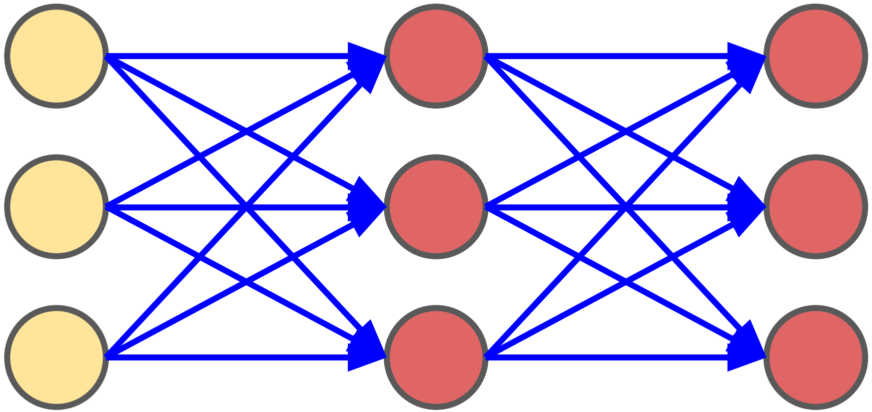
Neural Networks

1. Single Layer
2. Single Neuron
3. Multiple Layers
4. Input and Output



What does a single Neural Network Layer do ?

$$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 3 & 7 \\ 0 & 7 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 18 \\ 24 \\ 27 \end{bmatrix}$$

What does a single Neural Network Layer do

$$\begin{bmatrix} 18 \\ 24 \\ 27 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$$

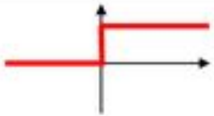
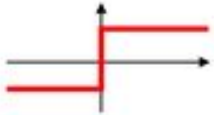
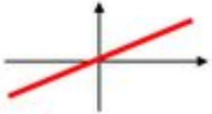
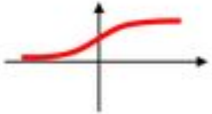
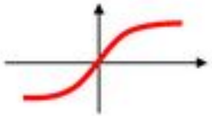
What does a single Neural Network Layer do

$$F\left(\begin{bmatrix}18\\24\\27\end{bmatrix}+\begin{bmatrix}-3\\0\\2\end{bmatrix}\right)$$

What does a single Neural Network Layer do

$$F\left(\begin{bmatrix}18\\24\\27\end{bmatrix}+\begin{bmatrix}-3\\0\\2\end{bmatrix}\right)$$

Activation Function

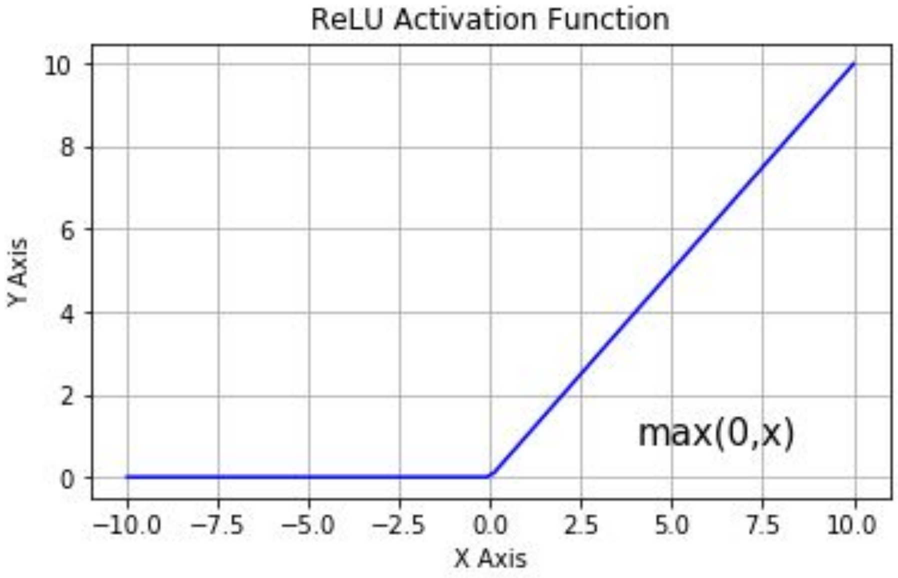
Activation function	Equation	Example	1D Graph
Unit step (Heaviside)	$\phi(z) = \begin{cases} 0, & z < 0, \\ 0.5, & z = 0, \\ 1, & z > 0, \end{cases}$	Perceptron variant	
Sign (Signum)	$\phi(z) = \begin{cases} -1, & z < 0, \\ 0, & z = 0, \\ 1, & z > 0, \end{cases}$	Perceptron variant	
Linear	$\phi(z) = z$	Adaline, linear regression	
Piece-wise linear	$\phi(z) = \begin{cases} 1, & z \geq \frac{1}{2}, \\ z + \frac{1}{2}, & -\frac{1}{2} < z < \frac{1}{2}, \\ 0, & z \leq -\frac{1}{2}, \end{cases}$	Support vector machine	
Logistic (sigmoid)	$\phi(z) = \frac{1}{1 + e^{-z}}$	Logistic regression, Multi-layer NN	
Hyperbolic tangent	$\phi(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$	Multi-layer NN	

https://en.wikipedia.org/wiki/Activation_function

ReLU Activation function

It stands for Rectified Linear Unit

$$\mathrm{ReLU}(x) = \max(x,0)$$



What does a single Neural Network Layer do

$$F\left(\begin{bmatrix} 18 \\ 24 \\ 27 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} F(15) \\ F(24) \\ F(29) \end{bmatrix}$$

