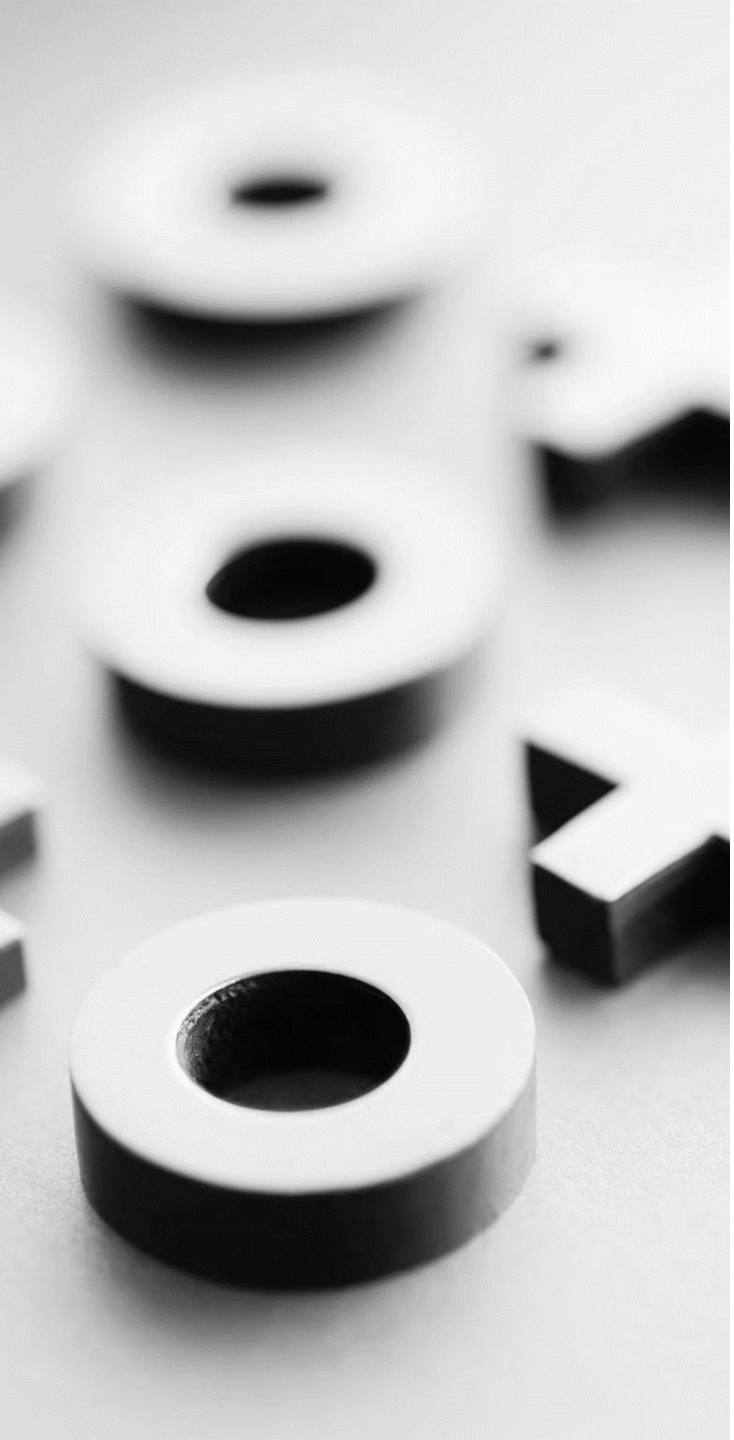
CS 5/7320
Artificial Intelligence

Adversarial Search and Games

AIMA Chapter 5

Slides by Michael Hahsler
with figures from the AIMA textbook

"Reflected Chess pieces" by Adrian Askew



Games

- Games typically confront the agent with a competitive (adversarial) environment affected by an opponent.
- We will focus on deterministic two-player zero-sum games with perfect information.
- We call the two players:
  1) **Max** tries to maximize his utility.
  2) **Min** tries to minimize Max’s utility since it is a zero-sum game.

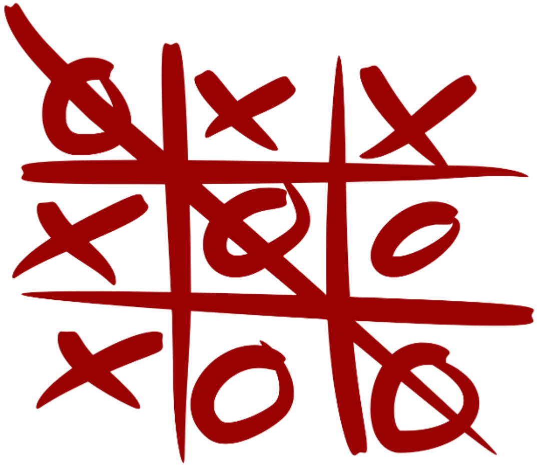
Definition of a Game

- **Definition:**

s_0	The initial state (position, board).
$Actions(s)$	Legal moves in state s.
$Result(s, a)$	Transition model.
$Terminal(s)$	Test for terminal states.
$Utility(s)$	Utility for player Max.

- **State space**: a graph defined by the initial state and the transition function containing all reachable states (e.g., chess positions).
- **Game tree**: a search tree superimposed on the state space. A complete game tree follows every sequence from the current state to the terminal state (the game ends).

