

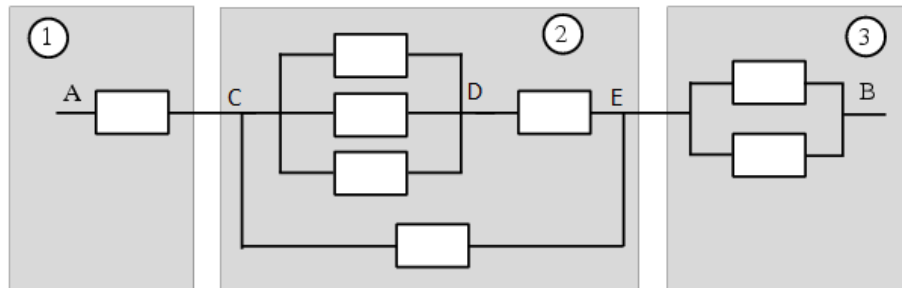
Tutorial 1 Solutions
September 16/17, 2010

1. If $A \subset B$, then $\mathbf{P}(B \cap A) = \mathbf{P}(A)$. But we know that in order for A and B to be independent, $\mathbf{P}(B \cap A) = \mathbf{P}(A)\mathbf{P}(B)$. Therefore, A and B are independent if and only if $\mathbf{P}(B) = 1$ or $\mathbf{P}(A) = 0$. This could happen, for example, if B is the universe or if A is empty.
2. This problem is similar in nature to Example 1.24, page 40. In order to compute the success probability of individual sub-systems, we make use of the following two properties, derived in that example:
 - If a *serial* sub-system contains m components with success probabilities p_1, p_2, \dots, p_m , then the probability of success of the entire sub-system is given by

$$\mathbf{P}(\text{whole system succeeds}) = p_1 p_2 p_3 \dots p_m$$

- If a *parallel* sub-system contains m components with success probabilities p_1, p_2, \dots, p_m , then the probability of success of the entire sub-system is given by

$$\mathbf{P}(\text{whole system succeeds}) = 1 - (1 - p_1)(1 - p_2)(1 - p_3) \dots (1 - p_m)$$



Let $\mathbf{P}(X \rightarrow Y)$ denote the probability of a successful connection between node X and Y . Then,

$$\mathbf{P}(A \rightarrow B) = \mathbf{P}(A \rightarrow C)\mathbf{P}(C \rightarrow E)\mathbf{P}(E \rightarrow B) \text{ (since they are in series)}$$

$$\mathbf{P}(A \rightarrow C) = p$$

$$\mathbf{P}(C \rightarrow E) = 1 - (1 - p)(1 - \mathbf{P}(C \rightarrow D)\mathbf{P}(D \rightarrow E))$$

$$\mathbf{P}(E \rightarrow B) = 1 - (1 - p)^2$$

The probabilities $\mathbf{P}(C \rightarrow D)$, $\mathbf{P}(D \rightarrow E)$ can be similarly computed as

$$\mathbf{P}(C \rightarrow D) = 1 - (1 - p)^3$$

$$\mathbf{P}(D \rightarrow E) = p$$

The probability of success of the entire system can be obtained by substituting the subsystem success probabilities:

$$\mathbf{P}(A \rightarrow B) = p(1 - (1 - p)(1 - (1 - (1 - p)^3)p)(1 - (1 - p)^2).$$

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3. The Chess Problem.

(a) i. $P(2nd \text{ Rnd Req}) = (0.6)^2 + (0.4)^2 = 0.52$

ii. $P(\text{Bo Wins 1st Rnd}) = (0.6)^2 = 0.36$

iii. $P(\text{Al Champ}) = 1 - P(\text{Bo Champ}) - P(\text{Ci Champ})$
 $= 1 - (0.6)^2 * (0.5)^2 - (0.4)^2 * (0.3)^2 = 0.8956$

(b) i. $P(\text{Bo Challenger} | 2nd \text{ Rnd Req}) = \frac{(0.6)^2}{0.52} = \frac{0.36}{0.52} = 0.6923$

ii. $P(\text{Al Champ} | 2nd \text{ Rnd Req}) = \underbrace{P(\text{Al} \cap \text{Bo} | 2nd)}_{=0} + \underbrace{P(\text{Al} \cap \text{Ci} | 2nd)}_{=0} + \underbrace{P(\text{Al} \cap \text{Ind} | 2nd)}_{=0} \quad [\text{Total Probability Theorem}]$
 $= P(\text{Al Champ} | \text{Bo Challenger, 2nd Rnd Req}) \times P(\text{Bo Challenger} | 2nd \text{ Rnd Req})$
 $+ P(\text{Al Champ} | \text{Ci Challenger, 2nd Rnd Req}) \times P(\text{Ci Challenger} | 2nd \text{ Rnd Req}) \quad [\text{Conditional Probability in sub-sample space}]$
 $= (1 - (0.5)^2) \times 0.6923 + (1 - (0.3)^2) \times 0.3077$
 $= 0.7992$

(c) $P((\text{Bo Challenger}) | \{(2nd \text{ Rnd Req}) \cap (\text{One Game})\}) = \frac{(0.6)^2 * (0.5)}{(0.6)^2 * (0.5) + (0.4)^2 * (0.7)}$

$= \frac{(0.6)^2(0.5)}{0.2920} = 0.6164$

$= \frac{P(\text{Bo} \cap 2nd \cap One)}{P(2nd \cap One)} = \frac{P(2nd \cap One | \text{Bo}) \cdot P(\text{Bo})}{P(2nd \cap One | \text{Bo}) \cdot P(\text{Bo}) + P(2nd \cap One | \text{Ci}) \cdot P(\text{Ci}) + \underbrace{P(2nd \cap One | \text{Ind}) \cdot P(\text{Ind})}_{=0}}$
 $= \frac{0.5 \cdot (0.6)^2}{0.5 \cdot (0.6)^2 + 0.7 \cdot (0.4)^2}$

$\text{Bo} \subset 2nd, \text{Ci} \subset 2nd$

$\text{Bo} \cup \text{Ci} = 2nd$

$\text{One} \subset 2nd$

$\text{Ind} = 2nd^c$

$\text{Ind} + 2nd = \text{Ind} + \text{Bo} + \text{Ci} = \Omega$

$P(\text{Bo} | 2nd \cap \text{One}) = P(\text{Bo} | \text{One})$

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