22-1. When a 20-lb weight is suspended from a spring, the spring is stretched a distance of 4 in. Determine the natural frequency and the period of vibration for a 10-lb weight attached to the same spring.

$$k = \frac{20}{\frac{4}{12}} = 60 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{60}{\frac{10}{32.2}}} = 13.90 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = 0.452 \text{ s} \qquad \textbf{Ans}$$

$$f = \frac{1}{\tau} = 2.21 \text{ Hz} \qquad \textbf{Ans}$$

22-2. A spring has a stiffness of 600 N/m. If a 4-kg block is attached to the spring, pushed 50 mm above its equilibrium position, and released from rest, determine the equation which describes the block's motion. Assume that positive displacement is measured downward.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{4}} = 12.25 \text{ rad/s}$$

$v = 0, \quad x = -0.05$ m at $t = 0$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$$-0.05 = 0 + B$$

$$B = -0.05$$

$$v = A\omega_n \cos \omega_n t - Bp \sin \omega_n t$$

$$0 = A(12.25) - 0$$

$$A = 0$$

Thus, $x = -0.05 \cos(12.2t)$ m **Ans**

22-3. When a 3-kg block is suspended from a spring, the spring is stretched a distance of 60 mm. Determine the natural frequency and the period of vibration for a 0.2-kg block attached to the same spring.

$$k = \frac{F}{\Delta x} = \frac{3(9.81)}{0.060} = 490.5 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{490.5}{0.2}} = 49.52 \text{ rad/s}$$

$$f = \frac{\omega_n}{2\pi} = \frac{49.52}{2\pi} = 7.88 \text{ Hz} \qquad \textbf{Ans}$$

$$\tau = \frac{1}{f} = \frac{1}{7.88} = 0.127 \text{ s} \qquad \textbf{Ans}$$

***22-4.** An 8-kg block is suspended from a spring having a stiffness $k = 80$ N/m. If the block is given an upward velocity of 0.4 m/s when it is 90 mm above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that positive displacement is measured downward.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{80}{8}} = 3.162 \text{ rad/s}$$

$v = -0.4$ m/s, $\quad x = -0.09$ m at $t = 0$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$$-0.09 = 0 + B$$

$$B = -0.09$$

$$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$-0.4 = A(3.162) - 0$$

$$A = -0.126$$

Thus, $x = -0.126 \sin(3.16t) - 0.09 \cos(3.16t)$ m **Ans**

$C = \sqrt{A^2 + B^2} = \sqrt{(-0.126)^2 + (-0.09)^2} = 0.155$ m **Ans**

22-5. A 2-lb weight is suspended from a spring having a stiffness $k = 2$ lb/in. If the weight is pushed 1 in. upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What is the amplitude and the natural frequency of the vibration?

$$k = 2(12) = 24 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{24}{\frac{2}{32.2}}} = 19.66 \text{ rad/s}$$

$f = \dfrac{\omega_n}{2\pi} = 3.13$ Hz **Ans**

$y = -\dfrac{1}{12}, \quad v = 0$ at $t = 0$

From Eqs. 22-3 and 22-4.

$$-\frac{1}{12} = 0 + B$$

$$B = -0.0833$$

$$0 = A\omega_n + 0$$

$$A = 0$$

$C = \sqrt{A^2 + B^2} = 0.0833$ ft $= 1$ in. **Ans**

Position equation,

$y = (0.0833 \cos 19.7t)$ ft **Ans**

22-6. A 6-lb weight is suspended from a spring having a stiffness $k = 3$ lb/in. If the weight is given an upward velocity of 20 ft/s when it is 2 in. above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the weight, measured from the equilibrium position. Assume positive displacement is downward.

$k = 3(12) = 36$ lb/ft

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{36}{\frac{6}{32.2}}} = 13.90 \text{ rad/s}$$

$t = 0, \quad v = -20$ ft/s, $\quad y = -\frac{1}{6}$ ft

From Eq. 22-3,

$$-\frac{1}{6} = 0 + B$$

$$B = -0.167$$

From Eq. 22-4,

$$-20 = A(13.90) + 0$$

$$A = -1.44$$

Thus,

$y = [-1.44\sin(13.9t) - 0.167\cos(13.9t)]$ ft **Ans**

From Eq. 22-10,

$C = \sqrt{A^2 + B^2} = \sqrt{(1.44)^2 + (-0.167)^2} = 1.45$ ft **Ans**

22-7. A 6-kg block is suspended from a spring having stiffness of $k = 200$ N/m. If the block is given an upward velocity of 0.4 m/s when it is 75 mm above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that positive displacement is downward.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{6}} = 5.774$$

$x = A\sin\omega_n t + B\cos\omega_n t$

$x = -0.075$ m when $t = 0$,

$-0.075 = 0 + B; \quad B = -0.075$

$v = A\omega_n\cos\omega_n t - B\omega_n\sin\omega_n t$

$v = -0.4$ m/s when $t = 0$,

$-0.4 = A(5.774) - 0; \quad A = -0.0693$

Thus,

$x = -0.0693\sin(5.77t) - 0.075\cos(5.77t)$ **Ans**

$C = \sqrt{A^2 + B^2} = \sqrt{(-0.0693)^2 + (-0.075)^2} = 0.102$ m **Ans**

***22-8.** A 3-kg block is suspended from a spring having a stiffness of $k = 200$ N/m. If the block is pushed 50 mm upward from its equilibrium position and then released from rest, determine the equation that describes the motion. What are the amplitude and the natural frequency of the vibration? Assume that positive displacement is downward.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{3}} = 8.165$$

$$f = \frac{\omega_n}{2\pi} = \frac{8.165}{2\pi} = 1.299 = 1.30 \text{ Hz} \quad \textbf{Ans}$$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$x = -0.05$ m when $t = 0$,

$$-0.05 = 0 + B; \quad B = -0.05$$

$$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$v = 0$ when $t = 0$,

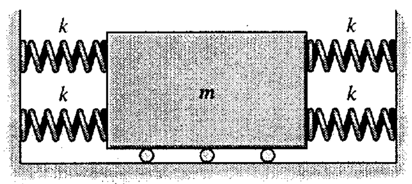
$$0 = A(8.165) - 0; \quad A = 0$$

Hence,

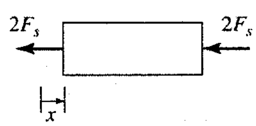
$$x = -0.05 \cos(8.16t) \qquad \textbf{Ans}$$

$$C = \sqrt{A^2 + B^2} = \sqrt{(0)^2 + (-0.05)^2} = 0.05 \text{ m} = 50 \text{ mm} \quad \textbf{Ans}$$

22-9. Determine the frequency of vibration for the block. The springs are originally compressed Δ.



$$\xrightarrow{+} \sum F_x = ma_x; \quad -4kx = m\ddot{x}$$



$$\ddot{x} + \frac{4k}{m}x = 0$$

$$f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$$

$$f = \frac{1}{\pi}\sqrt{\frac{k}{m}} \qquad \textbf{Ans}$$

22-10. A pendulum has a 0.4-m-long cord and is given a tangential velocity of 0.2 m/s toward the vertical from a position $\theta = 0.3$ rad. Determine the equation which describes the angular motion.

See Example 22-1.

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.81}{0.4}} = 4.95$$

$$\theta = A \sin \omega_n t + B \cos \omega_n t$$

$\theta = 0.3$ rad when $t = 0$,

$$0.3 = 0 + B; \quad B = 0.3$$

$$\dot{\theta} = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

Since $s = \theta l$, $\dot{v} = \dot{\theta} l$. Hence,

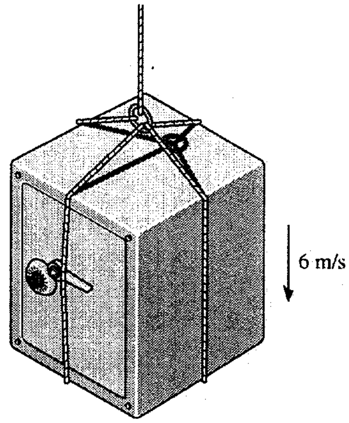
$-0.2 = \dot{\theta}(0.4)$, $\dot{\theta} = -0.5$ when $t = 0$,

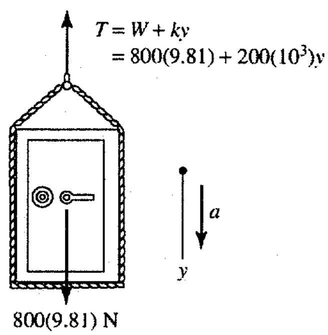
$$-0.5 = A(4.95); \quad A = -0.101$$

Thus,

$\theta = -0.101 \sin(4.95t) + 0.3 \cos(4.95t)$ **Ans**

22-11. A cable is used to suspend the 800-lb safe. If the safe is being lowered at 6 m/s when the motor controlling the cable suddenly jams (stops), determine the maximum tension in the cable and the frequency of vibration of the safe. Neglect the mass of the cable and assume it is elastic such that it stretches 20 mm when subjected to a tension of 4 kN.





Freebody Diagram: Here the stiffness of the cable is $k = \dfrac{4000}{0.02} = 200(10^3)$ N/m. When the safe is being displaced by an amount y downward vertically from its equilibrium position, the *restoring force* that developed in the cable $T = W + ky = 800(9.81) + 200(10^3)y$.

Equation of Motion:

$$+\uparrow \sum F_x = 0; \quad 800(9.81) + 200(10^3)y - 800(9.81) = -800a \quad [1]$$

Kinematics: Since $a = \dfrac{d^2y}{dt^2} = \ddot{y}$, then substituting this value into Eq. [1], we have

$$200(10^3)y = -800\ddot{y}$$

$$\ddot{y} + 250x = 0 \qquad [2]$$

From Eq. [2], $\omega_n^2 = 250$, thus, $\omega_n = 15.81$ rad/s. Applying Eq. 22-14, we have

$$f = \frac{\omega_n}{2\pi} = \frac{15.81}{2\pi} = 2.52 \text{ Hz} \quad \textbf{Ans}$$

The solution of the above differential equation (Eq. [2]) is in the form of

$$y = C\sin(15.81t + \phi) \quad [3]$$

Taking the time derivative of Eq. [3], we have

$$\dot{y} = 15.81C\cos(15.81t + \phi) \quad [4]$$

Applying the initial condition of $y = 0$ and $\dot{y} = 6$ m/s at $t = 0$ to Eqs. [3] and [4] yields

$$0 = C\sin\phi \qquad [5]$$

$$6 = 15.81C\cos\phi \quad [6]$$

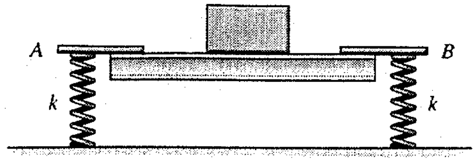
Solving Eqs. [5] and [6] yields

$$\phi = 0^\circ \quad C = 0.3795 \text{ m}$$

Since $y_{max} = C = 0.3795$ m, the maximum cable tension is given by

$$T_{max} = W + ky_{max} = 800(9.81) + 200(10^3)(0.3795) = 83.7 \text{ kN} \quad \textbf{Ans}$$

***22-12.** The uniform beam is supported at its ends by two springs A and B, each having the same stiffness k. When nothing is supported on the beam, it has a period of vertical vibration of 0.83 s. If a 50-kg mass is placed at its center, the period of vertical vibration is 1.52 s. Compute the stiffness of each spring and the mass of the beam.



$$\tau = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{\tau^2}{(2\pi)^2} = \frac{m}{k}$$

$$\frac{(0.83)^2}{(2\pi)^2} = \frac{m_B}{2k} \qquad (1)$$

$$\frac{(1.52)^2}{(2\pi)^2} = \frac{m_B + 50}{2k} \qquad (2)$$

Eqs. (1) and (2) become

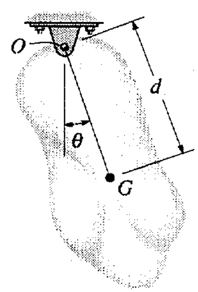
$$m_B = 0.03490k$$

$$m_B + 50 = 0.1170k$$

$$m_B = 21.2 \text{ kg} \qquad \textbf{Ans}$$

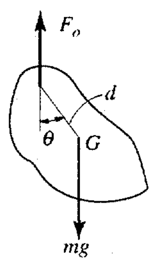
$$k = 609 \text{ N/m} \qquad \textbf{Ans}$$

22-13. The body of arbitrary shape has a mass m, mass center at G, and a radius of gyration about G of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration.



