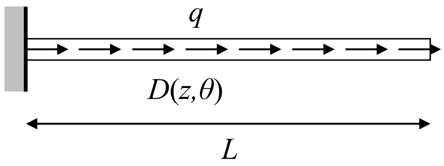
Project: Stochastic Finite Element Methods

Within this project the students will write a stochastic finite element program. The scope of the program is to compute the description of the stochastic response of the bar shown in the figure below.



The length of the bar is L = 2m and the distributed axial load is q = 1kN/m. The axial resistance of the bar $D(z,\theta)$ is described by a homogeneous random field with lognormal marginal distribution, mean $\mu_D = 100\,\text{kN}$ and coefficient of variation δ_D. The auto-correlation coefficient function of the underlying Gaussian random field is given by the following exponential model:

$$\rho_{UU}(\Delta z) = \exp\left(-\frac{\Delta z}{l}\right)$$

where l is the correlation length. The random field is to be discretized by the Karhunen-Loève expansion. The stochastic response of the tip displacement will be evaluated using the polynomial chaos expansion (PCE). The coefficients of the expansion will be estimated by least-square regression applying a random experimental design (e.g. Latin Hypercube sampling and/or quasi-random sampling).

In addition, the students should investigate the following aspects:

(a) The influence of the order of the polynomial chaos on the quality of the moment estimates obtained using the coefficients of the PCE.

(b) The influence of the correlation length and coefficient of variation of the input random field δ_D on the moments of the tip displacement.

(c) The influence of the number of points in the experimental design used to determine the coefficients on the quality of the moment estimates.

Finally, the students will perform the reliability analysis using the polynomial chaos approximation of the tip displacement for the case where $\delta_D = 0.2$ and l = 1m to estimate the probability that the tip displacement will exceed 4cm. The influence of the order of the polynomial chaos on the quality of the reliability estimate should be investigated.