# Module 2
Conductors and Resistors

2.1 INTRODUCTION

In solids, electrons in the outermost orbit of atoms determine its electrical properties. Electron theory is applicable to all solids, both metals and nonmetals. In addition, it explains the electrical, thermal and magnetic properties of solids. The structure and properties of solids are explained employing their electronic structure by the electron theory of solids.

There are three main stages for development of this theory, namely, classical free electron theory, quantum free electron theory and zone theory.

1. **Classical free electron theory:** Classical free electron theory states that metal contains free electrons which are responsible for the electrical conductivity and metals obey the laws of classical mechanics.

2. **Quantum free electron theory**: According to Quantum free electron theory, the free electrons move with a constant potential. This theory obeys the quantum laws.

3. **Brillouin zone or Band theory**: According to Brillouin zone or Band theory free electrons move in periodic potential provided by the lattice.

2.2 ELECTRICAL CONDUCTION

Schematic of the circuit for determining the conductivity of a solid
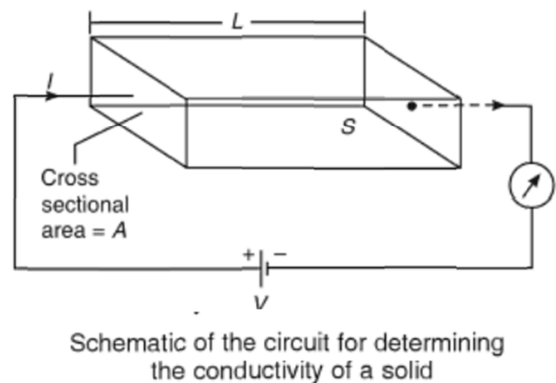
When an electronic potential difference is applied across materials, it conducts electricity. Electrical conductivity is defined as the quantity of electricity flowing through a material per unit area per unit time maintained at unit potential gradient. The material with this property is called a conducting material. The most important property of a material is electrical resistance, and it is

used to characterise the electrical properties of the material. Let R be the resistance of the material. A and l are the area and length of the material, respectively.

Therefore, the resistivity of the material is

$$\text{Resistivity } (\rho) = Resistance \; x \; \frac{Area}{Length}$$

$$\rho = R\frac{A}{l}$$

The reciprocal of the electrical resistivity is known as electrical conductivity (σ),

$$\sigma = \frac{1}{\rho}$$

$$= \frac{1}{R}\frac{l}{A} = \frac{l}{RA} \; \Omega^{-1}m^{-1}$$

The electrical conductivity of a material depends only on the presence of free electrons or conduction electrons. These free electrons move freely in the metal and do not correspond to any atom. This group of electrons is also called electron gas.

2.3 CLASSIFICATION OF CONDUCTING MATERIALS

Based on the electrical conductivity, conducting materials are classified into three major categories, namely,

(1) Zero resistivity materials

(2) Low resistivity materials, and

(3) High resistivity materials

(1) Zero Resistivity Materials The material which conducts electricity at zero resistance is known as a zero resistivity material. Superconducting materials like alloys of aluminium, zinc, gallium, nichrome, nibium, etc., conduct electricity almost at zero resistance below the transition temperature. The above superconducting materials are perfect diamagnets. The applications of these materials are energy saving in power systems, superconducting magnets, memory storage, etc.

(2) Low Resistivity Materials The resistivity of high electrical conductivity materials is very low. The electrical conductivity of metals and alloys like silver, and aluminium is very high, of the order of 10^8 Ω^{-1} m^{-1}. Low resistivity materials are used as resistors, conductors, electrical

contacts, etc., in electrical devices. In addition, these materials are also used in electrical power transmission and distribution lines, winding wires in motors and transformers.

(3) High Resistivity Materials Generally, materials with high resistivity will have low temperature coefficient of resistivity. For example, tungsten, platinum, nichrome, etc., have high resistivity and low temperature coefficient of resistance. High resistivity materials are used in the manufacturing of resistors, heating elements, resistance thermometers, etc.

