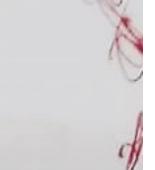


## National Institute of Technology Goa

Programme Name: B.Tech
End Semester Examinations, December 2022



Course Name: Mathematics-III

Date: 09/12/2022 Duration: 3 Hours Course Code: MA200

Time: 02:00PM - 05:00 PM

Max. Marks: 100

- 1. Answer All Questions.
- 2. No marks will be given if the explanation of your answer is missing.
- 3. The question paper consists of 2 pages.

## Part A: Complex Analysis

- 1. If f(z) = u + iv is an analytic function of z' and if  $u + v = \frac{\sin 2x}{\cosh 2y \cos 2x}$ . If f(z) (8M)
- 2. Find the Laurent series of the function  $f(z) = \frac{1}{z^2 + 4z + 3}$  in (10M) (a) 1 < |z| < 3 (b) |z| > 3 (c) 0 < |z + 1| < 2 (d) 1 > |z|.
- 3. Evaluate using Cauchy's Residue theorem  $\int_C \frac{e^{zt}}{z^2(z^2+2z+2)} dz$ , where C: |z| = 3. (6M)
  - 4. Verify Cauchy's theorem for the function  $5 \sin 2z$ , if C is the square with vertices  $1 \pm i$  and  $-1 \pm i$ . (6M)
  - 5. Evaluate the following improper integrals using Residue theorem
    (a)  $\int_0^\infty \frac{dx}{(x^2+9)(x^2+4)^2}$ (b)  $\int_0^\infty \frac{x \sin x}{(x^2+1)(x^2+4)} dx$ (c)  $\int_0^{2\pi} \frac{d\theta}{(5-3\sin\theta)^2}$ .
  - 6. Find the bi-linear transformation that maps the points -1, i, 1 + i onto 0, 2i, 1 i respectively. (5M)

## Part B: Differential Equations

- 7. Use the Frobenius method to solve the differential equation  $(x^2 x)y'' + 3y' 2y = 0$  near x = 0. (10M)
- 8. Find all the eigenvalues and eigenfunctions of the Strum-Liouville problem  $\frac{d}{dx}\left(x\frac{dy}{dx}\right) + \lambda \frac{y}{x} = 0$ , y'(1) = 0,  $y'(e^{2\pi}) = 0$ . And check the orthogonality of eigenfunctions. (10M)
- 9. A bar of 10 cm long with its ends A and B kept at 20° and 40° respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 50° and at the same time at B is lowered to 10°. Find the subsequent temperature distribution. (10M)

- 10. Solve Laplace's equation in rectangle with u(0, y) = 0, u(a, y) = 0, u(x, 0) = 0 and  $u(x, b) = \sin\left(\frac{n\pi x}{a}\right)$ , 0 < x < a. (10M)
- 11. A string of length L is stretched and fastened to two fixed points. Find the solution of the wave equation  $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$  when initial displacement  $y(x,0) = b \sin \frac{\pi x}{L}$  (10M)