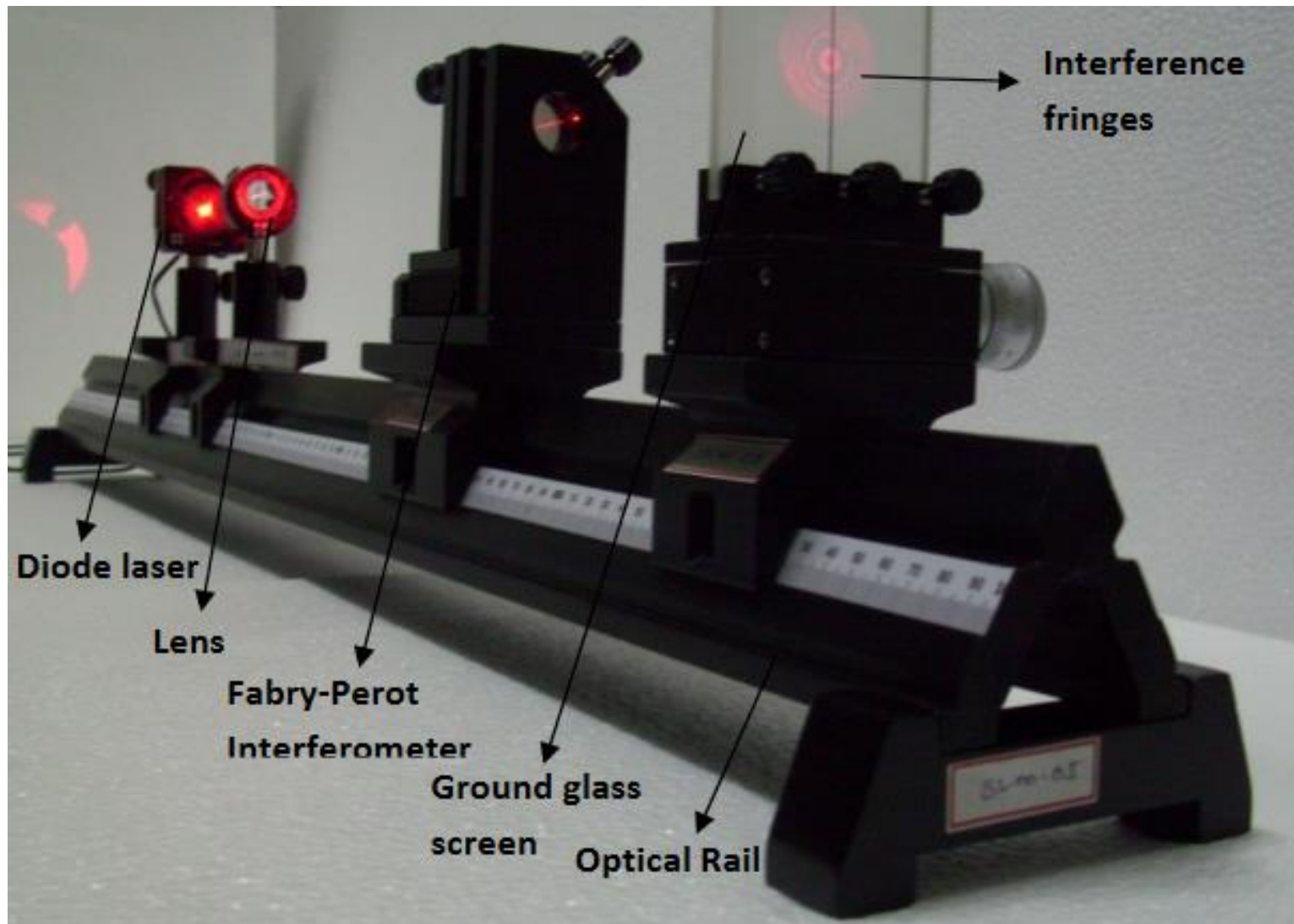


**PH 301**

**ENGINEERING OPTICS**

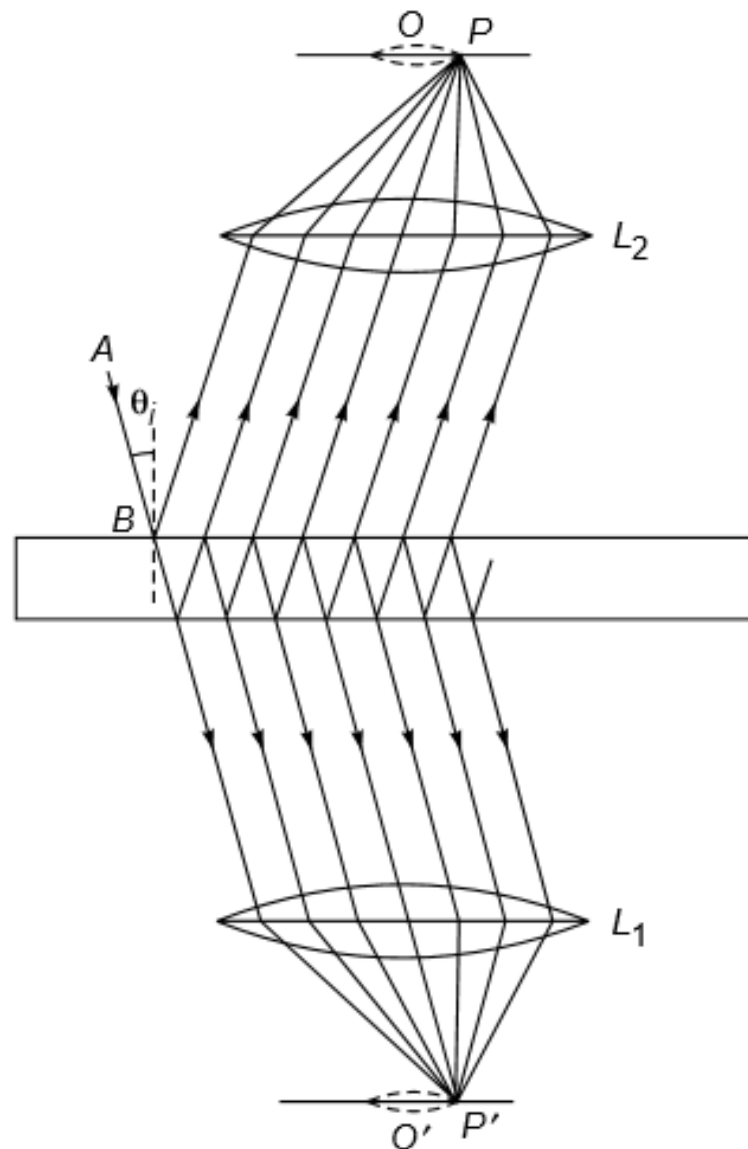
**Lecture\_Interferometers\_14**



**Fabry-Perot set-up**

$$R = r_1^2 = r_2^2$$

$$\tau = t_1 t_2 = 1 - R$$



$$\mathcal{R} = \frac{F \sin^2 \delta/2}{1 + F \sin^2 \delta/2}$$

$$F = \frac{4R}{(1 - R)^2}$$

$$T = \frac{1}{1 + F \sin^2 \delta/2}$$

Any ray parallel to AB will focus at the same point P. If ray AB is rotated about normal at B, then point P will rotate on the circumference of a circle centered at O; this circle will be bright or dark depending on the value of  $\theta_i$ . Rays incident at different angles will focus at different distances from O, & one will obtain concentric bright & dark rings for an extended source.

**Reflectivity & Transmittivity of Fabry-Perot etalon add to unity.**

$$\mathfrak{R} + T = \frac{F \sin^2 \delta / 2}{1 + F \sin^2 \delta / 2} + \frac{1}{1 + F \sin^2 \delta / 2} = \frac{1 + F \sin^2 \delta / 2}{1 + F \sin^2 \delta / 2} = 1$$

$$T = \frac{1}{1 + F \sin^2 \delta / 2} = 1 \quad \text{if} \quad \delta = 2m\pi, \quad m = 1, 2, 3, \dots$$

