Roll No

NATIONAL INSTITUTE OF TECHNOLOGY GOA

Farmagudi, Ponda, Goa, 403401

Programme Name: B.Tech.

End Semester Examinations, December-2022

Course Name: Discrete Mathematics | Course Code: CS 203

Date: 13th December | Time: 2.00 PM

Duration: 3 Hours | Max. Marks: 100

ANSWER ALL QUESTIONS

Q1. Determine whether these system specifications are consistent or not: **[5 M]**

a. "The diagnostic message is stored in the buffer or it is retransmitted"
"If the diagnostic message is stored in the buffer, then it is not retransmitted"
"The diagnostic message is not stored in the buffer"

b. "The router can send packets to the edge system only if it supports the new address space"
"For the router to support the new address space it is necessary that the latest software release be installed"
"The router can send packets to the edge system if the latest software released is installed"
"The router does not support the new address space" **[5 M]**

Q2.

a. What Boolean search would you use to look for web pages about hiking in West Virginia? What if you want to find web pages about hiking in Virginia not West?

b. What is the value of x after the given statement "if (1+1=2) XOR (2+2=5) THEN x:=x+1" is encountered in a computer program if x=1 before the statement is reached.

c. What the sentence means if the or is an inclusive or (disjunction) versus an exclusive or:
"When you buy a new car, you get Rs. 5000 cashback or a 2% discount"

d. Find the negation of this statement:
"Ram is smart and hard working"

e. Prove or disprove $(p \Rightarrow q) \Rightarrow r$ and $p \Rightarrow (q \Rightarrow r)$ are logically equivalent. **[10 M]**

Q3. Express the below specifications into logical expression

a. If it is hot outside you buy an ice cream, and if you buy an ice cream it is hot outside

b. The message was sent from an unknown system but it was not scanned for virus.

c. No one is perfect

d. At least one of your friend is perfect (Hint: P(x) be x is perfect, F(x) be x is your friend)

e. No directories in the file system can be opened and no files can be closed when system errors have been detected. **[10 M]**

Q4.

a. Show that there exist irrational number x and y such that x^y is rational

b. Prove that $n^2+1 >= 2^n$ when n is a positive integer with $1 <= n <= 4$.

c. Prove that if n is an integer, then $n^2 >= n$. (Hint: Proof by cases)

d. Find the successors of the following sets

i. {ǿ} ii. {ǿ, {ǿ}}

e. Draw the Venn diagram of $A - (B \cap C \cap D)$ **[10 M]**

Q5.

a. f: R → R where $f(x) = \sqrt{x}$. Is it function? Justify your answer

b. f: R → R where $f(x) = \pm\sqrt{(x^2+1)}$? Justify your answer

c. Suppose that g is function from A to B and f is a function from B and C. Show that if both f and g are onto function, then f o g is also onto.

d. Show that the function $f(x) = ax + b$ from R to R is invertible, where a and b are constants with $a \neq 0$ and find the inverse of f.

[10 M]

Q6.

a. Use mathematical induction to show that if S is a finite set with n elements where n is a nonnegative integer, then S has 2^n subsets.

b. Use mathematical induction to prove that $3^n < n!$ if n is an integer greater than 6.

[10 M]

Q7.

a. How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen.

b. How many must be selected to guarantee that at least three hearts are selected

[15 M]

Q 8.

a. How many reflexive relations are there on a set with n elements? Explain in details.

b. How many Symmetric relations are there on a set with n elements? Explain in details.

c. Let R1 and R2 be the "congruent modulo 3" and the "congruent module 4" relations, respectively on the set of integers. That is $R1 = \{(a,b)|a \equiv b\ (mod\ 3)\}\ and\ R2 = \{(a,b)|\ a \equiv b\ (mod\ 4)\}$. Find $R1 \cup R2$ and $R1 \cap R2$

Q9. [15 M]

Let the Figure 1 shows the cost of communication from one city to other.

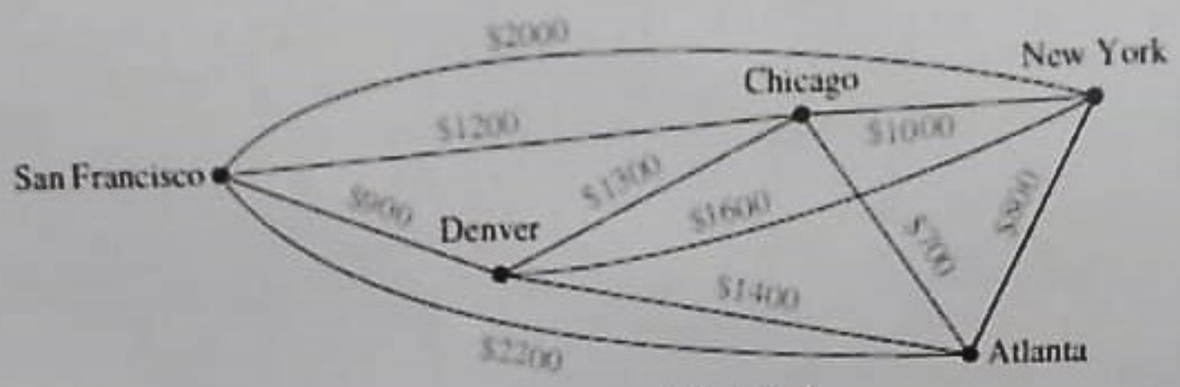

Figure 1

a. Find the least expensive communication network connecting all these cities.

b. Find the second least expensive communication network connecting all these cities.

c. Find the most expensive communication network connecting all these cities.

[10 M]

Q10.

a. Consider the rotation of geometric figures in a plane. Let R denotes the set of degrees, i.e., $R = \{0, 60, 120, 180, 240, 300\}$ denotes the six possible ways to rotate geometric figures drawn on a plane. Let © is the binary relation operation on R such that a © b (a, b ∈ R), is the overall angular rotation corresponding to the successive rotation by **a** and then **b.** Justify, is (R, ©) a group or not?

b. Let $\sigma \in S_n$ and $S_n = \{f : N \to N \mid f$ is one to one and onto$\}$. Let σ is a K-cycle. Verify that (1456)(152) = (16)(245)

-----All The Best-----