5-7

$$sI-A=\begin{pmatrix} s+1 & 2 & 2 \\ 0 & s+1 & -1 \\ -1 & 0 & s+1 \end{pmatrix} \qquad \det(sI-A)=s^3+3s^2+5s+5.$$

$C=[1\ 1\ 0]$

$CA=[-1\ -3\ -1]$

$CA^2=[0\ 5\ 0]$

$$P=\begin{pmatrix} 5 & 3 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} 1 & 1 & 0 \\ -1 & -3 & -1 \\ 0 & 5 & 0 \end{pmatrix}=\begin{pmatrix} 2 & 1 & -3 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix} \qquad P^{-1}=\frac{1}{-5}\begin{pmatrix} 1 & -3 & -1 \\ -1 & 3 & -4 \\ 2 & -1 & -2 \end{pmatrix}$$

期望多项式：$(s+2)(s+2)(s+3)=s^3+7s^2+16s+12.$

故 $\bar{g}=[12-5,\ 16-5,\ 7-3]^T=[7.\ 11.\ 4]^T$

$$g=P^{-1}\bar{g}=[6,-2,1]^T \qquad A-gC=\begin{pmatrix} -7 & -8 & -2 \\ 2 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix}$$

∴观测器方程为
$$\begin{cases} \dot{\hat{x}}=(A-gC)\hat{x}+Bu+gy=\begin{bmatrix} -7 & -8 & -2 \\ 2 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}\hat{x}+\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}u+\begin{bmatrix} 6 \\ -2 \\ 1 \end{bmatrix}y \\ w=\hat{x} \end{cases}$$

5-8 rank C=1，且系统可控可观，故可设计 3-1=2 维的状态观测器.

$$T=\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad T^{-1}=\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\bar{A}=TAT^{-1}=\left[\begin{array}{c|cc} -1 & -2 & -1 \\ \hline 0 & -1 & 1 \\ 1 & -1 & -1 \end{array}\right] \qquad \bar{B}=TB=\begin{bmatrix} 2 \\ \hline 0 \\ 1 \end{bmatrix} \qquad \bar{C}=CT^{-1}=[1 \mid 0\ 0]$$

$$\bar{A}_{22}-G\bar{A}_{12}=\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}-\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}[-2\ \ -1]=\begin{pmatrix} -1+2g_1 & 1+g_1 \\ -1+2g_2 & -1+g_2 \end{pmatrix}$$

$|\lambda I-(\bar{A}_{22}-G_2\bar{A}_{12})|=(\lambda+2)(\lambda+3)$ 解得 $g_1=-1,\ g_2=-1$， $F=\bar{A}_{22}-G_2\bar{A}_{12}=\begin{bmatrix} -3 & 0 \\ -3 & -2 \end{bmatrix}$

再计算 $N=\bar{B}_2-G_2\bar{B}_1=\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $G=\bar{A}_{21}-G_2\bar{A}_{11}+(\bar{A}_{22}-G_2\bar{A}_{12})G_2=\begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

$$E=\begin{bmatrix} -C_1^{-1}C_2 \\ I_{n-q} \end{bmatrix}=\begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad M=\begin{bmatrix} C_1^{-1}(I_q-C_2G_2) \\ G_2 \end{bmatrix}=\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$$

∴观测器为 $\dot{z}=Fz+Nu+Gy$， $w=Ez+My$

5-9

$y=\begin{bmatrix}0 & I\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ 设x_2为q维，可由输出直接估计.

$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u = A_{11}x_1 + A_{12}y + B_1u$

$\dot{y} = \dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2u = A_{21}x_1 + A_{22}y + B_2u$. 可写为 $\bar{y} = \dot{y} - A_{22}y - B_2u = A_{21}x_1$

$$\begin{cases}\dot{x}_1 = A_{11}x_1 + \begin{bmatrix}A_{12} & B_1\end{bmatrix}\begin{bmatrix}y\\ u\end{bmatrix}\\ \bar{y} = A_{21}x_1\end{cases}$$

故观测器可写为 $\dot{\hat{x}}_1 = (A_{11}-G_1A_{21})\hat{x}_1 + (A_{12}y + B_1u) + G_1(\dot{y} - A_{22}y - B_2u)$

令 $z = \hat{x}_1 - G_1y$，则观测器为：

$$\dot{z} = (A_{11}-G_1A_{21})z + (B_1 - G_1B_2)u + \left[(A_{12}-G_1A_{22}) + (A_{11}-G_1A_{21})G_1\right]y$$

