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现代控制理论

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5.1.1 问题描述

考虑如下非线性时滞系统：

$$\dot{x}_i(t) = x_{i+1}(t), i = 1,2,\cdots,n-1$$
$$\dot{x}_n(t) = f\left(t, x_1(t-\tau_1(t)), x_2(t-\tau_2(t)), \cdots, x_n(t-\tau_n(t))\right) + u(t)$$
$$y(t) = x_1(t) \tag{5.1.1}$$

其中$x(t) = [x_1(t),\cdots,x_n(t)]^T \in \mathbb{R}^n$，$u(t) \in \mathbb{R}$和$y(t) \in \mathbb{R}$分别是系统的状态、输入和输出，$\tau_i(t) \geq 0$代表未知时延，$f \in \mathbb{R}$为不确定函数。

5.1 状态时滞系统控制

□ 控制目的

在$x(t)$可得的条件下，设计无记性控制器u使得闭环系统内所有信号有界，同时被控对象的输出跟踪给定的期望轨迹$y_d(t)$。

□ 假设

- 假设1：存在未知常数$\bar{\tau}_i$使得$\dot{\tau}_i(t) \le \bar{\tau}_i < 1, i = 1, \cdots, n$。
- 假设2：存在已知光滑的$\mathcal{K}$类函数$\alpha_i(\xi)$: $[0, +\infty) \to [0, +\infty)$和未知常数$\theta_i > 0$及$g > 0$使得$\Big|f\Big(t, x_1\big(t - \tau_1(t)\big), x_2\big(t -$

5.1.2 控制器设计

定　义　$E_i(t) = x_i(t) - y_d^{(i-1)}(t)(i = 1, \cdots, n)$　，　$E(t) = [E_1(t), E_2(t), \cdots, E_n(t)]^T \in \mathbb{R}^n$。根据式(5.1.1)有

$$\dot{E}(t) = AE(t) + B\left[f + u(t) - y_d^{(n)}(t)\right] \tag{5.1.2}$$

其中

$$A = \begin{bmatrix} 0 & & \\ \vdots & I_{n-1} & \\ 0 & \dots & 0 \end{bmatrix} \in \mathbb{R}^{n\times n}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbb{R}^n \tag{5.1.3}$$

由式(5.1.3)可知，存在增益阵 $K \in \mathbb{R}^{1\times n}$ 使得 $A + BK$ 为Hurwitz矩阵。进而，存在正定对称矩阵 $P \in \mathbb{R}^{n\times n}$ 使得 $(A + BK)^T P + P(A + BK) = -I_n$。

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令

$$u(t) = KE(t) + y_d^{(n)}(t) + \bar{u}(t) \tag{5.1.4}$$

其中$\bar{u}(t) \in \mathbb{R}$待设计。

将式(5.1.4)代入(5.1.2)有

$$\dot{E}(t) = (A + BK)E(t) + B[f + \bar{u}(t)] \tag{5.1.5}$$

定义$W = E^T PE$，则有

$$\dot{W} = -E^T(t)E(t) + 2E^T(t)PBf + 2E^T(t)PB\bar{u}(t) \tag{5.1.6}$$

对于光滑$\mathcal{K}$类函数$\alpha_i(\xi)$，存在连续函数$\bar{\alpha}_i(\xi)$使得$\alpha_i(\xi) = \xi\bar{\alpha}_i(\xi), \forall \xi \geq 0$。定义

$$\rho(W) = c_1 + c_2 \sum_{i=1}^{n} \bar{\alpha}_i^2 \left(2\sqrt{\frac{W}{\lambda_{\min}(P)}}\right) \tag{5.1.7}$$

其中$c_1 > 0$和$c_2 > 0$为设计参数。

令

$$V_1(t) = \int_0^{W(t)} \rho(\xi)d\xi$$

可以验证

$$\begin{aligned}\dot{V}_1 &= \rho(W)\dot{W} \\ &= -\rho(W)E^T E + 2\rho(W)E^T PBf + 2\rho(W)E^T PB\bar{u}(t)\end{aligned} \tag{5.1.8}$$

利用假设2，有

$$2\rho(W)E^T PBf \le 2\rho(W)|E^T PB|\left[\sum_{i=1}^{n} \theta_i\alpha_i\left(\left|x_i\left(t-\tau_i(t)\right)\right|\right) + g\right] \tag{5.1.9}$$

5.1 状态时滞系统控制

令 $h_i=\sup_{t\geq 0}\left|y_d^{(i-1)}(t)\right|, i=1,\cdots,n-1$ 。 由 $\left|x_i\big(t-\tau_i(t)\big)\right|\leq \max\left\{2\left|E_i\big(t-\tau_i(t)\big)\right|,2h_i\right\}$及$\mathcal{K}$类函数的性质可知

$$\alpha_i\left(\left|x_i\big(t-\tau_i(t)\big)\right|\right)\leq \alpha_i\left(2\left|E_i\big(t-\tau_i(t)\big)\right|\right)+\alpha_i(2h_i) \tag{5.1.10}$$

此外

$$2\rho(W)|E^TPB|\sum_{i=1}^{n}\theta_i\alpha_i\left(2\left|E_i\big(t-\tau_i(t)\big)\right|\right)$$

$$\leq \sum_{i=1}^{n}\left[\frac{\theta_i^2}{\delta(1-\bar{\tau}_i)}\rho^2(W)|E^TPB|^2+\delta(1-\bar{\tau}_i)\alpha_i^2\left(2\left|E_i\big(t-\tau_i(t)\big)\right|\right)\right] \tag{5.1.11}$$

