CS 225: Switching Theory

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Extra Class Time

Monday! Time

Previous Class

Switching Algebra

This Class

- Switching Algebra
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  - Switching circuit
  - Propositional calculus

Evaluating Boolean Equations

- Evaluate the Boolean equation **F = (a AND b) OR (c AND d)** for the given values of variables a, b, c, and d:
  - Q1: a=1, b=1, c=1, d=0.
    - Answer: F = (1 AND 1) OR (1 AND 0) = 1 OR 0 = 1.
  - Q2: a=0, b=1, c=0, d=1.
    - Answer: F = (0 AND 1) OR (0 AND 1) = 0 OR 0 = 0.
  - Q3: a=1, b=1, c=1, d=1.
    - Answer: F = (1 AND 1) OR (1 AND 1) = 1 OR 1 = 1.

a	b	AND
0	0	0
0	1	0
1	0	0
1	1	1

a	b	OR
0	0	0
0	1	1
1	0	1
1	1	1

a	NOT
0	1
1	0

Simplification of Expressions

Example 1: Simplify $T(A,B,C,D) = A'C' + ABD + BC'D + AB'D' + ABCD'$

- Apply consensus theorem $A'C' + ABD + BC'D = A'C' + ABD$
- $T = A'C' + ABD + AB'D' + ABCD'$ [place as $x=A'$, $y=C'$, $z=BD$]
- Apply distributive law: $AD'(B' + BC) \rightarrow AD'(B' + C)$

- Thus, $T = A'C' + A[BD + D'(B' + C)]$

Example 2: Simplify $T(A,B,C,D) = A'B + ABD + AB'CD' + BC$

$A'B + ABD = B(A' + AD) = B(A' + D)$

- $AB'CD' + BC = C(B + AB'D') = C(B + AD')$
- Thus, $T = A'B + BD + ACD' + BC$
- $T = A'B + BD + ACD' + ABC + A'BC$
- Use absorption law: $A'B + A'BC = A'B$
- Using consensus theorem: $BD + ACD' + ABC = BD + ACD'$

- So $T = A'B + BD + ACD$

- $X=D$, $Y=B$, $Z=AC$

Canonical Forms

Deriving an expression from a truth table:

- Find the sum of all terms that correspond to combinations for which function is 1
- Each term is a product of the variables on which the function depends
- Variable x_i appears in uncomplemented (complemented) form in the product if has value 1 (0) in the combination
- Truth table for $f = x'y'z' + x'yz' + x'yz + xyz' + xyz$

Decimal code	x	y	z	f
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1

Decimal code	x	y	z	f
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

Canonical Sum-of-products

Minterm: a product term that contains each of the n variables as factors in either complemented or uncomplemented form

- It assumes value 1 for exactly one combination of variables

Canonical sum-of-products: sum of all minterms derived from combinations for which function is 1

- Also called disjunctive normal expression

Compact representation of switching functions: $\sum(0,2,3,6,7)$

Decimal code	x	y	z	f
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

Canonical Product-of-sums

Maxterm: a sum term that contains each of the n variables in either complemented or uncomplemented form

- It assumes value 0 for exactly one combination of variables
- Variable xi appears in uncomplemented (complemented) form in the sum if it has value 0 (1) in the combination

Canonical product-of-sums: product of all maxterms derived from combinations for which function is 0

- Also called conjunctive normal expression

Compact representation of switching functions: $\prod(1,4,5)$

$$f = (x + y + z')(x' + y + z)(x' + y + z')$$

Decimal code	x	y	z	f
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

Shannon's Expansion to Obtain Canonical Forms

Shannon's expansion theorem:

$$f(x1, x2, \ldots, xn) = x1 \,.\, f(1, x2, \ldots, xn) + x1' \,.\, f(0, x2, \ldots, xn)$$

$$f(x1, x2, \ldots, xn) = [x1 + f(0, x2, \ldots, xn)] \,.\, [x1' + f(1, x2, \ldots, xn)]$$

