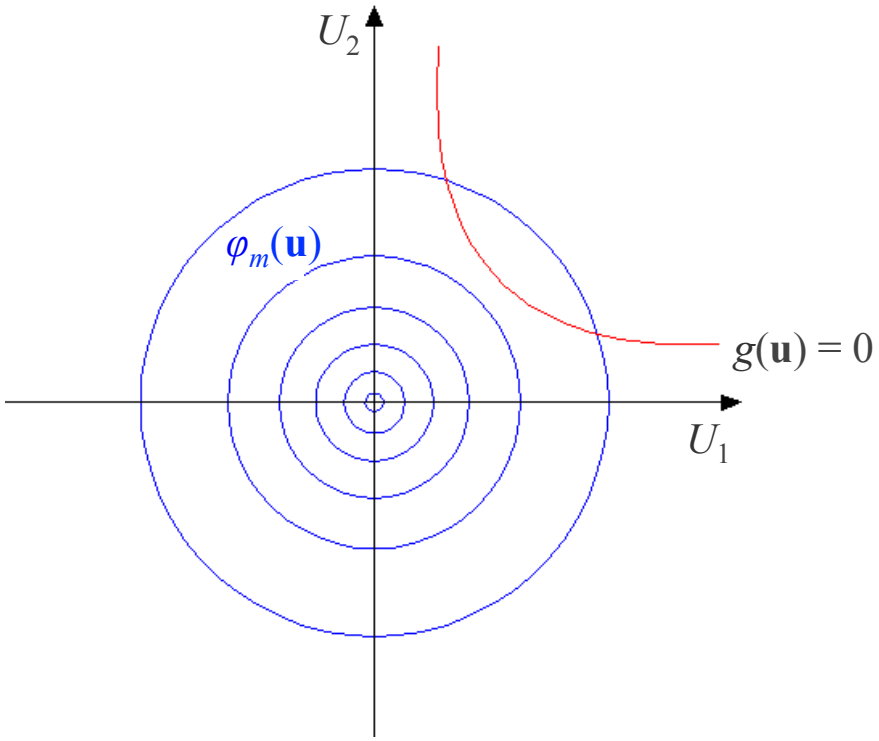
Reliability problem



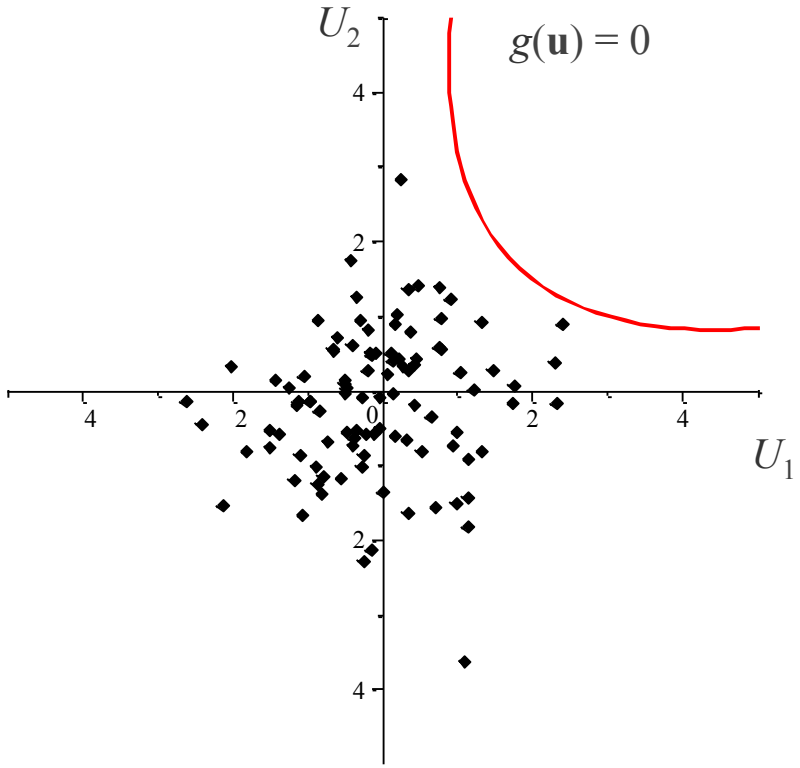
Probability of failure: $$P(F) = \int_{g(\mathbf{u}) \le 0} \varphi_m(\mathbf{u}) d\mathbf{u}$$

