PH 201

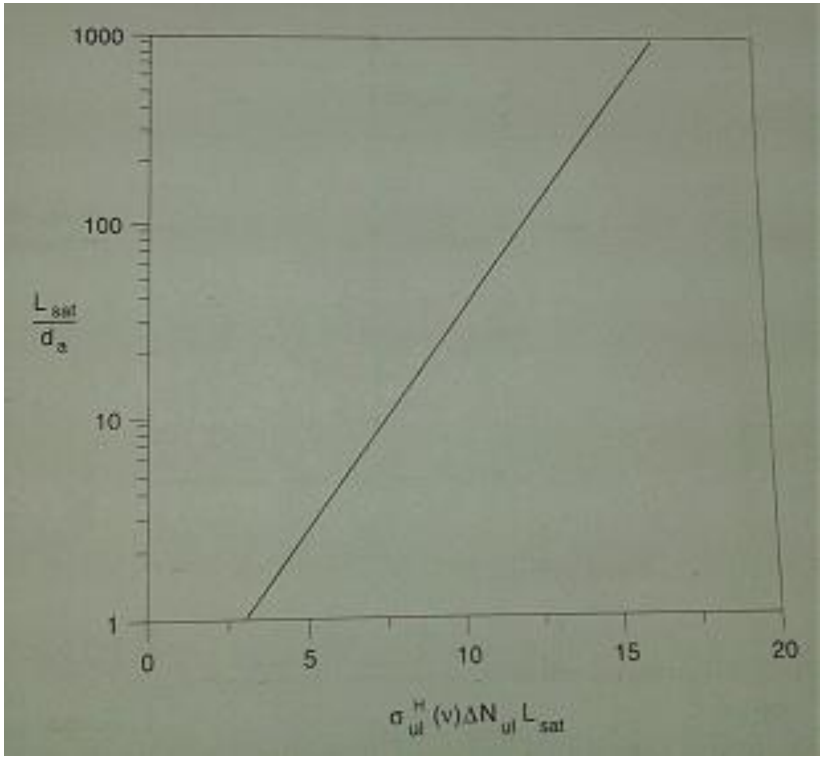
OPTICS & LASERS

Lecture_Lasers_6

Shape or Geometry of Amplifying Medium

$$e^{\sigma_{ul}^{H}(\upsilon)N_u} = 16\left(\frac{L_{sat}}{d_a}\right)^2 \qquad e^{x} = 16\left(\frac{L_{sat}}{d_a}\right)^2$$

Solution to above Eq. can be graphed in the form of either L_{sat}/d_a vs. x.



Graph of ratio of saturated gain length L_{sat} to diameter d_a versus exponential gain coefficient, $\sigma.\Delta N.L_{sat}$

Ratio of length of amplifier to its diameter (L_{sat}/d_a) is an important factor in how much gain is needed to reach saturation.

In most cases, it is difficult to generate a large gain factor in an amplifying medium of any reasonable length.