输出 $w = \hat{x} = \begin{pmatrix}\hat{x}_1\\ \hat{x}_2\end{pmatrix} = \begin{bmatrix}z + G_1y\\ y\end{bmatrix} = \begin{pmatrix}I_{n-q}\\ 0\end{pmatrix}z + \begin{bmatrix}G_1\\ I_q\end{bmatrix}y$

5-10 $\operatorname{rank}C = 2$，故变换阵 $T = \begin{bmatrix}1&0&1&1\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$，$T^{-1} = \begin{bmatrix}1&0&-1&-1\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$

$\bar{A} = TAT^{-1} = \left[\begin{array}{cc|cc}1&0&-1&-1\\1&0&0&-1\\ \hline 0&0&0&0\\1&0&-1&-1\end{array}\right]$ $\quad \bar{B} = TB = \left[\begin{array}{cc}1&1\\0&0\\ \hline 0&1\\0&0\end{array}\right]$ $\quad \bar{C} = CT^{-1} = \left[\begin{array}{cc|cc}1&0&0&0\\0&1&0&0\end{array}\right]$

$G = \begin{bmatrix}g_1 & g_2\\ g_3 & g_4\end{bmatrix}$ 代入式(5-45)(5-46)得：

$$\dot{z} = \begin{bmatrix}g_1 & g_1+g_2\\ g_3-1 & g_3+g_4-1\end{bmatrix}z + \begin{bmatrix}-g_1 & -g_1+1\\ -g_3 & -g_4\end{bmatrix}u + \begin{bmatrix}-g_1-g_2+g_1^2+g_1g_3+g_2g_3 & g_1g_2+g_1g_4+g_2g_4\\ 1-2g_3-g_4-g_1+g_1g_3+g_3^2+g_3g_4 & -g_2+g_2g_3-g_4+g_3g_4+g_4^2\end{bmatrix}y.$$

$$w = \begin{bmatrix}-1&-1\\0&0\\1&0\\0&1\end{bmatrix}z + \begin{pmatrix}1-g_1-g_3 & -g_2-g_4\\ 0 & 1\\ g_1 & g_2\\ g_3 & g_4\end{pmatrix}y.$$

b. (1) $g_1=g_4=0$. $\quad F = \begin{bmatrix}0 & g_2\\ g_3-1 & g_3-1\end{bmatrix}$ $\quad |\lambda I - F| = \lambda^2 + (1-g_3)\lambda + g_2(1-g_3) = (\lambda+2)^2$

故 $g_3=-3$，$g_2=1$

$\therefore \dot{z} = \begin{bmatrix}0&1\\-4&-4\end{bmatrix}z + \begin{bmatrix}0&1\\3&0\end{bmatrix}u + \begin{bmatrix}-4&0\\16&-4\end{bmatrix}y$

$w = \begin{bmatrix}-1&-1\\0&0\\1&0\\0&1\end{bmatrix}z + \begin{bmatrix}4&-1\\0&1\\0&1\\-3&0\end{bmatrix}y.$

(b). $g_2=g_3=0$. $\quad F = \begin{bmatrix}g_1 & g_1\\ -1 & g_4-1\end{bmatrix}$ $\quad |\lambda I - F| = \lambda^2 + (1-g_1-g_4)\lambda + g_1g_4 = \lambda^2+4\lambda+4$

$\begin{cases}g_1+g_4 = -3\\ g_1g_4 = 4\end{cases}$ 方程无解??? 不能配置.

11. 解. 先设计状态反馈阵 $K=[k_1\ \ k_2]$

$\therefore A+BK=\begin{bmatrix}1+k_1 & 1+k_2\\ k_1 & k_2-2\end{bmatrix}$ $\therefore |\lambda I-(A+BK)|=\begin{vmatrix}\lambda-k_1-1 & -k_2-1\\ -k_1 & \lambda+2-k_2\end{vmatrix}=\lambda^2+(1-k_1-k_2)\lambda-(k_1+1)(2-k_2)-k_1k_2-k_1$

化简得 $\lambda^2+(1-k_1-k_2)\lambda-3k_1+k_2-2$

期望特征多项式为 $\lambda^2+2\lambda+2$

$\therefore \begin{cases}1-k_1-k_2=2\\ -3k_1+k_2-2=2\end{cases}$ 解得 $\begin{cases}k_1=-\frac{5}{4}\\ k_2=\frac{1}{4}\end{cases}$ $\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}-\frac{5}{4} & \frac{1}{4}\end{bmatrix}$

再设计状态观测器. 其结构为 $\dot{\hat{x}}=(A-GC)\hat{x}+Bu+Gy$.

