Mechanism and Machine Theory 75 (2014) 97–106

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

General approach for inverse kinematics of nR robots

Yanhui Wei, Shengqi Jian, Shuang He, Zhepeng Wang

College of Automation, Harbin Engineering University, Harbin 150001, China

ARTICLE INFO

Article history:
Received 26 October 2013
received in revised form 3 January 2014
accepted 9 January 2014

Keywords:
nR serial robots
Inverse kinematics
Conformal space geometry
Gradient projection method

ABSTRACT

Usually finding the method to use to solve the inverse kinematics of a nR robot is a difficult problem as no effective analytic method has been identified so far. This article uses a semi-analytic method and a general method to solve the spatial nR robot inverse kinematics problem. It overcomes the numerical method's limitations related to accuracy with a real-time aspect. Initially, conformal geometric space theory was used to establish general kinematic equations. Based on that, the weighted space vector projection method was used to analyze the relationship between the robot spatial rotation angles and the value of the space vector projection. The weighted value of every joint's projection on the end-effector vector was treated as the basis for changing the robot end's orientation. By determining the weighted value of every joint's projection on the end-effector vector, it was possible to achieve the semi-analytic inverse kinematic solution. Finally, to prove the validity and feasibility of the theory it was tested with a special 6R robot.

1. Introduction

With the current wide range of applications of industrial robot techniques, the application of the 6R serial mechanisms is of great significance. The inverse kinematics in serial mechanisms, which plays a crucial role in the robotics, is a prerequisite of serial robot control. It is directly related to tasks like off-line programming, path planning, and real time control. The final position and orientation can only be fully programmed and controlled through the using of inverse kinematics to transform the position and orientation of the joint variables.[1]

The inverse kinematics of 6R serial mechanisms, which belongs to the same question as the inverse kinematics of 7R, is the most difficult issue in the kinematics of serial mechanisms and is regarded as the Mount Everest in kinematic analysis of spatial mechanisms [1]. Researchers worldwide have carried out much useful exploration and research. The approaches used in the study of inverse kinematics of 6R serial mechanisms can be divided into the analytical and numerical methods. Generally, the analytic solutions are difficult to obtain due to the multi-parameters, nonlinearity and coupling of the solutions, and the algebra equations involved in the inverse kinematics of 6R serial manipulators [2]. The analytical method is suitable for 6R serial manipulators with special geometry parameters and the theoretical solution can be achieved through vector, spiral and Lie algebraic methods. The analytical method is accurate and able to find all solutions. However, it needs massive algebra and matrix operations as well as complicated derivation. Furthermore, it is necessary for the manipulator' position and orientation to have decoupling characteristics or the characteristic polynomial to have an order of less than four. Ref. [3] brought quaternions into kinematics research of spatial serial manipulators. It solved a classic problem of 6R robot inverse kinematics. A decomposable method was presented in Ref. [4]. However, it is only suitable for certain special circumstances, such as decoupling. The approaches used in the mathematical modeling

[1] This work was supported by the National Natural Science Foundation (Grant No. 51205074), Specialized Research Fund for the Doctoral Program of Higher Education (Grant No. 20112304120007), Postdoctoral Foundation of Heilongjiang Province (Grant No. LBH-Q10129), Harbin Specialized Research Foundation for Innovation Talents (Grant No. RC2012QN009037), and the Fundamental Research Funds for the Central Universities (Grant No. HEUCF041405).

0094-114X/$ – see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.mechmachtheory.2014.01.008

of the inverse kinematics of serial mechanisms mainly consist of the D–H matrix method, spherical trigonometry, real matrices method, dual numbers method, and so on. These methods have no versatility and lead to different inverse kinematics algorithms. Raghavan and Roth [5] constructed 14 basic equations with six inverse kinematics equations through vector operation. After an elimination operation, a 24-order equation of one variable and up to 16 sets of inverse kinematics solutions were obtained. However, eight extraneous roots existed. Raghavan's method was improved by Manocha [6] who enhanced its stability and accuracy by adopting the matrix eigenvalue decomposition method. In order to settle the displacement analysis of a general spatial 7R mechanism, Ref. [7,8] eliminated extraneous roots by employing a plural method and matrix operation respectively. Based on early research, Ref. [9] classified the inverse kinematics of 6R serial mechanisms into two categories. In the first category, the solution to the inverse kinematics of 6R robots meets the Pieper criteria when using the closed solution. In the other, the orders of the target matrix obtained by Manocha were lowered from 24 to 16. When this was done the efficiency and stability were improved. In addition, the Newton–Raphson iterative algorithm was employed to solve the inverse kinematics issues that did not satisfy the Pieper Criterion.

