



National Institute of Technology Goa

Programme Name: B.Tech
End Semester Examinations, December 2022

Course Name: **Mathematics-III**

Date: 09/12/2022

Duration: 3 Hours

Course Code: **MA200**

Time: 02:00PM - 05:00 PM

Max. Marks: 100

1. Answer All Questions.
2. No marks will be given if the explanation of your answer is missing.
3. The question paper consists of 2 pages.

Part A: Complex Analysis

1. If $f(z) = u + iv$ is an analytic function of z and if $u + v = \frac{\sin 2x}{\cosh 2y \cos 2x}$, find $f(z)$ (8M)
2. Find the Laurent series of the function $f(z) = \frac{1}{z^2 + 4z + 3}$ in (10M)
 - (a) $1 < |z| < 3$ (b) $|z| > 3$ (c) $0 < |z + 1| < 2$ (d) $1 > |z|$.
- ★ 3. Evaluate using Cauchy's Residue theorem $\int_C \frac{e^{zt}}{z^2(z^2 + 2z + 2)} dz$, where $C : |z| = 3$. (6M)
4. Verify Cauchy's theorem for the function $5 \sin 2z$, if C is the square with vertices $1 \pm i$ and $-1 \pm i$. (6M)
5. Evaluate the following improper integrals using Residue theorem (15M)
 - (a) $\int_0^\infty \frac{dx}{(x^2 + 9)(x^2 + 4)^2}$ (b) $\int_0^\infty \frac{x \sin x}{(x^2 + 1)(x^2 + 4)} dx$ (c) $\int_0^{2\pi} \frac{d\theta}{(5 - 3 \sin \theta)^2}$.
- ★ 6. Find the bi-linear transformation that maps the points $-1, i, 1 + i$ onto $0, 2i, 1 - i$ respectively. (5M)

Part B: Differential Equations

7. Use the Frobenius method to solve the differential equation $(x^2 - x)y'' + 3y' - 2y = 0$ near $x = 0$. (10M)
8. Find all the eigenvalues and eigenfunctions of the Sturm-Liouville problem $\frac{d}{dx} \left(x \frac{dy}{dx} \right) + \lambda \frac{y}{x} = 0$, $y'(1) = 0$, $y'(e^{2\pi}) = 0$. And check the orthogonality of eigenfunctions. (10M)
9. A bar of 10 cm long with its ends A and B kept at 20° and 40° respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 50° and at the same time at B is lowered to 10° . Find the subsequent temperature distribution. (10M)

10. Solve Laplace's equation in rectangle with $u(0, y) = 0$, $u(a, y) = 0$, $u(x, 0) = 0$ and $u(x, b) = \sin\left(\frac{n\pi x}{a}\right)$, $0 < x < a$. (10M)
11. A string of length L is stretched and fastened to two fixed points. Find the solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ when initial displacement $y(x, 0) = b \sin \frac{\pi x}{L}$ (10M)