CE111: Engineering Drawing

Lecture 6: Engineering curves-II

Roulettes

Roulettes

- Roulettes are curves generated by the rolling contact of one curve or line on another curve or line.
- There are various types of roulettes.
- The most common types of roulettes used in engineering practice are: Cycloids, trochoids, and Involutes.

ENGINEERING CURVES

Part-II Roulettes

(Point undergoing two types of displacements)

CYCLOID

1. General Cycloid
2. Trochoid (superior)
3. Trochoid (Inferior)
4. Epi-Cycloid
5. Hypo-Cycloid

INVOLUTE

1. Involute of a circle
   a)String Length = πD
   b)String Length > πD
   c)String Length < πD
2. Pole having Composite shape.
3. Rod Rolling over a Semicircular Pole.

SPIRAL

1. Spiral of One Convolution.
2. Spiral of Two Convolutions.

HELIX

1. On Cylinder
2. On a Cone

AND

Methods of Drawing Tangents & Normals To These Curves.

DEFINITIONS

CYCLOID:

It is a locus of a point on the periphery of a circle which rolls on a straight line path.

EPI-CYCLOID

If the circle is rolling on another circle from outside

HYPO-CYCLOID.

If the circle is rolling from inside the other circle

SUPERIOR TROCHOID:

If the point in the definition of cycloid is outside the circle

INFERIOR TROCHOID

If it is inside the circle

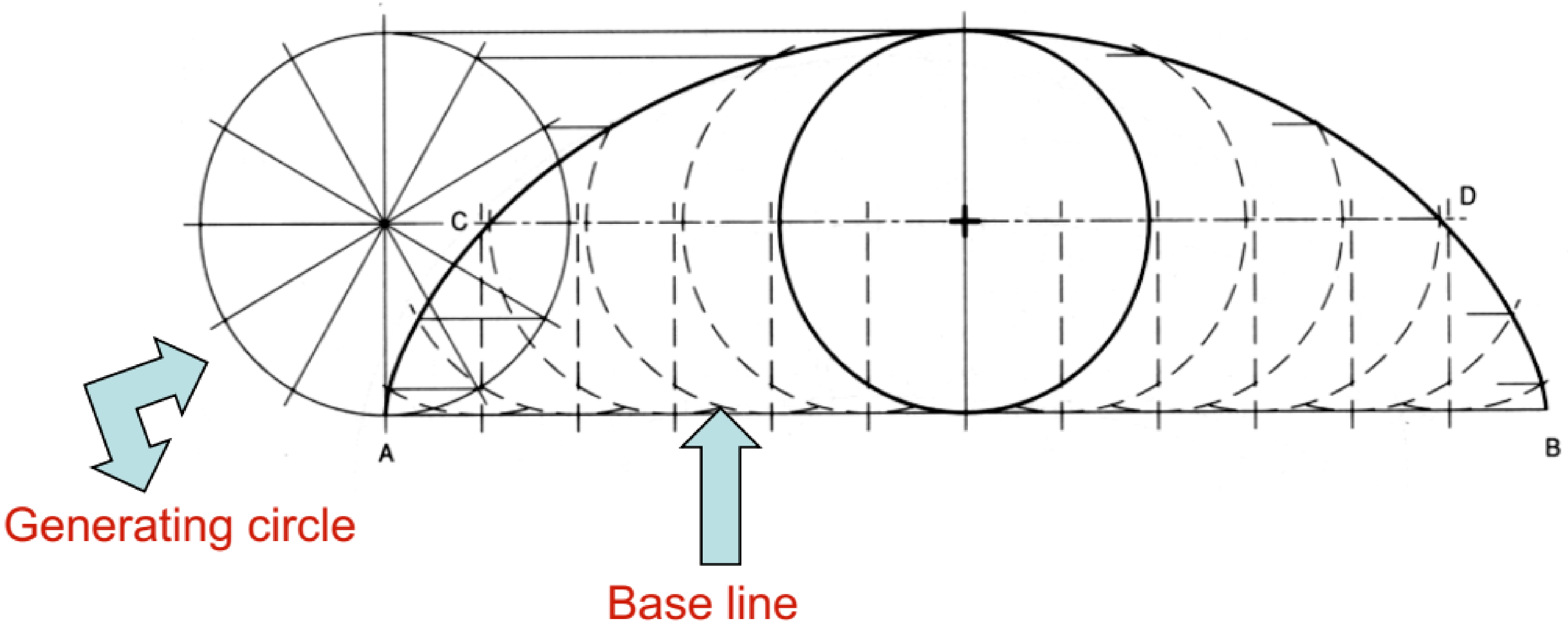
INVOLUTE:

It is a locus of a free end of a string when it is wound round a circular pole

SPIRAL:

It is a curve generated by a point which revolves around a fixed point and at the same moves towards it.

Cycloid

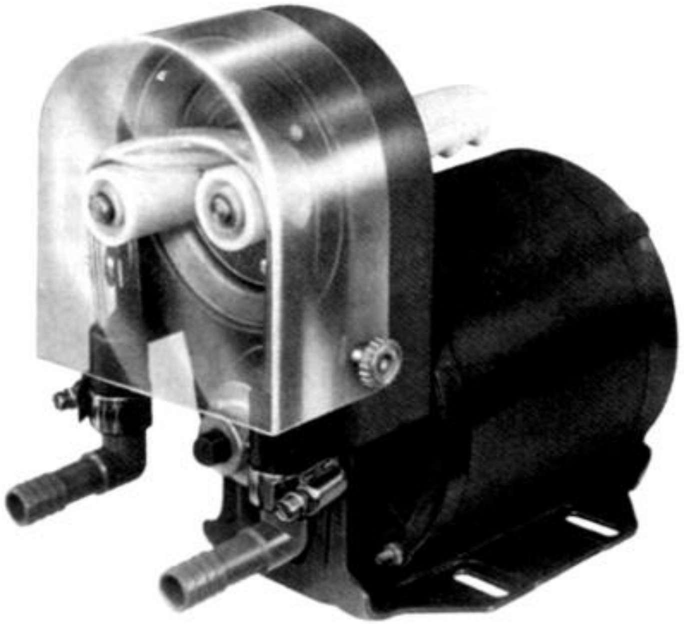


A Cycloid is generated by a point on the circumference of a circle rolling along a straight line without slipping

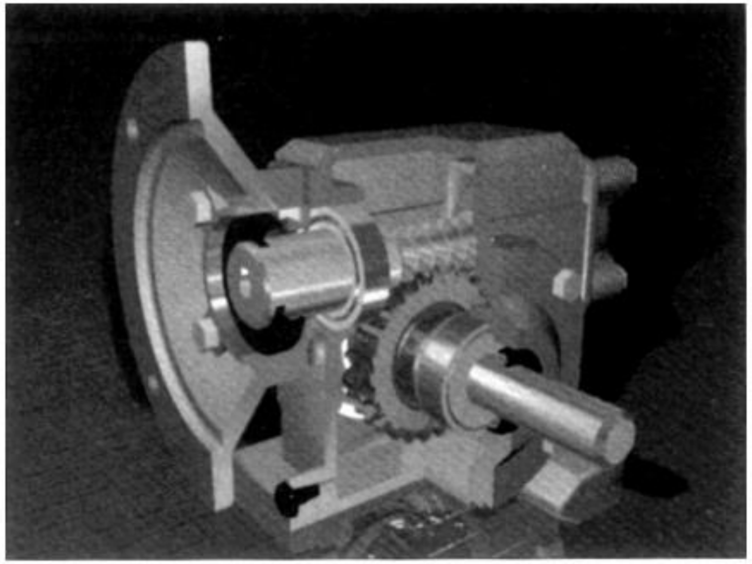
The rolling circle is called the Generating circle

The straight line is called the Directing line or Base line

Applications



Cycloids find application in gears for rotary pumps, watches, etc



High power transmission gear teeth profiles are involutes

The curved history of cycloids, from Galileo to cycle gears

Imagine a small light fixed to the rim of a bicycle wheel. As the bike moves, the light rises and falls in a series of arches. A long-exposure nocturnal photograph would show a cycloid, the curve traced out by a point on a circle as it rolls along a straight line. A light at the wheel-hub traces out a straight line. If the light is at the mid-point of a spoke, the curve it follows is a curtate cycloid. A point outside the rim traces out a prolate cycloid, with a backward loop.

Cycloids were studied by many leading mathematicians over the past 500 years. The name cycloid originates with Galileo, who studied the curve in detail. The story of Galileo dropping objects from the Leaning Tower of Pisa is well-known. Although he could not have known it, a falling object traces out an arc of an inverted cycloid. This is due to the tiny deflection caused by the Earth's rotation. Moreover, an object thrown straight upward follows the loop of a prolate cycloid, landing slightly to the west of its launch point.

Blaise Pascal, who had abandoned mathematics for theology, found relief from a toothache by contemplating the properties of cycloids. Taking this to be a sign from above, he resumed his mathematical researches.

Pascal proposed some problems on the cycloid and one of the respondents was Christopher Wren, better known as the architect of St Paul's Cathedral in London.

Wren proved that the length of a cycloid arch is four times the diameter of the circle that generates it. Today, this is an easy problem in integral calculus but in 1658 it was a formidable achievement.

