Indian Institute of Technology Patna

MA-101: Mid Semester Exam

Autumn Semester: 2009-10

M.M.: 30 Time: 2 Hrs

Note: Attempt all the questions.

1. Show that the point $(\frac{1}{2}, \frac{3\pi}{2})$ lies on the curve $r = -\sin(\theta/3)$ and hence find the slope of the curve at this point. **[1 + 1]**

2. Let $f : [0, 2] \to R$ be a continuous function and $f(0) = f(2)$. Prove that there exist real numbers $x_1, x_2 \in [0, 2]$ such that $x_2 - x_1 = 1$ and $f(x_2) = f(x_1)$. **[3]**

3. By finding s_n, the partial sum of $n-$ terms, test the convergence of the series:
$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots$$ [2]

4. For what values of x do the following series converge (a) absolutely (b) conditionally?
$$1 - \frac{2x}{2!} + \frac{3^2x^2}{3!} - \frac{4^3x^3}{4!} + \cdots$$ [4]

5. Prove that an increasing sequence is convergent *iff* it is bounded from above. Use this result, to show that the sequence $x_n = \frac{1}{1^2} + \frac{1}{2^2} + \cdots + \frac{1}{n^2}$ for each $n \in N$ is convergent. **[3 + 2]**

6. Using Dominant terms, Concavity and Asymptotes, sketch a graph of
$$y = \frac{x^3 - x^2 - 8}{x - 1}$$ [5]

7. Suppose f is continuous on $[a, b]$, differentiable on (a, b) and satisfy $(f(a))^2 - (f(b))^2 = a^2 - b^2$. Then show that the equation $f'(x)f(x) = x$ has at least one root in (a, b). [2]

8. Show that the ellipse $x = a\cos(t)$, $y = b\sin(t)$, $a > b > 0$, has its largest curvature on its major axis and its smallest curvature on its minor axis. [3]

9. Let $f : [a, b] \to R$ be defined by $f(x) = 1$ for $x \in Q \bigcap [a, b]$ and $f(x) = -1$ for $x \in (R - Q) \bigcap [a, b]$. Use Riemann integrability condition to show that f is not Riemann integrable. [2]

10. Find the following limit, if it exists
$$\lim_{(x,y)\to(0,0)} \frac{x^3y^2}{x^6 + y^4}$$ [2]