# SOLUTION MANUAL FOR PATTERN RECOGNITION AND MACHINE LEARNING

EDITED BY

ZHENGQI GAO

Information Science and Technology School Fudan University Nov.2017

# 0.1 Introduction

### **Problem 1.1 Solution**

We let the derivative of *error function E* with respect to vector  $\mathbf{w}$  equals to  $\mathbf{0}$ , (i.e.  $\frac{\partial E}{\partial \mathbf{w}} = 0$ ), and this will be the solution of  $\mathbf{w} = \{w_i\}$  which minimizes *error function E*. To solve this problem, we will calculate the derivative of E with respect to every  $w_i$ , and let them equal to 0 instead. Based on (1.1) and (1.2) we can obtain:

$$\frac{\partial E}{\partial w_{i}} = \sum_{n=1}^{N} \{y(x_{n}, \mathbf{w}) - t_{n}\} x_{n}^{i} = 0$$

$$= > \sum_{n=1}^{N} y(x_{n}, \mathbf{w}) x_{n}^{i} = \sum_{n=1}^{N} x_{n}^{i} t_{n}$$

$$= > \sum_{n=1}^{N} (\sum_{j=0}^{M} w_{j} x_{n}^{j}) x_{n}^{i} = \sum_{n=1}^{N} x_{n}^{i} t_{n}$$

$$= > \sum_{n=1}^{N} \sum_{j=0}^{M} w_{j} x_{n}^{(j+i)} = \sum_{n=1}^{N} x_{n}^{i} t_{n}$$

$$= > \sum_{j=0}^{M} \sum_{n=1}^{N} x_{n}^{(j+i)} w_{j} = \sum_{n=1}^{N} x_{n}^{i} t_{n}$$

If we denote  $A_{ij} = \sum_{n=1}^{N} x_n^{i+j}$  and  $T_i = \sum_{n=1}^{N} x_n^i t_n$ , the equation above can be written exactly as (1.222), Therefore the problem is solved.

## **Problem 1.2 Solution**

This problem is similar to Prob.1.1, and the only difference is the last term on the right side of (1.4), the penalty term. So we will do the same thing as in Prob.1.1:

$$\frac{\partial E}{\partial w_{i}} = \sum_{n=1}^{N} \{y(x_{n}, \mathbf{w}) - t_{n}\} x_{n}^{i} + \lambda w_{i} = 0$$

$$= > \sum_{j=0}^{M} \sum_{n=1}^{N} x_{n}^{(j+i)} w_{j} + \lambda w_{i} = \sum_{n=1}^{N} x_{n}^{i} t_{n}$$

$$= > \sum_{j=0}^{M} \{\sum_{n=1}^{N} x_{n}^{(j+i)} + \delta_{ji} \lambda\} w_{j} = \sum_{n=1}^{N} x_{n}^{i} t_{n}$$

where

$$\delta_{ji} \begin{cases} 0 & j \neq i \\ 1 & j = i \end{cases}$$

### **Problem 1.3 Solution**

This problem can be solved by Bayes' theorem. The probability of selecting an apple P(a):

$$P(a) = P(a|r)P(r) + P(a|b)P(b) + P(a|g)P(g) = \frac{3}{10} \times 0.2 + \frac{1}{2} \times 0.2 + \frac{3}{10} \times 0.6 = 0.34$$

Based on Bayes' theorem, the probability of an selected orange come from the green box P(g|o):

$$P(g|o) = \frac{P(o|g)P(g)}{P(o)}$$

We calculate the probability of selecting an orange P(o) first:

$$P(o) = P(o|r)P(r) + P(o|b)P(b) + P(o|g)P(g) = \frac{4}{10} \times 0.2 + \frac{1}{2} \times 0.2 + \frac{3}{10} \times 0.6 = 0.36$$

Therefore we can get:

$$P(g|o) = \frac{P(o|g)P(g)}{P(o)} = \frac{\frac{3}{10} \times 0.6}{0.36} = 0.5$$

### **Problem 1.4 Solution**