46TH AUSTRALASIAN COMBINATORICS CONFERENCE

CMSA
Combinatorial Mathematics
Society of Australasia
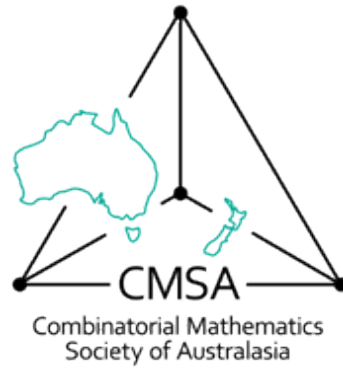
The University of Queensland, 2–6 December, 2024

The Institute
ICA of Combinatorics
and its Applications

THE UNIVERSITY
OF QUEENSLAND
AUSTRALIA

Welcome!

Welcome to 46ACC and welcome to Brisbane. This is the seventh time a conference in this series has been hosted at the University of Queensland. The first hosting here (3rd ACCM) was 50 years ago in 1974. In addition to oversight and assistance provided by the Combinatorial Mathematics Society of Australasia, we gratefully acknowledge support from the Institute of Combinatorics and its Applications, and the School of Mathematics and Physics at the University of Queensland. We are delighted to have 70 registrants in attendance, and we wish you an enjoyable stay in Brisbane.

The organisers:
Sara Davies
Barbara Maenhaut
Darryn Bryant

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Working Space

Room 67-146 (Level 1 of the Priestley Building) is available for use by conference attendees from 8am - 6pm Monday to Friday.

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2 Invited talks

On partial linear spaces and rank 3 groups

Alice Devillers

University of Western Australia

A partial linear space is an incidence structure consisting of points and lines such that every line contains at least 3 points and every pair of points is in at most a line. I will assume partial linear spaces to be finite, not graphs nor linear spaces. Going all the way back to my PhD dissertation, I have been interested in partial linear spaces with varying degrees of symmetry, called k-ultrahomogeneity. The weakest level of symmetry among these is when $k = 2$: when the automorphism group is transitive on the ordered pairs of collinear points and on the ordered pairs of non-collinear points. This is equivalent to the automorphism group having rank 3.

Primitive rank 3 groups are classified. In 2005 I classified partial linear spaces admitting a primitive almost simple rank 3 group, and in 2008 the ones with a primitive rank 3 group of grid type. The case of primitive affine rank 3 groups was much harder to tackle, but was finally done in 2021, with Joanna Fawcett, John Bamberg and Cheryl Praeger (except for a few 'hopeless' cases.)

Imprimitive rank 3 groups in general have not been classified, but in 2006, with Jonathan Hall, we managed to do the easiest case, when all lines have size 3. Recent work provided classification of imprimitive rank 3 groups with some extra assumptions. Rank 3 quasiprimitive groups were classified in 2011 (AD, Michael Giudici, Cai Heng Li, Geoffrey Pearce, Cheryl Praeger) and rank 3 innately transitive in 2023 (Anton Baykalov, AD and Cheryl Praeger). With Anton Baykalov and Cheryl Praeger, we have now classified the partial linear spaces with such groups, finding some nice infinite families and a small number of sporadic examples.

Graphs on finite groups

Melissa Lee

Monash University

Groups are often used to quantify the symmetries of graphs and other geometric objects. In recent decades, there has been accelerating interest in constructions in the opposite direction, namely graphs defined from group properties. In this talk, we will discuss several types of such constructions, demonstrate how deep group-theoretic properties can be translated into the language of graphs, and outline some recent work and open problems in the area.

Groups acting on trees and tree-like graphs

Florian Lehner

University of Auckland

The study of groups acting on infinite trees plays a foundational role in geometric group theory, and is instrumental in many other branches of mathematics such as algebraic topology (e.g. via the Seifert-van Kampen Theorem), and topological group theory (in particular, the study of totally disconnected, locally compact groups).

Bass-Serre theory is perhaps the most important tool for analysing group actions on trees. It relates group actions on trees to graphs of groups, allowing a description of the groups as iterated amalgamated free products and HNN extensions. Conversely, given a graph of groups, it allows us to construct a group action on a tree. Unfortunately, it is rather difficult to use this construction to obtain interesting new examples of groups acting on trees since the actions of vertex and edge stabilisers must satisfy strong compatibility conditions.

To overcome this issue, Reid and Smith recently introduced the theory of local action diagrams. This theory eliminates the need for compatibility conditions, but only allows for the construction of very specific groups. More precisely, it covers all groups satisfying Tits' property (P), in other words, groups for which there is no interaction between stabilisers of disjoint subtrees.

This talk consists of two parts. In the first part we give an overview of some key ideas in Bass-Serre theory and the theory of local action diagrams, highlighting the advantages and disadvantages of both approaches. In the second part, we introduce a new ("amalgamated") version of local action diagrams. The resulting theory can be thought of as "in between" Bass-Serre theory and the theory of local action diagrams in the sense that it allows us to model some interaction between stabilisers of disjoint subtrees, at the expense of re-introducing some weak compatibility conditions.

Although the motivation for much of the presented research comes from topological group theory, our methods are purely combinatorial. No background in topological group theory will be assumed.

The talk is based on joint work (at various stages of completion) with M. Chan, M. Hamann, C. Lindorfer, B. Miraftab, R. Möller, T. Rühmann, and W. Woess.

Ramsey with purple edges

Anita Liebenau

UNSW Sydney

The classical Ramsey number $R(s,t)$ is the smallest number n such that every red-blue colouring of the edges of the complete graph on n vertices contains a complete red subgraph on s vertices, or a complete blue subgraph on t vertices. Since the introduction of Ramsey numbers by Erdős and Szekeres in 1935, the quest for finding Ramsey numbers has not only inspired powerful methods in graph theory and probabilistic combinatorics but also revealed profound connections to logic, computer science, and discrete geometry.

Motivated by a question of David Angell, we study a variant of Ramsey numbers where some edges are coloured both red and blue, or: purple. Specifically, we are interested in the largest number $g = g(s,t,n)$, for some s and t and $n < R(s,t)$, such that there exists a red-blue-purple colouring of the edges of K_n with g purple edges, without a red-purple K_s and without a blue-purple K_t. We determine g asymptotically for a large family of parameters. The talk will be introductory in nature. Joint work with Thomas Lesgourgues and Nye Taylor.

