PH 301

ENGINEERING OPTICS

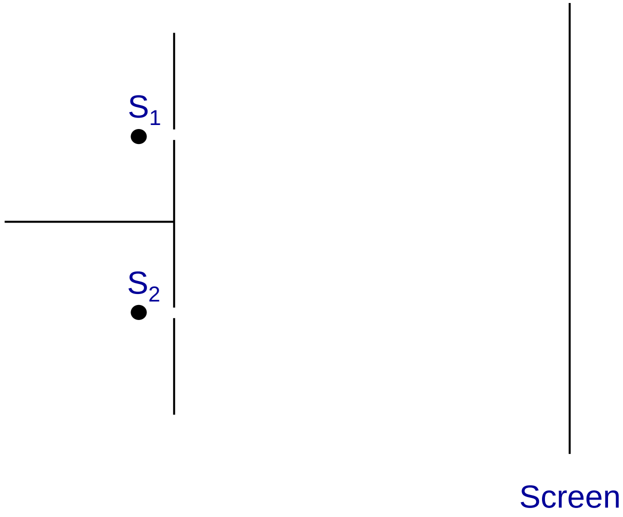
Lecture_Two-beam Interference_13

Interference of Light Waves

If we use two conventional light sources, like two sodium lamps, illuminating two pin holes, we will not observe any interference pattern on screen.

In a conventional light source, light comes from a large no. of independent atoms; each atom emitting light for about 10^{-10} sec. i.e., light emitted by an atom is essentially a pulse lasting for only 10^{-10} sec.

Even if atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases.



Light sources from holes S_1 & S_2 will have a fixed phase relationship for a period of about 10^{-10} sec, hence interference pattern will keep on changing every billionth of a second.

Eye can notice intensity changes which last at least for a tenth of a second & hence we will observe uniform intensity over screen.

It is difficult to observe interference pattern even with two laser beams unless they are phase locked.

Thus, one tries to derive interfering waves from a single wave so that phase relationship is maintained.

Method to achieve phase relationship:

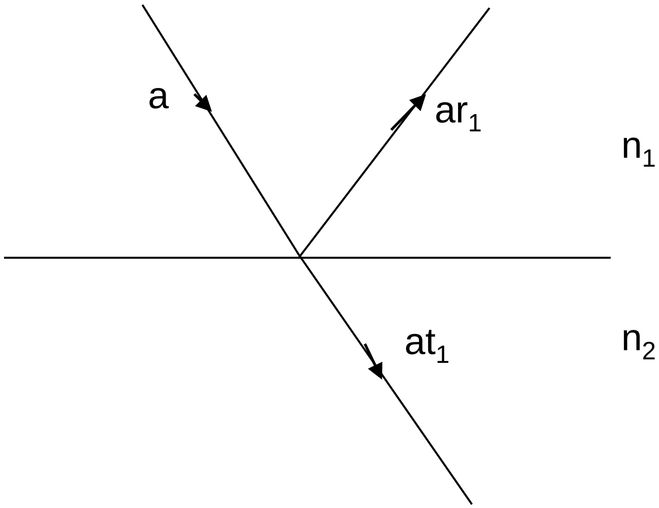
- **Division of wavefront:** **A beam is allowed to fall on two closely spaced holes & two beams emanating from holes interfere.**
- **Division of amplitude:** **A beam is divided at two or more reflecting surfaces & reflected beams interfere.**

Phase change on reflection

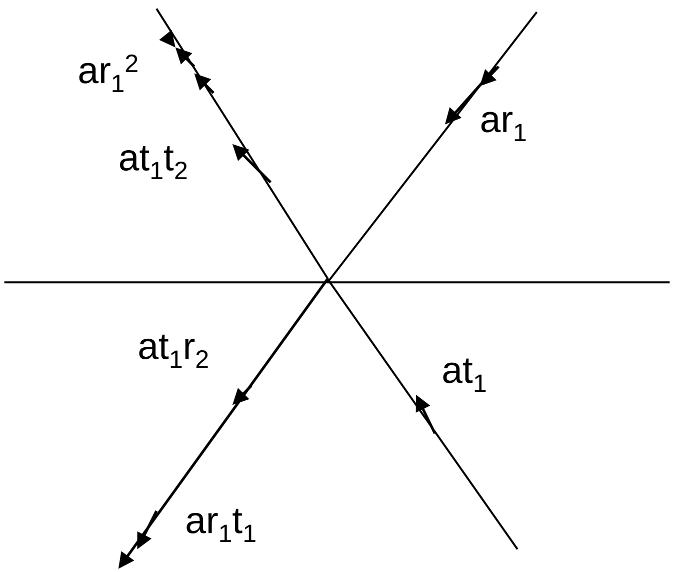
Principle of optical reversibility: Reflection of light at an interface between two media.

