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Chapter: **Mathematics (maths) - Z-Transforms and Difference Equations**

Important Questions and Answers: Z-Transforms and Difference Equations

Mathematics (maths) - Z-Transforms and Difference Equations

Z -TRANSFORMS AND DIFFERENCE EQUATIONS

PART –A

1. Prove that $Z[a^n] = \dfrac{z}{Z-a}$ **is** $|z| > |a|$.

We know that $Z\{x(n)\} = \displaystyle\sum_{n=0}^{\infty} x(n) z^{-n}$

$$Z[a^n] = \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left[\frac{a}{z}\right]^n \qquad = 1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 + \dots$$

$$= \left[1 - \frac{a}{z}\right]^{-1} \qquad\qquad \left[\because = (1-x)^{-1} = 1 + x + x^2 + \dots\right]$$

$$= \left[\frac{z-a}{z}\right]^{-1} \qquad = \frac{z}{z-a}, |z| > |a|.$$

2. Prove that $Z(n) = \dfrac{z}{(z-1)^2}$.

We know that $Z\{x(n)\} = \displaystyle\sum_{n=0}^{\infty} x(n) z^{-n}$

$$Z[n] = \sum_{n=0}^{\infty} n z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{n}{z^n} \qquad = 0 + \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^2} + \ldots$$

$$= \frac{1}{z}\left[1 + 2\left(\frac{1}{z}\right) + 3\left(\frac{1}{z}\right)^2 + \ldots \right]$$

$$= \frac{1}{z}\left[\left(1 - \frac{1}{z}\right)^{-2} \right] \qquad \left[\because (1-x)^{-2} = 1 + 2x + 3x^2 + \ldots \right]$$

$$= \frac{1}{z}\left[\left(\frac{z-1}{z}\right)^{-2} \right] \qquad = \frac{1}{z}\left[\frac{z}{z-1} \right]^2 \qquad = \frac{z}{(z-1)^2} .$$

3. Find $Z\left\{a^{|n|}\right\}$.

We know that $Z\{x(n)\} = \displaystyle\sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$$Z\left\{a^{|n|}\right\} = \sum_{n=-\infty}^{\infty} a^{|n|} z^{-n}$$

$$= \sum_{n=-\infty}^{-1} a^{-n} z^{-n} + \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \left[\ldots\ldots + a^3 z^3 + a^2 z^2 + az \right] + z\left[a^n \right]$$

$$\left[\because z[a^n] = \frac{z}{z-a}\right]$$

$$= \frac{az}{1-az} + \frac{z}{z-a} \qquad\qquad \left[\because G.P. = \frac{a}{1-r}\right]$$

$$= \frac{z - a^2 z}{(1-az)(z-a)}.$$

4. Find $Z\{e^{-an}\}$.

We know that $Z[a^n] = \dfrac{z}{z-a}$

$$Z[e^{-an}] = Z[(e^{-a})^n] = \dfrac{z}{z-e^{-a}} \quad \text{Here } a = e^{-a}.$$

5. Find $Z[a^{n-1}]$.

We know that $\quad Z[a^n] = \dfrac{z}{z-a}$

We know that $\quad Z[a^n] = \dfrac{z}{z-a}$

$$Z[a^{n-1}] = Z[a^n a^{-1}]$$

$$= a^{-1} Z[a^n]$$

$$= a^{-1}\left[\dfrac{z}{z-a}\right] \qquad = \dfrac{1}{a}\left[\dfrac{z}{z-a}\right].$$

6. Find $Z\left[\dfrac{1}{n(n+1)}\right]$.

$$\dfrac{1}{n(n+1)} = \dfrac{A}{n} + \dfrac{B}{n+1}$$

$1 = A(n+1) + B(n)$

Put $n = 0$ we get, $1 = A$

Put $n = -1$ we get, $1 = -B$ (i.e) $B = -1$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

We know that $\quad Z\left[\frac{1}{n}\right] = \log \frac{z}{z-1}$

$$Z\left[\frac{1}{n+1}\right] = z \log \frac{z}{z-1}$$

$$\therefore \quad Z\left[\frac{1}{n(n+1)}\right] = z\left[\frac{1}{n}\right] - Z\left[\frac{1}{n+1}\right] \quad \text{by linearity}$$

$$= \log \frac{z}{z-1} - z \log \frac{z}{z-1}$$

$$= (1-z)\log \frac{z}{z-1}.$$

7. Find $Z[a^n \cos n\theta]$.

We know that $Z[a^n f(n)] = F\left[\dfrac{z}{a}\right]$

$$Z[a^n \cos n\theta] = [Z[\cos n\theta]]_{z \to z/a}$$

$$= \left[\frac{z(z - \cos\theta)}{z^2 - 2z\cos\theta + 1}\right]_{z \to z/a}$$

$$= \frac{\dfrac{z}{a}\left[\dfrac{z}{a} - \cos\theta\right]}{\dfrac{z^2}{a^2} - 2\dfrac{z}{a}\cos\theta + 1}$$

$$= \frac{z[z - a\cos\theta]}{z^2 - 2az\cos\theta + a^2}$$

8. Find $Z\left[\dfrac{a^n}{n!}\right]$.

Sol:

