MA101 Real Analysis
Integration in Vector Fields: Green's, Stoke's and Divergence Theorems

1. What is the mathematical and Geometrical meaning of the following?
   (a) ∇f (or grad f),
   (b) $\nabla \cdot \mathbf{F}$ (or $Div(F)$) and
   (c) $\nabla \times \mathbf{F}$ (or $Curl(F)$)?

   Here f is scalar function, $F = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector valued function.

2. Evaluate $\int\limits_C (xy + y + z)ds$ along the curve $r(t) = 2t\mathbf{i} + t\mathbf{j} + (2 - 2t)\mathbf{k}$, $0 \leq t \leq 1$.

3. Work Find the work done by the force $F = xy\mathbf{i} + (y - x)\mathbf{j}$ over the straight line from $(1, 1)$ to $(2, 3)$.

4. Work Find the work done by the gradient of $f(x, y) = (x + y)^2$ counterclockwise around the circle $x^2 + y^2 = 4$ from $(2, 0)$ to itself.

5. Find the circulation and flux of the fields $F_1 = x\mathbf{i} + y\mathbf{j}$ and $F_2 = -y\mathbf{i} + x\mathbf{j}$ around and across each of the following curves.
   (a) $r(t) = (cost)\mathbf{i} + (sint)\mathbf{j},\ 0 \leq t \leq 2\pi$,
   (b) $r(t) = (cost)\mathbf{i} + (4sint)\mathbf{j},\ 0 \leq t \leq 2\pi$.

6. Show that $F = (e^x cosy + yz)\mathbf{i} + (xz - e^x siny)\mathbf{j} + (xy + z)\mathbf{k}$ is conservative Over its natural domain and find a potential function for it.

7. Verify the Green's Theorem (Flux form and Circulation form) for the field $F = -y\mathbf{i} + x\mathbf{j}$. Take the domains of integration in each case to be the disk $R : x^2 + y^2 = a^2$ and its bounding circle $C : r = (acost)\mathbf{i} + (asint)\mathbf{j},\ 0 \leq t \leq 2\pi$.

8. Use Green's Theorem to find the counterclockwise circulation and outward flux for the field F and curve C.
   (a) $F = (x^2 + 4y)\mathbf{i} + (x + y^2)\mathbf{j}$; C: The square bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$.
   (b) $F = (y^2 - x^2)\mathbf{i} + (x^2 + y^2)\mathbf{j}$; C: The triangle hounded by $y = 0$, $x = 3$, and $y = x$

9. If a simple closed curve C in the plane and the region R it encloses satisfy the hypotheses of Green's Theorem, the area of R is given by

$$Area\ of\ R = \frac{1}{2} \oint_C xdy - ydx.$$

10. How to compute surface area of curved surface in 3D by using double integral?

11. What is formula for Flux across a surface in 3D?

12. What is the statement of Stokes theorem?

13. What is the statement of Divergence theorem?

14. Integrate $G(x, y, z) = x + y + z$ aver the portion of the plane $2x + 2y + z = 2$ that lies in the first octant.

15. Use a parametrization to find the flux $\int_S \mathbf{F}\cdot\mathbf{n}d\sigma$ where $\mathbf{F} = y^2\mathbf{i}+xz\mathbf{j}-\mathbf{k}$ across the Cone $z = 2\sqrt{x^2+y^2}$, $0 \le z \le 2$ (outward : normal away from the z-axis).

16. Use the Divergence Theorem to find the outward flux ofF across the boundary of the region D.

    (a) $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$, D : The region cut from the solid cylinder $x^2 + y^2 \le 4$ by the planes $z = 0$ and $z = 1$.

    (b) $\mathbf{F} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})/\sqrt{x^2+y^2+z^2}$, D : The region : $1 \le x^2 + y^2 + z^2 \le 4$.

17. Use the surface integral in Stokes' Theorem to calculate the circulation of the field $\mathbf{F}$ around the curve C in the indicated direction.

    (a) $\mathbf{F} = x^2y^3\mathbf{i} + \mathbf{j} + z\mathbf{k}$. C : The intersection of the cylinder $x^2 + y^2 \le 4$ and the hemisphere $x^2 + y^2 + z^2 = 16$, $z \ge 0$, counter clockwise when viewed from above.

    (b) $\mathbf{F} = (y^2 + z^2)\mathbf{i} + (x^2 + z^2)\mathbf{j} + (x^2 + y^2)\mathbf{k}$. C : The boundary of the triangle cut from the plane $x + y + z = 1$ by the first octant, counterclockwise when viewed from above.