PH 301

ENGINEERING OPTICS

Lecture_8

Lens design

- **Design of optical imaging systems is an engineering discipline.**
- **Optical design is both a science & an art & for this reason this is a technology that can cause problems if it is not done properly.**
- **Closer association with electronics in devices such as digital cameras, enhanced machine vision systems, MEMS & microoptical systems for telecommunications, & other related applications.**

- **Rigorous way of treating light is to accept its electromagnetic wave nature & solve Maxwell's Eqs. However no. of configurations for which exact solutions can be found is very limited & most practical cases require approxs.**

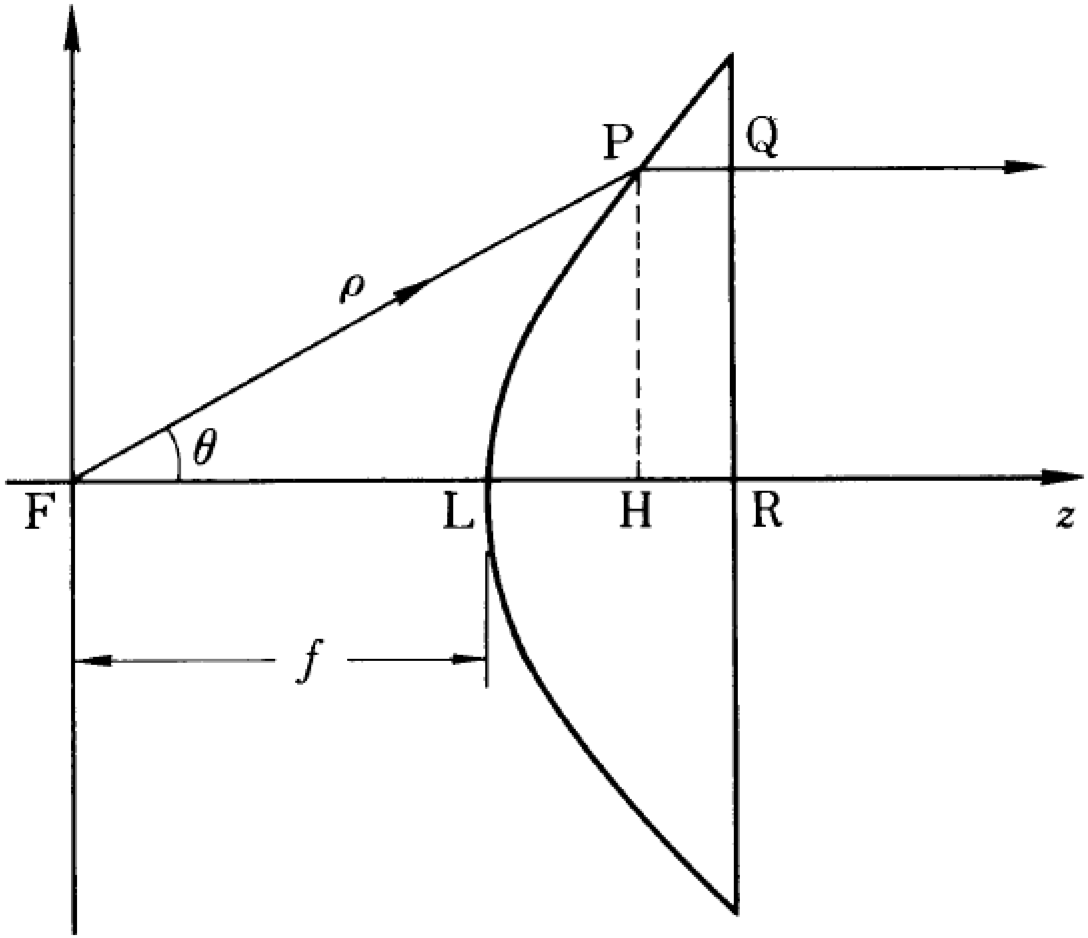
- **Based on specific method of approx., optics has been broadly divided into two categories;**

***Geometrical Optics* (*Ray Optics*)** *&* ***Wave Optics* (*Physical Optics*)**

- **Approx. used in geometrical optics puts emphasis on finding light path; it is especially useful for tracing path of propagation in *inhomogeneous media or in designing optical instruments.***

- **Approx. used in physical optics, puts emphasis on analyzing interference & diffraction & gives a more accurate determination of light distributions.**

Design of Plano-Convex Lens



Design of curvature of a plano-convex lens

- **Contour of a plano-convex lens is made in such a way that light from a point source becomes a parallel beam after passing through lens.**
- **Optical paths F – P – Q & F – L – R are identical.**
- **Let H be point of projection from P to optical axis.**
- **Since PQ = HR, optical paths will be identical if FP equals FH.**

$$Optical\ path\ of\ FP = \rho$$

$$Optical\ path\ of\ FH = f + n(\rho\cos\theta - f)$$

where *n* = refractive index, *f* = focal length FL.

- **When optical path of FP is set equal to FH, formula for contour of plano-convex lens is,**

$$\rho = \frac{(n-1)f}{n\cos\theta - 1}$$

Converting contour expression into rectangular coordinates.

Consider only *y* = 0 plane.