What does a single Neural Network Layer do ?

$$F\left(\begin{bmatrix} 2 & 4 & 2 \\ 1 & 3 & 7 \\ 0 & 7 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}\right)$$

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Weight Matrix

What does a single Neural Network Layer do ?

$$F\left(\begin{bmatrix} 2 & 4 & 2 \\ 1 & 3 & 7 \\ 0 & 7 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}\right)$$

Bias Vector

What does a single Neural Network Layer do ?

$$F\left(\begin{bmatrix} 2 & 4 & 2 \\ 1 & 3 & 7 \\ 0 & 7 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}\right)$$

Input Vector

What does a single Neural Network Layer do ?

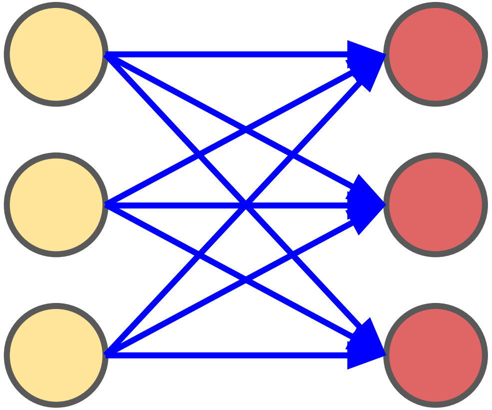
$$F\left(\begin{bmatrix} 2 & 4 & 2 \\ 1 & 3 & 7 \\ 0 & 7 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}\right)$$

Trainable Parameters

What does a single Neural Network Layer do ?

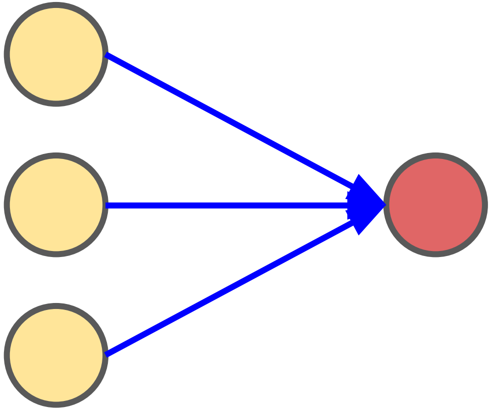
$$F(\mathbf{W}x + \mathbf{b})$$

What does a single Neural Network Layer do ?



$$F(\, \mathbf{W}x + \mathbf{b} \,)$$

What does a single Neuron do ?

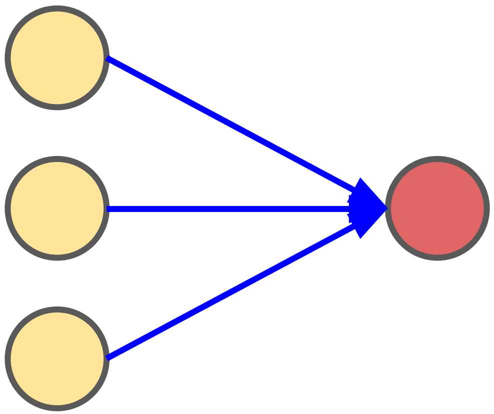


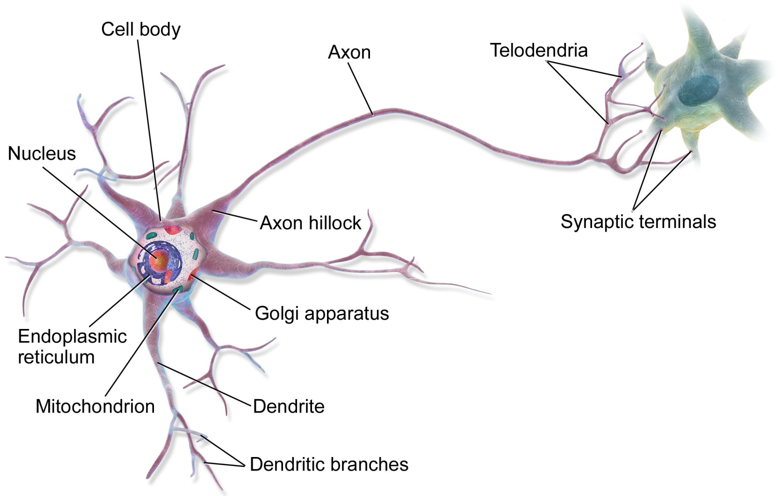
$$F(\, w_1^{\,T} x + b_1 \,)$$

Weights for each neuron

$$W = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 3 & 7 \\ 0 & 7 & 3 \end{bmatrix} = \begin{bmatrix} w_1^T \\ w_2^T \\ w_3^T \end{bmatrix}$$

What does a single Neuron do ?