In absence of any absorption, a light ray that is reflected or refracted will retrace its original path if its direction is reversed.

Consider a light ray incident on an interface of two media of r.i., n_1 & n_2.



r_1 = Amplitude reflection coefficient; t_1 = Amplitude transmission coefficient; a = Amplitude of incident ray; ar_1 = Amplitude of reflected ray; at_1 = Amplitude of refracted ray. r_2 & t_2 = Amplitude reflection & transmission coefficient when a ray is incident from medium 2 to medium 1.



Reverse the rays:

Consider a light ray of amplitude at_1 incident on medium 1 & a ray of amplitude ar_1 incident on medium 2.

- **Ray of amplitude at_1 will give rise to a reflected ray of amplitude at_1r_2 & a transmitted ray of amplitude at_1t_2.**

- **Ray of amplitude ar_1 will give rise to a ray of amplitude ar_1^2 & a refracted ray of amplitude ar_1t_1.**
- **According to principle of optical reversibility, two rays of amplitudes ar_1^2 & at_1t_2 must combine to give incident ray.**

$$ar_1^2 + at_1t_2 = a$$

$$\Rightarrow t_1t_2 = 1 - r_1^2 \qquad \textit{Stoke's relation}$$

Further, the two rays of amplitudes at_1r_2 & ar_1t_1 must cancel each other,

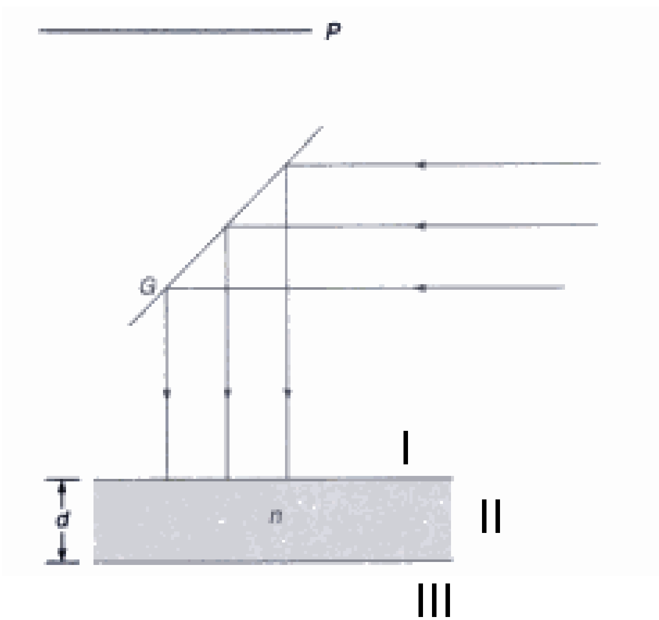
$$at_1r_2 + ar_1t_1 = 0$$

$$\Rightarrow r_2 = -r_1 \qquad \textit{Stoke's relation}$$

- **An abrupt phase change of π occurs when light gets reflected by a denser medium.**
- **No such abrupt phase change occurs when light gets reflected by a rarer medium.**

Interference by division of amplitude

If a plane wave is incident normally on a thin film of uniform thickness *d* then waves reflected from upper surface interfere with waves reflected from lower surface. Wave reflected from lower surface of film traverses an additional optical path of *2nd*. If film is placed in air, then wave reflected from upper surface of film will undergo a sudden change in phase of *π*.

