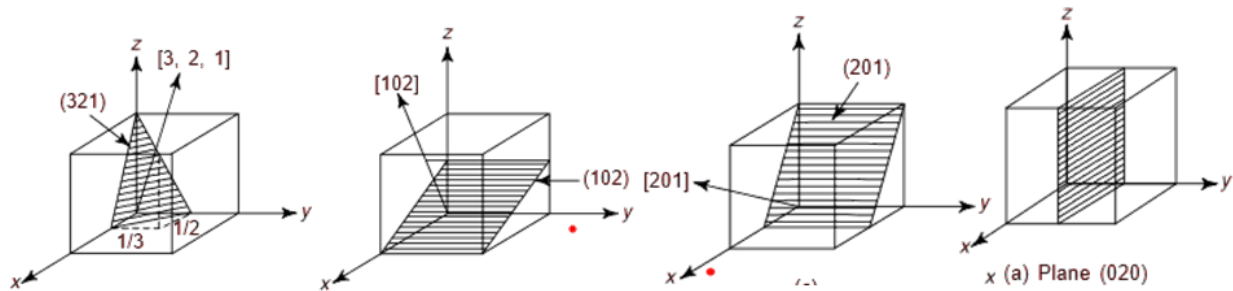


**1a. Draw the planes and directions of FCC structures (321), (102), (201) and (020). (4M)**



**1b. Determine the packing efficiency and density of sodium chloride from the following data: (i) radius of the sodium ion = 1.02 Å, (ii) radius of chlorine ion = 1.61 Å (iii) atomic mass of sodium = 22.99 amu and atomic mass of chlorine = 35.45 amu. (3 M)**

The unit cell structure of NaCl is FCC. We can see that the Na<sup>+</sup> and Cl<sup>-</sup> ions touch along the cube edges.

Lattice parameter,  $a = 2(\text{radius of Na}^+ + \text{radius of Cl}^-) = 2(1.02 + 1.61) = 5.26 \text{ Å}$

Atomic Packing Fraction = Volume of ions present in the unit cell / Volume of the unit cell

$$= \frac{4\left(\frac{4}{3}\right)\pi r_{\text{Na}^+}^3 + 4\left(\frac{4}{3}\right)\pi r_{\text{Cl}^-}^3}{a^3}$$

$$= \frac{16\pi [(1.02^3 + 1.61^3)]}{5.26^3}$$

$$= 0.602 \text{ or } 60.2\% \text{ (1.5 M)}$$

Density = Mass of the unit cell / Volume of the unit cell

$$= 4(22.99 + 35.45) \times 1.66 \times 10^{-27} / (5.26 \times 10^{-10})^3$$

$$= 2465 \text{ kg/m}^3 \text{ or } 2.465 \text{ gm/cm}^3 \text{ (1.5 M)}$$

**1c. Aluminium has FCC structure. Its density is 2100 kg/m<sup>3</sup>. Find the unit cell dimensions and atomic diameter. Given at. weight of Al = 23.98. (3 M)**

