Distances to planes and lines

1. Using vector methods, find the distance from the point (1,0,0) to the plane $2x + y - 2z = 0$. Include a 'cartoon' sketch illustrating your solution.

Answer: The sketch shows the plane and the point $P = (1, 0, 0)$. $Q = (0, 0, 0)$ is a point on the plane. R is the point on the plane closest to P.

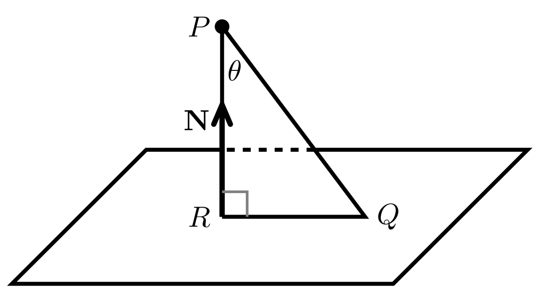
As usual, our sketches are merely suggestive and we do not actually find the point R.

The figure shows that

$$\text{distance} = |PR| = \left|\overrightarrow{\mathbf{PQ}}\right| \cos\theta = \left|\overrightarrow{\mathbf{PQ}} \cdot \frac{\mathbf{N}}{|\mathbf{N}|}\right|.$$

Computing $\overrightarrow{\mathbf{PQ}} = \langle 1, 0, 0 \rangle$ gives

$$\text{distance} = \left|\overrightarrow{\mathbf{PQ}} \cdot \frac{\mathbf{N}}{|\mathbf{N}|}\right| = \left|\langle 1, 0, 0 \rangle \cdot \frac{\langle 2, 1, -2 \rangle}{3}\right| = \frac{2}{3}.$$



2. Using vector methods, find the distance from the point (0,0) to the line $2x + y = 2$. Include a sketch.

Answer: Finding the distance from a point to a line in the plane is just like finding the distance from a point to a plane in space.

The normal to the line is $\mathbf{N} = \langle 2, 1 \rangle$ and a point on the line is $Q = (1, 0)$. We have

$$\text{distance} = \left|\overrightarrow{\mathbf{PQ}} \cdot \frac{\mathbf{N}}{|\mathbf{N}|}\right| = \left|\langle -1, 0 \rangle \cdot \frac{\langle 2, 1 \rangle}{\sqrt{5}}\right| = \frac{2}{\sqrt{5}}.$$