14-1. A woman having a mass of 70 kg stands in an elevator which has a downward acceleration of 4 m/s^2 starting from rest. Determine the work done by her weight and the work of the normal force which the floor exerts on her when the elevator descends 6 m. Explain why the work of these forces is different.

$+\downarrow \Sigma F_y = ma_y; \qquad 70(9.81) - N_P = 70(4)$

$N_P = 406.7$ N

$U_W = 6(686.7) = 4.12$ kJ **Ans**

$U_{N_P} = -6(406.7) = -2.44$ kJ **Ans**

The difference accounts for a change in kinetic energy. **Ans**

Note: $v^2 = v_0^2 + 2a_c(s - s_0)$

$v^2 = 0 + 2(4)(6 - 0)$

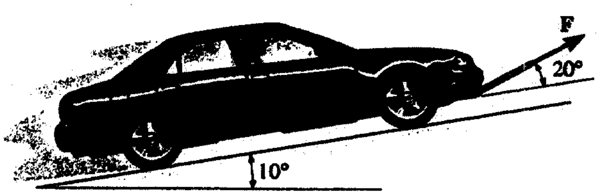
$v = 6.928$ m/s

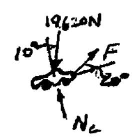
$\Delta T = \frac{1}{2}(70)(6.928)^2 = 1.68$ kJ

Also, $T_1 + \Sigma U_{1-2} = T_2$

$\Delta T = \Sigma U_{1-2} = 4.12 - 2.44 = 1.68$ kJ

14-2. The car having a mass of 2 Mg is originally traveling at 2 m/s. Determine the distance it must be towed by a force F = 4 kN in order to attain a speed of 5 m/s. Neglect friction and the mass of the wheels.



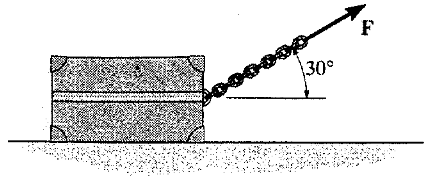


$T_1 + \Sigma U_{1-2} = T_2$

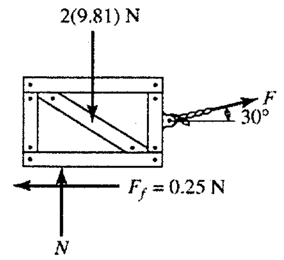
$\frac{1}{2}(2000)(2)^2 + [\,4000\cos 20°(s) - 19\,620\sin 10°(s)\,] = \frac{1}{2}(2000)(5)^2$

$s = 59.7$ m **Ans**

14-3. The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N, where s is measured in meters. When $s = 15$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.25$ N. Applying Eq. 13-7, we have



$$+\uparrow \sum F_y = ma_y; \quad N + \left(\frac{300}{1+s}\right)\sin 30° - 2(9.81) = 2(0)$$

$$N = \left(-\frac{150}{1+s} + 19.62\right) \text{N}$$

Principle of Work and Energy: The horizontal component of force F which acts in the direction of displacement does *positive* work, whereas the friction force $F_f = 0.25\left(-\frac{150}{1+s} + 19.62\right) = \left(-\frac{37.5}{1+s} + 4.905\right)$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force F and the weight of the crate do not displace hence do no work. Applying Eq. 14-7, we have

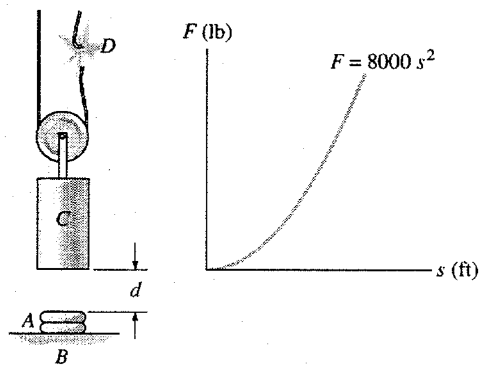
$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(2)(8^2) + \int_{15\text{ m}}^{25\text{ m}} \left(\frac{300}{1+s}\right)\cos 30°\, ds$$

$$- \int_{15\text{ m}}^{25\text{ m}} \left(-\frac{37.5}{1+s} + 4.905\right) ds = \frac{1}{2}(2)v^2$$

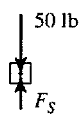
$v = 12.6$ m/s **Ans**

***14-4.** The "air spring" A is used to protect the support structure B and prevent damage to the conveyor-belt tensioning weight C in the event of a belt failure D. The force developed by the spring as a function of its deflection is shown by the graph. If the weight is 50 lb and is suspended a height $d = 1.5$ ft above the top of the spring, determine the maximum deformation of the spring in the event the conveyor belt fails. Neglect the mass of the pulley and belt.



$$T_1 + \sum U_{1-2} = T_2$$

$$0 + \left[50(1.5 + s) - \int_0^s 8000 s^2 ds\right] = 0$$

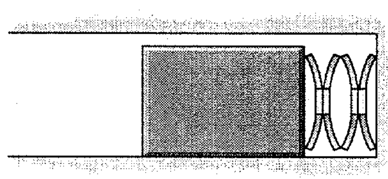


$$50(1.5 + s) - \frac{8000 s^2}{3} = 0$$

$$8000 s^2 - 150 s - 225 = 0$$

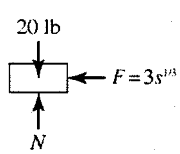
$z = 0.3246$ ft $= 3.90$ in. **Ans**

14-5. The smooth plug has a weight of 20 lb and is pushed against a series of Belleville spring washers so that the compression in the spring is $s = 0.05$ ft. If the force of the spring on the plug is $F = (3s^{1/3})$ lb, where s is given in feet, determine the speed of the plug after it moves away from the spring. Neglect friction.



$$T_1 + \sum U_{1-2} = T_2$$

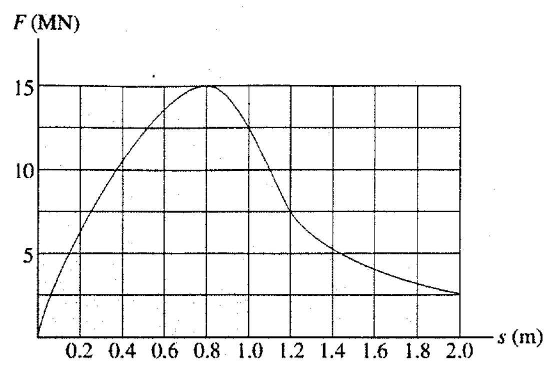
$$0 + \int_0^{0.05} 3s^{\frac{1}{3}} ds = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$



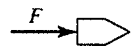
$$3\left(\frac{3}{4}\right)(0.05)^{\frac{4}{3}} = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$

$v = 0.365$ ft/s **Ans**

14-6. When a 7-kg projectile is fired from a cannon barrel that has a length of 2 m, the explosive force exerted on the projectile, while it is in the barrel, varies in the manner shown. Determine the approximate muzzle velocity of the projectile at the instant it leaves the barrel. Neglect the effects of friction inside the barrel and assume the barrel is horizontal.



The work done is measured as the area under the force—displacement curve. This area is approximately 31.5 squares. Since each square has an area of $2.5(10^6)(0.2)$

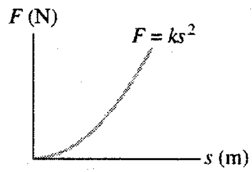


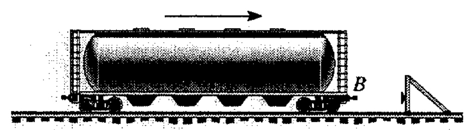
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + [(31.5)(2.5)(10^6)(0.2)] = \frac{1}{2}(7)(v_2)^2$$

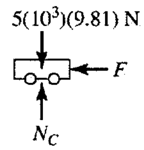
$v_2 = 2121$ m/s $= 2.12$ km/s (approx.) **Ans**

14-7. Design considerations for the bumper B on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.





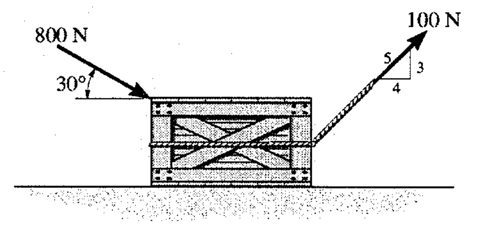
$$\frac{1}{2}(5000)(4)^2 - \int_0^{0.2} ks^2\,ds = 0$$



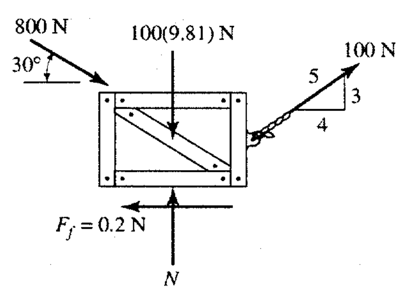
$$40\,000 - k\frac{(0.2)^3}{3} = 0$$

$k = 15.0$ MN/m^2 **Ans**

***14-8.** The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.2$ N. Applying Eq. 13-7, we have



$$+\uparrow \sum F_y = ma_y; \quad N + 100\left(\frac{3}{5}\right) - 800\sin 30° - 100(9.81) = 100(0)$$

$$N = 1321 \text{ N}$$

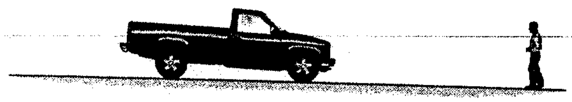
Principle of Work and Energy: The horizontal components of forces 800 N and 1000 N which act in the direction of displacement do *positive* work, whereas the friction force $F_f = 0.2(1321) = 264.2$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical components of the 800 N and 1000 N forces and the weight of the crate do not displace hence do no work. Since the crate is originally at rest, $T_1 = 0$. Applying Eq. 14-7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 800\cos 30°(s) + 100\left(\frac{4}{5}\right)s - 264.2s = \frac{1}{2}(100)(6^2)$$

$s = 3.54$ m **Ans**

14-9. When the driver applies the brakes of a light truck traveling 40 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?



$40 \text{ km/h} = \frac{40(10^3)}{3600} = 11.11 \text{ m/s} \qquad 80 \text{ km/h} = 22.22 \text{ m/s}$

$T_1 + \Sigma U_{1-2} = T_2$

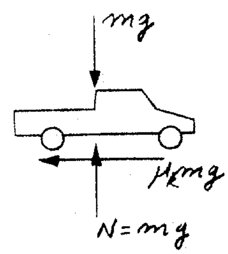
$\frac{1}{2}m(11.11)^2 - \mu_k mg(3) = 0$

$\mu_k g = 20.576$

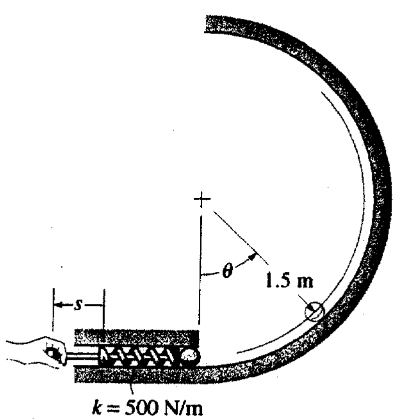
$T_1 + \Sigma U_{1-2} = T_2$

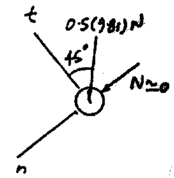
$\frac{1}{2}m(22.22)^2 - (20.576)m(d) = 0$

$d = 12 \text{ m}$ **Ans**



14-10. The 0.5-kg ball of negligible size is fired up the vertical circular track using the spring plunger. The plunger keeps the spring compressed 0.08 m when $s \doteq 0$. Determine how far s it must be pulled back and released so that the ball will begin to leave the track when $\theta = 135°$.





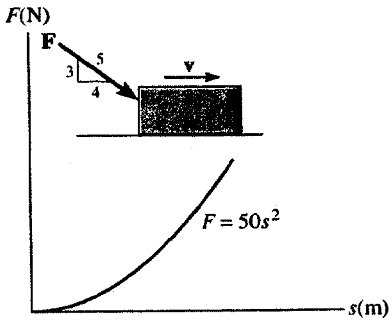
$\Sigma F_n = ma_n; \quad 0.5(9.81)\sin 45° = 0.5\left(\frac{v^2}{1.5}\right) \qquad v^2 = 10.41 \text{ m}^2/\text{s}^2$

$T_1 + \Sigma U_{1-2} = T_2$

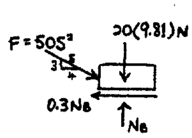
$0 + \left\{\left(\frac{1}{2}(500)(s+0.08)^2 - \frac{1}{2}(500)(0.08)^2\right) - 0.5(9.81)(1.5+1.5\sin 45°)\right\} = \frac{1}{2}(0.5)(10.41)$

$s = 0.1789 \text{ m} = 179 \text{ mm}$ **Ans**

■**14-11.** The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position s of the block. Determine how far the block slides before its velocity becomes 5 m/s. When $s = 0$ the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



$+\uparrow \Sigma F_y = 0; \qquad N_B - 20(9.81) - \frac{3}{5}(50\,s^2) = 0$



$N_B = 196.2 + 30\,s^2$

$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}(20)(2)^2 + \frac{4}{5}\int_0^s 50\,s^2\,ds - 0.3(196.2)(s) - 0.3\int_0^s 30\,s^2\,ds = \frac{1}{2}(20)(5)^2$$

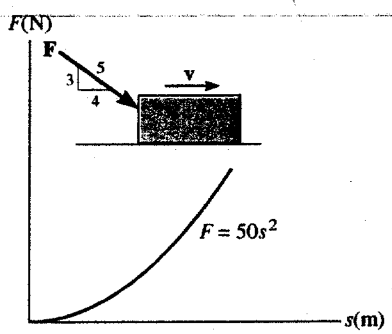
$40 + 13.33\,s^3 - 58.86\,s - 3\,s^3 = 250$

$s^3 - 5.6961\,s - 20.323 = 0$

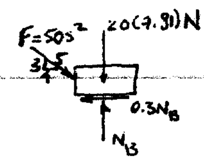
Solving for the real root yields

$s = 3.41$ m **Ans**

***14-12.** The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with position s of the block. Determine the speed of the block after it slides 3 m. When $s = 0$ the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



$+\uparrow \Sigma F_y = 0; \qquad N_B - 20(9.81) - \frac{3}{5}(50\,s^2) = 0$



$N_B = 196.2 + 30\,s^2$

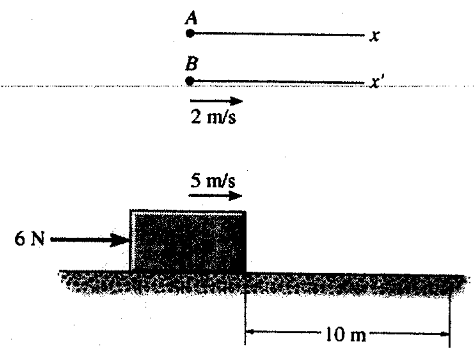
$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}(20)(2)^2 + \frac{4}{5}\int_0^3 50\,s^2\,ds - 0.3(196.2)(3) - 0.3\int_0^3 30\,s^2\,ds = \frac{1}{2}(20)(v)^2$$

$40 + 360 - 176.58 - 81 = 10\,v^2$

$v = 3.77$ m/s **Ans**

14-13. As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of 5 m/s and travels 10 m, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of 2 m/s relative to A. *Hint:* The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.



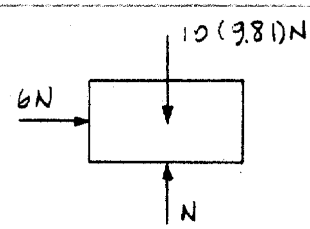
Observer A :

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10)(5)^2 + 6(10) = \frac{1}{2}(10)v_2^2$$

$v_2 = 6.08$ m/s   Ans

Observer B :



$F = ma$

$6 = 10a \qquad a = 0.6\ \text{m/s}^2$

$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$10 = 0 + 5t + \frac{1}{2}(0.6)t^2$$

$$t^2 + 16.67t - 33.33 = 0$$

$$t = 1.805\ \text{s}$$

At $v = 2$ m/s, $s' = 2(1.805) = 3.609$ m

Block moves $10 - 3.609 = 6.391$ m

Thus

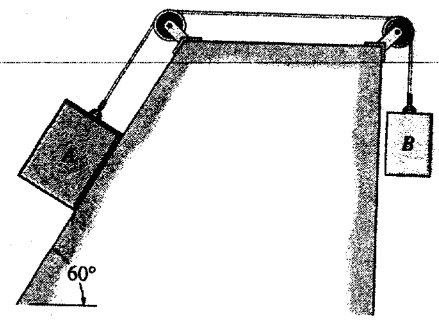
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10)(3)^2 + 6(6.391) = \frac{1}{2}(10)v_2^2$$

$v_2 = 4.08$ m/s   Ans

Note that this result is 2m/s less than that observed by A.

