

## Taylor's Formula

Taylor's formula describes how to get power series representations of functions.

The function  $e^x$  doesn't look like a polynomial; we have to figure out what the values of  $a_i$  have to be in order to describe  $e^x$  as a series.

*Taylor's formula* says that given any function  $f$  for which the  $n^{th}$  derivative  $f^{(n)}(x)$  exists for  $x$  near 0,

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n.$$

We'll learn how to use it soon.

Why should this work? Suppose that:

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots$$

Then:

$$f'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + \dots \quad \text{and}$$

$$f''(x) = 2a_2 + 3 \cdot 2a_3x + 4 \cdot 3a_4x^2 + \dots \quad \text{and}$$

$$f^{(3)}(x) = 3 \cdot 2a_3 + 4 \cdot 3 \cdot 2a_4x + \dots$$

Evaluating each of these at 0 we see that:  $f(0) = a_0$ ,  $f'(0) = a_1$ ,  $f''(0) = 2a_2$

and  $f^{(3)}(0) = 3 \cdot 2a_3$ . Solving for  $a_3$  we get  $a_3 = \frac{f^{(3)}(0)}{3 \cdot 2 \cdot 1}$  and in general:

$$a_n = \frac{f^{(n)}(0)}{n!},$$

where:

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 1.$$

We define  $0! = 1$  because that makes our formulas work nicely.

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