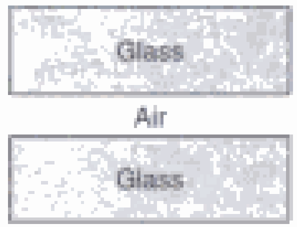


$$2nd = m\lambda \qquad \text{Destructive interference}$$

$$= (m + \frac{1}{2})\lambda \qquad \text{Constructive interference}$$

$$m = 0, 1, 2, 3.....$$

Amplitudes of waves reflected from upper & lower surfaces will, in general, be slightly different, & as such interference will not be completely destructive. However, with appropriate choice of r.i. of media II & III, two amplitudes can be made nearly equal.



Thin film of air formed between two glass plates.

For an air film between two glass plates no phase change will occur on reflection at glass-air interface, but a phase change of *π* will occur on reflection at air-glass interface & conditions for maxima & minima will remain same.

If medium I is crown glass, *n* = 1.52

medium II is oil, *n* = 1.60

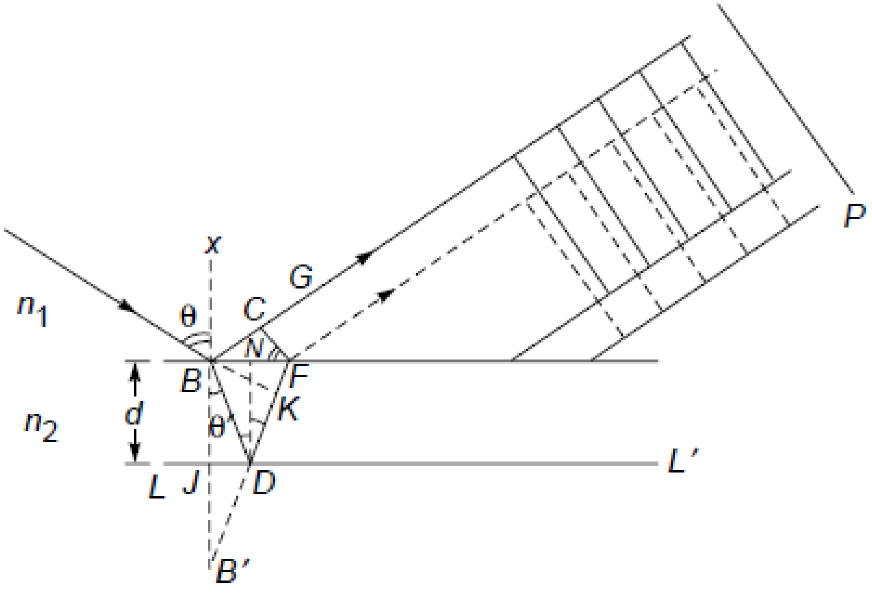
medium III is flint glass *n* = 1.66

Then a phase change of *π* will occur at both reflections & conditions for maxima & minima would be

$$2nd = (m + \frac{1}{2})\lambda \qquad Minima$$

$$= m\lambda \qquad Maxima$$

Oblique incidence of plane wave on thin film



Additional optical path, Δ

$$\Delta = n_2(BD + DF) - n_1 BC$$
$$= 2n_2 d\cos\theta'$$

For a film placed in air, a phase change of *π* will occur when reflection takes place at B.

$$\Delta = 2n_2 d\cos\theta' = m\lambda \qquad \textit{Minima}$$
$$= (m + \frac{1}{2})\lambda \qquad \textit{Maxima}$$

Cosine law: Wave reflected from lower surface of film traverses an additional optical path,

$$\Delta = 2n_2 d\cos\theta'$$

$$\Delta\ [= n_2(BD + DF) - n_1 BC] = 2n_2 d \cos\theta'$$

Let θ and θ' denote the angles of incidence and refraction, respectively. We drop a perpendicular BJ from point B on the lower surface LL' and extend BJ and FD to point B' where they meet (see Fig. 15.5). Clearly,

$$\angle JBD = \angle BDN = \angle NDF = \theta'$$

where N is the foot of the perpendicular drawn from point D on BF. Now

$$\angle BDJ = \frac{\pi}{2} - \theta'$$

and $$\angle B'DJ = \pi - \left[\left(\frac{\pi}{2} - \theta'\right) + \theta' + \theta'\right] = \frac{\pi}{2} - \theta'$$