Monte Carlo

Example

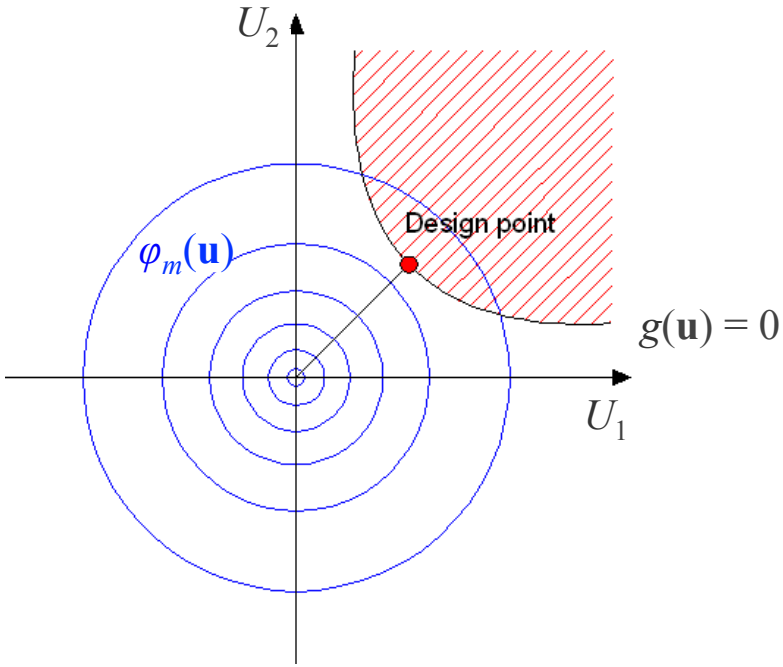
$$g(\mathbf{u}) = 0.1(u_1 - u_2)^2 - \frac{1}{\sqrt{2}}(u_1 - u_2) + 2.5$$

Estimation with 100 samples



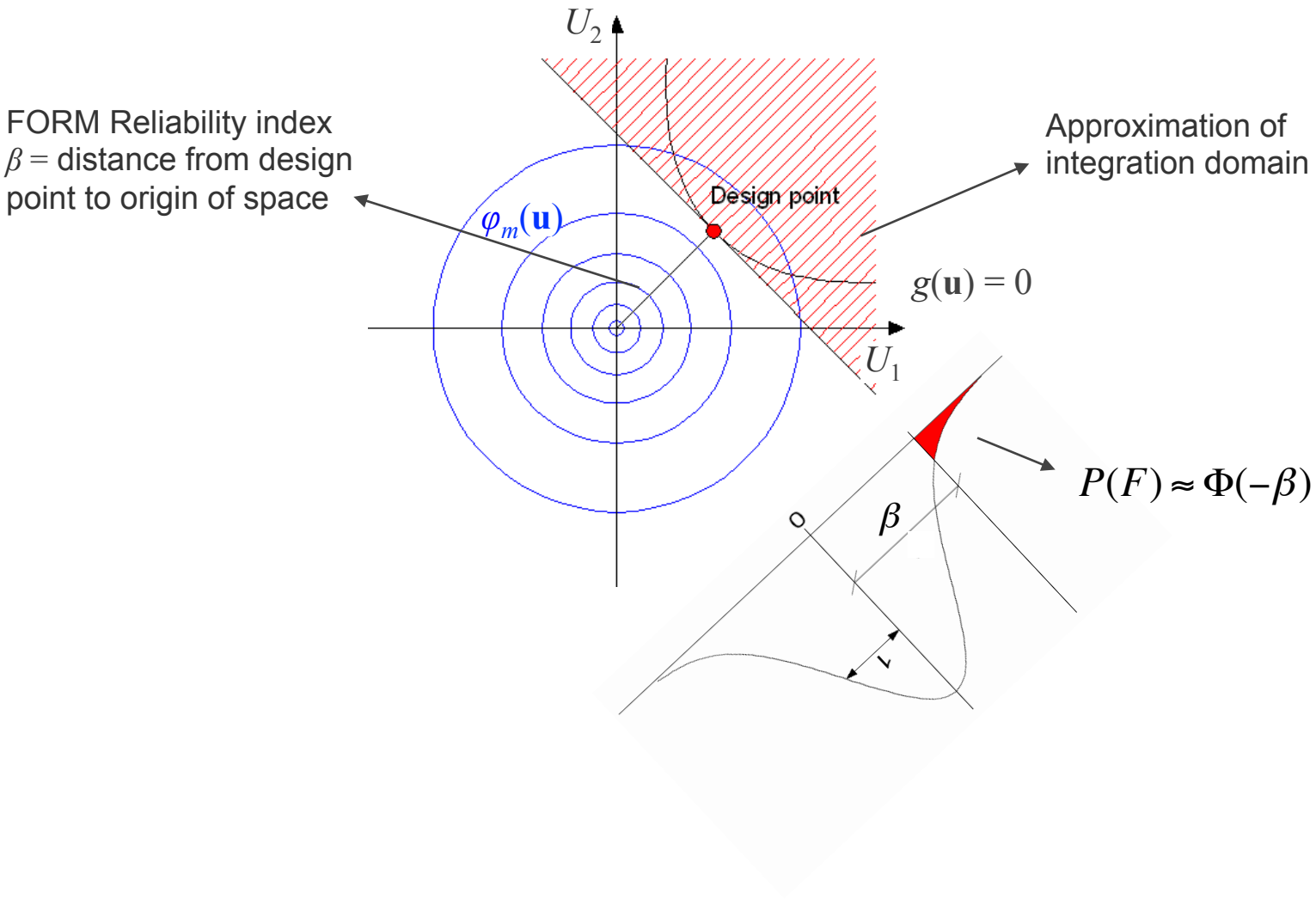
FORM (First Order Reliability Method)

