Computer Organization and Architecture

Arithmetic Operation

Veena Thenkanidiyoor
National Institute of Technology Goa

1

1

Recap

- Need for number and character representation
- Unsigned number representation
- Signed number representation
  - Sign-and-magnitude
  - 1's complement
  - 2's complement
- Character representation

2

Addition of 1-bit Numbers

0	1	0	1
+ 0	+ 0	+ 1	+ 1
0	1	1	10

2 - base.

$$\begin{array}{r} 253 \\ +\ 77 \\ \hline 30 \end{array}$$

Sum is 0
Carry-out is 1

- To add 2 n-nit numbers
  - Add bit pairs starting from the LSB
  - Propagate carries toward the MSB

3

2's Complement Arithmetic

n = 4

7	0111	4	0100	-4	1100	4	0100	-4	1100
+ (-3)	+ 1101	+ 3	0011	+ (-3)	1101	+ (- 3)	1101	+ 3	0011
4	10100	7	0111	-7	11001	1	10001	-1	1111

- Ignoring the carry-out from the MSB in the above addition(s), gives the correct answer

4

Addition and Subtraction of n-bit Signed Numbers Represented in 2's Complement

- To add two numbers, add their n-bit representations, ignoring the carry-out signal from the most significant bit (MSB) position. The sum will be algebraically correct value in the 2's complement representation as long as the answer is in the range $-(2^{n-1})\ \ to\ (2^{n-1}-1)$

- To subtract two numbers X and Y, that is, to perform X-Y, form the 2's complement of Y and then add it to X, as in rule 1. Again the result will be algebraically correct value in the 2's complement representation as long as the answer is in the range $-(2^{n-1})\ \ to\ (2^{n-1}-1)$

Example

(a)

```
  0010    (+2)
+ 0011    (+3)
------    ----
  0101    (+5)
```

(b)

```
  0100    (+4)
+ 1010    (- 6)
------    -----
  1110    (- 2)
```

(c)

```
  1011    (- 5)
+ 1110    (- 2)
------    -----
  1001    (- 7)
```

(d)

```
  0111    (+ 7)
+ 1101    (- 3)
------    -----
  0100    (+ 4)
```

Rule 1

Example

(e)

$$\begin{array}{r r} 1101 & (-3) \\ -\ 1001 & -(-7) \\ \hline & \end{array} \Rightarrow \begin{array}{r r} 1101 & \\ +\ 0111 & \\ \hline 0100 & (+4) \end{array}$$

X − Y

X

Y

X

Y =

+4.

(f)

$$\begin{array}{r r} 0010 & (+2) \\ -\ 0100 & -(+4) \\ \hline & \end{array} \Rightarrow \begin{array}{r r} 0010 & \\ +\ 1100 & \\ \hline 1110 & (-2) \end{array}$$

X

Y

Y =

Rule 2

7

Example

(g)

$$\begin{array}{r r} 0110 & (+6) \\ -\ 0011 & -(+3) \\ \hline & \end{array} \Rightarrow \begin{array}{r r} 0110 & \\ +\ 1101 & \\ \hline 0011 & (+3) \end{array}$$

(h)

$$\begin{array}{r r} 1001 & (-7) \\ -\ 1011 & -(-5) \\ \hline & \end{array} \Rightarrow \begin{array}{r r} 1001 & \\ +\ 0101 & \\ \hline 1110 & (-2) \end{array}$$

Rule 2

8

Example

(i)	1 0 0 1	(- 7)	⇒	1 0 0 1	
	- 0 0 0 1	(+1)		+ 1 1 1 1	
				1 0 0 0	(- 8)
(j)	0 0 1 0	(+2)	⇒	0 0 1 0	
	- 1 1 0 1	(- 3)		+ 0 0 1 1	
				0 1 0 1	(+ 5)

Rule 2

2's Complement Representation

- Subtrahend to be 2's-complemented
- Subtraction is proceeded like how addition done
- Simplicity of adding or subtracting signed numbers in 2's-complement representation

(i)	1 0 0 1	(- 7)	⇒	1 0 0 1	
	- 0 0 0 1	(+1)		+ 1 1 1 1	
				1 0 0 0	(- 8)
(j)	0 0 1 0	(+2)	⇒	0 0 1 0	
	- 1 1 0 1	(- 3)		+ 0 0 1 1	
				0 1 0 1	(+ 5)

Arithmetic Overflow in Integer Arithmetic

- When the result of an arithmetic operation is outside the representable range, an arithmetic overflow has occurred
- Addition
- Unsigned integers: $0 \ \ to \ (2^n - 1)$

$n=4$ $\quad 0$ to 2^4-1

0 to 15

1) 8 + 9 = 17

$$\begin{array}{r} 1000 \\ 1001 \\ \hline 1)\,0001 \end{array}$$

2) 7 + 7 = 14 →

$$\begin{array}{r} 0111 \\ 0111 \\ \hline 1110 \end{array}$$

3) 15 + 3 = 18

$$\begin{array}{r} 1111 \\ 0011 \\ \hline 1)\,0010 \end{array}$$

Carry - MSB ⇒ Arithmetic Overflow.

11

Arithmetic overflow-2's Complement Addition

- Range: $-(2^{n-1}) \ \ to \ \ (2^{n-1} - 1)$

$n=4$ $\quad -8 \rightarrow 7$

① (+7) + (+4) = 11 →

$$\begin{array}{r} 0111 \\ 0100 \\ \hline \underbrace{1011}_{-5} \end{array}$$

② (−4) + (−6) = −10

$$\begin{array}{r} 1100 \\ 1010 \\ \hline 1)\,\underbrace{0110}_{+6} \end{array}$$

12

Arithmetic overflow-2’s Complement Addition

- Range: $-(2^{n-1})\ \ to\ \ (2^{n-1}-1)$

3) (-8) $+(+7)$ = -1

$$\begin{array}{r} 1000 \\ 0111 \\ \hline 1111 \\ \hline -1 \end{array}$$

4) (-7) $+(+7)$ = 0

$$\begin{array}{r} 1001 \\ 0111 \\ \hline 1)0000 \\ \hline 0 \end{array}$$

13

Arithmetic overflow-2’s Complement Addition

- Range: $-(2^{n-1})\ \ to\ \ (2^{n-1}-1)$
- The carry-out signal from the sign-bit (MSB) position is not a sufficient indicator of overflow when adding signed numbers
- Overflow can occur only when adding two numbers that have the same sign

14

Arithmetic overflow-2's Complement Addition

- Examine the signs of 2 summands *X* and *Y* and the sign of the result
  - When both operands *X* and *Y* have same sign, an overflow occurs when the sign of sum *S* is not the same as the signs of *X* and *Y*

indicate

$$\begin{array}{r} +2 \\ +(+2) \\ \hline +4 \end{array} \qquad \begin{array}{r} 0010 \\ 0010 \\ \hline 0100 \end{array}$$

- Observe carry to *C(n-1)* and *C(n)*
  - Different: Overflow
  - Same: No overflow

15

To Summarize

- Adding 1- bit numbers
- Addition operation on signed numbers represented in 2's complement
- Addition and Subtraction of n-bit Signed Numbers Represented in 2's Complement
- Arithmetic overflow

16

Reference

- **Carl Hamacher, Zvonko Vranesic and Safwat Zaky, "Computer Organization", 5th Edition, Tata McGraw Hill, 2002**

Thank You