Thus $$BD = BD' \quad \text{and} \quad BJ = JB' = d$$

or $$BD + DF = B'D + DF = B'F$$

Hence $$\Delta = n_2 B'F - n_1 BC \tag{7}$$

Now $$\angle CFB = \angle CBX = \theta$$

$$BC = BF \sin\theta = \frac{KF}{\sin\theta'} \sin\theta = \frac{n_2}{n_1} KF \tag{8}$$

where K is the foot of the perpendicular from B on $B'F$. Substituting the above expression for BC in Eq. (7), we get

$$\Delta = n_2 B'F - n_2 KF = n_2 B'K$$

or $$\Delta = 2n_2 d \cos\theta' \tag{9}$$

which is known as the *cosine law.*

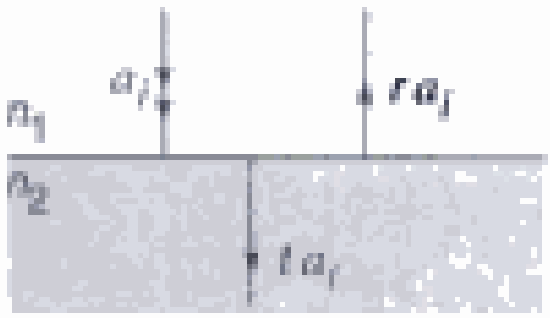
Non-reflecting films

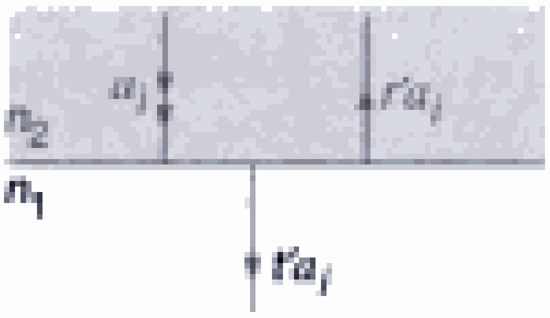
Thin film interference phenomenon reduces reflectivity of lens surfaces.

When a light beam (propagating in a medium of *r.i.*, n_1) is incident normally on a dielectric of refractive index n_2 then amplitudes of reflected & transmitted beams are

$$a_r = \frac{n_1 - n_2}{n_1 + n_2} a_i \qquad\qquad a_t = \frac{2n_1}{n_1 + n_2} a_i$$

where a_i, a_r, & a_t are amplitudes of incident, reflected, & transmitted beams, respectively.





Amplitude reflection & transmission coefficients *r* & *t* are given by

$$r = \frac{n_1 - n_2}{n_1 + n_2} \qquad\qquad t = \frac{2n_1}{n_1 + n_2}$$

In many optical instruments (telescope) there are many interfaces & loss of intensity due to reflections can be severe.

Ex: Reflectivity of crown glass surface in air is

$$\left(\frac{n-1}{n+1}\right)^2 = \left(\frac{1.5-1}{1.5+1}\right)^2 = 0.04$$

i.e. 4% of incident light is reflected.

For a dense flint glass *n* = 1.67 about 6% of light is reflected. Thus, if we have a large no. of surfaces, losses at interfaces can be considerable.

To reduce losses, lens surfaces are often coated with a *λ/4n* thick non-reflecting film; refractive index of film being less than that of lens.

Ex. **Glass (n = 1.5) may be coated with MgF_2 film & film thickness d should be such that (considering normal incidence, $\cos\theta'$ = 1, n_f = *r.i.* of film)**