In 1696, Johann Bernoulli posed a problem that he called the brachistochrone – or shortest time – problem: find the path along which gravity brings a mass from one point to another one not directly below it. The five mathematicians who responded included Newton, Leibniz and Johann's brother Jakob. The desired path is a cycloid.

The story goes that Newton received the problem one evening upon returning from the Royal Mint, where he was master. He stayed up late working on it and by 4am he had obtained a solution, which he mailed that morning. Although his solution was anonymous, Bernoulli perceived its authority and brilliance, giving his reaction in the classic phrase "*ex ungue leonem*", the lion is recognised from his claw.

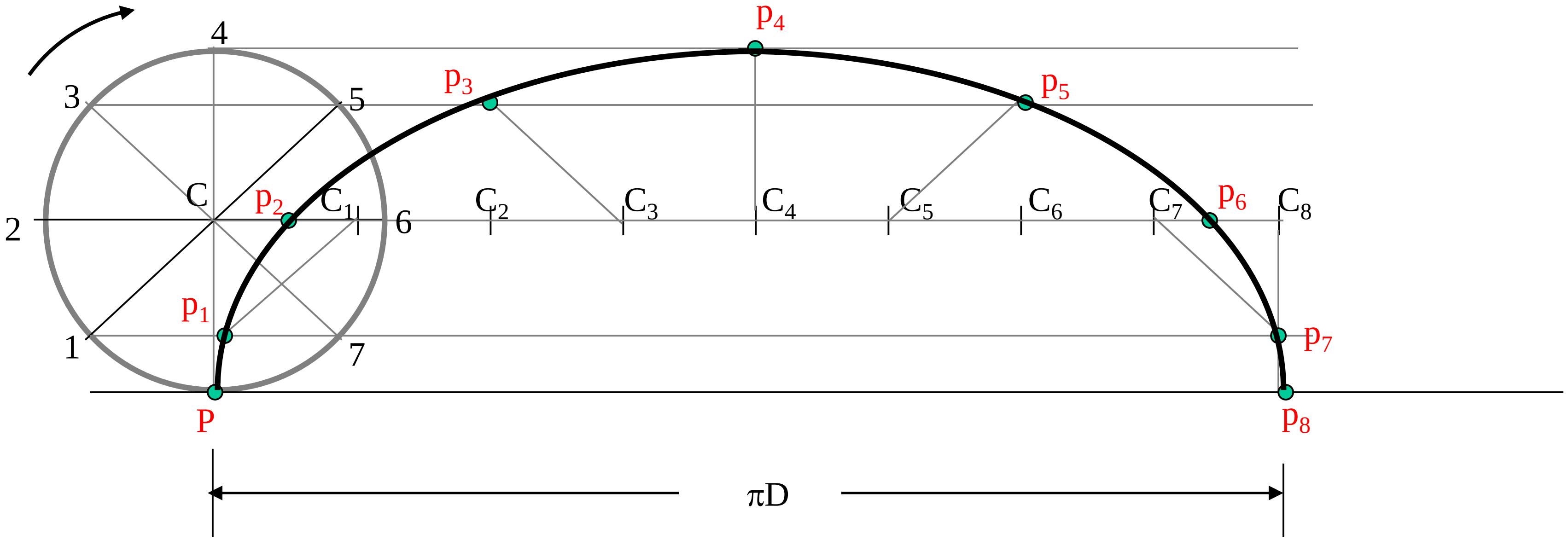
Cycloid arches have been used in some modern buildings, a notable example being the Kimbell Art Museum in Fort Worth, Texas, designed by the renowned architect Louis I Kahn.

Parallel units

In the atmosphere the rotation of the Earth generates cycloidal motion: icebergs and floating buoys have been seen to trace multiple loops of a prolate cycloid. Finally, epicycloids and hypocycloids are used in modern gear systems as they provide good contact between meshed gear teeth giving efficient energy transmission.

PROBLEM : DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON STRAIGHT LINE PATH. Take Circle diameter as 50 mm

CYCLOID



Solution Steps:

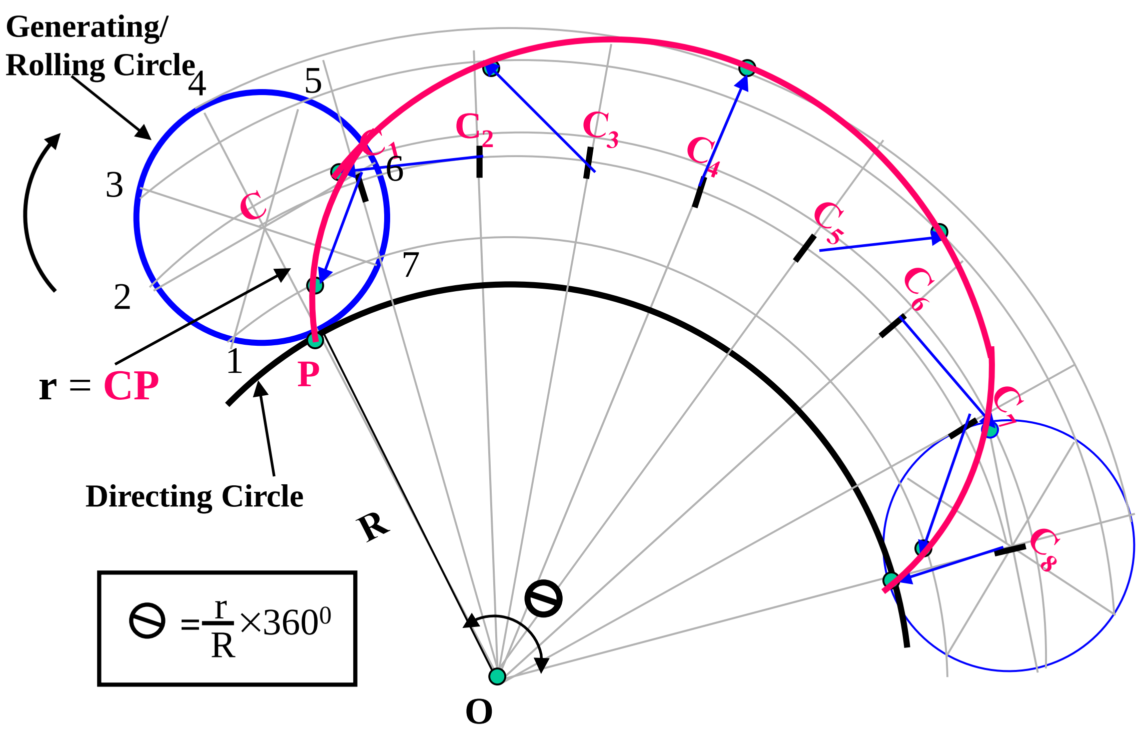
1) From center C draw a horizontal line equal to πD distance.
2) Divide πD distance into 8 number of equal parts and name them C1, C2, C3__ etc.
3) Divide the circle also into 8 number of equal parts and in clock wise direction, after P name 1, 2, 3 up to 8.
4) From all these points on circle draw horizontal lines. (parallel to locus of C)
5) With a fixed distance C-P in compass, C1 as center, mark a point on horizontal line from 1. Name it P.
6) Repeat this procedure from C2, C3, C4 upto C8 as centers. Mark points P2, P3, P4, P5 up to P8 on the horizontal lines drawn from 2, 3, 4, 5, 6, 7 respectively.
7) Join all these points by curve. **It is Cycloid**.

PROBLEM : DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON A CURVED PATH. **Take diameter of rolling Circle 50 mm And radius of directing circle i.e. curved path, 75 mm.**

EPI CYCLOID :

Solution Steps:

1) When smaller circle will roll on larger circle for one revolution it will cover Π D distance on arc and it will be decided by included arc angle θ.
2) Calculate θ by formula $\theta = (r/R) \times 360$ degrees.
3) Construct angle θ with radius OC and draw an arc by taking O as center OC as radius and form sector of angle θ.
4) Divide this sector into 8 number of equal angular parts. And from C onward name them C1, C2, C3 up to C8.
5) Divide smaller circle (Generating circle) also in 8 number of equal parts. And next to P in clockwise direction name those 1, 2, 3, up to 8.
6) With O as center, O-1 as radius draw an arc in the sector. Take O-2, O-3, O-4, O-5 up to O-8 distances with center O, draw all concentric arcs in sector. Take fixed distance C-P in compass, C1 center, cut arc of 1 at P1. Repeat procedure and locate P2, P3, P4, P5 unto P8 (as in cycloid) and join them by smooth curve. This is EPI – CYCLOID.

