Theorem 1.1. Yoneda If A be a locally small category, then,

$$[\mathcal{A}^{op}, Set](H_A, X) \cong X(A)$$
 naturally in $A \in \mathcal{A}$ and $X \in [\mathcal{A}^{op}, Set]$

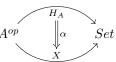
Explaination:

First of all, we fix any category, A. Now we choose two things (independent of each other):

i an object, A from the category $\mathcal{A} = \mathcal{A}^{op}$

ii an object, X of the category $[\mathcal{A}^{op}, Set]$ which is precisely a functor $X: \mathcal{A}^{op} \to Set$

Here, $[\mathcal{A}^{op}, Set](H_A, X)$ denotes arrows $H_A \to X$ in $[\mathcal{A}^{op}, Set]$ i.e. natural transformations, $\alpha: A^{op}$



X(A) is precisely a set. As X(A) is the image of (our chosen object,) A, under (our chosen functor,) X.

So, the idea is that our choice of A and X completely determines all possible maps (i.e. natural transformations) from H_A to X. This answers out big question of "what are all the maps $H_A \to X$ " or "how does H_A see other presheaves on \mathcal{A} ".

More than the above must be true, as it's not just that the two are isomorphic, but they're **naturally** isomorphic. This more is supposed to be that both $[A^{op}, Set](H_A, X)$ and X(A) are functorial in both A and X