Shannon's expansion around two variables:

$$\begin{aligned} f(x1, x2, \ldots, xn) = {} & x1x2f(1, 1, x3, \ldots, xn) + x1x2'f(1, 0, x3, \ldots, xn) \\ & + x1'x2f(0, 1, x3, \ldots, xn) + x1'x2'f(0, 0, x3, \ldots, xn) \end{aligned}$$

Similar Shannon's expansion around all n variables yields the canonical sum-of-products

Repeated expansion of the dual form yields the canonical product-of-sums

Simpler Procedure for Canonical Sum-of-products

1. Examine each term: if it is a minterm, retain it; continue to next term
2. In each product which is not a minterm: check the variables that do not occur; for each x_i that does not occur, multiply the product by $(x_i + x_i')$
3. Multiply out all products and eliminate redundant terms

Example: $T(x,y,z) = x'y + z' + xyz$

$$
\begin{aligned}
&= x'y(z + z') + (x + x')(y + y')z' + xyz \\
&= x'yz + x'yz' + xyz' + xy'z' + x'yz' + x'y'z' + xyz \\
&= x'yz + x'yz' + xyz' + xy'z' + x'y'z' + xyz
\end{aligned}
$$

Canonical product-of-sums obtained in a dual manner

Example:

$$
\begin{aligned}
T &= x'(y' + z) \\
&= (x' + yy' + zz')(y' + z + xx') \\
&= (x' + y + z)(x' + y + z')(x' + y' + z)(x' + y' + z')(x + y' + z)(x' + y' + z) \\
&= (x' + y + z)(x' + y + z')(x' + y' + z)(x' + y' + z')(x + y' + z)
\end{aligned}
$$

Transforming One Form to Another

Example: Find the canonical product-of-sums for

$$T(x,y,z) = x'y'z' + x'y'z + x'yz + xyz + xy'z + xy'z'$$

$$\begin{aligned} T &= (T')' \\ &= [(x'y'z' + x'y'z + x'yz + xyz + xy'z + xy'z')']' \\ &= [x'yz' + xyz']' \qquad \text{//Complement } T' \text{ consists of minterms not contained in } T. \\ &= (x + y' + z)(x' + y' + z) \end{aligned}$$

Canonical forms are unique

Two switching functions are equivalent if and only if their corresponding canonical forms are identical

Functional Properties

Let binary constant a_i be the value of function f(x1, x2, …, xn) for the combination of variables whose decimal code is i. Thus,

$$f(x1, x2, \ldots, xn)= a_0x_1'x_2'\ldots x_n' + a_1x_1'x_2'\ldots x_n + \ldots + a_rx_1x_2\ldots x_n$$

The coefficient a_i is set to 1 (0) if the corresponding minterm is (is not) in the canonical form

Since there are 2^n coefficients, each of which can have two values, 0 and 1, there are 2^2^n possible switching functions of n variables

Example: Canonical sum-of-products form for two variables

$$f(x,y) = a_0x'y' + a_1x'y + a_2xy' + a_3xy$$

Thus 2^2^2 =16 functions corresponding to the 16 possible assignments of 0's and 1's to a_0, a_1, a_2, and a_3

List of Functions of Two Variables

a3	a2	a1	a0	f(x,y)	Name of Function	Symbol
0	0	0	0	0	Inconsistency	
0	0	0	1	$x'y'$	NOR	$x \downarrow y$
0	0	1	0	$x'y$		
0	0	1	1	x'	NOT	x'
0	1	0	0	xy'		
0	1	0	1	y'		
0	1	1	0	$x'y+xy'$	Exclusive OR	$x \oplus y$
0	1	1	1	$x'+y'$	NAND	$x \mid y$

a3	a2	a1	a0	f(x,y)	Name of Function	Symbol
1	0	0	0	xy	AND	$x.y$
1	0	0	1	$xy+x'y'$	Equivalence	$x \equiv y$
1	0	1	0	y		
1	0	1	1	$x'+y$	Implication	$x \longrightarrow y$
1	1	0	0	x		
1	1	0	1	$x+y'$	Implication	$y \longrightarrow x$
1	1	1	0	$x+y$	OR	$x + y$
1	1	1	1	1	Tautology	

Thanks