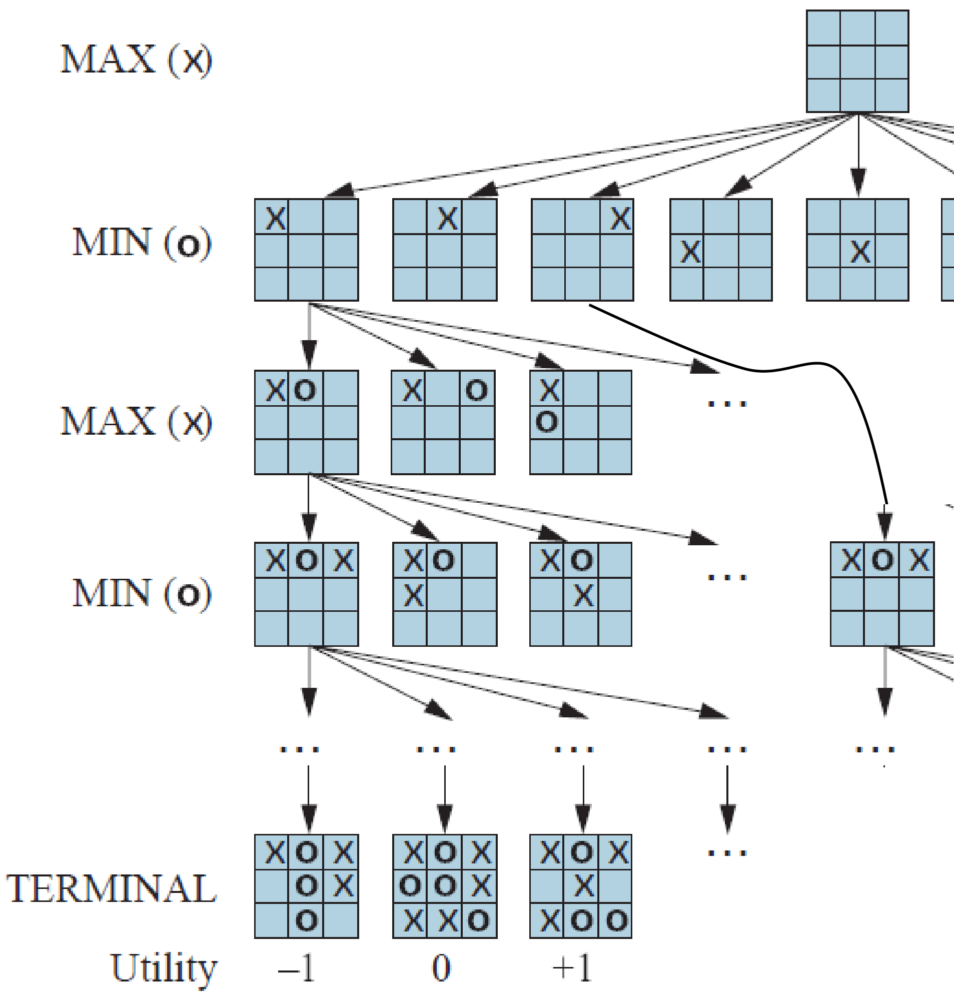
Example: Tic-tac-toe



s_0	Empty board.
$Actions(s)$	Empty squares.
$Result(s, a)$	Place symbol (x/o) on empty square.
$Terminal(s)$	Did a player win or is the game a draw?
$Utility(s)$	+1 if x wins, -1 if o wins and 0 for a draw.
	Utility is only defined for terminal states.

Here player x is Max and player o is Min.

Tic-tac-toe: Partial Game Tree



The state space is a little smaller than:

$$3^9 = 19{,}683$$

Why is the complete game tree size much larger? A little smaller than:

$$9! = 362{,}880$$

What can we do?

Methods for Adversarial Games

Exact Methods

- **Model as nondeterministic actions**: The opponent is seen as part of an environment with nondeterministic actions. Non-determinism is the result of the unknown moves by the opponent. We **consider all possible moves** by the opponent.
- **Find optimal decisions**: Minimax search and Alpha-Beta pruning where **each player plays optimal** to the end of the game.

Heuristic Methods

(game tree is too large)

- **Heuristic Alpha-Beta Tree Search**:
  a. Cut off game tree and use heuristic for utility.
  b. Forward Pruning: ignore poor moves.
- **Monte Carlo Tree search**: Estimate utility of a state by simulating complete games and average the utility.

Nondeterministic Actions

- Recall And-Or Search from AIMA Chapter 4

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Nondeterministic Actions

Each action consists of the move by the player and all possible (i.e., nondeterministic) responses by the opponent.

Outcome of actions in the environment is nondeterministic = **transition model need to describe uncertainty about the opponent's behavior.**

Example transition:

$$Results(s_1, a) = \{s_2, s_4, s_5\}$$

i.e., action a in s_1 can lead to one of several states.

AND-OR DFS Search Algorithm

= nested If-then-else statements

function AND-OR-SEARCH($problem$) **returns** a conditional plan, or $failure$
  return OR-SEARCH($problem$, $problem$.INITIAL, [ ])

function OR-SEARCH($problem, state, path$) **returns** a $conditional$ $plan$, or $failure$
  if $problem$.IS-GOAL($state$) **then return** the empty plan
  if IS-CYCLE($path$) **then return** $failure$ // don't follow loops
  for each $action$ **in** $problem$.ACTIONS($state$) **do** // check all possible actions
    $plan \leftarrow$ AND-SEARCH($problem$, RESULTS($state, action$), $[state] + path$])
    if $plan \neq failure$ **then return** $[action] + plan]$
  return $failure$

