Part I Problems and Solutions

Problem 1: *Find* $\mathcal{L}\left(t^4 e^{\pi t}\right)$

Solution:

$$\mathcal{L}\left(t^4\right) = \frac{24}{s^5} \rightarrow \mathcal{L}\left(t^4 e^{\pi t}\right) = \frac{24}{(s-\pi)^5}$$

Problem 2: *Find* $\mathcal{L}^{-1}\left(\frac{3}{2s-4}\right)$

Solution:

$$\frac{3}{2s-4} = \frac{3}{2}\frac{1}{s-2} \xrightarrow{\mathcal{L}^{-1}} \frac{3}{2}e^{2t}$$

Problem 3: *Find* $\mathcal{L}^{-1}\left(\frac{1}{s^2+4s+4}\right)$

Solution:

$$\frac{1}{s^2+4s+4} = \frac{1}{(s+2)^2} \xrightarrow{\mathcal{L}^{-1}} te^{-2t}$$

(since $\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) = t$)

Problem 4: *Find* $\mathcal{L}^{-1}\left(\frac{s+2}{s^2+4s+5}\right)$

Solution:

$$\frac{s+2}{s^2+4s+5} = \frac{s+2}{(s+2)^2+1} \xrightarrow{\mathcal{L}^{-1}} e^{-2t}\cos t$$

(since $\mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) = \cos t$)

Problem 5: *Find* $\mathcal{L}^{-1}\left(\frac{5s-6}{s^2-3s}\right)$

Solution:

$$\frac{5s-6}{s^2-3s} = \frac{2}{s} + \frac{3}{s-3} \xrightarrow{\mathcal{L}^{-1}} 2 + 3e^{3t}$$

Problem 6: *Find* $\mathcal{L}^{-1}\left(\frac{5-2s}{s^2+7s+10}\right)$

Solution:

$$\frac{5-2s}{s^2+7s+10} = \frac{5-2s}{(s+2)(s+5)} = \frac{3}{s+2} - \frac{5}{s+2} \xrightarrow{\mathcal{L}^{-1}} 3e^{-2t} - 5e^{-5t}$$

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