Indian Institute of Technology Patna
MA102: Mathematics II
B. Tech. I Year (Spring Semester)
End Semester Examination April- 2016

Time: 3 hours | Total Marks: 50

<u>Note:</u> There are **Five** questions. Attempt all the questions. Give precise and brief answer. Notations have their usual meaning. Standard formulae may be used.

Q1. a. Prove or disprove the following statements:

(i). If $A = [a_{ij}]$ is skew symmetric, then $a_{jj} = 0$ for each j.

(ii). If $A = [a_{ij}]$ is skew Hermitian, then a_{jj} is purely imaginary for each j.

(iii). If A is real and symmetric, then $B = iA$ is skew Hermitian. [2+2+2=6]

b. Suppose that V is a finite dimensional vector space and S and T are linear maps from V to V. Prove that $ST = I$ if and only if $TS = I$. [2]

c. In a real inner product space, show that $(x + y)$ is orthogonal to $(x - y)$ if and only if $||x|| = ||y||$. [2]

Q2. a. Solve the differential equation
$(2x + \tan y)dx + (x - x^2 \tan y)dy = 0.$ [3]

b. Solve the differential equation
$(8x^2y^3 - 2y^4)dx + (5x^3y^2 - 8xy^3)dy = 0.$ [4]

c. Prove that the necessary condition for the differential equation $M(x, y)dx+N(x, y)dy = 0$ to be an exact differential equation is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. [2]

Q3. a. Use <u>Undetermined Coefficients Method</u> to solve $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = \sin x$. [3]

b. Use <u>D-Operator Method</u> to solve $(D - 1)^2(D^2 + 1)^2y = \sin x$. [4]

c. Use <u>Variation of Parameters Method</u> to solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 4\ln(x)$. [4]

Q4. a. Find the set of ordinary points, regular singular points and irregular singular points of the following differential equation

$$(x^5 + x^4 - 6x^3)\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + (x-2)y = 0.$$

[3]

OR

a. Show that e^x is a part of complementary function of the differential equation

$$xy'' - (2x+1)y' + (x+1)y = (x^2 + x - 1)e^{2x}.$$

Also find its complete solution. [3]

b. Find a **Power Series Solution** of the differential equation

$$(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2xy = 0,$$

at $x = 0$. [6]

Q5. a. $J_n(x)$ is a Bessel function of first kind and order n, prove that $J_n'(x) = \frac{1}{2}\left\{J_{(n-1)}(x) - J_{(n+1)}(x)\right\}$. [4]

OR

a (i). Solve the following differential equation by reducing into Clairaut's form:

$$e^{3x}(p-1) + p^3 e^{2y} = 0.$$

a (ii). Solve $x^2p^2 + xyp - 6y^2 = 0$, where $p = \frac{dy}{dx}$. [2+2=4]

b. If $P_n(x)$ denotes Legendre's polynomial of degree n. Prove that

$$(2n+1)xP_n(x) = (n+1)P_{(n+1)}(x) + nP_{(n-1)}(x).$$

[4]

c. Obtain the approximate solution of the following problem by using Picard's method upto third iterations:

$$\frac{dy}{dx} = 2x + y^3, \quad y(0) = 0.$$

[3]
