Department of Humanities and Sciences
National Institute of Technology Goa
Farmagudi, Ponda, Goa - 403 401

Module 2

Conductors and Resistors : The resistivity range, The free electron theory, Conduction by free electrons, Conductor and resistor materials, Superconducting materials

OBJECTIVES

- To understand the electrical conduction in materials.
- To understand the resistivity range of conductors and resistors
- To explain the conductivity of a material using free electron theory.
- To derive the equation for electrical conductivity of a solid using free electron theory.
- To understand the Conductor and resistor materials, Superconducting materials

INTRODUCTION

- In solids, electrons in the outermost orbit of atoms determine its electrical properties.
- The structure and properties of solids are explained employing their electronic structure by the electron theory of solids.
- Electron theory is applicable to all solids, both metals and nonmetals.
- Electron theory explains the electrical, thermal and magnetic properties of solids.

Classical free electron theory: Metal contains free electrons which are responsible for the electrical conductivity and metals obey the laws of classical mechanics

Quantum free electron theory: The free electrons move with a constant potential and obeys the quantum laws

Zone theory: Free electrons move in periodic potential provided by the lattice. This theory is known as **band theory of solids**

The Electrical Conduction

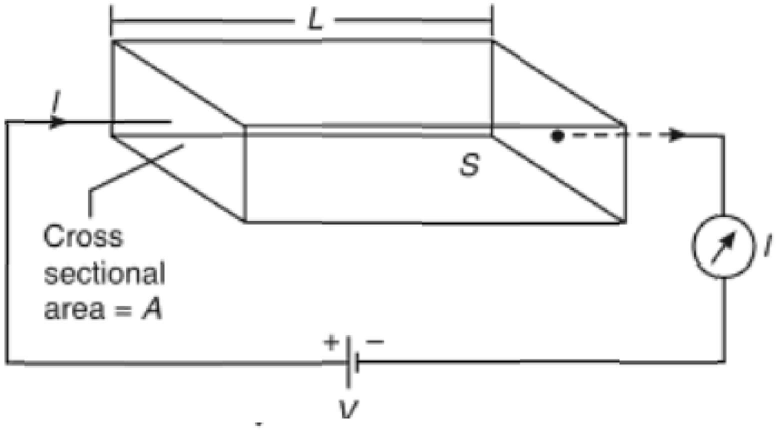
The electrical resistance of a solid

$$R \propto \frac{L}{A}$$

where L is the length and A cross-section area of the solid.

$$\mathbf{R\ (Electrical\ resistance)} = \rho \frac{L}{A}$$

ρ is called electrical resistivity.



Schematic of the circuit for determining the conductivity of a solid

Therefore, the resistivity of the material is

$$Resistivity\ (\rho) = Resistance \times \frac{Area}{Length}$$

$$\rho = R\frac{A}{L}\,\Omega\,\mathrm{m}$$

The reciprocal of the electrical resistivity is known as electrical conductivity (σ),

$$\sigma = \frac{1}{\rho}$$

$$\sigma = \left(\frac{1}{R}\right)\left(\frac{L}{A}\right)\Omega^{-1}\mathrm{m}^{-1}$$

The electrical conductivity of a material depends only on the presence of free electrons or conduction electrons.

CLASSIFICATION OF CONDUCTING MATERIALS

Based on the electrical conductivity, conducting materials are classified into three major categories

(1) Zero Resistivity Materials

- The material which conducts electricity at zero resistance.
- Eg. Superconducting materials like alloys of aluminium, zinc, gallium, nichrome, nibium, etc., conduct electricity almost at zero resistance below the transition temperature. T
- **Applications** of these materials are energy saving in power systems, superconducting magnets, memory storage, etc.

(2) Low Resistivity Materials

- The resistivity is very low (high electrical conductivity)
- The electrical conductivity of metals and alloys like silver, and aluminium is very high **($10^8\ \Omega^{-1}\ m^{-1}$)**
- Low resistivity materials are used as resistors, conductors, electrical contacts, etc.,

(3) High Resistivity Materials

- The materials have high resistivity (low temperature coefficient).
- Example, tungsten, platinum, nichrome, etc., have high resistivity and low temperature coefficient of resistance.
- High resistivity materials are used in **resistors**, **heating elements**, **resistance thermometers**, etc.