As for 6R serial robots in real operations, an inverse kinematics solution, which can meet certain task requirements and the position and orientation requirements of the end-effector, is needed. To this end, a variety of approaches has been presented. A common method is separating a_i, α_i and s_i, θ_i, which are in the D–H parameters of each joint of a 6R series manipulator. For the 6R series manipulator kinematics structures, the solution matrix is composed of two independent homogeneous and linear equations that use the methods of dual quaternion and Lie algebra. The 16 inverse kinematics solutions for each joint angle are obtained by iterating and eliminating the two equations. Qiao et al. [10] and Rocco et al. [11–14] obtained the numerical solution for the inverse kinematics of a general 6R series manipulator using dual quaternion theory and Lie groups, and Lie algebras and other methods, respectively. The introduction of genetic algorithms and neural networks is another numerical method that is commonly used with the issue of inverse kinematics. The constraints, such as the feed value of the angle of the joint, will be set. The target function is the difference between the solution and the target. Based on that, the algorithms mentioned above can achieve the best fit of the feed value. Chiddarwar and Babu [15] compared the effect of forecast and conventional neural network algorithms on the solving efficiency; while, K Köker et al. [16] proposed a neural network algorithm for the inverse kinematics of 3 DOF robots by taking velocity and acceleration into consideration. Kalra et al. [17] and Chapelle et al. [18] presented an algorithm for the inverse kinematics of 6 DOF robots based on a genetic algorithm. Hammour et al. [19] planned the trajectory of the 6R manipulator using a continuous genetic algorithm; and Zha [20] employed a genetic algorithm to search for the minimum eigenvalue of the surface that the position and orientation vector of the end of the actuator formed to obtain the optimal trajectory planning.

Based on the research experiences of other scholars, this paper focuses on the multi-rotating joint robot and proposes an approach for the fast solving of the inverse kinematics issues utilizing conformal geometric space theory and the space vector projection method.

2. Establishment of kinematic equations using conformal geometric space theory

2.1. Geometry representation in conformal space

Conformal geometric space theory was proposed by Li [21], and it has become a mainstream part of the international geometric algebra. Conformal geometric algebra (CGA), a new branch of Clifford algebra, is an algebra representation and computing system that is based on advanced geometric invariants. Liao [22] established the kinematic equations of the 6R manipulator and simplified the equations using CGA and Dixon resultant elimination, respectively, and then the 6R robot inverse position issue was successfully solved.

In 4-dimensional projective geometric algebra, and its extension 5-dimensional conformal geometric algebra, both spheres and circles can be used in the computing as basic algebraic variables. The representation of conformal geometric algebra entities is shown in Table 1.

Where x and n denote geometries in 3D space,

$$\begin{cases} x = x_1e_1 + x_2e_2 + x_3e_3 \\ n = n_1e_1 + n_2e_2 + n_3e_3 \end{cases}. \tag{1}$$

$e_i(i = 1, 2, 3)$ denotes three basic unit vectors in 3D and where e_0 represents the origin while e_∞ refers to the infinity; r is radius of sphere; d is the distance from plane to origin.

Table 1
Representation of conformal geometric algebra entities [23].

Geometry	Representation
Point	$P = x + x^2e_\infty/2 + e_0$
Sphere	$s = P - r^2e_\infty/2$
Plane	$\pi = n + de_\infty$
Circle	$z = s_1 \wedge s_2$
Line	$l = \pi_1 \wedge \pi_2$
Point pair	$Pp = s_1 \wedge s_2 \wedge s_3$

Table 2
Definitions of e_0 and e_∞.

	$s_5' = 0$	$s_5' \neq 0$
$s_4' = 0$	Plane through the origin	Sphere or point through the origin
$s_4' \neq 0$	Plane	Sphere or point

2.2. Computation of distance and angle in conformal space

The geometries presented in Table 1 can be represented by vectors. The distance can be derived based on the fact that the inner product of vectors is a scalar.

The vector form of the conformal geometric algebra can be written as:

$$S = s_1 e_1 + s_2 e_2 + s_3 e_3 + s_4 e_\infty + s_5 e_0 \tag{2}$$

where $s = s_1 e_1 + s_2 e_2 + s_3 e_3$, while s_4 and s_5 are defined in Table 2.

s_i denotes different spherical, and $\{\pi_i\}$ denotes different plane. Both definitions in the table are dual.

The algorithm for the unit vector in the conformal geometric algebra is shown as follows:

$$\begin{cases} e_1^2 = e_2^2 = e_3^2 = 1 \\ e_0^2 = e_\infty^2 = 0 \\ e_0 \cdot e_\infty = -1 \end{cases} . \tag{3}$$

The inner product of S and S' can be obtained by:

$$S \cdot S' = (s + s_4 e_\infty + s_5 e_0)\left(s' + s_4' e_\infty + s_5' e_0\right) = s \cdot s' - s_4 \cdot s_5' - s_5 \cdot s_4'. \tag{4}$$

The standardized inner product of the dual geometry can be employed to represent the angle between the two geometries.

$$\cos\theta = \frac{o_1^* \cdot o_2^*}{|o_1^*||o_2^*|} \tag{5}$$

In conformal geometric algebra, the angle between two objects o_1, o_2 like two lines or two planes can be computed using inner product of their normalized dualities.

2.3. General kinematic equations of the robot

The kinematic model of the nR serial robot is shown in Fig. 1. It is composed from n revolute joints and rigid linkages.

