PH 201

OPTICS & LASERS

Lecture_Lasers_12

Ref.: William T. Silfvast, *Laser Fundamentals*, 2nd ed., Cambridge Univ. Press (2004)

Transverse Laser Cavity Modes

$$U(x, y) = e^{-\rho^2/\omega^2} = e^{-(x^2+y^2)/\omega^2}$$

ω **= scaling factor &** ***ρ*** **is radial distance to any location on mirror from central point on mirror.**

Function *U*(*x*,*y*) gives variation of distribution of electric field amplitude over mirror at various locations (*x*,*y*).

Functions that are their own Fourier transforms can be written as products of Hermite polynomials & Gaussian distribution function:

$$U_{pq}(x, y) = H_p\left(\frac{\sqrt{2}x}{\omega}\right) H_q\left(\frac{\sqrt{2}y}{\omega}\right) e^{-(x^2+y^2)/\omega^2}$$

For *x*-*y* symmetry, *p* & *q* are integers that designate order of Hermite polynomials.

Every set of (*p*,*q*) represents a specific stable distribution of wave amplitude at one of the mirrors, or a specific transverse mode of open-walled cavity.

Hermite polynomials:

$$H_0(u) = 1$$

$$u \;\; can \;\; be \;\; \frac{\sqrt{2}x}{\omega} \;\; or \;\; \frac{\sqrt{2}y}{\omega}$$

$$H_1(u) = 2u$$

$$H_2(u) = 2(2u^2 - 1)$$

$$H_m(u) = (-1)^m e^{u^2} \frac{d^m(e^{-u^2})}{du^m}$$

Transverse mode distributions are designated as TEM$_{pq}$, where TEM stands for "transverse electromagnetic".

Lowest-order mode is designated as TEM$_{00}$ & is just a simple Gaussian distribution. $e^{-(x^2+y^2)/\omega^2}$

$$U_{pq}(x, y) = H_p\left(\frac{\sqrt{2}x}{\omega}\right)H_q\left(\frac{\sqrt{2}y}{\omega}\right)e^{-(x^2+y^2)/\omega^2}$$

Hermite-Gaussian solutions of this Eq. are obtained by using *x*-*y* coordinates whose spatial distributions lead to *x*-*y* symmetry.

It is also possible to solve Fresnel-Kirchhoff integral using cylindrical coordinates, & solutions have complete circular symmetry.

Hermite-Gaussian solutions require slight astigmatism in cavity, which is provided with one or more Brewster angle windows within cavity.

