Indian Institute of Technology Patna

MA-102 (Mathematics II)

B.Tech. I year (Spring Semester: 2015-16)

Mid Semester Examination-2016

Maximum Marks: 30 Total Time: 2 Hours

Attempt all questions:

1. Is the set $W = \{f(x) \in P(\mathbb{F}) : f(x) = 0 \text{ or has degree } n\}$ a subspace of $P(\mathbb{F})$ if $n \geq 1$? Justify your answer, where $P(\mathbb{F})$ is the vector space of all polynomials over field $\mathbb{F}$ in inderminate x. **[1]**

2. Prove or disprove: there exists a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $T(1,1) = (1,0,2)$ and $T(2,3) = (1,-1,4)$. **[1]**

3. Let A and B be two $n \times n$ matrices. Then show that $det(AB) = det(A)det(B)$. **[3]**

4. Solve the following system of linear equations by Gauss elimination method or indicate non existence of solution: $7y + 3z = -12;\ 2x + 8y + z = 0;\ -5x + 2y - 9z = 26$. **[3]**

5. Find inverse of the following matrix by applying elementary transformations:
$$A = \begin{pmatrix} 1 & 3 & -3 \\ -3 & -5 & 2 \\ -4 & 4 & -6 \end{pmatrix}.$$
[2]

6. Determine whether $(1,1,1,1), (1,2,3,2), (2,5,6,4), (2,6,8,5)$ forms a basis of $\mathbb{R}^4$. If not, find the dimension of the subspace spanned by these vectors. **[3]**

7. For what value of a, the given system of linear equations has a solution:
$$\begin{aligned} x + 2y + z &= 1 \\ -x + 4y + 3z &= 2 \\ 2x - 2y + az &= 3. \end{aligned}$$
[2]

8. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a map defined by $T(a,b,c) = (a+b+c, -a-c, b)$. Show that T is a linear transformation. Also find range space, null space, rank and nullity of T. **[3]**

9. Let W be the subspace of $\mathbb{R}^6$ composed of all vectors $[a_1, ..., a_6]^t$ satisfying $\sum_{i=1}^{6} a_i = 0$. Does there exist a one-one mapping from W to $\mathbb{R}^4$? **[2]**

10. Find the matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ over $\mathbb{R}$, defined as $T(x, y) = (2x - 3y, x, y + 5x)$ with respect to the bases $B = \{(1, 1), (1, -1)\}$ and $B_1 = \{(1, -1, 0), (1, 1, 1), (1, 1, -2)\}$ of $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. **[3]**

11. Find the non-singular matrix P which diagonalize the matrix

$$A = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix}.$$

[3]

12. Let A and P be both $n \times n$ matrices and P be a nonsingular matrix. Then show that A and $P^{-1}AP$ have the same eigenvalues. **[1]**

13. State Cayley-Hamilton theorem and verify it for the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$. **[3]**