$$gL_{sat} = \sigma_{ul}^{H}(\upsilon)\Delta N_{ul}L_{sat} = 12$$

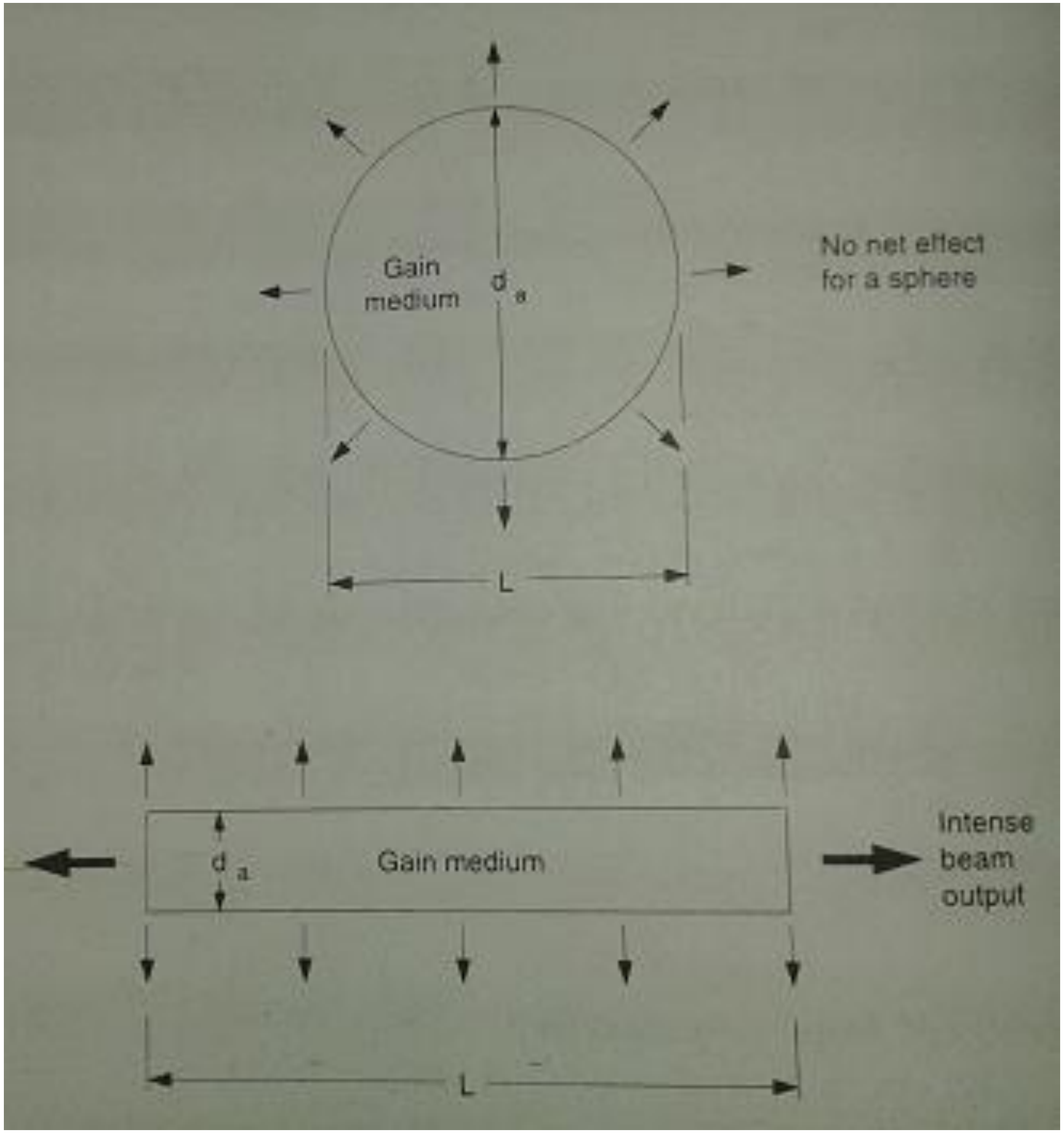
An amplifier with a low value of L_{sat}/d_a, would require less gain.
This reason turns out to be erroneous.

Consider two differently shaped gain media: long cylinder & sphere

For a long cylinder, say $\frac{L_{sat}}{d_a} = 100$

spontaneous emission will originate at one end of medium & will emerge at other end in an elongated shape. This could certainly be considered a beam. Value of gL_{sat} would be approx. 12 & thus beam would have grown by an amount $e^{12} = 1.6 \times 10^5$!

This is an extremely large increase resulting in a very intense beam with an extremely low divergence.



Two possible types of gain media

Consider the case of a sphere (diameter d_a).

