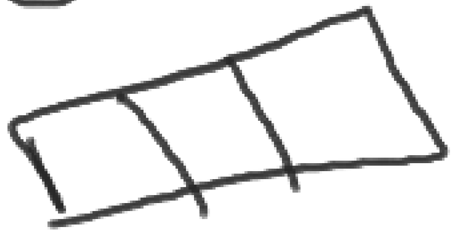
10^{24} atoms/m^3

B.E. Statistics

000 = 10



O O → indistinguishable

A A

B B

|A|A| |AA| | |AA| → 3

$$\frac{(3+3-1)!}{3!\,(3-1)!} = \frac{5!}{3!\,2!} = 10$$

$$W(n) = \prod_{i=1} \frac{(n_i+s_i-1)!}{n_i!\,(s_i-1)!} = \frac{(2+2-1)!}{2!\,1!} = \frac{3!}{2!} = 3$$

$$w(\bar{N}) = \prod_{i=0}^{i=r} \frac{(n_i + s_i - 1)!}{n_i!\,(s_i - 1)!}$$

Take log on both sidy

$$\ln(\omega(\bar{N})) = \sum_{i=0}^{i=r} \left[\ln(n_i + s_i - 1)! - \ln n_i! - \ln(s_i - 1)! \right]$$

stirling approximation

$$\ln x! = x \ln x - x$$

$$\ln(n_i + s_i - 1) \Rightarrow n_i + s_i - 1$$

large systems 10^{24} atoms/m^3

$$\Rightarrow n_i + s_i - 1 \cong (n_i + s_i)$$

$$= \sum_{i=0}^{i=r} (n_i + s_i)\ln(n_i + s_i) - (\cancel{n_i} + \cancel{s_i}) - \{n_i \ln n_i - \cancel{n_i}\}$$

$$+ (s_i - 1)\ln(s_i - 1) + (\cancel{s_i} - 1)\Big]$$

$$= \sum_{i=0}^{i=r} (n_i + s_i)\ln(n_i + s_i) - n_i \ln n_i - (s_i - 1)\ln(s_i - 1) - 1\Big]$$

$$\delta \ln(\omega(\bar{n})) = 0$$

$$\ln \omega(\bar{n}) = \sum_i \Big[(n_i + s_i) \ln(n_i + s_i) - n_i \ln n_i - \underbrace{(s_i - 1)\ln(s_i - 1)}_{=0} - 1 \Big]$$

$$= \sum_i \Big[\cancel{(n_i + s_i)} \frac{1}{\cancel{(n_i + s_i)}} \delta n_i + \delta n_i \ln(n_i + s_i) - \cancel{n_i} \frac{1}{\cancel{n_i}} \delta n_i$$

$$- \delta n_i \ln n_i \Big] = 0$$

$$= \sum_i \Big[\delta n_i \ln(n_i + s_i) - \delta n_i \ln n_i \Big] = 0$$

$$= \sum_i \Big[\ln\left(\frac{n_i + s_i}{n_i}\right) \delta n_i = 0 \qquad (1)$$

$$\sum_i \ln\left(\frac{n_i+s_i}{n_i}\right)\delta n_i = 0 \quad — ①$$

Total no. of Particly in the system remain constant

$$\sum_i n_i = n = \text{const} \Rightarrow \boxed{\sum_{i=0}^{i=r} \delta n_i = 0} \quad — ②$$

$$\sum_{i=0}^{i=r} \varepsilon_i n_i = E = \text{cont} \Rightarrow \boxed{\sum_{i=0}^{r} \varepsilon_i \delta n_i = 0} \quad — 3$$

lagrange method of undeterminent multipliy

$$\sum_{i=0}^{r} \ln\left(\frac{n_i+s_i}{n_i}\right)\delta n_i = 0 \quad \text{— (1)}$$

