

Time Invariance

In the case of constant coefficient operators $p(D)$, there is an important and useful relationship between solutions to $p(D)x = q(t)$ for input signals $q(t)$ which start at different times t . The following result shows why these operators are called “Linear Time Invariant” (or LTI).

Translation invariance. If $p(D)$ is a constant-coefficient differential operator and $p(D)x = q(t)$, then $p(D)y = q(t - c)$, where $y(t) = x(t - c)$.

This is the “time invariance” of $p(D)$. Here is an example of its use.

Example. Suppose that we know that $x_p(t) = \sqrt{2} \sin(t/2 - \pi/4)$ is a solution to the DE

$$2\ddot{x} + \dot{x} + x = \sin(t/2) \quad (1)$$

Find a solution y_p to

$$2\ddot{x} + \dot{x} + x = \sin(t/2 - \pi/3) \quad (2)$$

Solution. By translation-invariance, we have immediately that

$$y_p = \sqrt{2} \sin(t/2 - \pi/4 - \pi/3) = \sqrt{2} \sin(t/2 - 7\pi/12).$$

$p(D)x(t) = q(t)$
 $t = t - c$
 $\Rightarrow p(D)x(t - c) = q(t - c)$
Just variable substitution

$\Downarrow t/2 \rightarrow t/2 - \pi/3$

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18.03SC Differential Equations
Fall 2011

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