Taking origin as focus of lens, coordinates of point P are

$$\rho = \sqrt{x^2 + z^2}$$

$$\cos\theta = \frac{z}{\sqrt{x^2 + z^2}}$$

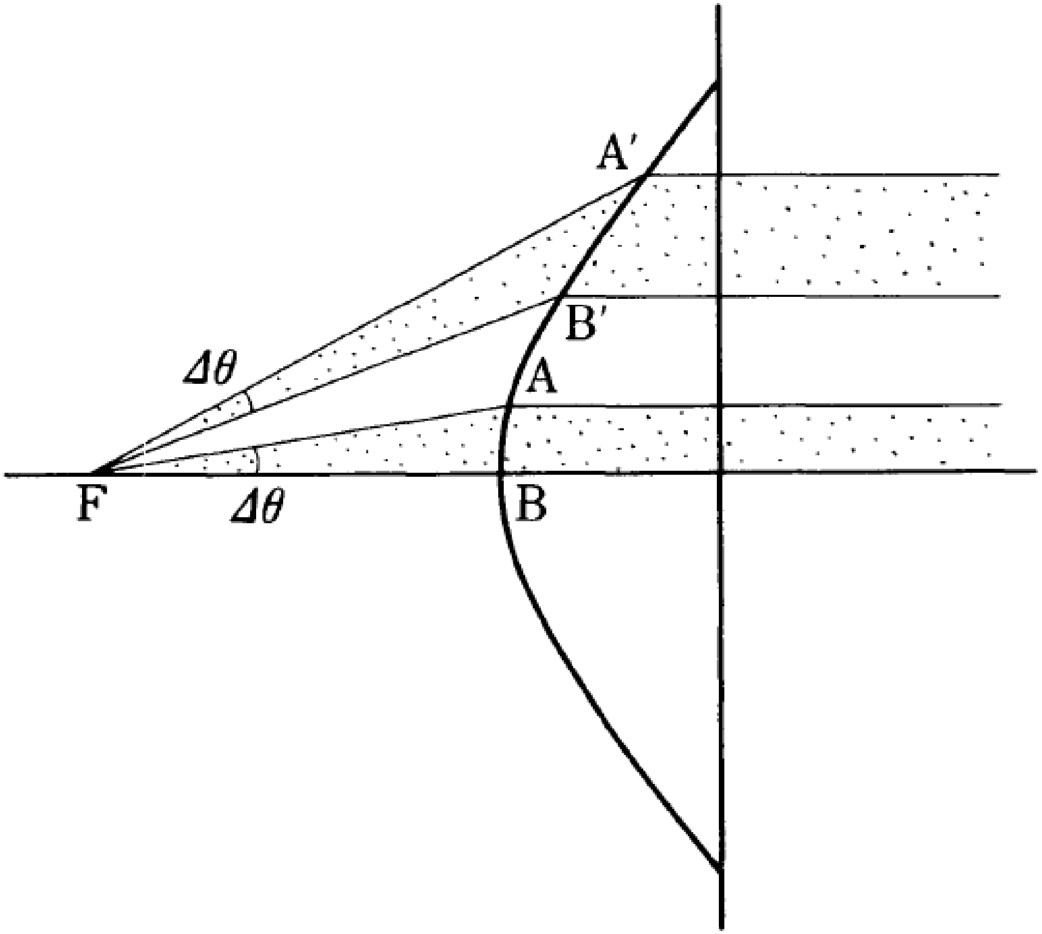
Substituting values of *ρ* & *cosθ*,

$$\frac{(z-c)^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$a = \frac{f}{n+1}, \qquad b = \sqrt{\frac{n-1}{n+1}}\, f, \qquad c = \frac{n}{n+1}\, f$$

It can be seen that contour of lens is a hyberbola.

- Even though light from a point source located at the focus becomes a parallel beam after passing through lens, intensity distribution across this beam is not uniform.



- Solid angle ∟AFB is made equal to solid angle ∟A’FB’.
- Light beam inside latter angle is spread more & is therefore weaker than that of the former.
- Inhomogeneity becomes a problem for large lens diameters.

Illustration of cause of intensity non-uniformity existing in a parallel beam emerging from a plano-convex lens.

Domain of Geometrical Optics

- **If λ is imagined to become vanishingly small, domain of geometrical optics suffice to analyze optical systems.**

- **While actual λ is always finite, nontheless provided all variations or changes of amplitude & phase of a wavefield take place on spatial scales that are very large compared with a wavelength, predictions of geometrical optics will be accurate.**

- **Examples for situations for which geometrical optics does not yield accurate predictions occur when we insert a sharp edge or a sharply defined aperture in a beam of light,**

  or

  when we change phase of a wave by a significant fraction of 2π radians over spatial scales that are comparable with λ.

- If we imagine a periodic phase grating for which a smooth change of phase by 2π radians takes place only over a distance of many wavelengths, predictions of geometrical optics for amplitude distribution behind grating will be reasonably accurate.

- If changes of 2π radians take place in only a few wavelengths, or take place very abruptly, then diffraction effects cannot be ignored, & a full wave optics (physical optics) treatment is needed.