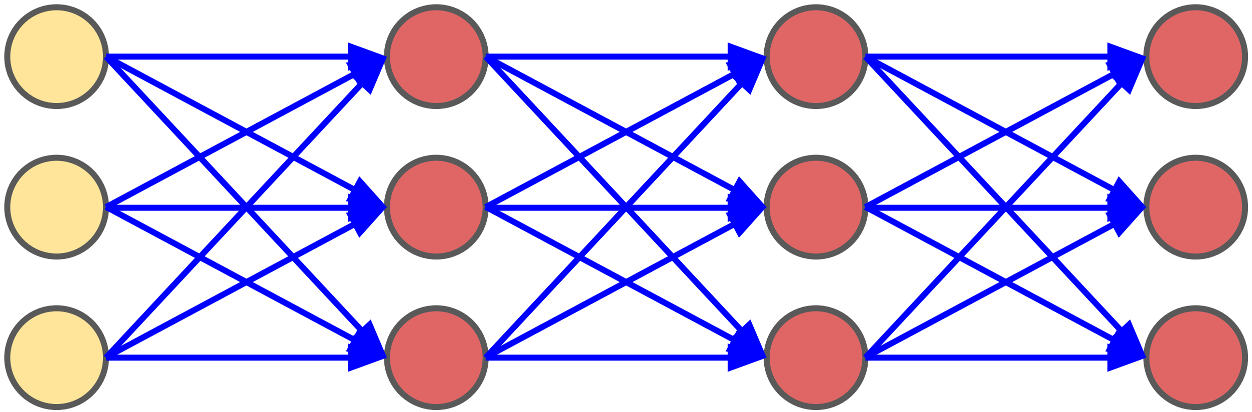




$$F(w_1^T x + b_1)$$

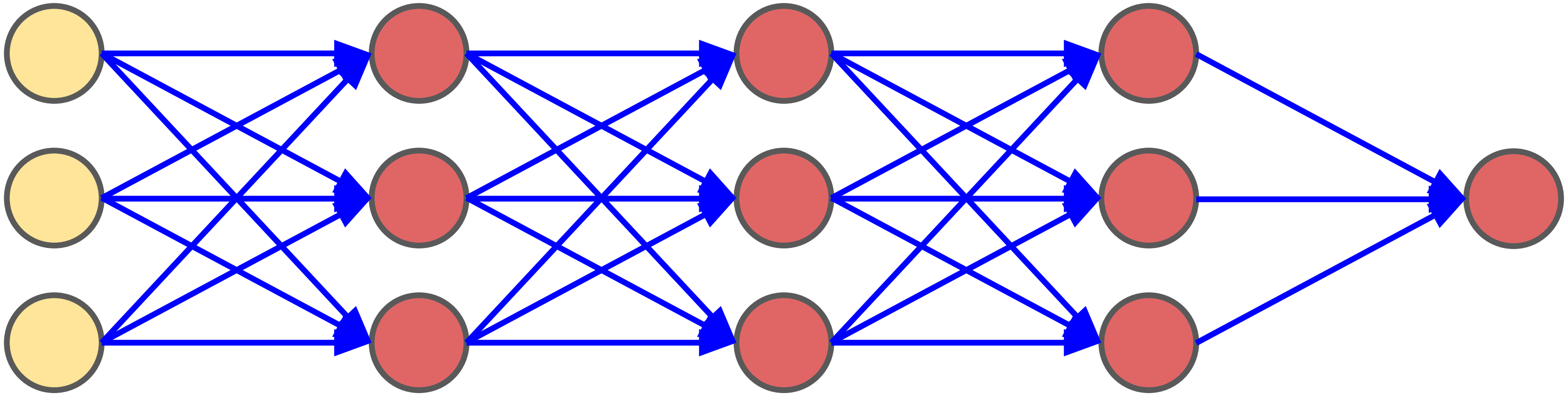
Playground

Stacking multiple layers



It's called a *deep* neural network when you use multiple layers

What are the inputs and outputs?



Gradient Descent

1. The Idea is to go downhill according to the linear approximation of your function
2. But not too much as your approximation will not be true
3. This means updating each weight in proportional to the gradient

$$w' = w - \alpha g$$

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Learning Rate

How to compute gradients - Backpropagation

Numerically approximating gradients is too slow and computationally expensive

The neural network is a well defined mathematical function, we should be able to differentiate it and calculate the derivatives

This is also too tedious so we instead use an algorithm called backpropagation

Computational Graph

$$F(x_1, x_2, x_3) = \max(\ w_1 x_1 + w_2 x_2,\ x_3\ )$$

Consider this simple function, imagine that this is the cost function and we want to find the gradients with respect to the two weights

