Indian Institute of Technology Patna
MA - 102: B.Tech. I year(Spring Semester: 2009-10)

End Semester Examination

Time: 3 Hours Maximum Marks: 50

Roll No.: Name:

There are **Sixteen** questions in this paper. Attempt all questions. No mark will be awarded for answers without proper justification. For disproving any statement give proper reasoning with counterexample.

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1. Solve the IVP: $\dfrac{dy}{dx} + \dfrac{y}{x} - \sqrt{y} = 0;\quad y(1) = 0$ [3]

2. For the IVP: $\dfrac{dy}{dx} = f(x, y);\quad y(x_0) = y_0$, write Picard's and 'simple' Euler method equations for the nth iteration ('simple' means basic Euler's method of linear order). For Euler's equations take step size 'h'. [2]

3. Determine the dependency of interval of validity for the solution of IVP: $\dfrac{dy}{dx} = y^2;\quad y(0) = y_0$ on the value of y_0. [3]

4. Consider the two sol'ns of the ODE $y'' + py' + qy = 0$ on $[a, b]$, where p, q are constants. Using Wronskian, prove that these sol'ns are LD if both sol'ns have maxima or minima at the same point $x_0 \in [a, b]$. [3]

5. With out solving the differential equations $y'' + \cos(t)y' = 0$, find the value of its Wronskian (up to a constant). [2]

6. A circuit has in series a register of R ohm, an inductor of L henry, and a capacitor of C farad. If the initial (means at time $t = 0$) current is zero and initial charge on the capacitor is Q_0, find a condition on R, C and L such that the current is a damped oscillatory function of time (*i.e.* like underdamped motion of spring-mass-dashpot system). [3]

7. Using the variation of parameter method, find a particular sol'n of the Differential Eq'n: $x\dfrac{d^2y}{dx^2} - (x+1)\dfrac{dy}{dx} + y = x^2$. It is given that the $e^x; x+1$ and $e^x + x + 1$ are three solutions of corresponding homogeneous Differential Eq'n. [3]

8. Using the operator or any other method, find general sol'n of the Differential Eq'n ($D \equiv \frac{d}{dx}$): $(D-1)^2(D^2+1)^2 y = \sin^2(x/2) + xe^x$ [5]

9. Express $\sin^{-1}(x)$ in the form of a power series (write first three non-zero terms only) by solving $y' = (1-x^2)^{-\frac{1}{2}}$ by two different methods (separable method and power series method). [3]

10. It is given that Differential Eq'n $x^2y'' + x(x+a)y' + by = 0$, where a and b are constants, is solved by Frobenious series method. Find the values of constants a and b such that the indicial equation has roots 1 and $-\frac{1}{2}$. Write first three terms of one Frobenius series solution. Does second LI solution contain the 'log' term always. [5]

11. Using Laplace Transform technique solve: $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = 2e^{-x};\ x \geq 0,\ y(0) = 1,\ y'(0) = 0.$ [3]

12. Express the function $f(t) = \begin{cases} \sin t, & \text{if } 0 < t \leq \pi, \\ 0, & \text{if } \pi < t \end{cases}$ in terms of unit step function and then find its Laplace transform. [2]

13. Prove or Disprove: There exists some value of constant k such that $F(p) = k + \tan^{-1}(p)$ is Laplace transform of some function $f(t)$. [2]

14. Find values for a and b such that the real matrix $A = \begin{pmatrix} 2 & a \\ 1 & b \end{pmatrix}$ has eigenvalues -1 and 4. Also find general solution of the system of differential equations $\begin{pmatrix} x' \\ y' \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix}$, where x' and y' represent the derivative of x and y with respect to t respectively. [3]

15. Show that the basis $\{1, x, x^2\}$ of the space $P_2(x)$ of all real polynomials of degree ≤ 2, equipped with the inner product $\langle f(x), g(x) \rangle = \int_0^1 f(t).g(t)dt$ is not an orthogonal basis. Find an orthogonal basis of this inner product space. [2+3]

16. Check whether the matrix $\begin{pmatrix} 15 & -12 & -16 \\ 4 & -2 & -5 \\ 9 & -8 & -9 \end{pmatrix}$ is diagonalizable or not. [3]