Pattern Recognition Based on Fuzzy Three-Valued Logic Reasoning

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***Abstract*—Fuzzy pattern recognition is an application of fuzzy mathematic theory in the field of pattern recognition. Theoretic basis of this method is fuzzy multiple-valued logic whose truth-value is determined depending on membership function. Now, however, the membership function definition of fuzzy set has not formed uniform method and general format yet. Determination of membership function is mainly affected by subjective factors and is easy to become troublesome problem. For solving this problem, this paper firstly proposes fuzzy three-valued logic [0, 0.5, 1] on the basis of research fruits about physiological function of "synapse" of human's cerebra and objective situation of truth-values of proposition existing. According to L. A. Zadeh's fuzzy set theory, synthesizing operation rules of fuzzy relationship and fuzzy reasoning theory, the paper introduces and discusses the explanation for fuzzy three-valued logic and the scientificalness and rationality for applying it into pattern recognition. Application examples in this paper indicate the feasibility and maneuverability of the method proposed in this paper. Certain practical value has been presented with the fact that the recognition results comply with the actual situation.**

Keywords- fuzzy pattern recognition; fuzzy three-valued logic; fuzzy set; membership function; logic true value

I. INTRODUCTION

Pattern recognition which has become a fairly perfect theoretic system is a subject rapidly developed in 1960's. As a subject, pattern recognition belongs to machine intelligence so as to belong to the artificial intelligence category. Theoretic development and wide application of pattern recognition greatly promote the development and application of artificial intelligence.

Recently, fuzzy mathematics theory has been applied in pattern recognition and formed unique fuzzy pattern recognition methodology. Theoretic basis of this methodology is fuzzy multiple-valued logic or infinite logic. Critical problem for taking multiple value of logic truth-value is to precisely determine the membership function with which the calculated precision of membership grade directly affects the precision of pattern recognition. However, to determine the membership function, consideration should be taken not only on mathematic theory but also on practice. Thus, applying fuzzy pattern recognition in practice issue is not easy.

To be convenient for extending application of fuzzy pattern recognition, this paper firstly proposes a novel pattern recognition methodology based on fuzzy three-valued logic [0, 0.5, 1] according to background of three-valued logic situation existing in real life and result found by cerebra physiologist that synapse which is the basic unit of cerebra has three functional states including opening, closing and partly closing. Some examples are appended for reference.

II. BASIC METHODS FOR PATTERN RECOGNITION

Basic methods for pattern recognition include statistics pattern recognition, structural pattern recognition, fuzzy pattern recognition, neural network pattern recognition, multiple classifier fusion, etc.

By analyzing above five kinds of pattern recognition methods, we consider on view of thinking logic that they should roughly categorized into two kinds of logic, two-valued logic and fuzzy logic. Fuzzy pattern recognition thought belongs to fuzzy logic and the others belong to two-valued logic.

III. FUZZY THREE-VALUED LOGIC

A. Concept of fuzzy three-valued logic

Typically, fuzzy logic is an infinite one. Fuzzy three-valued logic which is called three value logic defines the truth-value of fuzzy proposition with three values within interval [0, 0.5, 1]. Meaning of three-valued logic adopted in this paper is:

When truth-value of proposition is "0", it is a false proposition;

When truth-value of proposition is "1", it is a true proposition;

When truth-value of proposition is "0.5", it is considered half-true and half-false (intermediate state) proposition, or mixed proposition. Truth-values of all propositions with incomplete true or false deem to be 0.5 no matter how much

978-1-4244-4994-1/09/$25.00 ©2009 IEEE

truth-value proportions occupied. Purpose of this provision is to simplify the problem and easy to calculate.

B. Fuzzy rules and their representations

L. A. Zadeh's fuzzy rules depict the relationship between fuzzy set in precondition and that in conclusion of fuzzy proposition, namely fuzzy relationship. This relationship can be represented by fuzzy set so as to convert fuzzy rules into fuzzy set. And then natural language rules are converted into numerical set (matrix).

