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现代控制理论

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2.3.1 问题描述

考虑如下一类具有输出反馈形式的非线性系统：

$$
\begin{aligned}
\dot{x}_1 &= x_2 + \sum_{j=1}^{q} a_j \varphi_{j,1}(y) \\
&\vdots \\
\dot{x}_{\rho-1} &= x_\rho + \sum_{j=1}^{q} a_j \varphi_{j,\rho-1}(y) \\
\dot{x}_\rho &= x_{\rho+1} + \sum_{j=1}^{q} a_j \varphi_{j,\rho}(y) + b_m \eta(y) u \\
&\vdots \\
\dot{x}_{n-1} &= x_n + \sum_{j=1}^{q} a_j \varphi_{j,n-1}(y) + b_1 \eta(y) u \\
\dot{x}_n &= \sum_{j=1}^{q} a_j \varphi_{j,n}(y) + b_0 \eta(y) u \\
y &= x_1
\end{aligned}
\tag{2.3.1}
$$

2.3 输出反馈自适应控制

其中$x=[x_1,\cdots,x_n]^T\in\mathbb{R}^n$为系统状态，$u\in\mathbb{R}$为输入，$y\in\mathbb{R}$为输出，$\varphi_{j,i}(y)$和$\eta(y)$为已知光滑函数，$a=\left[a_1,\cdots,a_q\right]^T\in\mathbb{R}^q$和$b=[b_m,\cdots,b_0]^T\in\mathbb{R}^{m+1}$为未知常量， $b_m\neq 0$， $\rho=n-m$。式（2.3.1）可写成如下紧凑形式

$$\dot{x}=Ax+\varphi(y)a+\begin{bmatrix}0_{\rho-1}\\ b\end{bmatrix}\eta(y)u$$

其中

$$A=\begin{bmatrix}0 & & \\ \vdots & I_{n-1} & \\ 0 & \dots & 0\end{bmatrix}\in\mathbb{R}^{n\times n}$$

$$\varphi(y)=\begin{bmatrix}\varphi_{1,1}(y) & \dots & \varphi_{q,1}(y)\\ \vdots & & \vdots\\ \varphi_{1,n}(y) & \dots & \varphi_{q,n}(y)\end{bmatrix}\in\mathbb{R}^{n\times q}$$

2.3 输出反馈自适应控制

□ 控制目的

在仅有输出y可测的条件下，设计控制信号u使得

- 闭环系统内所有信号有界；
- 被控对象输出$y(t)$跟踪给定的期望轨迹$y_d(t)$。

□ 假设

- 假设1：b_m的符号已知。
- 假设2：$B(s) \coloneqq b_m s^m + \cdots + b_1 s + b_0$为Hurwitz多项式。
- 假设3：$\eta(y) \neq 0, \forall y \in \mathbb{R}$。
- 假设4：$y_d(t)$及其前ρ阶导数已知且有界。

2.3.2 滤波器设计

为估计系统的不可测状态，构造如下一组K滤波器

$$\dot{\xi} = A_0\xi + Ky \tag{2.3.2}$$

$$\dot{\Xi} = A_0\Xi + \varphi(y) \tag{2.3.3}$$

$$\dot{\lambda} = A_0\lambda + E_n\eta(y)u \tag{2.3.4}$$

其中$A_0 = A - KE_1^T$，$K = [k_1, \ldots, k_n]^T \in \mathbb{R}^n$由设计人员选取使得$A_0$为Hurwitz矩阵，$E_i$表示$\mathbb{R}^n$中的第$i$个坐标向量。引入信号$v_j = A_0^j\lambda(j = 0, \ldots, m)$，其导数满足

$$\dot{v}_j = A_0v_j + E_{n-j}\eta(y)u \tag{2.3.5}$$

x的估计$\hat{x}$可表示为

$$\hat{x} = \xi + \Xi a + \sum_{j=0}^{m} b_jv_j \tag{2.3.6}$$

定义$\varepsilon = x - \hat{x}$，则有

2.3 输出反馈自适应控制

$$\begin{aligned}\dot{\varepsilon} &= Ax + \varphi(y)a + \begin{bmatrix} 0_{\rho-1} \\ b \end{bmatrix}\eta(y)u - A_0\xi - Ky - A_0\Xi a - \varphi(y)a \\ &\quad - \textstyle\sum_{j=0}^{m} b_j A_0 v_j - \sum_{j=0}^{m} b_j E_{n-j}\eta(y)u \\ &= (Ax - Ky) - A_0\left(\xi + \Xi a + \textstyle\sum_{j=0}^{m} b_j v_j\right) \\ &= A_0 x - A_0\hat{x} \\ &= A_0\varepsilon \end{aligned} \tag{2.3.7}$$