For $$\frac{L_{sat}}{d_a} = 1 \qquad\qquad \sigma_{ul}^{H}(\upsilon) N_u L_{sat} = 2.7$$

This would represent an exponential growth factor of $e^{2.7} = 15$, which is significantly lower than the value of 1.6×10^5 obtained for elongated medium.

While such a laser would be just as intense at the surface of sphere as that from elongated medium (since it, too, would reach I_{sat}), intensity would be rapidly reduced as one moves away from sphere.

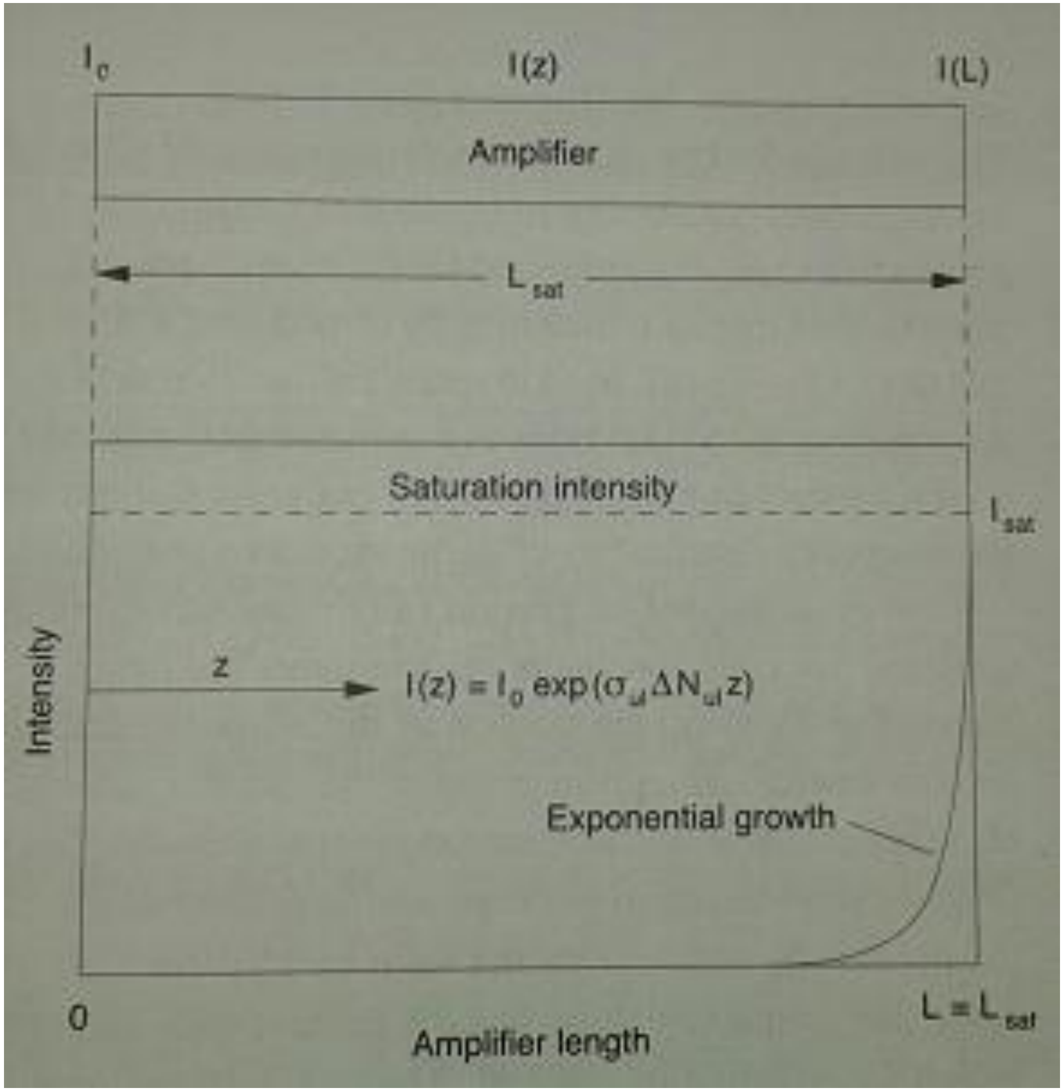
In other words, radiation originating from different locations within sphere would cause the beam to diverge rapidly in same manner as radiation emitting from a spherically shaped incoherent source.

