

1. (a) Prove that any comparison SORT algorithm requires  $\Omega(n \lg n)$  comparison in worst case. [4]  
(b) What is a stability property in Counting Sort algorithm? [2]  
(c) Why the time complexity of Counting Sort algorithm beats the lower bound of sorting? [2]
2. (a) Suppose the partitioning algorithm of Quick sort always produces a 9-to-1 proportional split (unbalanced case). Obtain the recurrence on the running time of Quick sort and find out its running time. [5]  
(b) Is the array with values  $\langle 23, 17, 14, 6, 13, 10, 1, 5, 7, 12 \rangle$  a max-heap? [2]
3. (a) State the Matrix-Chain-Multiplication problem. [1]  
(b) Using Amortized Analysis, show that for  $n$  increment operations in incrementing a binary counter, the total amortized cost is  $O(n)$ . [4]
4. Consider the directed graph  $G$ .  
(a) Call  $\text{dfs}(G)$  to compute finishing time for each vertex  $u.f$ . [3]  
(b) Compute transpose of  $G$ , say  $G^T$ . Call  $\text{dfs}(G^T)$ , considering vertices in order of decreasing  $u.f$  (obtained in step(a)). [3]  
(c) Obtain the strongly connected components (SCC) of  $G$ . [3]  
(d) Obtain the component graph  $G^{\text{SCC}}(V^{\text{SCC}}, E^{\text{SCC}})$ . [2]  
(e) Obtain the topological sorting of  $G^{\text{SCC}}$ . [3]
5. Consider the following graph  $G$  and use your dfs algorithm [2]  
(a) Find out all the articulation points of  $G$ . [4]  
(b) Find out all the biconnected components of  $G$ . [4]