Resistivity range in Ohm m ⇨ *25 orders of magnitude*

$$R = \rho \frac{L}{A}$$

The Resistivity of Materials (ohm m)

Metallic materials ⟷ Semi-conductors

10^{-9}	10^{-7}	10^{-5}	10^{-3}	10^{-1}	10^{1}	10^{3}
Ag Cu Al Au	Ni Pb	Sb Bi Graphite	Ge (doped)	Ge		Si

\- *Solid electrolytes* -

Insulators

10^{5}	10^{7}	10^{9}	10^{11}	10^{13}	10^{15}	10^{17}
Window glass *Ionic conductivity*		Bakelite	Porcelain Diamond Rubber Polyethylene	Lucite Mica	PVC	SiO_2 (pure)

Resistivities and conductivities of some solids

Material	Resistivity Ohm.m	Conductivity S/m
Silver	1.47×10^{-8}	68×10^{6}
Copper	1.78×10^{-8}	58×10^{6}
Aluminium	2.63×10^{-8}	38×10^{6}
Steel	20.00×10^{-8}	5×10^{6}
Lead	22.00×10^{-8}	4×10^{6}
Carbon	3500×10^{-8}	0.03×10^{6}
Germanium	6.00×10^{-1}	1.67
Silicon	2300.00	4.35×10^{-4}
Aluminium glass	10^{10} to 10^{12}	10^{-10} to 10^{-12}
Borosilicate glass	10^{13}	10^{-13}
Polyethylene	10^{13} to 10^{13}	10^{-13} to 10^{-15}

The Free Electron Theory

- Outermost electrons of the atoms take part in conduction.
- These electrons are assumed to be free to move through the whole solid
  ⇨ Free electron cloud / gas, Fermi gas.
- Potential field due to ion-cores is assumed constant $\Rightarrow$ potential energy of electrons is not a function of the position (constant negative potential).
- The kinetic energy of the electron is much lower than that of bound electrons in an isolated atom.

Wave particle duality of electrons

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.62\ x\ 10^{-34}\ J\ s}{(9.109\ x\ 10^{-31}\ kg)\ v} = \frac{7.27x10^{-4}}{v} m$$

- λ → de Broglie wavelength
- v → velocity of the electrons
- h → Planck's constant

Wave number vector (k)

Non relativistic

$$k = \frac{2\pi}{\lambda} \qquad E = \frac{1}{2}mv^2 \qquad E = \left(\frac{h^2}{8\pi^2 m}\right)k^2$$

$$k = 2\pi\frac{mv}{h} \qquad \frac{kh}{2\pi m} = v$$

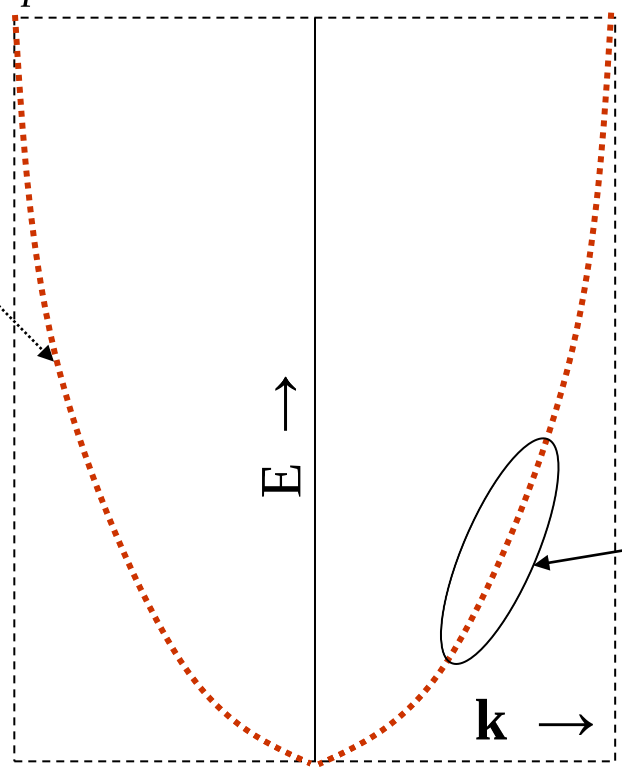
$$E = \frac{1}{2}m\left(\frac{kh}{2\pi m}\right)^2 = \frac{h^2k^2}{8\pi^2 m}$$