Concept of a Ray

- **Consider a monochromatic disturbance traveling in a medium with *r.i.* (*n*) that varies slowly on scale of an optical wavelength. Such a disturbance can be described by an amplitude & phase distribution,**

$$U(\vec{r}) = A(\vec{r})\exp[jk_0 s(\vec{r})]$$

$A(\vec{r})$ **= amplitude**

$k_0 S(\vec{r})$ **= phase**

$$k_0 = \frac{2\pi}{\lambda}$$

$S(\vec{r})$ **is called *Eikonal* function.**

Surfaces defined by

$S(\vec{r})$ **= constant**

are called wavefronts of disturbance. Direction of power flow & direction of wave vector are both normal to wavefronts at each point *r* in an isotropic medium.

- **A ray is defined as a trajectory or a path through space that starts at any particular point on a wavefront & moves through space with wave, always remaining perpendicular to wavefront at every point on trajectory.**

- **Thus a ray traces out path of power flow in an isotropic medium.**

Helmholtz Eq.

$$\left(\nabla^2 + k_0^2\right)U = 0$$

$$k_0^2\left[n^2 - \left|\nabla S\right|^2\right]A + \nabla^2 A - jk_0\left[2\nabla S.\nabla A + A\nabla^2 S\right] = 0$$

Real & imaginary parts of this Eq. must vanish independently. For real part to vanish, we require

$$\left|\nabla S\right|^2 = n^2 + \left(\frac{\lambda_0}{2\pi}\right)^2 \frac{\nabla^2 A}{A}$$

Using artifice of allowing wavelength to approach zero to recover geometrical optics limit of this Eq., last term is seen to vanish, leaving so-called *Eikonal Equation*, which is perhaps the most fundamental description of behaviour of light under approximations of geometrical optics.

$$\left|\nabla S(\vec{r})\right|^2 = n^2(\vec{r})$$

This Eq. serves to define wavefront *S*. Once wavefronts are known, trajectories defining rays can be determined.

Refraction, Snell's Law & Paraxial Approximation

Rays traveling in a medium with constant *r.i.* always travel in st. lines. However, when wave travels trough a medium having *r.i.* that changes in space, ray directions will undergo changes that depend on changes of *r.i.*

When changes of *r.i.* are gradual, ray trajectories will be smoothly changing curves in space. Such bending of rays is called *Refraction*.

However, when a wave encounters an abrupt boundary between two media having different *r.i.*, ray directions are changed suddenly as they pass through interface. Angles of incidence & refraction are related by Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Problem of interest here, is changes of *r.i.* on passage through a lens, will always be abrupt, so Snell's law will form the basis of analysis.

Paraxial approximation: **Rays traveling close to optical axis & at small angles to that axis. In such a case, Snell's law reduces to a simple linear relationship between angles of incidence & refraction,**

$$n_1\theta_1 = n_2\theta_2$$

& in addition cosines of these angles can be replaced by unity. The product ,

$$\widehat{\theta} = n\theta$$

r.i. **(*n*) & angle *θ* within that medium is called *reduced angle*. Thus paraxial version of Snell's law states that reduced angle remains constant as light passes through a sharp interface between media of different refractive indices.**

$$\widehat{\theta}_1 = \widehat{\theta}_2$$

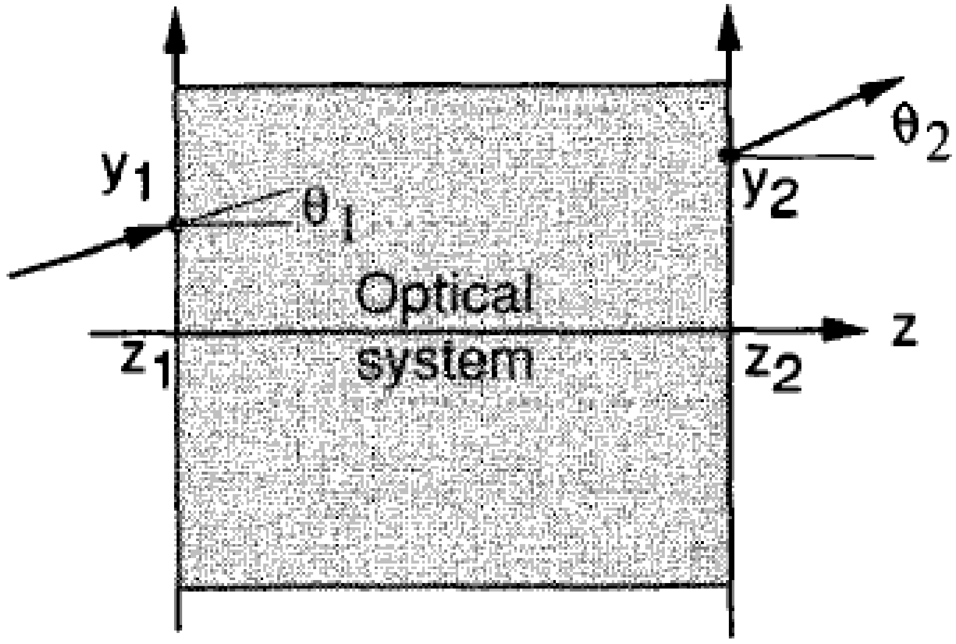
Ray Transfer Matrix

Under paraxial conditions, properties of rays in optical systems can be treated with an elegant matrix formalism, which in many respects is the geometrical-optics equivalent of operator method of wave optics.