$\circlearrowleft + \sum M_O = I_O\alpha; \qquad -mgd\sin\theta = [mk_G^2 + md^2]\ddot{\theta}$

$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\sin\theta = 0$$



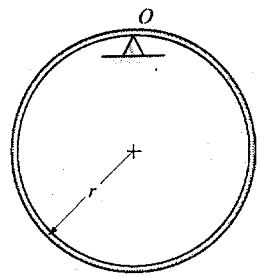
However, for small rotation $\sin\theta = \theta$. Hence

$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\theta = 0$$

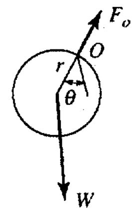
From the above differential equation, $\omega_n = \sqrt{\frac{gd}{k_G^2 + d^2}}$.

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{g^d}{k_G^2 + d^2}}} = 2\pi\sqrt{\frac{k_G^2 + d^2}{g^d}} \qquad \textbf{Ans}$$

22-14. The thin hoop of mass m is supported by a knife-edge. Determine the natural period of vibration for small amplitudes of swing.



$I_O = mr^2 + mr^2 = 2mr^2$

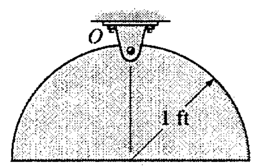


$$\circlearrowleft + \sum M_O = I_O\alpha; \quad -mgr\theta = (2mr^2)\ddot{\theta}$$

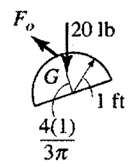
$$\ddot{\theta} + \left(\frac{8}{2r}\right)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2r}{g}} \quad \textbf{Ans}$$

22-15. The semicircular disk weighs 20 lb. Determine the natural period of vibration if it is displaced a small amount and released.



Moment of inertia about O:



$I_A = I_G + md^2$

$$\frac{1}{2}\left(\frac{20}{32.2}\right)(1)^2 = I_G + \left(\frac{20}{32.2}\right)\left[\frac{4(1)}{3\pi}\right]^2 \quad I_A = 0.1987 \text{ slug} \cdot \text{ft}^2$$

$I_O = I_G + md^2$

$$= 0.1987 + \left(\frac{20}{32.2}\right)\left[I - \frac{4(1)}{3\pi}\right]^2 = 0.4045 \text{ slug} \cdot \text{ft}^2$$

Equation of motion:

$$\Sigma M_O = I_{Ox_s}\alpha; \quad 20\left[I - \frac{4(1)}{3k}\right]\sin\theta = -0.4045\ddot{\theta}$$

$\ddot{\theta} + 28.462\sin\theta = 0$

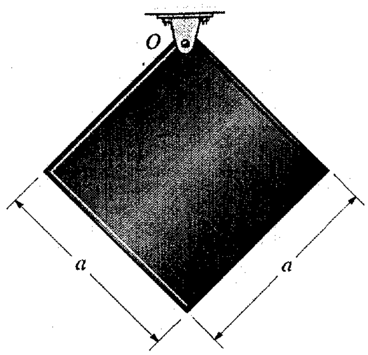
However, for small rotation $\sin\theta = \theta$. Hence

$\ddot{\theta} + 28.462\theta = 0$

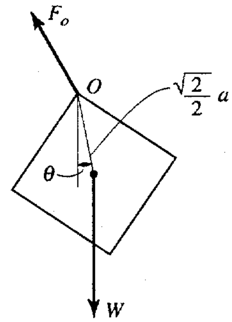
From the above differential equation. $\omega_n = \sqrt{28.462} = 5.335$.

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{5.335} = 1.18 \text{ s} \quad \textbf{Ans}$$

***22-16.** The square plate has a mass m and is suspended at its corner by the pin O. Determine the natural period of vibration if it is displaced a small amount and released.



$$I_O = \frac{1}{12}m(a^2 + a^2) + m\left(\frac{\sqrt{2}}{2}a\right)^2 = \frac{1}{6}ma^2 + \frac{1}{2}ma^2 = \frac{2}{3}ma^2$$



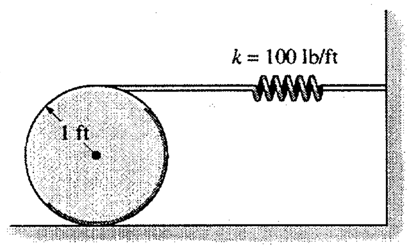
$$\circlearrowleft + \sum M_O = I_O\alpha; \qquad -mg\left(\frac{\sqrt{2}}{2}a\right)\theta = \left(\frac{2}{3}ma^2\right)\ddot{\theta}$$

$$\ddot{\theta} + \left(\frac{3\sqrt{2}g}{4a}\right)\theta = 0$$

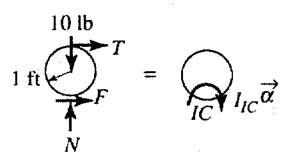
$$\omega_n = \sqrt{\frac{3\sqrt{2}g}{4a}}$$

$$\tau = \frac{2\pi}{\omega_n} = 6.10\sqrt{\frac{a}{g}} \qquad \textbf{Ans}$$

22-17. The disk has a weight of 10 lb and rolls without slipping on the horizontal surface as it oscillates about its equilibrium position. If the disk is displaced, by rolling it counterclockwise 0.4 rad, determine the equation which describes its oscillatory motion when it is released.



$$I_{IC} = \frac{1}{2}\left(\frac{10}{32.2}\right)(1)^2 + \frac{10}{32.2}(1)^2 = 0.4658 \text{ slug}\cdot\text{ft}^2$$



$$\circlearrowleft + \sum M_{IC} = I_{IC}\alpha; \quad -100(2\theta) = 0.4658\ddot{\theta}$$

$$\ddot{\theta} + 429.3\theta = 0$$

$$\omega_n = \sqrt{429.3} = 20.72 \text{ rad/s}$$

$$\theta = A\sin\omega_n t + B\cos\omega_n t$$

$$\omega = 0, \quad \theta = 0.4, \quad t = 0$$

$$0.4 = 0 + B$$

$$B = 0.4$$

$$\omega = \dot{\theta} = A\omega_n\cos\omega_n t - B\omega_n\sin\omega_n t$$

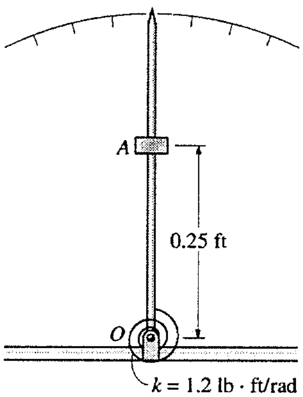
$$0 = A\omega_n - 0$$

$$A = 0$$

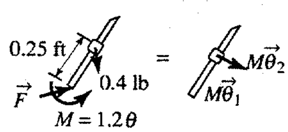
Thus,

$\theta = 0.4\cos(20.7t)$ **Ans**

22-18. The pointer on a metronome supports a 0.4-lb slider A, which is positioned at a fixed distance from the pivot O of the pointer. When the pointer is displaced, a torsional spring at O exerts a restoring torque on the pointer having a magnitude $M = (1.2\theta)$ lb · ft, where θ represents the angle of displacement from the vertical, measured in radians. Determine the natural period of vibration when the pointer is displaced a small amount θ and released. Neglect the mass of the pointer.



$$I_O = \frac{0.4}{32.2}(0.25)^2 = 0.7764(10^{-3})\ \text{slug} \cdot \text{ft}^2$$



$$\circlearrowleft + \sum M_O = I_O\alpha; \quad -1.2\theta + 0.4(0.25)\sin\theta = 0.7764(10^{-3})\ddot{\theta}$$

For small θ, $\sin\theta = \theta$

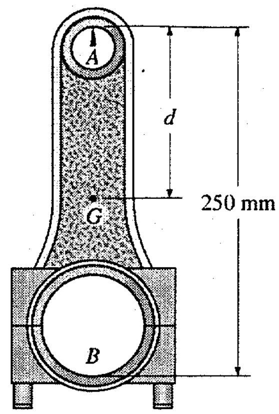
So that

$$\ddot{\theta} + 1417.5\theta = 0$$

$$\omega_n = \sqrt{1417.5} = 37.64 \text{ rad/s}$$

$$\tau = \frac{2\pi}{37.64} = 0.167 \text{ s}$$ **Ans**

22-19. The connecting rod is supported by a knife edge at A and the period of vibration is measured as $\tau_A = 3.38$ s. It is then removed and rotated 180° so that it is supported by the knife edge at B. In this case the period of vibration is measured as $\tau_B = 3.96$ s. Determine the location d of the center of gravity G, and compute the radius of gyration k_G.



Freebody Diagram: When an object of arbitary shape having a mass m is pinned at O and is displaced by an angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum moment about point O to eliminate O_x and O_y,

$$+\sum M_O = I_O\alpha; \qquad -mg\sin\theta(l) = I_O\alpha \qquad [1]$$

Kinematics: Since $\alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta = \theta$ if θ is small, then substitute these values into Eq. [1], we have

$$-mgl\theta = I_O\ddot{\theta} \quad \text{or} \quad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \qquad [2]$$

From Eq. [2], $\omega_n^2 = \frac{mgl}{I_O}$, thus, $\omega_n = \sqrt{\frac{mgl}{I_O}}$. Applying Eq. 22-12, we have

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_O}{mgl}} \qquad [3]$$

When the rod is rotating about A, $\tau = \tau_A = 3.38$ s and $l = d$. Substitute these values into Eq. [3], we have

$$3.38 = 2\pi\sqrt{\frac{I_A}{mgd}} \qquad I_A = 0.2894mgd$$

When the rod is rotating about B, $\tau = \tau_B = 3.96$ s and $l = 0.25 - d$. Substitute these values into Eq. [3], we have

$$3.96 = 2\pi\sqrt{\frac{I_B}{mg(0.25-d)}} \qquad I_B = 0.3972mg(0.25-d)$$

However, the mass moment inertia of the rod about its mass center is

$$I_G = I_A - md^2 = I_B - m(0.25-d)^2$$

Then,

$$0.2894mgd - md^2 = 0.3972mg(0.25-d) - m(0.25-d)^2$$

$$d = 0.1462 \text{ m} = 146 \text{ mm} \qquad \textbf{Ans}$$

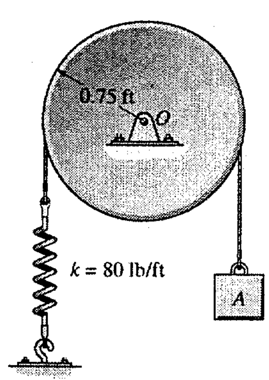
Thus, the mass moment inertia of the rod about its mass center is

$$I_G = I_A - md^2 = 0.2894m(9.81)(0.1462) - m(0.1462^2) = 0.3937m$$

The radius of gyration is

$$k_G = \sqrt{\frac{I_G}{m}} = \sqrt{\frac{0.3937m}{m}} = 0.627 \text{ m} \qquad \textbf{Ans}$$

***22-20.** The disk, having a weight of 15 lb, is pinned at its center O and supports the block A that has a weight of 3 lb. If the belt which passes over the disk is not allowed to slip at its contacting surface, determine the natural period of vibration of the system.