2.4 THE RESISTIVITY RANGE

Table 1: The Resistivity of Materials (ohm m)

10^{-9}	10^{-7}	10^{-5}	10^{-3}	10^{-1}	10^{1}	10^{3}
Ag Cu Al Au	Ni Fe	Sb Bi Graphite	Ge (doped)	Ge (pure)		Si (pure)
← Metals →			← Semiconductors →			

10^{5}	10^{7}	10^{9}	10^{11}	10^{13}	10^{15}	10^{17}
Window glass		Bakelite	Porcelain Diamond Rubber, Nylon Polyethylene	Lucite Mica	PVC	SiO_2 (pure)
← Insulators →						

Electrical resistivity (or conductivity) is probably the most remarkable of all physical properties, in that *it varies over 25 orders of magnitude*. To get a feel for this wide range, Table.1 lists the electrical resistivity, at room temperature, of a number of materials which are important from the engineering point of view. The materials fall into three broad categories

Conductors are metals and alloys. Gold, silver and copper are among the best conductors of electricity. Therefore, their electrical resistivities are the lowest, as shown in Table 1. They are followed by aluminium whose resistivity is 60% higher than that of copper. Transition metals such as iron and nickel are not as good conductors as the above. Still poorer conductors are the semimetals of the fifth column, e.g., antimony and bismuth. Graphite, with one of its bonding electrons resonating between the (sp^2) bonds, also fall in this category of semimetals. The electrical resistivity of conductors ranges from 10^{-9} to 10^{-3} ohm m. The electrical conductivity, being the reciprocal of resistivity, ranges from 10^{9} ohm^{-1} m^{-1} to 10^{3} ohm^{-1} m^{-1}.

When the resistivity is in the range 10^{-3}–10^3 ohm m, we have the second category of materials known as *semiconductors*. They form the base for a number of solid state devices. Here, the resistivity is *a strong function of small concentrations of impurities*. Doped germanium, with an impurity content of a few tens per million, can have a resistivity about two orders of magnitude lower than that of pure germanium, see Table 1. Pure silicon has a higher resistivity than pure germanium.

The third category of materials are *insulators*. Common electrical insulating materials such as polyethylene, bakelite, lucite, mica, PVC, rubber and porcelain fall in this category. The resistivity range for this category extends from 10^4 to beyond 10^{17} ohm m. Here, a difference in resistivity of some twelve orders of magnitude is noticeable between a silica glass and soda-lime-silica (window glass). This striking difference is a result of the ionic conductivity of window glass. The relatively loosely-bound sodium and calcium cations in soda-lime glass diffuse and conduct much more readily, as compared to the tightly-bound immobile silicon cations in pure silica. Ionic conduction and ionic diffusivity are closely related phenomena.

2.5 THE FREE ELECTRON THEORY

The conducting properties of a solid are not a function of the total number of electrons in the solid, as only the *outermost electrons* of the atoms can take part in conduction. In the free electron model, the outermost electrons of an atom are not bound to that atom, but are *free to move through the whole solid*. These electrons have been variously called the free electron cloud, the free electron gas or the Fermi gas. In the free electron theory, the basic assumption is that *the potential field due to the ion cores is uniform* throughout the solid. The free electrons have the same potential energy everywhere in the solid. Due to the electrostatic attraction between a free electron and the ion cores, this potential energy will be a finite negative value. As we are interested only in energy differences, we can assume this constant potential to be zero. Then the only energy that we have to consider is the *kinetic energy*. This kinetic energy is substantially lower than that of the bound electrons in an isolated atom, as the field of motion for the free electron is considerably enlarged in the solid as compared to the field around an isolated atom. This effect is explained below. Electrons have both particle-like and wave-like characteristics. The de Broglie wavelength of an electron λ is related to its momentum *mv* as

$$\lambda = \frac{h}{mv} \quad (2.1)$$

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where h is Planck's constant, m is the mass of the free electron and v is its velocity. The wavelength is inversely related to the magnitude of the wave number vector **k:**

$$k = \frac{2\pi}{\lambda} \quad (2.2)$$

As the velocity of the free electrons is much smaller than that of light, we can ignore relativistic effects and use the classical relation for kinetic energy E:

$$E = \frac{1}{2}mv^2 \quad (2.3)$$

Substituting Eqs. (2.1) and (2.2) into Eq. (2.3), we obtain

$$E = \frac{h^2 K^2}{8\pi^2 m} \quad (2.4)$$

The kinetic energy E increases as the square of the wave number. This parabolic relationship between E and k is shown in Fig. 2.1. As λ is inversely related to k, the electron with the largest de Broglie wavelength will have the lowest kinetic energy. With the enlarged field of motion in the solid, the electrons can have larger wavelengths and hence lower kinetic energies.
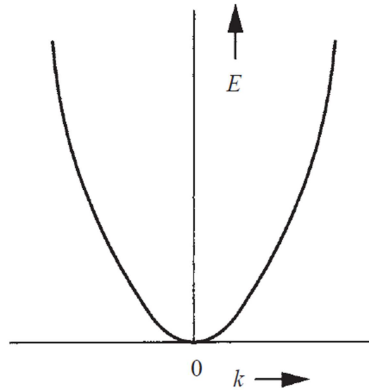

Fig. 2.1 The parabolic relationship between the kinetic energy E of a free electron and its wave number k.

