N1.19

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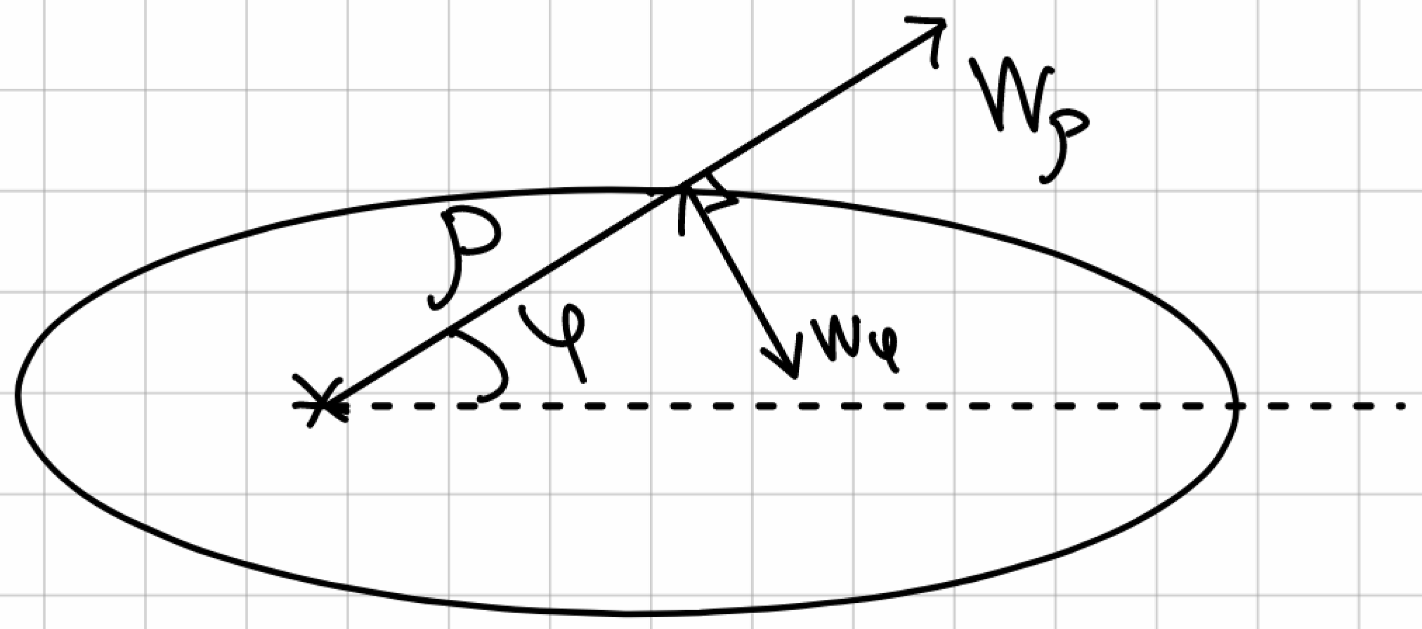


$W_\varphi,\ W_\rho = ?$

$$\rho = \frac{p}{1+e\cos\varphi} \Rightarrow 1 + e\cos\varphi = \frac{p}{\rho} \Rightarrow \boxed{\cos\varphi = \frac{\frac{p}{\rho}-1}{e}}$$

$$\rho^2\dot\varphi = 2C = \text{const} \Rightarrow 2\cancel{\rho}\dot\rho\dot\varphi + \rho^{\cancel{2}}\ddot\varphi = 0 \qquad (*)$$

$$\dot\rho = \underbrace{\frac{p e \sin\varphi\cdot\dot\varphi}{(1+e\cos\varphi)^2}}_{=\frac{\rho^2}{p}} = \frac{\cancel{\rho^2}}{p}\cdot e\sin\varphi\cdot\frac{2c}{\cancel{\rho^2}} = \frac{2ec}{p}\cdot\sin\varphi$$