For equilibrium:

$T_{st} = 3 \text{ lb}$

$\circlearrowleft + \sum M_O = I_O\alpha + ma(0.75)$

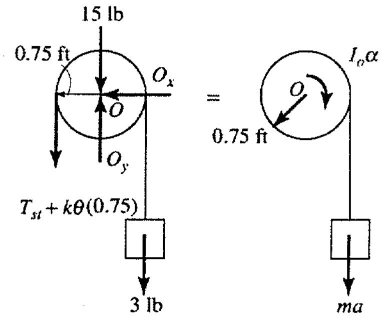
$a = 0.75\alpha$

$$-T_{st}(0.75) - (80)(\theta)(0.75)(0.75) + (3)(0.75) = \left[\frac{1}{2}\left(\frac{15}{32.2}\right)(0.75)^2\right]\ddot{\theta} + \left(\frac{3}{32.2}\right)(0.75)\ddot{\theta}(0.75)$$

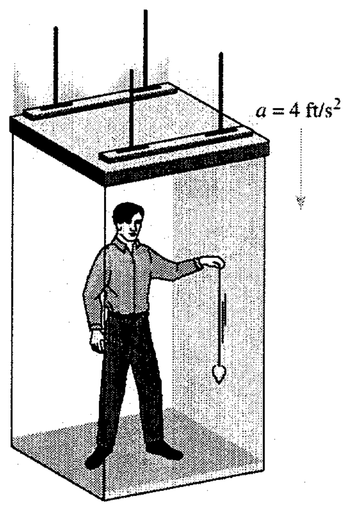
$$-2.25 - 45\theta + 2.25 = 0.131\ddot{\theta} + 0.05241\ddot{\theta}$$

$$\ddot{\theta} + 245.3\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{245.3}} = 0.401 \text{ s} \qquad \textbf{Ans}$$



22-21. While standing in an elevator, the man holds a pendulum which consists of an 18-in. cord and a 0.5-lb bob. If the elevator is descending with an acceleration $a = 4 \text{ ft/s}^2$, determine the natural period of vibration for small amplitudes of swing.



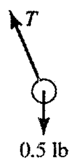
Since the acceleration of the pendulum is $(32.2 + 4) = 36.2 \text{ ft/s}^2$

Using the result of Example 22-1,

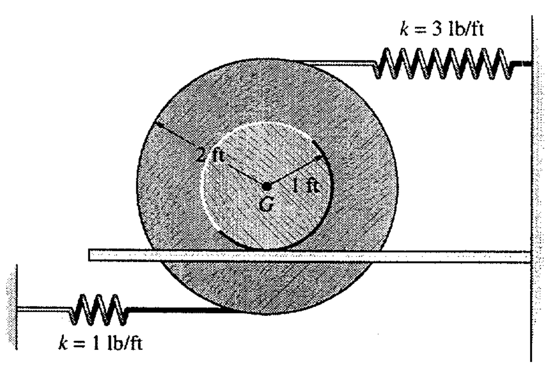
We have

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{36.2}{18/12}} = 4.91 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{4.91} = 1.28 \text{ s} \qquad \textbf{Ans}$$



22-22. The 50-lb spool is attached to two springs. If the spool is displaced a small amount and released, determine the natural period of vibration. The radius of gyration of the spool is $k_G = 1.5$ ft. The spool rolls without slipping.



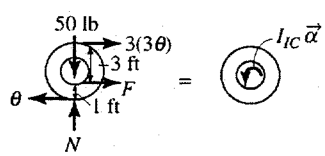
$$I_{IC} = \frac{50}{32.2}(1.5)^2 + \frac{50}{32.2}(1)^2 = 5.047 \text{ slug} \cdot \text{ft}^2$$

$\circlearrowleft + \sum M_{IC} = I_{IC}\alpha;\quad -3(3\theta)(3) - 1(\theta) = 5.047\ddot{\theta}$

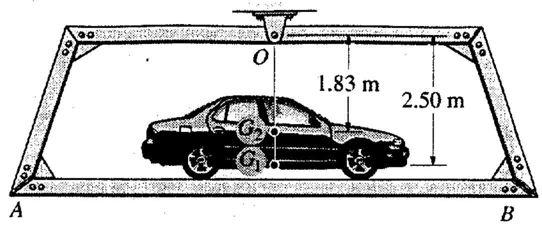
$$\ddot{\theta} + 5.5483\theta = 0$$

$$\omega_n = \sqrt{5.5483}$$

$$\tau = \frac{2\pi}{\sqrt{5.5483}} = 2.67 \text{ s} \qquad \textbf{Ans}$$



22-23. The platform AB when empty has a mass of 400 kg, center of mass at G_1, and natural period of oscillation $\tau_1 = 2.38$ s. If a car, having a mass of 1.2 Mg and center of mass at G_2, is placed on the platform, the natural period of oscillation becomes $\tau_2 = 3.16$ s. Determine the moment of inertia of the car about an axis passing through G_2.



Freebody Diagram: When an object of arbitary shape having a mass m is pinned at O and being displaced by an angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum moment about point O to eliminate O_x and O_y.

$$\circlearrowleft + \sum M_O = I_O\alpha: \quad -mg\sin\theta(l) = I_O\alpha \qquad [1]$$

Kinematics: Since $\alpha = \dfrac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta = \theta$ if θ is small, then substitute these values into Eq. [1], we have

$$-mgl\theta = I_O\ddot{\theta} \quad \text{or} \quad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \qquad [2]$$

From Eq. [2], $\omega_n^2 = \dfrac{mgl}{I_O}$, thus, $\omega_n = \sqrt{\dfrac{mgl}{I_O}}$, Applying Eq. 22-12, we have

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_O}{mgl}} \qquad [3]$$

When the platform is empty, $\tau = \tau_1 = 2.38$ s, $m = 400$ kg and $l = 2.50$ m. Substitute these values into Eq. [3], we have

$$2.38 = 2\pi\sqrt{\frac{(I_O)_p}{400(9.81)(2.50)}} \quad (I_O)_p = 1407.55 \text{ kg}\cdot\text{m}^2$$

When the car is on the platform, $\tau = \tau_2 = 3.16$ s, $m = 400$ kg $+ 1200$ kg $= 1600$ kg, $l = \dfrac{2.50(400) + 1.83(1200)}{1600} = 1.9975$ m and $I_O = (I_O)_C + (I_O)_p = (I_O)_C + 1407.55$. Substitute these values into Eq. [3], we have

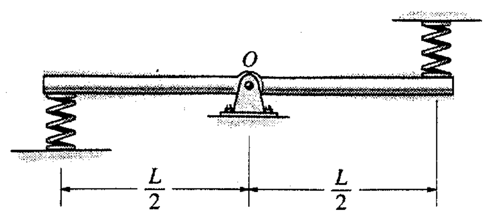
$$3.16 = 2\pi\sqrt{\frac{(I_O)_C + 1407.55}{1600(9.81)(1.9975)}} \quad (I_O)_C = 6522.76 \text{ kg}\cdot\text{m}^2$$

Thus, the mass moment inertia of the car about its mass center is

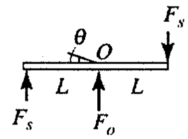
$$(I_G)_C = (I_O)_C - m_C d^2$$

$$= 6522.76 - 1200(1.83^2) = 2.50(10^3) \text{ kg}\cdot\text{m}^2 \quad \textbf{Ans}$$

***22-24.** The uniform rod has a mass m and is supported by the pin O. If the rod is given a small displacement and released, determine the natural period of vibration. The springs are unstretched when the rod is in the position shown.



$$F_s = k\left(\frac{L}{2}\right)\theta$$



$$\circlearrowleft + \sum M_O = I_O\alpha; \quad -F_s(L) = I_O\ddot{\theta}$$

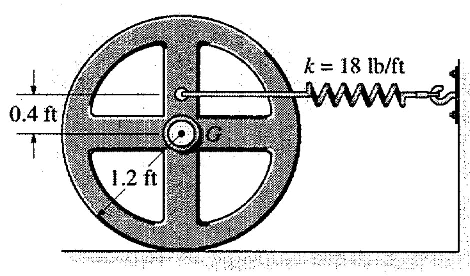
$$-k\left(\frac{L}{2}\right)(L)(\theta) = \frac{1}{12}mL^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{6k}{m}\theta = 0$$

$$\omega_n = \sqrt{\frac{6k}{m}}$$

$$\tau = \frac{2\pi}{\omega_n} = \pi\sqrt{\frac{2m}{3k}} \qquad \textbf{Ans}$$

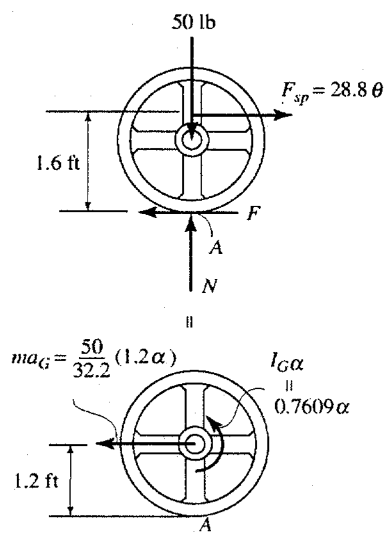
***22-25.** The 50-lb wheel has a radius of gyration about its mass center G of $k_G = 0.7$ ft. Determine the frequency of vibration if it is displaced slightly from the equilibrium position and released. Assume no slipping.



Kinematics: Since the wheel rolls without slipping, then $a_G = \alpha r = 1.2\alpha$. Also when the wheel undergoes a small angular displacement θ about point A, the spring is stretched by $x = 1.6\sin\theta$. Since θ is small, then $\sin\theta = \theta$. Thus, $x = 1.6\theta$.

Freebody Diagram: The spring force $F_{sp} = kx = 18(1.6\theta) = 28.8\theta$ will create the *restoring moment* about point A.

