भारतीय प्रौद्योगिकी संस्थान पटना

INDIAN INSTITUTE OF TECHNOLOGY PATNA

PH101 (Physics-I) **Mid-Semester Examination (September 14, 2015)**

[Full Marks: 40] **[Time: 120 minutes]**

• All the questions are compulsory. • Answers must be to the point (refrain from writing essays!. • Answers to all parts of a given question must be answered together. • Marks for the questions are given in bold within square brackets.

1. Fill in the blanks using most suitable words/short phrases/mathematical expressions (wherever applicable): **[10]**
(a) In the case of a mass moving under a central force $\vec{F}(\vec{r})$ given by $\vec{F}(\vec{r}) = -\nabla V(\vec{r})$, the effective potential $V_{eff}(\vec{r})$ is given by __________.
(b) Elemental volume $d\tau$ in spherical polar coordinate system is given by __________.
(c) For a conservative force $\vec{F}(\vec{r})$ defined by $\vec{F}(\vec{r}) = -\nabla V(\vec{r})$, $V(\vec{r})$ satisfies the equation __________.
(d) A chain of mass M and length L is suspended vertically with its lowest end touching a weighing scale. The chain is released and starts falling onto the weighing scale. The reading of the scale when a length of chain, x, has fallen is given by __________.
(e) If ω is the angular speed of earth's rotation about its axis, the Coriolis force acting on an object of mass m located at the equator moving towards west with a velocity $\vec{v}$ is __________Newton.
(f) Rotations through finite angles __________.
(g) An overdamped oscillator can cross the origin __________.
(h) The principal axes of a system composed of two point masses m_1 and m_2 connected by a massless rod of length l are located along __________.
(i) The moment of inertia tensor relative to the origin of a mass m located at (2, 1, -3) is given by __________.

2. An object of mass m moves under the influence of a 1-d potential $V(x) = \frac{a}{x^3} - \frac{b}{x^2}$, where $a,\ b > 0$.
(a) Sketch the potential, marking the locations of maxima/minima (if any). Also mark the equilibrium point.
(b) Obtain frequency of small oscillations about the equilibrium point.
(c) What happens when $a,\ b < 0$? Explain using a suitable plot. **[5]**

3. For a damped harmonic oscillator, $m\ddot{x} = -\alpha x - \beta\dot{x}$, or alternatively, $\ddot{x} + 2\Gamma\dot{x} + \Omega^2 x = 0$, where, $\alpha = m\Omega^2$ and $\beta = 2m\Gamma$.
(a) Illustrate graphically the difference between underdamped, overdamped and the critically damped motions.
(b) Show that $\frac{dU}{dt} = -2m\Gamma\dot{x}^2$, where, U is the total energy. **[5]**

4. In free space, a single stage rocket of initial mass M, of which $M - m_{\mathrm{R}}$ is fuel, burns its fuel at a constant rate k and ejects the exhaust gases with constant speed u. The rocket starts from rest and moves through a medium that exerts the resistance force ϵkv, where v is the forward velocity of the rocket, and ϵ is a small positive constant. The Rocket equation in this case is given by, $m\frac{dv}{dt} = -\dot{m}u - \epsilon kv$.
(a) Show that the maximum speed attained by the rocket is given by $v_{max} = \frac{u}{\epsilon}[1 - \gamma^{-\epsilon}]$, where, $\gamma = \frac{M}{m_{\mathrm{R}}}$.
(b) Find the maximum speed v_{max} achieved by the rocket if the initial mass of the rocket is 50 Tonnes of which 49.5 Tonnes is fuel with $u = 1000\,m/s$ and $\epsilon = 0.01$. **[5]**

5. The Euler's equations of motion can be written in a concise form as $\vec{\tau} = \frac{d\vec{L}}{dt} = \bar{I}\,\dot{\vec{\omega}} + \vec{\omega} \times \vec{L}$.
(a) Explain the physical significance of each of the terms and the condition under which the above equation is valid for a rigid body.
(b) Write down the three Euler's equations (which correspond to the individual components of the above equation).
(c) Using Euler's equations, prove the following statement: "The rotational motion of a rigid body is stable about the axis about which the moment of inertia is either a maximum or a minimum". **[5]**

6. Using a method of your choice, show that the first (amplitude dependent) correction term to the time period of a simple pendulum of mass m and length l is given by $2\pi\sqrt{\frac{l}{g}} \cdot \frac{\theta_0^2}{16}$, where, θ_0 is the amplitude of oscillations and g is the acceleration due to gravity. **[5]**

7. A particle of mass m which moves in a plane is acted upon by a force $\vec{F} = m\dot{r}\dot{\theta}\hat{\theta}$. Show that $\dot{r} = \sqrt{A\ln r + B}$, where A and B are constants which can be determined from initial conditions of motion. **[5]**

END OF EXAM PAPER