We will first build the computational graph that this function represents

Computational Graph

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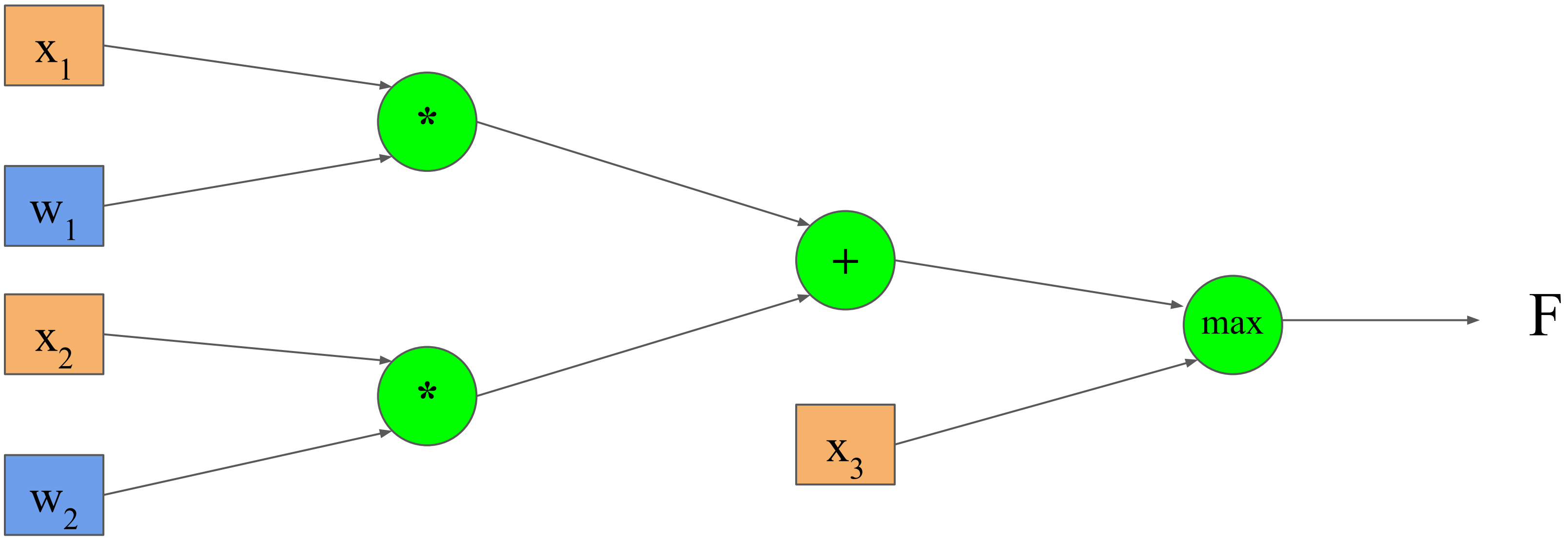
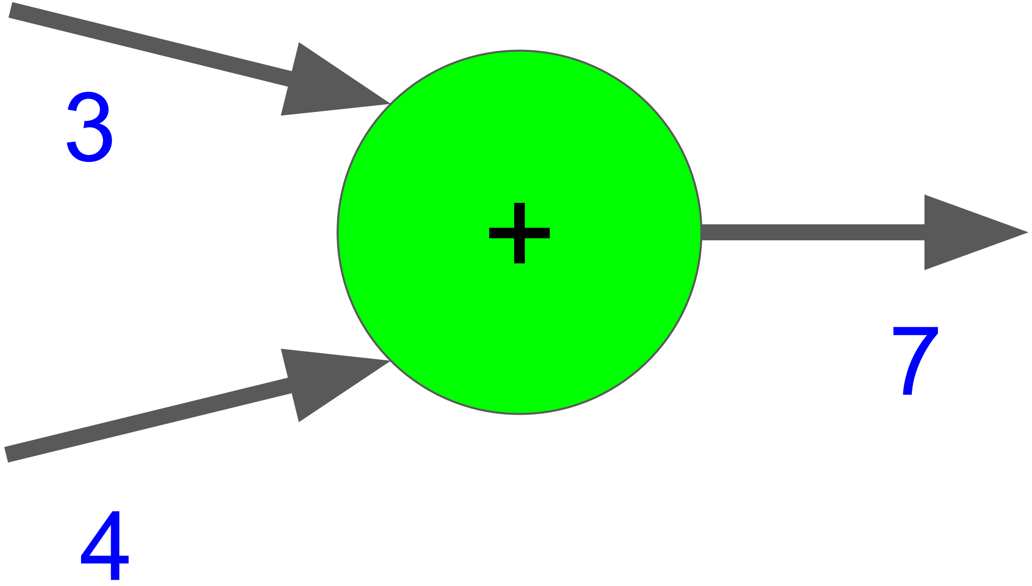


