National Institute of Technology Goa

Programme Name: **B.Tech.**

Minor Test, May 2022

Course Name: **Probability and Statistics** Course Code: **MA250**

Duration : 1 Hr Max. Marks: **30**

ANSWER ALL QUESTIONS

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1. The random variables X and Y have joint pdf:

$$f_{X,Y}(x,y) = c\,\sin(x+y) \quad 0 \le x \le \pi/2,\ 0 \le y \le \pi/2$$

   (a) Find the value of the constant c.

   (b) Find the joint cdf of X and Y.

   (c) Find the marginal pdf's of X and of Y.

   (d) Find the mean, variance, and covariance of X and Y.

2. Let $Z = X/Y$. Find the pdf of Z if X and Y are independent and both exponentially distributed with mean one.

3. Let $W = X + Y$ and $Z = X - Y$.

   (a) Find an expression for the joint pdf of W and Z.

   (b) Find $f_{WZ}(z,w)$ if it is given that X and Y are independent exponential random variables with parameter $\lambda = 1$.

4. Suppose U and V are independent zero-mean, unit-variance Gaussian random variables, and let

$$X = U + V \quad Y = 2U + V.$$

   Find the joint characteristic function of X and Y, and using it evaluate $E[XY]$.

5. Suppose $X_1, X_2, \ldots, X_n$ are jointly Gaussian random variables with $\text{COV}(X_i, X_j) = 0$ for $i \ne j$. Show that $X_1, X_2, \ldots, X_n$ are independent random variables.

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6. Evaluate differential entropy of a Gaussian Random Variable with mean μ and variance σ^2.

7. A company self-insures its large fleet of cars against collisions. To determine its mean repair cost per collision, it has randomly chosen a sample of 16 accidents. If the average repair cost in these accidents is $2,200$ with a sample standard deviation of 800, find a 90 percent confidence interval estimate of the mean cost per collision.

8. In a certain chemical process, it is very important that a particular solution that is to be used as a reactant have a pH of exactly 8.20. A method for determining pH that is available for solutions of this type is known to give measurements that are normally distributed with a mean equal to the actual pH and with a standard deviation of .02. Suppose 10 independent measurements yielded the

following pH values:
$$
\begin{array}{cc}
8.18 & 8.17 \\
8.16 & 8.15 \\
8.17 & 8.21 \\
8.22 & 8.16 \\
8.19 & 8.18
\end{array}
$$

   (a) What conclusion can be drawn at the $\alpha = .10$ level of significance?

   (b) What about at the $\alpha = .05$ level of significance?

9. Suppose we perform n independent observations of a Poisson random variable with mean α. Find the maximum likelihood estimate for α

10. State and prove Central Limit Theorem.

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