Indian Institute of Technology Patna
MA102: Mathematics II
End Semester Exam (26-4-2017)

Time: 3hrs

Max. Marks: 50

Note: There are total **9** questions. Answer all questions. Give precise and brief answer. Standard formulae may be used. **Do not write anything on the question paper. Write your Roll at the end.** Notations are standard and same as used in class.

Que 1. Answer all parts of this question at one place.

(a.) Consider the ODE: $y' + p(x)y = 0$, where $p(x)$ is continuous function on some interval I. Let $\phi_1(x)$ and $\phi_2(x)$ be any two solutions of this equation. If $\phi_1(x_0) = \phi_2(x_0)$ for some $x_0 \in I$ then show that $\phi_1(x) = \phi_2(x)$ for all $x \in I$. [2]

(b.) Solve $(y \log y - 2xy)dx + (x + y)dy = 0$. [2]

(c.) Solve $y' = (x + y)^2$. [2]

(d.) Solve $y - xy' = y'y^2e^y$ [1]

(e.) Let $J_p = \sum_{n=0}^{\infty}(-1)^n \frac{(x/2)^{2n-p}}{n!(p+n)!}$ be solution of Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$ for integer p. Then show that between any two consecutive zeros of $J_1(x)$ there exists a zero of $J_0(x)$ [2]

(f.) Find the Laplace transform of $f(x) = e^{3x}x^4$. [2]

Que 2. a) Solve the following differential equation: [1]

$$y'' + 2y' + 3y = 0$$ [2]

b) Given that $y_1 = x$ is a solution of the differential equation $(1-x^2)y''-2xy'+2y = 0$, find the other LI solution y_2. [2]

Que 3. a) Solve the following differential equation: [3]

$$x^2y'' + 4xy' + y = 0, x > 0.$$

b) Using <u>variation of parameters,</u> find the particular solution of following ODE: [3]

$$y'' + y = sec\,x\ cosec\,x$$

Que 4. Use Runge Kutta method (RK-4, <u>formula given at end</u>) to solve the IVP: $y' = x^2 + y^2, y(0) = 0$ for $x \in [0, 1]$ choosing $h = 0.5$. [4]

Que 5. a) Find the series solutions of Legendre Equation $(1 - x^2)y'' - 2xy' + m(m + 1)y = 0$. Show that the solutions obtained are LI. [3+1]
b) Show that $\int_{-1}^{1} P_2(x)P_3(x)dx = 0$. Here $P_2(x)$ and $P_3(x)$ are Polynomial solutions of Legendre equation such that $P_m(1) = 1,\ m = 2, 3$. [2]

Que 6. Using Laplace Transform (<u>Do Not use any other method</u>) solve the following IVP: [4]

$$y'' + 2y' + 5y = e^{-x}\sin x,\ y(0) = 0,\ y'(0) = 1.$$

Que 7. a) Solve the following system of first order linear equations:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -4 & -1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Find two LI solutions and hence find the general solution. [4]

b) What is critical point for the system $\begin{cases} x' = & -3x + 2y, \\ y' = & -2x + 2y. \end{cases}$ Find whether this critical point is stable or unstable. [2]

Que 8. Find e^{2A} when

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{bmatrix}.$$

It is given that one of the eigenvalues of A is 5 with eigenvector $\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$. Note that, in solution, find inverse of the eigenvector matrix by Gauss-Jordan method only. [6]

Que 9. Suppose you want to solve system $Ax = b$ by using iterative solvers, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

Prove that Jacobi method does not work for the given system. [4]
(Recall! Jacobi does not work if and only if $|\lambda| < 1$, where λ is a eigenvalue of corresponding error matrix)

Important Formulae:
RK-4 Method for $y' = f(x, y)$.

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where, $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$

$k_3 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$, $k_4 = f(x_n + h, y_n + hk_3)$

Good Luck

ROLL NUMBER: