

## Solutions to Polynomial Input 2

**Quiz:** Which of the following are true about the differential equation  $3x^{(4)} + 2x^{(3)} + x'' = 2t^2 + 1$ ?

$$p(s) = 3s^4 + 2s^3 + s^2 \Rightarrow p(0) = 0 \Rightarrow \text{bump all degrees up by order of lowest derivative}$$

**Choices:**

- a) It has no polynomial solutions.
- b) It has exactly one polynomial solution.
- c) It has many polynomial solutions.
- d) All its solutions are polynomials.
- e) We can't say from the information given.

**Answer:** The answer is c.

Because the smallest derivative in the differential operator is 2, the method of undetermined coefficients says we should look for a particular solution of the form  $x_p = At^4 + Bt^3 + Ct^2$ . Therefore there is at least one polynomial solution.

But, for any  $D, E$  the function  $Dt + E$  is a homogenous solution. (You can see this directly or because 0 is a double root of the characteristic equation.) Thus, there are a lots of polynomial solutions.

Since there are nonzero roots of the characteristic equation not every solution is a polynomial.

$$\text{By the way, } x_p = \frac{1}{12}t^4 - \frac{2}{3}t^3 + \frac{3}{2}t^2.$$

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