PH 201

OPTICS & LASERS

Lecture_Lasers_2

Radiation & Thermal Equilibrium

Absorption & Stimulated Emission

Thermal equilibrium can be achieved between two masses by one or more possible heat transfer processes:

- **Conduction**
- **Convection, &**
- **Radiation**

- **Convection process takes much longer time to reach equilibrium than does conduction process.**

- **For radiation, two effects must take place:**
  - **masses must be radiating energy, &**
  - **they must be capable of absorbing radiation from other body.**

Radiating Bodies

If a collection of atoms is at temp. *T* then, according to Boltzmann Eqn., probability distribution function f_i that any atom has a discrete energy E_i;

$$f_i(E_i) = C_1 g_i e^{-E_i/kT}$$

***k* = Boltzmann's const. = 8.6164 × 10^{-5} eV/K**
g_i = statistical weight of level *i*
C_1 = normalizing const. It's value is same for all energy levels & is subject to constraint that

$$\sum_i f_i = \sum_i C_1 g_i e^{-E_i/kT} = 1$$

which suggests that electron must exist in one of the *i* energy levels.

If *N* is the total no. of atoms per unit volume of this species & N_i is population density occupying a specific energy level *i*, then

$$\sum_i N_i = N$$

$$N_i = f_i N = C_1 g_i e^{-E_i/kT} N$$

Radiating Bodies

For a high-density material such as solid, energy levels are usually continuously distributed.

Thus distribution function would be expressed as a <u>probability per unit energy</u> *g*(*E*) such that probability of finding a fraction of that material excited to a specific energy *E* within an energy width *dE* would be given by (ignoring statistical wt.)

$$g(E)dE = C_2 e^{-E/kT} dE$$

This probability would also be subject to normalizing constraint,

$$\int_0^\infty g(E)dE = \int_0^\infty C_2 e^{-E/kT} dE = 1$$

$$C_2 = \frac{1}{kT}, \qquad g(E) = \frac{1}{kT} e^{-E/kT}$$

If *N* denotes total no. of atoms per unit volume in solid & if we refer to no. of atoms per unit volume within a specific energy range *dE* as *N*(*E*), the normalizing condition requires,

$$N = \int_0^{\infty} N(E)dE$$

No. of atoms at energy *E* within a specific energy range *dE*,

$$N(E)dE = \frac{N}{kT} e^{-E/kT} dE$$

Ratio of populations that exists at two specific energies:

For discrete energy levels, ratio of population densities N_u & N_l (no. of particles per unit volume) of atoms with electrons occupying energy levels *u* & *l* (with corresponding energies E_u & E_l),

$$\frac{N_u}{N_l} = \frac{g_u}{g_l} e^{-(E_u - E_l)/kT} = \frac{g_u}{g_l} e^{-\Delta E_{ul}/kT}$$

Assuming E_u is higher than E_l, & $\Delta E_{ul} = E_u - E_l$

Similarly, ratio of population densities of a dense material (such as solid) at energies E_u & E_l within an energy interval dE,

$$\frac{N(E_u)dE}{N(E_l)dE} = \frac{N(E_u)}{N(E_l)} = e^{-\Delta E_{ul}/kT}$$

In dense materials, there are so many sublevels within small ranges of energy that statistical weights for most levels are essentially same; hence they would effectively cancel.

When a collection of atoms – whether in the form of a <u>gas</u>, a <u>liquid</u>, or a <u>solid</u> – are assembled together & reach equilibrium, not only kinetic energies related to their motion will be in thermal equilibrium; the distribution of their internal energies associated with specific energy levels they occupy will also be in thermal equilibrium.

- **Determine temp. required to excite electrons of atoms within a solid to energies sufficient to produce radiation in the visible portion of the electromagnetic spectrum when the electrons decay from those excited levels.**

Assuming a typical solid material with density $N = 5 \times 10^{28}$ atoms/m^3 in ground state.

For several different temps. we will compute how many of those atoms would occupy energy levels high enough to radiate that energy as visible light.

Visible radiation: 700 nm (red) to 400 nm (violet)

Photons with energies: 1.7 eV to 3.1 eV

We would thus be interested in energy levels above 1.7 eV that are populated within solid, since any electron having energy higher than 1.7 eV will have potential of radiating a visible photon.