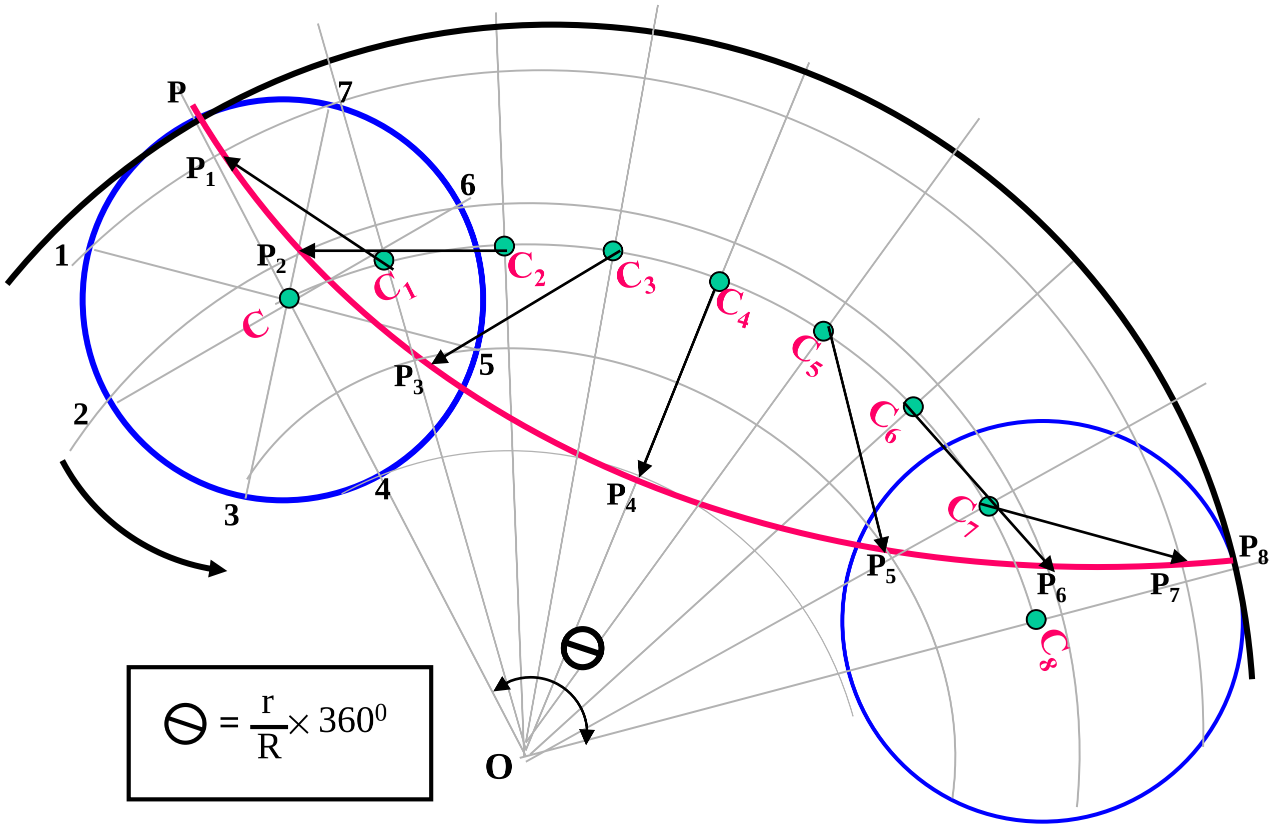


PROBLEM : DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS FROM THE INSIDE OF A CURVED PATH. **Take diameter of rolling circle 50 mm and radius of directing circle (curved path) 75 mm.**

HYPO CYCLOID

Solution Steps:

1) Smaller circle is rolling here, inside the larger circle. It has to rotate anticlockwise to move ahead.
2) Same steps should be taken as in case of EPI – CYCLOID. Only change is in numbering direction of 8 number of equal parts on the smaller circle.
3) From next to P in anticlockwise direction, name 1,2,3,4,5,6,7,8.
4) Further all steps are that of epi – cycloid. **This is called HYPO – CYCLOID.**



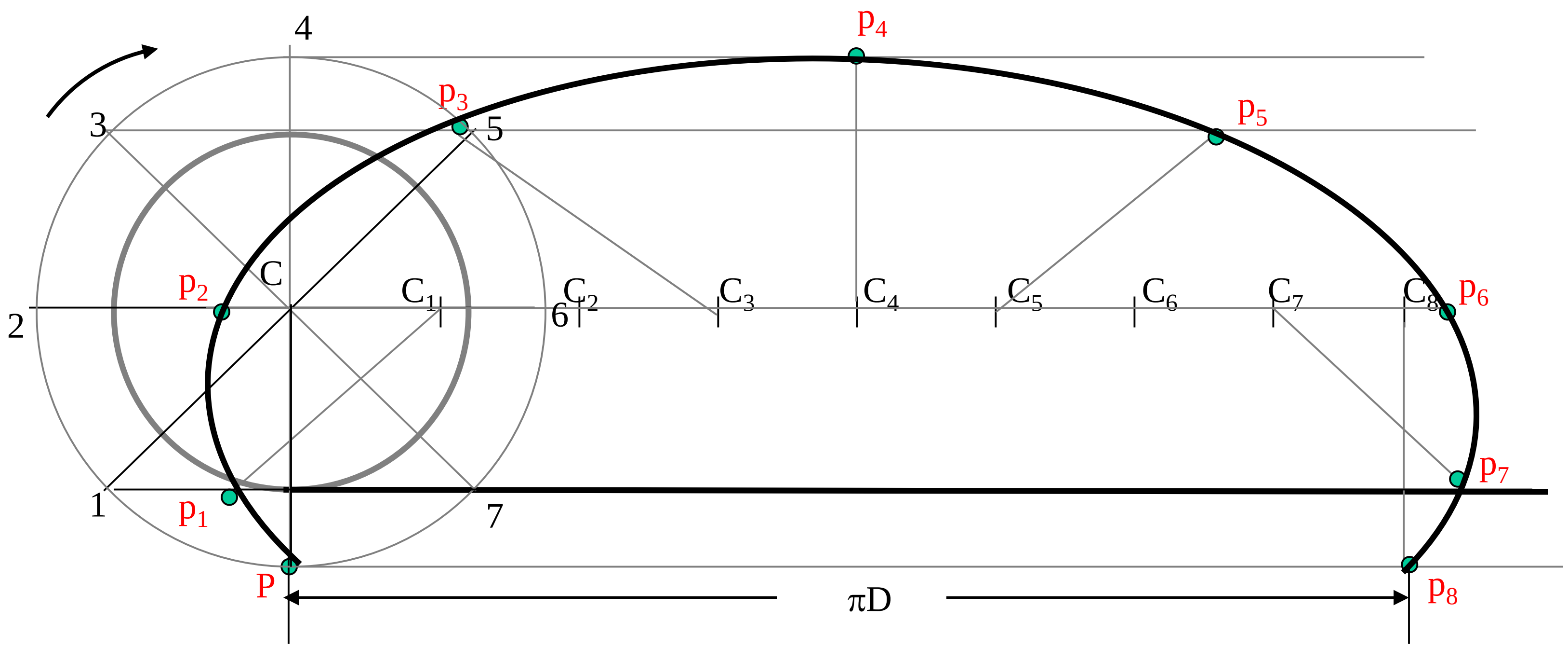
$$\theta = \frac{r}{R} \times 360^0$$

OC = R (Radius of Directing Circle)

CP = r (Radius of Generating Circle)

PROBLEM : DRAW LOCUS OF A POINT , 5 MM AWAY FROM THE PERIPHERY OF A CIRCLE WHICH ROLLS ON STRAIGHT LINE PATH. Take Circle diameter as 50 mm

SUPERIOR TROCHOID

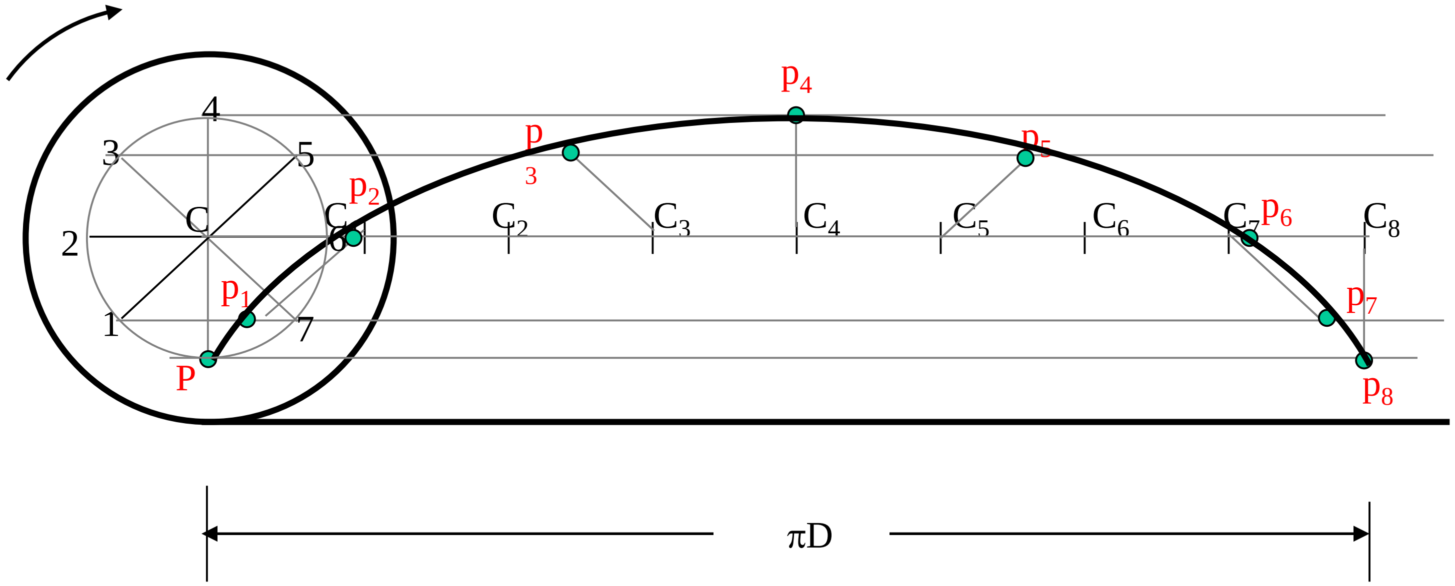


Solution Steps:

1) Draw circle of given diameter and draw a horizontal line from it's center C of length Π D and divide it in 8 number of equal parts and name them C1, C2, C3, up to C8.
2) Draw circle by CP radius, as in this case CP is larger than radius of circle.
3) Now repeat steps as per the previous problem of cycloid, by dividing this new circle into 8 number of equal parts and drawing lines from all these points parallel to locus of C and taking CP radius wit different positions of C as centers, cut these lines and get different positions of P and join
4) This curve is called **Superior Trochoid**.