**To get an estimate of width of transmission resources, let**

$$T = \frac{1}{1 + F \sin^2 \delta / 2} = \frac{1}{2} \quad \text{if} \quad \delta = 2m\pi \pm \frac{\Delta\delta}{2}$$

$$\frac{1}{1 + F \sin^2 \delta / 2} = \frac{1}{2}$$

$$1 + F \sin^2 \delta / 2 = 2$$

$$F \sin^2 \delta / 2 = 1$$

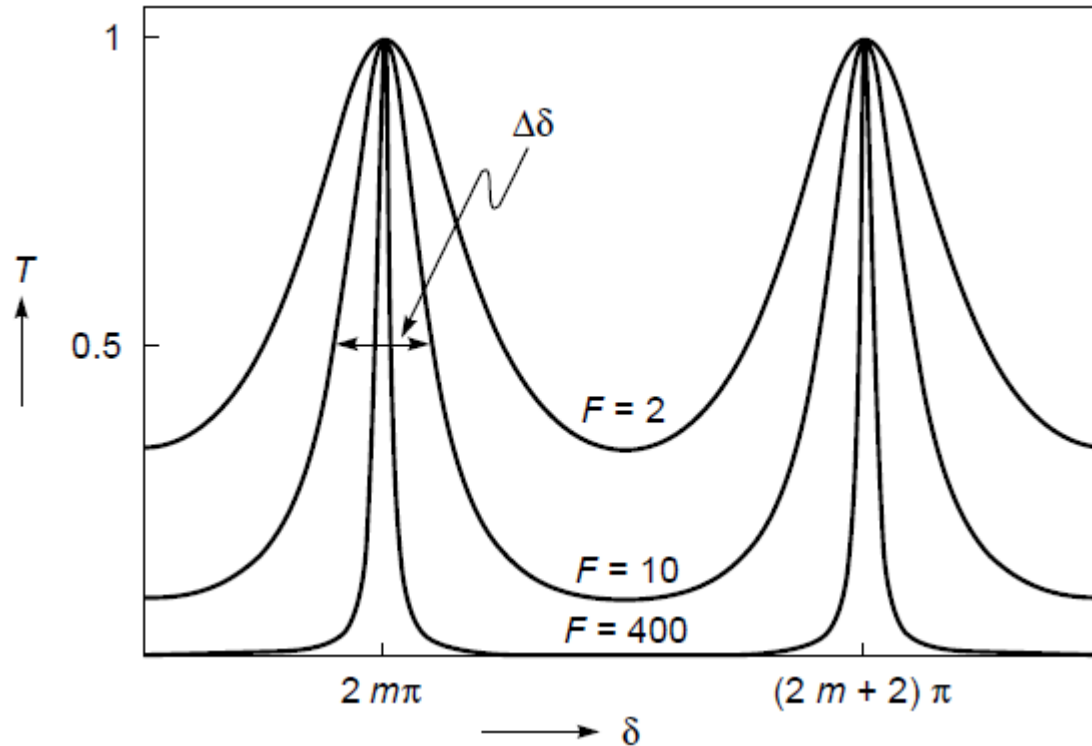
$$\sin^2 \delta / 2 = \frac{1}{F}$$

$$T = \frac{1}{2} \quad \text{if} \quad \delta = 2m\pi \pm \frac{\Delta\delta}{2}$$

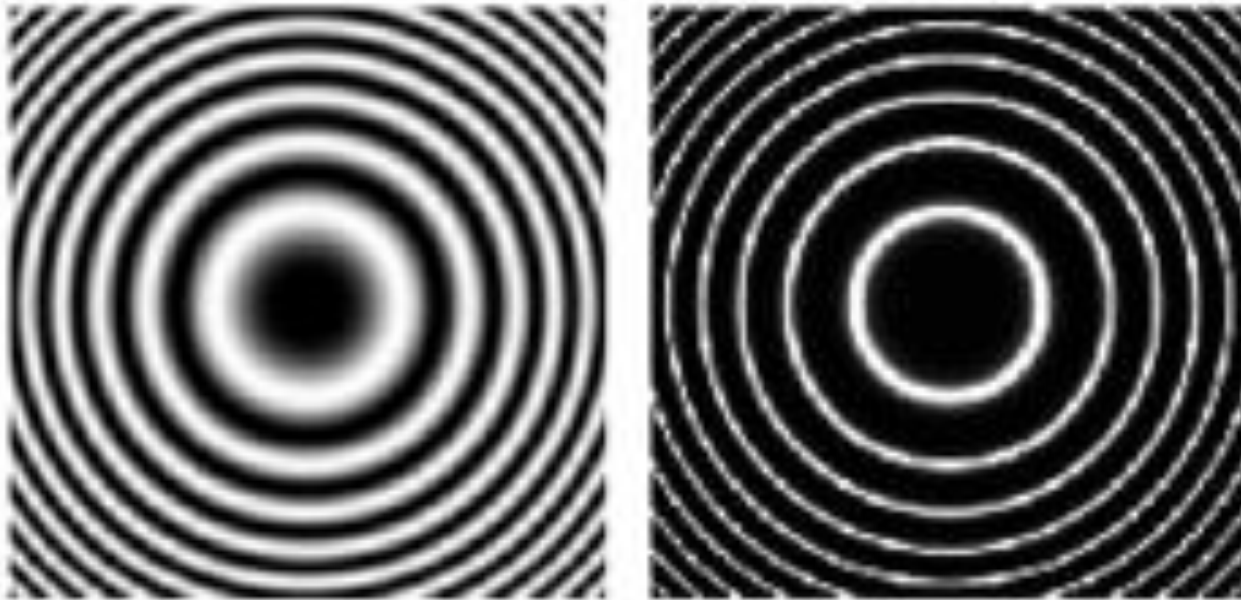
$$\sin\left(\frac{\Delta\delta}{4}\right) = \frac{1}{\sqrt{F}}$$

$$\frac{\Delta\delta}{4} = \frac{1}{\sqrt{F}} \quad \Rightarrow \quad \Delta\delta = \frac{4}{\sqrt{F}} = \frac{2(1-R)}{\sqrt{R}}$$

**Thus, transmission resources become sharper as value of  $F$  increases.**



Transmittivity of a Fabry-Perot etalon as a function of  $d$  for different values of  $F$ ; value of  $m$  is usually large. Transmission resonances become sharper as we increase  $F$ . FWHM is  $\Delta\delta$ .



Low finesse vs. high finesse

**Ex.** Assume an etalon with  $n_2 = 1$ ,  $h = 1$  cm, and  $F = 400$  ( $F = 400$  implies  $R \approx 0.905$ ; i.e., each mirror of the etalon has about 90% reflectivity).  $\lambda_1 = 5000 \text{ \AA}$  &  $\lambda_2 = 4999.98 \text{ \AA}$ .

$$\delta = \frac{4\pi}{\lambda} n_2 h \cos \theta_2 = 2m\pi$$

$$\cos \theta_2 = \frac{m\lambda}{2n_2 h}$$

$$\theta_2 = \cos^{-1} \left( \frac{m\lambda}{2n_2 h} \right)$$

For  $\lambda_1 = 5000 \text{ \AA}$ ,

$$\theta_2 = \cos^{-1} \left( \frac{m}{40000} \right)$$

$$\theta_2 = 0^\circ, 0.41^\circ, 0.57^\circ, 0.70^\circ, \dots$$

$$m = 40000; 39999; 39998; 39997; \dots$$



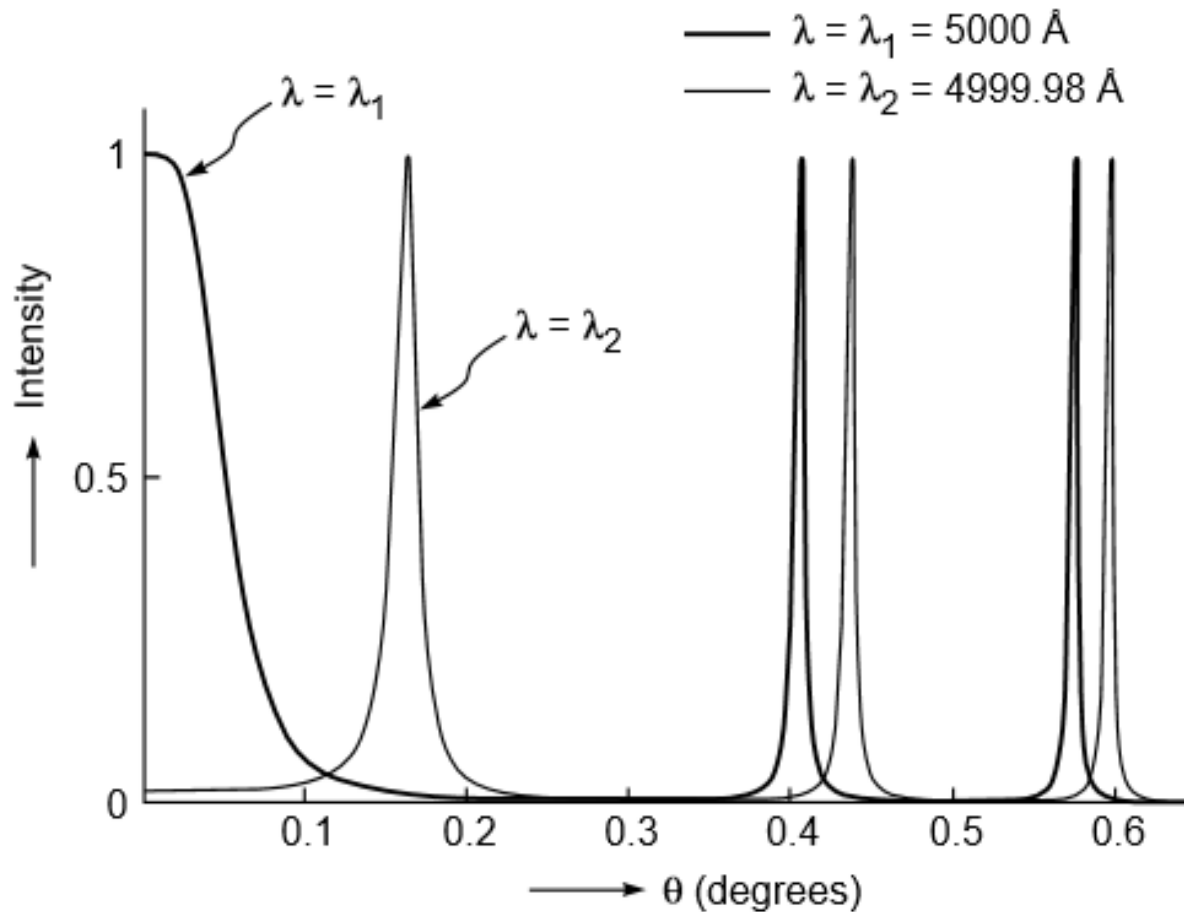
For  $\lambda_2 = 4999.98 \text{ \AA}$ ,

