Event control action rules for Ampersand

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Abstract

Ampersand Tarski is a tool used to produce functional software documents based on business process requirements. At times, logical discrepancies arise when system changes occur which violate the restrictions set forth by the user. When a system violation occurs, one of two things can happen: the change that is meant to take place is adjusted so it no longer violates the restrictions or the changes are discarded. The purpose of Event condition action rules for Ampersand (EFA) was to replace the exec-engine that is currently used to deal with violations; unlike the exec-engine, EFA is automated and provides proof of correctness embedded in the code, it able to type SQL statements and assure no "dead-ends" occur when queries are executed.

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1 Introduction

This document is a meant as a guide for EFA that includes the motivations taken from a business perspective, the mathematical and software foundations that resulted in the logical flow of EFA's design, and the testing that took place to assure EFA's functionality and correctness.

Currently, Ampersand is readily accessible to the public through Github and it is equipped with the ability to assess logical discrepancies on sets of data based on user-specified restrictions. Logical discrepancies arise when system changes occur which violate the restrictions set forth by the user. When a system violation occurs, one of two things can happen: the change that is meant to take place is adjusted so it no longer violates the restrictions or the changes are discarded. Ampersand is used to manipulate data and generate prototypes, although there is a debugger, certain errors still slip through. When the system rules are changed by the user, all data which are inconsistent with the new system must be eliminated or rehabilitated so it can be returned back into the system. Data inconsistencies are persistent bugs that can distort the product that Ampersand seeks to provide.

These data inconsistencies are corrected through ECA rules which use process algebra (PA) to correct or discard data using violations. EFA is used to translate these ECA rules, execute SQL queries to correct violations and safeguards the database from illegal transactions. ¡¡¡¡¡¡¡ HEAD

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1.4 Naming Conventions and Terminology

ECA Stands for Event-Condition Action. The rule structure used for data bases and commonly used in market ready business rule engines. ECA rules are used in Ampersand to describe how a database should be modified in response to a system constraint becoming untrue.

ADL Stands for "Abstract Data Language" ((**?** , 13)). From a given set of formally defined business requirements, Ampersand generates a functional specification consisting of a data model, a service catalog, a formal specification of the services, and a function point analysis. An ADL script acts as an input for Ampersand. An ADL file consists of a plain ASCII text file.

Ampersand Ampersand is the name of this project. It is used to refer to both the method of generating functional specification from formalized business requirements, and the software tool which implements this method.

Business requirements Requirements which exist due to some real world constraints (i.e. financial, logistic, physical or safety constraints).

Business rules See *Business Requirements*.

EFA Stands for "ECA (see above) for Ampersand". This term is used to refer to the contribution of this project.

Functional specification A *formal* document which details the operation, capabilities, and appearance of a software system.

Natural language Language written in a manner similar to that of human communication; language intended to be interpreted and understood by humans, as opposed to machines.

Requirements engineering The process of translating business requirements into a functional specification.

Prototype Ampersand generates a prototype for the user that provides a front-end interface that connects to a back-end database.

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Ampersand

Objectives

Document Guide

2 Business

Ampersand follows a *rule based* design principle. Rules are integral to an organization and these are based on some principles and guidelines set by an organization. Ampersand uses an ECA (Event - Condition - Action) approach to make sure all rules are satisfied. An ideal information infrastructure supports employees and other stakeholders to maintain the rules of the business. To maintain a rule means to prevent or correct all violations that might occur due to any external or internal factor.

This problem is addressed in Ampersand using AMMBR (Joo). The role of the AMMBR method in Ampersand is to maintain these business rules. In AMMBR, human involvement is only limited to representing rules (in the ADL files). While the Ampersand software still remains in development phase, there is no way of checking the correctness of the AMMBR method at a low level. Previously the ECA rule generated by AMMBR was directly fed into the Exec Engine to maintain these business rules.

With the extension of Ampersand by the EFA project, we allow means of checking the correctness of the Action associated with an ECA rule. The ECA rules, which act as an input to the EFA project are translated to human-readable SQL. This can later be viewed in the command line using the `''~-~-print~-eca~-info''` flag written out in one of the artifacts. The generated SQL, not only allows for checking the correctness of AMMBR but also helps in identifying patterns and unreachable states of the method. EFA project will utilized to complete the AMMBR algorithm and also to test out future modification to the AMMBR algorithm. Since AMMBR is an essential element of the Ampersand software, EFA will compliment future modification to this method.

3 System Architecture and Module Hierarchy

3.1 System Architecture

This section provides an overview of system architecture and module hierarchy. The initial section introduces term and tools used in the making of each EFA module. The module design is detailed with UML-like class diagrams. However, UML class diagrams are typically used to describe the module systems of object-oriented programs, as opposed to functional programs. Many of the components of the traditional UML class diagram are inapplicable to functional programs; therefore, we detail our modifications to the UML class diagram syntax in section

Furthermore, the syntax used to describe types and data declarations is not actual Haskell syntax. The syntax shares many similarities, but several changes to the syntax are made in this document in order to present the module hierarchy in a clear manner. These changes are also detailed, in section

3.2 Partner or Collaborative Applications

This section provides a brief description of the modules that EFA uses. More details are provided in the Module Guide for EFA.