PROBLEM : DRAW LOCUS OF A POINT , 5 MM INSIDE THE PERIPHERY OF A CIRCLE WHICH ROLLS ON STRAIGHT LINE PATH. Take Circle diameter as 50 mm

INFERIOR TROCHOID

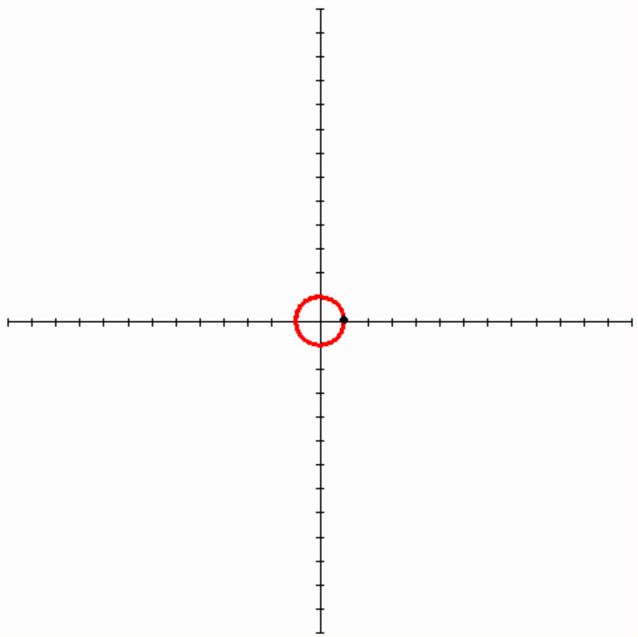


Solution Steps:

1) Draw circle of given diameter and draw a horizontal line from it's center C of length Π D and divide it in 8 number of equal parts and name them C1, C2, C3, up to C8.
2) Draw circle by CP radius, as in this case CP is SHORTER than radius of circle.
3) Now repeat steps as per the previous problem of cycloid, by dividing this new circle into 8 number of equal parts and drawing lines from all these points parallel to locus of C and taking CP radius with different positions of C as centers, cut these lines and get different positions of P and join those in curvature.
4) This curve is called **Inferior Trochoid**.

INVOLUTE:

It is a locus of a free end of a string when it is wound round a circular pole

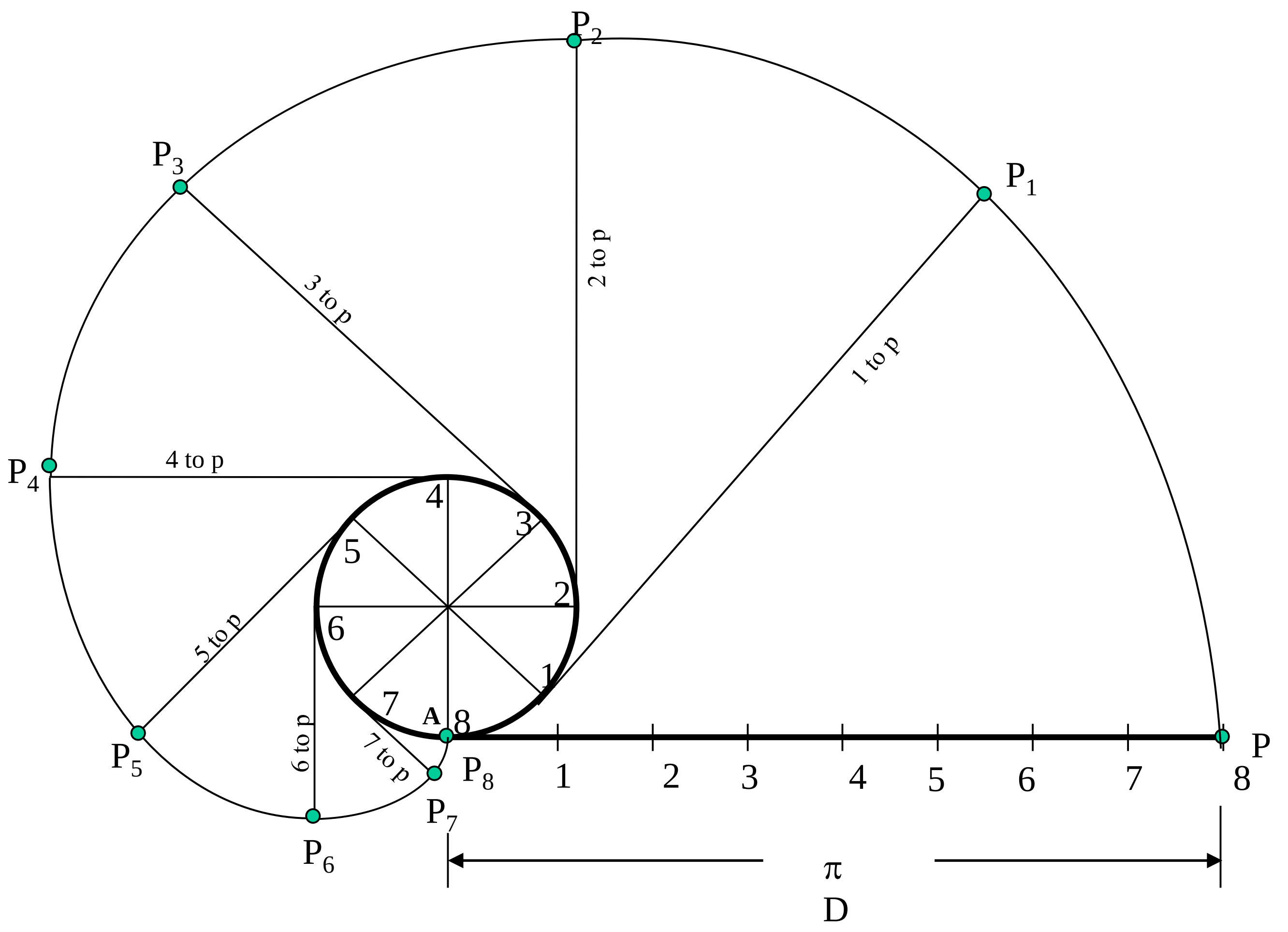


Problem no : Draw Involute of a circle.
String length is equal to the circumference of circle.

INVOLUTE OF A CIRCLE

Solution Steps:

1) Point or end P of string AP is exactly πD distance away from A. Means if this string is wound round the circle, it will completely cover given circle. B will meet A after winding.
2) Divide πD (AP) distance into 8 number of equal parts.
3) Divide circle also into 8 number of equal parts.
4) Name after A, 1, 2, 3, 4, etc. up to 8 on πD line AP as well as on circle (in anticlockwise direction).
5) To radius C-1, C-2, C-3 up to C-8 draw tangents (from 1,2,3,4,etc to circle).
6) Take distance 1 to P in compass and mark it on tangent from point 1 on circle (means one division less than distance AP).
7) Name this point P1
8) Take 2-B distance in compass and mark it on the tangent from point 2. Name it point P2.
9) Similarly take 3 to P, 4 to P, 5 to P up to 7 to P distance in compass and mark on respective tangents and locate P3, P4, P5 up to P8 (i.e. A) points and join them in smooth curve it is an INVOLUTE of a given circle.