$\therefore$ 需设计 $G=\begin{bmatrix}g_1\\g_2\end{bmatrix}$. 使 $(A-GC)$ 的特征值为 $\{-3,-4\}$.

$\therefore A-GC=\begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix}-\begin{bmatrix}g_1\\g_2\end{bmatrix}[2\ \ 1]=\begin{bmatrix}1-2g_1 & 1-g_1\\ -2g_2 & -2-g_2\end{bmatrix}$

$\therefore |\lambda I-(A-GC)|=\begin{vmatrix}\lambda+2g_1-1 & g_1-1\\ 2g_2 & \lambda+g_2+2\end{vmatrix}=\lambda^2+(2g_1+g_2+1)\lambda+(2g_1-1)(g_2+2)-2g_1g_2+2g_2$

$=\lambda^2+(1+2g_1+g_2)\lambda+4g_1+g_2-2$

期望特征多项式为 $\lambda^2+7\lambda+12$

$\therefore \begin{cases}1+2g_1+g_2=7\\ 4g_1+g_2-2=12\end{cases}$ 解得 $\begin{cases}g_1=4\\ g_2=-2\end{cases}$ $\begin{bmatrix}4\\-2\end{bmatrix}[2\ \ 1]$

$\therefore$ 观测器方程为 $\dot{\hat{x}}=\begin{bmatrix}-7 & -3\\ 4 & 0\end{bmatrix}\hat{x}+\begin{bmatrix}1\\1\end{bmatrix}u+\begin{bmatrix}4\\-2\end{bmatrix}y$ $u=K\hat{x}+v$.

状态反馈方程为 $\dot{x}=\begin{bmatrix}-\frac{1}{4} & \frac{5}{4}\\ -\frac{5}{4} & -\frac{7}{4}\end{bmatrix}x+\begin{bmatrix}1\\1\end{bmatrix}v$

$\therefore$ 联立方程得到闭环系统方程为

$$\begin{bmatrix}\dot{x}\\ \dot{\hat{x}}\end{bmatrix}=\boxed{\begin{bmatrix}1 & 1 & -\frac{5}{4} & \frac{1}{4}\\ 0 & -2 & -\frac{5}{4} & \frac{1}{4}\\ 8 & 4 & -\frac{33}{4} & -\frac{11}{4}\\ -4 & -2 & \frac{11}{4} & \frac{1}{4}\end{bmatrix}}\begin{bmatrix}x\\ \hat{x}\end{bmatrix}+\begin{bmatrix}1\\1\\1\\1\end{bmatrix}v$$

$\begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix}-\begin{bmatrix}8 & 4\\ -4 & -2\end{bmatrix}$

$\begin{bmatrix}-7 & -3\\ 4 & 0\end{bmatrix}+\begin{bmatrix}-\frac{5}{4} & \frac{1}{4}\\ -\frac{5}{4} & \frac{1}{4}\end{bmatrix}$

$$y=[2\ \ 1\ \ 0\ \ 0]\begin{bmatrix}x\\ \hat{x}\end{bmatrix}$$

基于观测器的状态反馈的输入-输出反馈形式

就是把状态反馈 $u = kx + v$

变成 $u = k\hat{x} + v$

老师说去年只有一位同学算对了，那这种类型的题目基本就是必考题了，大家一定要明白题目的意思以及正确的化简。

$$
\begin{cases}
\dot{x} = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \\
y = \begin{bmatrix} 2 & 1 \end{bmatrix} x \\
\dot{\hat{x}} = \begin{bmatrix} -7 & -3 \\ 4 & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u + \begin{bmatrix} 4 \\ -2 \end{bmatrix} y \\
u = \begin{bmatrix} -\frac{5}{4} & \frac{1}{4} \end{bmatrix} \hat{x} + v
\end{cases}
$$

消去 u 和 y.

① $\dot{x} = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} -\frac{5}{4} & \frac{1}{4} \end{bmatrix} \hat{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v$

$= \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} -\frac{5}{4} & \frac{1}{4} \\ -\frac{5}{4} & \frac{1}{4} \end{bmatrix} \hat{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v$

② $\dot{\hat{x}} = \begin{bmatrix} -7 & -3 \\ 4 & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} -\frac{5}{4} & \frac{1}{4} \end{bmatrix} \hat{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v + \begin{bmatrix} 4 \\ -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} x.$

⇒ 联立①②

$$
\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} 1 & 1 & -5/4 & 1/4 \\ 0 & -2 & -5/4 & +1/4 \\ 8 & 4 & -33/4 & -11/4 \\ -4 & -2 & 11/4 & 1/4 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} v
$$

$$
y = \begin{bmatrix} 2 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}
$$