现代控制理论

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5.2.1 问题描述

考虑如下系统：

$$y(t)=\frac{B(s)}{\bar{A}(s)}(u(t)+\mu\Delta(s)u(t-\tau)) \tag{5.2.1}$$

其中s代表$\frac{d}{dt}$，$\tau>0$代表未知常量延时，$u\in\mathbb{R}$为输入，$y\in\mathbb{R}$为输出，$\Delta(s)$为未知有理传递函数，$\bar{A}(s)=s^n+a_{n-1}s^{n-1}+\cdots+a_1s+a_0$，$B(s)=b_ms^m+\cdots+b_1s+b_0$，$a_i(i=0,\dots,n-1)$、$b_j(j=0,\dots,m)$和$\mu>0$为未知常数，$b_m\neq 0$，$\rho:=n-m$。

5.2 输入时滞系统控制

□ 控制目的

在仅有输出y可测的条件下，设计控制信号u，使得闭环系统内所有信号有界，同时被控对象输出收敛至0。

□ 假设

- 假设1：b_m的符号已知且$B(s) = b_m s^m + \cdots + b_1 s + b_0$为Hurwitz多项式;
- 假设2：Δ(s)稳定、严格正则。

5.2 输入时滞系统控制

令E_i表示$\mathbb{R}^n$中的第i 个坐标向量，系统（5.2.1）可用状态空间形式表示如下：

$$\dot{x} = Ax - ax_1 + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b \end{bmatrix} u$$

$$y = x_1 + \mu\Delta(s)x_1(t-\tau) \tag{5.2.2}$$

其中$x = [x_1, \cdots, x_n]^T \in \mathbb{R}^n$,

$$A = \begin{bmatrix} 0 & & \\ \vdots & I_{n-1} & \\ 0 & \dots & 0 \end{bmatrix} \in \mathbb{R}^{n\times n}, a = \begin{bmatrix} a_{n-1} \\ \vdots \\ \vdots \\ a_0 \end{bmatrix} \in \mathbb{R}^n, b = \begin{bmatrix} b_m \\ \vdots \\ \vdots \\ b_0 \end{bmatrix} \in \mathbb{R}^{m+1} \tag{5.2.3}$$

5.2.2 滤波器设计

构造如下一组K滤波器:

$$\begin{aligned}\dot{\eta} &= A_0\eta + E_n y \\ \dot{\lambda} &= A_0\lambda + E_n u\end{aligned} \tag{5.2.4}$$

其中$A_0 = A - KE_1^T$，$K = [k_1, \cdots, k_n]^T \in \mathbb{R}^n$由设计人员选取使得$A_0$为Hurwitz矩阵。引入信号

$$\xi_n = -A_0^n\eta, \xi_i = A_0^i\eta(i = 0, \dots, n-1)，\ v_j = A_0^j\lambda(j = 0, \dots, m) \tag{5.2.5}$$

可以验证

$$\begin{aligned}\dot{\xi}_n &= A_0\, \xi_n + Ky \\ \dot{\xi}_i &= A_0\, \xi_i + E_{n-i}y \\ \dot{v}_j &= A_0 v_j + E_{n-j}u\end{aligned} \tag{5.2.6}$$

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x的估计$\hat{x}$可表示为

$$\hat{x} = \xi_n - \sum_{i=0}^{n-1} a_i \xi_i + \sum_{j=0}^{m} b_j v_j \tag{5.2.7}$$

定义$\varepsilon = x - \hat{x}$，则有

$$\dot{\varepsilon} = A_0 \varepsilon + (a - K)\mu\delta_1 \tag{5.2.8}$$

其中$\delta_1 = \Delta(s) x_1(t-\tau)$.

定义$V_\varepsilon = \frac{1}{2}\varepsilon^T H_1 \varepsilon$，其中正定对称矩阵$H_1$满足$A_0^T H_1 + H_1 A_0 = -2I_n$。$V_\varepsilon$的导数满足

$$\dot{V}_\varepsilon = -\varepsilon^T\varepsilon + \varepsilon^T H_1 (a-K)\mu\delta_1 \le -\frac{1}{2}\varepsilon^T\varepsilon + \frac{1}{2}\|H_1(a-K)\|^2 \mu^2 \delta_1^2 \tag{5.2.9}$$

令$\xi_n,\xi_i,v_j,\varepsilon$的第$r$个元素分别记为$\xi_{n,r},\xi_{i,r},v_{j,r},\varepsilon_r$，则

$$
\begin{aligned}
\dot{y} &= \dot{x_1} + s\mu\Delta(s)x_1(t-\tau) \\
&= x_2 - a_{n-1}[y-\mu\Delta(s)x_1(t-\tau)] + s\mu\Delta(s)x_1(t-\tau) \\
&= \xi_{n,2} - \sum_{i=0}^{n-1} a_i\,\xi_{i,2} + \sum_{j=0}^{m} b_j\,v_{j,2} - a_{n-1}y + \varepsilon_2 + \delta_2
\end{aligned}
\tag{5.2.10}
$$

其中$\delta_2 = (s+a_{n-1})\mu\Delta(s)x_1(t-\tau)$.