We will calculate no. of species that have an electron in an excited energy level that lies higher than 1.7 eV above ground state for several different temps. by taking integral of $N(E)dE$ over energy range from 1.7 eV to infinity.

For a typical solid, most of atoms will decay nonradiatively from excited levels, but a certain portion could emit visible radiation depending upon radiation efficiency of material.

At room temp., T ~ 300 K & kT = 0.026 eV

$$N_{VIS} = \frac{N}{kT}\int_{1.7eV}^{\infty} e^{-E/kT}\,dE = -N\left[e^{-E/kT}\right]_{1.7eV}^{\infty}$$

$$= (-5\times 10^{28})[0 - 4\times 10^{-29}] \cong \frac{0}{m^3}$$

Thus, there are essentially no atoms in this energy range from which visible photons could radiate.

For this reason we can see nothing when we enter a room that has no illumination, even though human is very sensitive & can detect as only a few photons. No thermal radiation in visible spectrum could be emitted from walls, floors, ceiling, or furniture when those various masses are at room temp.

Consider temp. of 1000 K & kT = 0.086 eV

$$N_{VIS} = \frac{N}{kT} \int_{1.7eV}^{\infty} e^{-E/kT} dE = -N \left[e^{-E/kT} \right]_{1.7eV}^{\infty}$$

$$= (-5\times10^{28})[0 - 2.6\times10^{-9}] = \frac{1.3\times10^{20}}{m^3}$$

Thus, increasing temp. by more than a factor of 3, we have gone from essentially no atoms in those excited levels to an appreciable no. in those levels.

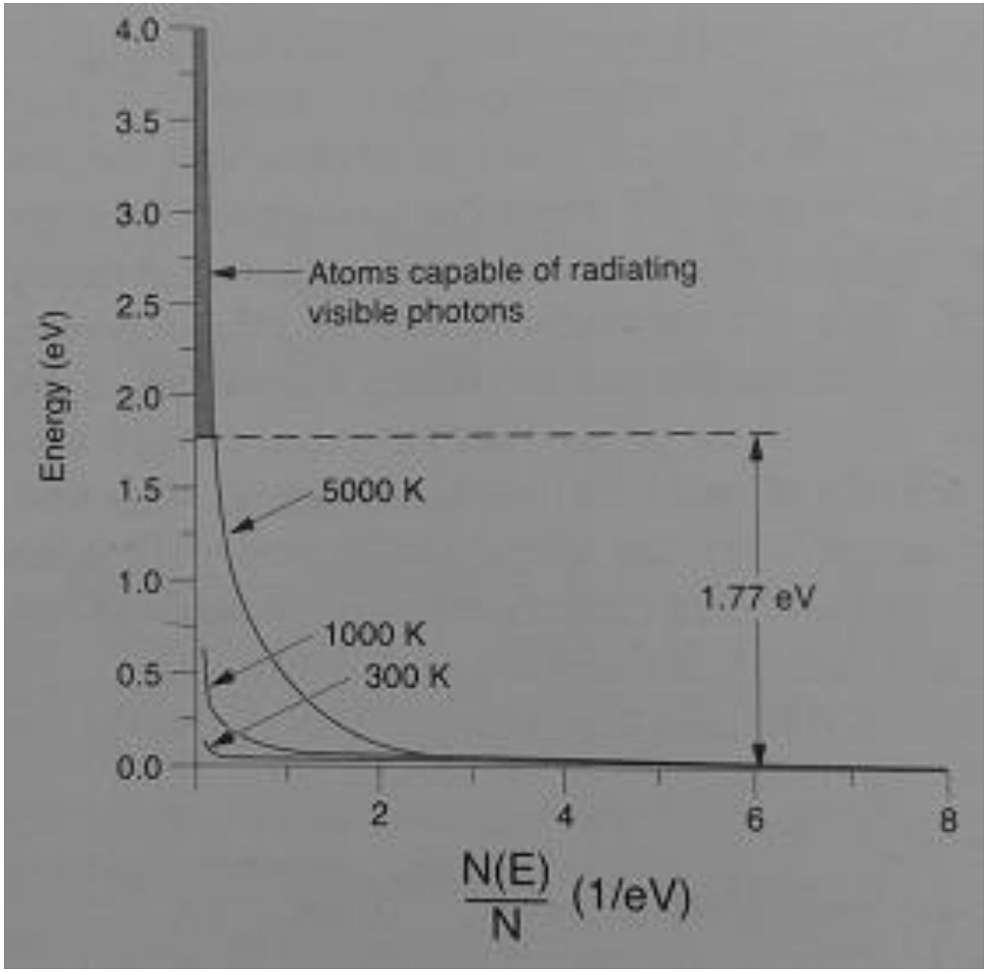
Ex. In glowing coals of a campfire, in glowing briquettes of a barbecue fire, or from heating elements of an electric stove; all are at temps. of approx. 1,000 K.

