Roll No									

National Institute of Technology Goa

Programme Name: B.Tech.

Mid Semester Examinations, March 2021

Course Name: Mathematics - III (Probability and Statistics) | Course Code: MA250

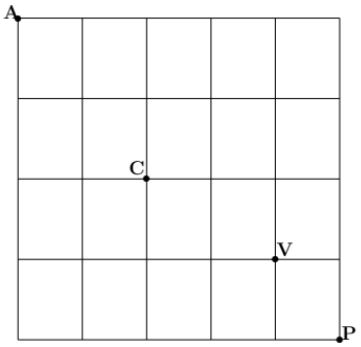
Date: 11/03/2021 | Time: 10.30 AM - 12.00 PM

Duration: 90 Minutes | Max. Marks: 50

ANSWER ALL QUESTIONS $(10 \times 5 = 50)$

1. In a certain factory, machines I, II, and III are all producing springs of the same length. Machines I, II, and III produce 1%, 4%, and 2% defective springs, respectively. Of the total production of springs in the factory, Machine I produces 30%, Machine II produces 25%, and Machine III produces 45%. (a) If one spring is selected at random from the total springs produced in a given day, determine the probability that it is defective. (b) Given that the selected spring is defective, find the conditional probability that it was produced by Machine II. [5M]

2. A streetmap of Goa is given below. You arrive at the Madgaon railway station at A and wish to take a taxi to NIT GOA at P. The taxi driver, being an honest sort, will take a route from A to P with no backtracking, always traveling south or east.



   (a) How many such routes are possible from A to P?

   (b) If you insist on stopping off at the Ponda at C, how many routes can the taxi driver take from A to P?

   (c) If wish to stop off at both the Ponda at C and the Farmagudi at V, how many routes can your taxi driver take?

   (d) If you wish to stop off at either C or V(at least one), how many routes can the taxi driver take?

3. Three students took an equivalency test at different times with following results. Baiju has a score of 35 when the mean of the test was 40 and standard deviation is 4.2. Mahesh has a score of 125 when the mean of the test was 120 and standard deviation is 5.0. Manimaran has a z-score of 1.15 when the mean of the test was 235 and standard deviation is 25.0.

   (a) What is the z- score of Mahesh?.

   (b) Who has relatively better score amoung the three?

4. Chances of getting a head every time you toss a coin is $\frac{2}{3}$ independently from one toss to next. Suppose we toss the coin 200 times then calculate the probability that we get 90 heads.

5. Let A and B be two finite sets, with $\mid A \mid = m$ and $\mid B \mid = n$.

   (a) How many distinct functions (mappings) can you define from set A to set B ie., $f : A \to B$?

   (b) How many distinct one-to-one functions (mappings) can you define from set A to set B ie., $f : A \to B$?

   (c) With the above two results, what is the probability of arbitrarily chosen function is one-to-one for m =3 and n=3?

6. A random variable X is continuous variable related to the amount of antibodies produced in response to covid-19 vaccine injected in a person. It has been theorized that said variable should approximately follow a Probability density function of the form :

$$f(x) = \begin{cases} \frac{cx}{1+x^2} & , 0 \leq x \leq 5 \\ 0 & otherwise \end{cases}$$

   (a) Determine the value of c.

   (b) Calculate mean and variance.

   (c) Calculate $P(1 \leq x \leq 3)$.

7. Let $X \sim Exponential(2)$ and $Y = 2 + 3X$. Find $P(X > 2)$. Find EY and $Var(Y)$. Find $P(X > 2|Y < 11)$. [5M]

8. Two balls are selected at random from a box containing three red, two green and four white. If $X$and Y are the number of red balls and green balls respectively included among the two balls drawn from the box. Find [5M]

   (a) Joint probability of X and Y.

   (b) Marginal probability of X and Y.

   (c) Conditional distribution of X given $Y = 1$.

9. The joint density of X and Y is given by:

$$f(x,y) = 2e^{-(x+y)}, 0 < x < y < \infty,$$

Compute $E[X|Y=y]$ and $Var[X|Y=y]$. [5M]

10. Write R program and find answers via simulation for the following situations.

    (a) What is the probability of having the same side come up 7 times in a row in 1000 coin tosses?

    (b) In six coin tosses, what's the chance of making a different side come up for each toss, that's, you never get two tails or two heads in a row?

    (c) Say a glass rod of 0.10 unit length drops and breaks into 5 random pieces. Write an R-program to find the probability that the smallest piece has length below 0.02. [5M]

$***$SMILE$***$