5.2.3 控制器设计

定义$z_1 = y$，$z_i = v_{m,i} - \alpha_{i-1}, (i = 2, \dots, \rho)$ (5.2.11)

其中α_{i-1}是将在第$(i-1)$步设计的镇定函数。令

$$\alpha_\rho := u + v_{m,\rho+1}, z_{\rho+1} := 0 \tag{5.2.12}$$

由式(5.2.6)、(5.2.11)和(5.2.12)可得

$$\dot{v}_{m,i} = -k_i v_{m,1} + z_{i+1} + \alpha_i \tag{5.2.13}$$

第1步：z_1的导数可表示为

$$\dot{z}_1 = b_m z_2 + b_m \alpha_1 + \xi_{n,2} + \theta^T \omega_1 + \varepsilon_2 + \delta_2 \tag{5.2.14}$$

其中 $\theta = [-a^T, b^T]^T \in \mathbb{R}^{n+m+1}$，$\omega_1 = [\xi_{n-1,2} + y, \xi_{n-2,2}, \dots, \xi_{0,2}, 0, v_{m-1,2}, \cdots, v_{0,2}]^T \in \mathbb{R}^{n+m+1}$。令$\hat{\theta}$和$\hat{p}$分别是$\theta$和$p = \dfrac{1}{b_m}$的估计，定义

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{|b_m|}{2\gamma} \tilde{p}^2$$

其中正定对称矩阵$\Gamma \in \mathbb{R}^{(n+m+1)\times(n+m+1)}$和标量$\gamma > 0$ 为设计参数，$\tilde{\theta} = \hat{\theta} - \theta$，$\tilde{p} = \hat{p} - p$，微分$V_1$有

$$
\begin{aligned}
\dot{V}_1 &= z_1 b_m z_2 + z_1 b_m \alpha_1 + z_1 \xi_{n,2} + z_1 \theta^T \omega_1 + z_1 \varepsilon_2 \\
&\quad + z_1 \delta_2 + \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} + \frac{|b_m|}{\gamma} \tilde{p} \dot{\hat{p}} \\
&\le z_1 b_m z_2 + z_1 b_m \alpha_1 + z_1 \xi_{n,2} + z_1 \theta^T \omega_1 + d_1 z_1^2 \\
&\quad + \frac{1}{2d_1} \varepsilon^T \varepsilon + \frac{1}{2d_1} \delta_2^2 + \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} + \frac{|b_m|}{\gamma} \tilde{p} \dot{\hat{p}} \\
&\le -c_1 z_1^2 + z_1 b_m z_2 + z_1 b_m \alpha_1 - z_1 \bar{\alpha}_1 + \tilde{\theta}^T \Gamma^{-1} \left(\dot{\hat{\theta}} - \Gamma \omega_1 z_1 \right) \\
&\quad + \frac{|b_m|}{\gamma} \tilde{p} \dot{\hat{p}} + \frac{1}{2d_1} \varepsilon^T \varepsilon + \frac{1}{2d_1} \delta_2^2
\end{aligned}
\tag{5.2.15}
$$

5.2 输入时滞系统控制

其中 $\bar{\alpha}_1 = -c_1 z_1 - \hat{\theta}^T \omega_1 - \xi_{n,2} - d_1 z_1$，$c_1 > 0$和$d_1 > 0$为设计参数。令

$$\tau_1 = \Gamma \omega_1 z_1 \tag{5.2.16}$$

$$\alpha_1 = \hat{p} \bar{\alpha}_1 \tag{5.2.17}$$

$$\dot{\hat{p}} = -\mathrm{sign}(b_m) \gamma z_1 \bar{\alpha}_1 \tag{5.2.18}$$

将式(5.2.16)-(5.2.18) 代入式(5.2.15)，有

$$\dot{V}_1 \le -c_1 z_1^2 + z_1 b_m z_2 + \tilde{\theta}^T \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_1 \right) + \frac{1}{2d_1} \varepsilon^T \varepsilon + \frac{1}{2d_1} \delta_2^2 \tag{5.2.19}$$