$J_i(i = 1{,}2\cdots n)$ denotes the rotational axis of the i-th joint; $L_i(i = 0{,}1{,}2\cdots n)$ is the i-th rigid body; $\Sigma_i(i = 1{,}2\cdots n)$ represents the coordinate system fixed with L_i; Σ_0 stands for the base coordinate system; $a_i(i = 0{,}1{,}2\cdots n)$ is the vector which start from the i-th rotational joint center and points to the $(i + 1)$-th rotational joint center; β_i is the angle between a_{i-1} and a_i; and θ_i is the rotation

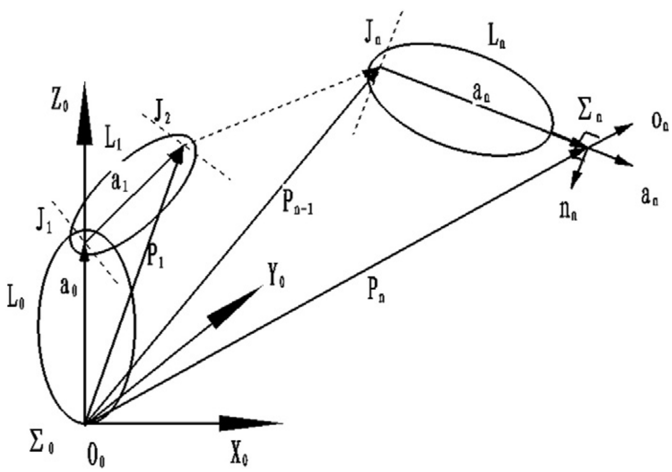

Fig. 1. Kinematic model of nR serial robot.

angle of the i-th rotational joint. Due to the fixed joint parameters and the fixed numerical relation between β_i and θ_i, the norm of a_i, determined by the shape of rigid body L_i, stays unchanged. While it changes with the rotation angle, it can be represented as $a_i(\beta_i)$; $P_i(i = 1,2\cdots n)$is the space vector of the intersection point of vector a_i and vector J_i.

As shown in Fig. 1, the position vector of the end of the robot can be represented as:

$$p_n = a_0 + \sum_{k=1}^{n} a_k(\beta_k). \tag{6}$$

Since the end of the robot meets not only the location requirements but also the orientation requirements, $a_n(\beta_n)$ remains fixed and the vector of the rotation axis of the joint, J_n, can be determined. Besides, a_0 can be determined due to the fixity of L_0. Therefore, the issue of inverse kinematics is transformed into how to determine the vector angle $a_k(k = 1, 2 \cdots n - 1)$.

The inverse kinematics equation can be established based on conformal geometric space theory and Eq. (5):

$$\cos\beta_i = \frac{a_{i-1} \cdot a_i}{|a_{i-1}||a_i|}(i = 1, 2, \cdots n). \tag{7}$$

The relation between β_i and θ_i is shown in Fig. 2, and Eq. (8) can be obtained.

$$\cos\theta_i = \frac{h_i^2 + h_{i-1}^2 + k_i^2 - a_i^2 - a_{i-1}^2 + 2a_i a_{i-1} \cos\beta_i}{2h_i h_{i-1}} \tag{8}$$

h_i, $k_i(i = 1,2\cdots n)$ are fixed structure parameters and can be measured.

3. Weighted space vector projection method

3.1. Space vector projection method

Using the space vector projection method, the vector value can be determined intuitively and quickly. According to Eq. (6), p_n, a_0 and a_n are known. Therefore, Eq. (9) can be established based on the space vector projection method:

$$\|p_n\| = \kappa_0\|a_0\| + \kappa_n\|a_n\| + \sum_{k=1}^{n-1} \kappa_{n-1}\|a_k\| \tag{9}$$

where $\kappa_i(i = 0,1,\cdots n)$ is the ratio of the projection of $a_i(i = 0,2,\cdots n)$ on p_n and the length of $a_i(i = 0,1,\cdots n)$, which is a signed number. When the structure, position, and orientation of the robot are determined, $\kappa_i(i = 1,2,\cdots n)$ is relevant to $\beta_i(i = 1,\cdots n)$ and κ_{i-1}. Then, Eq. (10) can be derived:

$$\kappa_i = f(\beta_i, \kappa_{i-1})(i = 1, 2\cdots n). \tag{10}$$

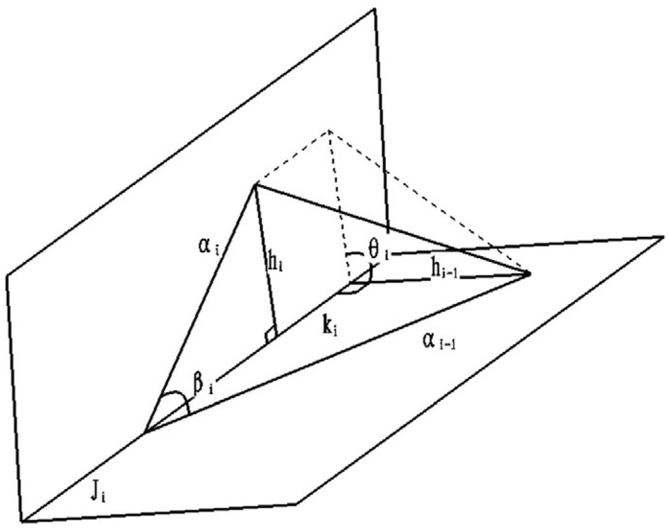

Fig. 2. Relation between β_i and θ_i.

