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CS303 Tutorial 10

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Ans1:

To prove: All CFL's are not closed under complementations.

Proof:

Assume CFL's are closed under complementation.

Consider two CFL's L_1 and L_2. By our assumption $\overline{L_1}$ and $\overline{L_2}$ are also CFLs

Since, CFL's are closed under union, $\overline{L_1} \cup \overline{L_2}$ is a CFL.

By our assumption $\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2$ is also CFL, for all CFL's L_1, L_2

But we know $L_1 \cap L_2$ is not always a CFL since CFLs are not closed under complentation. This is a contradiction due to our initial assumption that CFL's are closed under complementation.

∴ All CFL's are not closed under complementation.

Hence Proved!!

(2.) Given: If L is CFL and R is Regular language

To prove: $L \cup R$ is CFL

Proof: Regular languages are CFL's.

∴ R is CFL.

We know that CFLs are closed under union.

Hence, $L \cup R$ is also a CFL

Eg: if $L = \{a^n b^n \mid n \geq 0\}$ and $R = a^* b^*$

are CFL and regular language over the alphabet $\{a, b\}$ respectively.

$L \cup R = a^* b^*$ which is regular, hence a CFL as well.

∴ $L \cup R$ is CFL. Hence Proved!!

③

ⓐ All strings over $\{0,1\}$ with the substring 0101

$$(0+1)^* \, 0101 \, (0+1)^*$$

ⓑ All strings beginning with 11 and ending with ab.

$$11(1+a+b)^* ab$$

ⓒ Set of all strings over $\{a,b\}$ with 3 consecutive b's

$$(a+b)^* \, bbb \, (a+b)^*$$

ⓓ Set of all strings that end with '1' and has no substring '00'

$$(1+01)^+$$

④ For a given string of length 'n'. No. of substrings of

$$\boxed{S = a_1 a_2 a_3 \cdots a_{n-1} a_n}$$

length 1, $= n$

2, $= n-1$

3, $= n-2$

$\vdots$

$n-1 = 2$

$n = 1$

Total no. of substrings are

$$\Rightarrow 1+2+3+\dots(n-1)+n$$

$$= \boxed{\frac{n(n+1)}{2}}$$

This is the total no. of substrings that can be formed from a string of length n.