- Design point



FORM (First Order Reliability Method)

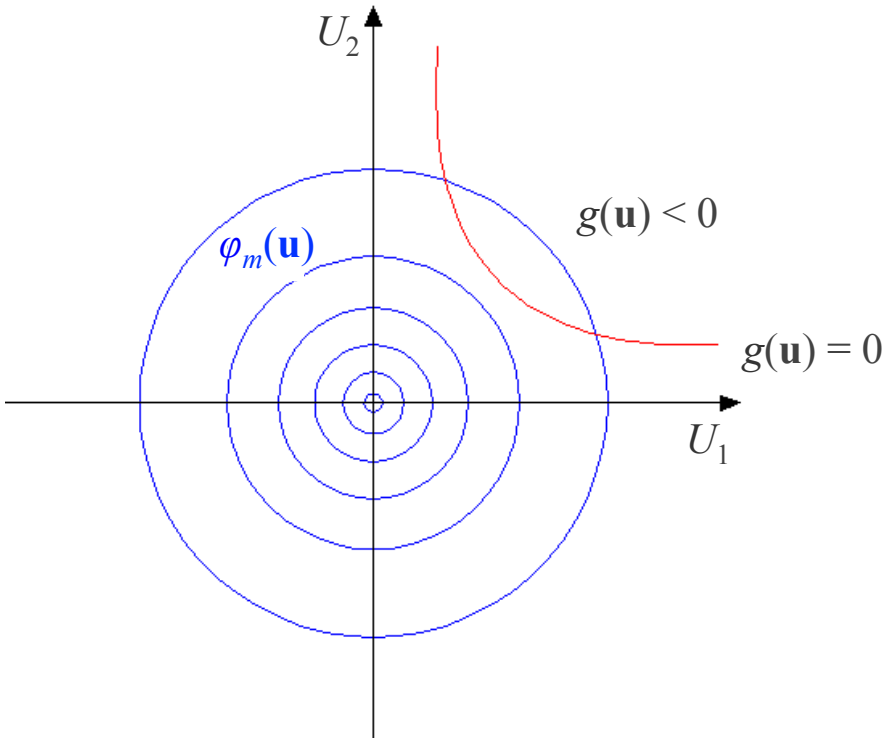
- Design point

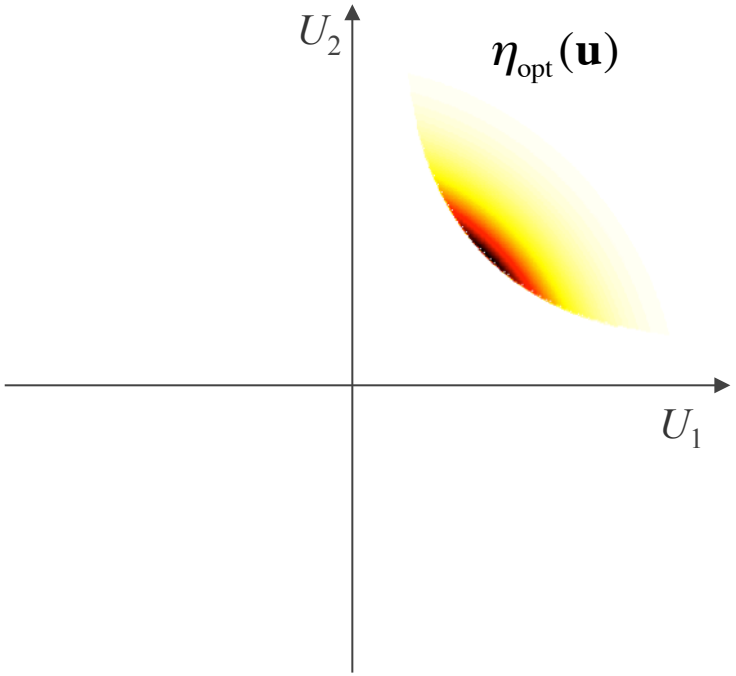


Importance sampling

Optimal importance sampling density

$$\eta_{\text{opt}}(\mathbf{u}) = \frac{1}{P(F)} I\big(g(\mathbf{u}) \le 0\big)\varphi_m(\mathbf{u})$$