To apply this methodology, it is necessary to consider only *meridional rays*, which are rays traveling in paths that are completely contained in a single plane containing z-axis.

We call transverse axis in this plane; *y* axis, & therefore plane of reference is (*y*, *z*) plane.

A ray with transverse coordinate y_1 enters optical system at angle θ_1 & same ray now with transverse coordinate y_2, leaves the system with angle θ_2.



Input & output of an optical system

Problem: To determine position y_2 & angle θ_2 of output ray for every possible y_1 & θ_1 associated with an input ray.

Under paraxial condition, relationships between (y_2, θ_2) & (y_1, θ_1) are linear & can be written explicitly as,

$$y_2 = Ay_1 + B\overset{\frown}{\theta}_1$$

$$\overset{\frown}{\theta}_2 = Cy_1 + D\overset{\frown}{\theta}_1$$

The Eq. can be expressed more compactly in matrix notation,

$$\begin{bmatrix} y_2 \\ \overset{\frown}{\theta}_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \overset{\frown}{\theta}_1 \end{bmatrix}$$

Matrix,

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

is called ***ray-transfer matrix*** or ***ABCD matrix***.

In (*y*, *z*) plane, under paraxial conditions, reduced ray angle $\hat{\theta}$ with respect to *z* axis is related to local spatial frequency f_l.

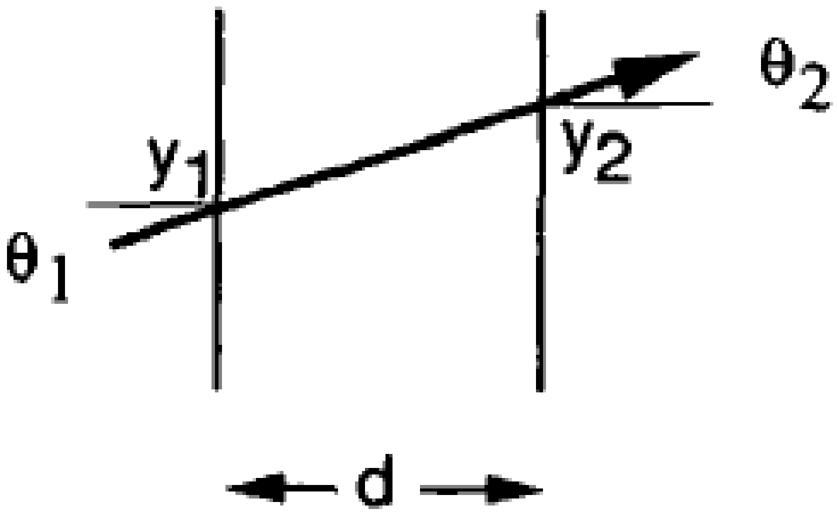
$$f_l = \frac{\theta}{\lambda} = \frac{\hat{\theta}}{\lambda_\theta}$$

Therefore, ray transfer-matrix can be regarded as specifying a transformation between spatial distribution of local spatial frequency at input & corresponding distribution at output.

Elementary ray-transfer matrices

❖ **Propagation through free space of r.i. *n***:

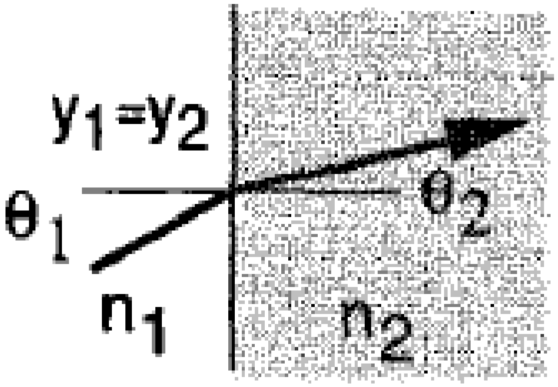
Geometrical rays travel in st. lines in a medium with constant *r.i*. Therefore effect of propagation through free space is to translate the location of ray in proportion to angle at which it travels & to leave angle of the ray unchanged. Ray-transfer matrix describing propagation over distance *d*,



$$M = \begin{bmatrix} 1 & d/n \\ 0 & 1 \end{bmatrix}$$

❖ **Refraction at a planar interface**:

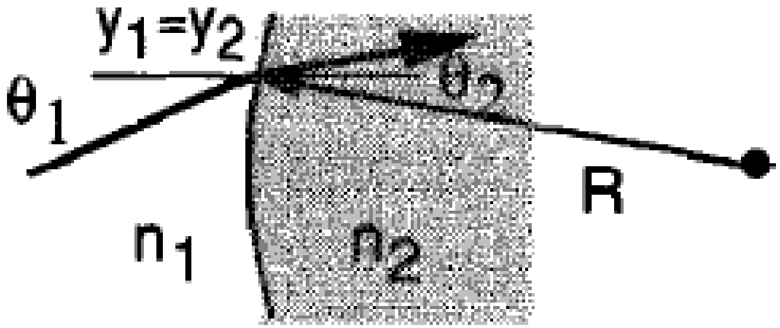
At a planar interface (n_1 & n_2) position of ray is unchanged but angle of ray is transformed according to Snell's law; the reduced angle remains unchanged.



