Problems: Normal Form of Green's Theorem

Use geometric methods to compute the flux of $\mathbf{F}$ across the curves C indicated below, where the function $g(r)$ is a function of the radial distance r.

1. $\mathbf{F} = g(r)\langle x, y\rangle$ and C is the circle of radius a centered at the origin and traversed in a clockwise direction.

Answer: (Radial field) $\mathbf{F}$ is parallel to $\mathbf{n}$ with $\langle x, y\rangle = a\mathbf{n}$ on C, so we have $\mathbf{F} \cdot \mathbf{n} = g(a) \cdot a$
$\Rightarrow$ Flux $= g(a)2\pi a^2$.

2. $\mathbf{F} = g(r)\langle -y, x\rangle$; C as above.

Answer: (Tangential field) Since $\mathbf{F}$ is orthogonal to $\mathbf{n}$ the flux is 0.

3. $\mathbf{F} = 3\langle 1, 1\rangle$; C is the line segment from $(0, 0)$ to $(1, 1)$.

Answer: Since $\mathbf{F}$ is parallel to the line segment C we have $\mathbf{F} \cdot \mathbf{n} = 0$. $\Rightarrow$ flux $= 0$.

4. $\mathbf{F} = 3\langle -1, 1\rangle$; C is the line segment from $(0, 0)$ to $(1, 1)$.

Answer: $\mathbf{F}$ is orthogonal to C. $\mathbf{F}$ points in the opposite direction from $\mathbf{n}$ because $\mathbf{n}$ is clockwise from the direction vector for C.
$\Rightarrow$ flux $= \int \mathbf{F} \cdot \mathbf{n}\, dS = \boxed{\int 3\sqrt{2}\, ds} = 6$.