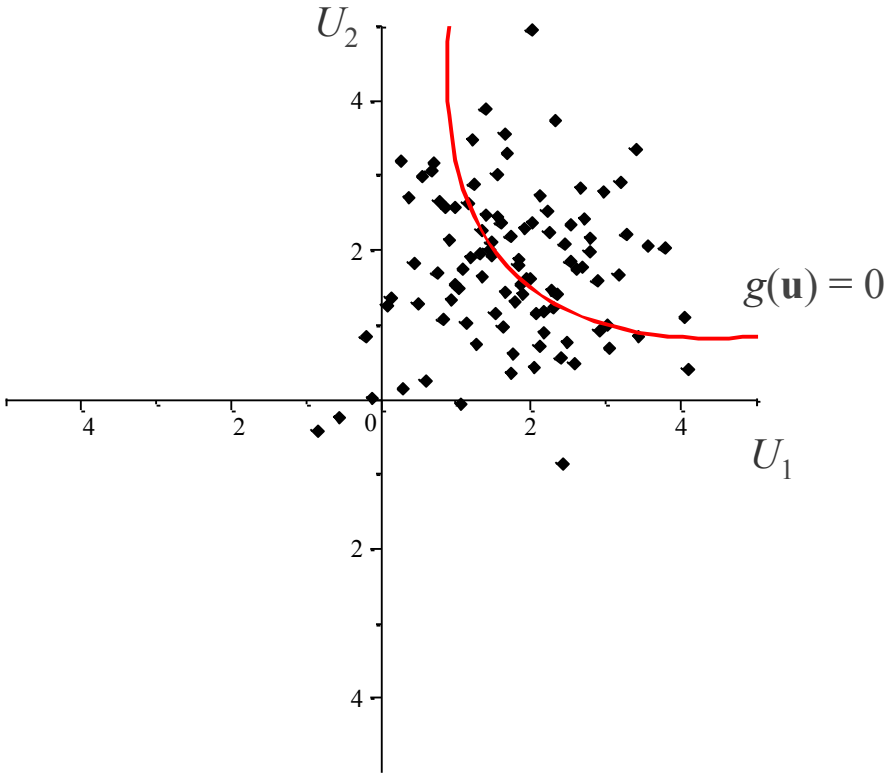
→ Requires the knowledge of $P(F)$

Importance sampling

Example

$$g(\mathbf{u}) = 0.1\left(u_1 - u_2\right)^2 - \frac{1}{\sqrt{2}}\left(u_1 - u_2\right) + 2.5$$

IS density centered at design point



Adaptive importance sampling