Even though the variation in the kinetic energy E is shown to be continuous in Fig. 2.1, these are in fact *very closely spaced discrete energy levels*, as the quantum restrictions arising from the Pauli exclusion principle apply to the free electrons as well. Let us consider a solid, in which the electron motion is unidirectional. Let L be the length of this axis of motion. The longest wavelength that is permissible for the free electron is equal to twice this length as shown in Fig. 2.2, as the amplitude of the wave should be zero *at both ends of this length*. The next smaller

wavelength that is permissible is equal to L, the third one is equal to $(2/3)L$, and so on. The corresponding values of the wavenumber k are π/L, $2\pi/L$,...., as obtained from Eq. (2.2).
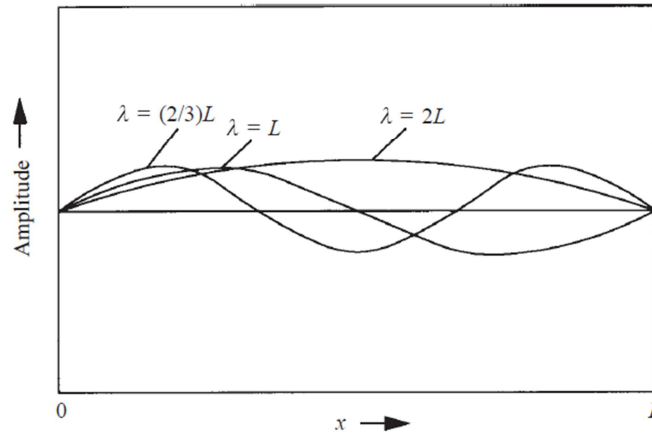

Fig. 2.2 The de Broglie wavelengths of the first few electrons moving along x.

In a solid that is electrically neutral, a free electron having a certain speed in one direction will always be associated with another electron having the same speed but moving in the opposite direction. (If this were not so, there will be a net flow of electrons in one direction, even in the absence of an externally applied field.) In other words, the wave number k takes on both positive and negative values. Thus, for the first two states, we have k equal to $\pm\pi/L$ and $\pm 2\pi/L$. In general, $k = \pm n\pi/L$, where the quantum number n takes on successively increasing integer values. Substituting $k = k = \pm n\pi/L$ in Eq. (2.4), we obtain

$$E = \frac{h^2 n^2}{8mL^2} \qquad (2.5)$$

In a three-dimensional solid, the free electron can move in any direction in space. So, three quantum numbers n_x, n_y and n_z corresponding to the three coordinate axes are used to define each quantum state. n_x, n_y and n_z take on successively-increasing integer values. $(n_x^2 + n_y^2 + n_z^2)$ is substituted in place of n^2 used for the unidirectional motion. Equation (14.5) can be rewritten for three dimensional motion as

$$E = h^2 \left(\frac{n_x^2 + n_y^2 + n_z^2}{8mL^2} \right) \qquad (2.6)$$

Each distinct combination of n_x, n_y and n_z corresponds to a quantum state, where two electrons of opposite spins can reside. Several combinations of n_x, n_y and n_z can result in the same value of E. Quantum states with the same energy are said to be *degenerate*. The successive energy levels

calculated from Eq. (2.6) are so close to one another that E can be considered to be varying continuously as illustrated for unidirectional motion in Fig. 2.1.

2.6 CLASSICAL FREE ELECTRON THEORY OF METALS

Very high electrical and thermal conductivity are the outstanding properties of metals. The positive ion cores and the valence electrons are the charge carriers in metals and hence, good conductivity means the presence of a mobile charge carrier. The observed high conductivity due to conduction by the drift ion cores is difficult to understand. If this happens, then conductivity must increase with increase in temperature. Hence, it may be concluded that mobile charges in metals are valence electrons. The classical free electron theory reveals that the free electrons are fully responsible for electrical conduction.

