Roll No								

National Institute of Technology Goa

Programme Name: B.Tech

Online Mid Semester Examinations, June 2021

Course Name: **Mathematics II** | Course Code: **MA150**

Date: 11/06/2021 | Time: 10:00 A.M - 11:30 A.M

Duration: $1\frac{1}{2}$ Hours | Max. Marks: 50

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1. Answer All Questions.
2. **No marks will be given if the explanation of your answer is missing**.
3. The question paper consists of **three** pages.
4. Upload the answer sheet with file name your roll number in .pdf format (Eg. 20MCE1001) on or before 11:45 AM.

Part A Linear Algebra

1. Verify the following with necessary justifications or by counter example [10M]

   (a) If $u = (1, 1, 1)$ is perpendicular to v and w, then v is parallel to w.

   (b) All Permutation matrices are symmetric.

   (c) If A is not symmetric then A^{-1} is not symmetric.

   (d) If B is the inverse of A^2, then BA is the inverse of A.

   (e) If A is a 3×3 matrix with rank 2 then the system of equations $Ax = b$ has unique solution.

   (f) An m *by* n matrix has no more than m pivot variables.

   (g) A square matrix has no free variables.

   (h) The column space of $A - I$ equals the column space of A.

   (i) If $A^T = -A$ then the row space of A equals the column space.

   (j) If $AB = B$ then $A = I$.

2. (a) Prove that A is invertible if a≠0 and a≠b (find the pivots or A^{-1}). Then find three numbers of c so that C is not invertible. [5M]

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$$

(b) Find the inverses (in any legal way) of

$$A = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 2 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \end{bmatrix}$$

3. Reduce the following matrices to an upper triangular matrix U, pre-multiplying it by a sequence of elementary matrices, then find lower triangular matrix L such that $A = LU(S = LU)$ and also find D such that A = LDU where [5M]

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix} \text{ and } S = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

4. Suppose the matrix is given by $A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 0 & 3 & 8 & 7 \\ 0 & 0 & 4 & 2 \end{bmatrix}$, [5M]

(a) Find all the special solutions to $AX = 0$ and describe in words the whole nullspace of A.

(b) Describe the column space of this particular matrix A. "All combinations of the four columns" is not a sufficient answer.

(c) Find the rank of A.

(d) What is the reduced row echelon form $R^* = rref(B)$ when B is the 6 by 8 block matrix $B = \begin{bmatrix} A & A \\ A & A \end{bmatrix}$ using the same matrix A.

Part B Ordinary Differential Equations

1. (a) Verify the nature of ODE $y' = \dfrac{2y^4 + x^4}{xy^3}$ and then solve the equation using suitable method.

(b) Solve the first order ODE of the form $y' + xy = xy^2$ [5M]

2. Check whether a given ODE $\dfrac{dy}{dx} = \dfrac{3yx^2}{x^3 + 2y^4}$ is exact. Further, solve the ODE after obtaining the integrating factor. [5M]

3. (a) Obtain the solutions of ODE $y'' + 4y' + 5y = 0$. Further, also verify that obtained solutions are linearly independent, if so find the general solution of the equation.

(b) Solve $x^2y'' - 3xy' + 4y = 0$. [5M]

4. Solve $(D^2 + 2)y = x^2e^{3x} + e^x \cos 2x$. [5M]

5. Use method of variation of parameters to find a unique solution the ODE

$$3y'' + 4y' + y = (\sin t)e^{-t}$$

with following initial conditions $y(0) = 1,\ y'(0) = 0$. [5M]

$* * *$ALL THE BEST $* * *$