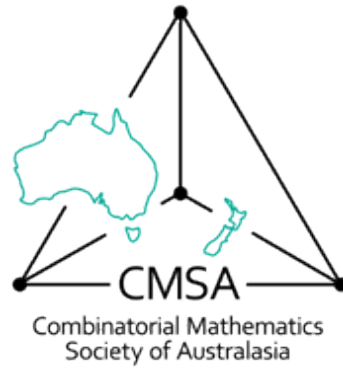


46TH AUSTRALASIAN COMBINATORICS CONFERENCE



The University of Queensland, 2–6 December, 2024



© 2024, the organisers:

Sara Davies

Barbara Maenhaut

Darryn Bryant

46acc.github.io/

Welcome!

Welcome to 46ACC and welcome to Brisbane. This is the seventh time a conference in this series has been hosted at the University of Queensland. The first hosting here (3rd ACCM) was 50 years ago in 1974. In addition to oversight and assistance provided by the Combinatorial Mathematics Society of Australasia, we gratefully acknowledge support from the Institute of Combinatorics and its Applications, and the School of Mathematics and Physics at the University of Queensland. We are delighted to have 70 registrants in attendance, and we wish you an enjoyable stay in Brisbane.

The organisers:

Sara Davies

Barbara Maenhaut

Darryn Bryant

Contents

Welcome!	iii
1 Schedule	1
2 Invited talks	5
3 Contributed talks	15
4 List of participants	65

1

Schedule

Sunday

	Prentice 42 - outside room 216
16:00 – 18:00	Welcome reception and registration

Working Space

Room 67-146 (Level 1 of the Priestley Building) is available for use by conference attendees from 8am - 6pm Monday to Friday.

Monday

	Prentice 42-216	Prentice 42-115
8:00 – 8:45	Registration	
8:45 – 9:00	Opening address (Barbara Maenhaut)	
9:00 – 10:00	Anita Liebenau 10	
10:00 – 10:30	Morning tea	
10:30 – 11:00	Đorđe Mitrović* 47	Thomas Lesgourgues 44
11:00 – 11:30	Lei Chen* 23	Katie Clinch 24
11:30 – 12:00	Cheryl E. Praeger 50	Jack Allsop* 17
12:00 – 12:30	Jonathan Klawitter 41	Tara Kemp* 40
12:30 – 14:30	Lunch break	
14:30 – 15:30	Florian Lehner 9	
15:30 – 16:00	Afternoon tea	
16:00 – 16:30	Semin Oh 49	Lander Verlinde* 61
16:30 – 17:00	Sakander Hayat 33	Shiksha Shiksha* 55
17:00 – 17:30	M. Tariq Rahim 51	Ian Seong* 54

Tuesday

	Prentice 42-216	Prentice 42-115
9:00 – 10:00	<i>David Wood</i> 14	
10:00 – 10:30	Morning tea	
10:30 – 11:00	Marston Conder 25	Keita Yasufuku* 64
11:00 – 11:30	Kevin Hendrey 34	Takatomo Yamasaki* 63
11:30 – 12:00	Marc Distel* 27	Koji Imamura 37
12:00 – 12:30	Dickson Annor* 18	Paul Leopardi 43
12:30 – 14:30	Lunch break	
14:30 – 15:30	<i>Alice Devillers</i> 7	
15:30 – 16:00	Afternoon tea	
16:00 – 16:30	Ian Roberts 52	Matthew Slattery-Holmes* 56
16:30 – 17:00	Adam Mammoliti 45	Hung Hoang 35
17:00 – 17:30	CMSA AGM	

Wednesday

	Prentice 42-216
9:00 – 10:00	<i>Anita Pasotti</i> 13
10:00 – 10:30	Morning tea
11:00 – 17:30	<i>Excursion</i>

Thursday

	Prentice 42-216	Prentice 42-115
9:00 – 10:00	<i>Sam Mattheus</i> 12	
10:00 – 10:30	Morning tea	
10:30 – 11:00	Muhammad Talha Farooq* 29	Biplab Basak 19
11:00 – 11:30	James Lefevre 42	Raju Kumar Gupta* 32
11:30 – 12:00	Yudhistira Andersen Bunjamin 22	Sourav Sarkar* 53
12:00 – 12:30	Jack Neubecker* 48	Abdul Basit 20
12:30 – 14:00	Lunch break	
14:00 – 15:00	<i>Jie Ma</i> 11	
15:00 – 15:30	Brendan McKay 46	Pawaton Kaemawichanurat 39
15:30 – 16:00	Afternoon tea	
16:00 – 16:30	Nick Wormald 62	Nick Brettell 21
16:30 – 17:00	Mikhail Isaev 38	Graham Farr 30

18:00: Conference dinner (Transcontinental Hotel, 482 George St, Brisbane City)

Friday

	Prentice 42-216	Prentice 42-115
9:00 – 10:00	<i>Melissa Lee</i> 8	
10:00 – 10:30	Morning tea	
10:30 – 11:00	I Nengah Suparta 57	Geertrui Van de Voorde 60
11:00 – 11:30	Abdullahi Umar 59	Gary Greaves 31
11:30 – 12:00	Daniel Horsley 36	Brhane Gebremichel Tnsau 58
12:00 – 12:30	Ajani De Vas Gunasekara 26	Christopher Duffy 28

2

Invited talks

<i>Alice Devillers – On partial linear spaces and rank 3 groups</i>	7
<i>Melissa Lee – Graphs on finite groups</i>	8
<i>Florian Lehner – Groups acting on trees and tree-like graphs</i>	9
<i>Anita Liebenau – Ramsey with purple edges</i>	10
<i>Jie Ma – A hypergraph bipartite Turán problem</i>	11
<i>Sam Mattheus – Forbidden subgraphs: past, present and future</i>	12
<i>Anita Pasotti – Heffter Spaces</i>	13
<i>David Wood – The global structure of planar graphs</i>	14

On partial linear spaces and rank 3 groups

Alice Devillers

University of Western Australia

A partial linear space is an incidence structure consisting of points and lines such that every line contains at least 3 points and every pair of points is in at most a line. I will assume partial linear spaces to be finite, not graphs nor linear spaces. Going all the way back to my PhD dissertation, I have been interested in partial linear spaces with varying degrees of symmetry, called k -ultrahomogeneity. The weakest level of symmetry among these is when $k = 2$: when the automorphism group is transitive on the ordered pairs of collinear points and on the ordered pairs of non-collinear points. This is equivalent to the automorphism group having rank 3.

Primitive rank 3 groups are classified. In 2005 I classified partial linear spaces admitting a primitive almost simple rank 3 group, and in 2008 the ones with a primitive rank 3 group of grid type. The case of primitive affine rank 3 groups was much harder to tackle, but was finally done in 2021, with Joanna Fawcett, John Bamberg and Cheryl Praeger (except for a few ‘hopeless’ cases.)

Imprimitive rank 3 groups in general have not been classified, but in 2006, with Jonathan Hall, we managed to do the easiest case, when all lines have size 3. Recent work provided classification of imprimitive rank 3 groups with some extra assumptions. Rank 3 quasiprimitive groups were classified in 2011 (AD, Michael Giudici, Cai Heng Li, Geoffrey Pearce, Cheryl Praeger) and rank 3 innately transitive in 2023 (Anton Baykalov, AD and Cheryl Praeger). With Anton Baykalov and Cheryl Praeger, we have now classified the partial linear spaces with such groups, finding some nice infinite families and a small number of sporadic examples.

Monash University

Groups are often used to quantify the symmetries of graphs and other geometric objects. In recent decades, there has been accelerating interest in constructions in the opposite direction, namely graphs defined from group properties. In this talk, we will discuss several types of such constructions, demonstrate how deep group-theoretic properties can be translated into the language of graphs, and outline some recent work and open problems in the area.

