Principles of Data Communications

Reference Book: Data Communications and Networking by Behrouz A. Forouzan

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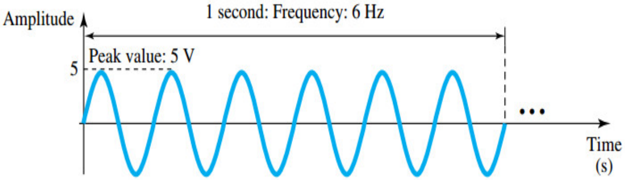
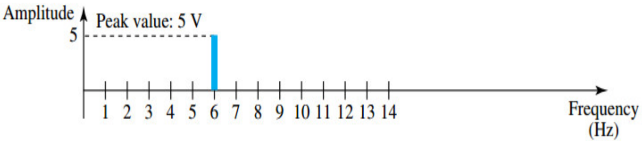
Time and Frequency Domains

The time-domain and frequency-domain plots of a sine wave



a. A sine wave in the time domain (peak value: 5 V, frequency: 6 Hz)



b. The same sine wave in the frequency domain (peak value: 5 V, frequency: 6 Hz)

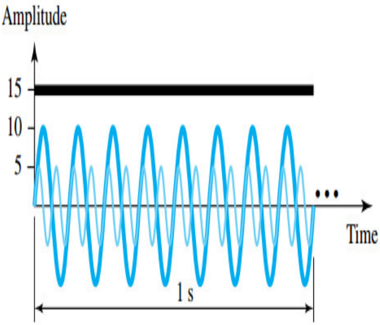
Time and Frequency Domains

- A sine wave is comprehensively defined by its amplitude, frequency, and phase.
- The time-domain plot shows changes in signal amplitude with respect to time (it is an amplitude-versus-time plot). Phase is not explicitly shown on a time-domain plot.
- To show the relationship between amplitude and frequency, we can use what is called a frequency-domain plot. A frequency-domain plot is concerned with only the peak value and the frequency.
- Changes of amplitude during one period are not shown.

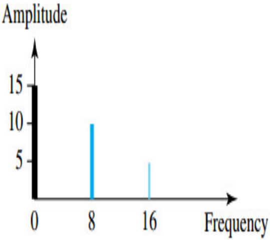
- It is obvious that the frequency domain is easy to plot and conveys the information that one can find in a time domain plot.
- The advantage of the frequency domain is that we can immediately see the values of the frequency and peak amplitude.
- A complete sine wave is represented by one spike.
- The position of the spike shows the frequency; its height shows the peak amplitude.
- **A complete sine wave in the time domain can be represented by one single spike in the frequency domain.**

- The frequency domain is more compact and useful when we are dealing with more than one sine wave. For example, th below figure shows three sine waves, each with different amplitude and frequency. All can be represented by three spikes in the frequency domain.

The time domain and frequency domain of three sine waves



a. Time-domain representation of three sine waves with frequencies 0, 8, and 16



b. Frequency-domain representation of the same three signals

Composite Signals

- Simple vs Composite
- So far, we have focused on simple sine waves. Simple sine waves have many applications in daily life. We can send a single sine wave to carry electric energy from one place to another.
- For example, the power company sends a single sine wave with a frequency of 60 Hz to distribute electric energy to houses and businesses.
- As another example, we can use a single sine wave to send an alarm to a security center when a burglar opens a door or window in the house. In the first case, the sine wave is carrying energy; in the second, the sine wave is a signal of danger.

Composite Signals

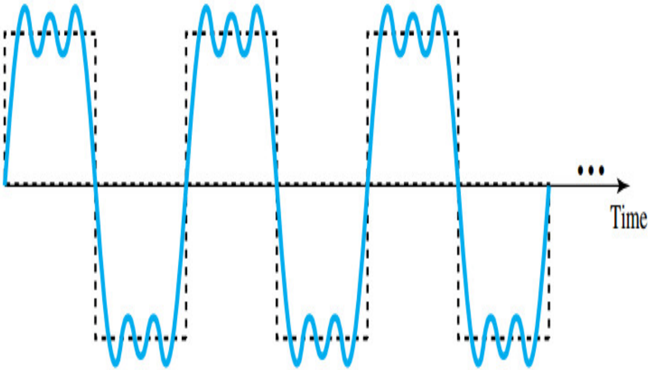
- If we had only one single sine wave to convey a conversation over the phone, it would make no sense and carry no information. We would just hear a buzz.
- We need to send a composite signal to communicate data. A composite signal is made of many simple sine waves.
- A single-frequency sine wave is not useful in data communications; we need to send a composite signal, a signal made of many simple sine waves.