The issue of inverse kinematics becomes how to determine $\kappa_i(i = 0,1,2\cdots n)$ while meeting the constraints of the rotation axis vector of the joints. When the solution is unique, $\kappa_i(i = 0,1,2\cdots n)$ is also unique. For other situations, like multi solutions and redundant robots, obstacle avoidance and path planning can be implemented by adjusting $\kappa_i(i = 0,1,2\cdots n)$. Therefore, $\kappa_i(i = 0,1,2\cdots n)$ is defined as the weighted value of the space vector to adjust the configuration of the robot. This paper focuses on the basic inverse kinematics method, therefore, obstacle avoidance and path planning are beyond the discussion.

3.2. Weighted value of space vector

According to Eq. (10), there is an iterative relation among $\kappa_i(i = 0,1,2\cdots n)$ under the constraints of $\beta_i(i = 0,1,\cdots n)$, which is shown in Fig. 3. The projection of a_{i-1} on P_n, $\kappa_{i-1}||a_{i-1}||$, and the rotation axis vector of the i-th joint can be determined once a_{i-1} is known. Besides, based on the range of $\kappa_i||a_i||$, which is known, and the constraints, a_i can be obtained. Along with β_i and θ_i, the inverse kinematics can be implemented.

With regard to nR(n < 5) robots, they can directly obtain the analytical form of the inverse kinematics solutions based on the space vector projection method. However, the inverse kinematics for nR(n > 4) robots requires the combination of the space vector projection and numerical methods.

3.3. The relationship between vector projection and joint angle

The relationship between vector projection and joint angle is usually used in joint angle calculation such as, getting joint angle, finding robot's max projection configuration and matching work configuration etc. Fig. 4 shows the relationship between vector projection and joint angle.

Points O_i are on the revolute axis of each i revolute joint. They are calculated by D–H method. Plane τ is combined by initial joint vector a_i^0 and joint vector a_i, which rotated specific degrees by joint axis J_i (joint vector a_i rotated by joint axis J_i forms to a conical surface). Because robot's rigid link may not be vertical to joint axis J_i, we can assume that point B and point F have the same projection point A on joint axis J_i. So that the value of AB and AF are known. When joint angle is zero, a_i^0 equals to a_i, that is, O_iB. The projection points of Point B and Point F are point D and point G on orientation vector P_n respectively. Draw the GF's parallel line passing point D and we get point of intersection C. It's known that DG is difference value Δ between a_i's projection value on vector P_n' and a_i^0's projection value on vector P_n'. The projection vector on plane τ of position and orientation vector P_n intersects BC at point E. Since O_iB, P_n and plane τ are all known, we can get O_iB and P_n's intersection angle α, P_n and plane τ's intersection angle γ by space geometry theory.

Δ is known, and we can build the following equations when we are solving intersection angle θ.

$$O_iD = a_i^0 \cos\alpha \tag{11}$$

$$O_iF = a_i^0 \tag{12}$$

$$O_iC = \frac{O_iD * O_iF}{O_iD + \Delta} \tag{13}$$

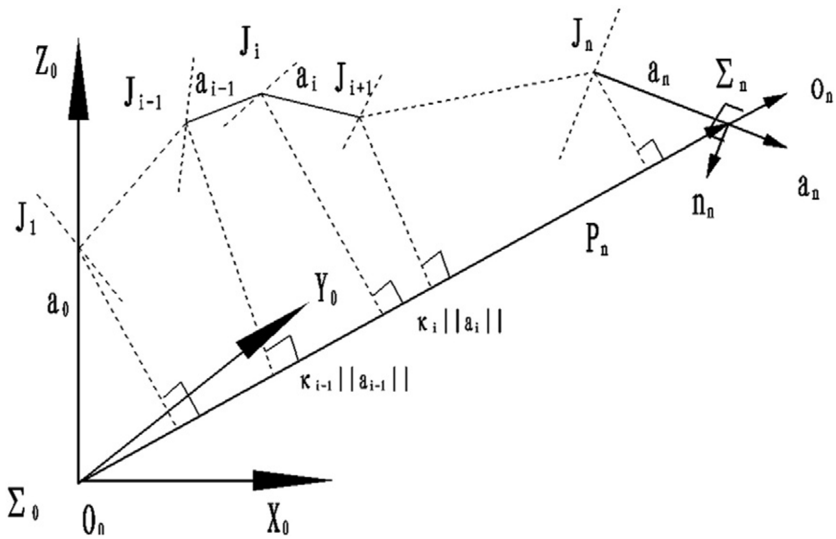

Fig. 3. Projection relations among space vector.

