CS225 Switching Theory

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(Before-mid) Quiz Test On Next Week

Previous Class

Minimization/ Simplification **of Switching Functions**

K-map (SOP)

Quine-McCluskey (Tabular) Minimization

This Class

Minimization/ Simplification **of Switching Functions**

Quine-McCluskey (Tabular) Minimization

Tabulation Procedure for Obtaining the Set of All Prime implicants

Systematic Quine-McCluskey tabulation procedure: for functions with a large number of variables

- Fundamental idea: repeated application of the combining theorem $Aa + Aa' = A$ on all adjacent pairs of terms yields the set of all prime implicants

Example: minimize $f_1(w,x,y,z) = \sum(0,1,8,9) = w'x'y'z' + w'x'y'z + wx'y'z' + wx'y'z$

- Combine first two and last two terms to yield

$$f_1(w,x,y,z) = w'x'y'(z' + z) + wx'y'(z' + z) = w'x'y' + wx'y'$$

- Combine this expression in turn to yield

$$f_1(w,x,y,z) = x'y'(w' + w) = x'y'$$

- Similar result can be obtained by initially combining the first and third and the second and fourth terms

Tabulation Procedure (Contd.)

Two k-variable terms can be combined into a single (k-1)-variable term if and only if they have k-1 identical literals in common and differ in only one literal

- *Using the binary representation of minterms: two minterms can be combined if their binary representations differ in only one position*

Example: w'x'y'z' (0000) and w'x'y'z (0001) can be combined into 000-,

indicating z has been absorbed and the combined term is w'x'y'

Tabulation Procedure (Contd.)

Procedure:

1. *Arrange all minterms in groups, with all terms in the same group having the same number of 1's. Start with the least number of 1's (called the index) and continue with groups of increasing numbers of 1's.*

2. *Compare every term of the lowest-index group with each term in the successive group. Whenever possible, combine them using the combining theorem. Repeat by comparing each term in a group of index i with every term in the group of index i + 1. Place a check mark next to every term which has been combined with at least one term.*

3. *Compare the terms generated in step 2 in the same fashion: generate a new term by combining two terms that differ by only a single 1 and whose dashes are in the same position. Continue until no further combinations are possible. The remaining unchecked terms constitute the set of prime implicants.*

Example

Example: apply procedure to $f_2 \sum(w, x, y, z)$ = (0,1,2,5,7,8,9,10,13,15)

Step (i)

	w	x	y	z	
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
5	0	1	0	1	✓
9	1	0	0	1	✓
10	1	0	1	0	✓
7	0	1	1	1	✓
13	1	1	0	1	✓
15	1	1	1	1	✓

Step (ii)

	w	x	y	z	
0,1	0	0	0	--	✓
0,2	0	0	--	0	✓
0,8	--	0	0	0	✓
1,5	0	--	0	1	✓
1,9	--	0	0	1	✓
2,10	--	0	1	0	✓
8,9	1	0	0	--	✓
8,10	1	0	--	0	✓
5,7	0	1	--	1	✓
5,13	--	1	0	1	✓
9,13	1	--	0	1	✓
7,15	--	1	1	1	✓
13,15	1	1	--	1	✓

Step (iii)

	w	x	y	z	
0,1,8,9	--	0	0	--	A
0,2,8,10	--	0	--	0	B
1,5,9,13	--	--	0	1	C
5,7,13,15	--	1	--	1	D

P= {x′y′, x′z′, y′z, xz}

Find the K-map and match with this result

Prime Implicant Chart

Prime implicant chart: pictorially displays covering relationships between prime implicants and minterms

Example: prime implicant chart for $f_2(w,x,y,z) = \sum(0,1,2,5,7,8,9,10,13,15)$

	✓0	1	✓2	✓5	✓7	✓8	9	✓10	✓13	✓15
$A = x'y'$	×	×				×	×			
✓ $B = x'z'$	×		⊗			×		⊗		
$C = y'z$		×		×			×		×	
✓ $D = xz$				×	⊗				×	⊗

Cover: a row is said to cover the columns in which it has x's

Problem: select a minimal subset of prime implicants such that each column contains at least one x in the rows corresponding to the selected subset and the total number of literals in the prime implicants selected is as small as possible

Essential rows: if a column contains a single x, the prime implicant corresponding to the row in which the x appears is essential, e.g., B, D

Cover remaining minterms 1 and 9 using A or C: thus, two minimal expressions:

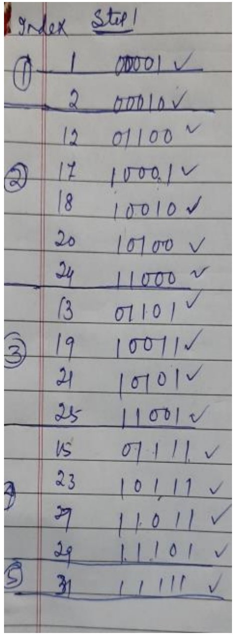
$$f_2 = x'z' + xz + x'y' \text{ or } f_2 = x'z' + xz + y'z$$

