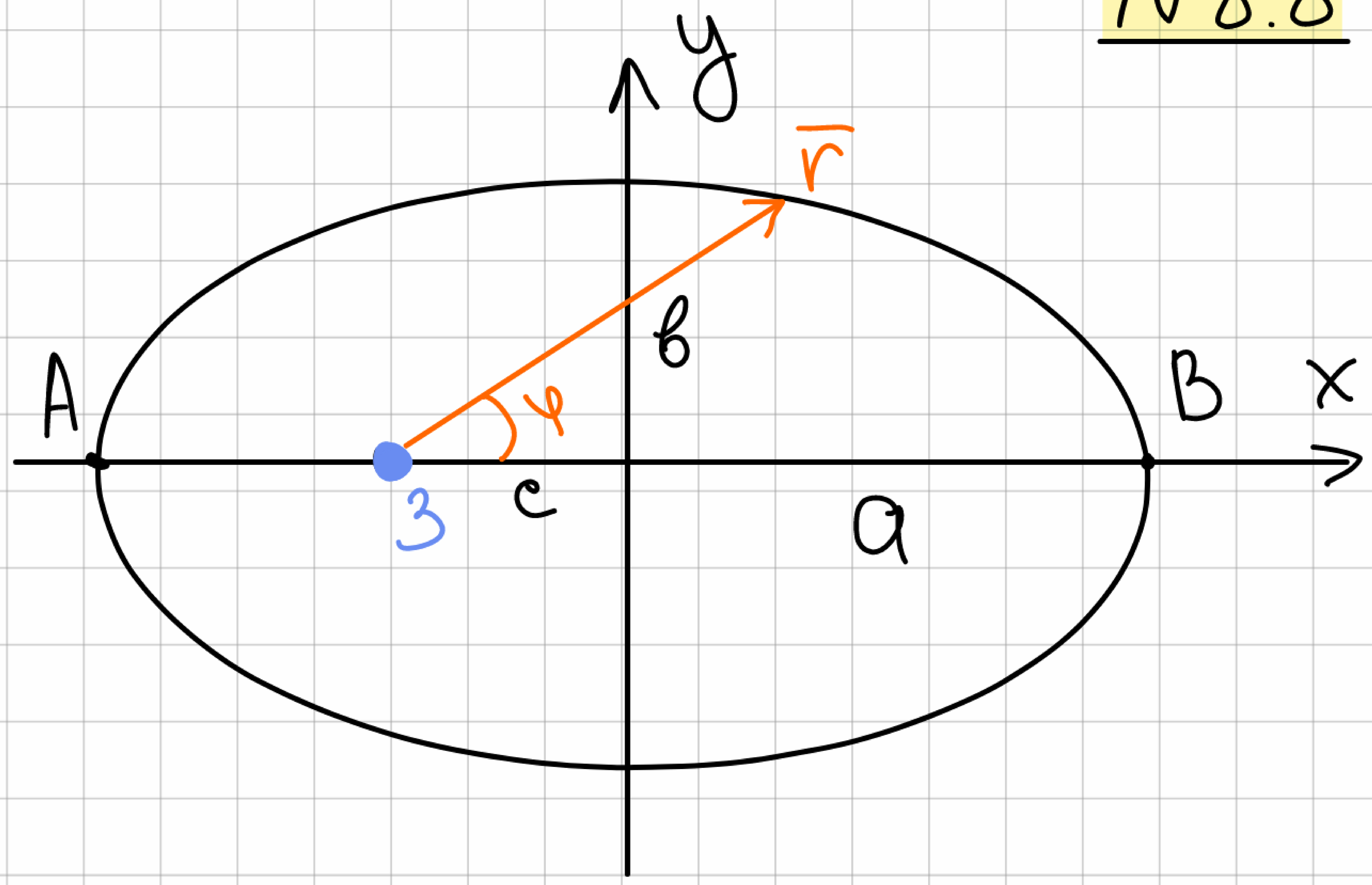
VIII

№ 8.8



$$\frac{\omega_{max}}{\omega_{min}} = ?$$

$$e = \frac{c}{a} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

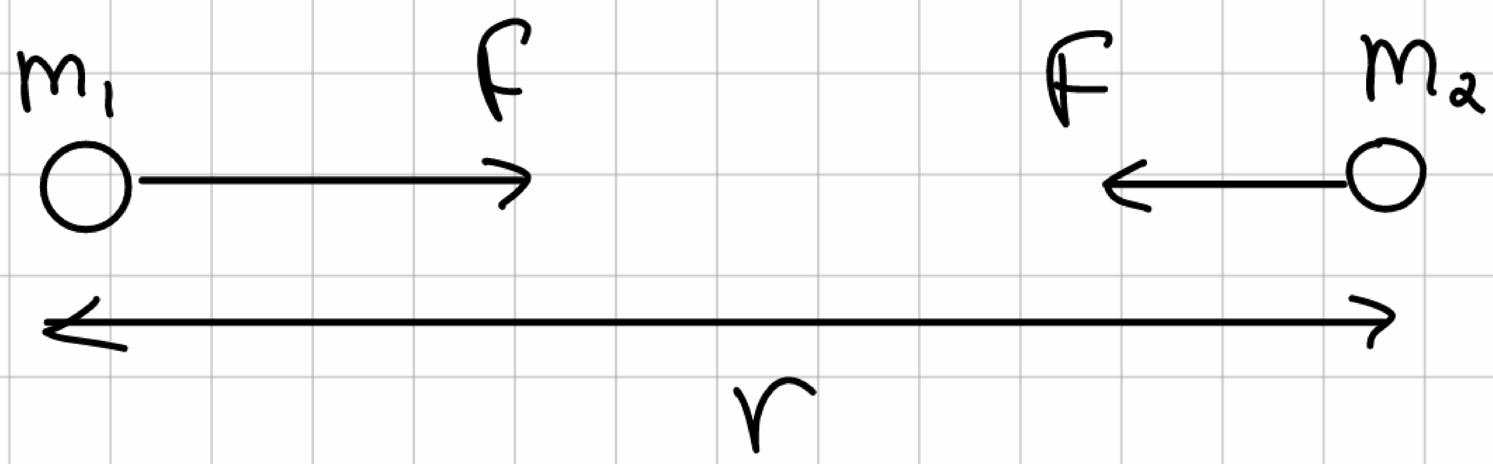
По II з. Кеплера: $r^2 \dot{\varphi} = const \Rightarrow \omega \propto \frac{1}{r^2}$

$\Rightarrow \omega_{max} = \omega_A$; $\omega_{min} = \omega_B$

$$r_{3A} = a - c = a - ae = a(1-e) \Rightarrow \omega_{max} \propto \frac{1}{a^2(1-e)^2}$$