令ξ，λ， v_j和ε的第i个元素分别记为ξ_i， λ_i， $v_{j,i}$和ε_i，$\varphi(y)$和Ξ的第i行分别记为$\varphi_i(y)$和Ξ_i。由式(2.3.1)、(2.3.6)和(2.3.7)可知

$$\begin{aligned}\dot{y} &= x_2 + \textstyle\sum_{j=1}^{q} a_j \varphi_{j,1}(y) \\ &= \xi_2 + (\varphi_1(y) + \Xi_2)a + \textstyle\sum_{j=0}^{m} b_j v_{j,2} + \varepsilon_2 \end{aligned} \tag{2.3.8}$$

2.3.3 控制器设计

第1步：根据式(2.3.8)，跟踪误差$z_1 = y - y_d$的导数可表示为

$$\dot{z}_1 = b_m v_{m,2} + \xi_2 + \theta^T \omega_1 + \varepsilon_2 - \dot{y}_d \tag{2.3.9}$$

其中$\theta = [a^T, b^T]^T \in \mathbb{R}^{q+m+1}$， $\omega_1 = [\varphi_1(y) + \Xi_2, 0, v_{m-1,2}, \cdots, v_{0,2}]^T \in \mathbb{R}^{q+m+1}$。定义第1个准Lyapunov函数：

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{|b_m|}{2\gamma} \tilde{p}^2 + \frac{1}{4d_1} \varepsilon^T H \varepsilon \tag{2.3.10}$$

其中正定对称矩阵$\Gamma \in \mathbb{R}^{(q+m+1)\times(q+m+1)}$，$\gamma > 0$和$d_1 > 0$为设计参数，$\tilde{\theta} := \hat{\theta} - \theta$， $\tilde{p} := \hat{p} - p$，$\hat{\theta}$和$\hat{p}$分别为θ和$p = \frac{1}{b_m}$的估计，正定对称矩阵H是Lyapunov方程$A_0^T H + H A_0 = -I_n$的解。微分V_1有

2.3 输出反馈自适应控制

$$\dot{V}_1 = z_1\left(b_m v_{m,2} + \xi_2 + \theta^T \omega_1 + \varepsilon_2 - \dot{y}_d\right) + \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} + \frac{|b_m|}{\gamma} \tilde{p} \dot{\hat{p}} - \frac{1}{4d_1} \varepsilon^T \varepsilon \tag{2.3.11}$$

定义

$$z_2 = v_{m,2} - \alpha_1 \tag{2.3.12}$$

其中α_1为第1个待设计的镇定函数。由Young不等式可得

$$z_1 \varepsilon_2 \leq d_1 z_1^2 + \frac{1}{4d_1} \varepsilon_2^2 \tag{2.3.13}$$

将式(2.3.13)和(2.3.12)代入(2.3.11)，可证

2.3 输出反馈自适应控制

$$\begin{aligned}\dot{V}_1 &\le z_1\left(b_m z_2 + b_m\alpha_1 + \xi_2 + \hat{\theta}^T\omega_1 + d_1 z_1 - \dot{y}_d\right)\\ &\quad +\tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} - \Gamma\omega_1 z_1\right) + \frac{|b_m|}{\gamma}\tilde{p}\dot{\hat{p}}\\ &\le -c_1 z_1^2 + z_1 b_m z_2 + z_1 b_m \alpha_1 - z_1\bar{\alpha}_1\\ &\quad +\tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} - \Gamma\omega_1 z_1\right) + \frac{|b_m|}{\gamma}\tilde{p}\dot{\hat{p}}\end{aligned} \tag{2.3.14}$$