Groups acting on trees and tree-like graphs

Florian Lehner

University of Auckland

The study of groups acting on infinite trees plays a foundational role in geometric group theory, and is instrumental in many other branches of mathematics such as algebraic topology (e.g. via the Seifert-van Kampen Theorem), and topological group theory (in particular, the study of totally disconnected, locally compact groups).

Bass-Serre theory is perhaps the most important tool for analysing group actions on trees. It relates group actions on trees to graphs of groups, allowing a description of the groups as iterated amalgamated free products and HNN extensions. Conversely, given a graph of groups, it allows us to construct a group action on a tree. Unfortunately, it is rather difficult to use this construction to obtain interesting new examples of groups acting on trees since the actions of vertex and edge stabilisers must satisfy strong compatibility conditions.

To overcome this issue, Reid and Smith recently introduced the theory of local action diagrams. This theory eliminates the need for compatibility conditions, but only allows for the construction of very specific groups. More precisely, it covers all groups satisfying Tits' property (P), in other words, groups for which there is no interaction between stabilisers of disjoint subtrees.

This talk consists of two parts. In the first part we give an overview of some key ideas in Bass-Serre theory and the theory of local action diagrams, highlighting the advantages and disadvantages of both approaches. In the second part, we introduce a new ("amalgamated") version of local action diagrams. The resulting theory can be thought of as "in between" Bass-Serre theory and the theory of local action diagrams in the sense that it allows us to model some interaction between stabilisers of disjoint subtrees, at the expense of re-introducing some weak compatibility conditions.

Although the motivation for much of the presented research comes from topological group theory, our methods are purely combinatorial. No background in topological group theory will be assumed.

The talk is based on joint work (at various stages of completion) with M. Chan, M. Hamann, C. Lindorfer, B. Mirafteb, R. Möller, T. Rühmann, and W. Woess.

UNSW Sydney

Motivated by a question of David Angell, we study a variant of Ramsey numbers where some edges are coloured both red and blue, or: purple. Specifically, we are interested in the largest number $g = g(s, t, n)$, for some s and t and $n < R(s, t)$, such that there exists a red-blue-purple colouring of the edges of K_n with g purple edges, without a red-purple K_s and without a blue-purple K_t . We determine g asymptotically for a large family of parameters. The talk will be introductory in nature. Joint work with Thomas Lesgourgues and Nye Taylor.

Sam Matthews

The last few years have seen explosive progress on decades-old questions in extremal combinatorics, especially in problems involving forbidden subgraphs. While the probabilistic method has traditionally led the way on the constructive side of this area, recent advances show that ideas from algebra and geometry have become equally influential. However, despite intensive study of such objects in design theory, finite geometry, and algebraic graph theory, there has been relatively little interaction between these domains and extremal combinatorics. In this talk, we will explore the strengths and weaknesses of algebro-geometric constructions and highlight a few open problems.

Heffter Spaces

Anita Pasotti

University of Brescia - Italy

A *half-set* of a group $(G, +)$ of odd order is a complete system of representatives for the set of all pairs of opposite elements of $G \setminus \{0\}$.

Let $(G, +)$ be an abelian group of order $2v + 1 \geq 7$. A (v, k) *Heffter system* on G is a partition \mathcal{P} of a half-set of G into zero-sum parts, called *blocks*, of size k . Two Heffter systems \mathcal{P} and \mathcal{Q} on the same half-set are *orthogonal* if every block of \mathcal{P} intersects every block of \mathcal{Q} in at most one element.

In 2015 Archdeacon [1] introduced the notion of a Heffter array as an interesting link between combinatorial designs and topological graph theory. In a few words a Heffter array is equivalent to a pair of orthogonal Heffter systems on a cyclic group. We refer to [6] for an extensive survey on these arrays, their variants and generalizations, and their connections to other topics.

In [2] we proposed the more general problem of constructing “many” mutually orthogonal Heffter systems, which led us to introduce a new combinatorial design, that we called a *Heffter space*. A $(v, k; r)$ Heffter space is a resolvable partial linear space of degree r whose point-set is a half-set of an abelian group G of order $2v + 1$ and whose blocks are zero-sum k -subsets of G . One of the motivations for studying Heffter spaces is that every $(v, k; r)$ Heffter space with suitable properties gives rise to r mutually orthogonal k -cycle systems of order $2v + 1$, a topic recently studied in [4, 5].

References

- [1] D.S. Archdeacon, *Heffter arrays and biembedding graphs on surfaces*, Electron. J. Combin. **22** (2015), #P1.74.
- [2] M. Buratti, A. Pasotti, *Heffter spaces*, Finite Fields Appl. **98** (2024), 102464.
- [3] M. Buratti, A. Pasotti, *More Heffter spaces via finite fields*, preprint available at <https://arxiv.org/abs/2408.12412>.
- [4] A.C. Burgess, N.J. Cavenagh, D.A. Pike, *Mutually orthogonal cycle systems*, Ars Math. Con-temp. **23** (2023), #P2.05.
- [5] S. Küçükçifçi, E.Ş. Yazıcı, *Orthogonal cycle systems with cycle length less than 10*, J. Combin. Des. **32** (2024), 31–45.
- [6] A. Pasotti, J.H. Dinitz, *A survey of Heffter arrays*, Fields Inst. Commun. **86** (2024), 353–392.

David Wood

Planar graphs are of fundamental importance in graph theory, for historical reasons and as a springboard into graph minor theory. This talk is about the global structure of planar graphs. The goal is to describe planar graphs in terms of much simpler tree-like graphs. I will describe several recent advances in this direction, and give sample applications that solve old open problems. Extensions to other minor-closed graph classes will also be presented.

3

Contributed talks

<i>Jack Allsop*</i> – Subsquares in random Latin squares	17
<i>Dickson Annor*</i> – Graph Covers and Minors	18
<i>Biplab Basak</i> – Minimal simplicial degree d self-maps of $\mathbb{S}^{n-1} \times \mathbb{S}^1$	19
<i>Abdul Basit</i> – Partitioning problems in discrete geometry	20
<i>Nick Brettell</i> – Detachable pairs in 3-connected simple graphs and 3-connected matroids	21
<i>Yudhistira Andersen Bunjamin</i> – Different ways of constructing infinite families of group divisible designs with two group sizes	22
<i>Lei Chen*</i> – S -Arc-Transitivity of Vertex-Transitive Digraphs	23
<i>Katie Clinch</i> – The Peaceable Queens Problem	24
<i>Marston Conder</i> – Recent discoveries about degree-diameter-girth issues	25
<i>Ajani De Vas Gunasekara</i> – Completing partial k -star designs	26
<i>Marc Distel*</i> – Proper Minor-Closed Classes as $O(\sqrt{n})$ -blowups	27
<i>Christopher Duffy</i> – Mixed Graphs, Quasi-transitivity, Square Roots and the Oriented Chromatic Number	28
<i>Muhammad Talha Farooq*</i> – k -Matching Configurations in Octagonal and Octagonal-Quadrilateral Structures	29
<i>Graham Farr</i> – Interpretations of some transforms on binary functions	30
<i>Gary Greaves</i> – Edge-regular graphs with regular cliques	31
<i>Raju Kumar Gupta*</i> – A characterization of normal 3-pseudomanifolds with at most two singularities	32
<i>Sakander Hayat</i> – Graphs whose generalized adjacency matrix has few distinct eigenvalues	33
<i>Kevin Hendrey</i> – The Planar Graph Product Structure Theorem With Shorter Paths	34
<i>Hung Hoang</i> – Signotopes with few plus signs	35
<i>Daniel Horsley</i> – On decomposition thresholds for odd length cycles	36
<i>Koji Imamura</i> – On the Critical Problem for codes over $\mathbb{Z}/q\mathbb{Z}$	37
<i>Mikhail Isaev</i> – On the maximum number of common neighbours in dense random regular graphs	38
<i>Pawaton Kaemawichanurat</i> – Degree-based function index of trees, unicyclic graphs and bicyclic graphs with given bipartition	39
<i>Tara Kemp*</i> – Latin squares with disjoint subsquares	40
<i>Jonathan Klawitter</i> – Rectangular Duals on the Cylinder and the Torus	41
<i>James Lefevre</i> – Equitable colourings of Uniform Group Divisible Designs and Maximum Packings	42
<i>Paul Leopardi</i> – Big data in combinatorics: is it feasible?	43
<i>Thomas Lesgourgues</i> – Odd-Ramsey numbers of complete bipartite graphs	44

