

Partial derivatives

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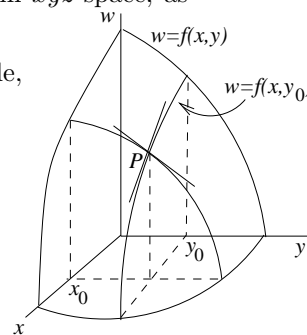
Let $w = f(x, y)$ be a function of two variables. Its graph is a surface in xyz -space, as pictured.

Fix a value $y = y_0$ and just let x vary. You get a function of *one* variable,

$$(1) \quad w = f(x, y_0), \quad \text{the **partial function** for } y = y_0.$$

Its graph is a curve in the vertical plane $y = y_0$, whose slope at the point P where $x = x_0$ is given by the derivative

$$(2) \quad \left. \frac{d}{dx} f(x, y_0) \right|_{x_0}, \quad \text{or} \quad \left. \frac{\partial f}{\partial x} \right|_{(x_0, y_0)}.$$



We call (2) the **partial derivative** of f with respect to x at the point (x_0, y_0) ; the right side of (2) is the standard notation for it. **The partial derivative is just the ordinary derivative of the partial function** — it is calculated by holding one variable fixed and differentiating with respect to the other variable. Other notations for this partial derivative are

$$f_x(x_0, y_0), \quad \left. \frac{\partial w}{\partial x} \right|_{(x_0, y_0)}, \quad \left(\frac{\partial f}{\partial x} \right)_0, \quad \left(\frac{\partial w}{\partial x} \right)_0;$$

the first is convenient for including the specific point; the second is common in science and engineering, where you are just dealing with relations between variables and don't mention the function explicitly; the third and fourth indicate the point by just using a single subscript.

Analogously, fixing $x = x_0$ and letting y vary, we get the partial function $w = f(x_0, y)$, whose graph lies in the vertical plane $x = x_0$, and whose slope at P is the *partial derivative of f with respect to y* ; the notations are

$$\left. \frac{\partial f}{\partial y} \right|_{(x_0, y_0)}, \quad f_y(x_0, y_0), \quad \left. \frac{\partial w}{\partial y} \right|_{(x_0, y_0)}, \quad \left(\frac{\partial f}{\partial y} \right)_0, \quad \left(\frac{\partial w}{\partial y} \right)_0.$$

The partial derivatives $\partial f / \partial x$ and $\partial f / \partial y$ depend on (x_0, y_0) and are therefore functions of x and y .

Written as $\partial w / \partial x$, the partial derivative gives the rate of change of w with respect to x alone, at the point (x_0, y_0) : it tells how fast w is increasing as x increases, when y is held constant.

For a function of three or more variables, $w = f(x, y, z, \dots)$, we cannot draw graphs any more, but the idea behind partial differentiation remains the same: **to define the partial derivative with respect to x , for instance, hold all the other variables constant and take the ordinary derivative with respect to x** ; the notations are the same as above:

$$\frac{d}{dx} f(x, y_0, z_0, \dots) = f_x(x_0, y_0, z_0, \dots), \quad \left(\frac{\partial f}{\partial x} \right)_0, \quad \left(\frac{\partial w}{\partial x} \right)_0.$$

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