Consider temp. of 5,000 K (temp. of sun) & kT = 0.43 eV

$$N_{VIS} = \frac{N}{kT} \int_{1.7eV}^{\infty} e^{-E/kT} dE = -N\left[e^{-E/kT}\right]_{1.7eV}^{\infty}$$

$$= (-5\times10^{28})[0 - 1.9\times10^{-2}] = \frac{9.5\times10^{26}}{m^3}$$

At this temp., nearly 10% of atoms would be excited to an energy of 1.7 eV or higher, & material would be radiating with an intensity that is too bright to look at.



Rapid increase in population at higher energies with increase in temp. Energy levels that are high enough to produce visible radiation are populated only when temp is significantly above room temp.

When observing thermal radiation (such as glowing coals), there are two effects:

- **Stefan-Boltzmann law: More energy is radiated from object as temp is increased. Total radiated intensity emitted from a body at temp *T* is proportional to 4th power of temp.**

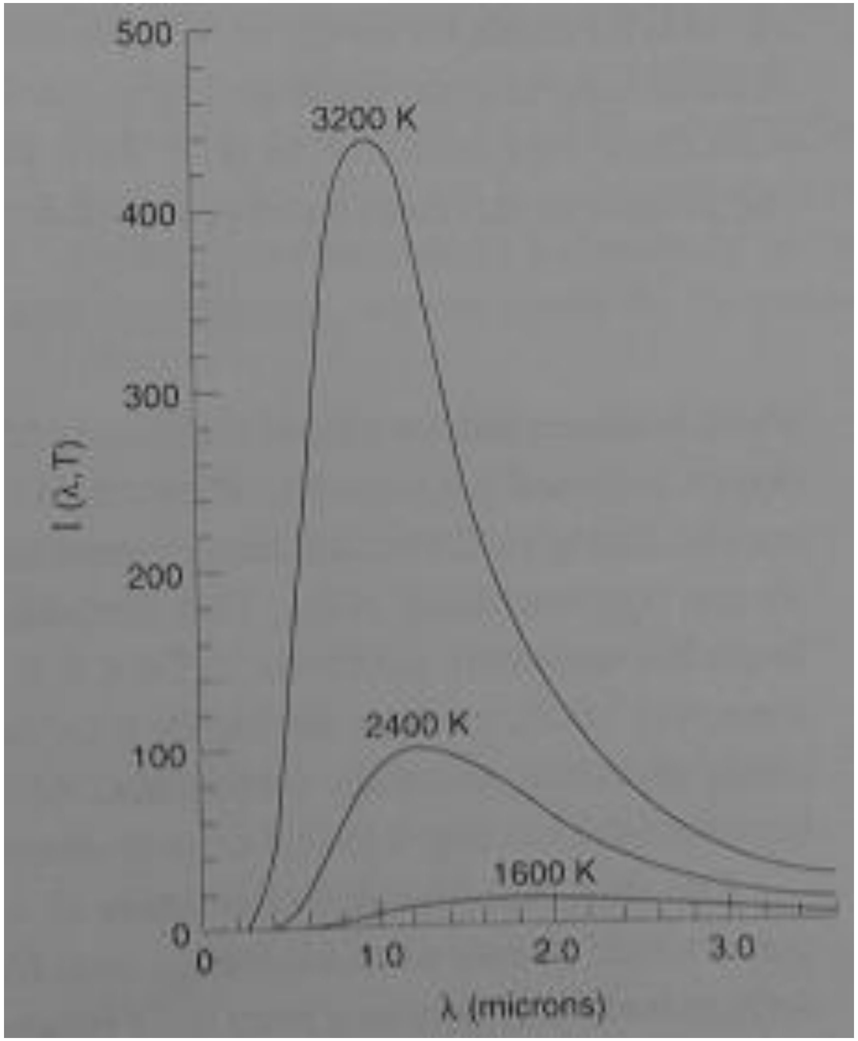
$$I = e_M \sigma T^4$$

σ = 5.6 × 10^{-8} W/m^2-K^4, e_M = emissivity, it's value is specific for a given material. Its value varies between 0 & 1.

