{k} + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$+ (1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}) \times [(1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})]$$

$$-0.6a_B\mathbf{i} + 0.8a_B\mathbf{j} = (-\alpha_y - 2\alpha_z)\mathbf{i} + (\alpha_x - 5.00625)\mathbf{j}$$

$$+ (2\alpha_x + 0.503125)\mathbf{k}$$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ components, we have

$$-0.6a_B = -\alpha_x - 2\alpha_z \qquad [5]$$

$$0.8a_B = \alpha_x - 5.00625 \qquad [6]$$

$$0 = 2\alpha_x + 0.503125 \qquad [7]$$

Solving Eqs. [6] and [7] yields

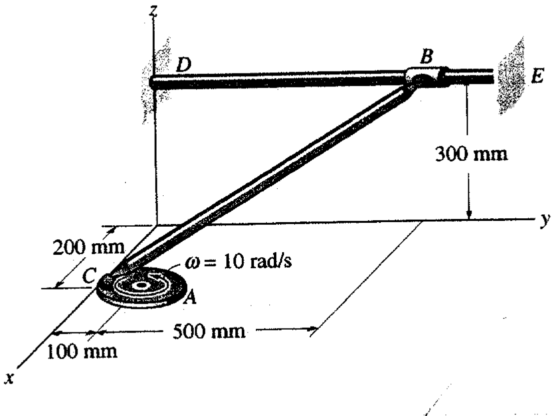
$$\alpha_x = -0.2515\ \text{rad/s}^2$$

$$a_B = -6.57\ \text{m/s}^2 \qquad \textbf{Ans}$$

Negative sign indicates that $\mathbf{a}_B$ is directed in the opposite direction to that of the above assumed direction.

Note: In order to determine α_y and α_z, one should obtain another equation by specifying the direction of α which acts *perpendicular* to the axis of rod AB.

20-24. The rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a velocity of $v_A = 5$ ft/s, determine the angular velocity of the rod and the velocity of collar B at the instant shown. Assume the angular velocity of the rod is directed perpendicular to the rod.



$\mathbf{v}_A = \{5\mathbf{i}\}$ ft/s

$\mathbf{v}_B = -v_B\mathbf{j}$

$\mathbf{r}_{B/A} = \{-3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}\}$

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$

$$-v_B\mathbf{j} = 5\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -3 & 4 & -5 \end{vmatrix}$$

$5 - 5\omega_y - 4\omega_z = 0$

$5\omega_x - 3\omega_z = -v_B$

$4\omega_x + 3\omega_y = 0$

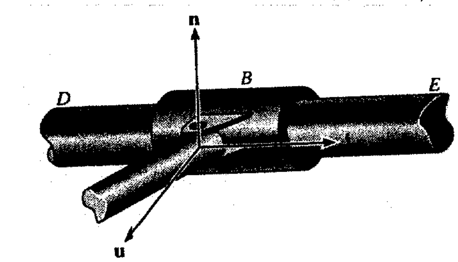
$\boldsymbol{\omega} \cdot \mathbf{r}_{B/C} = 0$

$-3\omega_x + 4\omega_y - 5\omega_z = 0$

$\boldsymbol{\omega} = \{-0.375\mathbf{i} + 0.5\mathbf{j} + 0.625\mathbf{k}\}$ rad/s **Ans**

$\mathbf{v}_B = \{-3.75\mathbf{j}\}$ ft/s **Ans**

20-25. Solve Prob. 20-24 if the connection at B consists of a pin as shown in the figure below, rather than a ball-and-socket joint. *Hint:* The constraint allows rotation of the rod both along bar DE ($\mathbf{j}$ direction) and along the axis of the pin ($\mathbf{n}$ direction). Since there is no rotational component in the $\mathbf{u}$ direction, i.e., perpendicular to $\mathbf{n}$ and $\mathbf{j}$ where $\mathbf{u} = \mathbf{j} \times \mathbf{n}$, an additional equation for solution can be obtained from $\boldsymbol{\omega} \cdot \mathbf{u} = 0$. The vector $\mathbf{n}$ is in the same direction as $\mathbf{r}_{B/C} \times \mathbf{r}_{D/C}$.



$\mathbf{v}_C = \{1\mathbf{i}\}$ m/s $\quad \mathbf{v}_B = -v_B\mathbf{j} \quad \boldsymbol{\omega}_{BC} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$\mathbf{r}_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}$ m

$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$

$$-v_B\mathbf{j} = 1\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -0.2 & 0.6 & 0.3 \end{vmatrix}$$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ components

$1 + 0.3\omega_y - 0.6\omega_z = 0 \qquad (1)$

$0.3\omega_x + 0.2\omega_z = v_B \qquad (2)$

$0.6\omega_x + 0.2\omega_y = 0 \qquad (3)$

Also,

$\mathbf{r}_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}$ m

$\mathbf{r}_{D/C} = \{-0.2\mathbf{i} + 0.3\mathbf{k}\}$ m

$$\mathbf{r}_{B/C} \times \mathbf{r}_{D/C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.2 & 0.6 & 0.3 \\ -0.2 & 0 & 0.3 \end{vmatrix} = \{0.18\mathbf{i} + 0.12\mathbf{k}\}\ \text{m}^2$$

$$\mathbf{n} = \frac{0.18\mathbf{i} + 0.12\mathbf{k}}{\sqrt{0.18^2 + 0.12^2}} = 0.8321\mathbf{i} + 0.5547\mathbf{k}$$

$\mathbf{u} = \mathbf{j} \times \mathbf{n} = \mathbf{j} \times (0.8321\mathbf{i} + 0.5547\mathbf{k}) = 0.5547\mathbf{i} - 0.8321\mathbf{k}$

$\boldsymbol{\omega}_{BC} \cdot \mathbf{u} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (0.5547\mathbf{i} - 0.8321\mathbf{k})$

$0.5547\omega_x - 0.8321\omega_z = 0 \qquad (4)$

Solving Eqs.(1) to (4) yields :

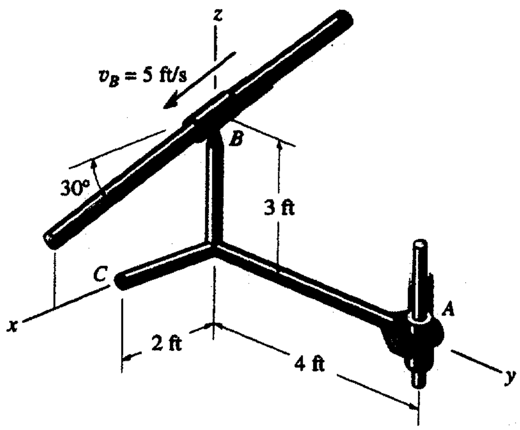
$\omega_x = 0.769$ rad/s $\quad \omega_y = -2.31$ rad/s $\quad \omega_z = 0.513$ rad/s $\quad v_B = 0.333$ m/s

Then

$\boldsymbol{\omega}_{BC} = \{0.769\mathbf{i} - 2.31\mathbf{j} + 0.513\mathbf{k}\}$ rad/s **Ans**

$\mathbf{v}_B = \{-0.333\mathbf{j}\}$ m/s **Ans**

20-26. The rod assembly is supported at B by a ball-and-socket joint and at A by a clevis. If the collar at B moves in the x–z plane with a speed $v_B = 5$ ft/s, determine the velocity of points A and C on the rod assembly at the instant shown. *Hint:* See Prob. 20–25.

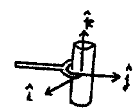


$\mathbf{v}_B = \{5\cos 30°\mathbf{i} - 5\sin 30°\mathbf{k}\}$ ft/s $\quad \mathbf{v}_A = -v_A\mathbf{k} \quad \boldsymbol{\omega}_{ABC} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$\mathbf{r}_{A/B} = \{4\mathbf{j} - 3\mathbf{k}\}$ ft $\quad \mathbf{r}_{C/B} = \{2\mathbf{i} - 3\mathbf{k}\}$ ft

$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{ABC} \times \mathbf{r}_{A/B}$

$$-v_A\mathbf{k} = 5\cos 30°\mathbf{i} - 5\sin 30°\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0 & 4 & -3 \end{vmatrix}$$



Equating **i**, **j** and **k** components

$5\cos 30° - 3\omega_y - 4\omega_z = 0 \quad [1]$

$3\omega_x = 0 \quad [2]$

$4\omega_x - 5\sin 30° = -v_A \quad [3]$

Also, since there is no rotation abouthe y axis

$\boldsymbol{\omega}_{ABC} \cdot \mathbf{j} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (\mathbf{j}) = 0$

$\omega_y = 0 \quad [4]$

Solving Eqs.[1] to [4] yields :

$\omega_x = \omega_y = 0 \quad \omega_z = 1.083$ rad/s $\quad v_A = 2.5$ ft/s ↓

Then $\quad \boldsymbol{\omega}_{ABC} = \{1.083\mathbf{k}\}$ rad/s

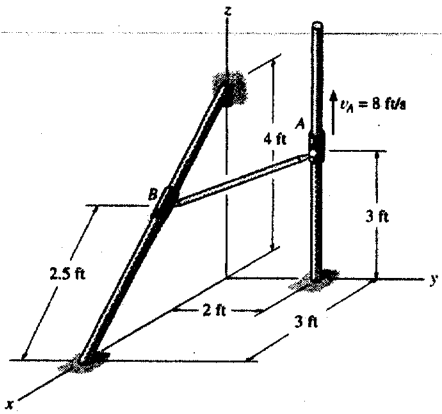
$\mathbf{v}_A = \{-2.50\mathbf{k}\}$ ft/s **Ans**

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{ABC} \times \mathbf{r}_{C/B}$

$$= 5\cos 30°\mathbf{i} - 5\sin 30°\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1.083 \\ 2 & 0 & -3 \end{vmatrix}$$

$= \{4.33\mathbf{i} + 2.17\mathbf{j} - 2.50\mathbf{k}\}$ ft/s **Ans**

20-27. Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A moves upward with a velocity of 8 ft/s, determine the angular velocity of the rod and the speed of collar B at the instant shown. Assume that the rod's angular velocity is directed perpendicular to the rod.



$\mathbf{v}_A = \{8\mathbf{k}\}$ ft/s $\quad \mathbf{v}_B = -\frac{3}{5}v_B\mathbf{i} + \frac{4}{5}v_B\mathbf{k} \quad \boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$\mathbf{r}_{B/A} = \{1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}\}$ ft

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\frac{3}{5}v_B\mathbf{i} + \frac{4}{5}v_B\mathbf{k} = 8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1.5 & -2 & -1 \end{vmatrix}$$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$

$$-\omega_y + 2\omega_z = -\frac{3}{5}v_B \qquad (1)$$

$$\omega_x + 1.5\omega_z = 0 \qquad (2)$$

$$8 - 2\omega_x - 1.5\omega_y = \frac{4}{5}v_B \qquad (3)$$

Since ω_{AB} is perpendicular to the axis of the rod,

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}) = 0$$

$$1.5\omega_x - 2\omega_y - \omega_z = 0 \qquad (4)$$

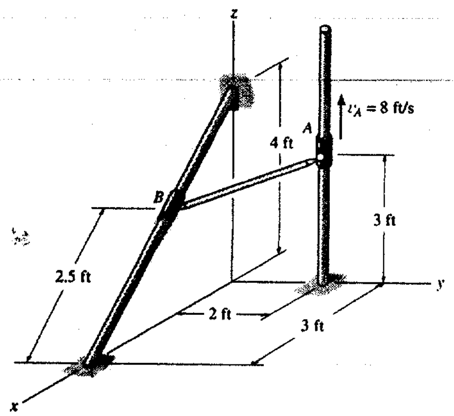
Solving Eqs.(1) to(4) yields :

$\omega_x = 1.1684$ rad/s $\quad \omega_y = 1.2657$ rad/s $\quad \omega_z = -0.7789$ rad/s

$v_B = 4.71$ ft/s **Ans**

Then $\quad \boldsymbol{\omega}_{AB} = \{1.17\mathbf{i} + 1.27\mathbf{j} - 0.779\mathbf{k}\}$ rad/s **Ans**

***20-28.** Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A moves upward with an acceleration of $a_A = 4\ \text{ft/s}^2$, determine the angular acceleration of rod AB and the magnitude of acceleration of collar B. Assume that the rod's angular acceleration is directed perpendicular to the rod.



