Proofs using vectors

1. The median of a triangle is a vector from a vertex to the midpoint of the opposite side. Show the sum of the medians of a triangle $= \mathbf{0}$.

<u>**Answer:**</u> The median of side AB is the vector from vertex C to the midpoint of AB. Label this midpoint as P. As usual we write $\mathbf{P}$ for the origin vector $\overrightarrow{\mathbf{OP}}$.

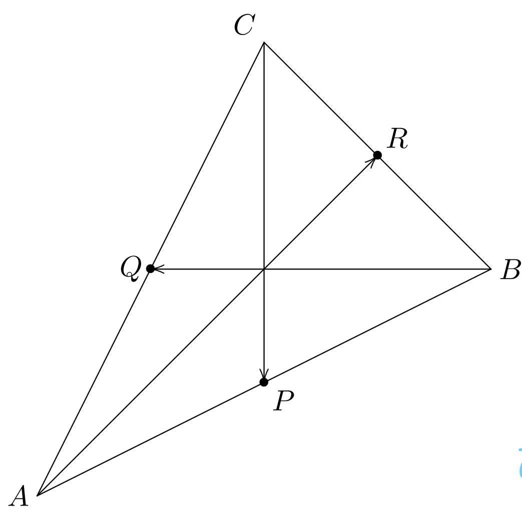
The midpoint $\mathbf{P} = \frac{1}{2}(\mathbf{A}+\mathbf{B}) \;\Rightarrow\; \overrightarrow{\mathbf{CP}} = \frac{1}{2}(\mathbf{B}+\mathbf{A}) - \mathbf{C}$.

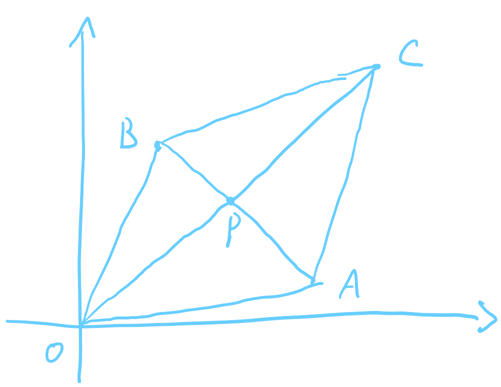
$\overrightarrow{CP} = \overrightarrow{OP} - \overrightarrow{OC}$

Likewise: $\overrightarrow{\mathbf{BQ}} = \frac{1}{2}(\mathbf{A}+\mathbf{C}) - \mathbf{B}$ and $\overrightarrow{\mathbf{AR}} = \frac{1}{2}(\mathbf{B}+\mathbf{C}) - \mathbf{A}$.

$\Rightarrow$ sum of medians is

$$\overrightarrow{\mathbf{CP}} + \overrightarrow{\mathbf{BQ}} + \overrightarrow{\mathbf{AR}} = \left(\frac{1}{2}(\mathbf{B}+\mathbf{A}) - \mathbf{C}\right) + \left(\frac{1}{2}(\mathbf{A}+\mathbf{C}) - \mathbf{B}\right) + \left(\frac{1}{2}(\mathbf{B}+\mathbf{C}) - \mathbf{A}\right) = 0.$$





$\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$?

~~$\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$~~

$\overrightarrow{OB} = \overrightarrow{OP} + \overrightarrow{PB}$

$\overrightarrow{OA} = \overrightarrow{OP} + \overrightarrow{PA}$

$\overrightarrow{OB} + \overrightarrow{OA} = 2\overrightarrow{OP} + \overrightarrow{PB} + \overrightarrow{PA}$

$\overrightarrow{PB} + \overrightarrow{PA} = 0$

$\overrightarrow{OB} + \overrightarrow{OA} = 2\overrightarrow{OP}$

$\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$

Vector Proof

Can also use geometric proof

Diagonal of Parallelogram bisect each other