Ampersand Core Modules	
Module Name	**Description**
AbstractSyntaxTree	Ampersand's abstract representation of input from ADL.
FSpec	A module that represents the F-Spec structure.
Basics	An Ampersand internal module that provides basic functions for data manipulation.
ParseTree	Ampersand parse tree for ADL script; concrete representation of the input, and retains all information of the input.

External Haskell Modules	
Module Name	**Description**
GHC.TypeLits	Internal GHC module used in the implementation of type-level natural numbers (hac)
Data.List	A module that provides support for operations on list structures.
Data.Char	A module that provides support for characters and operations on characters.
Data.Coerce	Provides safe coercions between data types; allows user to safely convert between values of type that have the same representation with no run-time overhead (hac).
Debug.Trace	Interface for tracing and monitoring execution, used for investigating bugs and other performance issues (hac).
GHC.TypeLits	Internal GHC module that declares the constants used in type-level implementation of natural numbers (hac).
SimpleSQL.Syntax	The AST for SQL queries (hac)
Text.PrettyPrint .Leijen	A pretty printer module based off of Philip Wadler's 1997 "A prettier printer", used to show SQL queries in a readable manner to humans (hac).
GHC.Exts	This module provides access to types, classes, and functions necessary to use GHC extensions.
Unsafe.Coerce	A helper module that converts a value from any type to any other type.This is used in the translation of ECA rules to SQL using user-defined data types.
System.IO.Unsafe	IO computation must be free of side effects and independent of its environment to be considered safe. Any I/O computation that is wrapped in unsafePerformIO performs side effects.

EFA Modules	
Module Name	**Description**
ECA2SQL	The top-level module that takes ECA rules from FSpec and converts it into SQL queries.
Equality	Various utilities related to type level equality
PrettyPrinterSQL	Prints SQl queries in human-readable format
Singletons	Module for Singleton datatypes, the module defines singleton for a kind in terms of an isomorphism between a type and a type representation
Trace	Provides trace messages and various utilities used by ECA2SQL
TSQLCombinators	Uses an overloaded operator for indexing and implements SQL as a primitive data type.
TypedSQL	Contains basic SQL types represented in Haskell.
Utils	Provides utility functions for ECA2SQL and contains type families

3.3 A Description of Haskell-Similar Syntax

This section details the syntax used to describe the module system of Ampersand. This syntax largely borrows from actual Haskell syntax, and from the Agda programming languageAgda is a dependently typed functional language, and since a large part of our work deals with "faking" dependent types, the syntax of Agda is conducive to easy communication of our module system. The principle of faking dependent types in Haskell is detailed in Hasochism (LM13) (a portmanteau of Haskell and masochism, because purportedly wanting to fake dependent types in Haskell is masochism). While the implementation has since been refined many times over, the general approach is still the same, and will not be detailed here. While the changes made to the Haskell syntax are reasonably complex, the ensuing module description becomes vastly simplified. This section is meant to be used as a reference - in many cases, the meaning of a type is self-evident.

3.4 Description of Types and Kinds

In the way that a type classifies a set of values, a kind classifies a set of types. Haskell permits one to define algebraic data types, which are then "promoted" to the kind level (YWC$^+$12). This permits the type constructor of the datatype to be used as a kind constructor, and for the value constructors to be used as type constructors. In every case in our system, when we define a datatype and use the promoted version, we never use the *unpromoted* version. That is, we define types which are never used as types, only as kinds, and constructors which are never used as value constructors,only type constructors. We write $\mathrm{X} : \mathrm{A} \rightarrow \mathrm{B} \rightarrow \ldots \rightarrow \mathrm{Type}$ to denote a regular data type, and $\mathrm{Y} : \mathrm{A} \rightarrow \mathrm{B} \rightarrow \ldots \rightarrow \mathrm{Kind}$ to denote a datatype which is used exclusively as a kind.

3.5 Description of Dependant types

The syntax used to denote a "fake" dependent type in our model is the same as used to denote a real dependent type in Agda. $(\mathrm{x} : \mathrm{A}) \rightarrow \mathrm{B}$ is the function from x to some value of type B, where B can mention x. This nearly looks like a real Haskell type - in Haskell, the syntax would be `forall (x :: A) . B`. However, the semantics of these two types are vastly different - the former can pattern match on the value of x, while the latter cannot.

In certain cases, it may be elucidating to see the *real* Haskell type of an entity (function, datatype, etc.). To differentiate the two, they are typeset differently, as in this example.

The real type of a function whose type is given as $(\mathrm{x} : \mathrm{A}) \rightarrow \mathrm{B}$ in our model is `forall (x :: A) . SingT x -> B`. `SingT :: A -> Type` denotes the singleton type for the kind A, which is inhabited by precisely one value for each type which inhabits A. The role and use of singleton types is detailed further on, in section 3.8.6.

The syntax $\forall\ (\mathrm{x} : \mathrm{A}) \rightarrow \mathrm{B}$ is used to denote the regular Haskell type `forall (x :: A) . B`. As is customary in Haskell, the quantification may be dropped when the kind A is clear from the context: $\forall\ (\mathrm{x} : \mathrm{A}) \rightarrow \mathrm{P\ x}$ and $\forall\ \mathrm{x} \rightarrow \mathrm{P\ x}$ denote the type `forall (x :: A) . P x`.

