

Indian Institute of Technology Patna  
MA-225: B.Tech. II year  
Spring Semester: 2018-19  
Mid Semester Examination

Maximum Marks: 30

Total Time: 2 Hours

**Note:** Answer all eight questions. You can use scientific calculator.

1. Consider the function  $f_X(x) = b(2x - x^3)$ ,  $0 < x < \frac{2}{5}$ , otherwise 0. Then find  $b$  so that it is a valid density function. Also compute the conditional probability  $P(X < 0.2 \mid X > 0.1)$ . [1+2]
2. A large lot of tires contains 5% defectives. Suppose 4 good tires are to be chosen from the lot for a car. Determine the probability of finding at least 2 defective tires before 4 good ones are selected. Compute the expected number of defective tires before finding 4 good tires. Compute the moment generating function of the corresponding distribution. (derive expressions for mean and mgf) [1+2+2]
3. Let  $X$  be a random variable which represents the height of a group of population in a city. Suppose that  $X$  has mean value 129 cm and standard deviation 19.8 cm. Using the Chebyshev inequality, obtain a lower bound on the probability that height of this population will take values between 89.4 and 168.6 cm. Further if this  $X$  is normally distributed then obtain the corresponding probability in terms of the standard normal CDF. [1.5+1.5]
4. (a) State and prove the Bayes theorem. [2]  
(b) Is it possible that an event is independent of itself? If so, when is this the case? [1]
5. The future lifetime  $X$  for a particular product can be described using the probability function  $P(X > x) = e^{-(x/5)^3}$  for  $x > 0$ , otherwise 0; also  $x$  is measured in years. Compute the expected life of such a product. Determine the conditional probability that a product will survive less than 10 years given that it is in working condition for more than 5 years. [2+3]
6. A batch of 100 printed circuit cards is populated with semiconductor chips. Assume that this batch contains 20 defective cards. Now if a random sample of 10 circuit cards is taken out from the batch without replacement for function testing. Then write probability mass function of  $X$  (hypergeometric variable) which denotes the number of defective cards in the selected sample. Determine the probability of getting at least 2 defective circuit cards in the sample. Compute this probability using the binomial random variable approximation. [1+2+2]
7. Let  $X$  follow a normal distribution with both mean and variance being 2. Then find the cumulative distribution function of  $-1.5X - 0.5$ . Also obtain the corresponding moment generating function. Properly write required steps in both the cases. [2+2]
8. Three horses  $x, y$  and  $z$  enter in a race. If an outcome of the form  $yxz$  means that  $y$  finishes first,  $x$  second and  $z$  third then write the sample space. Consider the event  $A$  as ' $x$  finishes before  $y$ ' and  $B$  as ' $x$  finishes before  $z$ '. Do events  $A$  and  $B$  form a partition of the sample space. [1+1]