<i>Adam Mammoliti</i> – On Minimising the Number of Subsets and Supersets of a Family of Sets	45
<i>Brendan McKay</i> – Some thoughts on Eulerian orientations	46
<i>Dorđe Mitrović*</i> – Exponential graph growth via eigenspaces of graphs over finite fields	47
<i>Jack Neubecker*</i> – Factor-critical hypergraphs	48
<i>Semin Oh</i> – On Q -integral graphs with small Q -spectral radius	49
<i>Cheryl E. Praeger</i> – Flag-transitive, imprimitive, 2-designs	50
<i>M. Tariq Rahim</i> – On Radenković and Gutman conjecture for trees	51
<i>Ian Roberts</i> – Marginal Analysis on Finite Sets	52
<i>Sourav Sarkar*</i> – An Upper Bound Theorem for Homology 4-Manifolds	53
<i>Ian Seong*</i> – A generalization of the Askey-Wilson relations using a projective geometry	54
<i>Shiksha Shiksha*</i> – A study of floor plans through graphs	55
<i>Matthew Slattery-Holmes*</i> – Pattern-Avoiding Peak Functions	56
<i>I Nengah Suparta</i> – Characterization of Strongly Graceful Unicyclic Graphs	57
<i>Brhane Gebremichel Tnsau</i> – Towards a classification of 1-homogeneous distance-regular graphs with positive intersection number a_1	58
<i>Abdullahi Umar</i> – Lattice Paths and Order-Preserving Transformations of a Finite Chain	59
<i>Geertrui Van de Voorde</i> – Anzahl theorems for classical spaces	60
<i>Lander Verlinde*</i> – Nearly-orthogonal sets over finite fields	61
<i>Nick Wormald</i> – Asymptotic normality for submaps of maps using high moments	62
<i>Takatomo Yamasaki*</i> – On covering radii of rank metric codes	63
<i>Keita Yasufuku*</i> – On the minimum length of linear codes	64

*Jack Allsop**

(Joint work with Ian Wanless)

We prove that with probability $1 - o(1)$ as $n \rightarrow \infty$, a uniformly random Latin square of order n contains no subsquare of order 4 or more, resolving a conjecture of McKay and Wanless. We also show that the expected number of subsquares of order 3 is bounded.

Minimal simplicial degree d self-maps of $\mathbb{S}^{n-1} \times \mathbb{S}^1$

Biplab Basak

Indian Institute of Technology Delhi

(Joint work with Anshu Agarwal and Sourav Sarkar)

In topology, the degree of a map between orientable manifolds is a crucial invariant that provides deep insights into the structural and geometric properties of the manifolds involved, as well as the relationships between them. Understanding how to construct maps of a given degree between manifolds has been the focus of extensive research, especially within the context of orientable topological spaces.

In this talk, we present a novel construction of degree d simplicial maps between orientable manifolds, specifically focusing on the product manifold $\mathbb{S}^{n-1} \times \mathbb{S}^1$. For each integer $d \in \mathbb{Z}$, we construct a simplicial map of degree d from a colored triangulation of $\mathbb{S}^{n-1} \times \mathbb{S}^1$ with $2(n+1) \max |d| + 1$ facets to the standard $2(n+1)$ -facet colored triangulation of $\mathbb{S}^{n-1} \times \mathbb{S}^1$. Our results demonstrate that these colored triangulations are minimal in the sense that they use the smallest possible number of facets necessary to support a degree d simplicial self-map of $\mathbb{S}^{n-1} \times \mathbb{S}^1$, where $n \geq 2$.

Furthermore, we extend our construction to provide a minimal degree d simplicial map from any closed orientable n -manifold to the standard n -sphere \mathbb{S}^n , for $n \geq 1$. These constructions not only shed light on the interplay between combinatorial and topological properties in the context of colored triangulations but also offer new tools for the study of simplicial maps and manifold topology.

Abdul Basit

Equipartition problems study how partitions of Euclidean spaces split families of measures. The quintessential example is the Ham Sandwich Theorem, which states that any d finite measures in \mathbb{R}^d , each absolutely continuous with respect to the Lebesgue measure, there exists a hyperplane that divides \mathbb{R}^d into two half-spaces of the same size with respect to each measure.

We will survey some old and new equipartitioning results, and discuss some fascinating open problems. This talk is based on joint work with Boris Aronov, Indu Ramesh, Gianluca Tasinato and Uli Wagner.

Detachable pairs in 3-connected simple graphs and 3-connected matroids

Nick Brettell

Victoria University of Wellington

(Joint work with Charles Semple and Gerry Toft)

Tutte (1961) proved that a simple 3-connected graph G has an edge e such that either the deletion or contraction of e from G results in a graph that remains simple and 3-connected, unless G is a wheel. What if we instead ask for a pair of edges such that deleting both or contracting both retains simplicity and 3-connectedness? We call a pair of edges with this property *detachable*. In recent joint work with Gerry Toft and Charles Semple, building on work of Alan Williams (2014), we characterised the simple 3-connected graphs with no detachable pairs. In fact, we obtain this as a corollary of a more general result regarding detachable pairs in 3-connected matroids. In this talk, I will discuss this result, the motivation behind this work, and some potential applications.

S -Arc-Transitivity of Vertex-Transitive Digraphs

Lei Chen*

University of Western Australia

The investigation of s -arc-transitivity can be dated back to 1947. Tutte [4] studied cubic graphs and showed that a cubic graph can be at most 5-arc-transitive. A more general result for s -arc-transitivity of graphs was obtained by Weiss [5] and it turns out that finite undirected graphs of valency at least 3 that are not cycles can be at most 7-arc-transitive. In stark contrast with the situation in undirected graphs, Praeger [3] showed that for each s and d there are infinitely many finite s -arc-transitive digraphs of valency d that are not $(s + 1)$ -arc-transitive.

However, once we add the condition of primitivity the situation gets quite different. Since the lack of evidence of existence of vertex-primitive 2-arc-transitive digraphs, Praeger [3] asked if there exists any vertex-primitive 2-arc-transitive digraph. The question was then answered in [1] and [2] by constructing infinite families of G -vertex-primitive $(G, 2)$ -arc-transitive digraphs such that G is AS and SD types, respectively. In [2] Giudici and Xia then asked for a G -vertex-primitive (G, s) -arc-transitive digraph that is not a directed cycle, what is the upper bound on s . A reasonable conjecture is that $s \leq 2$. At the same time, Giudici and Xia [2] showed that to answer that question it suffices for us to consider the case when G is almost simple.

In this talk, I will introduce the current progress of the study of the s -arc-transitivity of vertex-primitive digraphs of various almost simple groups. Indeed, all of the studied almost simple groups follow the conjecture that $s \leq 2$. Moreover, I will also discuss a bit about the s -arc-transitive vertex-quasiprimitive digraphs and show that for vertex-quasiprimitive digraphs, s can also be unbounded.