Suppose that A and B are two fuzzy sets, R is a kind of relationship between A and B, U and V respectively represent the domain to which A or B belongs, $\mu_R(u_i, v_j)$ (i,j=1,2,……)is membership grade of element (u_i, v_j) corresponding to R, then :

$$R = \mu_R(\mu_1, v_1)/(\mu_1, v_1) + \mu_R(\mu_1, v_2)/(\mu_1, v_2) + \ldots + \mu_R(\mu_i, v_j)/(\mu_i, v_j) + \ldots = \int_{U \times V} \mu_R(u, v)/(u, v) \quad (1)$$

Suppose a rule that if x is A, then y is B. Here, A and B are two language values. So, A and B are two fuzzy sets on the view of L. A. Zadeh. This rule represents a fuzzy relationship R, and R can also be represented as a fuzzy set.

For calculating membership grade $\mu_R(u_i, v_j)$ (i, j=1, 2, …), L. A. Zadeh proposed a lot of methods among which a representative one is:

$$\mu_R(u_i, v_j) = (\mu_A(u_i) \wedge \mu_B(v_j)) \vee (1 - \mu_A(u_i)) \quad (i, j=1, 2, \ldots) \quad (2)$$

Where, $\wedge$ and $\vee$ represent taking minimal value and maximal value, i.e. min and max.

Then the original natural language rule becomes a numerical set (matrix), namely $A \to B = R$

C. Essentiality of three-valued logic

Thinking process of human being often faces a large number of ambiguous fuzzy states such intermediate states as half-true and half-false or coexistence of true and false and so on. Obviously, this real problem could not be settled using two-valued logic.

To overcome the limitation of two-valued logic, Professor L. A. Zadeh proposed fuzzy set theory followed by fuzzy logic which took truth-value within the interval [0, 1] and introduced multiple-valued logic or infinite logic. On view of Logic, this solves ambiguous states in thinking process. The critical problem for taking multiple logic truth-value (theoretically infinite values can be taken) within [0, 1] depends on the rational determination of membership grade.

It is worth to point out that the definition of membership function of fuzzy set does not have a uniform method and general format. Basically, it is subjectively given by human. Although some specific formats for membership function have been proposed in some references, certain theoretic foundation is absent with these formats. They are mainly gotten from human's imagination. Even, some of them are given purely on the view of mathematic esthetics which bring about large amount of randomicity.

Since determination of truth-value of fuzzy logic within [0, 1] has large amount of randomicity, thus, it is inevitable for simplifying intermediate states in fuzzy logic, i.e. simplifying fuzzy logic from multiple-value logical states to three-valued logic states. After simplification, the fuzzy logic still maintains certain or the lowest limit which can be only three-valued logic. Hence, the fuzzy three-valued logic proposed in this paper is reasonable and scientific.

D. Some Common Mistakes

Actually, fuzzy three-valued logic is a special example of fuzzy logic or multiple-valued logic. Under satisfying the definition and operation rules of fuzzy logic, fuzzy logic becomes fuzzy three-valued logic only when one intermediate value, 0.5, is chosen within truth-value interval [0, 1] of fuzzy logic. Number "3" was deemed as majority in ancient China. By this, three-valued logic is actually multiple-valued logic. The explanation for the background and significance of fuzzy three-valued logic proposed in this paper is presented below.

1) Objectively existing of ambiguous and half-true and half-false reality

Many fuzzy phenomena and states exist which correspond to intermediate value between the truth-values, 0 and 1, of fuzzy logic. Ambiguous events and languages always happen. Examples include vague and ambiguous statements in articles in daily life and work. There are some other examples such that it will rain tomorrow or will not; students often say "half-right-half-wrong" while discussing the solution of a question after exam, people sometimes say "true-or-false can not be distinguished, halt-trust and half-doubt or six of one, half a dozen of other" while talking about some matters. Other similar situations exist.

Thus it can be seen that large amount of "intermediate stares" exist within the status interval [0, 1] in fuzzy logic. Comparably, the "third state" is universal. Hence, existing of objective reality is background of fuzzy three-valued logic proposed in this paper.