A hypergraph bipartite Turán problem

Jie Ma

University of Science and Technology of China

In this talk, we investigate the hypergraph Turán number $\mathrm{ex}(n, K_{s,t}^{(r)})$. Here, $K_{s,t}^{(r)}$ denotes the r-uniform hypergraph with vertex set $\left(\cup_{i \in [t]} X_i\right) \cup Y$ and edge set $\{X_i \cup \{y\} : i \in [t], y \in Y\}$, where $X_1, X_2, \cdots, X_t$ are t pairwise disjoint sets of size $r-1$ and Y is a set of size s disjoint from each X_i. This study was initially explored by Erdős and has since received substantial attention in research. Recent advancements by Bradač, Gishboliner, Janzer and Sudakov have greatly contributed to a better understanding of this problem. They proved that $\mathrm{ex}(n, K_{s,t}^{(r)}) = O_{s,t}(n^{r-\frac{1}{s-1}})$ holds for any $r \geq 3$ and $s, t \geq 2$. They also provided constructions illustrating the tightness of this bound if $r \geq 4$ is *even* and $t \gg s \geq 2$. Furthermore, they proved that $\mathrm{ex}(n, K_{s,t}^{(3)}) = O_{s,t}(n^{3-\frac{1}{s-1}-\varepsilon_s})$ holds for $s \geq 3$ and some $\epsilon_s > 0$. Addressing this intriguing discrepancy between the behavior of this number for $r = 3$ and the even cases, Bradač et al. post a question of whether

$$\mathrm{ex}(n, K_{s,t}^{(r)}) = O_{r,s,t}(n^{r-\frac{1}{s-1}-\varepsilon}) \text{ holds for } \textit{odd } r \geq 5 \text{ and any } s \geq 3.$$

We provide an affirmative answer to this question, utilizing novel techniques to identify regular and dense substructures. This result highlights a rare instance in hypergraph Turán problems where the solution depends on the parity of the uniformity. This is joint work with Tianchi Yang.

Forbidden subgraphs: past, present and future

Sam Mattheus

Vrije Universiteit Brussel

The last few years have seen explosive progress on decades-old questions in extremal combinatorics, especially in problems involving forbidden subgraphs. While the probabilistic method has traditionally led the way on the constructive side of this area, recent advances show that ideas from algebra and geometry have become equally influential. However, despite intensive study of such objects in design theory, finite geometry, and algebraic graph theory, there has been relatively little interaction between these domains and extremal combinatorics. In this talk, we will explore the strengths and weaknesses of algebro-geometric constructions and highlight a few open problems.

Heffter Spaces

Anita Pasotti

University of Brescia - Italy

A *half-set* of a group $(G, +)$ of odd order is a complete system of representatives for the set of all pairs of opposite elements of $G \setminus \{0\}$.

Let $(G, +)$ be an abelian group of order $2v + 1 \geq 7$. A (v, k) *Heffter system* on G is a partition $\mathcal{P}$ of a half-set of G into zero-sum parts, called *blocks*, of size k. Two Heffter systems $\mathcal{P}$ and $\mathcal{Q}$ on the same half-set are *orthogonal* if every block of $\mathcal{P}$ intersects every block of $\mathcal{Q}$ in at most one element.

In 2015 Archdeacon [1] introduced the notion of a Heffter array as an interesting link between combinatorial designs and topological graph theory. In a few words a Heffter array is equivalent to a pair of orthogonal Heffter systems on a cyclic group. We refer to [6] for an extensive survey on these arrays, their variants and generalizations, and their connections to other topics.

In [2] we proposed the more general problem of constructing "many" mutually orthogonal Heffter systems, which led us to introduce a new combinatorial design, that we called a *Heffter space*. A $(v, k; r)$ Heffter space is a resolvable partial linear space of degree r whose point-set is a half-set of an abelian group G of order $2v+1$ and whose blocks are zero-sum k-subsets of G. One of the motivations for studying Heffter spaces is that every $(v, k; r)$ Heffter space with suitable properties gives rise to r mutually orthogonal k-cycle systems of order $2v + 1$, a topic recently studied in [4, 5].

References

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The global structure of planar graphs

David Wood

Monash University

Planar graphs are of fundamental importance in graph theory, for historical reasons and as a springboard into graph minor theory. This talk is about the global structure of planar graphs. The goal is to describe planar graphs in terms of much simpler tree-like graphs. I will describe several recent advances in this direction, and give sample applications that solve old open problems. Extensions to other minor-closed graph classes will also be presented.

3 Contributed talks

Subsquares in random Latin squares

Jack Allsop*

Monash University

(Joint work with Ian Wanless)

We prove that with probability $1 - o(1)$ as $n \to \infty$, a uniformly random Latin square of order n contains no subsquare of order 4 or more, resolving a conjecture of McKay and Wanless. We also show that the expected number of subsquares of order 3 is bounded.

Graph Covers and Minors

Dickson Annor*

La Trobe University

(Joint work with Michael Payne, David Wood)

A cover graph G of a graph H is a graph that 'locally looks like H' in the sense that there is a projection map from G to H that preserves neighbourhoods. Given a minor closed class $\mathcal{M}$. Define $\mathcal{F}_{\mathcal{M}}$ to be the class of graphs that have a cover in $\mathcal{M}$. In this talk we will discuss our work on certain classes of planar graphs $\mathcal{M}$. This is joint work with Michael Payne and David Wood.

Minimal simplicial degree d self-maps of $\mathbb{S}^{n-1} \times \mathbb{S}^1$

Biplab Basak

Indian Institute of Technology Delhi

(Joint work with Anshu Agarwal and Sourav Sarkar)

In topology, the degree of a map between orientable manifolds is a crucial invariant that provides deep insights into the structural and geometric properties of the manifolds involved, as well as the relationships between them. Understanding how to construct maps of a given degree between manifolds has been the focus of extensive research, especially within the context of orientable topological spaces.

In this talk, we present a novel construction of degree d simplicial maps between orientable manifolds, specifically focusing on the product manifold $\mathbb{S}^{n-1} \times \mathbb{S}^1$. For each integer $d \in \mathbb{Z}$, we construct a simplicial map of degree d from a colored triangulation of $\mathbb{S}^{n-1} \times \mathbb{S}^1$ with $2(n+1)\max|d|, 1$ facets to the standard $2(n+1)$-facet colored triangulation of $\mathbb{S}^{n-1} \times \mathbb{S}^1$. Our results demonstrate that these colored triangulations are minimal in the sense that they use the smallest possible number of facets necessary to support a degree d simplicial self-map of $\mathbb{S}^{n-1} \times \mathbb{S}^1$, where $n \geq 2$.

Furthermore, we extend our construction to provide a minimal degree d simplicial map from any closed orientable n-manifold to the standard n-sphere $\mathbb{S}^n$, for $n \geq 1$. These constructions not only shed light on the interplay between combinatorial and topological properties in the context of colored triangulations but also offer new tools for the study of simplicial maps and manifold topology.

Partitioning problems in discrete geometry

Abdul Basit

Monash University

Equipartition problems study how partitions of Euclidean spaces split families of measures. The quintessential example is the Ham Sandwich Theorem, which states that any d finite measures in $\mathbb{R}^d$, each absolutely continuous with respect to the Lebesgue measure, there exists a hyperplane that divides $\mathbb{R}^d$ into two half-spaces of the same size with respect to each measure.

We will survey some old and new equipartitioning results, and discuss some fascinating open problems. This talk is based on joint work with Boris Aronov, Indu Ramesh, Gianluca Tasinato and Uli Wagner.