my moves

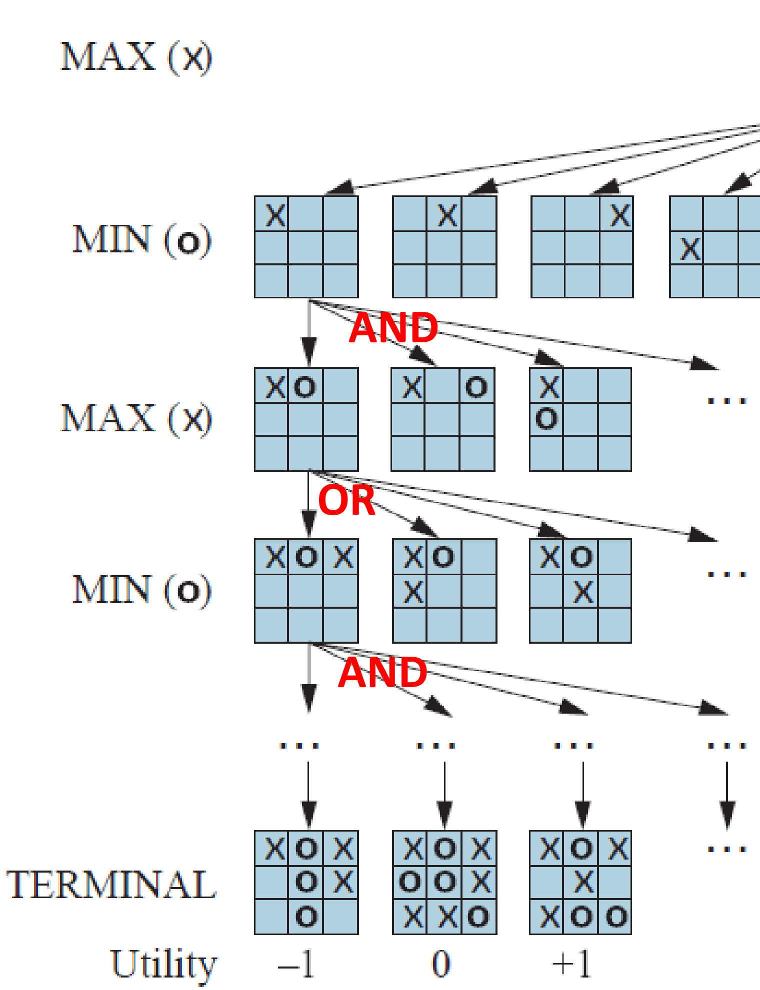
function AND-SEARCH($problem, states, path$) **returns** a $conditional$ $plan$, or $failure$
  for each s_i **in** $states$ **do** // check all states in belief state
    $plan_i \leftarrow$ OR-SEARCH($problem, s_i, path$)
    if $plan_i = failure$ **then return** $failure$
  return [**if** s_1 **then** $plan_1$ **else if** s_2 **then** $plan_2$ **else** ... **if** s_{n-1} **then** $plan_{n-1}$ **else** $plan_n$]

opponent moves

- And-Or Search searches the whole tree till it finds a subtree that leads only to goal nodes.
- BFS and A* search can also be used to search an AND-OR tree.

Tic-tac-toe: AND-OR Search

We play MAX and decide on our actions (OR).
MIN's actions introduce non-determinism (AND).



Objective: Find a subtree that has only win leaf nodes (utility +1).

We consider all opponent's moves in the AND stage. This includes MIN's best move. We call playing always the best move **playing optimally**.

Optimal Decisions

Minimax Search and Alpha-Beta Pruning

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Heuristic Methods
(game tree is too large)

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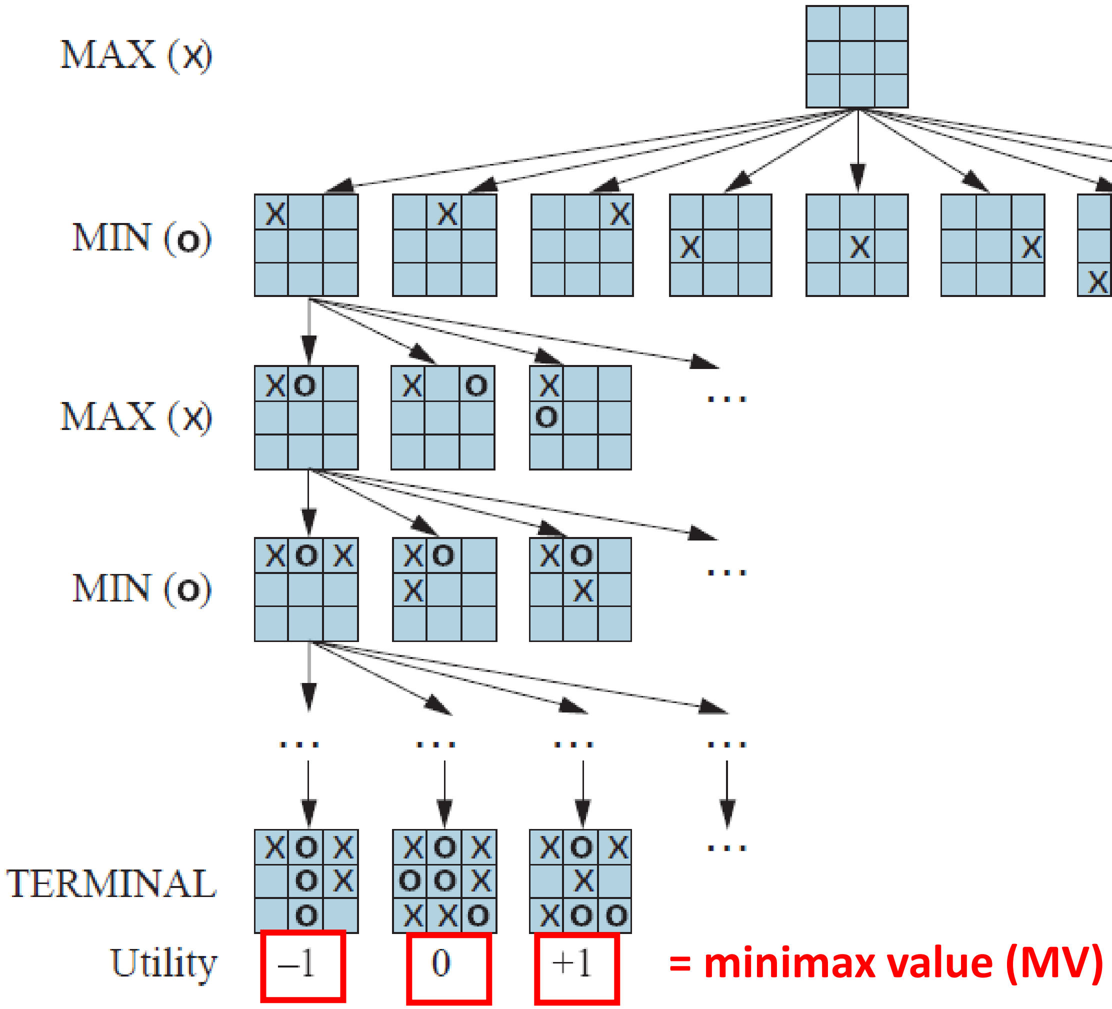
Idea: Minimax Decision

- Assign each state a **minimax value** that reflects how much Max prefers the state (= Min dislikes the state).

