Indian Institute of Technology Patna

Physics Department

PH 201: Tutorial I

1. White light falls normally on a transmission grating that contains 1000 lines per centimetre. At what angle will red light (λ_0 = 650 nm) emerge in the second order spectrum?

Sol^n: Grating Eq., $d\sin\theta = n\lambda$

$n = 2$

λ= 650 nm = 6.5×10^{-5}cm

d = 1/1,000 lines per cm = 0.001 cm per line

$\sin\theta = n\lambda/d = (2 \times 6.5 \times 10^{-5})/0.001 = 0.13$

$\theta = \sin^{-1}(0.13) = 7.4695 = 7.5^o$

2. Light having a frequency of 4.0×10^{14} Hz is incident on a grating formed with 10,000 lines per centimetre. What is the highest order spectrum that can be seen with this device?

Sol^n: Grating Eq., $d\sin\theta = n\lambda$

The largest value of n occurs when the sine function is equal to one, making the left side of the Eq. as large as possible.

$d = n\lambda$

$n = d/\lambda$

d = 1/10,000 lines per cm = 0.0001 cm per line

$\lambda = c/\upsilon = (3 \times 10^{10}$ cm$)/(4 \times 10^{14}$ Hz$) = 0.75 \times 10^{-4}$ cm

$n = d/\lambda$ =0.0001 cm per line/0.75×10^{-4} cm = 1.333

Only the 1storder spectrum is visible.

3. What is the total number of lines a grating must have in order just to separate the sodium doublet (λ_1 = 5896 Å, λ_2 = 5890 Å) in the third order?

Sol^n: $\Delta\lambda$ = (5896 – 5890)Å = 6 Å

$R = \lambda/\Delta\lambda = 5893/6 = 982.167$

$n = 3$

$R = nN$

$N = R/n = 982.167/3 = 327.389$

$N = 327$

4. Consider a plane wave incident normally on a long narrow slit of width 0.02 cm. The Fraunhofer diffraction pattern is observed on the focal plane of a lens whose focal length is 20 cm. Assuming λ = 6000Å determine the positions of the first and second minima. Also determine the positions of the first and second maxima.

Sol^n: b = 0.02 cm

λ = 6000 Å = 6.0×10^{-5} cm

f = 20 cm

$I = I_0\,(\sin^2\beta/\beta^2)$ $\qquad\qquad \beta = \pi b\sin\theta/\lambda$

Positions of 1st & 2nd minima.

$$I = 0, \quad when \quad \beta = m\pi, \quad m \neq 0$$

$$When \quad \beta = 0, \quad \frac{\sin\beta}{\beta} = 1 \qquad \Rightarrow I = I_0$$

$$\Rightarrow b\sin\theta = m\lambda; \qquad m = \pm 1, \pm 2, \pm 3, \ldots (Minima)$$

$$I_{\min}\ \theta = \pm\sin^{-1}(\lambda/b)$$

$$II_{\min}\ \theta = \pm\sin^{-1}(2\lambda/b)$$

$I_{\min}\ \theta = 0.17^{o}$ $\qquad\qquad II_{\min}\ \theta = 0.34^{o}$

Positions of 1st & 2nd maxima.

$\tan\beta = \beta$ Maxima

Root β = 0 corresponds to central maximum & other roots can be found by determining points of intersections of curves $y = \beta$&$y = tan\beta$. Intersections occur at $\beta = 1.43\pi$, $\beta = 2.46\pi$, etc. & are known as 1st maximum, 2nd maximum, etc.

$\beta = \pi b sin\theta/\lambda$

I_{max}; $1.43\pi = \pi b sin\theta/\lambda$

$I_{max}\theta = sin^{-1}(1.43 \times 6.0 \times 10^{-5}/0.02) = 0.24^o$

II_{max}; $2.46\pi = \pi b sin\theta/\lambda$

$II_{max}\theta = sin^{-1}(2.46 \times 6.0 \times 10^{-5}/0.02) = 0.42^o$

5. Consider a diffraction grating with 8000 lines per inch and assume that light of wavelength 5460 Å and 5460.072 Å illuminates the grating over a region of 2 inch.
    a. Calculate the number of orders in the diffracted spectrum.
    b. Calculate the dispersion in the third order.
    c. In which diffraction orders will the two wavelength components be resolved?

