

## Part II Problems and Solutions

**Problem 1:** Find the general real solution of

$$\frac{d^3x}{dt^3} - \frac{dx}{dt} = t^2 + 1$$

*Indirect solving using substitution*  
*Just another view*

**Solution:** characteristic polynomial  $p(s) = s^3 - s$ ,  $p(0) = 0$  and so we can't apply undetermined coefficients directly. Let  $u = \dot{x}$ , so  $\ddot{u} - u = t^2 + 1$ . Try  $u = at^2 + bt + c$ , so  $\ddot{u} = 2a$  and  $t^2 + 1 = \ddot{u} - u = -at^2 - bt + (2a - c)$  implies  $a = -1$ ,  $b = 0$ ,  $2a - c = 1$  or  $c = -3$ : so  $u_p = -t^2 - 3$ . Then  $x_p$  is the integral of  $u_p$ :

$$x_p = -\frac{1}{3}t^3 - 3t.$$

To solve the homogeneous equation, factor  $p(s) = s(s-1)(s+1)$  so  $x_h = c_1 + c_2e^t + c_3e^{-t}$ . General solution:  $x = x_p + x_h$ .

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