14-14. Determine the velocity of the 20-kg block A after it is released from rest and moves 2 m down the plane. Block B has a mass of 10 kg and the coefficient of kinetic friction between the plane and block A is $\mu_k = 0.2$. Also, what is the tension in the cord?



Block A :

$+\nwarrow \Sigma F_y = 0; \quad N_A - 20(9.81)\cos 60^\circ = 0$

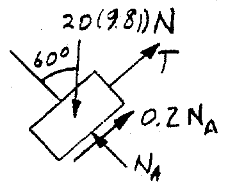
$N_A = 98.1 \text{ N}$

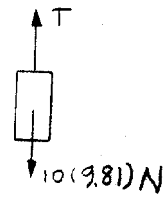
System :

$T_1 + \Sigma U_{1-2} = T_2$

$(0+0) + 20(9.81)(\sin 60^\circ)2 - 0.2(98.1)(2) - 10(9.81)(2) = \frac{1}{2}(20)v^2 + \frac{1}{2}(10)v^2$

$v = 2.638 = 2.64 \text{ m/s}$ **Ans**





Block B :

$T_1 + \Sigma U_{1-2} = T_2$

$0 + T(2) - 10(9.81)(2) = \frac{1}{2}(10)(2.638)^2$

$T = 115 \text{ N}$ **Ans**

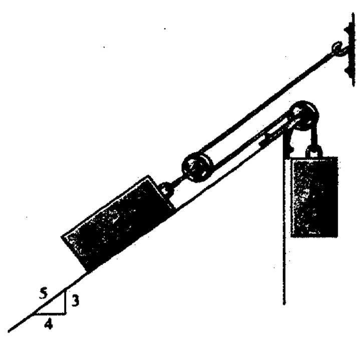
Also, block A :

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 20(9.81)(\sin 60^\circ)(2) - T(2) - 0.2(98.1)(2) = \frac{1}{2}(20)(2.638)^2$

$T = 115 \text{ N}$ **Ans**

14-15. Block A has a weight of 60 lb and block B has a weight of 10 lb. Determine the speed of block A after it moves 5 ft down the plane, starting from rest. Neglect friction and the mass of the cord and pulleys.



$2s_A + s_B = l$

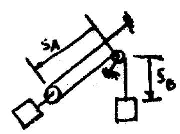
$2\Delta s_A + \Delta s_B = 0$

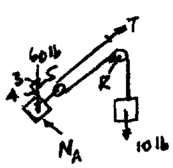
$2v_A + v_B = 0$

$T_1 + \Sigma U_{1-2} = T_2$

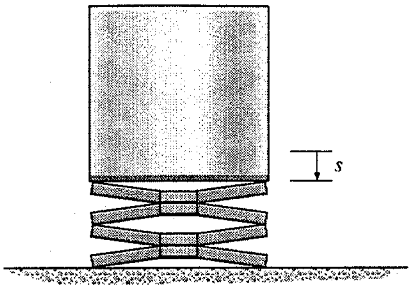
$0 + 60(\frac{3}{5})(5) - 10(10) = \frac{1}{2}(\frac{60}{32.2})v_A^2 + \frac{1}{2}(\frac{10}{32.2})(2v_A)^2$

$v_A = 7.18 \text{ ft/s}$ **Ans**

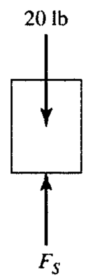




*14-16. The smooth plug has a weight of 20 lb and is pushed against a series of Belleville spring washers so that the compression in the spring is $s = 0.05$ ft. If the force of the spring on the plug is $F = (100s^{1/3})$ lb, where s is given in feet, determine the speed of the plug just after it moves away from the spring, i.e., at $s = 0$.



Principle of Work and Energy: The spring force which acts in the direction of displacement does *positive* work, whereas the weight of the block does *negative* work since it acts in the opposite direction to that of displacement. Since he block is initially at rest, $T_1 = 0$. Applying Eq. 14-7, we have

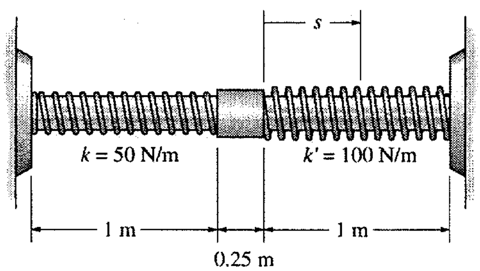


$$T_1 + \sum U_{1-2} = T_2$$

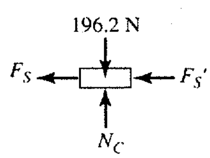
$$0 + \int_0^{0.05\text{ ft}} 100s^{1/3}\,ds - 20(0.05) = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$

$V = 1.11$ ft/s **Ans**

14-17. The collar has a mass of 20 kg and rests on the smooth rod. Two springs are attached to it and the ends of the rod as shown. Each spring has an uncompressed length of 1 m. If the collar is displaced $s = 0.5$ m and released from rest, determine its velocity at the instant it returns to the point $s = 0$.



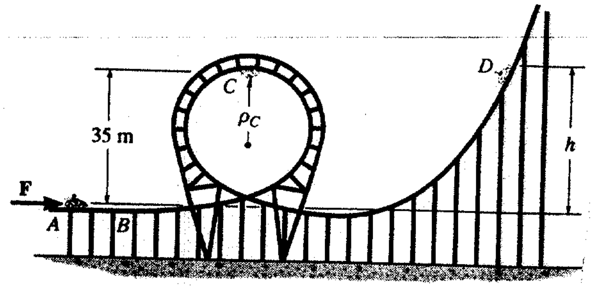
$$T_1 + \sum U_{1-2} = T_2$$



$$0 + \frac{1}{2}(50)(0.5)^2 + \frac{1}{2}(100)(0.5)^2 = \frac{1}{2}(20)v_C^2$$

$v_C = 1.37$ m/s **Ans**

14-18. Determine the height h to the top of the incline D to which the 200-kg roller coaster car will reach, if it is launched at B with a speed just sufficient for it to round the top of the loop at C without leaving the track. The radius of curvature at C is $\rho_C = 25$ m.



Equation of Motion : Here, it is required that $N = 0$. Applying Eq. 13−8 to FBD(a), we have

$$\Sigma F_n = ma_n; \quad 200(9.81) = 200\left(\frac{v_C^2}{25}\right) \quad v_C^2 = 245.25 \text{ m}^2/\text{s}^2$$

Principle of Work and Energy : The weight of the roller coaster car and passengers do *negative* work since they act in the opposite direction to that of displacement. When the roller coaster car travels from B to C, applying Eq. 14−7, we have

$$T_B + \sum U_{B-C} = T_C$$

$$\frac{1}{2}(200)\,v_B^2 - 200(9.81)(35) = \frac{1}{2}(200)(245.25)$$

$$v_B = 30.53 \text{ m/s}$$

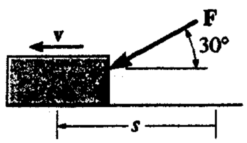
When the roller coaster car travels from B to D, it is required that the car stops at D, hence $T_D = 0$.

$$T_B + \sum U_{B-D} = T_D$$

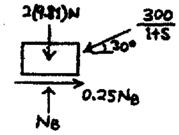
$$\frac{1}{2}(200)\left(30.53^2\right) - 200(9.81)(h) = 0$$

$$h = 47.5 \text{ m} \qquad \textbf{Ans}$$

14-19. The 2-kg block is subjected to a force having a constant direction and a magnitude $F = (300/(1+s))$ N, where s is in meters. When $s = 4$ m, the block is moving to the left with a speed of 8 m/s. Determine its speed when $s = 12$ m. The coefficient of kinetic friction between the block and the ground is $\mu_k = 0.25$.



$$+\uparrow \Sigma F_y = 0; \quad N_B = 2(9.81) + \frac{150}{1+s}$$



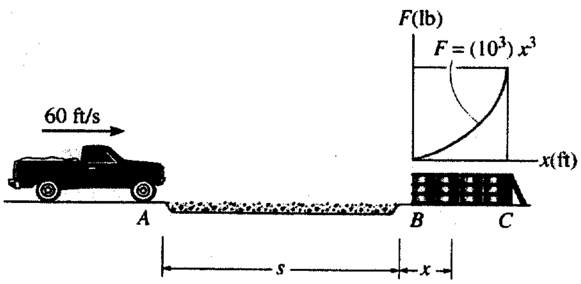
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(2)(8)^2 - 0.25[2(9.81)(12-4)] - 0.25\int_4^{12} \frac{150}{1+s}\,ds + \int_4^{12} \frac{300}{1+s}\,ds \cos 30^\circ = \frac{1}{2}(2)(v_2^2)$$

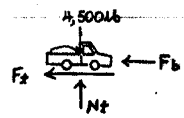
$$v_2^2 = 24.76 - 37.5 \ln\left(\frac{1+12}{1+4}\right) + 259.81 \ln\left(\frac{1+12}{1+4}\right)$$

$$v_2 = 15.4 \text{ m/s} \qquad \textbf{Ans}$$

***14-20.** The motion of a truck is arrested using a bed of loose stones AB and a set of crash barrels BC. If experiments show that the stones provide a rolling resistance of 160 lb per wheel and the crash barrels provide a resistance as shown in the graph, determine the distance x the 4500-lb truck penetrates the barrels if the truck is coasting at 60 ft/s when it approaches A. Take $s = 50$ ft and neglect the size of the truck.



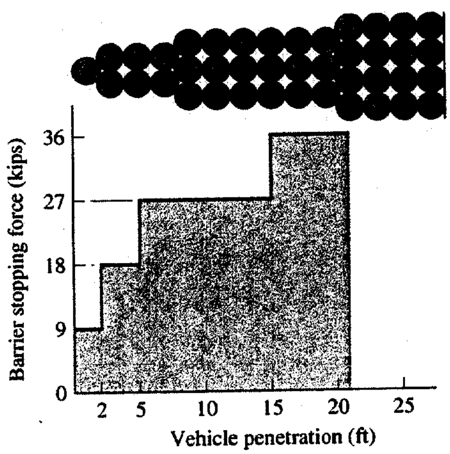
$$\frac{1}{2}\left(\frac{4500}{32.2}\right)(60)^2 - 4(160)(50) - \int_0^x (10)^3 x^3\,dx = 0$$



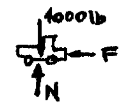
$$219\,552.80 - \frac{10^3}{4}x^4 = 0$$

$x = 5.44$ ft **Ans**

14-21. The crash cushion for a highway barrier consists of a nest of barrels filled with an impact-absorbing material. The barrier stopping force is measured versus the vehicle penetration into the barrier. Determine the distance a car having a weight of 4000 lb will penetrate the barrier if it is originally traveling at 55 ft/s when it strikes the first barrel.



$$T_1 + \Sigma U_{1-2} = T_2$$



$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(55)^2 - Area = 0$$

$Area = 187.89$ kip·ft

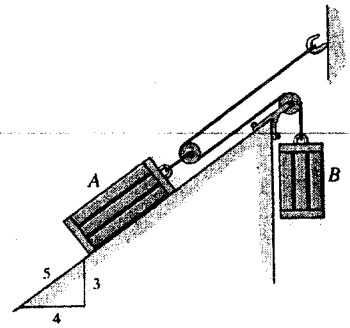
$$2(9) + (5-2)(18) + x(27) = 187.89$$

$x = 4.29$ ft $< (15-5)$ ft **(O.K!)**

Thus

$s = 5$ ft $+ 4.29$ ft $= 9.29$ ft **Ans**

14-22. The two blocks A and B have weights $W_A = 60$ lb and $W_B = 10$ lb. If the kinetic coefficient of friction between the incline and block A is $\mu_k = 0.2$, determine the speed of A after it moves 3 ft down the plane starting from rest. Neglect the mass of the cord and pulleys.

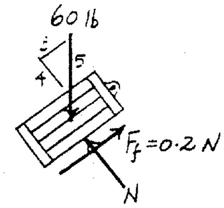


Kinematics : The speed of the block A and B can be related by using position coordinate equation.

$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A = 2(3) = 6 \text{ ft}$$

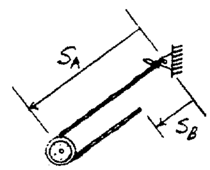
$$2v_A - v_B = 0 \qquad [1]$$



Equation of Motion : Applying Eq. 13 – 7, we have

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 60\left(\frac{4}{5}\right) = \frac{60}{32.2}(0) \qquad N = 48.0 \text{ lb}$$

Principle of Work and Energy : By considering the whole system, W_A which acts in the direction of the displacement does *positive* work. W_B and the friction force $F_f = \mu_k N = 0.2(48.0) = 9.60$ lb does *negative* work since they act in the opposite direction to that of displacement. Here, W_A is being displaced vertically (downward) $\frac{3}{5}\Delta s_A$ and W_B is being displaced vertically (upward) Δs_B. Since blocks A and B are at rest initially, $T_1 = 0$. Applying Eq. 14 – 7, we have



$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_A\left(\frac{3}{5}\Delta s_A\right) - F_f\Delta s_A - W_B\Delta s_B = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

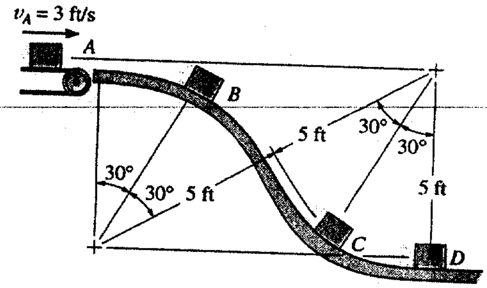
$$60\left[\frac{3}{5}(3)\right] - 9.60(3) - 10(6) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$1236.48 = 60v_A^2 + 10v_B^2 \qquad [2]$$

qs. [1] and [2] yields

$$v_A = 3.52 \text{ ft/s} \qquad \textbf{Ans}$$
$$v_B = 7.033 \text{ ft/s}$$

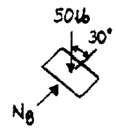
14-23. Packages having a weight of 50 lb are delivered to the chute at $v_A = 3$ ft/s using a conveyor belt. Determine their speeds when they reach points B, C, and D. Also calculate the normal force of the chute on the packages at B and C. Neglect friction and the size of the packages.



$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5)(1 - \cos 30°) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_B^2$$

$v_B = 7.221 = 7.22$ ft/s **Ans**



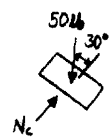
$$+\swarrow \Sigma F_n = ma_n; \qquad -N_B + 50\cos 30° = \left(\frac{50}{32.2}\right)\left[\frac{(7.221)^2}{5}\right]$$

$N_B = 27.1$ lb **Ans**

$$T_A + \Sigma U_{A-C} = T_C$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5\cos 30°) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_C^2$$

$v_C = 16.97 = 17.0$ ft/s **Ans**



$$+\nearrow \Sigma F_n = ma_n; \qquad N_C - 50\cos 30° = \left(\frac{50}{32.2}\right)\left[\frac{(16.97)^2}{5}\right]$$

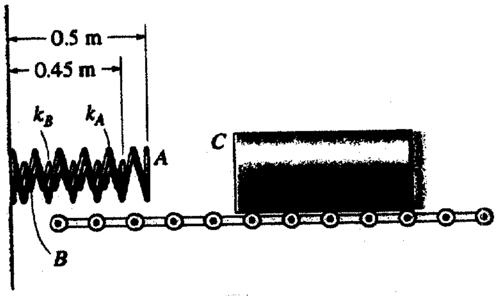
$N_C = 133$ lb **Ans**

$$T_A + \Sigma U_{A-D} = T_D$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_D^2$$

$v_D = 18.2$ ft/s **Ans**

*14-24. The steel ingot has a mass of 1800 kg. It travels along the conveyor at a speed $v = 0.5$ m/s when it collides with the "nested" spring assembly. Determine the maximum deflection in each spring needed to stop the motion of the ingot. Take $k_A = 5$ kN/m, $k_B = 3$ kN/m.