- **Wien's law (analysis of spectral content of radiation): Radiation increases with decreasing wavelength to a maximum value at a specific wavelength & then it decreases relatively rapidly at even shorter wavelengths.**

Wavelength at which maximum emission occurs for any given temp.

$$\lambda_m T = 2.898 \times 10^{-3}\, mK$$

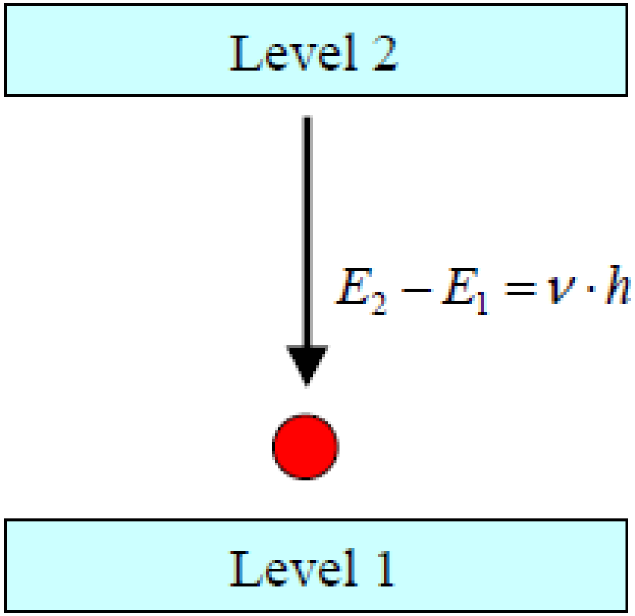


Spectrum of radiation versus wavelength of a heated mass (blackbody) of several temps.

Atomic transitions

The frequency of the emitted photon going from Level 2 to 1 is given by:

$$\nu = \frac{E_2 - E_1}{h}$$



Defining *Ni* as the electron population of level *i* and considering the Boltzman equation which describes the relation between the electrons in level 1 and 2 at thermal equilibrium:

$$N_2 - N_1 = \exp\left(-\frac{E_2 - E_1}{k_B T}\right)$$

(k_B = Boltzman constant)

giving that $E_2 > E_1$ and $T>0$ => $N_1>N_2$!

Atomic transitions

To amplify light, the stimulated emission must by stronger than the absorption ($N_2 > N_1$)… but how is this possible???

An electromagnetic wave with frequency $\boldsymbol{\nu}$ traveling in $\boldsymbol{z}$-direction through the media with 2 atom levels is normally exponentially absorbed:

$$I = I_0 \exp(-\mu z)$$

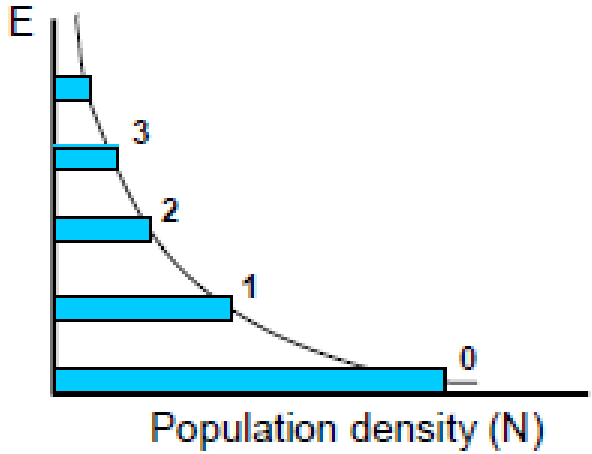
$\mu > \mathbf{0}$ is the lineal Absorption coefficient.

