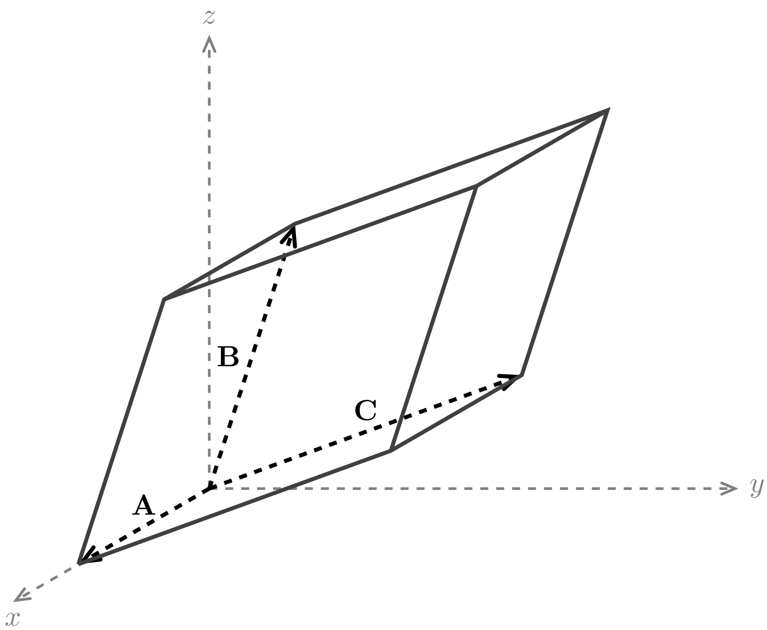
Volumes and determinants

1. a) Find the volume of the parallelepiped with edges given by the origin vectors $\langle 1, 2, 4\rangle$, $\langle 2, 0, 0\rangle$, $\langle 1, 5, 2\rangle$

Answer: The figure below shows the box.

The volume is $|\det(\mathbf{A}, \mathbf{B}, \mathbf{C})| = \left|\det\begin{pmatrix} 1 & 2 & 4 \\ 2 & 0 & 0 \\ 1 & 5 & 2 \end{pmatrix}\right| = |-2 \cdot (-16)| = 32.$



2. We know $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0.$

What does this say about the origin vectors $\langle 1, 2, 3\rangle$, $\langle 4, 5, 6\rangle$ and $\langle 7, 8, 9\rangle$?

Answer: Call the three vectors $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$. Since $\det(\mathbf{A}, \mathbf{B}, \mathbf{C}) = 0$ the volume of the parallelepiped with these vectors as edges is 0. This means all three origin vectors lie in a plane.

To see this consider the figure in problem 1. It shows the opposite case, when the vectors are not in a plane the resulting parallelepiped is really three dimensional and has non-zero volume.