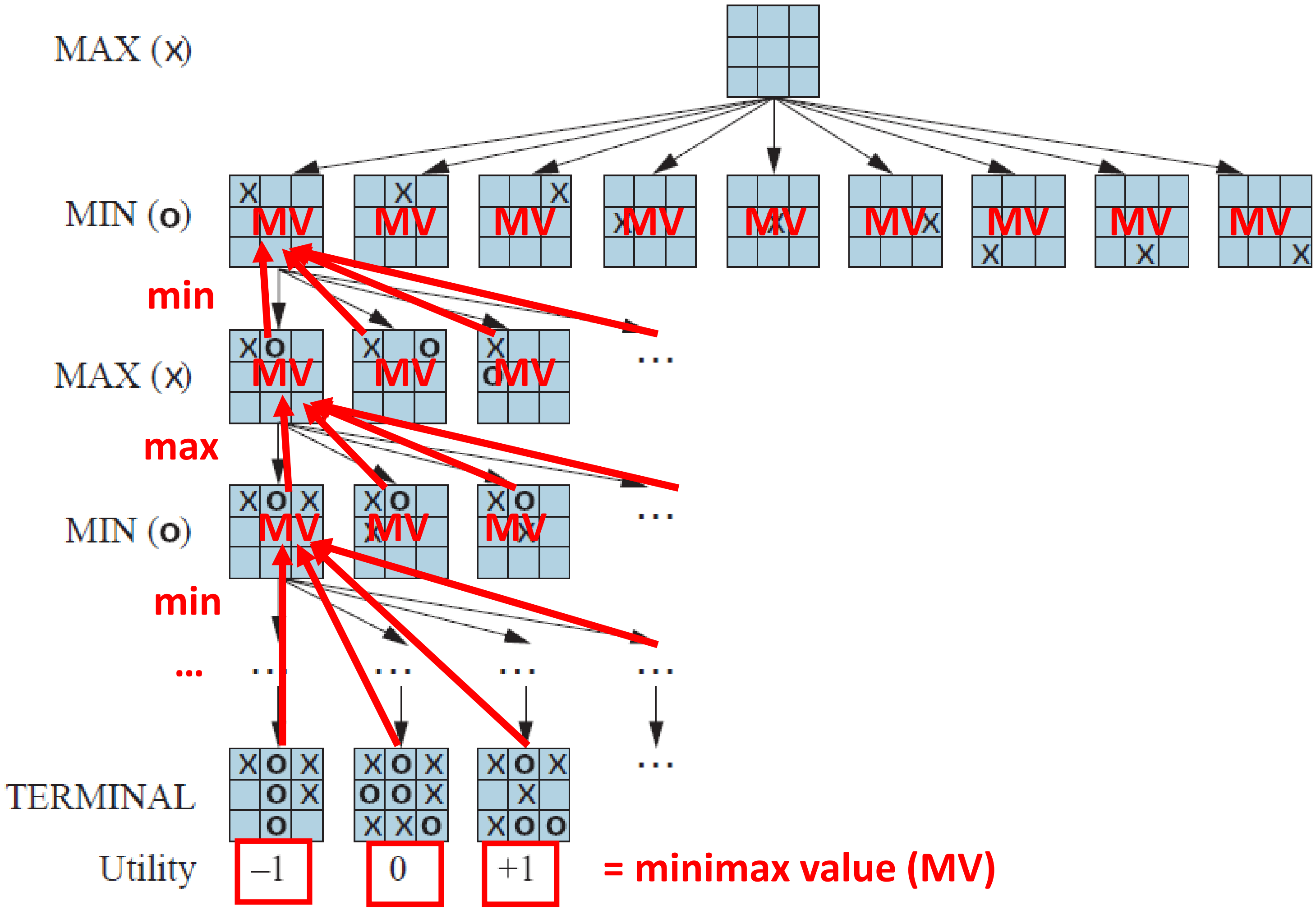
$$Minimax(s) = \begin{cases} Utility(s) & \text{if } terminal(s) \\ \max_{a \in Actions(s)} Minimax\big(Result(s,a)\big) & \text{if } move = Max \\ \min_{a \in Actions(s)} Minimax\big(Result(s,a)\big) & \text{if } move = Min \end{cases}$$

- The minimax value is the utility for Max in state s assuming that **both players play optimally** from s to the end of the game.
- The **optimal decision** for Max is the action that leads to the state with the largest minimax value.