2) Supporting of physical research fruits on cerebra

A Finn cerebral physiologist ever proposed that the basic components for constructing cerebra and computer were completely different. The basic components of computer are transistors, whereas basic components of cerebra are synapses. This physiologist considered that a significant difference between transistors and synapses is that the latter not only possessed two states (open and close) used by computers, but also can be opened or closed partly. So, synapses maybe possess intermediate value such as nine tenth, two tenth, etc. except for two states, 0 and 1, possessed by computers. In language of fuzzy logic, the truth-value can be not only taken as two extreme situations, 0 and 1, but also taken as intermediate values between 0 and 1. Thus, it can be seen that there are sufficiently scientific foundations on which

affirmation of existence of three-valued logic and further proposed three-valued logic [0, 0.5, 1] are based.

E. Rationality of definition of truth-value '0.5'

1) It satisfies the Sufficient Reason Law for taking 0.5 as intermediate value

Equation of Sufficient Reason Law is that A is true because B is true, and B can induce A. physical research shows that synapse of cerebra possess one state among intermediate values between 0 and 1 under these two extreme states. 0.5 is a number between 0 and 1; hence, it is a truth-value of three-valued logic. Set [0, 0.5, 1] meets the requirement of three kinds of opening and closing functions possessed by synapse. This means that intermediate value, 0.5, meets the requirement of three-valued logic.

We can conclude that two conditions should be satisfied as sufficient reason: (a) the reason must be true, i.e. B is true; (b) reason and conclusion must have certain logical relationship.

2) 0.5 is the arithmetic mean value of truth-value interval [0, 1]. i.e. (0+1) ÷2 =0.5. On the view of mathematics, this can better reflect the intermediate value between [0, 1].

3) The ambiguous intermediate state has the biggest fuzzy grade. Truth-value selection of 0.5 can be more correct reflection of the fuzzy grade of fuzzy logic than other random values selected within 0 and 1. This depends on the following definition of fuzzy grade proposed by DeLuca in 1972.

Mapping f : P(E)→[0, 1] is called fuzzy grade on P(E) , if it satisfies following conditions:

$$A \in \widetilde{P}(E) \Leftrightarrow f(A) = 0 \tag{3}$$

$$\mu_{\widetilde{F}}(x) \equiv 0.5 \Leftrightarrow f(\widetilde{F}) = 1 \tag{4}$$

$$\forall x \in E, \text{且} \widetilde{A}, \widetilde{B} \in \widetilde{P}(E) \tag{5}$$

If $\mu_{\widetilde{A}}(x) \geq \mu_{\widetilde{B}}(x) \geq 0.5$,or $\mu_{\widetilde{A}}(x) \leq \mu_{\widetilde{B}}(x) \leq 0.5$,then $f(\widetilde{A}) \leq f(\widetilde{B})$

Apparently, according to equation (3), fuzzy grade of ordinary set is 0, i.e. the ordinary set is unequivocal.

According to equation (4), fuzzy set whose all fuzzy grade equal to 0.5 is the fuzziest in which all elements are ambiguous.

According to equation (5), we can easily know that the more fuzzy set close to the fuzziest set $\widetilde{F}$, the bigger the fuzzy grade is.

To illuminate the fact that truth-value of 0.5 can mostly reflect the fuzzy grade of ambiguous intermediate states, two truth-values, 0.8 and 0.3 are randomly selected for calculating their Hamming fuzzy grades. The calculated fuzzy grades are 0.14 and 0.2. They are all less than 0.34, the Hamming fuzzy grade of 0.5.

