PH 301

ENGINEERING OPTICS

Lecture_5_Polarization-2

Production of Polarized Light

1. Wire Grid Polarizer & Polaroid
2. Polarization by Reflection
3. Polarization by Double Refraction
4. Polarization by Scattering

Wire Grid Polarizer & Polaroid

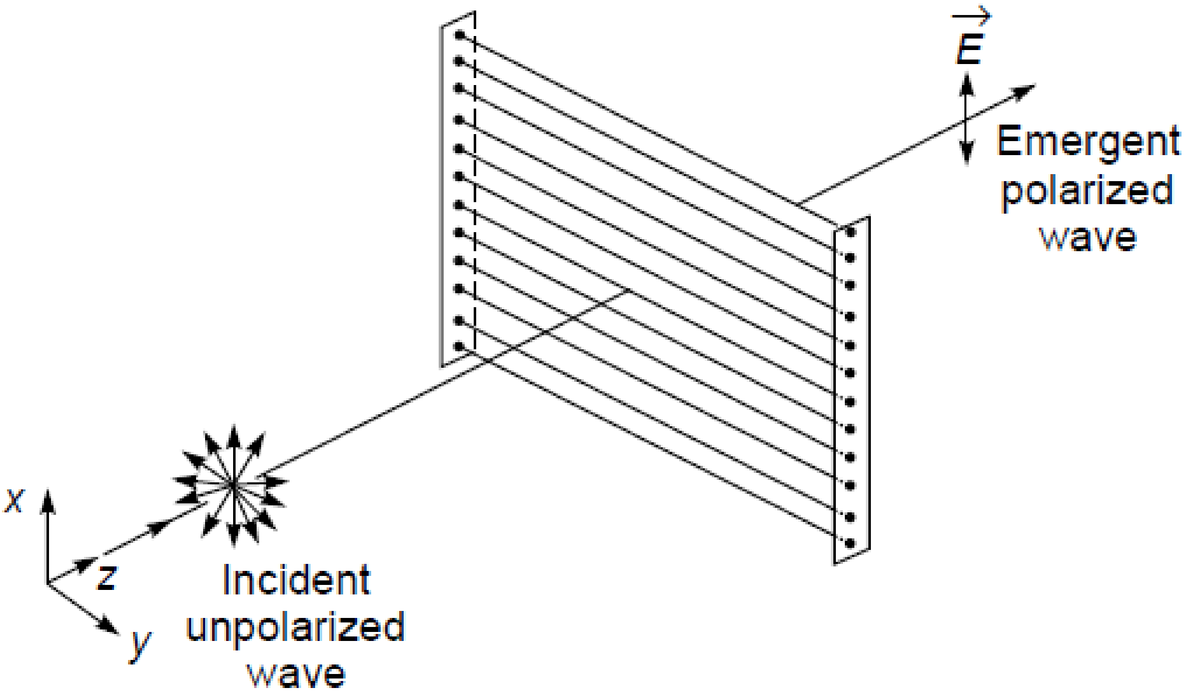
- It consists of a large number of thin copper wires placed parallel to one another. When an unpolarized *em* wave is incident on it, then **component of electric vector along length of wire is absorbed**.

- This is so because electric field does work on electrons inside thin wires, & energy associated with electric field is lost in Joule heating of wires. On the other hand, since wires are assumed to be very thin, component of electric vector along x axis passes through without much attenuation.

- Emergent wave is linearly polarized with electric vector along x axis.

- However, for system to be effective (for E_y component to be almost completely attenuated) spacing between wires should be <~ λ.

- Fabrication of such a polarizer for a 3 *cm* microwave is relatively easy because spacing has to be <~ 3 *cm*.



Wire grid polarizer

- On the other hand, since light waves are associated with a very small wavelength (~ 5 $\times 10^{-5}$ cm) fabrication of a polarizer in which wires are placed at distances ~ 5 $\times 10^{-5}$ cm is extremely difficult.
- Bird & Parrish [1950] did succeed in putting about 30,000 wires in about 1".
- It is extremely difficult to fabricate a wire grid polarizer which would be effective for visible light.
- However, instead of long, thin wires, one may employ long chain polymer molecules that contain atoms (such as iodine) which provide high conductivity along length of chain.
- Long chain molecules are aligned so that they are almost parallel to one another.
- Because of high conductivity provided by iodine atoms, electric field parallel to molecules gets absorbed.

- A sheet containing such long chain polymer molecules (which are aligned parallel to one another) is known as a **Polaroid**.

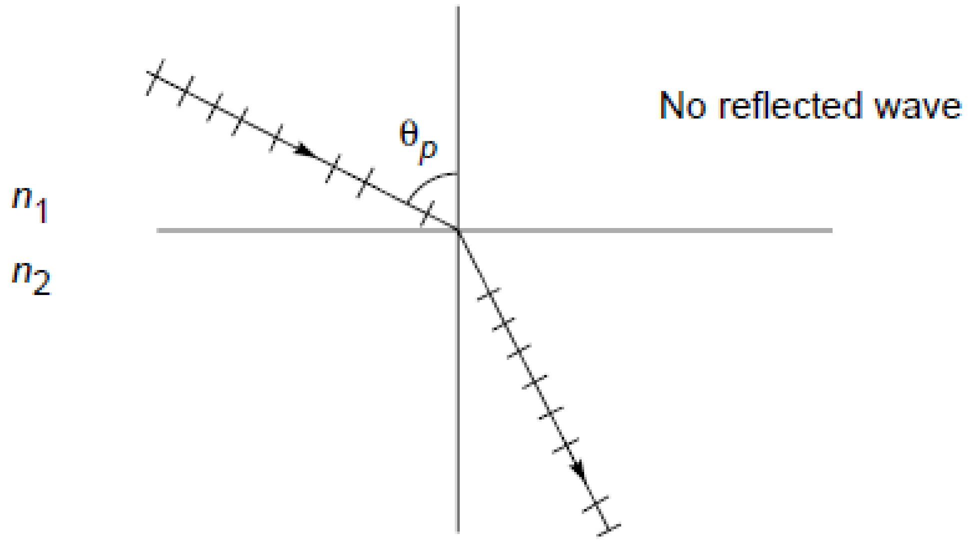
- When a light beam is incident on such a Polaroid, molecules (aligned parallel to one another) absorb component of electric field which is parallel to direction of alignment because of high conductivity provided by iodine atoms; component perpendicular to it passes through.

- Aligned conducting molecules act similar to wires in wire grid polarizer, & since spacing between two adjacent long chain molecules is small compared to optical wavelength, Polaroid is usually very effective in producing linearly polarized light.

- Aligning of long chain conducting molecules is not very difficult.

Polarization by Reflection

- Consider incidence of a plane wave on a dielectric assuming that electric vector associated with incident wave lies in plane of incidence as shown.



If a linearly polarized wave (with its *E* in plane of incidence) is incident on interface of two dielectrics with angle of incidence equal to θ_p [= $\tan^{-1}(n_2/n_1)$], then reflection coefficient is zero.

