Indian Institute of Technology Patna
MA - 201: Mathematics III
B.Tech II year (Autumn Semester: 2009-10)

End Semester Examination

Maximum Marks: 50 | Total Time: 3 Hours

Note:

1. This question paper has **TWO** pages and contains **NINE** questions. Please check all pages and report the discrepancy, if any.
2. Answer all questions.

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1 (i) Find Laplace transform of $t\cos(t) + t^2\sin(3t)$. [2]

(ii) Solve using Laplace transform technique $\frac{dx}{dt} + x(t) = f(t)$, $x(0) = 1$, where $f(t) = t$ for $0 \leq t < 4$, and $f(t) = 1$ for $t \geq 4$. Write the solution explicitly for both range of t. [4]

2 (i) Evaluate the complex integral $\int_{|z|=1} \frac{(z^2-1)^2}{z^2(2z^2+5z+2)}\, dz$. [5]

(ii) Find the analytic function $f(z) = u(x,y) + iv(x,y)$ where $u(x,y) = e^x(x\cos(y) - y\sin(y))$. Write the function in terms of z. [4]

3 (i) Find a first order PDE from two parameter (a, b are parameters) family $z = (x+a)(y+b)$. [2]

(ii) Let $u(x,y) = f(xe^y) + g(y^2\cos(y))$, where f and g are infinitely differentiable functions. Derive the PDE of minimum order satisfied by u. [2]

4 (i) Find the general solution of $x^2p + y^2q = (x+y)z$. [2]

(ii) Find a complete integral of $z^2 - pqxy = 0$ by Charpit's method. [3]

(iii) Find a complete integral of $z^2(p^2z^2 + q^2) = 1$ without using Charpit's method directly (use some special form). [2]

5 (i) It is given that a complete integral of the equation $p^2x + qy - z = 0$ is $(ay - z + x + b)^2 = 4bx$. Using the given data, derive the equation of the integral surface containing the line $y = 1, x + z = 0$. [2]

(ii) Find the general equation of surfaces orthogonal to the family given by $x(x^2 + y^2 + z^2) = c_1y^2$ (c_1 is a parameter). [2]

6 (i) Let $f(x) = 0$ for $-5 < x < 0$, $f(x) = 3$ for $0 < x < 5$ and be periodic of period 10. How should $f(x)$ be defined at $x = -5$, 0 and 5 in order that the Fourier series will converge to $f(x)$ for $-5 \leq x \leq 5$. [2]

6 (ii) Using the separation of variable method, solve the Dirichlet problem:

$$\begin{aligned} u_{xx} + u_{yy} &= 0,\ 0 < x < a,\ 0 < y < b, \\ u(x,b) = u(a,y) &= 0, \\ u(0,y) &= 0, \\ u(x,0) &= f(x). \end{aligned}$$

[4]

7 (i) If $F_c(\alpha)$ and $G_c(\alpha)$ are Fourier cosine transforms of $f(x)$ and $g(x)$ respectively, then prove that

$$\int_0^\infty F_c(\alpha)G_c(\alpha)d\alpha = \int_0^\infty f(x)g(x)dx$$

and hence by taking $f(x) = e^{-ax}$ and $g(x) = e^{-bx}$, show that

$$\int_0^\infty \frac{1}{(a^2+\alpha^2)(b^2+\alpha^2)}d\alpha = \frac{\pi}{2ab(a+b)}.$$

[2]

(ii) Using the Fourier transform technique, find the solution of following mathematical model of heat flow:

$$\begin{aligned} \frac{\partial u}{\partial t} &= k\frac{\partial^2 u}{\partial x^2},\ -\infty < x < \infty,\ t > 0, \\ u(x,t) &= u_t(x,t) \to 0, \text{as } |x| \to \infty \\ u(x,0) &= f(x). \end{aligned}$$

[4]

8 (i) Find the normal form of one dimensional linear wave equation for the transverse vibration of a string and hence find its d'Alembert solution in infinite medium (i.e., $-\infty < x < \infty$) under the given initial displacement and velocity $f(x)$ and $g(x)$ respectively. [4]

(ii) Find the normal form of PDE: $x^2r - 2xys + y^2t - xp + 3yq = \frac{8y}{x}$. [2]

9 (i) Write the Neumann Boundary Value Problem of two dimensional Laplace equation and write the necessary condition for the existence of its solution. [1]

(ii) Write the Maximum Principle for Dirichlet Boundary Value problem for two dimensional Laplace equation. [1]

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