CS225 Switching Theory

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Previous Class

- Switching Algebra
    - Switching circuit
    - Propositional calculus

This Class

Minimization/ Simplification **of Switching Functions**

Last class

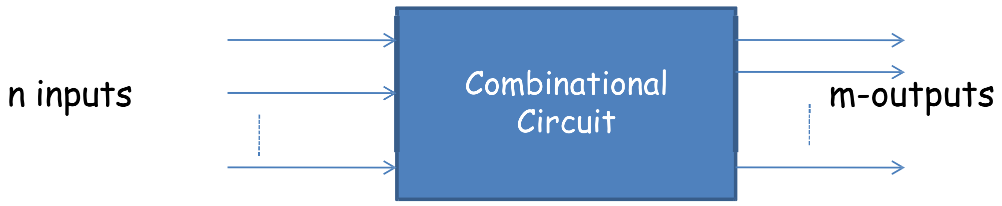
Q1. Solve the following expression

$$(x + y'z')(y + x'z')(z + x'y')$$

Ans:

$$\begin{aligned}
&(x + y'z')(y + x'z')(z + x'y') \\
&= (xy + xx'z' + y'z'y + y'z'x'z')(z + x'y') \\
&= (xy + 0 + 0 + x'y'z')(z + x'y') \\
&= xyz + xyx'y' + x'y'z'z + x'y'z'x'y' \\
&= xyz + 0 + 0 + x'y'z' \\
&= xyz + x'y'z'
\end{aligned}$$

Combinational Circuit



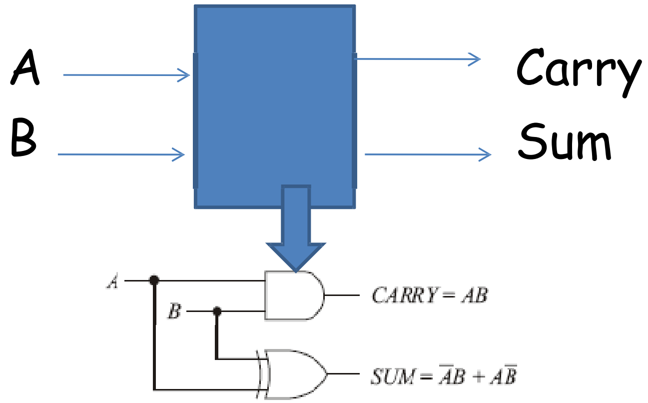
Design procedure:

from the specification

1. Determine the required number of inputs and outputs
2. Derive the truth table
3. Find the Simplified Boolean expression for each output as a function of the input variable
4. Draw the logic diagram
5. Verify the correctness

EX.1:Binary Adder

- Half Adder:
- Four cases to remember (Two single-bit addition)
  - 0+0=0
  - 0+1=1
  - 1+0=1
  - 1+1=10 (Carry has been generated)



Combinational Circuit

- Analysis

1. Find the Boolean functions for each gate and obtain the output
2. Repeat step 1 until the output(s) of the circuit is obtained
3. Obtain the output Boolean function in terms of input variables

Find the simplified Boolean expression!
with minimum terms\ literals

Definitions

- Literals x_i or x_i'
- Product Term $x_2x_1'x_0$
- Sum Term $x_2 + x_1' + x_0$

- Minterm of n variables: A product of n literals in which every variable appears exactly once.

- Maxterm of n variables: A sum of n literals in which every variable appears exactly once.

- Adjacency of minterms (maxterms): Two minterms (maxterms) are adjacent if they differ by only one variable.

Implementation

Specification → Schematic Diagram
Net list,
Switching expression

Obj min cost → Search in solution space
(max performance)

Cost: wires, gates → Literals, product terms, sum terms

For two level logic (sum of products or product of sums), we want to minimize # of terms, and # of literals

Simplifying Switching Functions

Finding an equivalent switching expression that minimizes some cost criteria:

1. Minimize literal count
2. Minimize literal count in sum-of-products (or product-of-sums) expression
3. Minimize number of terms in a sum-of-products expression provided no other expression exists with the same number of terms and fewer literals

Example: $f(x,y,z) = x'yz' + x'y'z + xy'z' + x'yz + xyz + xy'z$

$= x'z' + y'z' + yz + xz$

Implementation: Specification => Logic Diagram

Karnaugh Map: A 2-dimensional truth table
Boolean expressions can be minimized by combining terms
K-maps minimize equations graphically

Flow 1: Boolean Algebra

1. Specification
2. Truth table
3. Sum of products (SOP) or product of sums(POS) canonical form
4. Reduced expression using Boolean algebra
5. Schematic diagram of two level logic

Flow 2: K Map

1. Specification
2. Truth Table
3. Karnaugh Map (truth table in two dimensional space)
4. Reduce using K-Maps
5. Reduced expression (SOP or POS)
6. Schematic diagram of two level logic

K-Map: Truth Table in 2 Dimensions

2- Variables Truth Table

ID	A	B	f(A,B)
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	1

2- Variables K-Map

	B = 0	B = 1
A = 0	0 0	1 1
A = 1	2 1	3 1

Algebraic procedure to combine terms using the *Aa + Aa' = A* rule

f(A,B) = A + B

The Map Method

Karnaugh map: modified form of truth table

z \ xy	00	01	11	10
0	0	2	6	4
1	1	3	7	5

(a) Location of minterms in a three variable map.

z \ xy	00	01	11	10
0		1	1	
1			1	

(b) Map for function $f(x, y, z) = \sum(2,6,7) =$ $yz' + xy$

yz \ wx	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

(c) Location of minterms in a four-variable map.

yz \ wx	00	01	11	10
00		1	1	1
01		1	1	
11			1	
10			1	

(d) Map for function $f(w, x, y, z) = \sum(4,5,8,12,13,14,15) =$ $wx + xy' + wy'z'$

Simplification and Minimization of Functions

Cube: collection of 2^m cells, each adjacent to *m* cells of the collection

- Cube is said to **cover** these cells
- Cube expressed by a product of *n-m* literals for a function containing *n* variables
- *m* literals not in the product said to be **eliminated**

Example: $w'xy'z' + w'xy'z + wxy'z' + wxy'z$

$= xy'(w'z' + w'z + wz' + wz)$

$= xy'$

yz \ wx	00	01	11	10
00		1	1	1
01		1	1	
11			1	
10			1	

(d) Map for function f(w, x, y, z) = $\sum(4, 5, 8, 12, 13, 14, 15) =$

$wx + xy' + wy'z'$

Minimization (Contd.)

Example:

Use of cell 6 in forming both cubes justified by idempotent law

z \ xy	00	01	11	10
0	0	2	6	4
1	1	3	7	5

(a) Location of minterms in a three-variable map.

z \ xy	00	01	11	10
0		1	1	
1			1	

(b) Map for function

f(x, y, z) = $\sum(2, 6, 7) = yz' + xy$

Corresponding algebraic manipulations:

$$
\begin{aligned}
f &= x'yz' + xyz' + xyz \\
&= x'yz' + xyz' + xyz' + xyz \\
&= yz'(x' + x) + xy(z' + z) \\
&= yz' + xy
\end{aligned}
$$

Thanks