

NATIONAL INSTITUTE OF TECHNOLOGY GOA

Farmagudi, Ponda, Goa, 403401

Programme Name: B.Tech.

Mid Semester Examinations, October-2022

Course Name: Discrete Mathematics

Date: 17th October

Duration: 1 Hour 30 Minutes

Course Code: CS 203

Time: 9.30 AM

Max. Marks: 50

ANSWER ALL QUESTIONS

Q1. Express the below specifications into logical expression

(5 M)

a. Getting elected follows from knowing the right people $P \rightarrow Q$ D. You can access the website iff you have paid the subscription fee

c. A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.

d. Winds from the south imply a spring thaw

Carol gets seasick only if she is on a boat

Q2. State the converse, contrapositive and inverse of each of these conditional statements

(5 M)

a. I will go to beach whenever it is a sunny day

When I stay up late, it is necessary that I sleep until noon

~ Fr (F(M) ~ P(M))

Express the predicate logic of the below specifications

(10 M)

None of my friend is perfect, (Hint: X is a person, F(x) is your friend, P(x) represents) perfect

b. What is wrong with this "proof"?

"Theorem": If n² is positive, then n is positive.

"Proof": Suppose that n² is positive. Because the conditional statement "If n is positive, then n² is positive" is true, we can conclude that n is positive

c. $\forall x[P(x) \ V \ Q(x)] = \forall x \ P(x) \ V \ \forall x \ Q(x)$ Is it valid? Give your answer with justification

d. $\exists x [P(x) \ V \ Q(x)] = \exists x P(x) \ V \ \exists x \ Q(x)$ Is it valid? Give your answer with justification

Q4. Prove using method of induction that 7 divides $2^{(4n+2)} + 3^{(2n+1)}$ for all nonnegative integers n. (5 M)Q5. (5 M)

a. Let $U = \{1, 2, ..., n\}$. Let $A = \{(x, X) \mid x \in X, X \subseteq U\}$. Find the |A|?

b. How many number of elements in the power set P(S) if $S = \{\{(0), 1, 2, 3, \{1,2\}\}\}$? Give your answer with justification.

Q6. Use quantifiers to express the below statements:

use quantifiers to express the below statements: (1) a. If a person is female and is a parent, then this person is someone's mother. (Hint: Quantifiers with a domain consisting of all people, and logical connectives)

b. Everyone has exactly one best friend. (Hint: Domain consisting of all people)

c. Everyone in your class with an internet connection has chatted over the internet with at least one other student in your class. (Hint: Domain consisting of all student of your class)

 $\mathbf{Q7}$. Find the POSET for the divisibility for the given set ($\{3, 4, 12, 24, 48, 72\}$, /) and draw the Hasse diagram (5 M)

Q8. In a group of 100 students, 72 students can speak English and 43 students can speak Hindi. Based on these data, answer the following questions: (5 M)

a. Find the number of students who can speak English only.

b. Find the number of students who can speak Hindi only.

Find the number of students who can speak both English and Hindi.