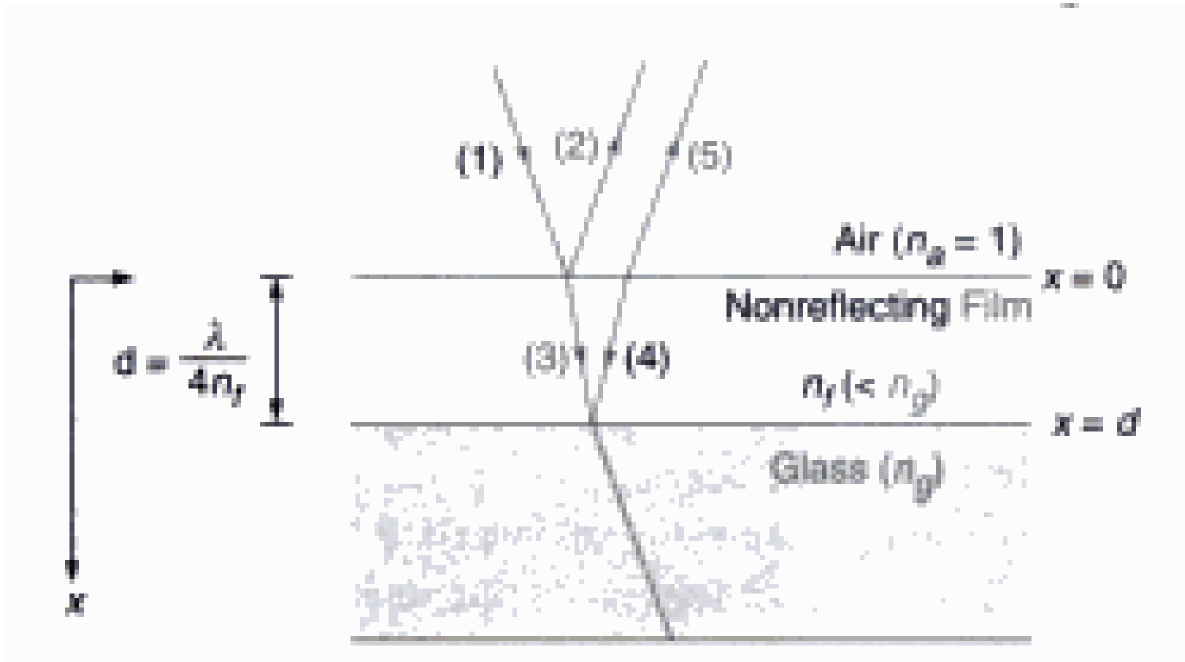
$$\Delta = 2n_2 d\cos\theta' = (m+\frac{1}{2})\lambda$$

$$2n_f d = \frac{1}{2}\lambda$$

$$d = \frac{\lambda}{4n_f}$$

For MgF_2, n_f = 1.38, λ = 5.0 x 10^{-5} cm

$$d = \frac{\lambda}{4n_f} = \frac{5.0\times10^{-5}cm}{4\times1.38} = 0.9\times10^{-5}cm$$



Let n_a, n_f, & n_g be *r.i.* of air, non-reflecting film, & glass respectively. If a is amplitude of incident wave then amplitudes of reflected & refracted waves would be

$$-\frac{n_f - n_a}{n_f + n_a}a \qquad \frac{2n_a}{n_f + n_a}a$$

Amplitudes of waves corresponding to rays (4) & (5) would be

$$-\frac{2n_a}{n_f + n_a}\frac{n_g - n_f}{n_g + n_f}a \qquad -\frac{2n_a}{n_f + n_a}\frac{n_g - n_f}{n_g + n_f}\frac{2n_f}{n_f + n_a}a$$

For complete destructive interference, waves corresponding to rays (2) & (5) should have same amplitude.

$$-\frac{n_f - n_a}{n_f + n_a} a \quad = \quad -\frac{2n_a}{n_f + n_a} \frac{n_g - n_f}{n_g + n_f} \frac{2n_f}{n_f + n_a} a$$

$$or \quad \frac{n_f - n_a}{n_f + n_a} = \frac{n_g - n_f}{n_g + n_f}$$

$$\Rightarrow \quad n_f = \sqrt{n_a n_g}$$

$$\frac{4 n_f n_a}{(n_f + n_a)^2} = \sim unity \;\; (0.97)$$

$$for \; n_a = 1, n_f = 1.34$$

For a *λ*/4*n* thick film, reflectivity will be about $\left[\frac{n_f - n_a}{n_f + n_a} - \frac{n_g - n_f}{n_g + n_f}\right]^2$

For n_a = 1, n_f = 1.38, & n_g = 1.5, reflectivity will be about 1.3%.

In absence of film, reflectivity would have been about 4%.

Reduction of reflectivity is much more pronounced for dense flint glass. This technique of reducing reflectivity is known as blooming.

High reflectivity by thin film deposition

Reflectivity of glass surfaces can be increased by coating glass surface by a thin film of suitable material.