References

- [1] M. Giudici, C.H. Li and B. Xia, An infinite family of vertex-primitive 2-arc-transitive digraphs, *J. Combin. Theory Ser. B* 127(2017), 1-13.
- [2] M. Giudici and B. Xia, Vertex-quasiprimitive 2-arc-transitive digraphs, *Ars Math. Contemp.* 14 (2018) no. 1, 67-82.
- [3] C.E. Praeger, Highly arc-transitive digraphs, *European J. Combin.* 10 (3) (1989) 281–292.
- [4] W.T. Tutte, A family of cubical graphs, *Proc. Cambridge Philos. Soc.* 43(1947), 459–474.
- [5] R. Weiss, The non-existence of 8-transitive graphs, *Combinatorica* 1 (3) (1981) 309–311.

The Peaceable Queens Problem

Katie Clinch

UNSW Sydney

(Joint work with Matthew Drescher, Tony Huynh, and Abdallah Saffidine)

The *peaceable queens problem* asks to determine the maximum number $a(n)$ such that there is a placement of $a(n)$ white queens and $a(n)$ black queens on an $n \times n$ chessboard so that no queen can capture any queen of the opposite color.

This problem is first mentioned in Stephen Ainley’s 1977 book *Mathematical Puzzles*, in which Ainley provides a general construction, giving the lower bound $a(n) \geq 0.1458n^2$. Perhaps surprisingly, this remains an optimal construction, despite many computational efforts at improvement. In comparison, the question of finding upper bounds on $a(n)$ has received little attention.

In this talk, we provide new asymptotic upper and lower bounds for $a(n)$, and for the variant problem on the toroidal board. This is joint work with Matthew Drescher, Tony Huynh, and Abdallah Saffidine.

Recent discoveries about degree-diameter-girth issues

Marston Conder

University of Auckland

(Joint work with [Various])

The well-known *Moore graphs* (including odd-length cycles, the complete graphs, the Petersen graph and the Hoffman-Singleton graph) are regular graphs of maximum conceivable order with given degree and diameter, or equivalently, regular graphs of minimum conceivable order with given degree and girth (sometimes called *cages*), using the Moore bound.

More generally, the task of finding the largest regular graph with given degree and diameter is called the *degree-diameter problem*, and the corresponding one for given degree and girth is called the *cage problem*. Various people in the combinatorics community in Australia and New Zealand have contributed answers or partial answers to these problems.

At a BIRS workshop at Banff in May 2023, some investigations were made into the broader notion of a *degree-diameter-girth problem*, namely finding the largest regular graph with given degree, diameter and girth. I will report on some of what was discovered.

Proper Minor-Closed Classes as $O(\sqrt{n})$ -blowups

Marc Distel*

Monash University

The strong product of graphs G and H is the graph with vertex set $V(G) \boxtimes V(H)$ and edges between distinct pairs of vertices if they are adjacent or equal in each coordinate. In this talk, we investigate for which simple graphs H are n -vertex K_t -minor graphs G contained in $H \boxtimes K_b$, with b roughly $O(\sqrt{n})$.

In this talk we explore the structure of graphs arising from the transitive closure of an oriented graph, providing a connection between the classical notion of quasi-transitivity of directed graphs and the study of graph square roots. In doing so we find a full classification for graphs that arise as an undirected square of an orientation of a tree, and prove that the problem of deciding if a graph admits a mixed quasi-transitive orientation is NP-complete. We relate our work to on-going work in the study of the oriented chromatic number, defined using homomorphisms of oriented graphs.

k-Matching Configurations in Octagonal and Octagonal-Quadrilateral Structures

*Muhammad Talha Farooq**

King Mongkut's University of Technology Thonburi Thailand, Macquarie University
Sydney Australia

(Joint work with Pawaton Kaemawichanurat, Thap Panitanarak)

The Hosoya index of graph G is determined by counting the total number of k -matching, for all possible values of k . The Hosoya index holds significant importance in the realm of mathematical chemistry due to its correlation with various thermodynamic properties of hydrocarbons. Consequently, it is important to calculate the number of k -matchings of different chemical structures. In this work, we introduce a novel approach utilizing the transfer matrix technique to enumerate the number of k -matching, denoted as $p(G, k)$, within octagonal and octagonal-quadrilateral linear and random chains. Consequently, for all $k \geq 0$, the enumeration of $p(G, k)$ in arbitrary octagonal and octagonal-quadrilateral chains is achieved by employing a suitable combination of six transfer matrices, three matrices for each chain, with a dimension of $4(k+1) \times 4(k+1)$, along with a k -matching vector with dimension $4(k+1) \times 1$. This approach offers a comprehensive framework for determining $p(G, k)$ for a wide range of molecular structures, contributing to advancements in chemical graph theory and computational chemistry.

Interpretations of some transforms on binary functions

Graham Farr

Monash University

A *binary function* is a function $f : 2^E \rightarrow \mathbb{C}$ for which $f(\emptyset) = 1$, where E is a finite ground set. Binary functions generalise binary matroids in the sense that any indicator function of a linear space over $\text{GF}(2)$ is a $\{0, 1\}$ -valued binary function (using the natural correspondence between subsets of E and their characteristic vectors in $\text{GF}(2)^E$). The author showed in 1993 that binary functions have deletion and contraction operations and extend arbitrary matroids, with duality corresponding to the Hadamard transform, and admit a generalisation of the Whitney rank generating function (a close relative of the Tutte polynomial). Subsequent papers (2004–2019) provided a family of transforms $L^{[\mu]}$ and associated minor operations, indexed by complex numbers μ , and developed their theory, with the identity transform and Hadamard transform corresponding to $\mu = 1$ and $\mu = -1$ respectively.

In this talk, we look at properties of transforms $L^{[\mu]}$ when $|\mu| = 1$. We use these transforms to characterise those binary functions for which the Hadamard transform is just the elementwise complex conjugate. We then give an interpretation of $L^{[\mu]}f$, for $|\mu| = 1$: it yields an appropriate quantum superposition of all the partial Hadamard transforms of f . We discuss the interpretation for the special case of plane graphs.

Edge-regular graphs with regular cliques

Gary Greaves

Nanyang Technological University, Singapore

In 1981, Arnold Neumaier posed the problem “Is every edge-regular graph with a regular clique strongly regular?” In 2018, Greaves and Koolen found an infinite family of edge-regular graphs that are not strongly regular but have regular cliques, thus answering Neumaier’s question. Since 2018, many more families of such graphs (now known as Neumaier graphs) have been discovered.

In this talk, we will give a brief overview of the study of Neumaier graphs. We will then consider Neumaier graphs that are very close to being strongly regular in the sense of having a small coherent rank. Lastly, we will present a newly discovered family of Neumaier graphs, with cyclotomic constructions.

Sakander Hayat

(Joint work with Ximing Cheng, Muhammad Javaid, Jack Koolen)

The generalized adjacency matrix of a graph Γ is defined as $M(x, y, z) := xI + yA + z(J - I - A)$, where x, y and z are real numbers satisfying $y \neq z$ and J (resp. I) is the all-ones matrix (resp. identity matrix) of suitable dimension. For a real number h , we define the matrix B_h by, $B_h := A + h(J - I)$. This implies that M is an affine transformation of B_h . Note that the Seidel matrix S of a graph defined as $S := J - I - 2A$, then $S = -2B_{-\frac{1}{2}}$. The adjacency matrix of a graph is $A = B_0$ and adjacency matrix of its complement is $\bar{A} := J - I - A$ and $\bar{A} = -B_{-1}$. Note that the B_h matrix is always irreducible if $h \notin \{0, -1\}$. In this talk, I will present some results regarding graphs with few distinct B_h -eigenvalues which we have proven recently. Graphs with two distinct B_h -eigenvalues have been characterized. Various results on connected graphs with three distinct B_h -eigenvalues have been proven. Disconnected graphs with three distinct B_h -eigenvalues are also characterized. As a by-product, we obtain the first example of non-regular non-bipartite graphs with three distinct distance eigenvalues.

