Total No. of Printed Pages:3

F.E. Semester- I (Revised Course 2019-20)
EXAMINATION MARCH 2021
Mathematics-I

[Duration : Two Hours] **[Total Marks :60]**

Instructions:-
1) Answer THREE FULL QUESTIONS with ONE QUESTION FROM EACH PART.
2) Assume suitable data, if necessary.
3) Figures to right indicate full marks.

Part A

Q1) a) Evaluate $\int_0^\infty \frac{x^4}{4^x} dx$ using gamma function. **(4 Marks)**

a) Use Taylor's theorem, find $\sqrt{9.12}$ **(4 Marks)**

b) Test the convergence of the following series **(12 Marks)**

i) $\sum_{n=1}^{n=\infty} sin\left(\frac{1}{n}\right)$

ii) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$

ii) $\frac{3}{4}+\frac{3.4}{4.6}+\frac{3.4.5}{4.6.8}+\cdots\infty$

Q2) a) If $y = \mathrm{acos}(\log x) + \mathrm{bsin}(\log x)$ show that $\mathrm{x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n} = 0$ **(8 marks)**

b) Find the interval of convergence of the following series **(6 Marks)**
$\frac{1}{2}x + x^2 + \frac{9}{8}x^3 + x^4 + \frac{25}{32}x^5 + \cdots$

c) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{tan\theta}\, d\theta \; \int_0^{\frac{\pi}{2}} \sqrt{cot\,\theta}\, d\theta$ **(6 Marks)**

Q3) a) Evaluate **(12 Marks)**

1) $\lim_{x\to\frac{\pi}{2}}(tanx)^{\cos x}$

2) $\text{Lim}_{x\to 0}\frac{\log(1-x^2)}{\log(cosx)}$

3) $\text{Lim}_{x\to 0}\frac{\sin x-\log(e^x\,cosx)}{x\,sinx}$

(4 Marks)

b) Show that $\int_0^1 \left(1-x^{1/n}\right)^m dx = \frac{n!\,m!}{(m+n)!}$

c) Find the expansion of e^{cos2x} up to x^5 **(4 Marks)**

Part B

Q4) a) Solve the following differential equations **(12 Marks)**

i) $\frac{dy}{dx}=\frac{xy^2}{\sqrt{1+x^2}}$

ii) $x\frac{dy}{dx}+y=y^2\log_e x$

iii) $(x^2\,y^2+1)ydx+(3-2x^2y^2)xdy=0$

b) If $\mathrm{Z}=\mathrm{f(u,v)}$ where $\mathrm{u}=\mathrm{x}-\mathrm{y}$ and $\mathrm{v}=\mathrm{xy}$, prove that $x\frac{\partial^2 Z}{\partial x^2}+y\frac{\partial^2 Z}{\partial y^2}=(x+y)\left(\frac{\partial^2 Z}{\partial u^2}+xy\frac{\partial^2 Z}{\partial v^2}\right)$

(8 Marks)

Q5) a) Use the method of Lagrange's multiplier to find the point on the surface $\mathrm{z}^2=\mathrm{xy}+4$ nearest to the origin. **(8 Marks)**

(6 Marks)

b) If $u=\sin^{-1}\left[\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{6}}+y^{\frac{1}{6}}}\right]$. Evaluate $x^2\frac{\partial^2 z}{\partial^2 x}+2xy\frac{\partial^2 z}{\partial y\partial x}+y^2\frac{\partial^2 z}{\partial^2 y}$

c) Solve $\frac{dy}{dx}=\frac{3x+2y-5}{2x-3y+4}$ **(6 Marks)**

Q6) a) Find the extreme values of the function **(06 Marks)**
$\mathrm{f(x,y)}=\mathrm{x}^3+\mathrm{y}^3-3\mathrm{x}-12\mathrm{y}+20$

b) Verify Euler's Theorem for **(08 Marks)**

$$u = x^4 y^2 \sin^{-1}\left(\frac{y}{x}\right)$$

c) Solve the differential equation $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$ **(06 Marks)**

Part C

Q7 a) Prove that $\beta(m,n) = \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx$ **(07 Marks)**

b) If $u = \frac{x^2+y^2}{\sqrt{x}+\sqrt{y}} + \frac{1}{5}\sin^{-1}\left(\frac{x^2+y^2}{x^2+2xy}\right)$, **(08 Marks)**

find $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$

c) Evaluate $\mathrm{Lim}_{x\to 0}(\cos x)^{1/x^2}$ **(05 Marks)**

Q8 a) If $y = e^{\tan^{-1} x}$ show that $(1 + \mathrm{x}^2)\mathrm{Y}_{\mathrm{n}+2} + [2(\mathrm{n}+1)\mathrm{x} - 1]\mathrm{y}_{\mathrm{n}+1} + \mathrm{n}(\mathrm{n}+1)_{\mathrm{Yn}} = 0$ **(7 Marks)**

c) Prove that $\log_e(1 - x + x^2) = -x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4 - \frac{1}{5}x^5 + \cdots$ **(7 Marks)**

d) Solve $x^4 \frac{dy}{dx} + x^3 y = -\sec(xy)$ **(06 Marks)**