$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

❖ Refraction at a spherical interface:

At a spherical interface (n_1 & n_2) position of a ray is not changed but angle is changed. However, at a point on interface at distance y from optical axis, the normal to interface is not parallel to optical axis, but rather is inclined with respect to optical axis.



$$\psi = \arcsin\frac{y}{R} \approx \frac{y}{R}$$

R is radius of spherical surface.

If angles θ_1 & θ_2 are measured with respect to optical axis, Snell's law at transverse coordinate *y* becomes,

$$n_1\theta_1 + n_1\frac{y}{R} = n_2\theta_2 + n_2\frac{y}{R}$$

$$\overset{\frown}{\theta}_1 + n_1\frac{y}{R} = \overset{\frown}{\theta}_2 + n_2\frac{y}{R}$$

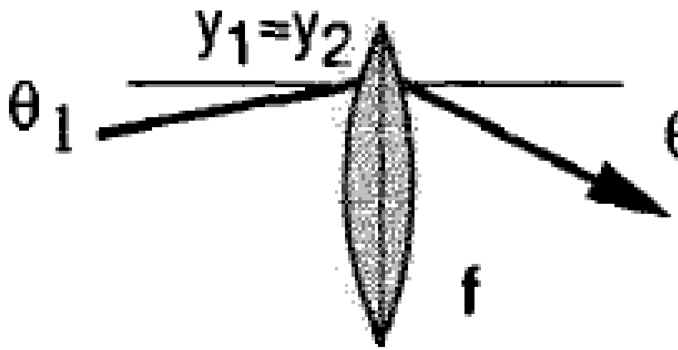
$$\overset{\frown}{\theta}_2 = \overset{\frown}{\theta}_1 + n_2\frac{n_1 - n_2}{R}y$$

Ray-transfer matrix for a spherical interface can be written as,

$$M = \begin{bmatrix} 1 & 0 \\ \dfrac{n_1 - n_2}{R} & 1 \end{bmatrix}$$

A positive value for *R* signifies a convex surface encountered from left to right, while a negative value for *R* signifies a concave surface.

❖ **Passage through a thin lens**:



A thin lens (index n_2 embedded in a medium of index n_1) can be treated by cascading two spherical interfaces.

Roles of n_1 & n_2 are interchanged for two surfaces.

Representing ray-transfer matrices of surfaces on left & right by M_1 & M_2, respectively, ray-transfer matrix for sequence of two surfaces,

$$M = M_2 M_1$$

$$M = M_2 M_1$$

$$M = \begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R_1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -(n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) & 1 \end{bmatrix}$$

Focal length of a lens is defined by,

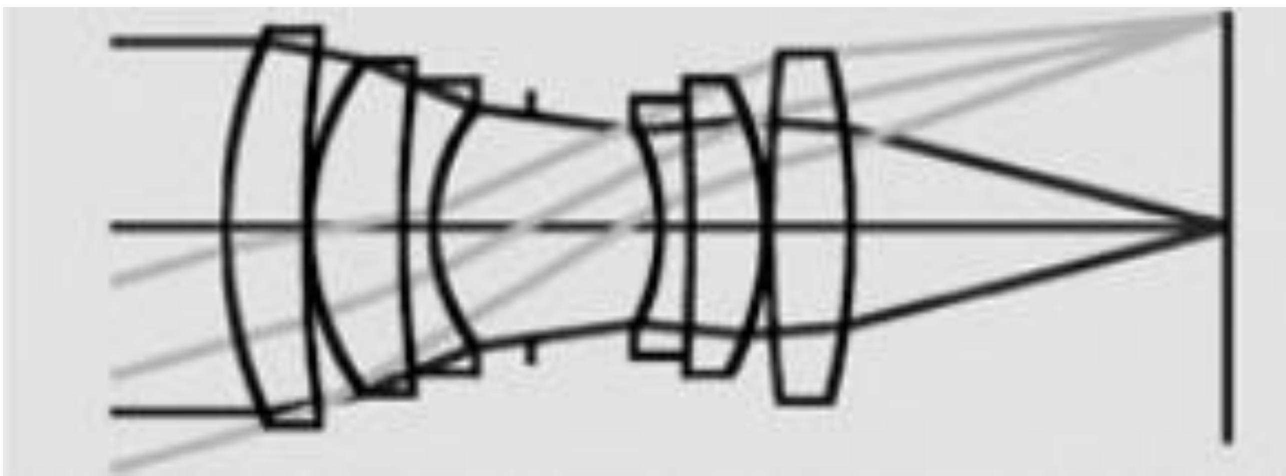
$$\frac{1}{f} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$M = \begin{bmatrix} 1 & 0 \\ \frac{-n_1}{f} & 1 \end{bmatrix}$$