3.6 Constraints

The Haskell syntax `A -> B` denotes a function from A to B. However, we use the arrow to additionally denote constraints. For example, the function `Show a => a -> String` would be written simply as $\mathrm{Show\ a} \rightarrow \mathrm{a} \rightarrow \mathrm{String}$. In certain casees, a constraint is intended to be used only in an implicit fashion (i.e. as an actual constraint), in which case the constraint is written with the typical $\Rightarrow$ syntax.

3.6.1 Existential quantification

The type $\exists\ (\mathrm{x} : \mathrm{A})\ (\mathrm{P\ x})$ indicates that there exists some x of kind A which satisfies the predicate P. Unfortunately, Haskell does not have first class existential quantification. It must be encoded in one of two ways:

3.6.2 Types, kinds, and type synonyms

Type synonyms are written in the model as $\text{Ty} : \text{K} = \text{X}$, where Ty is the name of the type synonym, K is its kind, and X its implementation. This is to differentiate from type families, which are written as $\text{Ty} : \text{K}$ **where** $\text{Ty} \ldots = \ldots$.

3.6.3 Overloading

Haskell supports overloaded function names through type classes. When we use a type class to simply overload a function name, we simply write the function name multiple times with different types. The motivation for this is that often the real type will be exceeding complex, because it must be so to get good type inference.

3.6.4 Omitted implementations

When the implementation of a type synonym, or any other entity, is omitted, it is replaced by "...". This is to differentiate from a declaration of the form $\text{Ty} : \text{Type}$, which is an abstract type whose constructors cannot be accessed. Furthermore, types may have pattern-match-only constructors; that is, constructors which can only be used in the context of a pattern match, and not to construct a value of that type. This is denoted by the syntax "**pattern** Ctr : Ty". Furthermore, it is not a simple matter of convention - the use of this constructor in expressions will be strictly forbidden by Haskell.

- With a function (by DeMorgan's law):
  `(forall (x :: A) . P x -> r) -> r`
- With a datatype:
  `data Exists p where Exists :: p x -> Exists p`

Choice of form is based on the circumstances in which the function will most likely be used, since whether one form is more convenient than the other depends largely on the intended use. However, these two forms are completely interchangeable (albeit with some syntactic noise) so the syntax presented here does not distinguish between the two.

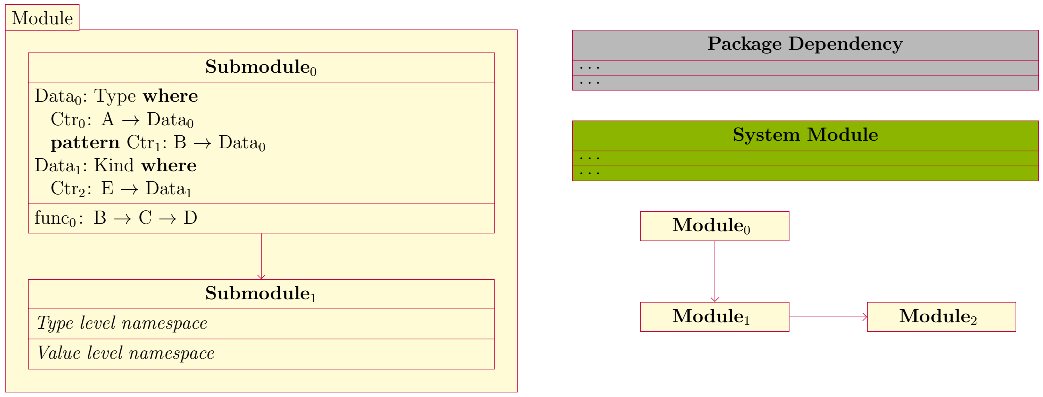

Figure 1: Example of module diagram syntax

3.7 A Description of Module Diagram Syntax

The module hierarchy is broken down into multiple levels to better describe the system. A coarse module hierarchy is given, and each module is further broken into submodules. A dependency between two modules A and B indicates that each submodule in A depends on all of B. There is no necessity to break down modules into submodule, if they do not have any interesting submodule structure. Arrows between modules and submodules denote a dependency.

External dependencies, which are modules which come from an external pacakge, are indicated in grey. System modules, which are modules part of Ampersand, but not written specifically for EFA (or, on which EFA depends, but few or no changes have been made from the original module before the existence of EFA), are indicated in green. The module hierarchy of these modules is not described here; they are included simply to indicate which symbols are imported from these modules. An example of the syntax is found in figure

3.8 Module Hierarchy

This section contains a hierarchal breakdown of each module, as well as a brief explanation of each modules' elements. The module heiarchy of EFA as a whole is given in figure 2. Note that every module which is part of EFA depends on the Haskell `base` package (which is the core libraries of Haskell). Also note that for the `base` package, we only include primitive definitions (i.e. those not defined in real Haskell) which may be difficult to track down in the documentation. The kinds $\mathbb{N}$ and Symbol correspond to type level natural number and string literals, respectively.

The kind Constraint is the kind of class and equality constraints, for example, things like Show x and Int $\sim$ Bool. Note that `Show` itself does *not* have kind Constraint – its kind is Type $\rightarrow$ Constraint. The detailed semantics of these primitive entities can be found in the GHC user guide (otUoG). While many modern features of GHC are used in the actual implementation, they are not mentioned in, nor required to understand, the module description.