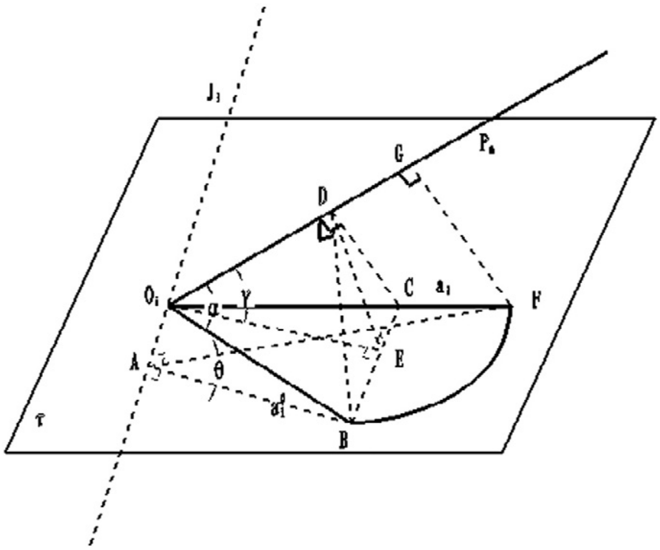

Fig. 4. The relation between vector projection and joint angle.

$$O_iE = \frac{O_iD}{\cos\gamma} \tag{14}$$

$$BC = \sqrt{(O_iB)^2 - (O_iE)^2} + \sqrt{(O_iC)^2 - (O_iE)^2} \tag{15}$$

Assuming $\angle FO_iB$ is θ_i', then:

$$\theta_i' = \arccos\left(\frac{O_iE}{O_iB}\right) + \arccos\left(\frac{O_iE}{O_iC}\right) \tag{16}$$

$$\left\|a_i^0\right\|^2 + \|a_i\|^2 - 2\left\|a_i^0\right\| * \|a_i\| * \cos\theta_i' = (BF)^2 \tag{17}$$

$$(AB)^2 + (AF)^2 - 2(AB) * (AF) * \cos\theta = (BF)^2. \tag{18}$$

By Eqs. (11)–(18), we can build the relationship between Δ and θ. When θ is known, we can get Δ by the relation showing in Fig. 4.

4. Determination process of weighted value of space vector

As mentioned previously, the issue of inverse kinematics for nR robots turns into the determination of the weighted value of the space vector.

4.1. Determination of space vector projection ratio based on the maximum

nR robot's configuration on the target point, which has the max space vector projection value, is defined as max space vector projection working configuration. By Eq. (9), the max space vector projection working configuration's confirmation can lead to the confirmation of the weighted value of numerical max space vector.

Eq. (9) indicates the relationship between the weighted value of the space vector and the position and orientation vector, while Eq. (10) denotes the constraint relation between the vector of the joint motion and the link. Neither of these equations can solve the issue of inverse kinematics for the nR(n > 4) robots.

According to Eq. (10), the maximum projection of each joint vector on P_n, $\max(\kappa_i)(i = 1, 2 \cdots n-1)$, can be achieved. Based on that, the new projection of the position and orientation vectors can be obtained and is shown in the following equation:

$$\left\|P'\right\| = \kappa_0\|a_0\| + \sum_{k=1}^{n-1} \max(\kappa_{n-1})\|a_k\|. \tag{19}$$

P' and P_n share the same vector direction and the following relation:

$$\left\|P'\right\| \geq \|P_n\| - \kappa_n \|a_n\| \tag{20}$$

where δ is the total projection margin of the joint vector on the position and orientation vectors, and the following equation is obtained:

$$\delta = \left\|P'\right\| - \|P_n\| + \kappa_n \|a_n\|. \tag{21}$$

4.2. Determination of work configuration of $\delta \to 0$

The work configuration of the maximum projection $\max(\kappa_i)(i = 1, 2 \cdots n - 1)$ can be obtained based on the topological relations of the robot, which is defined as the initial configuration here.

The simplest way to find intermediate configurations and make $\delta \to 0$ is to make δ divide equally into $(n - 2)$ copies. The projection of each robot space link on P_n reduces $^{\delta}/_{n-2}$ through the movement of the robot joint angles. However, due to the constrains placed on the joints, the method of adjacent joint compensation has to be adopted as an alternative.

Fig. 5 shows the maximum projection of the rod vector a_i on P_n. Based on the equal division method introduced above, the projection of a_i on P_n becomes $b_i - ^{\delta}/_{n-2}$. However, the projection may have some difference because of the motion range of the i-th joint's joint angle, we define b_i' as the minimum projection and get the following equation:

$$\Delta_i = b_i' - b_i + {}^{\delta}\big/_{n-2}. \tag{22}$$

Therefore, the projection of a_{i-1} and a_{i+1} on P_n reduces $^{\delta}/_{n-2} + ^{\Delta_i}/_2$. The new projection value on the position and orientation vectors can be obtained in turn using this method to achieve $\delta \to 0$. Then, the new work configuration can be derived through the initial configuration.

4.3. Compensation and correction of orientation

The work configuration obtained in the previous section uses the vector projection method and satisfies the space projection constraints. However, for real situations, the space orientation constraints must also be met.