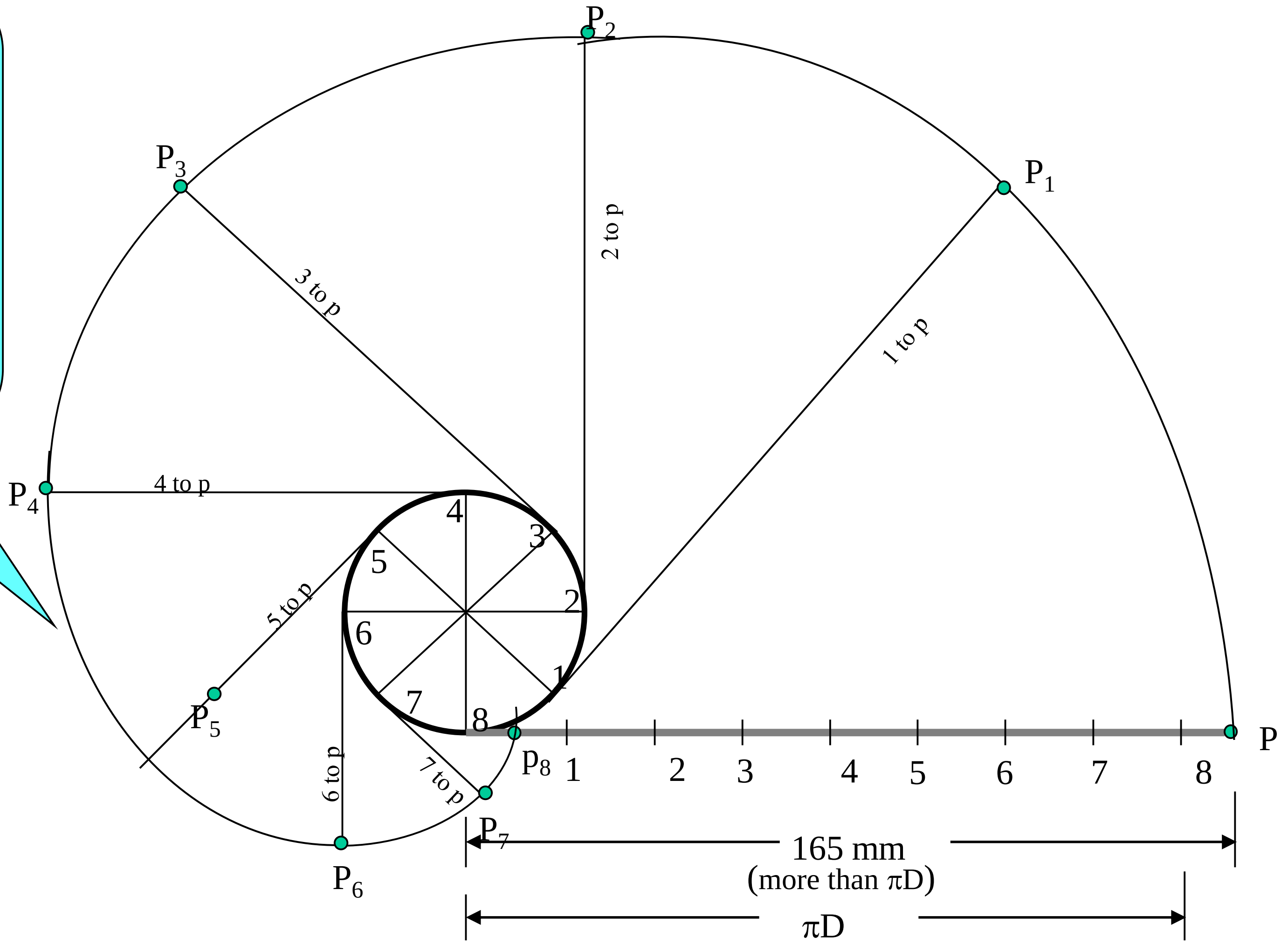
INVOLUTE OF A CIRCLE

String length MORE than πD

Problem : Draw Involute of a circle.
String length is MORE than the circumference of circle.

Solution Steps:
In this case string length is more than Π D.

But remember!
Whatever may be the length of string, mark Π D distance horizontal i.e.along the string and divide it in 8 number of equal parts, and not any other distance. Rest all steps are same as previous INVOLUTE. Draw the curve completely.



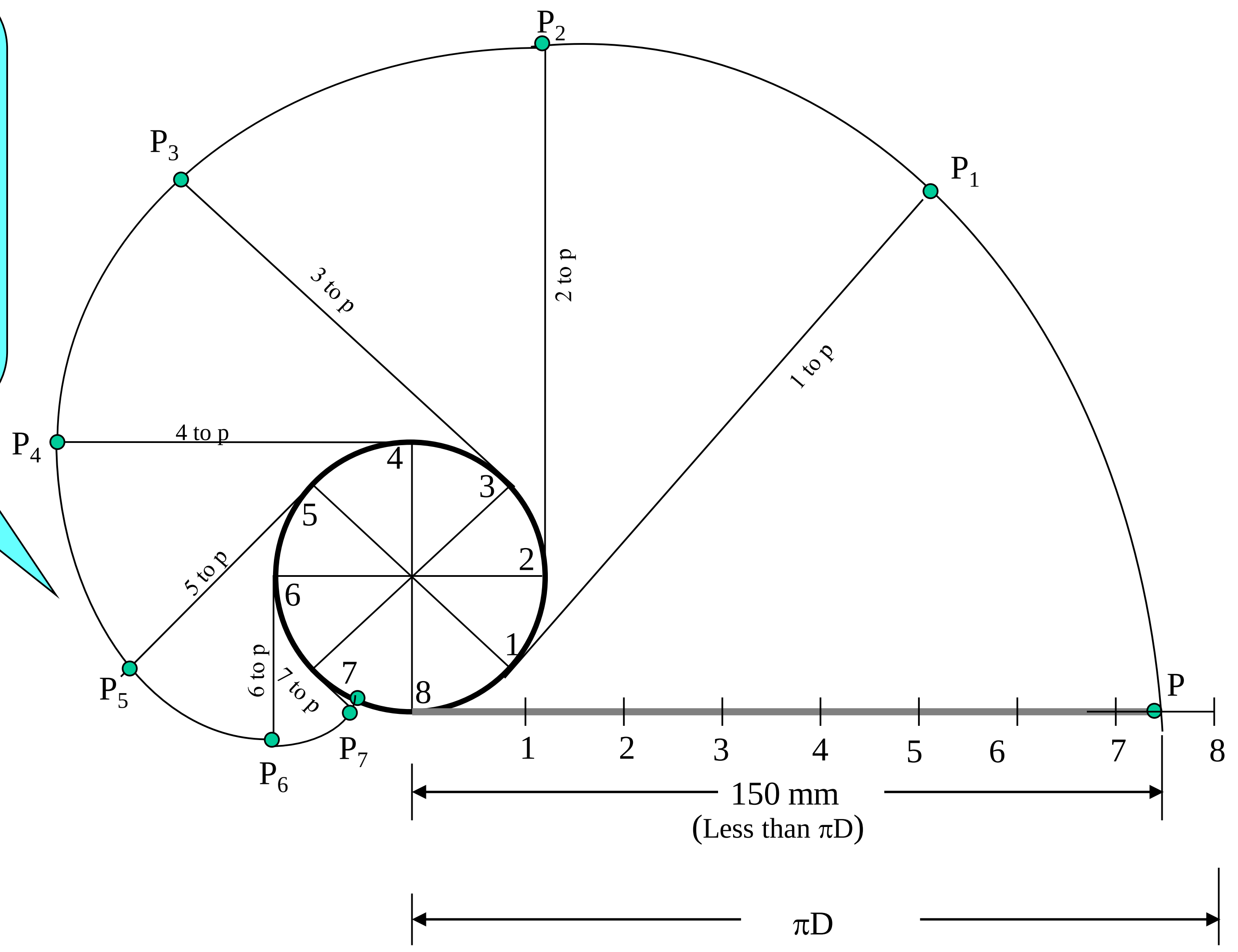
Problem : Draw Involute of a circle.
String length is LESS than the circumference of circle.

INVOLUTE OF A CIRCLE

String length LESS than πD

Solution Steps:
In this case string length is Less than Π D.

But remember!
Whatever may be the length of string, mark Π D distance horizontal i.e.along the string and divide it in 8 number of equal parts, and not any other distance. Rest all steps are same as previous INVOLUTE. Draw the curve completely.

