Riemann Sums

We haven't yet finished with approximating the area under a curve using sums of areas of rectangles, but we won't use any more elaborate geometric arguments to compute those sums.

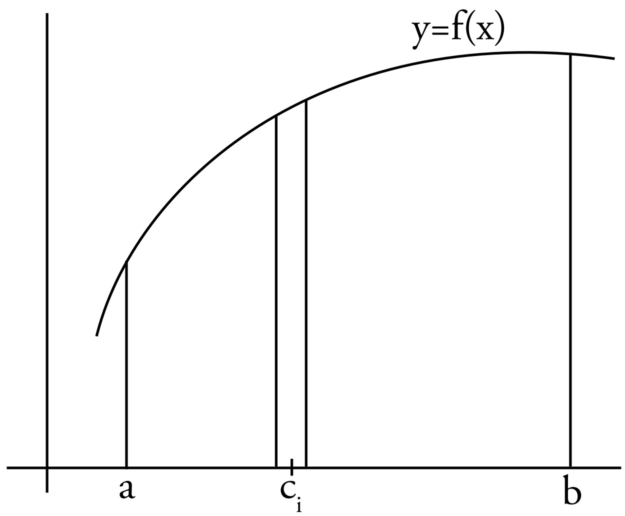

Figure 1: Area under a curve

The general procedure for computing the definite integral $\int_a^b f(x)\,dx$ is:

- Divide $[a, b]$ into n equal pieces of length $\Delta x = \dfrac{b-a}{n}$.

- Pick *any* value c_i in the i^{th} interval and use $f(c_i)$ as the height of the rectangle.

- Sum the areas of the rectangles:

$$f(c_1)\Delta x + f(c_2)\Delta x + \cdots + f(c_n)\Delta x = \sum_{i=1}^{n} f(c_i)\Delta x$$

The sum $\sum_{i=1}^{n} f(c_i)\Delta x$ is called a *Riemann Sum.*

This notation is supposed to be reminiscent of Leibnitz' notation. In the limit as n goes to infinity, this sum approaches the value of the definite integral:

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x = \int_a^b f(x)\,dx$$

Which is the area under the curve $y = f(x)$ above $[a, b]$.