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第2步：注意到α_1可表示为y，$\hat{\theta}$和$X_1 = [\eta^T, \lambda_1, \ldots, \lambda_{m+1}, \hat{p}]^T$的光滑函数，$z_2 = v_{m,2} - \alpha_1$的导数可表示为

$$\dot{z}_2 = z_3 + \alpha_2 + \beta_2 + \theta^T\omega_2 - b_m z_1 - \frac{\partial\alpha_1}{\partial y}\varepsilon_2 - \frac{\partial\alpha_1}{\partial y}\delta_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} \tag{5.2.20}$$

其中$\beta_2 = -k_2 v_{m,1} - \frac{\partial\alpha_1}{\partial y}\xi_{n,2} - \frac{\partial\alpha_1}{\partial X_1}\dot{X}_1$，$\omega_2 = \Big[-\frac{\partial\alpha_1}{\partial y}\big(\xi_{n-1,2} +$

定义

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{5.2.21}$$

其导数满足

$$\dot{V}_2 \le -c_1 z_1^2 + z_2\left(z_3 + \alpha_2 + \beta_2 + \hat{\theta}^T\omega_2 + d_2\left(\frac{\partial\alpha_1}{\partial y}\right)^2 z_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}\right)$$

$$+\tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} - \tau_1 - \Gamma\omega_2 z_2\right) + \sum_{j=1}^{2}\frac{1}{2d_j}\left(\varepsilon^T\varepsilon + \delta_2^2\right) \tag{5.2.22}$$

其中$d_2 > 0$为设计参数。令

$$\tau_2 = \tau_1 + \Gamma\omega_2 z_2 \tag{5.2.23}$$

$$\alpha_2 = -c_2 z_2 - \beta_2 - \hat{\theta}^T\omega_2 - d_2\left(\frac{\partial\alpha_1}{\partial y}\right)^2 z_2 + \frac{\partial\alpha_1}{\partial\hat{\theta}}\tau_2 \tag{5.2.24}$$

其中$c_2 > 0$为设计参数。然后有

$$\dot{V}_2 \leq -c_1 z_1^2 - c_2 z_2^2 + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \left(\tau_2 - \dot{\hat{\theta}} \right) + \tilde{\theta}^T \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_2 \right) + \sum_{j=1}^{2} \frac{1}{2d_j} \left(\varepsilon^T \varepsilon + \delta_2^2 \right) \tag{5.2.25}$$

第 i 步 $(3 \leq i \leq \rho)$： 注意到 α_{i-1} 是 y，$\hat{\theta}$ 和 $X_{i-1} = [\eta^T, \lambda_1, \ldots, \lambda_{m+i-1}, \hat{p}]^T$的光滑函数，$z_i = v_{m,i} - \alpha_{i-1}$的导数可表示为

$$\dot{z}_i = z_{i+1} + \alpha_i + \beta_i + \theta^T \omega_i - \frac{\partial \alpha_{i-1}}{\partial y} \varepsilon_2 - \frac{\partial \alpha_{i-1}}{\partial y} \delta_2 - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \tag{5.2.26}$$

其　中　$\beta_i=-k_i v_{m,1}-\frac{\partial\alpha_{i-1}}{\partial y}\xi_{n,2}-\frac{\partial\alpha_{i-1}}{\partial X_{i-1}}\dot{X}_{i-1}$，$\omega_i=-\frac{\partial\alpha_{i-1}}{\partial y}\left[\xi_{n-1,2}+y,\xi_{n-2,2},\ldots,\xi_{0,2},v_{m,2},\ldots,v_{0,2}\right]^T\in\mathbb{R}^{n+m+1}$。

定义

$$V_i=V_{i-1}+\frac{1}{2}z_i^2 \tag{5.2.27}$$

其中

$$\dot{V}_{i-1}\le-\sum_{k=1}^{i-1}c_k z_k^2+z_{i-1}z_i+\sum_{k=2}^{i-1}z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\left(\tau_{i-1}-\dot{\hat{\theta}}\right)+\tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}}-\tau_{i-1}\right)+\sum_{j=1}^{i-1}\frac{1}{2d_j}\left(\varepsilon^T\varepsilon+\delta_2^2\right) \tag{5.2.28}$$