其中$c_1 > 0$为设计参数，$\bar{\alpha}_1 = -c_1 z_1 - \hat{\theta}^T\omega_1 - \xi_2 - d_1 z_1 + \dot{y}_d$。针对$\hat{\theta}$，定义第1个调节函数

$$\tau_1 = \Gamma\omega_1 z_1 \tag{2.3.15}$$

选取

$$\alpha_1 = \hat{p}\bar{\alpha}_1 \tag{2.3.16}$$

其中$\hat{p}$对应的自适应律为

$$\dot{\hat{p}} = -\mathrm{sign}(b_m)\gamma z_1\bar{\alpha}_1 \tag{2.3.17}$$

2.3 输出反馈自适应控制

将式(2.3.15)-(2.3.17)及关系式$b_m\hat{p}-b_m\tilde{p}=b_mp=1$代入式(2.3.14)，得

$$\dot{V}_1 \le -c_1z_1^2+z_1b_mz_2+\tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}}-\tau_1\right) \tag{2.3.18}$$

第2步：由关系式$v_j=A_0^j\lambda$及A_0的结构可以验证：

$$v_{j,i}=\ [*,\dots,*,1]\begin{bmatrix}\lambda_1\\ \vdots\\ \lambda_{j+i}\end{bmatrix} \tag{2.3.19}$$

从式(2.3.16)和(2.3.19)出发，可知α_1可表示为y，$\hat{\theta}$和$X_1=[y_d,\dot{y}_d,\xi,\Xi_1,\dots,\Xi_n,\lambda_1,\dots,\lambda_{m+1},\hat{p}]^T$的光滑函数。于是$z_2=v_{m,2}-\alpha_1$的导数可表示为

$$\begin{aligned}\dot{z}_2 &= -k_2v_{m,1}+v_{m,3}\\ &\quad -\frac{\partial\alpha_1}{\partial y}\xi_2+\theta^T\omega_2-b_mz_1-\frac{\partial\alpha_1}{\partial y}\varepsilon_2-\frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}-\frac{\partial\alpha_1}{\partial X_1}\dot{X}_1\end{aligned}$$

2.3 输出反馈自适应控制

$$= v_{m,3} + \beta_2 + \theta^T\omega_2 - b_m z_1 - \frac{\partial\alpha_1}{\partial y}\varepsilon_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} \tag{2.3.20}$$

其中

$\omega_2 = \left[-\frac{\partial\alpha_1}{\partial y}(\varphi_1(y) + \Xi_2), z_1 - \frac{\partial\alpha_1}{\partial y}v_{m,2}, -\frac{\partial\alpha_1}{\partial y}v_{m-1,2}, \dots, -\frac{\partial\alpha_1}{\partial y}v_{0,2}\right]^T \in \mathbb{R}^{q+m+1}$，$\beta_2 = -k_2 v_{m,1} - \frac{\partial\alpha_1}{\partial y}\xi_2 - \frac{\partial\alpha_1}{\partial X_1}\dot{X}_1$。

定义第2个准Lyapunov函数

$$V_2 = V_1 + \tfrac{1}{2}z_2^2 + \frac{1}{4d_2}\varepsilon^T H\varepsilon \tag{2.3.21}$$

其导数满足

$$\dot{V}_2 \le -c_1 z_1^2 + z_2\left(v_{m,3} + \beta_2 + \theta^T\omega_2 - \frac{\partial\alpha_1}{\partial y}\varepsilon_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}\right)$$
$$+\tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} - \tau_1\right) - \frac{1}{4d_2}\varepsilon^T\varepsilon \tag{2.3.22}$$

2.3 输出反馈自适应控制

定义

$$z_3 = v_{m,3} - \alpha_2 \tag{2.3.23}$$

其中α_2为第2个待设计的镇定函数。可以证明

$$-z_2 \frac{\partial \alpha_1}{\partial y} \varepsilon_2 \le d_2 \left(\frac{\partial \alpha_1}{\partial y}\right)^2 z_2^2 + \frac{1}{4d_2} \varepsilon_2^2 \tag{2.3.24}$$

