

Introduction to Related Rates

We're continuing with a related rates problem from last class.

Example: Police are 30 feet from the side of the road. Their radar sees your car approaching at 80 feet per second when your car is 50 feet away from the radar gun. The speed limit is 65 miles per hour (which translates to 95 feet per second). Are you speeding?

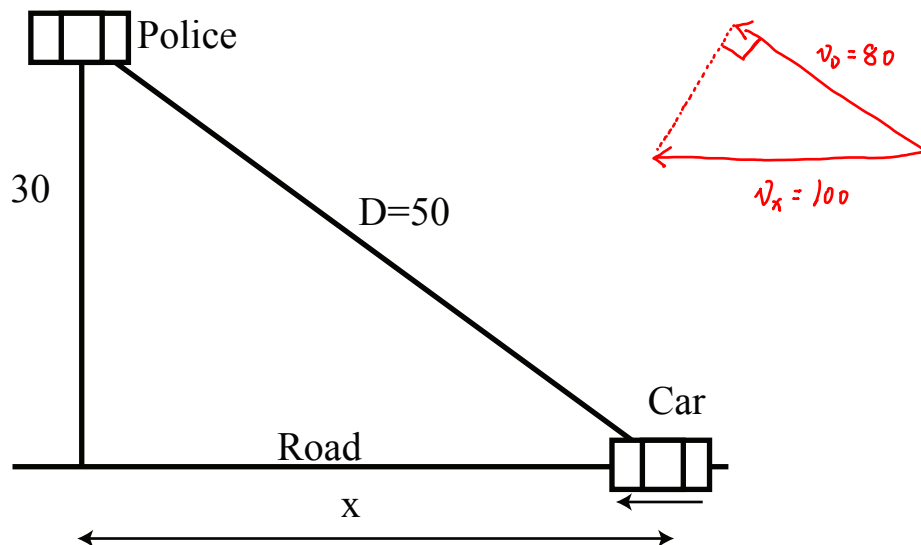


Figure 1: Illustration of example 1: triangle with the police, the car, the road, D and x labeled.

We chose t to stand for time in seconds, x to represent the distance along the road from your car to the police car, and D to represent the straight line distance between your car and the police car.

We know that $\frac{dD}{dt} = -80\text{ft/sec}$ and want to find out whether $\frac{dx}{dt} < -95\text{ft/sec} \cong 65\text{mi/hr}$.

To answer this question we need to understand how x is related to D . First, we know from the Pythagorean theorem that:

$$30^2 + x^2 = D^2$$

We'll differentiate this equation with respect to time using implicit differentiation; we could solve for x , but this would take longer.

While we do this, we must be careful not to replace a variable like D by a constant like 50. The number 50 is a constant — its rate of change is 0. The rate of change of D is -80ft/sec . We must differentiate first before plugging in values.

$$\frac{d}{dt}(30^2 + x^2) = \frac{d}{dt}(D^2) \implies 2xx' = 2DD' \implies x' = \frac{2DD'}{2x}$$

Now we can plug in the instantaneous numerical values:

$$x' = \frac{2 \cdot 50 \cdot (-80)}{2 \cdot 40} = -100 \frac{\text{feet}}{\text{s}} \cong -68 \frac{\text{mi}}{\text{hr}}$$

This exceeds the speed limit of 95 feet per second. You are, in fact, speeding.

There is another, longer, way of solving this problem. Start with:

$$D = \sqrt{30^2 + x^2} = (30^2 + x^2)^{1/2}$$

$$\frac{d}{dt}D = \frac{1}{2}(30^2 + x^2)^{-1/2}(2x \frac{dx}{dt})$$

Plug in the values:

$$-80 = \frac{1}{2}(30^2 + 40^2)^{-1/2}(2)(40) \frac{dx}{dt}$$

and solve to find:

$$\frac{dx}{dt} = -100 \frac{\text{feet}}{\text{s}}$$

A third strategy is to differentiate $x = \sqrt{D^2 - 30^2}$. It is easiest to differentiate the equation in its simplest algebraic form $30^2 + x^2 = D^2$, which was our first approach.

The general strategy for these types of problems is:

1. Draw a picture. Set up variables and equations.
2. Take derivatives.
3. Plug in the given values. Don't plug the values in until *after* taking the derivatives.

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