k is a vector in 2D/3D and is represented as a scalar in 1D

$$E = \left(\frac{h^2}{8\pi^2 m}\right)k^2$$

λ ↑ → k ↓ → E ↓

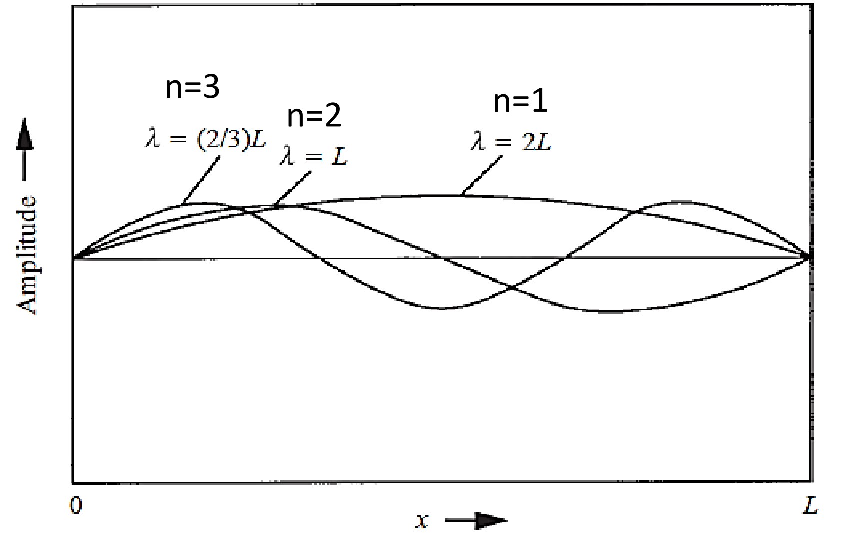
Free electrons



Discrete energy levels
(Pauli's exclusion principle)

Confined Electron

Electron in an 1D box



The de Broglie wavelengths of the first few electrons moving along ***x***

If the length of the box is L (e.g. a crystal)

Number of electrons moving from left to right equals the number in the opposite direction

$$n_x \frac{\lambda}{2} = L \qquad k = \frac{n_x \pi}{L}$$

n → integer *(quantum number)*

$$E_{n_x} = \frac{h^2}{8\pi^2 m}\left(\frac{n_x \pi}{L}\right)^2 = \frac{n_x^2 h^2}{8mL^2}$$

$$n_x \frac{\lambda}{2} = L$$

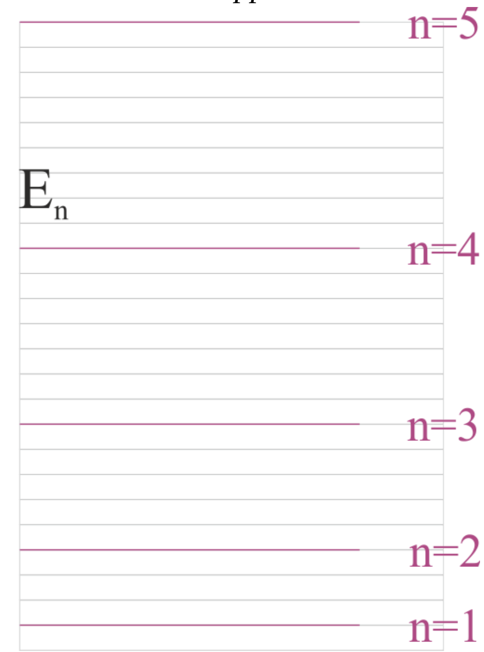
$$n_x \frac{1}{2}\left(\frac{2\pi}{k}\right) = L \qquad \left(\frac{n_x \pi}{L}\right) = k$$

$$k = \frac{2\pi}{\lambda} \qquad \lambda = \frac{2\pi}{k}$$

$$E_{n_x} = n_x^2 \left(\frac{h^2}{8mL^2}\right)$$

Quantization of Energy levels

$$n = \left[\frac{8mL^2}{n_x^2 h^2} E_{n_x}\right]^{\frac{1}{2}} = \left[8mE_{n_x}\right]^{\frac{1}{2}} \frac{L}{n_x h}$$



In 3D

$$E_n = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right) = \frac{h^2}{8mL^2}\left(n^2\right) \qquad n^2 = \frac{8mL^2}{h^2}E_n \qquad E = \frac{h^2k^2}{8\pi^2 m}$$

Problem: Calculate the energy difference between the $n_x = n_y = n_z = 1$ level and the next higher energy level for free electrons in a solid cube of 10 mm 10 mm 10 mm.

$$L = 10 \text{ mm} = 10^{-2} \text{ m}$$

$$E = \frac{(6.626 \times 10^{-34})^2 \, (1^2 + 1^2 + 1^2)}{8 \times 9.109 \times 10^{-31} \times (10^{-2})^2}$$

$$= 1.81 \times 10^{-33} \text{ J}$$

There are many equal energy quantum states above this energy level, with values of *nx*, *ny* and *nz* as (1,1,2), (1,2,1), (2,1,1), . For these states,

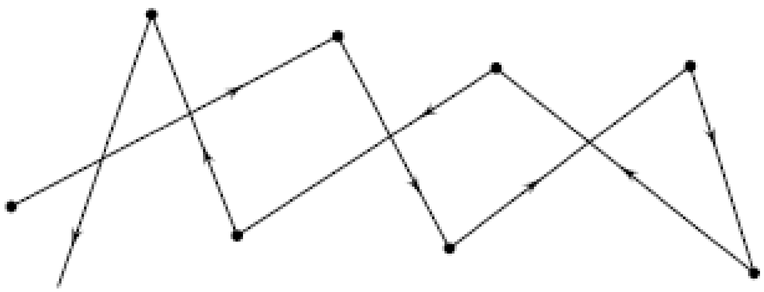
$$E = \frac{(6.626 \times 10^{-34})^2 \, (1^2 + 1^2 + 2^2)}{8 \times 9.109 \times 10^{-31} \times (10^{-2})^2}$$

$$= 3.62 \times 10^{-33} \text{ J}$$

The energy difference between the first and the next higher energy levels is extremely small, only **1.81 10^{-33} J**

Conduction by Free Electrons

Electrons moving towards the positive end of the applied field obtain extra velocity, while those moving in the opposite direction lose some velocity.