The Planar Graph Product Structure Theorem With Shorter Paths

Kevin Hendrey

Monash University

(Joint work with David Wood)

The Planar Graph Product Structure Theorem of Dujmović, Joret, Micek, Morin, Ueckerdt and Wood (2020) shows that every planar graph is a subgraph of the strong product of a path, a graph of treewidth 3 and clique of bounded size. This breakthrough result has been used to resolve or make progress on long standing open problems in a variety of areas including book embeddings of graphs, nonrepetitive colours and efficient constructions of universal graphs. Currently, there are no nontrivial bounds in the literature for the lengths of the paths in this result. We prove that every planar graph G is a subgraph of the strong product of a path P , a graph H of treewidth 3 and a bounded size clique, where the length of the path is $o(|V(G)|)$.

TU Wien

Signotopes are a combinatorial structure generalising binary strings, permutations, and simple pseudoline arrangements. Manin and Schechtman (1989) introduced the higher Bruhat order $B(n, r)$, which is a natural order of the r -signotopes on n elements and a generalisation of the weak order of permutations. We show that the lower (and by symmetry upper) levels of the higher Bruhat order contain the same number of elements for a fixed difference $n - r$.

On decomposition thresholds for odd length cycles

Daniel Horsley

Monash University

(Joint work with Darryn Bryant, Peter Dukes, Barbara Maenhaut, Richard Montgomery)

An (edge) *decomposition* of a graph G is a set of subgraphs of G whose edge sets partition the edge set of G . I will discuss our recent proof that, for each odd $\ell \geq 5$, any graph G of sufficiently large order n with minimum degree at least $(\frac{1}{2} + \frac{1}{2\ell-4} + o(1))n$ has a decomposition into ℓ -cycles if and only if ℓ divides $|E(G)|$ and each vertex of G has even degree. This threshold cannot be improved beyond $\frac{1}{2} + \frac{1}{2\ell-2}$. It was previously shown that the thresholds approach $\frac{1}{2}$ as ℓ becomes large, but our thresholds do so significantly more rapidly. Our methods can be applied to tripartite graphs more generally and we also obtain some bounds for decomposition thresholds of other tripartite graphs.

On the maximum number of common neighbours in dense random regular graphs

Mikhail Isaev

UNSW Sydney

(Joint work with Maksim Zhukovskii)

We derive the distribution of the maximum number of common neighbours of a pair of vertices in a dense random regular graph. The proof involves two important steps. One step is to establish the extremal independence property: the asymptotic equivalence with the maximum component of a vector with independent marginal distributions. The other step is to prove that the distribution of the number of common neighbours for each pair of vertices can be approximated by the binomial distribution. The talk is based on <https://arxiv.org/pdf/2312.15370>

Degree-based function index of trees, unicyclic graphs and bicyclic graphs with given bipartition

Pawaton Kaemawichanurat

King Mongkut's University of Technology Thonburi

(Joint work with Tomas Vetrik)

We investigate the degree-based function index $I_f(G) = \sum_{vw \in E(G)} f(d_G(v), d_G(w))$ of a graph G , where $E(G)$ is the set of edges of G , $d_G(v)$ and $d_G(w)$ are the degrees of vertices v and w in G , respectively, and f is a symmetric function of two variables which satisfies some conditions. We obtain sharp upper bounds on I_f for trees, unicyclic graphs and bicyclic graphs with given bipartition. Then, among trees and unicyclic graphs with given bipartition, we present graphs with the largest values of the general reduced second Zagreb index $GRM_a(G) = \sum_{vw \in E(G)} (d_G(v) + a)(d_G(w) + a)$ for $a > -1$, general Randić index $R_a(G) = \sum_{vw \in E(G)} [d_G(v)d_G(w)]^a$ and first general Gourava index $FGO_a(G) = \sum_{vw \in E(G)} [d_G(v)d_G(w) + d_G(v) + d_G(w)]^a$ for $a \geq 1$, general Sombor index $SO_{a,b}(G) = \sum_{vw \in E(G)} ([d_G(v)]^a + [d_G(w)]^a)^b$, generalized Zagreb index $GZ_{a,b}(G) = \sum_{vw \in E(G)} ([d_G(v)]^a [d_G(w)]^b + [d_G(v)]^b [d_G(w)]^a)$ and one other general index $M_{a,b}(G) = \sum_{vw \in E(G)} [d_G(v)d_G(w)]^a [d_G(v) + d_G(w)]^b$ for $a \geq 1$ and $b \geq 1$.

Tara Kemp*

A latin square is an n by n array with each of n symbols occurring exactly once in each row and column, and a subsquare is a smaller latin square within a latin square. Given a partition $n = n_1 + n_2 + \cdots + n_k$, a realization is a latin square with disjoint subsquares of orders n_1, n_2, \dots, n_k . The existence of a realization for a given partition is a partially solved problem, and much of the difficulty of the problem comes from finding general constructions. In this talk, we will discuss how realizations can be found for large families of partitions, with the help of timetabling, graph theory, and rational numbers.

Rectangular Duals on the Cylinder and the Torus

Jonathan Klawitter

University of Auckland

(Joint work with Therese Biedl, Philipp Kindermann)

A rectangular dual of a plane graph G is a contact representation of G by interior-disjoint rectangles such that (i) no four rectangles share a point, and (ii) the union of all rectangles is a rectangle. In this talk, we look at when and how we can construct a rectangular dual of a graph embedded on the cylinder or on the torus. While we can give a full characterisation and construction algorithm for cylindrical rectangular duals, the toroidal case has proven more challenging. Yet when we are also given a combinatorial description of rectangle adjacencies (a rectangular edge labelling), then we can test whether a graph embedded on the flat torus admits a toroidal rectangular dual.

Equitable colourings of Uniform Group Divisible Designs and Maximum Packings

James Lefevre

The University of Queensland

(Joint work with Andrea C. Burgess, Peter Danziger, Diane Donovan, Tara Kemp, David A. Pike, E. Şule Yazici)

An *equitable c -colouring* of a block design consists of a mapping from the underlying points to a set of c colours such that, for any block B and any two colours c_1 and c_2 , the number of points of B mapped to c_1 differs from the number mapped to c_2 by at most one. A recent application arises from the theory of DNA data storage systems. The proposed system encodes binary sequences as “nicks” in the sugar-phosphate backbone of native DNA such as *E. coli*. Biological constraints on the nick positions imply that optimal efficiency can be obtained through the use of equitably 2-colourable maximum packings. We survey known results and open problems alongside related results for BIBDs and GDDs. We include new necessary and sufficient conditions for the existence of equitably colourable uniform GDDs, and outline an alternative, direct construction for equitably 2-colourable maximum packings with block size 4.

Paul Leopardi

This talk updates the talk given at 42 ACCMCC in 2019 on FAIR databases in combinatorics. It describes work done by Bercic and others to build, catalog and expand FAIR mathematical databases, and presents ideas and use cases for a proposed multi-terabyte database of strongly regular graphs.

Odd-Ramsey numbers of complete bipartite graphs

Thomas Lesgourgues

University of Waterloo

(Joint work with Simona Boyadzhyska, Shagnik Das, and Kalina Petrova)

In his study of graph codes, Alon introduced the “odd-Ramsey” number of a graph H , defined as the minimum number of colours needed to colour the edges of K_n so that every copy of H intersects some colour class in an odd number of edges. Focusing on the case where H is a complete bipartite graph, I will present a surprising link with a well-studied coding-theory parameter, maximising the dimension of a linear binary code avoiding codewords of given weights.

