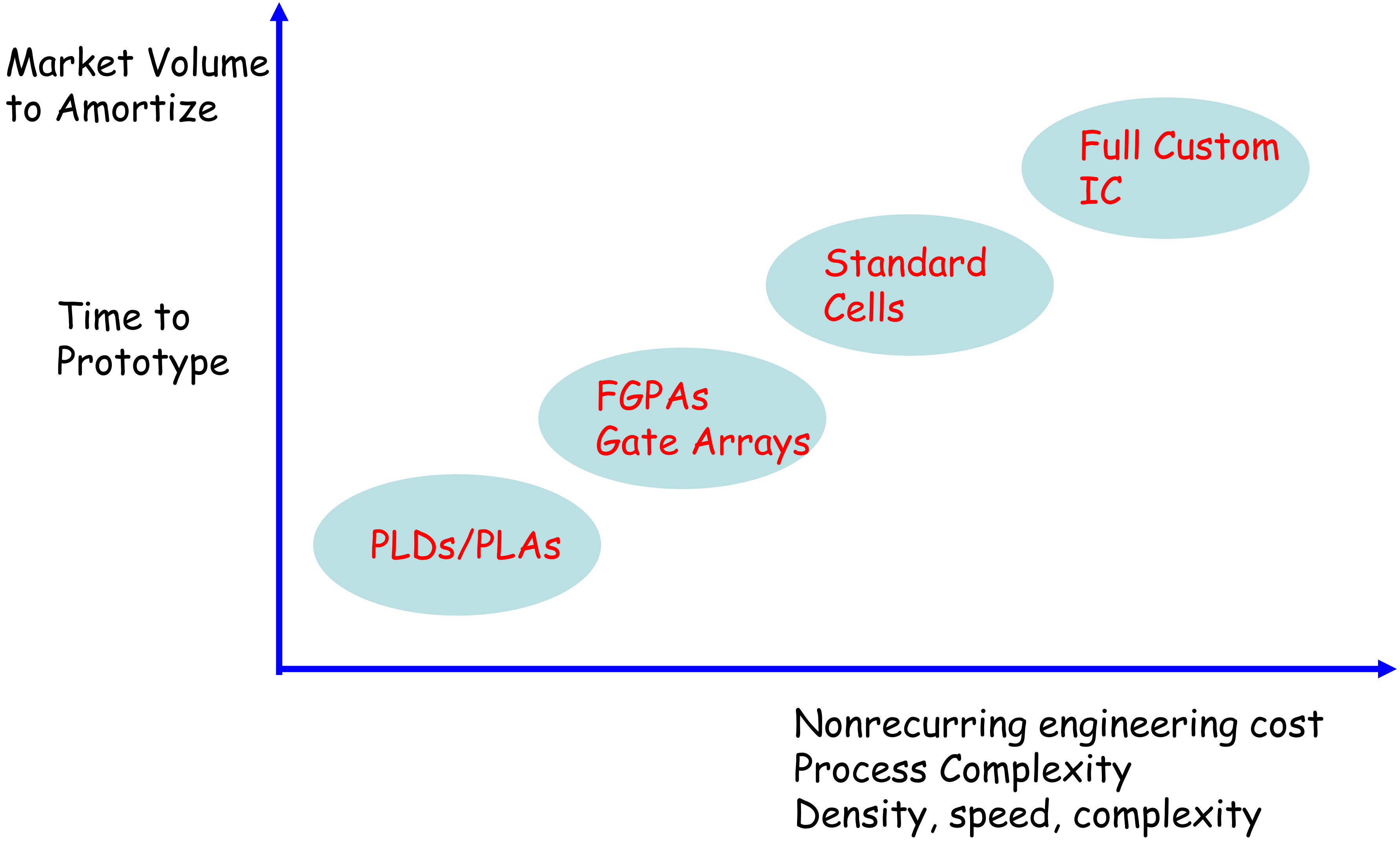