$$\text{Density} = \frac{nm}{a^3} N_A$$

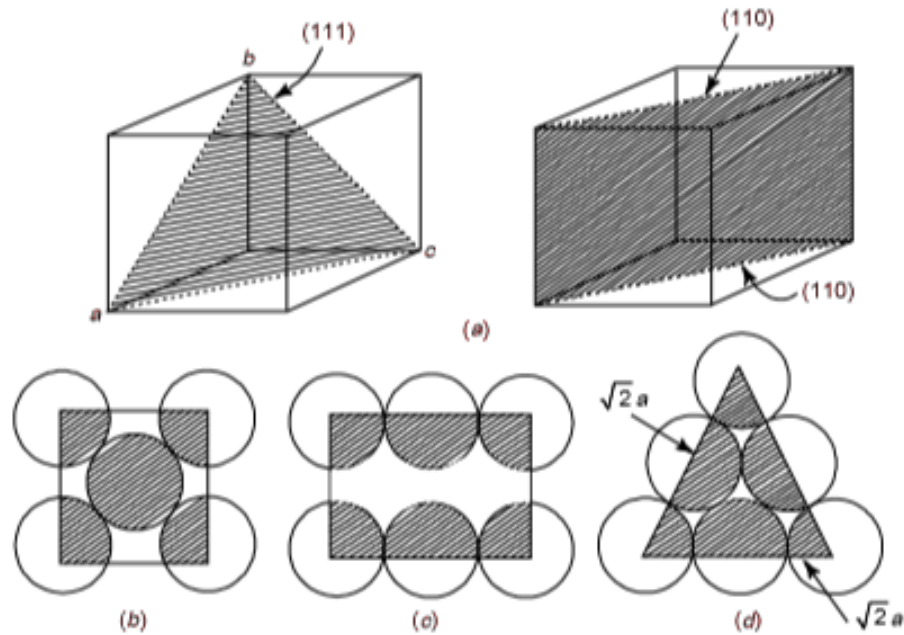
$$= 2100 \text{ Kg/m}^3 = 2.1 \text{ gm/cm}^3$$

$$2.1 = \frac{4 \times 23.98}{a^3 \times 6.023 \times 10^{23}} \Rightarrow a = 4.23 \times 10^{-10} \text{ or } 4.23 \text{ Å}$$

For FCC structure,  $r = a/2 \times 1.414 = 4.23/2 \times 1.414 = 1.852988 \text{ \AA}$

$$\text{Diameter} = 2r = 3.70 \text{ \AA}$$

2a. Calculate the planar atomic densities of planes (100), (110) and (111) in FCC unit cell and apply your result for lead (FCC form). (3 M)



**Fig. 3.44 Distribution of atoms in planes (100), (110) and (111) in FCC unit cell**

Number of atoms contained in (100) plane is  $4 \times \frac{1}{4} + 1 = 2$

Let  $a$  be the edge of the unit cell and  $r$  the radius of the atom, then

$$a = 2\sqrt{2} r$$

$\therefore$  Planar density of plane (100)

$$= \frac{2}{4 \times 2r^2} = \frac{0.25}{r^2}$$

The radius of lead atom is  $1.75 \text{ \AA}$ . The planar density of (100) plane of lead

$$= \frac{0.25}{(1.75 \times 10^{-7})^2} = 8.2 \times 10^{12} \text{ atoms/mm}^2$$

$$= 8.2 \times 10^{18} \text{ atoms/m}^2$$

(ii) Plane (110): From Fig. 3.44(c), we have the number of atoms contained in plane (110)

$$= 4 \times \frac{1}{4} + 2 \times \frac{1}{2} = 2$$

The top edge of the plane (110) is  $4r$ , whereas the vertical edge  $= a = 2\sqrt{2} r$ . Thus the planar density of (110)

$$= \frac{2}{8\sqrt{2}r^2} = 0.177/r^2$$

In case of (110) plane in lead, we have planar density  $= \frac{0.177}{(1.75 \times 10^{-2})^2}$

(iii) From Fig. 3.44(d), we have the number of atoms contained in the plane (111)

$$= 3 \times \frac{1}{6} + \frac{3}{2} = 2$$

$$\text{Area of (111) plane} = \frac{1}{2} \sqrt{\frac{3}{2}} a \sqrt{2} a = 4\sqrt{3}r^2$$

$$\therefore \text{Planar density of (111)} = \frac{2}{4\sqrt{3}r^2} = \frac{0.29}{r^2}$$

For lead crystal, we obtain the value  $9.5 \times 10^{12}$  atoms/mm<sup>2</sup>.

