





CS 32 Week 10

Discussion 1B

  last discussion of the quarter!!  

this week's topics:

heap, priority_queue, final review!

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a couple things before we start...

- ***First, please find a partner / form a group!***
(Yes, I mean right now.) We'll be doing some practice problems.
- After that, please fill out the LA feedback form if you haven't already now that it's the end of the quarter: <https://tinyurl.com/S22LAFeedback>
 - You can also use the QR code aha >>
 - It really helps me improve as an LA, and improve our section! I want to provide resources that y'all need/want
- Be sure to select the ***Student to LA End of Quarter Feedback*** option!



Topics

Complete binary tree

Binary Heap

- insertion
- Deletion
- Get maximum(minimum) from max(min) heap
- Heap sort
- Time complexity

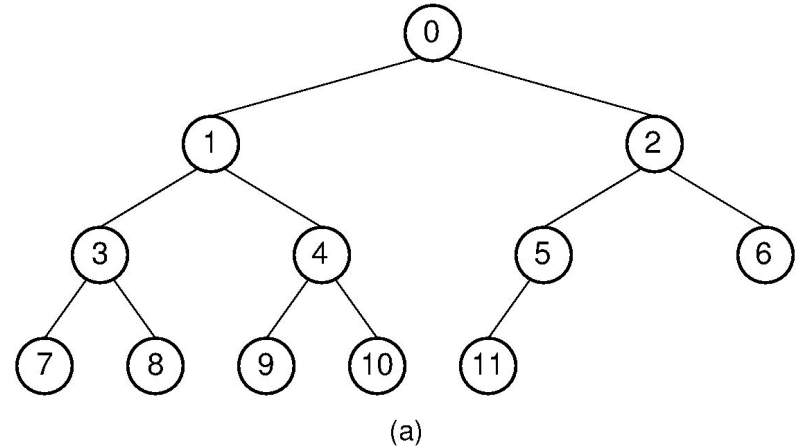
C++ STL priority queue

C++ STL summary

Final Review

Complete binary tree

A complete binary tree is a binary tree in which all the levels are **completely filled** except possibly the **lowest one**, which is filled **from the left**.

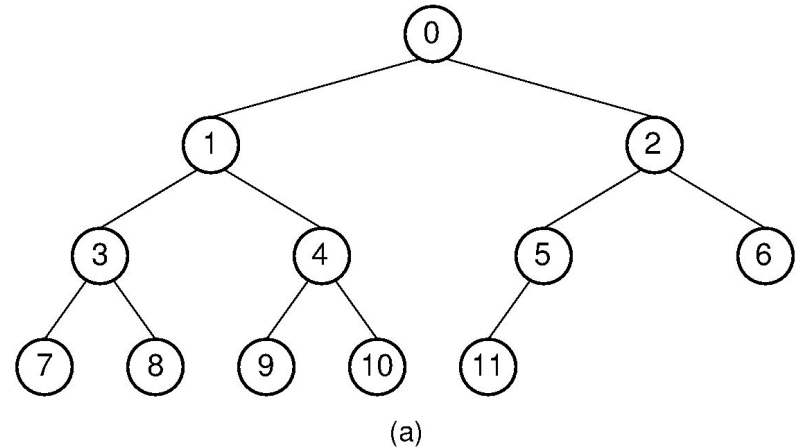


Complete binary tree: array implementation

If the complete binary tree has N nodes with values saved in an array: $a[N]$.

Then for all node i , its parent is $\lfloor (i-1)/2 \rfloor$. Its children are $2i+1$ and $2i+2$.

We also save the size of the array (or use a dynamically allocated array like vector).

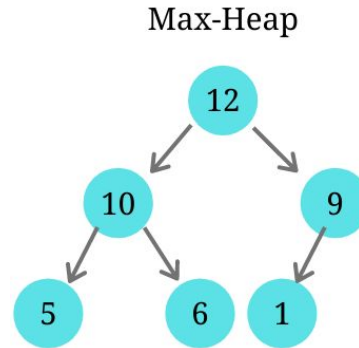


Heap: definition

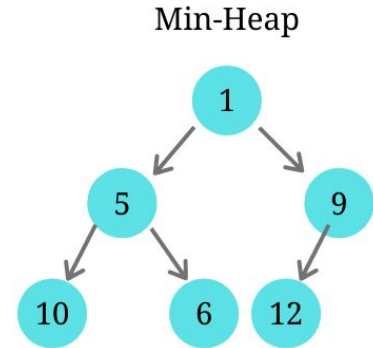
A max(min) Heap is a complete binary tree such that all parent nodes have values greater(less) than their children.