Detachable pairs in 3-connected simple graphs and 3-connected matroids

Nick Brettell

Victoria University of Wellington

(Joint work with Charles Semple and Gerry Toft)

Tutte (1961) proved that a simple 3-connected graph G has an edge e such that either the deletion or contraction of e from G results in a graph that remains simple and 3-connected, unless G is a wheel. What if we instead ask for a pair of edges such that deleting both or contracting both retains simplicity and 3-connectedness? We call a pair of edges with this property *detachable*. In recent joint work with Gerry Toft and Charles Semple, building on work of Alan Williams (2014), we characterised the simple 3-connected graphs with no detachable pairs. In fact, we obtain this as a corollary of a more general result regarding detachable pairs in 3-connected matroids. In this talk, I will discuss this result, the motivation behind this work, and some potential applications.

Different ways of constructing infinite families of group divisible designs with two group sizes

Yudhistira Andersen Bunjamin

UNSW Sydney

(Joint work with R. Julian R. Abel, Thomas Britz, Diana Combe and Changyuan Wang)

A k-GDD, or group divisible design with block size k, is a triple $(X, G, \mathcal{B})$ where X is a set of points, G is a partition of X into subsets (called groups) and $\mathcal{B}$ is a collection of k-element subsets of X (called blocks) such that any two points from distinct groups appear together in exactly one block and no two distinct points from any group appear together in any block. There are a number of known necessary conditions for the existence of a GDD. However, these conditions are not sufficient.

Over the past five years, we have constructed infinite families of 3-GDDs and 4-GDDs with only two group sizes for several pairs of group sizes. For each of these pairs of group sizes, the work usually requires piecing together several recursive constructions for smaller infinite families of k-GDDs. More recent work focused on 4-GDDs with only groups of size 4 and 10. This family of 4-GDDs was constructed quite differently from those that were constructed in the past.

In this talk, we will compare the different ways in which these constructions of smaller families have been pieces together to obtain constructions for the much larger infinite families. We will discuss the insights on the advantages and disadvantages these different methods that have been gained through the recent work on 4-GDDs with groups of size 4 and 10.

Joint work with R. Julian R. Abel, Thomas Britz, Diana Combe and Changyuan Wang

S-**Arc-Transitivity of Vertex-Transitive Digraphs**

Lei Chen*

University of Western Australia

The investigation of s-arc-transitivity can be dated back to 1947. Tutte [4] studied cubic graphs and showed that a cubic graph can be at most 5-arc-transitive. A more general result for s-arc-transitivity of graphs was obtained by Weiss [5] and it turns out that finite undirected graphs of valency at least 3 that are not cycles can be at most 7-arc-transitive. In stark contrast with the situation in undirected graphs, Praeger [3] showed that for each s and d there are infinitely many finite s-arc-transitive digraphs of valency d that are not $(s + 1)$-arc-transitive.

However, once we add the condition of primitivity the situation gets quite different. Since the lack of evidence of existence of vertex-primitive 2-arc-transitive digraphs, Praeger [3] asked if there exists any vertex-primitive 2-arc-transitive digraph. The question was then answered in [1] and [2] by constructing infinite families of G-vertex-primitive $(G, 2)$-arc-transitive digraphs such that G is AS and SD types, respectively. In [2] Giudici and Xia then asked for a G-vertex-primitive (G, s)-arc-transitive digraph that is not a directed cycle, what is the upper bound on s. A reasonable conjecture is that $s \leqslant 2$. At the same time, Giudici and Xia [2] showed that to answer that question it suffices for us to consider the case when G is almost simple.

In this talk, I will introduce the current progress of the study of the s-arc-transitivity of vertex-primitive digraphs of various almost simple groups. Indeed, all of the studied almost simple groups follow the conjecture that $s \leqslant 2$. Moreover, I will also discuss a bit about the s-arc-transitive vertex-quasiprimitive digraphs and show that for vertex-quasiprimitive digraphs, s can also be unbounded.

References

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The Peaceable Queens Problem

Katie Clinch

UNSW Sydney

(Joint work with Matthew Drescher, Tony Huynh, and Abdallah Saffidine)

The *peaceable queens problem* asks to determine the maximum number $a(n)$ such that there is a placement of $a(n)$ white queens and $a(n)$ black queens on an $n \times n$ chessboard so that no queen can capture any queen of the opposite color.

This problem is first mentioned in Stephen Ainley's 1977 book *Mathematical Puzzles*, in which Ainley provides a general construction, giving the lower bound $a(n) \geq 0.1458n^2$. Perhaps surprisingly, this remains an optimal construction, despite many computational efforts at improvement. In comparison, the question of finding upper bounds on $a(n)$ has received little attention.

In this talk, we provide new asymptotic upper and lower bounds for $a(n)$, and for the variant problem on the toroidal board. This is joint work with Matthew Drescher, Tony Huynh, and Abdallah Saffidine.

Recent discoveries about degree-diameter-girth issues

Marston Conder

University of Auckland

The well-known *Moore graphs* (including odd-length cycles, the complete graphs, the Petersen graph and the Hoffman-Singleton graph) are regular graphs of maximum conceivable order with given degree and diameter, or equivalently, regular graphs of minimum conceivable order with given degree and girth (sometimes called *cages*), using the Moore bound.

More generally, the task of finding the largest regular graph with given degree and diameter is called the *degree-diameter problem*, and the corresponding one for given degree and girth is called the *cage problem*. Various people in the combinatorics community in Australia and New Zealand have contributed answers or partial answers to these problems.

At a BIRS workshop at Banff in May 2023, some investigations were made into the broader notion of a *degree-diameter-girth problem*, namely finding the largest regular graph with given degree, diameter and girth. I will report on some of what was discovered.

Completing partial k-star designs

Ajani De Vas Gunasekara

The University of Notre Dame Australia

(Joint work with Daniel Horsley)

A *k-star* is a complete bipartite graph $K_{1,k}$. A *partial k-star design of order n* is a pair $(V, \mathcal{A})$ where V is a set of n vertices and $\mathcal{A}$ is a set of edge-disjoint k-stars whose vertex sets are subsets of V. If each edge of the complete graph with vertex set V is in some star in $\mathcal{A}$, then $(V, \mathcal{A})$ is a (complete) *k-star design*. We say that $(V, \mathcal{A})$ is *completable* if there is a k-star design $(V, \mathcal{B})$ such that $\mathcal{A} \subseteq \mathcal{B}$. In this talk, I will discuss the problem of completing a partial design in general and present our result, which determines, for all values of k and n, the minimum number of stars in an uncompletable partial k-star design of order n. This is joint work with Daniel Horsley.

Proper Minor-Closed Classes as $O(\sqrt{n})$-blowups

Marc Distel*

Monash University

The strong product of graphs G and H is the graph with vertex set $V(G) \boxtimes V(H)$ and edges between distinct pairs of vertices if they are adjacent or equal in each co-ordinate. In this talk, we investigate for which simple graphs H are n-vertex K_t-minor graphs G contained in $H \boxtimes K_b$, with b roughly $O(\sqrt{n})$.