**2b. The force of attraction between ions of Na and Cl is  $2.02 \times 10^{-9}$  N when the two ions just touch each other. Given: ionic radius of  $\text{Na}^+$  ion is  $1.1 \text{ \AA}$ ,  $e = 1.6 \times 10^{-19} \text{ C}$ ,  $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N} - \text{m}^2$ . Find the radius of  $\text{Cl}^-$  ion. (2 M)**

$$\text{We have } F_1 = -\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r_2^2} \text{ (force of attraction)}$$

$$r = 3.37 \times 10^{-9} \text{ m}$$

$$r = r_{\text{Na}} + r_{\text{Cl}}$$

$$r_{\text{Cl}} = 3.37 - 1.1 \text{ \AA} = 2.27 \text{ \AA}$$

**2c. The empty electron states are available immediately above the fermi level in a material. What type of material it is? And find the flux per unit potential gradient (5 M)**

**Ans: Conductor; Conduction by Free Electrons theory**

As conductivity  $\sigma$  is by definition the flux per unit potential gradient, we have

$$\sigma = \frac{ne^2\tau}{m}$$

**3a. There are  $10^9$  electrons/m<sup>3</sup>, which serves as carriers in a material. The conductivity of material is  $0.01 \text{ Ohm}^{-1}/\text{m}$ . Find the drift velocity of these carriers, when  $0.17 \text{ Volt}$  is applied across  $0.27 \text{ mm}$  distance with the material. Given:  $e = 1.602 \times 10^{-19} \text{ C}$  and  $m = 9.1 \times 10^{-31} \text{ kg}$ . (3M)**

$$E = V/d$$

$$= 630 \text{ V/m}$$

Let  $v$  be the drift velocity.

$$\text{The conductivity } \sigma = \frac{nev}{E}$$

$$0.01 = 2.54 \times 10^{-13} \cdot v$$

$$V = 3.93 \times 10^{10} \text{ m/s}$$

**3b. Find the conductivity of copper at 300 K. The collision time for electron scattering in copper at 300 K is  $4 \times 10^{-14}$  sec. Given that density of copper =  $8960 \text{ kg/m}^3$ , atomic weight of copper =  $53.54 \text{ amu}$  and mass of an electron =  $9.1 \times 10^{-31} \text{ kg}$ . (3M)**

$$\text{We know that the carrier concentration} = \frac{\text{Avogadro's number} \times \text{Density}}{\text{Atomic weight}}$$

$$n = 6.023 \times 10^{23} \cdot 8960 / 53.54$$

$$= 1.0 \times 10^{26} \text{ m}^{-3}$$

$$\sigma = \frac{e^2 n t}{m}$$

$$= (1.6 \times 10^{-19})^2 \times 10^{26} \times 4 \times 10^{-14} / 9.1 \times 10^{-31}$$

$$= 1.13 \times 10^5 \text{ ohm}^{-1} \text{m}^{-1} \text{ or } 1.13 \times 10^5 \text{ mho/m}$$

**3c. If someone were to give you a poly crystalline material of  $\text{NaFePO}_4$ , how would you go about discovering the crystal structure, and what theory and principle would you use to do so? Explain your method with a neat sketch. (4M)**

**Ans:** Powder crystal x-ray diffraction technique used to determine the structure of the material

**4a. Calculate the resistance of a Cu wire 100 cm long and having cross-sectional area of 3 sq. mm at  $20^\circ\text{C}$ . Given, the resistivity of Al at  $20^\circ\text{C}$  =  $2.66 \times 10^{-8} \text{ ohm-m}$  (2M)**

$$R = \frac{\rho l}{A}$$

$$= 2.66 \times 10^{-8} \times 1 / 3 \times 10^{-6}$$

$$= 8.87 \times 10^{-3} \text{ ohm}$$

**4b. The critical temperature of mercury is 5.2 K. Calculate the wavelength of a photon whose energy is just sufficient to break up Cooper pairs in mercury at T = 0. In what region of the electromagnetic spectrum are such photons found? (4M)**

The Cooper pair binding energy, or gap energy, is  $E_g = 3kT_c$

$$= 3 \times 1.4 \times 10^{-23} \times 5.2$$

$$= 2.184 \times 10^{-22} \text{ J} = 1.36 \times 10^{-3} \text{ eV}$$

$$E_g = h\nu = hc/\lambda$$

$$\lambda = 6.6 \times 10^{-34} \times 3 \times 10^8 / 2.18 \times 10^{-22}$$

$$= 9 \times 10^{-4} \text{ m}$$

Obviously, these photons are in the very short wavelength part of the microwave region