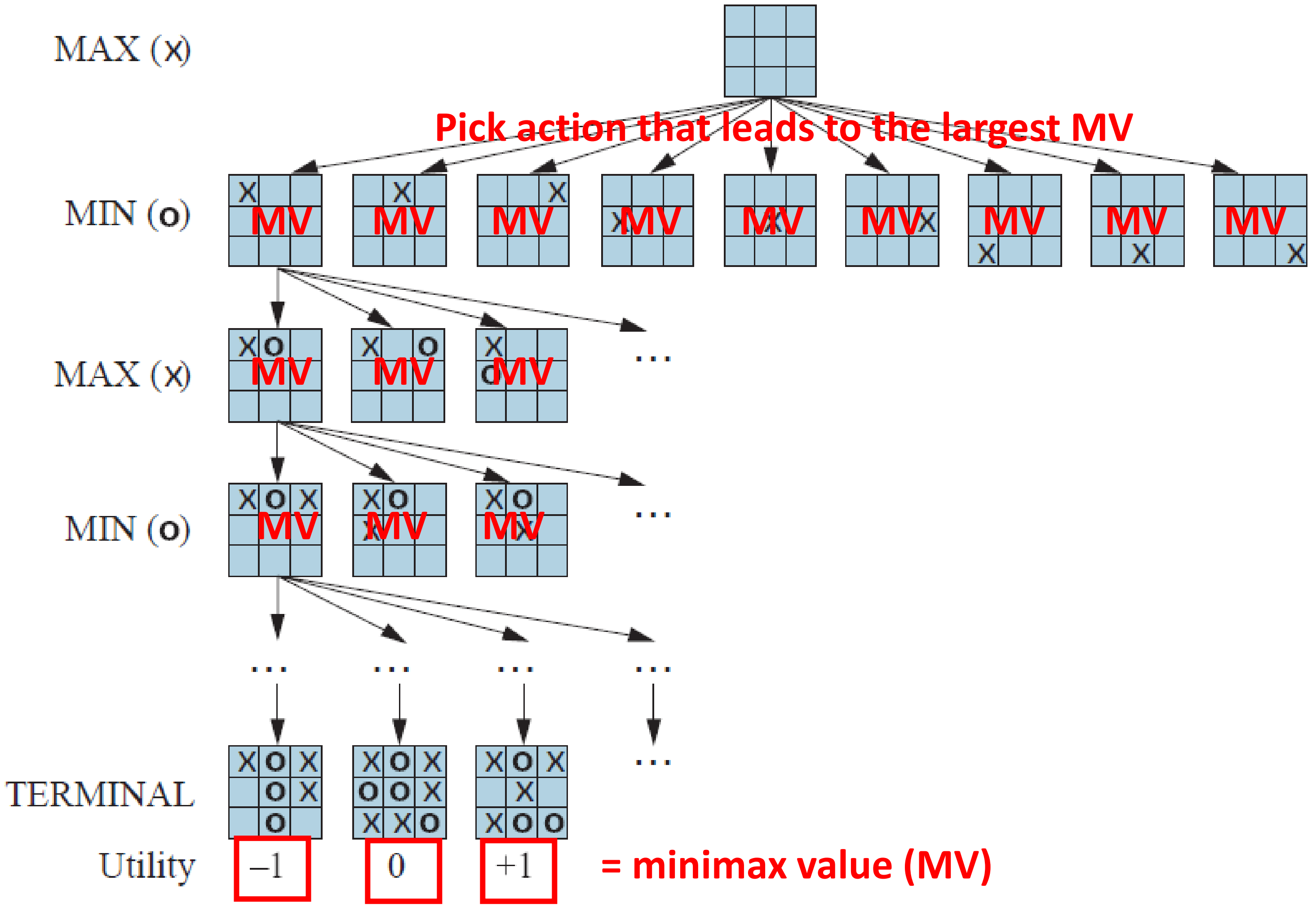
Minimax Search



Minimax Search : Back-up Minimax Values



Minimax Search: Decision



Approach: Follow tree to each terminal node and back up minimax value.

Note: This is just a generalization of the AND-OR Tree Search and returns first action of the conditional plan.

```
function MINIMAX-SEARCH(game, state) returns an action
  player ← game.TO-MOVE(state)
  value, move ← MAX-VALUE(game, state)
  return move

function MAX-VALUE(game, state) returns a (utility, move) pair
  if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null
  v ← −∞
  for each a in game.ACTIONS(state) do
    v2, a2 ← MIN-VALUE(game, game.RESULT(state, a))
    if v2 > v then
      v, move ← v2, a
  return v, move

function MIN-VALUE(game, state) returns a (utility, move) pair
  if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null
  v ← +∞
  for each a in game.ACTIONS(state) do
    v2, a2 ← MAX-VALUE(game, game.RESULT(state, a))
    if v2 < v then
      v, move ← v2, a
  return v, move
```

Represents OR Search

Represents AND Search

Issue: Game Tree Size

b: branching factor
m: max depth of tree

- This traverses the complete game tree using DFS!

Time complexity: $O(b^m)$

- Only feasible for very simple games!

- Example: Tic-tac-toe
  $b = 9, m = 9 \rightarrow O(9^9) = O(387{,}420{,}489)$
  b decreases from 9 to 8, 7, …
  $\rightarrow$ we get less than $O(9!) = O(362{,}880)$

- We need to reduce the search space! → **Game tree pruning**

Alpha-Beta Pruning

- **Idea**: Do not search parts of the tree if they do not make a difference to the outcome.

- **Observations**:
  - $\min(3, x, y)$ can never be more than 3
  - $\max(5, \min(3, x, y, \dots))$ does not depend on the values of x or y.
  - Minimax search applies alternating min and max.

- **Approach**: maintain for each node bounds for the minimax value $[alpha, beta]$ and prune subtrees that cannot be part of the solution.
  - Alpha is used by Max and means "$Minimax(s)$ is at least alpha."
  - Beta is used by Min and means "$Minimax(s)$ is at most beta."

= minimax search + pruning

```
function ALPHA-BETA-SEARCH(game, state) returns an action
   player ← game.TO-MOVE(state)
   value, move ← MAX-VALUE(game, state, −∞, +∞)
   return move

function MAX-VALUE(game, state, α, β) returns a (utility, move) pair
   if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null
   v ← −∞
   for each a in game.ACTIONS(state) do
      v2, a2 ← MIN-VALUE(game, game.RESULT(state, a), α, β)
      if v2 > v then
         v, move ← v2, a
         α ← MAX(α, v)
      if v ≥ β then return v, move
   return v, move

function MIN-VALUE(game, state, α, β) returns a (utility, move) pair
   if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null
   v ← +∞
   for each a in game.ACTIONS(state) do
      v2, a2 ← MAX-VALUE(game, game.RESULT(state, a), α, β)
      if v2 < v then
         v, move ← v2, a
         β ← MIN(β, v)
      if v ≤ α then return v, move
   return v, move
```

Notes:

- Pruning can be made more effective by **move ordering**: Check known good moves first to get a good bound early.
- Optimal decision algorithms still scale poorly!

Heuristic Alpha-Beta Tree Search

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Heuristic Methods
(game tree is too large)

- **Heuristic Alpha-Beta Tree Search**:
    a. Cut off game tree and use heuristic for utility.
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- **Monte Carlo Tree search**: Estimate utility of a state by simulating complete games and average the utility.

Idea: Cutting off search

Stop search at a node before the terminal node is reached.
Use a heuristic evaluation function $Eval(s)$ to approximate the utility for that node/state.

