PH 201

OPTICS & LASERS

Lecture_Lasers_5

Conditions for Producing a Laser

- ❖ **Population inversion (necessary condition)**
- ❖ **Saturation intensity (sufficient condition)**

Saturation Intensity

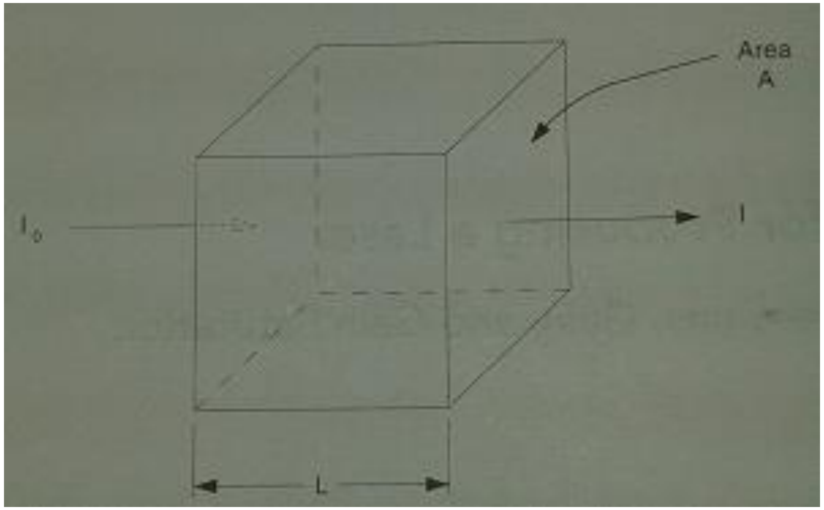
$$I = I_0 e^{\sigma_{ul}[N_u - (g_u/g_l)N_l]z}$$

This Eq. suggests possibility of rapid growth of a beam as it passes through a medium of length *z* if exponential coefficient is positive & of sufficient magnitude.

Determine requirements for magnitude of that coefficient for various physical geometries.

Consider effect of saturation of laser beam as it grows exponentially within a gain medium.

Assume there is a medium in which population inversion exists, & assume that value of gain is large enough to provide significant amplification.



Light of intensity I_0 incident upon an absorptive material of length L & area A.

If we increase length of medium to allow intensity (I) to increase exponentially,

$$I = I_0 e^{\sigma_{ul}[N_u - (g_u/g_l)N_l]z}$$

then the beam could eventually reach an intensity (at some specific length z) such that energy stored in upper level is not sufficient to satisfy exponential growth demands of beam.

Hence, there must be a limiting expression to estimate intensity at which this saturation process occurs.

Length (z) of medium at which saturation effect occurs is referred to as saturation length L_{sat}.

Intensity achieved by beam when $z = L_{sat}$ is referred to as saturation intensity I_{sat}.

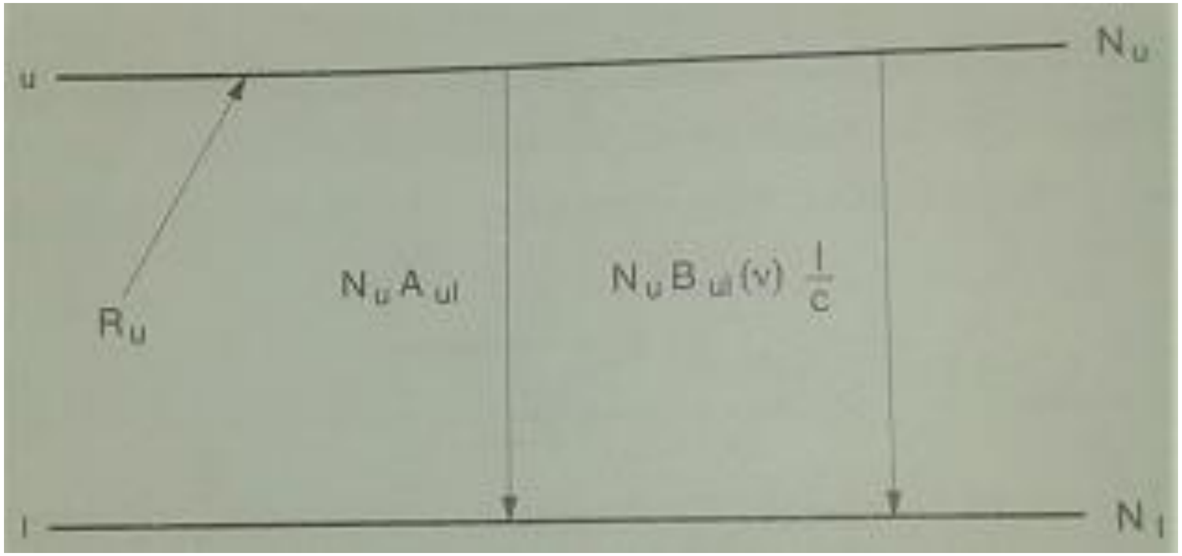
Consider levels u & l in a case where a steady state population density N_u exists in upper level. Population inversion exists between u & l. Assume there is a pumping flux R_u (no. of excitations per unit volume per unit time) that is populating u (which has a lifetime τ_u) & would therefore decay at a rate of $1/\tau_u$.