- If angle of incidence θ is such that

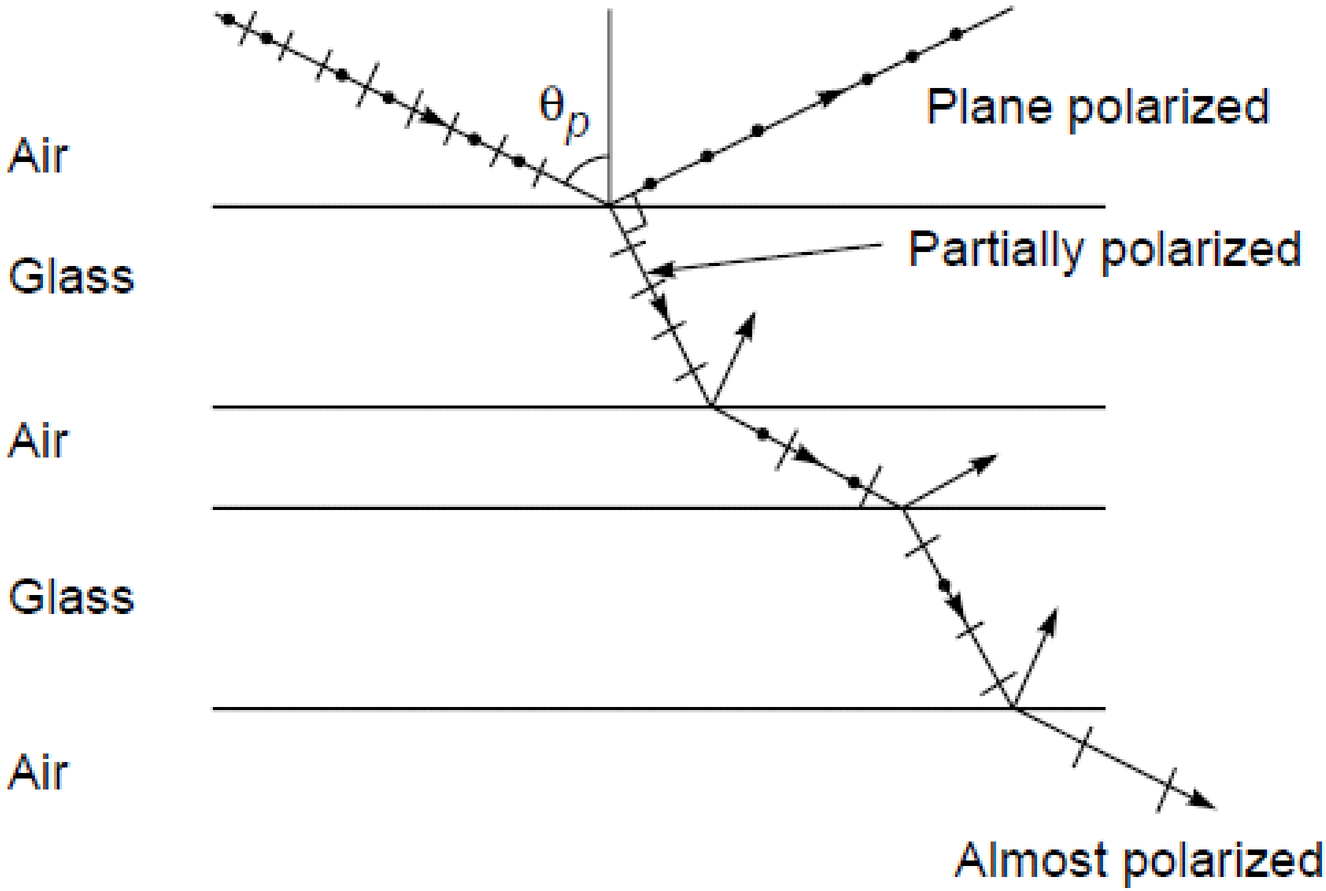
$$\theta = \theta_p = tan^{-1}(n_2/n_1)$$

then reflection coefficient is zero.

- If an unpolarized beam is incident at this angle, then reflected beam will be linearly polarized with its electric vector perpendicular to plane of incidence.

- Above Eq. is called **Brewster's law** & at this angle of incidence, reflected & transmitted rays are at right angles to one another; angle θ_p is known as **polarizing angle** or **Brewster angle**.

- For air-glass interface, $n_1 = 1$ & $n_2 \approx 1.5$, giving $\theta_p \approx 57^o$. The transmitted beam is partially polarized, & if one uses a large number of reflecting surfaces, one obtains an almost plane polarized transmitted beam.



If an unpolarized beam is incident with an angle of incidence θ_p, reflected beam is plane polarized whose electric vector is perpendicular to plane of incidence. Transmitted beam is partially polarized, & if this beam is made to undergo several reflections, then emergent beam is almost plane polarized with its electric vector in plane of incidence.

- For air-water interface,

  $n_1 \approx 1$ & $n_2 \approx 1.33$ & polarizing angle $\theta_p \approx 53^o$.

- If sunlight is incident on sea at an angle close to polarization angle, then reflected light will be almost polarized.

- If we now view through a rotating Polaroid, sea will appear more transparent when Polaroid blocks the reflected light.

- If Polaroid allows the (almost polarized) reflected beam to pass through, we see the glare from water surface; the glare can be blocked by using a vertical polarizer, & one can see inside of water.

(a)

(b)

If sunlight is incident on water surface at an angle close to polarizing angle, then reflected light will be almost polarized.

(a) If Polaroid allows the (almost polarized) reflected beam to pass through, we see glare from water surface.

(b) Glare can be blocked by using a vertical polarizer, & one can see inside the water.

Polarization by Double Refraction

- **Double Refraction**: When an unpolarized beam enters an anisotropic crystal, it splits up into two beams, each being characterized by a certain state of polarization.

