

Solutions to Polynomial Input

Quiz: Which of the following are true about the differential equation $3x^{(4)} + 2x^{(3)} + x'' - x' + 4x = 2t^2 + 1$?

Choices:

- a) It has no polynomial solutions.
- b) It has exactly one polynomial solution.
- c) It has many polynomial solutions.
- d) All its solutions are polynomials.
- e) We can't say from the information given.

Answer: The answer is b.

The method of undetermined coefficients says there will be a particular solution of the form $x_p = At^2 + Bt + C$. Therefore there is at least one polynomial solution.

The general solution is of the form $x = x_p + x_h$, where x_h is a homogeneous solution. Since 0 is not a root of the characteristic equation, every (nonzero) homogeneous solution is a combination of exponentials and/or sinusoidal functions. Therefore x is a polynomial only for the case $x_h = 0$. That is, x_p is the only polynomial solution.

By the way, $x_p = \frac{1}{2}t^2 + \frac{1}{4}t + \frac{1}{16}$.

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