Postulates of Classical Free Electron Theory

The free electrons, or electron gas, available in a metal move freely here and there during the absence of an electronic field similar to the gas molecules moving in a vessel as shown in Fig. 2.3. These free electrons collide with other free electrons or positive ion cores or the walls of the container. Collisions of this type are known as elastic collisions. The total energy of an electron is assumed to be purely Kinetic Energy (KE).
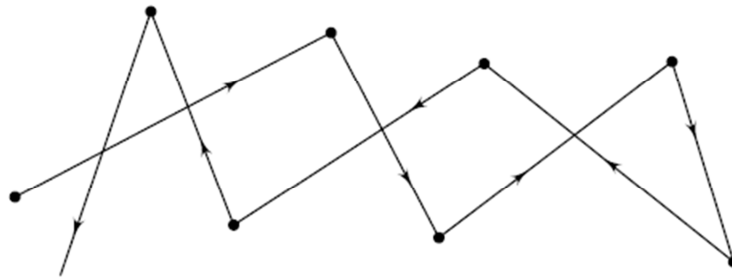

Fig 2.3 Absence of the field—Free electron

On the other hand, suppose an electric field is applied to the material through an external arrangement as shown in Fig. 2.4. The free electrons available in the metal gain some amount of energy and are directed to move towards a higher potential. These electrons acquire a constant velocity known as drift velocity <v>. The velocity of the electron obeys the Maxwell–Boltzmann studies.
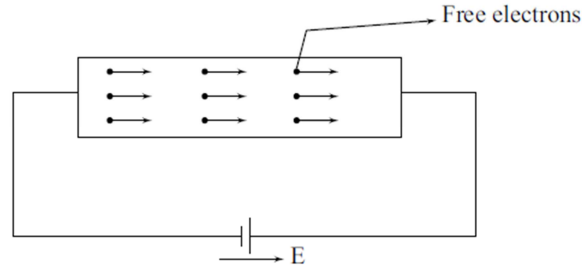
Fig. 2.4 Presence of field—free electron

Drift Velocity

It is defined as the average velocity acquired by the free electron in a particular direction during the presence of an electric field.

Relaxation Time

The relaxation time is defined as the time taken by a free electron to reach its equilibrium position from its disturbed position, during the presence of an applied field.

$$\tau = \frac{l}{<v>}$$

where l is the distance travelled by the electron.

2.7 Conduction by Free Electrons

We have already noted that the wave number *k* takes both positive and negative values. For every electron moving with a certain speed in a direction, there is another electron moving with the same speed in the opposite direction. This equal and opposite velocity distribution in a neutral solid can be biased by an *externally applied electric field* to yield a *net velocity* in one direction. With this biasing, the solid conducts electricity. Under an applied field, the *E–k* relationship of Fig. 2.1 gets modified to the distribution shown in Fig. 2.5. The negatively charged electrons are accelerated towards the positive end of the field. The velocity of the fastest electron moving in the direction of the positive end has a larger magnitude than that of the fastest electron moving towards the negative end of the field. Such redistribution is possible, *only when empty electron states are available immediately above the Fermi level*. This availability is a basic characteristic of conductors, as opposed to semiconductors and insulators.
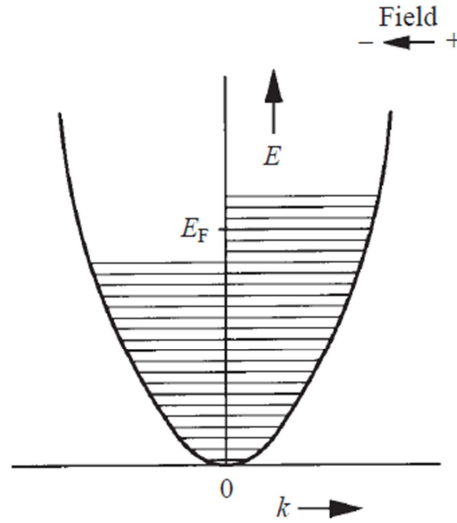
Fig. 2.4 Electrons moving towards the positive end of the applied field acquire extra velocity, while those moving in the opposite direction lose some velocity.