$$\ddot\rho = \frac{2ec}{p}\cdot\cos\varphi\cdot\dot\varphi = \frac{2\cancel{e}c}{p}\cdot\frac{\frac{p}{\rho}-1}{\cancel{e}}\cdot\frac{2c}{\rho^2} = \frac{4c^2}{p\rho^2}\left(\frac{p}{\rho}-1\right)$$

$$\boxed{W_\rho = \ddot\rho - \rho\dot\varphi^2} = \frac{4c^2}{p\rho^2}\cdot\left(\frac{p}{\rho}-1\right) - \cancel{\rho}\cdot\frac{4c^2}{\rho^{\cancel{4}3}} =$$

$$= \cancel{\frac{4c^2}{\rho^3}} - \frac{4c^2}{p\rho^2} - \cancel{\frac{4c^2}{\rho^3}} = -\frac{4c^2}{p\rho^2}$$

$$\boxed{W_\varphi = 2\dot\rho\dot\varphi + \rho\ddot\varphi} = 0 \quad \text{из } (*)$$

N 1.37 (б)

$v;\ W_r;\ W_\varphi;\ W_\theta = ?$

$$\begin{cases} x = r\cos\varphi\sin\theta; \\ y = r\sin\varphi\sin\theta; \\ z = r\cos\theta; \end{cases}$$

$$\boxed{v^2 = \sum_{i=1}^{3}\sum_{j=1}^{3}\left(\frac{\partial \varphi_i}{\partial q^j}\cdot \dot q^j\right)^2} = \sum_{i=1}^{3}\left[\left(\frac{\partial\varphi_i}{\partial r}\cdot\dot r\right)^2 + \right.$$

$$\left. + \left(\frac{\partial\varphi_i}{\partial\varphi}\cdot\dot\varphi\right)^2 + \left(\frac{\partial\varphi_i}{\partial\theta}\dot\theta\right)^2\right] =$$

$$= \dot r^2\Big(\underbrace{\underbrace{\cos^2\varphi\cdot\sin^2\theta + \sin^2\varphi\cdot\sin^2\theta}_{=\sin^2\theta} + \cos^2\theta}_{=1}\Big) +$$

$$+ \dot\varphi^2 r^2\Big(\underbrace{+\sin^2\varphi\cdot\sin^2\theta + \cos^2\varphi\cdot\sin^2\theta}_{=\sin^2\theta}\Big) +$$

$$+ \dot\theta^2 r^2\Big(\underbrace{\underbrace{\cos^2\varphi\cdot\cos^2\theta + \sin^2\varphi\cdot\cos^2\theta}_{\cos^2\theta} + \sin^2\theta}_{=1}\Big) =$$

$$= \boxed{\dot r^2 + \dot\varphi^2 r^2\cdot\sin^2\theta + \dot\theta^2 r^2 = v^2}$$

$$\boxed{W_j = \left[\sum_{i=1}^{3}\left(\frac{\partial\varphi_i}{\partial q_j}\right)^2\right]^{-\frac12}\cdot\left[\frac{d}{dt}\frac{\partial}{\partial\dot q^j}\left(\frac{v^2}{2}\right) - \frac{\partial}{\partial q^j}\left(\frac{v^2}{2}\right)\right]}$$

$$\left(\frac{\partial x}{\partial r}\right)^2 + \left(\frac{\partial y}{\partial r}\right)^2 + \left(\frac{\partial z}{\partial r}\right)^2 = 1;$$

$$\left(\frac{\partial x}{\partial\varphi}\right)^2 + \left(\frac{\partial y}{\partial\varphi}\right)^2 + \left(\frac{\partial z}{\partial\varphi}\right)^2 = r^2\cdot\sin^2\theta;$$

$$\left(\frac{\partial x}{\partial\theta}\right)^2+\left(\frac{\partial y}{\partial\theta}\right)^2+\left(\frac{\partial z}{\partial\theta}\right)^2 = r^2;$$

