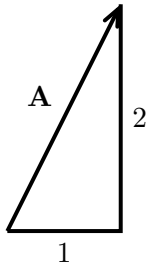
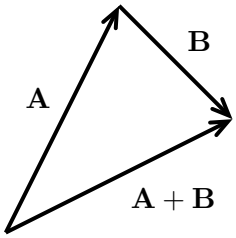
Vector lengths

1. Let $\mathbf{A} = \langle 1, 2\rangle$, $\mathbf{B} = \langle 1, -1\rangle$ and $\mathbf{C} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. Find the lengths of $\mathbf{A}$, $\mathbf{A} + \mathbf{B}$ and $\mathbf{C}$.

Answer: Length is just an expression of the Pythagorean theorem. The picture below shows $|\mathbf{A}| = \sqrt{1 + 2^2} = \sqrt{5}$



Likewise $\mathbf{A} + \mathbf{B} = \langle 2, 1\rangle \;\Rightarrow\; |\mathbf{A} + \mathbf{B}| = \sqrt{5}$. Perhaps, it's surprising that $\mathbf{A}$ and $\mathbf{A} + \mathbf{B}$ can have the same length.



$\mathbf{C}$ is a vector in space, but the length is computed the same way, except there are 3 terms under the radical sign.

$$|\mathbf{C}| = \sqrt{1 + 2^2 + 3^2} = \sqrt{14}.$$

$C = \vec{i} + 2\vec{j} + 3\vec{k}$

$= (\vec{i} + 2\vec{j}) + 3\vec{k}$

$|C| = \sqrt{\left(\sqrt{1^2 + 2^2}\right)^2 + 3^2}$

$= \sqrt{1^2 + 2^2 + 3^2}$

18.02SC Multivariable Calculus
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