

Roll No								
---------	--	--	--	--	--	--	--	--



National Institute of Technology Goa

Programme Name: B.Tech

Online Mid Semester Examinations, October 2021

Course Name: **Mathematics-III**

Date: 09/10/2021

Duration: 90 Minutes

Course Code: **MA200**

Time: 10:00AM - 11:30 AM

Max. Marks: 50

-
1. Answer All Questions.
 2. **No marks will be given if the explanation of your answer is missing.**
 3. The question paper consists of **two** pages.
-

1. Use the Frobenius method to solve the ODE $9x(1-x)y'' - 12y' + 4y = 0$. [8M]

2. Consider the ODE $xy'' + y' + 2y = 0$, $y(1) = 1$, $y'(1) = 2$. [7M]

- (a) Find the analytic and singular points of the above ODE.
- (b) Solve the above ODE near $x = 1$ using the power series method.
- (c) Also find the general solution of the above ODE using the given initial conditions.

3. Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$. [8M]

4. Evaluate the limits if they exists?

(a) $\lim_{z \rightarrow (-i)} \frac{iz^3 + 1}{z^2 + 1}$

(b) $\lim_{z \rightarrow 0} \frac{\operatorname{Im}(z)}{z}$ [5M]

5. Verify Cauchy's theorem for the function $f(z) = 3z^2 + iz - 4$ if C is the square with vertices at $1 \pm i$ and $-1 \pm i$. [7M]

6. Evaluate the line integral $\int_C \bar{z} dz$, where C is given by:

$$\left. \begin{array}{l} x = 3t \\ y = t^2 \end{array} \right\}; -1 \leq t \leq 4 \quad [5M]$$

7. If a function $F(z)$ is defined to be

[5M]

$$F(\zeta) = \oint_{\gamma} \frac{4z^2 + z + 5}{z - \zeta} dz,$$

where ' γ ' is the ellipse $(\frac{x}{2})^2 + (\frac{y}{3})^2 = 1$, then evaluate:

- (a) $F(3.5)$
- (b) $F(i)$
- (c) $F''(-1)$ and
- (d) $F''(-i)$

8. (a) Evaluate the contour integral over the closed curve C given by $|z| = 2$:

[2M]

$$\oint_C \frac{e^{z^2}}{(z - i)^4} dz.$$

(b) Express the value of the given trigonometric function in the cartesian($a + ib$) form;

- i. $\cos(i)$
- ii. $\sin(2 + i)$
- iii. $\tan(\pi - 2i)$

[3M]

ALL THE BEST