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ECE556
Power Market Operations

Problem 1

Completion of MILP Formulation

Variables:

There are 14 variables in my generator model problem: x1, x2, x3, x4, x5, x6, x7, x8,x9,x10,x11,x12,x13,x14. They are defined as given below:

Variable	Definition
x1	G1 Accepted Quantity 20 MW Step
x2	G1 Accepted Quantity 30 MW Step
x3	G1 Accepted Quantity 15 MW Step
x4	G1 = x1 + x2 + x3 or total MW to be accepted by G1
x5	G2 Indicator Variable
x6	G2 Indicator Variable
x7	G2 Indicator Variable
x8	G2 Indicator Variable
x9	G2 Variable
x10	G2 Variable
x11	G2 Variable
x12	G2 = x5 + x6 + x7 + x8 + x9 + x10 + x11 or total MW to be accepted by G2
x13	Unit commitment status of G1
x14	Unit commitment status of G2

Objective Function:

The objective of the problem is to minimize cost of combined bids of both generators.

$$\min \sum (20\times x1) + (25\times x2) + (30\times x3) + (x5\times 200) + (x6\times 620) + (x7\times 1660) + (x8\times 2460)$$
$$+ (28\times x9) \quad + (26\times x10) \quad + (32\times x11) + (x13\times 100) + (x14\times 200)$$

The objective function is the sum of the unit price of the MW multiplied by the quantity to be accepted added to the other ranges and their respective products plus the no-load cost.

Note: In the MATLAB portion of the model, the x4 and x12 variables are set equal to 0 since it is not in the problem definition to minimize these.

Constraints

The constraints of the problem are shown below:

$$x1 + x2 + x3 = x4$$
$$x4 + x12 = 60$$
$$x = \delta_0 \times 0 + \delta_1 \times 15 + \delta_2 \times 55 + \delta_3 \times 80 + z_1 + z_2 + z$$
$$y = \delta_0 \times 200 + \delta_1 \times 620 + \delta_2 \times 1660 + \delta_3 \times 2460 + 28z_1 + 26z_2 + 32z_3$$
$$\delta_0 + \delta_1 + \delta_2 + \delta_3 = \delta$$
$$z_1 - 15\delta_0 \leq 0$$
$$z_2 - 40\delta_1 \leq 0$$
$$z_3 - 25\delta_2 \leq 0$$
$$15x13 \leq x4 \leq 65x13$$
$$10x14 \leq x12 \leq 80x14$$

Bounds

The bounds of the model are defined in the problem statement and are shown below:

Variable	Lower Bound	Upper Bound
x1	0	20
x2	0	30
x3	0	15
x4	15	65
x5	0	1
x6	0	1
x7	0	1
x8	0	1
x9	0	Inf
x10	0	Inf
x11	0	Inf
x12	10	80
x13	0	1
x14	0	1

MATLAB CODE

```
% Week 4 Assignment
% Kathleen Williams
% Set Input data

% Objective Function
f = [20 25 30 0 200 620 1660 2460 28 26 32 0 100 200]';

% Quantity
```

```
Aeq = [1 1 1 -1 0 0 0 0 0 0 0 0 0 0;
       0 0 0 0 1 1 1 1 1 1 1 -1 0 0;
       0 0 0 1 0 0 0 0 0 0 0 1 0 0];
beq = [0 0 60]';

lb = [0 0 0 0 0 0 0 0 0 0 0 0 0 0]';
ub = [20 30 15 65 1 1 1 1 inf inf inf 80 1 1]';

A = [0 0 0 0 15 0 0 0 -1 0 0 0 0 0;
     0 0 0 0 0 40 0 0 0 -1 0 0 0 0;
     0 0 0 0 0 0 25 0 0 0 -1 0 0 0;
     0 0 0 1 0 0 0 0 0 0 0 0 -15 0;
     0 0 0 0 0 0 0 0 0 0 0 1 0 -10;
     0 0 0 -1 0 0 0 0 0 0 0 0 65 0;
     0 0 0 0 0 0 0 0 0 0 0 -1 0 80];
A = -A;
 b = [0 0 0 0 0 0 0]';

xint=[0 0 0 0 1 1 1 1 0 0 0 0 1 1]'; %specifying whether a variable is integer: 1->integer, 0->continuous

% call MIP solver
[x,fval,exitflag,output,lambda]=mipprog(f,A,b,Aeq,beq,lb,ub,xint);


% Output results
%1. Optimal solution
x
%2. Objective function value
fval
%3. Shadow price for inequality constraints
lambda.ineqlin
%4. Reduced cost for lower bounds and upper bounds
lambda.lower
lambda.upper
```