Forward propagation



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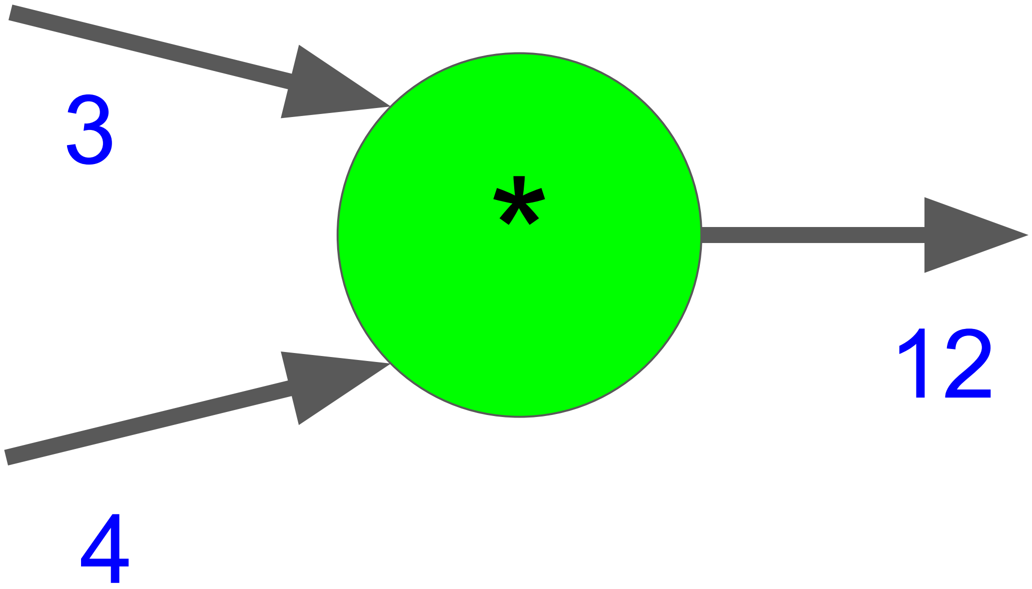

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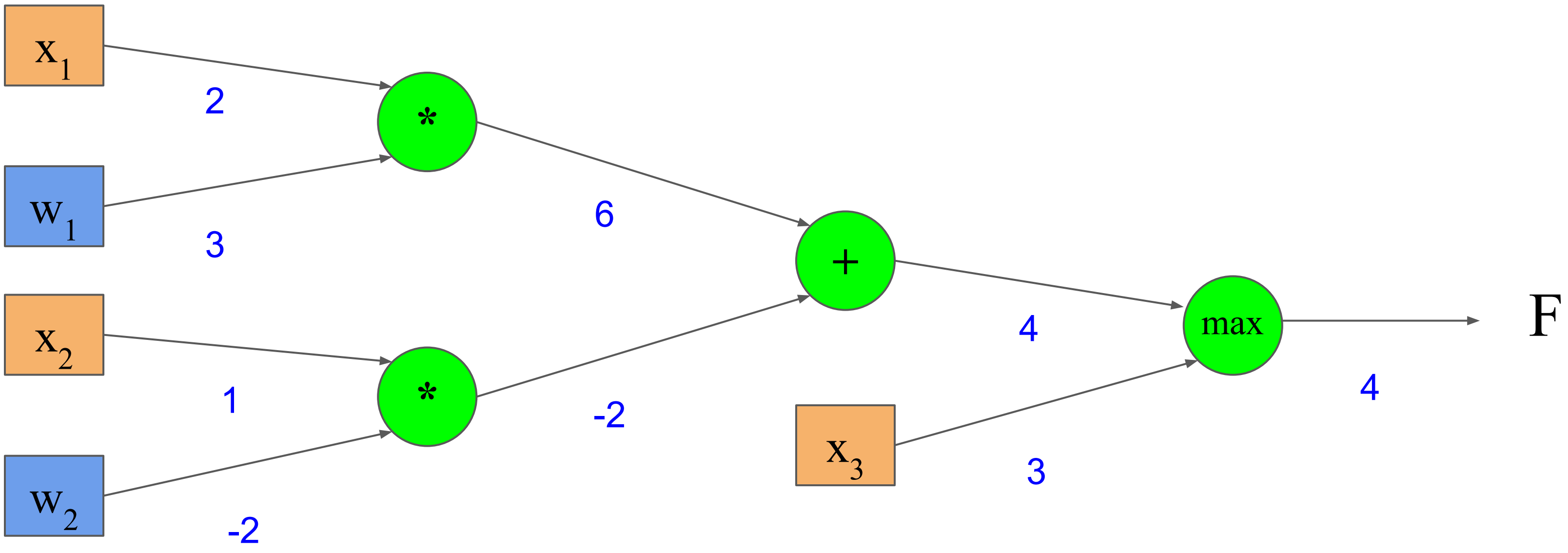