Mixed Graphs, Quasi-transitivity, Square Roots and the Oriented Chromatic Number

Christopher Duffy

University of Melbourne

In this talk we explore the structure of graphs arising from the transitive closure of an oriented graph, providing a connection between the classical notion of quasi-transitivity of directed graphs and the study of graph square roots. In doing so we find a full classification for graphs that arise as an undirected square of an orientation of a tree, and prove that the problem of deciding if a graph admits a mixed quasi-transitive orientation is NP-complete. We relate our work to on-going work in the study of the oriented chromatic number, defined using homomorphisms of oriented graphs.

k-Matching Configurations in Octagonal and Octagonal-Quadrilateral Structures

Muhammad Talha Farooq*

King Mongkut's University of Technology Thonburi Thailand, Macquarie University Sydney Australia

(Joint work with Pawaton Kaemawichanurat, Thap Panitanarak)

The Hosoya index of graph G is determined by counting the total number of k-matching, for all possible values of k. The Hosoya index holds significant importance in the realm of mathematical chemistry due to its correlation with various thermodynamic properties of hydrocarbons. Consequently, it is important to calculate the number of k-matchings of different chemical structures. In this work, we introduce a novel approach utilizing the transfer matrix technique to enumerate the number of k-matching, denoted as $p(G, k)$, within octagonal and octagonal-quadrilateral linear and random chains. Consequently, for all $k \geqslant 0$, the enumeration of $p(G, k)$ in arbitrary octagonal and octagonal-quadrilateral chains is achieved by employing a suitable combination of six transfer matrices, three matrices for each chain, with a dimension of $4(k+1) \times 4(k+1)$, along with a k-matching vector with dimension $4(k+1) \times 1$. This approach offers a comprehensive framework for determining $p(G, k)$ for a wide range of molecular structures, contributing to advancements in chemical graph theory and computational chemistry.

Interpretations of some transforms on binary functions

Graham Farr

Monash University

A *binary function* is a function $f : 2^E \to \mathbb{C}$ for which $f(\emptyset) = 1$, where E is a finite ground set. Binary functions generalise binary matroids in the sense that any indicator function of a linear space over GF(2) is a $\{0, 1\}$-valued binary function (using the natural correspondence between subsets of E and their characteristic vectors in $\mathrm{GF}(2)^E$). The author showed in 1993 that binary functions have deletion and contraction operations and extend arbitrary matroids, with duality corresponding to the Hadamard transform, and admit a generalisation of the Whitney rank generating function (a close relative of the Tutte polynomial). Subsequent papers (2004–2019) provided a family of transforms $L^{[\mu]}$ and associated minor operations, indexed by complex numbers μ, and developed their theory, with the identity transform and Hadamard transform corresponding to $\mu = 1$ and $\mu = -1$ respectively.

In this talk, we look at properties of transforms $L^{[\mu]}$ when $|\mu| = 1$. We use these transforms to characterise those binary functions for which the Hadamard transform is just the elementwise complex conjugate. We then give an interpretation of $L^{[\mu]}f$, for $|\mu| = 1$: it yields an appropriate quantum superposition of all the partial Hadamard transforms of f. We discuss the interpretation for the special case of plane graphs.

Edge-regular graphs with regular cliques

Gary Greaves

Nanyang Technological University, Singapore

In 1981, Arnold Neumaier posed the problem "Is every edge-regular graph with a regular clique strongly regular?" In 2018, Greaves and Koolen found an infinite family of edge-regular graphs that are not strongly regular but have regular cliques, thus answering Neumaier's question. Since 2018, many more families of such graphs (now known as Neumaier graphs) have been discovered.

In this talk, we will give a brief overview of the study of Neumaier graphs. We will then consider Neumaier graphs that are very close to being strongly regular in the sense of having a small coherent rank. Lastly, we will present a newly discovered family of Neumaier graphs, with cyclotomic constructions.

A characterization of normal 3-pseudomanifolds with at most two singularities

Raju Kumar Gupta*

Indian Institute of Technology Delhi

(Joint work with Biplab Basak, Sourav Sarkar)

Characterizing face-number-related invariants of a given class of simplicial complexes has been a central topic in combinatorial topology. In this regard, one of the well-known invariants is g_2. Let K be a normal 3-pseudomanifold such that $g_2(K) \leq g_2(\text{lk}\,(v)) + 9$ for some vertex v in K. Suppose either K has only one singularity or K has two singularities (at least) one of which is an $\mathbb{RP}^2$-singularity. In this talk, we prove that K is obtained from some boundary complexes of 4-simplices by a sequence of operations of types connected sums, bistellar 1-moves, edge contractions, edge expansions, vertex foldings, and edge foldings. In case K has one singularity, $|K|$ is a handlebody with its boundary coned off. Further, we prove that the above upper bound is sharp for such normal 3-pseudomanifolds.

Graphs whose generalized adjacency matrix has few distinct eigenvalues

Sakander Hayat

Universiti Brunei Darussalam

(Joint work with Ximing Cheng, Muhammad Javaid, Jack Koolen)

The generalized adjacency matrix of a graph Γ is defined as $M(x, y, z) := xI + yA + z(J - I - A)$, where x, y and z are real numbers satisfying $y \neq z$ and J (resp. I) is the all-ones matrix (resp. identity matrix) of suitable dimension. For a real number h, we define the matrix B_h by, $B_h := A + h(J - I)$. This implies that M is an affine transformation of B_h. Note that the Seidel matrix S of a graph defined as $S := J - I - 2A$, then $S = -2B_{-\frac{1}{2}}$. The adjacency matrix of a graph is $A = B_0$ and adjacency matrix of its complement is $\overline{A} := J - I - A$ and $\overline{A} = -B_{-1}$. Note that the B_h matrix is always irreducible if $h \notin \{0, -1\}$. In this talk, I will present some results regarding graphs with few distinct B_h-eigenvalues which we have proven recently. Graphs with two distinct B_h-eigenvalues have been characterized. Various results on connected graphs with three distinct B_h-eigenvalues have been proven. Disconnected graphs with three distinct B_h-eigenvalues are also characterized. As a by-product, we obtain the first example of non-regular non-bipartite graphs with three distinct distance eigenvalues.

The Planar Graph Product Structure Theorem With Shorter Paths

Kevin Hendrey

Monash University

(Joint work with David Wood)

The Planar Graph Product Structure Theorem of Dujmović, Joret, Micek, Morin, Uekerdt and Wood (2020) shows that every planar graph is a subgraph of the strong product of a path, a graph of treewidth 3 and clique of bounded size. This breakthrough result has been used to resolve or make progress on long standing open problems in a variety of areas including book embeddings of graphs, nonrepetetive colours and efficient constructions of universal graphs. Currently, there are no nontrivial bounds in the literature for the lengths of the paths in this result. We prove that every planar graph G is a subgraph of the strong product of a path P, a graph H of treewidth 3 and a bounded size clique, where the length of the path is $o(|V(G)|)$.

Signotopes with few plus signs

Hung Hoang

TU Wien

(Joint work with Helena Bergold, Lukas Egeling)

Signotopes are a combinatorial structure generalising binary strings, permutations, and simple pseudoline arrangements. Manin and Schechtman (1989) introduced the higher Bruhat order B(n,r), which is a natural order of the r-signotopes on n elements and a generalisation of the weak order of permutations. We show that the lower (and by symmetry upper) levels of the higher Bruhat order contain the same number of elements for a fixed difference $n - r$.