As demonstrated in Fig. 6, P_{n-1}' is the space vector of the n-th joint center in the work configuration of $\delta \to 0$ and this configuration can only fulfill position requirement but not orientation requirement. While P_{n-1}'' is the space vector of the n-th joint center in the target configuration, this work configuration can fulfill them both. P_{n-1}'' can be obtained by rotating P_{n-1}' around the space vector axis P_n by a certain degree ϕ, because both P_{n-1}'' and P_{n-1}' have the same projection on P_n. Based on that, the work configuration of $\phi \to 0$ was searched while regarding the work configuration of $\delta \to 0$ as the work configuration.

The input, $\beta_i(i = 1, 2, \cdots n - 1)$, is used to adjust the robot link space vector to make $\phi \to 0$.

The joint axis of the n-th joint, J_n, is determined once a_n is determined. Based on that, the vector of the plane that contains a_{n-1} can be fixed. However, the rod vector a_{n-1}'' in the work configuration of $\phi \to 0$ is likely to be excluded in the plane that J_n determines. Under such a circumstance, a_{n-1}'' has only one point of intersection with the plane, which is located in the center of the i-th joint. Therefore, the work configuration of $\varphi \to 0$ needs further orientation compensation to meet of the robot's orientation requirements.

The relationship between the work configuration of $\phi \to 0$ and the initial work configuration is illustrated in Fig. 7.

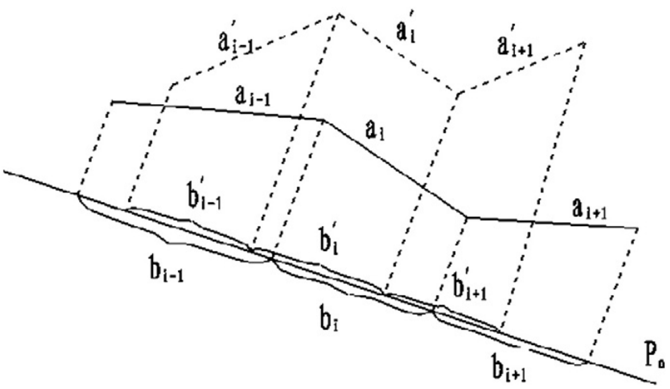

Fig. 5. Adjacent joint compensation.

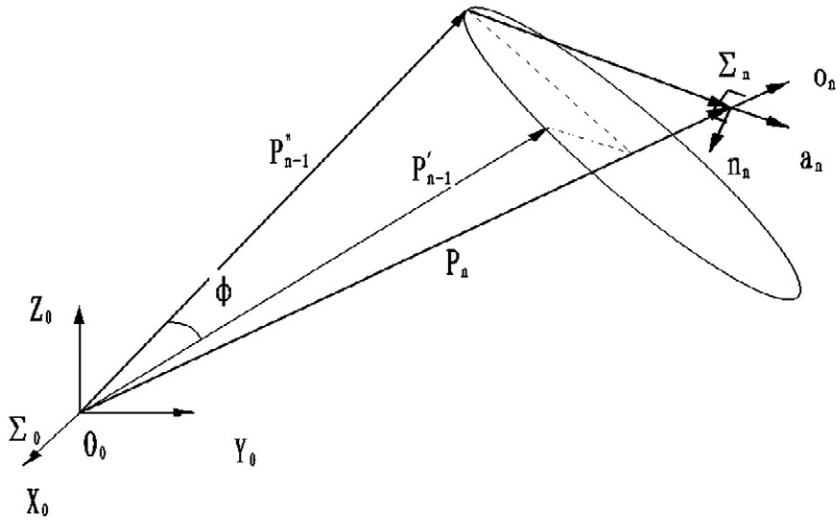

Fig. 6. Orientation compensation.

In Fig. 7, the work configuration of $\phi \to 0$ guarantees the superposition of the n-th joint center in it with the n-th joint center in the real work configuration. However, J''_n in the work configuration of $\phi \to 0$ does not coincide with J_n in the target configuration, which means the end of the robot's orientation does not meet the task requirements.

While J_n is known, a_{n-1} and the plane it is in, η_{n-1}, can be determined based on the topological relations of the robot. Then the compensation issue is to search appropriate a_{n-1} in η_{n-1} by adjusting $\beta_i(i = 1, 2, \cdots n-1)$ and meeting the constraints of the rods.

5. Example verification

This article used a general 6R robot model, as in Fig. 8, to prove the validity of this method.

At this moment the robot's orientation matrix is obtained:

$$T = \begin{bmatrix} -0.8673 & 0.4894 & -0.0908 & 50.0142 \\ 0.2884 & 0.3456 & -0.8929 & -1003 \\ -0.4056 & -0.8007 & -0.4409 & -471.074 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{23}$$