Radiation emitted from entire sphere will be emitted equally in all directions, since sphere is completely symmetrical.

Energy that would normally be radiated in all directions is concentrated into a specific direction - LASER

How a beam would grow as length is added to the amplifier?

It grows exponentially over length L_{sat}.



Exponential growth & saturation of a laser beam as a function of amplifier length

As it approaches saturation intensity, it can no longer grow at that rate. It will then begin to extract significant energy from amplifier, because stimulated emission rate will exceed spontaneous emission rate.

But before I_{sat} is reached, intensity is so low that developing beam has very little effect on gain medium since N_u is not significantly altered.

It would be useful to make L_{sat}/d_a reasonably large so that the beam can develop a well-defined direction before I_{sat} is reached.

Typical lasers require L_{sat}/d_a ratio ranging from about 10 to 1000, which suggests a desirable range of gain from 7 to 17 in order to reach saturation.

$$g_{th} L_{sat} = \sigma_{ul}^{H}(\upsilon) \Delta N_{ul} L_{sat} \cong 12 \pm 5$$

$\sigma_{ul}^{H}(\upsilon)$ **Stimulated emission cross-section (Homogeneous broadening)**

$\sigma_{ul}^{D}(\upsilon)$ **Stimulated emission cross-section (Doppler broadening)**

Exponential Growth Factor (GAIN)

Examine various components of gain to produce a gain of 12.

- **Transition probability is fixed for a given laser.**
- **Mirrors are required to enable most lasers to reach I_{sat}.**

Components of a gain

- **Stimulated emission cross-section**
- **Population difference**
- **Gain length**

- **Stimulated emission cross-section: Longer wavelength laser can be made since cross-section depends upon square of wavelength. It is difficult to make short-wavelength lasers.**

$$\sigma_{ul}^{H}(\upsilon_0) = \frac{1}{4\pi^2}\left(\frac{\lambda_{ul}^2 A_{ul}}{\eta^2 \Delta\upsilon^H}\right)$$

LASER	Wavelength (nm)	Cross-section (σ_{ul}) (m^2)
He-Ne	632.8	3.0×10^{-17}
Argon	488.0	2.5×10^{-16}
He-Cd	441.6	9.0×10^{-18}
Copper (CVL)	510.5	8.6×10^{-18}
CO_2	10,600.0	3.0×10^{-22}
Excimer	248.0	2.6×10^{-20}
Dye (Rh6G)	577.0	2.5×10^{-20}
Semiconductor	800.0	1.0×10^{-22}
Nd:YAG	1064.1	2.8×10^{-23}
Nd:Glass	1062.3	3.0×10^{-24}
Ti:Al_2O_3	800.0	3.4×10^{-23}
Cr:LiSrAlF	850.0	4.8×10^{-24}

Stimulated emission cross-section for a variety of lasers

- **Population difference:**

$$\Delta N_{ul} = N_u - \left(\frac{g_u}{g_l}\right) N_l$$

$$N_u > \left(\frac{g_u}{g_l}\right) N_l$$

Necessity for a relatively large population density difference ΔN_{ul}

Since gain must be of the order of 12 & since σ_{ul} is typically of order 10^{-16} m^2, it would be desirable to have population density difference of order of $12/\sigma_{ul}$, or in this case approx. 1.2×10^{17} m^{-3} for a laser with L ~ 1m. Such a density is not easy to achieve.

- **Gain length:** Requirements of length L of gain medium. Lasers have been made with gain media lengths ranging from less than 1 mm to longer than 10 meters.