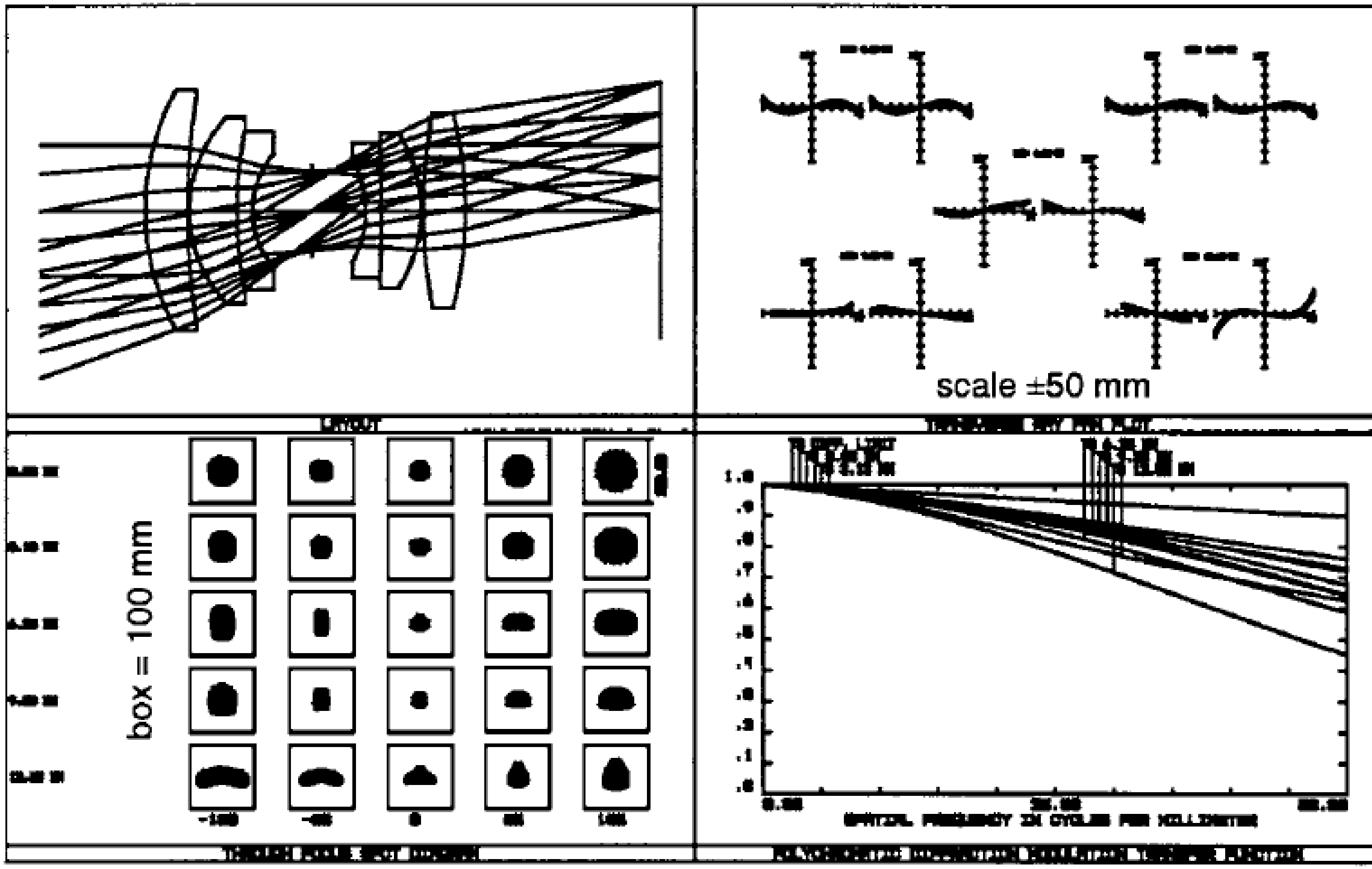
If light propagates first through a structure with ray-transfer matrix $\boldsymbol{M}_1$, then through a structure with ray-transfer matrix $\boldsymbol{M}_2$, etc., with a final structure having ray-transfer matrix $\boldsymbol{M}_n$, then overall ray-transfer matrix for entire system is,