Stimulated emission will reduce population, $\Delta E = h\upsilon_{ul}$

Stimulated emission rate from u to l at frequency v,

$$B_{ul}(\upsilon)\Delta\upsilon.u(\upsilon) = B_{ul}(\upsilon)I(\upsilon)\Delta\upsilon / c = B_{ul}(\upsilon)I / c$$

Using $u(\upsilon) = I(\upsilon) / c$



Terms associated with flux input & decay from energy level *u*.

Since we have assumed that level *u* is in a steady state equilibrium, so we can write a rate Eq. taking into account all of population changes affecting level *u* & equate it to zero. Steady state implies that there are no net changes in N_u.

$$\frac{dN_u}{dt} = R_u - N_u\left[\left(\frac{1}{\tau_u}\right) + \frac{B_{ul}(\upsilon)I}{c}\right] = 0$$

$$N_u = \frac{R_u}{\dfrac{1}{\tau_u} + \dfrac{B_{ul}(\upsilon)I}{c}}$$

When $I = 0$, value of population density,

$$\boxed{N_u = R_u \tau_u}$$

Different lasers have different lifetime (τ_u), they vary over many orders of magnitude. For upper laser levels most visible lasers, value of τ_u ranges from 10^{-3} to 10^{-9} s.

Value of $1/\tau_u$ would be quite large ranging from 10^3/s to 10^9/s.

Examine following expressions,

$$N_u = \frac{R_u}{\frac{1}{\tau_u} + \frac{B_{ul}(\upsilon)I}{c}} \qquad I = I_0 e^{\sigma_{ul}[N_u - (g_u/g_l)N_l]z}$$

As *I* increases the beam traverses a long gain medium, a value of *z* (*z* = L_{sat}) would eventually be reached such that the intensity *I*, & consequently stimulated emission term in denominator could eventually become as large as the term associated with level lifetime.

$$\frac{1}{\tau_u} = \frac{B_{ul}(\upsilon)I}{c} \qquad N_u = \frac{R_u \tau_u}{2}$$

When saturation occurs, population of level *u* would decrease by a factor of 2 owing to stimulated emission.

Exponential growth factor, $\sigma_{ul}(\upsilon)\Delta N_{ul} z$

for *z* = L_{sat} (gain) would also decrease by approximately a factor of 2.

Further increase in I would further decrease gain in regions where beam further propagates into medium.

Thus, we define I_{sat} as that intensity at which stimulated emission rate downward equals normal derivative decay rate:

$$\frac{I_{sat} B_{ul}(\upsilon)\eta}{c} = A_{ul} = \frac{1}{\tau_u} \qquad I_{sat} = \frac{c}{B_{ul}(\upsilon)\eta\tau_u}$$

Using Einstein relationship between $B_{ul}(v)$ & $A_{ul}(v)$,

$$I_{sat} = \frac{c8\pi h\upsilon_{ul}^3\eta^3}{c^3 A_{ul}(\upsilon)\eta\tau_u} = \frac{8\pi\upsilon_{ul}^2\eta^2}{c^2 A_{ul}(\upsilon)}\frac{h\upsilon_{ul}}{\tau_u}$$

Using following expression, $\sigma_{ul}^H(\upsilon) = \dfrac{c^2}{8\pi\eta^2\upsilon^2}A_{ul}(\upsilon)$

$$\boxed{I_{sat} = \frac{h\upsilon_{ul}}{\sigma_{ul}^H(\upsilon)\tau_u}}$$

Development & Growth of a Laser Beam

Growth of Beam for a Gain Medium with Homogeneous Broadening

Determine value of gain at which the beam would reach saturation intensity?