The force experienced by an electron of charge e in an applied field of gradient can be equated to the force as defined in the classical law:

$$\varepsilon e = ma \qquad (2.7)$$

where m is the mass of the electron and a is the acceleration due to the applied field. The electrons that are accelerated towards the positive end of the field do not continue to increase their velocity indefinitely. They collide with obstacles on their way. Depending on the time interval between two successive collisions, the electrons acquire an average increment of velocity called *drift velocity*, all of which they lose during a collision, as illustrated in Fig. 2.5. The drift velocity is the extra velocity that electrons acquire over and above their normal velocity in the absence of a field.
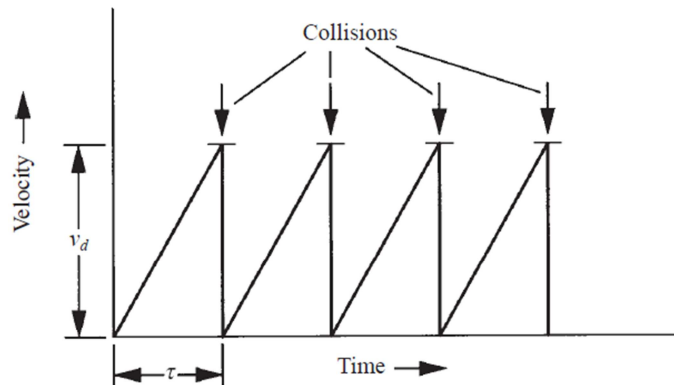

Fig. 2.5 The extra velocity acquired by an electron due to an applied field is lost on collision with an impurity, imperfection or phonon.

If the average collision time is t and v_d is the drift velocity acquired by the electrons, Eq. (2.7) can be rewritten as

$$m(v_d/\tau) = \varepsilon e \qquad (2.8)$$

$$v_d = \frac{\varepsilon e \tau}{m} \qquad (2.9)$$

The flux J_e due to the flow of electrons is called the *current density*:

$$J_e = nev_d = \frac{ne^2\tau\varepsilon}{m} \qquad (2.10)$$

where *n* is the number of free electrons of charge *e*. This is in the form of Ohm's law. As conductivity σ is by definition the flux per unit potential gradient, we have

$$\sigma = \frac{ne^2\tau}{m} \qquad (2.11)$$

The electrical resistivity ρ is the reciprocal of conductivity.

2.8 Conductor and Resistor Materials

For use as **conductors** in applications such as transmission lines and distribution lines, low I^2R loss is the primary consideration and the choice would be from amongst the best conductors, keeping in view the cost, fabricability and mechanical strength. *Copper and aluminium* are the most likely choices. For long distance transmission lines, aluminium is chosen. As a large cross section would reduce the I^2R loss, thick cables are preferred. If the elastic modulus of the aluminium cables is improved by reinforcement with steel as in ACSR (aluminium conductor steel reinforced) cables, the distance between successive poles along the transmission line can be substantially increased. More expensive copper is used for distribution lines, busbars and other energy conversion applications. OFHC (oxygen-free high conductivity) copper is often specified. Among the common solutes in copper, Fe, P and As are the most harmful in impairing the electrical conductivity.

For **electrical contacts** used in switches, brushes and relays, the material must possess high electrical conductivity, high thermal conductivity, high melting point and good oxidation resistance. High thermal conductivity helps to dissipate the heat effectively. High melting point is desirable so that any accidental overheating does not fuse together the contact points. Good oxidation resistance is necessary to keep the contact clean and free of insulating oxides. Copper and silver largely satisfy the above requirements. For low cost, copper is commonly used. For critical contacts such as those used in aircrafts, silver is preferred. The low mechanical strength

of pure silver is increased by the dispersion of fine particles of CdO. Dislocations moving in silver have to bend around the dispersed CdO particles and a fine dispersion increases the strength. CdO improves the wear resistance of silver. It also decomposes at the melting point of silver, thereby absorbing much of the heat generated by arcing and minimizing the loss of expensive silver by evaporation.

For *resistor* applications, the primary requirements are uniform resistivity (achieved in a homogeneous alloy), stable resistance (achieved by avoiding metallurgical changes such as ageing and relaxation of residual stresses), small temperature coefficient of resistance (α) and low thermoelectric potential with respect to copper. A small α minimizes the error in measurements due to variations in the ambient temperature. α is defined as

$$\alpha = \frac{1}{R}\frac{dR}{dT}$$

where R is the resistance of the alloy at temperature T. For manganin alloy (87% Cu and 13% Mn), α is only 20×10^{-6} K^{-1} as against 4000×10^{-6} K^{-1} for pure copper. Constantan (60% Cu and 40% Ni) is another such alloy. These alloys have also good resistance to atmospheric corrosion, another desirable property in a resistor.

