Derivative of a Power Series

We can differentiate power series. For example, $\cos(x) = \sin'(x)$ so we can find a power series for $\cos(x)$ by differentiating the power series for $\sin(x)$ term by term — the same way we differentiate polynomials.

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\begin{aligned} \cos(x) &= \sin'(x) \\ &= 1 - 3\frac{x^2}{3!} + 5\frac{x^4}{5!} - 7\frac{x^6}{7!} + \cdots \\ \cos(x) &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \end{aligned}$$

Notice how $3\frac{x^2}{3!}$ became $\frac{x^2}{2!}$ when we canceled the 3's. This happens with each term of the power series.

The radius of convergence of the derivative of a power series is the same as the radius of convergence of the power series you started with. Here $R = 1.\ \infty$

Of course, you could get this same formula using Taylor's formula and the derivatives of the cosine function.

18.01SC Single Variable Calculus
Fall 2010