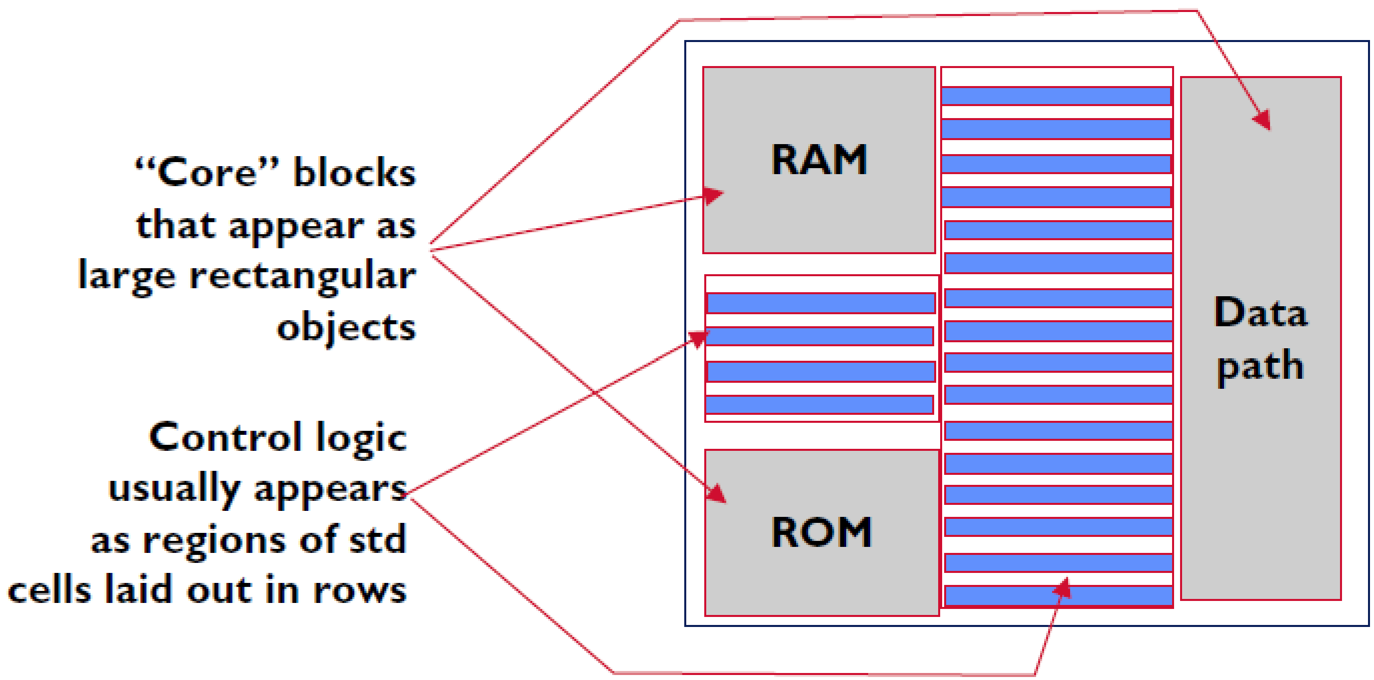
# FPGA - Field Programmable Gate Array