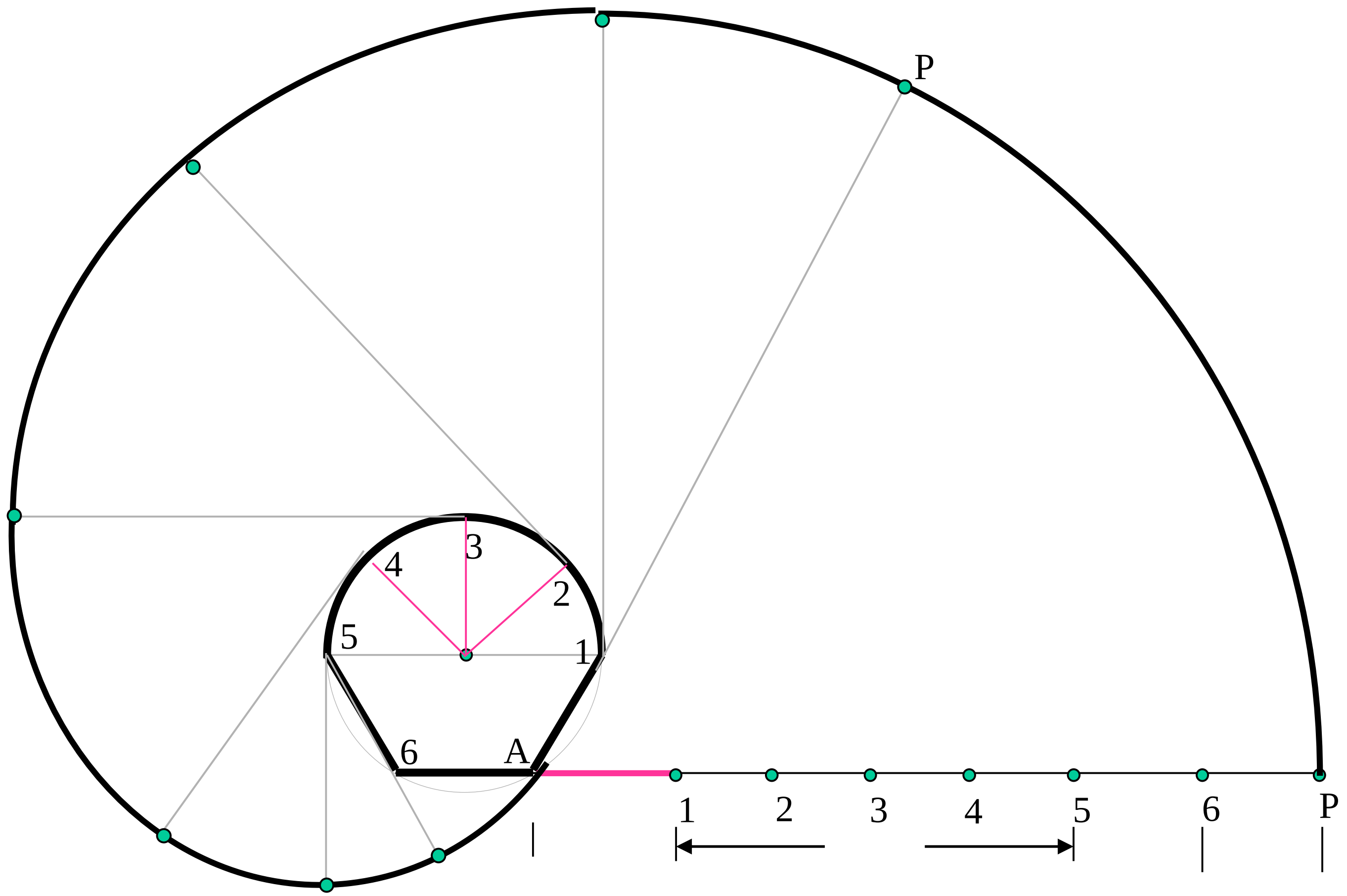


PROBLEM : A POLE IS OF A SHAPE OF HALF HEXABON AND SEMICIRCLE. ASTRING IS TO BE WOUND HAVING LENGTH EQUAL TO THE POLE PERIMETER DRAW PATH OF FREE END *P* OF STRING WHEN WOUND COMPLETELY.
(Take hex 30 mm sides and semicircle of 60 mm diameter.)

INVOLUTE OF COMPOSIT SHAPED POLE

SOLUTION STEPS:

Oops!! Similar problem will there be in the tutorial.. :P :P

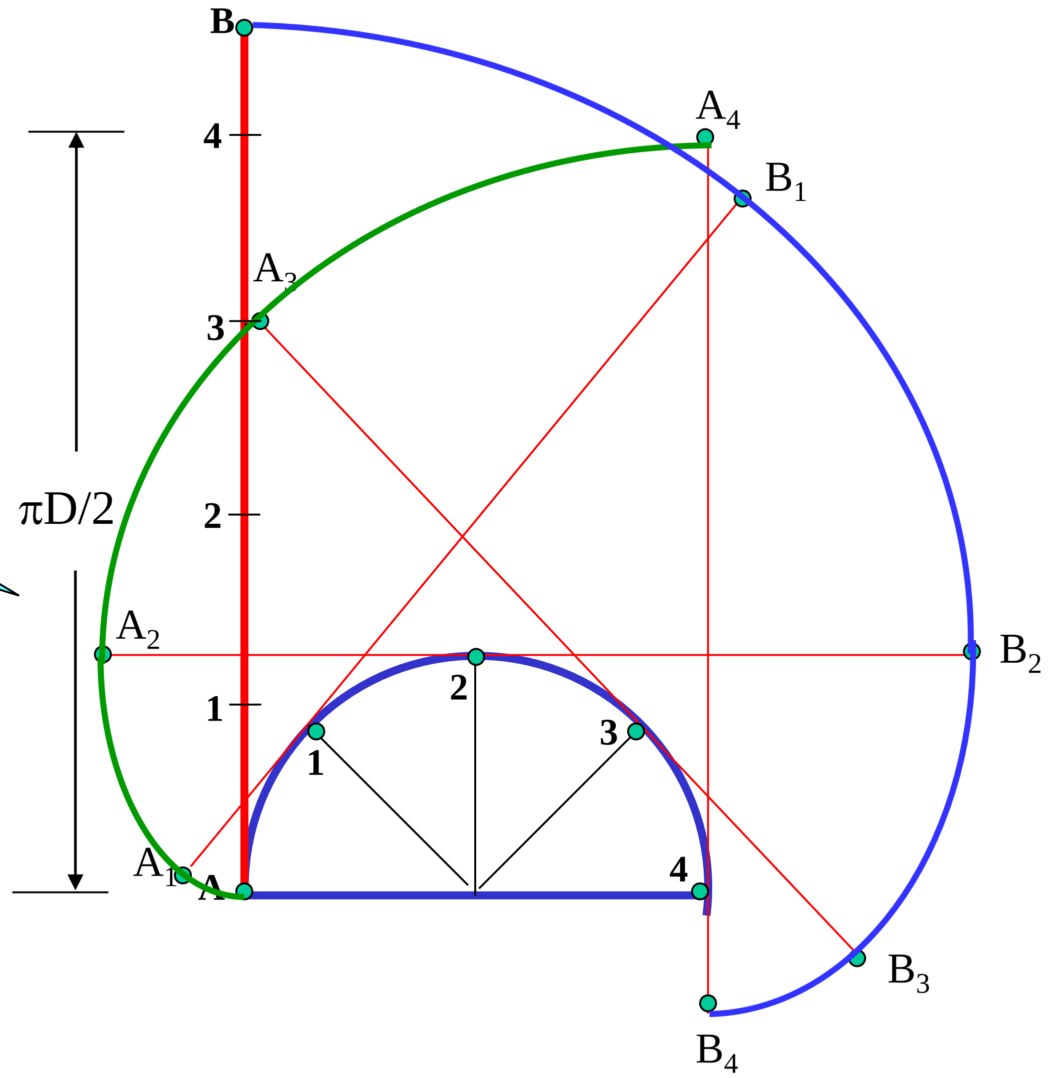


PROBLEM : Rod AB 85 mm long rolls over a semicircular pole without slipping from it's initially vertical position till it becomes up-side-down vertical.
Draw locus of both ends A & B.

Solution Steps?

If you have studied previous problems properly, you can surely solve this also.
Simply remember that this being a rod, it will roll over the surface of pole.
Means when one end is approaching, other end will move away from poll.

OBSERVE ILLUSTRATION CAREFULLY!



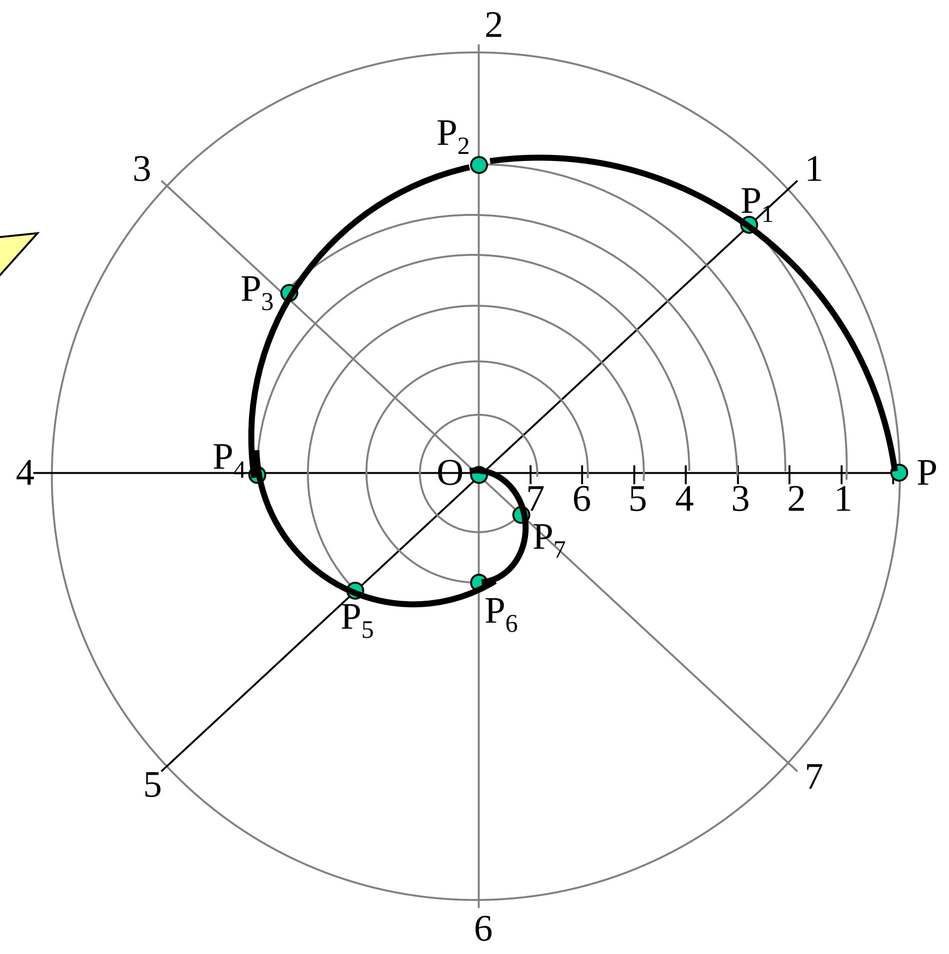
Problem: Draw a spiral of one convolution. Take distance PO 40 mm.