利用式(5.2.26)-(5.2.28)，可以证明

$$\dot{V}_i \le -\sum_{k=1}^{i-1} c_k z_k^2 + z_i \left(z_{i-1} + \alpha_i + \beta_i + \hat{\theta}^T \omega_i \right.$$
$$\left. + d_i \left(\frac{\partial \alpha_{i-1}}{\partial y} \right)^2 z_i - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \right) + \sum_{k=1}^{i-1} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \left(\tau_{i-1} - \dot{\hat{\theta}} \right)$$
$$+ \tilde{\theta}^T \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_{i-1} - \Gamma_i \omega_i z_i \right) + \sum_{j=1}^{i} \frac{1}{2d_j} (\varepsilon^T \varepsilon + \delta_2^2) \tag{5.2.29}$$

其中$d_i > 0$为设计参数。令

$$\tau_i = \tau_{i-1} + \Gamma_i \omega_i z_i \tag{5.2.30}$$

$$\alpha_i = -c_i z_i - z_{i-1} - \beta_i - \hat{\theta}^T \omega_i - d_i \left(\frac{\partial \alpha_{i-1}}{\partial y} \right)^2 z_i + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \tau_i$$
$$+ \sum_{k=2}^{i-1} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma \omega_i \tag{5.2.31}$$

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其中$c_i > 0$为设计参数。然后有

$$\dot{V}_i \leq -\sum_{k=1}^{i} c_k z_k^2 + z_i z_{i+1} + \sum_{k=2}^{i} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \left(\tau_i - \dot{\hat{\theta}}\right) + \tilde{\theta}^T \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_i\right) + \sum_{j=1}^{i} \frac{1}{2d_j} (\varepsilon^T \varepsilon + \delta_2^2) \tag{5.2.32}$$

在得到τ_ρ和α_ρ后，令

$$\dot{\hat{\theta}} = \tau_\rho \tag{5.2.33}$$

由式（5.2.12）可知

$$u = \alpha_\rho - \nu_{m,\rho+1} \tag{5.2.34}$$

在式（5.2.32）中，令$i = \rho$并利用式（5.2.33），有

$$\dot{V}_\rho \leq -\sum_{k=1}^{\rho} c_k z_k^2 + \frac{1}{2} d_0 \varepsilon^T \varepsilon + \frac{1}{2} d_0 \delta_2^2 \tag{5.2.35}$$

其中$d_0 = \sum_{j=1}^{\rho} \frac{1}{d_j}$。

5.2.4 稳定性分析

定义$\bar{V}_\rho = V_\rho + d_0 V_\varepsilon$，由式（5.2.9）和式（5.2.35）有

$$\dot{\bar{V}}_\rho \le -\sum_{k=1}^{\rho} c_k z_k^2 + \frac{1}{2} d_0 \|H_1(a-K)\|^2 \mu^2 \delta_1^2 + \frac{1}{2} d_0 \delta_2^2 \quad (5.2.36)$$

基于假设2，令$(A_1, b_h, \bar{E}_1^T)$是$\Delta(s)$的可观标准型实现，则有

$$\dot{h} = A_1 h + b_h x_1,\ \Delta(s) x_1 = \bar{E}_1^T h = h_1 \quad (5.2.37)$$

其中h_1表示h的第一个元素。于是有

$$[\Delta(s) x_1]^2 \le \|h\|^2 \quad (5.2.38)$$

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$$[(s+a_{n-1})\Delta(s)x_1]^2 \le 2[s\Delta(s)x_1]^2 + 2a_{n-1}^2[\Delta(s)x_1]^2$$
$$\le 2\|\dot{h}\|^2 + 2a_{n-1}^2\|h\|^2$$
$$\le 4\|A_1\|^2\|h\|^2 + 4\|b_h\|^2 x_1^2 + 2a_{n-1}^2\|h\|^2$$
$$\le (4\|A_1\|^2 + 2a_{n-1}^2)\|h\|^2 + 4\|b_h\|^2 x_1^2 \quad (5.2.39)$$

于是

$$\delta_1^2 \le \|h(t-\tau)\|^2$$
$$\delta_2^2 \le \mu^2\left(4\|A_1\|^2 + 2a_{n-1}^2\right)\|h(t-\tau)\|^2 + 4\mu^2\|b_h\|^2 x_1^2(t-\tau) \quad (5.2.40)$$

另外，由式(5.2.2)可知

$$x_1^2 \le 2z_1^2 + 2\mu^2\delta_1^2 \le 2z_1^2 + 2\mu^2\|h(t-\tau)\|^2 \quad (5.2.41)$$