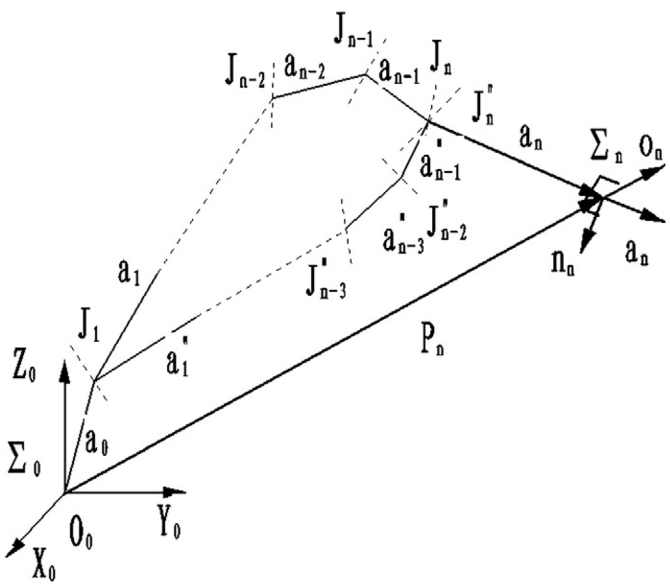

Fig. 7. Relationship between the work configuration of $\phi \to 0$ and the initial work configuration.

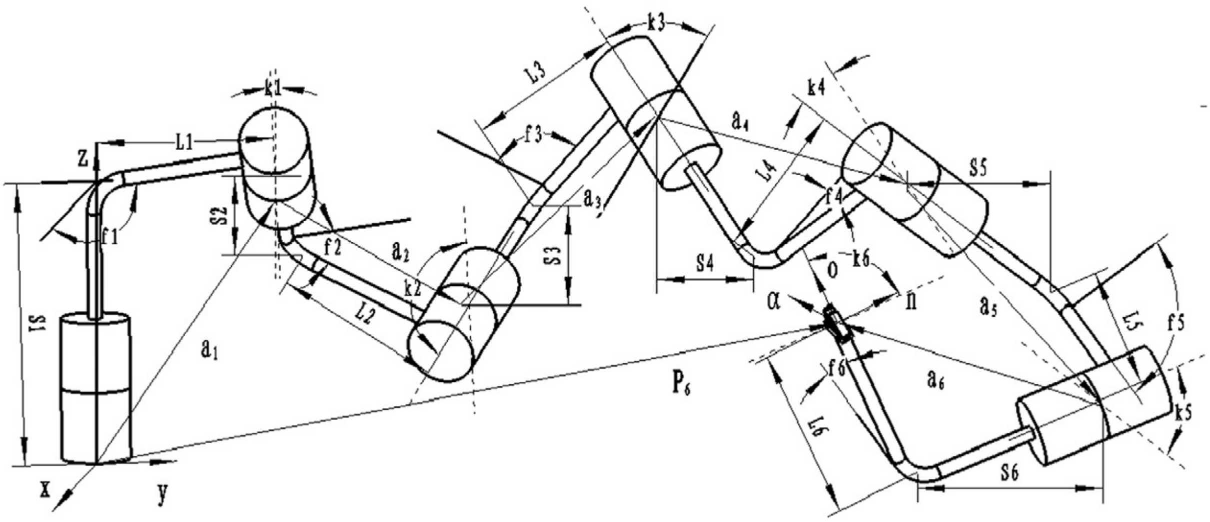

Fig. 8. General 6R robot model.

5.1. Work configuration through the maximum value of space vector projection

According to Eq. (23) the robot end orientation vector is known:

$$P_n = [50.0142, -1003, -471.0744]. \tag{24}$$

Its vector length equals 1109.2 mm.
The value of the biggest projection's space vector equals:

$$\max P_n = [-58.80, 1820.90, -23.70]. \tag{25}$$

Its vector length equals 1822 and the vector projection difference equals 712.8 mm.

5.2. Work configuration of $\delta \to 0$

Depending on the equal division method, the work configuration's rotation angles were found to be the same as in Table 5, and its gradient projection equals zero. However, the position requirement is not satisfied by the value of the robot work configuration's end. So regarding the rotation angles in Table 5 as the original configuration, the spatial vector analytic method is used to obtain the new work configuration, as shown in Table 6. (See Tables 3 and 4.)

Table 3
Robot structure parameters.

i	*Si* (mm)	*Fi* (°)	*Li* (mm)	*Ki* (°)
1	400	110	300	100
2	200	50	300	80
3	300	120	300	50
4	200	30	200	−70
5	200	70	150	60
6	270	50	270	−40

Table 4
Rotation angles of work configuration by the maximum value of space vector projection.

1	2	3	4	5	6
−87.145°	25.717°	0.843°	0.114°	0.089°	25.430°

Table 5
Rotation angles of δ→0 work configuration.

1	2	3	4	5	6
−4.7768°	−0.0401°	−9.5465°	−10.3131°	0.1691°	4.662°

Table 6
Rotation angles of new work configuration.

1	2	3	4	5	6
52.9848°	−8.7168°	−17.7382°	−20.9123°	−30.6762°	−50.8271°

Table 7
Rotation angles of final work configuration.

1	2	3	4	5	6
110°	50°	120°	30°	70°	50°

5.3. Modification of orientation

The robot work configuration should not only satisfy the need of the end position, but also its orientation, therefore, modifications on the orientation should be made. Based on the method presented in Section 3.3, the final work configuration can be determined, as shown in Table 7.

