

Laplace Inverse by Table Lookup

The first thing we need to be able to do is to use the Laplace table to find the inverse Laplace transform. We will illustrate this entirely by examples.

Notation: The inverse Laplace transform will be denoted \mathcal{L}^{-1} .

Example 1. Find $\mathcal{L}^{-1}(1/(s-2))$.

Solution. Use the table entry $\mathcal{L}(e^{at}) = 1/(s-a)$:

$$\mathcal{L}^{-1}(1/(s-2)) = e^{2t}.$$

Example 2. Find $\mathcal{L}^{-1}(1/(s^2+9))$.

Solution. Use the table entry $\mathcal{L}(\sin(\omega t)) = \omega/(s^2 + \omega^2)$ and linearity:

$$\mathcal{L}^{-1}\left(\frac{1}{s^2+9}\right) = \frac{1}{3}\mathcal{L}^{-1}\left(\frac{3}{s^2+3^2}\right) = \frac{1}{3}\sin(3t).$$

Example 3. Find $\mathcal{L}^{-1}(4/s^2)$.

Solution. Use the table entry $\mathcal{L}(t) = 1/s^2$: $\mathcal{L}^{-1}(4/s^2) = 4t$.

Example 4. Find $\mathcal{L}^{-1}(4/(s-2)^2)$.

Solution. Use the s -shift formula $\mathcal{L}(e^{zt}f(t)) = F(s-z)$, where, in this case,

$$F(s) = 4/s^2 \Rightarrow f(t) = 4t \text{ by example (3).}$$

Therefore, $\mathcal{L}^{-1}(4/(s-2)^2) = \mathcal{L}^{-1}(F(s-2)) = e^{2t}f(t) = e^{2t}4t$.

Example 5. Find $\mathcal{L}^{-1}\left(\frac{1}{s^2+4s+13}\right)$.

Solution. We first need to *complete the square*

$$s^2 + 4s + 13 = s^2 + 4s + 4 + 9 = (s+2)^2 + 9.$$

We have a shifted function $F(s+2)$, where $F(s) = 1/(s^2+9)$. Using example (2), we know that $f(t) = \sin(3t)/3$, so using the s -shift rule we get

$$\mathcal{L}^{-1}\left(\frac{1}{s^2+4s+13}\right) = \mathcal{L}^{-1}(F(s+2)) = e^{-2t}\frac{\sin(3t)}{3}.$$

Example 6. Find $\mathcal{L}^{-1} \left(\frac{s}{(s^2 + \omega^2)^2} \right)$.

Solution. We haven't seen this formula yet, but there is a table entry, which gives: $\frac{t}{2\omega} \sin(\omega t)$.

Example 7. Find $\mathcal{L}^{-1} \left(\frac{1}{(s^2 + \omega^2)^2} \right)$.

Solution. This is also a table entry, answer: $\frac{1}{2\omega^3} (\sin(\omega t) - \omega t \cos(\omega t))$.

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