12

	Indian Institute of Technology Patna Mathematics - I (MA101) B. Tech 1st Year (Autumn) 2017 - 18 End Sem Exam (Subjective Type)	
Max Time 2 Hour	Maximum Marks 30	21st Nov 2017

Note: This part consists of a total 8 questions, printed on both sides. All are compulsory. Notations have their usual meanings.

1. Let f be a bounded function defined on $[a, b]$.

   (a) Define $U(f, P)$ and $L(f, P)$. [1M]

   (b) Define $U(f)$ and $L(f)$. [1M]

   (c) Discuss existence of $\int_0^1 f(x)dx$ where [1.5M]

   $$f(x) = \begin{cases} x, & x \in \mathbb{Q} \\ 0, & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}.$$

2. Let f be an integrable function on $[a, b]$. For $x \in [a, b]$ let $F(x) = \int_a^x f(t)dt$. Then prove that

   (a) $F(x)$ is continuous on $[a, b]$. [1.5M]

   (b) If f is continuous at x_0 on (a, b), then $F(x)$ is differential at x_0 and [2M]

   $$F'(x_0) = f(x_0).$$

3. Answer the following:

   (a) Given

   $$\int_{(1,2,1)}^{(2,1,1)} (2x \ln y - yz)\, dx + \left(\frac{x^2}{y} - xz\right) dy - (xz)\, dz \quad .$$

   i. Find out the potential function. [1.5M]

   ii. Hence evaluate the above integral. [0.5M]

   (b) Find the flux of the Field $\vec{F_1} = -y\hat{i} + x\hat{j}$ across the curve $r(\vec{t}) = \cos t\ \hat{i} + 4 \sin t\ \hat{j},\ 0 \leq t \leq 2\pi$. [1.5M]

4. Answer the following:

   (a) Given field $\vec{G} = M\ \hat{\imath} + N\ \hat{\jmath}$ and a simple closed curve C state the following:

      i. Flux-Divergence form of Green's Theorem. [1M]

      ii. Circulation-Curl form of Green's Theorem. [1M]

   (b) State and use Divergence Theorem to Calculate outward flux of the Field $\vec{G_1} = x^2\ \hat{\imath} + xz\ \hat{\jmath} + 3z\ \hat{k}$ across the boundary of the solid sphere $x^2 + y^2 + z^2 \leq 4$. [2.5M]

5. (a) Let $f(x,y) = \begin{cases} xsin(\frac{1}{y}) + \frac{x^2-y^2}{x^2+y^2}, & y \neq 0 \\ 0, & y = 0 \end{cases}$

   test the existence of $\lim\limits_{(x,y)\to(0,0)} f(x,y)$. [1.5]

   (b) Show that if $f(x,y)$ is differentiable at (a,b), then partial derivatives $f_x(a,b)$ and $f_y(a,b)$ exist. [2].

6. (a) Examine whether $f(x) = 1 - |x|$ is minimum or maximum at $x = 0$?. [2]

   (b) Derive the equation of the tangent plane to the ellipsoid

$$x^2 + \frac{y^2}{4} + \frac{z^2}{10} = 3$$

   at $(1, 2, \sqrt{10})$. [2]

7. Use Lagrangian method to find the extrema of $f(x,y) = xy + 14$ subject to $x^2 + y^2 = 18$ and also extreme points. [3.5]

8. (a) Use triple integration to find the volume of Ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

   [2]

   (b) Solve

$$\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x - y}{2}\, dx\, dy$$

   by transforming it to the $u - v$ plane. [2]

End