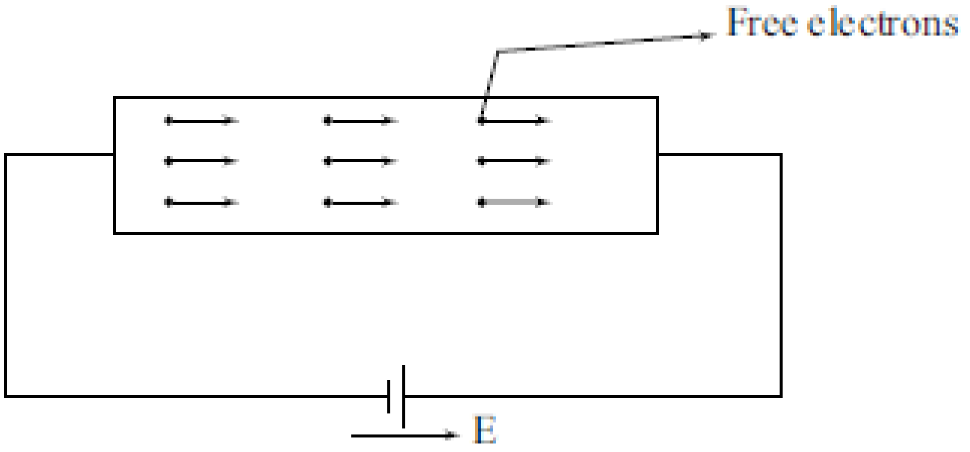
Absence of the field—Free electron

Drift Velocity

The average velocity acquired by the free electron in a particular direction during the presence of an electric field.

Relaxation Time

The relaxation time is defined as the time taken by a free electron to reach its equilibrium position from its disturbed position, during the presence of an applied field



Presence of field—free electron

Conduction by Free Electrons…..

The force experienced by an **electron of charge *e*** in an **applied field of gradient** can be equated to the force as defined in the classical law:

$$\varepsilon e = ma$$

If the average collision time is τ and v_d is the drift velocity acquired by the electrons

$$\varepsilon e = m(v_d/t)$$

$$v_d = \frac{\varepsilon e \tau}{m}$$

The flux J_e due to the flow of electrons is called the *current density*

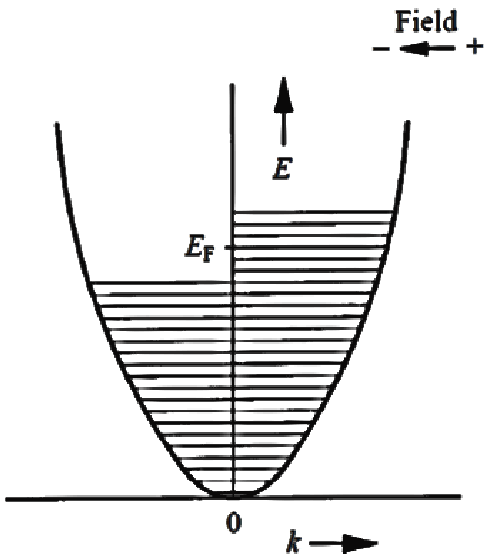
$$J_e = nev_d = \frac{ne^2\tau\varepsilon}{m}$$

conductivity is by definition the flux per unit potential gradient

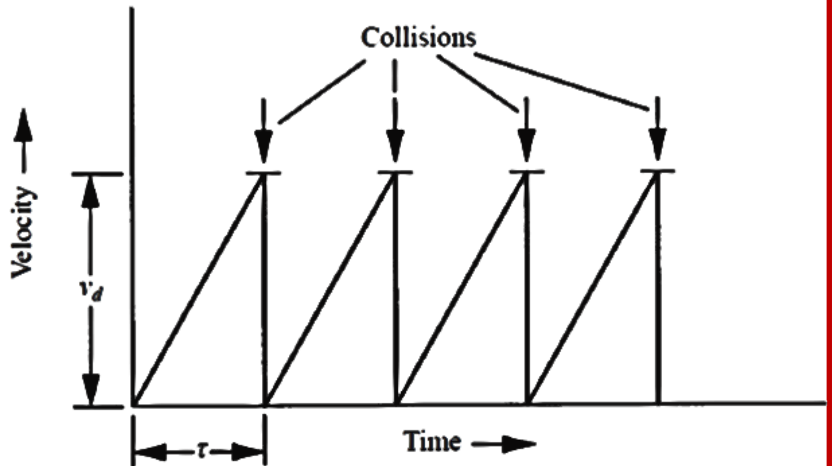
$$\frac{1}{R} = \frac{ne^2\tau\varepsilon}{m}\frac{1}{\varepsilon}$$

$$I = \frac{V}{R}$$
$$\frac{1}{R} = \frac{I}{V}$$

$$\boxed{\sigma = \frac{ne^2\tau}{m}}$$



Electrons moving towards the positive end of the applied field acquire extra velocity, while those moving in the opposite direction lose some velocity.