From Prob. 20 − 27

$$\omega_{AB} = \{1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k}\}\ \text{rad/s}$$

$$\mathbf{r}_{B/A} = \{1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}\}\ \text{ft}$$

$$\alpha_{AB} = \alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}$$

$$\mathbf{a}_A = \{4\mathbf{k}\}\ \text{ft/s}^2 \qquad \mathbf{a}_B = -\frac{3}{5}a_B\mathbf{i} + \frac{4}{5}a_B\mathbf{k}$$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A})$$

$$-\frac{3}{5}a_B\mathbf{i} + \frac{4}{5}a_B\mathbf{k} = 4\mathbf{k} + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k})$$
$$+ (1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k})$$
$$\times [(1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k}) \times (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k})]$$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ components

$$-\alpha_y + 2\alpha_z - 5.3607 = -\frac{3}{5}a_B \qquad (1)$$

$$\alpha_x + 1.5\alpha_z + 7.1479 = 0 \qquad (2)$$

$$7.5737 - 2\alpha_x - 1.5\alpha_y = \frac{4}{5}a_B \qquad (3)$$

Since α_{AB} is perpendicular to the axis of the rod,

$$\alpha_{AB} \cdot \mathbf{r}_{B/A} = (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \cdot (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}) = 0$$

$$1.5\alpha_x - 2\alpha_y - \alpha_z = 0 \qquad (4)$$

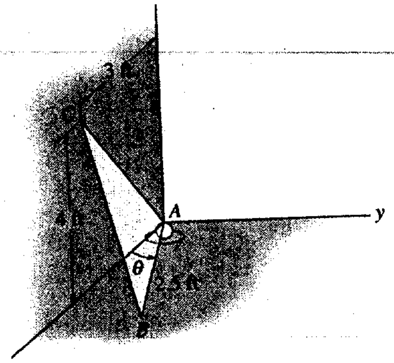
Solving Eqs.(1) to (4) yields :

$\alpha_x = -2.7794\ \text{rad/s}^2 \quad \alpha_y = -0.6285\ \text{rad/s}^2 \quad \alpha_z = -2.91213\ \text{rad/s}^2$

$a_B = 17.6\ \text{ft/s}^2$ **Ans**

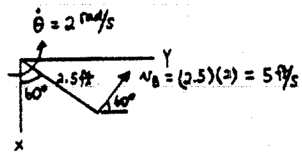
Then $\alpha_{AB} = \{-2.78\mathbf{i} - 0.628\mathbf{j} - 2.91\mathbf{k}\}\ \text{rad/s}^2$ **Ans**

20-29. The triangular plate ABC is supported at A by a ball-and-socket joint and at C by the x–z plane. The side AB lies in the x–y plane. At the instant $\theta = 60°$, $\dot{\theta} = 2$ rad/s and point C has the coordinates shown. Determine the angular velocity of the plate and the velocity of point C at this instant.



$$\mathbf{v}_B = -5\sin 60°\mathbf{i} + 5\cos 60°\mathbf{j}$$

$$= \{-4.33\mathbf{i} + 2.5\mathbf{j}\} \text{ ft/s}$$



$$\mathbf{v}_C = (v_C)_x\mathbf{i} + (v_C)_z\mathbf{k}$$

$$\mathbf{r}_{C/A} = \{3\mathbf{i} + 4\mathbf{k}\} \text{ ft}$$

$$\mathbf{r}_{B/A} = \{1.25\mathbf{i} + 2.165\mathbf{j}\} \text{ ft}$$

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$-4.33\mathbf{i} + 2.5\mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1.25 & 2.165 & 0 \end{vmatrix}$$

$$-2.165\omega_z = -4.33\,; \quad \omega_z = 2 \text{ rad/s}$$
$$2.165\omega_x - 1.25\omega_y = 0; \quad \omega_y = 1.732\omega_x$$

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_{C/A}$$

$$(v_C)_x\mathbf{i} + (v_C)_z\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & 2 \\ 3 & 0 & 4 \end{vmatrix}$$

$$(v_C)_x = 4\omega_y$$

$$0 = 4\omega_x - 6; \quad \omega_x = 1.5 \text{ rad/s}$$

$$(v_C)_z = -3\omega_y$$

Solving,

$$\omega_y = 2.5981 \text{ rad/s}$$

$$(v_C)_x = 10.392 \text{ ft/s}$$

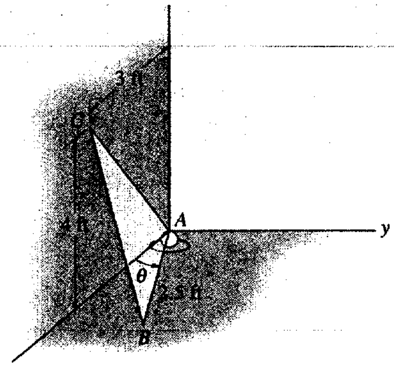
$$(v_C)_z = -7.7942 \text{ ft/s}$$

Thus,

$$\boldsymbol{\omega} = \{1.50\mathbf{i} + 2.60\mathbf{j} + 2.00\mathbf{k}\} \text{ rad/s}$$ **Ans**

$$\mathbf{v}_C = \{10.4\mathbf{i} - 7.79\mathbf{k}\} \text{ ft/s}$$ **Ans**

20-30. The triangular plate ABC is supported at A by a ball-and-socket joint and at C by the x–z plane. The side AB lies in the x–y plane. At the instant $\theta = 60°$, $\dot{\theta} = 2$ rad/s, $\ddot{\theta} = 3$ rad/s^2, and point C has the coordinates shown. Determine the angular acceleration of the plate and the acceleration of point C at this instant.



~~From Prob. 20–29,~~

$\omega = 1.5\mathbf{i} + 2.5981\mathbf{j} + 2\mathbf{k}$

$\mathbf{r}_{B/A} = 1.25\mathbf{i} + 2.165\mathbf{j}$

$\mathbf{v}_B = -4.33\mathbf{i} + 2.5\mathbf{j}$

$(a_B)_t = 3(2.5) = 7.5 \text{ ft/s}^2$

$(a_B)_n = (2)^2(2.5) = 10 \text{ ft/s}^2$

$\mathbf{a}_B = -7.5\sin 60°\mathbf{i} + 7.5\cos 60°\mathbf{j} - 10\cos 60°\mathbf{i} - 10\sin 60°\mathbf{j}$

$\mathbf{a}_B = -11.4952\mathbf{i} - 4.91025\mathbf{j}$

$\mathbf{a}_B = \alpha \times \mathbf{r}_{B/A} + \omega \times \mathbf{v}_{B/A}$

$$-11.4952\mathbf{i} - 4.91025\mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 1.25 & 2.165 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 2.5981 & 2 \\ -4.33 & 2.5 & 0 \end{vmatrix}$$

$-11.4952 = -2.165\alpha_z - 5$

$-4.91025 = 1.25\alpha_z - 8.66$

$\alpha_z = 3 \text{ rad/s}^2$

$0 = 2.165\alpha_x - 1.25\alpha_y + 15 \qquad (1)$

$\mathbf{a}_C = \alpha \times \mathbf{r}_{C/A} + \omega \times \mathbf{v}_{C/A}$

$\mathbf{v}_{C/A} = 10.39\mathbf{i} - 7.794\mathbf{k}$

$$\mathbf{a}_C = (a_C)_x\mathbf{i} + (a_C)_z\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 3 & 0 & 4 \end{vmatrix} + \begin{vmatrix} \end{vmatrix}$$

$(a_C)_x = 4\alpha_y - 20.25 \qquad (2)$

$0 = 3\alpha_z - 4\alpha_x + 32.4760 \qquad (3)$

$(a_C)_z = -3\alpha_y - 27 \qquad (4)$

Solving Eqs. (1) – (4),

$\alpha_x = 10.369 \text{ rad/s}^2$

$\alpha_y = 29.96 \text{ rad/s}^2$

$(a_C)_x = 99.6 \text{ ft/s}^2$

$(a_C)_z = -117 \text{ ft/s}^2$

$\mathbf{a}_C = \{99.6\mathbf{i} - 117\mathbf{k}\} \text{ ft/s}^2$ **Ans**

$\alpha = \{10.4\mathbf{i} + 30.0\mathbf{j} + 3\mathbf{k}\} \text{ rad/s}^2$ **Ans**

20-31. Solve Example 20–5 such that the x, y, z axes move with curvilinear translation, $\mathbf{\Omega} = \mathbf{0}$, in which case the collar appears to have both an angular velocity $\mathbf{\Omega}_{xyz} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$ and radial motion.

Relative to XYZ, let xyz have

$\Omega = \mathbf{0} \qquad \dot{\Omega} = \mathbf{0}$

$\mathbf{r}_B = \{-0.5\mathbf{k}\}$ m

$\mathbf{v}_B = \{2\mathbf{j}\}$ m/s

$\mathbf{a}_B = \{0.75\mathbf{j} + 8\mathbf{k}\}$ m/s^2

Relative to xyz, let $x'y'z'$ be coincident with xyz and be fixed to BD. Then

$\Omega_{xyz} = \omega_1 + \omega_2 = \{4\mathbf{i} + 5\mathbf{k}\}$ rad/s $\qquad \dot{\Omega}_{xyz} = \dot{\omega}_1 + \dot{\omega}_2 = \{1.5\mathbf{i} - 6\mathbf{k}\}$ rad/s^2

$(\mathbf{r}_{C/B})_{xyz} = \{0.2\mathbf{j}\}$ m

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\mathbf{r}_{C/B})_{xyz}$$

$$= 3\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})$$

$$= \{-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = (\ddot{\mathbf{r}}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{x'y'z'}\right]$$

$$+\left[(\dot{\omega}_1 + \dot{\omega}_2) \times (\mathbf{r}_{C/B})_{xyz}\right] + \left[(\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right]$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[2\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times 3\mathbf{j}\right] + \left[(1.5\mathbf{i} - 6\mathbf{k}) \times 0.2\mathbf{j}\right] + \left[(4\mathbf{i} + 5\mathbf{k}) \times (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})\right]$$

$$= \{-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + \mathbf{0} + (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})$$

$= \{-1.00\mathbf{i} + 5.00\mathbf{j} + 0.800\mathbf{k}\}$ m/s **Ans**

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \mathbf{0} + \mathbf{0} + \mathbf{0} + (-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k})$$

$= \{-28.8\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}$ m/s^2 **Ans**

***20-32.** Solve Example 20–5 by fixing x, y, z axes to rod BD so that $\mathbf{\Omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$. In this case the collar appears only to move radially outward along BD; hence $\mathbf{\Omega}_{xyz} = \mathbf{0}$.