Assume both springs compress

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(1800)(0.5)^2 - \frac{1}{2}(5000)s^2 - \frac{1}{2}(3000)(s-0.05)^2 = 0$$

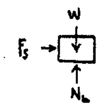
$$225 - 2500\,s^2 - 1500(s^2 - 0.1\,s + 0.0025) = 0$$

$$s^2 - 0.0375\,s - 0.05531 = 0$$

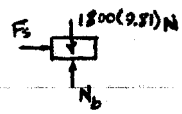
$s = 0.2547$ m > 0.05 m (O.K!)

$s_A = 0.255$ m Ans

$s_B = 0.205$ m Ans



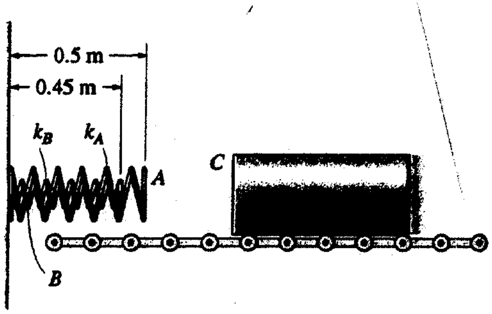
14-25. The steel ingot has a mass of 1800 kg. It travels along the conveyor at a speed $v = 0.5$ m/s when it collides with the "nested" spring assembly. If the stiffness of the outer spring is $k_A = 5$ kN/m, determine the required stiffness k_B of the inner spring so that the motion of the ingot is stopped at the moment the front, C, of the ingot is 0.3 m from the wall.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(1800)(0.5)^2 - \frac{1}{2}(5000)(0.5 - 0.3)^2 - \frac{1}{2}(k_B)(0.45-0.3)^2 = 0$$

$k_B = 11.1$ kN/m Ans



14-26. Block A has a weight of 60 lb and block B has a weight of 10 lb. Determine the distance A must descend from rest before it obtains a speed of 8 ft/s. Also, what is the tension in the cord supporting block A? Neglect the mass of the cord and pulleys.

$$2\,s_A + s_B = l$$

$$2\,\Delta s_A = -\Delta s_B$$

$$2\,v_A = -v_B$$

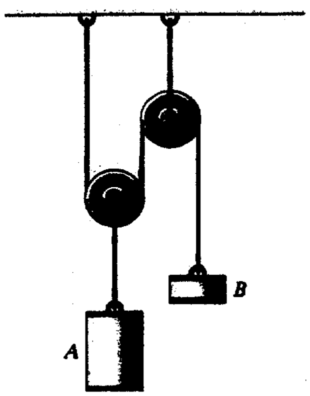
For $v_A = 8$ ft/s, $v_B = -16$ ft/s

For the system:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$[0+0] + [60(s_A) - 10(2s_A)] = \frac{1}{2}\left(\frac{60}{32.2}\right)(8)^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(-16)^2$$

$s = 2.484 = 2.48$ ft Ans



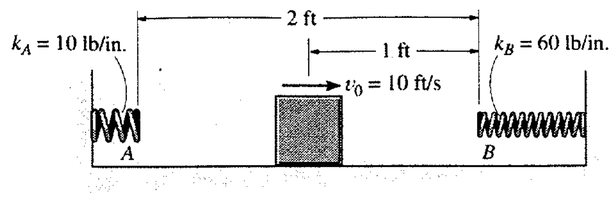
For block A:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 60(2.484) - T_A(2.484) = \frac{1}{2}\left(\frac{60}{32.2}\right)(8)^2$$

$T_A = 36.0$ lb Ans

14-27. The 25-lb block has an initial speed of $v_0 = 10$ ft/s when it is midway between springs A and B. After striking spring B, it rebounds and slides across the horizontal plane toward spring A, etc. If the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the total distance traveled by the block before it comes to rest.



Principle of Work and Energy: Here, the friction force $F_f = \mu_k N = 0.4(25) = 10.0$ lb. Since the friction force is always opposite the motion, it does negative work. When the block strikes spring B and stops momentarily, the spring force does *negative* work since it acts in the opposite direction to that of displacement. Applying Eq. 14-7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{25}{32.2}\right)(10^2) - 10(1 + s_1) - \frac{1}{2}(60)s_3^2 = 0$$

$$s_1 = 0.8275 \text{ ft}$$

Assume the block bounces back and stops without striking spring A. The spring force does *positive* work since it acts in the direction of displacement. Applying Eq. 14-7, we have

$$T_2 + \sum U_{2-3} = T_3$$

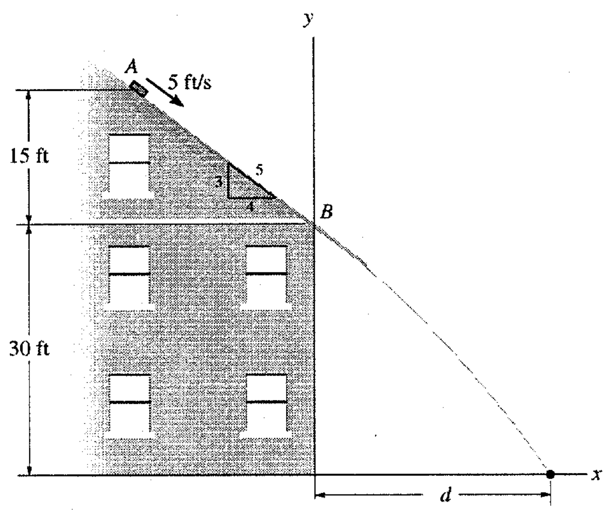
$$0 + \frac{1}{2}(60)(0.8275^2) - 10(0.8275 + s_2) = 0$$

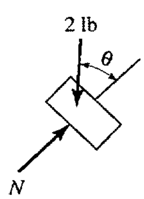
$$s_2 = 1.227 \text{ ft}$$

Since $s_2 = 1.227$ ft < 2 ft, the block stops before it strikes spring A. Therefore, the above assumption was correct. Thus, the total distance traveled by the block before it stops is

$$s_{\text{Tot}} = 2s_1 + s_2 + 1 = 2(0.8275) + 1.227 + 1 = 3.88 \text{ ft} \quad \textbf{Ans}$$

***14-28.** The 2-lb brick slides down a smooth roof, such that when it is at A it has a velocity of 5 ft/s. Determine the speed of the block just before it leaves the surface at B, the distance d from the wall to where it strikes the ground, and the speed at which it hits the ground.





$$T_A + \sum U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(15) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

$$v_B = 31.48 \text{ ft/s} = 31.5 \text{ ft/s} \quad \textbf{Ans}$$

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$$d = 0 + 31.48\left(\frac{4}{5}\right)t$$

$(+\downarrow)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$30 = 0 + 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(32.2)t^2$$

$$16.1t^2 + 18.888t - 30 = 0$$

Solving for the positive root.

$$t = 0.89916 \text{ s}$$

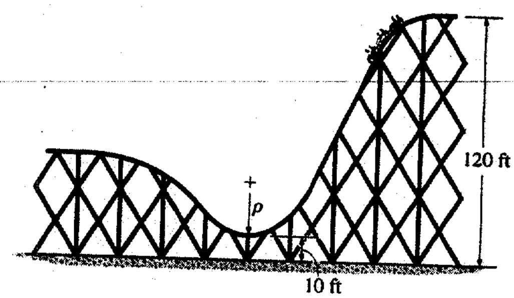
$$d = 31.48\left(\frac{4}{5}\right)(0.89916) = 22.6 \text{ ft} \quad \textbf{Ans}$$

$$T_A + \sum U_{A-C} = T_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(45) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2$$

$$v_C = 54.1 \text{ ft/s} \quad \textbf{Ans}$$

14-29. Roller coasters are designed so that riders will not experience more than 3.5 times their weight as a normal force against the seat of the car. Determine the smallest radius of curvature ρ of the track at its lowest point if the car has a speed of 5 ft/s at the crest of the drop. Neglect friction.



Principle of Work and Energy : Here, the rider is being displaced vertically (downward) by $s = 120 - 10 = 110$ ft and does *positive* work. Applying Eq. 14 – 7 we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{W}{32.2}\right)(5^2) + W(110) = \frac{1}{2}\left(\frac{W}{32.2}\right)v^2$$

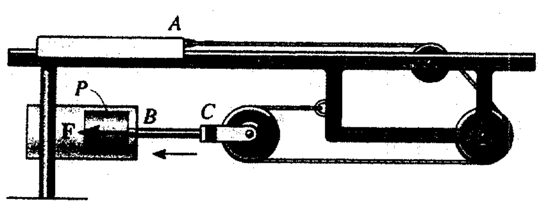
$$v^2 = 7109 \text{ ft}^2/\text{s}^2$$

Equation of Motion : It is required that $N = 3.5W$. Applying Eq. 13 – 7, we have

$$\Sigma F_n = ma_n; \quad 3.5W - W = \left(\frac{W}{32.2}\right)\left(\frac{7109}{\rho}\right)$$

$$\rho = 88.3 \text{ ft} \qquad \textbf{Ans}$$

14-30. The catapulting mechanism is used to propel the 10-kg slider A to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies a constant force $F = 20$ kN to rod BC such that it moves it 0.2 m, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.

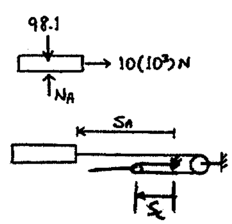


$$2s_C + s_A = l$$

$$2\Delta s_C + \Delta s_A = 0$$

$$2(0.2) = -\Delta s_A$$

$$-0.4 = \Delta s_A$$

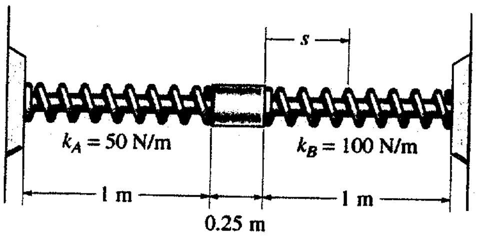


$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (10\,000)(0.4) = \frac{1}{2}(10)(v_A)^2$$

$$v_A = 28.3 \text{ m/s} \qquad \textbf{Ans}$$

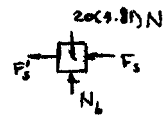
14-31. The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.



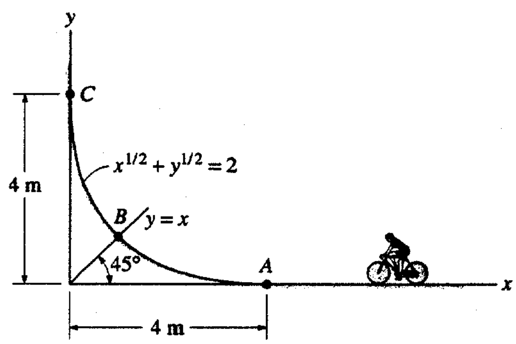
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(2)^2 - \frac{1}{2}(50)(s)^2 - \frac{1}{2}(100)(s)^2 = 0$$

$s = 0.730$ m Ans



***14-32.** The cyclist travels to point A, pedaling until he reaches a speed $v_A = 8$ m/s. He then coasts freely up the curved surface. Determine the normal force he exerts on the surface when he reaches point B. The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.



$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

For $y = x$,

$$2x^{\frac{1}{2}} = 2$$

$x = 1, y = 1$ (Point B)

Thus,

$$\tan\theta = \frac{dy}{dx} = -1$$

$$\theta = -45°$$

$$\frac{dy}{dx} = (-x^{-\frac{1}{2}})(y^{\frac{1}{2}})$$

$$\frac{d^2y}{dx^2} = y^{\frac{1}{2}}(\frac{1}{2}x^{-\frac{3}{2}}) - x^{-\frac{1}{2}}(\frac{1}{2})(y^{-\frac{1}{2}})(\frac{dy}{dx})$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}y^{\frac{1}{2}}x^{-\frac{3}{2}} + \frac{1}{2}(\frac{1}{x})$$

For $x = y = 1$.

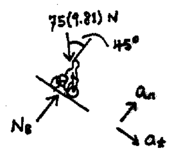
$$\frac{dy}{dx} = -1, \quad \frac{d^2y}{dx^2} = 1$$

$$\rho = \frac{[1+(-1)^2]^{3/2}}{1} = 2.828 \text{ m}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(75)(8^2) - 75(9.81)(1) = \frac{1}{2}(75)(v_B^2)$$

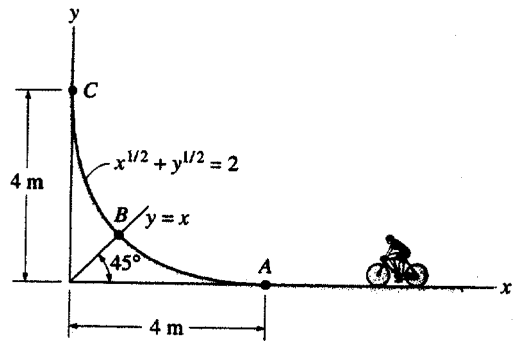
$$v_B^2 = 44.38$$



$\nearrow + \Sigma F_n = m\,a_n;\quad N_B - 9.81(75)\cos 45° = 75(\frac{44.38}{2.828})$

$N_B = 1.70$ kN Ans

14-33. The cyclist travels to point A, pedaling until he reaches a speed $v_A = 4$ m/s. He then coasts freely up the curved surface. Determine how high he reaches up the surface before he comes to a stop. Also, what are the resultant normal force on the surface at this point and his acceleration? The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.



$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(75)(4)^2 - 75(9.81)(y) = 0$$

$$y = 0.81549 \text{ m} = 0.815 \text{ m} \qquad \textbf{Ans}$$

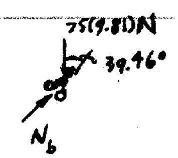
$$x^{1/2} + (0.81549)^{1/2} = 2$$

$$x = 1.2033 \text{ m}$$

$$\tan\theta = \frac{dy}{dx} = \frac{-(1.2033)^{-1/2}}{(0.81549)^{-1/2}} = -0.82323$$

$$\theta = -39.46°$$

$$\nearrow + \Sigma F_n = m\,a_n; \quad N_b - 9.81(75)\cos 39.46° = 0$$

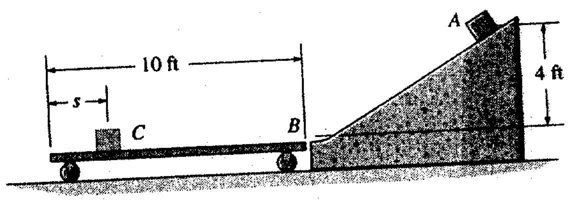


$$N_b = 568 \text{ N} \qquad \textbf{Ans}$$

$$+\searrow \Sigma F_t = m\,a_t; \quad 75(9.81)\sin 39.46° = 75\,a_t$$

$$a = a_t = 6.23 \text{ m/s}^2 \qquad \textbf{Ans}$$

14-34. The 30-lb box A is released from rest and slides down along the smooth ramp and onto the surface of a cart. If the cart is *fixed from moving*, determine the distance s from the end of the cart to where the box stops. The coefficient of kinetic friction between the cart and the box is $\mu_k = 0.6$.



Principle of Work and Energy : W_A which acts in the direction of the vertical displacement does *positive* work when the block displaces 4 ft vertically. The friction force $F_f = \mu_k N = 0.6(30) = 18.0$ lb does *negative* work since it acts in the opposite direction to that of displacement. Since the block is at rest initially and is required to stop, $T_A = T_C = 0$. Applying Eq. 14–7, we have

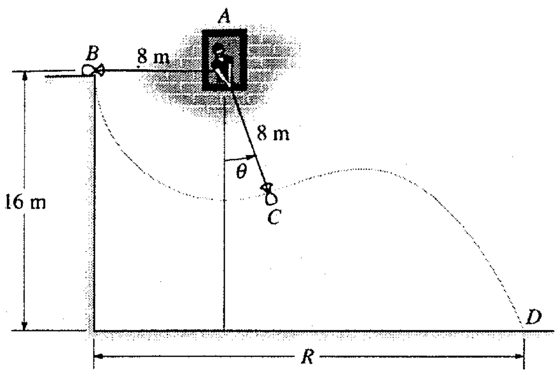
$$T_A + \sum U_{A-C} = T_C$$

$$0 + 30(4) - 18.0s' = 0 \qquad s' = 6.667 \text{ ft}$$

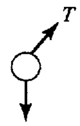
Thus,

$$s = 10 - s' = 3.33 \text{ ft} \qquad \textbf{Ans}$$

14-35. The man at the window A wishes to throw the 30-kg sack on the ground. To do this he allows it to swing from rest at B to point C, when he releases the cord at $\theta = 30°$. Determine the speed at which it strikes the ground and the distance R.