This is part of a more general article, joint work with Simona Boyadzhiyska, Shagnik Das, and Kalina Petrova.

On Minimising the Number of Subsets and Supersets of a Family of Sets

Adam Mammoliti

UNSW Sydney

(Joint work with Daniel Horsley and Adam Gowty)

Let \mathcal{F} be a family of k -subsets of $[n]$. The lower shadow of \mathcal{F} is the set of all $(k-1)$ -subsets of $[n]$ that are a subset of at least one member of \mathcal{F} and the upper shadow of \mathcal{F} is the set of all $(k+1)$ -subsets of $[n]$ that are a superset of at least one member of \mathcal{F} . The Kruskal-Katona Theorem states that over all k -uniform families \mathcal{F} of subsets of $[n]$ with $|\mathcal{F}| = m$ the minimum size of the lower shadow of \mathcal{F} is that of the lower shadow of the first m k -subsets of $[n]$ in the colexicographical order. An analogous result is true if we instead minimise the size of the upper shadow of \mathcal{F} .

In this talk I present a generalisation of the Kruskal-Katona Theorem where the sets in the family \mathcal{F} need not be the same size and we consider minimising the size of \mathcal{F}^\uparrow , the set of all subsets of $[n]$ that are either a subset or a superset of at least one member of \mathcal{F} , over all families $\mathcal{F} \subseteq 2^{[n]}$ with m members. I present a recursive formula for the minimum size of \mathcal{F}^\uparrow for any n and $m \leq 2^n$. I also present a relative simple lower bound for the minimum size of $|\mathcal{F}^\uparrow|$, namely it is true that $|\mathcal{F}^\uparrow| \geq \sqrt{2^{n+2m}} - m$ where $\mathcal{F} \subseteq 2^{[n]}$ has $m \leq 2^n$ members. This bound is tight when it is an integer and is asymptotically correct as $n \rightarrow \infty$ with $m = \omega(1)$.

This is based on joint work with Daniel Horsley and Adam Gowty.

Given an undirected graph with even degrees, an Eulerian orientation is an orientation of every edge such that each vertex has an equal number of incoming and outgoing edges. In addition to being combinatorially pleasant objects, Eulerian orientations have physical applications, most famously in the study of the behaviour of ice. Our focus will be on the number of Eulerian orientations. We will explain an asymptotic formula for graphs of sufficient density. The formula contains the inverse square root of the number of spanning trees, for which we do not have a heuristic explanation. We will also show a strong experimental correlation between the number of spanning trees and the number of Eulerian orientations even for graphs of bounded degree. This leads us to propose a new heuristic for the number of Eulerian orientations which performs much better than previous heuristics for graphs of chemical interest.

Exponential graph growth via eigenspaces of graphs over finite fields

*Đorđe Mitrović**

University of Auckland

(Joint work with Gabriel Verret, Jeroen Schillewaert, Florian Lehner)

Let Γ be a finite connected graph and G a vertex-transitive group of its automorphisms. The pair (Γ, G) is called *locally- L* if the group induced by the action of the vertex-stabiliser G_v on the neighbourhood of a vertex v is permutation isomorphic to L . The graph growth of a permutation group L describes the growth of the order of G_v as a function of the order of Γ across locally- L pairs (Γ, G) .

In this talk, we present new constructions of symmetric graphs with “large” eigenspaces over finite fields, notably infinite graphs that admit eigenvectors with finite support. As a corollary, we obtain that the graph growth of certain imprimitive permutation groups is exponential.

(Joint work with Darryn Bryant, Sara Davies)

We call a hypergraph *factor-critical* if it has no 1-factor but the deletion of any single vertex leaves a hypergraph with a 1-factor. This is a natural generalisation of the well-studied notion of factor-criticality of graphs. We are interested in the minimum number of hyperedges in a factor-critical hypergraph of a given order. We prove that the minimum number of hyperedges in a factor-critical k -uniform hypergraph of order n is exactly n for all $n \equiv 1 \pmod{k}$, $n > 1$. However, we require non-linear hypergraphs to achieve this bound. We then focus on factor-critical linear 3-uniform hypergraphs and establish some results on the number of hyperedges in such hypergraphs.

Semin Oh

(Joint work with Jeong Rye Park, Jongyook, Yoshio Sano Park)

For a graph, the sum of the adjacency matrix and the degree matrix is called the signless Laplacian matrix and denoted by Q . If Q of a graph G has only integer eigenvalues, G is called a Q -integral graph. We show classifications of Q -integral graphs with fixed Q -spectral radius up to 6.

On Radenković and Gutman conjecture for trees

M. Tariq Rahim

Department of Mathematics, Abbottabad University of Science and Technology

(Joint work with Afeefa Maryam)

Consider a simple graph G of order n and size m with Laplacian eigenvalues $\mu_1, \mu_2, \dots, \mu_{n-1}$ and $\mu_n = 0$. The average degree of the graph G is defined as $\bar{d} = \frac{2m}{n}$. The Laplacian energy of the graph G , denoted by $LE(G)$, is defined as $LE(G) = \sum_{i=1}^n |\mu_i - \bar{d}|$. Radenković and Gutman conjectured that among all trees of order n , the path graph P_n has the smallest Laplacian energy. Let $T_n(d)$ denote the class of all trees of order n and diameter d . In this paper, we establish the Radenković and Gutman conjecture for $T \in T_n(5)$.

Marginal Analysis on Finite Sets

Ian Roberts

Charles Darwin University

An overview of a new mathematical approach is provided. The approach is called *Marginal Analysis* and it is applicable to a range of problems involving ordered collections of sets. The approach simplifies the analysis of some difficult contemporary problems and provides insight and solutions to a range of new or unsolved problems. These problems include the analysis of functions defined on collections of sets such as the Kruskal-Katona function and the Lubell function, and on various types of antichains and associated functions in the Boolean Lattice.

Marginal Analysis can be considered as a form of Discrete Calculus. On the surface, it is simple in concepts and requires a small mathematical tool-box, but one has to be discerning in the choice and blending of the parts. An overview of the approach will be described in a visual and non-technical form. Researchers in other areas may find some ideas and results of interest to them, and an opening into potential new work.

Marginal analysis uses the language of sets but the results can be applied to Hypergraphs, Graphs, Projective Geometry, Coding Theory, and Combinatorial Design. Antichains and the Kruskal-Katona function are core concerns, and thus the talk informs the study of independent sets, blocking sets, vertex covers, intersecting set systems, combinatorial designs, and more.

Useful background ideas and knowledge include antichains, the Kruskal-Katona Theorem and the associated squashed (or colex) order, and the BLYM (or LYM) inequality.

An Upper Bound Theorem for Homology 4-Manifolds

*Sourav Sarkar**

Indian Institute of Technology Delhi

(Joint work with Biplab Basak)

The g -vector of a simplicial complex contains significant information about the combinatorial and topological structure of the complex. Several classification results concerning the structure of normal pseudomanifolds and homology manifolds have been established in relation to the value of g_2 , the third component of the g -vector. It is known that when $g_2 = 0$, all normal pseudomanifolds of dimensions at least three are stacked spheres. Walkup proved that a homology 3-manifold with $g_2 \leq 9$ is a triangulated sphere. We demonstrate that homology 4-manifolds with $g_2 \leq 5$ are triangulated spheres and are derived from triangulated 4-spheres with $g_2 \leq 2$ by a series of connected sum, bistellar 1- and 2-moves, edge contraction, edge expansion, and edge flipping operations. Furthermore, we establish that this inequality is optimally attainable, i.e., it cannot be extended to $g_2 = 6$.