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定义$V_h = q_1 h^T H_2 h$，其中正定对称矩阵H_2满足$A_1^T H_2 + H_2 A_1 = -I$，$q_1 \le \frac{c_1}{8\|H_2 b_h\|^2}$。由式(5.2.36)-(5.2.41)可以证明

$$
\begin{aligned}
\dot{\vec{V}}_\rho + \dot{V}_h \le & -c_1 z_1^2(t) + \frac{1}{2} d_0 \|H_1(a-K)\|^2 \mu^2 \, \|h(t-\tau)\|^2 \\
& + d_0 \mu^2 \left[\left(2\|A_1\|^2 + a_{n-1}^2 \right) \|h(t-\tau)\|^2 + 2\|b_h\|^2 x_1^2(t-\tau) \right] \\
& - q_1 h^T h + \frac{q_1}{2} h^T h + 4 q_1 \|H_2 b_h\|^2 \left(z_1^2(t) + \mu^2 \|h(t-\tau)\|^2 \right) \\
\le & -\frac{1}{2} c_1 z_1^2(t) + \frac{1}{2} d_0 \|H_1(a-K)\|^2 \mu^2 \|h(t-\tau)\|^2 \\
& + d_0 \mu^2 \left[\left(2\|A_1\|^2 + a_{n-1}^2 \right) \|h(t-\tau)\|^2 + 2\|b_h\|^2 x_1^2(t-\tau) \right] \\
& - \frac{1}{2} q_1 h^T h + 4 q_1 \|H_2 b_h\|^2 \mu^2 \|h(t-\tau)\|^2
\end{aligned}
\tag{5.2.42}
$$

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令

$$q_2 = \frac{1}{2}d_0\|H_1(a-K)\|^2 + d_0\left[\left(2\left\|A_1\right\|^2 + a_{n-1}^2\right)\right] + 4q_1\|H_2 b_h\|^2 \tag{5.2.43}$$

$$q_3 = 2d_0\|b_h\|^2 \tag{5.2.44}$$

$$\bar{V}_h = (q_2\mu^2 + 2q_3\mu^4)\int_{t-\tau}^{t} h^T(l)h(l)dl \tag{5.2.45}$$

则

$$\dot{\bar{V}}_\rho + \dot{V}_h + \dot{\bar{V}}_h \le -\frac{1}{2}c_1 z_1^2(t) + (q_2\mu^2 + 2q_3\mu^4)h^T(t)h(t) - 2q_3\mu^4 h^T(t-\tau)h(t-\tau) - \frac{1}{2}q_1 h^T(t)h(t) + q_3\mu^2 x_1^2(t-\tau) \tag{5.2.46}$$

令

$$\bar{V}_x = q_3\mu^2 \int_{t-\tau}^{t} x_1^2(l)dl,\ \ V = \bar{V}_\rho + V_h + \bar{V}_h + \bar{V}_x \tag{5.2.47}$$

然后有

$$\begin{aligned}\dot{V} &\le -\frac{1}{2}c_1 z_1^2(t) + (q_2\mu^2 + 2q_3\mu^4)h^T(t)h(t) \\ &\quad -2q_3\mu^4 h^T(t-\tau)h(t-\tau) - \frac{1}{2}q_1 h^T(t)h(t) \\ &\quad +q_3\mu^2[2z_1^2(t) + 2\mu^2 h^T(t-\tau)h(t-\tau)] \\ &\le -\left(\frac{1}{2}c_1 - 2q_3\mu^2\right)z_1^2(t) - [\frac{1}{2}q_1 - (2q_3\mu^4 + q_2\mu^2)]h^T(t)h(t)\end{aligned} \tag{5.2.48}$$

令

$$\mu^* = \min\left\{\sqrt{\frac{c_1}{4q_3}}, \sqrt{\frac{\sqrt{q_2^2+4q_1q_3}-q_2}{4q_3}}\right\} \tag{5.2.49}$$

则当$\mu < \mu^*$时，$\frac{1}{2}c_1 - 2q_3\mu^2 > 0$，$\frac{1}{2}q_1 - (2q_3\mu^4 + q_2\mu^2) > 0$，进而

$$\dot{V} \leq -\left(\frac{1}{2}c_1 - 2q_3\mu^2\right)z_1^2(t) \tag{5.2.50}$$

定理5.2：考虑由被控对象(5.2.1)、滤波器(5.2.4)、自适应律(5.2.18)及(5.2.33)和控制律(5.2.34)组成的闭环系统。假定假设1和2成立，对于任意满足$\mu < \mu^*$的μ，闭环系统内所有信号全局一致有界，且$y(t)$收敛至0。