

Assignment-III

- 1 Which three matrices E_{21} , E_{31} , E_{32} put A into triangular form U ?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \quad \text{and} \quad E_{32}E_{31}E_{21}A = U.$$

Multiply those E 's to get one matrix M that does elimination: $MA = U$.

- 2 (a) What 3 by 3 matrix E_{13} will add row 3 to row 1?
(b) What matrix adds row 1 to row 3 and *at the same time* row 3 to row 1?
(c) What matrix adds row 1 to row 3 and *then* adds row 3 to row 1?

- 3 Multiply these matrices in the orders EF and FE :

$$E = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix}.$$

Also compute $E^2 = EE$ and $F^3 = FFF$. You can guess F^{100} .

- 4 The equations $Ax = b$ and $Ax^* = b^*$ have the same matrix A . What double augmented matrix should you use in elimination to solve both equations at once?

Solve both of these equations by working on a 2 by 4 matrix :

$$\begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Challenge Problems

- 1 Find the triangular matrix E that reduces “*Pascal’s matrix*” to a smaller Pascal:

Elimination on column 1

$$E \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}.$$

Which matrix M (multiplying several E ’s) reduces Pascal all the way to I ?

- 2 Find elimination matrices E_{21} then E_{32} then E_{43} to change K into I :

$$E_{43} E_{32} E_{21} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -a & 1 & 0 & 0 \\ 0 & -b & 1 & 0 \\ 0 & 0 & -c & 1 \end{bmatrix} = I.$$