$$\theta_2 = \cos^{-1}\left(\frac{m}{40000.16}\right)$$

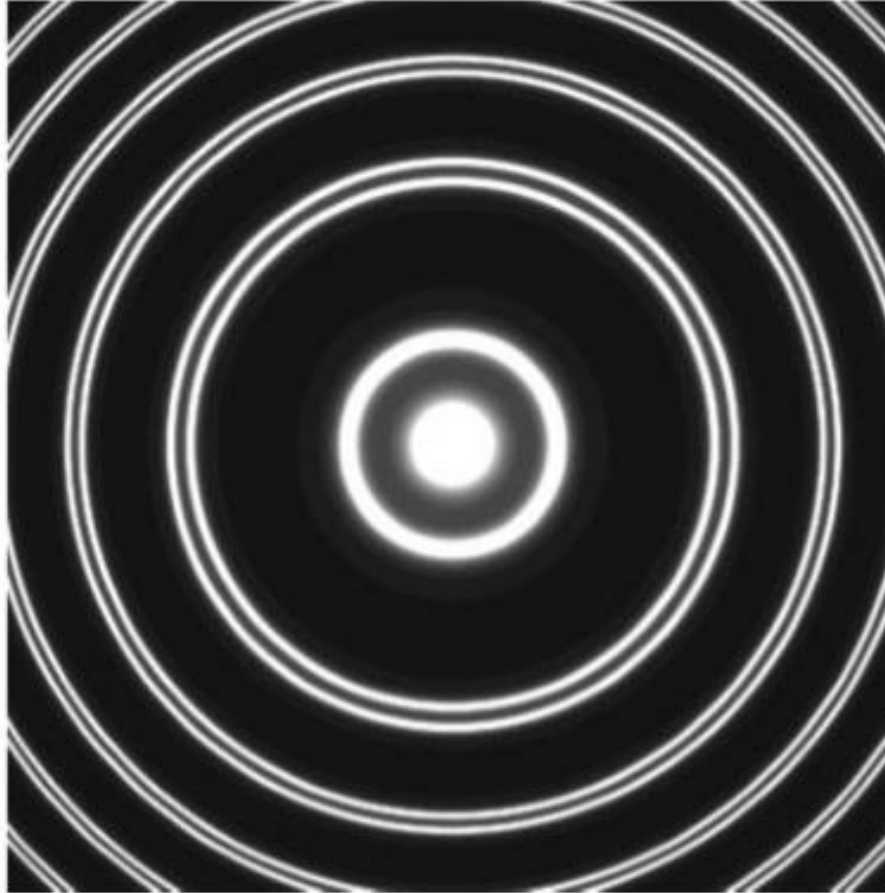
$$\theta_2 = 0.162^\circ, 0.436^\circ, 0.595^\circ, \dots$$

