

2.4 证明:

$\because Y$  为第 1 列为 0 的矩阵

$$\text{设 } Y \text{ 的形式为 } Y = \begin{pmatrix} y_{11} & \cdots & y_{n1} \\ y_{21} & & \vdots \\ \vdots & & \vdots \\ y_{m1} & \cdots & y_{mn} \end{pmatrix} = \begin{pmatrix} 0 & \cdots & y_{n1} \\ 0 & & \vdots \\ \vdots & & \vdots \\ 0 & \cdots & y_{mn} \end{pmatrix}$$

$$\therefore \det Y = 0 \times A_{11} + 0 \times A_{21} + \cdots + 0 \times A_{m1} = 0$$

$\because I$  为单位矩阵,  $Z$  为零矩阵

$$\text{设 } A \text{ 的形式为 } A = \begin{pmatrix} a_{11} & \cdots & a_{1j} \\ a_{21} & & \vdots \\ \vdots & & \vdots \\ a_{i1} & \cdots & a_{ij} \end{pmatrix} = \left( \begin{array}{c|ccc} 1 & 0 & \cdots & a_{1j} \\ 0 & 1 & & a_{2j} \\ \vdots & \vdots & & \vdots \\ 0 & a_{i1} & \cdots & a_{ij} \end{array} \right)$$

可知  $A$  为分块三角矩阵, 设右下角矩阵为  $B$

$$\therefore \det A = 1 \times \det B$$

$$\begin{aligned} \text{而 } \det B &= 1 \times A_{22} + 0 \times A_{32} + 0 \times A_{42} + \cdots + 0 \times A_{i2} \\ &= 1 \times \det Y + 0 = 0 \end{aligned}$$

$$\therefore \det A = 0$$

$\therefore A$  为奇异矩阵