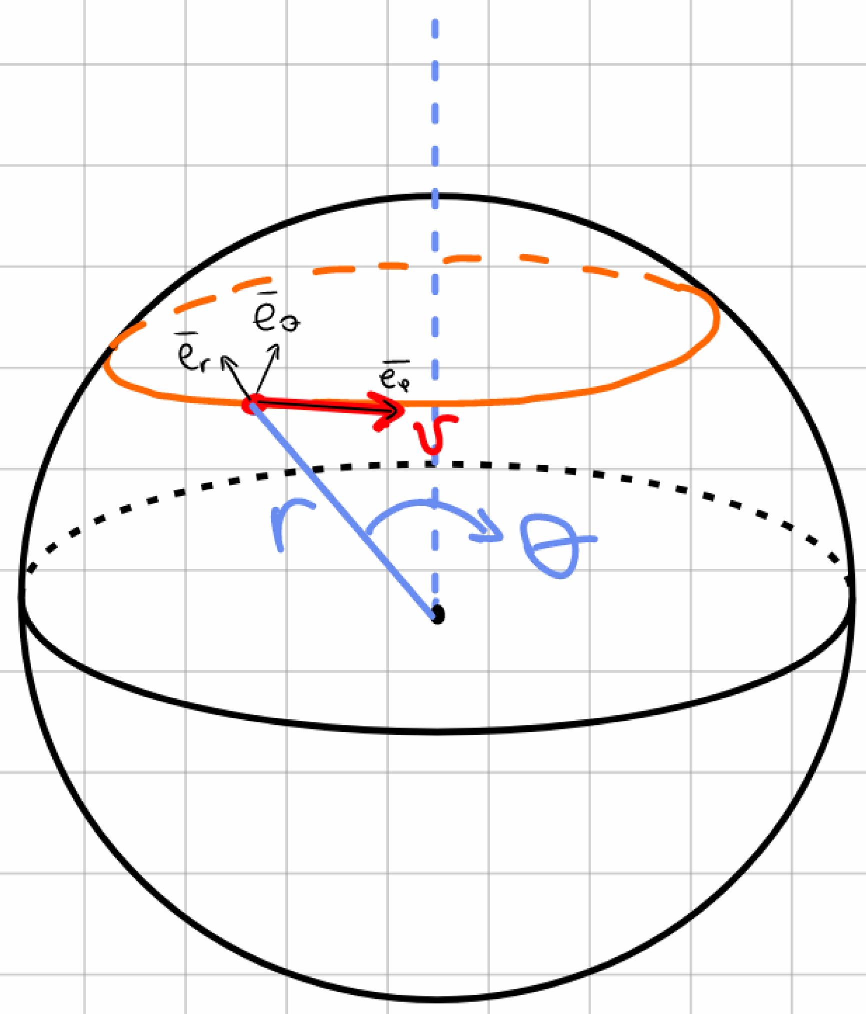
$$\Rightarrow W_r = \frac{d}{dt}\cdot\frac{\partial\left(\frac{1}{2}\left(\dot r^2+\dot\varphi^2 r^2\sin^2\theta + r^2\dot\theta^2\right)\right)}{\partial \dot r} - \frac{\partial}{\partial r}(-//-) =$$

$$= \boxed{\ddot r - \left(r\dot\varphi^2\sin^2\theta + r\dot\theta^2\right)}$$

$$W_\varphi = \frac{1}{r\sin\varphi}\cdot\frac{\partial}{\partial\dot\varphi}(-//-) - \cancel{\frac{\partial}{\partial\varphi}(-//-)}^{=0} = \boxed{\ddot\varphi r\sin\theta + 2\dot\varphi\dot r\sin\theta + 2\dot\varphi r\cos\theta\cdot\dot\theta}$$

$$W_\theta = \frac{1}{r}\cdot\frac{\partial}{\partial\dot\theta}(-//-) - \frac{\partial}{\partial\theta}(-//-) = \boxed{\left(2\dot r\dot\theta + r\ddot\theta\right) - \dot\varphi r\sin\theta\cos\theta}$$