Film thickness is again $\lambda/4n_f$ where n_f represents *r.i.* of film; however, film is such that its *r.i.* is greater than that of glass; consequently an abrupt phase change of π occurs only at air-film interface & beams reflected from air-film interface & film-glass interface constructively interfere.

Ex. Consider a film of refractive index 2.37 (Zinc Sulphide) then reflectivity is about 16%.

$$\frac{(2.37-1)^2}{(2.37+1)^2}$$

In presence of a glass surface of *r.i.* 1.5, reflectivity will become about 35%

$$\left[-\frac{2.37-1}{2.37+1}-\frac{4\times1\times2.37}{(3.37)^2}\times\frac{2.37-1.5}{2.37+1.5}\right]^2$$

If difference between *r.i.* of film & glass is increased, then reflectivity will also increase.

Two Beam Interference

Division of Amplitude

- Michelson interferometer
- Twyman & Green interferometer
- Jamin interferometer
- Mach-Zehnder interferometer
- Wavefront shearing interferometer
- Gauge measuring interferometer

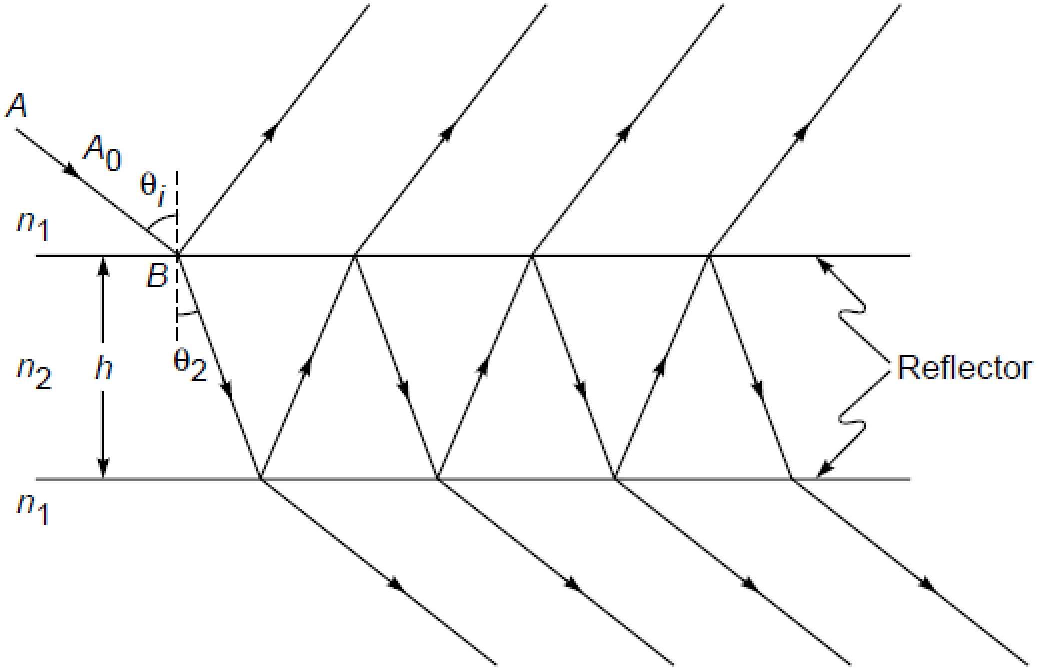
Multiple Beam Interference

- Fabry-Perot interferometer
- Spherical Fabry-Perot interferometer
- Lumer-Gehrcke interferometer

Multiple Beam Interferometry

Marie Fabry & Jean Perot invented Fabry-Perot interferometer (1899) which is characterized by a very high resolving power.

Multiple reflections from a plane parallel film



Let A_0 be (complex) amplitude of incident wave. Let r_1 & t_1 represent amplitude reflection & transmission coefficients, respectively, when wave is incident from n_1 toward n_2, & let r_2 & t_2 represent corresponding coefficients when wave is incident from n_2 toward n_1.