The extra velocity acquired by an electron due to an applied field is lost on collision.

THERMAL CONDUCTIVITY

The **thermal conductivity** is ability to conduct heat.
Thermal conductivity (K) of a material is equal to the amount of heat energy (Q) conducted per unit area of cross-section per second to the temperature gradient (dT/dx).

$$Q \propto dT/dx$$

$$Q = K \frac{dT}{dx}$$

$$K = -\frac{Q}{(dT/dx)}$$

In solids, the conduction takes place both by available free electrons and thermally excited lattice vibrations known as phonons.

the total conductivity is

$$K_{total} = K_{electrons} + K_{phonons}$$

Metals $K_{total} = K_{electrons}$

Semiconductors

$$K_{total} = K_{electrons} + K_{phonons}$$

Insulators $K_{total} = K_{phonons}$

Conductor and Resistor Materials

Properties of Typical Conductors and Resistors at Room Temperature

Material	Resistivity, 10^{-8} ohm m	Temperature coefficient α, K^{-1}	Density, 10^3 kg m^{-3}	Tensile strength*, MN m^{-2}
Silver	1.5	0.0040	10.49	125
Copper	1.7	0.0043	8.96	210
Gold	2.2	0.0035	19.32	138
Aluminium	2.8	0.0042	2.70	60
Tungsten wire	5.5	0.0045	19.3	2800
Molybdenum wire	4.9	0.0050	10.2	700
Platinum wire	10.9	0.0037	21.45	350
Tantalum wire	15.5	0.0032	16.6	490
Nichrome wire	108	0.0001	8.41	1000
Manganin	48	0.00002	8.2	420
Kanthal wire	135	0.00003	7.2	800

Conductors

- applications: transmission lines and distribution lines
- Low I^2R loss
- *Eg. Copper and aluminium*

Electrical contacts:

- Switches, brushes and relays
- High electrical conductivity, high thermal conductivity, high melting point and good oxidation resistance

Resistors

- primary requirements are uniform resistivity

$$\alpha = \frac{1}{R}\frac{dR}{dT}$$

Heating Elements

- high melting point, high electrical resistance, good oxidation resistance, good creep strength, low elastic modulus and low thermal expansion
- Nichrome (80% Ni and 20% Cr), Kanthal (69% Fe, 23% Cr, 6% Al and 2% Co)

Resistance Thermometers

resistance temperature detectors (RTDs), are sensors used to measure temperature.

- High temperature coefficient of resistance for good sensitivity
- Pure metals (Pt)

