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National Institute of Technology Goa

End Semester Examination, May-2015

Department of Humanities and Sciences

Course Name: **MATHEMATICS-II** Course Code: MA 150

Date: May 1, 2015 Time:9:30 AM

Duration: 3Hours. Max. Marks: 100

ANSWER ALL QUESTIONS

PART - A $(10 \times 2 = 20)$

1. Prove or disprove: If $S = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ is linearly dependent set of vectors in R^5 then each vector of S must be a linear combination of the remaining vectors of S.
2. Let A be an $n \times n$ matrix such that the system of equations $AX = 0$ has a non-trivial solution. Is it possible that the system of equation $A^tX = b$ has a unique solution for some $b \in R^n$? Justify.
3. Find a 2×2 matrix A that has $u = [2,\ 2]^t$ and $v = [4,\ 5]^t$ as eigenvectors with associated eigenvalues 3 and 1 respectively.
4. Prove or disprove: There exist 2×2 matrices A and B such that $AB - BA = I_2$.
5. Consider the square matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$
   (a) If 3 is an eigenvalue of A then find the eigenvalue of $adj(A)$
   (b) Find the nature of eigen vectors.
6. Prove that the eigen values of a skew-Hermitian matrix are purely imaginary.
7. Find the Laplace transform of the following functions
   (a) e^{t^2} (b) $(t^2 - 1)H(t - 1)$
8. Find the relation between $H(t - a)$ and $\delta(t - a)$
9. If $\int_1^5 t^n \delta(t - 2)dt = 8$, what is the exponent n?
10. Compute $t * t * t$

$(8 \times 10 = 80)$

PART - B

11. (a) Solve: $y' = e^{x-y}(e^x - e^y)$ [5M]

(b) Find the orthogonal trajectories of the family of curve $x^2 - y^2 = c$ and also sketch the relevant graphs [5M]

12. Apply the variation of parameters method to solve the differential equation: [10M]

$$x^3y''' - 3x^2y'' + 6xy' - 6y = x^4 \ln x.$$

13. (a) Let λ_1 and λ_2 be two distinct eigenvalues of a matrix A and let u_1 and u_2 be eigenvectors of A corresponding to λ_1 and λ_2, respectively. Show that $u_1 + u_2$ is not an eigenvector of A. [5M]

(b) Determine the eigenvalues and eigenvectors of $B = 2A^2 - (A/2) + 3I$ where $A = \begin{pmatrix} 8 & -4 \\ 2 & 2 \end{pmatrix}$ [5M]

14. Reduce the following quadratic form to standard form by making an approximate change of variables, $X = QY$ where Q is an orthogonal matrix

$$6x^2 + 3y^2 + 3z^2 - 2yz + 4zx - 4xy$$

15. (a) Find the Laplace inverse transform of [5M]

$$ln\left[1 + \frac{\omega^2}{s^2}\right]$$

(b) Using the Laplace transform, solve the integral equation [5M]

$$y(t) = 1 - \sinh t + \int_0^t (1+\tau)y(t-\tau)d\tau$$

16. (a) Let S be the subspace of $\mathbb{R}^5$ spanned by $\langle 1, 1, -1, 0, 0\rangle$, $\langle 0, 2, 1, 0, 0\rangle$ and $\langle 0, 1, -2, 0, 0\rangle$. Find the vector in S closest to $\langle 3, 0, 0, 1, 4\rangle$.

(b) Determine whether the following points are collinear: $\langle -1, 1, 6\rangle$, $\langle 2, 0, 1\rangle$ and $\langle 3, 0, 0\rangle$. If not, determine the equation for the plane containing these points. [5M]

17. (a) Solve the following the linear system of equation: [5M]

$$\begin{cases} 7x_1 - 3x_2 + 4x_3 = -7 \\ 2x_1 + x_2 - x_3 + 4x_4 = 6 \\ x_2 - 3x_4 = -5 \end{cases}$$ [5M]

(b) Find the least square line for the data points given below: [1, 3.8], [3, 11.7], [5, 20.6], [7, 26.5], and [9, 35.2]. [5M]

18. Consider the differential equation $y''' - 2y'' - y' + 2y = 0$. Convert given differential equal to three first order differential equations and solve them by matrix method. [5M]

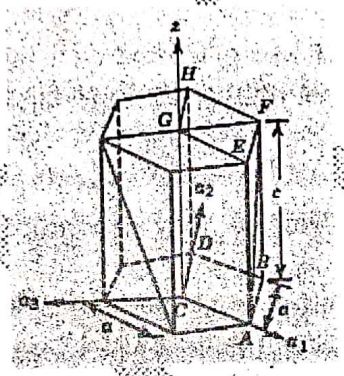
[10M]