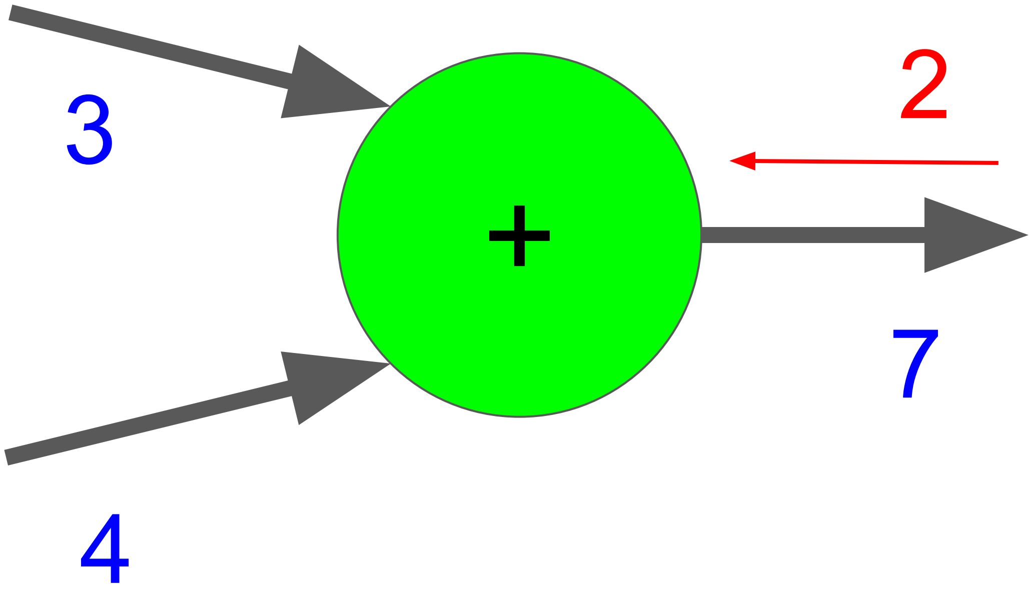
Forward propagation

1. Each Circle represents a computational operation
2. Each **arrow** is an intermediate quantity that is computed
3. Each box contains some value
4. Orange boxes are inputs
5. Blue boxes are parameters
6. Compute the inputs to an operation first then apply the operation to compute its output

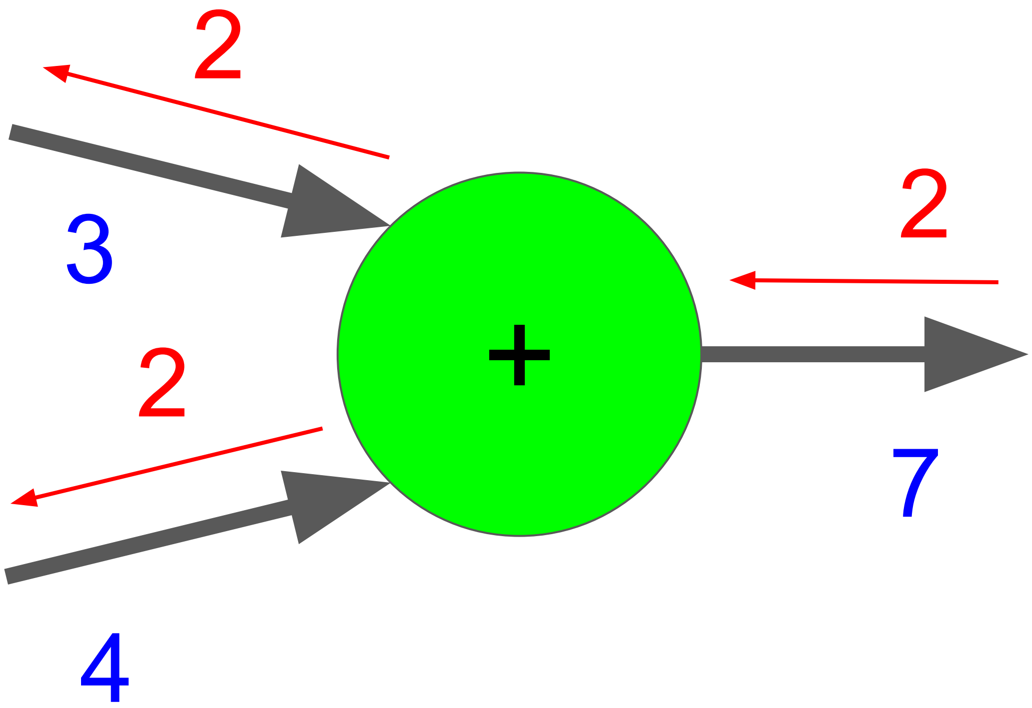
Backward propagation

1. The numbers flow backwards along the arrows
2. At each arrow the quantity represents the gradient of some final function with respect to the quantity at that arrow