Properties of the evaluation function:

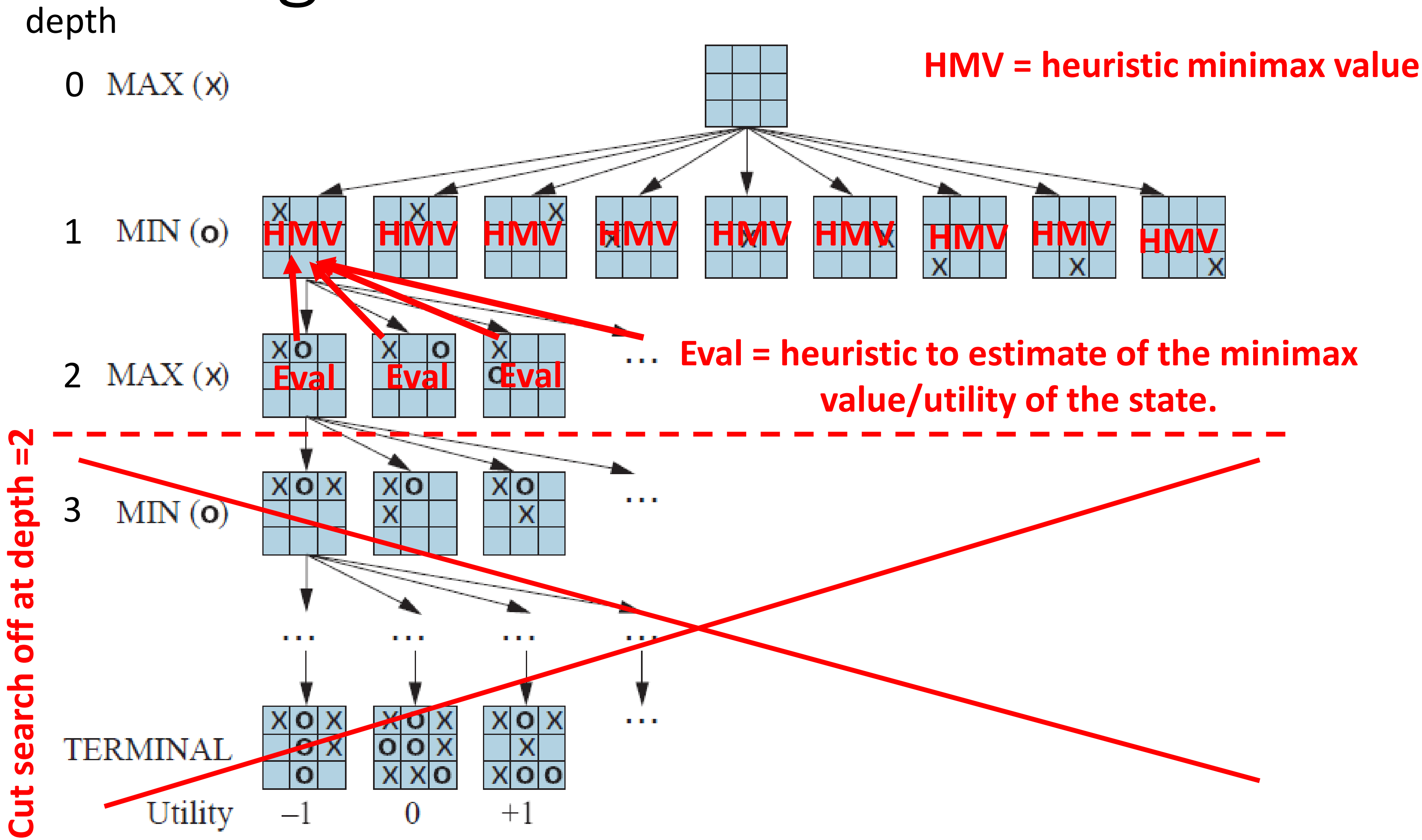
- Fast to compute.
- $Eval(s) \in [Utility(loss), Utility(win)]$
- Correlated with the actual chance of winning (e.g., using features of the state).

Example: A weighted linear function

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \cdots + w_n f_n(s)$$

where f_i is a feature of the state (e.g., # of pieces captured in chess).