MATLAB RESULTS

```
EDU>> week4prob1

x =

   20
   30
   10
   60
    0
    0
    0
    0
    0
    0
    0
    0
    1
    0


fval =

      1550
```

Optimal Solution

Accepted Quantity

The accepted quantity from each generator range is shown below and in the MATLAB results (attached):

G1 Accepted Quantities Information

Quantity (MW)	Price ($/MWH)
20	20
30	25
10	30

G2 Accepted Quantities Information

Quantity (MW)	Price ($/MWH)
0	28
0	26
0	32

Cost to Supply Load

Unit G2 is not committed, but G1 is committed with a total cost of
20X20+30X25+10X30+100= $1550

The total minimized cost to supply load is **$1550**

Problem 2

x1 + x2 – z = 0
3*x1 -2*2 – z = 0
-10 <= z <= 15

Constraint 1 x1 + x2 - z

x1	x2	Z	total
0	0	-10	10
0	0	15	-15
0	5	-10	15
0	5	15	-10
5	0	-10	15
5	0	15	-10
5	5	-10	20
5	5	15	-5
		max	20

		min	-15

Constraint 2 3*x1 – 2*x2 - z

x1	x2	z	total
0	0	-10	10
0	0	15	-15
0	5	-10	0
0	5	15	-25
5	0	-10	25
5	0	15	0
5	5	-10	15
5	5	15	-10
		max	25
		min	-25

MILP Formulation

x1 + x2 – z m = -15 M = 20
3*x1 -2*x2 – z m = -25 M = 25

max z
s.t.

$$x1 + x2 - z + (-15)\delta_1 \geq -15 + 0$$

$$x1 + x2 - z + 20\delta_1 \leq 20 + 0$$

$$3x1 - 2x2 - z + (-25)\delta_2 \geq -25 + 0$$

$$3x1 - 2x2 - z + 25\delta_2 \leq 25 + 0$$

$$\delta_1 + \delta_2 \geq 1$$

$$0 \leq x1, x2 \leq 5$$

$$-10 \leq z \leq 15$$

MATLAB CODE

```
% Week 4 Assignment
% Kathleen Williams
% Set Input data

% Objective Function
f = [0 0 -1 0 0]';

% Quantity
Aeq = [];
beq = []';

lb = [0 0 -10 0 0]';
ub = [5 5 15 1 1]';
```

```
A = [-1 -1 1 15 0;
    1 1 -1 20 0;
    -3 2 1 0 25;
    3 -2 -1 0 25;
    0 0 0 -1 -1];
b = [15 20 25 25 -1]';

xint=[0 0 0 1 1]'; %specifying whether a variable is integer: 1->integer, 0->continuous

% call MIP solver
[x,fval,exitflag,output,lambda]=mipprog(f,A,b,Aeq,beq,lb,ub,xint);


% Output results
%1. Optimal solution
x
%2. Objective function value
fval
%3. Shadow price for inequality constraints
lambda.ineqlin
%4. Reduced cost for lower bounds and upper bounds
lambda.lower
lambda.upper
```

RESULTS

```
EDU>> week4prob2

x =

     5
     0
    15
     0
     1


fval =

   -15
```

Optimal Solution

x1	5
x2	0
z	15
Delta1	0
Delta2	1

Problem 3a

Individual Iterations

<u>Iteration 1</u>

Solve MP1

Min z_{lower}
S.t. $z_{lower} \geq 6y$
y is an element of {0,1,2…10}

MATLAB CODE

```
f=[0 1]';
Aeq=[];
beq=[];
A=[6 -1];
b=[0];
lb=[0 0]';
ub=[10 inf]';
xint=[1 0]';

% Call LP solver
[x,fval,exitflag,output,lambda]=mipprog(f,A,b,Aeq,beq,lb,ub,xint);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval
```

RESULTS

```
x =

     0
     0


fval =

     0
```

$y=0$, $z_{lower}=0$

Solve SP1

Use y=0 from the master problem.