$-1 \times$

$$\sum_{i=0}^{r} \delta n_i = 0 \quad \text{— 2}$$

$\alpha \times$

$$\sum_{i=0}^{r} \varepsilon_i \delta n_i = 0 \quad \text{— 3}$$

$\beta \times$

$$\sum_{i=0}^{r}\left[-\ln\left(\frac{n_i+s_i}{n_i}\right)+\alpha+\beta\varepsilon_i\right]\delta n_i = 0 \quad \text{— (4)}$$

$$-\ln\left(\frac{n_i+s_i}{n_i}\right)+\alpha+\beta\epsilon_i=0$$

$$\ln\left(\frac{n_i+s_i}{n_i}\right)=\alpha+\beta\epsilon_i$$

$$\frac{n_i+s_i}{n_i}=e^{\alpha+\beta\epsilon_i}$$

$$1+\frac{s_i}{n_i}=e^{\alpha+\beta\epsilon_i} \Rightarrow \frac{s_i}{n_i}=e^{\alpha+\beta\epsilon_i}-1$$

$$\boxed{\frac{n_i}{s_i} = \frac{1}{e^{\alpha+\beta\epsilon_i} - 1}}$$

$$\beta = \frac{1}{kT}$$

$i = 0$

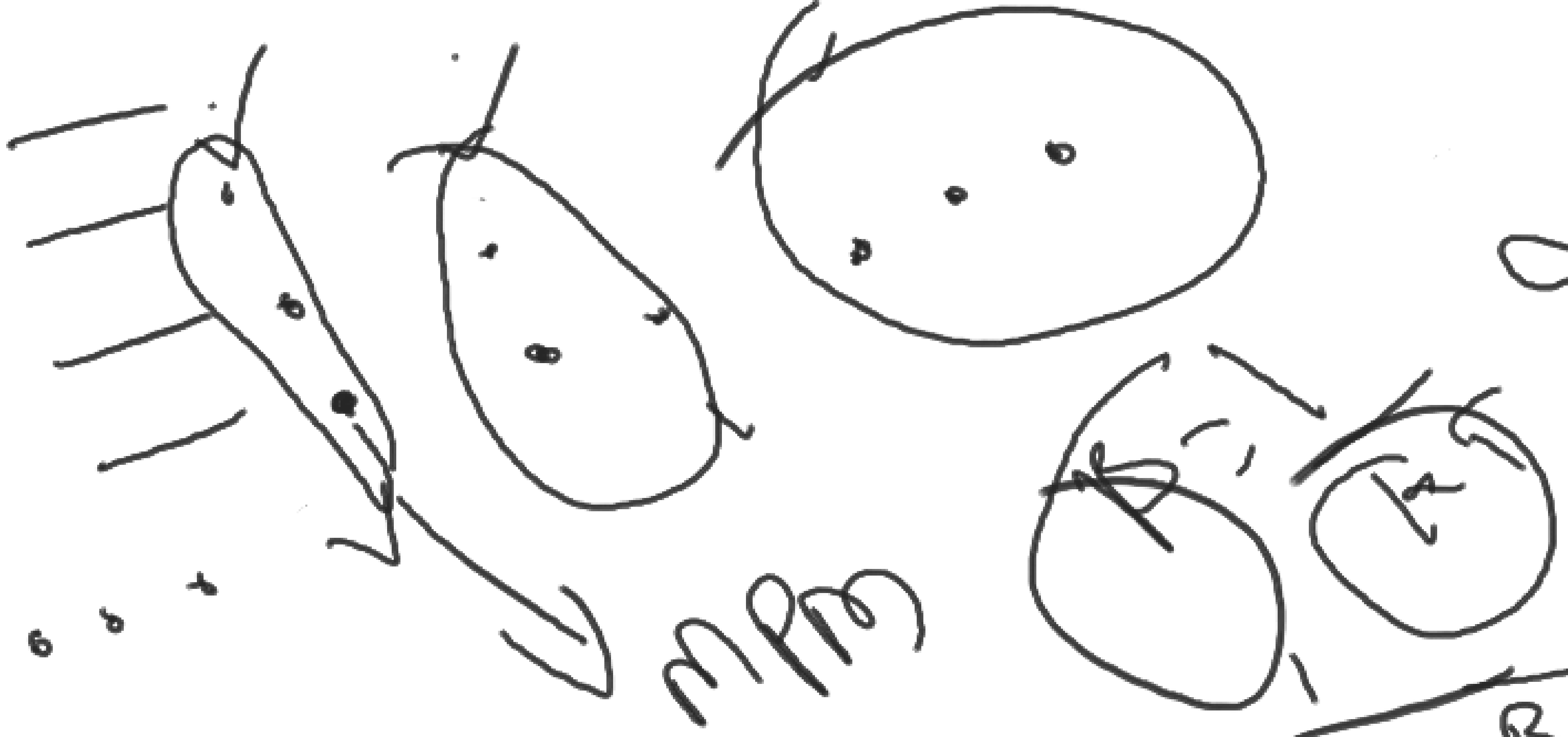
$$\frac{n_0}{s_0} = \frac{1}{e^{\alpha+\beta\epsilon_1} - 1}$$