**4c. What are type-I and type-II superconducting materials? Give three examples of each why type-II materials are preferred for applications of superconductivity. (4M)**

**5a. Find the shortest wavelength of the x-rays emitted by an x ray tube operating at 30 KV (3M)**

$$\frac{hc}{\lambda_{min}} = eV$$

$$\lambda_{min} = \frac{hc}{eV} = \frac{12375}{V} = \frac{12375}{30 \times 10^3} = 0.4125 \text{ \AA}$$

**5b. A certain crystal has axial units x : y : z of 0.424:1:0.367. Find the Miller indices of crystal faces whose intercepts are 0.212:1:0.183.**

$$h = x/a; k = y/b; l = z/c$$

$$h = 0.424/0.212 = 2; k = 1/1 = 1; l = 0.367/0.183$$

$$(2 \ 1 \ 2)$$

**5c. What are polar and non-polar dielectrics? Derive Clausius-Mosotti equation for a solid dielectric exhibiting electronic polarizability. (4 M)**

**6a. Find the total polarizability of CO<sub>2</sub>, if its susceptibility is  $0.985 \times 10^{-3}$  and density is 1.977 kg/ m<sup>3</sup> (4M)**

**Ans:  $3.22 \times 10^{-40}$**

$$\chi = \frac{N_A \rho \alpha_e}{M \epsilon_0}$$

Here  $\chi = 0.985 \times 10^{-3}$ ,  $N_A = 6.023 \times 10^{26}$ ,  $\rho = 1.977$  and  $M$  = Molecular weight  $\text{CO}_2$  is 44.01 g/mol

$$\text{The total polarizability } \alpha_e = \frac{\chi M \epsilon_0}{N_A \rho}$$

$$= 0.985 \times 10^{-3} \times 44.01 \times 8.85 \times 10^{-12} / (6.023 \times 10^{26} \times 1.977) \\ = 3.22 \times 10^{-40} \text{ F.m}^2.$$

**6b. A solid elemental dielectric having density of  $3 \times 10^{28}$  atoms/m<sup>3</sup> shows an electronic polarizability of  $10^{-40}$  F.m<sup>2</sup>. Assuming the internal electric field to be a Lorentz field, find the dielectric constant of the material. (4M)**

**Ans: 1.339**

$$\epsilon_r = \frac{\alpha_e N}{\epsilon_0} + 1 \\ = \frac{10^{-40} \times 3 \times 10^{28}}{8.85 \times 10^{-12}} + 1 = 1.339$$

**6c. With usual notations show that  $\mathbf{P} = \epsilon_0(\epsilon_r - 1) \mathbf{E}$  (2M)**

$$\mathbf{P} = N \alpha_e \left[ \mathbf{E} + \frac{\mathbf{P}}{3\epsilon_0} \right]$$

We know that,

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$\frac{\mathbf{P}}{\mathbf{E}} = \frac{\mathbf{D}}{\mathbf{E}} - \epsilon_0$$

From the definition of electric displacement vector,

$$\mathbf{D} = \epsilon \mathbf{E}$$

Therefore,

$$\frac{\mathbf{P}}{\mathbf{E}} = \epsilon - \epsilon_0 = \epsilon_r \epsilon_0 - \epsilon_0$$

where

$$\epsilon_r = \epsilon / \epsilon_0$$

$$\frac{\mathbf{P}}{\mathbf{E}} = \epsilon_0(\epsilon_r - 1)$$

or,

$$\mathbf{P} = \epsilon_0 (\epsilon_r - 1) \mathbf{E}$$

**7a. Write a short note on Dia, Para, Ferromagnetic materials and their applications (5M)**

7b. The index of refraction for LiF is 1.395, its density is  $2.635 \times 10^3 \text{ kg/m}^3$ , and its molecular weight is  $26 \times 10^{-3} \text{ kg/mol}$ . (5M)

Recall that  $\epsilon_0 = 8.854 \times 10^{-12} \text{ C/V-m}$ .

1: Calculate the total polarizability for LiF.