N1.41



$r = const;\ \theta = const;$

$$k = \frac{\bar n}{\rho}$$

из $(1.37\ б)$: $W_r = -r\dot\varphi^2\sin^2\theta;\ W_\varphi = 0;\ W_\theta = -r\dot\varphi^2\sin\theta\cos\theta$

$$\bar n = \frac{\bar W}{W} = \frac{W_r\cdot\bar e_r + W_\theta\cdot\bar e_\theta}{\sqrt{W_r^2+W_\theta^2}} \Rightarrow \bar k = \frac{\bar n}{r\cdot\sin\theta}$$

$$\Rightarrow \boxed{\bar k = -\frac{\bar e_r}{r} - \frac{\bar e_\theta\cdot ctg\,\theta}{r}}$$

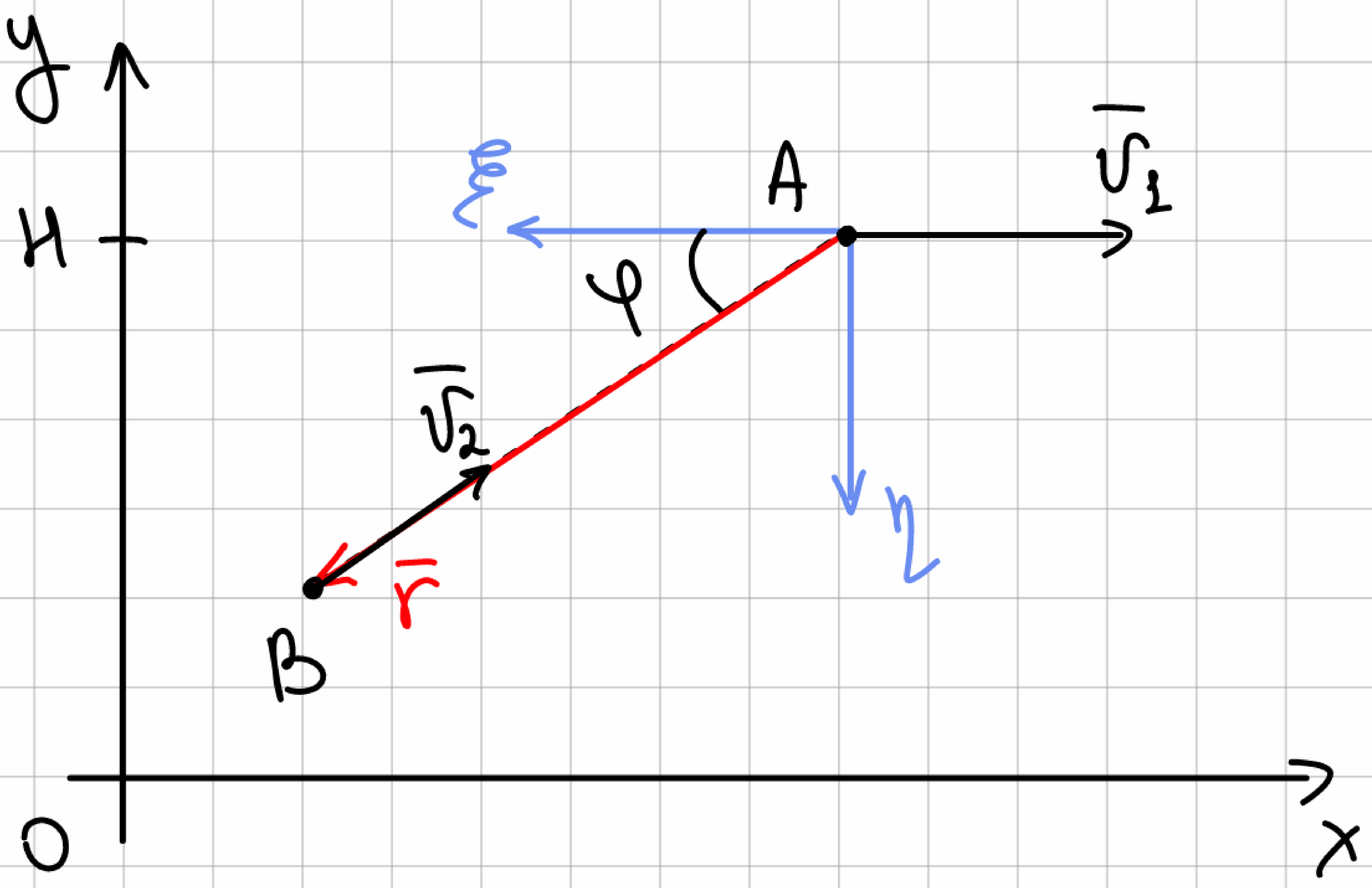