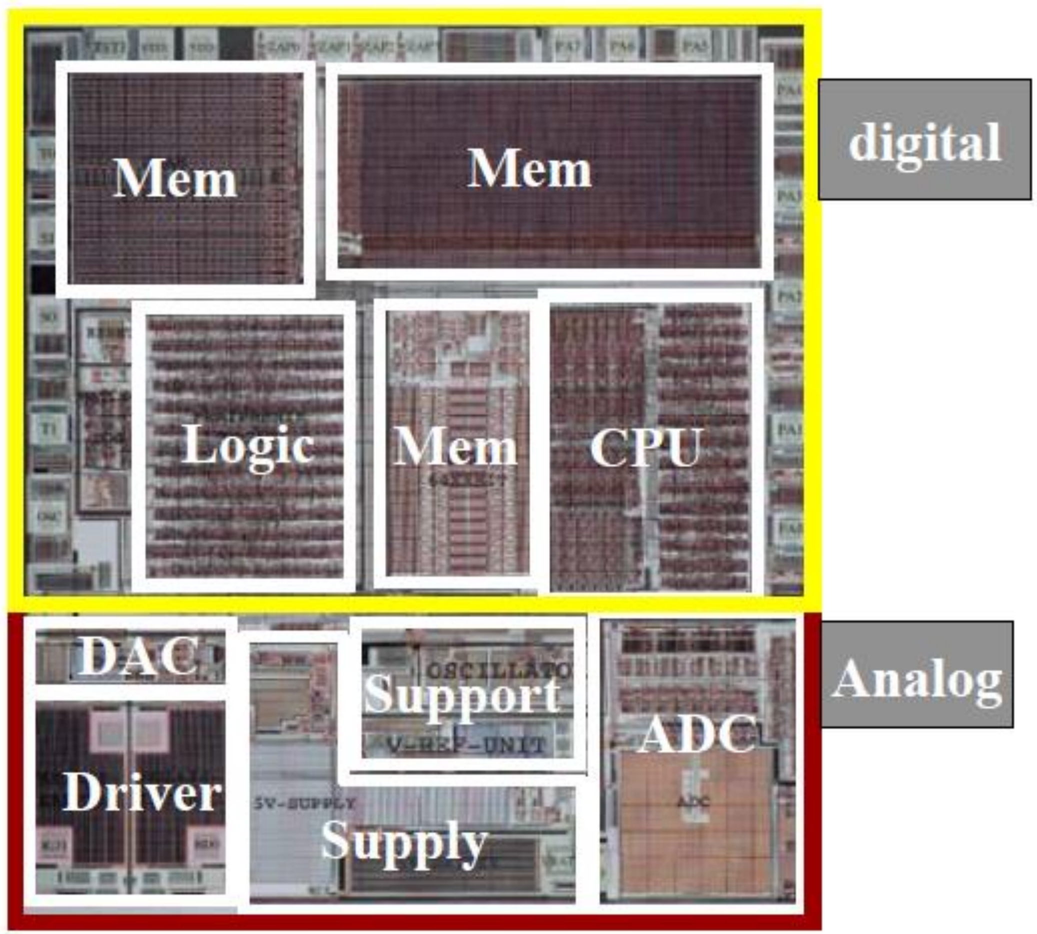
# Alternative Technologies for IC Implementation