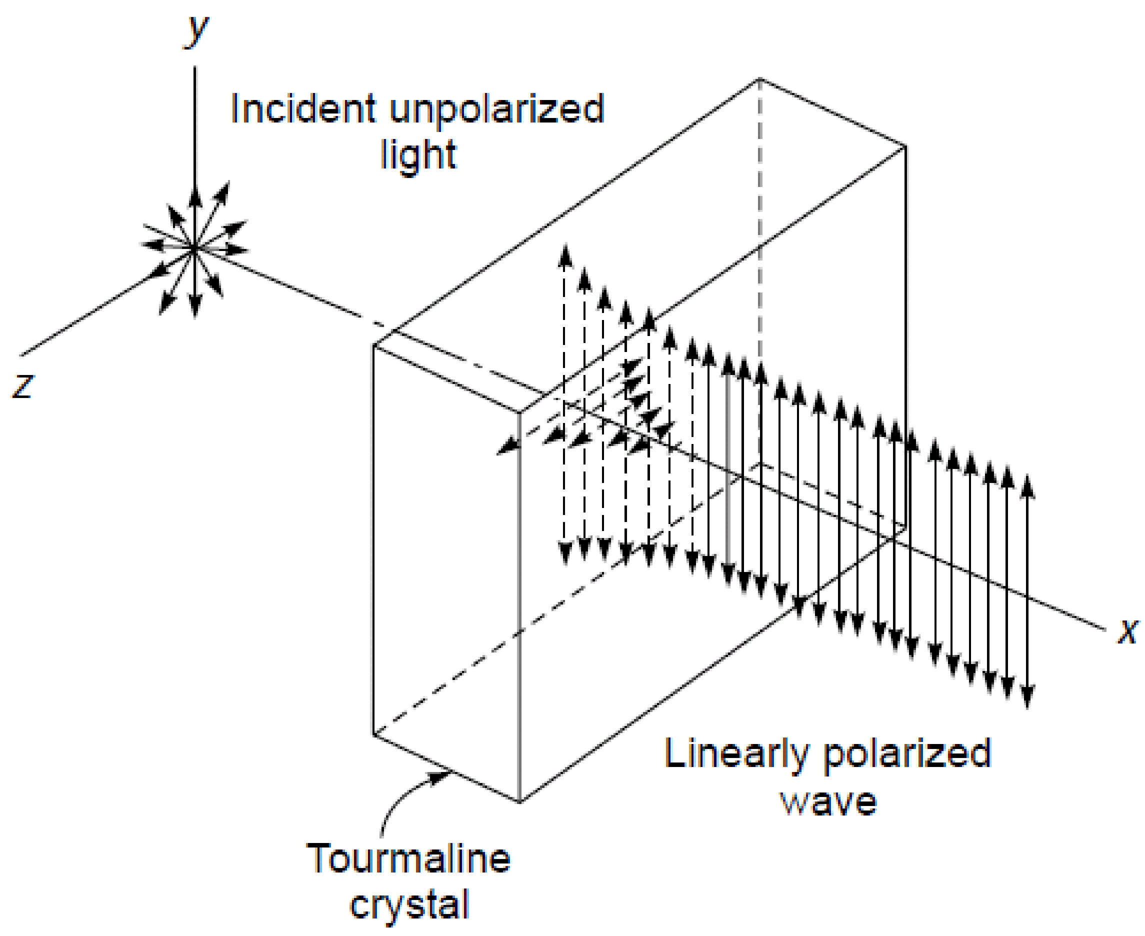
- If, by some method, we could eliminate one of the beams, then we would obtain a linearly polarized beam.

- Eliminating one of the beams is through selective absorption, this property is known as **dichroism**.

- A crystal, tourmaline has different coefficients of absorption for the two linearly polarized beams into which the incident beam splits up.

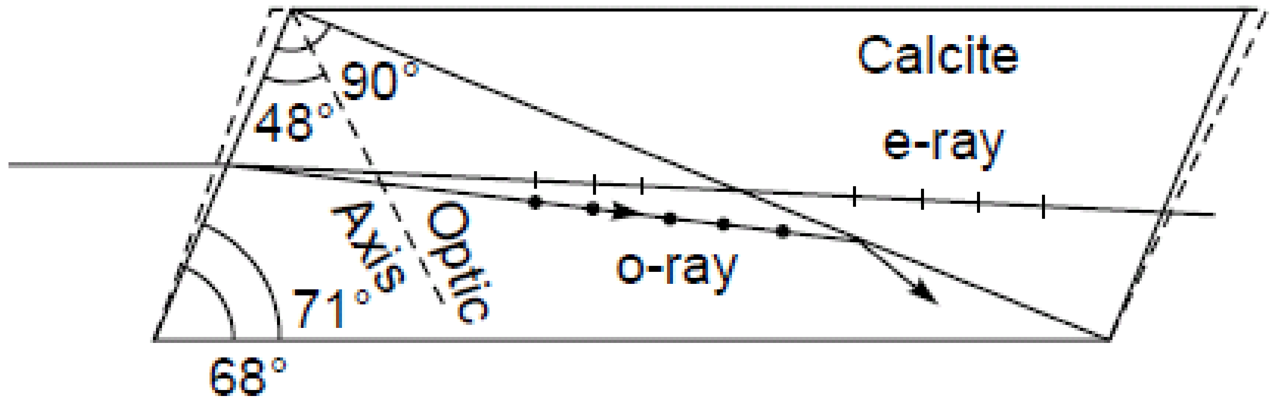
- Consequently, one of the beams gets absorbed quickly, & the other component passes through without much attenuation.

- If an unpolarized beam is passed through a tourmaline crystal, emergent beam will be linearly polarized.



When an unpolarized beam enters a dichroic crystal (tourmaline), it splits up into two linearly polarized components. One of the components gets absorbed quickly, & other component passes through without much attenuation.

- Another method for eliminating one of the polarized beams is through **total internal reflection**.
- The two beams have different velocities, & as such corresponding refractive indices will be different.
- If one can sandwich a layer of a material whose refractive index lies between the two, then for one of the beams, the incidence will be at a rarer medium & for the other it will be at a denser medium.
- This principle is used in a **Nicol prism** which consists of a calcite crystal cut in such a way that for the beam, for which the sandwiched material is a rarer medium, the angle of incidence is greater than the critical angle. Thus this particular beam will be eliminated by total internal reflection.
- Ordinary ray undergoes total internal reflection & extraordinary component passes through, & beam emerging from the crystal is linearly polarized.



Nicol prism. Dashed outline corresponds to natural crystal which is cut in such a way that ordinary ray undergoes total internal reflection at Canada balsam layer.