Equation of Motion: The mass moment inertia of the wheel about its mass center is $I_G = mk_G^2 = \frac{50}{32.2}(0.7^2) = 0.7609$ slug · ft².



$$\circlearrowleft + \sum M_A = (M_k)_A; -28.8\theta(1.6) = \frac{50}{32.2}(1.2\alpha)(1.2) + 0.7609\alpha$$

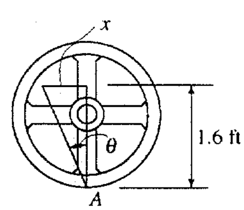
$$\alpha + 15.376\theta = 0 \qquad [1]$$

Since $\alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$, then substitute this values into Eq. [1], we have

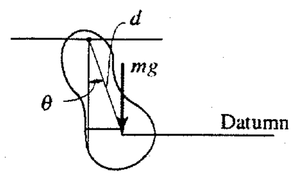
$$\ddot{\theta} + 15.376\theta = 0 \qquad [2]$$

From Eq. [2], $\omega_n^2 = 15.376$, thus, $\omega_n = 3.921$ rad/s. Applying Eq. 22-14, we have

$$f = \frac{\omega_n}{2\pi} = \frac{3.921}{2\pi} = 0.624 \text{ Hz} \qquad \textbf{Ans}$$



22-26. Solve Prob. 22-13 using energy methods.



$$T + V = \frac{1}{2}[mk_G^2 + md^2]\theta^2 + mg(d)(1 - \cos\theta)$$

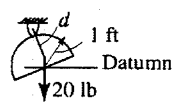
$$(k_G^2 + d^2)\dot{\theta}\ddot{\theta} + gd(\sin\theta)\dot{\theta} = 0$$

$$\sin\theta \approx \theta$$

$$\ddot{\theta} + \frac{gd}{(k_G^2 + d^2)}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{(k_G^2 + d^2)}{gd}}$$ **Ans**

22-27. Solve Prob. 22-15 using energy methods.



$$I_A = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{20}{32.2}\right)(1)^2 = 0.3106 \text{ slug} \cdot \text{ft}^2$$

$$I_O = I_G + md^2$$

$$= (I_A - m\bar{r}^2) + md^2$$

$$= 0.3106 - \frac{20}{32.2}\left[\frac{4(1)}{3\pi}\right]^2 + \frac{20}{32.2}\left[1 - \frac{4(1)}{3\pi}\right]^2$$

$$= 0.4045 \text{ slug} \cdot \text{ft}^2$$

$$T_{max} = \frac{1}{2}I_O\omega_n^2\theta_{max}^2$$

$$V_{max} = mgd(1 - \cos\theta_{max})$$

For small θ_{max}, $\cos\theta_{max} \approx 1 - \frac{\theta_{max}^2}{2}$

$$V_{max} = mgd\left(\frac{\theta_{max}^2}{2}\right)$$

$$T_{max} = V_{max}$$

$$\frac{1}{2}(0.4045)\omega_n^2\theta_{max}^2 = 20\left[1 - \frac{4(1)}{3\pi}\right]\left(\frac{\theta_{max}^2}{2}\right)$$

$$\omega_n = 5.33 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{5.33} = 1.18 \text{ s}$$ **Ans**

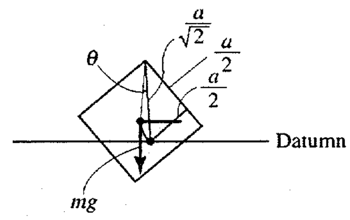
***22-28.** Solve Prob. 22-16 using energy methods.

$$T+V=\frac{1}{2}\left[\frac{1}{12}m(a^2+a^2)+m\left(\frac{a}{\sqrt{2}}\right)^2\right]\dot{\theta}^2+mg\left(\frac{a}{\sqrt{2}}\right)(1-\cos\theta)$$

$$\frac{2}{3}ma^2\dot{\theta}\ddot{\theta}+mg\left(\frac{a}{\sqrt{2}}\right)(\sin\theta)\dot{\theta}=0$$

$$\sin\theta=\theta\ddot{\theta}+\frac{3g}{2\sqrt{2}a}\theta=0$$

$$\tau=\frac{2\pi}{\omega_n}=\frac{2\pi}{1.0299}\left(\sqrt{\frac{a}{g}}\right)=6.10\sqrt{\frac{a}{g}}\qquad \textbf{Ans}$$



22-29. Solve Prob. 22-20 using energy methods.

$$s=0.75\theta,\quad s=0.75\theta$$

$$T+V=\frac{1}{2}\left[\frac{1}{2}\left(\frac{15}{32.2}\right)(0.75)^2\right]\dot{\theta}^2+\frac{1}{2}\left(\frac{3}{32.2}\right)(0.75\dot{\theta})^2$$

$$+\frac{1}{2}(80)(s_{eq}+0.75\theta)^2-3(0.75\theta)$$

$$0=0.1834\dot{\theta}\ddot{\theta}+80(s_{eq}+0.75\theta)(0.75\dot{\theta})-2.25\dot{\theta}$$

$$F_{eq}=80s_{eq}=3$$

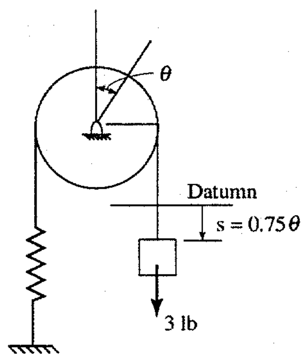
$$s_{eq}=0.0375\text{ ft}$$

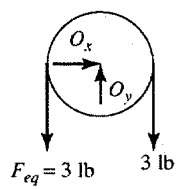
Thus,

$$0.1834\ddot{\theta}+45\theta=0$$

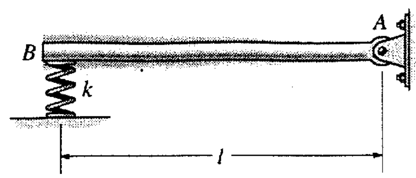
$$\ddot{\theta}+245.3=0$$

$$\tau=\frac{2\pi}{\omega_n}=\frac{2\pi}{\sqrt{245.3}}=0.401\text{ s}\qquad \textbf{Ans}$$





22-30. The uniform rod of mass m is supported by a pin at A and a spring at B. If the end B is given a small downward displacement and released, determine the natural period of vibration.



$$T = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\dot{\theta}^2$$

$$V = \frac{1}{2}k(y_{eq} + y_2)^2 - mgy_1$$

$$= \frac{1}{2}k(l\theta_{eq} + l\theta)^2 - mg\left(\frac{1}{2}\right)\theta$$

$$T + V = \frac{1}{6}ml^2\dot{\theta}^2 + \frac{1}{2}k(l\theta_{eq} + l\theta)^2 - mg\left(\frac{l\theta}{2}\right)$$

Time derivative

$$0 = \frac{1}{3}ml^2\dot{\theta}\ddot{\theta} + kl(\theta_{eq} + \theta)\dot{\theta} - mgl\frac{\dot{\theta}}{2}$$

For equilibrium

$$k(l\theta_{eq}) = mgl/2, \theta_{eq} = \frac{mg}{2k}$$

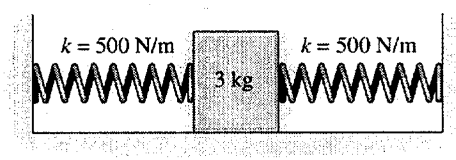
Thus,

$$0 = \frac{1}{3}ml\ddot{\theta} + k\theta$$

$$\ddot{\theta} + (3k/m)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{3k}} \quad \textbf{Ans}$$

22-31. Determine the differential equation of motion of the 3-kg block when it is displaced slightly and released. The surface is smooth and the springs are originally unstretched.



$$T + V = \text{const.}$$

$$T = \frac{1}{2}(3)\dot{x}^2$$

$$V = \frac{1}{2}(500)x^2 + \frac{1}{2}(500)x^2$$

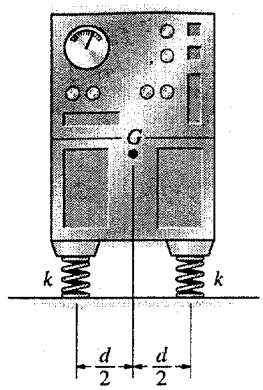
$$T + V = 1.5\dot{x}^2 + 500x^2$$

$$1.5\,(2\dot{x})\,\ddot{x} + 1000x\dot{x} = 0$$

$$3\ddot{x} + 1000x = 0$$

$$\ddot{x} + 333x = 0 \qquad \textbf{Ans}$$

***22-32.** The machine has a mass m and is uniformly supported by *four* springs, each having a stiffness k. Determine the natural period of vertical vibration.



$T + V = \text{const.}$

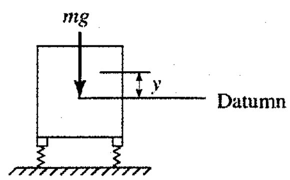
$T = \frac{1}{2}m(\dot{y})^2$

$V = mgy + \frac{1}{2}(4k)(\Delta s - y)^2$

$T + V = \frac{1}{2}m(\dot{y})^2 + mgy + \frac{1}{2}(4k)(\Delta s - y)^2$

$m\dot{y}\ddot{y} + mg\dot{y} - 4k(\Delta s - y)\dot{y} = 0$

$m\ddot{y} + mg + 4ky - 4k\Delta s = 0$



Since $\Delta s = \dfrac{mg}{4k}$

Then,

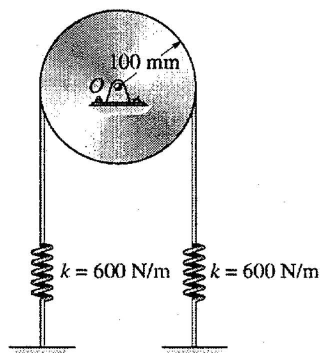
$m\ddot{y} + 4ky = 0$

$\ddot{y} + \dfrac{4k}{m}y = 0$

$\omega_n = \sqrt{\dfrac{4k}{m}}$

$\tau = \dfrac{2\pi}{\omega_n} = \pi\sqrt{\dfrac{m}{k}}$ **Ans**

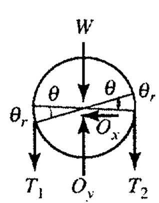
22-33. The 7-kg disk is pin-connected at its midpoint. Determine the natural period of vibration of the disk if the springs have sufficient tension in them to prevent the cord from slipping on the disk as it oscillates. *Hint:* Assume that the initial stretch in each spring is δ_O. This term will cancel out after taking the time derivative of the energy equation.



$E = T + V$

$= \frac{1}{2}k(\theta r + \delta_O)^2 + \frac{1}{2}k(\theta r - \delta_O)^2 + \frac{1}{2}I_O(\dot{\theta})^2$

$E = k(\theta r + \delta_O)\dot{\theta}r + k(\theta r - \delta_O)\dot{\theta}r + I_O\dot{\theta}\ddot{\theta} = 0$



Thus,

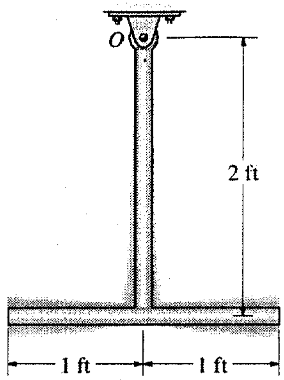
$\ddot{\theta} + \dfrac{2kr^2}{I_O}\theta = 0$

$\omega_n = \sqrt{\dfrac{2kr^2}{I_O}}$

$\tau = \dfrac{2\pi}{\omega_n} = 2\pi\sqrt{\dfrac{I_O}{2kr^2}}$

$\tau = 2\pi\sqrt{\dfrac{\frac{1}{2}(7)(0.1)^2}{2(600)(0.1)^2}} = 0.339 \text{ s}$ **Ans**

22-34. Determine the natural period of vibration of the pendulum. Consider the two rods to be slender, each having a weight of 8 lb/ft.

