

## Another Example of Logarithmic Differentiation

This example could be done equally well by converting to base  $e$ , but we're going to do it using logarithmic differentiation. Recall that the rule we use for logarithmic differentiation is  $(\ln u)' = u'/u$ .

Here we have a "moving" (non-constant) exponent and a moving base.

**Example:** Let  $v = x^x$ . Find  $v'$ .

First, we take the natural log of both sides to see that  $\ln v = \ln(x^x) = x \ln x$ .

Next, we differentiate both sides of the equation, using the product rule and the rule for the derivative of  $\ln x$  on the right hand side:

$$(\ln v)' = \ln x + x \cdot \frac{1}{x}.$$

Now apply the formula  $(\ln u)' = u'/u$ . to get:

$$v'/v = 1 + \ln x$$

Plugging in  $x^x$  for  $v$  and solving for  $v'$ , we get:

$$\begin{aligned} \frac{v'}{x^x} &= 1 + \ln x \\ v' &= x^x(1 + \ln x) \\ \frac{d}{dx} x^x &= x^x(1 + \ln x) \end{aligned}$$

$$x^x = (e^{x \ln x})^x$$

$$= e^{x^2 \ln x}$$

$$\frac{d}{dx} x^x = \frac{d}{dx} e^{x^2 \ln x}$$

$$= e^{x^2 \ln x} \cdot (2x \ln x + 1)$$

$$\ln x^x = x \ln x$$

$$\frac{d}{dx} \ln x^x = \frac{d}{dx} x \ln x$$

$$\frac{1}{x^x} \cdot \frac{d}{dx} x^x = \ln x + 1$$

MIT OpenCourseWare  
<http://ocw.mit.edu>

18.01SC Single Variable Calculus  
Fall 2010

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.