Heuristic Alpha-Beta Tree Search: Cutting off search



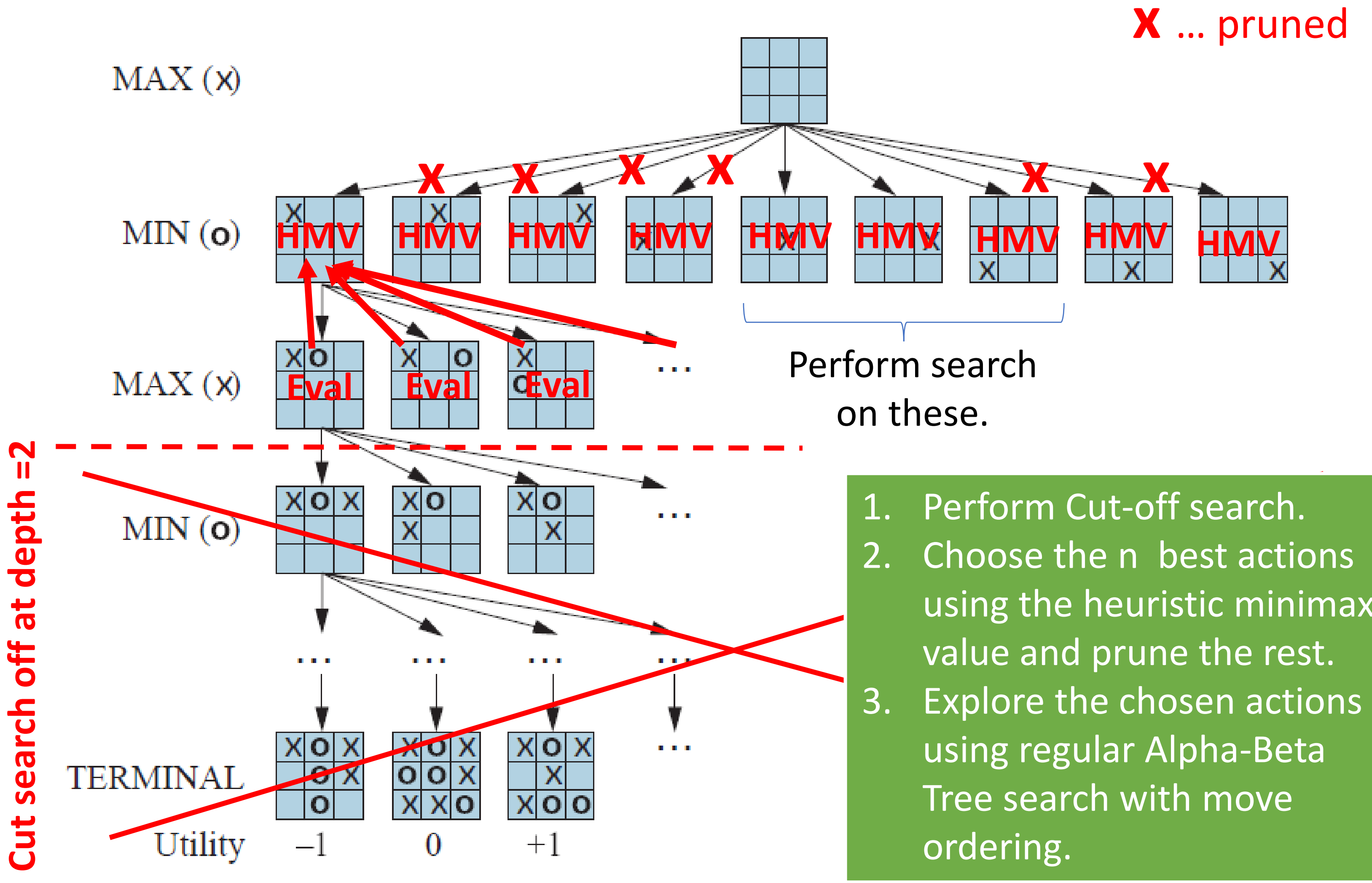
Idea: Forward pruning

Prune moves that appear poor. Poor can be evaluated in several ways:

- Low evaluation value after shallow search.
- Past experience.

Issue: May prune important moves.

Heuristic Alpha-Beta Tree Search: Forward Pruning Example



1. Perform Cut-off search.
2. Choose the n best actions using the heuristic minimax value and prune the rest.
3. Explore the chosen actions using regular Alpha-Beta Tree search with move ordering.

Monte Carlo Tree Search (MCTS)

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Idea

- **Approximate $\boldsymbol{Eval(s)}$** as the average utility of several simulation runs to the terminal state (called playouts).

- **Playout policy**: How to choose moves during the simulation runs? Example policies:
  - Random.
  - Heuristics for good moves developed by experts.
  - Learn good moves from self-play (e.g., with deep neural networks).

- Typically used for problems with
  - High branching factor (many possible moves).
  - Unknown or hard to define good evaluation functions.

Pure Monte Carlo Search

Find the next best move.

- Method
  1. Simulate N playouts from **current state**.
  2. Select the move with the highest win percentage.

- Converges to optimal play for stochastic games as N increases.
- **Do as many playouts as you can** given the available time.

Monte Carlo Tree Search

- Plan ahead and build a game tree using simulation.
- **Select the starting state** for playouts to focus on important parts of the game tree. It is a tradeoff between:

  a) **Exploration**: search from states that currently have few playouts.

  b) **Exploitation**: more playouts for states that have done well to get more accurate estimates.

Selection using Upper Confidence Bounds applied to Trees (UCT)

Tradeoff constant ($\approx \sqrt{2}$) can be optimizes using experiments

$$UCB1(n) = \frac{U(n)}{N(n)} + C\sqrt{\frac{\log(N(Parent(n)))}{N(n)}}$$

Average utility (=exploitation)

High for nodes with few playouts (=exploration)

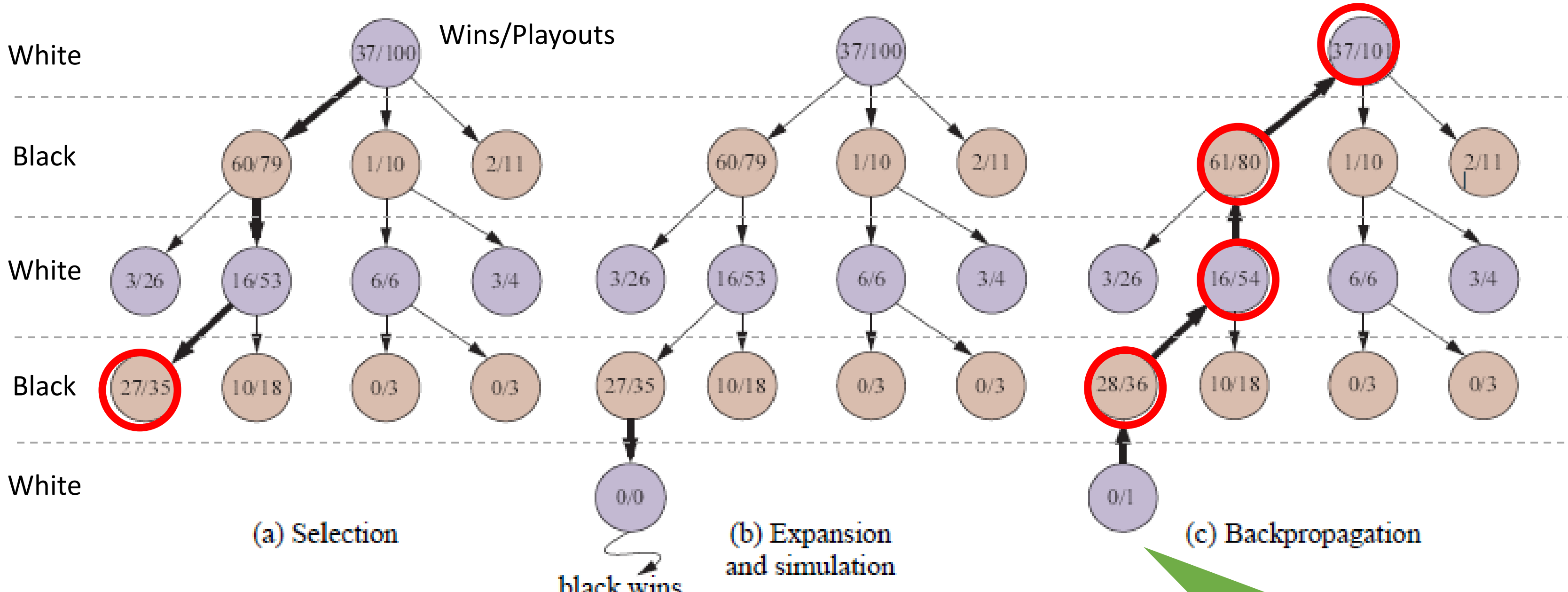
$U(n)$ … total utility of all playouts going through node n
$N(n)$ … number of playouts through n

Policy: Select leaf with highest UCB1 score.

function MONTE-CARLO-TREE-SEARCH(*state*) **returns** *an action*
  tree ← NODE(*state*)
  while IS-TIME-REMAINING() **do**
    leaf ← SELECT(*tree*)
    child ← EXPAND(*leaf*)
    result ← SIMULATE(*child*)
    BACK-PROPAGATE(*result*, *child*)
  return the move in ACTIONS(*state*) whose node has highest number of playouts

Highest UCB1 score

UCB1 selection favors win percentage more and more.



