

Shortest Path (SPFA)

Pittsford Sutherland Programming Club

What is shortest path algorithm?

An algorithm that finds the shortest distance between any two nodes in a graph (either directed or undirected).

BFS(Breadth-first search) on a graph

A searching algorithm on a graph that save all the possible nodes in a queue.

Bfs node k , queue q

1. Pushes all unvisited nodes that can be reached from k into q
2. Bfs the first element of q if q isn't empty.

Example of a shortest path problem

An undirected graph G . We define edge set, $E(u,v, w)$ (u and v are the nodes attached to $E(u, v)$, w is the weight of $E(u, v, w)$), and vertex set, V .

Find the shortest distance from v_1 to v_2 ($v_1, v_2 \in V$).

Most popular algorithm - Dijkstra

Dijkstra is a $O(E \log V)$ algorithm that uses Priority Queue(STL in C++).

Dijkstra Algorithm link:

https://www.geeksforgeeks.org/dijkstras-shortest-path-algorithm-using-priority_queue-stl/

SPFA(Shortest Path Faster Algorithm)

Key idea: update the shortest path while doing BFS.

```
procedure Shortest-Path-Faster-Algorithm( $G, s$ )
1   for each vertex  $v \neq s$  in  $V(G)$ 
2        $d(v) := \infty$ 
3    $d(s) := 0$ 
4   offer  $s$  into  $Q$ 
5   while  $Q$  is not empty
6        $u := \text{poll } Q$ 
7       for each edge  $(u, v)$  in  $E(G)$ 
8           if  $d(u) + w(u, v) < d(v)$  then
9                $d(v) := d(u) + w(u, v)$ 
10              if  $v$  is not in  $Q$  then
11                  offer  $v$  into  $Q$ 
```

Sample Question

<https://drive.google.com/open?id=1QozDH0sN-YARbVBRwEsPwHxCnJ1CZv908TxakJHNRw>