Amplitude of successive reflected waves,

$$A_0 r_1,\ A_0 t_1 r_2 t_2 e^{i\delta},\ A_0 t_1 r_2^3 t_2 e^{2i\delta},\ldots\ldots\ldots\ldots$$

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{4\pi n_2 h\cos\theta_2}{\lambda_0}$$

Resultant (complex) amplitude of reflected waves,

$$A_r = A_0\left[r_1 + t_1 r_2 t_2 e^{i\delta}\left(1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \ldots\ldots\ldots\ldots\right)\right]$$

$$= A_0\left(r_1 + \frac{t_1 t_2 r_2 e^{i\delta}}{1 - r_2^2 e^{i\delta}}\right)$$

If the reflectors are lossless,

$$R = r_1^2 = r_2^2$$

$$T = t_1 t_2 = 1 - R$$

Using the fact $r_2 = -r_1$

$$A_r = A_0\left(r_1 + \frac{t_1 t_2 (-r_1) e^{i\delta}}{1 - \mathrm{Re}^{i\delta}}\right) = A_0 r_1\left(1 - \frac{t_1 t_2 e^{i\delta}}{1 - \mathrm{Re}^{i\delta}}\right) = A_0 r_1\left(1 - \frac{(1-R)e^{i\delta}}{1 - \mathrm{Re}^{i\delta}}\right)$$

$$\frac{A_r}{A_0} = r_1\left(1 - \frac{(1-R)e^{i\delta}}{1 - \mathrm{Re}^{i\delta}}\right)$$

Reflectivity of Fabry-Perot etalon,

$$\Re = \left|\frac{A_r}{A_0}\right|^2 = r_1^2\left(1 - \frac{(1-R)e^{i\delta}}{1-\mathrm{Re}^{i\delta}}\right)^2 = R\left|\frac{1-\mathrm{Re}^{i\delta} - e^{i\delta} + \mathrm{Re}^{i\delta}}{1-\mathrm{Re}^{i\delta}}\right|^2 = R\left|\frac{1-e^{i\delta}}{1-\mathrm{Re}^{i\delta}}\right|^2$$

$$\Re = R\frac{(1-\cos\delta)^2 + \sin^2\delta}{(1-R\cos\delta)^2 + R^2\sin^2\delta} = \frac{4R\sin^2\delta/2}{(1-R)^2 + 4R\sin^2\delta/2}$$

$$\Re = \frac{F\sin^2\delta/2}{1+F\sin^2\delta/2} \qquad F = \frac{4R}{(1-R)^2}$$

Coefficient of Finesse

When R << 1, F is small,

$$\Rightarrow \Re \propto \sin^2\delta/2$$

~ same interference pattern is obtained in 2 beam interference.

Amplitude of successive transmitted waves, (assuming 1st transmitted wave to have zero phase)

$$A_0 t_1 t_2,\ A_0 t_1 t_2 r_2^2 e^{i\delta},\ A_0 t_1 t_2 r_2^4 e^{2i\delta}, \ldots\ldots\ldots\ldots$$

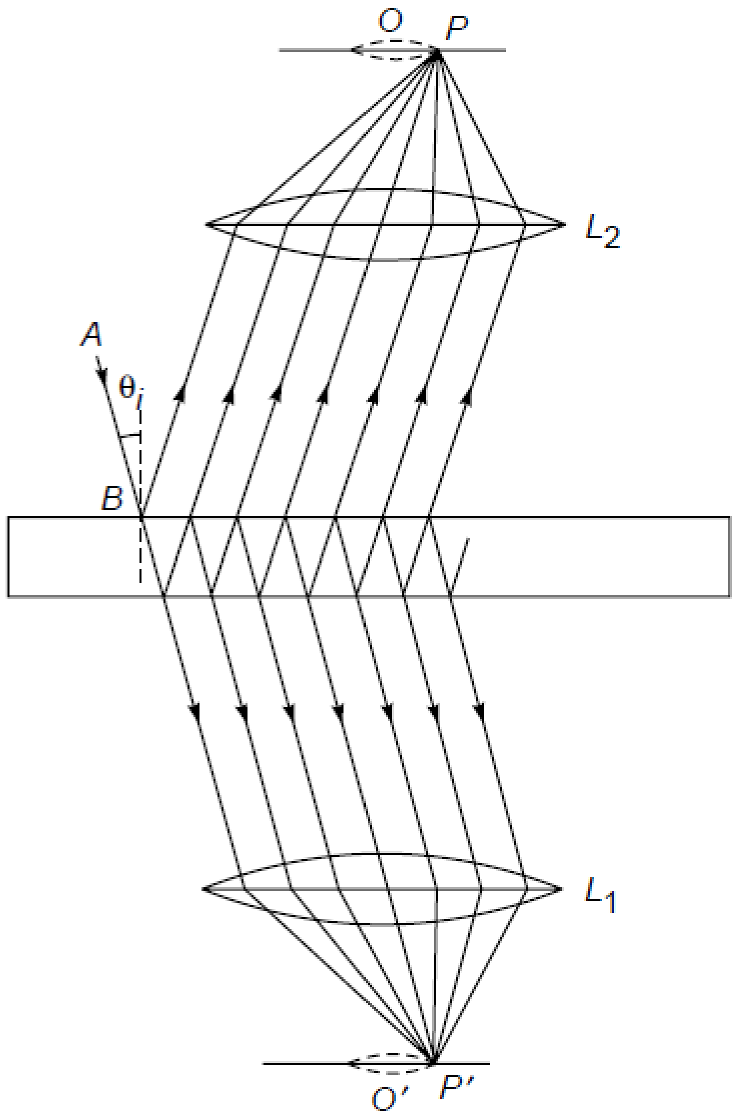
Resultant (complex) amplitude of transmitted waves,