We know that $Z[a^n f(n)] = F\left[\dfrac{z}{a}\right]$

$$Z\left[a^n \frac{1}{n!}\right] = \left[Z\left[\frac{1}{n!}\right]\right]_{z \to z/a}$$

$$= \left[e^{1/z}\right]_{z \to z/a} \qquad \left[\because Z\left[\frac{1}{n!}\right] = e^{1/z}\right]$$

$$= e^{\frac{1}{(z/a)}}$$

$$= e^{\frac{a}{z}}$$

9. Prove that $Z[nf(n)] = -z\dfrac{d}{dz}F(Z).$

Give., $F(Z) = Z[f(n)]$

$$F(Z) = \sum_{n=0}^{\infty} f(n)z^{-n}$$

$$\frac{d}{dz}[F(Z)] = \sum_{n=0}^{\infty}(-n)f(n)z^{-n-1}$$

$$= -\sum_{n=0}^{\infty} nf(n)\frac{z^{-n}}{z}$$

$$z\frac{d}{dz}F(Z) = -\sum_{n=0}^{\infty} nf(n)z^{-n}$$

$$= -Z[nf(n)]$$

$$Z[nf(x)] = -z\frac{d}{dz}F(Z)$$

10. Find $Z(n^2)$.

We know that $Z[nf(n)] = -Z\dfrac{d}{dz}F(Z)$

$$Z[n^2] = Z[nn] = -z\frac{d}{dz}[Z(n)]$$

$$= -z\frac{d}{dz}\left[\frac{z}{(z-1)^2}\right]$$

$$= -z\left[\frac{(z-1)^2(1) - z[2(z-1)]}{(z-1)^2}\right]$$

$$= -z\left[\frac{z-1-2z}{(z-1)^2}\right]$$

$$= -z\left[\frac{-1-z}{(z-1)^2}\right]$$

$$= z\frac{(z+1)}{(z-1)^3} = \frac{z^2+z}{(z-1)^3}$$

11. Find the Z-transform of $\{nC_k\}$.

$$= 1 + nC_1 z^{-1} + nC_2 z^{-2} + \ldots\ldots\ldots + nC_n z^{-n}$$

This is the expansion of binominal theorem.

$$= (1 + z^{-1})^n$$

12. Find $Z[e^{-t}t^2]$.

We know that $Z[e^{-at}f(t)] = Z[f(t)]_{z \to ze^{aT}}$

$Z[e^{-t}t^2] = [Z[t^2]_{z \to ze^T}$

We know that $Z\left[e^{-at}f(t)\right]=Z[f(t)]_{z\to ze^{aT}}$

$$Z\left[e^{-t}t^2\right]=\left[Z\left[t^2\right]\right]_{z\to ze^T}$$

$$=\frac{T^2ze^T\left(ze^T+1\right)}{\left(ze^T-1\right)^3}$$

13. Define Unit Sample sequence.

The unit sample sequence $\delta(n)$ is defined the sequence with values

$$u(n)=\begin{cases}1 & for \quad n>0 \\ 0 & for \quad n<0\end{cases}$$

14. Define Unit step sequence.

The unit step sequence $u(n)$ has values.

$$u(n)=\begin{cases}1 & for \quad n>0 \\ 0 & for \quad n<0\end{cases}$$

15. Find $Z\left[2^n\delta(n-2)\right].$

$$Z\left[2^n\delta(n-2)\right]=Z[\delta(n-2)]z\to_{z/2}$$

$$=\left[\frac{1}{z^2}\right]_{z\to z/2}=\frac{1}{\left(\frac{z}{2}\right)^2}=\frac{4}{z}$$

16. If $Z[f(n)]=F(z),,$ then

$Z[f(n)] = F(z),$, **then** $f(0) = \lim_{z \to \infty} F(z)$.

