Department of Mathematics
Indian Institute of Technology Patna
MA - 201: B.Tech. II year
Autumn Semester: 2019-20

Assignment-5: Complex Analysis

1. Locate the kind of singularities for the following functions at the indicated point z.
   (i) $\frac{\cot z\pi}{(z-a)^2}$, $z = 0$ (ii) $\sin\frac{1}{1-z}$, $z = 1$ (iii) $\tan\frac{1}{z}$, $z = 0$

2. For each of the function given below write the **principal part** of the function at its isolated singular points and classify whether that point is a pole, removable singularity, or an essential singular point:
   (i) $\frac{\exp z}{z+\sin z}$ (ii) $\frac{z^2}{1+z}$ (iii) $ze^{\frac{1}{z}}$
   (iv) $\frac{\sin z}{z}$ (v) $\frac{1}{(2-z)^3}$ (vi) $(z-1)\cos(\frac{1}{z+2})$

3. Determine the poles and their order for the following functions:
   (i) $\frac{1-\cosh z}{z^3}$ (ii) $\frac{1-e^{2z}}{z^4}$ (iii) $\frac{e^{2z}}{(z-1)^2}$ (iv) $\frac{1}{z^6+1}$
   (v) $\frac{1}{3z^4+10z^2+3}$ (vi) $z\cot z$ (vii) $\frac{1}{1-e^z}$ (viii) $\frac{\sinh z}{z^4}$ (ix) $\frac{\cot \pi z}{z^2}$

4. Determine zeros and their order for given functions.
   (i) $(z+2-i)^2$ (ii) $z^4 + z^2$ (iii) $e^{2z} - e^z$ (iv) $e^{\tan z}$
   (v) $(1+z^2)^4$ (vi) $z^2(e^z-1)$ (vii) z^6+2z^3+1 (viii) $z\sin z^2$

5. Compute residue at all singular points of following functions.
   (i) $\frac{1}{z^5+z^3}$ (ii) $z\cos\frac{1}{z}$ (iii) $\tanh z$ (iv) $z\sec z$ (v) $\frac{e^z}{z^2+\pi^2}$
   (vi) $e^{1+(1/z)}$ (vii) $\frac{\csc z}{z}$ (viii) $\frac{e^{4z}-1}{\sin^2 z}$ (ix) $\frac{\sin z}{z^2+1}$ (x) $z^n\sin\frac{1}{z}$, n integer

6. Evaluate $\int_C f(z)dz$ (C is positively oriented) using Residue Theorem when:
   (i) $f(z) = \frac{3z^3+2}{(z-1)(z^2+9)}$, $C: |z-2| = 2$ (ii) $f(z) = \frac{1}{z^3(z+4)}$,, $C: |z+2| = 3$
   (iii) $f(z) = \frac{\cosh \pi z}{z(z^2+1)}$, $C: |z| = 2$ (iv) $\frac{z^3 e^{1/z}}{1+z^3}$, $C: |z| = 3$
   (v) $f(z) = \frac{1}{4+z^4}$, $C: |z+1-i| = 1$ (vi) $f(z) = \frac{\sin z}{4z^2-\pi^2}$, $C: |z| = 2$
   (vii) $f(z) = \frac{1}{z\sin^2 z}$, $C: |z| = 1$ (viii) $f(z) = \frac{\sin z}{4z^2-\pi^2}$, $C: |z| = 2$
   (ix) $f(z) = \frac{1}{3z^4+10z^2+3}$, $C: |z-i\sqrt{3}| = 1$ (x) $f(z) = \frac{\sin z}{4z^2-\pi^2}$, $C: |z| = 2$
   (xi) $f(z) = \frac{\sin z e^{1/(z-1)^2}}{z^2(z^2-1)}$, $C: |z+(1/2)| = 1$ (xii) $f(z) = \tan \pi z$, $C: |z| = 2$
   (xiii) $f(z) = e^{1/z}\sin(1/z)$, $C: |z| = 1$ (xiv) $f(z) = z^3 e^{(-1/z^2)}$, $C: |z+i| = 2$

7. Evaluate following integral.
   (i) $\int_0^{2\pi} \frac{\cos\theta}{3+\sin\theta} d\theta$ (ii) $\int_0^{\pi} \frac{1}{2-\cos\theta} d\theta$
   (iii) $\int_0^{2\pi} \frac{1}{a+\cos\theta} d\theta$, $-1 < a < 1$ (iv) $\int_0^{2\pi} \frac{\cos^2 3\theta}{5-4\cos 2\theta} d\theta$
   (v) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+16)^2} dx$ (vi) $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2(x^2+4)} dx$
   (vii) $\int_{-\infty}^{\infty} \frac{1}{x^2-2x+2} dx$ (viii) $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+a^2)(x^2+c^2)} dx$