

Problems: Normal Form of Green's Theorem



Use geometric methods to compute the flux of \mathbf{F} across the curves C indicated below, where the function $g(r)$ is a function of the radial distance r .

1. $\mathbf{F} = g(r)\langle x, y \rangle$ and C is the circle of radius a centered at the origin and traversed in a clockwise direction.

Answer: (Radial field) \mathbf{F} is parallel to \mathbf{n} with $\langle x, y \rangle = a\mathbf{n}$ on C , so we have $\mathbf{F} \cdot \mathbf{n} = g(a) \cdot a \Rightarrow \text{Flux} = g(a)2\pi a^2$.

2. $\mathbf{F} = g(r)\langle -y, x \rangle$; C as above.

Answer: (Tangential field) Since \mathbf{F} is orthogonal to \mathbf{n} the flux is 0.

3. $\mathbf{F} = 3\langle 1, 1 \rangle$; C is the line segment from $(0, 0)$ to $(1, 1)$.

Answer: Since \mathbf{F} is parallel to the line segment C we have $\mathbf{F} \cdot \mathbf{n} = 0 \Rightarrow \text{flux} = 0$.

4. $\mathbf{F} = 3\langle -1, 1 \rangle$; C is the line segment from $(0, 0)$ to $(1, 1)$.

Answer: \mathbf{F} is orthogonal to C . \mathbf{F} points in the opposite direction from \mathbf{n} because \mathbf{n} is clockwise from the direction vector for C .

$\Rightarrow \text{flux} = \int \mathbf{F} \cdot \mathbf{n} dS = \int 3\sqrt{2} ds = 6$.

MIT OpenCourseWare
<http://ocw.mit.edu>

18.02SC Multivariable Calculus
Fall 2010

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.