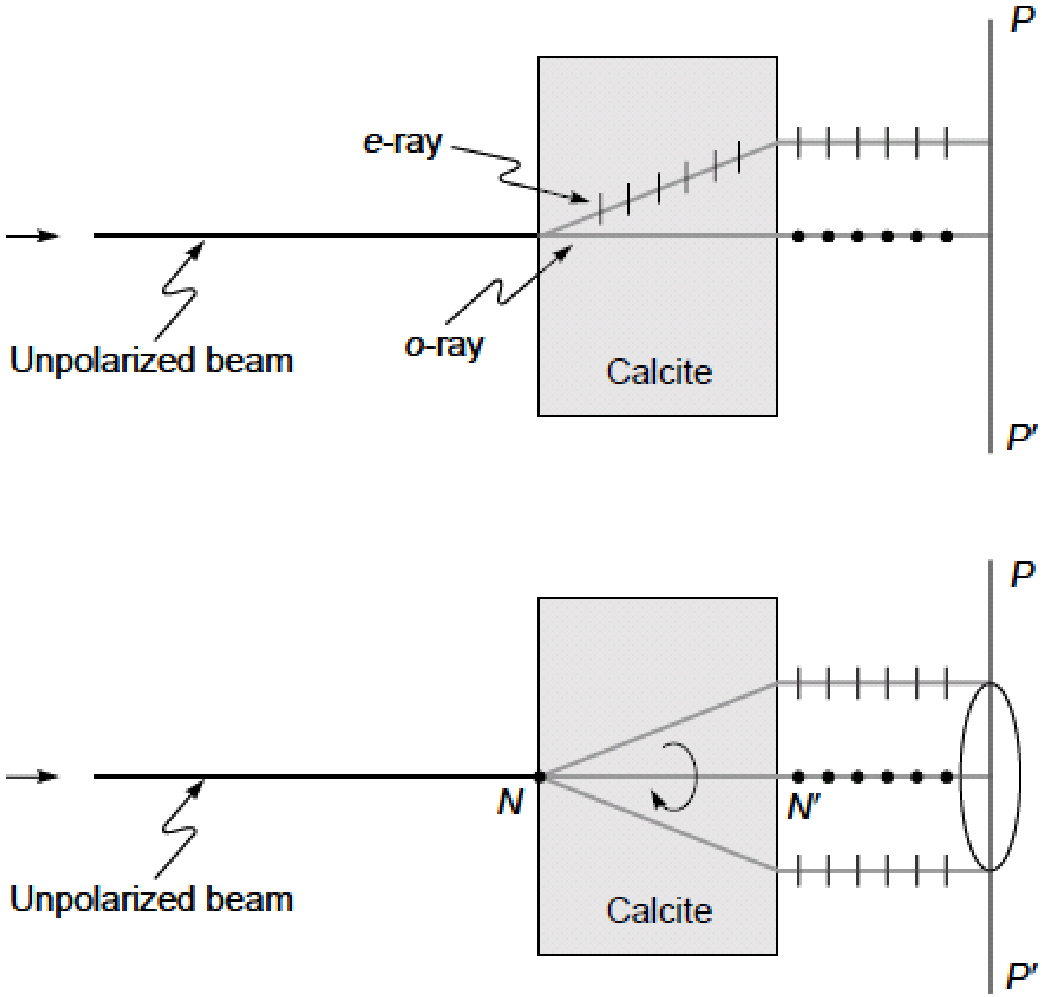
Double Refraction

- When an unpolarized light beam is incident normally on a calcite crystal, it would in general, split up into two linearly polarized beams.

- Beam which travels undeviated is known as **ordinary ray** (*o*-ray) & obeys Snell's laws of refraction.

- 2nd beam, which in general does not obey Snell's laws, is known as **extraordinary ray** (*e*-ray).

- Appearance of two beams is due to the phenomenon of **double refraction**, & a crystal such as calcite is usually referred to as a double-refracting crystal.

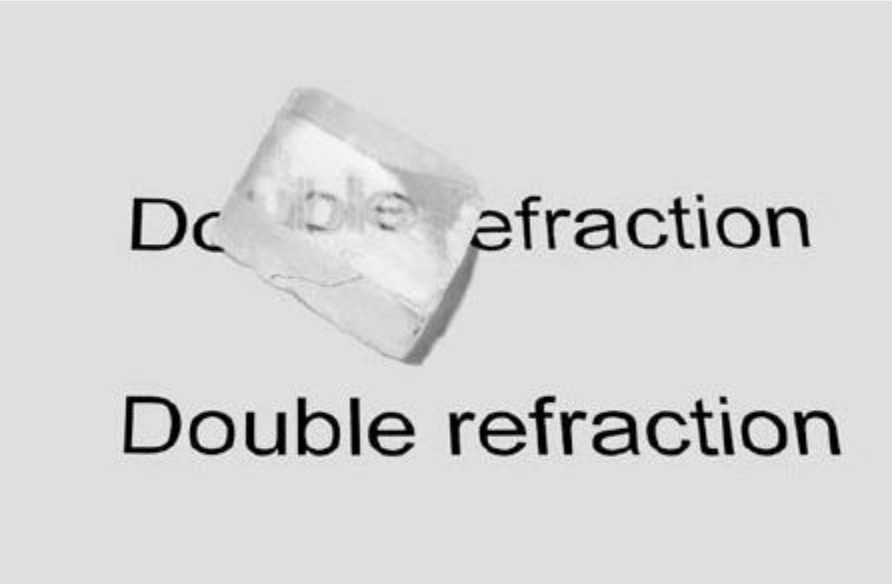


(a) When an unpolarized light beam is incident normally on a calcite crystal, it would in general, split up into two linearly polarized beams. (b) If we rotate crystal about NN‘, then *e*-ray will rotate about NN‘.

- If we put a Polaroid PP' behind calcite crystal & rotate Polaroid (about NN'), then for two positions of Polaroid (when pass axis is perpendicular to plane of paper) the *e*-ray will be completely blocked & only *o*-ray will pass through.

- When pass axis of Polaroid is in plane of paper (i.e., along line PP'), then *o*-ray will be completely blocked & only *e*-ray will pass through.

- If we rotate the crystal about NN', then *e*-ray will rotate about the axis.



A calcite crystal showing double refraction.

- Velocity of *o*-ray is same in all directions & velocity of *e*-ray is different in different directions.

- Anisotropic substances: Calcite, Quartz

- Along a particular direction (fixed in the crystal), the two velocities are equal; this direction is known as the **optic axis of crystal**.

- **Uniaxial crystals**: two rays have same speed only along one direction (which is optic axis).

- **Biaxial crystals**: there may be two directions along which the two rays have same speed.

- Velocities of ordinary & extraordinary rays are given by following equations:

$$v_{ro} = \frac{c}{n_0} \qquad \textit{ordinary ray}$$

$$\frac{1}{v_{re}^2} = \frac{\sin^2\theta}{(c/n_e)^2} + \frac{\cos^2\theta}{(c/n_o)^2} \qquad \textit{extraordinary ray}$$

where n_o & n_e are constants of crystal & θ is the angle that the ray makes with optic axis; we have assumed the optic axis to be parallel to z axis.

- Thus, c/n_o & c/n_e are velocities of extraordinary ray when it propagates parallel & perpendicular to optic axis.

❖ Equation of an ellipse (in *zx* plane) is given by:

$$\frac{z^2}{a^2}+\frac{x^2}{b^2}=1$$

❖ If (ρ,θ) represents the polar coordinates, then $z = \rho\cos\theta$ & $x = \rho\sin\theta$, & equation of ellipse can be written in the form:

$$\frac{1}{\rho^2}=\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}$$

❖ In three dimensions this Eq. will represent an ellipsoid of revolution with the optic axis as the axis of revolution.

❖ Thus if we plot v_{re} as a function of θ, we obtain an ellipsoid of revolution; on the other hand, since v_{ro} is independent of θ, if we plot v_{ro} (as a function of θ), we obtain a sphere. Along the optic axis, $\theta = 0$ &

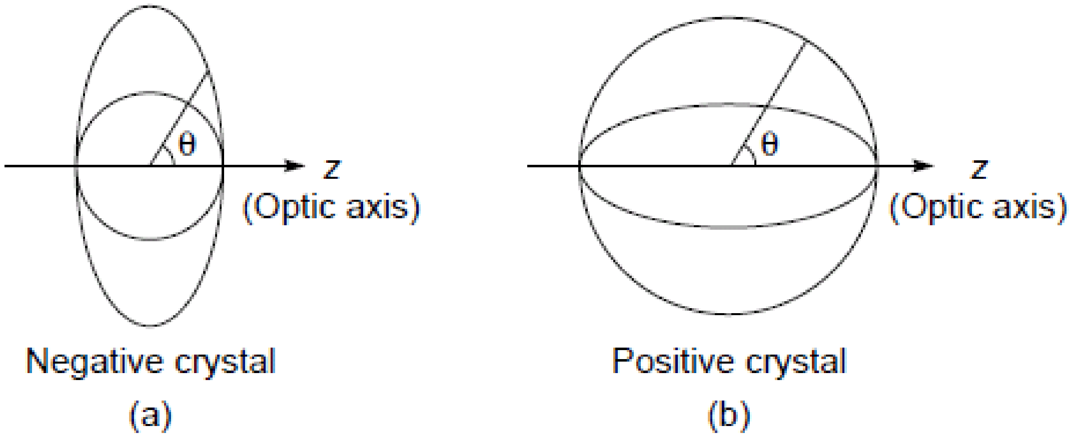
$$v_{ro}=v_{re}=\frac{c}{n_0}$$

❖ Consider the value of v_{re} perpendicular to optic axis (i.e., for $\theta = \pi/2$).