Therefore, the root of a max(min) heap has the greatest(smallest) value in the entire heap.

Heap is not totally sorted!!!



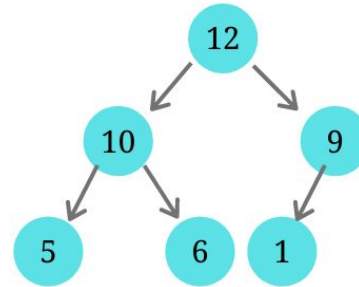
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Heap: Definition

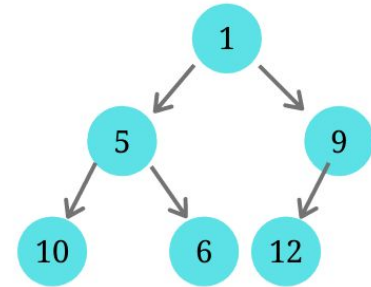
```
//heap of valuetype double  
//may also use fixed-size array for heap  
vector<double> heap;
```

Max-Heap



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Min-Heap



Heap: insertion

```
void insert(vector<double>& heap, const double& val) {  
    //insert a value val to heap  
}
```


Heap: insertion

```
void insert(vector<double>& heap, const double& val) {  
    heap.push_back(val);  
    int cur_ind = heap.size() - 1;  
    while(cur_ind != 0  
        && heap[cur_ind] > heap[(cur_ind-1)/2]) {  
        swap(heap[cur_ind], heap[(cur_ind-1)/2]);  
        cur_ind = (cur_ind-1)/2;  
    }  
}
```

Heap: deletion

```
void remove(vector<double>& heap) {  
    //remove the root element of the heap  
}
```

Heap: deletion

```
void remove(vector<double>& heap) {
    swap(heap[0], heap[heap.size() - 1]); //swap root to leaf
    heap.pop_back();
    int sz = heap.size();
    int cur = 0;
    while (2 * cur + 1 < sz) {
        if (2 * cur + 2 >= sz) { //only left child exists
            if (heap[2 * cur + 1] > heap[cur]) {
                swap(heap[2 * cur + 1], heap[cur]);
                cur = 2 * cur + 1;
            }
            else break;
        }
        else {
            //larger than both left and right
            if (heap[cur] > heap[2 * cur + 1] && heap[cur] > heap[2 * cur + 2])
                break;
            //pick the larger element of left and right
            if (heap[2 * cur + 1] > heap[2 * cur + 2]) {
                swap(heap[cur], heap[2 * cur + 1]);
                cur = 2 * cur + 1;
            }
            else {
                swap(heap[cur], heap[2 * cur + 2]);
                cur = 2 * cur + 2;
            }
        }
    }
}
```

Heap: get maximum(minimum) of max(min) heap

```
double get_max(const vector<double>& heap) {  
    //return maximal element of the heap  
}
```

Heap: get maximum (minimum)

```
double get_max(const vector<double>& heap) {  
    return heap[0];  
}
```

Heap sort

```
void heap_sort(vector<double>& heap) {  
    //sort the heap  
}
```

Heap sort

```
void heap_sort(vector<double>& heap) {  
    if (heap.size() <= 1) return;  
    double val = heap[0]; //save the largest  
    remove(heap); //remove the largest  
    heap_sort(heap); //sort the rest  
    heap.push_back(val); //add largest back  
}
```

Heap: complexity

| | Average | worst |
|-----------------------|---------------|---------------|
| Insertion: | $O(\log N)$ | $O(\log N)$ |
| Deletion: | $O(\log N)$ | $O(\log N)$ |
| Get_max for max heap: | $O(1)$ | $O(1)$ |
| Heap_sort: | $O(N \log N)$ | $O(N \log N)$ |

STL: priority_queue (#include <queue>)

A linear data structure.

