Indian Institute of Technology Patna
MA101 (Mathematics-I)
B.Tech -I year
Autumn Semester: 2013-2014
(End Semester Examintaion)

Maximum Marks: 50 Time: 3 Hours

Note:

(i) This question paper has TWO pages and contain seventeen questions. Please check all pages and report the discrepancy, if any.

(ii) Attempt all questions.

1. Find $\lim_{x\to 0} \frac{x + x\cos x}{\sin x \cos x}$. [2.5]

2. Determine the value of a for which the function

$$f(x) = \begin{cases} x^2 - 1, & x < 3; \\ 2ax, & x \geq 3 \end{cases}$$

is continuous at every x. [2.5]

3. Find the area of the region in the first quadrant that is bounded above by the curve $y = 2\sqrt{x}$ and below by the line $y = x - 3$. [2.5]

4. Determine $\int sin^2 x\, dx$. Hence determine the area between the graph of $\sin^2 x$ and the x-axis over $[0, 2\pi]$. [2.5]

5. Show that the repeated limits exist but simultaneous limit does not exist at origin for the function defined by

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & when\ (x,y) \neq (0,0) \\ 0, & when\ (x,y) = (0,0) \end{cases}.$$

[3]

6. Suppose the function $f(x,y)$ is defined by

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & when\ (x,y) \neq (0,0) \\ 0, & when\ (x,y) = (0,0) \end{cases}.$$

Is $f(x,y)$ continuous at $(0,0)$? [2]

7. Prove that $f(x,y) = \sqrt{|xy|}$, is not differentiable at $(0,0)$, but both the partial derivatives exist at $(0,0)$ and have the values zero. [4]

8. If

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & when\ (x,y) \neq (0,0) \\ 0, & when\ (x,y) = (0,0) \end{cases}.$$

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$. [3]

9. If $x^x y^y z^z = c$, then show that at $x = y = z$,

$$\frac{\partial^2 z}{\partial x \partial y} = -(x(logex))^{-1}.$$

[3]

10. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s, if $w = x + 2y + z^2$; $x = \frac{r}{s}$; $y = r^2 \log s$; $z = 2r$. [3]

11. Find the derivative of $f(x, y) = xe^y + \cos(xy)$, at the point $P(2, 0)$ in the direction of the vector $\vec{u} = 3\hat{i} - 4\hat{j}$. [2]

12. Show that minimum value of $u = xy + \frac{a^3}{x} + \frac{a^3}{y}$, is $3a^2$. [2]

13. Use Lagrange Multiplier method to find the shortest distance from origin to the hyperbola $x^2 + 8xy + 7y^2 = 225, z = 0$. [4]

14. Use an appropriate transformation to find the integral $\iint_D y^3(2x - y)e^{(2x-y)^2} dxdy$ where $D : \{0 \leq y \leq 2, y/2 \leq x \leq (y + 4)/2\}$. [4]

15. Evaluate $\iiint_D xdxdydz$; where D is the region in space bounded by the plane $x = 0$, $y = 0$, $z = 2$ and the surface $z = x^2 + y^2$. [4]

16. Change the order of the following double integral $\int_0^2 [\int_{\sqrt{2x-x^2}}^{\sqrt{2x}} f(x, y)dydx$. [3]

17. Let $F(x, y, z) = \frac{xi+yj+zk}{(x^2+y^2+z^2)^{3/2}}$ $(x, y, z) \neq (0, 0, 0)$. Find div(F). [3]