

Proofs using vectors

1. The median of a triangle is a vector from a vertex to the midpoint of the opposite side.

Show the sum of the medians of a triangle = $\mathbf{0}$.

Answer: The median of side AB is the vector from vertex C to the midpoint of AB . Label this midpoint as P . As usual we write \mathbf{P} for the origin vector \overrightarrow{OP} .

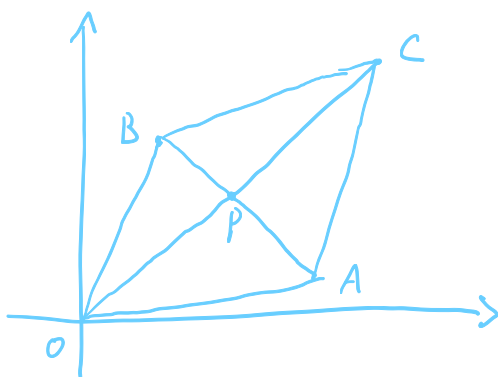
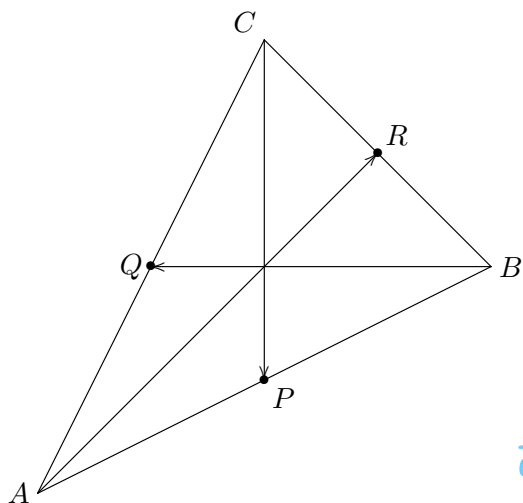
The midpoint $\mathbf{P} = \frac{1}{2}(\mathbf{A} + \mathbf{B}) \Rightarrow \overrightarrow{CP} = \frac{1}{2}(\mathbf{B} + \mathbf{A}) - \mathbf{C}$.

$$\overrightarrow{CP} = \overrightarrow{OP} - \overrightarrow{OC}$$

Likewise: $\overrightarrow{BQ} = \frac{1}{2}(\mathbf{A} + \mathbf{C}) - \mathbf{B}$ and $\overrightarrow{AR} = \frac{1}{2}(\mathbf{B} + \mathbf{C}) - \mathbf{A}$.

\Rightarrow sum of medians is

$$\overrightarrow{CP} + \overrightarrow{BQ} + \overrightarrow{AR} = \left(\frac{1}{2}(\mathbf{B} + \mathbf{A}) - \mathbf{C} \right) + \left(\frac{1}{2}(\mathbf{A} + \mathbf{C}) - \mathbf{B} \right) + \left(\frac{1}{2}(\mathbf{B} + \mathbf{C}) - \mathbf{A} \right) = \mathbf{0}.$$



$$\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}) ?$$

~~$$\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$$~~

$$\overrightarrow{OB} = \overrightarrow{OP} + \overrightarrow{PB}$$

$$\overrightarrow{OA} = \overrightarrow{OP} + \overrightarrow{PA}$$

$$\overrightarrow{OB} + \overrightarrow{OA} = 2\overrightarrow{OP} + \overrightarrow{PB} + \overrightarrow{PA}$$

$$\overrightarrow{PB} + \overrightarrow{PA} = \mathbf{0}$$

$$\overrightarrow{OB} + \overrightarrow{OA} = 2\overrightarrow{OP}$$

$$\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$$

Vector Proof

Can also use geometric proof

Diagonal of Parallelogram bisect each other

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