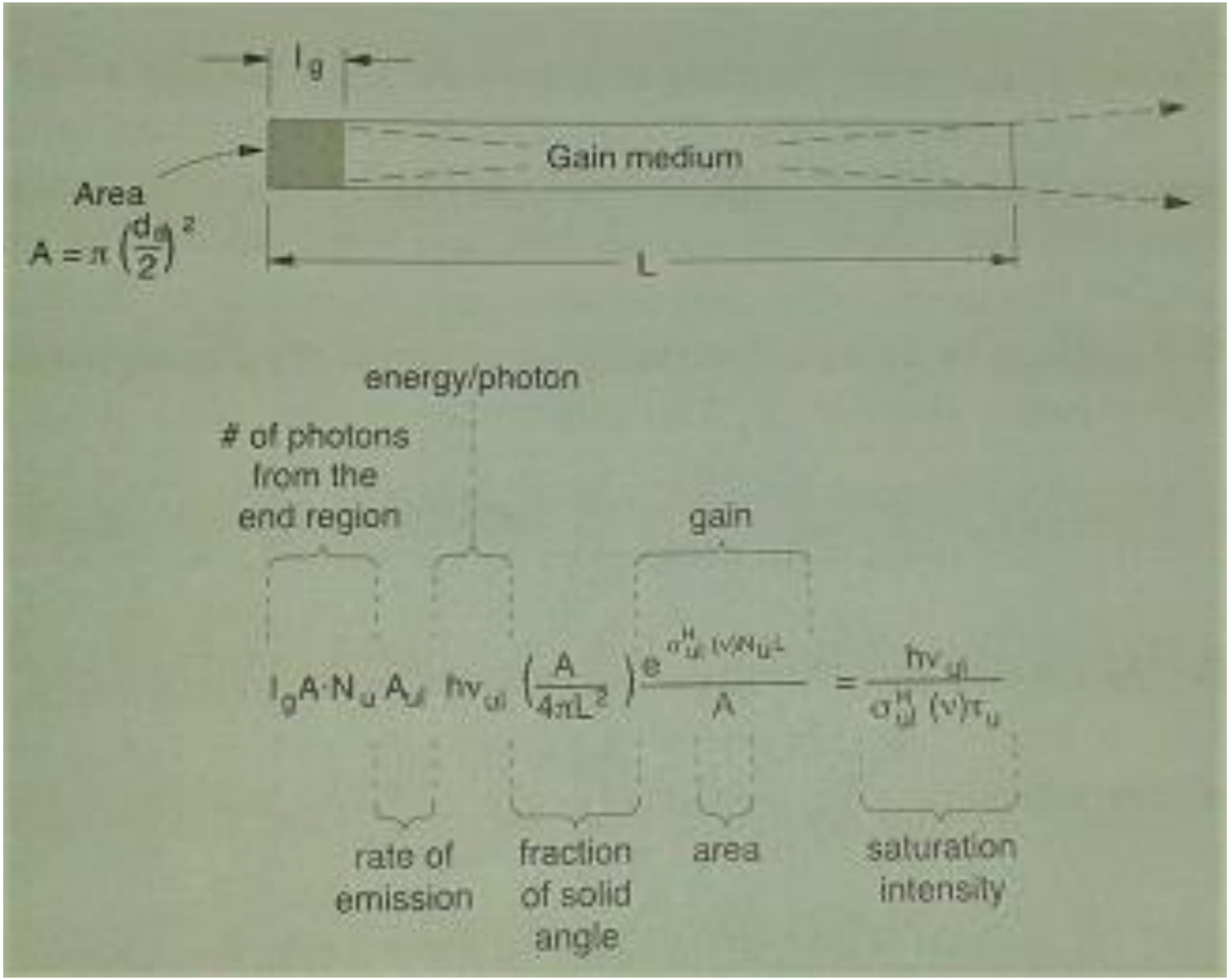
$$\sigma_{ul}(\upsilon)\Delta N_{ul} z = ?$$

Above I_{sat} laser beam can no longer grow exponentially. Here we will obtain an approximate range of values that are dependent on length & width of gain medium.