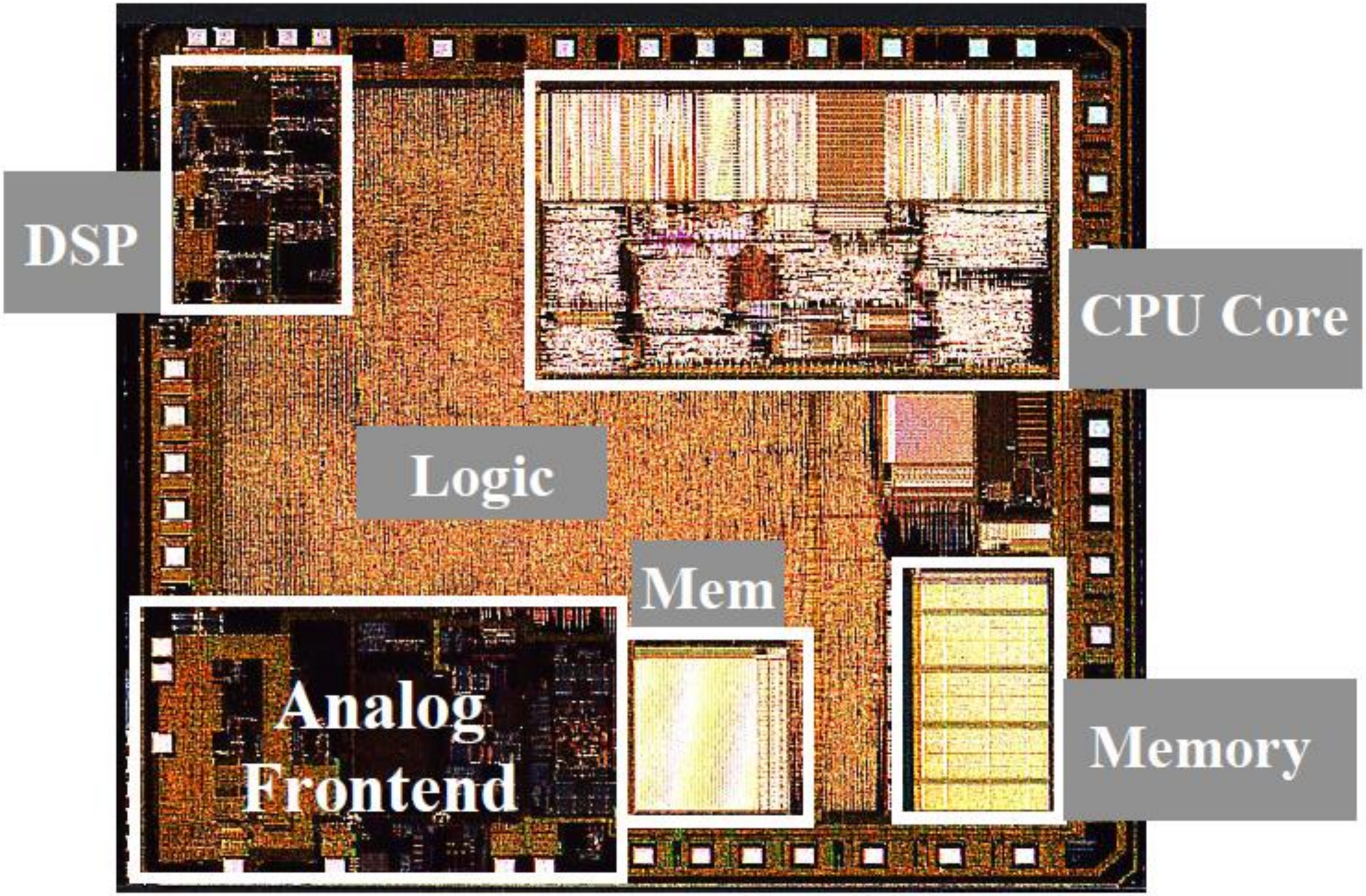
# System on Chip



# System on chip – example(automotive)

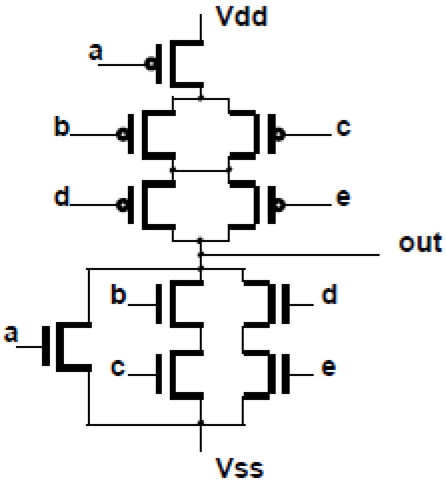


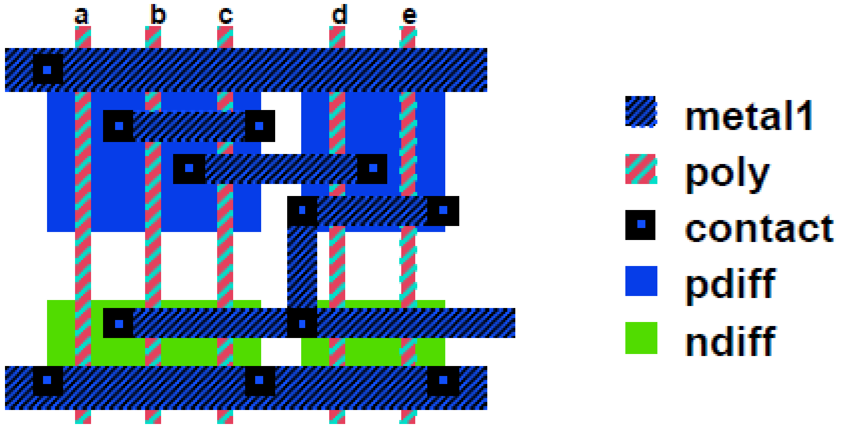
# System on chip –example(Telecom)