It can be demonstrated that $\mu \sim N_1 - N_2 > 0$

The amplification of light is only possible if $N_1 - N_2 < 0$ ($\Rightarrow \mu < \mathbf{0}$)

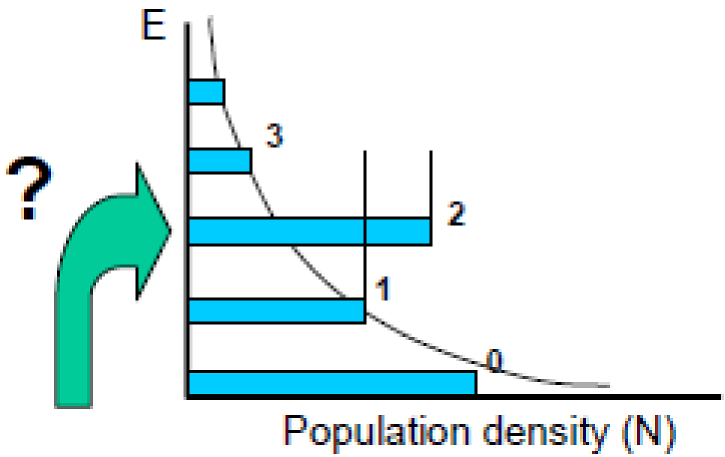
That means… **THE MEDIA IS ACTIVE**!