On decomposition thresholds for odd length cycles

Daniel Horsley

Monash University

(Joint work with Darryn Bryant, Peter Dukes, Barbara Maenhaut, Richard Montgomery)

An (edge) *decomposition* of a graph G is a set of subgraphs of G whose edge sets partition the edge set of G. I will discuss our recent proof that, for each odd $\ell \geq 5$, any graph G of sufficiently large order n with minimum degree at least $(\frac{1}{2} + \frac{1}{2\ell-4} + o(1))n$ has a decomposition into ℓ-cycles if and only if ℓ divides $|E(G)|$ and each vertex of G has even degree. This threshold cannot be improved beyond $\frac{1}{2} + \frac{1}{2\ell-2}$. It was previously shown that the thresholds approach $\frac{1}{2}$ as ℓ becomes large, but our thresholds do so significantly more rapidly. Our methods can be applied to tripartite graphs more generally and we also obtain some bounds for decomposition thresholds of other tripartite graphs.

On the Critical Problem for codes over $\mathbb{Z}/q\mathbb{Z}$

Koji Imamura

Kumamoto University

(Joint work with Norihiro Nakashima and Takuya Saito)

The *Critical Problem* posed by H. Crapo and G.-C. Rota in 1970 is one of the most significant problems in matroid theory. Let $\mathbb{F}_q$ be a finite field of q elements. For any subset $S \subseteq \mathbb{F}_q^k$, the *critical exponent* of S is defined as follows:

$$c(S; q) := k - \max\{\dim D : D \leq \mathbb{F}_q^k \text{ and } D \cap S = \emptyset\}.$$

Then the problem is to find the critical exponent for a given subset S.

They also proved the *Critical Theorem*, which provides the matroid-theoretic approach to the Critical Problem. Let $p(M; \lambda)$ denote the characteristic polynomial of a matroid M and let M/X denote the contraction of M by $X \subseteq E$. Then the theorem is described as follows.

Theorem *Let C be a k-dimensional subspace of $\mathbb{F}_q^n$ and set $E := \{1, \ldots, n\}$. For any $X \subseteq E$ and any $\ell \in \mathbb{Z}^+$, the number of ordered ℓ-tuples $(\boldsymbol{v}_1, \ldots, \boldsymbol{v}_\ell)$ of integers of vectors in C with* $\mathrm{supp}(\boldsymbol{v}_1) \cup \cdots \cup \mathrm{supp}(\boldsymbol{v}_m) = X$ *is* $p(M_C/(E - X); q^\ell)$, *where* $\mathrm{supp}(\boldsymbol{v}) = \{i \in E \mid v_i \neq 0\}$ *for $\boldsymbol{v} = (v_1, \ldots, v_n) \in \mathbb{F}_q^n$.*

In this talk, we examine the Critical Problem for codes over the ring of integers modulo $q \in \mathbb{Z}^+$. For the purpose, we consider the theory of characteristic quasi-polynomial introduced by H. Kamiya, A. Takemura, and H. Terao in 2008. As an application, we study the weight enumerator of codes over $\mathbb{Z}/q\mathbb{Z}$.

On the maximum number of common neighbours in dense random regular graphs

Mikhail Isaev

UNSW Sydney

(Joint work with Maksim Zhukovskii)

We derive the distribution of the maximum number of common neighbours of a pair of vertices in a dense random regular graph. The proof involves two important steps. One step is to establish the extremal independence property: the asymptotic equivalence with the maximum component of a vector with independent marginal distributions. The other step is to prove that the distribution of the number of common neighbours for each pair of vertices can be approximated by the binomial distribution. The talk is based on https://arxiv.org/pdf/2312.15370

Degree-based function index of trees, unicyclic graphs and bicyclic graphs with given bipartition

Pawaton Kaemawichanurat

King Mongkut's University of Technology Thonburi

(Joint work with Tomas Vetrik)

We investigate the degree-based function index $I_f(G) = \sum_{vw \in E(G)} f(d_G(v), d_G(w))$ of a graph G, where $E(G)$ is the set of edges of G, $d_G(v)$ and $d_G(w)$ are the degrees of vertices v and w in G, respectively, and f is a symmetric function of two variables which satisfies some conditions. We obtain sharp upper bounds on I_f for trees, unicyclic graphs and bicyclic graphs with given bipartition. Then, among trees and unicyclic graphs with given bipartition, we present graphs with the largest values of the general reduced second Zagreb index $GRM_a(G) = \sum_{vw \in E(G)}(d_G(v) + a)(d_G(w) + a)$ for $a > -1$, general Randić index $R_a(G) = \sum_{vw \in E(G)}[d_G(v)d_G(w)]^a$ and first general Gourava index $FGO_a(G) = \sum_{vw \in E(G)}[d_G(v)d_G(w) + d_G(v) + d_G(w)]^a$ for $a \geqslant 1$, general Sombor index $SO_{a,b}(G) = \sum_{vw \in E(G)}([d_G(v)]^a + [d_G(w)]^a)^b$, generalized Zagreb index $GZ_{a,b}(G) = \sum_{vw \in E(G)}([d_G(v)]^a[d_G(w)]^b + [d_G(v)]^b[d_G(w)]^a)$ and one other general index $M_{a,b}(G) = \sum_{vw \in E(G)}[d_G(v)d_G(w)]^a[d_G(v) + d_G(w)]^b$ for $a \geqslant 1$ and $b \geqslant 1$.

Latin squares with disjoint subsquares

Tara Kemp*

The University of Queensland

A latin square is an n by n array with each of n symbols occurring exactly once in each row and column, and a subsquare is a smaller latin square within a latin square. Given a partition $n = n_1 + n_2 + \cdots + n_k$, a realization is a latin square with disjoint subsquares of orders $n_1, n_2 \ldots, n_k$. The existence of a realization for a given partition is a partially solved problem, and much of the difficulty of the problem comes from finding general constructions. In this talk, we will discuss how realizations can be found for large families of partitions, with the help of timetabling, graph theory, and rational numbers.

Rectangular Duals on the Cylinder and the Torus

Jonathan Klawitter

University of Auckland

(Joint work with Therese Biedl, Philipp Kindermann)

A rectangular dual of a plane graph G is a contact representation of G by interior-disjoint rectangles such that (i) no four rectangles share a point, and (ii) the union of all rectangles is a rectangle. In this talk, we look at when and how we can construct a rectangular dual of a graph embedded on the cylinder or on the torus. While we can give a full characterisation and construction algorithm for cylindrical rectangular duals, the toroidal case has proven more challenging. Yet when we are also given a combinatorial description of rectangle adjacencies (a rectangular edge labelling), then we can test whether a graph embedded on the flat torus admits a toroidal rectangular dual.