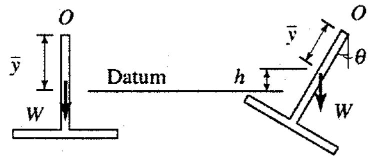


$$\bar{y} = \frac{1(8)(2) + 2(8)(2)}{8(2) + 8(2)} = 1.5 \text{ ft}$$

$$I_O = \frac{1}{32.2}\left[\frac{1}{12}(2)(8)(2)^2 + 2(8)(1)^2\right]$$

$$+ \frac{1}{32.2}\left[\frac{1}{12}(2)(8)(2)^2 + 2(8)(2)^2\right] = 2.8157 \text{ slug} \cdot \text{ft}^2$$

$$h = \bar{y}(1 - \cos\theta)$$



$$T + V = \text{const.}$$

$$T = \frac{1}{2}(2.8157)\left(\dot{\theta}\right)^2 = 1.4079\dot{\theta}^2$$

$$V = 8(4)(1.5)(1 - \cos\theta) = 48(1 - \cos\theta)$$

$$T + V = 1.4079\dot{\theta}^2 + 48(1 - \cos\theta)$$

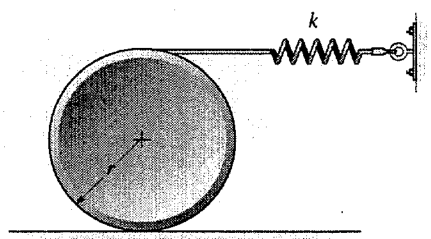
$$1.4079\left(2\dot{\theta}\right)\ddot{\theta} + 48(\sin\theta)\dot{\theta} = 0$$

For small θ, $\sin\theta = \theta$, then

$$\ddot{\theta} + 17.047\theta = 0$$

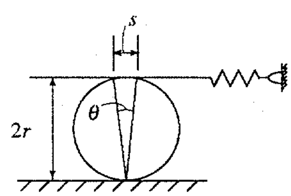
$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{17.047}} = 1.52 \text{ s} \quad \textbf{Ans}$$

22-35. Determine the natural period of vibration of the disk having a mass m and radius r. Assume the disk does not slip on the surface of contact as it oscillates.



$$T + V = \text{const.}$$

$$s = (2r)\theta$$



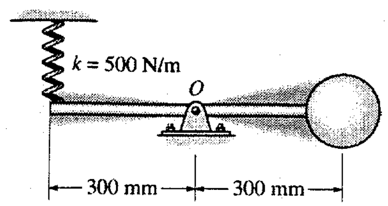
$$T + V = \frac{1}{2}\left[\frac{1}{2}mr^2 + mr^2\right]\dot{\theta}^2 + \frac{1}{2}k(2r\theta)^2$$

$$0 = \frac{3}{2}mr^2\dot{\theta}\ddot{\theta} + 4kr^2\theta\dot{\theta}$$

$$\ddot{\theta} + \frac{8k}{3m}\theta = 0$$

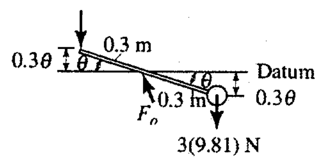
$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{8k}{3m}}} = 3.85\sqrt{\frac{m}{k}} \qquad \textbf{Ans}$$

***22-36.** Determine the natural period of vibration of the 3-kg sphere. Neglect the mass of the rod and the size of the sphere.



$E = T + V$

$= \frac{1}{2}(3)(0.3\dot{\theta})^2 + \frac{1}{2}(500)(\delta_{st} + 0.3\theta)^2 - 3(9.81)(0.3\theta)$



$E = \dot{\theta}[3(0.3)^2\ddot{\theta} + 500(\delta_{st} + 0.3\theta)(0.3) - 3(9.81)(0.3)] = 0$

By statics,

$T(0.3) = 3(9.81)(0.3)$

$T = 3(9.81)$ N

$\delta_{st} = \frac{3(9.81)}{500}$

Thus,

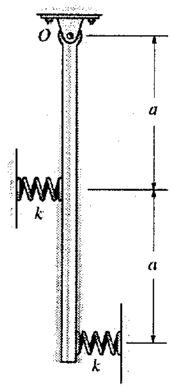
$3(0.3)^2\ddot{\theta} + 500(0.3)^2\theta = 0$

$\ddot{\theta} + 166.67\theta = 0$

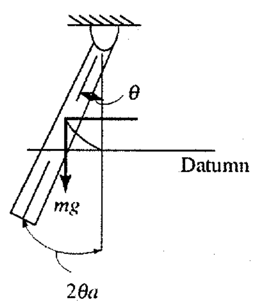
$\omega_n = \sqrt{166.67} = 12.91$ rad/s

$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{12.91} = 0.487$ s **Ans**

22-37. The slender rod has a mass m and is pinned at its end O. When it is vertical, the springs are unstretched. Determine the natural period of vibration.



$T + V = \frac{1}{2}\left[\frac{1}{3}m(2a)^2\right]\dot{\theta}^2 + \frac{1}{2}k(2\theta a)^2 + \frac{1}{2}k(\theta a)^2 + mga(1 - \cos\theta)$



$0 = \frac{4}{3}ma^2\dot{\theta}\ddot{\theta} + 4ka^2\theta\dot{\theta} + ka^2\theta\dot{\theta} + mga\sin\theta\dot{\theta}$

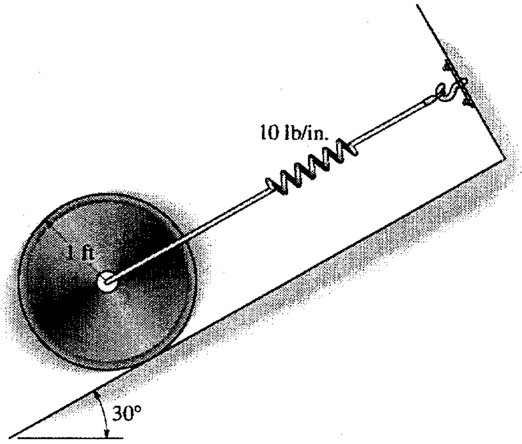
$\sin\theta = \theta$

$\frac{4}{3}ma^2\ddot{\theta} + 5ka^2\theta + mga\theta = 0$

$\ddot{\theta} + \left(\frac{15ka + 3mg}{4ma}\right)\theta = 0$

$\tau = \frac{2\pi}{\omega_n} = \frac{4\pi}{\sqrt{3}}\left(\frac{ma}{5ka + mg}\right)^{\frac{1}{2}}$ **Ans**

22-38. Determine the natural frequency of vibration of the 20-lb disk. Assume the disk does not slip on the inclined surface.



θ is the displacement of the disk.

The disk rolls a distance $s = r\theta$

$\Delta k = r\theta \sin 30°$

$E = T + V$

$$= \frac{1}{2} I_{IC}(\dot{\theta})^2 + \frac{1}{2} k[\delta_{st} + \theta r]^2 - W(r\theta \sin 30°)$$

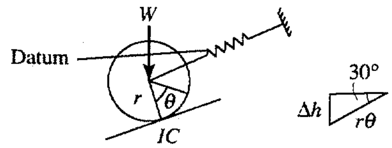
$$E = \dot{\theta}(I_{IC}\ddot{\theta} + k\delta_{st} r + k\theta r^2 - W r \sin 30°) = 0$$

Since $k\delta_{st} = W r \sin 30°$

$$\ddot{\theta} + \frac{kr^2}{I_{IC}}\theta = 0$$

$$I_{IC} = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$$

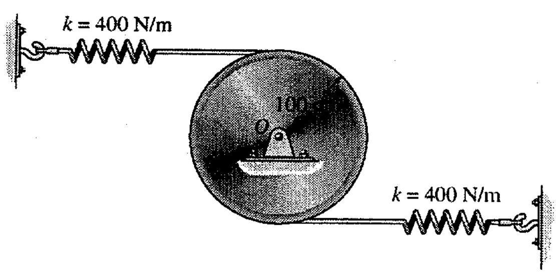
Thus,



$$f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{kr^2}{I_{IC}}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{10(12)(1)^2}{\frac{3}{2}\left(\frac{30}{32.2}\right)(1)^2}} = 1.81 \text{ Hz} \quad \textbf{Ans}$$

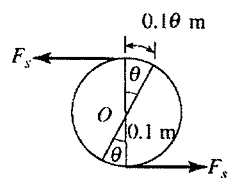
22-39. If the disk has a mass of 8 kg, determine the natural frequency of vibration. The springs are originally unstretched.



$$I_O = \frac{1}{2}(8)(0.1)^2 = 0.04$$

$T_{max} = V_{max}$

$$\frac{1}{2} I_O (p\theta_{max})^2 = 2\left[\frac{1}{2}k(r\theta_{max})^2\right]$$

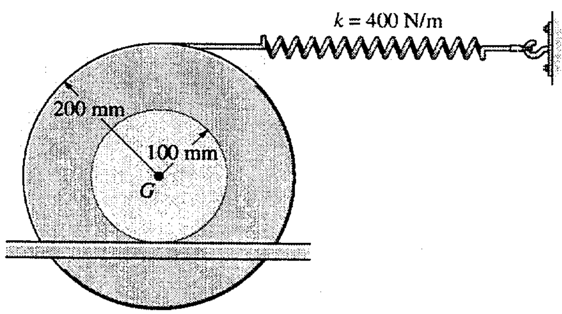


Thus,

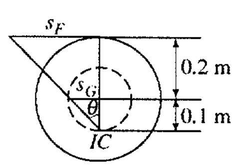
$$\omega_n = \sqrt{\frac{2kr^2}{I_O}}$$

$$f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2(400)(0.1)^2}{0.04}} = 2.25 \text{ Hz} \quad \textbf{Ans}$$

***22-40.** Determine the differential equation of motion of the 3-kg spool. Assume that it does not slip at the surface of contact as it oscillates. The radius of gyration of the spool about its center of mass is $k_G = 125$ mm.



Kinematics: Since no slipping occurs, $s_G = 0.1\theta$ hence $s_F = \frac{0.3}{0.1}s_G = 0.3\theta$. Also,



$v_G = 0.1\dot{\theta}$.

$E = T + V$

$$E = \frac{1}{2}[(3)(0.125)^2]\dot{\theta}^2 + \frac{1}{2}(3)(0.1\dot{\theta})^2 + \frac{1}{2}(400)(0.3\theta)^2 = \text{const.}$$

$$= 0.038444\dot{\theta}^2 + 18\theta^2$$

$$0.0768759\dot{\theta}\ddot{\theta} + 36\theta\dot{\theta} = 0$$

$$0.076875\dot{\theta}(\ddot{\theta} + 468.29\theta) = 0 \text{ Since } 0.076875\dot{\theta} \neq 0$$

$$\ddot{\theta} + 468\theta = 0 \quad \textbf{Ans}$$

22-41. The block shown in Fig. 22-16 has a mass of 20 kg, and the spring has a stiffness $k = 600$ N/m. When the block is displaced and released, two successive amplitudes are measured as $x_1 = 150$ mm and $x_2 = 87$ mm. Determine the coefficient of viscous damping, c.