- In the early 1900s, the French mathematician Jean-Baptiste Fourier showed that any composite signal is actually a combination of simple sine waves with different frequencies, amplitudes, and phases.
- **According to Fourier analysis, any composite signal is a combination of simple sine waves with different frequencies, amplitudes, and phases.**

- A composite signal can be periodic or nonperiodic. A periodic composite signal can be decomposed into a series of simple sine waves with discrete frequencies frequencies that have integer values (1, 2, 3, and so on). A nonperiodic composite signal can be decomposed into a combination of an infinite number of simple sine waves with continuous frequencies, frequencies that have real values.
- **If the composite signal is periodic, the decomposition gives a series of signals with discrete frequencies; if the composite signal is nonperiodic, the decomposition gives a combination of sine waves with continuous frequencies.**

Time and Frequency Domains

A composite periodic signal

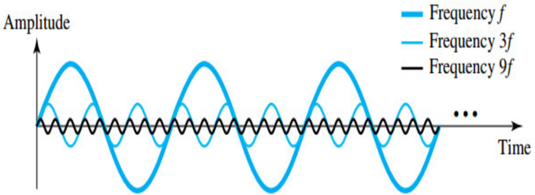


Time and Frequency Domains

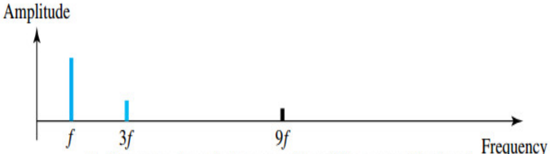
- Previous figure shows a periodic composite signal with frequency f. This type of signal is not typical of those found in data communications. We can consider it to be three alarm systems, each with a different frequency. The analysis of this signal can give us a good understanding of how to decompose signals.
- It is very difficult to manually decompose this signal into a series of simple sine waves. However, there are tools, both hardware and software, that can help us do the job. We are not concerned about how it is done; we are only interested in the result.

Time and Frequency Domains

Decomposition of a composite periodic signal in the time and frequency domains



a. Time-domain decomposition of a composite signal



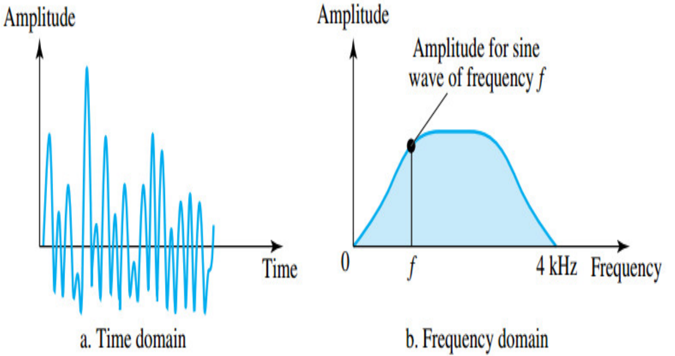
b. Frequency-domain decomposition of the composite signal

Time and Frequency Domains

- Previous Figure shows the result of decomposing the above signal in both the time and frequency domains.
- The amplitude of the sine wave with frequency f is almost the same as the peak amplitude of the composite signal. The amplitude of the sine wave with frequency $3f$ is one-third of that of the first, and the amplitude of the sine wave with frequency $9f$ is one-ninth of the first.
- The frequency of the sine wave with frequency f is the same as the frequency of the composite signal; it is called the fundamental frequency, or first harmonic.
- The sine wave with frequency $3f$ has a frequency of 3 times the fundamental frequency; it is called the third harmonic. The third sine wave with frequency $9f$ has a frequency of 9 times the fundamental frequency; it is called the ninth harmonic.

