The Fundamental Theorem of Calculus

The fundamental theorem of calculus is probably the most important thing in this entire course. There will be two versions of it; when we need to abbreviate we'll refer to the first as FTC1 and the second as FTC2.

Theorem: If $f(x)$ is continuous and $F'(x) = f(x)$, then:

$$\int_a^b f(x)dx = F(b) - F(a).$$

This may look familiar; when we talked about antiderivatives we wrote:

$$F(x) = \int f(x)\,dx.$$

The output of an indefinite integral is a function or family of functions; the output of a definite integral is a real number. The fundamental theorem of calculus is the connection between definite and indefinite integrals.

Notation: We need not always name the antiderivative function; we can use the following abbreviation:

$$F(b) - F(a) = F(x)|_a^b = F(x)|_{x=a}^{x=b}.$$

The later form is useful when you wish to emphasize which variable you will substitute the values for.

This allows us to rewrite the fundamental theorem of calculus as:

$$\int_a^b f(x)dx = F(x)|_a^b.$$

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