Min $20x_1 + 24x_2 + 10x_3$

S.t. $x_1 + 2x_2 + x_3 \geq 15 - 2y$
$4x_1 + 4x_2 + x_3 \geq 18 - y$
$x_1, x_2, x_3 \geq 0$

MATLAB CODE

```
f=[20 24 10]';
Aeq=[];
beq=[];
A=[-1 -2 -1;
  -4 -4 -1];
ys=0;
b=[-15+2*ys -18+ys]';
lb=[0 0 0]';
ub=[inf inf inf]';

% Call LP solver
[x,fval,exitflag,output,lambda]=linprog(f,A,b,Aeq,beq,lb,ub);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval

%3. Dual variables
lambda.ineqlin
```

RESULTS

Optimization terminated successfully.

```
x =

    0.0000
    1.5000
   12.0000


fval =

  156.0000


ans =

    8.0000
    2.0000
```

$x_1=0, x_2=1.5, x_3=12 \quad u_1=8, u_2=2$

$z_{UPPER} = 6y + 156 = 156$

$z_{UPPER} = 156 > z_{LOWER} = 0$, continue with another iteration

Feasibility Cut

$F^T u^P = 18$

$z_{lower} \geq 6y + 156 - 18(y\text{-}0) = -12y + 156$

$z_{lower} \geq -12y + 156$

Iteration 2

Solve MP2

Min z_{lower}
S.t. $z_{lower} \geq 6y$
$z_{lower} \geq -12y + 156$
y is an element of $\{0,1,2\ldots10\}$

MATLAB CODE

```
f=[0 1]';
Aeq=[];
beq=[];
A=[6 -1;
   -12 -1];
b=[0 -156];
lb=[0 0]';
ub=[10 inf]';
xint=[1 0]';

% Call LP solver
[x,fval,exitflag,output,lambda]=mipprog(f,A,b,Aeq,beq,lb,ub,xint);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval
```

RESULTS

```
x =

    9
   54


fval =

   54
```

$y = 9$, $z_{lower} = 54$

Solve SP1

Use y=9 from the master problem.

Min $20x_1 + 24x_2 + 10x_3$
S.t. $x_1+ 2x_2+x_3 \geq 15 - 2y$
$4x_1+4x_2+x_3\geq 18\text{-}y$
$x_1,x_2,x_3 \geq 0$

MATLAB CODE

```
f=[20 24 10]';
Aeq=[];
beq=[];
A=[-1 -2 -1;
  -4 -4 -1];
ys=9;
b=[-15+2*ys -18+ys]';
lb=[0 0 0]';
ub=[inf inf inf]';

% Call LP solver
[x,fval,exitflag,output,lambda]=linprog(f,A,b,Aeq,beq,lb,ub);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval

%3. Dual variables
lambda.ineqlin
```

RESULTS

```
Optimization terminated successfully.

x =

   2.2500
   0.0000
   0.0000


fval =

  45.0000


ans =

   0.0000
   5.0000
```

$x_1=2.25,x_2=0,x_3=0$ $u_1=0,u_2=5$

$z_{UPPER}=6y+45= 99$

$z_{UPPER} = 99 > z_{LOWER} = 54$, continue with another iteration

Feasibility Cut

$F^T u^P = 5$

$z_{lower} \geq 6y + 45 - 5(y-9) = y+90$

$z_{lower} \geq y+90$

Iteration 3

Solve MP3

Min z_{lower}
S.t. $z_{lower} \geq 6y$
$z_{lower} \geq -12y + 156$
$z_{lower} \geq y+90$
y is an element of {0,1,2…10}

MATLAB CODE

```
f=[0 1]';
Aeq=[];
beq=[];
A=[6 -1;
   -12 -1;
   1 -1];
b=[0 -156 -90]';
lb=[0 0]';
ub=[10 inf]';
xint=[1 0]';

% Call LP solver
[x,fval,exitflag,output,lambda]=mipprog(f,A,b,Aeq,beq,lb,ub,xint);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval
```

RESULTS

```
x =

    5
   96


fval =

   96
```

$y = 5$, $z_{lower} = 96$

Solve SP1

Use y=5 from the master problem.

Min $20x_1 + 24x_2 + 10x_3$
S.t. $x_1 + 2x_2 + x_3 \geq 15 - 2y$
$4x_1 + 4x_2 + x_3 \geq 18 - y$
$x_1, x_2, x_3 \geq 0$

MATLAB CODE

```
f=[20 24 10]';
Aeq=[];
beq=[];
A=[-1 -2 -1;
  -4 -4 -1];
ys=5;
b=[-15+2*ys -18+ys]';
lb=[0 0 0]';
ub=[inf inf inf]';

% Call LP solver
[x,fval,exitflag,output,lambda]=linprog(f,A,b,Aeq,beq,lb,ub);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval

%3. Dual variables
lambda.ineqlin
```

