Fourier Coefficients: Complex with Sound

Caution: If you listen to the sound through headphones, start by setting the volume **low** and increase it *slowly*. Only let the volume get high enough to hear the sounds. This is especially important when playing pure sine waves because brain is used to auditory input of many frequencies at once. To get the same apparent loudness from a single frequency requires a large amplitude. Prolonged exposure to a high amplitude sound of a given frequency can damage your hearing at that frequency.

As usual, first open the applet and play with it.

The applet makes a Fourier series of a function out of complex exponentials instead of sines and cosines. It then graphs the real part of the function plays the periodic sound whose pressure wave corresponds to the graph.

Using the applet.

1. The fundamental (base) frequency is given ν in kilohertz (kHz). Therefore, the fundamental angular frequency is $\omega = 2000\pi\nu$. There is a slider for setting the value of ν.

2. The function $f(t)$ is given by $f(t) = \sum c_n e^{in\omega t}$, where the c_n are complex coefficients. Therefore $f(t)$ is a complex-valued function.

3. There is a parameter ϕ with a slider. Leave it at 0 for now.

4. The graph shows $\mathrm{Re}(e^{i\phi} f(t))$. The sound played corresponds to this graph.

5. To select one of coefficients c_n, click on the yellow dot for that coefficient in either of the windows on the lower right. There are two ways to adjust the value of c_n. First, you can use the mouse to drag the white dot representing the value of c_n around the complex plane displayed on the lower left. The second method is to use the sliders on the lower right. The top slider represents the magnitude $|c_n|$ and the bottom slider the polar angle $\mathrm{Arg}(c_n)$. Notice that as you move the sliders the point in the complex plane also moves.

Exploring the Applet

Now click the reset button to set all the coefficients to 0.

Set ϕ to 0.

Make sure the '$f(t)$ real' checkbox is not selected.
Set the frequency to the lower A value. (This is the musical note A440 (Hz).)
Select the first coefficient (c_1) and set the magnitude to 2. The graph should be a sinusoid.

Now start the sound, you should hear a steady pure tone. While the sound is playing adjust the $\mathrm{Arg}(c_1)$. What happened to the graph? What happened to the sound?

The graph should slide left or right as you change the phase angle. The sound shouldn't change: you ear does not detect phase.

Play with setting the higher harmonics, i.e. setting the other coefficients. How does the sound change? Does the pitch that you're hearing change?

If the higher harmonics have much lower amplitudes than the fundamental frequency, then the fundamental pitch will stay the same but the quality of the sound will change. If the amplitude of a higher harmonic approaches that of the fundamental you may begin to hear it as a separate note.

Make sure the '$f(t)$ real' checkbox is not selected and $\phi = 0$. Try to adjust the coefficients to get a square wave.
Hint: Which coefficients of the square wave are nonzero? How do you get a sine function out of complex exponentials.

As we found in an earlier session, the Fourier series of the square wave with fundamental angular frequency $\omega = 2\pi\nu$ and amplitude 1 is

$$\frac{4}{\pi}\left(\sin(\omega t) + \sin(3\omega t)/3 + \sin(5\omega t)/5 + \ldots\right).$$

Since $\sin(x)$ *is the real part of* $-ie^{ix}$ *and we selected amplitude 2, we should set*

$$c_1 = -\frac{8i}{\pi},\ c_3 = -\frac{8i}{3\pi},\ c_5 = -\frac{8i}{5\pi},\ c_7 = -\frac{8i}{7\pi},\ c_9 = -\frac{8i}{9\pi}.$$

(All the even coefficients are 0.)

Could you get the square wave using the coefficients c_{-1}, c_{-3} etc.? Could you get the square wave using real coefficients and adjusting the value of ϕ ?