  Long lasers have not turned out to be very useful. Long lasers are too cumbersome, too difficult to setup & operate, & hence not very practical.

He-Ne laser (632.8 nm):

Cross-section = 3×10^{-17} m^2

$\Delta N_{ul} \sim 5 \times 10^{15}$ m^{-3}

Therefore, L = 80 m

Making a laser with such a gain length is not realistic.

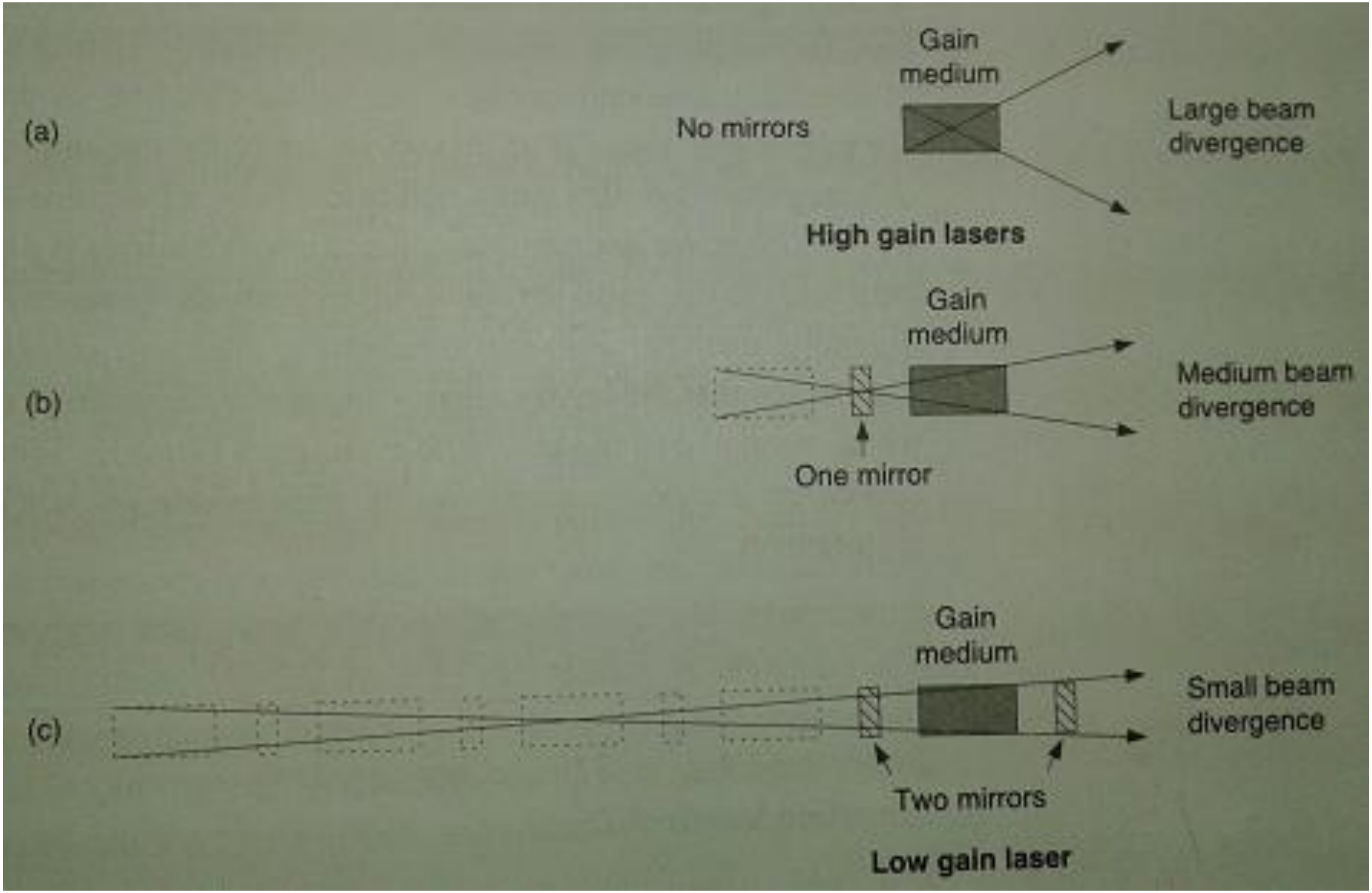
Threshold Requirements for a Laser

- **Most lasers have limitations on population density that can be achieved in any particular energy level, which in turn places limits on maximum ΔN_{ul} that can be obtained for a specific laser transition.**

- **Also, stimulated emission cross-section is essentially a constant for a specific laser transition. Therefore, only factor of gain that can be increased in *effective* length of amplifier, which can be realized with use of mirrors.**

- **Laser with No Mirrors: Beam would emerge if length L were sufficient for beam to reach I_{sat} at end of amplifier. Beam would then meet threshold gain requirement**

$$g_{th} L_{sat} = \sigma_{ul}^{H}(\upsilon) \Delta N_{ul} L_{sat} \cong 12 \pm 5$$



Laser beam divergence for an amplifier with (a) no mirrors, (b) one mirror, & (c) two mirrors

❖ Laser with One Mirror:

If length L were not sufficient to reach saturation, we could add a mirror behind amplifier. Adding a mirror (assuming 100% reflectivity at laser wavelength) can be thought of as adding a 2nd amplifier behind 1st one, since mirror effectively produces an image of existing amplifier located behind that amplifier.