Equitable colourings of Uniform Group Divisible Designs and Maximum Packings

James Lefevre

The University of Queensland

(Joint work with Andrea C. Burgess, Peter Danziger, Diane Donovan, Tara Kemp, David A. Pike, E. Şule Yazici)

An *equitable c-colouring* of a block design consists of a mapping from the underlying points to a set of c colours such that, for any block B and any two colours c_1 and c_2, the number of points of B mapped to c_1 differs from the number mapped to c_2 by at most one. A recent application arises from the theory of DNA data storage systems. The proposed system encodes binary sequences as "nicks" in the sugar-phosphate backbone of native DNA such as E. coli. Biological constraints on the nick positions imply that optimal efficiency can be obtained through the use of equitably 2-colourable maximum packings. We survey known results and open problems alongside related results for BIBDs and GDDs. We include new necessary and sufficient conditions for the existence of equitably colourable uniform GDDs, and outline an alternative, direct construction for equitably 2-colourable maximum packings with block size 4.

Big data in combinatorics: is it feasible?

Paul Leopardi

ACCESS-NRI, ANU

This talk updates the talk given at 42 ACCMCC in 2019 on FAIR databases in combinatorics. It describes work done by Bercic and others to build, catalog and expand FAIR mathematical databases, and presents ideas and use cases for a proposed multi-terabyte database of strongly regular graphs.

Odd-Ramsey numbers of complete bipartite graphs

Thomas Lesgourgues

University of Waterloo

(Joint work with Simona Boyadzhiyska, Shagnik Das, and Kalina Petrova)

In his study of graph codes, Alon introduced the "odd-Ramsey" number of a graph H, defined as the minimum number of colours needed to colour the edges of K_n so that every copy of H intersects some colour class in an odd number of edges. Focusing on the case where H is a complete bipartite graph, I will present a surprising link with a well-studied coding-theory parameter, maximising the dimension of a linear binary code avoiding codewords of given weights.

This is part of a more general article, joint work with Simona Boyadzhiyska, Shagnik Das, and Kalina Petrova.

On Minimising the Number of Subsets and Supersets of a Family of Sets

Adam Mammoliti

UNSW Sydney

(Joint work with Daniel Horsley and Adam Gowty)

Let $\mathcal{F}$ be a family of k-subsets of $[n]$. The lower shadow of $\mathcal{F}$ is the set of all $(k-1)$-subsets of $[n]$ that are a subset of at least one member of $\mathcal{F}$ and the upper shadow of $\mathcal{F}$ is the set of all $(k+1)$-subsets of $[n]$ that are a superset of at least one member of $\mathcal{F}$. The Kruskal-Katona Theorem states that over all k-uniform families $\mathcal{F}$ of subsets of $[n]$ with $|\mathcal{F}| = m$ the minimum size of the lower shadow of $\mathcal{F}$ is that of the lower shadow of the first m k-subsets of $[n]$ in the colexicographical order. An analogous result is is true if we instead minimise the size of the upper shadow of $\mathcal{F}$.

In this talk I present a generalisation of the Kruskal-Katona Theorem where the sets in the family $\mathcal{F}$ need not be the same size and we consider minimising the size of $\mathcal{F}^{\updownarrow}$, the set of all subsets of $[n]$ that are either a subset or a superset of at least one member of $\mathcal{F}$, over all families $\mathcal{F} \subseteq 2^{[n]}$ with m members. I present a recursive formula for the minimum size of $\mathcal{F}^{\updownarrow}$ for any n and $m \leq 2^n$. I also present a relative simple lower bound for the minimum size of $|\mathcal{F}^{\updownarrow}|$, namely it is true that $|\mathcal{F}^{\updownarrow}| \geq \sqrt{2^{n+2}m} - m$ where $\mathcal{F} \subseteq 2^{[n]}$ has $m \leq 2^n$ members. This bound is tight when it is an integer and is asymptotically correct as $n \to \infty$ with $m = \omega(1)$.

This is based on joint work with Daniel Horsley and Adam Gowty.

Some thoughts on Eulerian orientations

Brendan McKay

Australian National University

(Joint work with Mikhail Isaev, Rui-Ray Zhang)

Given an undirected graph with even degrees, an Eulerian orientation is an orientation of every edge such that each vertex has an equal number of incoming and outgoing edges. In addition to being combinatorially pleasant objects, Eulerian orientations have physical applications, most famously in the study of the behaviour of ice. Our focus will be on the number of Eulerian orientations. We will explain an asymptotic formula for graphs of sufficient density. The formula contains the inverse square root of the number of spanning trees, for which we do not have a heuristic explanation. We will also show a strong experimental correlation between the number of spanning trees and the number of Eulerian orientations even for graphs of bounded degree. This leads us to propose a new heuristic for the number of Eulerian orientations which performs much better than previous heuristics for graphs of chemical interest.

Exponential graph growth via eigenspaces of graphs over finite fields

Đorđe Mitrović*

University of Auckland

(Joint work with Gabriel Verret, Jeroen Schillewaert, Florian Lehner)

Let Γ be a finite connected graph and G a vertex-transitive group of its automorphisms. The pair (Γ, G) is called locally-L if the group induced by the action of the vertex-stabiliser G_v on the neighbourhood of a vertex v is permutation isomorphic to L. The graph growth of a permutation group L describes the growth of the order of G_v as a function of the order of Γ across locally-L pairs (Γ, G).

In this talk, we present new constructions of symmetric graphs with "large" eigenspaces over finite fields, notably infinite graphs that admit eigenvectors with finite support. As a corollary, we obtain that the graph growth of certain imprimitive permutation groups is exponential.

Factor-critical hypergraphs

Jack Neubecker*

The University of Queensland

(Joint work with Darryn Bryant, Sara Davies)

We call a hypergraph $f$actor-critical if it has no 1-factor but the deletion of any single vertex leaves a hypergraph with a 1-factor. This is a natural generalisation of the well-studied notion of factor-criticality of graphs. We are interested in the minimum number of hyperedges in a factor-critical hypergraph of a given order. We prove that the minimum number of hyperedges in a factor-critical k-uniform hypergraph of order n is exactly n for all $n \equiv 1 \pmod{k}$, $n > 1$. However, we require non-linear hypergraphs to achieve this bound. We then focus on factor-critical linear 3-uniform hypergraphs and establish some results on the number of hyperedges in such hypergraphs.

On Q-integral graphs with small Q-spectral radius

Semin Oh

Kyungpook National University

(Joint work with Jeong Rye Park, Jongyook, Yoshio Sano Park)

For a graph, the sum of the adjacency matrix and the degree matrix is called the signless Laplacian matrix and denoted by Q. If Q of a graph G has only integer eigenvalues, G is called a Q-integral graph. We show classifications of Q-integral graphs with fixed Q-spectral radius up to 6.

Flag-transitive, imprimitive, 2-designs

Cheryl E. Praeger

University of Western Australia

(Joint work with C. Amarra, A. Devillers, A. Montinaro)

A 2-(v, k, λ) *design* $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ consists of a set $\mathcal{P}$ of v *points* and a set $\mathcal{B}$ of *blocks* such that each block is a k-subset of $\mathcal{P}$ and each pair of distinct points is contained in λ blocks. A *flag* of $\mathcal{D}$ is an incident point-block pair, and a group G of automorphisms of $\mathcal{D}$ is *flag-transitive* if it acts transitively on the set of flags. Also G, and $\mathcal{D}$, are called *imprimitive* if G leaves invariant a nontrivial partition of the point set $\mathcal{P}$ (where the number of parts and the size of a part are each greater than 1). I will discuss new restrictions on the parameters v, k, λ for flag-transitive imprimitive designs, and new constructions of these designs.