$T_B + \sum U_{B-C} = T_C$



$$0 + 30(9.81)8\cos 30° = \frac{1}{2}(30)v_C^2$$

$v_C = 11.659$ m/s

$T_B + \sum U_{B-D} = T_D$

$$0 + 30(9.81)(16) = \frac{1}{2}(30)v_D^2$$

$v_D = 17.7$ m/s **Ans**

During free flight:

$$(+\downarrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$16 = 8\cos 30° - 11.659\sin 30° t + \frac{1}{2}(9.81)t^2$$

$$t^2 - 1.18848t - 1.8495 = 0$$

Solving for the positive root:

$$t = 2.0784 \text{ s}$$

$$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$$

$$s = 8\sin 30° + 11.659\cos 30°(2.0784)$$

$$s = 24.985 \text{ m}$$

Thus,

$R = 8 + 24.985 = 33.0$ m **Ans**

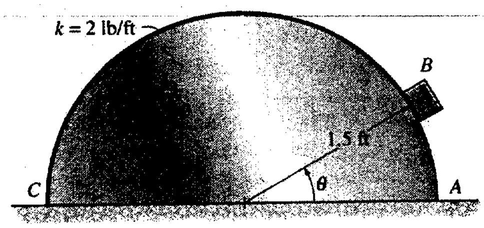
Also,

$$(v_D)_x = 11.659\cos 30° = 10.097 \text{ m/s}$$

$$(+\downarrow) \quad (v_D)_y = -11.659\sin 30° + 9.81(2.0784) = 14.559 \text{ m/s}$$

$v_D = \sqrt{(10.097)^2 + (14.559)^2} = 17.7$ m/s **Ans**

***14-36.** A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at $A(\theta = 0°)$, determine the unstretched length of the cord so the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.



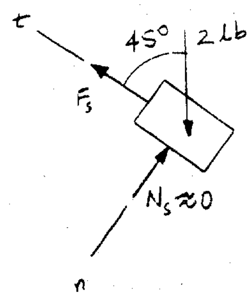
$+\swarrow \Sigma F_n = ma_n; \quad 2\sin 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right)$

$v = 5.844$ ft/s

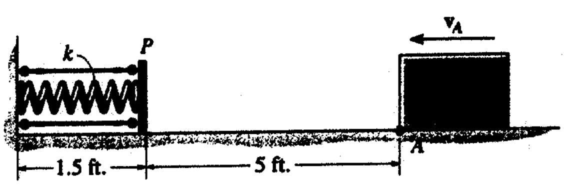
$T_1 + \Sigma U_{1-2} = T_2$

$$0 + \frac{1}{2}(2)[\pi(1.5) - l_0]^2 - \frac{1}{2}(2)\left[\frac{3\pi}{4}(1.5) - l_0\right]^2 - 2(1.5\sin 45°) = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2$$

$l_0 = 2.77$ ft Ans



14-37. The spring bumper is used to arrest the motion of the 4-lb block, which is sliding toward it at $v = 9$ ft/s. As shown, the spring is confined by the plate P and wall using cables so that its length is 1.5 ft. If the stiffness of the spring is $k = 50$ lb/ft, determine the required unstretched length of the spring so that the plate is not displaced more than 0.2 ft after the block collides into it. Neglect friction, the mass of the plate and spring, and the energy loss between the plate and block during the collision.



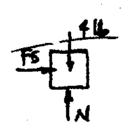
$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}\left(\frac{4}{32.2}\right)(9)^2 - \left[\frac{1}{2}(50)(s-1.3)^2 - \frac{1}{2}(50)(s-1.5)^2\right] = 0$

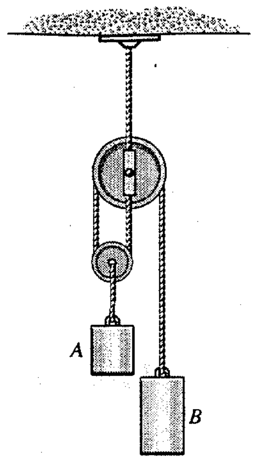
$0.20124 = s^2 - 2.60s + 1.69 - (s^2 - 3.0s + 2.25)$

$0.20124 = 0.4s - 0.560$

$s = 1.90$ ft Ans



14-38. Cylinder A has a mass of 3 kg and cylinder B has a mass of 8 kg. Determine the speed of A after it moves upwards 2 m starting from rest. Neglect the mass of the cord and pulleys.



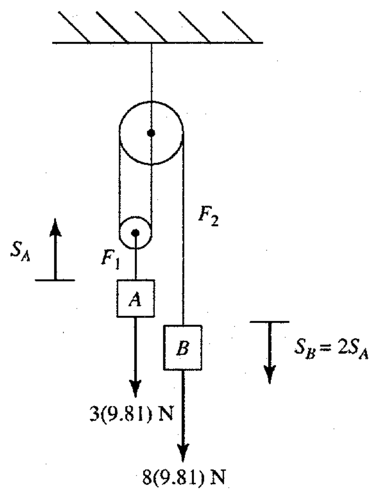
$$\sum T_1 + \sum U_{1-2} = \sum T_2$$

$$0 + 2[F_1 - 3(9.81)] + 4[8(9.81) - F_2] = \frac{1}{2}(3)v_A^2 + \frac{1}{2}(8)v_B^2$$

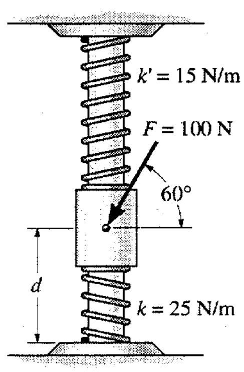
Also, $v_B = 2v_A$, and because the pulleys are massless, $F_1 = 2F_2$. The F_1 and F_2 terms drop out and the work-energy equation reduces to

$$255.06 = 17.5v_A^2$$

$v_A = 3.82$ m/s **Ans**



14-39. The collar has a mass of 20 kg and is supported on the smooth rod. The attached springs are undeformed when $d = 0.5$ m. Determine the speed of the collar after the applied force $F = 100$ N causes it to be displaced so that $d = 0.3$ m. When $d = 0.5$ m the collar is at rest.

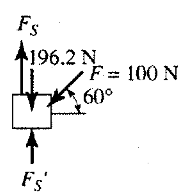


$$T_1 + \sum U_{1-2} = T_2$$

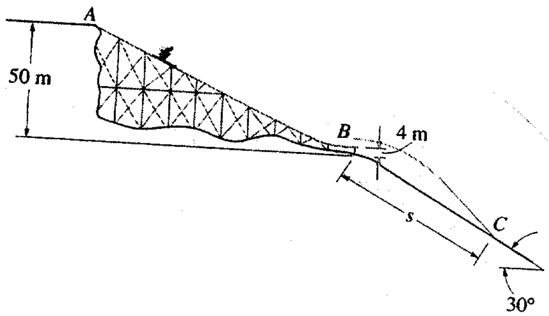
$$0 + 100 \sin 60°(0.5 - 0.3) + 196.2(0.5 - 0.3) - \frac{1}{2}(15)(0.5 - 0.3)^2$$

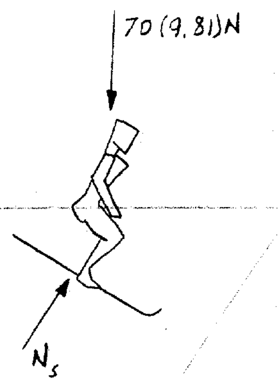
$$-\frac{1}{2}(25)(0.5 - 0.3)^2 = \frac{1}{2}(20)v_C^2$$

$v_C = 2.36$ m/s **Ans**



14-40. The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.





$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 70(9.81)(46) = \frac{1}{2}(70)(v_B)^2$$

$$v_B = 30.04 \text{ m/s} = 30.0 \text{ m/s} \qquad \textbf{Ans}$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$$

$$s\cos 30^\circ = 0 + 30.04t$$

$$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s\sin 30^\circ + 4 = 0 + 0 + \frac{1}{2}(9.81)t^2$$

Eliminating t,

$$s^2 - 122.67s - 981.33 = 0$$

Solving for the positive root

$$s = 130 \text{ m} \qquad \textbf{Ans}$$

14-41. The diesel engine of a 400-Mg train increases the train's speed uniformly from rest to 10 m/s in 100 s along a horizontal track. Determine the average power developed.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + U_{1-2} = \frac{1}{2}(400)(10^3)(10)^2$$

$$U_{1-2} = 20(10^6) \text{ J}$$

$$P_{avg} = \frac{U_{1-2}}{t} = \frac{20(10^6)}{100} = 200 \text{ kW} \qquad \textbf{Ans}$$

Also,

$$v = v_0 + a_c t$$

$$10 = 0 + a_c(100)$$

$$a_c = 0.1 \text{ m/s}^2$$

$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \qquad F = 400(10^3)(0.1) = 40(10^3) \text{ N}$$

$$P_{avg} = \mathbf{F}\cdot\mathbf{v}_{avg} = 40(10^3)\left(\frac{10}{2}\right) = 200 \text{ kW} \qquad \textbf{Ans}$$

14-42. Determine the power input for a motor necessary to lift 300 lb at a constant rate of 5 ft/s. The efficiency of the motor is $\epsilon = 0.65$.

Power: The power output can be obtained using Eq. 14-10.

$$P = \mathbf{F} \cdot \mathbf{v} = 300(5) = 1500 \text{ ft} \cdot \text{lb/s}$$

Using Eq. 14-11, the required power input for the motor to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{1500}{0.65} = 2307.7 \text{ ft} \cdot \text{lb/s} = 4.20 \text{ hp} \quad \textbf{Ans}$$

14-43. An electric streetcar has a weight of 15 000 lb and accelerates along a horizontal straight road from rest such that the power is always 100 hp. Determine how much time it takes to reach a speed of 40 ft/s.

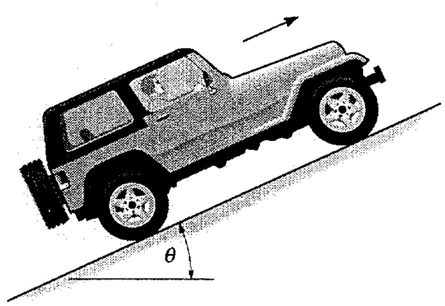
$$P = \frac{\Delta U}{\Delta t}$$

$$T_F = \Delta U = P\Delta t$$

$$\frac{1}{2}\left(\frac{15\,000}{32.2}\right)(40)^2 = 100(550)\Delta t$$

$$\Delta t = 6.78 \text{ s} \qquad \textbf{Ans}$$

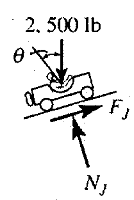
***14-44.** The jeep has a weight of 2500 lb and an engine which transmits a power of 100 hp to all the wheels. Assuming the wheels do not slip on the ground, determine the angle θ of the largest incline the jeep can climb at a constant speed $v = 30$ ft/s.



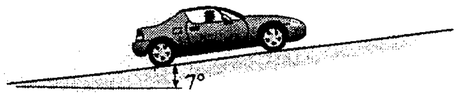
$$P = F_J v$$

$$100(550) = 2500 \sin\theta(30)$$

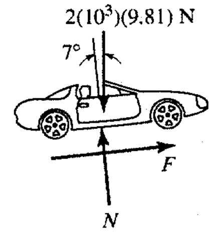
$$\theta = 47.2^\circ \qquad \textbf{Ans}$$



14-45. An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100$ km/h. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\epsilon = 0.65$.



Equation of Motion: The force F which is required to maintain the car's constant speed up the slope must be determined first.



$$+\sum F_{x'} = ma_{x'}; \quad F - 2(10^3)(9.81)\sin 7° = 2(10^3)(0)$$

$$F = 2391.08 \text{ N}$$

Power: Here, the speed of the car is $v = \left[\frac{100(10^3)\text{ m}}{\text{h}}\right] \times \left(\frac{1\text{ h}}{3600\text{ s}}\right) =$ 27.78 m/s. The power output can be obtained using Eq. 14-10.

$$P = \mathbf{F}\cdot\mathbf{v} = 2391.08(27.78) = 66.418(10^3)\text{ W} = 66.418\text{ kW}$$

Using Eq. 14-11, the required power input from the engine to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{66.418}{0.65} = 102\text{ kW} \quad \textbf{Ans}$$

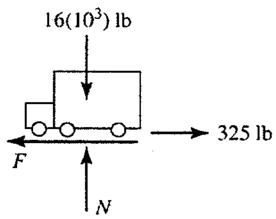
14-46. A loaded truck weighs $16(10^3)$ lb and accelerates uniformly on a level road from 15 ft/s to 30 ft/s during 4 s. If the frictional resistance to motion is 325 lb, determine the maximum power that must be delivered to the wheels.

$$a = \frac{\Delta v}{\Delta t} = \frac{30-15}{4} = 3.75\text{ ft/s}^2$$

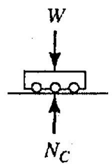
$$\overset{+}{\leftarrow}\sum F_x = ma_x; \quad F - 325 = \left(\frac{16(10^3)}{32.2}\right)(3.75)$$

$$F = 2188.35\text{ lb}$$

$$P_{max} = \mathbf{F}\cdot\mathbf{v}_{max} = \frac{2188.35(30)}{550} = 119\text{ hp} \quad \textbf{Ans}$$



14-47. An electric streetcar has a weight of 15 000 lb and accelerates along a horizontal straight road from rest such that the power is always 100 hp. Determine how far it must travel to reach a speed of 40 ft/s.



$$F = ma = \frac{W}{g}\left(\frac{v\,dv}{ds}\right)$$

$$P = Fv = \left(\frac{W}{g}\frac{v\,dv}{ds}\right)v$$

$$\int_0^s P\,ds = \int_0^v \frac{W}{g}v^2\,dv$$

$P = \text{constant}$

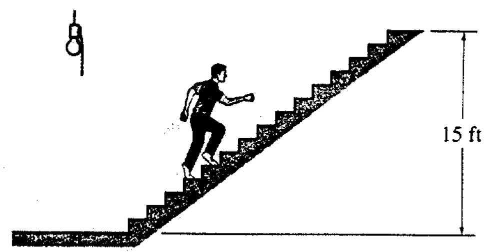
$$Ps = \frac{W}{g}\left(\frac{1}{3}\right)v^3$$

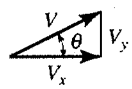
$$s = \frac{W}{3gP}v^3$$

$$s = \frac{15\,000(40)^3}{3(32.2)(100)(550)}$$

$s = 181$ ft **Ans**

***14-48.** The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.





Step height: 0.125 m

The number of steps: $\frac{4}{0.125} = 32$

Total load: $32(150)(9.81) = 47\,088$ N

If load is placed at the center height, $h = \frac{4}{2} = 2$ m, then

$$U = 47\,088\left(\frac{4}{2}\right) = 94.18 \text{ kJ}$$

$$v_y = v\sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683 \text{ m/s}$$

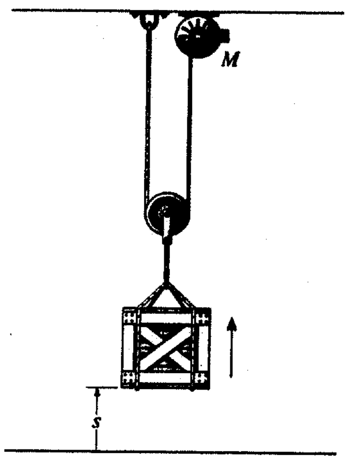
$$t = \frac{h}{v_y} = \frac{2}{0.2683} = 7.454 \text{ s}$$

$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6 \text{ kW} \quad \textbf{Ans}$$

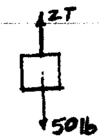
Also,

$$P = \mathbf{F}\cdot\mathbf{v} = 47\,088(0.2683) = 12.6 \text{ kW} \quad \textbf{Ans}$$

14-49. The 50-lb crate is given a speed of 10 ft/s in $t = 4$ s starting from rest. If the acceleration is constant, determine the power that must be supplied to the motor when $t = 2$ s. The motor has an efficiency $\epsilon = 0.76$. Neglect the mass of the pulley and cable.