Assuming that beam just reaches I_{sat} after two passes through amplifier.

$$g_{th}L_{sat} = g_{th}(2L) = g_{th}L_{eff} = \sigma_{ul}\Delta N_{ul}(2L) \cong 12 \pm 5$$

In this case, L_{sat} = 2L, effective saturation length or L_{eff}

❖ Laser with Two Mirrors:

If length *L* were still not sufficient for beam to reach I_{sat} after two passes through amplifier, we could add a 2nd mirror in front of amplifier. Slight "leak" out of end is allowed by using a mirror with only 99% reflectivity, so that a portion of beam can escape & provide an observable signal.

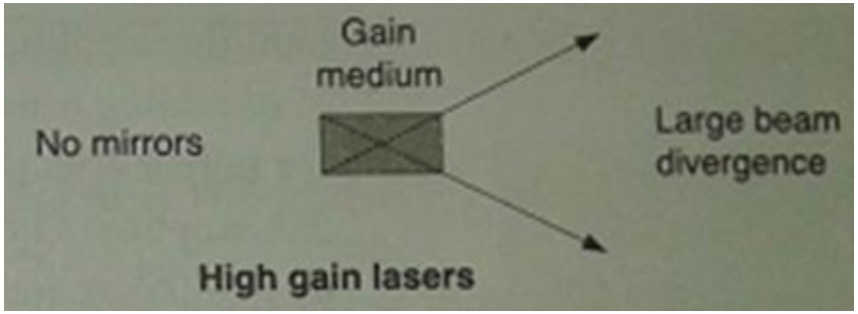
For such an arrangement, beam emerges with a very narrow angular divergence.

Two factors determine if a two-mirror laser can reach I_{sat}.

1. **Net gain per round trip** – There must be a net gain per round trip through amplifier. All of losses (transmission losses, scattering losses, & absorption losses) must be lower than gain or increase per round trip through amplifier.

2. **Sufficient gain duration** – Gain duration is amount of time that gain exists in amplifier. This is relevant to pulse lasers only. Gain duration ts must last long enough to allow a sufficient no. of passes of beam through amplifier in order for beam to reach I_{sat}.