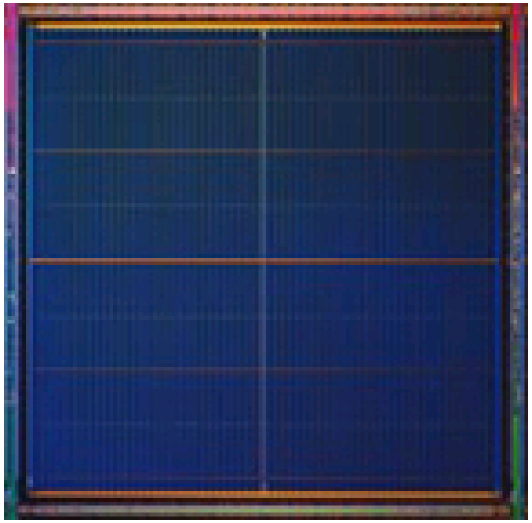

Courtesy Frank Op't Eynde, Alcatel

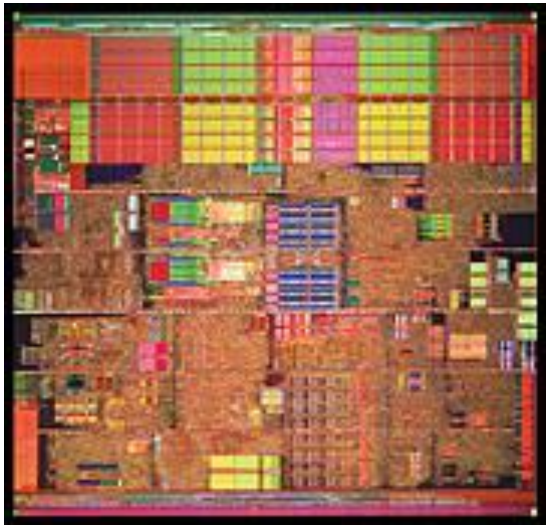
# Full custom- for Implementation



# Die Photos: Vertex vs. Pentium IV





- FGPA Vertex chip looks remarkably well structured
  - Very dense, very regular structure
- Full Custom Pentium chip somewhat more random in structure
  - Large on-chip memories (caches) are visible

# Shannon’s Expansion Theorem

# Review

| $x_2$ | $x_1$ | $x_0$ | $F(x_2, x_1, x_0)$ |
|---|---|---|---|
| 0 | 0 | 0 | $0 \;=\; F(0,0,0)$ |
| 0 | 0 | 1 | $1 \;=\; F(0,0,1)$ |
| 0 | 1 | 0 | $0 \;=\; F(0,1,0)$ |
| 0 | 1 | 1 | $0 \;=\; F(0,1,1)$ |
| 1 | 0 | 0 | $1 \;=\; F(1,0,0)$ |
| 1 | 0 | 1 | $1 \;=\; F(1,0,1)$ |
| 1 | 1 | 0 | $1 \;=\; F(1,1,0)$ |
| 1 | 1 | 1 | $0 \;=\; F(1,1,1)$ |

Shannon showed that the logic expression for the above F can be obtained by the following expansion

$$\begin{aligned} F(x_2, x_1, x_0) \;\; = \;\; & \overline{x_2} \cdot \overline{x_1} \cdot \overline{x_0} \cdot F(0,0,0) + \overline{x_2} \cdot \overline{x_1} \cdot x_0 \cdot F(0,0,1) + \overline{x_2} \cdot x_1 \cdot \overline{x_0} \cdot F(0,1,0) \\ & + \overline{x_2} \cdot x_1 \cdot x_0 \cdot F(0,1,1) + x_2 \cdot \overline{x_1} \cdot \overline{x_0} \cdot F(1,0,0) + x_2 \cdot \overline{x_1} \cdot x_0 \cdot F(1,0,1) \\ & + x_2 \cdot x_1 \cdot \overline{x_0} \cdot F(1,1,0) + x_2 \cdot x_1 \cdot x_0 \cdot F(1,1,1) \end{aligned}$$

$$F(x_2, x_1, x_0) = \overline{x_2} \cdot \overline{x_1} \cdot x_0 + x_2 \cdot \overline{x_1} \cdot \overline{x_0} \cdot + x_2 \cdot \overline{x_1} \cdot x_0 \cdot + x_2 \cdot x_1 \cdot \overline{x_0} = \sum m(1,4,5,6)$$

# POS form

$$
\begin{aligned}
F(x_2, x_1, x_0) \;=\; & \left(x_2 + x_1 + x_0 + F(0,0,0)\right) \cdot \left(x_2 + x_1 + \overline{x_0} + F(0,0,1)\right) \cdot \left(x_2 + \overline{x_1} + x_0 + F(0,1,0)\right) \\
& \cdot \left(x_2 + \overline{x_1} + \overline{x_0} + F(0,1,1)\right) \cdot \left(\overline{x_2} + x_1 + x_0 + F(1,0,0)\right) \cdot \left(\overline{x_2} + x_1 + \overline{x_0} + F(1,0,1)\right) \\
& \cdot \left(\overline{x_2} + \overline{x_1} + x_0 + F(1,1,0)\right) \cdot \left(\overline{x_2} + \overline{x_1} + \overline{x_0} + F(1,1,1)\right)
\end{aligned}
$$

$$
F(x_2, x_1, x_0) = (x_2 + x_1 + x_0) \cdot (x_2 + \overline{x_1} + x_0) \cdot (x_2 + \overline{x_1} + \overline{x_0}) \cdot (\overline{x_2} + \overline{x_1} + \overline{x_0}) = \prod M(0,2,3,7)
$$