Looks like a queue, but totally different. (queue uses linked list, priority_queue uses heap).

For standard types, the priority is **larger** values (max heap), but like set and map, one can overload the < operator or define a priority comparator.

Like a heap, a priority_queue is not totally sorted. But **its top element is guaranteed** to have the **highest priority** among all elements. It **automatically adjust** the heap after each pop and push.

fx Member functions

| | |
|------------------------------|---|
| (constructor) | Construct priority queue (public member function) |
| empty | Test whether container is empty (public member function) |
| size | Return size (public member function) |
| top | Access top element (public member function) |
| push | Insert element (public member function) |
| emplace <small>C++11</small> | Construct and insert element (public member function) |
| pop | Remove top element (public member function) |
| swap <small>C++11</small> | Swap contents (public member function) |

STL priority_queue: define priority comparator

```
struct LessThanByAge
{
    bool operator()(const Person& lhs, const Person& rhs) const
    {
        return lhs.age < rhs.age;
    }
};
```

then instantiate the queue like this:

```
std::priority_queue<Person, std::vector<Person>, LessThanByAge> pq;
```

One particular useful method is low priority first (min heap):

```
priority_queue <int, vector<int>, greater<int>> g
```

STL priority_queue example

```
priority_queue<int> g1;
priority_queue<int, vector<int>, greater<int>> g2;
int b[5] = {3, 2, 6, 1, 8};
for (int i = 0; i < 5; ++i) {
    g1.push(b[i]);
    g2.push(b[i]);
}
while(!g1.empty()) {
    cout << g1.top() << endl;
    g1.pop();
}
while(!g2.empty()) {
    cout << g2.top() << endl;
    g2.pop();
}
```

Output:

STL priority_queue example

```
priority_queue<int> g1;
priority_queue<int, vector<int>, greater<int>> g2;
int b[5] = {3, 2, 6, 1, 8};
for (int i = 0; i < 5; ++i) {
    g1.push(b[i]);
    g2.push(b[i]);
}
while(!g1.empty()) {
    cout << g1.top() << endl;
    g1.pop();
}
while(!g2.empty()) {
    cout << g2.top() << endl;
    g2.pop();
}
```

Output:

8
6
3
2
1
1
2
3
6
8

STL priority_queue: complexity

| | Average | worst |
|-------|-------------|-------------|
| push: | $O(\log N)$ | $O(\log N)$ |
| pop: | $O(\log N)$ | $O(\log N)$ |
| top: | $O(1)$ | $O(1)$ |

Q: How to use priority_queues(heaps) to keep track of the median of a data stream?

C++ STL data structure summary

Unordered_set (Hash): fast for look-up, **unsorted**. $O(1)$ for insertion, deletion, look-up.

Set (BST): for look-up, **sorted**. $O(\log N)$ for insertion, deletion, look-up.

Unordered_map (Hash): fast for mapping, **unsorted**. $O(1)$ for insertion, deletion, map by key.

Map (BST): for mapping, **sorted**. $O(\log N)$ for insertion, deletion, map by key.

Priority_queue (heap): for knowing extreme values, **unsorted**. $O(1)$ for knowing the max(min) from max(min) heap. $O(\log N)$ for insertion, deletion. $O(N)$ for look-up.

review problems!

Exercise 1

AVL tree is a self-balancing BST where for each node, the heights of its left subtree and right subtree differ by at most 1. Please implement a function `isAVL()` to check if a BST is an AVL tree. You may use any helper functions.

```
struct Node{
    Node *left, *right;
    int val;
};

bool isAVL(Node* root) {

}
```


Exercise 2

Implement the destructor for a binary tree class defined as

```
class BT {                                BT::~~BT() {
public:                                    }
    struct Node {
        Node *left, *right;
        vector<int*> v;
    };
    Node* root;
    ~BT();
};
```

You may define and use helper functions.

Exercise 3

Consider the following array that has just been partitioned by the first step of the quicksort algorithm:

3 0 2 4 5 8 7 6 9

Could 5 be the initial pivot? What about 7? Explain why or why not.

Things I didn't cover but you probably want to know.
Final review.

Ask me anything!

let's take a break, then...

kahoot time!
