INDIAN INSTITUTE OF TECHNOLOGY PATNA

MSE, PH 101 (PHYSICS I), Sept. 2013

Time: 2 hrs, Full Marks: 50

(Answer all questions. You should answer according to the marks allotted for each question)

1. i) For three non-coplanar vectors **A, B** and **C**; reciprocal vectors (**A′, B′** and **C′**) are defined as **A.A′=B.B′=C.C′=1.**
   a) Find the expression of the three reciprocal vectors. [2]
   b) Check that **A′.B**=0 and **A′.C**=0 [1]
   c) Prove that if **A.B×C**=V, then **A′.B′×C′**=$1/V$. [2]

   ii) Prove that the diagonals of an equilateral parallelogram are always perpendicular. [2]

2. a) Prove the 'law of sines' using the formula of cross product, where three vectors, **A, B** and **C**, having angles between them α, β, and γ, respectively: [2]
   b) Write down the three most important properties of a conservative force. [1]
   c) A rocket of mass M moves in space with a velocity **v**. Consider the rocket at time t. Between t & $t+\Delta t$ a mass of fuel Δm is burned and expelled as gas with velocity **u** relative to the rocket. **u** is independent of the velocity of the rocket. Derive the expression of the external force. [4]

3. a) N freight cars each having mass M are pulled with a force F by a locomotive (friction is neglected). What is the force exerted on each car? [2]
   b) Find the expression of velocity and acceleration in plane polar coordinate. [2]
   c) Prove Newton's second law for the dynamics of a system of particle. [3]

4. a) Prove that for an elastic collision of two particles, total momentum about the center of mass vanishes. [2]
   b) Show that speed of each particle in center of mass system is same before and after collision and the velocity vectors simply rotate in the scattering plane. [2]
   c) A raindrop of initial mass M_0 starts falling from rest under the influence of gravity. Assume that the drop gains mass from the cloud at a rate proportional to the product of its instantaneous mass and its instantaneous velocity. Show that the speed of the drop eventually becomes effectively constant, and give an expression for the terminal speed. Neglect air resistance. [3]

5. a) Prove that the force is everywhere perpendicular to the constant energy surfaces and points from higher to lower potential energy. [2]
   b) For a potential energy function:

$$U = \epsilon\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right]$$

   i) What are the radius at the potential minimum and the depth of the potential well? [2]
   ii) Find the frequency of small oscillations about equilibrium for two identical atoms of mass m bound to each other inside the above potential. [2]

6. a) Prove that, motion of a body under central force field always lies in a plane. Also show that the area swept out by the body per unit time is constant. [2+2]
   b) Find the angular momentum of a system of particle with both translation and rotation. [3]
   c) For a general rotation with an angular velocity w, around an arbitrary axis, prove that the velocity, **v**=w × **r**, where **r** is the radius vector from the origin of the coordinate system. [2]

7. a) Write down the expression of angular momentum for the general rotation of a rigid body and define inertia tensor. What do you mean by principal moment of inertia? [3+1]
   b) A particle of mass m is located at $x = 2$, $y = 0$, $z = 3$. Find the inertia tensor relative to the origin. [3]