2: Calculate the electronic contribution to the total polarizability.

Combine your information to calculate the ionic polarizability,  $\alpha_i$

$$\begin{aligned} \text{Answers: } \alpha &= [3(8.854 \times 10^{-12})(26 \times 10^{-3})/(6.02 \times 10^{23})(2.635 \times 10^3)][(9 - 1)/(9 + 2)] = 3.166 \times 10^{-40} \text{ F-m}^2; \alpha_e = \\ &= [3(8.854 \times 10^{-12})(26 \times 10^{-3})/2(6.02 \times 10^{23})(2.635 \times 10^3)][(1.95 - 1)/(1.95 + 2)] = \\ &= 5.2 \times 10^{-41} \text{ F-m}^2. \alpha_i = \alpha - \alpha_e = 3.16 \times 10^{-40} - 5.2 \times 10^{-41} = 2.64 \times 10^{-40} \text{ F-m}^2. \end{aligned}$$

8a. A magnetic material has a magnetization of 3000 A/m and a flux density of 0.005 wb/m<sup>2</sup>. Calculate the magnetic force and the relative permeability of the material (4M)

Ans: 977.72 A/m and 4.068

$$H = \frac{B}{\mu_0} - M = \frac{0.005}{12.57 \times 10^{-7}} - 3000 = 977.72 \text{ Am}^{-1}$$

$$\mu = \frac{B}{\mu_0 H} = 0. \frac{005}{12.57 \times 10^{-7} \times 977.72} = 4.068$$

8b. Assume that iron atoms have magnetic moment of two Bohr magnetons. Calculate the Curie constant if its density is 7150 kg/m<sup>3</sup> and atomic weight is 55.8 (4M)

Ans: 0.00241

$$\begin{aligned} C &= N\beta^2\mu_0/k \\ C &= \frac{6.022 \times 10^{23} \times 7150 \times (2 \times 9.27 \times 10^{-24})^2 \times 4 \times 3.1415 \times 10^{-7}}{55.8 \times 1.38 \times 10^{-23}} \\ &= 0.00241 \text{ or } 2.41 \times 10^{-3} \end{aligned}$$

8c. If a material have  $\epsilon < 0$ ,  $\mu < 0$ , What will happen electromagnetic radiation fall on surface? Why? (2M)

Ans: If a material have  $\epsilon < 0$ ,  $\mu < 0$ , resultant refractive index is NAGATIVE, So that electromagnetic radiation completely observed by the material and usually found phenomena in meta materials.

9a. Critical temperature of a superconductor at zero magnetic field is  $T_c$ . Determine the temperature at which the critical field becomes half of its value at 0K (4M)

Ans: 0.707  $T_c$

$$\text{Super conductor critical field } H_c = H_0[1 - (T/T_c)^2] \Rightarrow 1/2 = 1 - (T/T_c)^2 \Rightarrow T_{1/2} = \sqrt{T_c^2/2} \Rightarrow T_{1/2} = 0.707 T_c$$

**9b. For a certain metal the critical magnetic field is  $4 \times 10^3$  A/m at 6K and  $2 \times 10^4$  A/m at 0K. Determine its transition temperature (4M)**

$$T_c = \frac{T}{\left[1 - \left(\frac{H_c(T)}{H_c(0)}\right)\right]^{1/2}}$$

$$T_c = \frac{6}{\left[1 - \left(\frac{4 \times 10^3}{2 \times 10^4}\right)\right]^{1/2}} = 6.7 \text{ K}$$

**10a. Explain Size effects in nano materials (4M)**

**10b. How to characterize nano materials? Explain any one method with neat sketch (4M)**

Ans: SEM, TEM, AFM

**10c. A material has completely filled electronic states and possess a small value of induced magnetic moment, when there is an applied magnetic field. What type materials they are? Find the susceptibility of that material? (4M)**

**Langevin's Theory of Diamagnetism**

Diamagnetic material has completely filled electronic states and possess a small value of induced magnetic moment.

$$\chi = -\frac{Ne^2 \mu_0 \mu_r r_o^2}{6m}$$