A low thermoelectric potential with respect to copper, to which the resistor is commonly connected, reduces errors due to temperature differences between junctions. For high precision, dissimilar junctions should be maintained at the same temperature so that no thermoelectric potential develops.

Ballast resistors are used to maintain constant current in some industrial circuits. If the flow of current increases, the temperature increases and the resistance of the ballast increases. This in turn decreases the current in the circuit towards the initial value. An iron–nickel alloy with 71% Fe and 29% Ni with excellent oxidation resistance and a high α of 4500×10^{-6} K^{-1} is used for this application.

For *heating elements*, the primary requirements are high melting point, high electrical resistance, good oxidation resistance, good creep strength, low elastic modulus and low thermal expansion. The last two requirements help in reducing thermal fatigue due to repeated heating and cooling. The heating elements should be designed in a way as to allow unhindered expansion and contraction, for example, in the form of a coil of wire. Nichrome (80% Ni and 20% Cr) and Kanthal (69% Fe, 23% Cr, 6% Al and 2% Co) are used for heating elements up to 1300°C. SiC

and $MoSi_2$ can be used at higher temperatures up to 1700°C. Graphite, by virtue of its high sublimation temperature and good fabrication properties, is also widely used up to 1800°C. Molybdenum and tantalum need protective atmospheres at high temperatures, as their oxidation resistance is poor. By virtue of its very high melting point (3410°C), tungsten is used for filaments in incandescent lamps. Its creep resistance at white heat (above 1500°C) can be improved by dispersion hardening with thoria (ThO_2).

Resistance thermometers should have a high temperature coefficient of resistance for good sensitivity. A pure metal is obviously the choice for this application. Platinum, obtainable in very pure form, is used.
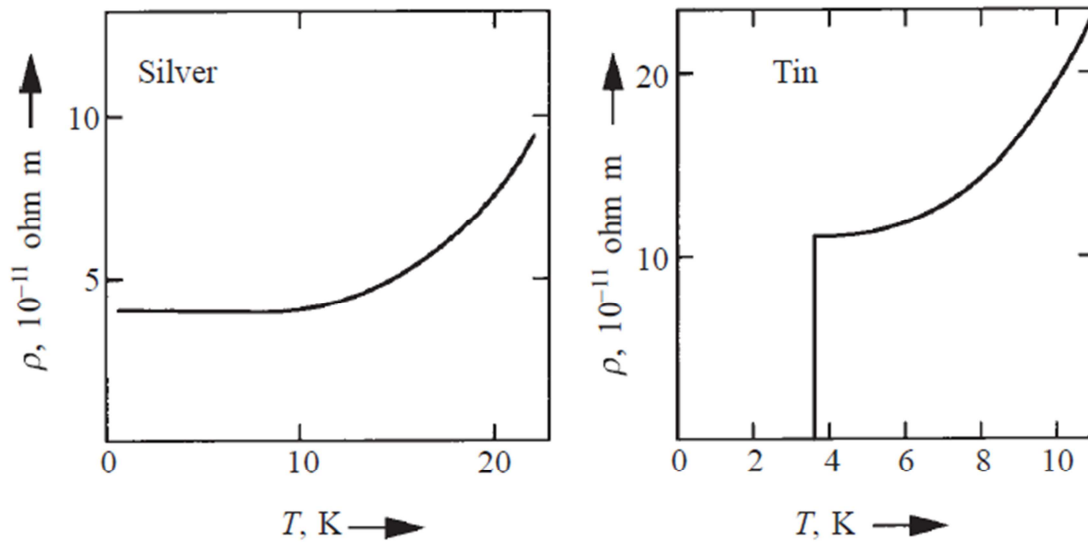
2.9 Superconducting Materials

Fig. 6 The electrical resistivity ρ of (a) pure silver, and (b) tin, as a function of temperature near 0 K.

Referring to Fig.6, the resistivity of pure silver decreases gradually to a low but measurable value of $\sim 10^{-11}$ ohm m at 0 K. Silver is not a superconductor. The resistivity of tin, on the other hand, falls abruptly at about 4 K to a value below the measurable limit of 10^{-20} ohm m. The temperature at which this abrupt fall occurs is called the *superconducting transition temperature* T_c. Tin is in the normal state above Tc and in the superconducting state below T_c. The resistivity in the superconducting state is zero for all practical purposes. A current induced in a superconducting ring persists for years with negligible decay.

