भारतीय प्रौद्योगिकी संस्थान पटना

INDIAN INSTITUTE OF TECHNOLOGY PATNA

PH103 (Physics-I) End-Semester Exam (November 20, 2017)

Important instructions: All Questions are compulsory. Questions 1-4 carry equal marks (8 each) and Questions 5-10 carry equal marks (3 each). The alphabets corresponding to your choices for Questions 5-10 must be written clearly at a common place. Please show the necessary rough work as appropriate separately (preferably in last four pages of the answer-script). Symbols have their usual meaning as per usage during the course lectures. Please follow all instructions carefully to avoid any penalty.

1. The wave function for a particle is given by:
$\psi(x) = \begin{cases} 2\alpha\sqrt{\alpha}xe^{-\alpha x}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$
Obtain the value of Δx. Show all the steps involved in the calculation.

2. Obtain the S-matrix for a quantum mechanical object of mass m and energy E ($E > 0$) in an attractive delta function potential $V(x) = -\xi\delta(x)$. Show all the steps involved in the calculation.

3. A particle of mass m is in the state
$\Psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]}$, where, A and a are positive real constants.
Obtain A and find the potential energy function $V(x)$ for which $\Psi(x,t)$ satisfies the corresponding time dependent Schrödinger equation. Show all the steps involved in the calculation.

4. A quantum mechanical system in a harmonic oscillator potential is prepared in the state. $|\psi> = \frac{1+i}{2}|3> + \frac{1-i}{2}|4>$.
Obtain the expectation value of $<x>$ in this state. Show all the steps involved in the calculation.

5. The values of [x, p_x], [a, $a^\dagger$], [N, a], [y, p_x] respectively, are:
(a) i$\hbar$, 1, -a, 0 (b) i$\hbar$, -1, a, 0 (c) i$\hbar$, 1, $a^\dagger$, 0 (d) i$\hbar$, 1, a, i$\hbar$
(e) -i$\hbar$, 1, a, 0 (f) -i$\hbar$, -1, a, 0 (f) -i$\hbar$, -1, -a, 0

6. Consider the infinite potential box in 3-d (as discussed and solved in class) given by:
$V(x,y,z) = \begin{cases} 0, & \text{if } 0 \leq x \leq a; 0 \leq y \leq a; 0 \leq z \leq a \\ \infty, & \text{otherwise} \end{cases}$. The degeneracy of 4^{th} and 6^{th} excited states respectively, are:
(a) 2, 3 (b) 3, 6 (c) 3, 3 (d) 1, 6 (e) 4, 5 (f) 1, 1 (g) 1, 3

7. The de Broglie wavelengths of a proton of kinetic energy 100 MeV and a bullet of mass 0.1 kg traveling at 1 km/s, respectively are:
(a) 28.57 fm and 6.626×10^{-36} m (b) 2.857 fm and 6.626×10^{-36} m
(c) 2.857 $\mathring{A}$ and 6.626×10^{-36} m (d) 2.857 pm and 6.626×10^{-36} m
(e) 2.857 fm and 6.626×10^{-33} m (f) 28.57 fm and 6.626×10^{-33} m
(g) 2.857 $\mathring{A}$ and 6.626×10^{-33} m

8. When light of wavelength 110 nm falls on a metal, electrons with kinetic energy 7.154 eV are emitted. The work function of the metal, the cut-off frequency and the cut-off wavelength are respectively,
(a) 4.14 eV, 10^{15}Hz, 300 nm (b) 2.07 eV, 5×10^{14}Hz, 600 nm
(c) 8.28 eV, 2×10^{15}Hz, 150 nm (d) 1.656 eV, 4×10^{14}Hz, 750 nm
(e) 3.312 eV, 8×10^{14}Hz, 375 nm (f) 6.21 eV, 1.5×10^{15}Hz, 200 nm

9. A dumb-bell shaped rigid body with two masses (m each) connected by a rigid mass-less rod of length 2a is rotating about an axis passing through its center with angular frequesncy ω, such that the mass-less rigid rod makes an angle ϕ with the axis. The magnitude of torque involved in the process is given by:
(a) $\mathrm{ma}^2\omega^2\sin(\phi)$ (b) $\mathrm{ma}^2\omega^2\sin(\phi)\cos(\phi)$
(c) $\mathrm{ma}^2\omega^2\sin(2\phi)$ (d) $\mathrm{ma}^2\omega^2\cos(\phi)$
(e) $\mathrm{ma}^2\omega^2\cos(2\phi)$ (f) $2\mathrm{ma}^2\omega^2\sin(2\phi)$

10. $P_{ab}(t)$ is the probability of finding a quantum mechanical object in the range $a < x < b$ at a time t and J is the probability current as discussed in class. The value of $\frac{dP_{ab}}{dt}$ is
(a) $J(b,t) - J(a,t)$ (b) $J(a,t) - J(b,t)$ (c) $J(a,t) + J(b,t)$
(d) $J^2(a,t) - J^2(b,t)$ (e) $J^2(b,t) - J^2(a,t)$ (f) none of the above

End of paper

Physical Constants
Electronic charge, $e = 1.6 \times 10^{-19}$ Coulomb.
Planck's constant, $h = 6.626 \times 10^{-34}$ J-s; $\hbar = \frac{h}{2\pi}$.
$1 \text{ fm} = 1 \times 10^{-15}$ m; $1 \text{ pm} = 1 \times 10^{-12}$ m $1\ \mathring{A} = 1 \times 10^{-10}$ m.
$1 \text{ MeV} = 10^6$ eV.