# Shannon's Expansion Theorem

- C. E. Shannon, "A Symbolic Analysis of Relay and Switching Circuits," *Trans. AIEE*, vol. 57, pp. 713-723, 1938.
- Consider:
    - Boolean variables, X1, X2, . . . , Xn
    - Boolean function, F(X1, X2, . . . , Xn)
- Then $F = X_i\ F(Xi{=}1) + X_i'\ F(Xi{=}0)$
- Where
    - $X_i'$ is complement of $X_i$
    - Cofactors, F(Xi=j) = F(X1, X2, . . . , Xi=j, . . . , Xn), j = 0 or 1

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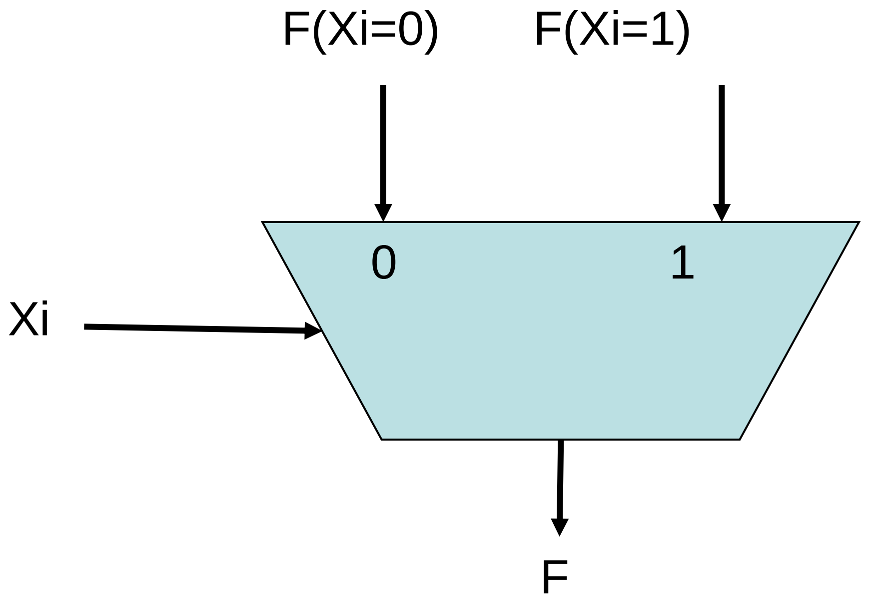
# Theorem

(1) $F = X_i \cdot F(X_i=1) + X_i' \cdot F(X_i=0) \quad \forall\, i=1,2,3,\ldots n$

(2) $F = (X_i + F(X_i=0)) \cdot (X_i' + F(X_i=1)) \quad \forall\, i=1,2,3,\ldots n$



# Example

$$\begin{aligned}
f(a,b,c,d) &= ab + b'cd + acd \\
f(a,1,c,d) &\overset{b=1}{=} a + acd = \underbrace{a + acd}_{covering} = a \\
f(a,0,c,d) &\overset{b=0}{=} cd + acd = \underbrace{cd + acd}_{covering} = cd \\
f(a,b,c,d) &= b \cdot f(a,1,c,d) + b' \cdot f(a,0,c,d) \quad \text{Shannon's expansion} \\
&= b \cdot a + b' \cdot cd \\
&= ab + b'cd
\end{aligned}$$

# Example-POS

$$
\begin{aligned}
f(a,b,c,d) &= (a+b+c)(a'+d)(b+c+d) \\
f(1,b,c,d) &\overset{a=1}{=} (1+b+c)(0+d)(b+c+d) = \underbrace{(d)(a+c+d)}_{covering} = d \\
f(0,b,c,d) &\overset{a=0}{=} (0+b+c)(1+d)(a+c+d) = \underbrace{(b+c)(b+c+d)}_{covering} = (b+c) \\
f(a,b,c,d) &= (a+f(0,b,c,d))(a'+f(1,b,c,d)) \quad \text{Shannon's expansion} \\
&= (a+b+c)(a'+d)
\end{aligned}
$$