Tabulation Procedure using Decimal Notation in the Presence of Don't-cares

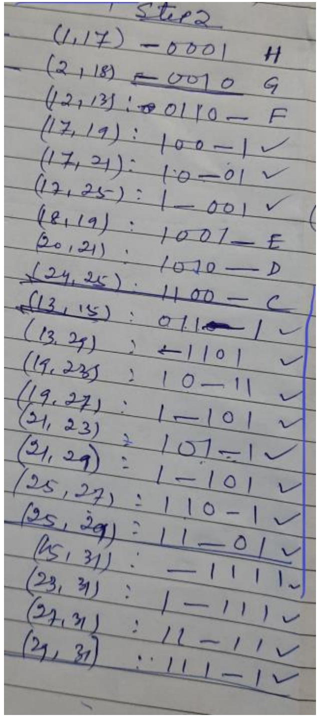
Example: apply procedure to $f_3(v,w,x,y,z) = \sum(13,15,17,18,19,20,21,23,25,27,29,31)$

$+ \sum_{\phi}(1,2,12,24)$

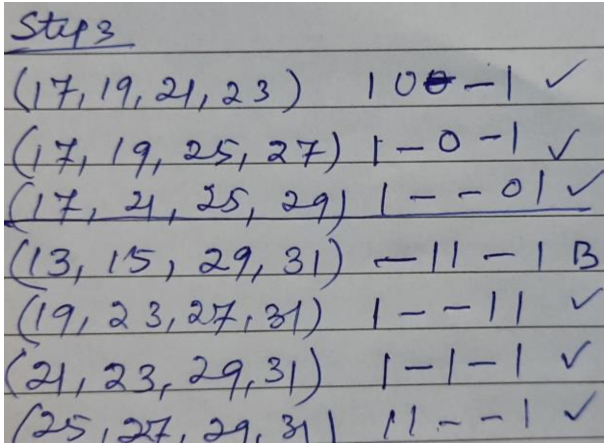
(a)



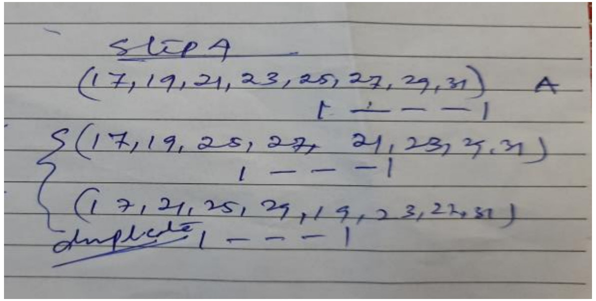
(b)



(c)



(d)



P= {vz, wxz, vwx'y', vw'xy', vw'x'y, v'wxy', w'x'yz', w'x'y'z'}

Don't-care Combinations

Don't-cares: not listed as column headings in the prime implicant chart

Example: $f_3(v,w,x,y,z) = \sum(13,15,17,18,19,20,21,23,25,27,29,31) + \sum_{\phi}(1,2,12,24)$

	✓ 13	✓ 15	✓ 17	18	✓ 19	✓ 20	✓ 21	✓ 23	✓ 25	✓ 27	✓ 29	✓ 31
✓ $A = vz$			×		×		×	×	×	×	×	×
✓ $B = wxz$	×	×									×	×
$C = vwx'y'$									×			
✓ $D = vw'xy'$						×	×					
$E = vw'x'y$				×	×							
$F = v'wxy'$	×											
$G = w'x'yz'$				×								
$H = w'x'y'z$			×									

Selection of nonessential prime implicants facilitated by listing prime implicants in decreasing order of the number of minterms they cover

Essential prime implicants: A, B, and D. They cover all minterms except 18, which can be covered by E or G, giving rise to two minimal expressions

Obtain the result using K-map and match

Determining the Set of All Irredundant Expressions

Deriving the minimal sum-of-products through prime implicant chart inspection: difficult for more complex cases Example: $f_4(v,w,x,y,z) = \sum(0,1,3,4,7,13,15,19,20,22,23,29,31)$

	0	1	✓ 3	4	✓ 7	✓ 13	✓ 15	✓ 19	20	22	✓ 23	✓ 29	✓ 31
✓A = wxz						×	×					×	×
B = xyz					×		×				×		×
✓C = w′yz			×		×			×			×		
D = vw′xy										×	×		
E = vw′xz′									×	×			
F = w′xy′z′				×					×				
G = v′w′x′z		×	×										
H = v′w′y′z′	×			×									
I = v′w′x′y′	×	×											

(*a*) Prime implicant chart.

	0	1	4	20	22
D					×
E				×	×
F			×	×	
G		×			
H	×		×		
I	×	×			

(*b*) Reduced prime implicant chart.

While every irredundant expression must contain A and C, none of them may contain B since it covers minterms already covered by A and C. The reduced chart, obtained after removing A, B, and C, has two x's in each column

Example (Contd.)

Use propositional calculus: define prime implicant function p to be 1 if each

column is covered by at least one of the chosen prime implicants, and 0 if not

$$p = (H + I)(G + I)(F + H)(E + F)(D + E)$$
$$= EHI + EFI + DFI + EGH + DFGH$$

At least three rows are needed to cover the reduced chart:

E, H, and I, or E, F, and I, and so on

Since all prime implicants in the reduced chart have the same literal count, there are four minimal sum-of-products:

$$f_4(v,w,x,y,z) = A + B + E + H + I = wxz + w'yz + vw'xz' + v'w'y'z' + v'w'x'y'$$
$$f_4(v,w,x,y,z) = A + B + E + F + I = wxz + w'yz + vw'xz' + w'xy'z' + v'w'x'y'$$
$$f_4(v,w,x,y,z) = A + B + D + F + I = wxz + w'yz + vw'xy + w'xy'z' + v'w'x'y'$$
$$f_4(v,w,x,y,z) = A + B + E + G + H = wxz + w'yz + vw'xz' + v'w'x'z + v'w'y'z'$$