Relative to XYZ, let $x'y'z'$ be coincident with XYZ and have $\Omega' = \omega_1$ and $\dot{\Omega}' = \dot{\omega}_1$.

$$\mathbf{\Omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 = \{4\mathbf{i} + 5\mathbf{k}\}\ \text{rad/s}$$

$$\dot{\mathbf{\Omega}} = \dot{\boldsymbol{\omega}}_1 + \dot{\boldsymbol{\omega}}_2 = \left[\left(\dot{\boldsymbol{\omega}}_1\right)_{x'y'z'} + \boldsymbol{\omega}_1 \times \boldsymbol{\omega}_1\right] + \left[\left(\dot{\boldsymbol{\omega}}_2\right)_{x'y'z'} + \boldsymbol{\omega}_1 \times \boldsymbol{\omega}_2\right]$$

$$= (1.5\mathbf{i} + \mathbf{0}) + \left[-6\mathbf{k} + (4\mathbf{i}) \times (5\mathbf{k})\right] = \{1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}\}\ \text{rad/s}^2$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\}\ \text{m}$$

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = \left(\dot{\mathbf{r}}_B\right)_{x'y'z'} + \boldsymbol{\omega}_1 \times \mathbf{r}_B = \mathbf{0} + (4\mathbf{i}) \times (-0.5\mathbf{k}) = \{2\mathbf{j}\}\ \text{m/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[\left(\ddot{\mathbf{r}}_B\right)_{x'y'z'} + \boldsymbol{\omega}_1 \times \left(\dot{\mathbf{r}}_B\right)_{x'y'z'}\right] + \dot{\boldsymbol{\omega}}_1 \times \mathbf{r}_B + \boldsymbol{\omega}_1 \times \dot{\mathbf{r}}_B$$

$$= \mathbf{0} + \mathbf{0} + \left[(1.5\mathbf{i}) \times (-0.5\mathbf{k})\right] + (4\mathbf{i} \times 2\mathbf{j}) = \{0.75\mathbf{j} + 8\mathbf{k}\}\ \text{m/s}^2$$

Relative to $x'y'z'$, let xyz have

$$\mathbf{\Omega}_{x'y'z'} = \mathbf{0}; \qquad \dot{\mathbf{\Omega}}_{x'y'z'} = \mathbf{0};$$

$$\left(r_{C/B}\right)_{xyz} = \{0.2\mathbf{j}\}\ \text{m}$$

$$(\mathbf{v}_{C/B})_{xyz} = \{3\mathbf{j}\}\ \text{m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \{2\mathbf{j}\}\ \text{m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{\Omega} \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 3\mathbf{j}$$

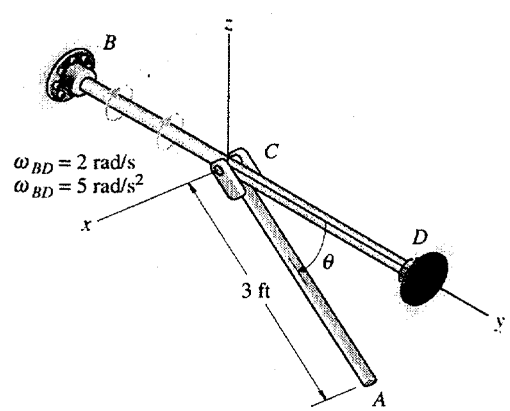
$$= \{-1\mathbf{i} + 5\mathbf{j} + 0.8\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\mathbf{\Omega}} \times \mathbf{r}_{C/B} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{C/B}) + 2\mathbf{\Omega} \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \left[(1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}) \times (0.2\mathbf{j})\right] + (4\mathbf{i} + 5\mathbf{k}) \times \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 2\left[(4\mathbf{i} + 5\mathbf{k}) \times (3\mathbf{j})\right] + 2\mathbf{j}$$

$$\mathbf{a}_C = \{-28.8\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}\ \text{m/s}^2 \qquad \textbf{Ans}$$

20-33. At a given instant, rod BD is rotating about the y axis with an angular velocity $\omega_{BD} = 2$ rad/s and an angular acceleration $\dot{\omega}_{BD} = 5$ rad/s^2. Also, when $\theta = 60°$ link AC is rotating downward such that $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 8$ rad/s^2. Determine the velocity and acceleration of point A on the link at this instant.



$$\Omega = -2\mathbf{i} - 2\mathbf{j}$$

$$\mathbf{r}_{A/C} = 3\cos 60°\mathbf{j} - 3\sin 60°\mathbf{k} = 1.5\mathbf{j} - 2.5980762\mathbf{k}$$

$$\dot{\Omega} = -5\mathbf{j} - 8\mathbf{i} + (-2\mathbf{j}) \times (-2\mathbf{i}) = \{-8\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{v}_A = \mathbf{v}_C + \Omega \times \mathbf{r}_{A/C} + (\mathbf{v}_{A/C})_{xyz}$$

$$\mathbf{v}_A = (-2\mathbf{i} - 2\mathbf{j}) \times (1.5\mathbf{j} - 2.5980762\mathbf{k}) + 0$$

$$\mathbf{v}_A = 5.19615\mathbf{i} - 5.19615\mathbf{j} - 3\mathbf{k} + 0$$

$$\mathbf{v}_A = \{5.20\mathbf{i} - 5.20\mathbf{j} - 3.00\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{A/C} + \Omega \times (\Omega \times \mathbf{r}_{A/C})$$

$$+ 2(\Omega \times (\mathbf{v}_{A/C})_{xyz}) + (a_{A/C})_{xyz}$$

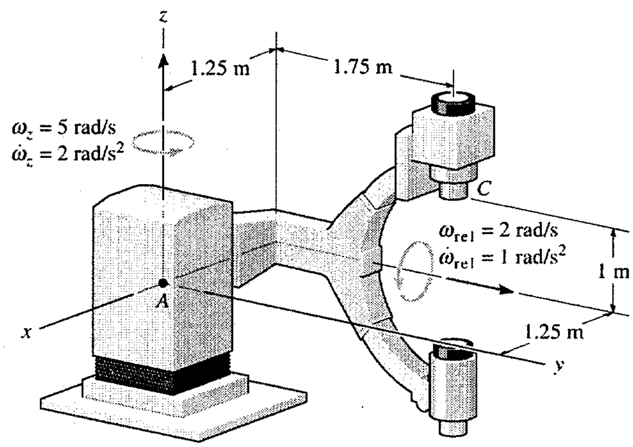
$$= \mathbf{0} + (-8\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}) \times (1.5\mathbf{j} - 2.5980762\mathbf{k})$$

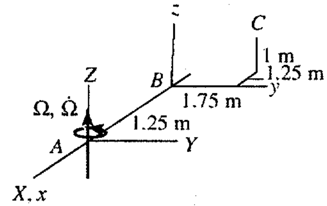
$$+ (-2\mathbf{i} - 2\mathbf{j}) \times (5.19615\mathbf{i} - 5.19615\mathbf{j} - 3\mathbf{k}) + \mathbf{0} + \mathbf{0}$$

$$\mathbf{a}_A = 24.9904\mathbf{i} - 26.7846\mathbf{j} + 8.7846\mathbf{k}$$

$$\mathbf{a}_A = \{25\mathbf{i} - 26.8\mathbf{j} + 8.78\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans}$$

20-34. During the instant shown the frame of the X-ray camera is rotating about the vertical axis at $\omega_z = 5$ rad/s and $\dot{\omega}_z = 2$ rad/s^2. Relative to the frame the arm is rotating at $\omega_{\text{rel}} = 2$ rad/s and $\dot{\omega}_{\text{rel}} = 1$ rad/s^2. Determine the velocity and acceleration of the center of the camera C at this instant.





$$\Omega = \{5\mathbf{k}\} \text{ rad/s}$$

$$\dot{\Omega} = \{2\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{r}_B = \{-1.25\mathbf{i}\} \text{ m}$$

$$\omega_B = \mathbf{0} + 5\mathbf{k} \times (-1.25\mathbf{i}) = -6.25\mathbf{j}$$

$$\mathbf{a}_B = \mathbf{0} + 2\mathbf{k} \times (-1.25\mathbf{i}) + \mathbf{0} + 5\mathbf{k} \times (-6.25\mathbf{j})$$

$$= 31.25\mathbf{i} - 2.5\mathbf{j}$$

$$\Omega_{xyz} = \{2\mathbf{j}\} \text{ rad/s}$$

$$\dot{\Omega}_{xyz} = \{1\mathbf{j}\} \text{ rad/s}^2$$

$$\mathbf{r}_{C/B} = \{1.75\mathbf{j} + 1\mathbf{k}\} \text{ m}$$

$$(\mathbf{v}_{C/B})_{xyz} = \dot{\mathbf{r}}_{C/B} = \mathbf{0} + (2\mathbf{j}) \times (1.75\mathbf{j} + 1\mathbf{k}) = 2\mathbf{i}$$

$$(\mathbf{a}_{C/B})_{xyz} = \ddot{\mathbf{r}}_{C/B} = \mathbf{0} + (1\mathbf{j}) \times (1.75\mathbf{j} + 1\mathbf{k}) + \mathbf{0} + (2\mathbf{j}) \times (2\mathbf{i})$$

$$= 1\mathbf{i} - 4\mathbf{k}$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/A})_{xyz}$$

$$\mathbf{v}_C = -6.25\mathbf{j} + 5\mathbf{k} \times (1.75\mathbf{j} + 1\mathbf{k}) + 2\mathbf{i}$$

$$\mathbf{v}_C = \{-6.75\mathbf{i} - 6.25\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans}$$

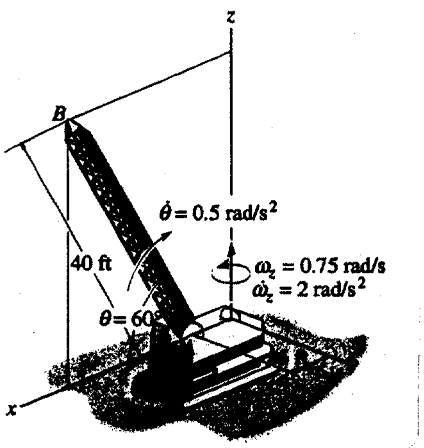
$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2(\Omega \times (\mathbf{v}_{C/B})_{xyz}) + (a_{C/B})_{xyz}$$

$$= (31.25\mathbf{i} - 2.5\mathbf{j}) + (2\mathbf{k}) \times (1.75\mathbf{j} + 1\mathbf{k}) + 5\mathbf{k}[(5\mathbf{k}) \times (1.75\mathbf{j} + 1\mathbf{k})]$$

$$+ 2(5\mathbf{k}) \times (2\mathbf{i}) + (1\mathbf{i} - 4\mathbf{k})$$

$$\mathbf{a}_C = \{28.75\mathbf{i} - 26.25\mathbf{j} - 4\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

20-35. The boom AB of the crane is rotating about the z axis with an angular velocity $\omega_z = 0.75$ rad/s, which is increasing at $\dot{\omega}_z = 2$ rad/s^2. At the same instant, $\theta = 60°$ and the boom is rotating upward at a constant rate $\dot{\theta} = 0.5$ rad/s^2. Determine the velocity and acceleration of the tip B of the boom at this instant.



