P_M5_1

October 10, 2021

1 Module 5 Peer Review Assignment

2 Problem 1

Roll two six-sided fair dice. Let X denote the larger of the two values. Let Y denote the smaller of the two values.

a) Construct a table that gives the joint probability mass function for X and Y.

YOUR ANSWER HERE

	y=1	y=2	y=3	y=4	y=5	y=6	Sum (y)
x=1	1/36	0	0	0	0	0	1/36
x=2	2/36	1/36	0	0	0	0	3/36
x=3	2/36	2/36	1/36	0	0	0	5/36
x=4	2/36	2/36	2/36	1/36	0	0	7/36
x=5	2/36	2/36	2/36	2/36	1/36	0	9/36
x=6	2/36	2/36	2/36	2/36	2/36	1/36	11/36
Sum(x)	11/36	9/36	7/36	5/36	3/36	1/36	

b) What is $P(X \geq 3, Y = 1)$?

YOUR ANSWER HERE

	y=1
x=3	2/36
x=4	2/36
x=5	2/36
x=6	2/36
Sum	8/36

$P(X \geq 3, Y = 1)$

$= P(X = 3, Y = 1) + P(X = 4, Y = 1) + P(X = 5, Y = 1) + P(X = 6, Y = 1)$

$= 2/36 + 2/36 + 2/36 + 2/36$

$= 8/36 = 2/9$

c) What is $P(X \geq Y + 2)$?

YOUR ANSWER HERE

	y=1	y=2	y=3	y=4	Sum
x=3	2/36				2/36
x=4	2/36	2/36			4/36
x=5	2/36	2/36	2/36		6/36
x=6	2/36	2/36	2/36	2/36	8/36

From the table we can see

$P(X \geq Y + 2)$

$= P(X = 3, Y = 1) + P(X = 4, Y \leq 2) + P(X = 5, Y \leq 3) + P(X = 6, Y \leq 4)$

= 2/36 + 4/36 + 6/36 + 8/36

= 20/36 = 5/9

d) Are X and Y independent? Explain.

YOUR ANSWER HERE

No, X and Y are not independent.

X and Y are independent rv if P(X=x, Y=y) = P(X=x)*P(Y=y) for all possible values of x and y.

but P(X=1,y=1) = 1/36

P(X=1) * P(Y=1) = (1/36) * (11/36) = 11/1296

3 Problem 2

Let (X, Y) be continuous random variables with joint PDF:

$$f(x, y) = \begin{cases} cxy^2 & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{else} \end{cases}$$

Part a)

Solve for c. Show your work.

YOUR ANSWER HERE

$\int_0^1 \int_0^1$ cxy^2 dx dy =1

C * ($\frac{1}{2}$ * x^2) $|_0^1$ * ($\frac{1}{3}$ * y^3) $|_0^1$ = 1

C * ($\frac{1}{2}$ - 0) * ($\frac{1}{3}$ - 0) = 1

C*$\frac{1}{6}$ =1

C = 6

Part b)

Find the marginal distributions $f_X(x)$ and $f_Y(y)$. Show your work.

YOUR ANSWER HERE

$f_X(x)$ = $\int_0^1$ $6xy^2$ dy

= $6x$ ($\frac{1}{3}$ * y^3)$|_0^1$

= $6x$ ($\frac{1}{3}$ - 0)

= 2x, $0 \leq x \leq 1$

$f_Y(y)$ = $\int_0^1$ $6xy^2$ dx

= $6y^2$ ($\frac{1}{2}$ * x^2)$|_0^1$

= $6y^2$ ($\frac{1}{2}$ - 0)

= $3y^2$, $0 \leq y \leq 1$

Part c)

Solve for $E[X]$ and $E[Y]$. Show your work.

YOUR ANSWER HERE

E[X] = $\int_0^1$ x 2x dx

= 2* ($\frac{1}{3}$ * x^3)$|_0^1$

= 2 * ($\frac{1}{3}$ - 0) = $\frac{2}{3}$

E[y]= $\int_0^1$ y $3y^2$ dy

= 3* ($\frac{1}{4}$ * y^4)$|_0^1$

= 3* ($\frac{1}{4}$ - 0) = $\frac{3}{4}$

Part d)

Using the joint PDF, solve for $E[XY]$. Show your work.

YOUR ANSWER HERE

E[XY]=$\int_0^1\int_0^1$ xy $6xy^2$ dx dy

= $\int_0^1\int_0^1$ $6x^2y^3$ dx dy

= 6 * ($\frac{1}{3}$ * x^3) $|_0^1$ * ($\frac{1}{4}$ * y^4) $|_0^1$

= 6 * ($\frac{1}{3}$ - 0) * ($\frac{1}{4}$ - 0)

= 6 *$\frac{1}{12}$

$=\frac{1}{2}$

Part e)

Are X and Y independent?

YOUR ANSWER HERE

Yes, X and Y are independent.

Because f(x,y)=f(x)*f(y) for all possible values of x and y.

f(X=0,Y=0) = f(0) * f(0) = 0

f(X=1,Y=1) = f(1)* f(1) = 2*3 =6

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