$\dfrac{d}{dx}a^x$?

We now want to learn to differentiate *any* exponential a^x. There are two roughly equivalent methods we can use:

Method 1: Convert a^x to something with base e and use the chain rule.

Because $\ln x$ is the inverse function to e^x we can rewrite a as $e^{\ln(a)}$. Thus:

$$a^x = \left(e^{\ln(a)}\right)^x = e^{x\ln(a)}$$

That looks like it might be tricky to differentiate. Let's work up to it:

$$\begin{aligned}\frac{d}{dx}e^x &= e^x \\ \frac{d}{dx}e^{3x} &= 3e^{3x} \quad \text{(by the chain rule)}\end{aligned}$$

$$\begin{aligned}\frac{d}{dx}e^{3x} &= \frac{d}{dx}e^t \quad (t=3x) \\ &= \frac{d}{dt}e^t \cdot \frac{d}{dx}t \\ &= e^t \cdot 3 \\ &= 3e^t = 3e^{3x}\end{aligned}$$

Remember, $\ln(a)$ is just a constant like 3, not a variable. Therefore:

$$\frac{d}{dx}e^{(\ln a)x} = (\ln a)e^{(\ln a)x}$$

or

$$\frac{d}{dx}a^x = (\ln a)a^x$$

This is a common type of calculation; you should practice it until you are comfortable with it. You may either memorize formulas for $\frac{d}{dx}e^{kx}$ and $\frac{d}{dx}a^x$ or re-derive them every time you need them.

Recall that $\dfrac{d}{dx}a^x = M(a)\cdot a^x$. So finally we know the value of $M(a)$:

$$M(a) = \ln(a)$$