$+\uparrow \Sigma F_y = m a_y; \qquad 2T - 50 = \frac{50}{32.2}a$



$(+\uparrow)\ v = v_0 + a_c t$

$10 = 0 + a(4)$

$a = 2.5 \text{ ft/s}^2$

$T = 26.94 \text{ lb}$

In $t = 2$ s,

$(+\uparrow) \quad v = v_0 + a_c t$

$v = 0 + 2.5(2) = 5 \text{ ft/s}$

$s_C + (s_C - s_P) = l$

$2v_C = v_P$

$2(5) = v_P = 10 \text{ ft/s}$

$P_O = 26.94(10) = 269.2$

$P_{in} = \frac{269.2}{0.76} = 354.49 \text{ ft}\cdot\text{lb/s}$

$P_{in} = 0.644 \text{ hp}$ **Ans**

14-50. A car has a mass m and accelerates along a horizontal straight road from rest such that the power is always a constant amount P. Determine how far it must travel to reach a speed of v.

Power: Since the power ouput is constant, then the traction force F varies with v. Applying Eq. 14–10, we have

$$P = \mathbf{F}\cdot\mathbf{v}$$

$$P = Fv \qquad F = \frac{P}{v}$$

Equation of Motion:

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad \frac{P}{v} = ma \qquad a = \frac{P}{mv}$$

Kinematics: Applying equation $ds = \dfrac{v\,dv}{a}$, we have

$$\int_0^s ds = \int_0^v \frac{mv^2}{P}dv \qquad s = \frac{mv^3}{3P} \qquad \textbf{Ans}$$

14-51. To dramatize the loss of energy in an automobile, consider a car having a weight of 5 000 lb that is traveling at 35 mi/h. If the car is brought to a stop, determine how long a 100-W light bulb must burn to expend the same amount of energy. (1 mi = 5280 ft.)

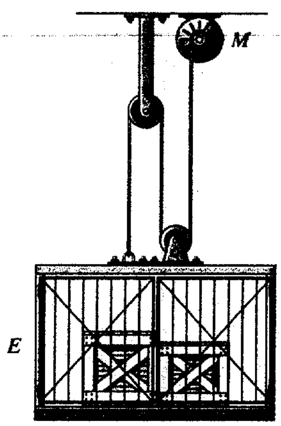
Energy : Here, the speed of the car is $v = \left(\frac{35\text{ mi}}{\text{h}}\right) \times \left(\frac{5280\text{ ft}}{1\text{ mi}}\right) \times \left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 51.33$ ft/s. Thus, the kinetic energy of the car is

$$U = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{5000}{32.2}\right)(51.33^2) = 204.59(10^3)\ \text{ft}\cdot\text{lb}$$

The power of the bulb is $P_{bulb} = 100\text{ W} \times \left(\frac{1\text{ hp}}{746\text{ W}}\right) \times \left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right)$ $= 73.73\ \text{ft}\cdot\text{lb/s}$. Thus,

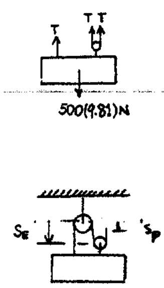
$$t = \frac{U}{P_{bulb}} = \frac{204.59(10^3)}{73.73} = 2774.98\text{ s} = 46.2\text{ min} \qquad \textbf{Ans}$$

***14-52.** The motor M is used to hoist the 500-kg elevator upward with a constant velocity $v_E = 8$ m/s. If the motor draws 60 kW of electrical power, determine the motor's efficiency. Neglect the mass of the pulleys and cable.



$+\uparrow \Sigma F_y = 0; \qquad 3T - 500(9.81) = 0$

$T = 1635$ N



$2s_E + (s_E - s_P) = l$

$3\,v_E = v_P$

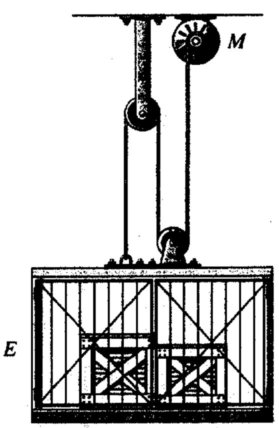
$v_P = 24$ m/s

$P_O = 1635(24) = 39.24$ kW

$P_i = 60$ kW

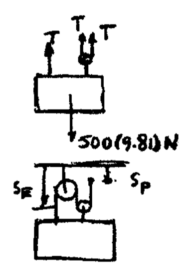
$$\varepsilon = \frac{P_O}{P_i} = \frac{39.24}{60} = 0.654 \qquad \textbf{Ans}$$

14-53. The 500-kg elevator starts from rest and travels upward with a constant acceleration $a_c = 2\text{ m/s}^2$. Determine the power output of the motor M when $t = 3$ s. Neglect the mass of the pulleys and cable.



$+\uparrow \Sigma F_y = m\,a_y; \qquad 3T - 500(9.81) = 500(2)$

$T = 1968.33$ N



$3s_E - s_P = l$

$3\,v_E = v_P$

When $t = 3$ s,

$(+\uparrow)\ v = v_0 + a_c\,t$

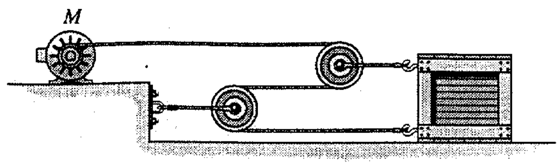
$v_E = 0 + 2(3) = 6$ m/s

$v_P = 3(6) = 18$ m/s

$P_O = 1968.33(18)$

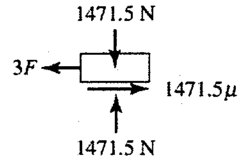
$P_O = 35.4$ kW $\qquad$ **Ans**

14-54. The crate has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. If the motor M supplies a cable force of $F = (8t^2 + 20)$ N, where t is in seconds, determine the power output developed by the motor when $t = 5$ s.



Time to start motor. $\mu_s = 0.3$

$$\overset{+}{\leftarrow}\sum F_x = 0: \quad 3F - 1471.5(0.3) = 0$$



$$F = 147.15 \text{ N}$$

$$F = 8t^2 + 20$$

$$147.15 = 8t^2 + 20$$

$$t = 3.987 \text{ s}$$

Motion: $\mu_k = 0.2$

$$\overset{+}{\leftarrow}\sum F_x = m\,a_x; \qquad 3F - 1471.5(0.2) = 150a$$

$$3(8t^2 + 20) - 294.3 = 150a$$

$$a = 0.160t^2 - 1.5620$$

$$\int_0^v dv = \int_{3.987}^{5} (0.160t^2 - 1.5620)dt$$

When $t = 5$ s,

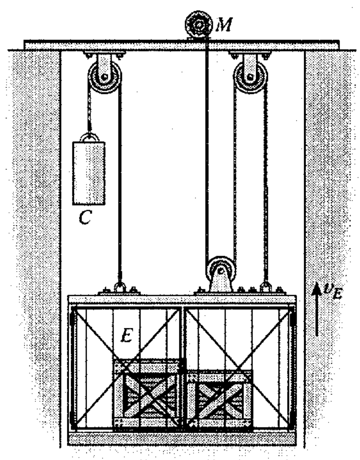
$$v = 0.160\left(\frac{t^3}{3}\right)\Bigg|_{3.987}^{5} - 1.5620\,t\Bigg|_{3.987}^{5}$$

$$v = 1.7045 \text{ ft/s}$$

$$P_O = 3[8(5)^2 + 20](1.7045) = 1124.97 \text{ N} \cdot \text{m/s}$$

$$P_O = 1.12 \text{ kW} \qquad \textbf{Ans}$$

14-55. The elevator E and its freight have a total mass of 400 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\epsilon = 0.6$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 4$ m/s.



Elevator:

Since $a = 0$,

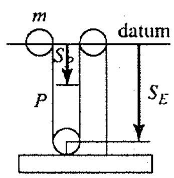
$$+\uparrow \sum F_y = 0; \qquad 60(9.81) + 3T - 400(9.81) = 0$$

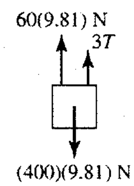
$$T = 1111.8 \text{ N}$$

$$2s_E + (s_E - s_P) = l$$

$$3v_E = v_P$$

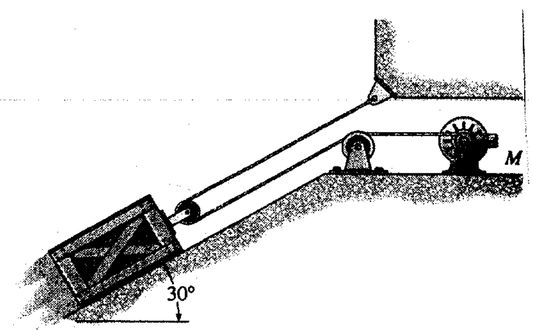
Since $v_E = -4$ m/s, $\quad v_P = -12$ m/s





$$P_i = \frac{\mathbf{F} \cdot \mathbf{v}_P}{e} = \frac{(1111.8)(12)}{0.6} = 22.2 \text{ kW} \quad \textbf{Ans}$$

***14-56.** The 50-kg crate is hoisted up the 30° incline by the pulley system and motor M. If the crate starts from rest and by constant acceleration attains a speed of 4 m/s after traveling 8 m along the plane, determine the power that must be supplied to the motor at this instant. Neglect friction along the plane. The motor has an efficiency of $\epsilon = 0.74$.

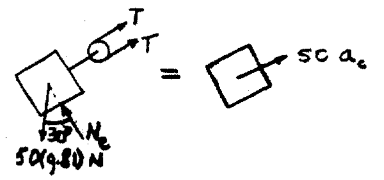


$v^2 = v_0^2 + 2a_c(s - s_0)$

$(4)^2 = 0 + 2a_c(8 - 0)$

$a_c = 1\ \text{m/s}^2$

$+\nearrow \Sigma F_x = ma_x;\quad 2T - 50(9.81)\sin 30° = (50)(1) \quad T = 147.6\ \text{N}$

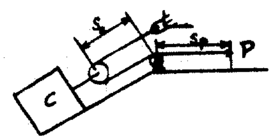


$2s_C + s_P = l$

$2v_C = -v_P$

$(2)(-4) = -v_P$

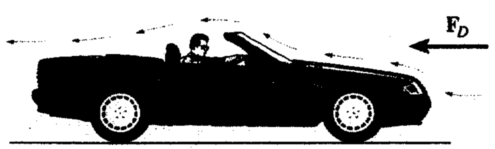
$v_P = 8\ \text{m/s}$



$P_o = \mathbf{T} \cdot \mathbf{v}_P = 147.6(8) = 1181\ \text{W}$

$P_i = \dfrac{P_o}{e} = \dfrac{1181}{0.74} = 1595.9\ \text{W} = 1.60\ \text{kW}$ **Ans**

14-57. The sports car has a mass of 2.3 Mg, and while it is traveling at 28 m/s the driver causes it to accelerate at 5 m/s². If the drag resistance on the car due to the wind is $F_D = (0.3v^2)$ N, where v is the velocity in m/s, determine the power supplied to the engine at this instant. The engine has a running efficiency of $\epsilon = 0.68$.

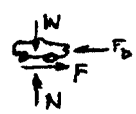


$\xrightarrow{+} \Sigma F_x = m\,a_x;\quad F - 0.3v^2 = 2.3(10^3)(5)$

$F = 0.3v^2 + 11.5(10^3)$

At $v = 28$ m/s

$F = 11\,735.2\ \text{N}$



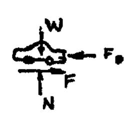
$P_o = (11\,735.2)(28) = 328.59\ \text{kW}$

$P_i = \dfrac{P_o}{e} = \dfrac{328.59}{0.68} = 483\ \text{kW}$ **Ans**

14-58. The sports car has a mass of 2.3 Mg and accelerates at 6 m/s², starting from rest. If the drag resistance on the car due to the wind is $F_D = (10v)$ N, where v is the velocity in m/s, determine the power supplied to the engine when $t = 5$ s. The engine has a running efficiency of $\epsilon = 0.68$.

$\xrightarrow{+} \Sigma F_x = m\,a_x;\quad F - 10v = 2.3(10^3)(6)$

$F = 13.8(10^3) + 10v$



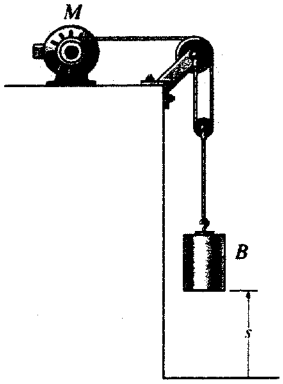
$(\xrightarrow{+})\ v = v_0 + a_c t$

$v = 0 + 6(5) = 30\ \text{m/s}$

$P_o = \mathbf{F} \cdot \mathbf{v} = [13.8(10^3) + 10(30)](30) = 423.0\ \text{kW}$

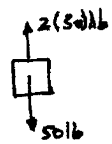
$P_i = \dfrac{P_o}{e} = \dfrac{423.0}{0.68} = 622\ \text{kW}$ **Ans**

14-59. The 50-lb load is hoisted by the pulley system and motor M. If the motor exerts a constant force of 30 lb on the cable, determine the power that must be supplied to the motor if the load has been hoisted $s = 10$ ft starting from rest. The motor has an efficiency of $\epsilon = 0.76$.



$$+\uparrow \Sigma F_y = m\,a_y; \quad 2(30) - 50 = \frac{50}{32.2}a_B$$

$$a_B = 6.44 \text{ m/s}^2$$



$$(+\uparrow)\ v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v_B^2 = 0 + 2(6.44)(10-0)$$

$$v_B = 11.349 \text{ ft/s}$$

$$2s_B + s_M = l$$

$$2v_B = -v_M$$

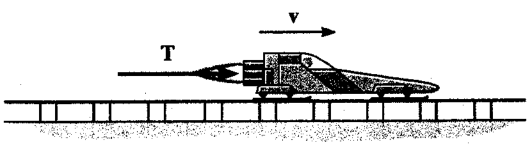
$$v_M = -2(11.349) = -22.698 \text{ ft/s}$$

$$P_O = \mathbf{F}\cdot\mathbf{v} = 30(22.698) = 680.94 \text{ ft}\cdot\text{lb/s}$$

$$P_i = \frac{680.94}{0.76} = 895.97 \text{ ft}\cdot\text{lb/s}$$

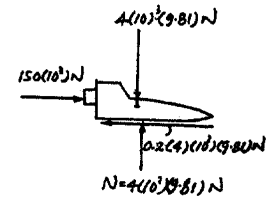
$$P_i = 1.63 \text{ hp} \qquad \textbf{Ans}$$

***14-60.** The rocket sled has a mass of 4 Mg and travels from rest along the horizontal track for which the coefficient of kinetic friction is $\mu_k = 0.20$. If the engine provides a constant thrust $T = 150$ kN, determine the power output of the engine as a function of time. Negiect the loss of fuel mass and air resistance.



$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad 150(10)^3 - 0.2(4)(10)^3(9.81) = 4(10)^3 a$$

$$a = 35.54 \text{ m/s}^2$$

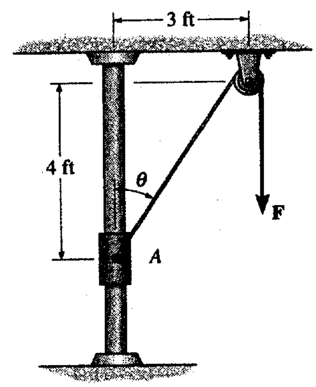


$$(\overset{+}{\rightarrow})\ v = v_0 + a_c t$$

$$= 0 + 35.54t = 35.54t$$

$$P = \mathbf{T}\cdot\mathbf{v} = 150(10)^3(35.54t) = 5.33t \text{ MW} \qquad \textbf{Ans}$$

14-61. The 10-lb collar starts from rest at A and is lifted by applying a constant force of $F = 25$ lb to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = 60°$.