$$M = M_n \text{.........} M_2 M_1$$

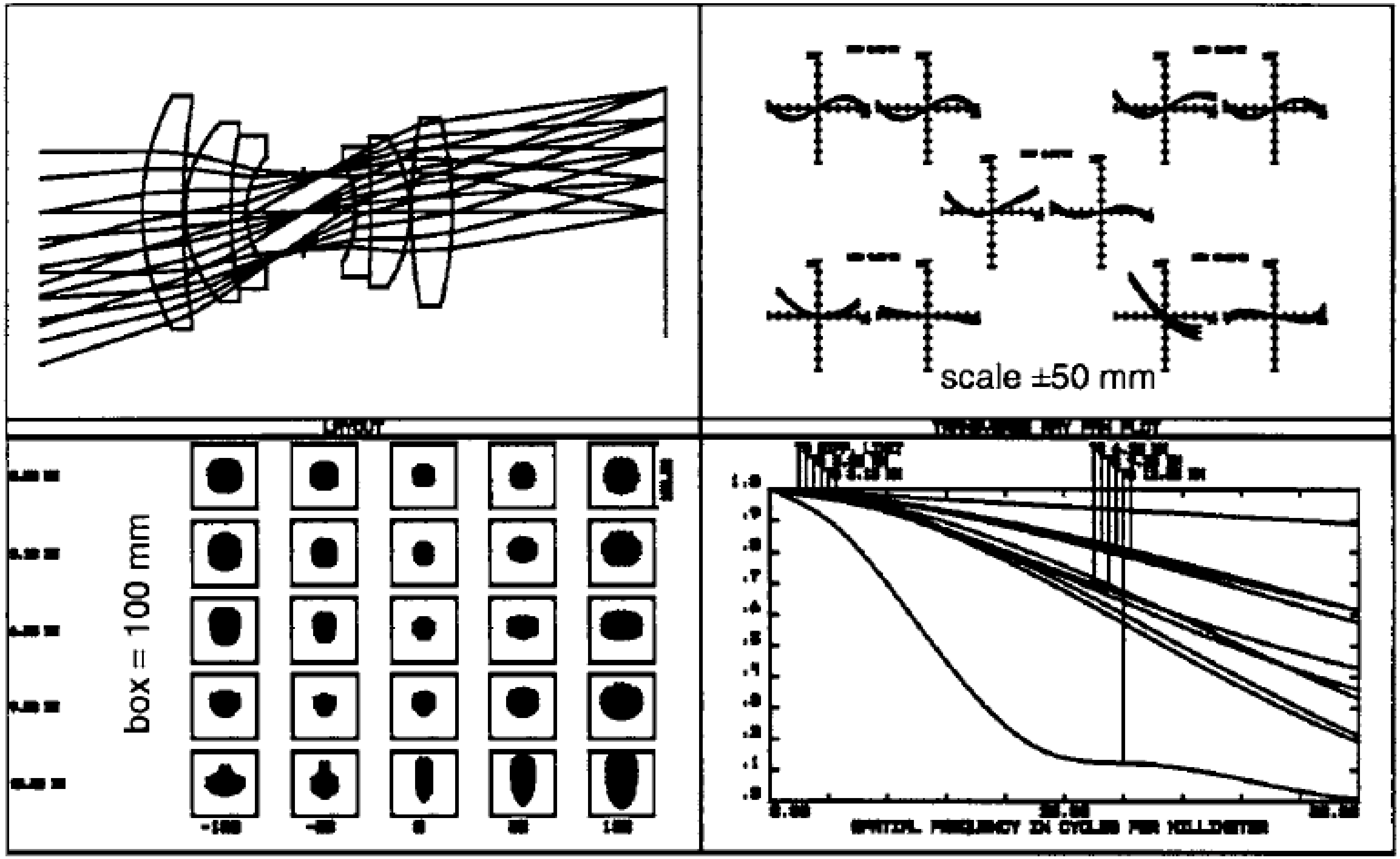
All of the elementary matrices presented have a determinant that is unity.



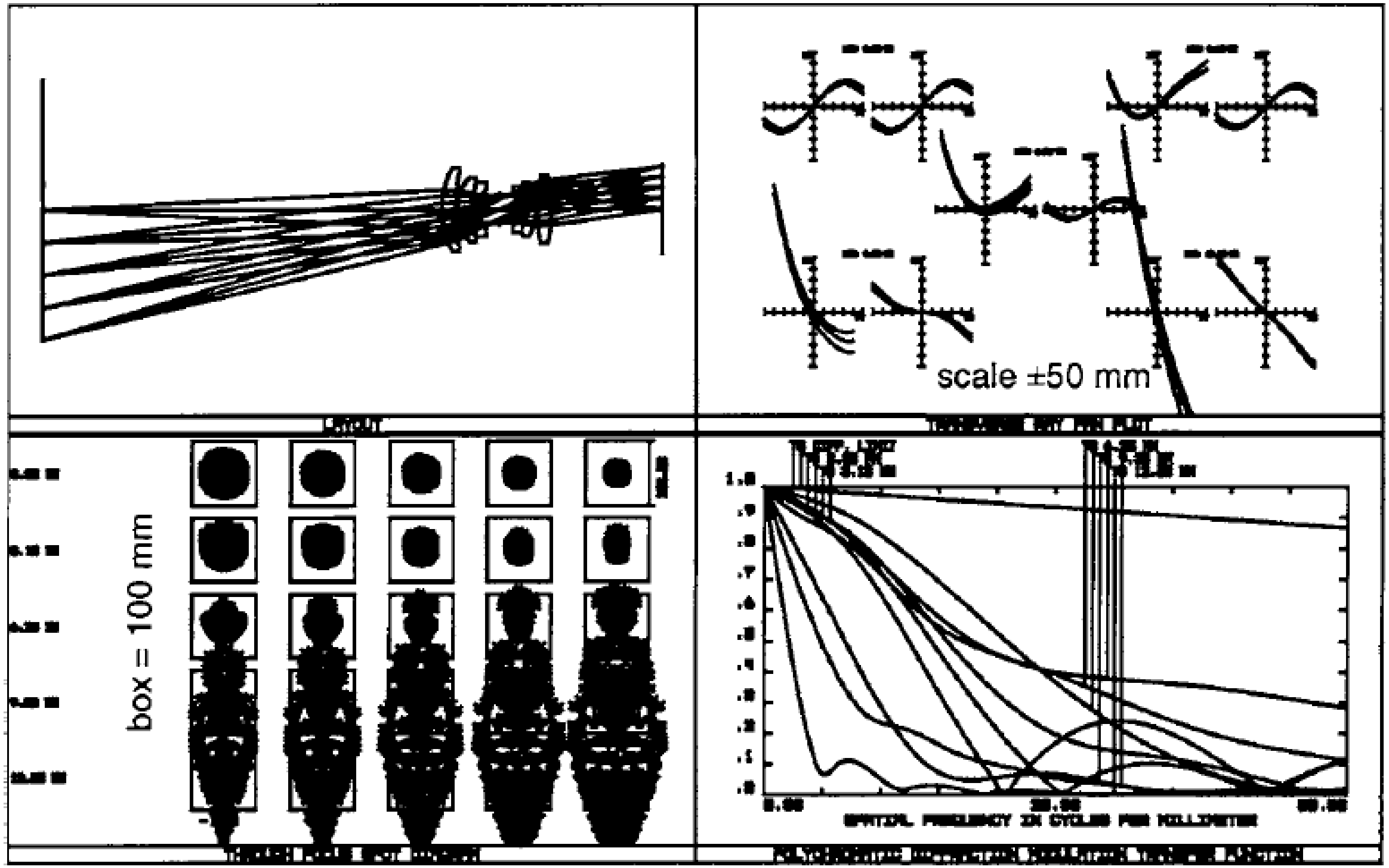
Modulation transfer function (MTF) data are plotted to 50 line pairs/mm.