Inversion of energy levels



$$N_2 - N_1 = \exp\left(-\frac{E_2 - E_1}{k_B T}\right)$$

$$N_2 - N_1 < 0$$



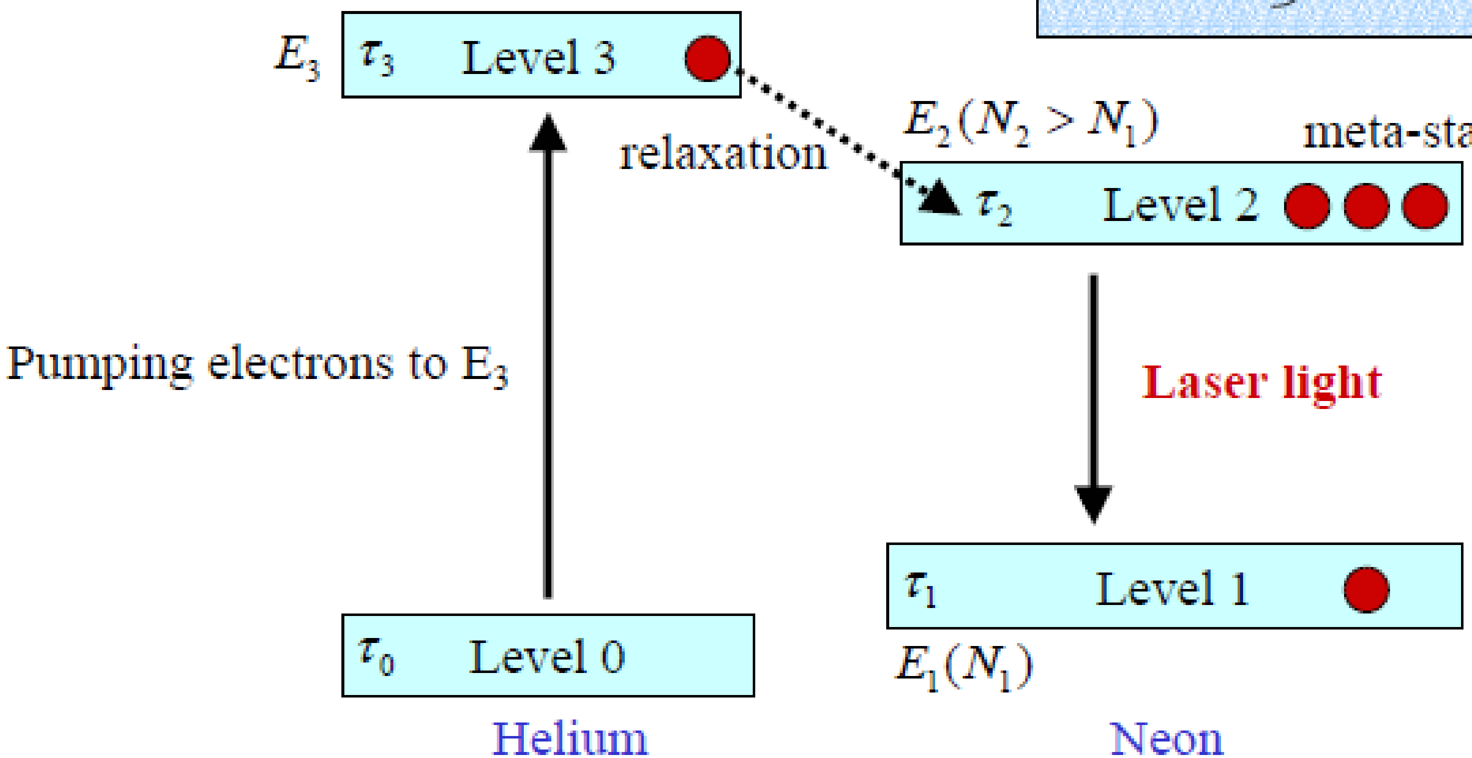
$$\Delta N = N_2 - N_1 > 0$$

$$\Delta E = h\gamma = hc / \lambda$$

Inversion of energy levels

One solution to this problem is to use three energy levels (example He-Ne laser):

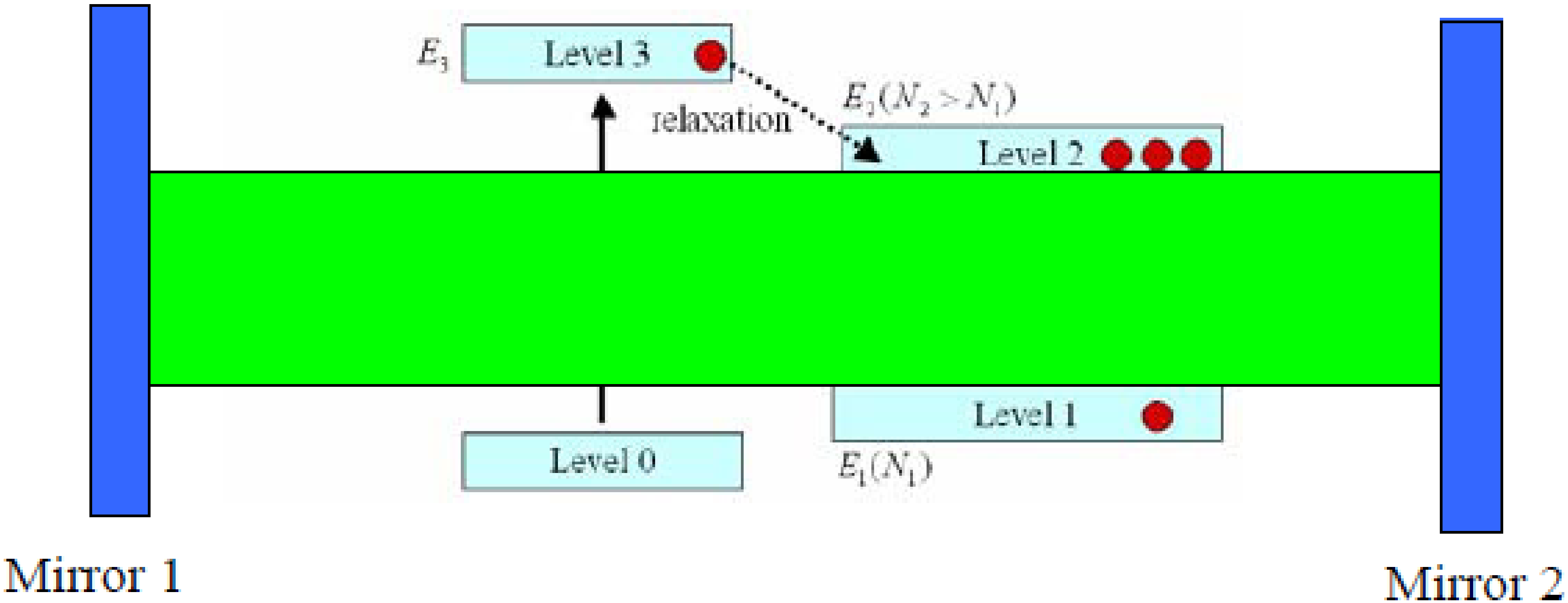
$$\left.\begin{array}{l} \tau_2 > \tau_3 \\ \tau_2 \quad long \\ \tau_1 \quad short \end{array}\right\} N_2 > N_1!!!$$



Stimulated emission

Now $N_2 > N_1$, however… we must amplify the intensity of our beam!

➔ We must build a LASER resonator!



However, with this set-up the intensity will grow up to infinite!

What can we do to obtain the LASER beam?