Equation of a Plane

1. Find the equation of the plane containing the three points $P_1 = (1, 0, 1)$, $P_2 = (0, 1, 1)$, $P_3 = (1, 1, 0)$.

Answer: The vectors $\overrightarrow{\mathbf{P_1P_2}}$ and $\overrightarrow{\mathbf{P_1P_3}}$ are in the plane, so

$$\mathbf{N} = \overrightarrow{\mathbf{P_1P_2}} \times \overrightarrow{\mathbf{P_1P_3}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \mathbf{i}(-1) - \mathbf{j}(1) + \mathbf{k}(-1) = \langle -1, -1, -1 \rangle.$$

is a normal to the plane. In point-normal form the equation for the plane is

$$-(x-1) - y - (z-1) = 0 \;\Leftrightarrow\; x + y + z = 2.$$

2. Find the equation of the line through $(1, 2)$ and $(3, 1)$ in point-normal form.

Answer: A vector along the line is $\mathbf{v} = \langle 3, 1 \rangle - \langle 1, 2 \rangle = \langle 2, -1 \rangle$, so a normal to the line is $\mathbf{N} = \langle 1, 2 \rangle$. Thus, in point-normal form the line has equation

$$1(x-1) + 2(y-2) = 0 \;\Leftrightarrow\; x + 2y = 5.$$