$$m = 40000; 39999; 39998; 39997; \dots$$

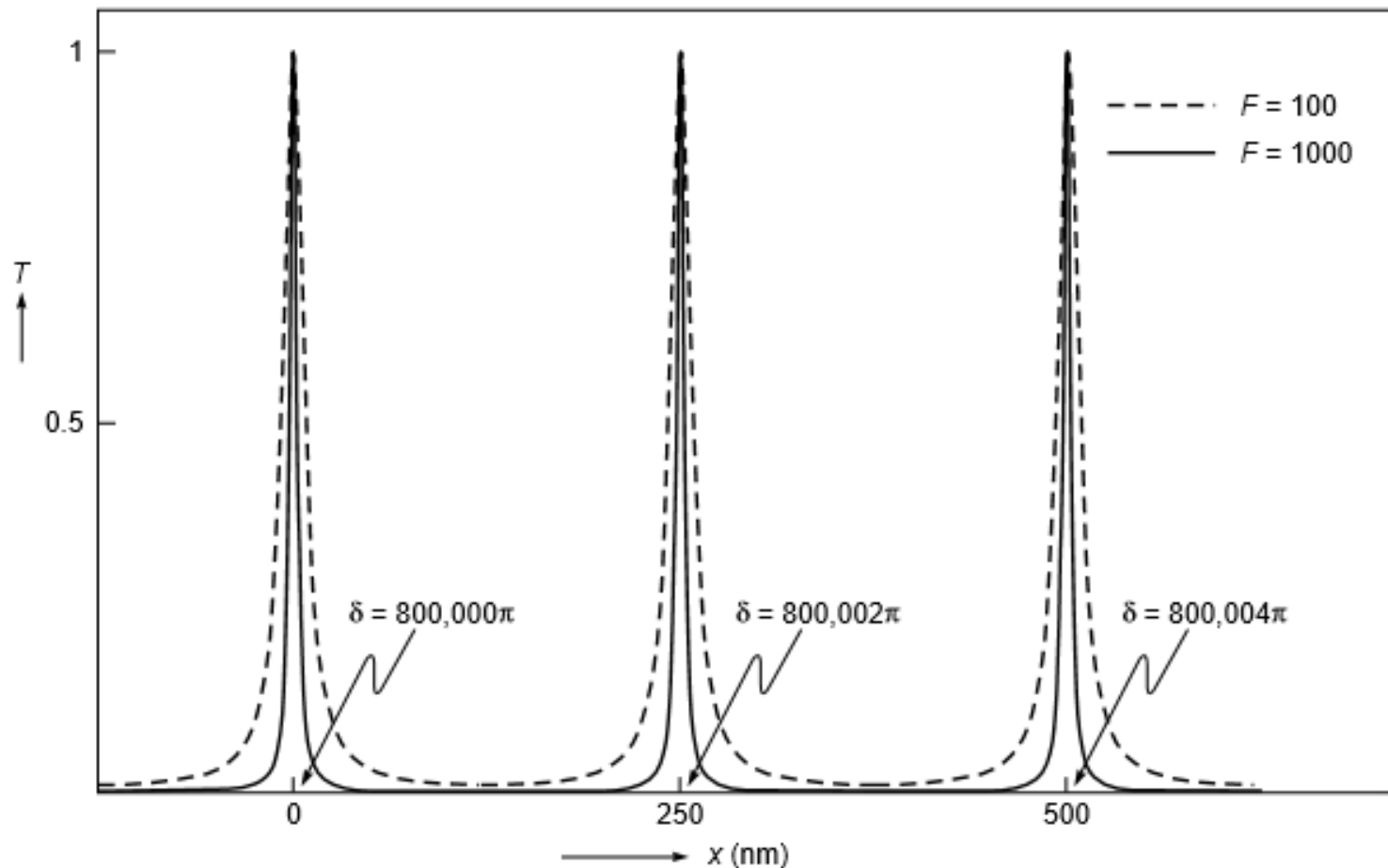
$$\Delta\lambda_2 = 0.02 \text{ \AA}$$



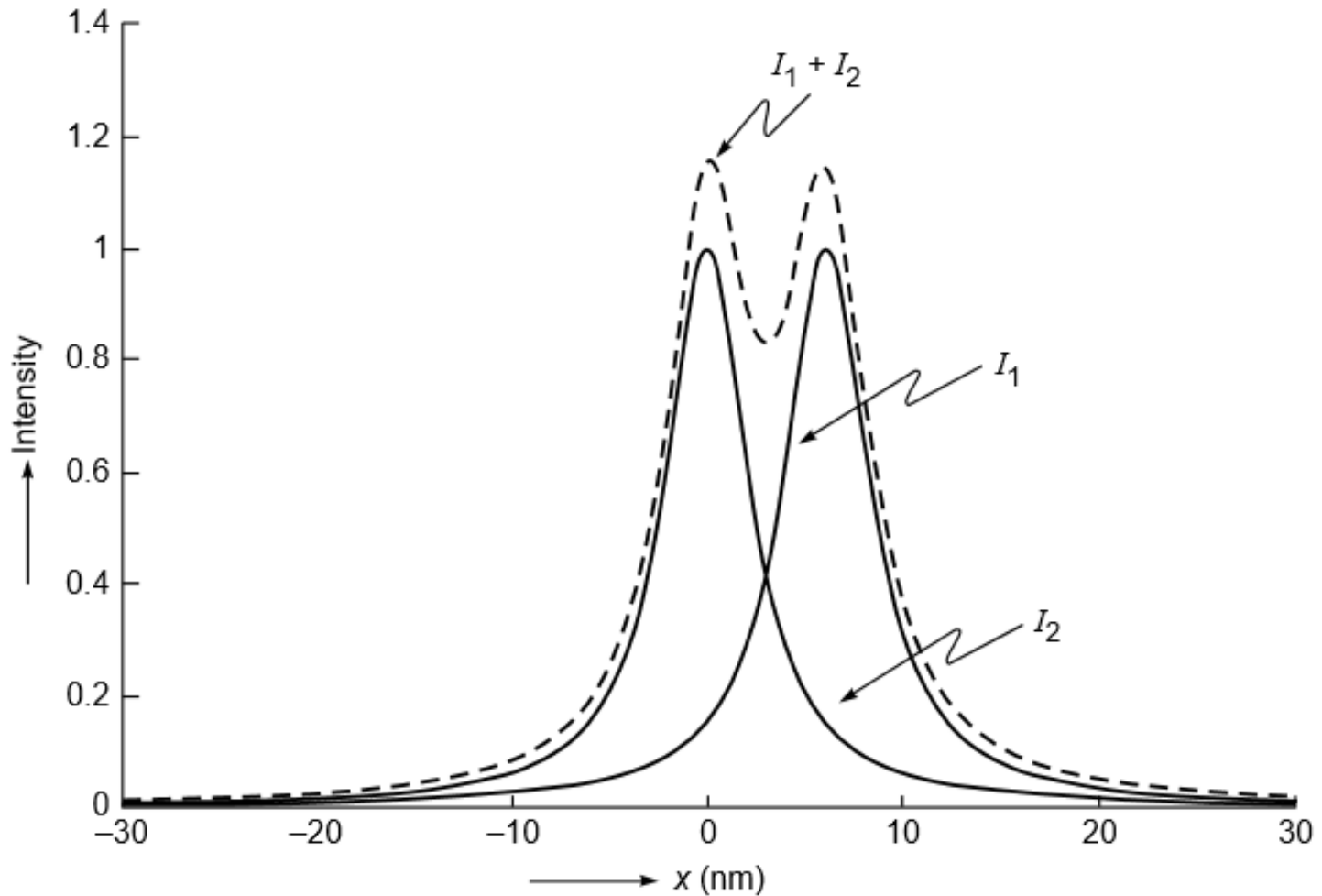
Variation of intensity with  $\theta$  for a Fabry-Perot interferometer with  $n_2 = 1$ ,  $h = 1.0 \text{ cm}$ , &  $F = 400$ ,  $\lambda_1 = 5000 \text{ \AA}$  &  $\lambda_2 = 4999.98 \text{ \AA}$ .



Computer-generated ring pattern in a Fabry-Perot etalon with  $n_2 = 1$ ,  $h = 1.0$  cm,  $F = 400$  corresponding to  $\lambda_1 = 5000$  Å &  $\lambda_2 = 4999.98$  Å.



Variation of intensity at point P with  $x$  for a monochromatic beam incident normally on a scanning Fabry-Perot interferometer; solid curve corresponds to  $F = 1000$  & dashed curve corresponds to  $F = 100$ .



Individual intensity variations  $I_1$  &  $I_2$  in presence of two frequencies  $\nu_1$  &  $\nu_2$  & total intensity variation  $I_1 + I_2$  when two frequencies are just resolved.