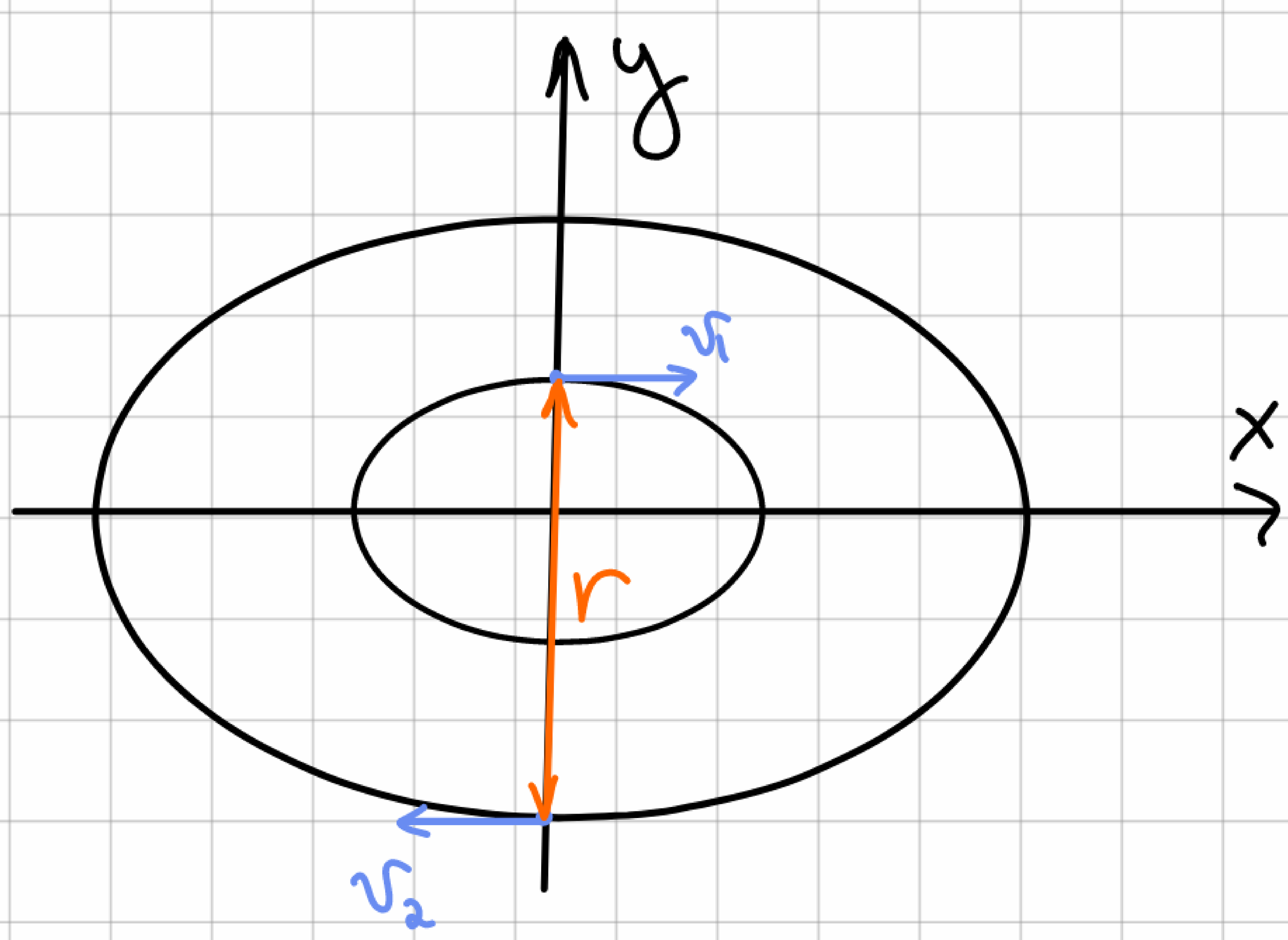
$$r_{3B} = a(1+e) \Rightarrow \omega_{min} \propto \frac{1}{a^2(1+e)^2} \Rightarrow \boxed{\frac{\omega_{max}}{\omega_{min}} = \left(\frac{1+e}{1-e}\right)^2}$$

№ 8.14



$$F = \gamma \frac{m_1 \cdot m_2}{r^2}$$

v_1, v_2 – ?



$$m(\ddot{\rho} - \rho \cdot \dot{\varphi}^2) = F(\rho) \quad , \rho = const \Rightarrow$$

$$\begin{cases} m_1 r_1 \dot{\varphi}^2 = \gamma \dfrac{m_1 m_2}{r^2}; & (1) \\ m_2 r_2 \dot{\varphi}^2 = \gamma \dfrac{m_1 m_2}{r^2}; & (2) \end{cases} \Rightarrow \boxed{r_1 = r_2 \cdot \frac{m_2}{m_1}}$$

$$r = r_1 + r_2 = r_1\left(1 + \frac{m_1}{m_2}\right) \Rightarrow \begin{cases} r_1 = \dfrac{m_2}{m_1 + m_2} \cdot r; \\ r_2 = \dfrac{m_1}{m_1 + m_2} \cdot r; \end{cases}$$

в (1) и (2)

$$v = \rho \dot{\varphi} \Rightarrow \dot{\varphi} = \frac{v}{\rho}$$

$$(1): \quad m_1 \cdot \frac{m_2}{m_1 + m_2} \cdot r \cdot \frac{v_1^2}{\frac{m_2^2}{(m_1 + m_2)^2}} = \gamma \frac{m_1 m_2}{r^2} \Rightarrow \boxed{v_1^2 = \gamma \frac{m_2^2}{(m_1 + m_2) r}}$$

аналогично из (2): $\boxed{v_2^2 = \gamma \dfrac{m_1^2}{(m_1 + m_2) r}}$

N 8.45

8.45. Частица массы m, несущая положительный заряд q, движется в поле неподвижного положительного заряда Q. Найти уравнение траектории.

