Indian Institute of Technology Patna
Mathematics - I (MA101)
B. Tech 1st Year (Autumn) 2016 - 17
End Semester Examination (Objective Type)

Max Time 1 Hours | Maximum Marks 18 | 22 Nov 2016

Name and Roll Number:____________________

Note: This part consists of a total 18 questions, printed on both sides and each question is of 1 Mark. All are compulsory. Notations have their usual meanings. Marks will not be awarded if over writing is found. Fill the answers only when it is final as over writing is not allowed.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

1. The series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$ is

   (A) divergent
   (B) convergent for $-1 < x < 1$
   (C) convergent for $0 < x < 1$
   (D) convergent for all values of x.

2. Let $g(x, y) = x^3 - 3xy^2 \ \forall \ (x, y) \in \mathbb{R}^2$. Then $(0, 0)$ is

   (A) a point of local maxima
   (B) a point of local minima
   (C) saddle point
   (D) none of these.

3. The function $f(x, y) = |x| + |y| \ \forall \ (x, y) \in \mathbb{R}^2$ is

   (A) differentiable every where
   (B) not continuous everywhere
   (C) not differentiable at (0,0)
   (D) None of these.

4. The direction in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases most rapidly at the point $(1, 1)$ is

   (A) $\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$
   (B) $-\frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j$
   (C) $\frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j$
   (D) None of these.

5. If $w = x^2 + y^2, x = r - s, y = r + s$, then $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial s}$ in terms of r and s are

   (A) $4r, 4s$
   (B) $2r, 2s$
   (C) none of these
   (D) $4r, 2s$.

6. The plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. The slope of the tangent to the parabola at $(1, 2, 5)$ is

   (A) 4 (B) 5 (C) 2 (D) none of these.

7. The equation of the tangent plane for the sphere $S = \{(x, y, z)|x^2 + y^2 + z^2 = 1\}$ at the point $(0, 0, 1)$ is

   (A) $z = 1$ (B) $z = -1$ (C) $x = 1$ (D) none of these.

8. $\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6 + y^2}$

   (A) 2
   (B) does not exist
   (C) 1
   (D) 0.

9. Function $f(x, y) = \frac{xy}{x^2+y^2}$ for $(x, y) \neq 0$, $f(0, 0) = 0$

(A) is continuous in everywhere

(B) is discontinuous at $(0, 0)$

(C) none of these

(D) is differentiable every where.

10. If f is a bounded function on $[a, b]$, and if P and Q are partitions of $[a, b]$, then which option given below is wrong.

(A) $L(f, P) \leq U(f, Q)$

(B) $L(f, Q) \leq U(f, P)$

(C) $L(f, P) \leq L(f, P \cup Q)$

(D) $U(f, P) \leq U(f, P \cup Q)$.

11. If f and g are Riemann Integrable on $[a, b]$ then, which option given below is not correct

(A) $f + g$ is Riemann Integrable on $[a, b]$

(B) $f - g$ is Riemann Integrable on $[a, b]$

(C) fg is Riemann Integrable on $[a, b]$

(D) $\frac{f}{g+1}$ is Riemann Integrable on $[a, b]$.

12. The value of $\int_0^{\pi/2} \int_0^{x} \frac{\sin x}{x} dA$ is

(A) 1

(B) $\pi/2$

(C) 2

(D) 3.

13. Value of the integral $\iint_R \frac{y}{1+x^2y^2}$ dA, where $R : 0 \leq x \leq 1$ and $0 \leq y \leq 1$ is

(A) $\frac{1}{8}(\pi - \log(4))$

(B) $\frac{1}{2}(\pi - \log(4))$

(C) $\frac{1}{4}(\pi - \log(4))$

(D) $\frac{1}{16}(\pi - \log(4))$.

14. Given $x = r\cos\theta$ and $y = r\sin\theta$, the Jacobian, $J(r, \theta)$ of the transformation from x-y system to r-θ system is

(A) $2r$

(B) $r/2$

(C) r

(D) $\sqrt{r}$.

15. If $\mathbf{F} = (y - x^2)\mathbf{i} + (z - y^2)\mathbf{j} + (x - z^2)\hat{\mathbf{k}}$, then $\nabla \cdot \mathbf{F}$ or $Div(\mathbf{F})$ is

(A) $2x + 2y + 2z$

(B) 3

(C) $x + y + z$

(D) $-2x - 2y - 2z$.

16. If $\mathbf{F} = (y - x^2)\mathbf{i} + (z - y^2)\mathbf{j} + (x - z^2)\hat{\mathbf{k}}$, then $\nabla \times \mathbf{F}$ or $Curl(\mathbf{F})$ is

(A) none of these

(B) $\mathbf{i} - \mathbf{j} - \mathbf{k}$

(C) $-\mathbf{i} - \mathbf{j} - \mathbf{k}$

(D) $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

17. Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$ and C is the square $\{(x, y) : |x| = 1, |y| = 1\}$ in counterclockwise direction.

(A) 2

(B) 1

(C) 0

(D) 4.

18. Green's theorem (C is counter clockwise simple closed curve and R is region bounded by C) says that

(A) $\oint_C Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$

(B) $\oint_C Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} + \frac{\partial M}{\partial y}\right) dA$

(C) $\oint_C Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial y} + \frac{\partial M}{\partial x}\right) dA$

(D) none of these.

End

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End Semester Examination (Subjective)

Max Time 2 Hours | Maximum Marks 32 | 22 Nov 2016

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Note: This part consists of total 8 questions. All are compulsory. Notations have their usual meanings. Solve the questions in given sequence.

1. Find the limit if it exists
$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x-y}.$$
[4]

2. Let function $f(x,y)$ be defined as
$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & ; \quad (x,y) \neq (0,0) \\ 0, & ; \quad (x,y) = (0,0). \end{cases}$$
Compute $f_{xy}(0,0)$. **[4]**

3. Use Lagrange's multipliers to Minimize :
$$f(x,y,z) = x^2+y^2+z^2$$
Subject to :
$$x^2 - z^2 - 1 = 0.$$
[4]

4. Find the equation to normal to the Paraboloid
$$2z = x^2 + y^2$$
at $(1,1,1)$. **[4]**

5. Use triple integral to find the volume of the cylinder $x^2+y^2=4$ bounded between the planes $z=0$ and $z=4-y$. **[4]**

6. Use double integral and the transformation $u = xy$ and $v = xy^2$ to find the area of the region R in the x-y plane bounded by the hyperbolas $xy=1$, $xy=2$ and the curves $xy^2=3$ and $xy^2=4$. **[4]**

7. State Green's theorem and use it to evaluate
$$\oint_C e^{-x}\sin y\,dx + e^{-x}\cos y\,dy$$
in counter clock wise direction along C, where C is a rectangle with the vertices $(0,0)$, $(\pi/2,0)$, $(\pi/2,\pi/2)$ and $(0,\pi/2)$. **[1+3]**

8. Consider the function $f:[0,1]\to\mathbb{R}$ given by
$$f(x) = \begin{cases} 1, & \text{when } x \text{ is rational} \\ 0, & \text{when } x \text{ is irrational} \end{cases}$$
Prove from definition that $f(x)$ is not a Riemann integrable function. **[4]**

____________ End ____________