❖ For a negative crystal, $n_e < n_o$ &

$$v_{re}\left(\theta = \frac{\pi}{2}\right) = \frac{c}{n_e} > v_{ro}$$

❖ Minor axis will now be along optic axis, & ellipsoid of revolution will lie outside the sphere.



(a) In a negative crystal, ellipsoid of revolution (which corresponds to the extra ordinary ray) lies outside sphere; the sphere corresponds to ordinary ray. (b) In a positive crystal, ellipsoid of revolution (which corresponds to extraordinary ray) lies inside sphere.

- For a positive crystal, $n_e > n_o$ &

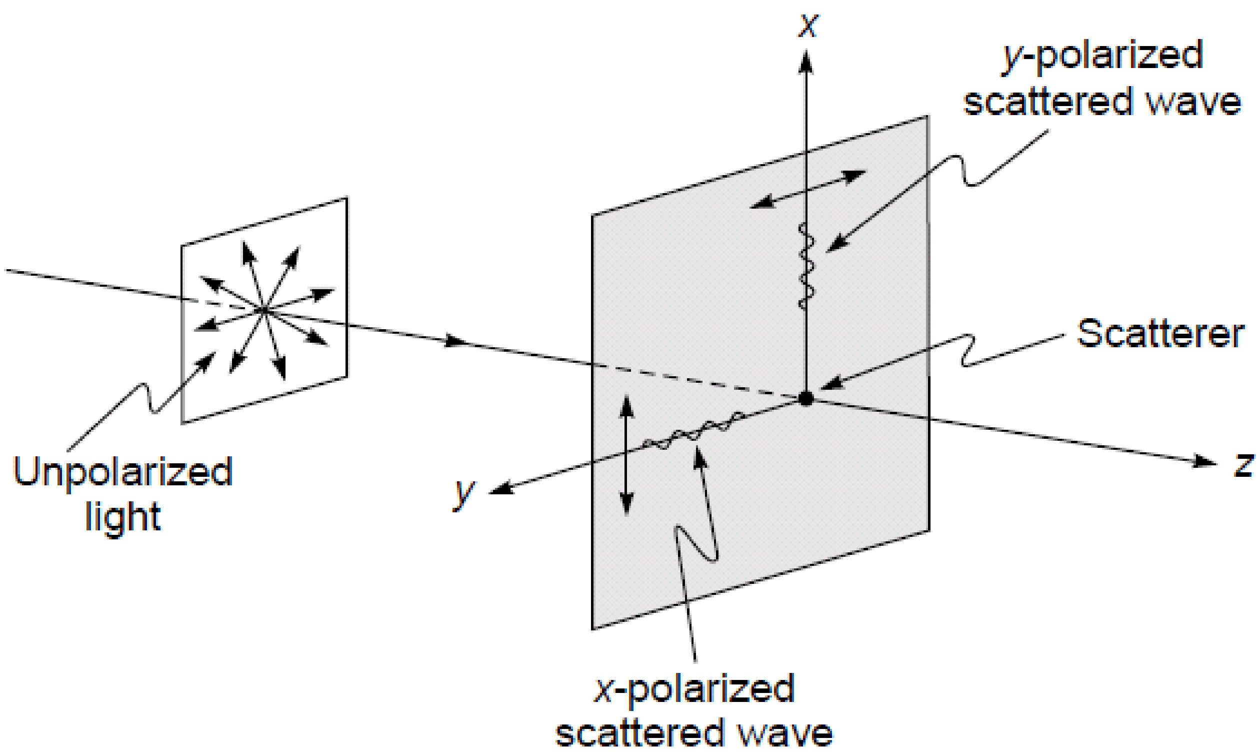
$$v_{re}\left(\theta = \frac{\pi}{2}\right) = \frac{c}{n_e} < v_{ro}$$

- Major axis will now be along optic axis, & ellipsoid of revolution will lie inside the sphere.

- Ellipsoid of revolution & sphere are known as **ray velocity surfaces**.