$$F = \frac{qQ}{r^2}; \quad \bar{F}(r) = F(r) \cdot \frac{\bar{r}}{r}$$

$$\dot{\bar{K}}_0 = \bar{M}_0 = [\bar{r} \times \bar{F}] = \frac{F(r)}{r}[\bar{r} \times \bar{r}] = 0$$

$$\bar{v} = \begin{bmatrix} \dot{r} \\ r\dot{\varphi} \end{bmatrix}; \quad \bar{w} = \begin{bmatrix} \ddot{r} - r\dot{\varphi}^2 \\ \frac{1}{r}\frac{d}{dt}(r^2\dot{\varphi}) \end{bmatrix} \Rightarrow r^2 \dot{\varphi} = C = \text{const}$$

$$m(\ddot r - r\dot\varphi^2) = F(r) \Rightarrow \ddot r - \frac{c^2}{r^3} = \frac{F(r)}{m} \quad (*)$$

Решим дифф.ур., введя замену $u = \frac{1}{r}$:

$$\dot\varphi = \frac{c}{r^2} = c\cdot u^2 \; ; \; \ddot r = -c^2u^2u''_{\varphi\varphi}$$

$$(*) : -c^2\cancel{u^2}u''_{\varphi\varphi} - c^2u^{\cancel{3}} = \frac{F(r)}{m\cdot u^2}$$

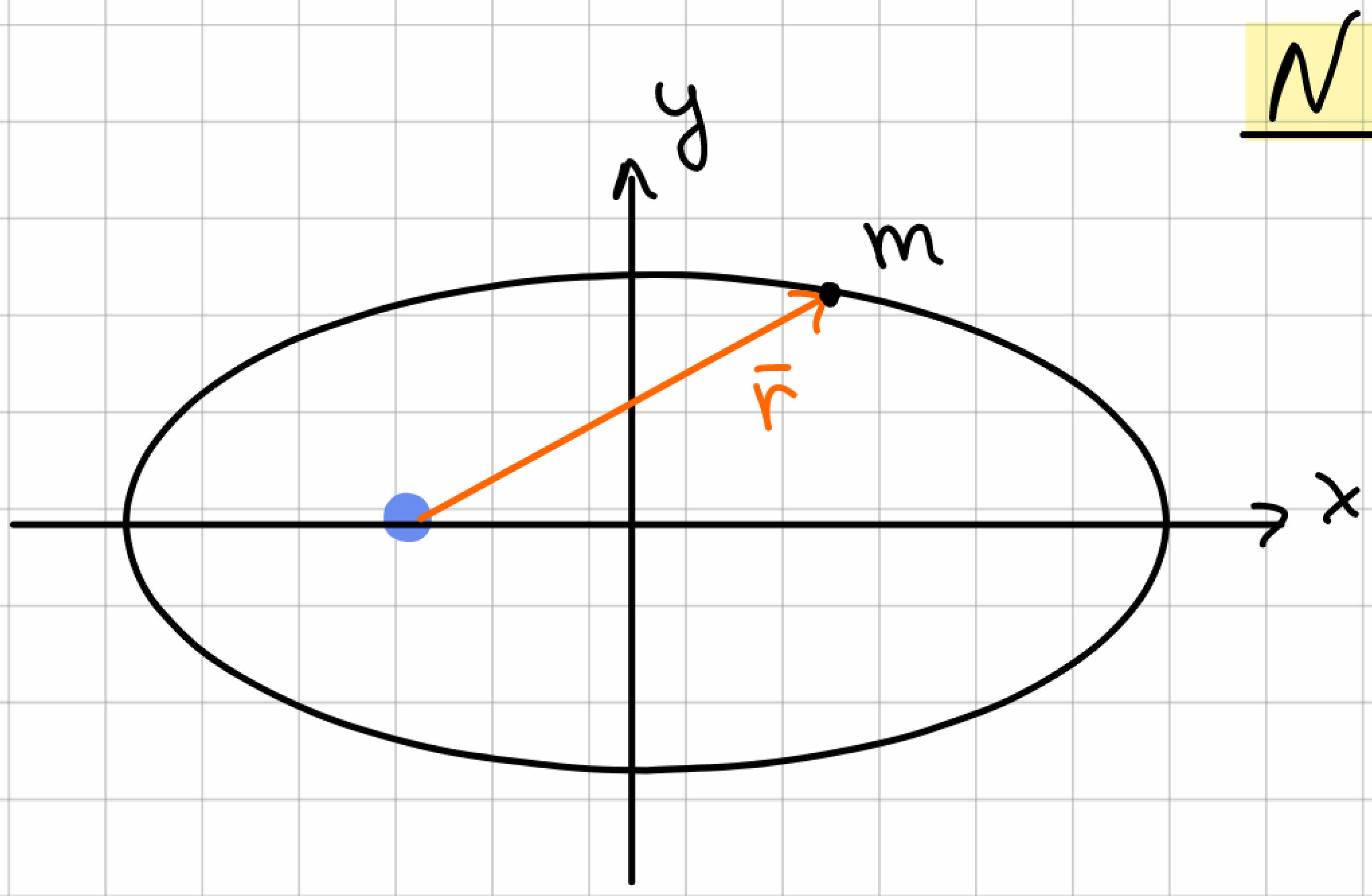
$$\Rightarrow u''_{\varphi\varphi} + u = \frac{\cancel{F(r)}\; qQ}{mc^2\cancel{u^2}}$$