Motion of moving reference:

$\mathbf{\Omega} = \omega_z = \{0.75\mathbf{k}\}$ rad/s

$\dot{\mathbf{\Omega}} = (\dot{\omega})_{xyz} = \{2\mathbf{k}\}$ rad/s^2

$\mathbf{r}_O = \{5\mathbf{i}\}$ ft

$\mathbf{v}_O = \dot{\mathbf{r}}_O = (\dot{\mathbf{r}}_O)_{xyz} + \mathbf{\Omega} \times \mathbf{r}_O$

$= 0 + (0.75\mathbf{k}) \times (5\mathbf{i})$

$= \{3.75\mathbf{j}\}$ ft/s

$\mathbf{a}_O = \ddot{\mathbf{r}}_O = [(\ddot{\mathbf{r}}_O)_{xyz} + \mathbf{\Omega} \times (\dot{\mathbf{r}}_O)_{xyz}] + \dot{\mathbf{\Omega}} \times \mathbf{r}_O + \mathbf{\Omega} \times \dot{\mathbf{r}}_O$

$= 0 + 0 + (2\mathbf{k}) \times (5\mathbf{i}) + (0.75\mathbf{k}) \times (3.75\mathbf{j})$

$= \{-2.8125\mathbf{i} + 10\mathbf{j}\}$ ft/s^2

Motion of B with respect to moving reference:

$\mathbf{\Omega}_{B/O} = \{-0.5\mathbf{j}\}$ rad/s

$\dot{\mathbf{\Omega}}_{B/O} = 0$

$\mathbf{r}_{B/O} = 40\cos 60°\mathbf{i} + 40\sin 60°\mathbf{k} = \{20\mathbf{i} + 34.64\mathbf{k}\}$ ft

$(\mathbf{v}_{B/O})_{xyz} = \dot{\mathbf{r}}_{B/O} = (\dot{\mathbf{r}}_{B/O})_{xyz} + \mathbf{\Omega}_{B/O} \times \mathbf{r}_{B/O}$

$= 0 + (-0.5\mathbf{j}) \times (20\mathbf{i} + 34.64\mathbf{k})$

$= \{-17.32\mathbf{i} + 10\mathbf{k}\}$ ft/s

$(\mathbf{a}_{B/O})_{xyz} = \ddot{\mathbf{r}}_{B/O} = [(\ddot{\mathbf{r}}_{B/O})_{xyz} + \mathbf{\Omega}_{B/O} \times (\dot{\mathbf{r}}_{B/O})_{xyz}] + \dot{\mathbf{\Omega}}_{B/O} \times \mathbf{r}_{B/O} + \mathbf{\Omega}_{B/O} \times \dot{\mathbf{r}}_{B/O}$

$= 0 + 0 + 0 + (-0.5\mathbf{j}) \times (-17.32\mathbf{i} + 10\mathbf{k})$

$= \{-5\mathbf{i} - 8.66\mathbf{k}\}$ ft/s^2

$\mathbf{v}_B = \mathbf{v}_O + \mathbf{\Omega} \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz}$

$= (3.75\mathbf{j}) + (0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (-17.32\mathbf{i} + 10\mathbf{k})$

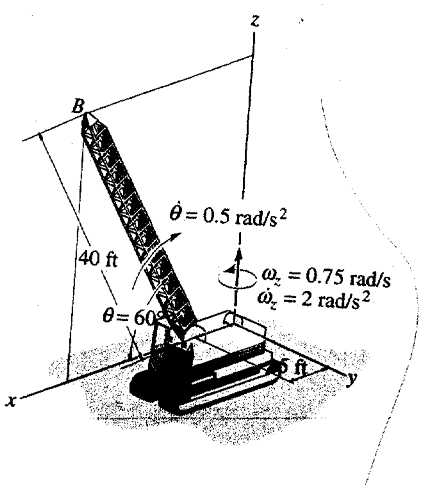
$\mathbf{v}_B = \{-17.3\mathbf{i} + 18.8\mathbf{j} + 10.0\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_B = \mathbf{a}_O + \dot{\mathbf{\Omega}} \times \mathbf{r}_{B/O} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{B/O}) + 2\mathbf{\Omega} \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz}$

$= (-2.8125\mathbf{i} + 10\mathbf{j}) + (2\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (0.75\mathbf{k}) \times [(0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k})]$
$+ 2(0.75\mathbf{k}) \times (-17.32\mathbf{i} + 10\mathbf{k}) + (-5\mathbf{i} - 8.66\mathbf{j})$

$\mathbf{a}_B = \{-19.1\mathbf{i} + 24.0\mathbf{j} - 8.66\mathbf{k}\}$ ft/s^2 **Ans**

*20-36. The boom AB of the crane is rotating about the z axis with an angular velocity of $\omega_z = 0.75$ rad/s, which is increasing at $\dot{\omega}_z = 2$ rad/s^2. At the same instant, $\theta = 60°$ and the boom is rotating upward at $\dot{\theta} = 0.5$ rad/s^2, which is increasing at $\ddot{\theta} = 0.75$ rad/s^2. Determine the velocity and acceleration of the tip B of the boom at this instant.



Motion of moving reference:

$$\Omega = \omega_z = \{0.75\mathbf{k}\}\ \text{rad/s}$$

$$\dot{\Omega} = (\dot{\omega})_{xyz} = \{2\mathbf{k}\}\ \text{rad/s}^2$$

$$\mathbf{r}_O = \{5\mathbf{i}\}\ \text{ft}$$

$$\mathbf{v}_O = \dot{\mathbf{r}}_O = (\dot{\mathbf{r}}_O)_{xyz} + \Omega \times \mathbf{r}_O$$
$$= 0 + (0.75\mathbf{k}) \times (5\mathbf{i})$$
$$= \{3.75\mathbf{j}\}\ \text{ft/s}$$

$$\mathbf{a}_O = \ddot{\mathbf{r}}_O = [(\ddot{\mathbf{r}}_O)_{xyz} + \Omega \times (\dot{\mathbf{r}}_O)_{xyz}] + \dot{\Omega} \times \mathbf{r}_O + \Omega \times \dot{\mathbf{r}}_O$$
$$= 0 + 0 + (2\mathbf{k}) \times (5\mathbf{i}) + (0.75\mathbf{k}) \times (3.75\mathbf{j})$$
$$= \{-2.8125\mathbf{i} + 10\mathbf{j}\}\ \text{ft/s}^2$$

Motion of B with respect to moving reference:

$$\Omega_{B/O} = \{-0.5\mathbf{j}\}\ \text{rad/s}$$

$$\dot{\Omega}_{B/O} = \{-0.75\mathbf{j}\}\ \text{rad/s}^2$$

$$\mathbf{r}_{B/O} = 40\cos 60°\mathbf{i} + 40\sin 60°\mathbf{k} = \{20\mathbf{i} + 34.64\mathbf{k}\}\ \text{ft}$$

$$(\mathbf{v}_{B/O})_{xyz} = \dot{\mathbf{r}}_{B/O} = (\dot{\mathbf{r}}_{B/O})_{xyz} + \Omega_{B/O} \times \mathbf{r}_{B/O}$$
$$= 0 + (-0.5\mathbf{j}) \times (20\mathbf{i} + 34.64\mathbf{k})$$
$$= \{-17.32\mathbf{i} + 10\mathbf{k}\}\ \text{ft/s}$$

$$(\mathbf{a}_{B/O})_{xyz} = \ddot{\mathbf{r}}_{B/O} = [(\ddot{\mathbf{r}}_{B/O})_{xyz} + \Omega_{B/O} \times (\dot{\mathbf{r}}_{B/O})_{xyz}] + \dot{\Omega}_{B/O} \times \mathbf{r}_{B/O} + \Omega_{B/O} \times \dot{\mathbf{r}}_{B/O}$$
$$= 0 + 0 + (-0.75\mathbf{j}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (-0.5\mathbf{j}) \times (-17.32\mathbf{i} + 10\mathbf{k})$$
$$= \{-30.98\mathbf{i} + 6.34\mathbf{k}\}\ \text{ft/s}^2$$

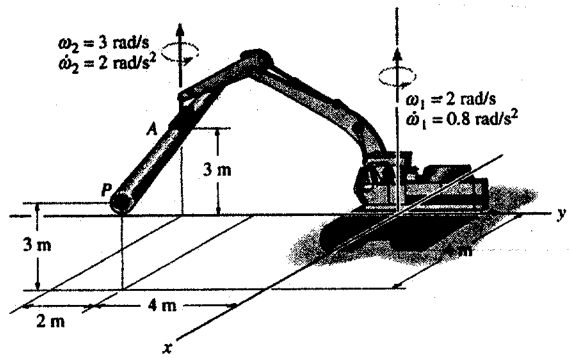
$$\mathbf{v}_B = \mathbf{v}_O + \Omega \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz}$$
$$= (3.75\mathbf{j}) + (0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (-17.32\mathbf{i} + 10\mathbf{k})$$

$$\mathbf{v}_B = \{-17.3\mathbf{i} + 18.8\mathbf{j} + 10.0\mathbf{k}\}\ \text{ft/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz}$$
$$= (-2.8125\mathbf{i} + 10\mathbf{j}) + (2\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (0.75\mathbf{k}) \times [(0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k})]$$
$$+ 2(0.75\mathbf{k}) \times (-17.32\mathbf{i} + 10\mathbf{k}) + (-30.98\mathbf{i} + 6.34\mathbf{j})$$

$$\mathbf{a}_B = \{-45.0\mathbf{i} + 24.0\mathbf{j} + 6.34\mathbf{k}\}\ \text{ft/s}^2 \qquad \textbf{Ans}$$

20-37. At the instant shown, the boom is rotating about the z axis with an angular velocity $\omega_1 = 2$ rad/s and angular acceleration $\dot{\omega}_1 = 0.8$ rad/s^2. At this same instant the swivel is rotating at $\omega_2 = 3$ rad/s when $\dot{\omega}_2 = 2$ rad/s^2, both measured relative to the boom. Determine the velocity and acceleration of point P on the pipe at this instant.



Relative to XYZ, let xyz have

$\Omega = \{2\mathbf{k}\}$ rad/s $\quad \dot{\Omega} = \{0.8\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_A = \{-6\mathbf{j} + 3\mathbf{k}\}$ m ($\mathbf{r}_A$ changes direction relative to XYZ.)