Time and Frequency Domains

The time and frequency domains of a nonperiodic signal



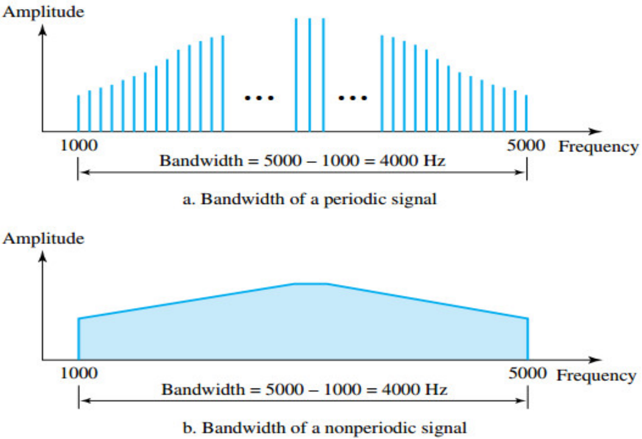
Time and Frequency Domains

- Previous Figure shows a nonperiodic composite signal. It can be the signal created by a microphone or a telephone set when a word or two is pronounced. In this case, the composite signal cannot be periodic, because that implies that we are repeating the same word or words with exactly the same tone.
- In a time-domain representation of this composite signal, there are an infinite number of simple sine frequencies. Although the number of frequencies in a human voice is infinite, the range is limited. A normal human being can create a continuous range of frequencies between 0 and 4 kHz.
- Note that the frequency decomposition of the signal yields a continuous curve. There are an infinite number of frequencies between 0.0 and 4000.0 (real values). To find the amplitude related to frequency f, we draw a vertical line at f to intersect the envelope curve. The height of the vertical line is the amplitude of the corresponding frequency.

Bandwidth

- The range of frequencies contained in a composite signal is its bandwidth. The bandwidth is normally a difference between two numbers. For example, if a composite signal contains frequencies between 1000 and 5000, its bandwidth is 5000-1000, or 4000.
- The bandwidth of a composite signal is the difference between the highest and the lowest frequencies contained in that signal.

The bandwidth of periodic and nonperiodic composite signals



- Figure shows the concept of bandwidth. The figure depicts two composite signals, one periodic and the other nonperiodic.

- A periodic signal has a bandwidth of 20 Hz. The highest frequency is 60 Hz. What is the lowest frequency? Draw the spectrum if the signal contains all frequencies of the same amplitude.

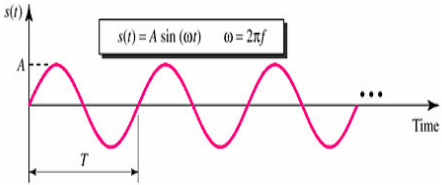
Trigonometric Functions

We can mathematically describe a sine wave as

$$s(t) = A\sin(2\pi f t) = A\sin\left(\frac{2\pi}{T}t\right)$$

where s is the instantaneous amplitude, A is the peak amplitude, f is the frequency, and T is the period (phase will be discussed later). Figure C.1 shows a sine wave.

Figure C.1 A sine wave



Find the peak value, frequency, and period of the following sine waves.

a. $s(t) = 5 \sin(10\pi t)$

b. $s(t) = \sin(10t)$

Solution

a. Peak amplitude: $A = 5$ Frequency: $10\pi = 2\pi f$, so $f = 5$ Period: $T = 1/f = 1/5$ s

b. Peak amplitude: $A = 1$ Frequency: $10 = 2\pi f$, so $f = 10/(2\pi) = 1.60$ Period: $T = 1/f = 1/1.60 = 0.628$ s

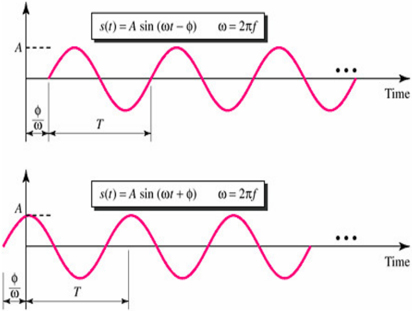
C.1.3. ***Example C.2***

Show the mathematical representation of a sine wave with a peak amplitude of 2 and a frequency of 1000 Hz.

Solution

The mathematical representation is $s(t) = 2 \sin(2000\pi t)$.