SPIRAL

IMPORTANT APPROACH FOR CONSTRUCTION!
FIND TOTAL ANGULAR AND TOTAL LINEAR DISPLACEMENT AND DIVIDE BOTH IN TO SAME NUMBER OF EQUAL PARTS.

Solution Steps

1. With PO radius draw a circle and divide it in EIGHT parts. Name those 1,2,3,4, etc. up to 8
2. .Similarly divided line PO also in EIGHT parts and name those 1,2,3,-- as shown.
3. Take o-1 distance from op line and draw an arc up to O1 radius vector. Name the point P_1
4. Similarly mark points P_2, P_3, P_4 up to P_8
   And join those in a smooth curve.
   It is a SPIRAL of one convolution.

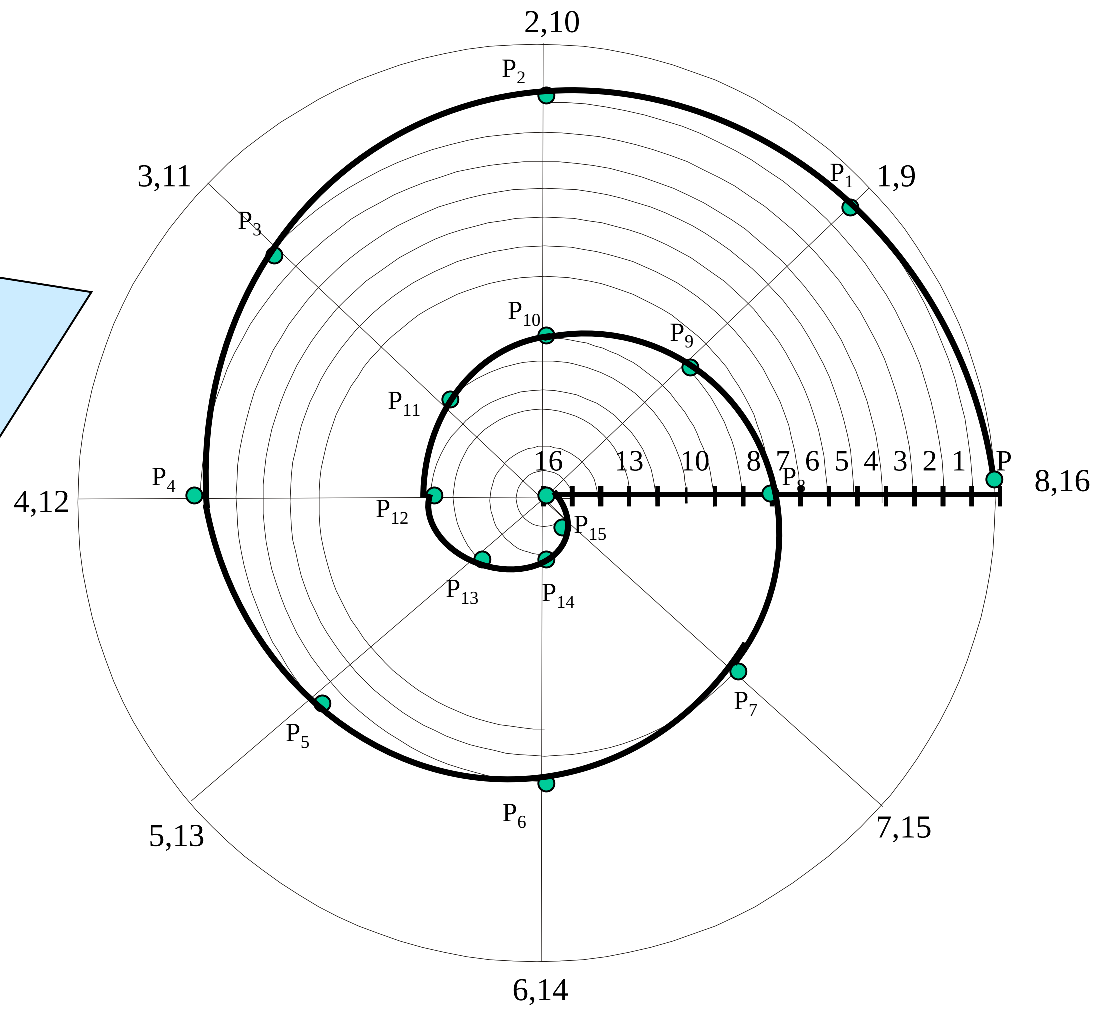


Problem :
Point P is 80 mm from point O. It starts moving towards O and reaches it in two revolutions around.it Draw locus of point P (To draw a Spiral of TWO convolutions).

SPIRAL of two convolutions

IMPORTANT APPROACH FOR CONSTRUCTION!
FIND TOTAL ANGULAR AND TOTAL LINEAR DISPLACEMENT AND DIVIDE BOTH IN TO SAME NUMBER OF EQUAL PARTS.

SOLUTION STEPS:
Total angular displacement here is two revolutions And Total Linear displacement here is distance PO.
Just divide both in same parts i.e. Circle in EIGHT parts.
(means total angular displacement in SIXTEEN parts)
Divide PO also in SIXTEEN parts.
Rest steps are similar to the previous problem.



STEPS:
DRAW INVOLUTE AS USUAL.

MARK POINT **Q** ON IT AS DIRECTED.

JOIN **Q** TO THE CENTER OF CIRCLE **C**.
CONSIDERING **CQ** DIAMETER, DRAW
A SEMICIRCLE AS SHOWN.

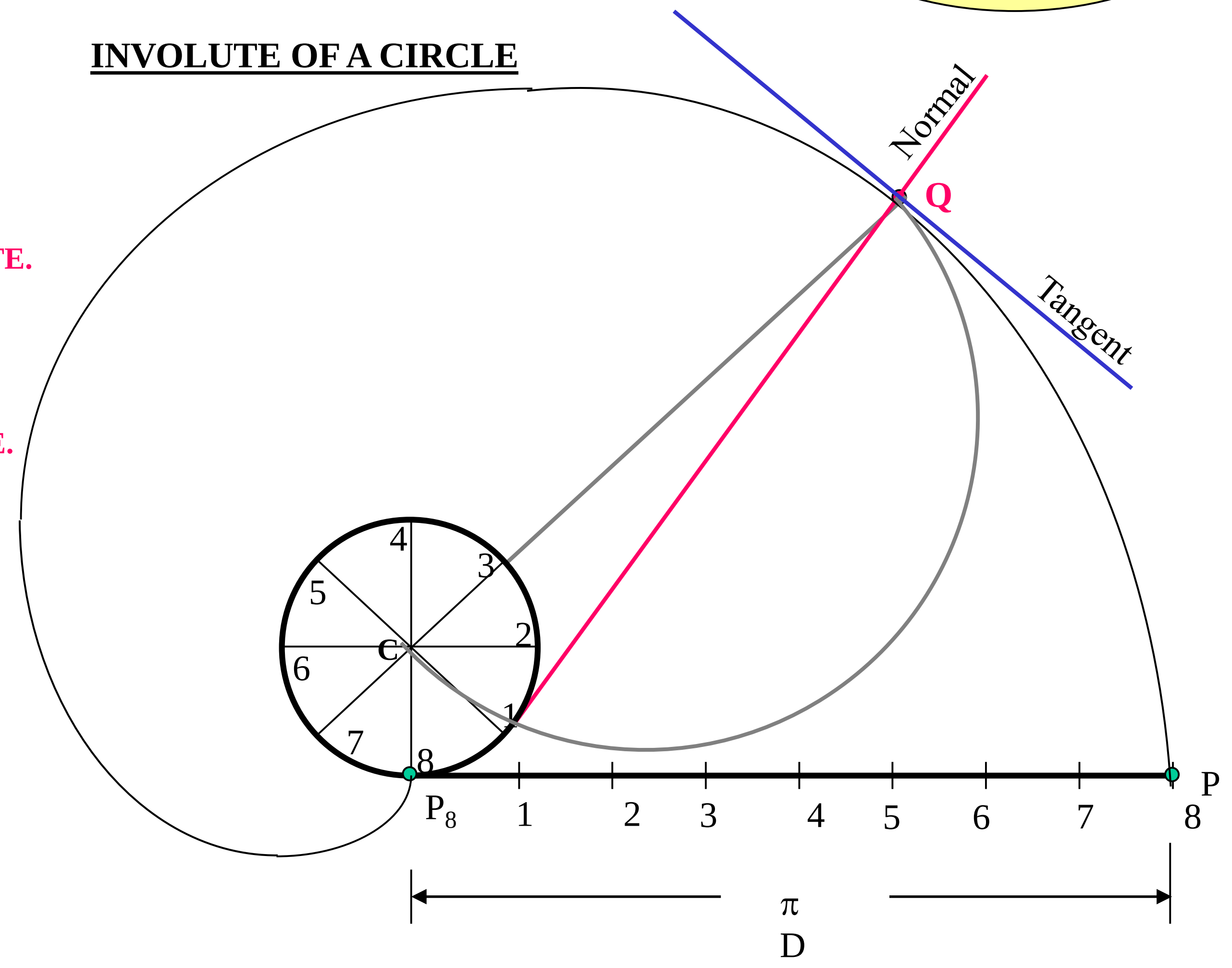
MARK POINT OF INTERSECTION OF
THIS SEMICIRCLE AND POLE CIRCLE
AND JOIN IT TO **Q.**

THIS WILL BE **NORMAL TO INVOLUTE.**

DRAW A LINE AT RIGHT ANGLE TO
THIS LINE FROM **Q.**

IT WILL BE TANGENT TO INVOLUTE.