Backward propagation



Backward propagation



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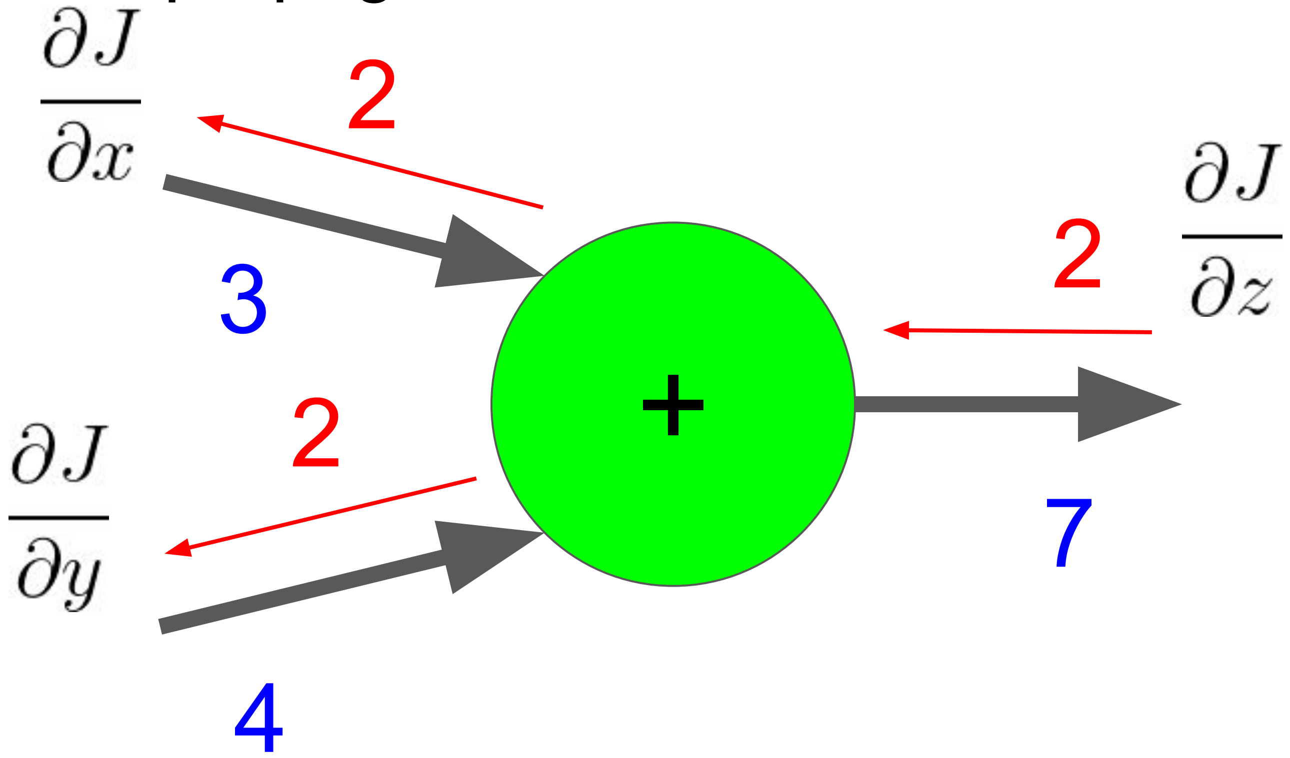
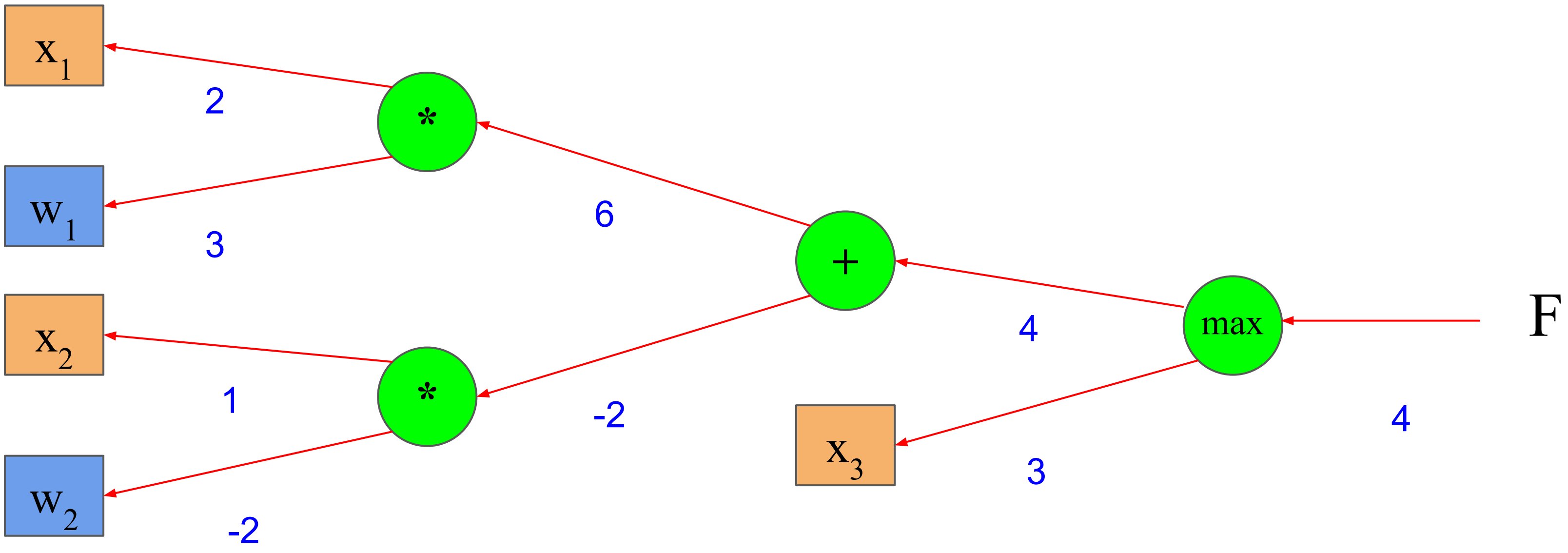