*** ALL THE BEST ***

1. a) Draw the following Miller Planes and Directions of the unit cell.
   (i) Planes: [210],[1$\bar{1}$0], [231], [101] and (ii) Directions: [$\bar{1}$00] and[111]. (6M)

   b) The spacing between the principle planes in sodium chloride crystal is 2.82 Å. It is found that the first order Bragg reflection of a monochromatic beam of X-rays occur at an angle of 10°. (a)What is the wavelength of X-rays? (b) At what angle would be second order reflection occur? (4M)

   c) Determine the Miller–Bravais indices for the plane shown in the hexagonal unit cell. (4M)



   d) If the average energy required to produce a Frenkel defect in an ionic crystal is 1.4 eV. Find out the ratio of the number of Frenkel defects at 20°C and 300°C in one gram of the crystal. (4M)

2. a) Calculate the Fermi energy of sodium at 0K assuming that it has one free electron per atom and density of sodium is 970 kg/m^3 and atomic weight is 23. (3M)

   b) Discuss the Kronig-Penney model for the motion of an electron in a periodic potential and derive an expression for the density of energy states and carrier concentration in a solid material (Metal) by using Fermi distribution function. (7M)

   c) Show that the probability of finding an electron of energy ΔE above the Fermi level is same as probability of not finding an electron at energy ΔE below the Fermi level. (4M)

   d) In a Hall co-efficient experiment, a current of 0.25 Amp is sent through a metal strip having thickness 0.2 mm and width 5 mm. The Hall voltage is found to be 0.15 mv when a magnetic field of 2000 gauss is used. (a)What is carrier concentration (b) What is the drift velocity of carrier. (3M)

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3. a) Derive the expression $n = (2N_D)^{1/2}\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/4} exp\left[\left(\frac{E_D - E_C}{2kT}\right)\right]$ for majority carrier concentration in n-type semiconductor at low temperatures. (5M)

b) Draw the energy band diagrams of extrinsic semiconductors (n-type & p-type) at temperatures 0 K and 300K. (3M)

c) What fraction of the conductivity of intrinsic silicon at room temperature is due to (a) electrons and b) Holes? The electron and hole mobilities are 0.135 m^2/v.s and 0.048 m^2/v.s respectively. (4M)

d) The resistivity ρ of the two sides of an abrupt germanium diode are 2Ω cm on p-side and 1Ω cm on n-side. Calculate the height of the potential energy barrier at room temperature. $[n_i = 2.25 \times 10^{18}/cm^3, \mu_e = 3800 cm^2/V.s\ \mu_h = 1800 cm^2/V.s\ and\ k = 1.38 \times 10^{23}\ J/K]$ (4M)

e) i) Explain the operation of p-n-p transistor and ii) In an n-p-n transistor circuit, the collector current is 15 mA. If 95% of the electrons emitted by the emitter reach the collector, what is the base current? (6M)

4. a) A sphere of radius 'a' is polarized along the radius vector such that $\vec{P} = P_0\vec{r}$. Determine ρ, σ, total charge, $\vec{D}$ and $\vec{E}$ (4M)

b) Consider an electron of charge '–e ' moving in a circular orbit of radius 'a' about a charge '+e' in a field directed at right angles, to the plane of the orbit. Show that the polarizability α is approximately $4\pi\varepsilon_0 a^3$. (4M)

c) Derive the Clausius-Mosotti equation for non-polar solids having cubic crystal structure. (5M)

d) A dielectric material has $\varepsilon_r = 4.94\ and\ n^2 = 2.69$. Calculate the ratio between electronic and ionic polarizability of this material. (4M)

5. a) What are intrinsically conducting polymers? How they are classified and explain them with an example each. (4M)

b) Define nanotechnology and explain the effect on physical properties due to nanoscale. (4M)

c) Write the applications of SEM (Scanning Electron Microscope) and TEM (Transmission Electron Microscope) (3M)

6. a) What is meant by Hysteresis? Explain hysteresis loss. How would you use the hysteresis curves to select material for the construction of permanent magnets? (4M)

b) A magnetic material has a magnetization of 2300 A/m and produces a flux density of 0.00314 Wb/m^2. Calculate magnetizing force and relative permittivity of the material. $[\mu_0 = 12.57 \times 10^{-7}\ H/m]$ (3M)

c) What is the significance of critical temperature, critical magnetic field and critical current density for superconductors? Explain Meissner effect. (4M)

d) Calculate the critical current for a wire of lead having a diameter of 1mm at 4.2 K. The critical temperature for lead is 7.18 K and $H_C(0) = 6.5 \times 10^4 A/m$. (4M)

*** *All the best***