Assuming that the system is underdamped.

$$x_1 = De^{-\left(\frac{c}{2m}\right)t_1} \quad (1)$$

$$x_2 = De^{-\left(\frac{c}{2m}\right)t_2} \quad (2)$$

Divide Eq. (1) by Eq. (2) $\quad \frac{x_1}{x_2} = \frac{e^{-\left(\frac{c}{2m}\right)t_1}}{e^{-\left(\frac{c}{2m}\right)t_2}}$

$$\ln\left(\frac{x_1}{x_2}\right) = \left(\frac{c}{2m}\right)(t_2 - t_1) \quad (3)$$

However, $\quad t_2 - t_1 = \tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\left(\frac{c}{C_c}\right)^2}}$ and $\omega_n = \frac{C_c}{2m}$

$$t_2 - t_1 = \frac{4m\pi}{C_c\sqrt{1-\left(\frac{c}{C_c}\right)^2}} \quad (4)$$

Substitute Eq. (4) into Eq. (3) yields:

$$\ln\left(\frac{x_1}{x_2}\right) = \left(\frac{c}{2m}\right)\frac{4m\pi}{C_c\sqrt{1-\left(\frac{C}{C_c}\right)^2}}$$

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{C}{C_c}\right)}{\sqrt{1-\left(\frac{C}{C_c}\right)^2}} \quad (5)$$

From Eq. (5)

$x_1 = 0.15$ m $\quad x_2 = 0.087$ m $\quad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{20}} = 5.477$ rad/s

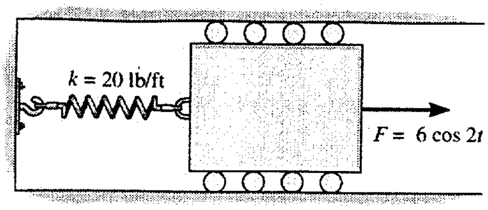
$C_c = 2m\omega_n = 2(20)(5.477) = 219.09$ N · s/m

$$\ln\left(\frac{0.15}{0.087}\right) = \frac{2\pi\left(\frac{c}{219.09}\right)}{\sqrt{1-\left(\frac{c}{219.09}\right)^2}}$$

$c = 18.9$ N · s/m $\qquad$ **Ans**

Since $C < C_c$, the system is underdamped. Therefore, the assumption is OK!

22-42. The 20-lb block is attached to a spring having a stiffness of 20 lb/ft. A force $F = (6\cos 2t)$ lb, where t is in seconds, is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.



$$C = \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{20}{\frac{20}{32.2}}} = 5.6745 \text{ rad/s}$$

$$C = \frac{\frac{6}{20}}{1 - \left(\frac{2}{5.6745}\right)^2} = 0.343 \text{ ft}$$

$$x_p = C\cos 2t$$

$$\dot{x}_p = -C(2)\sin 2t$$

Maximum velocity is

$v_{max} = C(2) = 0.343(2) = 0.685$ ft/s **Ans**

22-43. A 4-kg block is suspended from a spring that has a stiffness of $k = 600$ N/m. The block is drawn downward 50 mm from the equilibrium position and released from rest when $t = 0$. If the support moves with an impressed displacement of $\delta = (10\sin 4t)$ mm, where t is in seconds, determine the equation that describes the vertical motion of the block. Assume positive displacement is downward.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{4}} = 12.25$$

The general solution is defined by Eq. 22-23 with $k\delta_0$ substituted for F_0.

$$y = A\sin\omega_n t + B\cos\omega_n t + \left(\frac{\delta_0}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]}\right)\sin\omega t$$

$\delta = (0.01\sin 4t)$m, hence $\delta_0 = 0.01$, $\omega = 4$, so that

$$y = A\sin 12.25t + B\cos 12.25t + 0.0112\sin 4t$$

$y = 0.05$ when $t = 0$,

$0.05 = 0 + B + 0;\quad B = 0.05$ m

$$\dot{y} = A(12.25)\cos 12.25t - B(12.25)\sin 12.25t + 0.0112(4)\cos 4t$$

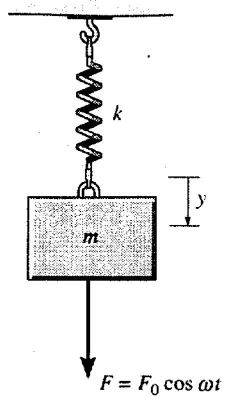
$v = \dot{y} = 0$ when $t = 0$,

$0 = A(12.25) - 0 + 0.0112(4);\quad A = -0.00366$ m

Expressing the result in mm, we have

$y = (-3.66\sin 12.25t + 50\cos 12.25t + 11.2\sin 4t)$ mm **Ans**

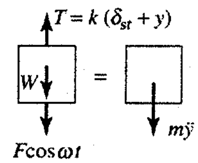
***22-44.** If the block is subjected to the impressed force $F = F_0 \cos \omega t$, show that the differential equation of motion is $\ddot{y} + (k/m)y = (F_0/m)\cos \omega t$, where y is measured from the equilibrium position of the block. What is the general solution of this equation?



$$+\downarrow \sum F_y = ma_y; \quad F_0 \cos \omega t + W - k\delta_{st} - ky = m\ddot{y}$$

Since $W = k\delta_{st}$,

$$\ddot{y} + \left(\frac{k}{m}\right) y = \frac{F_0}{m} \cos \omega t \qquad (1) \quad \textbf{Q.E.D.}$$



$y_c = A \sin \omega_n y + B \cos \omega_n y$ (complementary solution)

$y_p = C \cos \omega t$ (particular solution)

Substitute y_p into Eq. (1),

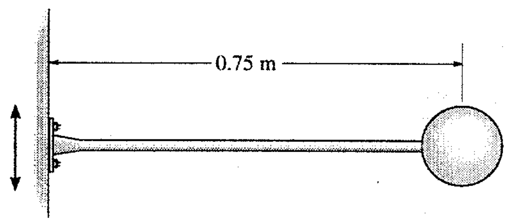
$$C\left(-\omega^2 + \frac{k}{m}\right)\cos \omega t = \frac{F_0}{m} \cos \omega t$$

$$C = \frac{\frac{F_0}{m}}{\left(\frac{k}{m} - \omega^2\right)}$$

$$y = y_c + y_p$$

$$y = A \sin \omega_n t + B \cos \omega_n t + \left(\frac{F_0}{(k - m\omega^2)}\right) \cos \omega t \quad \textbf{Ans}$$

22-45. The light elastic rod supports a 4-kg sphere. When an 18-N vertical force is applied to the sphere, the rod deflects 14 mm. If the wall oscillates with harmonic frequency of 2 Hz and has an amplitude of 15 mm, determine the amplitude of vibration for the sphere.



$$k = \frac{F}{\Delta y} = \frac{18}{0.014} = 1285.71 \text{ N/m}$$

$$\omega = 2 \text{ Hz} = 2(2\pi) = 12.57 \text{ rad/s}$$

$$\delta_0 = 0.015 \text{ m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1285.71}{4}} = 17.93$$

Using Eq. 22-22, the amplitude is

$$(x_p)_{max} = \left| \frac{\delta_0}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right| = \left| \frac{0.015}{1 - \left(\frac{12.57}{17.93}\right)^2} \right|$$

$$(x_p)_{max} = 0.0295 \text{ m} = 29.5 \text{ mm} \qquad \textbf{Ans}$$

22-46. A block having a mass of 0.8 kg is suspended from a spring having a stiffness of 120 N/m. If a dashpot provides a damping force of 2.5 N when the speed of the block is 0.2 m/s, determine the period of free vibration.

$$F = cv \quad c = \frac{F}{v} = \frac{2.5}{0.2} = 12.5 \text{ N}\cdot\text{s/m}$$

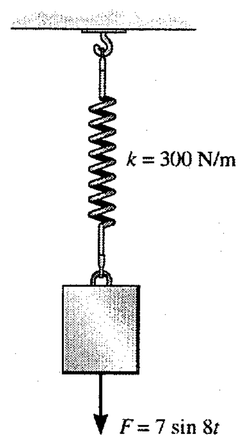
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{120}{0.8}} = 12.247 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2(0.8)(12.247) = 19.60 \text{ N}\cdot\text{s/m}$$

$$\omega_d = \omega_n\sqrt{1-\left(\frac{c}{c_c}\right)^2} = 12.247\sqrt{1-\left(\frac{12.5}{19.6}\right)^2} = 9.432 \text{ rad/s}$$

$$\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{9.432} = 0.666 \text{ s}$$ **Ans**

22-47. A 5-kg block is suspended from a spring having a stiffness of 300 N/m. If the block is acted upon by a vertical force $F = (7\sin 8t)$ N, where t is in seconds, determine the equation which describes the motion of the block when it is pulled down 100 mm from the equilibrium position and released from rest at $t = 0$. Assume that positive displacement is downward.



The general solution is defined by:

$$y = A\sin\omega_n t + B\cos\omega_n t + \left(\frac{\frac{F_0}{k}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)\sin\omega t$$

Since

$F = 7\sin 8t, \qquad F_0 = 7$ N, $\qquad \omega = 8$ rad/s, $\qquad k = 300$ N/m

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{5}} = 7.746 \text{ rad/s}$$

Thus,

$$y = A\sin 7.746t + B\cos 7.746t + \left(\frac{\frac{7}{300}}{1-\left(\frac{8}{7.746}\right)^2}\right)\sin 8t$$

$y = 0.1$ m when $t = 0$,

$0.1 = 0 + B - 0; \qquad B = 0.1$ m

$\dot{y} = A(7.746)\cos 7.746t - B(7.746)\sin 7.746t - (0.35)(8)\cos 8t$

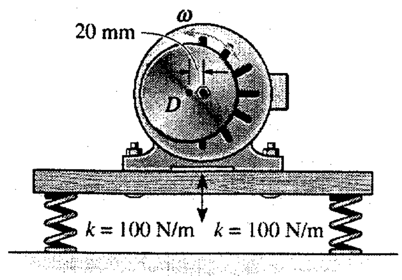
$y = \dot{y} = 0$ when $t = 0$.

$\dot{y} = A(7.746) - 2.8 = 0; \qquad A = 0.361$

Expressing the results in mm, we have

$y = (361\sin 7.75t + 100\cos 7.75t - 350\sin 8t)$ mm **Ans**

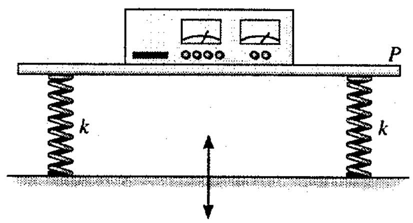
*22-48. The electric motor has a mass of 50 kg and is supported by *four springs*, each spring having a stiffness of 100 N/m. If the motor turns a disk D which is mounted eccentrically, 20 mm from the disk's center, determine the angular rotation ω at which resonance occurs. Assume that the motor only vibrates in the vertical direction.



Resonance occurs when $\omega = \omega_n$.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4(100)}{50}} = 2.83 \text{ rad/s}$$

22-49. The instrument is centered uniformly on a platform P, which in turn is supported by *four* springs, each spring having a stiffness $k = 130$ N/m. If the floor is subjected to a vibration $\omega = 7$ Hz, having a vertical displacement amplitude $\delta_0 = 0.17$ ft, determine the vertical displacement amplitude of the platform and instrument. The instrument and the platform have a total weight of 18 lb.



$k = 4(130) = 520$ lb/ft

$\delta_0 = 0.17$ ft

$\omega = 7 \text{ Hz} = 7(2\pi) = 43.98$ rad/s

Using Eq. 22-22, the amplitude is

$$(x_p)_{max} = \left| \frac{\delta_0}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right|$$

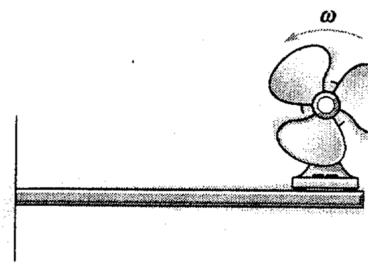
Since $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{520}{\frac{18}{32.2}}} = 30.50$ rad/s

Then,

$$(x_p)_{max} = \left| \frac{0.17}{1 - \left(\frac{43.98}{30.50}\right)^2} \right| = 0.157 \text{ ft}$$

$(x_p)_{max} = 1.89$ in. **Ans**

22-50. The fan has a mass of 25 kg and is fixed to the end of a horizontal beam that has a negligible mass. The fan blade is mounted eccentrically on the shaft such that it is equivalent to an unbalanced 3.5-kg mass located 100 mm from the axis of rotation. If the static deflection of the beam is 50 mm as a result of the weight of the fan, determine the amplitude of steady-state vibration of the fan when the angular velocity of the fan is 10 rad/s. *Hint:* See the first part of Example 22-8.



