Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2017-18
Mid Semester Examination

Maximum Marks: 30 Total Time: 2 Hours

Note: Answer all eight questions. You can use scientific calculator.

1. (*i*) Let A, B, C and D be some events then using the definition of conditional probability show that $P(A \cap B \cap C \cap D) = P(A \mid B \cap C \cap D)P(B \mid C \cap D)P(C \mid D)P(D)$. [1]
(*ii*) An urn contains 10 identical balls of which five are black, three are red and two are white. Four balls are drawn without replacement. Find the probability that the first ball is black, the second red, the third white and the fourth ball is black. [1]

2. (*i*) Let B, A_1, A_2, A_3 are independent events. Then show that B^c, A_1, A_2, A_3 are mutually independent events. [1]
(*ii*) Consider a sample space $\{1, 2, 3, 4, 5\}$ with probabilities as $P(1) = 1/8, P(5) = 5/16, P(2) = P(3) = P(4) = p$. Compute p and let $A = \{1, 3, 4\}, B = \{1, 2, 4\}, C = \{1, 2, 3\}$. Verify independence of events A, B and C. [2]

3. An urn contains 3 white and 10 black balls. Another urn contains 5 white and 3 black balls. Two balls are drawn at random from the first urn and placed in the second urn and then a ball is selected from the second urn. Find the probability that it is a white ball. [3]

4. Consider a Bernoulli experiment where it is known 5% defective components are produced by the company. Let X denote the number of components that are to examined in order to get 3 defective components. Write probability mass function of X. Determine the probability that at least 5 components are to be examined in order to get 3 defective. Also compute $E(X^2)$ and the corresponding moment generating function. [1+2+2+2]

5. In the manufacture of car tyres, a particular production process is known to yield 10 tyres with defective walls in every lot of 100 tyres produced. From a production of 100 tyres, a random sample of 4 tyres is selected for testing and let X denotes number of defective tyres. Assuming hypergeometric distribution find the probability that sample contains at least one defective tyre. Compute this probability using the Poisson approximation. Also determine the expected value of (hypergeometric random variable) X^2. [2+1+2]

6. Suppose that it is known that a certain surgery has a 90% chance to succeed. This surgery is going to be performed on 8 patients. Assuming Bernoulli experiment let random variable X denotes the number of successful surgery. Write probability mass functions of X and $8 - X$. Determine the probability that surgery will fail exactly three times. Compute the variance of X. [1+1+2]

7. A random variable X has the density function given by $f_X(x) = 9xe^{-9x^2/2}, x \geq 0$. Determine the interquartile range of the distribution. [3]

8. Suppose that random variable X follows a normal distribution with mean 1 and standard deviation 0.5. Find the moment generating function of $0.5X - \frac{1}{3}$. Using this write probability density and cumulative distribution functions of $0.5X - \frac{1}{3}$. [3]