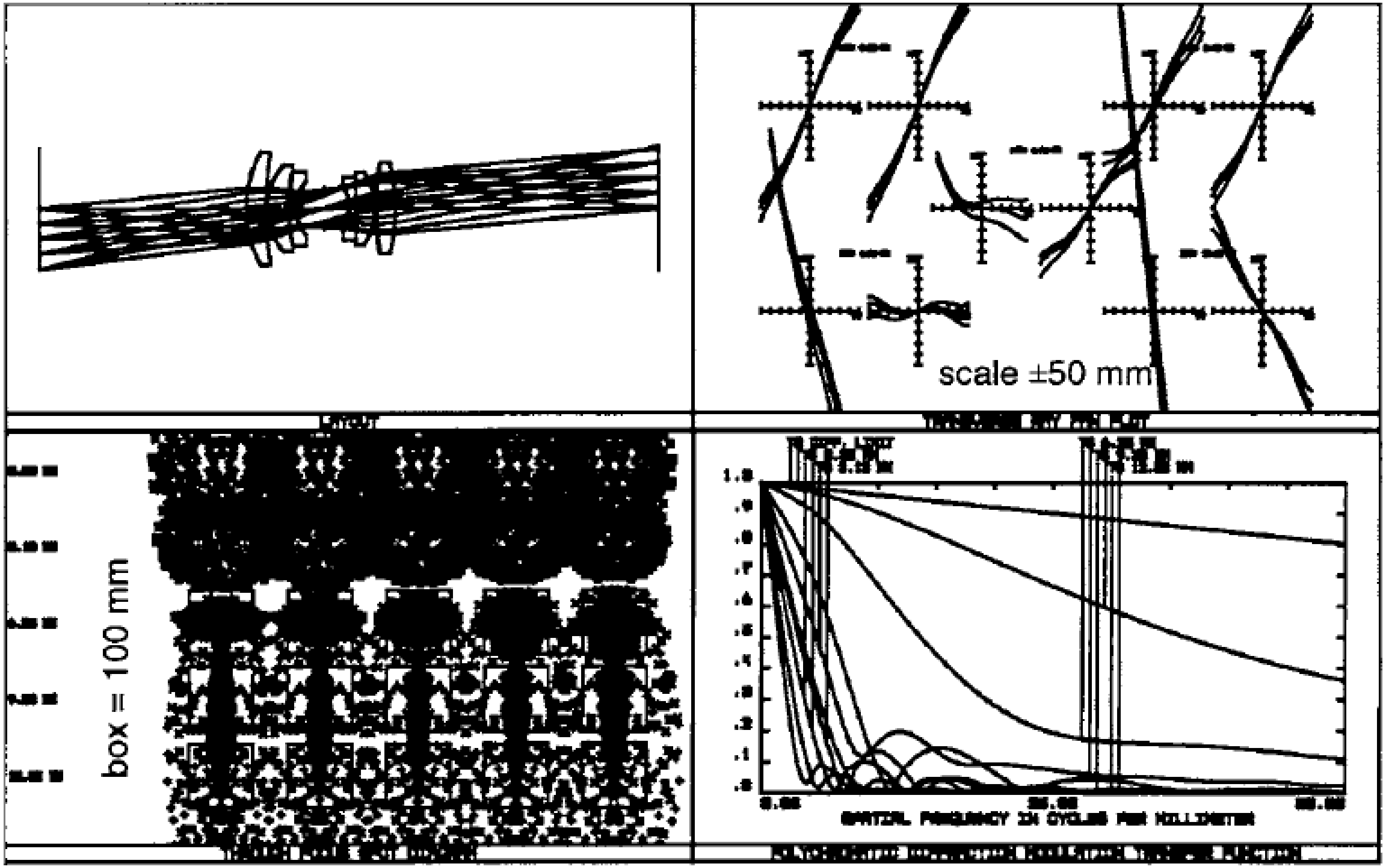
A 35 mm focal length *f*/2.8 lens at infinity



A 35 mm focal length *f*/2.8 lens at 500 mm object distance



A 35 mm focal length *f*/2.8 lens at 100 mm object distance



A 35 mm focal length *f*/2.8 lens at unit magnification (32.41 mm) object distance