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Tutorial Sheet-5

1. Let $X \sim Geo(p)$ distribution. Find the probabilities that X is even, it is odd.

2. Suppose that a trainee soldier shoots a target in an independent manner. Let the probability that target is shot in any one shot is 0.7. Find the probability that (i) target would be hit on 10th attempt (ii) target would be hit in less than 4 shots (iii) target would be hit in an even number of shots. Also find the average number of shots needed to hit the target. (0.000013, 0.973, 0.23, 1.43)

3. Derive a recurrence formula formula to find the the k^{th} central moments of a $NB(r, p)$ distribution.

4. Suppose the probability is 0.10 that any person will believe a rumor about the private life of a certain celebrity. What is the probability that the sixth person to hear this rumor will be the first one to believe it? (0.06)

5. To avoid detection at customs, a traveler places 6 narcotic tablets in a bottle containing 9 tablets that are similar to narcotic tablets. If the custom official selects 3 of the tablets at random for analysis, what is the probability that the traveler will be arrested for illegal possession of narcotics? (53/65)

6. From a lot of 10 missiles, 4 are selected at random and fired. If the lot contains 3 defective missiles that will not fire, what is the probability that (i) all 4 will fire? (ii) at most 2 will not fire? Finally, use Chebyshev theorem to interpret the interval $\mu \pm 2\sigma$ and $\mu \pm 3\sigma$. (1/6, 29/30, (-1.04, 3.44))

7. A life insurance salesman regularly calls on families that have recently moved to Patna even though odds are 9 to 1 against his making a sale. (i) What is the probability that he will make his first sale of the week to the fourth family on which he calls (ii) What is the probability that he will still be waiting to make his first sale of the week after having visited 10 families. (0.073, 0.349)

8. A company produces 10000 bulbs, among which 300 are found to be defective. A sample of 30 bulbs is randomly selected and the number of defective is checked. Let X be the RV representing the number of bulbs selected that are defective. Find the probability that there is at least one defective bulb in the selected sample. (0.599)

9. A scientist inoculates several mice, one at a time, with a disease germ until he finds 2 that have contracted the disease. If the probability of contracting the disease is 1/6, what is the probability that 8 mice are required. (0.0651)

10. Three people toss a fair coin and the odd man pays for coffee. If the coins all turn up the same they are tossed again. Find the probability that fewer than 4 tosses are needed. (63/64)

11. Refusal rate in telephone polls is known to be approximately 20%. A newspaper report indicates that 50 people were interviewed before the first refusal. (i) Comment on the validity of the report using probability argument. (ii) Find the expected number of people interviewed before a refusal. (0.00001, 4)

12. An machine produces a product of which 5% are defective. Find the probability that at least 4 items are to be examined in order to get 2 defective. (0.995)

13. Cotton linters used in the production of rocket propellent are subjected to a nitration process that enables the cotton fibers to go into solution. The process is 90% effective in that the material produced can be shaped as desired in a later processing stage with probability 0.9. What is the probability that exactly 20 lots will be produced in order to obtain third defective lot. Find the expected value and the variance of the corresponding random variable.

14. A company receives a shipment of twenty items. Because inspection of each individual item is expensive, it has policy of checking a random sample of six items from such a shipment, accepting delivery if no more than one item is defective. What is the probability that a shipment with five defective items will be accepted?

15. An oil drilling company ventures into various locations, and its success or failure is independent from one location to another. Suppose the probability of a success at any particular location is 0.25. (*i*) What is the probability that a driller drills 10 locations and finds one success? (*ii*) The driller feels that he will go bankrupt if he drills 10 times before the first success occurs. What are the drillers prospects of bankruptcy? (0.187, 0.056, 0.014)

16. The acceptance scheme for purchasing lots containing a large number of batteries is to test no more than 75 randomly selected batteries and to reject a lot if a single battery fails. Suppose the prob of a failure is 0.001. (*i*) What is the probability that a lot is accepted? (*ii*) What is the probability that a lot is rejected on 20th test? (*iii*) What is the probability that it is rejected in 10 or less trials? (0.9277, 0.000981, 0.01)

17. Certain items are purchased in lots of size 20. The policy is to check 6 items randomly from lot and then accept it if all 6 are non defective. Assume that 40 percent of the lots have 5 defective items and 60 percent have only 2, find the proportion of lots that purchaser reject.

18. Let X be the number of births in a family until the second daughter is born. If the probability of having a male child is 0.5 find the probability that sixth child in the family is the second daughter. (0.078)

19. Suppose that in a large lot containing X manufactured items, 30% of the items are defective and 70% are non defective. Suppose that 10 items are selected at random without replacement from that lot. Find (*i*) an exact expression for the probability distribution that not more than one defective item will be obtained. (*ii*) an approximate expression for this probability distribution based on binomial distribution.