Consider a cylindrical gain medium that has a length L, a cross-sectional area A, & a diameter d_a. Within that gain medium we assume that a population inversion exists.

Assume that upper level u is instantaneously populated by some pumping process to achieve a density N_u, & radiative decay rate from u to l at frequency v_{ul} is A_{ul}.

$$\Delta N_{ul} \cong N_u$$



Growth & development of a laser beam from an elongated gain medium

Consider beam is starting at one end of medium in a region of length l_g. We define l_g as one gain length such that

$$\sigma_{ul}^{H}(\upsilon)\Delta N_{ul} l_g \cong \sigma_{ul}^{H}(\upsilon) N_u l_g = 1$$

Here, $l_g < L$

We assume that atoms in level *u* within that region are radiating at a rate A_{ul} & with an energy per photon hv_{ul}. Some of these photons are emitted in elongated direction of amplifier & would therefore be enhanced by stimulated emission as they transit through length *L* of medium. Thus, a beam would evolve as radiation propagates down the length of medium & intensity grows exponentially.

There will be similar condition when beam arrives at other end of medium for a beam travels in opposite direction.

Calculations would be same for both directions.

Energy radiated per unit time into a 4*π* solid angle from within volume ($A.l_g$),

$$N_u (A.l_g) A_{ul} h\upsilon_{ul}$$

Fraction of energy radiating within a solid angle *dΩ* that would reach opposite end of medium: $d\Omega / 4\pi$

Fraction of total solid angle: $\frac{A}{L^2}\left(\frac{1}{4\pi}\right) = \frac{A}{4\pi L^2}$

Energy radiated from that volume element per unit time is amplified by an amount,

$$e^{\sigma_{ul}^{H}(\upsilon) N_u L}$$

by the time it reaches other end of medium. Energy per unit time divided by area would give intensity, *I*. Equating this intensity to saturation intensity I_{sat}.

$$(N_u . A . l_g) A_{ul} h\upsilon_{ul} \frac{A}{4\pi L^2} \frac{e^{\sigma_{ul}^{H}(\upsilon) N_u L}}{A} = I_{sat} = \frac{h\upsilon_{ul}}{\sigma_{ul}^{H}(\upsilon)\tau_u}$$

Decay time: $\tau_u = \frac{1}{A_{ul}}$

$$(N_u . A . l_g) A_{ul} h\upsilon_{ul} \frac{A}{4\pi L^2} \frac{e^{\sigma_{ul}^{H}(\upsilon) N_u L}}{A} = \frac{h\upsilon_{ul} A_{ul}}{\sigma_{ul}^{H}(\upsilon)}$$

$$N_u \left[\pi \left(\frac{d_a}{2} \right)^2 \right] . \left(\frac{1}{\sigma_{ul}^{H}(\upsilon) N_u} \right) . \frac{1}{4\pi L^2} e^{\sigma_{ul}^{H}(\upsilon) N_u L} = \frac{1}{\sigma_{ul}^{H}(\upsilon)}$$

where

$$A = \pi \left(\frac{d_a}{2} \right)^2 \quad \& \quad l_g = \left(\frac{1}{\sigma_{ul}^{H}(\upsilon) N_u} \right)$$

Further simplification,

$$\boxed{e^{\sigma_{ul}^{H}(\upsilon) N_u} = 16 \left(\frac{L_{sat}}{d_a} \right)^2}$$

Choosing a length l_g leads to simplified result.

If a very much shorter region than l_g were chosen then a significant amount of energy that might eventually be amplified would be left out of the calculation.

If a significantly longer region than l_g were chosen, then a much shorter exponential growth length than L_{sat} would have to be used & the beam would not gain as much energy through amplification.

Minor changes do not alter results significantly.