$$Z[f(n)] = \sum_{n=0}^{\infty} f(n) z^{-n}$$

$$= f(0) + \frac{f(1)}{z} + \frac{f(2)}{z^2} + \dots$$

$$\lim_{x \to \infty} Z[f(n)] = \lim_{z \to \infty} \left[f(0) + \frac{f(1)}{z} + \frac{f(2)}{z^2} + \dots \right]$$

$$\lim_{x \to \infty} F(z) = f(0)$$

17. Find the Z-transform of $na^n\, u(n)$.

$$na^n u(n).$$

$$Z[na^n u(n)] = z^{-1} \frac{d}{dz^{-1}} \left[\frac{z}{z-a} \right] \text{ by def. of u (n)}$$

$$= z^{-1} \frac{d}{dz^{-1}} \left[1 - az^{-1} \right]^{-1}$$

$$= z^{-1}(-1)\left(1 - az^{-1} \right)^{-2} \left[-a \right]$$

$$= az^{-1} \left(1 - az^{-1} \right)^{-2}$$

$$= \frac{az^{-1}}{\left(1 - az^{-1} \right)^2}.$$

18. Define convolution of sequences.

　　i) The convolution of two sequences $\{x(n)\}$ and $\{y(n)\}$ is defined as

a. $\{x(n) * y(n)\} = \sum\limits_{K=-\infty}^{\infty} f(K)g(n-K)$ if the sequences are non – causal and

b. $\{x(n) * y(n)\} = \sum\limits_{K=0}^{n} f(K)g(n-K)$ if the sequences are causal.

ii)The convolution of two functions f(t) and g (t) is defined as

$$\{f(t) * g(t)\} = \sum\limits_{K=0}^{n} f(KT)g(n-K)T,$$ where is T is the sampling period.

PART B

1. Using the Z transforms, Solve $u_{n+2} + 3u_{n+1} + 2u_n = 0$ *given* $u_0 = 1, u_1 = 2.$

Solution:

Given $u_{n+2} + 3u_{n+1} + 2u_n = 0$

$$Z[u_{n+2}] + Z[3u_{n+1}] + Z[2u_n] = 0$$

$$[z^2 U(z) - z^2 u(0) - zu(1)] + 3[zU(z) - zu(0)] + 2 U(z) = 0$$

$$(z^2 + 3z + 2) U(z) - z^2 - 2z - 3z = 0 \quad [u_0 = 1, u_1 = 2]$$

$$U(z) = \frac{z^2 + 5z}{z^2 + 3z + 2}$$

$$U(z) = \frac{z(z+5)}{z^2 + 3z + 2}$$

$$\frac{U(z)}{z} = \frac{(z+5)}{(z+1)(z+2)} = \frac{A}{(z+1)} + \frac{B}{(z+2)} \quad \cdots\cdots\cdots\cdots (1)$$

Then $z + 5 = A(z+2) + B(z+1)$

Put z = -1, we get	Put z = -2, we get
4 = A	3 = -B
A = 4	B = -3

(1)
$$\frac{U(z)}{z} = \left[\frac{4}{(z+1)}\right] - \left[\frac{3}{(z+2)}\right]$$

$$\Rightarrow \quad U(z) = 4\left[\frac{z}{(z+1)}\right] - 3\left[\frac{z}{(z+2)}\right]$$

$$Z[u(n)] = 4\left[\frac{z}{(z+1)}\right] - 3\left[\frac{z}{(z+2)}\right]$$

$$u(n) = 4Z^{-1}\left[\frac{z}{(z+1)}\right] - 3Z^{-1}\left[\frac{z}{(z+2)}\right]$$

$$= 4(-1)^n - 3(-2)^n$$

$$= [4 - 3(2^n)](-1)^n$$

2. Solve the difference equation $y(n+3) - 3y(n+1) + 2y(n) = 0$

given that $y(0) = 4, y(1) = 0$ *and* $y(2) = 8$

Solution:

Given $y(n+3) - 3y(n+1) + 2y(n) = 0$

$Z[y(n+3)] - 3Z[y(n+1)] + 2Z[y(n)] = 0$

$[z^3 Y(z) - z^3 y(0) - z^2 y(1) - zy(2)] - 3[zY(z) - 2y(0)] + 2Y(z) = 0$

$[z^3 Y(z) - 4z^3 - 8z] - 3[zY(z) - 4z] + 2Y(z) = 0$ $[y(0) = 4, y(1) = 0, y(2) = 8]$

