

Power Series Multiplication

Once you have one power series, there are ways to get new power series from it. One thing you can do is multiply — can we use power series to multiply x by $\sin(x)$?

We have a power series for $\sin(x)$:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

Can we get one for x ? Yes!

$$x = 0 + x + 0x^2 + 0x^3 + 0x^4 + \cdots$$

We can treat power series just like polynomials and multiply them together:

$$\begin{aligned} x \cdot \sin(x) &= x \cdot \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \right) \\ &= x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \cdots \end{aligned}$$

The radius of convergence will be the smaller of the two radii of convergence; in this case $R = \infty$.

Power series multiplication is just like polynomial multiplication. It can get tedious if you're dealing with a lot of terms, but this example was pretty simple.

Notice that we just multiplied two odd functions, $\sin x$ and x , and so their product is even. That's reflected in the fact that all the terms in the power series expansion of $x \sin(x)$ have even degree. For an odd function, like $\sin(x)$, all the terms in the power series have odd degree. In general, power series of even functions will have only even degree terms and power series of odd functions will consist of all odd degree terms.

Question: Why is the radius of convergence the smaller of the two radii?

Answer: If $|x|$ is larger than the smallest radius of convergence then one of your functions isn't defined at x , so you can't expect multiplication by that function to make sense.

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