In this talk we present a generalization of the Askey-Wilson relations that involves a projective geometry. Let V denote a finite-dimensional vector space over a finite field. The corresponding projective geometry P is the poset consisting of the subspaces of V , with partial order by inclusion. We construct some matrices A, A^* that have rows and columns indexed by P . These matrices are generalizations of the adjacency matrix and the dual adjacency matrix for some distance-regular graph called the Grassmann graph. We show that A, A^* satisfy some relations that generalize the Askey-Wilson relations.

A study of floor plans through graphs

*Shiksha Shiksha**

La Trobe University, Bendigo, Australia

(Joint work with Krishnendra Shekhawat)

In this study, we delve into the characterization of architectural floor plans using graph-theoretic concepts. We investigate adjacency graphs, which not only define module adjacencies but also reveal essential properties of floor plans. Planarity ensures floor plan existence, while biconnected graphs facilitate rectangular boundaries. However, separating triangles lead to non-rectangular rooms, and more than four corner-implying paths necessitate L -shaped or other non-rectangular boundaries. Furthermore, flippable vertices and flippable edges in graph labelings allow for multiple topologically distinct layouts. By understanding these graph properties, we provide insights into constructing diverse and efficient floor plans, crucial for both architectural design and algorithmic applications.

Given a set of permutations Π , let $\mathfrak{S}_n(\Pi)$ denote the elements of the symmetric group \mathfrak{S}_n that avoid every element of Π . Hamaker, Pawlowski, and Sagan defined a function $R_n(\Pi)$ to be the sum of peak functions indexed by the peak sets of permutations in $\mathfrak{S}_n(\Pi)$. We investigate combinatorial properties of these functions, including when their expansion in terms of Schur Q-functions is symmetric and when it has positive coefficients for all $\Pi \subset \mathfrak{S}_3$. We make use of various combinatorial objects including Dyck paths and tableaux diagrams during our investigations.

I Nengah Suparta

(Joint work with M. Bača, M. Demange, A. Semaničová-Feňovčíková, N.L.D. Sintuari)

$$\{|f(u) - f(v)| : uv \in E\} = \{1, 2, \dots, |E|\},$$

then f is called *graceful labeling* for G , and the graph G is called *graceful*. A *matching* in G is a non empty subset M of E such that any two elements of M are not adjacent in G . The matching M is called *perfect* if every vertex of G is incident with an element of M . In this case, the graph G is called with *perfect matching*. Let G be a graceful graph with graceful labeling f and some perfect matching M . If in addition, we also have that $f(u) + f(v) = |E|$ for every $uv \in M$, then f is called *strongly graceful labeling* for G , and the graph G is called *strongly graceful*. In this talk we characterize unicyclic graphs as strongly graceful graphs.

Towards a classification of 1-homogeneous distance-regular graphs with positive intersection number a_1

Brhane Gebremichel Tnsau

University of Science and Technology of China

(Joint work with Jack H. Koolen , Mamoon Abdullah and Jae-Ho Lee)

Let Γ be a graph with diameter at least two. Then Γ is said to be 1-homogeneous (in the sense of Nomura) whenever for every pair of adjacent vertices x and y in Γ , the distance partition of the vertex set of Γ with respect to both x and y is equitable, and the parameters corresponding to equitable partitions are independent of the choice of x and y . Assume Γ is 1-homogeneous distance-regular with intersection number $a_1 > 0$ and $D \geq 5$. Define $b = b_1/(\theta_1 + 1)$, where b_1 is the intersection number and θ_1 is the second largest eigenvalue of Γ . In this talk we will show that if intersection number $c_2 \geq 2$, then $b \geq 1$ and one of the following (i)–(vi) holds: (i) Γ is a regular near $2D$ -gon, (ii) Γ is a Johnson graph $J(2D, D)$, (iii) Γ is a halved ℓ -cube where $\ell \in \{2D, 2D + 1\}$, (iv) Γ is a folded Johnson graph $\bar{J}(4D, 2D)$, (v) Γ is a folded halved $(4D)$ -cube, (vi) The valency of Γ is bounded by a function of b . Using this result, we will characterize 1-homogeneous graphs with classical parameters and $a_1 > 0$, as well as tight distance-regular graphs.

(Joint work with A. Laradji)

Let \mathcal{PO}_n be the monoid of order-preserving partial transformations on $[n] = \{1, 2, \dots, n\}$. It is known that there exist bijections between \mathcal{PO}_n and its subsemigroups with certain lattice paths that start from $(0, 0)$ and end at $(n-1, n-1)$ in the Cartesian plane (Laradji and Umar, 2016). In this talk we are going to discuss several consequences of these bijections.

(Joint work with Maarten De Boeck)

In this talk, I'll give a gentle introduction to the results obtained in our recent work.

In this talk, I'll give a gentle introduction to the results obtained in our recent work.

Nearly-orthogonal sets over finite fields

*Lander Verlinde**

University of Auckland

(Joint work with Rajko Nenadov)

For a field \mathbb{F} and integers d, k and ℓ , a set $A \subseteq \mathbb{F}^d$ is called (k, ℓ) -nearly orthogonal if all vectors in A are non-self-orthogonal and every $k + 1$ vectors in A contain $\ell + 1$ pairwise orthogonal vectors. Determining the maximal size of such sets has been studied extensively for different fields, showing some interesting differences depending on its characteristic. Recently, Haviv, Mattheus, Mitojević and Wigderson have improved the lower bound on nearly orthogonal sets over finite fields, using counting arguments and a hypergraph container lemma. We generalise this by showing the existence of a set $A \subseteq \mathbb{F}^d$ of essentially the same size, with a stronger property that given any family of subsets $A_1, \dots, A_{\ell+1} \subset A$, each of size $k + 1$, we can find a vector in each A_i such that they are all pairwise orthogonal. This was previously known only for $\ell = 1$. Joint work with Rajko Nenadov.

Nick Wormald

(Joint work with Michael Drmota and Eva Hainzl)

There is a species of koala that doesn't live in trees. These trees are of course graphs with no cycles; the koala is a graph properly embedded in the plane (i.e. a map) with two big ears at the side of its head. There are old results on the distribution of the number of copies of a given submap in a random planar map, and in particular these results imply that any given small submap appears with high probability. (As a consequence, to show that all maps are 4-colourable, it is enough to show that the number of large maps that are 4-colourable is not exponentially small.) But the actual asymptotic distribution of the number of copies of a given submap in a random map has only been pinned down for submaps which cannot overlap with each other. Koalas have nice big ears which can overlap in interesting ways, and to deal with this feature, the usual generating function approaches to showing asymptotic normality become intractible. We use instead a combination of generating function work and an extension of the "Poisson Paradigm" to distributions whose expected value tends to infinity.

On covering radii of rank metric codes

Takatomo Yamasaki*

Kumamoto University

(Joint work with Keisuke Shiromoto and Koji Imamura)

In classical coding theory, a linear code is a k -dimensional subspace C of \mathbb{F}_q^n . The Hamming distance between $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_n)$ in \mathbb{F}_q^n is defined by $d_H(x, y) := |\{i \mid x_i \neq y_i\}|$. Then, the *covering radius* $\rho(C)$ of a linear code C is defined as follows:

$$\rho(C) := \max_{x \in \mathbb{F}_q^n} \min_{c \in C} d_H(x, c).$$

The covering radius is one of the important parameters in a linear code for understanding the decoding and the error-detecting capabilities of the code. The following relationship between the covering radius and the weight distributions was introduced by Delsarte in 1973:

Theorem (Delsarte bound) *Let $A_i(C^\perp)$ be the number of vectors in the dual space C^\perp of a linear code C whose Hamming weight is i . Then $\rho(C) \leq s$, where s is the number of $i \in \{1, \dots, n\}$ such that $A_i(C^\perp) \neq 0$.*

In this talk, we study the Delsarte bound for a *Delsarte rank metric code*, a subspace of the \mathbb{F}_q -linear space consisting of the $n \times m$ matrices. The bound has already been proved in terms of algebraic graph theory. However, we provide an alternative proof for the bound from a coding-theoretic perspective by introducing coset structure in rank metric codes. As an application of our coset structure, we propose a construction of *maximum rank distance* (MRD) codes from almost MRD codes.