Involute Method of Drawing Tangent & Normal



CYCLOID
Method of Drawing Tangent & Normal

STEPS:

DRAW CYCLOID AS USUAL.
MARK POINT **Q** ON IT AS DIRECTED.

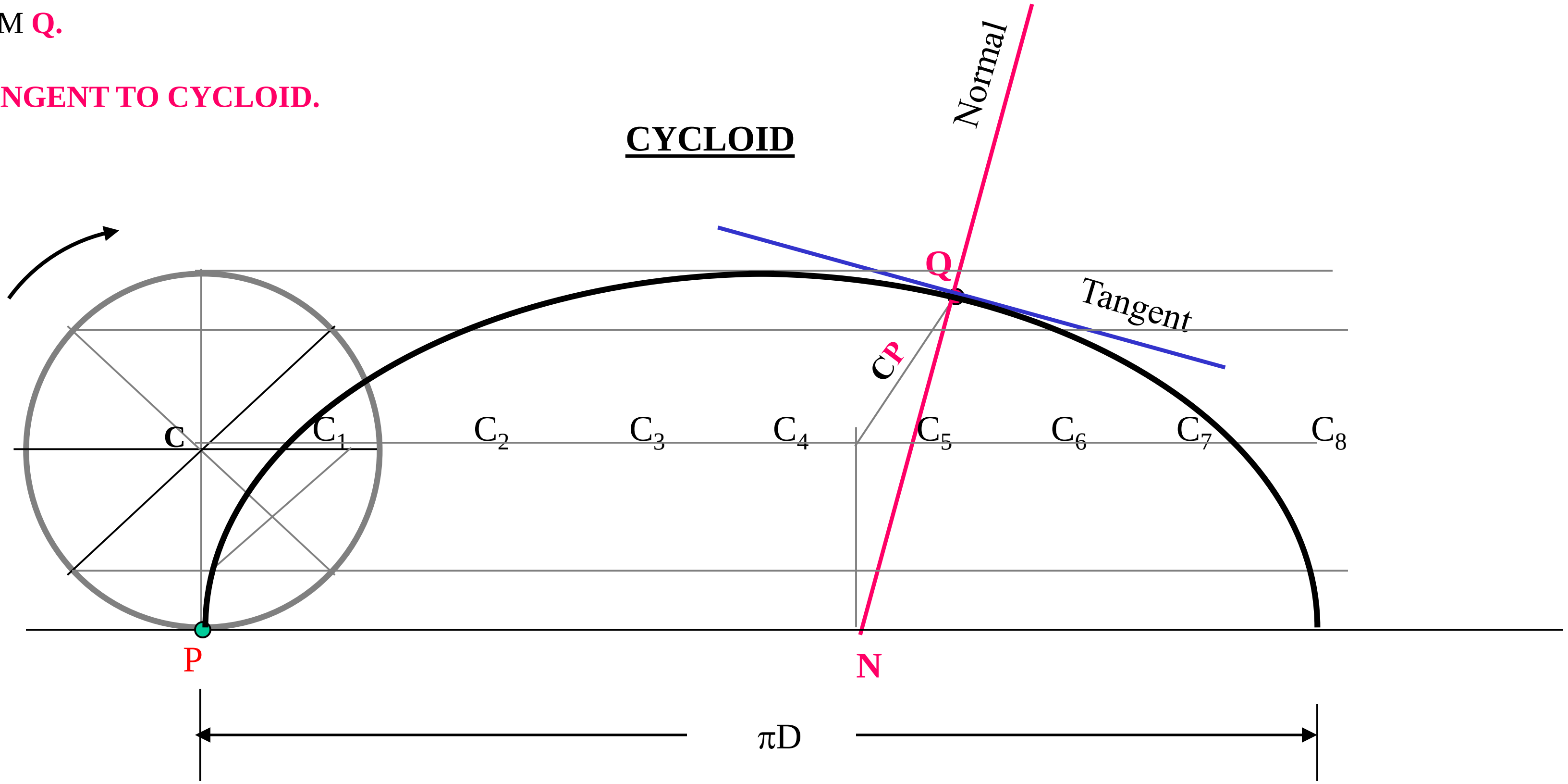
WITH CP DISTANCE, FROM **Q.** CUT THE POINT ON LOCUS OF **C** AND JOIN IT TO **Q.**

FROM THIS POINT DROP A PERPENDICULAR ON GROUND LINE AND NAME IT N

JOIN N WITH Q.THIS WILL BE **NORMAL TO CYCLOID.**

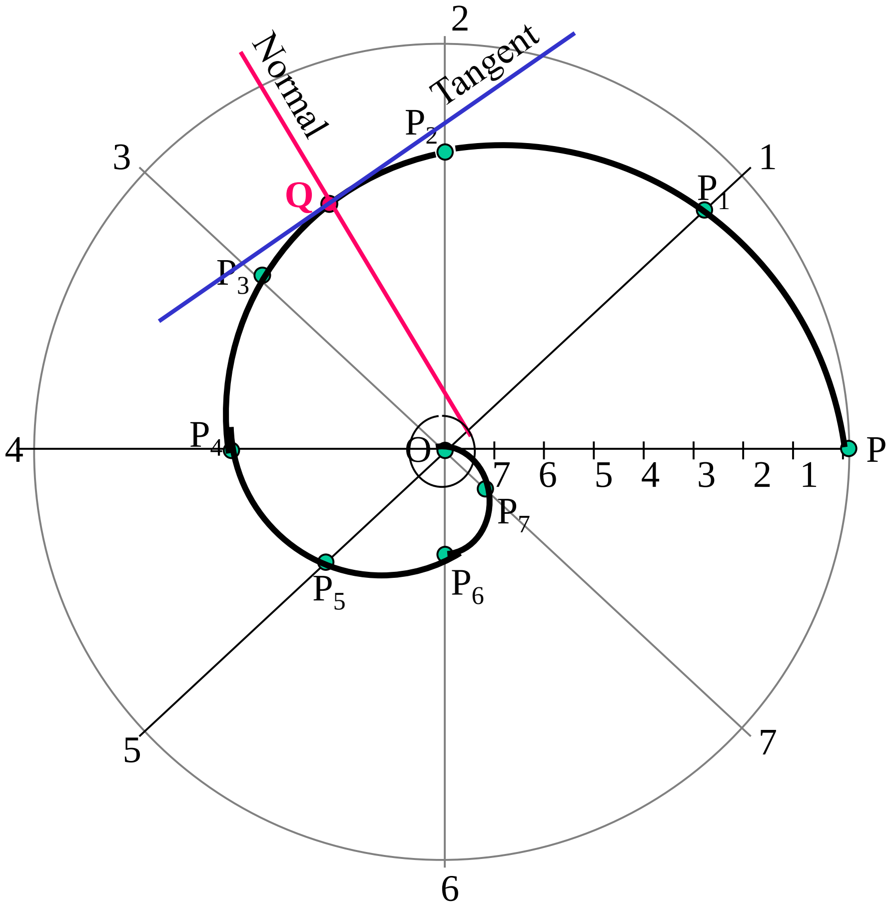
DRAW A LINE AT RIGHT ANGLE TO THIS LINE FROM **Q.**

IT WILL BE TANGENT TO CYCLOID.



Spiral.
Method of Drawing Tangent & Normal

SPIRAL (ONE CONVOLUSION.)



$$\text{Constant of the Curve} = \frac{\text{Difference in length of any radius vectors}}{\text{Angle between the corresponding radius vector in radian.}}$$

$$= \frac{OP - OP_2}{\pi/2} = \frac{OP - OP_2}{1.57}$$

$$= 3.185 \text{ m.m.}$$

STEPS:

*DRAW SPIRAL AS USUAL.
DRAW A SMALL CIRCLE OF RADIUS EQUAL TO THE CONSTANT OF CURVE CALCULATED ABOVE.

* LOCATE POINT **Q** AS DISCRIBED IN PROBLEM AND THROUGH IT DRAW A TANGENTTO THIS SMALLER CIRCLE.THIS IS A **NORMAL** TO THE SPIRAL.

*DRAW A LINE AT RIGHT ANGLE

*TO THIS LINE FROM **Q.**
IT WILL BE TANGENT TO CYCLOID.