CS 225: Switching Theory

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Announcement

- Assignment -1 will be uploaded soon
 - Deadline: 27th Jan 2020

- No CS225 classes on Next week
 - Will be adjusted later

Previous Class

- Number Systems and Codes
 - Different Number systems (positional)
 - Conversion
 - Representation (complement)
 - Binary Arithmetic

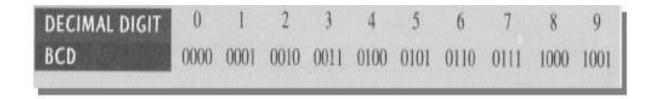
This Class

Number Systems and Codes

- Codes
 - BCD, cyclic code etc.
 - Gray code
 - Parity and Error correcting code
- Switching Algebra

Binary Coded Decimal (BCD)

- Use 4-bit binary to represent one decimal digit
- Easy conversion
- Wasting bits (4-bits can represent 16 different values, but only 10 values are used). Clearly, BCD requires more bits. BUT, it is easier to understand/interpret



BCD Addition

• Example: Add 378 and 539 in BCD.

0011 0111 1000 (378 in BCD)

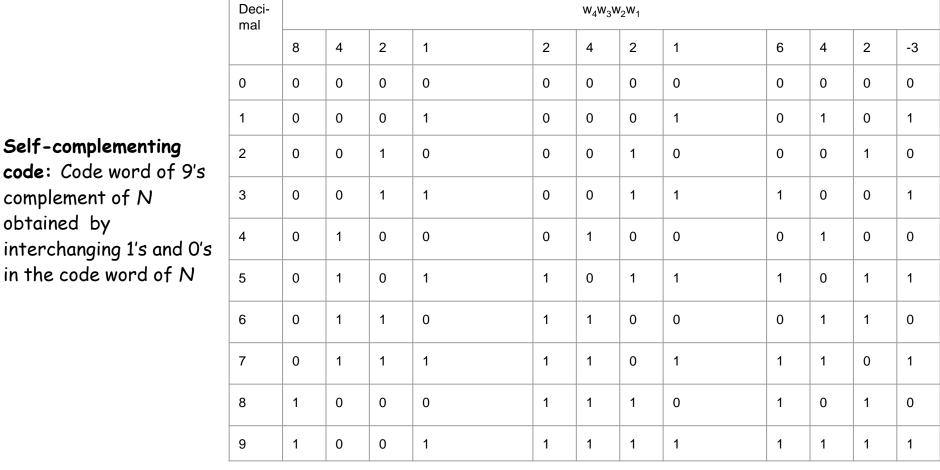
0101 0011 1001 (539 in BCD)

Decimal Codes

Self-complementing

complement of N

obtained by



BCD

Self-complementing Codes

Non-weighted Codes

Decimal Digit	Excess-3	Cyclic
0	0011	0000
1	0100	0001
2	0101	0011
3	0110	0010
4	0111	0110
5	1000	1110
6	1001	1010
7	1010	1000
8	1011	1100
9	1100	0100

Add 3 to BCD

Successive code words Differ in only one digit

Gray Code

Decimal number	Gray	Binary			
	g3 g2 g1 g0	b3 b2 b1 b0			
0	0 0 0 0	0 0 0 0			
1	0 0 0 1	0 0 0 1			
2	0 0 1 1	0 0 1 0			
3	0 0 1 0	0 0 1 1			
4	0 1 1 0	0 1 0 0			
5	0 1 1 1	0 1 0 1			
6	0 1 0 1	0 1 1 0			
7	0 1 0 0	0 1 1 1			

Decimal number	Gray				Binary			
	g3	g2	g1	g0	b3	b2	b1	b0
8	1	1	0	0	1	0	0	0
9	1	1	0	1	1	0	0	1
10	1	1	1	1	1	0	1	0
11	1	1	1	0	1	0	1	1
12	1	0	1	0	1	1	0	0
13	1	0	1	1	1	1	0	1
14	1	0	0	1	1	1	1	0
15	1	0	0	0	1	1	1	1

Conversion

Binary to Gray:

Start from right side LSB as: gi = bi+bi+1, gn = bn

Gray to Binary:

Start from left side MSB as: bn = gn and bi-1 = bi + gi-1

Convert

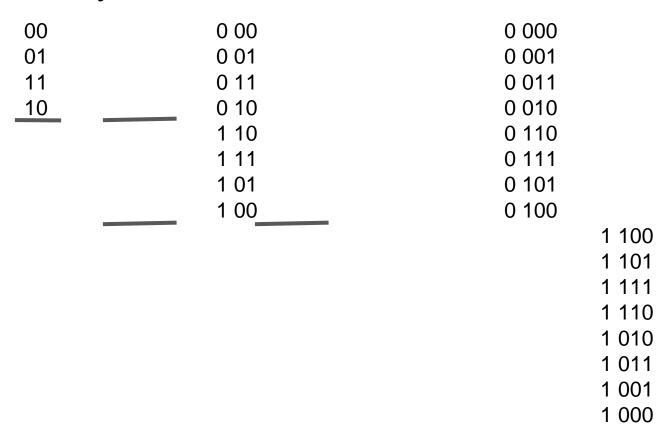
• Binary (1001) to gray

1 1 0 1

• Gray (1 1 0 0) to binary

1 0 0 0

Reflection of Gray Codes



Error-detecting Codes

p: parity bit;

Even parity used in codes.

Distance between codewords: no. of bits they differ in

Minimum distance of a code: smallest no. of bits in which any two code words differ

Minimum distance of above single errordetecting codes = 2

Decimal Digit	Even-parity BCD				2-out-of-5				;	
	8	4	2	1	р	0	1	2	4	7
0	0	0	0	0	0	0	0	0	1	1
1	0	0	0	1	1	1	1	0	0	0
2	0	0	1	0	1	0	1	1	0	0
3	0	0	1	1	0	0	1	1	0	0
4	0	1	0	0	1	1	0	0	1	0
5	0	1	0	1	0	0	1	0	1	0
6	0	1	1	0	0	0	0	1	1	0
7	0	1	1	1	1	1	0	0	0	1
8	1	0	0	0	1	0	1	0	0	1
9	1	0	0	1	0	0	0	1	0	1

Hamming Codes: Single Error-correcting

Minimum distance for SEC or double-error detecting (DED) codes = 3 Example: {000,111} Minimum distance for SEC and DED codes = 4

No. of information bits = m

No. of parity check bits, p1, p2, ..., pk = k No. of bits in the code word = m+k

Assign a decimal value to each of the m+k bits: from 1 to MSB to m+k to LSB

Perform k parity checks on selected bits of each code word: record results as 0 or 1

Form a binary number (called position number), c1c2...ck, with the k parity checks

Hamming Codes (Contd.)

No. of parity check bits, k, must satisfy: $2^k \ge m+k+1$

Example: if m = 4 then k = 3

Place check bits at the following locations: 1, 2, 4, ..., 2k-1

Example code word: 1100110

- Check bits: p1= 1, p2 = 1, p3 = 0
- Information bits: 0, 1, 1, 0

Hamming Code Construction

Select p_1 to establish even parity in positions: 1, 3, 5, 7

Select p_2 to establish even parity in positions: 2, 3, 6, 7

Select p_3 to establish even parity in positions: 4, 5, 6, 7

Error position	Position number				
	c1	c2	c3		
0 (no error)	0	0	0		
1	0	0	1		
2	0	1	0		
3	0	1	1		
4	1	0	0		
5	1	0	1		
6	1	1	0		
7	1	1	1		

Hamming Code Construction (Contd.)

Position:	1 p ₁	2 p ₂	3 m ₁	4 p ₃	5 m ₂	6 m ₃	7 m ₄
Original BCD message:			0		1	0	0
Parity Check in positions $1,3,5,7$ requires $p_1=1$	1		0		1	0	0
Parity Check in positions 2,3,6,7 requires p ₂ =0							
	1	0	0		1	0	0
_							
Parity Check in positions 4,5,6,7 requires p ₃ =1	1	0	0	1	1	0	0
Coded message	1	0	0	1	1	0	0

Hamming Code Construction

• Ex: If the original message is to be send is 0010

The message to be send is ?

0 1 0 1 0 1 0

If the received message is 0 1 0 1 0 1 1

Error position is:

111 (7)

SEC/DED Code

Add another parity bit such that all eight bits have even parity

- Two errors occur: overall parity check satisfied, but position number indicates error double error (cannot be corrected)
- Single error occurs: overall parity check not satisfied
 - o Position no. is 0: error in last parity bit
 - Else, position no. indicates erroneous bit
- No error occurs: all parity checks indicate even parities

. Thanks