(a) Selection
(b) Expansion and simulation
(c) Backpropagation

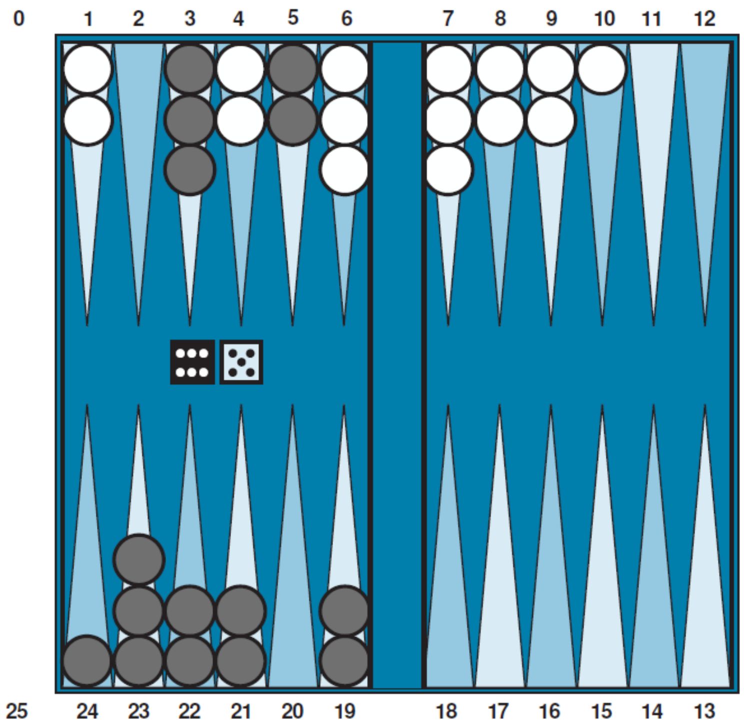
Note: the simulation path is not recorded!

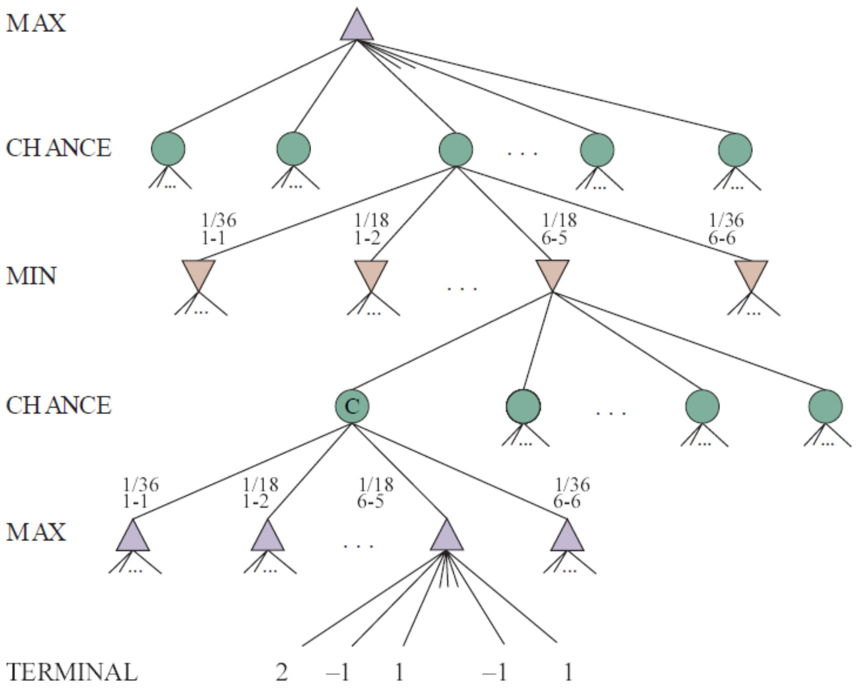
Stochastic Games

Games With Random Events

Stochastic Games

- Game includes a "random action" r (e.g., dice, dealt cards)
- Add **chance nodes** that calculate the expected value.





Backgammon

Expectiminimax

- Game includes a “random action” r (e.g., dice, dealt cards)
- For **chance nodes** we calculate the expected minimax value.

$$Expectiminimax(s) =$$

$$\begin{cases} Utility(s) & \text{if } terminal(s) \\ \max_{a \in Actions(s)} Expectiminimax\big(Result(s,a)\big) & \text{if } move = Max \\ \min_{a \in Actions(s)} Expectiminimax\big(Result(s,a)\big) & \text{if } move = Min \\ \boxed{\sum_r P(r) Expectiminimax\big(Result(s,r)\big) \quad \text{if } move = Chance} \end{cases}$$

- Options:
  - Use Minimax algorithm. Issue: Search tree size explodes if the number of “random actions” is large. Think of drawing cards for poker!
  - Approximate Expectiminimax with an evaluation function.
  - Perform Monte Carlo Tree Search.

Conclusion

Nondeterministic actions:

- The opponent is seen as part of an environment with nondeterministic actions. Non-determinism is the result of the unknown moves by the opponent. *All possible moves are considered*.

Optimal decisions:

- Minimax search and Alpha-Beta pruning where *each player plays optimal* to the end of the game.
- Choice nodes and Expectiminimax for stochastic games.

Scale only for tiny problems!

Heuristic Alpha-Beta Tree Search:

- Cut off game tree and use *heuristic evaluation function* for utility (based on state features).
- Forward Pruning: ignore poor moves.

Monte Carlo Tree search:

- Simulate complete games and calculate proportion of wins.
- Learn playout policy using self-play and deep learning.
- Use modified UCB1 scores to expand the game tree.

State of the Art