№1.47

$$\bar W = \frac{\gamma}{r^3}\cdot[\dot{\bar r}\times\bar r]\ ;\quad \gamma = const;\quad \bar v_0\ ;$$

$$(\bar W\cdot\bar r) = \frac{\gamma}{r^3}\left([\dot{\bar r}\times\bar r],\bar r\right) = 0 \Rightarrow \bar W\perp\bar r \quad (\bar W\perp\dot{\bar r})$$

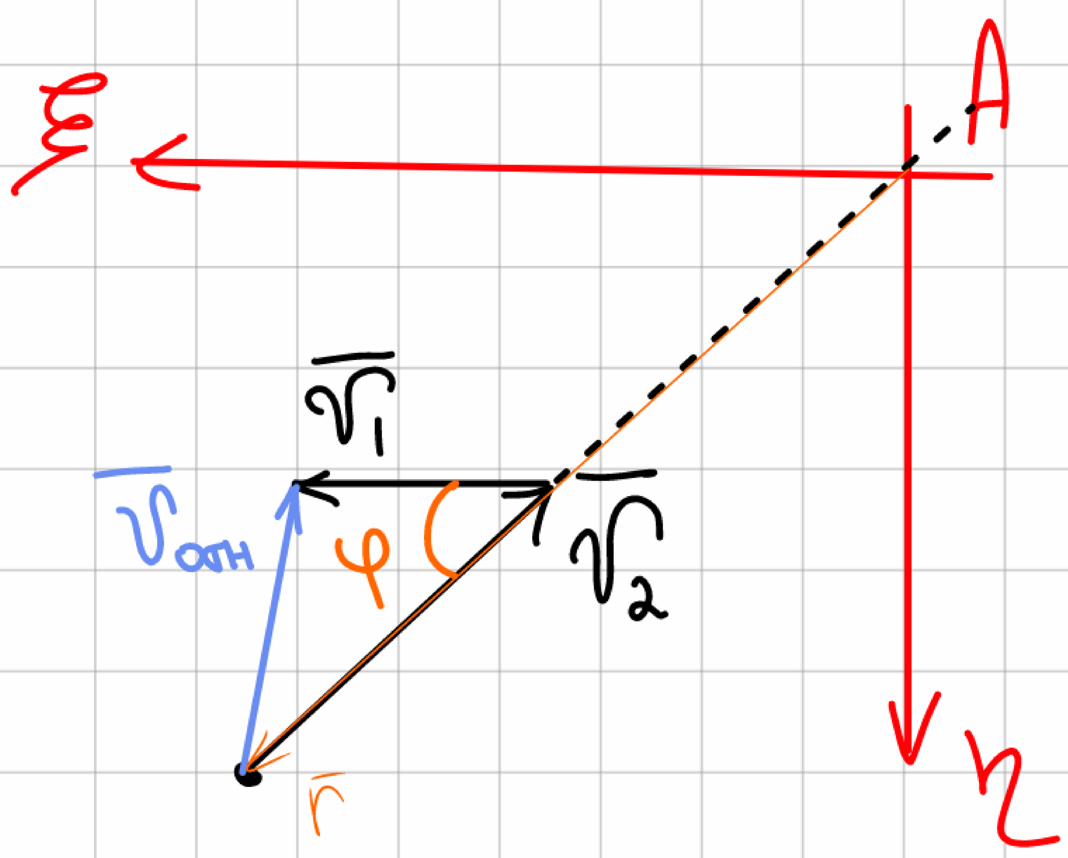
$$\rho_{кр} = \frac{v_0^2}{|\bar W|} = \frac{v_0^2 r^3}{|\gamma\cdot[\dot{\bar r}\times\bar r]|} = \boxed{\frac{v_0^2 r^3}{|\gamma\cdot[\bar v_0\times\bar r]|}}$$