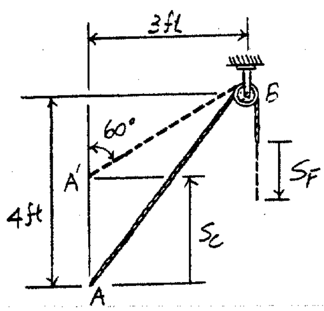


Principle of Work and Energy : The 25 - lb force which acts in the direction of displacement does *positive* work, whereas the weight of the collar does *negativ* work sinceit acts in the opposite direction to that of displacement. The 25 - lb force and the collar are being displaced vertically by $s_F = AB - A'B = \sqrt{3^2 + 4^2} - 3/\sin 60° = 1.5359$ ft and $s_C = 4 - 3/\tan 60° = 2.2679$ ft. Since the block is at rest initially, $T_1 = 0$. Applying Eq. 14−7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 25(1.5359) - 10(2.2679) = \frac{1}{2}\left(\frac{10}{32.2}\right)v^2$$

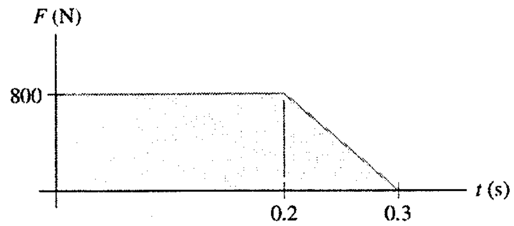
$$v = 10.06 \text{ ft/s}$$

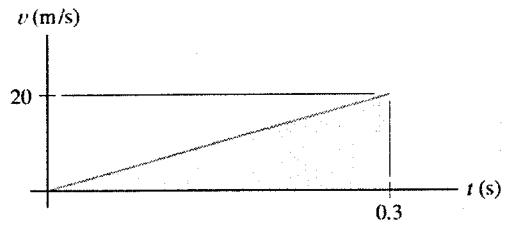


Power : The power output at the instant when $s = 10$ ft can be obtained using Eq. 14−10.

$$P = \mathbf{F}\cdot\mathbf{v} = 25\cos 60°(10.06) = 125.76 \text{ ft}\cdot\text{lb/s} = 0.229 \text{ hp} \qquad \textbf{Ans}$$

14-62. An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.3$ s.





For $0 \leq t \leq 0.2$

$F = 800$ N

$$v = \frac{20}{0.3}t = 66.67\,t$$

$P = \mathbf{F} \cdot \mathbf{v} = 53.3t$ kW **Ans**

For $0.2 \leq t \leq 0.3$

$F = 2400 - 8000t$

$v = 66.67\,t$

$P = \mathbf{F} \cdot \mathbf{v} = (160t - 533\,t^2)$ kW **Ans**

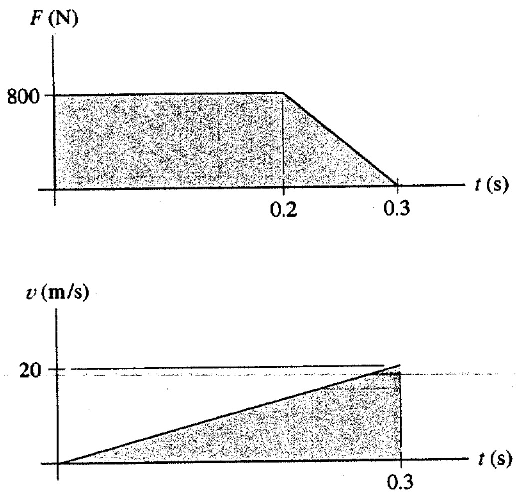
$$U = \int_0^{0.3} P\,dt$$

$$U = \int_0^{0.2} 53.3\,t\,dt + \int_{0.2}^{0.3} (160t - 533\,t^2)dt$$

$$= \frac{53.3}{2}(0.2)^2 + \frac{160}{2}[(0.3)^2 - (0.2)^2] - \frac{533}{3}[(0.3)^3 - (0.2)^3]$$

$= 1.69$ kJ **Ans**

14-63. An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the 0.3-second time period.



See solution to Prob. 14–62.

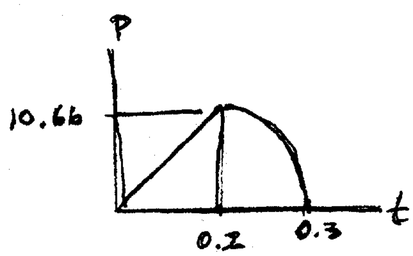
$P = 160\,t - 533\,t^2$

$\dfrac{dP}{dt} = 160 - 1066.6\,t = 0$

$t = 0.15 \text{ s} < 0.2 \text{ s}$

Thus maximum occurs at $t = 0.2$ s

$P_{max} = 53.3(0.2) = 10.7 \text{ kW}$ **Ans**



*14-64. Solve Prob. 14-18 using the conservation of energy equation.

At C, with normal force from the track $N \approx 0$,

$$+\downarrow \sum F_y = ma_y: \quad mg = m\frac{v_C^2}{\rho_C}$$

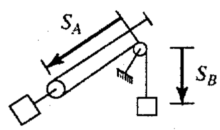
$$v_C^2 = (9.81)(25)\ (\text{m/s})^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m(9.81)(25) + m(9.81)(35) = 0 + m(9.81)h$$

$h = 47.5$ m **Ans**

14-65. Solve Prob. 14-15 using the conservation of energy equation.



$$2s_A + s_B = l$$

$$2\Delta s_A + \Delta s_B = 0$$

$$2v_A + v_B = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$[0+0] + [0+0] = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(2v_A)^2 + 10(10) - 60\left(\frac{3}{5}\right)(5)$$

$v_A = 7.18$ ft/s **Ans**

14-66. Solve Prob. 14-17 using the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(100)(0.5)^2 + \frac{1}{2}(50)(0.5)^2 = \frac{1}{2}(20)v^2 + 0$$

$v = 1.37$ m/s **Ans**

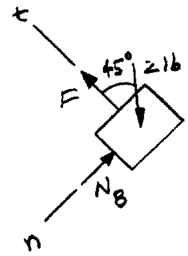
14-67. Solve Prob. 14-31 using the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(20)(2)^2 + 0 = 0 + \frac{1}{2}(50)s^2 + \frac{1}{2}(100)s^2$$

$s = 0.730$ m **Ans**

***14-68.** Solve Prob. 14-36 using the conservation of energy equation.



$$+\!\!\swarrow \Sigma F_n = ma_n; \quad 2\sin45° = \left(\frac{2}{32.2}\right)\left(\frac{v_B^2}{1.5}\right)$$

$$v_B = 5.844 \text{ ft/s}$$

Datum at A:

$$T_A + V_A = T_B + V_B$$

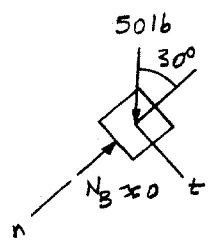
$$0+\frac{1}{2}(2)(\pi(1.5)-l_0)^2 = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2+\frac{1}{2}(2)\left[\pi(1.5)-\frac{\pi}{4}(1.5)-l_0\right]^2+2(1.5\sin45°)$$

$l_0 = 2.77$ ft **Ans**

14-69. Solve Prob. 14-23 using the conservation of energy equation.

Datum at A:

$$T_A + V_A = T_B + V_B$$



$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2+0 = \frac{1}{2}\left(\frac{50}{32.2}\right)v_B^2 - 50(5)(1-\cos30°)$$

$v_B = 7.221$ ft/s $= 7.22$ ft/s **Ans**

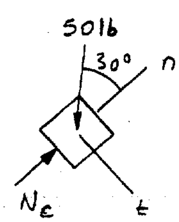
$$+\!\!\swarrow \Sigma F_n = ma_n; \quad -N_B + 50\cos30° = \left(\frac{50}{32.2}\right)\left(\frac{(7.221)^2}{5}\right)$$

$N_B = 27.1$ lb **Ans**

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2+0 = \frac{1}{2}\left(\frac{50}{32.2}\right)v_C^2 - 50(5\cos30°)$$

$v_C = 16.97$ ft/s $= 17.0$ ft/s **Ans**



$$+\!\!\nearrow \Sigma F_n = ma_n; \quad N_C - 50\cos30° = \left(\frac{50}{32.2}\right)\left(\frac{(16.97)^2}{5}\right)$$

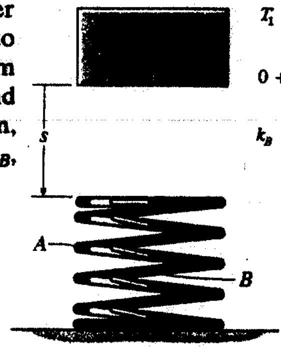
$N_C = 133$ lb **Ans**

$$T_A + V_A = T_D + V_D$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2+0 = \frac{1}{2}\left(\frac{50}{32.2}\right)v_D^2 - 50(5)$$

$v_D = 18.2$ ft/s **Ans**

14-70. Two equal-length springs are "nested" together in order to form a shock absorber. If it is designed to arrest the motion of a 2-kg mass that is dropped $s = 0.5$ m above the top of the springs from an at-rest position, and the maximum compression of the springs is to be 0.2 m, determine the required stiffness of the inner spring, k_B, if the outer spring has a stiffness $k_A = 400$ N/m.

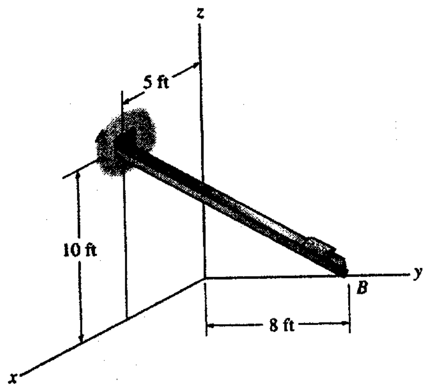


$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 - 2(9.81)(0.5+0.2) + \frac{1}{2}(400)(0.2)^2 + \frac{1}{2}(k_B)(0.2)^2$$

$k_B = 287$ N/m **Ans**

14-71. The block has a weight of 1.5 lb and slides along the smooth chute AB. It is released from rest at A, which has coordinates of $A(5\text{ ft}, 0, 10\text{ ft})$. Determine the speed at which it slides off at B, which has coordinates of $B(0, 8\text{ ft}, 0)$.



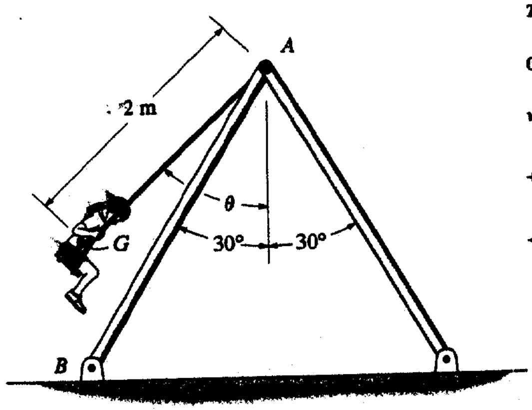
Datum at B :

$$T_A + V_A = T_B + V_B$$

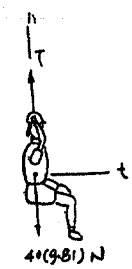
$$0 + 1.5(10) = \frac{1}{2}\left(\frac{1.5}{32.2}\right)(v_B)^2 + 0$$

$v_B = 25.4$ ft/s **Ans**

***14-72.** The girl has a mass of 40 kg and center of mass at G. If she is swinging to a maximum height defined by $\theta = 60°$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.



The maximum tension in the cable occurs when $\theta = 0°$.

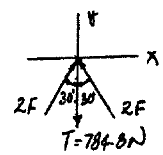


$$T_1 + V_1 = T_2 + V_2$$

$$0 + 40(9.81)(-2\cos 60°) = \tfrac{1}{2}(40)v^2 + 40(9.81)(-2)$$

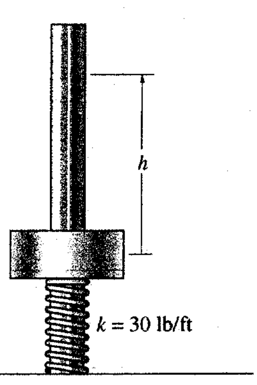
$v = 4.429$ m/s

$+\uparrow \Sigma F_n = ma_n;\quad T - 40(9.81) = (40)\left(\frac{4.429^2}{2}\right) \quad T = 784.8$ N



$+\uparrow \Sigma F_y = 0;\quad 2(2F)\cos 30° - 784.8 = 0 \quad F = 227$ N **Ans**

14-73. The collar has a weight of 8 lb. If it is pushed down so as to compress the spring 2 ft and then released from rest ($h = 0$), determine its speed when it is displaced $h = 4.5$ ft. The spring is not attached to the collar. Neglect friction.

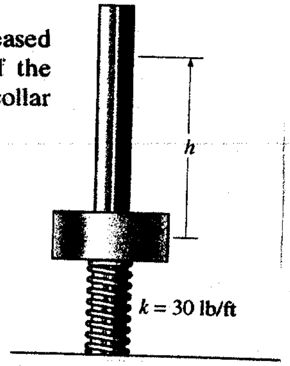


$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(30)(2)^2 = \frac{1}{2}\left(\frac{8}{32.2}\right)v_2^2 + 8(4.5)$$

$v_2 = 13.9$ ft/s **Ans**

14-74. The collar has a weight of 8 lb. If it is released from rest at a height of $h = 2$ ft from the top of the uncompressed spring, determine the speed of the collar after it falls and compresses the spring 0.3 ft.

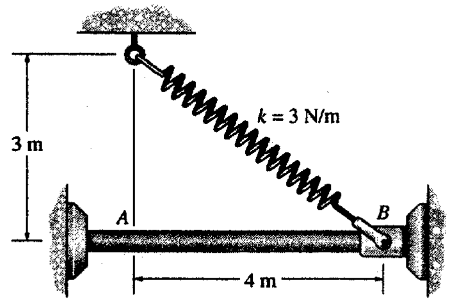


$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{8}{32.2}\right)v_2^2 - 8(2.3) + \frac{1}{2}(30)(0.3)^2$$

$v_2 = 11.7$ ft/s **Ans**

14-75. The 2-kg collar is attached to a spring that has an unstretched length of 3 m. If the collar is drawn to point B and released from rest, determine its speed when it arrives at point A.



Potential Energy : The initial and final elastic potential energy are $\frac{1}{2}(3)\left(\sqrt{3^2+4^2}-3\right)^2 = 6.00$ J and $\frac{1}{2}(3)(3-3)^2 = 0$, respectively. The gravitational potential energy remains the same since the elevation of collar does not change when it moves from B to A.

Conservation of Energy :

$$T_B + V_B = T_A + V_A$$

$$0 + 6.00 = \frac{1}{2}(2)\,v_A^2 + 0$$

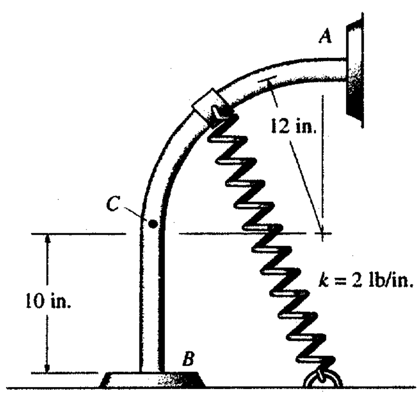
$v_A = 2.45$ m/s **Ans**

***14-76.** The 5-lb collar is released from rest at A and travels along the smooth guide. Determine the speed of the collar just before it strikes the stop at B. The spring has an unstretched length of 12 in.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 5\left(\frac{22}{12}\right) + \frac{1}{2}(24)\left(\frac{10}{12}\right)^2 = \frac{1}{2}\left(\frac{5}{32.2}\right)v_B^2 + 0 + 0$$

$v_B = 15.0$ ft/s **Ans**



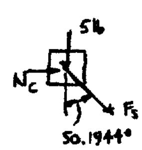
14-77. The 5-lb collar is released from rest at A and travels along the smooth guide. Determine its speed when its center reaches point C and the normal force it exerts on the rod at this point. The spring has an unstretched length of 12 in., and point C is located just before the end of the curved portion of the rod.

$$T_A + V_A = T_C + V_C$$

$$0 + 0 + \frac{1}{2}[2(12)]\left(\frac{10}{12}\right)^2 + 5\left(\frac{12}{12}\right) = \frac{1}{2}\left(\frac{5}{32.2}\right)v^2 + \frac{1}{2}[2(12)]\left(\sqrt{\left(\frac{12}{12}\right)^2 + \left(\frac{10}{12}\right)^2} - \frac{12}{12}\right)^2$$

$v = 12.556$ ft/s $= 12.6$ ft/s **Ans**

$\xrightarrow{+} \Sigma F_n = m a_n;$ $\quad N_C + F_s \sin 50.1944° = \frac{5}{32.2}\left(\frac{(12.556)^2}{1}\right)$

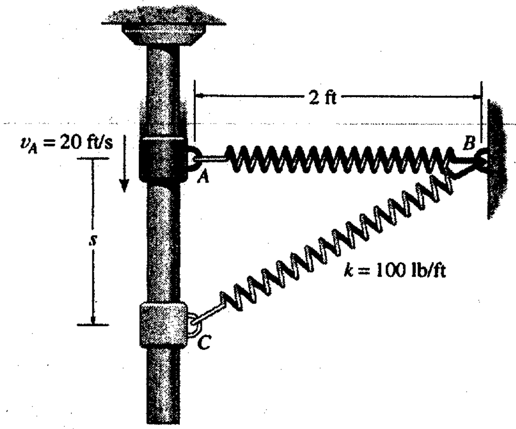


$F_s = ks;$ $\quad F_s = 2(12)\left[\sqrt{\left(\frac{12}{12}\right)^2 + \left(\frac{10}{12}\right)^2} - \frac{12}{12}\right] = 7.2410$ lb

Thus,

$N_C = 18.9$ lb **Ans**

14-78. The 2-lb block is given an initial velocity of 20 ft/s when it is at A. If the spring has an unstretched length of 2 ft and a stiffness of $k = 100$ lb/ft, determine the velocity of the block when $s = 1$ ft.



