Nonlinear Control: a Introduction

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Nonlinear control

Typical tasks in nonlinear control

- Stabilization
- Output regulation
- Trajectory tracking

Methodology

- State feedback: all states are measurable, static or dynamic feedback
- Output feedback: only the output is measurable

Other detailed approaches

- **Optimal control**: performance index to be optimized, constraints
- **Robust control**: influences on closed-loop stability due to uncertainties
- **Adaptive control**: quantitative influences on system performances due to parametric uncertainties

Robust control

- System plant belongs to an “uncertainty set”.
- The closed-loop system with any member in the “uncertainty set” can be stabilized.
- Example:

$$\dot{x} = f(x) + u \quad \text{with} \quad |f(x)| < k|x|, \; k > 0 \tag{1}$$

can be asymptotically stabilized by $u = -kx$, even though the detailed express of $f(x)$ is unknown.

Adaptive control

- Parametric uncertainties exist
- Feedback control with parameter estimation in the loop
- Example:

$$\dot{x} = px + u \quad \text{with uncertain } p \tag{2}$$

can be stabilized by $u = -kx$ with $\dot{k} = \beta x^2$ ($\beta > 0$)

Robust adaptive control

- Parametric uncertainties, un-modeled dynamics, etc.
- Combination of robust control and adaptive control
- Example:

$$\dot{x} = px + d(x) + u \tag{3}$$

where p is uncertain, and $d(x)$ is uncertain with known bound $|d(x)| < D(x)$. The Robust adaptive control can be designed by

$$u = -kx - (D(x) + \epsilon)\text{sgn}(x),\ \dot{k} = \beta x^2,\ \beta > 0,\ \epsilon > 0, \tag{4}$$

to stability the closed-loop system.

Categories of feedback control

- Static state feedback:

$$\dot{x} = f(t, x, u) \tag{5}$$

The static state feedback is in the form of $u = \gamma(t, x)$.

The static state feedback is "memoryless" with respect to x.

- Dynamic state feedback:

$$u = \gamma(t, x, z),\ \dot{z} = g(t, x, z). \tag{6}$$

Integral control and adaptive control are examples of dynamic state feedback.

Categories of feedback control

- Static output feedback:

$$\dot{x} = f(t, x, u),\ y = h(t, x, u) \tag{7}$$

The static state feedback is in the form of $u = \gamma(t, y)$.

Example: $\dot{x}_1 = -x_1 + x_2^2,\ \dot{x}_2 = u,\ y = x_2,\ u = -y$.

- Dynamic output feedback:

$$u = \gamma(t, y, z),\ \dot{z} = g(t, y, z). \tag{8}$$

Observers are usually applied in dynamic output feedback.

Several typical nonlinear control approaches

In this course,

- Approximate linearization
  - ⋆ Linearization around working points
  - ⋆ Pole assignment, LQR, etc.
  - ⋆ Local stability
  - ⋆ To enlarge stability region $\Rightarrow$ Gain scheduling.
- Feedback linearization
  - ⋆ Use feedback to cancel nonlinearities
  - ⋆ Global stability
  - ⋆ Usually combined with robust control

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Several typical nonlinear control approaches

In this course,

- Sliding model control (SMC)
  - ⋆ Use dis-continuous term to overcome the uncertainty
  - ⋆ Matched uncertainty
- Backstepping
  - ⋆ System plant in cascaded form, or “triangular form”
  - ⋆ Un-matched uncertainty can be handled
- Lyapunov redesign
  - ⋆ Additional terms in the controller to treat matched uncertainties
- Direct Lyapunov-based control