On Radenković and Gutman conjecture for trees

M. Tariq Rahim

Department of Mathematics, Abbottabad University of Science and Technology

(Joint work with Afeefa Maryam)

Consider a simple graph G of order n and size m with Laplacian eigenvalues $\mu_1, \mu_2, \cdots, \mu_{n-1}$ and $\mu_n = 0$. The average degree of the graph G is defined as $\bar{d} = \frac{2m}{n}$. The Laplacian energy of the graph G, denoted by $LE(G)$, is defined as $LE(G) = \sum_{i=1}^{n} |\mu_i - \bar{d}|$. Radenković and Gutman conjectured that among all trees of order n, the path graph P_n has the smallest Laplacian energy. Let $T_n(d)$ denote the class of all trees of order n and diameter d. In this paper, we establish the Radenković and Gutman conjecture for $T \in T_n(5)$.

Marginal Analysis on Finite Sets

Ian Roberts

Charles Darwin University

An overview of a new mathematical approach is provided. The approach is called *Marginal Analysis* and it is applicable to a range of problems involving ordered collections of sets. The approach simplifies the analysis of some difficult contemporary problems and provides insight and solutions to a range of new or unsolved problems. These problems include the analysis of functions defined on collections of sets such as the Kruskal-Katona function and the Lubell function, and on various types of antichains and associated functions in the Boolean Lattice.

Marginal Analysis can be considered as a form of Discrete Calculus. On the surface, it is simple in concepts and requires a small mathematical tool-box, but one has to be discerning in the choice and blending of the parts. An overview of the approach will be described in a visual and non-technical form. Researchers in other areas may find some ideas and results of interest to them, and an opening into potential new work.

Marginal analysis uses the language of sets but the results can be applied to Hypergraphs, Graphs, Projective Geometry, Coding Theory, and Combinatorial Design. Antichains and the Kruskal-Katona function are core concerns, and thus the talk informs the study of independent sets, blocking sets, vertex covers, intersecting set sytems, combinatorial designs, and more.

Useful background ideas and knowledge include antichains, the Kruskal-Katona Theorem and the associated squashed (or colex) order, and the BLYM (or LYM) inequality.

An Upper Bound Theorem for Homology 4-Manifolds

Sourav Sarkar*

Indian Institute of Technology Delhi

(Joint work with Biplab Basak)

The g-vector of a simplicial complex contains significant information about the combinatorial and topological structure of the complex. Several classification results concerning the structure of normal pseudomanifolds and homology manifolds have been established in relation to the value of g_2, the third component of the g-vector. It is known that when $g_2 = 0$, all normal pseudomanifolds of dimensions at least three are stacked spheres. Walkup proved that a homology 3-manifold with $g_2 \leq 9$ is a triangulated sphere. We demonstrate that homology 4-manifolds with $g_2 \leq 5$ are triangulated spheres and are derived from triangulated 4-spheres with $g_2 \leq 2$ by a series of connected sum, bistellar 1- and 2-moves, edge contraction, edge expansion, and edge flipping operations. Furthermore, we establish that this inequality is optimally attainable, i.e., it cannot be extended to $g_2 = 6$.

A generalization of the Askey-Wilson relations using a projective geometry

Ian Seong*

University of Wisconsin-Madison

In this talk we present a generalization of the Askey-Wilson relations that involves a projective geometry. Let V denote a finite-dimensional vector space over a finite field. The corresponding projective geometry P is the poset consisting of the subspaces of V, with partial order by inclusion. We construct some matrices A,A* that have rows and columns indexed by P. These matrices are generalizations of the adjacency matrix and the dual adjacency matrix for some distance-regular graph called the Grassmann graph. We show that A,A* satisfy some relations that generalize the Askey-Wilson relations.

A study of floor plans through graphs

Shiksha Shiksha*

La Trobe University, Bendigo, Australia

(Joint work with Krishnendra Shekhawat)

In this study, we delve into the characterization of architectural floor plans using graph-theoretic concepts. We investigate adjacency graphs, which not only define module adjacencies but also reveal essential properties of floor plans. Planarity ensures floor plan existence, while biconnected graphs facilitate rectangular boundaries. However, separating triangles lead to non-rectangular rooms, and more than four corner-implying paths necessitate L-shaped or other non-rectangular boundaries. Furthermore, flippable vertices and flippable edges in graph labelings allow for multiple topologically distinct layouts. By understanding these graph properties, we provide insights into constructing diverse and efficient floor plans, crucial for both architectural design and algorithmic applications.

Pattern-Avoiding Peak Functions

Matthew Slattery-Holmes*

University of Otago

Given a set of permutations Π, let $\mathfrak{S}_n(\Pi)$ denote the elements of the symmetric group $\mathfrak{S}_n$ that avoid every element of Π. Hamaker, Pawlowski, and Sagan defined a function $R_n(\Pi)$ to be the sum of peak functions indexed by the peak sets of permutations in $\mathfrak{S}_n(\Pi)$. We investigate combinatorial properties of these functions, including when their expansion in terms of Schur Q-functions is symmetric and when it has positive coefficients for all $\Pi \subset \mathfrak{S}_3$. We make use of various combinatorial objects including Dyck paths and tableaux diagrams during our investigations.

Characterization of Strongly Graceful Unicyclic Graphs

I Nengah Suparta

Mathematics Department, Ganesha University of Education

(Joint work with M. Bača, M. Demange, A. Semaničová-Feňovčiková, N.L.D. Sintiari)

A *graph* $G := G(V, E)$ we mean as a system which consists of a finite non-empty set V of *vertices* and a possibly empty set E of 2-element subsets of V called *edges*. For the sake of convenience we write uv for 2-element subset $\{u, v\}$, $u, v \in V$. The cardinality of V, $|V|$, and the cardinality of E, $|E|$, are called *order* and *size* of G, respectively. Let f be an *injective* function from the vertex set V into the set $\{0, 1, \dots, |E|\}$. If the set

$$\{|f(u) - f(v)| : uv \in E\} = \{1, 2, \dots, |E|\},$$

then f is called *graceful labeling* for G, and the graph G is called *graceful*. A *matching* in G is a non empty subset M of E such that any two elements of M are not adjacent in G. The matching M is called *perfect* if every vertex of G is incident with an element of M. In this case, the graph G is called with *perfect matching*. Let G be a graceful graph with graceful labeling f and some perfect matching M. If in addition, we also have that $f(u) + f(v) = |E|$ for every $uv \in M$, then f is called *strongly graceful labeling* for G, and the graph G is called *strongly graceful*. In this talk we characterize unicyclic graphs as strongly graceful graphs.