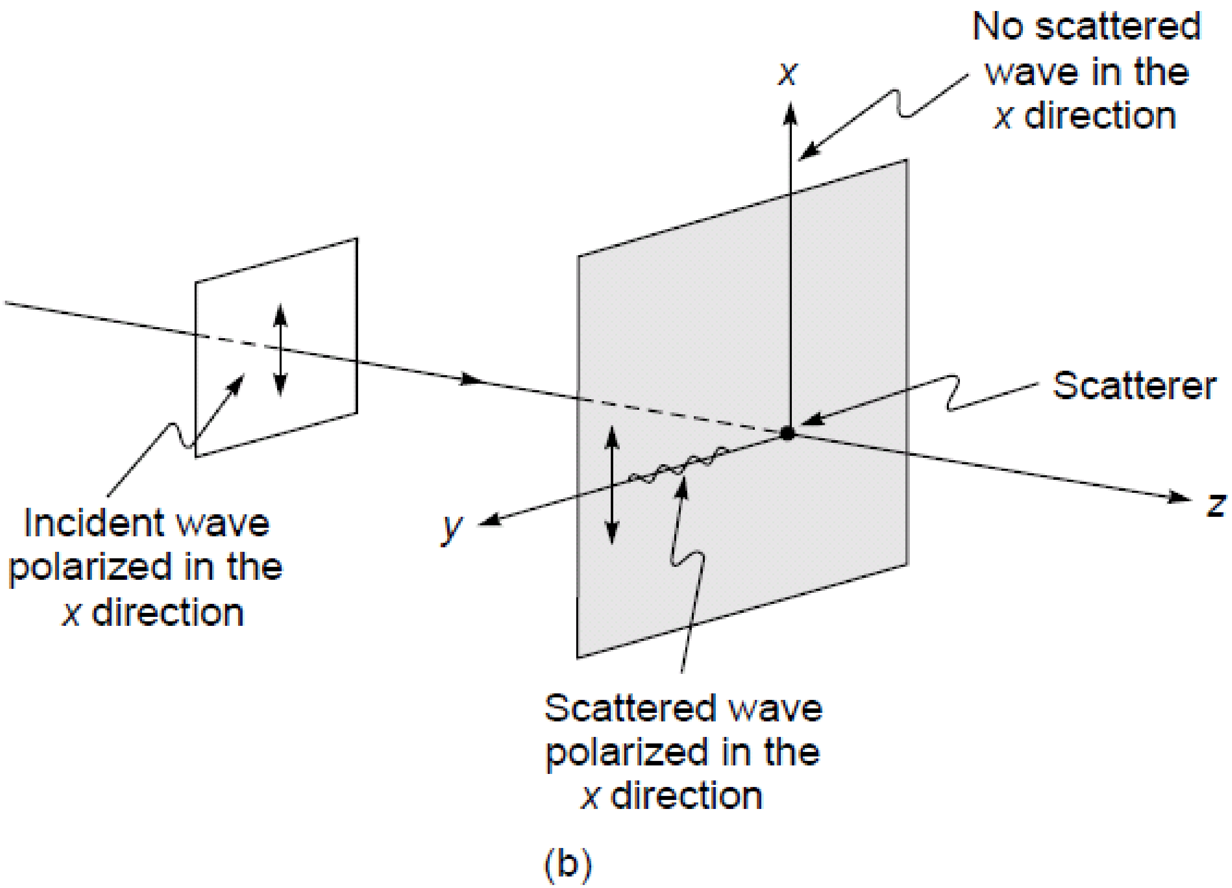
Polarization by Scattering

- If an unpolarized beam is allowed to fall on a gas, then beam scattered at 90° to incident beam is linearly polarized.
- This follows from the fact that the waves propagating in *y* direction are produced by *x* component of dipole oscillations & *y* component of dipole oscillations will produce no field in *y* direction.
- Clearly, if incident beam is linearly polarized with its electric vector along x direction, then there will be no scattered light along x axis.
- **Blue color of sky is due to Rayleigh scattering of sunlight by molecules in our atmosphere.**
- When sun is about to set, if we look vertically upward, light will have a high degree of polarization; this is so because angle of scattering will be very close to 90°.
- If we view blue sky (which is vertically above us) with a rotating Polaroid, we will observe considerable variation of intensity.



(a)

If electromagnetic wave is propagating along *z* direction, then scattered wave along any direction that is perpendicular to *z* axis will be linearly polarized.



If a linearly polarized wave (with its *E* oscillating along *x* direction) is incident on a dipole, then there will be no scattered wave in *x* direction.

Malus' Law

- Consider a polarizer P_1 which has a pass axis parallel to x axis; i.e., if an unpolarized beam propagating in z direction is incident on polarizer, then electric vector associated with the emergent wave will oscillate along x axis.

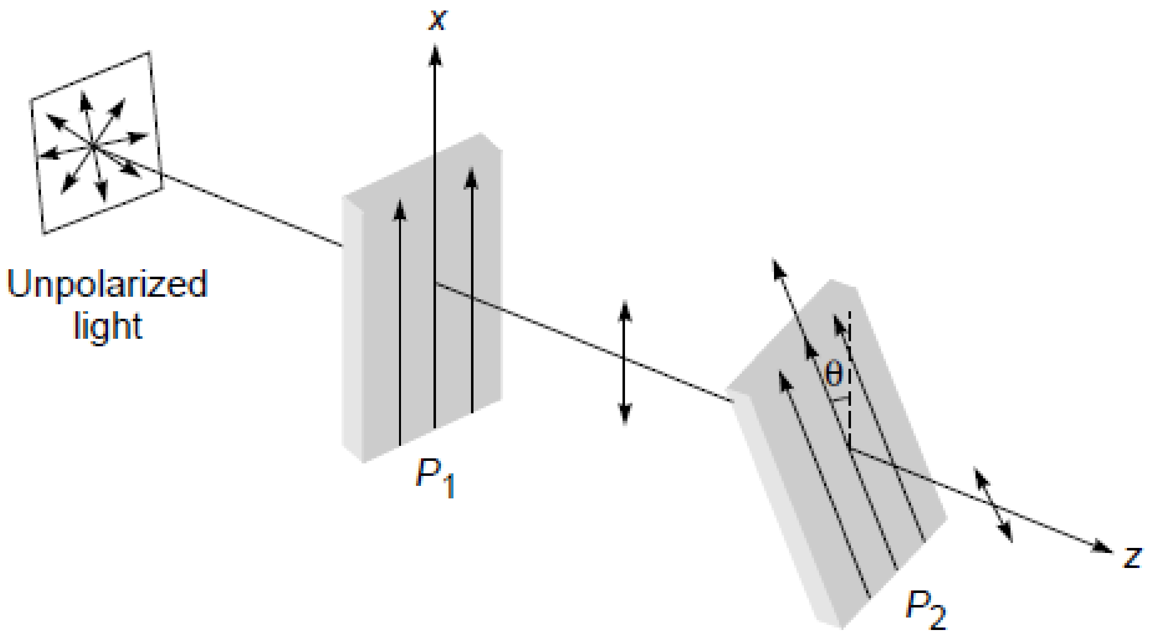
- If polarizer is a Polaroid, then for pass axis to be along the x direction, the long chain molecules must be aligned along the y axis.

- Consider incidence of the x-polarized beam on the Polaroid P_2 whose pass axis makes an angle θ with the x axis.

- If amplitude of incident electric field is E_0, then the amplitude of the wave emerging from the Polaroid P_2 will be $E_0 cos\theta$, & thus intensity of emerging beam will be

$$\boxed{I = I_0 cos^2\theta}$$

where I_0 represents intensity of emergent beam when pass axis of P_2 is along x axis (i.e., when $\theta = 0$). This equation is called **Malus' law**.



An unpolarized light beam gets x-polarized after passing through the polaroid P1, the pass axis of the second polaroid P2 makes an angle θ with the x axis. Intensity of emerging beam will vary as $\cos^2\theta$.