$i = 2$

$$\frac{n_2}{s_2} = \frac{1}{e^{\alpha+\beta\epsilon_2} - 1}$$

$$\boxed{\frac{n_i}{s_i} = \frac{1}{e^{\alpha+\frac{\epsilon_i}{kT}} - 1}}$$

$$\boxed{f_{BE} = \frac{n_i}{s_i} = \frac{1}{e^{\alpha+\beta\epsilon_i} - 1}}$$



macro

micro

MB FD BE

A B

A A

$$r_{BE} = \frac{n_i}{s_i} = \frac{1}{e^{\alpha+\beta/kT} - 1}$$

$$r_{FD} = \frac{n_i}{s_i} = \frac{1}{e^{\alpha+\beta/kT} + 1}$$

$n_1 + n_2 + n_3 + \dots + n_{30} = 90$

$\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3$

$s_1 \quad s_2 \quad s_3$

$n_1 \quad n_2 \quad n_3$

E_1, E_2, E_3

ε_4

E_{50} n_i

Fermi - Dirac statistic

$$\frac{s_i}{n_i!(s_i\,n_i)}$$

└→ B E statistic

$$\Rightarrow \frac{4!}{3!(4-3)!} = \frac{4!}{3!}$$ + PEL

O O → Indistinguishably

A A

B B

→ n_i

→ s_i

| A | A | → 1

F.D. Statistics

1. The Particles are indistinguishable so that there is no distinction b/w the different ways in which n_i Particles are chosen