# Sodium doublet

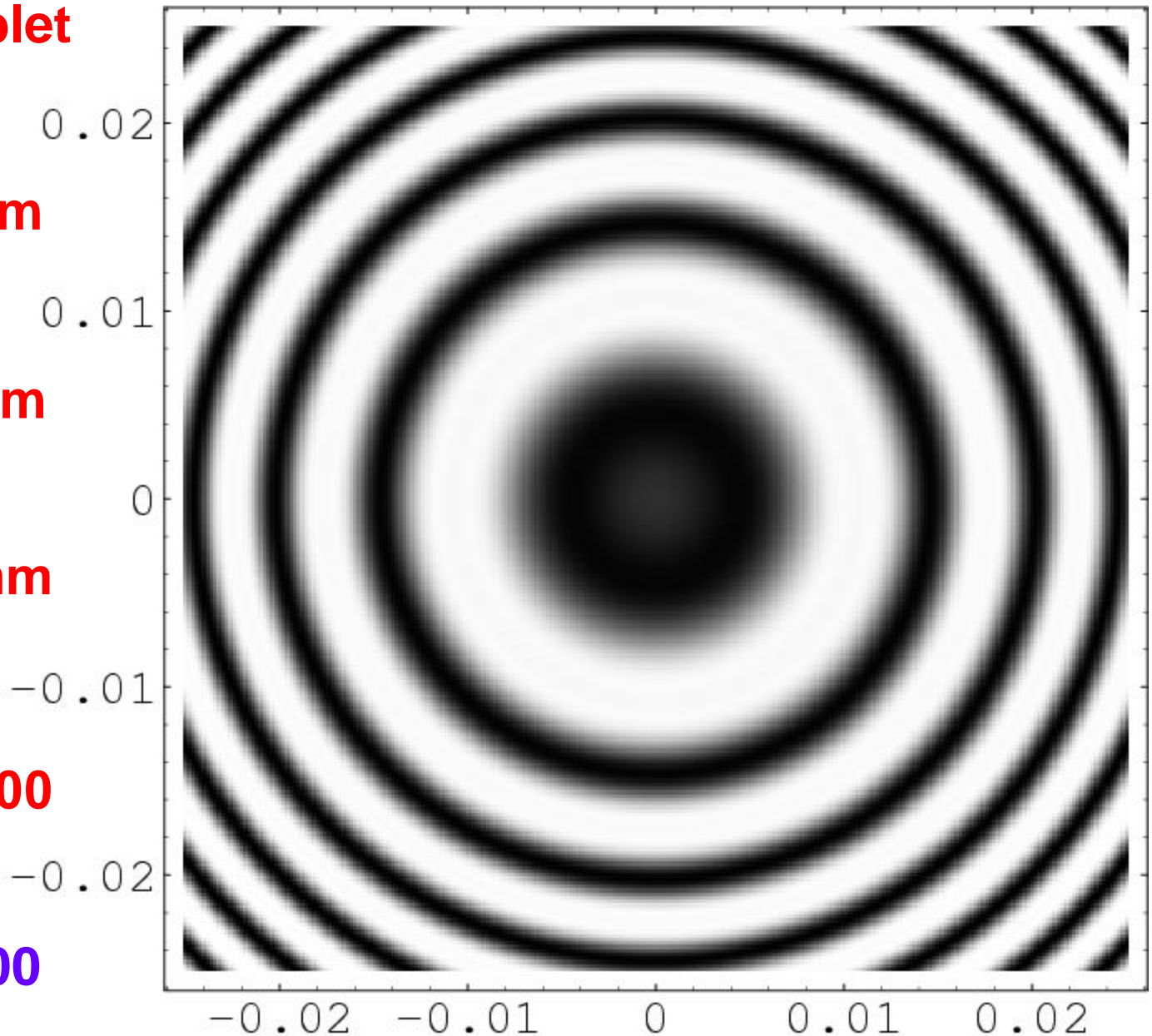
$\lambda_1 = 589.0 \text{ nm}$

$\lambda_2 = 589.6 \text{ nm}$

$\Delta\lambda = 0.6 \text{ nm}$

$\lambda/\Delta\lambda \sim 1000$

RP < 1000



# Sodium doublet

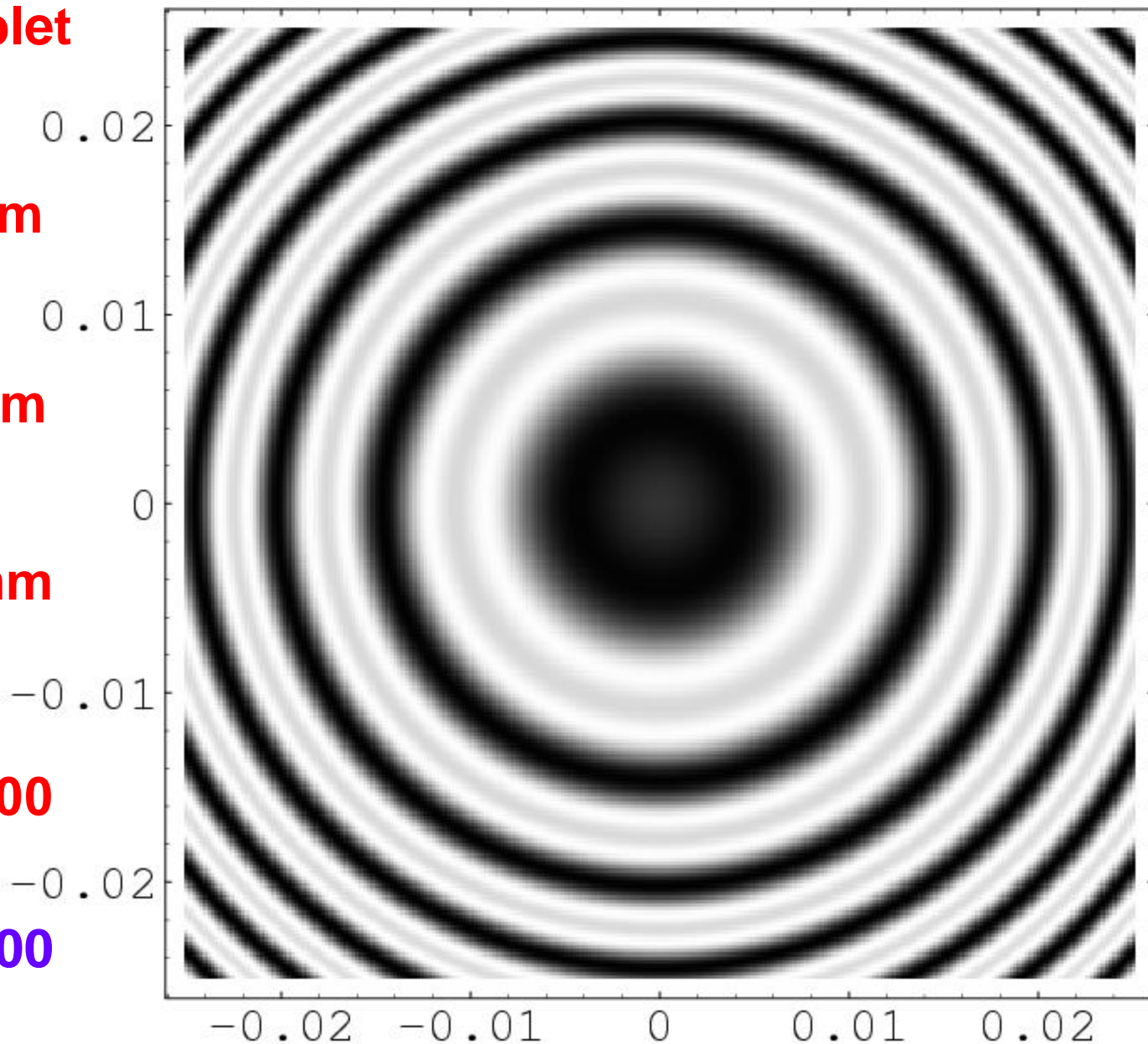
$\lambda_1 = 589.0 \text{ nm}$

$\lambda_2 = 589.6 \text{ nm}$

$\Delta\lambda = 0.6 \text{ nm}$

$\lambda/\Delta\lambda \sim 1000$

RP  $\sim 1000$



## Sodium doublet

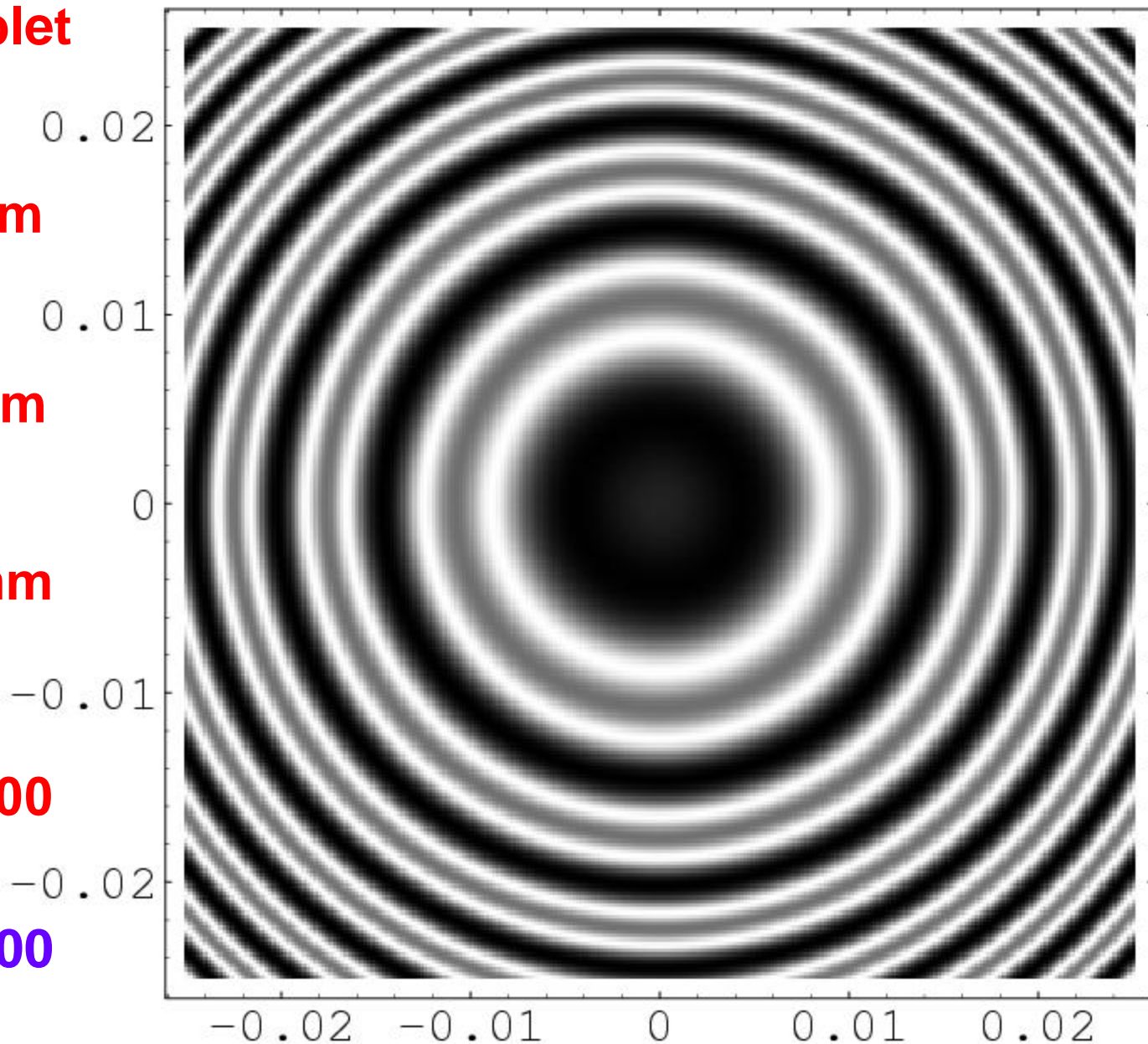
$$\lambda_1 = 589.0 \text{ nm}$$

$$\lambda_2 = 589.6 \text{ nm}$$

$$\Delta\lambda = 0.6 \text{ nm}$$

$$\lambda/\Delta\lambda \sim 1000$$

$$\text{RP} > 1000$$





