

This document will contain definitions and other trivial notes.

1 ‘Trivial’ definitons

Definiton 1.1 (Inclusion Map). For $A \subseteq B$, a map $f : A \rightarrow B$ that takes $x \in A$ to $x \in B$

Definiton 1.2 (Monomorphism). Injective homomorphism

Definiton 1.3 (Epimorphism). Surjective homomorphism

Definiton 1.4 (Endomorphism). A homomorphism with same domain and co-domain

Definiton 1.5 (Automorphism). An isomorphism with same domain and co-domain

Definiton 1.6 (Pre-order). A reflexive, transitive binary relation.

Definiton 1.7 (Partial Order). Pre-order that’s antisymmetric.

Definiton 1.8 (Total Order). Partial order with trichotomy.

Definiton 1.9 (Presheaf). A functor $\mathcal{A}^{op} \mapsto Set$

2 Group Theory

Definiton 2.1 (Monoid). A semi-group with identity.

Definiton 2.2 (Direct Product). Direct product of groups G and H is the group on the set $G \times H$ with the binary operation:

$$(a, b)(a', b') = (aa', bb') \text{ where } a, a' \in G; b, b' \in H$$

Definiton 2.3 (Congruence Relation). On a monoid M , an equivalence relation, \sim , such that $\forall a, a', b, b' \in M$,

$$a \sim a', b \sim b' \implies ab \sim a'b'$$

Definiton 2.4 ((left) G-Set). Let G be a group, and X a set. Then, f is a left group action on of G on X , or X is a left G -Set iff

$$f : G \times X \mapsto X : \forall x \in X, f(e_G, x) = x \text{ and } \forall a, b \in G, f(ab, x) = f(a, f(b, x))$$

Definiton 2.5 (Equivariant Map). A function is called an equivariant map if it’s domain and co-domain are acted on by the same symmetry group, and the function commutes with that group action.

$$f : X \rightarrow Y$$

Definiton 2.6 (Natural Projection). Let G be a group and $N \trianglelefteq G$. Natural projection, $\pi : G \rightarrow G/N$ defined as $g \mapsto gN$

3 Vector Spaces

Definiton 3.1 (Tensor Product).