The first application that was conceived of the superconducting effect was to produce a large permanent magnetic field by inducing a perpetual current in a superconductor. However, at temperatures below T_c, as the magnetic field strength reaches a critical value H_c, the superconductivity disappears. This is illustrated in Fig. 7. At $T = T_c$, $H_c = 0$. H_c increases as the temperature decreases below T_c. A similar graph relates the critical current density Jc that a superconductor can carry at temperatures below Tc. At $T = T_c$, $J_c = 0$. J_c increases as the temperature decreases below T_c in a manner similar to H_c in Fig. 7. In other words, the maximum current that a superconductor carries at a given temperature below Tc is limited by the magnetic field it produces at the surface of the superconductor.
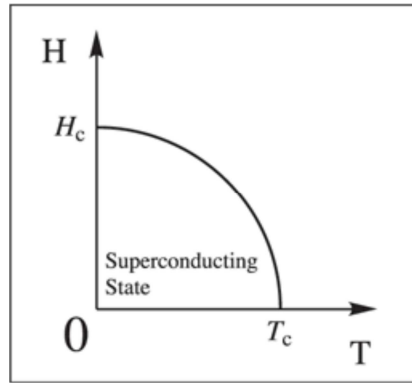

Fig. 7. The critical magnetic field H_c as a function of temperature for the superconducting-to-normal transition. $H_c = 0$ at T_c

In the superconducting state, the flux lines of a magnetic field are ejected out of the superconductor, as illustrated in Fig. 8. This effect is known as the *Meissner effect*. A superconductor is a perfect diamagnetic material, with the magnetic susceptibility χ equal to -1.
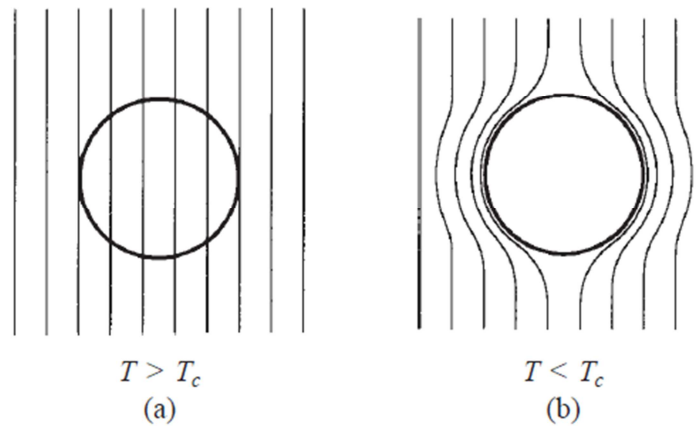

Fig. 8 The flux lines of an applied magnetic field are ejected out of the solid on crossing to the superconducting state.

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Types of superconductors

Superconductors are classified into two types. They are Type I superconductors and Type II superconductors. Type I superconductors are known as soft superconductors and type II superconductors are known as hard superconductors.

Type I Superconductors

Type I superconductors behave as perfect diamagnetic materials and obey the Meissner effect. Figure 9 shows the relation between the magnetisation produced and the applied magnetic field for Type I superconductors. A negative sign is introduced in the magnetisation value to represent the diamagnetic property of the superconductor. The material produces a repulsive force up to the critical field H_c. Therefore, due to the repulsive force, it does not allow the magnetic field to penetrate through it. Hence, the material behaves as a superconductor. At H_c, the repulsive force is zero and hence, the material behaves as a normal conductor. Sn, Hg, Nb, V, $C_{0.1}T_{0.3}V_{0.6}$ are some examples of Type I superconductors.
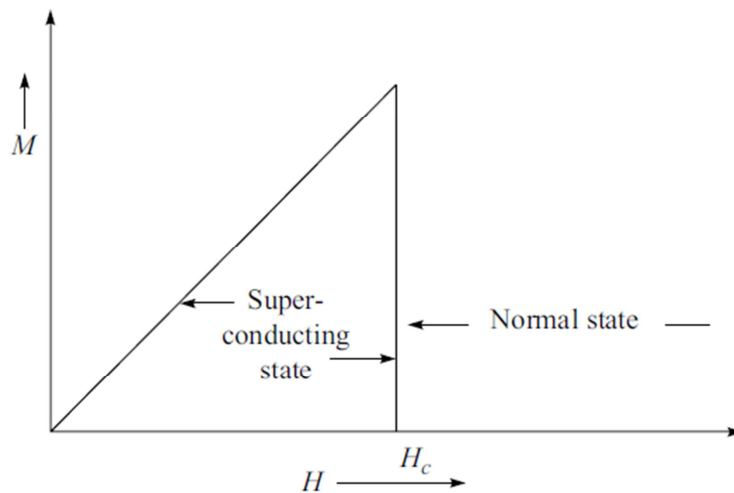