# Shannon's Expansion Theorem

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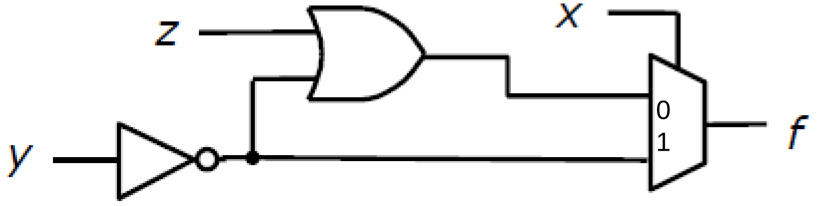
# More examples

| $x$ | $y$ | $z$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

$f=x'y'z'+x'y'z+x'yz+xy'z'+xy'z$

choose x as the expansion variable

$f=x'(y'z'+y'z+yz)+x(y'z'+y'z)$
$f=x'(y'+z)+x(y')$

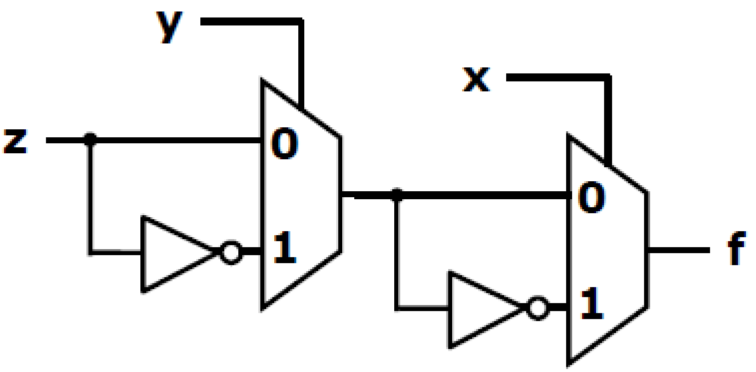


# More examples

| $x$ | $y$ | $z$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