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = (\dot{\mathbf{r}}_A)_{xyz} + \Omega \times \mathbf{r}_A = 0 + (2\mathbf{k}) \times (-6\mathbf{j} + 3\mathbf{k}) = \{12\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_A = \ddot{\mathbf{r}}_A = \left[(\ddot{\mathbf{r}}_A)_{xyz} + \Omega \times (\dot{\mathbf{r}}_A)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_A + \Omega \times \dot{\mathbf{r}}_A$$

$$= 0 + 0 + (0.8\mathbf{k}) \times (-6\mathbf{j} + 3\mathbf{k}) + (2\mathbf{k}) \times (12\mathbf{i})$$

$$= \{4.8\mathbf{i} + 24\mathbf{j}\} \text{ m/s}^2$$

Relative to xyz, let $x'y'z'$ have the origin at A and

$\Omega_{xyz} = \{3\mathbf{k}\}$ rad/s $\quad \dot{\Omega}_{xyz} = \{2\mathbf{k}\}$ rad/s^2 (Ω_{xyz} does not change direction relative to xyz.)

$(\mathbf{r}_{P/A})_{xyz} = \{4\mathbf{i} + 2\mathbf{j}\}$ m ($(\mathbf{r}_{P/A})_{xyz}$ changes direction relative to xyz.)

$$(\mathbf{v}_{P/A})_{xyz} = (\dot{\mathbf{r}}_{P/A})_{xyz} = (\dot{\mathbf{r}}_{P/A})_{x'y'z'} + \Omega_{xyz} \times (\mathbf{r}_{P/A})_{xyz}$$

$$= 0 + (3\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})$$

$$= \{-6\mathbf{i} + 12\mathbf{j}\} \text{ m/s}$$

$$(\mathbf{a}_{P/A})_{xyz} = (\ddot{\mathbf{r}}_{P/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{P/A})_{x'y'z'} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/A})_{x'y'z'}\right] + \left[\dot{\Omega}_{xyz} \times (\mathbf{r}_{P/A})_{xyz}\right] + \left[\Omega_{xyz} \times (\dot{\mathbf{r}}_{P/A})_{xyz}\right]$$

$$= 0 + 0 + \left[(2\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})\right] + \left[(3\mathbf{k}) \times (-6\mathbf{i} + 12\mathbf{j})\right]$$

$$= \{-40\mathbf{i} - 10\mathbf{j}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_P = \mathbf{v}_A + \Omega \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz}$$

$$= 12\mathbf{i} + \left[(2\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})\right] + (-6\mathbf{i} + 12\mathbf{j})$$

$$= \{2\mathbf{i} + 20\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans}$$

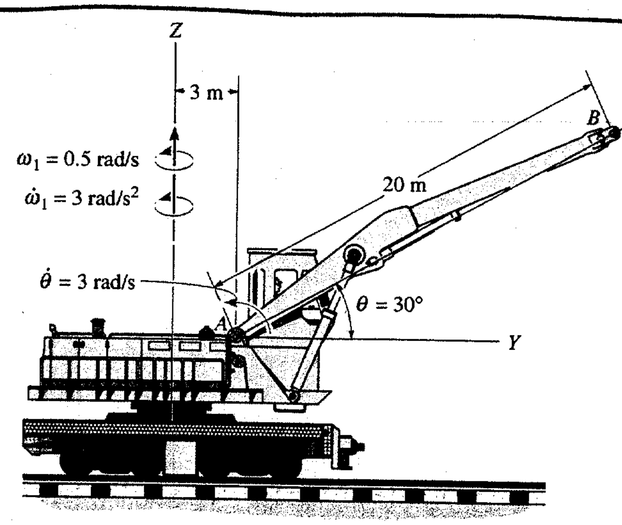
$$\mathbf{a}_P = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{P/A} + \Omega \times (\Omega \times \mathbf{r}_{P/A}) + 2\Omega \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$

$$= (4.8\mathbf{i} + 24\mathbf{j}) + \left[(0.8\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})\right] + 2\mathbf{k} \times \left[2\mathbf{k} \times (4\mathbf{i} + 2\mathbf{j})\right]$$

$$+ \left[2(2\mathbf{k}) \times (-6\mathbf{i} + 12\mathbf{j})\right] + (-40\mathbf{i} - 10\mathbf{j})$$

$$= \{-101\mathbf{i} - 14.8\mathbf{j}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

20-38. The boom AB of the locomotive crane is rotating about the Z axis with an angular velocity $\omega_1 = 0.5$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s². At this same instant, $\theta = 30°$ and the boom is rotating upward at a constant rate of $\dot{\theta} = 3$ rad/s. Determine the velocity and acceleration of the tip B of the boom at this instant.



$\mathbf{\Omega} = \{0.5\mathbf{k}\}$ rad/s $\quad \dot{\mathbf{\Omega}} = \{3\mathbf{k}\}$ rad/s² $\quad \mathbf{r}_A = \{3\mathbf{j}\}$ m

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = (\dot{\mathbf{r}}_A)_{xyz} + \mathbf{\Omega} \times \mathbf{r}_A = 0 + (0.5\mathbf{k}) \times (3\mathbf{j}) = \{-1.5\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_A = \ddot{\mathbf{r}}_A = [(\ddot{\mathbf{r}}_A)_{xyz} + \mathbf{\Omega} \times (\dot{\mathbf{r}}_A)_{xyz}] + \dot{\mathbf{\Omega}} \times \mathbf{r}_A + \mathbf{\Omega} \times \dot{\mathbf{r}}_A$$

$$= 0 + 0 + (3\mathbf{k}) \times (3\mathbf{j}) + (0.5\mathbf{k}) \times (-1.5\mathbf{i})$$

$$= \{-9\mathbf{i} - 0.75\mathbf{j}\} \text{ m/s}^2$$

$\mathbf{\Omega}_{xyz} = \{3\mathbf{i}\}$ rad/s $\quad \dot{\mathbf{\Omega}}_{xyz} = 0$

$$\mathbf{r}_{B/A} = 20\cos 30°\mathbf{j} + 20\sin 30° = \{17.32\mathbf{j} + 10\mathbf{k}\} \text{ m}$$

$$(\mathbf{v}_{B/A})_{xyz} = \dot{\mathbf{r}}_{B/A} = (\dot{\mathbf{r}}_{B/A})_{xyz} + \mathbf{\Omega}_{xyz} \times \mathbf{r}_{B/A}$$

$$= 0 + (3\mathbf{i}) \times (17.32\mathbf{j} + 10\mathbf{k})$$

$$= \{-30\mathbf{j} + 51.96\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{B/A})_{xyz} = \ddot{\mathbf{r}}_{B/A} = [(\ddot{\mathbf{r}}_{B/A})_{xyz} + \mathbf{\Omega}_{xyz} \times (\dot{\mathbf{r}}_{B/A})_{xyz}] + [\dot{\mathbf{\Omega}}_{xyz} \times \mathbf{r}_{B/A}] + [\mathbf{\Omega}_{xyz} \times \dot{\mathbf{r}}_{B/A}]$$

$$(\mathbf{a}_{B/A})_{xyz} = 0 + 0 + 0 + [(3\mathbf{i}) \times (-30\mathbf{j} + 51.96\mathbf{k})] = \{-155.88\mathbf{j} - 90\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= -1.5\mathbf{i} + [(0.5\mathbf{k}) \times (17.32\mathbf{j} + 10\mathbf{k})] + (-30\mathbf{j} + 51.96\mathbf{k})$$

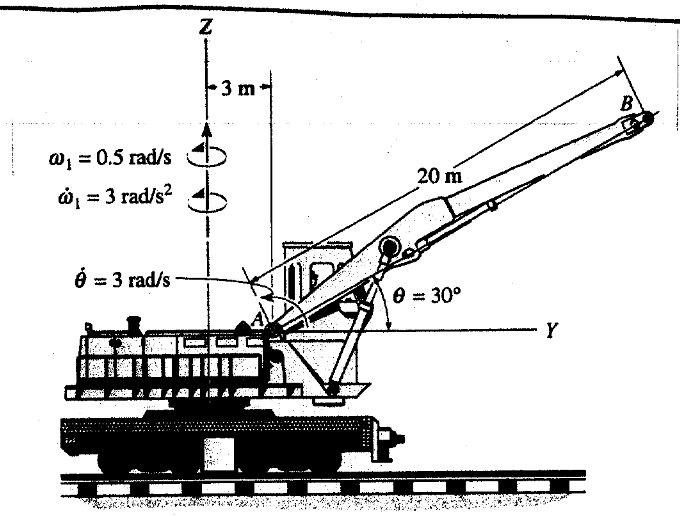
$$= \{-10.2\mathbf{i} - 30\mathbf{j} + 52.0\mathbf{k}\} \text{ m/s}$$ **Ans**

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\mathbf{\Omega}} \times \mathbf{r}_{B/A} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{B/A}) + 2\mathbf{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (-9\mathbf{i} - 0.75\mathbf{j}) + [(3\mathbf{k}) \times (17.32\mathbf{j} + 10\mathbf{k})] + 0.5\mathbf{k} \times [0.5\mathbf{k} \times (17.32\mathbf{j} + 10\mathbf{k})] + [2(0.5\mathbf{k}) \times (-30\mathbf{j} + 51.96\mathbf{k})] + (-155.88\mathbf{j} - 90\mathbf{k})$$

$$= \{-31.0\mathbf{i} - 161\mathbf{j} - 90\mathbf{k}\} \text{ m/s}^2$$ **Ans**

20-39. The locomotive crane is traveling to the right at 2 m/s and has an acceleration of 1.5 m/s², while the boom is rotating about the Z axis with an angular velocity $\omega_1 = 0.5$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s². At this same instant, $\theta = 30°$ and the boom is rotating upward at a constant rate $\dot{\theta} = 3$ rad/s. Determine the velocity and acceleration of the tip B of the boom at this instant.



$\boldsymbol{\Omega} = \{0.5\mathbf{k}\}$ rad/s $\quad \boldsymbol{\Omega} = \{3\mathbf{k}\}$ rad/s² $\quad \mathbf{r}_A = \{3\mathbf{j}\}$ m

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = (\dot{\mathbf{r}}_A)_{xyz} + \boldsymbol{\Omega}\times\mathbf{r}_A = 2\mathbf{j} + (0.5\mathbf{k})\times(3\mathbf{j}) = \{-1.5\mathbf{i} + 2\mathbf{j}\}\ \text{m/s}$$

$$\mathbf{a}_A = \ddot{\mathbf{r}}_A = [(\ddot{\mathbf{r}}_A)_{xyz} + \boldsymbol{\Omega}\times(\dot{\mathbf{r}}_A)_{xyz}] + \dot{\boldsymbol{\Omega}}\times\mathbf{r}_A + \boldsymbol{\Omega}\times\dot{\mathbf{r}}_A$$

$$= 1.5\mathbf{j} + (0.5\mathbf{k})\times(2\mathbf{j}) + (3\mathbf{k})\times(3\mathbf{j}) + (0.5\mathbf{k})\times(-1.5\mathbf{i} + 2\mathbf{j})$$

$$= \{-11\mathbf{i} + 0.75\mathbf{j}\}\ \text{m/s}^2$$

$\boldsymbol{\Omega}_{xyz} = \{3\mathbf{i}\}$ rad/s $\quad \dot{\boldsymbol{\Omega}}_{xyz} = 0$

$$\mathbf{r}_{B/A} = 20\cos30°\mathbf{j} + 20\sin30° = \{17.32\mathbf{j} + 10\mathbf{k}\}\ \text{m}$$

$$(\mathbf{v}_{B/A})_{xyz} = \dot{\mathbf{r}}_{B/A} = (\dot{\mathbf{r}}_{B/A})_{xyz} + \boldsymbol{\Omega}_{xyz}\times\mathbf{r}_{B/A}$$

$$= 0 + (3\mathbf{i})\times(17.32\mathbf{j} + 10\mathbf{k})$$

$$= \{-30\mathbf{j} + 51.96\mathbf{k}\}\ \text{m/s}$$

$$(\mathbf{a}_{B/A})_{xyz} = \ddot{\mathbf{r}}_{B/A} = [(\ddot{\mathbf{r}}_{B/A})_{xyz} + \boldsymbol{\Omega}_{xyz}\times(\dot{\mathbf{r}}_{B/A})_{xyz}] + [\dot{\boldsymbol{\Omega}}_{xyz}\times\mathbf{r}_{B/A}] + [\boldsymbol{\Omega}_{xyz}\times\dot{\mathbf{r}}_{B/A}]$$

$$(\mathbf{a}_{B/A})_{xyz} = 0 + 0 + 0 + [(3\mathbf{i})\times(-30\mathbf{j} + 51.96\mathbf{k})] = \{-155.88\mathbf{j} - 90\mathbf{k}\}\ \text{m/s}^2$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega}\times\mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= -1.5\mathbf{i} + 2\mathbf{j} + [(0.5\mathbf{k})\times(17.32\mathbf{j} + 10\mathbf{k})] + (-30\mathbf{j} + 51.96\mathbf{k})$$