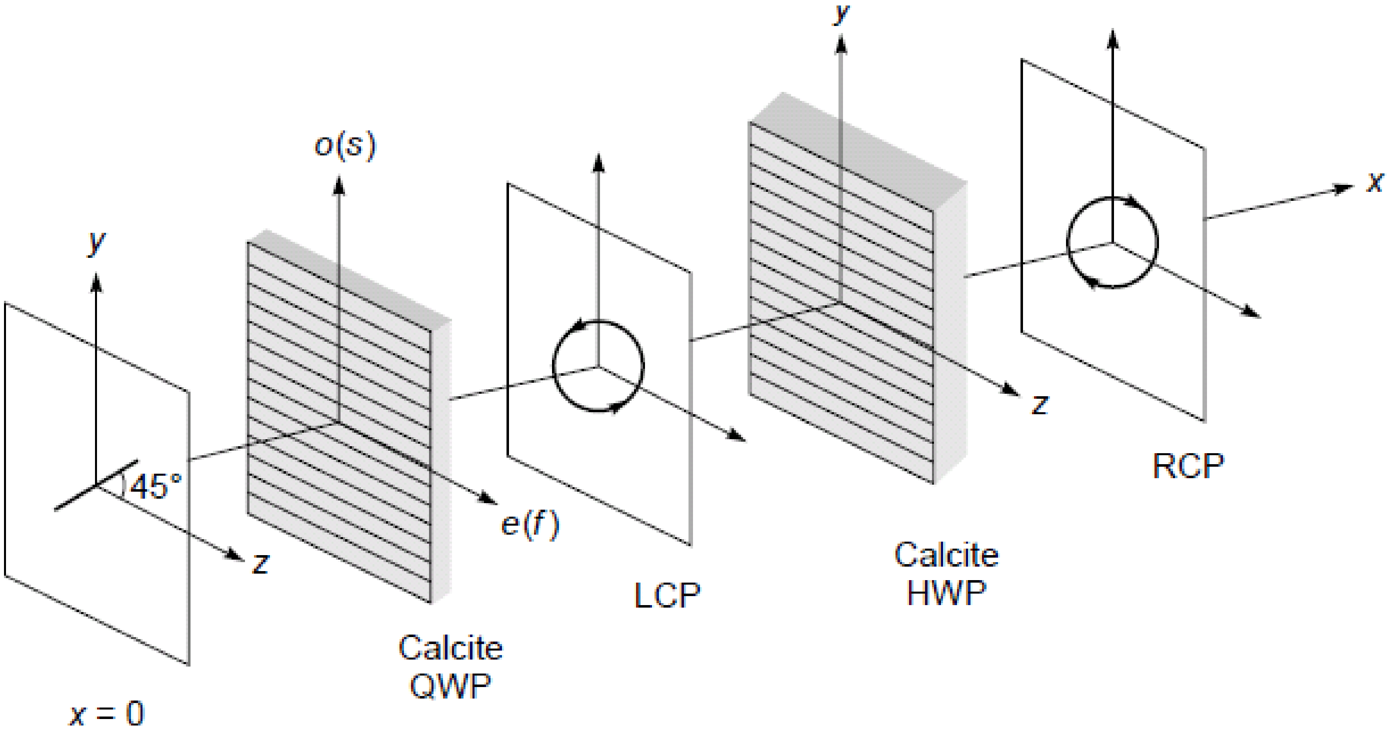
- If a linearly polarized beam is incident on a Polaroid & if Polaroid is rotated about z axis, then intensity of the emergent wave will vary according to **Malus' law**.

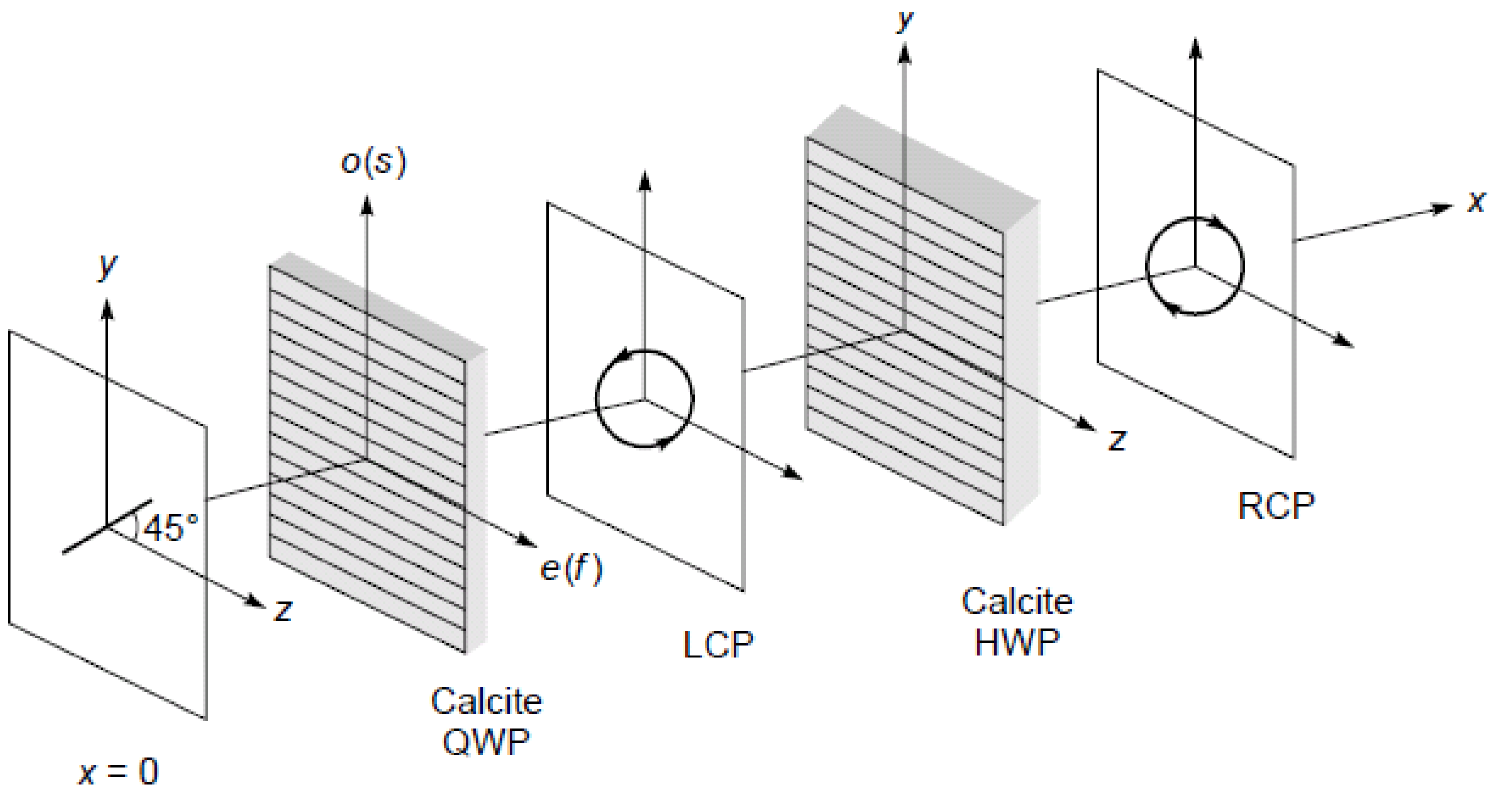
- If Polaroid P_2 is rotated in clockwise direction, then intensity will increase until pass axis is parallel to x axis; a further rotation will result in a decrease in intensity until the pass axis is parallel to the y axis, where intensity will be almost zero.

  If we further rotate it, it will pass through a maximum & again a minimum before it reaches its original position.

Interference of Polarized Light:
Quarter Wave Plates & Half Wave Plates

❖ Consider normal incidence of a plane polarized beam on a calcite crystal whose optic axis is parallel to surface of crystal. Assuming z axis to be along optic axis.





A linearly polarized light making an angle 45º with *z* axis gets converted to a LCP after propagating through a calcite QWP; further, a LCP gets converted to a RCP after propagating through a calcite HWP. Optic axis in QWP & HWP is along *z* direction. [*s*-slow, *f*-fast]

- If incident beam is *y*-polarized, beam will propagate as an *o*-wave & *e*-wave will be absent.

- If incident beam is *z*-polarized, beam will propagate as an *e*-wave & *o*-wave will be absent.

- For any other state of polarization of incident beam, both extraordinary & ordinary components will be present.

- For a negative crystal such as calcite $n_e < n_o$, & *e*-wave will travel faster than *o*-wave.