Fig. 9 Type I superconductor

Type II Superconductors

Type II superconductors do not perfectly obey the Meissner effect. These materials behave as a perfect superconductor up to H_{c1}. Above H_{c1}, the repulsive force decreases, resulting in decrease in the magnetization M and hence, the magnetic flux starts to penetrate through the material. The magnetic field penetrates up to the value H_{c2}. In the region up to H_{c2}, the material behaves as a superconductor as shown in Fig. 9.
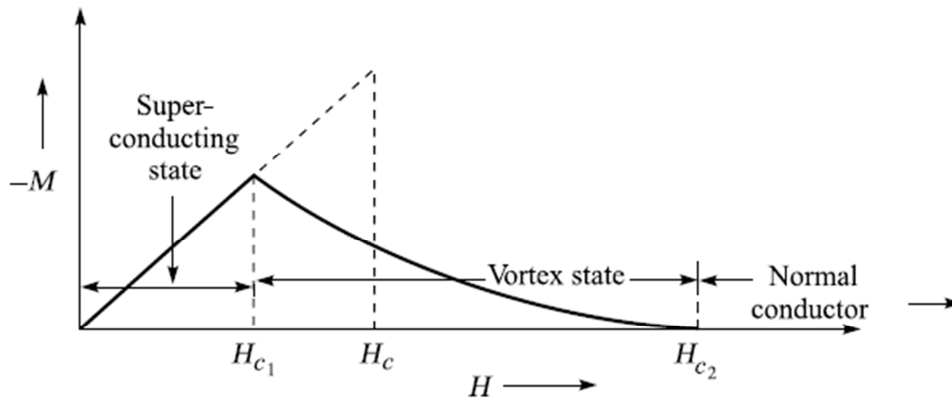
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Fig. 9 Type II superconductor

Let Hc1 and Hc2 be the lower and upper critical fields. The region between H_{c1} and H_{c2} is known as vortex state or mixed state. Above H_{c2}, the materials behave as normal conductors. Examples for type II superconductors are Nb_3Sn, Nb_3Ge, $YBa_2Cu_3O_7$.

Potential Applications of Superconducting materials

1. Electric generators: superconducting generators are very smaller in size and weight when compared with conventional generators. The low loss superconducting coil is rotated in an extremely strong magnetic field. Motors with very high powers could be constructed at very low voltage as low as 450V. This is the basis of new generation of energy saving power systems.

2. Low loss transmission lines and transformers: Since the resistance is almost zero at superconducting phase, the power loss during transmission is negligible. Hence electric cables are designed with superconducting wires. If superconductors are used for winding of a transformer, the power losses will be very small.

3. Magnetic Levitation : Diamagnetic property of a superconductor ie , rejection of magnetic flux lines is the basis of magnetic levitation. A superconducting material can be suspended in air against the repulsive force from a permanent magnet. This magnetic levitation effect can be used for high speed transportation.

4. Generation of high Magnetic fields: superconducting materials are used for producing very high magnetic fields of the order of 50Tesla. To generate such a high field, power consumed is only 10kW whereas in conventional method for such a high field power generator consumption is about 3MW. Moreover in conventional method, cooling of copper solenoid by water circulation is required to avoid burning of coil due to Joule heating.

5. Fast electrical switching: A superconductor possesses two states , the superconducting and normal. The application of a magnetic field greater than Hc can initiate a change from superconducting to normal and removal of field reverses the process. This principle is applied in development of switching element cryotron. Using such superconducting elements, one can develop extremely fast large scale computers.

6. Logic and storage function in computers: they are used to perform logic and storage functions in computers. The current – voltage characteristics associated with Josephson junction are suitable for memory elements.

7. SQUIDS (superconducting Quantum Interference Devices) : It is a double junction quantum interferometer. Two Josephson junctions mounted on a superconducting ring forms this interferometer. The SQUIDS are based on the flux quantization in a superconducting ring. Every minute magnetic signals are detected by these SQUID sensors. These are used to study tiny magnetic signals from the brain and heart. SQUID magnetometers are used to detect the paramagnetic response in the liver. This gives the information of iron held in the liver of the body accurately