其中$\delta=\frac{c_2\lambda_{\min}(P)}{4\lambda_{\max}(P)}$。

将式(5.1.9)-(5.1.11)代入式(5.1.8)，可得

$$\dot{V}_1 \le -\rho(W)E^TE + a_1\rho^2(W)|E^TPB|^2 + \sum_{i=1}^{n}\delta(1-\bar{\tau}_i)\alpha_i^2\left(2\left|E_i\left(t-\tau_i(t)\right)\right|\right) + 2a_2\rho(W)|E^TPB| + 2\rho(W)E^TPB\bar{u}(t) \tag{5.1.12}$$

其中

$$a_1 = \sum_{i=1}^{n}\frac{\theta_i^2}{\delta(1-\bar{\tau}_i)}, a_2 = \sum_{i=1}^{n}\theta_i\alpha_i(2h_i) + g \tag{5.1.13}$$

令$\hat{a}_1$和$\hat{a}_2$分别是a_1和a_2的估计，并定义$\tilde{a}_1 = \hat{a}_1 - a_1$，$\tilde{a}_2 = \hat{a}_2 - a_2$。

定义如下Lyapunov-Krasovskii泛函：

$$V_2 = V_1 + \sum_{j=1}^{2} \frac{1}{2\gamma_j} \tilde{a}_j^2 + \delta \sum_{i=1}^{n} \int_{t-\tau_i}^{t} \alpha_i^2(2|E_i(l)|)\, dl \qquad (5.1.14)$$

其中$\gamma_1 > 0$和$\gamma_2 > 0$为设计参数。微分式(5.1.14)可得

$$\dot{V}_2 \le -\rho(W)E^T E + a_1\rho^2(W)|E^T PB|^2 + \delta \sum_{i=1}^{n} \alpha_i^2(2|E_i|)$$

$$+2a_2\rho(W)|E^T PB| + 2\rho(W)E^T PB\bar{u}(t) + \sum_{j=1}^{2} \frac{1}{\gamma_j} \tilde{a}_j \dot{\hat{a}}_j$$

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$$\dot{V}_2 \le -\rho(W)E^T E + \hat{a}_1\rho^2(W)|E^T PB|^2 + \delta\sum_{i=1}^{n}\alpha_i^2(2|E_i|)$$
$$+2\hat{a}_2\rho(W)|E^T PB| + 2\rho(W)E^T PB\bar{u}(t)$$
$$+\frac{1}{\gamma_1}\tilde{a}_1\left[\dot{\hat{a}}_1 - \gamma_1\rho^2(W)|E^T PB|^2\right]$$
$$+\frac{1}{\gamma_2}\tilde{a}_2\left[\dot{\hat{a}}_2 - 2\gamma_2\rho(W)|E^T PB|\right] \tag{5.1.15}$$

选取

$$\dot{\hat{a}}_1 = \gamma_1\rho^2(W)|E^T PB|^2 \tag{5.1.16}$$
$$\dot{\hat{a}}_2 = 2\gamma_2\rho(W)|E^T PB| \tag{5.1.17}$$
$$\bar{u}(t) = -\frac{1}{2}\hat{a}_1\rho(W)E^T PB - \frac{\hat{a}_2^2\rho(W)|E^T PB|}{|\hat{a}_2\rho(W)E^T PB| + \sigma(t)} \tag{5.1.18}$$

其中$\sigma(t) = m_1 e^{-m_2 t}$，$m_1 > 0$和$m_2 > 0$为常量。

注意

$$
\begin{aligned}
&2\rho(W)E^T PB\bar{u} \\
&= -\hat{a}_1\rho^2(W)|E^T PB|^2 - \frac{2\hat{a}_2^2\rho^2(W)|E^T PB|^2}{|\hat{a}_2\rho(W)E^T PB| + \sigma(t)} \\
&\le -\hat{a}_1\rho^2(W)|E^T PB|^2 - 2\hat{a}_2\rho(W)|E^T PB| + 2\sigma(t)
\end{aligned}
\tag{5.1.19}
$$

将式(5.1.16)、(5.1.17)和(5.1.19)代入式(5.1.15)，有

$$
\begin{aligned}
\dot{V}_2 &\le -\rho(W)E^T E + \delta\sum_{i=1}^{n}\alpha_i^2(2|E_i|) + 2\sigma(t) \\
&\le -\rho(W)E^T E + \delta\sum_{i=1}^{n}\alpha_i^2\left(2\sqrt{\frac{W}{\lambda_{\min}(P)}}\right) + 2\sigma(t)
\end{aligned}
$$

$$\leq -\rho(W)E^TE + 4\delta\frac{W}{\lambda_{\min}(P)}\sum_{i=1}^{n}\bar{\alpha}_i^2\left(2\sqrt{\frac{W}{\lambda_{\min}(P)}}\right) + 2\sigma(t)$$

$$\leq -\left[c_1 + c_2\sum_{i=1}^{n}\bar{\alpha}_i^2\left(2\sqrt{\frac{W}{\lambda_{\min}(P)}}\right)\right]\frac{W}{\lambda_{\max}(P)}$$

$$+4\delta\frac{W}{\lambda_{\min}(P)}\sum_{i=1}^{n}\bar{\alpha}_i^2\left(2\sqrt{\frac{W}{\lambda_{\min}(P)}}\right) + 2\sigma(t) \tag{5.1.20}$$

注意到$\delta = \frac{c_2\lambda_{\min}(P)}{4\lambda_{\max}(P)}$，上式变为

$$\dot{V}_2 \leq -\frac{c_1}{\lambda_{\max}(P)}W + 2\sigma(t) \tag{5.1.21}$$

由上式可知闭环系统内所有信号全局一致有界，且$\lim\limits_{t\to+\infty}[y(t) - y_d(t)] = 0$。