Roll No								

National Institute of Technology Goa

Programme Name: B.Tech

Online Mid Semester Examinations, October 2020

Course Name: **Mathematics-III**
Date: 10/10/2020
Duration: 90 Minutes

Course Code: **MA200**
Time: 9:30AM - 11:00 AM
Max. Marks: 50

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1. Answer All Questions.
2. **No marks will be given if the explanation of your answer is missing**.
3. The question paper consists of **two** pages.

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1. (a) Find the roots of the polynomial $z^4 + 16$ and using it find the factors of degree two with real coefficient.

   (b) Find the value of real and imaginary parts of $(1 + \iota)^{2020}$, where $\iota = \sqrt{-1}$.

   [3+2=5]

2. Construct an analytic function $f(z) = u(x, y) + \iota v(x, y)$, where $z = x + \iota y$ given the imaginary part $v(x, y) = \sinh x \cos y$. Also obtain $u(x, y)$, the real part of the function $f(z)$. [5]

3. (a) Evaluate the line integral on the circumference of an unit circle γ with $|z| = 1$: $\oint\limits_{\gamma} |z - 1|\, |\mathrm{d}z|$.

   (b) Evaluate:

$$\oint\limits_{|z-a|=a} \frac{z}{z^4 - 1} dz \qquad a > 1.$$

   [5]

4. Evaluate:

$$\oint\limits_{|z|=7} \frac{\sin z}{(z^2 + 1)^7} dz.$$

   [5]

5. Classify the singularities of the following differential equation: [5]

   (a) $x\,(x - 1)\, y'' + (\sin x)\, y' + 2x\,(x - 1)\, y = 0$

   (b) $(x - 1)\, y'' + \cot(\pi x) y' + \operatorname{cosec}^2(\pi x) y = 0$

6. Consider the ordinary differential equation (ODE) [10]

$$xy''(x) + (2 + x)y'(x) - 2y(x) = 0$$

(a) Substitute $y = \sum_{n=0}^{\infty} A_n x^{m+n}$ into given ODE and express left hand side of the ODE as a power series with each term having the (common) factor x^n

(b) Write down the indicial equation for ODE and determine its roots

(c) Derive the recurrence formula for A_n

(d) Determine how many independent solutions this method gives. Give the first three non vanishing term of each infinite series involved.

(e) Write down the complete solution of ODE.

7. Obtain solution of the Cauchy problem $3u_x + 2u_y = 0$ with $u(x, 0) = \sin x$. [5]

8. Use separation of variable method to solve the one dimensional wave equation $u_{tt} = 4u_{xx}$ $0 < x < 1,\ t > 0$, with boundary and initial conditions as given below:

$$u_x(0, t) = u_x(1, t) = 0 \quad t > 0,$$
$$u(x, 0) = \cos^2 \pi x,\ \ u_t(x, 0) = \sin^2 \pi x \cos \pi x \quad 0 < x < 1.$$

[10]