RESULTS

```
Optimization terminated successfully.

x =

    1.5000
    1.7500
    0.0000


fval =

   72.0000


ans =

    4.0000
    4.0000
```

$x_1=1.50, x_2=1.75, x_3=0 \quad u_1=4, u_2=4$

z_{UPPER}=6y+72= 102

z_{UPPER} = 102 > z_{LOWER} = 96, continue with another iteration

Feasibility Cut

$F^T u^P$= 12

$z_{lower} \geq 6y + 45 – 12(y-5) = -6y+132$

$z_{lower} \geq -6y+132$

Iteration 4

Solve MP4

Min z_{lower}
S.t. $z_{lower} \geq 6y$
$z_{lower} \geq -12y + 156$
$z_{lower} \geq y+90$
$z_{lower} \geq -6y+132$
y is an element of {0,1,2…10}

MATLAB CODE

```
f=[0 1]';
Aeq=[];
beq=[];
A=[6 -1;
   -12 -1;
   1 -1;
   -6 -1];
b=[0 -156 -90 -132]';
lb=[0 0]';
ub=[10 inf]';
xint=[1 0]';

% Call LP solver
[x,fval,exitflag,output,lambda]=mipprog(f,A,b,Aeq,beq,lb,ub,xint);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval
```

RESULTS

```
x =
```

```
    6
   96


fval =

   96
```

$y = 6$, $z_{lower} = 96$

Solve SP1

Use y=6 from the master problem.

Min $20x_1 + 24x_2 + 10x_3$
S.t. $x_1 + 2x_2 + x_3 \geq 15 - 2y$
$4x_1 + 4x_2 + x_3 \geq 18 - y$
$x_1, x_2, x_3 \geq 0$

MATLAB CODE

```
f=[20 24 10]';
Aeq=[];
beq=[];
A=[-1 -2 -1;
   -4 -4 -1];
ys=6;
b=[-15+2*ys -18+ys]';
lb=[0 0 0]';
ub=[inf inf inf]';

% Call LP solver
[x,fval,exitflag,output,lambda]=linprog(f,A,b,Aeq,beq,lb,ub);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval

%3. Dual variables
lambda.ineqlin
```

RESULTS

```
x =

    3.0000
    0.0000
    0.0000


fval =

   60.0000
```

ans =

```
   0.0004
   4.9999
```

$x_1=3, x_2=0, x_3=0 \quad u_1=0.0004, u_2=4.9999$

$z_{UPPER}=6y+60= 96$

$z_{UPPER} = 96 = z_{LOWER} = 96$, THIS IS FINALLY DONE

Entire Process

```
% MIP Problem 3a
% min  20x1 + 24x2 + 10x3 + 6y
% s.t. x1 + 2x2 + x3 + 2y >= 15
%      4x1 + 4x2 + x3 + y >= 18
%      0 <= x1,x2,x3, continuous
%      0 <= y <= 10, integer

% set input data
f=[20 24 10 6]';
A=[-1 -2 -1 -2;
   -4 -4 -1 -1];
b=[-15 -18]';
Aeq=[];
beq=[];
lb=[0 0 0 0]';
ub=[inf inf 10 10]';
xint=[0 0 1 1]'; %specifying whether a variable is integer: 1->integer, 0->continuous

% call MIP solver
[x,fval,exitflag,output,lambda]=mipprog(f,A,b,Aeq,beq,lb,ub,xint);

% output results
x

% objective function value
fval

EDU>> homework3a

x =

     3
     0
     0
     6
```

```
fval =

   96
```

x1 = 3, x2 = 0, y1 = 0, y2 = 6
zupper = zlower = 96

```
% MIP Problem 3a
% min  x + y
% s.t. 2x - y <= 3
%      0 <= x continuous
%      -5 <= y <= 4, integer

% set input data
f=[1 1]';
A=[2 -1];
b=[3]';
Aeq=[];
beq=[];
lb=[0 -5]';
ub=[inf 4]';
xint=[0 1]'; %specifying whether a variable is integer: 1->integer, 0->continuous

% call MIP solver
[x,fval,exitflag,output,lambda]=mipprog(f,A,b,Aeq,beq,lb,ub,xint);

% output results
x

% objective function value
fval

EDU>> homework3a

x =

     3
     0
     0
     6


fval =

   96

EDU>> homework3b

x =

     0
    -3
```

fval =

-3

x = 0, y = -3
zupper = zlower = -3

Problem 3a

Individual Iterations

$$\begin{aligned} &Min\ x + y \\ &S.t. \quad 2x - y \leq 3 \\ &\qquad\quad x \geq 0,\ y \in \{-5, -4, \ldots, 3, 4\} \end{aligned}$$