由式(2.3.22)-(2.3.24) 可得

$$\dot{V}_2 \le -c_1 z_1^2 + z_2 \left(z_3 + \alpha_2 + \beta_2 + \hat{\theta}^T \omega_2 + d_2 \left(\frac{\partial \alpha_1}{\partial y}\right)^2 z_2 - \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}}\right)$$
$$+\tilde{\theta}^T \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_1 - \Gamma \omega_2 z_2\right) \tag{2.3.25}$$

令

$$\tau_2 = \tau_1 + \Gamma \omega_2 z_2 \tag{2.3.26}$$

$$\alpha_2 = -c_2 z_2 - \beta_2 - \hat{\theta}^T \omega_2 - d_2 \left(\frac{\partial \alpha_1}{\partial y}\right)^2 z_2 + \frac{\partial \alpha_1}{\partial \hat{\theta}} \tau_2 \tag{2.3.27}$$

2.3 输出反馈自适应控制

于是有

$$\dot{V}_2 \le -c_1 z_1^2 - c_2 z_2^2 + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \left(\tau_2 - \dot{\hat{\theta}} \right) + \tilde{\theta}^T \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_2 \right) \quad (2.3.28)$$

第 i 步 $(3 \le i \le \rho - 1)$： 注 意 到 α_{i-1} 是 y， $\hat{\theta}$ 和 $X_{i-1} = \left[y_d, \dot{y}_d, \ldots, y_d^{(i-1)}, \xi, \Xi_1, \ldots, \Xi_n, \lambda_1, \ldots, \lambda_{m+i-1}, \hat{p} \right]^T$ 的光滑函数，$z_i = v_{m,i} - \alpha_{i-1}$的导数可表示为

$$\dot{z}_i = v_{m,i+1} + \beta_i + \theta^T \omega_i - \frac{\partial \alpha_{i-1}}{\partial y} \varepsilon_2 - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \quad (2.3.29)$$

其中 $\beta_i = -k_i v_{m,1} - \frac{\partial \alpha_{i-1}}{\partial y} \xi_2 - \frac{\partial \alpha_{i-1}}{\partial X_{i-1}} \dot{X}_{i-1}, \omega_i = -\frac{\partial \alpha_{i-1}}{\partial y} [\varphi_1(y) +$

(2.3.30)

2.3 输出反馈自适应控制

其中V_{i-1}的导数满足

$$\dot{V}_{i-1} \le -\sum_{j=1}^{i-1} c_j z_j^2 + z_{i-1} z_i + \sum_{j=2}^{i-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \left(\tau_{i-1} - \dot{\hat{\theta}} \right) + \tilde{\theta}^T \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_{i-1} \right) \tag{2.3.31}$$

由式(2.3.29)-(2.3.31)可证

$$\dot{V}_i \le -\sum_{j=1}^{i-1} c_j z_j^2 + z_i \left(z_{i-1} + v_{m,i+1} + \beta_i + \theta^T \omega_i - \frac{\partial \alpha_{i-1}}{\partial y} \varepsilon_2 - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \right) + \sum_{j=2}^{i-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \left(\tau_{i-1} - \dot{\hat{\theta}} \right) + \tilde{\theta}^T \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_{i-1} \right) - \frac{1}{4 d_i} \varepsilon^T \varepsilon。 \tag{2.3.32}$$

2.3 输出反馈自适应控制

利用Young不等式可得

$$-z_i\frac{\partial\alpha_{i-1}}{\partial y}\varepsilon_2\le d_i\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i^2+\frac{1}{4d_i}\varepsilon_2^2 \tag{2.3.33}$$

定义

$$z_{i+1}=v_{m,i+1}-\alpha_i \tag{2.3.34}$$

其中α_i是第i个待设计的镇定函数。将式(2.3.33)和(2.3.34)代入式(2.3.32)，有

$$\begin{aligned}\dot{V}_i\le&-\sum_{j=1}^{i-1}c_jz_j^2+z_i\left(z_{i-1}+z_{i+1}+\alpha_i+\beta_i+\hat{\theta}^T\omega_i\right.\\&\left.+d_i\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i-\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\right)+\sum_{j=2}^{i-1}z_j\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\left(\tau_{i-1}-\dot{\hat{\theta}}\right)\\&+\tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}}-\tau_{i-1}-\Gamma\omega_iz_i\right)\end{aligned} \tag{2.3.35}$$