## Sodium doublet

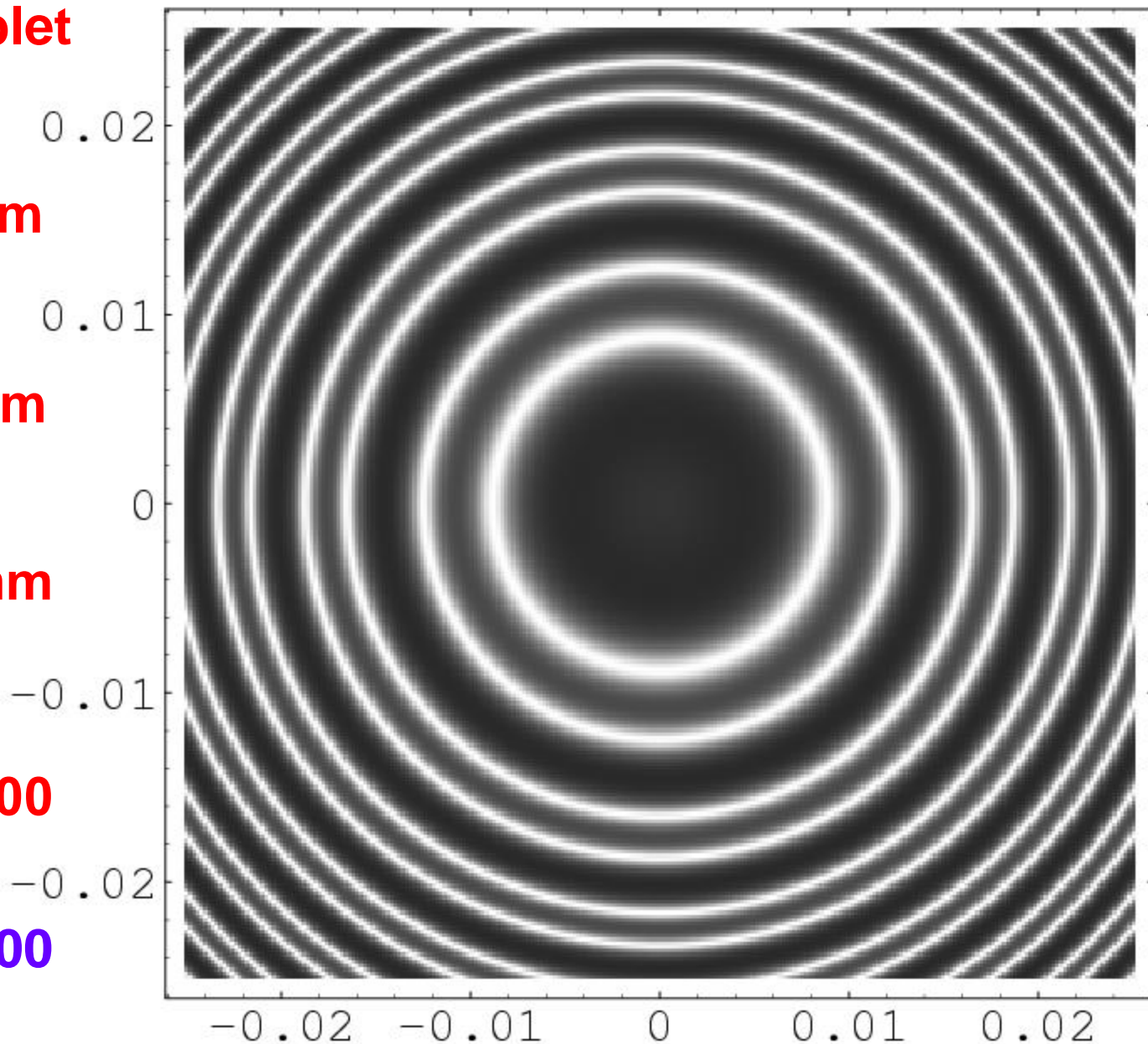
$$\lambda_1 = 589.0 \text{ nm}$$

$$\lambda_2 = 589.6 \text{ nm}$$

$$\Delta\lambda = 0.6 \text{ nm}$$

$$\lambda/\Delta\lambda \sim 1000$$

$$RP \gg 1000$$



# Sodium doublet

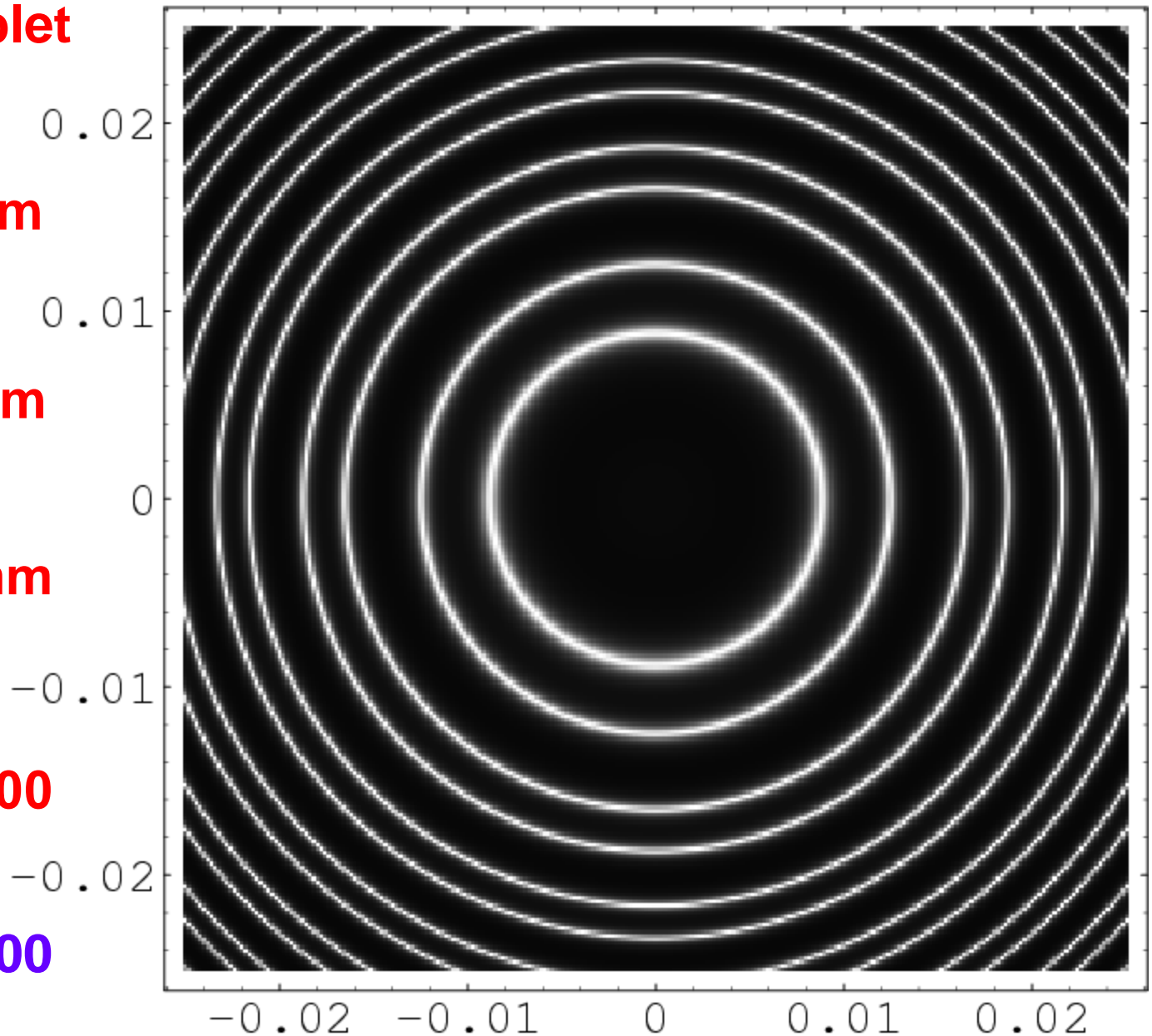
$\lambda_1 = 589.0 \text{ nm}$

$\lambda_2 = 589.6 \text{ nm}$

$\Delta\lambda = 0.6 \text{ nm}$

$\lambda/\Delta\lambda \sim 1000$

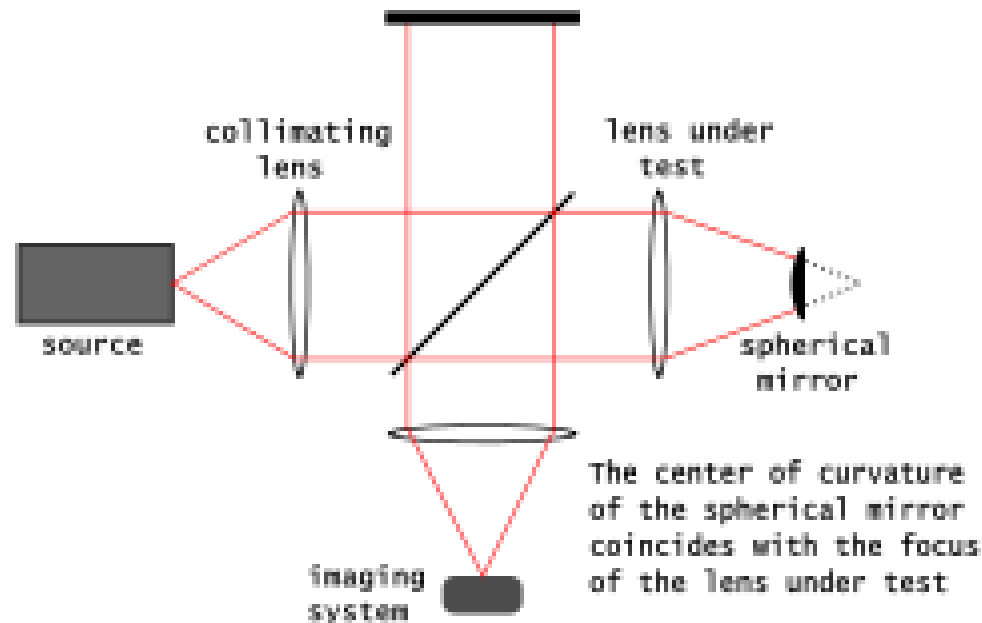
$RP \gg 1000$



# Twyman & Green Interferometer

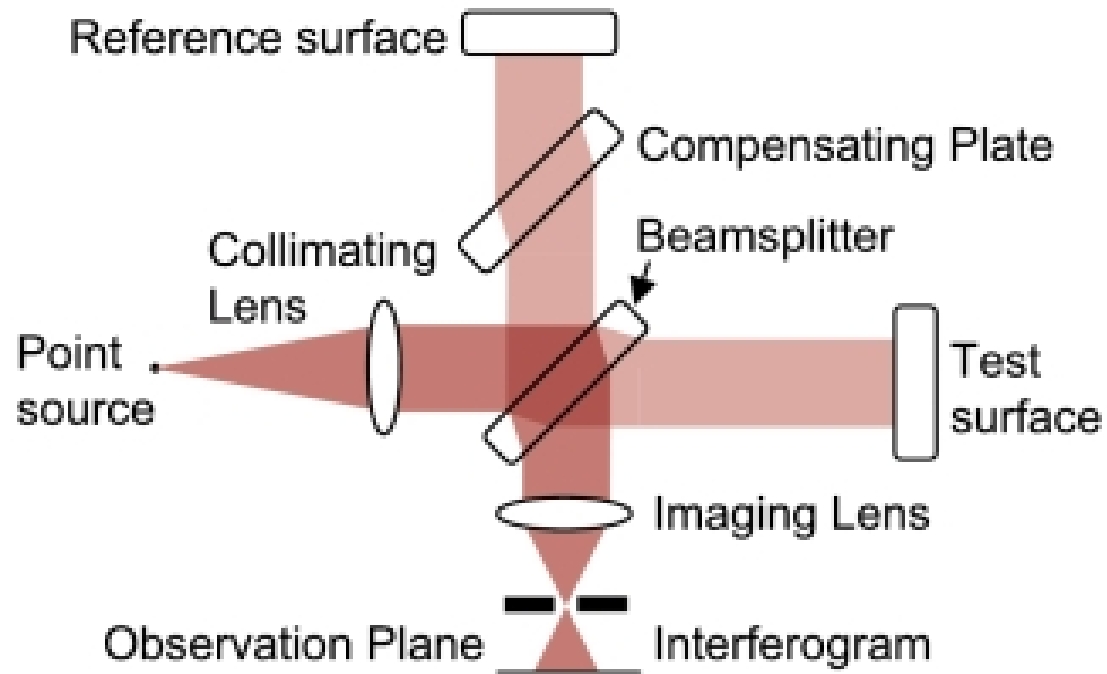
Frank Twyman & Arthur Green, 1916

It is a variant of Michelson interferometer principally used to test optical components.