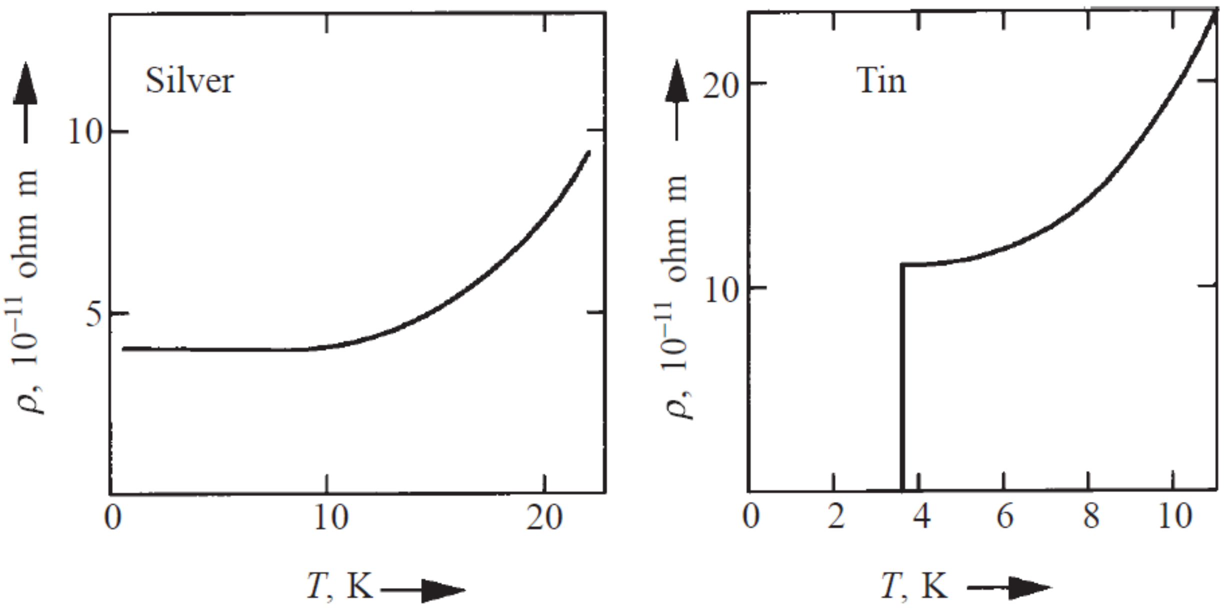
Properties of High Resistivity Metals

Metal	*Properties*	*Applications*
Tungsten	• Heavy metal • Quickly oxidisation in inert atmosphere. • Density ~19290 kg m^{-3} • Specific gravity ~ 19.6 • Melting point ~ 3683 K • Resistivity ~ $5.5 \times 10^{-8}\ \Omega$ m • Tensile strength ~ 3.45 GPa	Filament materials in electrical bulbs
Platinum	• Whitish metal with more ductility than silver, gold and copper • It has fcc structure • It is very ductile and malleable • Electrical conductivity is 16 % equal to copper • Oxidised even at high temperature • Specific gravity ~ 21.45 • Melting point ~ 2027 K • Resistivity ~ $10.5 \times 10^{-8}\ \Omega$ m • Temperature coefficient of resistivity ~ $3.93 \times 10^{-3}\ K^{-1}$ • Hardness (annealed) ~ 45 Brinell • Tensile strength ~117 MPa.	• Jewellry products • Resistance thermometer • Resistance wires • Thermocouples • Standard weights • Laboratory dishes

Composition, Properties and Applications of High Resistivity Alloys

Sr. No	Alloy	Composition	Properties	Applications
1.	Nichrome	80 % Ni 20 % Cr	• Higher ductility • Resistance ~ 108×10^{-8} Ωm • Temperature coefficient of resistance ~ 100×10^{-6} K^{-1}. • Maximum Working temperature ~ 1573 K.	Heating element in heaters and furnace
2.	Manganin	80-85 % Cu12- 15 % Mn 2-5 % Ni	• Good resistance to atmospheric corrosion • Higher ductility • Tensile strength ~ 482 MPa • Maximum working temperature ~ 343K • Resistivity ~ 48×10^{-8} Ωm • Temperature coefficient of resistivity is 20×10^{-6}	• Coil • Shunt wires in electrical instruments • Spring sheet
3.	Constantan	55 % Cu 45 % Ni	• High resistivity • High ductility • High corrosion resistance • Low temperature coefficient of resistance • High thermoelectric effect with either copper or ferrous • Tensile Strength ~ 965MPa • Maximum working temperature ~ 773K	• Thermocouple • Rheostats • Starters for electrical instruments
4.	Kanthal	69 % Fe 23 % Cr 6 % Al 2 % Co	• High oxidation resistance • High ductility and resistant to sulfuric acid • Resistivity ~ 139×10^{-8} Ω m Temperature coefficientresistivity ~ 30×10^{-6} K^{-1} • Maximum working temperature is 1573 K • Tensile strength ~ 813 MPa with elongation of 12-16 %	Heating element in heaters and furnace
5.	Alumel	94 % Ni 2.5 % Mn 0.5 % Fe Balance-other elements	• Maximum working temperature ~ 1366 K	Thermocouples
6.	Cromel	80 % Ni 20 % Cr	• Maximum working temperature ~ 1400 K • Resistance ~ 116×10^{-8} Ω • Temperature coefficient of resistivity ~ 0.58×10^{-3} K^{-1}	Thermocouples

Superconducting Materials

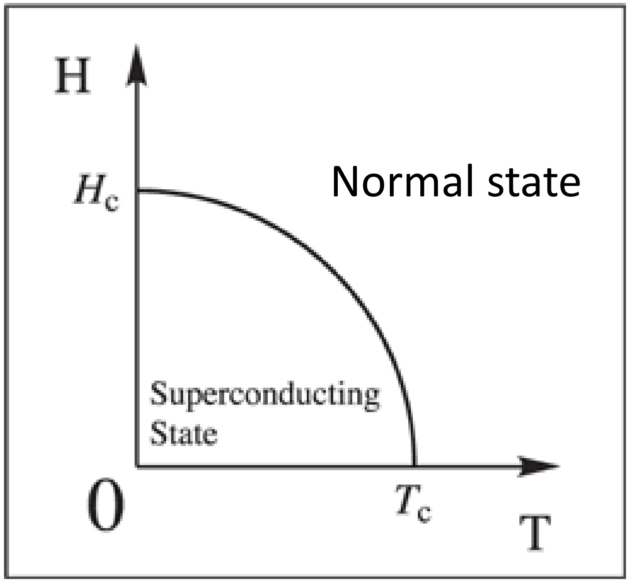


The electrical resistivity of (a) pure silver, and (b) tin, as a function of temperature near 0 K.