The primary interface to EFA is the function eca2PrettySQL, which takes an FSpec (the abstract syntax of Ampersand) and an ECA rule, and returns the pretty printed SQL code for that rule. Also note that while the dependencies within EFA modules is relatively complex, they depend on the rest of the Ampersand system in a simple manner. The modules Test and Prototype implement the testing framework and the prototype generation, respectively; these modules depend directly on only one module from EFA, namely ECA2SQL. Similarly, the majority of EFA itself does not depend directly on Ampersand modules outside of EFA. This makes EFA very resilient to changes in the core Ampersand system; in order to update EFA to work with a modification to Ampersand, only one EFA module – ECA2SQL – will generally need to be modified.

All functions named in the module hierarchy are total - they do not throw exceptions, or produce errors which are not handled or infinite loops. Therefore, no additional information past the type of the function is required to deduce the inputs and outputs of the function – they are precisely the inputs and outputs of the type.

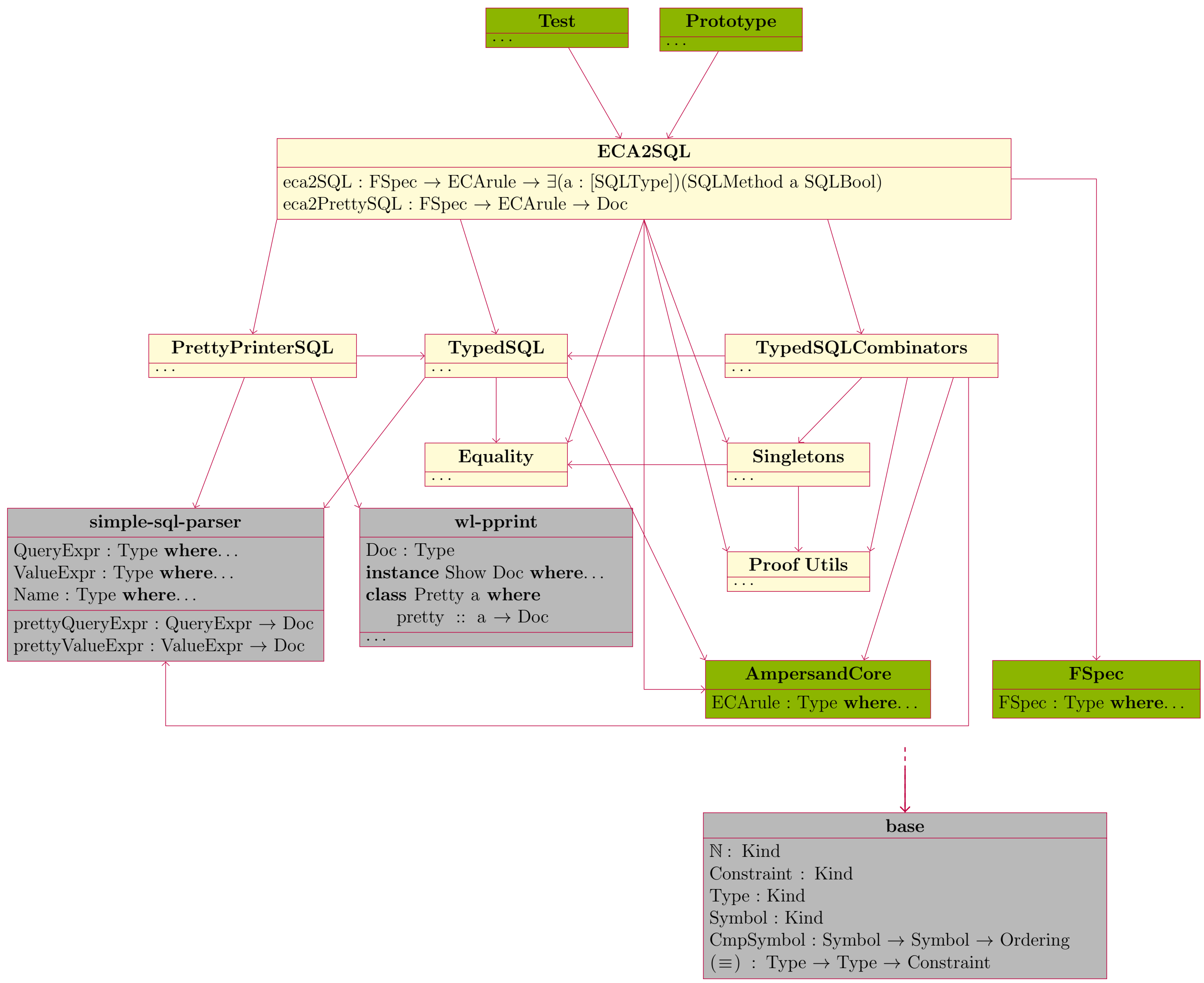

Figure 2: Module diagram for EFA as a whole

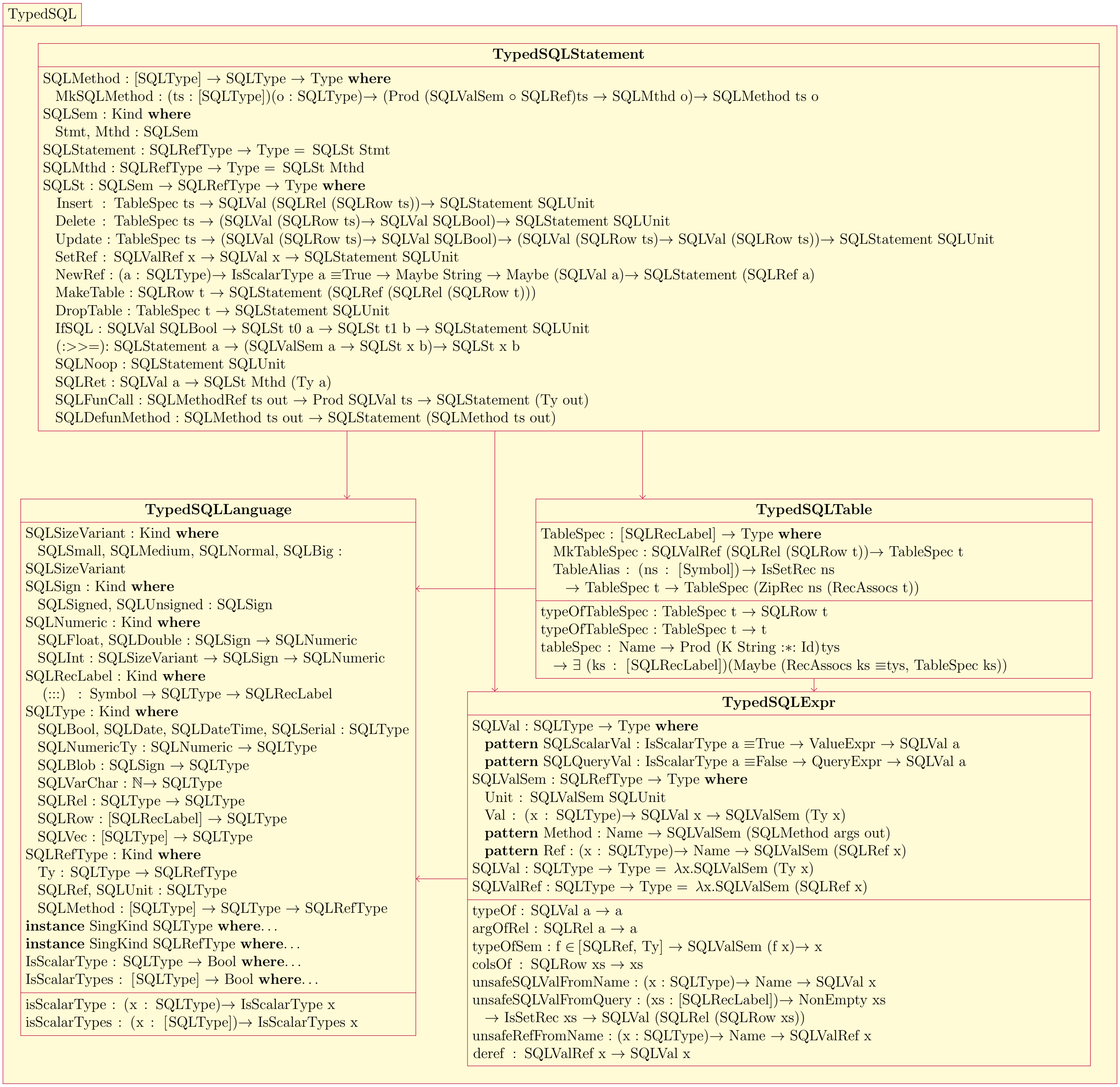

Figure 3: Module diagram for TypedSQL

3.8.1 TypedSQL

The module hierarchy for the TypedSQL module is shown in figure 3. The submodules of TypedSQL are quite large, however, the majority of the definitions within are type and kind definitions, which correspond precisely to entities defined by MySQL. The only exception is SQLRel, which distinguishes relations from scalar types - MySQL does not make this distinguishment. Only a few helper functions are defined in these modules – namely, only things which form the core interface to TypedSQL and in