6. Conclusion

This article used conformal geometric space theory and established general kinematic equations for the joints, which formed a baseline for solving the spatial nR robot inverse kinematics problem. Using a semi-analytic method to solve the spatial nR robot inverse kinematics problem, a weighted space vector projection method was developed. By solving the inverse kinematics of the spatial 6R robot in general form, it was proved that this method can calculate the inverse solution fast, which satisfies the position and orientation demanded in this paper. The method solves the real-time problem of general numerical method, thereby overcoming the limitation of the traditional analytical method.

This semi-analytic method can provide new research ideas for the determination of the planning and motion control of serial robots' trajectory.

References

[1] J. Duffy, Analysis of Mechanisms and Robot Manipulators, Edward Arnold, London, 1980.
[2] Y.J. Liu, T. Huang, Inverse kinematics and trajectory planning of 6R serial manipulators, J. Mech. Eng. 48 (3) (2012) 9–15.
[3] S.G. Qiao, Q.Z. Liao, S.M. Wei, H.J. Su, Inverse kinematic analysis of the general 6R serial manipulators based on double quaternions, Mech. Mach. Theory 45 (2010) 193–199.
[4] J. Zhao, W.Z. Wang, H.G. Cai, Generation of closed-form inverse kinematics for reconfigurable robots, Chin. J. Mech. Eng. 42 (8) (2006) 210–214.
[5] M. Raghavan, B. Roth, Kinematic analysis of the 6R manipulator of general geometry, International Journal of Robotics ResearchMIT Press, Tokyo, 1989, pp. 314–320.
[6] D. Manocha, J.F. Canny, Efficient inverse kinematics for general 6R manipulators, IEEE Trans. Robot. Autom. 5 (9) (1994) 648–657.
[7] Q.Z. Liao, C.G. Liang, Q.X. Zhang, A novel approach to the displacement analysis of general spatial 7R mechanism, Chin. J. Mech. Eng. 22 (3) (1986) 1–9.
[8] H.Y. Lee, C.G. Liang, Displacement analysis of the general spatial 7-link 7R mechanism, Mech. Mach. Theory 23 (3) (1988) 219–226.
[9] S.G. Liu, S.Q. Zhu, J.B. Li, X.Y. Wang, Research on real-time inverse kinematics algorithms for 6R robots, Control Theory Appl. 25 (6) (2008) 1037–1041.
[10] S. Qiao, Q. Liao, S. Wei, Inverse kinematic analysis of the general 6R serial manipulators based on double quaternions, Mech. Mach. Theory 45 (2) (2010) 193–199.
[11] S.D. Rocco, D. Eklund, A.J. Sommese, Algebraic C*-actions and the inverse kinematics of a general 6R manipulator, Appl. Math. Comput. 216 (9) (2010) 2512–2524.
[12] M.L. Husty, M. Pfurner, H.P. Schröcker, A new and efficient algorithm for the inverse kinematics of a general serial 6R manipulator, Mech. Mach. Theory 42 (1) (2007) 66–81.
[13] S. Regnier, F.B. Ouezdou, P. Bidaud, Distributed method for inverse kinematics of all serial manipulators, Mech. Mach. Theory 32 (7) (1997) 855–867.
[14] R. Manseur, K.L. Doty, Structural kinematics of 6-revolute-axis robot manipulators, Mech. Mach. Theory 31 (5) (1996) 647–657.
[15] S.S. Chiddarwar, N.R. Babu, Comparison of RBF and MLP neural networks to solve inverse kinematic problem for 6R serial robot by a fusion approach, Eng. Appl. Artif. Intell. 23 (7) (2010) 1083–1092.
[16] R. Köker, C. Öz, T. Çakar, A study of neural network based inverse kinematics solution for a three-joint robot, Robot. Auton. Syst. 49 (3) (2004) 227–234.
[17] P. Kalra, P.B. Mahapatra, D.K. Aggarwal, An evolutionary approach for solving the multimodal inverse kinematics problem of industrial robots, Mech. Mach. Theory 41 (10) (2006) 1213–1229.
[18] F. Chapelle, P. Bidaud, Closed form solutions for inverse kinematics approximation of general 6R manipulators, Mech. Mach. Theory 39 (3) (2004) 323–338.
[19] Z.S. Hammour, N.M. Mirza, S.M. Mirza, Cartesian path generation of robot manipulators using continuous genetic algorithms, Robot. Auton. Syst. 41 (4) (2002) 179–223.
[20] X.F. Zha, Optimal pose trajectory planning for robot manipulators, Mech. Mach. Theory 37 (10) (2002) 1063–1086.
[21] H.B. Li, D. Hestenes, A. Rockwood, Generalized Homogeneous Coordinates for Computational Geometry, Springer, Heidelberg, 2001. 27–60.
[22] Z.S. Ni, Q.Z. Liao, S.M. Wei, New algorithm for inverse kinematics analysis of general 6R serial robot, J. Beijing Univ. Posts Telecommun. 32 (2) (2009) 29–33.
[23] D. Hildenbrand, Geometric computing in computer graphics and robotics using conformal geometric algebra, Erlangung des akademischen Grades eines Doktor-Ingenieurs, Dissertation 18 (10) (2006).