Backward propagation

$$F(x_1, x_2, x_3) = \max(\ w_1x_1 + w_2x_2,\ x_3\ )$$



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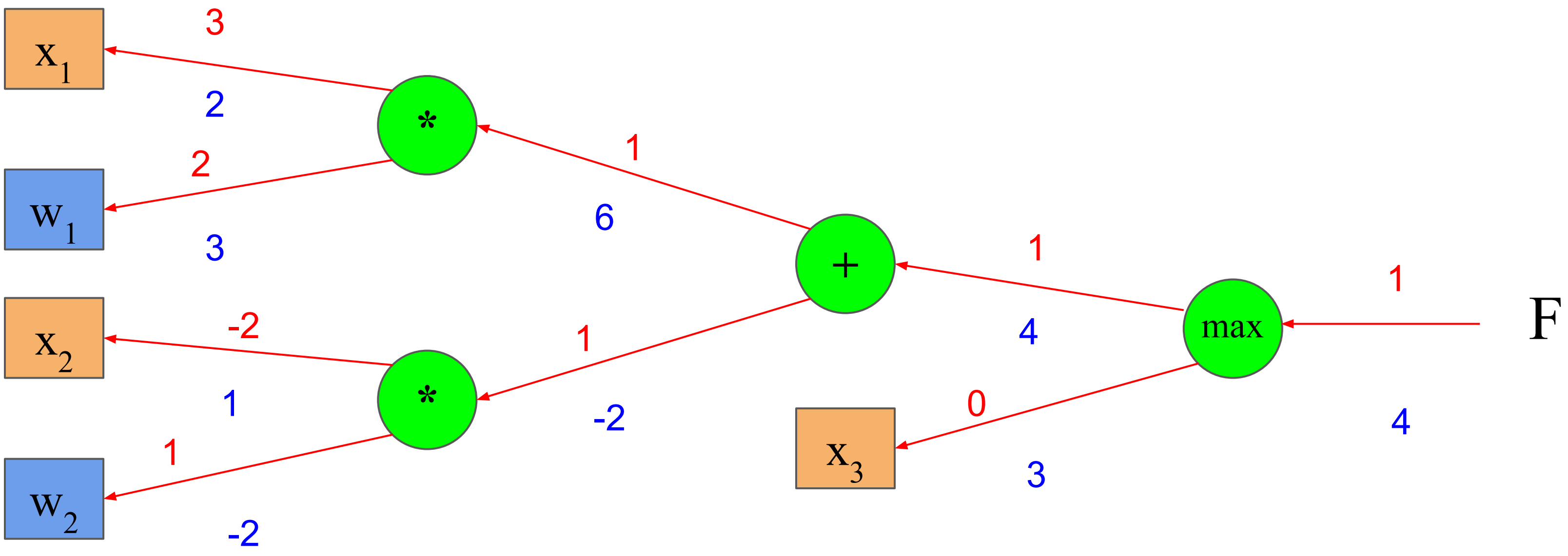
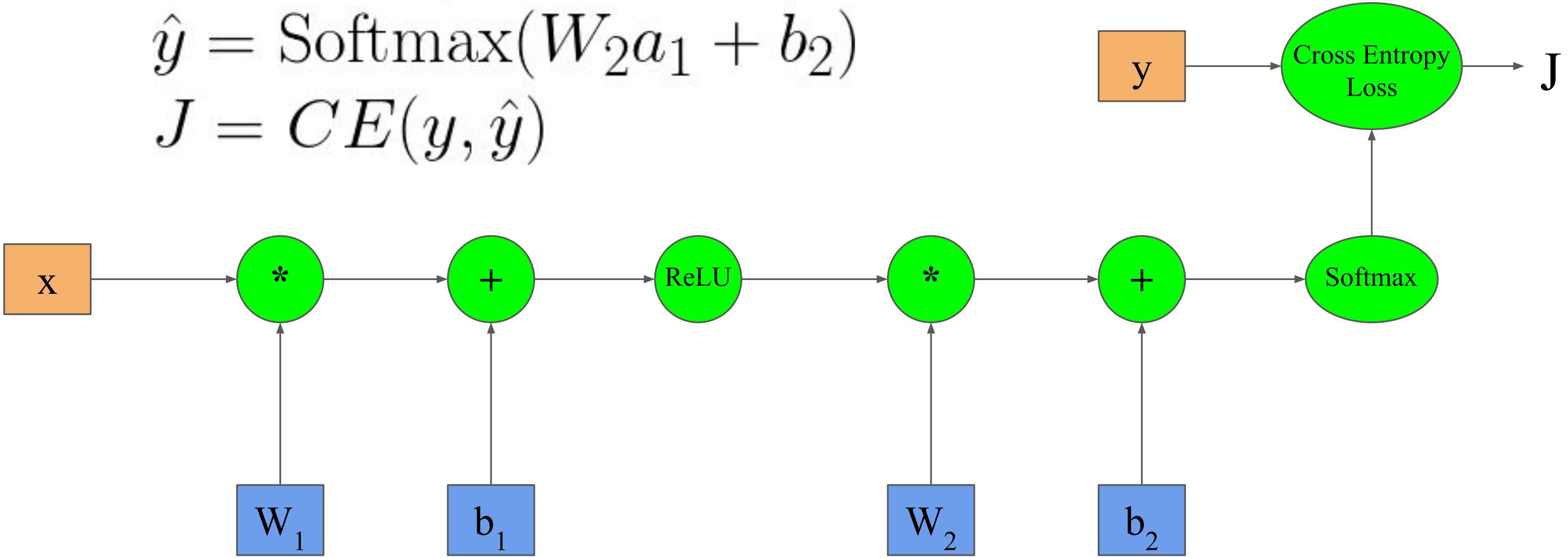


Two Layer Example

$$a_1 = \max(W_1 x + b_1, 0)$$
$$\hat{y} = \text{Softmax}(W_2 a_1 + b_2)$$
$$J = CE(y, \hat{y})$$



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