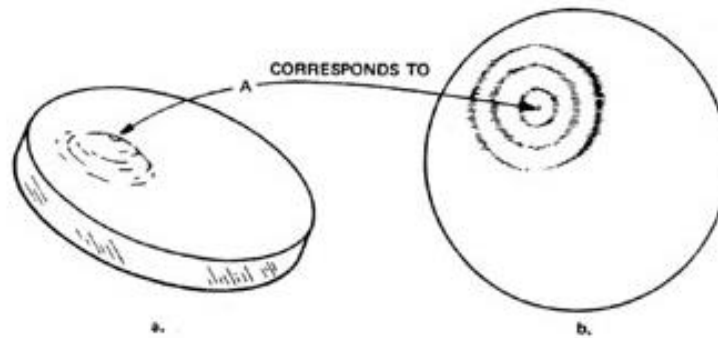
Twyman-Green interferometer set up to test a lens.

- ❖ Light from a laser is expanded & collimated into a parallel beam. A convex spherical mirror is positioned so that its center of curvature coincides with focus of the lens being tested. Emergent beam is recorded by an imaging system for analysis.
- ❖ Fixed mirror in Michelson interferometer is rotatable in TGI, & while light source is usually an extended source (although it can also be a laser) in a Michelson interferometer, light source is always a point-like source in TGI.
- ❖ Rotation of one mirror results in straight fringes appearing in interference pattern, a fringing which is used to test quality of optical components by observing changes in fringe pattern when component is placed in one arm of TGI.

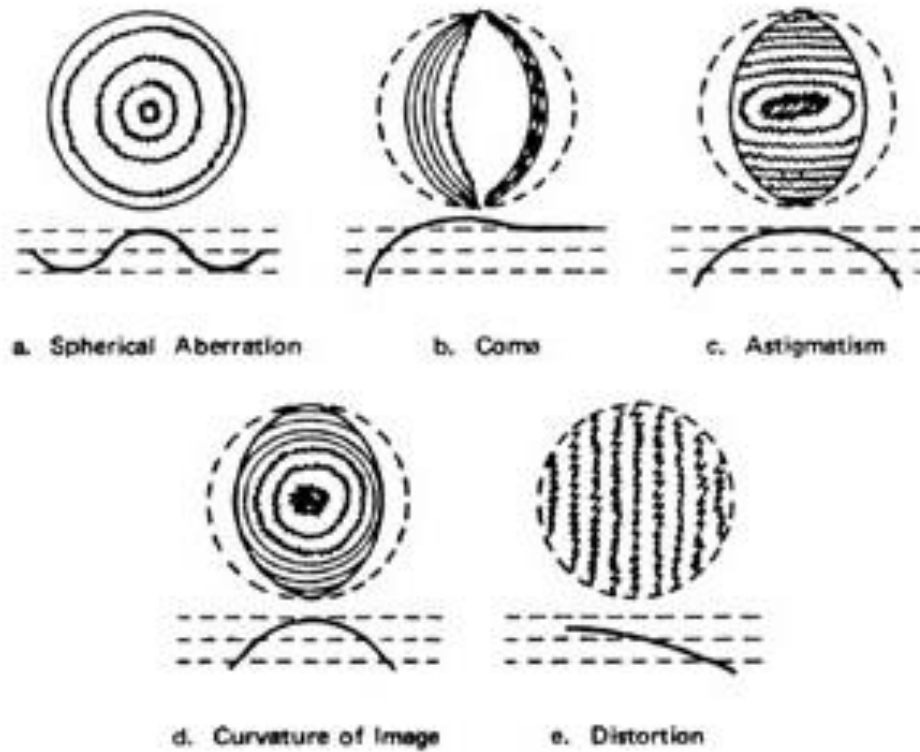


**Twyman-Green Interferometer**

- ❖ **TGI was invented & patented in 1916 & was originally intended for testing prisms & microscope objectives. Invention of laser increased its utility.**
- ❖ **Basic setup is for testing of flats, in which case reference beam reflects off of a known reference flat & returns to beamsplitter.**
- ❖ **Test beam is incident on unknown test part & also returns to beamsplitter.**
- ❖ **Beams are both split a second time, creating two complementary interferograms. One is projected towards point source, while more useful interferogram is relayed by an imaging lens to observation plane.**



Deformed mirror element & resulting fringe pattern seen with a TGI

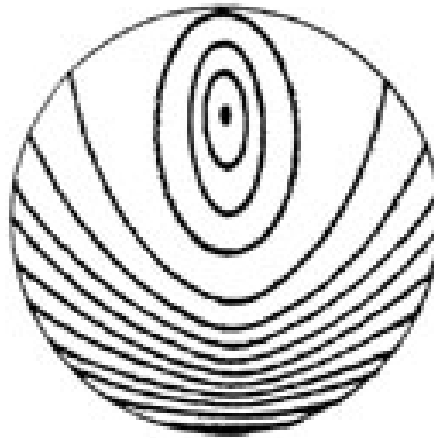


Typical fringe patterns from imperfect lenses as tested in TGI



Each fringe pattern typifies a particular lens aberration characterized as:

- ❖ Spherical aberration: Rays which pass through outer zones of lens do not meet the axis at same point as paraxial rays.
- ❖ Coma: Linear magnification of a very small object, located on the axis of the instrument, is different when different zones of lens are used to form the image.
- ❖ Astigmatism: A thin pencil of light which converges to two focal lines instead of one focal point is said to be astigmatic.
- ❖ Curvature of field: A plane object that does not result in a plane image of the focal plane is said to have curvature.
- ❖ Distortion: Image is distorted when object, which is supposed to be a plane figure at right angles to the axis of the lens, gives rise to an image which is not similar to itself geometrically.