$$\lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i \Rightarrow \overset{\frac{1}{r}}{u} = A\cos\varphi + \frac{qQ}{mc^2}$$

ЗСЭ: $\frac{mv^2}{2} + \frac{qQ}{r} = E$, $v^2 = \dot r^2 + r^2\dot\varphi^2$

$$\Rightarrow \boxed{r(\varphi) = \frac{p}{1 + A\cos\varphi}} \; ; \; p = \frac{\overset{mr^2\dot\varphi = mc}{k_0}}{mqQ} \; ; \; A = \sqrt{1 + \frac{2E}{m}\left(\frac{k_0}{qQ}\right)^2} \; ;$$

№8.53



$$r = \frac{p}{1 + e\cos\varphi}$$

$$m\left(v + \frac{q_1}{m}\right)R = m\left(v - \frac{q_2}{m}\right)R \Rightarrow q_1 = -q_2$$

$$\frac{\cancel{m}\left(v + \frac{q_1}{m}\right)^2}{\cancel{R}} = \gamma\frac{M\cancel{m}}{R^{\cancel{2}}} \qquad R = R_{max} \Rightarrow \cos\varphi = -1$$

$$\left(v + \frac{q_1}{m}\right)^2 = \frac{M(1-e)}{p} \Rightarrow \boxed{v = \sqrt{\frac{M}{p}\cdot(1-e)} - \frac{q_1}{m}} \quad (*)$$

$$r = \frac{p}{1+e\cos\varphi} \Rightarrow u\cdot p = 1 + e\cdot\cos\varphi$$

$$u = \frac{1}{p} + \frac{e}{p}\cdot\cos\varphi; \quad u'_\varphi = -\frac{e}{p}\sin\varphi; \quad u''_{\varphi\varphi} = -\frac{e}{p}\cos\varphi$$

$$u''_{\varphi\varphi} + u = \frac{F(u)}{mc\,u^2} \Rightarrow \cancel{-\frac{e}{p}\cos\varphi} + \frac{1}{p} + \cancel{\frac{e}{p}\cos\varphi} = \frac{Mm\cancel{u^2}}{mc^2\cancel{u^2}}$$

$$\Rightarrow c = \sqrt{Mp}$$

$$\begin{cases} r\dot\varphi = v; \\ r^2\dot\varphi = c; \end{cases} \Rightarrow v = \frac{c}{r} \Rightarrow \boxed{v\cdot r = c}$$

$$v\cdot\frac{p}{1-e} = \sqrt{Mp} \Rightarrow v = \sqrt{\frac{M}{p}}\cdot(1-e)$$

$$\text{в } (*): \quad \sqrt{\frac{M}{p}(1-e)} - \frac{q_1}{m} = \sqrt{\frac{M}{p}}\cdot(1-e)$$

$$\Rightarrow \boxed{q_1 = m\sqrt{\frac{M}{p}}\cdot\sqrt{1-e}\left(1-\sqrt{1-e}\right)}$$

$$t_{min} = \frac{S}{v+\frac{q_1}{m}} = \frac{\psi p}{(1-e)\cdot\left(v+\frac{q_1}{m}\right)} = \boxed{\frac{\psi p}{\sqrt{\frac{M}{p}(1-e)}\cdot(1-e)}}$$