$$A_t = A_0 t_1 t_2 \left(1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \ldots\ldots\ldots\ldots \right)$$

$$= A_0 \frac{t_1 t_2}{1 - r_2^2 e^{i\delta}} = A_0 \frac{1-R}{1-\mathrm{R}e^{i\delta}}$$

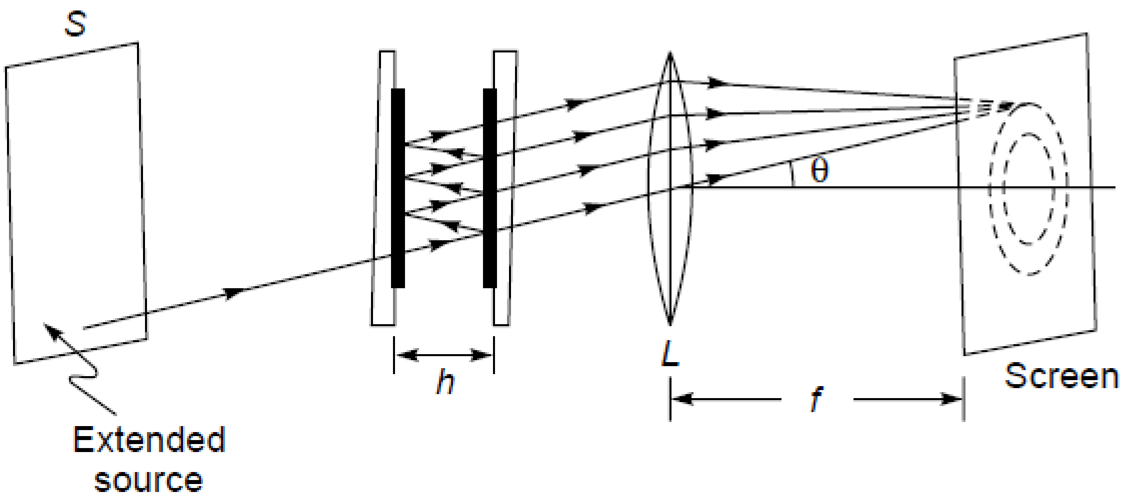
$$T = \left|\frac{A_t}{A_0}\right|^2 = \left|\frac{1-R}{1-\mathrm{R}e^{i\delta}}\right|^2 = \frac{(1-R)^2}{(1-R\cos\delta) + R^2\sin^2\delta}$$

$$T = \frac{1}{1 + F\sin^2\delta/2}$$



Any ray parallel to AB will focus at same point P.

If ray AB is rotated about normal at B, then P will rotate on circumference of a circle centered at point O; this circle will be bright or dark depending on θ_i. Rays incident at different angles will focus at different distances from point O, & one will obtain concentric bright & dark rings for an extended source.



Fabry-Perot etalon