Computer Organization and Architecture

Representation of Numbers, Characters

Veena Thenkanidiyoor

National Institute of Technology Goa

1

Recap

- Bus structure
- System software
- Performance of a computer
  - Basic performance equation
  - Factors affecting the performance
- Improving the performance
  - Pipelining
  - Superscalar processing
- Performance measurement

2

Numbers

- All computers deal with numbers
- Instructions that perform basic arithmetic operations use data operands
- How the numbers are represented? — Integers; Real numbers
- How the numbers are manipulated
  - Addition operation
  - Subtraction operation

3

Number Representation

- Information represented by two-values electrical signals
  - 0 and 1
  - Bit of information – binary digit
  - *n*-bit vector

$$B = b_{n-1}, \ldots b_1 \; b_0$$

Where $b_i = 0$ or 1 for $0 \le i \le n-1$

- Unsigned integer value V
- Need to represent both **positive and negative** numbers
  - Sign-and-magnitude
  - 1's-complement
  - 2's-complement
  - Left most bit (n-1) is 0 for positive numbers and 1 for negative numbers

4

Sign-and-Magnitude Representation

- n-bit binary vector: B
- Left most bit (n-1) is 0 for positive numbers and 1 for negative numbers in the binary vector B
- Rest of the n-1 bits in B correspond to the magnitude of the number
- +5 gets represented as 0101
- -5 gets represented as 1101
- Range: $-(2^{n-1}-1)$ *to* $+(2^{n-1}-1)$

5

5

n-bit Binary Representation of Signed numbers

n = 4

Range: −7 to +7

$-2^{n-1}+1$ to $2^{n-1}-1$ with +0, −0

Resolution: 1

$b_3b_2b_1b_0$	Sign and magnitude
0 1 1 1	+7
0 1 1 0	+6
0 1 0 1	+5
0 1 0 0	+4
0 0 1 1	+3
0 0 1 0	+2
0 0 0 1	+1
0 0 0 0	+0
1 0 0 0	-0
1 0 0 1	-1
1 0 1 0	-2
1 0 1 1	-3
1 1 0 0	-4
1 1 0 1	-5
1 1 1 0	-6
1 1 1 1	-7

* 2 Representations 0 +ve, -ve.

* +ve, -ve numbers → Different

6

6

1's Complement Representation

- n-bit binary vector: B
- Left most bit (n-1) is 0 for positive numbers and 1 for negative numbers in the binary vector B
- Rest of the n-1 bits in B correspond to the complement of the corresponding positive numbers
- +5 gets represented as 0101
- -5 gets represented as 1010
- Range: $-(2^{n-1}-1)$ $to+(2^{n-1}-1)$

7

7

n-bit Binary Representation of Signed numbers

$b_3b_2b_1b_0$	Sign and magnitude	1's complement
0 1 1 1	+7	+7
0 1 1 0	+6	+6
0 1 0 1	+5	+5
0 1 0 0	+4	+4
0 0 1 1	+3	+3
0 0 1 0	+2	+2
0 0 0 1	+1	+1
0 0 0 0	+0	+0
1 0 0 0	- 0	- 7
1 0 0 1	- 1	- 6
1 0 1 0	- 2	- 5
1 0 1 1	- 3	- 4
1 1 0 0	- 4	- 3
1 1 0 1	- 5	- 2
1 1 1 0	- 6	- 1
1 1 1 1	- 7	- 0

8

8

2's Complement Representation

- n-bit binary vector: B
- Left most bit (n-1) is 0 for positive numbers and 1 for negative numbers in the binary vector B
- Rest of the n-1 bits in B correspond to the complement of the corresponding positive numbers and plus 1
- +5 gets represented as 0101
- -5 gets represented as 1011
- Range: $-(2^{n-1})$ to $-(2^{n-1}-1)$

9

n-bit Binary Representation of Signed numbers

$b_3b_2b_1b_0$	Sign and magnitude	1's complement	2's complement
0111	+7	+7	+7
0110	+6	+6	+6
0101	+5	+5	+5
0100	+4	+4	+4
0011	+3	+3	+3
0010	+2	+2	+2
0001	+1	+1	+1
0000	+0	+0	+0
1000	-0	-7	-8
1001	-1	-6	-7
1010	-2	-5	-6
1011	-3	-4	-5
1100	-4	-3	-4
1101	-5	-2	-3
1110	-6	-1	-2
1111	-7	-0	-1

Resolution: 1

10

n-bit Binary Representation of Signed numbers

$b_3b_2b_1b_0$	Sign and magnitude	1's complement	2's complement
0 1 1 1	+7	+7	+7
0 1 1 0	+6	+6	+6
0 1 0 1	+5	+5	+5
0 1 0 0	+4	+4	+4
0 0 1 1	+3	+3	+3
0 0 1 0	+2	+2	+2
0 0 0 1	+1	+1	+1
0 0 0 0	+0	+0	+0
1 0 0 0	- 0	- 7	- 8
1 0 0 1	- 1	- 6	- 7
1 0 1 0	- 2	- 5	- 6
1 0 1 1	- 3	- 4	- 5
1 1 0 0	- 4	- 3	- 4
1 1 0 1	- 5	- 2	- 3
1 1 1 0	- 6	- 1	- 2
1 1 1 1	- 7	- 0	- 1

11

n-bit Binary Representation of Signed numbers

$b_3b_2b_1b_0$	Sign and magnitude	1's complement	2's complement
0 1 1 1	+7	+7	+7
0 1 1 0	+6	+6	+6
0 1 0 1	+5	+5	+5
0 1 0 0	+4	+4	+4
0 0 1 1	+3	+3	+3
0 0 1 0	+2	+2	+2
0 0 0 1	+1	+1	+1
0 0 0 0	+0	+0	+0
1 0 0 0	- 0	- 7	- 8
1 0 0 1	- 1	- 6	- 7
1 0 1 0	- 2	- 5	- 6
1 0 1 1	- 3	- 4	- 5
1 1 0 0	- 4	- 3	- 4
1 1 0 1	- 5	- 2	- 3
1 1 1 0	- 6	- 1	- 2
1 1 1 1	- 7	- 0	- 1

- Distinct representation for +0 and -0 in sign-and-magnitude and 1's complement representation
- -8 is representable in 2's complement representation
- Sign-and-magnitude representation is most natural
- 2's complement is highly unnatural
  - Most useful

12

Characters

- Computers must be able to handle text information consisting of characters
  - Alphabets, decimal digits, punctuation marks and so on
- 8 bit codes
  - American Standard Committee of Information Interchange (ASCII) code

13

To Summarize

- Need for number and character representation
- Unsigned number representation
- Signed number representation
  - Sign-and-magnitude
  - 1's complement
  - 2's complement
- Character representation

14

Reference

- **Carl Hamacher, Zvonko Vranesic and Safwat Zaky, "Computer Organization", 5th Edition, Tata McGraw Hill, 2002**

15

Thank You

16