Potential Energy : Datum is set along AB. The collar is 1 ft *below* the datum when it is at C. Thus, its gravitational potential energy at this point is $-2(1) = -2.00$ ft·lb. The initial and final elastic potential energy are $\frac{1}{2}(100)(2-2)^2 = 0$ and $\frac{1}{2}(100)\left(\sqrt{2^2+1^2}-2\right)^2 = 2.786$ ft·lb, respectively.

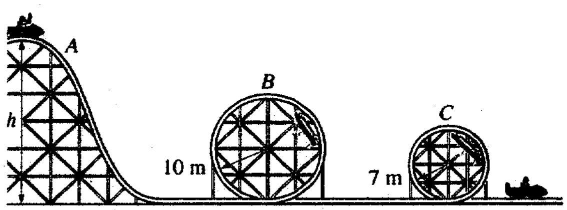
Conservation of Energy :

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(20^2) + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2 + 2.786 + (-2.00)$$

$$v_C = 19.4 \text{ ft/s} \qquad \textbf{Ans}$$

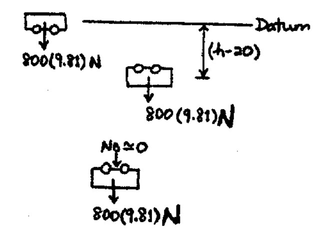
14-79. The roller-coaster car has a mass of 800 kg, including its passenger, and starts from the top of the hill A with a speed $v_A = 3$ m/s. Determine the minimum height h of the hill crest so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C?



Since friction is neglected, the car will travel around the 7 m-loop provided it first travels around the 10 m-loop.

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(800)(3)^2 + 0 = \frac{1}{2}(800)(v_B^2) - 800(9.81)(h-20)$$



$+\downarrow \Sigma F_n = m a_n;\qquad 800(9.81) = 800\left(\frac{v_B^2}{10}\right)$

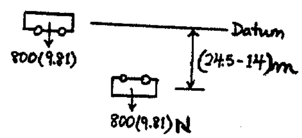
Thus, $\qquad v_B = 9.90$ m/s

$h = 24.5$ m $\qquad$ **Ans**

At B: $\qquad N_B = 0 \qquad$ **Ans** (For h to be minimum.)

$$T_A + V_A = T_C + V_C$$

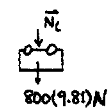
$$\frac{1}{2}(800)(3)^2 + 0 = \frac{1}{2}(800)(v_C)^2 - 800(9.81)(24.5-14)$$



$v_C = 14.69$ m/s

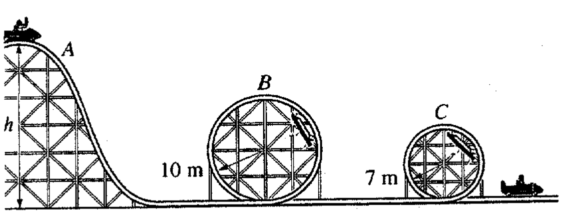
$a_n = \dfrac{14.69^2}{7}$

$+\downarrow \Sigma F_n = m a_n;\qquad N_C + 800(9.81) = 800\left(\frac{14.69^2}{7}\right)$



$N_C = 16.8$ kN $\qquad$ **Ans**

***14-80.** The roller-coaster car has a mass of 800 kg, including its passenger. If it is released from rest at the top of the hill A, determine the minimum height h of the hill crest so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C?

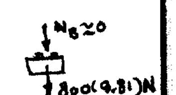


Since friction is neglected, the car will travel around the 7 m-loop provided it first travels around the 10 m-loop.

$$T_A + V_A = T_B + V_B$$

$$0 + 0 = \frac{1}{2}(800)(v_B^2) - 800(9.81)(h-20)$$

$+\downarrow \Sigma F_n = m a_n;\qquad 800(9.81) = 800\left(\frac{v_B^2}{10}\right)$



Thus, $\qquad v_B = 9.90$ m/s

$h = 25.0$ m $\qquad$ **Ans**

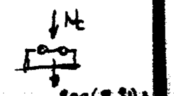
At B: $\qquad N_B = 0 \qquad$ **Ans** (For h to be minimum.)

$$T_A + V_A = T_C + V_C$$

$$0 + 0 = \frac{1}{2}(800)(v_C)^2 - 800(9.81)(25-14)$$

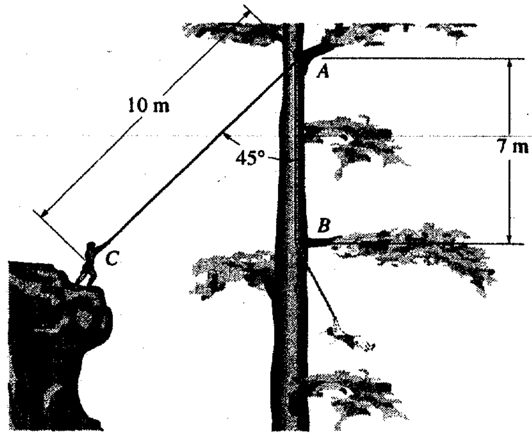
$v_C = 14.69$ m/s

$+\downarrow \Sigma F_n = m a_n;\qquad N_C + 800(9.81) = 800\left(\frac{(14.69)^2}{7}\right)$



$N_C = 16.8$ kN $\qquad$ **Ans**

14-81. Tarzan has a mass of 100 kg and from rest swings from the cliff by rigidly holding on to the tree vine, which is 10 m measured from the supporting limb A to his center of mass. Determine his speed just after the vine strikes the lower limb at B. Also, with what force must he hold on to the vine just before and just after the vine contacts the limb at B?

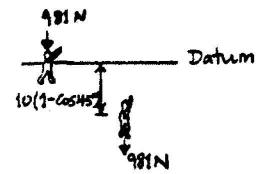


Datum at C:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(100)(v_C)^2 - 100(9.81)(10)(1 - \cos 45°)$$

$v_C = 7.581 = 7.58$ m/s **Ans**



Just before striking B, $\rho = 10$ m

$$+\uparrow \Sigma F_n = ma_n; \quad T - 981 = 100\left(\frac{(7.581)^2}{10}\right)$$

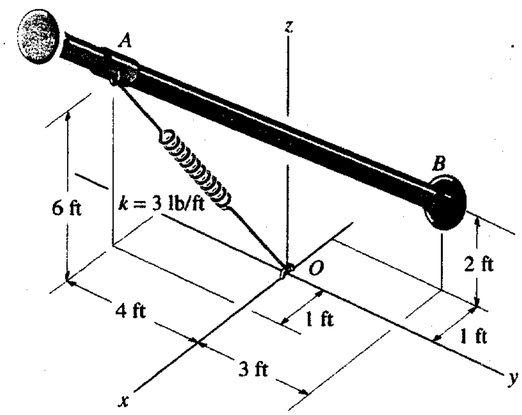
$T = 1.56$ kN **Ans**

Just after striking B, $\rho = 3$ m

$$+\uparrow \Sigma F_n = ma_n; \quad T - 981 = 100\left(\frac{(7.581)^2}{3}\right)$$

$T = 2.90$ kN **Ans**

14-82. The spring has a stiffness $k = 3$ lb/ft and an unstretched length of 2 ft. If it is attached to the 5-lb smooth collar and the collar is released from rest at A, determine the speed of the collar just before it strikes the end of the rod at B. Neglect the size of the collar.



Datum at B.

$$|r_{OA}| = \sqrt{(1)^2 + (4)^2 + (6)^2} = 7.28 \text{ ft}$$

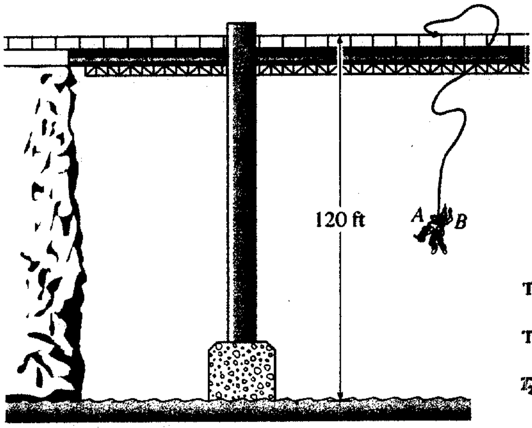
$$|r_{OB}| = \sqrt{(1)^2 + (3)^2 + (2)^2} = 3.74 \text{ ft}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (5)(6-2) + \frac{1}{2}(3)(7.28-2)^2 = \frac{1}{2}\left(\frac{5}{32.2}\right)v_B^2 + \frac{1}{2}(3)(3.74-2)^2$$

$v_B = 27.2$ ft/s **Ans**

14-83. Just for fun, two 150-lb engineering students A and B intend to jump off the bridge from rest using an elastic cord (bungee cord) having a stiffness $k = 80$ lb/ft. They wish to just reach the surface of the river, when A, attached to the cord, lets go of B at the instant they touch the water. Determine the proper unstretched length of the cord to do the stunt, and calculate the maximum acceleration of student A and the maximum height he reaches above the water after the rebound. From your results, comment on the feasibility of doing this stunt.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2(150)(120) = 0 + \frac{1}{2}(80)(x)^2$$

$$x = 30 \text{ ft}$$

Unstretched length of cord.

$$120 = l + 30$$

$$l = 90 \text{ ft} \qquad \textbf{Ans}$$

When A lets go of B.

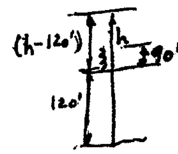
$$T_2 + V_2 = T_3 + V_3$$

$$0 + \frac{1}{2}(80)(30)^2 = 0 + (150)h$$

$$h = 240 \text{ ft}$$

This is not possible since 90 ft cord would have to stretch again, i.e., $h_{max} = 120 + 90 = 210$ ft

Thus $h > 120 + 90 = 210$ ft



$$T_2 + V_2 = T_3 + V_3$$

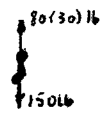
$$0 + \frac{1}{2}(80)(30)^2 = 0 + 150h + \frac{1}{2}(80)[(h-120) - 90]^2$$

$$36\,000 = 150h + 40(h^2 - 420h + 44\,100)$$

$$h^2 - 416.25h + 43\,200 = 0$$

Choosing the root > 210 ft

$$h = 219 \text{ ft} \qquad \textbf{Ans}$$

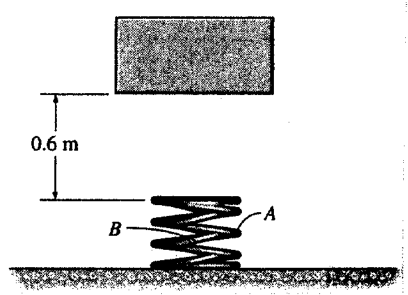


$$+\uparrow \Sigma F_y = m a_y; \quad 80(30) - 150 = \frac{150}{32.2}a$$

$$a = 483 \text{ ft/s}^2 \qquad \textbf{Ans}$$

It would not be a good idea to perform the stunt since $a = 15\ g$ which is excessive and A rises $219' - 120' = 99$ ft above the bridge!

***14-84.** Two equal-length springs having a stiffness $k_A = 300$ N/m and $k_B = 200$ N/m are "nested" together in order to form a shock absorber. If a 2-kg block is dropped from an at-rest position 0.6 m above the top of the springs, determine their deformation when the block momentarily stops.



Datum at initial position :

$$T_1 + V_1 = T_2 + V_2$$

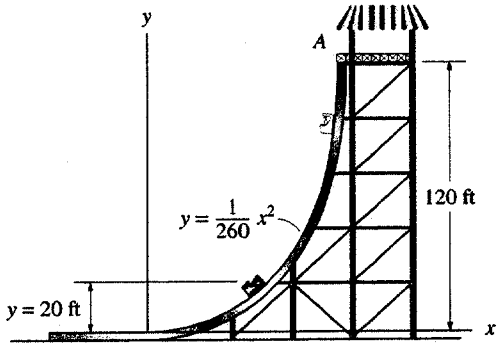
$$0 + 0 = 0 - 2(9.81)(0.6 + x) + \frac{1}{2}(300 + 200)(x)^2$$

$$250x^2 - 19.62x - 11.772 = 0$$

Solving for the positive root,

$$x = 0.260 \text{ m} \qquad \textbf{Ans}$$

14-85. The ride at an amusement park consists of a gondola which is lifted to a height of 120 ft at A. If it is released from rest and falls along the parabolic track, determine the speed at the instant $y = 20$ ft. Also determine the normal reaction of the tracks on the gondola at this instant. The gondola and passenger have a total weight of 500 lb. Neglect the effects of friction and the mass of the wheels.



$$y = \frac{1}{260}x^2$$

$$\frac{dy}{dx} = \frac{1}{130}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{130}$$

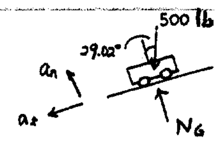
At $y = 120 - 100 = 20$ ft

$$x = 72.11 \text{ ft}$$

$$\tan\theta = \frac{dy}{dx} = 0.555, \qquad \theta = 29.02^\circ$$

$$\rho = \frac{[1+(0.555)^2]^{3/2}}{\frac{1}{130}} = 194.40 \text{ ft}$$

$$+\nwarrow \Sigma F_n = m\,a_n; \quad N_G - 500\cos 29.02^\circ = \frac{500}{32.2}\left(\frac{v^2}{194.40}\right) \qquad (1)$$



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{500}{32.2}\right)v^2 - 500(100)$$

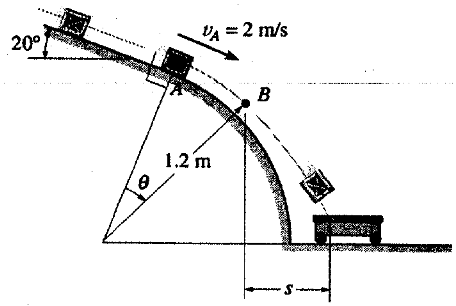
$$v^2 = 6440$$

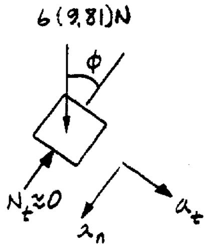
$v = 80.2$ ft/s **Ans**

Substituting into Eq. (1) yields

$N_G = 952$ lb **Ans**

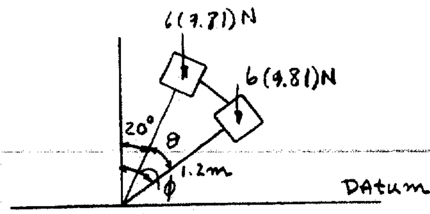
14-86. When the 6-kg box reaches point A it has a speed of $v_A = 2$ m/s. Determine the angle θ at which it leaves the smooth circular ramp and the distance s to where it falls into the cart. Neglect friction.