<u>Iteration 1</u>

Solve MP1

Min z_{lower}
S.t. $z_{lower} \geq y$
y is an element of {-5,-4,….3,4}

MATLAB CODE

```
f=[0 1]';
Aeq=[];
beq=[];
A=[1 -1];
b=[0];
lb=[-5 0]';
ub=[4 inf]';
xint=[1 0]';

% Call LP solver
[x,fval,exitflag,output,lambda]=mipprog(f,A,b,Aeq,beq,lb,ub,xint);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval
```

RESULTS

```
x =

   -5
    0


fval =

    0
```

y = -5, zlower = 0

Solve SP1

Use y=-5 from the master problem.

Min x
S.t. $2x \le 3+y$
$\quad x \ge 0$

MATLAB CODE

```
f=[1 0]';
Aeq=[];
beq=[];
A=[2 0];
ys=-5;
b=[3+ys]';
lb=[0 -5]';
ub=[inf 4]';

% Call LP solver
[x,fval,exitflag,output,lambda]=linprog(f,A,b,Aeq,beq,lb,ub);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval

%3. Dual variables
lambda.ineqlin
```

RESULTS

```
EDU>> week4_p3_2a

x =

   -5
    0


fval =
```

0

```
EDU>> week4_p3_2b
Exiting: One or more of the residuals, duality gap, or total relative error
        has grown 100000 times greater than its minimum value so far:
        the primal appears to be infeasible (and the dual unbounded).
        (The dual residual < TolFun=1.00e-008.)

x =

    0.0000
    0.1331


fval =

  2.6864e-010


ans =

  3.5217e+007
```

THIS IS INFEASIBLE

Solve SP2

Use y=-5 from the master problem.

Min S
S.t. $2x - S \leq 3+y$
$X,S \geq 0$

MATLAB

```
% Week 4: Problem #1 (Iteration 1, SP1)
% Set input data
f=[0 0 1]';
Aeq=[];
beq=[];
A=[2 0 -1];
ys=-5;
b=[3+ys]';
lb=[0 -5 0]';
ub=[inf 4 inf]';

% Call LP solver
[x,fval,exitflag,output,lambda]=linprog(f,A,b,Aeq,beq,lb,ub);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval

%3. Dual variables
lambda.ineqlin
```

RESULTS

```
EDU>> week4_p3_2b
Optimization terminated successfully.

x =

    0.0000
   -0.5000
    2.0000


fval =

    2.0000


ans =

    1.0000
```

$x = 0$, $S = 2$, $U = 1$

$x=0$ $s=2$, $u=1$

$2 - 1(y-(-5)) \leq 0$

$y \geq 3$

<u>Iteration 2</u>

Solve MP1

Min z_{lower}
S.t. $z_{lower} \geq y$
$y \geq -3$
y is an element of {-5,-4,….3,4}

MATLAB CODE

```
f=[0 1]';
Aeq=[];
beq=[];
A=[1 -1;
   -1 0];
b=[0 3]';
lb=[-5 -5]';
ub=[4 inf]';
xint=[1 0]';

% Call LP solver
[x,fval,exitflag,output,lambda]=mipprog(f,A,b,Aeq,beq,lb,ub,xint);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval
```

RESULTS

```
EDU>> week4_p3_2c

x =

   -3
   -3


fval =

   -3
```

$y = -3$, $z_{lower} = -3$

Solve SP1

Use y=-3 from the master problem.

Use y=-5 from the master problem.

Min x
S.t. $2x \leq 3+y$
$x \geq 0$

MATLAB CODE

```
f=[1 0]';
Aeq=[];
beq=[];
A=[2 0];
ys=-3;
b=[3+ys]';
lb=[0 -5]';
ub=[inf 4]';

% Call LP solver
[x,fval,exitflag,output,lambda]=linprog(f,A,b,Aeq,beq,lb,ub);

% Output results
%1. Optimal solution.
x

%2. Objective function value
fval

%3. Dual variables
lambda.ineqlin
```

MATLAB RESULTS

```
EDU>> week4_p3_2d
Optimization terminated successfully.

x =

    0.0000
   -0.5000
```

fval =

2.2370e-017

ans =

1.8038

ZUPPER = Y + 0 = -3

ZUPPER = ZLOWER,

THE PROBLEM IS CONVERGED