Sol^n: d = 8000 lines per inch = $2.54/8000 = 3.175 \times 10^{-4}$cm

λ_1 = 5460 Å; λ_2 = 5460.072 Å

$\lambda = (\lambda_1 + \lambda_2)/2 = 5.46 \times 10^{-5}$cm

(a) Grating Eq., $dsin\theta = n\lambda$

$d = n\lambda$

$n = d/\lambda = (3.175 \times 10^{-4}\text{cm})/5.46 \times 10^{-5}\text{cm} = 5$

$n = 5$

(b) Dispersion

$$\frac{\nabla\theta}{\nabla\lambda} = \frac{n}{d\cos\theta}$$

$$\sin\theta = \frac{n\lambda}{d} = \frac{3\times 5.46\times 10^{-5}}{3.175\times 10^{-4}} = 0.52$$

$$\theta = 31.33^\circ$$

$$\cos\theta = 0.85$$

$$n = 3$$

$$\frac{\nabla\theta}{\nabla\lambda} = \frac{3}{3.175\times 10^{-4}\times 0.85} \approx 1.06\ \mu m^{-1}$$

(c) Resolving power

$$R = \frac{\lambda}{\nabla\lambda} = \frac{5460.036\,\mathring{A}}{0.072\,\mathring{A}} = 8000$$

$$R = nN$$

$$N = 2\times 8000 = 16000$$

$$n = \frac{R}{N} = \frac{80000}{16000} = 5$$

The two wavelengths will be resolved only in 5^{th} order.

6. Consider a plane wave of wavelength 6×10^{-5} cm incident normally on a circular aperture of radius 0.01 cm. Calculate the positions of the brightest and the darkest points on the axis.

Sol^n: Fresnel diffraction due to a circular aperture.

$a = 0.01$ cm

$\lambda = 6.0 \times 10^{-5}$ cm

Fresnel half-period zones, radius, $a_n = \sqrt{\lambda d}$

$(a_n)^2 = n\lambda d$

As a increases, the intensity at point P will also increase until the circularaperture contains the 1^{st}half-period zone.

$a^2 = \lambda d$

The brightest point would be at a distance,

$a^2 = (2n+ 1)\lambda d$

For $n = 0$, $a^2 = \lambda d$, $d = a^2/\lambda$, $d = 0.0001/6.0 \times 10^{-5} = 10/6 = 1.66$ cm

For $n = 1$, $a^2 = 3\lambda d$, $d = a^2/3\lambda$, $d = 0.0001/3 \times 6.0 \times 10^{-5} = 10/18 = 0.56$ cm

For $n = 2$, $a^2 = 5\lambda d$, $d = a^2/5\lambda$, $d = 0.0001/3 \times 6.0 \times 10^{-5} = 10/30 = 0.33$ cm

The darkest point would be at a distance,

$$a^2 = 2n\lambda d$$

For $n = 1$, $a^2 = 2\lambda d$, $d = a^2/2\lambda$, $(0.01)^2/2 \times 6.0 \times 10^{-5} = 5/6 = 0.83$ cm

For $n = 2$, $a^2 = 4\lambda d$, $d = a^2/4\lambda$, $(0.01)^2/4 \times 6.0 \times 10^{-5} = 5/12 = 0.42$ cm

For $n = 3$, $a^2 = 6\lambda d$, $d = a^2/6\lambda$, $(0.01)^2/6 \times 6.0 \times 10^{-5} = 5/24 = 0.21$ cm

7. The output of a He-Ne laser (λ = 6328 Å) can be assumed to be Gaussian with plane phase front. For w_0 = 1 mm and w_0 = 0.2 mm, calculate the beam diameter at z = 20 m. [**Ans**. 2ω = 0.83 cm & 4.0 cm]

Sol^n.:

$$\gamma = \frac{\lambda z}{\pi w_0^2}$$

$$w(z) = w_0(1+\gamma^2)^{1/2} = w_0\left(1+\frac{\lambda^2 z^2}{\pi^2 w_0^4}\right)^{1/2}$$

$$R(z) \equiv z\left(1+\frac{1}{\gamma^2}\right) = z\left(1+\frac{\pi^2 w_0^4}{\lambda^2 z^2}\right)$$

8. A Gaussian beam is coming out of a laser. Assume λ = 6000 Å and that at z = 0, the beam width is 1 mm and the phase front is plane. After traversing 10 m through vacuum, what will be (a) the beam width and (b) the radius of curvature of the phase front? [**Ans.**2ω = 0.77 cm; R(z) = 1017 cm]

Sol^n.:

$$\gamma = \frac{\lambda z}{\pi w_0^2}$$

$$w(z) = w_0(1+\gamma^2)^{1/2} = w_0\left(1+\frac{\lambda^2 z^2}{\pi^2 w_0^4}\right)^{1/2}$$

$$R(z) \equiv z\left(1+\frac{1}{\gamma^2}\right) = z\left(1+\frac{\pi^2 w_0^4}{\lambda^2 z^2}\right)$$