2.3 输出反馈自适应控制

令

$$\tau_i = \tau_{i-1} + \Gamma \omega_i z_i \tag{2.3.36}$$

$$\alpha_i = -c_i z_i - z_{i-1} - \beta_i - \hat{\theta}^T \omega_i - d_i \left(\frac{\partial \alpha_{i-1}}{\partial y} \right)^2 z_i + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \tau_i + \sum_{j=2}^{i-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \omega_i \tag{2.3.37}$$

然后有

$$\dot{V}_i \le -\sum_{j=1}^{i} c_j z_j^2 + z_i z_{i+1} + \sum_{j=2}^{i} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \left(\tau_i - \dot{\hat{\theta}} \right) + \tilde{\theta}^T \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_i \right) \tag{2.3.38}$$

2.3 输出反馈自适应控制

第 ρ 步： 注 意 到 $\alpha_{\rho-1}$ 是 y ， $\hat{\theta}$ 和 $X_{\rho-1}=\left[y_d,\dot{y}_d,\ldots,y_d^{(\rho-1)},\xi,\Xi_1,\ldots,\Xi_n,\lambda_1,\ldots,\lambda_{n-1},\hat{p}\right]^T$ 的光滑函数，$z_\rho=v_{m,\rho}-\alpha_{\rho-1}$的导数可表示为

$$\dot{z}_\rho=\eta(y)u+\beta_\rho+\theta^T\omega_\rho-\frac{\partial\alpha_{\rho-1}}{\partial y}\varepsilon_2-\frac{\partial\alpha_{\rho-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} \tag{2.3.39}$$

其 中 $\beta_\rho=-k_\rho v_{m,1}+v_{m,\rho+1}-\frac{\partial\alpha_{\rho-1}}{\partial y}\xi_2-\frac{\partial\alpha_{\rho-1}}{\partial X_{\rho-1}}\dot{X}_{\rho-1}$， $\omega_\rho=-\frac{\partial\alpha_{\rho-1}}{\partial y}\left[\varphi_1(y)+\Xi_2,v_{m,2},\ldots,v_{0,2}\right]^T\in\mathbb{R}^{q+m+1}$。

令

$$V_\rho=V_{\rho-1}+\frac{1}{2}z_\rho^2+\frac{1}{4d_\rho}\varepsilon^T H\varepsilon \tag{2.3.40}$$

2.3 输出反馈自适应控制

由式(2.3.38)-(2.3.40)，有

$$\dot{V}_\rho \leq -\sum_{j=1}^{\rho-1} c_j z_j^2 + z_\rho\left(z_{\rho-1} + \eta(y)u + \beta_\rho + \theta^T\omega_\rho - \frac{\partial\alpha_{\rho-1}}{\partial y}\varepsilon_2 - \frac{\partial\alpha_{\rho-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\right) + \sum_{j=2}^{\rho-1} z_j\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\left(\tau_{\rho-1} - \dot{\hat{\theta}}\right) + \tilde{\theta}^T\Gamma^{-1}\left(\dot{\hat{\theta}} - \tau_{\rho-1}\right) + \frac{1}{4d_\rho}\varepsilon^T\varepsilon \tag{2.3.41}$$

根据Young不等式可得

$$-z_\rho\frac{\partial\alpha_{\rho-1}}{\partial y}\varepsilon_2 \leq d_\rho\left(\frac{\partial\alpha_{\rho-1}}{\partial y}\right)^2 z_\rho^2 + \frac{1}{4d_\rho}\varepsilon^T\varepsilon \tag{2.3.42}$$

2.3 输出反馈自适应控制

将式(2.3.42) 代入式(2.3.41)，有

$$\dot{V}_\rho \le -\sum_{j=1}^{\rho-1} c_j z_j^2 + z_\rho \left(z_{\rho-1} + \eta(y)u + \beta_\rho + \hat{\theta}^T \omega_\rho - d_\rho \left(\frac{\partial \alpha_{\rho-1}}{\partial y}\right)^2 z_\rho \right.$$

$$\left. - \frac{\partial \alpha_{\rho-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \right) + \sum_{j=2}^{\rho-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \left(\tau_{\rho-1} - \dot{\hat{\theta}} \right) + \tilde{\theta}^T \Gamma^{-1} \left(\dot{\hat{\theta}} - \tau_{\rho-1} - \Gamma \omega_\rho z_\rho \right) \tag{2.3.43}$$

