Indian Institute of Technology, Patna
MA102, B.Tech -I year
Spring Semester: 2013-2014
(Mid Semester Examination)

Maximum Marks: 30 **Time: 2 Hours**

Note:

(i) Please check all pages and report the discrepancy, if any.

(ii) Attempt all questions.

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1. Determine whether the following system of linear equations is consistent? Find the solutions in case the system is consistent.

$$\begin{aligned} x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 5x_1 - 8x_2 + 7x_3 &= 1 \end{aligned}$$

[2.5]

2. Find the inverse of the matrix A using the elementary row operations, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix}.$$

[2.5]

3. Let $V(C)$ be the vector space of all polynomials in x with complex coefficients. Let $p(x) \in V(C)$. Then $p(x)$ is called an even (odd) polynomial if $p(-x) = p(x)$ $(p(-x) = -p(x))$. Then prove that

   (a) The set W_1 of even polynomials in $V(C)$ and the set W_2 of odd polynomials in $V(C)$ are subspaces of $V(C)$. [3]

   (b) $V(C) = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$. [2]

4. Write the vector $x = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$, in the vector space of 2×2 real matrices, as a linear combination of

$$x_1 = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

[2]

5. Let W be the subspace generated by the polynomials

$$p_1 = x^3+2x^2-2x+1,\ \ p_2 = x^3+3x^2-x+4,\ \ p_3 = 2x^3+x^2-7x-7.$$

Find a basis and dimension of W. [3]

6. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(a,b,c) = (a-b, a+2b, b+3c), (a,b,c) \in \mathbb{R}^3$. Then show that T is invertible and determine T^{-1}. [3]

7. Let P_n denotes the vector space of all real polynomials in t of degree $<$ n. Differentiation is a linear transformation from P_n to P_{n-1} over R. Also integration defined by

$$\int (a_0 + a_1 t + \ldots + a_{n-2}t^{n-2}) = a_0 t + \frac{a_1}{2}t^2 + \ldots + \frac{a_{n-2}}{n-1}t^{n-1}$$

is a linear transformation from P_{n-1} to P_n over R. consider the differentiation transformation $f : P_4 \to P_3$ and the integration transformation $g : P_3 \to P_4$. Let X be the basis $\{1, t, t^2, t^3\}$ of P_4 and Y be the basis $\{1, t, t^2\}$ of P_3.

(a) Find the matrix of f with respect to X and Y. [1.5]

(b) Find the matrix of g with respect to Y and X. [1.5]

8. Let A be a square matrix of order n and P be a non-singular matrix of order n. Then eigen values of A and PAP^{-1} are same. [1]

9. For the matrix $A = \begin{bmatrix} a & 1 & -2 \\ -1 & 2 & b \\ 0 & c & -1 \end{bmatrix}$, one of the eigen value is 1 corresponding eigen vector is $(3,2,1)^T$. Also, given trace(A)=2.

(a) Find the values of a,b and c.

(b) If the matrix A is diagonalizable, then find a matrix P such that $P^{-1}AP$ is a diagonal matrix. [3]

10. Which one of the following statement is correct? Justify your answer. $[5 \times 1 = 5]$

(i) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(a,b,c) = (0, a, b)$. Then $KerT^2$ is equal to

(a) $\{(x, y, z) : x = 0\}$.

(b) $\{(x, y, z) : x = y = z\}$.

(c) $\{(x, y, z) : y = z = 0\}$.

(d) $\{(x, y, z) : y \neq z\}$.

(ii) Consider the basis $B = \{v_1, v_2, v_3\}$ of R^3 where $v_1 = (1, 1, 1)$,$v_2 = (1, 1, 0)$, $v_3 = (1, 0, 0)$ and $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $T(v_1) = (1, 0)$,$T(v_2) = (2, -1)$, $T(v_3) = (4, 3)$. Then $T(2, -3, 5)$ is equal to

(a) (-1,5) (b) (0,0) (c) (3,4) (d) (9,23)

(iii) Let A be an orthogonal matrix of order 2 having two distinct eigen values λ_1 and λ_2. Then the eigen values of A^{-1} are

(a) $\lambda_1^{-1}, \lambda_2^{-1}$ (b)λ_1, λ_2 (c)λ_1^2, λ_2^2 (d) $\lambda_1 + \lambda_2, \lambda_1 - \lambda_2$

(iv) If $A_{3\times3}$ be a matrix with α and β be only two eigen values ($\alpha \neq \beta$) then the characteristic polynomial of A is

(a) $(x - \alpha)^2(x - \beta)$ or $(x - \alpha)(x - \beta)^2$.

(b) $(x - \alpha)(x - \beta)$.

(c) $(x - \alpha)^3(x - \beta)$.

(d) $(x - \alpha)^2(x - \beta)^3$.

(v) If $B = P^{-1}AP$ for some invertible matrix P and X ($\neq 0$) be an eigen vector of the matrix A with respect to the eigen value λ. Then which one of the following is an eigen vector of the matrix B?

(a) PX (b) $P^{-1}X$ (c) X (d) $P^{-1}XP$