$[z^3 - 3z + 2]Y(z) - 4z^3 - 8z + 12z = 0$

$[z^3 - 3z + 2]Y(z) - 4z^3 - 4z = 0$

$[z^3 - 3z + 2]Y(z) = 4z^3 - 4z$

$$Y(z) = \frac{4z^3 - 4z}{z^3 - 3z + 2}$$

$$= \frac{4z(z^2 - 1)}{(z-1)^2(z+2)}$$

$$= \frac{(z+1)(z-1)}{(z-1)^2(z+2)}$$

$$= \frac{4Z(z+1)}{(z-1)(z+2)}$$

$$\frac{Y(z)}{z} = \frac{4(z+1)}{(z-1)(z+2)} = \frac{A}{z-1} + \frac{B}{z+2} \qquad \ldots\ldots\ldots(1)$$

$$4(z+1) = A(z+2) + B(z-1)$$

Put   z=1 , we get   Put z =-2 , we get

8 = 3A   -4 = -3B

A=8/3   B = 4/3

(1) $\quad \dfrac{Y(z)}{z} = \dfrac{8/3}{z-1} + \dfrac{4/3}{z+2}$

$$\dfrac{Y(z)}{z} = \dfrac{8}{3}\left\{\dfrac{z}{z-1}\right\} + \dfrac{4}{3}\left\{\dfrac{z}{z+2}\right\}$$

$$Z[y(n)] = \dfrac{8}{3}\left\{\dfrac{z}{z-1}\right\} + \dfrac{4}{3}\left\{\dfrac{z}{z+2}\right\}$$

$$y(n) = \dfrac{8}{3}Z^{-1}\left\{\dfrac{z}{z-1}\right\} + \dfrac{4}{3}Z^{-1}\left\{\dfrac{z}{z+2}\right\}$$

$$= \dfrac{8}{3}(1)^n + \dfrac{4}{3}(-2)^n$$

3. Using Z transforms, Solve $u_{n+2} - u_{n+1} + 6u_n = 4^n$ given that $u_0 = 0, u_1 = 1$

Solution:

Given $\quad u_{n+2} - u_{n+1} + 6u_n = 4^n$

$$Z[u_{n+2}] - Z[u_{n+1}] + 6Z[u_n] = Z[4^n]$$

$$[z^2 U(z) - z^2 u(0) - zu(1)] - 5[zU(z) - zu(0)] + 6U(z) = \dfrac{z}{z+4}$$

$(z^2 - 5z + 6)\,U(z) - z = \dfrac{z}{z+4}$ $\qquad\qquad\qquad$ $[u_0 = 0, u_1 = 1]$

$$(z^2 - 5z + 6)\,U(z) = z + \dfrac{z}{z+4}$$

$$(z-3)(z-2)\,U(z) = \dfrac{z^2 - 4z + z}{z-4}$$

$$(z-3)(z-2)\,U(z) = \dfrac{z^2 - 3z}{z-4}$$

$$U(z) = \dfrac{z(z-3)}{(z-3)(z-2)(z-4)}$$

$$U(z) = \frac{z}{(z-2)(z-4)}$$

$$\frac{U(z)}{z} = \frac{1}{(z-2)(z-4)}$$

$$\frac{U(z)}{z} = \frac{1}{(z-2)(z-4)} = \frac{A}{(z-2)} + \frac{B}{(z-4)} \quad \cdots\cdots\cdots\cdots\cdots (1)$$

$$1 = A(z-4) + B(z-2)$$

Put z = 2, we get   Put z = 4, we get

$$1 = -2A \qquad\qquad\qquad\qquad 1 = 2B$$

$$A = -\frac{1}{2} \qquad\qquad\qquad\qquad B = \frac{1}{2}$$

(2) $$\frac{U(z)}{z} = \left[\frac{-\frac{1}{2}}{(z-2)}\right] + \left[\frac{\frac{1}{2}}{(z-4)}\right]$$

$$\Rightarrow \quad U(z) = -\frac{1}{2}\left[\frac{z}{(z-2)}\right] + \left[\frac{z}{(z-4)}\right]$$

$$Z[u(n)] = -\frac{1}{2}\left[\frac{z}{(z-2)}\right] + \frac{1}{2}\left[\frac{z}{(z-4)}\right]$$

$$u(n) = -\frac{1}{2}Z^{-1}\left[\frac{z}{(z-2)}\right] + \frac{1}{2}Z^{-1}\left[\frac{z}{(z-4)}\right]$$

$$= -\frac{1}{2}(2)^n + \frac{1}{2}(4)^n$$

4.Using Z transforms, Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ *given that* $y_0 = 0, y_1 = 0$

Solution:

Given $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$

$$Z[y_{n+2}] + 6Z[y_{n+1}] + 9Z[y_n] = Z[2^n]$$

$$[z^2 Y(z) - z^2 y(0) - zy(1)] + 6[zY(z) - zy(0)] + 9Y(z) = \frac{z}{z-2}$$

$$(z^2 + 6z + 9) Y(z) = \frac{z}{z-2} \qquad\qquad y_0 = 0, y_1 = 0$$

$$(z^2 + 6z + 9) Y(z) = \frac{z}{z-2}$$

$$(z+3)^2 Y(z) = \frac{z}{z}$$

$$Y(z) = \frac{z}{(z-2)(z+3)^2}$$

$$\frac{Y(z)}{z} = \frac{1}{(z-2)(z+3)^2}$$

$$\frac{Y(z)}{z} = \frac{1}{(z-2)(z+3)^2} = \frac{A}{(z-2)} + \frac{B}{(z+3)} + \frac{C}{(z+3)^2} \quad\cdots\cdots\cdots\cdots (1)$$

$$1 = A(z+3)^2 + B(z-2)(z+3) + c(z-2)$$

Put z = 2, we get	Put z = -3, we get	Equating z^2 co-eff.
$1 = 25A$	$1 = -5C$	on both sides, we get
$A = \frac{1}{25}$	$C = -\frac{1}{5}$	$0 = A+B$
		$B = -A, B = -\frac{1}{25}$

(1) $\qquad \frac{Y(z)}{z} = \frac{1}{25}\frac{1}{z-2} - \frac{1}{25}\frac{1}{z+3} - \frac{1}{5}\frac{1}{(z+3)^2}$

$$Y(z) = \frac{1}{25}\frac{z}{z-2} - \frac{1}{25}\frac{z}{z+3} - \frac{1}{5}\frac{z}{(z+3)^2}$$

$$y(n) = \frac{1}{25} Z^{-1}\left[\frac{z}{z-2}\right] - \frac{1}{25} Z^{-1}\left[\frac{z}{z+3}\right] - \frac{1}{5} Z^{-1}\left[\frac{z}{(z+3)^2}\right]$$

ie,

$$y(n) = \frac{1}{25}(2)^n - \frac{1}{25}(-3)^n + \frac{1}{15}(-3)^n n$$

5. Find $Z^{-1}\left[\dfrac{z^2}{(z-a)(z-b)}\right]$.

Solution:

$$Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right] = Z^{-1}\left[\frac{z}{z-a} \cdot \frac{z}{z-b}\right]$$

$$= Z^{-1}\left[\frac{z}{z-a}\right] * Z^{-1}\left[\cdot\frac{z}{z-b}\right]$$

$$= a^n * b^n$$

$$= \sum_{n=0}^{n} a^m b^{n-m} = b^n \sum_{m=0}^{n} \left(\frac{a}{b}\right)^m$$

$$= b^n \frac{\left(\dfrac{a}{b}\right)^{n+1} - 1}{\dfrac{a}{b} - 1} \quad being \ a \ G.P$$

$$= \frac{a^{n+1} - b^{n+1}}{a - b}$$

Note: $\left[1 + a + a^2 + \ldots + a^{n-1} = \dfrac{a^n - 1}{a - 1}\right]$

6. Find $Z^{-1}\left[\dfrac{z^2}{(z-1)(z-3)}\right]$.

Solution:

$$Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right] = Z^{-1}\left[\frac{z}{z-1}\cdot\frac{z}{z-3}\right]$$

$$= Z^{-1}\left[\frac{z}{z-1}\right] * Z^{-1}\left[\frac{z}{z-3}\right]$$

$$= 1^n * 3^n$$

$$= \sum_{n=0}^{n} 1^m 3^{n-m} = 3^n \sum_{m=0}^{n}\left(\frac{1}{3}\right)^m$$

$$= 3^n \frac{\left(\dfrac{1}{3}\right)^{n+1} - 1}{\dfrac{1}{3} - 1} \quad being \ a \ G.P$$

$$= \frac{3^{n+1} - 1}{2}$$

Note: $\left[1 + a + a^2 + \ldots + a^{n-1} = \dfrac{a^n - 1}{a - 1}\right]$

Introduction

      The Z-transform plays a vital role in the field of communication Engineering and control Engineering, especially in digital signal processing. Laplace transform and Fourier transform are the most effective tools in the study of continuous time signals, where as Z –transform is used in discrete time signal analysis. The application of Z – transform in discrete analysis is similar to that of the Laplace transform in continuous systems. Moreover, Z-transform has many properties similar to those of the Laplace transform. But, the main difference is Z-transform operates only on sequences of the

discrete integer-valued arguments. This chapter gives concrete ideas about Z-transforms and their properties. The last section applies Z-transforms to the solution of difference equations.

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