particular, SQLStatement. These functions cannot be defined outside of the module, usually because they use an abstract constructor. By making the core interface to TypedSQL very small, maintaining the TypedSQL language definition separately from the implementation of EFA is simplified. All of the data types in TypedSQL are correct by construction, with the exception of the functions explicitly labeled "unsafe". These functions (unsafeSQLValFromName, unsafeSQLValFromQuery, and unsafeRefFromName) are required only when implementing a new SQL primitive on top of the SQL language - they are not intended for regular use.

The TypedSQLLanguage module models the SQL type language in Haskell with a series of kind declarations. The language being modeled is only a subset of the SQL type language, corresponding approximately to the subset which the core Ampersand system already uses. The meaning of each Haskell type corresponds exactly to the appropriate MySQL type, which are detailed in the MySQL manual (Cor). Similarly, the constructors of SQLSt all correspond to different varieties of SQL statements – for the majority of constructors, there is a one-to-one correspondence between the semantics of the constructor, and the semantics of the SQL statement with the same name. The exceptions are:

`SQLNoop` MySQL does not have a primitive no-op statement

`SQLDefunMethod` MySQL does not allow defining procedures within procedures; this constructor denotes that a method "defined" within another statement must first be loaded as a MySQL Stored Procedure (Cor).

`:>>=` This constructor corresponds to sequencing statements. This constructor embeds scope checking of MySQL statements in the Haskell compiler – ill-formed statements containing variables which are not defined (i.e. not in scope) will be rejected by the Haskell compiler.