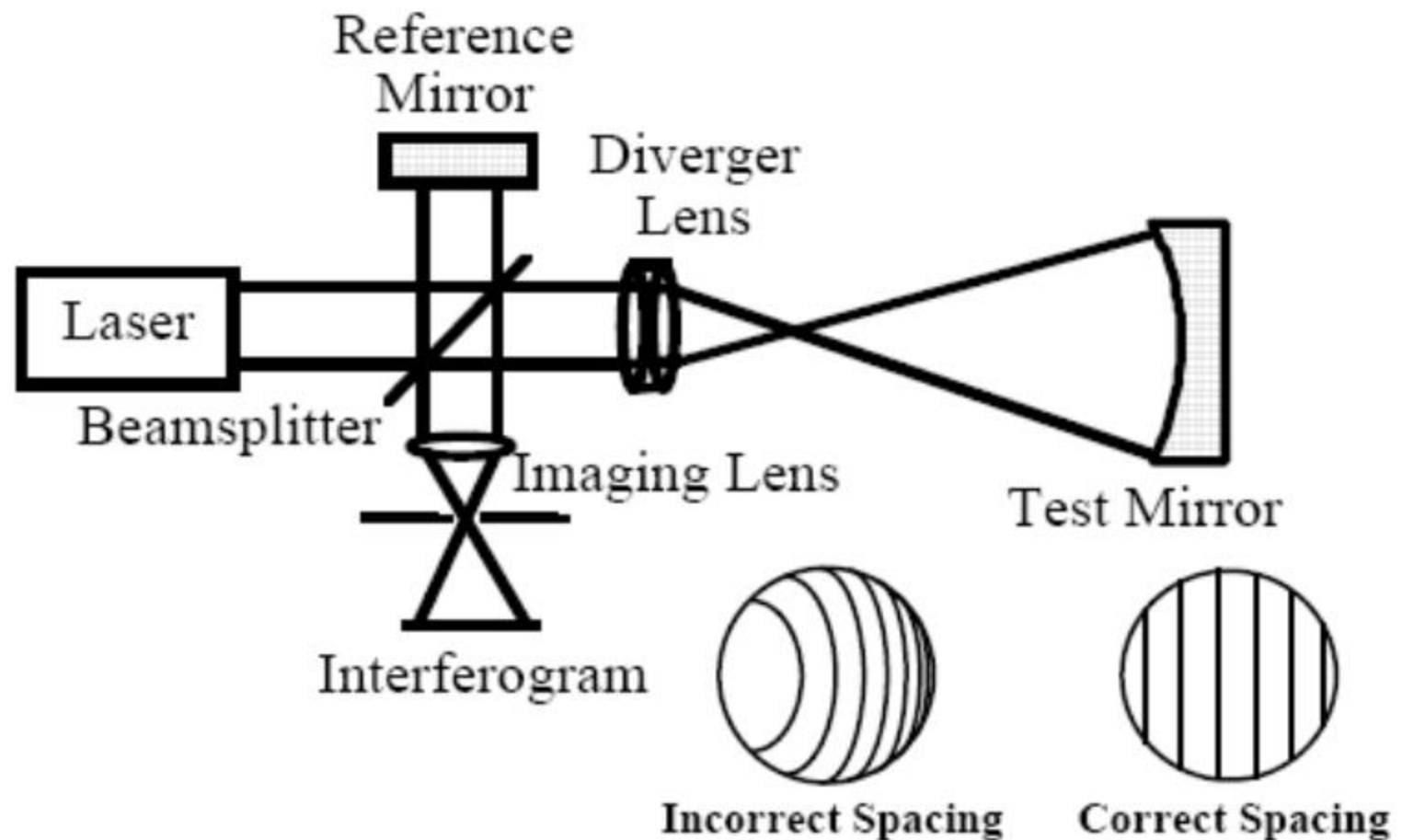


Fringe pattern generated by a lens as it is being inspected with a TGI. One side of the lens is perfectly flat.

### **Applications of TGI:**

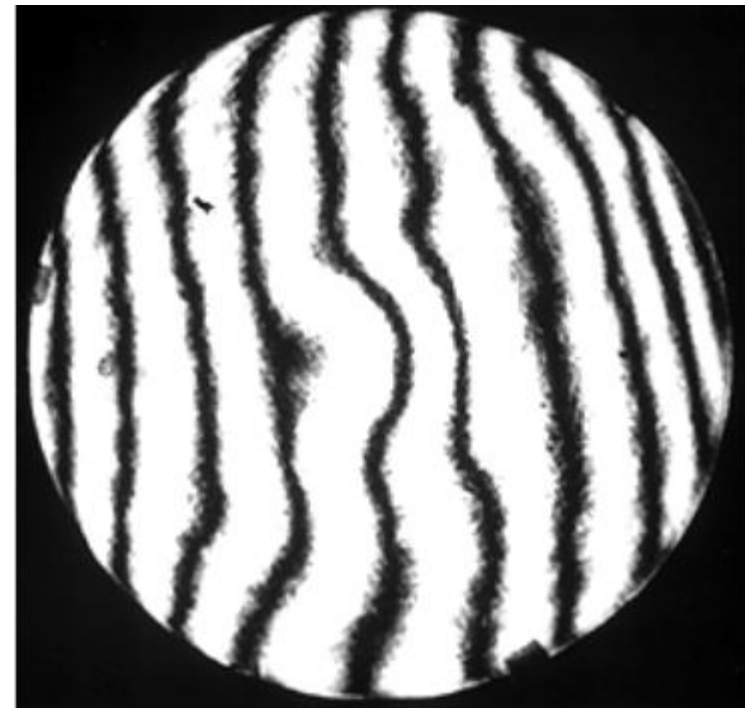
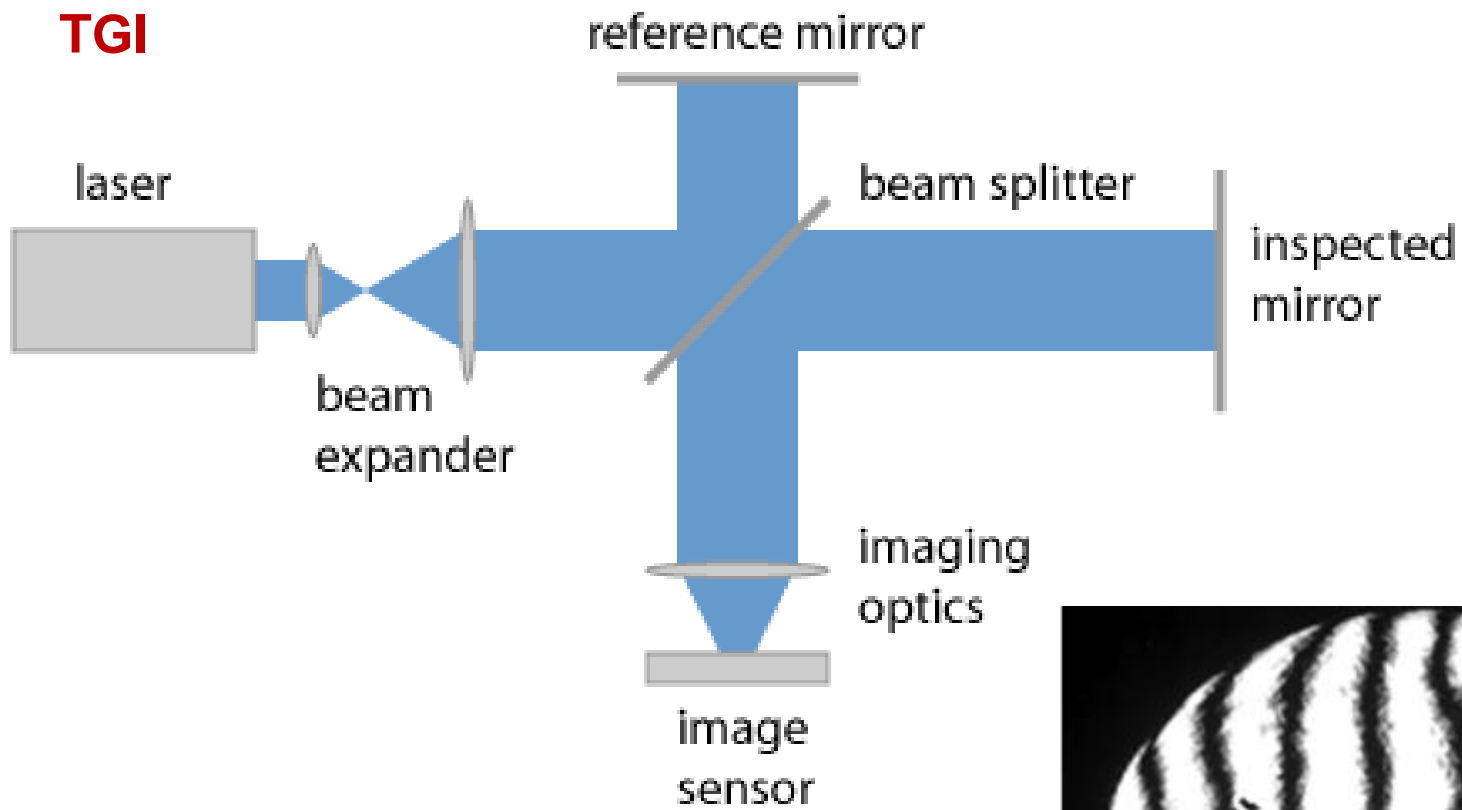
- ❖ Testing of prisms, microscope objectives, microspheres, & camera lenses.

## Spherical surfaces



Twyman-Green Interferometer

**TGI**



J Biomed Opt. 1999 Jan;4(1):176-82.

## **Application of twyman-green interferometer for evaluation of in vivo breakup characteristic of the human tear film.**

Licznerski TJ, Kasprzak HT, Kowalik W.

### **Abstract**

The paper presents an interferometric method of assessing the in vivo stability of the precorneal tear film. To observe dynamic effects on a human cornea the Twyman-Green interferometer with television frame speed digital registration synchronized with a laser flash was used. The instrument was applied to the human cornea in vivo. The results of the experiment, both tear film distribution and its dynamics, are presented. The proposed interferometric setup can be used to evaluate the breakup characteristics of the tear film, its distribution, and to examine its dynamic changes. The breakup profiles and their cross sections calculated from the interferogram analysis are presented. The depth of recorded breakup, calculated on the basis of interferogram analysis, amounts to about 1.5  $\mu\text{m}$ . The proposed method has the advantage of being noncontact and applies only a low-energy laser beam to the eye. This provides noninvasive viewing of human cornea in vivo and makes it possible to observe the kinetics of its tear film deterioration. © 1999 Society of Photo-Optical Instrumentation Engineers.

PMID: 23015183 DOI: [10.1117/1.429904](https://doi.org/10.1117/1.429904)



**National Television System Committee (NTSC): 25 fps (625 lines)**  
America, Canada, & Japan

**Phase Alternating Line (PAL): 30 fps (525 lines)**  
Europe, Australia, Middle East, Asia

**Outer layer of our tear film is an oil or lipid-based layer. Its main purpose is to seal the tear film which helps reduce evaporation of our natural tears.**

**WATER (AQUEOUS) LAYER: Middle layer is mostly comprised of water.**