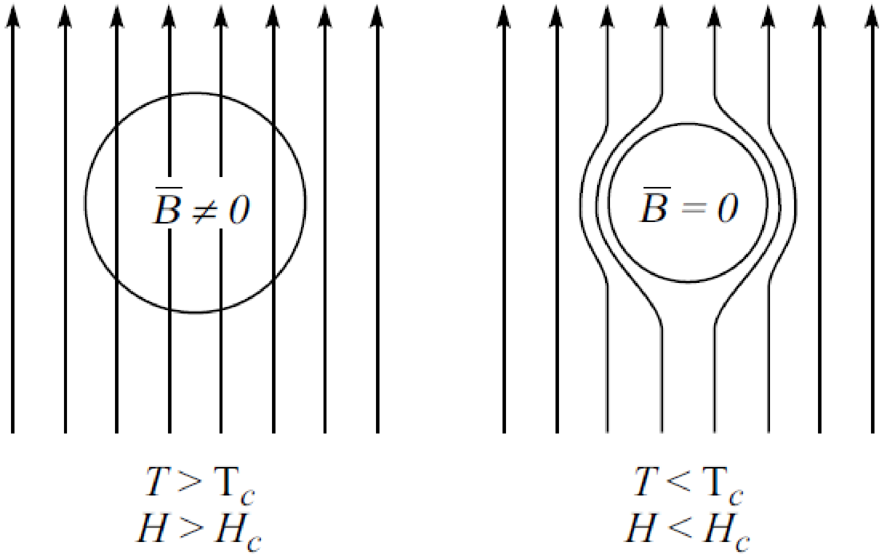
- **superconductor**, the resistivity suddenly drops to zero at very low temperatures
- **Critical temperature(T_c)** the temperature at which a normal conductor is converted into a superconductor.
- Eg. semiconductors, T_c varies from 0.3 K (GeTe) to 1.25 K (NbO); for metals, T_c varies from 0.35 K (Hafnium) to 9.22 K (Niobium); and for alloys, from 18.1 K (Nb_3Sn) to 22.65 K (Nb_3Ge).

Magnetic Field Effect:

- The **superconducting** state **can** be **destroyed** by a rise in the **applied magnetic field**
- **H_c** The minimum field required to destroy the superconducting property



- **Meissner effect :** When a small magnetic filed (below H_c) applied to superconducting material, the superconducting material behaves as a perfect diamagnetic material
- Magnetic lines of forces are **ejected** from the material