The SQLSt data type also distinguishes between two varieties of statements: SQLStatement and SQLMthd. The former is the type of regular statements, while the latter is the type of "almost" complete methods - methods whose formal parameters have not yet been bound. This is done in order to statically guarantee that a SQL method always returns a value. Due to the type of :>>=, this also rules out SQL programs which contain dead code – no code can follow SQLRet, which is always guaranteed to return from the function. While this does rule out some valid programs (for example, an if statement in which both branches end with a return, but there is no return following the if statement, will be rejected), these programs can be written in an equivalent way in our language without any loss of generality.

3.8.2 TypedSQLCombinators

The module TypedSQLCombinators, whose members are given in figure 4, implements a subset of primitive SQL functions on top of the TypedSQL expression type. The data type PrimSQLFunction encodes the specification of each function; the type and semantics of each function is that of the corresponding function in MySQL (refer to the MySQL manual (Cor) for details on each function). The only exception is the Alias function, which is a primitive syntactic constructor (not a named entity) in MySQL - rows can be aliased with a select statement. Aliasing a row means to change the name of each association in the row, but not the shape of the row (i.e. the types of each element of the row, as well as their ordering). The single function primSQL implements all of the primitive SQL functions. It takes as an argument a specification of the primitive function, a tuple of arguments of the correspond types, and returns a SQL value, again of the corresponding type.

TypedSQLCombinators

PrimSQLFunction : [SQLType] → SQLType → Type
  PTrue, PFalse : PrimSQLFunction [] SQLBool
  Not : PrimSQLFunction [SQLBool] SQLBool
  Or, And : PrimSQLFunction [SQLBool, SQLBool] SQLBool
  In, NotIn : PrimSQLFunction [a, SQLRel a] SQLBool
  Exists : PrimSQLFunction [SQLRel a] SQLBool
  GroupBy : PrimSQLFunction [SQLRel a, a] (SQLRel (SQLRel a))
  SortBy : PrimSQLFunction [SQLRel a, a] (SQLRel a)
  Max, Min, Sum, Avg : IsSQLNumeric a → PrimSQLFunction [SQLRel a] a
  Alias : (RecAssocs ts : RecAssocs ts') → PrimSQLFunction [SQLRel (SQLRow ts)] (SQLRel (SQLRow ts'))

primSQL : ∀(args : [SQLType])(out : SQLType)→ PrimSQLFunction args out → Prod SQLVal args → SQLVal out (!) : (xs : [SQLType])(i : Symbol)→ LookupRec xs i ≡ r → SQLVal (SQLRow xs)→ SingT i → SQLVal r (!) : (xs : [SQLType])(i : Symbol)→ LookupRec xs i ≡ r → SQLVal (SQLRel (SQLRow xs))→ SingT i → SQLVal r (!) : (xs : [SQLType])(i : ℕ) → LookupIx t i ≡ r → SQLVal (SQLVec t)→ SingT i → SQLVal r

Figure 4: Module diagram for TypedSQLCombinators

Equality

```
Dict : Constraint → Type
  Dict : p ⇒ Dict p
Exists : (k → Type)→ Type
  Ex : p x → Exists p
Not : Type → Type
class NFData a
doubleneg : NFData a ⇒ a → Not (Not a)
triviallyTrue : Not (Not ())
mapNeg : (NFData a, NFData b)⇒ (b → a)→ Not a → Not b
elimNeg : NFData a ⇒ Not a → a → Void
data Void where
data Dec : Type → Type
  Yes : !p → Dec p
  No : !(Not p) → Dec p
DecEq : k → k → Type = λa b.Dec (a ≡b)
```

```
cong : f ≡ g → a ≡ b → f a ≡ g b
cong2 : f ≡ g → a ≡ a' → b ≡ b' → f a b ≡ g a' b'
cong3 : f ≡ g → a ≡ a' → b ≡ b' → c ≡ c' → f a b c ≡ g a' b' c'
(#>>):: Exists p → (∀x → p x → r) → r
mapDec : (p → q)→ (Not p → Not q)→ Dec p → Dec q
liftDec2 :: Dec p → Dec q → (p → q → r) → (Not p → Not r) → (Not q → Not r) → Dec r
dec2bool :: DecEq a b → Bool
```

Figure 5: Module diagram for Equality

3.8.3 Equality

The Equality module (5) defines several utilities for working with proof-like values, including the existential quantification data type, proofs of congruence of propositional equality of various arities – cong, cong2, cong3 – and the Dec type, which encodes the concept of a decidable proposition. The most important element of this module, however is the abstract Not type. We must prove certain things about our program to the Haskell type system. For example, if one attempts to construct a scalar SQLVal for some SQLType S, one must first prove that that type is a scalar type. The main use of this is decidable equality, which is similar to regular equality, but additionally to giving a "yes" or "no" answer, it also stores a *proof* of that answer.

The view of propositions as types comes from the Curry-Howard isomorphism (Wad15); however, this is not quite sound in Haskell, because every type is inhabited by $\perp$, which corresponds to `undefined`, an exception, or non-termination. Due to laziness, an unevaluated $\perp$ can be silently ignored. At worst, this corresponds to a sound use of unsafeCoerce leaking into the "outside world", that is, allowing a user to accidently expose the use of an unsafeCoerce. One can usually work around this by evaluating all proof-like values to normal form before working with them (this is accomplished by the NFData class, which stands for normal form data). The normal form of most datatypes contains precisely one "type" of bottom – namely the value

$\perp$, as opposed to $\perp$ wrapped in a constructor, for example Just $\perp$. This bottom can then be removed with the Haskell primitive **seq**, producing a value which can soundly be used as a proof.

