Roll No								

National Institute of Technology Goa

Programme Name: B.Tech

Online End Semester Examinations, July 2021

Course Name: **Mathematics II** Course Code: **MA150**

Date: 28/07/2021 Time: 09:30 A.M - 12:30 P.M

Duration: 3 Hours Max. Marks: 100

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1. Answer All Questions.
2. **No marks will be given if the explanation of your answer is missing**.
3. The question paper consists of **Two** pages.
4. Upload the answer sheet with file name your roll number in .pdf format (Eg. 20MCE1001) on or before 12:45 PM.

Part A Linear Algebra

1. (a) Find the complete solution in the form of $x_p + x_n$ to these systems, [5M+5M]

   i. $x + y + z = 4$

   ii. $x + y + z = 4$

   $x - y + z = 4$

   (b) Construct a matrix for which $N(A)$= all combinations of (2,2,1,0) and (3,1,0,1).

2. Let $A = \begin{bmatrix} 1 & 3 & 1 \\ 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 5 \end{bmatrix}$. Factorize $A = ER$ and find bases and dimensions for the four fundamental subspaces. [5M]

3. Find $\hat{x}$ and p and P if $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ [5M]

4. Find the parabola $C + Dt + Et^2$ that comes closest to the values $b = 6, 0, 0$ when $t = 0, 1, 2$.

5. Decompose $A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}$ into Q and R and then solve $Ax = (1, 2, 7)$ by least square approximation. [5M]

6. (a) Let A be an $n \times n$ matrix such that the system of equations $AX = 0$ has a non-trivial solution. Is it possible that the system of equation $A^tX = b$ has a unique solution for some $b \in R^n$? Justify. [2M+2M+1M]

   (b) Find a 2×2 matrix A that has $u = [2,\ 2]^t$ and $v = [4,\ 5]^t$ as eigenvectors with associated eigenvalues 3 and 1 respectively.

   (c) Prove or disprove: There exist 2×2 matrices A and B such that $AB - BA = I_2$.

7. (a) Evaluate A^∞, where $A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$. [5M+5M]

   (b) Find an orthhonormal matrix Q that diagonalizes $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, and what is Λ?

8. Solve the differential equation $12y'' + 48y' + 36y = 0$ by linear algebra. [5M]

Part B Ordinary Differential Equations

1. Check whether a given ODE $y(y^2 - 2x^2)dx + x(2y^2 - x^2)dy = 0$ is exact. Further, solve the ODE after obtaining the integrating factor. [5M]

2. Solve the ODE $(x - \log y)dy + (y \log y)dx = 0$. [5M]

3. Find the largest interval of $\frac{dy}{dx} = \left(4y + e^{-x^2}\right) e^{2y},\ y(0) = 0$ in which Picard's theorem guaranties for solutions. [5M]

4. Find the Laplace transform of $t^2e^t \sin 4t$. [5M]

5. Find the inverse Laplace transform of $\frac{s+4}{s(s-1)(s^2+4)}$. [5M]

6. Use convolution property to find the inverse Laplace transform of $\frac{1}{s(s^2+4)^2}$. [5M]

7. Solve a given ODE $y'' - y = 2\left(1 - e^{-2x}\right)^{-\frac{1}{2}}$ by variation of parameters method. [10M]

8. Solve the ODE $y''' - 3y'' + 3y' - y = t^2e^t,\ y(0) = 1,\ y'(0) = 0,\ y''(0) = -2$ using Laplace transform. [10M]

$***$ALL THE BEST$***$