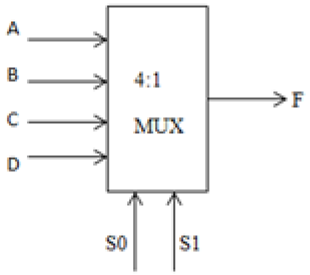
$y \oplus z$

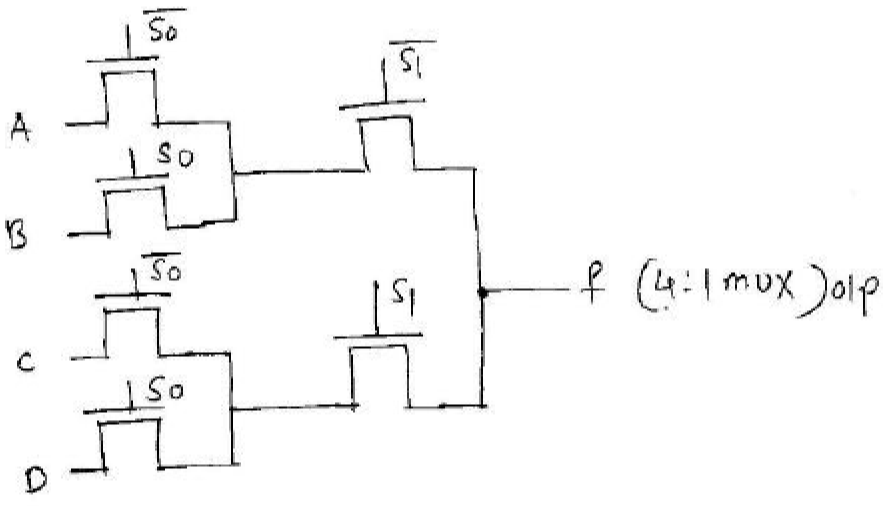
$(y \oplus z)'$



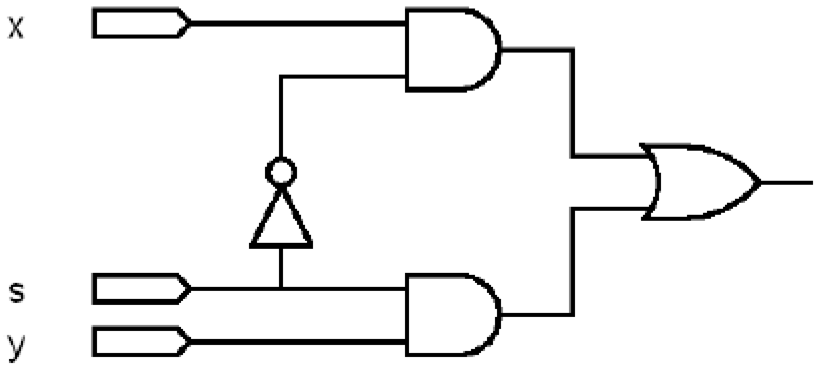
# MUX implementation

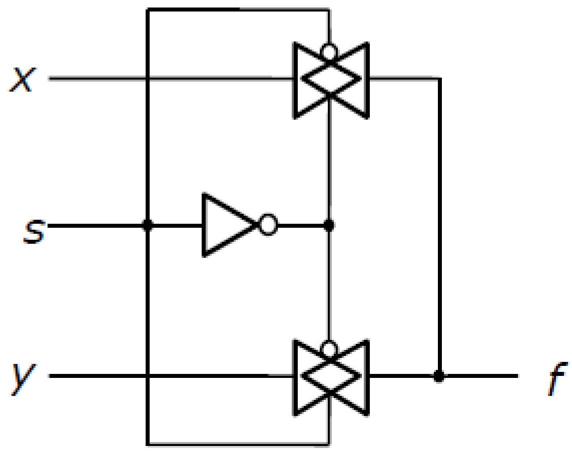
| S1 | S0 | F |
|---|---|---|
| 0 | 0 | A |
| 0 | 1 | B |
| 1 | 0 | C |
| 1 | 1 | D |



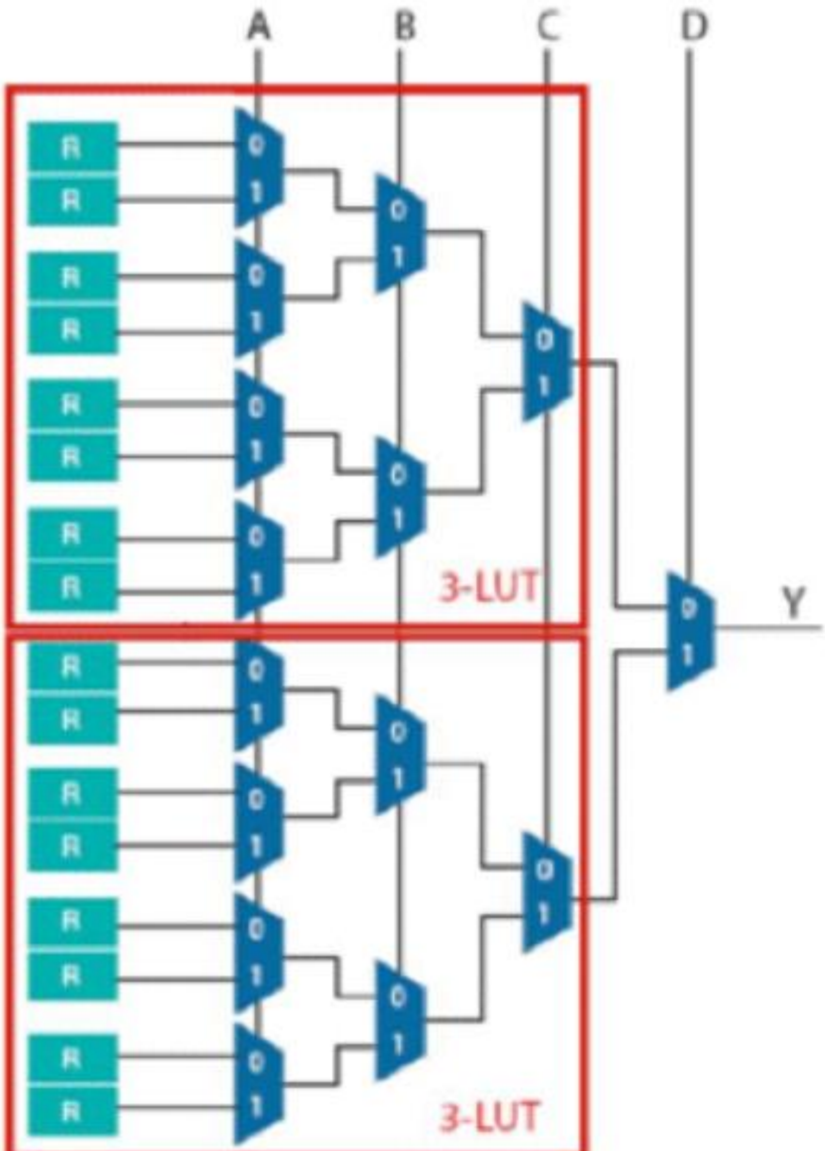



# MUX implementation




The preferred implementation


A
B
C
D
3-LUT
3-LUT
Y