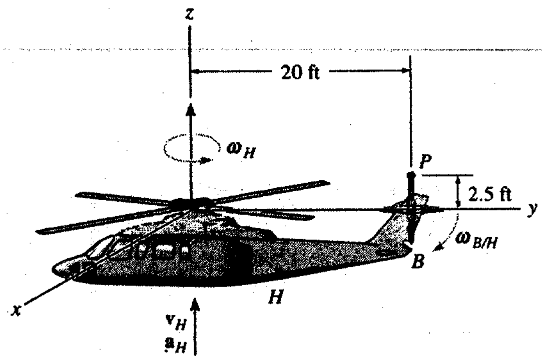
$$= \{-10.2\mathbf{i} - 28\mathbf{j} + 52.0\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}}\times\mathbf{r}_{B/A} + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{B/A}) + 2\boldsymbol{\Omega}\times(\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (-11\mathbf{i} + 0.75\mathbf{j}) + [(3\mathbf{k})\times(17.32\mathbf{j} + 10\mathbf{k})] + 0.5\mathbf{k}\times[0.5\mathbf{k}\times(17.32\mathbf{j} + 10\mathbf{k})] + [2(0.5\mathbf{k})\times(-30\mathbf{j} + 51.96\mathbf{k})] + (-155.88\mathbf{j} - 90\mathbf{k})$$

$$= \{-33.0\mathbf{i} - 159\mathbf{j} - 90\mathbf{k}\}\ \text{m/s}^2 \qquad \textbf{Ans}$$

*20-40. At the instant shown, the helicopter is moving upwards with a velocity $v_H = 4$ ft/s and has an acceleration $a_H = 2$ ft/s². At the same instant the frame H, *not* the horizontal blade, is rotating about a vertical axis with a constant angular velocity $\omega_H = 0.9$ rad/s. If the tail blade B is rotating with a constant angular velocity $\omega_{B/H} = 180$ rad/s, measured relative to H, determine the velocity and acceleration of point P, located on the tip of the blade, at the instant the blade is in the vertical position.



Relative to XYZ, let xyz have

$\Omega = \{0.9\mathbf{k}\}$ rad/s $\quad \dot{\Omega} = \mathbf{0}$ (Ω does not change direction relative to XYZ.)

$\mathbf{r}_B = \{20\mathbf{j}\}$ ft ($\mathbf{r}_B$ changes direction relative to XYZ.)

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \Omega \times \mathbf{r}_B = 4\mathbf{k} + (0.9\mathbf{k}) \times (20\mathbf{j}) = \{-18\mathbf{i} + 4\mathbf{k}\} \text{ ft/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times \left(\mathbf{r}_B\right)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$

$$= \left[2\mathbf{k} + \mathbf{0}\right] + \mathbf{0} + \left[(0.9\mathbf{k}) \times (-18\mathbf{i} + 4\mathbf{k})\right]$$

$$= \{-16.2\mathbf{j} + 2\mathbf{k}\} \text{ ft/s}^2$$

Relative to xyz, let $x'y'z'$ have

$\Omega_{xyz} = \{-180\mathbf{i}\}$ rad/s $\quad \dot{\Omega}_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to xyz.)

$(\mathbf{r}_{P/B})_{xyz} = \{2.5\mathbf{k}\}$ ft ($(\mathbf{r}_{P/B})_{xyz}$ changes direction relative to xyz.)

$$(\mathbf{v}_{P/B})_{xyz} = (\dot{\mathbf{r}}_{P/B})_{xyz} = (\dot{\mathbf{r}}_{P/B})_{x'y'z'} + \Omega_{xyz} \times (\mathbf{r}_{P/B})_{xyz} = \mathbf{0} + (-180\mathbf{i}) \times (2.5\mathbf{k}) = \{450\mathbf{j}\} \text{ ft/s}$$

$$(\mathbf{a}_{P/B})_{xyz} = (\ddot{\mathbf{r}}_{P/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{P/B})_{x'y'z'} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/B})_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times (\mathbf{r}_{P/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/B})_{xyz}$$

$$(\mathbf{a}_{B/A})_{xyz} = \left[\mathbf{0} + \mathbf{0}\right] + \mathbf{0} + (-180\mathbf{i}) \times (450\mathbf{j}) = \{-81\,000\mathbf{k}\} \text{ ft/s}^2$$

Thus,

$$\mathbf{v}_P = \mathbf{v}_B + \Omega \times \mathbf{r}_{P/B} + (\mathbf{v}_{P/B})_{xyz}$$

$$= (-18\mathbf{i} + 4\mathbf{k}) + \left[(0.9\mathbf{k}) \times (2.5\mathbf{k})\right] + (450\mathbf{j})$$

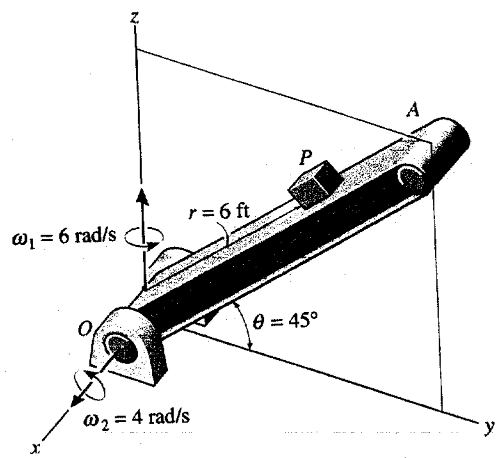
$$= \{-18\mathbf{i} + 450\mathbf{j} + 4\mathbf{k}\} \text{ ft/s}$$ **Ans**

$$\mathbf{a}_P = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{P/B} + \Omega \times (\Omega \times \mathbf{r}_{P/B}) + 2\Omega \times (\mathbf{v}_{P/B})_{xyz} + (\mathbf{a}_{P/B})_{xyz}$$

$$= (-16.2\mathbf{j} + 2\mathbf{k}) + \mathbf{0} + (0.9\mathbf{k}) \times \left[(0.9\mathbf{k}) \times (2.5\mathbf{k})\right] + \left[2(0.9\mathbf{k}) \times (450\mathbf{j})\right] + (-81\,000\mathbf{k})$$

$$= \{-810\mathbf{i} - 16.2\mathbf{j} - 81\,000\mathbf{k}\} \text{ ft/s}^2$$ **Ans**

20-41. At the instant shown, the arm OA of the conveyor belt is rotating about the z axis with a constant angular velocity $\omega_1 = 6$ rad/s, while at the same instant the arm is rotating upward at a constant rate $\omega_2 = 4$ rad/s. If the conveyor is running at a constant rate $\dot{r} = 5$ ft/s, determine the velocity and acceleration of the package P at the instant shown. Neglect the size of the package.



$\mathbf{\Omega} = \omega_1 = \{6\mathbf{k}\}$ rad/s

$\dot{\mathbf{\Omega}} = 0$

$\mathbf{r}_O = \mathbf{v}_O = \mathbf{a}_O = 0$

$\mathbf{\Omega}_{P/O} = \{4\mathbf{i}\}$ rad/s

$\dot{\mathbf{\Omega}}_{P/O} = 0$

$\mathbf{r}_{P/O} = \{4.243\mathbf{j} + 4.243\mathbf{k}\}$ ft

$(\mathbf{v}_{P/O})_{xyz} = (\dot{\mathbf{r}}_{P/O})_{xyz} + \mathbf{\Omega}_{P/O} \times \mathbf{r}_{P/O}$

$= (5\cos 45°\mathbf{j} + 5\sin 45°\mathbf{k}) + (4\mathbf{i}) \times (4.243\mathbf{j} + 4.243\mathbf{k})$

$= \{-13.44\mathbf{j} + 20.51\mathbf{k}\}$ ft/s

$(\mathbf{a}_{P/O}) = (\ddot{\mathbf{r}}_{P/O})_{xyz} + \mathbf{\Omega}_{P/O} \times (\dot{\mathbf{r}}_{P/O})_{xyz} + \dot{\mathbf{\Omega}}_{P/O} \times \mathbf{r}_{P/O} + \mathbf{\Omega}_{P/O} \times \dot{\mathbf{r}}_{P/O}$

$= 0 + (4\mathbf{i}) \times (3.536\mathbf{j} + 3.536\mathbf{k}) + 0 + (4\mathbf{i}) \times (-13.44\mathbf{j} + 20.51\mathbf{k})$

$= \{-96.18\mathbf{j} - 39.60\mathbf{k}\}$ ft/s^2

$\mathbf{v}_P = \mathbf{v}_O + \mathbf{\Omega} \times \mathbf{r}_{P/O} + (\mathbf{v}_{P/O})_{xyz}$

$= 0 + (6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k}) + (-13.44\mathbf{j} + 20.51\mathbf{k})$

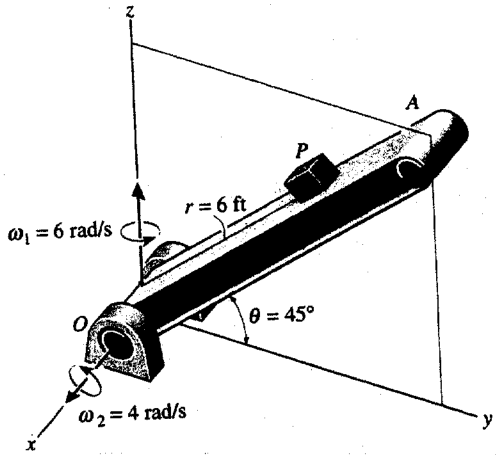
$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}$ ft/s **Ans**

$\mathbf{a}_P = \mathbf{a}_O + \dot{\mathbf{\Omega}} \times \mathbf{r}_{P/O} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{P/O}) + 2\mathbf{\Omega} \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$

$= 0 + 0 + (6\mathbf{k}) \times [(6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k})] + 2(6\mathbf{k}) \times (-13.44\mathbf{j} + 20.51\mathbf{k}) + (-96.18\mathbf{j} - 39.60\mathbf{k})$

$\mathbf{a}_P = \{161\mathbf{i} - 249\mathbf{j} - 39.6\mathbf{k}\}$ ft/s^2 **Ans**

20-42. At the instant shown, the arm OA of the conveyor belt is rotating about the z axis with a constant angular velocity $\omega_1 = 6$ rad/s, while at the same instant the arm is rotating upward at a constant rate $\omega_2 = 4$ rad/s. If the conveyor is running at a rate $\dot{r} = 5$ ft/s, which is increasing at $\ddot{r} = 8$ ft/s^2, determine the velocity and acceleration of the package P at the instant shown. Neglect the size of the package.