Towards a classification of 1-homogeneous distance-regular graphs with positive intersection number a_1

Brhane Gebremichel Tnsau

University of Science and Technology of China

(Joint work with Jack H. Koolen , Mamoon Abdullah and Jae-Ho Lee)

Let Γ be a graph with diameter at least two. Then Γ is said to be 1-homogeneous (in the sense of Nomura) whenever for every pair of adjacent vertices x and y in Γ, the distance partition of the vertex set of Γ with respect to both x and y is equitable, and the parameters corresponding to equitable partitions are independent of the choice of x and y. Assume Γ is 1-homogeneous distance-regular with intersection number $a_1 > 0$ and $D \geqslant 5$. Define $b = b_1/(\theta_1 + 1)$, where b_1 is the intersection number and θ_1 is the second largest eigenvalue of Γ. In this talk we will show that if intersection number $c_2 \geqslant 2$, then $b \geqslant 1$ and one of the following (i)–(vi) holds: (i) Γ is a regular near $2D$-gon, (ii) Γ is a Johnson graph $J(2D, D)$, (iii) Γ is a halved ℓ-cube where $\ell \in \{2D, 2D + 1\}$, (iv) Γ is a folded Johnson graph $\bar{J}(4D, 2D)$, (v) Γ is a folded halved $(4D)$-cube, (vi) The valency of Γ is bounded by a function of b. Using this result, we will characterize 1-homogeneous graphs with classical parameters and $a_1 > 0$, as well as tight distance-regular graphs.

Latttice Paths and Order-Preserving Transformations of a Finite Chain

Abdullahi Umar

Khalifa University

(Joint work with A. Laradji)

Let $\mathcal{PO}_n$ be the monoid of order-preserving partial transformations on $[n] = \{1, 2, \ldots, n\}$. It is known that there exist bijections between $\mathcal{PO}_n$ and its subsemigroups with certain lattice paths that start from $(0, 0)$ and end at $(n-1, n-1$ in the Cartesian plane (Laradji and Umar, 2016). In this talk we are going to discuss several consequences of these bijections.

Anzahl theorems for classical spaces

Geertrui Van de Voorde

University of Canterbury

(Joint work with Maarten De Boeck)

Glasby, Niemeyer and Praeger (and later Glasby, Ihringer and Mattheus) derived lower bounds for the probability of spanning a non-degenerate classical space by two non-degenerate subspaces. This problem is motivated by algorithms to recognise classical groups. More precisely, given a vector space V and a quadratic, symplectic, or unitary form f on V, these authors determine lower bounds on the proportion of pairs (U, U') of non-degenerate subspaces U, U' with respect to f, such that U and U' are trivially intersecting and $<U, U'>$ is a non-degenerate subspace of V among all such pairs of non- degenerate subspaces (U, U'). In recent joint work with Maarten De Boeck, we improve on those results by deriving the exact formulae for this proportion for symplectic, hermitian and odd characteristic quadratic forms.

In this talk, I'll give a gentle introduction to the results obtained in our recent work.

Nearly-orthogonal sets over finite fields

Lander Verlinde*

University of Auckland

(Joint work with Rajko Nenadov)

For a field $\mathbb{F}$ and integers d, k and ℓ, a set $A \subseteq \mathbb{F}^d$ is called (k, ℓ)-nearly orthogonal if all vectors in A are non-self-orthogonal and every $k + 1$ vectors in A contain $\ell + 1$ pairwise orthogonal vectors. Determining the maximal size of such sets has been studied extensively for different fields, showing some interesting differences depending on its characteristic. Recently, Haviv, Mattheus, Milojević and Wigderson have improved the lower bound on nearly orthogonal sets over finite fields, using counting arguments and a hypergraph container lemma. We generalise this by showing the existence of a set $A \subseteq \mathbb{F}^d$ of essentially the same size, with a stronger property that given any family of subsets $A_1, \ldots, A_{\ell+1} \subset A$, each of size $k + 1$, we can find a vector in each A_i such that they are all pairwise orthogonal. This was previously known only for $\ell = 1$. Joint work with Rajko Nenadov.

Asymptotic normality for submaps of maps using high moments

Nick Wormald

Monash University

(Joint work with Michael Drmota and Eva Hainzl)

There is a species of koala that doesn't live in trees. These trees are of course graphs with no cycles; the koala is a graph properly embedded in the plane (i.e. a map) with two big ears at the side of its head. There are old results on the distribution of the number of copies of a given submap in a random planar map, and in particular these results imply that any given small submap appears with high probability. (As a consequence, to show that all maps are 4-colourable, it is enough to show that the number of large maps that are 4-colourable is not exponentially small.) But the actual asymptotic distribution of the number of copies of a given submap in a random map has only been pinned down for submaps which cannot overlap with each other. Koalas have nice big ears which can overlap in interesting ways, and to deal with this feature, the usual generating function approaches to showing asymptotic normality become intractible. We use instead a combination of generating function work and an extension of the "Poisson Paradigm" to distributions whose expected value tends to infinity.

On covering radii of rank metric codes

Takatomo Yamasaki*

Kumamoto University

(Joint work with Keisuke Shiromoto and Koji Imamura)

In classical coding theory, a linear code is a k-dimensional subspace C of $\mathbb{F}_q^n$. The Hamming distance between $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ in $\mathbb{F}_q^n$ is defined by $d_{\mathrm{H}}(x, y) := \{i \mid x_i \neq y_i\}$. Then, the *covering radius* $\rho(C)$ of a linear code C is defined as follows:

$$\rho(C) := \max_{x \in \mathbb{F}_q^n} \min_{c \in C} d_{\mathrm{H}}(x, c).$$

The covering radius is one of the important parameters in a linear code for understanding the decoding and the error-detecting capabilities of the code. The following relationship between the covering radius and the weight distributions was introduced by Delsarte in 1973:

Theorem (Delsarte bound) *Let $A_i(C^\perp)$ be the number of vectors in the dual space $C^\perp$ of a linear code C whose Hamming weight is i. Then $\rho(C) \leq s$, where s is the number of $i \in \{1, \ldots, n\}$ such that $A_i(C^\perp) \neq 0$.*

In this talk, we study the Delsarte bound for a *Delsarte rank metric code*, a subspace of the $\mathbb{F}_q$-linear space consisting of the $n \times m$ matrices. The bound has already been proved in terms of algebraic graph theory. However, we provide an alternative proof for the bound from a coding-theoretic perspective by introducing coset structure in rank metric codes. As an application of our coset structure, we propose a construction of *maximum rank distance* (MRD) codes from almost MRD codes.

On the minimum length of linear codes

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(Joint work with Tatsuya Maruta)

A q-ary linear code of length n with dimension k (an $[n,k]_q$ code) is a k-dimensional subspace of the n-dimensional row vector space over the field of q elements. An $[n,k]_q$ code with minimum distance d is called an $[n,k,d]_q$ code.

We consider the problem to find optimal $[n,k,d]_q$ codes. This problem in coding theory is that of optimizing one of the parameters length n, dimension k, minimum distance d for given the other two which is referred to as the "optimal linear code problem"".

Also, it is known that the Griesmer bound is attained for all sufficiently large d for fixed q and k.

We tackle the problem to find $D_{q,k}$, the largest d such that the Griesmer bound is not attained for fixed k and q. This problem is solved when q is much larger than k.

In this presentation, we give a conjecture of $D_{q,k}$ and show that our conjecture on $D_{q,k}$ is valid for $q + 1 \leqslant k \leqslant 8$ with $4 \leqslant q \leqslant 7$.

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