№1.29



$v_1 = v;\ v_2 = 2v$

$r(\varphi) = ?$ в $A\xi\eta$



Коэфф. Ламе: $H_r = \left|\frac{\partial\bar r}{\partial r}\right| = 1;\ H_\varphi = \left|\frac{\partial\bar r}{\partial\varphi}\right| = r$

$\Rightarrow\ v_r = h_r\cdot\dot r = \dot r;\ v_\varphi = h_\varphi\cdot\dot\varphi = r\dot\varphi$

$$\begin{cases}\dot r = -v_2 + v_1\cos\varphi = \underline{v(\cos\varphi - 2)} & (1)\\ r\dot\varphi = v_\varphi = -v_1\sin\varphi = \underline{-v\sin\varphi} & (2)\end{cases}$$

$$(1):(2) \Rightarrow \int_H^r \frac{dr}{r} = \int_{\frac{\pi}{2}}^{\varphi} \frac{2-\cos\varphi}{\sin\varphi}\, d\varphi \Rightarrow \ln\frac{r}{H} = 2\ln\left|\frac{\sin\varphi}{(1+\cos\varphi)}\right| - \ln|\sin\varphi|$$

$$\Rightarrow \boxed{r = H\cdot\frac{\sin\varphi}{(1+\cos\varphi)^2}}$$

из (2): $r\dfrac{d\varphi}{dt} = -v\sin\varphi$;

$$H\cdot\frac{\sin\varphi}{(1+\cos\varphi)^2}\cdot\frac{d\varphi}{dt} = -v\sin\varphi \Rightarrow \int_{\pi/2}^{0}\frac{d\varphi}{(1+\cos\varphi)^2} = -\frac{v}{H}\int_0^T dt$$

$$\Rightarrow -\frac{2}{3} = -\frac{v}{H}T \Rightarrow \boxed{T = \frac{2}{3}\frac{H}{v}}$$

$$W_r = \ddot{r} - r\dot{\varphi}^2 = 0 \quad (\text{т.к. } v = const)$$

$$W_\varphi = r\ddot{\varphi} + 2\dot{r}\dot{\varphi} =$$

$$\dot{r} = v\cos\varphi - 2v \Rightarrow \ddot{r} = -v\sin\varphi\,\dot{\varphi}$$

$$\dot{\varphi} = -\frac{v\sin\varphi}{r};\quad \ddot{\varphi} = \frac{-v\cos\varphi\,\dot{\varphi}\cdot r + v\sin\varphi\,\dot{r}}{r^2} = -\frac{v\cos\varphi\,\dot{\varphi}}{r} +$$

$$+\frac{v^2\sin\varphi\cos\varphi}{r^2} - \frac{2v^2\sin\varphi}{r^2} \Rightarrow W_\varphi = \frac{2v^2\sin\varphi}{r}$$

$$\Rightarrow \boxed{W_\varphi = \frac{2v^2}{H}\cdot(1+\cos\varphi)^2}$$