Warning about units.

Previously, we calculated the volume of a parabolic "cauldron" to be $\frac{\pi}{2}a^2$. There's something fishy about this expression — it looks as if it has units of area, but it's describing a volume. In general, we must be very aware of what units we're using.

Suppose the height of the cauldron is $a = 100$cm. Then:

$$\begin{aligned} V &= \frac{\pi}{2}(100)^2\,\text{cm}^3 \\ &= \frac{\pi}{2}10^4\,\text{cm}^3 \\ &= \frac{\pi}{2}10 \sim 16\,\text{liters} \end{aligned}$$

Next, suppose that the height of the cauldron is $a = 1$m. Then:

$$\begin{aligned} V &= \frac{\pi}{2}(1)^2\,\text{m}^3 \\ &= \frac{\pi}{2}10^6\,\text{cm}^3 \\ &= \frac{\pi}{2}1000 \sim 1600\,\text{liters} \end{aligned}$$

But 100cm = 1m. Why are the answers different?

The problem is that we don't know the units in the equation $y = x^2$. If the units are centimeters, then 100cm = 10^2cm. If the units are meters then 1m = 1^2m. When we use centimeters as units, the cauldron is five times as tall as it is wide, so it looks like:

or 10000 cm = 100^2 cm

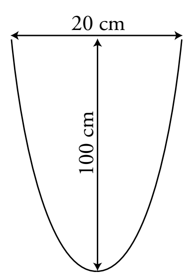

Figure 1: Cauldron cross section for units of centimeters.

When we interpret $y = x^2$ in meters, we find that the cauldron is twice as wide as it is tall, which seems more likely in the context of the problem.

This confusion about units arose because the equation $y = x^2$ is not scale-invariant.