1 Chapter 1

2 Functors(pg26)

Excercise (1.2.21)**.** As $A \cong$ A', there's a bijection between them in $\mathcal{A}$ i.e. $A \underset{f^{-1}}{\overset{f}{\longleftrightarrow}} A'$ with $f \in \mathcal{A}(A, A')$ and $f^{-1} \in \mathcal{A}(A', A)$. Thus, $F(f) \in \mathcal{B}(F(A), F(A'))$ and $F(f^{-1}) \in \mathcal{B}(F(A'), F(A))$. And as F is a functor,

$$F(f) \circ F(f^{-1}) = F(f \circ f^{-1}) = F(1_A) = 1_{F(A)} \text{ similarly, } F(f^{-1}) \circ F(f) = 1_{F(A')}$$

Hence, we have an isomorphism, $F(A) \underset{F(f^{-1})}{\overset{F(f)}{\longleftrightarrow}} A'$.

Excercise (1.2.22)**.** Let $a, a' \in A$ with $a \leq a'$, so that there's a morphism, $f : a \to a'$ in $\mathcal{A}$.
Now, $F : \mathcal{A} \to \mathcal{B}$ gives $F(a) \xrightarrow{F(f)} F(a')$ i.e. $F(a) \leq F(a')$.

Excercise (1.2.23)**.** (a) As $ob(G) = ob(G^{op})$,just need to ensure morphisms.

$$\forall f \in \mathcal{A}(A, B), \;\; f^{-1} \in \mathcal{A}(B, A) \implies f \in \mathcal{A}^{op}(B, A), \;\; f^{-1} \in \mathcal{A}^{op}(A, B)$$

Thus, for any two objects A, B , morphisms between them in $\mathcal{A}$ are also in $\mathcal{A}^{op}$. More precisely, define functors

$$G \xrightarrow{F} G^{op} \text{ and } G^{op} \xrightarrow{F^{-1}} G \text{ as } F(f) \mapsto f^{-1} \text{ and } G(f^{-1}) \mapsto f$$

Hence, $F^{-1} \circ F : G \to G$ and $F \circ F^{-1} : G^{op} \to G^{op}$. Giving an isomorphism, $G \cong G^{op}$ in **CAT**.

(b) Take the monoid, say M, consisting 2×2 matrices, $\left\{ I := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A := \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$ under matrix multiplication. So, M has two morphisms, the identity morphism, i and and a. Now, suppose $M \cong M^{op}$, then, a must have an inverse i.e. $k :=\in M : k.a = i$ i.e. a matrix $K : KA = I$ but this can't be as A is a singular matrix.

Excercise (1.2.24)**.**

TBD