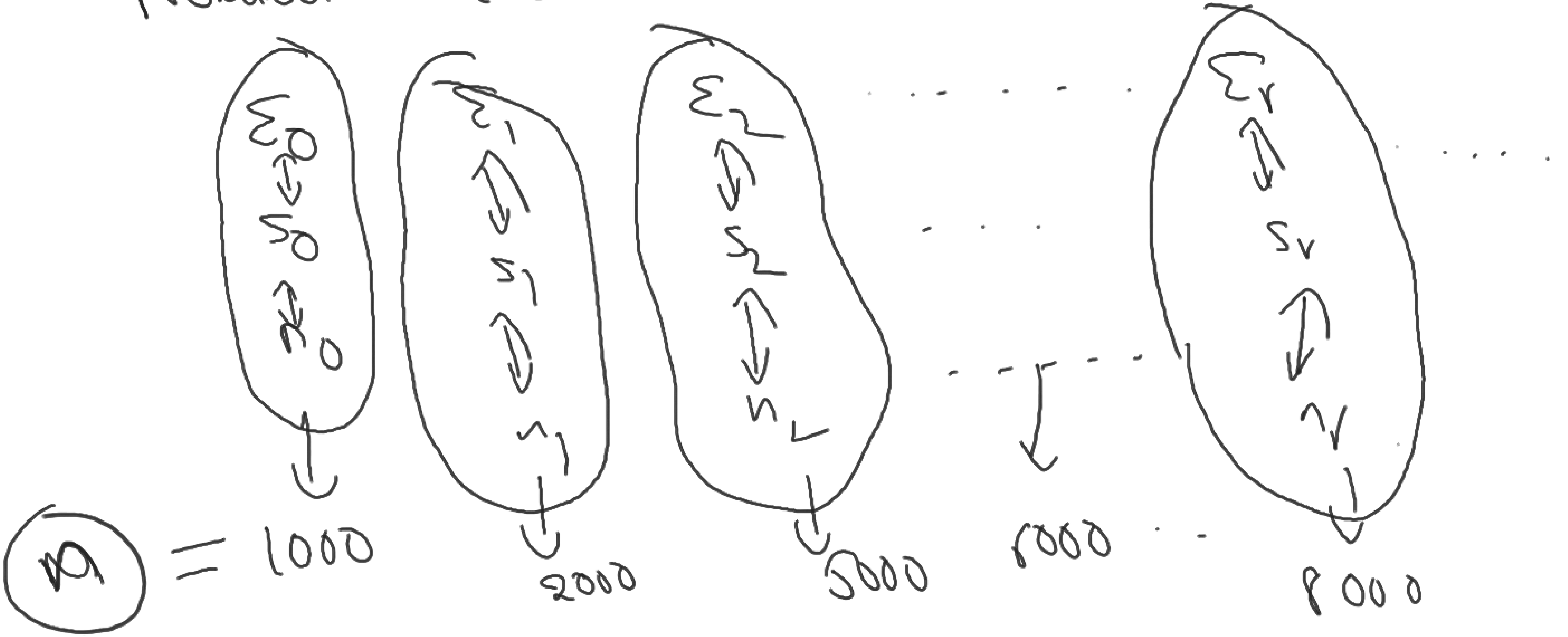
2. The Particles obey Pauli's Exclusion principle according to which each sublevel (or) cell may contain either 0 (or) 1

3. The total energy of the System remains Constant

4. The total no of particles in the either System is always constant $n = n_1 + n_2 + n_3 \cdots + n_r + \cdots$

$$= \sum^{r} n_i = n = \text{Const}$$

$W(\bar{n})$ = The no of ways in which the most Probable microstate $\bar{n}$ can be attained



S_i $\quad W(n) = \dfrac{S_i!}{n_i!\,(S_i-n_i)}$

ε_4, ε_3, ε_2, ε_1

s

$$W(n) = \frac{S_0!}{n_0!\,(S_0-n_0)}\;\frac{S_1!}{n_1!\,(S_1-n_1)}\;\frac{S_2!}{n_2!\,(S_2-n_2)}\cdots\frac{S_r!}{n_r!\,(S_r-n_r)}$$

$\dfrac{3}{3!}$ $\quad \dfrac{3!}{2!1!}$ $\quad \dfrac{3!}{1!1!1!}$

$$\delta \ln(W(n)) = 0 \Rightarrow W(n) = \prod_{i=0}^{r} \frac{S_i!}{n_i!\,(S_i-n_i)!}$$

most probable microstate

$6 \cdot 3 \cdot 10$

$60 \cdot 180$

$$\omega(\bar{n}) = \prod_{i=0}^{r} \frac{s_i!}{n_i!(s_i - n_i)}$$

$\downarrow$

Total no of ways in which the microstate $\bar{n}$ can be attained

Take ln on both sides

$$\ln(\omega(\bar{n})) = \sum_{i=0}^{i=r} \ln s_i! - \ln n_i! - \ln(s_i - n_i)!$$

$$\ln \omega(\bar{n}) = \sum_{i=0}^{v} \left[\ln s_i! - \ln n_i! - \ln (s_i - n_i)! \right]$$

stilling approximation

$$\ln x! = x \ln x - x$$

$$\ln \omega(\bar{n}) = \sum_{i=0}^{v} \left[s_i \ln s_i - \cancel{s_i} - n_i \ln n_i + \cancel{n_i} - (s_i - n_i) \ln (s_i - n_i) + \cancel{(s_i - n_i)} \right]$$

$$\ln \omega(\bar{n}) = \sum_{i=0}^{v} \left[s_i \ln s_i - n_i \ln n_i - (s_i - n_i) \ln (s_i - n_i) \right]$$