$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\omega^2 = 3.5(0.1)(10)^2 = 35 \text{ N}$$

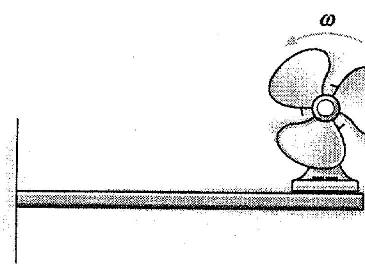
Using Eq. 22-22, the amplitude is

$$(x_p)_{max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right|$$

$$(x_p)_{max} = \left| \frac{\frac{35}{4905}}{1 - \left(\frac{10}{14.01}\right)^2} \right| = 0.0146 \text{ m}$$

$(x_p)_{max} = 14.6$ mm **Ans**

22-51. What will be the amplitude of steady-state vibration of the fan in Prob. 22-50 if the angular velocity of the fan is 18 rad/s? *Hint:* See the first part of Example 22-8.



$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\omega^2 = 3.5(0.1)(18)^2 = 113.4 \text{ N}$$

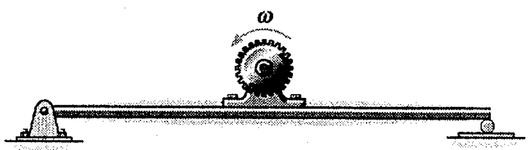
Using Eq. 22-22, the amplitude is

$$(x_p)_{max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right|$$

$$(x_p)_{max} = \left| \frac{\frac{113.4}{4905}}{1 - \left(\frac{18}{14.01}\right)^2} \right| = 0.0355 \text{ m}$$

$(x_p)_{max} = 35.5$ mm **Ans**

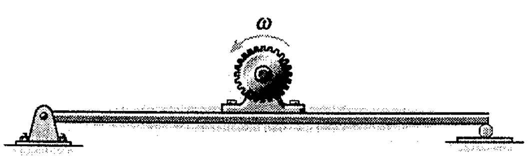
***22-52.** The electric motor turns an eccentric flywheel which is equivalent to an unbalanced 0.25-lb weight located 10 in. from the axis of rotation. If the static deflection of the beam is 1 in. because of the weight of the motor, determine the angular velocity of the flywheel at which resonance will occur. The motor weighs 150 lb. Neglect the mass of the beam.



$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.66$$

Resonance occurs when $\qquad \omega = \omega_n = 19.7 \text{ rad/s}$ **Ans**

22-53. What will be the amplitude of steady-state vibration of the motor in Prob. 22-52 if the angular velocity of the flywheel is 20 rad/s?



The constant value F_o of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_o = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)(20)^2 = 2.588 \text{ lb}$$

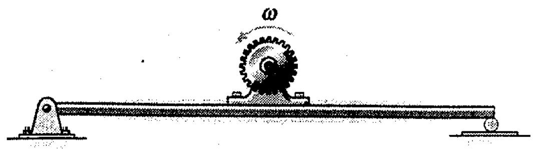
Hence $F = 2.588 \sin 20t$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

From Eq. 22-21, the amplitude of the steady state motion is

$$C = \left| \frac{F_0/k}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right| = \left| \frac{2.588/1800}{1 - \left(\frac{20}{19.657}\right)^2} \right| = 0.04085 \text{ ft} = 0.490 \text{ in.} \quad \textbf{Ans}$$

22-54. Determine the angular velocity of the flywheel in Prob. 22-52 which will produce an amplitude of vibration of 0.25 in.



The constant value F_o of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_o = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)\omega^2 = 0.006470\omega^2$$

$$F = 0.006470\omega^2 \sin \omega t$$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \quad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

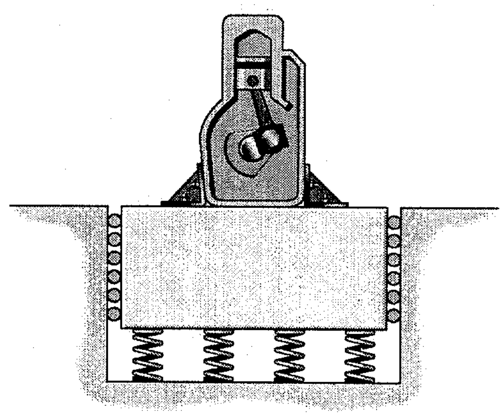
From Eq. 22.21, the amplitude of the steady state motion is

$$C = \left|\frac{F_0/k}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right|$$

$$\frac{0.25}{12} = \left|\frac{0.006470\left(\frac{\omega^2}{1800}\right)}{1-\left(\frac{\omega}{19.657}\right)^2}\right|$$

$\omega = 19.0$ rad/s **Ans**

22-55. The engine is mounted on a foundation block which is spring-supported. Describe the steady-state vibration of the system if the block and engine have a total weight of 1500 lb and the engine, when running, creates an impressed force $F = (50 \sin 2t)$ lb, where t is in seconds. Assume that the system vibrates only in the vertical direction, with the positive displacement measured downward, and that the total stiffness of the springs can be represented as $k = 2000$ lb/ft.



The steady-state vibration is defined by Eq. 22-22.

$$x_p = \frac{\frac{F_0}{k}}{1-\left(\frac{\omega}{\omega_n}\right)^2} \sin \omega t$$

Since $F = 50 \sin 2t$

Then $F_0 = 50$ lb, $\omega = 2$ rad/s

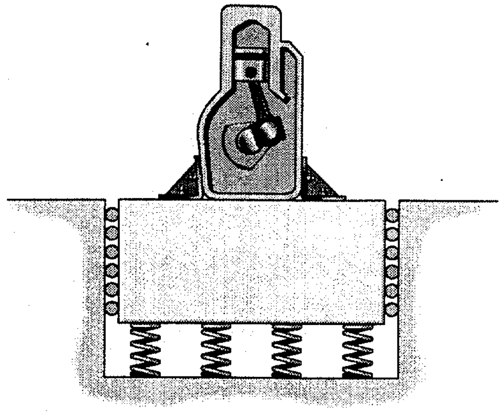
$k = 2000$ lb/ft

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2000}{\frac{1500}{32.2}}} = 6.55 \text{ rad/s}$$

Hence, $$x_p = \frac{\frac{50}{2000}}{1-\left(\frac{2}{6.55}\right)^2} \sin 2t$$

$x_p = (0.0276 \sin 2t)$ ft **Ans**

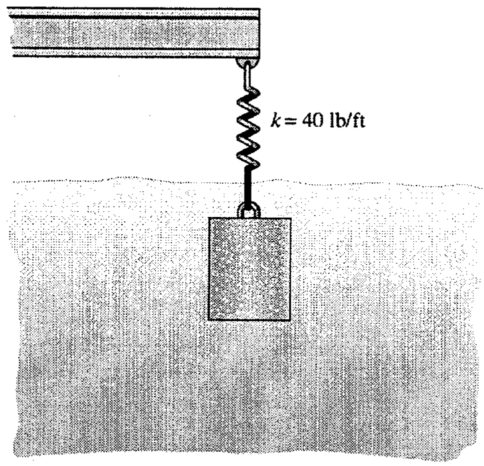
*22-56. Determine the rotational speed ω of the engine in Prob. 22-55 which will cause resonance.



Resonance occurs when

$$\omega = \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{\frac{2000}{1500}}{32.2}} = 6.55 \text{ rad/s} \quad \textbf{Ans}$$

22-57. The block, having a weight of 1.5 lb is immersed in a liquid such that the damping force acting on the block has a magnitude of $F = (0.8|v|)$ lb, where v is in ft/s. If the block is pulled down 0.8 ft and released from rest, determine the position of the block as a function of time. The spring has a stiffness of $k = 40$ lb/ft. Assume that positive displacement is downward.



Viscous Damped Free Vibration: Here $c = 0.8$ lb · s/ft, $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{15/32.2}} = 9.266$ rad/s and $c_c = 2m\omega_n = 2\left(\frac{15}{32.2}\right)(9.266) = 8.633$ lb · s/ft. Since $c < c_c$, the system is underdamped and the solution of the differential equation is in the form of

$$y = D[e^{-(c/2m)t}\sin(\omega_d t + \phi)] \qquad [1]$$

Taking the time derivative of Eq. [1], we have

$$v = \dot{y} = D\left[-\left(\frac{c}{2m}\right)e^{-(c/2m)t}\sin(\omega_d t + \phi) + \omega_d e^{-(c/2m)t}\cos(\omega_d t + \phi)\right]$$

$$= De^{-(c/2m)t}\left[-\left(\frac{c}{2m}\right)\sin(\omega_d t + \phi) + \omega_d\cos(\omega_d t + \phi)\right] \qquad [2]$$

Applying the initial condition $v = 0$ at $t = 0$ to Eq. [2], we have

$$0 = De^{-0}\left[-\left(\frac{c}{2m}\right)\sin(0 + \phi) + \omega_d\cos(0 + \phi)\right]$$

$$0 = D\left[-\left(\frac{c}{2m}\right)\sin\phi + \omega_d\cos\phi\right] \qquad [3]$$

Here, $\frac{c}{2m} = \frac{0.8}{2(15/32.2)} = 0.8587$ and $\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)} = 9.266\sqrt{1 - \left(\frac{0.8}{8.633}\right)^2} = 9.227$ rad/s. Substituting these values into Eq. [1] yields

$$0 = D[-0.8587\sin\phi + 9.227\cos\phi] \qquad [4]$$

Applying the initial condition $y = 0.8$ ft at $t = 0$ to Eq. [1], we have

$$0.8 = D[e^{-0}\sin(0 + \phi)]$$

$$0.8 = D\sin\phi \qquad [5]$$

Solving Eqs. [4] and [5] yields

$$\phi = 85.74° = 1.50 \text{ rad} \qquad D = 0.8022 \text{ ft}$$

Substituting these values into Eq. [1] yields

$$y = 0.802[e^{-0.859t}\sin(9.23t + 1.50)] \quad \textbf{Ans}$$

22-58. A 7-lb block is suspended from a spring having a stiffness of $k = 75$ lb/ft. The support to which the spring is attached is given simple harmonic motion which may be expressed as $\delta = (0.15 \sin 2t)$ ft, where t is in seconds. If the damping factor is $c/c_c = 0.8$, determine the phase angle ϕ of forced vibration.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$\delta = 0.15 \sin 2t$

$\delta_0 = 0.15, \omega = 2$

$$\phi' = \tan^{-1}\left(\frac{2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right) = \tan^{-1}\left(\frac{2(0.8)\left(\frac{2}{18.57}\right)}{1-\left(\frac{2}{18.57}\right)^2}\right)$$

$\phi' = 9.89°$ **Ans**

22-59. Determine the magnification factor of the block, spring, and dashpot combination in Prob. 22-58.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

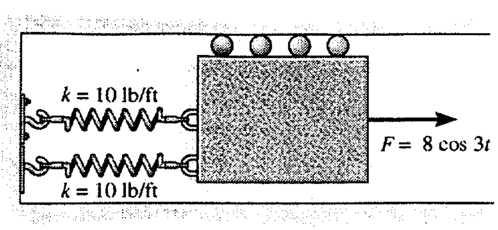
$\delta = 0.15 \sin 2t$

$\delta_0 = 0.15, \quad \omega = 2$

$$\mathrm{MF} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)\right]^2}} = \frac{1}{\sqrt{\left[1-\left(\frac{2}{18.57}\right)^2\right]^2+\left[2(0.8)\left(\frac{2}{18.57}\right)\right]^2}}$$

$\mathrm{MF} = 0.997$ **Ans**

***22-60.** The 30-lb block is attached to two springs having a stiffness of 10 lb/ft. A force $F = (8\cos 3t)$lb, where t is in seconds, is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.