Figure C.2 *Two horizontally shifted sine waves*



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When a signal is shifted to the left or right, its first zero crossing will be at a point in time other than the origin. To show this, we need to add or subtract another constant to ωt, as shown in the figure.

Figure C.3 Vertical shifting of sine waves

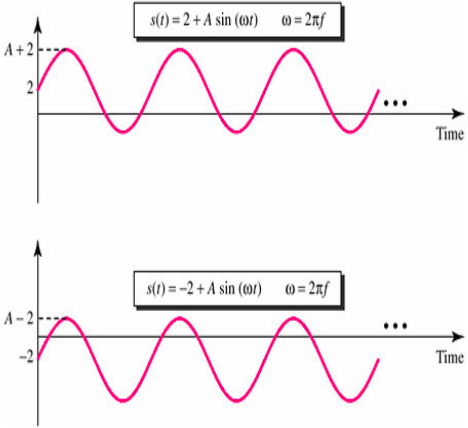
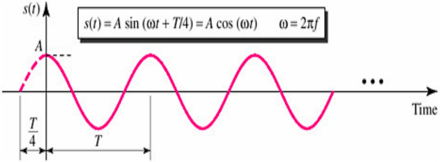


Figure C.4 *A cosine wave*



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C.1.7. Other Trigonometric Functions

There are many trigonometric functions; two of the more common are tan(ωt) and cot(ωt). They are defined as tan(ωt) = sin(ωt)/cos(ωt) and cot(ωt) = cos(ωt)/sin(ωt). Note that tan and cot are the inverse of each other.

Fourier Analysis

- Fourier Analysis is a tool that changes a time domain signal to a frequency domain signal and vice-versa.
- Fourier Series
  - Fourier proved that a composite periodic signal with period T (frequency f) can be decomposed into a series of sine and cosine functions in which each function is an integral harmonic of the fundamental frequency f of the composite signal. The result is called the Fourier series.
  - Using the series, we can decompose any periodic signal into its harmonics. Note that A_o is the average value of the signal over a period, A_n is the coefficient of the nth cosine component, and B_n is the coefficient of the nth sine component.

Figure C.5 Fourier series and coefficients of terms

Fourier series

$$s(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(2\pi n f t) + \sum_{n=1}^{\infty} B_n \cos(2\pi n f t)$$

$$A_0 = \frac{1}{T}\int_0^T s(t)\, dt \qquad A_n = \frac{2}{T}\int_0^T s(t)\cos(2\pi n f t)\, dt$$

$$B_n = \frac{2}{T}\int_0^T s(t)\sin(2\pi n f t)\, dt$$

Coefficients

- While the Fourier series gives the discrete frequency domain of a periodic signal, the Fourier transform gives the continuous frequency domain of a non periodic signal

Energy and Power Signals

- Energy Signal
  - Eg) Firecracker-rocket
  - Possesses a finite amount of energy and combustion stops once the chemical energy stored in it runs out.
  - If we attempt to compute the power delivered by the firecracker over a very long (infinite) period of time, it will be zero.
  - This is because power=energy/time; and since numerator is finite and denominator is infinite, power is zero.
- Power Signal
  - Eg) Sun
  - Energy-Infinite

Energy and power for continuous-time signals

The signal energy in the signal $x(t)$ is

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, \mathrm{d}t \, .$$

The signal power in the signal $x(t)$ is

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, \mathrm{d}t \, .$$

- If $0 < E < \infty$, then the signal $x(t)$ is called an energy signal. However, there are signals where this condition is not satisfied. For such signals we consider the power. If $0 < P < \infty$, then the signal is called a power signal. Note that the power for an energy signal is zero ($P = 0$) and the energy for a power signal is infinite ($E = \infty$).

Energy and power for discrete-time signals

- The signal energy in the discrete-time signal $x(n)$ is

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 .$$

The signal power in the signal $x(n)$ is

$$P = \lim_{N\to\infty}\left(\frac{1}{2N+1}\sum_{n=-N}^{N} |x(n)|^2\right).$$

- A discrete-time energy signal is defined as one for which $0 < E < \infty$ and a discrete-time power signal is defined as one for which $0 < P < \infty$.

Summary

- Time and Frequency Domains
- Composite Signals
- Bandwidth
- Mathematical Review
- Fourier Analysis
- Energy and Power Signals

THANK YOU