A problem arises when we consider the negation of a proposition. $\neg p$ istypically encoded in Haskell as p $\rightarrow \perp$, where the type $\perp$ can be represented by any uninhabited type, typically calledVoid. However, the normal form of a function can contain any number of $\perp$ hidden deep inside the function, because evaluating a function to normal form only evaluates up to the outermost binder. To prevent any unsoundness which this might cause, values of type Not p are reduced to normal form as they are built, starting with a canonical value which is known to be in normal form - the value trivially True. This is the role of NFData in the type signature of mapNeg. As mentioned previously, the type Not is abstract, so the provided interface, which is known to be sound, is the only way to construct and eliminate values of type Not p.

3.8.4 PrettySQL

The module PrettySQL defines pretty printers for each of the types corresponding to SQL entities, including SQL types, SQL values, SQL references and methods, and SQL statements. These pretty printers produce a value of type Doc (which comes from the wl-pprint package), which is like a string , but contains the layout and indentation of all lines of the document, allowing for easy composition of Doc values into larger documents, without worrying about layout. The SQL entities are pretty printed in a human readable format, complete with SQL comments which indicate the origin and motive of generated code.

PrettySQL
instance Pretty (SQLTypeS x) **where**...
instance Pretty (SQLVal x) **where**...
instance Pretty (SQLValSem x) **where**...
instance Pretty (SQLSt k x) **where**...
instance (str $\equiv$ String) $\Rightarrow$ Pretty (str, SQLMethod args out) **where**...

Figure 6: Module diagram for PrettySQL

3.8.5 Proof Utils

The ProofUtils modules, shown in figure 7 provides various utilities for proving compile time invariants, including the definitions of various predicates used in other modules, as well as the value-level functions which prove or disprove those predicates. Generally speaking, a predicate is a type level function which returns either a true or false value, or has kind Constraint, in which case truth corresponds to a satisfied constraint, while false to an unsatisfied one. Many predicates have value level witnesses as well; these are datatypes which are inhabited if and only if the predicate is true. Therefore, pattern matching on the predicate witness can be used to recover a proof of the predicate at run time.

The function of the most important predicates and types is briefly summarized as follows.

`Prod` The type Prod f xs represents the n-ary product of the type level list xs, with each $x \in xs$ being mapped to the type f x. This type is accompanied by the functions prod2sing, sing2prod which convert between singletons and products; the functions foldrProd, foldlProd, foldrProd', which are all eliminators for Prod (several eliminators are needed because the most general eliminator is not well typed in Haskell).

`Sum` The same as above, but the n-ary sum as opposed to product.

`All` The constraint All c xs holds if and only if c x holds for all $x \in xs$.

`Elem, IsElem` IsElem x xs holds precisely when $x \in xs$. Elem is the witness of IsElem.

`AppliedTo, Ap, :.:, Cmp, :*:, K, Id, Flip` Categorical data types which encode a generalized view of algebraic data types. This approach is largely standardized (and is only replicated here to avoid incurring a large dependency) – for more information, see (Swi08).

`RecAssocs, RecLabels, ZipRec` Type level functions for working with type level records. A record in this context is a list of types of some kind, each associated with a unique string label. RecAssocs, RecLabels retrieve the associations and labels of a record, respectively, while ZipRec constructs such a record from the associations and labels. It is the case that ZipRec (RecAssocs x)(RecLabels x)$\equiv$ x for all x .

`IsSetRec, SetRec` The predicate IsSetRec x holds precisely if x is a valid record type, whose labels are all unique. SetRec is the witness for IsSetRec.

`IsNotElem, NotElem` The predicate IsNotElem x xs holds precisely if $\neg x \in xs$. NotElem is the witnss for IsNotElem.

`&&,And` Binary and n-ary boolen conjunction, with the usual semantics.

ProofUtils

Prod : (f : k → Type)→ (xs : [k]) → Type
  PNil : Prod f []
  PCons : f x → Prod f xs → Prod f (x : xs)
Sum : (f : k → Type)→ (xs : [k]) → Type
  SHere : f x → Sum f (x : xs)
  SThere : Sum f xs → Sum f (x : xs)
class All (c : k → Constraint) (xs : [k]) **where**
  mkProdC : Proxy c → (∀a → c a ⇒ p a) → Prod p xs
instance All (c : k → Constraint) [] **where**. . .
instance (All c xs, c x) ⇒ All c (x : xs) **where**. . .
Elem : k → [k] → Type **where**
  MkElem : Sum (≡x)xs → Elem x xs
IsElem : (x : k) → (xs : [k]) → Constraint **where**. . .
AppliedTo : (x : k) → (f : k → Type)→ Type
  Ap : f x → x ‘AppliedTo‘ f
(:◦ :) : (f : k1 → Type)→ (g : k0 → k1) → (x : k0) → Type
  Cmp : f (g x) → (:◦ :) f g x
(:∗:) : (f : k0 → Type)→ (g : k0 → Type)→ (x : k0) → Type
  (:∗:) : f x → g x → (:∗:) f g x
K : (a : Type)→ (x : k) → Type
  K : a → K a x
Id : (a : Type)→ Id a
  Id : a → Id a
Flip : (f : k0 → k1 → Type)→ (x : k1) → (y : k0) → Type
  Flp : f y x → Flip f x y
(&&): (x : Bool) → (y : Bool) → Bool **where**. . .
RecAssocs : [RecLabel a b] → [b] **where**. . .
RecLabels : [RecLabel a b] → [b] **where**. . .
_IsSetRec : [RecLabel a b] → [a] → Constraint **where**. . .
IsSetRec : [RecLabel a b] → Constraint **where**. . .
SetRec : [a] → [RecLabel a b] → Type
SetRec : [RecLabel a b] → Type = SetRec []
LookupRecM : [RecLabel Symbol k] → Symbol → Maybe k **where**. . .
ZipRec : [a] → [b] → [RecLabel a b] **where**. . .
IsNotElem : [k] → k → Constraint **where**. . .
NotElem : [k] → k → Type
And : (xs : [Bool]) → Bool **where**. . .
NonEmpty : (xs : [k]) → Constraint **where**
  NonEmpty (x : xs)= ()

