CS204: Algorithms

Mid Semester, Autumn 2017,
IIT Patna

Duration:2 Hours | Full Marks: 30

1. [10]

   a. Solve following recurrence relation [4x1.5=6]

      i. $T(n) = \sqrt{n}\, T(\sqrt{n}) + n$

      ii. $T(n) = 2T(n/4) + \sqrt{n}$

      iii. $T(n) = T(n/4) + T(n/2) + cn^2$

      iv. $T(n) = 5T(n/5) + \sqrt{n}$, $T(1) = 1, T(0) = 0$

   b. Arrange entries of the following table to match [4x1=4]

Algorithm	*Recurrence*	*Run Time*
Binary Search	T(n) = 2 T(n/2) + O(n)	O(2^n)
Merge sort	T(n) = T(n-1) + n-1	*O(log n)*
Insertion sort	T(n) = 2T(n-1) + 1	*O(n log n)*
Tower of Hanoi	T(n) = T(n/2) + O(1)	O(n^2)

2. True/False with **very brief** justification [5x2=10]

   a. Comparison sort can be done in O(n) time

   b. Randomized partition algorithm partition the array in more balanced than 1:3 with 0.5 probability

   c. Quick sort is a stable sort

   d. A binary tree which is not full cannot represent any optimal prefix coding

   e. At most n/2 times partition procedure may be called in quick sort when n is the size of input.

3. A subsequence is defined as a sequence that appears in the same relative order, but not necessarily contiguous. For example "ace" is a subsequence of sequence "abcde". Answer any one of the following questions. [10]

   a. The Longest Increasing Subsequence (LIS) problem is to find the length of the longest subsequence of a given sequence such that all elements of the subsequence are sorted in increasing order. For example, the length of LIS for {10, 22, 9, 33, 21, 50, 41, 60, 80} is 6 and LIS is {10, 22, 33, 50, 60, 80}. [1+2+1+5+1=10]

      i. Show that brute force algorithm is exponential

      ii. Show that problem satisfy the optimal sub-structure property

      iii. Show that sub-problems are overlapping

      iv. Propose a dynamic programming algorithm (top-down/bottom up) to solve this problem

      v. Discuss about run time complexity of the proposed algorithm

b. Longest common subsequence problem is to find the length of a sequence which is a subsequence of two given sequences. [1+2+1+5+1=10]

i. Show that brute force algorithm is exponential
ii. Show that problem satisfy the optimal sub-structure property
iii. Show that sub-problems are overlapping
iv. Propose a dynamic programming algorithm (top-down/bottom up) to solve this problem
v. Discuss about run time complexity of the proposed algorithm