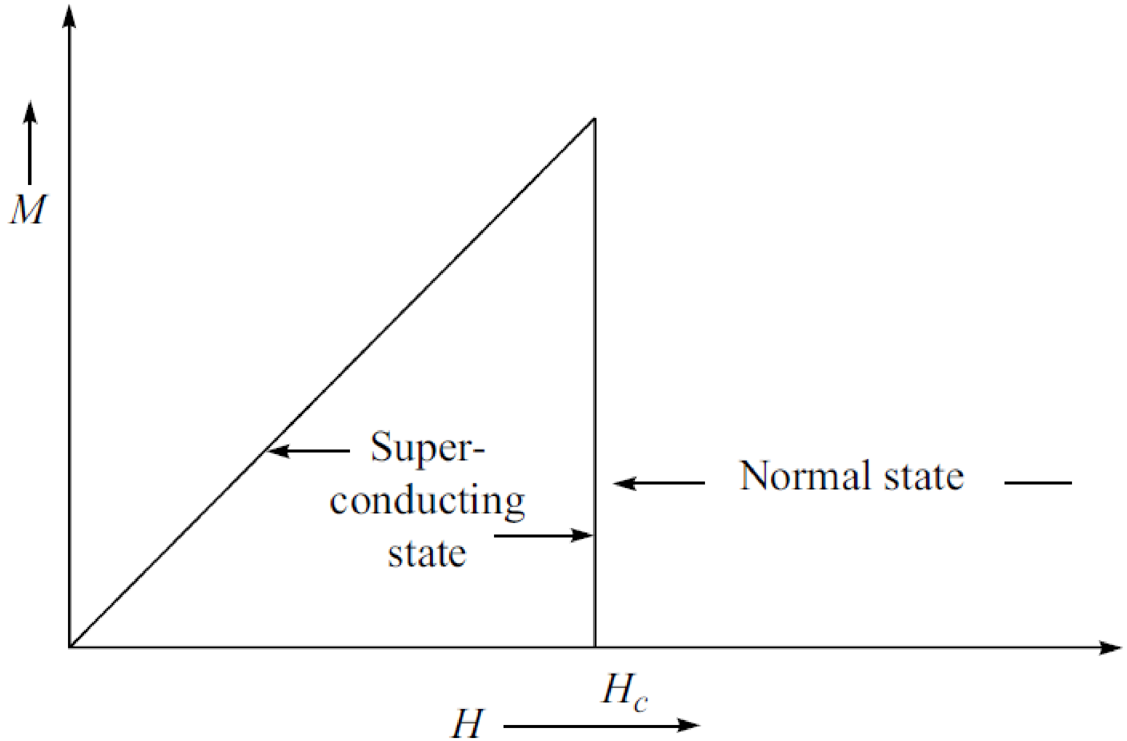


Meissner effect

TYPES OF SUPERCONDUCTORS

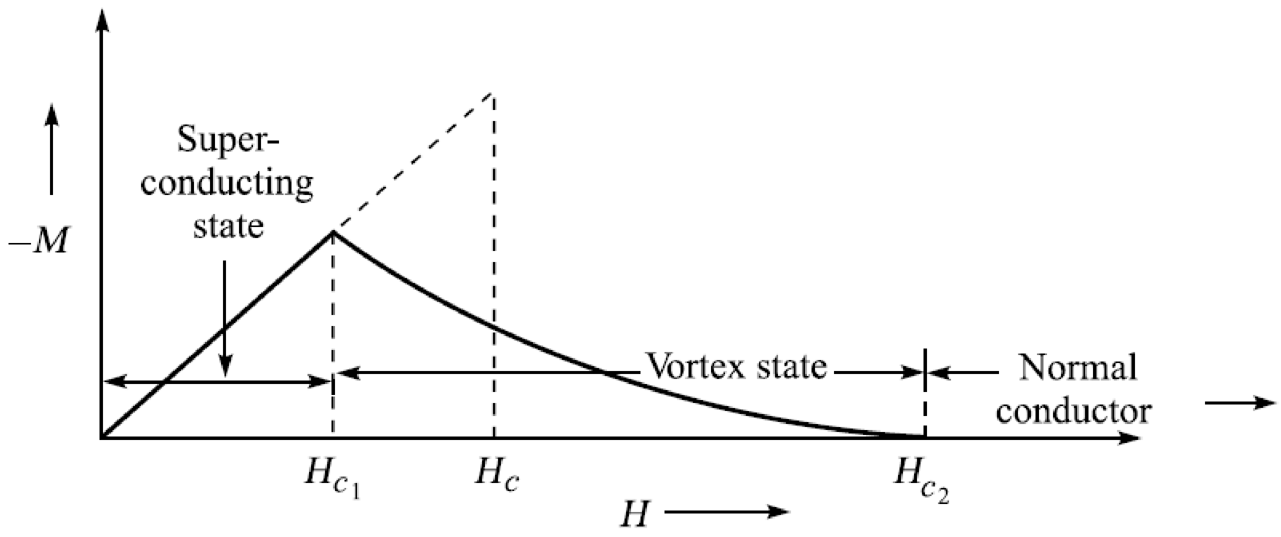
Type I Superconductors (soft superconductors)

- Type I superconductors behave as perfect diamagnetic materials and obey the Meissner effect.
- The material produces a repulsive force up to the critical field H_c
- At H_c, the repulsive force is zero
- Eg. Sn, Hg, Nb, V, $C_{0.1}T_{0.3}\,V_{0.6}$



Type I superconductor

Type II Superconductors (hard superconductors)



- Type II superconductors **do not** perfectly **obey** the **Meissner effect**
- These materials behave as a perfect superconductor up to $\mathbf{H_{c1}}$.
- Above H_{c1}, the repulsive force decreases, resulting in decrease in the magnetisation M
- The Meissner effect is incomplete in the region between Hc_1 and Hc_2; this region is known as the ***vortex region***
- Eg. Nb_3Sn, Nb_3Ge, $YBa_2Cu_3O_7$.

Potential applications of superconducting materials

- **Strong magenects:** producing very strong magnetic fields of about 50 Tesla.
- Superconductors can be used to perform logic and Memory / Storage element (persistent current) functions in Computers.
- **Maglev (magnetic levitation) trains**. These work because a superconductor repels a magnetic field so a magnet will float above a superconductor – this virtually eliminates the friction between the train and the track.
- **Superconducting cables (**Efficient Electricity Transportation): Large distance power transmission ($\rho = 0$)
- **SQUIDs** (Superconducting Quantum Interference Devices) are used to detect even the weakest magnetic field.
- **Switching device**
- Sensitive electrical equipment (small V variation $\longrightarrow$ large constant current)
- Highly efficient small sized electrical generator and transformer