$$\ln \omega(\bar{n}) = \sum_{i=0}^{v} \Big[\underbrace{s_i \ln s_i}_{=0} - n_i \ln n_i - (s_i - n_i)\ln(s_i - n_i) \Big]$$

$$\delta \ln \omega(\bar{n}) = 0$$

$$\delta \ln \omega(\bar{n}) = \sum_{i=0}^{v} -\Big[\delta n_i \ln n_i + \cancel{n_i} \frac{1}{\cancel{n_i}} \cancel{\delta n_i} \Big] - \Big[\cancel{(s_i - n_i)} \frac{1}{\cancel{(s_i - n_i)}} \cancel{\delta n} - \delta n_i \ln(s_i - n_i) \Big] = 0$$

$$\delta \ln \omega(\bar{n}) = \sum_{i=0}^{v} \Big[-\delta n_i \ln n_i + \delta n_i \ln(s_i - n_i) \Big] = 0$$

$$\Rightarrow \sum_{i=0}^{i=r} \boxed{\delta n_i \ln\left(\frac{s_i - n_i}{n_i}\right)} = 0 \quad - (1)$$

Total no. of particles in the system remains constant

$$\sum_{i=0}^{r} n_i = \text{const} = n \Rightarrow \boxed{\sum_{i=0}^{r} \delta n_i = 0} - (2)$$

Total energy of the system remains constant

$$\sum_{i=0}^{i=r} \varepsilon_i n_i = E = \text{const} \Rightarrow \boxed{\sum_{i=0}^{r} \varepsilon_i \delta n_i = 0} - (3)$$

$$\sum_{i=0}^{v} \delta n_i \left[\ln\left(\frac{s_i - n_i}{n_i}\right) \right] = 0 \quad — (1) \times -1$$

$$\sum_{i=0}^{v} \delta n_i = 0 \quad — (2) \times \alpha$$

$$\sum_{i=0}^{v} \varepsilon_i \delta n_i = 0 \quad — (3) \times \beta$$

using lagrange method of undetermined multipliers

$$-1^{(1)} * \alpha^{(2)} + \beta \epsilon_i^{(3)} = 0$$

$$\sum_{i=0}^{k} \delta n_i \left[-\ln\left(\frac{s_i - n_i}{n_i}\right) + \alpha + \beta \epsilon_i \right] = 0$$

$$-\ln\left(\frac{s_i - n_i}{n_i}\right) + \alpha + \beta \epsilon_i = 0$$

$$\ln\left(\frac{s_i - n_i}{n_i}\right) = \alpha + \beta \epsilon_i$$

$$\frac{s_i - n_i}{n_i} = e^{\alpha + \beta \epsilon_i}$$

n_i

$n = \text{cont}$

$i = 0, 1, 2, 3 \ldots$

$n_\smile$

$\epsilon_\smile$

$s_\smile$

$n_\smile$

$$\frac{s_i - n_i}{n_i} = e^{\alpha + \beta \epsilon_i}$$

$$\frac{s_i}{n_i} - 1 = e^{\alpha + \beta \epsilon_i}$$

$$\boxed{\frac{n_i}{s_i} = \frac{1}{e^{\alpha + \frac{\epsilon_i}{kT}} + 1}}$$

$$\frac{s_i}{n_i} = e^{\alpha + \beta \epsilon_i} + 1$$

$$\boxed{\text{prob } \frac{n_i}{s_i} = \frac{1}{e^{\alpha + \beta \epsilon_i} + 1}}$$

$$\frac{n_i}{s_i} = \frac{1}{e^{\alpha + \beta \epsilon_i} + 1}$$

$i = 1$

$$\frac{n_1}{s_1} = \frac{1}{e^{\alpha + \beta \epsilon_1} + 1}$$

$i = 2$

$$\frac{n_2}{s_2} = \frac{1}{e^{\alpha + \beta \epsilon_2} + 1}$$

$\beta = \frac{1}{kT}$