- Let electric vector (of amplitude E_0) associated with incident polarized beam make an angle Φ (arbitrary angle) with z axis.

- Such a beam can be assumed to be a superposition of two linearly polarized beams (vibrating in phase), polarized along y & z directions with amplitudes; $E_0 \sin\Phi$ & $E_0 \cos\Phi$, respectively.

- z component (whose amplitude is $E_0 \cos\Phi$) passes through as an *e*-beam propagating with wave velocity c/n_e.

- y component (whose amplitude is $E_0 \sin\Phi$) passes through as an *o*-beam propagating with wave velocity c/n_o.

- Since $n_e \neq n_o$, two beams will propagate with different velocities; as such, when they come out of crystal, they will not be in phase. Consequently, emergent beam (superposition of these two beams) will be, in general, **elliptically polarized**.

- Let plane $x = 0$ represent surface of crystal on which beam is incident. y & z components of incident beam can be written as

$$E_y = E_0 \sin\phi \cos(kx - \omega t)$$

$$E_z = E_0 \cos\phi \cos(kx - \omega t)$$

- where k (= ω/c) represents free space wave number. Thus, at $x = 0$,

$$E_y(x=0) = E_0 \sin\phi \cos\omega t$$

$$E_z(x=0) = E_0 \cos\phi \cos\omega t$$

- Inside crystal, two components will be given by

$$E_y = E_0 \sin\phi \cos(n_0 kx - \omega t) \qquad o - wave$$

$$E_z = E_0 \cos\phi \cos(n_e kx - \omega t) \qquad e - wave$$

- If thickness of crystal is d, then at emerging surface, we have

$$E_y = E_0 \sin\phi \cos(\omega t - \theta_o)$$

$$E_z = E_0 \cos\phi \cos(\omega t - \theta_e)$$

$$\theta_o = n_o kd, \quad \theta_e = n_e kd$$

- By appropriately choosing the instant t = 0, components may be rewritten as

$$E_y = E_0 \sin\phi \cos(\omega t - \theta)$$

$$E_z = E_0 \cos\phi \cos\omega t$$

- where $$\theta = \theta_o - \theta_e = kd(n_o - n_e) = \frac{\omega}{c}(n_o - n_e)d$$

  represents phase difference between *o*- & *e*-beams.

- If thickness *d* of crystal is such that θ = 2π, 4π, …. the emergent wave will have same state of polarization as incident beam.

- If thickness *d* of crystal is such that θ = π/2, the crystal is said to be a **quarter wave plate** (QWP) – a phase difference of π/2 implies a path difference of λ/4.

- If thickness *d* of crystal is such that θ = π, the crystal is said to be a **half wave plate** (HWP) – a phase difference of π implies a path difference of λ/2.

Example:

- Consider $\Phi = \pi/4$ & $\theta = \pi/2$, i.e., y & z components of incident wave have equal amplitudes, & crystal introduces a phase difference of $\pi/2$. For emergent wave we have

$$E_y = \frac{E_0}{\sqrt{2}} \sin \omega t \qquad\qquad E_z = \frac{E_0}{\sqrt{2}} \cos \omega t$$

which represents a circularly polarized wave because

$$E_y^2 + E_z^2 = \frac{E_0^2}{\sqrt{2}}$$

- To determine direction of rotation of electric vector, we note that at $t = 0$

$$E_y = 0 \qquad\qquad E_z = \frac{E_0}{\sqrt{2}}$$

& at $t = \Delta t$,

$$E_y \approx \frac{E_0}{\sqrt{2}}\omega\Delta t \qquad\qquad E_z \approx \frac{E_0}{\sqrt{2}}$$

These Eqs. show that as time increases, electric vector rotates in counterclockwise direction & hence beam is left circularly polarized.

- To introduce a phase difference of $\pi/2$, the thickness of crystal should have the value given by

$$d = \frac{c}{\omega(n_o - n_e)} \frac{\pi}{2} = \frac{1}{4} \frac{\lambda_0}{n_o - n_e}$$

where λ_0 is free space wavelength.

- For Calcite, n_0 = 1.65836, n_e = 1.48641 which correspond to λ_0 = 5893 Å at 18ºC. Substituting these values, we obtain

$$d = \frac{5893 \times 10^{-8}}{4 \times 0.17195} cm \approx 0.000857 \; mm$$

- Thus a Calcite QWP (at λ_0 = 5893 Å) will have a thickness of 0.000857 mm & will have its optic axis parallel to surface; such a QWP will introduce a phase difference of $\pi/2$ between *o*- & *e*-components at λ_0 = 5893 Å.

- If thickness is an odd multiple of above quantity, i.e., if

$$d = (2m+1)\frac{1}{4}\frac{\lambda_0}{n_o - n_e} \qquad m = 0,1,2\ldots.$$

then in Ex. considered above (i.e. when $\Phi = \pi/4$) it can be shown that emergent beam will be **left circularly polarized** for m = 0, 2, 4, … & **right circularly polarized** for m = 1, 3, 5, ….

y-polarized *o*-wave in Calcite has a smaller wave velocity (= c/n_o), & hence it is referred to as a slow wave *o*(*s*).

Similarly, *e*-wave is fast wave (in Calcite).

Analysis of Polarized Light

Different states of polarization:

- **Linearly polarized**
- **Circularly polarized**
- **Elliptically polarized**
- **Unpolarized**
- **Mixture of linearly polarized & unpolarized**
- **Mixture of circularly polarized & unpolarized**
- **Mixture of elliptically polarized & unpolarized**

To naked eyes, all states of polarization appear to be same.

Procedure for determining state of polarization of a light beam

- If we introduce a Polaroid in path of beam & rotate it about direction of propagation, then one of the following three possibilities can occur:

  1. If there is complete extinction at two positions of polarizer, then beam is linearly polarized.

Procedure for determining state of polarization of a light beam

2. **If there is no variation of intensity, then beam is unpolarized or circularly polarized or a mixture of unpolarized & circularly polarized light.**

   We now put a quarter wave plate on the path of beam followed by rotating Polaroid.

   If there is no variation of intensity, then incident beam is unpolarized.

   If there is complete extinction at two positions, then beam is circularly polarized (this is so because a quarter wave plate will transform a circularly polarized light into a linearly polarized light).

   If there is a variation of intensity (without complete extinction), then beam is a mixture of unpolarized & circularly polarized light.

Procedure for determining state of polarization of a light beam

3. **If there is a variation of intensity (without complete extinction), then beam is elliptically polarized or a mixture of linearly polarized & unpolarized or a mixture of elliptically polarized & unpolarized light.**

   We now put a quarter wave plate in front of Polaroid with its optic axis parallel to pass axis of Polaroid at position of maximum intensity.

   Elliptically Polarized light will transform to a linearly polarized light.

   Thus, if one obtains two positions of Polaroid where complete extinction occurs, then original beam is elliptically polarized.

   If complete extinction does not occur & position of maximum intensity occurs at same orientation as before, the beam is a mixture of unpolarized & linearly polarized light.

   Finally, if position of maximum intensity occurs at a different orientation of Polaroid, the beam is a mixture of elliptically polarized & unpolarized light.