On the minimum length of linear codes

*Keita Yasufuku**

Osaka Metropolitan University

(Joint work with Tatsuya Maruta)

A q -ary linear code of length n with dimension k (an $[n, k]_q$ code) is a k -dimensional subspace of the n -dimensional row vector space over the field of q elements. An $[n, k]_q$ code with minimum distance d is called an $[n, k, d]_q$ code.

We consider the problem to find optimal $[n, k, d]_q$ codes. This problem in coding theory is that of optimizing one of the parameters length n , dimension k , minimum distance d for given the other two which is referred to as the “optimal linear code problem”.

Also, it is known that the Griesmer bound is attained for all sufficiently large d for fixed q and k .

We tackle the problem to find $D_{q,k}$, the largest d such that the Griesmer bound is not attained for fixed k and q . This problem is solved when q is much larger than k .

In this presentation, we give a conjecture of $D_{q,k}$ and show that our conjecture on $D_{q,k}$ is valid for $q+1 \leq k \leq 8$ with $4 \leq q \leq 7$.

4

List of participants

Name	Affiliation	email address
Maliheh Alaei	The University of Queensland	m.alaeitazehkand@uq.edu.au
Jack Allsop	Monash University	jack.allsop@monash.edu
Maram Alqarni	The University of Queensland	maram.alqarni@student.uq.edu.au
Dickson Annor	La Trobe University	d.annor@latrobe.edu.au
Biplab Basak	Indian Institute of Technology Delhi	biplab@iitd.ac.in
Abdul Basit	Monash University	abdul.basit@monash.edu
Nick Brettell	Victoria University of Wellington	nick.brettell@vuw.ac.nz
Darryn Bryant	The University of Queensland	db@maths.uq.edu.au
Yudhistira Andersen Bunjamin	UNSW Sydney	yudhi@unsw.edu.au
Lei Chen	University of Western Australia	lei.chen@uwa.edu.au
Katie Clinch	UNSW Sydney	k.clinch@unsw.edu.au
Marston Conder	University of Auckland	m.conder@auckland.ac.nz
Sara Davies	The University of Queensland	sara.davies@uq.edu.au
Ajani De Vas Gunasekara	The University of Notre Dame, Australia	ajani.de.vas.gunasekara@nd.edu.au
Alice Devillers	University of Western Australia	alice.devillers@uwa.edu.au
Marc Distel	Monash University	Marc.Distel@monash.edu
Diane Donovan	The University of Queensland	dmd@maths.uq.edu.au
Christopher Duffy	University of Melbourne	christopher.duffy@unimelb.edu.au
Muhammad Talha Farooq	Macquarie University Sydney New South Wales	muhammadtalharao1@gmail.com
Graham Farr	Monash University	Graham.Farr@monash.edu
Afsane Ghafari Baghestani	Monash University	afsane.ghafaribaghestani@monash.edu
Gary Greaves	Nanyang Technological University, Singapore	grwgrvs@gmail.com
Raju Kumar Gupta	Indian Institute of Technology Delhi	rajugupta6174@gmail.com
Hao Chuien Hang	National Institute of Education, Singapore	hanghc@hotmail.com
Sakander Hayat	Universiti Brunei Darussalam	sakander.hayat@ubd.edu.bn
Kevin Hendrey	Monash University	kevin.hendrey1@monash.edu
Hung Hoang	TU Wien	phuchung.hoang@gmail.com
Daniel Horsley	Monash University	daniel.horsley@monash.edu
Koji Imamura	Kumamoto University	k-imamura@kumamoto-u.ac.jp
Mikhail Isaev	UNSW	isaev.m.i@gmail.com
Pawaton Kaemawichanurat	King Mongkut's University of Technology	pawaton.kae@kmutt.ac.th
Tara Kemp	The University of Queensland	t.kemp@uq.net.au
Jonathan Klawitter	University of Auckland	jonathan.klawitter@auckland.ac.nz
Sarah Lawson	The University of Queensland	sarah.lawson@uq.edu.au
Melissa Lee	Monash University	melissa.lee@monash.edu
James Lefevre	The University of Queensland	j.lefevre@uq.edu.au
Florian Lehner	University of Auckland	florian.lehner@auckland.ac.nz
Paul Leopardi	ACCESS-NRI	paul.leopardi@anu.edu.au
Thomas Lesgourgues	University of Waterloo	tlesgourgues@uwaterloo.ca
Anita Liebenau	UNSW Sydney	a.liebenau@unsw.edu.au
Jie Ma	University of Science and Technology of China	jiema@ustc.edu.cn
Barbara Maenhaut	The University of Queensland	bmm@maths.uq.edu.au
Adam Mammoliti	UNSW Sydney	adam.mammoliti@outlook.com.au
Sam Mattheus	Vrije Universiteit Brussel	sam.mattheus@vub.be
Brendan McKay	Australian National University	brendan.mckay@anu.edu.au
Đorđe Mitrović	University of Auckland	dmit755@aucklanduni.ac.nz
Jack Neubecker	The University of Queensland	j.neubecker@uq.edu.au
Semin Oh	Kyungpook National University	semin@knu.ac.kr
Anita Pasotti	Università degli Studi di Brescia	anita.pasotti@unibs.it
Tomasz Popiel	Monash University	tomasz.popiel@monash.edu
Cheryl Praeger	University of Western Australia	cheryl.praeger@uwa.edu.au
M.Tariq Rahim	Abbottabad University of Science and Technology Pakistan	tariqsms@gmail.com
Guang Rao	National University of Singapore	generalrao@hotmail.com
Ian Roberts	CDU	ian.roberts@cdu.edu.au
Sourav Sarkar	Indian Institute of Technology Delhi	sarkarsourav610@gmail.com

Ian Seong	University of Wisconsin-Madison	iseong@wisc.edu
Shiksha Shiksha	La Trobe University, Bendigo Campus, Bendigo	22044854@students.ltu.edu.au
Matthew Slattery-Holmes	University of Otago	slama077@student.otago.ac.nz
I Nengah Suparta	Ganesha University of Education	nengah.suparta@undiksha.ac.id
Nye Taylor	UNSW	nyem.taylor@gmail.com
Brhane Gebremichel Tnsau	University of Science and Technology of China	brhane@ustc.edu.cn
Abdullahi Umar	Khalifa University	abdullahi.umar@ku.ac.ae
Geertrui Van de Voorde	University of Canterbury	geertrui.vandevoorde@canterbury.ac.nz
Lander Verlinde	University of Auckland	lver263@aucklanduni.ac.nz
Ian Wanless	Monash University	ian.wanless@monash.edu
David Wood	Monash University	david.wood@monash.edu
Nick Wormald	Monash University	nick.wormald@monash.edu
Takatomo Yamasaki	Kumamoto University	230d8568@st.kumamoto-u.ac.jp
Keita Yasufuku	Osaka Metropolitan University	keita0125sve@icloud.com
Zhaorui Zhang	University of Queensland	zhaorui.zhang@uq.net.au