$\boldsymbol{\Omega} = \omega_1 = \{6\mathbf{k}\}$ rad/s

$\dot{\boldsymbol{\Omega}} = 0$

$\mathbf{r}_O = \mathbf{v}_O = \mathbf{a}_O = 0$

$\boldsymbol{\Omega}_{P/O} = \{4\mathbf{i}\}$ rad/s

$\dot{\boldsymbol{\Omega}}_{P/O} = 0$

$\mathbf{r}_{P/O} = \{4.243\mathbf{j} + 4.243\mathbf{k}\}$ ft

$(\mathbf{v}_{P/O})_{xyz} = (\dot{\mathbf{r}}_{P/O})_{xyz} + \boldsymbol{\Omega}_{P/O} \times \mathbf{r}_{P/O}$

$= (5\cos45°\mathbf{j} + 5\sin45°\mathbf{k}) + (4\mathbf{i}) \times (4.243\mathbf{j} + 4.243\mathbf{k})$

$= \{-13.44\mathbf{j} + 20.51\mathbf{k}\}$ ft/s

$(\mathbf{a}_{P/O})_{xyz} = 8\cos45°\mathbf{j} + 8\sin45°\mathbf{k} - 96.18\mathbf{j} - 39.60\mathbf{k}$

$= \{-90.52\mathbf{j} - 33.945\mathbf{k}\}$ ft/s^2

$\mathbf{v}_P = \mathbf{v}_O + \boldsymbol{\Omega} \times \mathbf{r}_{P/O} + (\mathbf{v}_{P/O})_{xyz}$

$= 0 + (6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k}) + (-13.44\mathbf{j} + 20.51\mathbf{k})$

$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}$ ft/s **Ans**

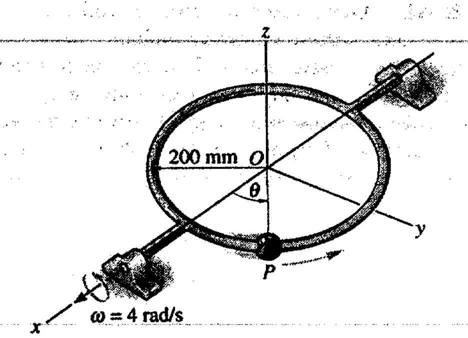
$\mathbf{a}_P = \mathbf{a}_O + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{P/O} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{P/O}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$

$= 0 + 0 + (6\mathbf{k}) \times [(6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k})] + 2(6\mathbf{k}) \times (-13.44\mathbf{j} + 20.51\mathbf{k}) + (-90.52\mathbf{j} - 33.945\mathbf{k})$

$= -152.75\mathbf{j} + 161.23\mathbf{i} - 90.52\mathbf{j} - 33.945\mathbf{k}$

$\mathbf{a}_P = \{161\mathbf{i} - 243\mathbf{j} - 33.9\mathbf{k}\}$ ft/s^2 **Ans**

20-43. The particle P slides around the circular hoop with a constant angular velocity of $\dot{\theta} = 6$ rad/s, while the hoop rotates about the x axis at a constant rate of $\omega = 4$ rad/s. If at the instant shown the hoop is in the x–y plane and the angle $\theta = 45°$, determine the velocity and acceleration of the particle at this instant.



Relative to XYZ, let xyz have

$\Omega = \omega = \{4\mathbf{i}\}$ rad/s, $\quad \dot{\Omega} = \dot{\omega} = \mathbf{0}$ (Ω does not change direction relative to XYZ.)

$\mathbf{r}_O = \mathbf{0}$; $\quad \mathbf{v}_O = \mathbf{0}$; $\quad \mathbf{a}_O = \mathbf{0}$

Relative to xyz, let coincident $x'y'z'$ have

$\Omega_{xyz} = \{6\mathbf{k}\}$ rad/s, $\quad \dot{\Omega}_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to XYZ.)

$(\mathbf{r}_{P/O})_{xyz} = 0.2\cos 45°\mathbf{i} + 0.2\sin 45°\mathbf{j} = \{0.1414\mathbf{i} + 0.1414\mathbf{j}\}$ m $\quad$ ($(\mathbf{r}_{P/O})_{xyz}$ changes direction relative to XYZ.)

$$(\mathbf{v}_{P/O})_{xyz} = \left(\dot{\mathbf{r}}_{P/O}\right)_{xyz} = \left(\dot{\mathbf{r}}_{P/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{P/O}\right)_{xyz} = 0 + (6\mathbf{k}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j})$$

$$= \{-0.8485\mathbf{i} + 0.8485\mathbf{j}\} \text{ m/s}$$

$$(\mathbf{a}_{P/O})_{xyz} = \left(\ddot{\mathbf{r}}_{P/O}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{P/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{P/O}\right)_{x'y'z'}\right] + \dot{\Omega} \times \left(\mathbf{r}_{P/O}\right)_{xyz} + \Omega \times \left(\dot{\mathbf{r}}_{P/O}\right)_{xyz}$$

$$= \left[0 + 0\right] + 0 + (6\mathbf{k}) \times (-0.8485\mathbf{i} + 0.8485\mathbf{j}) = \{-5.0912\mathbf{i} - 5.0912\mathbf{j}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{v}_{P/O})_{xyz} = 0 + (4\mathbf{i}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j}) - 0.8485\mathbf{i} + 0.8485\mathbf{j}$$

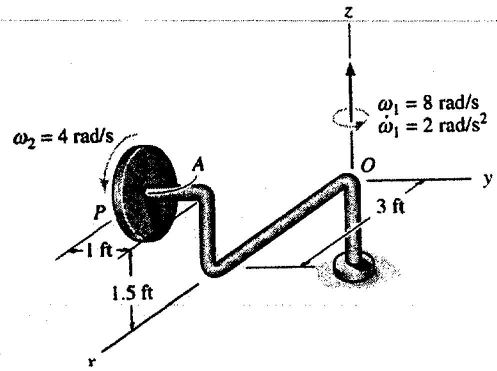
$$= \{-0.849\mathbf{i} + 0.849\mathbf{j} + 0.566\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans}$$

$$\mathbf{a}_P = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{P/O} + \Omega \times \left(\Omega \times \mathbf{r}_{P/O}\right) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$$

$$= 0 + 0 + (4\mathbf{i}) \times \left[(4\mathbf{i}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j})\right] + 2(4\mathbf{i}) \times (-0.8485\mathbf{i} + 0.8485\mathbf{j}) - 5.0912\mathbf{i} - 5.0912\mathbf{j}$$

$$= \{-5.09\mathbf{i} - 7.35\mathbf{j} + 6.79\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

*20-44. At the given instant, the rod is spinning about the z axis with an angular velocity $\omega_1 = 8$ rad/s and angular acceleration $\dot{\omega}_1 = 2$ rad/s^2. At this same instant, the disk is spinning at a constant rate $\omega_2 = 4$ rad/s, measured *relative* to the rod. Determine the velocity and acceleration of point P on the disk at this instant.



Coordinate Axes : The rotating x, y, z frame and fixed X, Y, Z frame are set with the origins at point A and O respectively.

Motion of A : Here, $\mathbf{r}_A$ changes direction with respect to X, Y, Z frame. The time derivatives of $\mathbf{r}_A$ can be found by setting another set of coordinate axis x', y', z' coincident with X, Y, Z rotating at $\Omega' = \omega_1 = \{8\mathbf{k}\}$ rad/s and $\dot{\Omega}' = \dot{\omega}_1 = \{2\mathbf{k}\}$ rad/s^2. Here, $\mathbf{r}_A = \{3\mathbf{i} - 1\mathbf{j} + 1.5\mathbf{k}\}$ ft.

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = (\dot{\mathbf{r}}_A)_{x'y'z'} + \Omega' \times \mathbf{r}_A = \mathbf{0} + 8\mathbf{k} \times (3\mathbf{i} - 1\mathbf{j} + 1.5\mathbf{k}) = \{8\mathbf{i} + 24\mathbf{j}\} \text{ ft/s}$$

$$\mathbf{a}_A = \ddot{\mathbf{r}}_A = \left[(\ddot{\mathbf{r}}_A)_{x'y'z'} + \Omega' \times (\dot{\mathbf{r}}_A)_{x'y'z'}\right] + \dot{\Omega}' \times \mathbf{r}_A + \Omega' \times \dot{\mathbf{r}}_A$$
$$= (\mathbf{0} + \mathbf{0}) + 2\mathbf{k} \times (3\mathbf{i} - 1\mathbf{j} + 1.5\mathbf{k}) + 8\mathbf{k} \times (8\mathbf{i} + 24\mathbf{j}) = \{-190\mathbf{i} + 70\mathbf{j}\} \text{ ft/s}^2$$

Motion of P with Respect to A : Let xyz axis rotate at $\Omega_{xyz} = \omega_2 = \{4\mathbf{j}\}$ rad/s and $\dot{\Omega}_{xyz} = \dot{\omega}_2 = \mathbf{0}$. Here, $\mathbf{r}_{P/A} = \{1\mathbf{i}\}$ ft.

$$(\mathbf{v}_{P/A})_{xyz} = \dot{\mathbf{r}}_{P/A} = (\dot{\mathbf{r}}_{P/A})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{P/A} = \mathbf{0} + (4\mathbf{j}) \times (1\mathbf{i}) = \{-4\mathbf{k}\} \text{ ft/s}$$

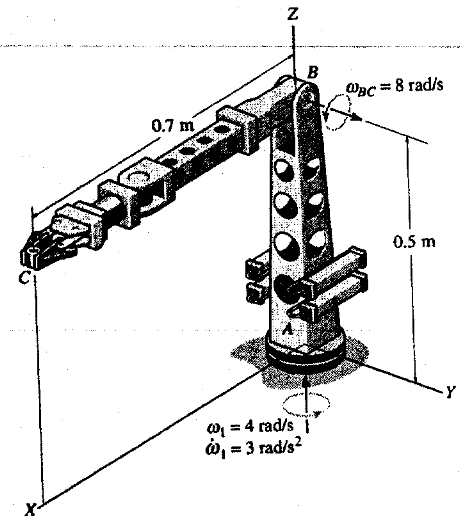
$$(\mathbf{a}_{P/A})_{xyz} = \ddot{\mathbf{r}}_{P/A} = \left[(\ddot{\mathbf{r}}_{P/A})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/A})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{P/A} + \Omega_{xyz} \times \dot{\mathbf{r}}_{P/A}$$
$$= \mathbf{0} + \mathbf{0} + \mathbf{0} + (4\mathbf{j}) \times (-4\mathbf{k}) = \{-16\mathbf{i}\} \text{ ft/s}^2$$

Motion of Point A : Here, $\Omega = \omega_1 = \{8\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \{2\mathbf{k}\}$ rad/s^2. Applying Eqs. 20 – 11 and 20 – 12, we have

$$\mathbf{v}_P = \mathbf{v}_A + \Omega \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz} = (8\mathbf{i} + 24\mathbf{j}) + 8\mathbf{k} \times 1\mathbf{i} + (-4\mathbf{k}) = \{8.00\mathbf{i} + 32.0\mathbf{j} - 4.00\mathbf{k}\} \text{ ft/s} \quad \textbf{Ans}$$

$$\mathbf{a}_P = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{P/A} + \Omega \times (\Omega \times \mathbf{r}_{P/A}) + 2\Omega \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$
$$= (-190\mathbf{i} + 70\mathbf{j}) + 2\mathbf{k} \times 1\mathbf{i} + 8\mathbf{k} \times (8\mathbf{k} \times 1\mathbf{i}) + 2(8\mathbf{k}) \times (-4\mathbf{k}) + (-16\mathbf{i}) = \{-270\mathbf{i} + 72.0\mathbf{j}\} \text{ ft/s}^2 \quad \textbf{Ans}$$

20-45. At the instant shown, the base of the robotic arm is turning about the z axis with an angular velocity of $\omega_1 = 4$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. Also, the boom segment BC is rotating at a constant rate of $\omega_{BC} = 8$ rad/s. Determine the velocity and acceleration of the part C held in its grip at this instant.



