

# 1

**Theorem 1.1. Yoneda** If  $\mathcal{A}$  be a locally small category, then,

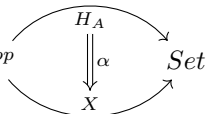
$$[\mathcal{A}^{op}, Set](H_A, X) \cong X(A) \text{ naturally in } A \in \mathcal{A} \text{ and } X \in [\mathcal{A}^{op}, Set]$$

**Explanation:**

First of all, we fix any category,  $\mathcal{A}$  . Now we choose two things (independent of each other):

- i an object,  $A$  from the category  $\mathcal{A} = \mathcal{A}^{op}$
- ii an object,  $X$  of the category  $[\mathcal{A}^{op}, Set]$  which is precisely a functor  $X : \mathcal{A}^{op} \rightarrow Set$

Here,  $[\mathcal{A}^{op}, Set](H_A, X)$  denotes arrows  $H_A \rightarrow X$  in  $[\mathcal{A}^{op}, Set]$  i.e. natural transformations,  $\alpha : \mathcal{A}^{op} \rightarrow Set$



$X(A)$  is precisely a set. As  $X(A)$  is the image of (our chosen object,)  $A$ , under (our chosen functor,)  $X$ .

So, the idea is that our choice of  $A$  and  $X$  completely determines all possible maps (i.e. natural transformations) from  $H_A$  to  $X$ . This answers our big question of "what are all the maps  $H_A \rightarrow X$  " or "how does  $H_A$  see other presheaves on  $\mathcal{A}$  ".

*More than the above must be true, as it's not just that the two are isomorphic, but they're **naturally** isomorphic.*  
 This more is supposed to be that both  $[\mathcal{A}^{op}, Set]$  and  $X(A)$  are *functorial* in both  $A$  and  $X$