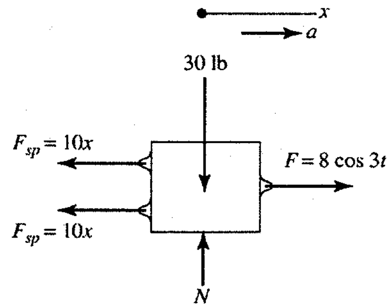


Freebody Diagram: When the block is being displaced by amount x to the right, the *restoring force* that develops in both springs is $F_{sp} = kx = 10x$.

Equation of Motion:

$$\xrightarrow{+} \sum F_x = 0; \qquad -2(10x) + 8\cos 3t = \frac{30}{32.2}a$$

$$a + 21.47x = 8.587\cos 3t \qquad [1]$$



Kinematics: Since $a = \frac{d^2x}{dt^2} = \ddot{x}$, then substituting this value into Eq. [1], we have

$$\ddot{x} + 21.47x = 8.587\cos 3t \qquad [2]$$

Since the friction will eventually dampen out the free vibration, we are only interested in the *particular solution* of the above differential equation which is in the form of

$$x_p = C\cos 3t$$

Taking second time derivative and substituting into Eq. [2], we have

$$-9C\cos 3t + 21.47C\cos 3t = 8.587\cos 3t$$

$$C = 0.6888 \text{ ft}$$

Thus, $\qquad x_p = 0.6888\cos 3t \qquad [3]$

Taking the time derivative of Eq. [3], we have

$$v_p = \dot{x}_p = -2.0663\sin 3t$$

Thus, $\qquad (v_p)_{max} = 2.07$ ft/s $\quad$ **Ans**

22-61. A block having a mass of 7 kg is suspended from a spring that has a stiffness $k = 600$ N/m. If the block is given an upward velocity of 0.6 m/s from its equilibrium position at $t = 0$, determine its position as a function of time. Assume that positive displacement of the block is downward and that motion takes place in a medium which furnishes a damping force $F = (50|v|)$N, where v is in m/s.

$c = 50$ N·s/m $\quad k = 600$ N/m $\quad m = 7$ kg

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{7}} = 9.258 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2(7)(9.258) = 129.6 \text{ N·s/m}$$

Since $c < c_c$ the system is underdamped,

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 9.258\sqrt{1 - \left(\frac{50}{129.6}\right)^2} = 8.542 \text{ rad/s}$$

$$\frac{c}{2m} = \frac{50}{2(7)} = 3.571$$

From Eq. 22-32

$$y = D\left[e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)\right]$$

$$v = \dot{y} = D\left[e^{-\left(\frac{c}{2m}\right)t}\omega_d\cos(\omega_d t + \phi) + \left(-\frac{c}{2m}\right)e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)\right]$$

$$v = De^{-\left(\frac{c}{2m}\right)t}\left[\omega_d\cos(\omega_d t + \phi) - \frac{c}{2m}\sin(\omega_d t + \phi)\right]$$

Applying the initial condition at $t = 0$, $y = 0$ and $v = -0.6$ m/s.

$$0 = D[e^{-0}\sin(0 + \phi)] \qquad \text{since} \quad D \neq 0$$

$$\sin\phi = 0 \qquad \phi = 0^\circ$$

$$-0.6 = De^{-0}[8.542\cos 0^\circ - 0]$$

$$D = -0.0702 \text{ m}$$

$y = \{-0.0702[e^{-3.57t}\sin(8.54t)]\}$ m $\quad$ **Ans**

22-62. The damping factor, c/c_c, may be determined experimentally by measuring the successive amplitudes of vibrating motion of a system. If two of these maximum displacements can be approximated by x_1 and x_2, as shown in Fig. 22-17, show that the ratio $\ln x_1/x_2 = 2\pi(c/c_c)/\sqrt{1-(c/c_e)^2}$. The quantity $\ln x_1/x_2$ is called the *logarithmic decrement.*

Using Eq. 22-32,

$$x = D\left[e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)\right]$$

The maximum displacement is

$$x_{max} = De^{-\left(\frac{c}{2m}\right)t}$$

At $t = t_1$, and $t = t_2$

$$x_1 = De^{-\left(\frac{c}{2m}\right)t_1}$$

$$x_2 = De^{-\left(\frac{c}{2m}\right)t_2}$$

Hence,

$$\frac{x_1}{x_2} = \frac{De^{-\left(\frac{c}{2m}\right)t_1}}{De^{-\left(\frac{c}{2m}\right)t_2}} = e^{-\left(\frac{c}{2m}\right)(t_1-t_2)}$$

Since $\omega_d t_2 - \omega_d t_1 = 2\pi$

then $t_2 - t_1 = \dfrac{2\pi}{\omega_d}$

so that $\ln\left(\dfrac{x_1}{x_2}\right) = \dfrac{c\pi}{m\omega_d}$

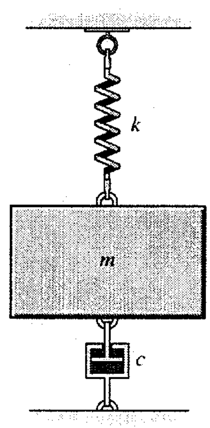
Using Eq. 22-33, $c_c = 2m\omega_n$

$$\omega_d = \omega_n\sqrt{1-\left(\frac{c}{c_c}\right)^2} = \frac{c_c}{2m}\sqrt{1-\left(\frac{c}{c_c}\right)^2}$$

So that,

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{c}{c_c}\right)}{\sqrt{1-\left(\frac{c}{c_c}\right)^2}} \qquad \textbf{Q.E.D.}$$

22-63. Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.

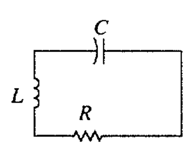


For the block

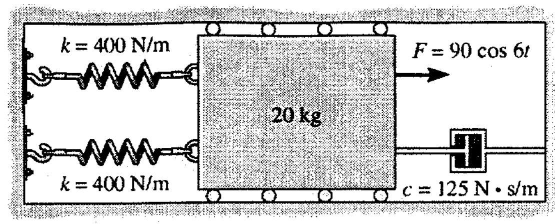
$$m\ddot{y} + c\dot{y} + ky = 0$$

Using Table 22-1

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = 0 \quad \textbf{Ans}$$



***22-64.** The 20-kg block is subjected to the action of the harmonic force $F = (90\cos 6t)$ N, where t is in seconds. Write the equation which describes the steady-state motion.



$F = 90\cos 6t$

$F_O = 90$ N, $\quad \omega = 6$ rad/s

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{20}} = 6.32 \text{ rad/s}$$

From Eq. 22-29,

$$c_c = 2m\omega_n = 2(20)(6.32) = 253.0$$

Using Eqs. 22-39,

$$x = C' \cos(\omega t - \phi)$$

$$C' = \frac{\frac{F_0}{k}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{C_c}\right)\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$= \frac{\frac{90}{800}}{\sqrt{[1-\left(\frac{6}{6.32}\right)^2]^2 + [2\left(\frac{125}{253.0}\right)\left(\frac{6}{6.32}\right)]^2}}$$

$$= 0.119$$

$$\phi = \tan^{-1}\left[\frac{\frac{c\omega}{k}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

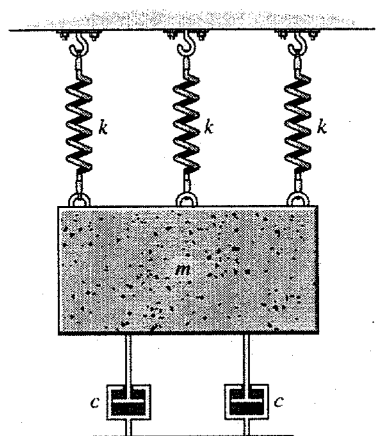
$$= \tan^{-1}\left[\frac{\frac{125(6)}{800}}{1-\left(\frac{6}{6.32}\right)^2}\right]$$

$$\phi = 83.9^\circ$$

Thus,

$x = 0.119\cos(6t - 83.9^\circ)$ m **Ans**

22-65. Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs? Take $k = 100$ N/m, $c = 200$ N · s/m, $m = 25$ kg.

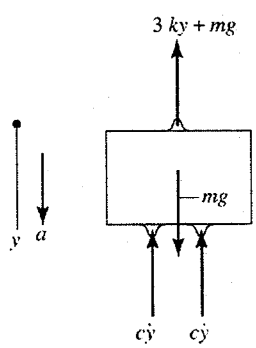


Freebody Diagram: When the block is being displaced by an amount y vertically downward, the *restoring force* is developed by the three springs attached to the block.

Equation of Motion:

$+\uparrow \sum F_x = 0; \qquad 3ky + mg + 2c\dot{y} - mg = -m\ddot{y}$

$$m\ddot{y} + 2c\dot{y} + 3ky = 0 \qquad [1]$$



Here, $m = 25$ kg, $c = 200$ N · s/m and $k = 100$ N/m. Substituting these values into Eq. [1] yields

$$25\ddot{y} + 400\dot{y} + 300y = 0$$

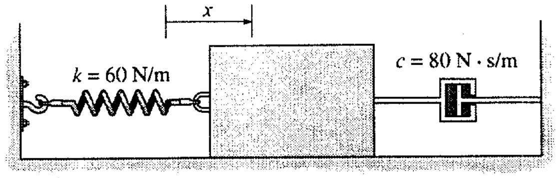
$\ddot{y} + 16\dot{y} + 12y = 0$ **Ans**

Comparing the above differential equation with Eq. 22-27, we have $m = 1$ kg, $c = 16$ N · s/m and $k = 12$ N/m. Thus, $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{12}{1}} = 3.464$ rad/s.

$$c_c = 2m\omega_n = 2(1)(3.464) = 6.928 \text{ N} \cdot \text{s/m}$$

Since $c > c_c$, the system will not vibrate. Therefore it is **overdamped**. **Ans**

22-66. The 10-kg block-spring-damper system is continually damped. If the block is displaced to $x = 50$ mm and released from rest, determine the time required for it to return to the position $x = 2$ mm.



$m = 10$ kg, $c = 80$, $k = 60$

$c_c = 2\sqrt{km} = 2\sqrt{600} = 49.0 < c$ (Overdamped)

$x = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$

$$\lambda_{1,2} = \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} = \frac{-80}{20} \pm \sqrt{\left(\frac{80}{20}\right)^2 - \frac{60}{10}}$$

$\lambda_1 = -0.8377$, $\lambda_2 = -7.1623$

At $t = 0$, $x = 0.05$, $\dot{x} = 0$

$0.05 = A + B$

$A = 0.05 - B$

$\dot{x} = A\lambda_1 e^{\lambda_1 t} + B\lambda_2 e^{\lambda_2 t}$

$0 = A\lambda_1 + B\lambda_2$

$0 = (0.05 - B)\lambda_1 + B\lambda_2$

$$B = \frac{0.05\lambda_1}{\lambda_1 - \lambda_2} = -6.6228(10^{-3})$$

$A = 0.056623$

$x = 0.056623e^{-0.8377t} - 6.6228(10^{-3})e^{-7.1623t}$

Set $x = 0.0002$ m and solve,

$t = 3.99$ s **Ans**