Relative to XYZ, let xyz have origin at B and have

$\Omega = \{4\mathbf{k}\}$ rad/s, $\quad \dot{\Omega} = \{3\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_B = \{0.5\mathbf{k}\}$ m ($\mathbf{r}_B$ does not change direction relative to XYZ.)

$\mathbf{v}_B = \mathbf{0}$

$\mathbf{a}_B = \mathbf{0}$

Relative to xyz, let coincident $x'y'z'$ have origin at B and have

$\Omega_{xyz} = \{8\mathbf{j}\}$ rad/s, $\quad \dot{\Omega}_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to xyz.)

$(\mathbf{r}_{C/B})_{xyz} = \{0.7\mathbf{i}\}$ m ($(\mathbf{r}_{C/B})_{xyz}$ changes direction relative to xyz.)

$$(\mathbf{v}_{C/B})_{xyz} = \left(\dot{\mathbf{r}}_{C/B}\right)_{xyz} = \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} = \mathbf{0} + (8\mathbf{j}) \times (0.7\mathbf{i}) = \{-5.6\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left(\ddot{\mathbf{r}}_{C/B}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + \mathbf{0} + (8\mathbf{j}) \times (-5.6\mathbf{k}) = \{-44.8\mathbf{i}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} = \mathbf{0} + (4\mathbf{k}) \times (0.7\mathbf{i}) + (-5.6\mathbf{k})$$

$$= \{2.80\mathbf{j} - 5.60\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans}$$

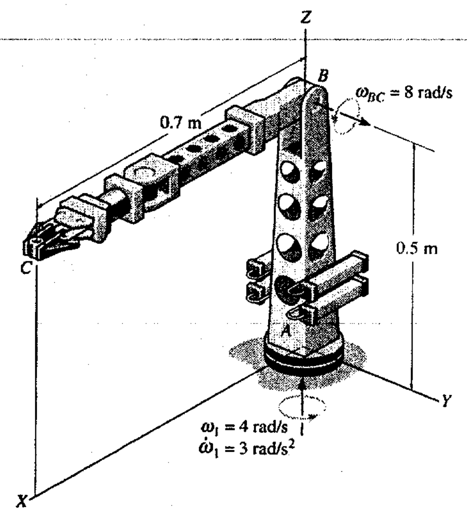
$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times \left(\Omega \times \mathbf{r}_{C/B}\right) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= \mathbf{0} + (3\mathbf{k}) \times (0.7\mathbf{i}) + (4\mathbf{k}) \times \left[(4\mathbf{k}) \times (0.7\mathbf{i})\right]$$

$$+ 2(4\mathbf{k}) \times (-5.6\mathbf{k}) - 44.8\mathbf{i}$$

$$= \{-56\mathbf{i} + 2.1\mathbf{j}\} \text{ m/s}^2 \qquad \textbf{Ans}$$

20-46. At the instant shown, the base of the robotic arm is turning about the z axis with an angular velocity of $\omega_1 = 4$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. Also, the boom segment BC is rotating at $\omega_{BC} = 8$ rad/s, which is increasing at $\dot{\omega}_{BC} = 2$ rad/s^2. Determine the velocity and acceleration of the part C held in its grip at this instant.



Relative to XYZ, let xyz with origin at B have

$\Omega = \{4\mathbf{k}\}$ rad/s, $\quad \dot{\Omega} = \{3\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_B = \{0.5\mathbf{k}\}$ m ($\mathbf{r}_B$ does not change direction relative to XYZ.)

$\mathbf{v}_B = 0$

$\mathbf{a}_B = 0$

Relative to xyz, let coincident $x'y'z'$ have origin at B and have

$\Omega_{xyz} = \{8\mathbf{j}\}$ rad/s, $\quad \dot{\Omega}_{xyz} = \{2\mathbf{j}\}$ rad/s^2 (Ω does not change direction relative to xyz.)

$(\mathbf{r}_{C/B})_{xyz} = \{0.7\mathbf{i}\}$ m (Ω does not change direction relative to xyz.)

$$(\mathbf{v}_{C/B})_{xyz} = \left(\dot{\mathbf{r}}_{C/B}\right)_{xyz} = \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} = 0 + (8\mathbf{j}) \times (0.7\mathbf{i}) = \{-5.6\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left(\ddot{\mathbf{r}}_{C/B}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{xyz}$$

$$= 0 + 0 + (2\mathbf{j}) \times (0.7\mathbf{i}) + (8\mathbf{j}) \times (-5.6\mathbf{k}) = \{-44.8\mathbf{i} - 1.40\mathbf{k}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} = 0 + (4\mathbf{k}) \times (0.7\mathbf{i}) + (-5.6\mathbf{k})$$

$$= \{2.80\mathbf{j} - 5.60\mathbf{k}\} \text{ m/s}$$ **Ans**

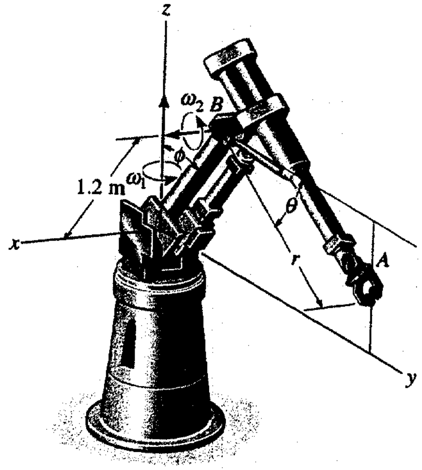
$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times \left(\Omega \times \mathbf{r}_{C/B}\right) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= 0 + (3\mathbf{k}) \times (0.7\mathbf{i}) + (4\mathbf{k}) \times \left[(4\mathbf{k}) \times (0.7\mathbf{i})\right]$$

$$+ 2(4\mathbf{k}) \times (-5.6\mathbf{k}) - 44.8\mathbf{i} - 1.40\mathbf{k}$$

$$= \{-56\mathbf{i} + 2.1\mathbf{j} - 1.40\mathbf{k}\} \text{ m/s}^2$$ **Ans**

20-47. At the instant shown, the industrial manipulator is rotating about the z axis at $\omega_1 = 5$ rad/s, and about joint B at $\omega_2 = 2$ rad/s. Determine the velocity and acceleration of the grip A at this instant, when $\phi = 30°$, $\theta = 45°$, and $r = 1.6$ m.



$\mathbf{\Omega} = \{5\mathbf{k}\}$ rad/s $\quad \dot{\mathbf{\Omega}} = \mathbf{0}$

$\mathbf{r}_B = 1.2\sin 30°\mathbf{j} + 1.2\cos 30°\mathbf{k} = \{0.6\mathbf{j} + 1.0392\mathbf{k}\}$ m

$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \mathbf{\Omega}\times\mathbf{r}_B = \mathbf{0} + (5\mathbf{k})\times(0.6\mathbf{j} + 1.0392\mathbf{k}) = \{-3\mathbf{i}\}$ m/s

$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \mathbf{\Omega}\times(\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\mathbf{\Omega}}\times\mathbf{r}_B + \mathbf{\Omega}\times\dot{\mathbf{r}}_B$

$= [\mathbf{0} + \mathbf{0}] + \mathbf{0} + [(5\mathbf{k})\times(-3\mathbf{i})]$

$= \{-15\mathbf{j}\}$ m/s^2

$\mathbf{\Omega}_{xyz} = \{2\mathbf{i}\}$ rad/s $\quad \dot{\mathbf{\Omega}}_{xyz} = \mathbf{0}$

$\mathbf{r}_{A/B} = 1.6\cos 45°\mathbf{j} - 1.6\sin 45°\mathbf{k} = \{1.1314\mathbf{j} - 1.1314\mathbf{k}\}$ m

$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \mathbf{\Omega}_{xyz}\times\mathbf{r}_{A/B}$

$= \mathbf{0} + (2\mathbf{i})\times(1.1314\mathbf{j} - 1.1314\mathbf{k})$

$= \{2.2627\mathbf{j} + 2.2627\mathbf{k}\}$ m/s

$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \mathbf{\Omega}_{xyz}\times(\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \left[\dot{\mathbf{\Omega}}_{xyz}\times\mathbf{r}_{A/B}\right] + \left[\mathbf{\Omega}_{xyz}\times\dot{\mathbf{r}}_{A/B}\right]$

$(\mathbf{a}_{A/B})_{xyz} = [\mathbf{0} + \mathbf{0}] + \mathbf{0} + [(2\mathbf{i})\times(2.2627\mathbf{j} + 2.2627\mathbf{k})]$

$= \{-4.5255\mathbf{j} + 4.5255\mathbf{k}\}$ m/s^2

$\mathbf{v}_A = \mathbf{v}_B + \mathbf{\Omega}\times\mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$

$= (-3\mathbf{i}) + [(5\mathbf{k})\times(1.1314\mathbf{j} - 1.1314\mathbf{k})] + (2.2627\mathbf{j} + 2.2627\mathbf{k})$

$= \{-8.66\mathbf{i} + 2.26\mathbf{j} + 2.26\mathbf{k}\}$ m/s **Ans**

$\mathbf{a}_A = \mathbf{a}_B + \dot{\mathbf{\Omega}}\times\mathbf{r}_{A/B} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r}_{A/B}) + 2\mathbf{\Omega}\times(\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$

$= (-15\mathbf{j}) + \mathbf{0} + (5\mathbf{k})\times[(5\mathbf{k})\times(1.1314\mathbf{j} - 1.1314\mathbf{k})] + [2(5\mathbf{k})\times(2.2627\mathbf{j} + 2.2627\mathbf{k})] + (-4.5255\mathbf{j} + 4.5255\mathbf{k})$

$= \{-22.6\mathbf{i} - 47.8\mathbf{j} + 4.53\mathbf{k}\}$ m/s^2 **Ans**