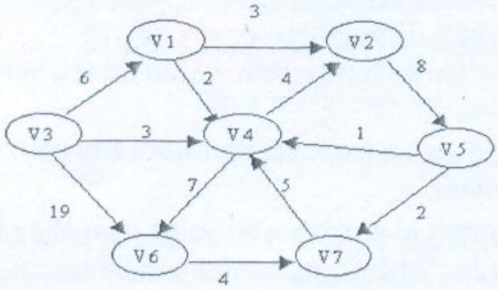
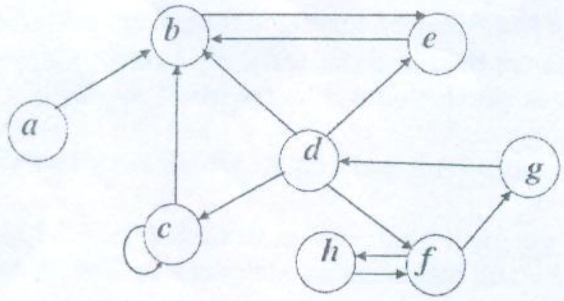
ESE CS204 Algorithms PART-A

Note:-Attempt Part A and Part B in separate main answer copies MM-Part-A:[50]



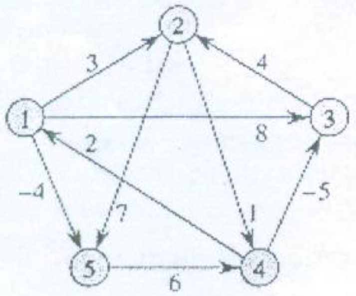
1. Consider the given the weighted directed graph (for unweighted directed graph, ignore the edge-weights in the given graph) and answer the following questions:-

i. Given a weighted graph and a vertex V1, find the shortest paths to all other vertices (run the algorithm on paper). [4]
ii. Explain the complexity of the shortest path algorithm for weighted graphs. [1]
iii. Given a unweighted graph and a vertex V1, find the shortest paths to all other vertices (run the algorithm on paper) [4]
iv. Explain the complexity of the shortest path algorithm for unweighted graphs.[1]
v. Explain the changes in the shortest path algorithms for unweighted directed graphs compared to that one for weighted directed graphs? [3]



2. Consider the directed graph G.

i. Label each vertex with discovery and finish time in depth-first search. [4]
ii. Classify edges of G into: Forward, Backward, Cross and Tree edges. [3]
iii. Find out strongly connected components (SCC) of G [4]
iv. Obtain the component graph G^{SCC} of G. [2]
v. Give the topological sort of G^{SCC}. [4]



3. Compute APSP using the Johnson's algorithm for the above given graph. [10]

4. (a) Discuss disjoint-set data structures. [3]

(b) Explain the minimum spanning tree algorithm which uses disjoint-set data structures. [7]

ESE CS204: Algorithms - Part B

Max Marks: 40 (excluding 8.c)

Instructions: Please **return** the question paper at the end of the exam

5. Let X be a set of n intervals on the real line. We say that a set of points P stabs X if every interval in X contains at least one point in P. The objective of this problem is to compute the smallest set of points P that stabs X . Assume that your input consists of two arrays XL[1 .. n] and XR[1 .. n], representing the left and right endpoints of the intervals in X . Devise an algorithm using greedy approach and prove that it is correct. **(15 pts)**

6. This question is relating to the potential method of amortized analysis. Suppose we have a potential function Φ such that $\Phi(D_i) \geqslant \Phi(D_0)$ for all i, but $\Phi(D_0) \neq 0$. Show that there exists a potential function Φ' such that Φ'(D_0) = 0 and $\Phi'(D_i) \geqslant 0$ for all $i \geqslant 1$. The amortized costs using Φ' is same as the amortized cost using Φ. **(5 pts)**

7. Show that if HAM-CYCLE is in P, then the problem of listing the vertices of a hamiltonian cycle, in order, is polynomial-time solvable.

A hamiltonian cycle of an undirected graph *G = (V, E)* is a simple cycle that contains each vertex in V. Please note that HAM-CYCLE problem is the decision version of the hamiltonian cycle problem. So the solution to HAM-CYCLE problem only says yes or no. **(10 pts)**

8. (a) Show that HAM-PATH problem is NP-Complete.

The HAM-PATH is defined as {*[G, u,v]*: there is a hamiltonian path from u to in graph G}. A hamiltonian path in a graph is a simple path that visits every vertex exactly once. **(7 pts)**

(b) Consider the CLIQUE problem restricted to graphs in which every vertex has degree at most 3. Call this problem CLIQUE-3.

Is the following NP-hardness proof for CLIQUE-3 correct?
We know that the CLIQUE problem in general graphs is NP-complete, so it is enough to present a reduction from CLIQUE-3 to CLIQUE. Given a graph G with vertices of degree ≤ 3, and a parameter g, the reduction leaves the graph and the parameter unchanged: clearly the output of the reduction is a possible input for the CLIQUE problem. Furthermore, the answer to both problems is identical. This proves the correctness of the reduction and, therefore, the NP-hardness of CLIQUE-3. **(3 pts)**

(c) **Extra Credit:** The k-SPANNING TREE problem is defined as follows

Input: An undirect graph G=(V,E)
Output: A spanning of tree of G in which each node has a degree less than or equal to *k*, if such a tree exists.

Show that for any k >= 2, the k-SPANNING TREE problem is NP-Complete. (Hint: Start with k=2. Then prove for k> 2). **(20 pts)**