At point B :

$$+\swarrow \Sigma F_n = ma_n: \quad 6(9.81)\cos\phi = 6\left(\frac{v_B^2}{1.2}\right) \quad (1)$$



Datum at bottom of curve :

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(6)(2)^2 + 6(9.81)(1.2\cos 20°) = \frac{1}{2}(6)(v_B)^2 + 6(9.81)(1.2\cos\phi)$$

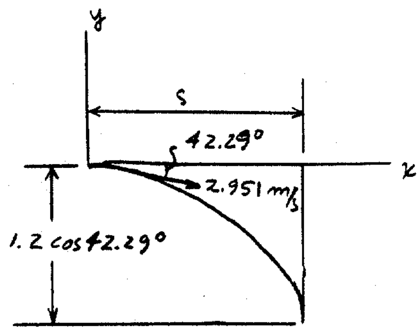
$$13.062 = 0.5v_B^2 + 11.772\cos\phi \quad (2)$$

Substitute Eq. (1) into Eq. (2), and solving for v_B,

$$v_B = 2.951 \text{ m/s}$$

Thus, $\quad \phi = \cos^{-1}\left(\frac{(2.951)^2}{1.2(9.81)}\right) = 42.29°$

$\theta = \phi - 20° = 22.3°$ **Ans**



$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$-1.2\cos 42.29° = 0 - 2.951(\sin 42.29°)t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 + 1.9857t - 0.8877 = 0$$

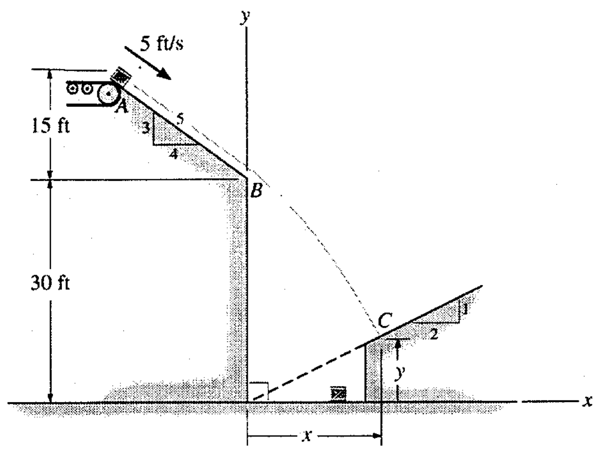
Solving for the positive root : $\quad t = 0.2687$ s

$(\xrightarrow{+}) \quad s = s_0 + v_0 t$

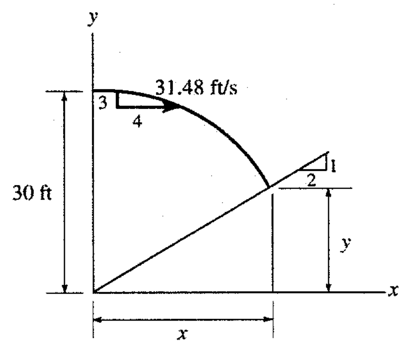
$$s = 0 + (2.951\cos 42.29°)(0.2687)$$

$s = 0.587$ m **Ans**

14-87. The 2-lb box has a velocity of 5 ft/s when it begins to slide down the smooth inclined surface at A. Determine the point $C(x, y)$ where it strikes the lower incline.



Datum at A:



$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2 - 2(15)$$

$$v_B = 31.48 \text{ ft/s}$$

$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$$x = 0 + 31.48\left(\frac{4}{5}\right)t \qquad (1)$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 30 - 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

Equation of inclined surface:

$$\frac{y}{x} = \frac{1}{2}; \qquad y = \frac{1}{2}x \qquad (2)$$

Thus,

$$30 - 18.888t - 16.1t^2 = 12.592t$$

$$-16.1t^2 - 31.480t + 30 = 0$$

Solving for the positive root,

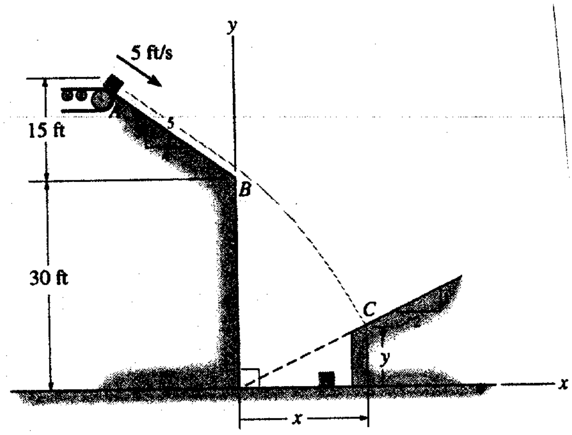
$$t = 0.7014 \text{ s}$$

From Eqs. (1) and (2):

$$x = 31.48\left(\frac{4}{5}\right)(0.7014) = 17.66 = 17.7 \text{ ft} \quad \textbf{Ans}$$

$$y = \frac{1}{2}(17.664) = 8.832 = 8.83 \text{ ft} \quad \textbf{Ans}$$

***14-88.** The 2-lb box has a velocity of 5 ft/s when it begins to slide down the smooth inclined surface at A. Determine its speed just before hitting the surface at C and the time to travel from A to C. The coordinates of point C are $x = 17.66$ ft, and $y = 8.832$ ft.



Datum at A :

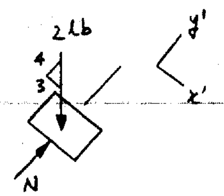
$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_C)^2 - 2[15 + (30 - 8.832)]$$

$v_C = 48.5$ ft/s **Ans**

$$+\searrow \Sigma F_{x'} = ma_{x'}; \quad 2\left(\frac{3}{5}\right) = \left(\frac{2}{32.2}\right)a_{x'}$$

$$a_{x'} = 19.32 \text{ ft/s}^2$$



$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2 - 2(15)$$

$v_B = 31.48$ ft/s

$(+\searrow)$ $\quad v_B = v_A + a_c t$

$$31.48 = 5 + 19.32 t_{AB}$$

$$t_{AB} = 1.371 \text{ s}$$

$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$$x = 0 + 31.48\left(\frac{4}{5}\right)t \qquad (1)$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 30 - 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

Equation of inclined surface :

$$\frac{y}{x} = \frac{1}{2}; \quad y = \frac{1}{2}x \qquad (2)$$

Thus

$$30 - 18.888t - 16.1t^2 = 12.592t$$

$$-16.1t^2 - 31.480t + 30 = 0$$

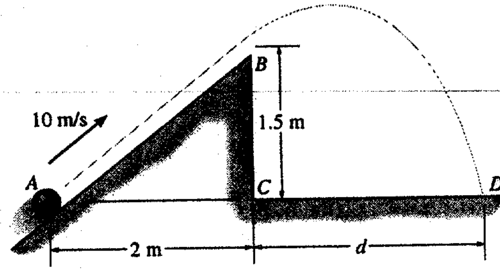
Solving for the positive root :

$t = 0.7014$ s

Total time is

$t = 1.371 + 0.7014 = 2.07$ s **Ans**

14-89. The 2-kg ball of negligible size is fired from point A with an initial velocity of 10 m/s up the smooth inclined plane. Determine the distance from point C to where it hits the horizontal surface at D. Also, what is its velocity when it strikes the surface?

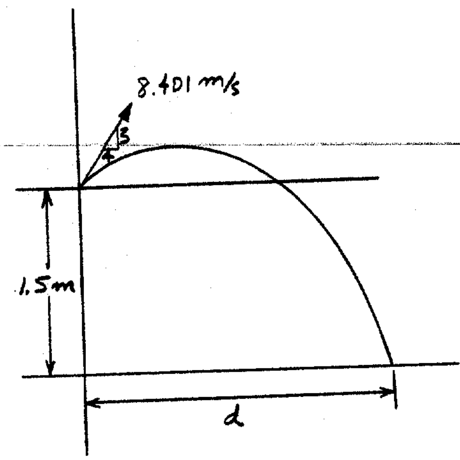


Datum at A:

$T_A + V_A = T_B + V_B$

$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_B)^2 + 2(9.81)(1.5)$

$v_B = 8.401$ m/s



$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$d = 0 + 8.401\left(\frac{4}{5}\right)t$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$-1.5 = 0 + 8.401\left(\frac{3}{5}\right)t + \frac{1}{2}(-9.81)t^2$

$-4.905t^2 + 5.040t + 1.5 = 0$

Solving for the positive root,

$t = 1.269$ s

$d = 8.401\left(\frac{4}{5}\right)(1.269) = 8.53$ m **Ans**

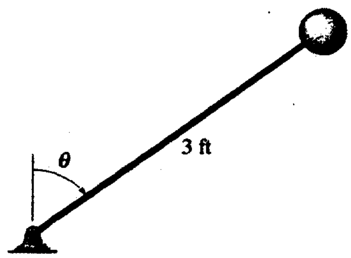
Datum at A:

$T_A + V_A = T_D + V_D$

$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_D)^2 + 0$

$v_D = 10$ m/s **Ans**

14-90. The ball has a weight of 15 lb and is fixed to a rod having a negligible mass. If it is released from rest when $\theta = 0°$, determine the angle θ at which the compressive force in the rod becomes zero.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{15}{32.2}\right)v^2 - 15(3)(1-\cos\theta)$$

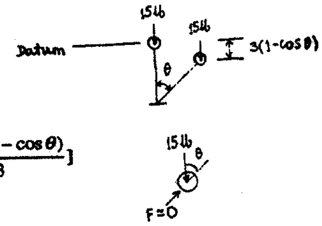
$$v^2 = 193.2(1-\cos\theta)$$

$$+\swarrow \Sigma F_n = m a_n; \qquad 15\cos\theta = \frac{15}{32.2}\left[\frac{193.2(1-\cos\theta)}{3}\right]$$

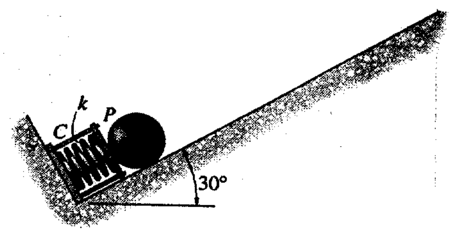
$$\cos\theta = 2 - 2\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\theta = 48.2° \qquad \text{Ans}$$



14-91. The 0.5-lb ball is shot from the spring device shown. The spring has a stiffness $k = 10$ lb/in. and the four cords C and plate P keep the spring compressed 2 in. when no load is on the plate. The plate is pushed back 3 in. from its initial position, so that $s = 30$ in. If it is then released from rest, determine the speed of the ball when it leaves the surface of the smooth inclined plane.



Potential Energy : The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30° = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625$ ft·lb. The initial and final elastic potential energy are $\frac{1}{2}(120)\left(\frac{2+3}{12}\right)^2 = 10.42$ ft·lb and $\frac{1}{2}(120)\left(\frac{2}{12}\right)^2 = 1.667$ ft·lb, respectively.

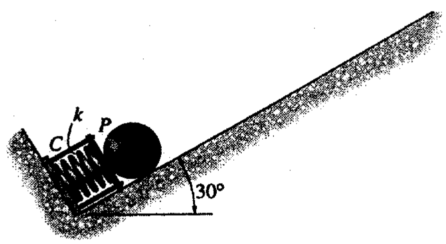
Conservation of Energy :

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 10.42 = \frac{1}{2}\left(\frac{0.5}{32.2}\right)v^2 + 0.625 + 1.667$$

$$v = 32.3 \text{ ft/s} \qquad \text{Ans}$$

***14-92.** The 0.5-lb ball is shot from the spring device shown. Determine the smallest stiffness k which is required to shoot the ball a maximum distance $s = 30$ in. up the plane after the spring is pushed back 3 in. and the ball is released from rest. The four cords C and plate P keep the spring compressed 2 in. when no load is on the plate.



Potential Energy : The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30° = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625$ ft·lb. The initial and final elastic potential energy are $\frac{1}{2}(k)\left(\frac{2+3}{12}\right)^2 = 0.08681k$ and $\frac{1}{2}(k)\left(\frac{2}{12}\right)^2 = 0.01389k$, respectively.

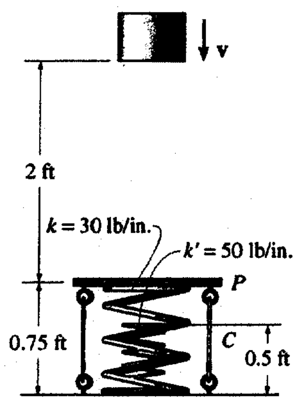
Conservation of Energy :

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 0.08681k = 0 + 0.625 + 0.01389k$$

$$k = 8.57 \text{ lb/ft} \qquad \text{Ans}$$

14-93. Four inelastic cables C are attached to a plate P and hold the 1-ft-long spring 0.25 ft in compression when *no weight* is on the plate. There is also an undeformed spring nested within this compressed spring. If the block, having a weight of 10 lb, is moving downward at $v = 4$ ft/s, when it is 2 ft above the plate, determine the maximum compression in each spring after it strikes the plate. Neglect the mass of the plate and spring and any energy lost in the collision.



$k = 30(12) = 360$ lb/ft

$k' = 50(12) = 600$ lb/ft

Assume both springs compress;

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(4)^2 + 0 + \frac{1}{2}(360)(0.25)^2 = 0 + \frac{1}{2}(360)(s+0.25)^2 + \frac{1}{2}(600)(s-0.25)^2 - 10(s+2)$$

$$13.73 = 180(s+0.25)^2 + 300(s-0.25)^2 - 10s - 20 \qquad (1)$$

$$33.73 = 180(s+0.25)^2 + 300(s-0.25)^2 - 10s$$

$$480s^2 - 70s - 3.73 = 0$$

Choose the positive root;

$s = 0.1873$ ft < 0.25 ft NG!

The nested spring does not deform.

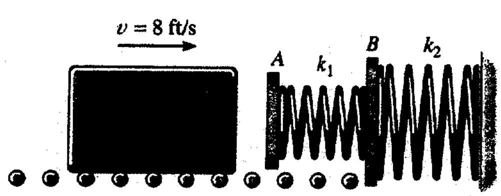
Thus Eq. (1) becomes

$$13.73 = 180(s+0.25)^2 - 10s - 20$$

$$180\,s^2 + 80\,s - 22.48 = 0$$

$s = 0.195$ ft **Ans**

14-94. The double-spring bumper is used to stop the 1500-lb steel billet in the rolling mill. Determine the maximum deflection of the plate A caused by the billet if it strikes the plate with a speed of 8 ft/s. Neglect the mass of the springs, rollers and the plates A and B. Take $k_1 = 3000$ lb/ft, $k_2 = 4500$ lb/ft.



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{1500}{32.2}\right)(8)^2 + 0 = 0 + \tfrac{1}{2}(3000)s_1^2 + \tfrac{1}{2}(4500)s_2^2 \qquad [1]$$

$$F_s = 3000s_1 = 4500s_2; \qquad [2]$$

$$s_1 = 1.5s_2$$

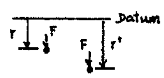
Solving Eqs. [1] and [2] yields :

$s_2 = 0.5148$ ft $\quad s_1 = 0.7722$ ft

$s_A = s_1 + s_2 = 0.7722 + 0.5148 = 1.29$ ft **Ans**

14-95. If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_em/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_em/r^2)$, Eq. 13–1. For the calculation, locate the datum at $r \rightarrow \infty$. Also, prove that F is a conservative force.

The work is computed by moving F from position r to a greater position r'.



$$V = -U = -\int F\,dr$$

$$= -G M_e m \int_r^{r'} \frac{dr}{r^2}$$

$$= -G M_e m\left(\frac{1}{r} - \frac{1}{r'}\right)$$

As $r' \rightarrow \infty$,

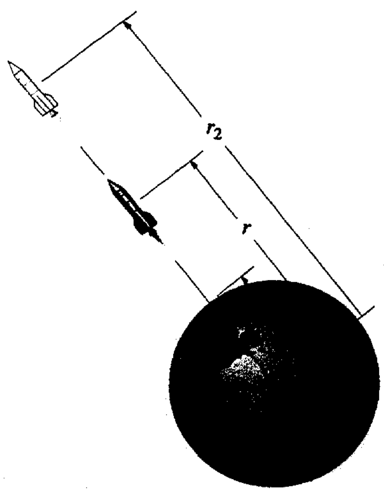
$$V = \frac{-G M_e m}{r}$$

To be conservative, require

$$F = -\nabla V = -\frac{\partial}{\partial r}\left(-\frac{G M_e m}{r}\right)$$

$$= \frac{-G M_e m}{r^2} \qquad \text{Q.E.D.}$$

***14-96.** A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming that no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_em/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.



$$F = G\frac{M_e m}{r^2}$$

$$U_{1-2} = \int F\,dr = -G M_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$U_{1-2} = G M_e m\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad \text{Ans}$$