(|&&): (a : Bool) → (b : Bool) → a && b
openSetRec : ∀(xs : [RecLabel k k’]) r0 → SingKind k ⇒ SetRec xs → (IsSetRec xs ⇒ r0)→ r0
decNotElem : ∀(xs : [a]) x → SingKind a ⇒ xs → x → Dec (NotElem xs x)
decSetRec : ∀ (xs : [RecLabel a b]) → SingKind a ⇒ xs → Dec (SetRec xs)
lookupRecM : ∀(xs : [RecLabel Symbol k])(x : Symbol)→ xs → x → LookupRecM xs x
and : (xs : [Bool]) → And xs
compareSymbol : (x : Symbol)→ (y : Symbol)→ CmpSymbol x y
prod2sing : ∀ (xs : [k]) → SingKind k ⇒ Prod SingT xs → SingT xs
sing2prod : ∀ (xs : [k]) → SingKind k ⇒ SingT xs → Prod SingT xs
foldrProd : ∀ acc (f : k → Type) xs → acc → (∀q → f q → acc → acc) → Prod f xs → acc
foldlProd : ∀ acc (f : k → Type) xs → acc → (∀q → f q → acc → acc) → Prod f xs → acc
mapProd : ∀(f : k → Type) g → (∀x → f x → g x) → Prod f xs → Prod g xs
foldrProd’ : ∀ (f : k → Type) xs1 → (∀x xs → f x → Prod g xs → Prod g (x : xs)) → Prod f xs1 → Prod g xs1
someProd : [Exists f] → Exists (Prod f)

Figure 7: Module diagram for ProofUtils

3.8.6 Singletons

The Singletons module (figure 8) is not fully detailed here; instead, a vastly simplified version of the Singletons module is presented. The SingT type denotes a generic singleton for any kind which implements SingKind – then, the main operation of interest on singletons is decidable equality, which is realized by the function %≡. The detailed implementation is omitted because the singletons approach in Haskell is well known and well documented (EW12). We re-implement singletons instead of using the well established `singletons` library because, while this library is very well written, it relies very heavily on Template Haskell (SJ02), which is essentially string-based metaprogramming. Template Haskell is extremely error prone and very difficult to maintain. As one of our primary goals is long-term maintainability, and Template Haskell changes, sometimes drastically, with every new release of GHC, including it in this project was deemed not worth the headache. Therefore, we have reimplemented singletons without Template Haskell, at the cost of having to write slightly more boilerplate.

Singletons
SingT : k → Type **class** SingKind (k : Kind) **where**. . .
(%≡) : ∀ (x : k) (y : k) → SingKind k ⇒ x → y → DecEq x y

Figure 8: Module diagram for Singletons

References

[Cor] Oracle Corporation. *MySQL 5.7 Reference Manual.*

[EW12] Richard A. Eisenberg and Stephanie Weirich. Dependently typed programming with singletons. *SIGPLAN Not.*, 47(12):117–130, September 2012.

[hac] Hackage.

[Joo] Stef Joosten. Deriving functional specications from business requirements with ampersand.

[LM13] Sam Lindley and Conor McBride. Hasochism: The pleasure and pain of dependently typed haskell programming. *SIGPLAN Not.*, 48(12):81–92, September 2013.

[otUoG] The University Court of the University of Glasgow. *The Glorious Glasgow Haskell Compilation System User's Guide.* Accessed: 2016-02-29.

[SJ02] Tim Sheard and Simon Peyton Jones. Template meta-programming for haskell. *SIGPLAN Not.*, 37(12):60–75, December 2002.

[Swi08] Wouter Swierstra. Data types la carte. *Cambridge University Press*, 2008.

[Wad15] Philip Wadler. Propositions as types. *Communications of the AC*, 58(12):7584, 2015.

[YWC+12] Brent A. Yorgey, Stephanie Weirich, Julien Cretin, Simon Peyton Jones, Dimitrios Vytiniotis, and José Pedro Magalhães. Giving haskell a promotion. In *Proceedings of the 8th ACM SIGPLAN Workshop on Types in Language Design and Implementation*, TLDI '12, pages 53–66, New York, NY, USA, 2012. ACM.