A laser medium has a gain coefficient of 60 m^{-1} at the center of the emission profile and an amplifier length of 0.2 m. For what diameter *d_a* of the gain medium would the saturation intensity be reached in a single pass through the amplifier? What would be the divergence half-angle of the beam exiting from the medium?



gain coefficient, *g* = 60 m^{-1} & *L* = 0.2 m
Exponential growth factor (gain) = 60 m^{-1} × 0.2 m = 12
Spontaneous emission originating from end of amplifier would grow as,

$$I = I_0 e^{12} = (1.67 \times 10^7) I_0$$

$$e^{\sigma_{ul}^{H}(\upsilon) N_u} = 16\left(\frac{L_{sat}}{d_a}\right)^2 \qquad\qquad e^{x} = 16\left(\frac{L_{sat}}{d_a}\right)^2$$

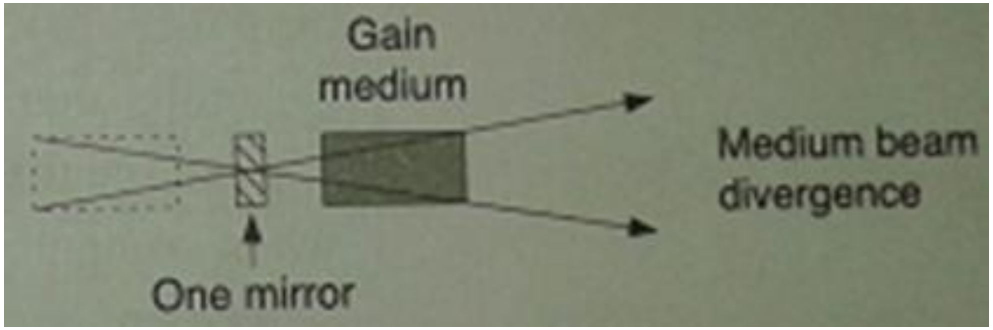
For a gain of 12, $\dfrac{L_{sat}}{d_a} = 100$

$$d_a = \frac{L_{sat}}{100} = \frac{0.2}{100} m = 2mm$$

Half-angle of emerging beam would have an angular divergence determined by length divided by diameter, since spontaneous emission originating from any radial position at the end of amplifier would emit radiation (toward other end of amplifier) that would reach saturation intensity when those rays were amplified as they traveled entire length of amplifier.

Half-angle divergence d_a/L_{sat} would be 1/100 = 0.01 rad (10 mrad)

A laser amplifier with a length of 0.12 m and a gain coefficient of 60 m^{-1} has 100% reflecting mirror at the laser wavelength coated on one end of the laser rod. For a road diameter of 5 mm, what would be the half-angle of the diverging beam exiting from the amplifier? Would the beam reach the saturation intensity as it emerges from the rod after having made a double pass through the rod?



With one mirror on the end of rod, this laser has an effective saturation length, $L_{eff} = 2L = 2 \times 0.12 = 0.24$ m

$$\frac{L_{sat}}{d_a} = \frac{0.24}{0.005} = 48$$

Half-angle,

$$\frac{d_a}{2L_{sat}} = \frac{0.005}{0.24 \times 2} = 2.1 \times 10^{-2}\, rad \quad (21\, mrad)$$

$$e^{x} = 16\left(\frac{L_{eff}}{d_a}\right)^2 = 16(48)^2 = 3.7\times10^4$$

$$x = g(\upsilon_0)L_{eff} = g(\upsilon_0)2L$$

$$L = \frac{x}{2g(\upsilon_0)}$$

gain coefficient, *g* = 60 m^{-1}

$$x = \sigma_{ul}^{H}(\upsilon)N_u L_{sat} \cong \sigma_{ul}^{H}(\upsilon)\Delta N_{ul} L_{sat} = 10.5$$

$$L = \frac{x}{2g} = \frac{10.5}{2(60)m^{-1}} = 0.087m$$

Thus, 0.12 m rod will be more than sufficient to reach saturation intensity.