选取

$$\dot{\hat{\theta}} = \tau_\rho = \tau_{\rho-1} + \Gamma \omega_\rho z_\rho \tag{2.3.44}$$

$$u = \frac{1}{\eta(y)} \left[-c_\rho z_\rho - z_{\rho-1} - \beta_\rho - \hat{\theta}^T \omega_\rho - d_\rho \left(\frac{\partial \alpha_{\rho-1}}{\partial y}\right)^2 z_\rho \right.$$

$$\left. + \frac{\partial \alpha_{\rho-1}}{\partial \hat{\theta}} \tau_\rho + \sum_{j=2}^{\rho-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \omega_\rho \right] \tag{2.3.45}$$

然后，式(2.3.43)变为

$$\dot{V}_\rho \leq -\sum_{j=1}^{\rho} c_j z_j^2 \tag{2.3.46}$$

2.3.4 稳定性分析

定理2.3：考虑由被控对象(2.3.1)、滤波器(2.3.2)-(2.3.4)、自适应律(2.3.17)和(2.3.44)以及控制律(2.3.45)组成的闭环系统。假定假设1-4成立，则闭环系统内所有信号全局一致有界且 $\lim_{t\to+\infty}[y(t)-y_d(t)]=0$。

2.3 输出反馈自适应控制

- 证明：由式(2.3.46)可知，$V_\rho(t) \le V_\rho(0), \forall t \ge 0$。因此$V_\rho, z_1, \ldots, z_\rho, \hat{\theta}$，$\hat{p}$和$\varepsilon$有界。由$z_1$的有界性和假设4可知$y = z_1 + y_d$有界，因此$\xi$和$\Xi$也有界。由式(2.3.4)有

$$\lambda_i = \frac{s^{i-1} + k_1 s^{i-2} + \cdots + k_{i-1}}{F(s)}[\eta(y)u] \tag{2.3.47}$$

其中$F(s) = s^n + k_1 s^{n-1} + \cdots + k_0$。由式(2.3.1)有

$$\frac{d^n y}{dt^n} - \sum_{i=1}^{n} \frac{d^{n-i}}{dt^{n-i}}[\varphi_i(y)a] = \sum_{i=0}^{m} b_i \frac{d^i}{dt^i}[\eta(y)u] \tag{2.3.48}$$

结合式(2.3.47)和(2.3.48)有

$$\lambda_i = \frac{(s^{i-1} + k_1 s^{i-2} + \cdots + k_{i-1})}{F(s)B(s)}\left[\frac{d^n y}{dt^n} - \sum_{i=1}^{n} \frac{d^{n-i}}{dt^{n-i}}[\varphi_i(y)a]\right] \tag{2.3.49}$$

2.3 输出反馈自适应控制

由此可知$\lambda_1, \dots, \lambda_{m+1}$有界，因此$X_1 = [y_d, \dot{y}_d, \Xi_1, \dots, \Xi_n, \lambda_1, \dots, \lambda_{m+1}, \hat{p}]^T$和$\alpha_1(y, \hat{\theta}, X_1)$有界。进而$v_{m,2} = z_2 + \alpha_1$有界，再由(2.3.19)可知$\lambda_{m+2}$有界。重复上述过程，可证明$\alpha_i$和$\lambda_{m+i+1}(i = 1, \dots, \rho - 1)$有界，最后可证明控制信号$u$有界。在得到$u$的有界性之后，可以验证闭环系统内所有信号全局一致有界。此外，由式(2.3.46)可知$\int_0^{+\infty} z_1^2(\tau)d\tau \le V_\rho(0)/c_1$；由式(2.3.9)可知$\dot{z}_1$有界。利用Barbalat引理，我们有$\lim_{t\to+\infty} z_1(t) = 0$，即$\lim_{t\to+\infty}[y(t) - y_d(t)] = 0$，证毕。