National Institute of Technology Goa

Programme Name: B.Tech
Online Mid Semester Examinations, October 2022

Course Name: **Mathematics-III**
Date: 17/10/2022
Duration: 90 Minutes

Course Code: **MA200**
Time: 02:00PM - 03:30 AM
Max. Marks: 50

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1. Answer All Questions.
2. **No marks will be given if the explanation of your answer is missing.**
3. The question paper consists of **two** pages.

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1. Determine $'p'$ such that the function $f(z) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{px}{y}\right)$ is analytic function. [4M]
2. Find whether the function $v = x^2 - y^2 + \frac{x}{x^2+y^2}$ can be imaginary part of an analytic function or not?. If so, find the analytic function and it's real part. [6M]
3. If $\tan(x+iy) = A + iB$ show that $A^2 + B^2 + 2A\cot 2x = 1$. [4M]
4. Determine all values and the principal value of (a) $\ln(-4)$, (b) $\ln(3i)$, (a) $\ln(\sqrt{3}-i)$. [6M]
5. Evaluate $\int_C (z^2+3z)\,dz$ along: [8M]
   (a) Circle $|z| = 2$ from $(2,0)$ to $(0,2)$ in counterclockwise direction.
   (b) The straight line from $(2,0)$ to $(0,2)$.
   (c) The straight lines from $(2,0)$ to $(0,2)$ and then from $(2,2)$ to $(0,2)$.
6. If a function $F(z)$ is defined to be [8M]

$$F(\alpha) = \oint_C \frac{5z^2 - 4z + 3}{z - \alpha}\,dz,$$

where 'C' is the ellipse $16x^2 + 9y^2 = 144$, then evaluate:

(a) $F(2)$
(b) $F(4)$
(c) $F(-3i)$
(d) $F'(i)$ and

(e) $F''(-2i)$

7. Discuss the type of singularity at singular points of $\sin z \left(\frac{e^z+1}{e^z-1}\right)$. [6M]

8. Evaluate $\int_{|z|=10} \frac{e^z}{e^z-2i} dz$ using Argument principle. [4M]

9. Evaluate $\oint_{|z|=1} e^{-\frac{1}{z}} \sin\left(\frac{1}{z}\right) dz$ using Residue theorem. [4M]

* * *ALL THE BEST * **

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