F. Reasoning Equations using fuzzy three-valued logic

Suppose that A' is fuzzy matrix (vector) of evidence facts, B' is the conclusion reasoned with fuzzy proposition, R is synthesized fuzzy relationship (set) or fuzz matrix synthesized using precondition fuzzy set and conclusion fuzzy set according to L. A. Zadeh fuzzy relationship. Then fuzzy reasoning equations or fuzzy relationship equations can be concluded as below:

1) Reasoning equation for B' under known A' and R is $B' = A' \circ R$

If it is represented using membership function, then

$$\mu_{B'}(y) = \bigvee_{x \in U} \{\mu_{A'}(x) \wedge \mu_R(x, y)\}$$

$$= \bigvee_{x \in U} \{\mu_{A'}(x) \wedge [\mu_A(x) \wedge \mu_B(y) \vee (1 - \mu_A(x))]\} \tag{6}$$

2) Reasoning equation for A' under known B' and R is $A' = R \circ B'$

IV. Application Example—Picture-Looking Recognition

"Ancient Temple Hiding behind the Remote Mountain" is a famous drawing of Song Dynasty. It was drawn in the period of Song Huizong according to same named proposition. Temple does not appear in it, but that bonze who is carrying water at the bank of brook amongst the forest makes admirer to imagine in mind along his foot prints that the ancient temple exists in the tree-lined mountain forest. This is reasoning result in art enjoying gained by using visualized thinking. Song Huizong made out the brilliant point in this drawing at a glance. He admired and said: "this drawing is precious by using a bonze to point out the word 'hiding'."

Convert above visualized thinking proposition which deduces ancient temple hiding behind the remote mountain from carrying water bonze into fuzzy proposition, then carry out picture-looking recognition by using the method proposed in this paper according to fuzzy three-valued logic reasoning method, i.e. recognize whether the temple hiding in the remote mountain using picture or drawing, and what is the result?

A. Statement of fuzzy proposition

It is known that if there is one bonze carrying water at the edge of brook, then there is a temple in the remote mountain; if there is a person looking like the bonze is walking into the remote mountain along the devious alley, whether the temple hides behind the remote mountain?

B. Three-valued logic reasoning

Suppose：

A→farmer,B→bonze

Fuzzy membership grades of A and B are:

μ_A = (1/house, 0/temple)

μ_B =(0.5/house,1/temple)

Note: "0.5" represents that the house may be occupied by the bonze or by the farmer. It is a "membership state" within "intermediate states".

μ_R (house,house)

$= \mu_A$ (house)$\wedge \mu_B$ (house)$\vee$(1- μ_A (house))

$=(1 \wedge 0.5) \vee (1-1)$

$=0.5$

μ_R (house,temple)

$= \mu_A$ (house)$\wedge \mu_B$ (temple)$\vee$(1- μ_A (house))

$=(1 \wedge 1) \vee (1-1)$

$=1$

μ_R (temple,house)

$= \mu_A$ (temple)$\wedge \mu_B$ (house)$\vee$(1- μ_A (temple))

$=(0 \wedge 0.5) \vee (1-0)$

$=1$

μ_R (temple,temple)

$= \mu_A$ (temple)$\wedge \mu_B$ (temple)$\vee$(1- μ_A (temple))

$=(0 \wedge 1) \vee (1-0)$

$=1$

$$\therefore R = \begin{bmatrix} 0.5 & 1 \\ 1 & 1 \end{bmatrix}$$

$$B' = (1,0) \circ \begin{bmatrix} 0.5 & 1 \\ 1 & 1 \end{bmatrix} = (0.5,1)$$

It represents (0.5/house, 1/temple).

The conclusion can be basically comprehended that there is a temple in the remote mountain or "ancient temple hiding behind the remote mountain".

V. CONCLUSION

Pattern recognition is one of the most important content in artificial intelligence. The research of the pattern recognition in this paper is based on fuzzy set, fuzzy logic and fuzzy reasoning theory. Fuzzy three-valued logic not only satisfies the requirement of functional situation of synapse in cerebra physiology, but also close to the thinking reality existing in daily life. So, it is feasible to research pattern recognition using fuzzy three-valued logic reasoning theory. Example indicates that the method proposed herein is maneuverable and can be preferably used in settlement of actual problems. However, recognition precision of this method needs to be further studied.

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