Transverse Modes using Curved Mirrors

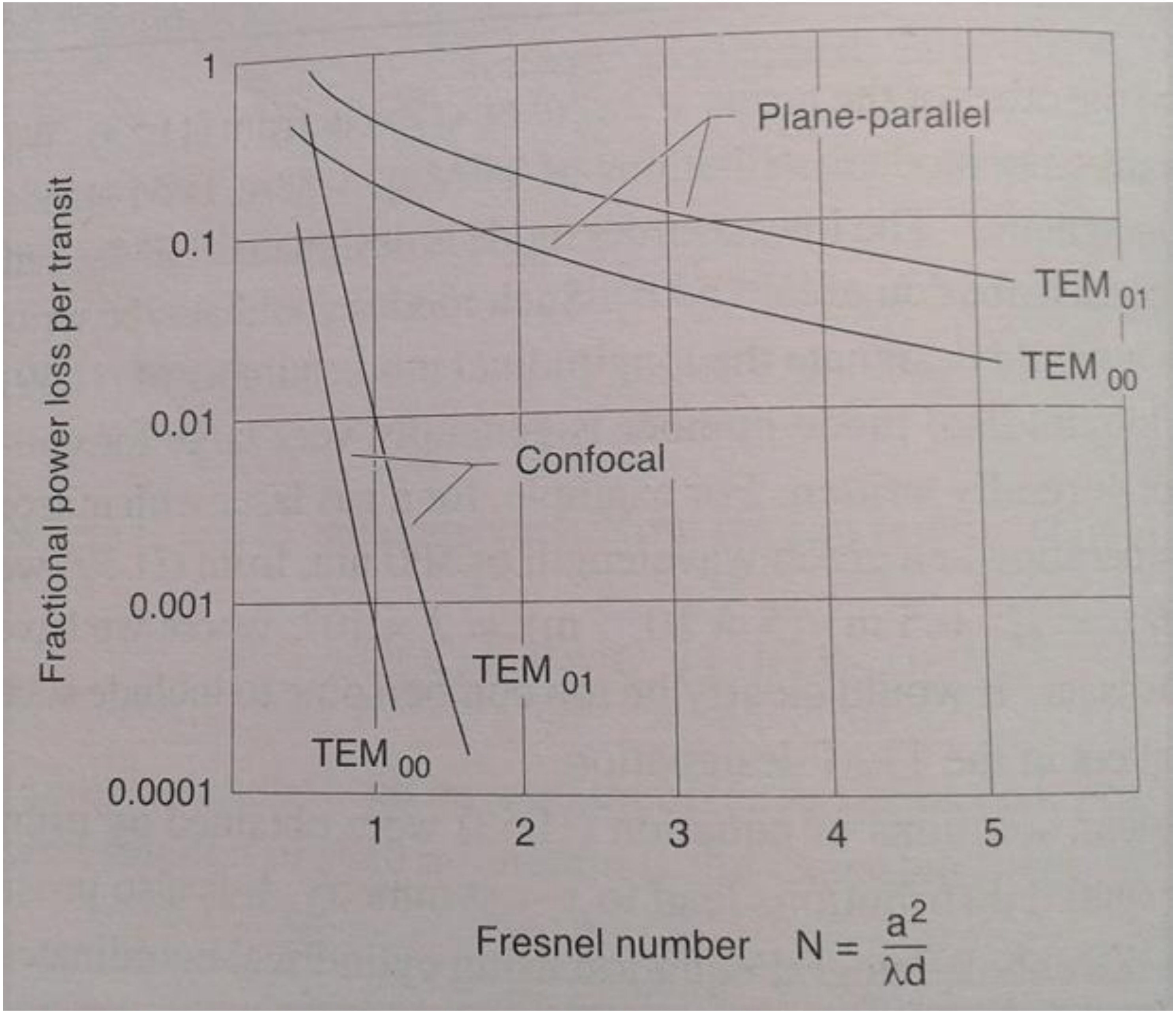
Using curved mirrors, beams are focused slightly after each reflection therefore beam amplitude near edges of mirror is reduced. This reduces diffraction losses.

Fresnel number or confocal parameter, $N = \dfrac{a^2}{\lambda d}$

a **= radius of mirror (or limiting aperture of mirror)**
λ **= laser wavelength**
d **= separation between mirrors**

Fractional losses per pass for a specific type of cavity (confocal cavity), in which radius of curvature of mirrors is equal to separation d between mirrors:

Diffraction losses are significantly lower with curved mirrors than plane-parallel resonator.



Fractional power loss per transit versus Fresnel number for a laser cavity

Write out the mode distributions at the mirrors for the TEM_{00}, the TEM_{01}, and the TEM_{11} modes in terms of the transverse variables *x*, *y*, and *ρ*.

$$\rho = \sqrt{x^2 + y^2}$$

$$U_{pq}(x, y) = H_p\left(\frac{\sqrt{2}x}{\omega}\right) H_q\left(\frac{\sqrt{2}y}{\omega}\right) e^{-(x^2+y^2)/\omega^2}$$

Distribution of electric field on the mirrors for TEM_{00} mode,

$$U_{00}(x, y) = TEM_{00} = H_0\left(\frac{\sqrt{2}x}{\omega}\right) H_0\left(\frac{\sqrt{2}y}{\omega}\right) e^{-(x^2+y^2)/\omega^2}$$

$$= e^{-(x^2+y^2)/\omega^2} = e^{-\rho^2/\omega^2}$$

Given that H_0 for both *x* & *y* is unity.

$$H_0(u) = 1$$

For TEM$_{01}$ mode,

$$U_{01}(x, y) = TEM_{01} = 1 \times H_1\left(\frac{\sqrt{2}y}{\omega}\right)e^{-(x^2+y^2)/\omega^2}$$

$$= \frac{2\sqrt{2}y}{\omega}\, e^{-(x^2+y^2)/\omega^2}$$

$$= \frac{2\sqrt{2}y}{\omega}\, e^{-\rho^2/\omega^2}$$

In this case, H_p for *x*-direction is unity.

$$H_1(u) = 2u$$

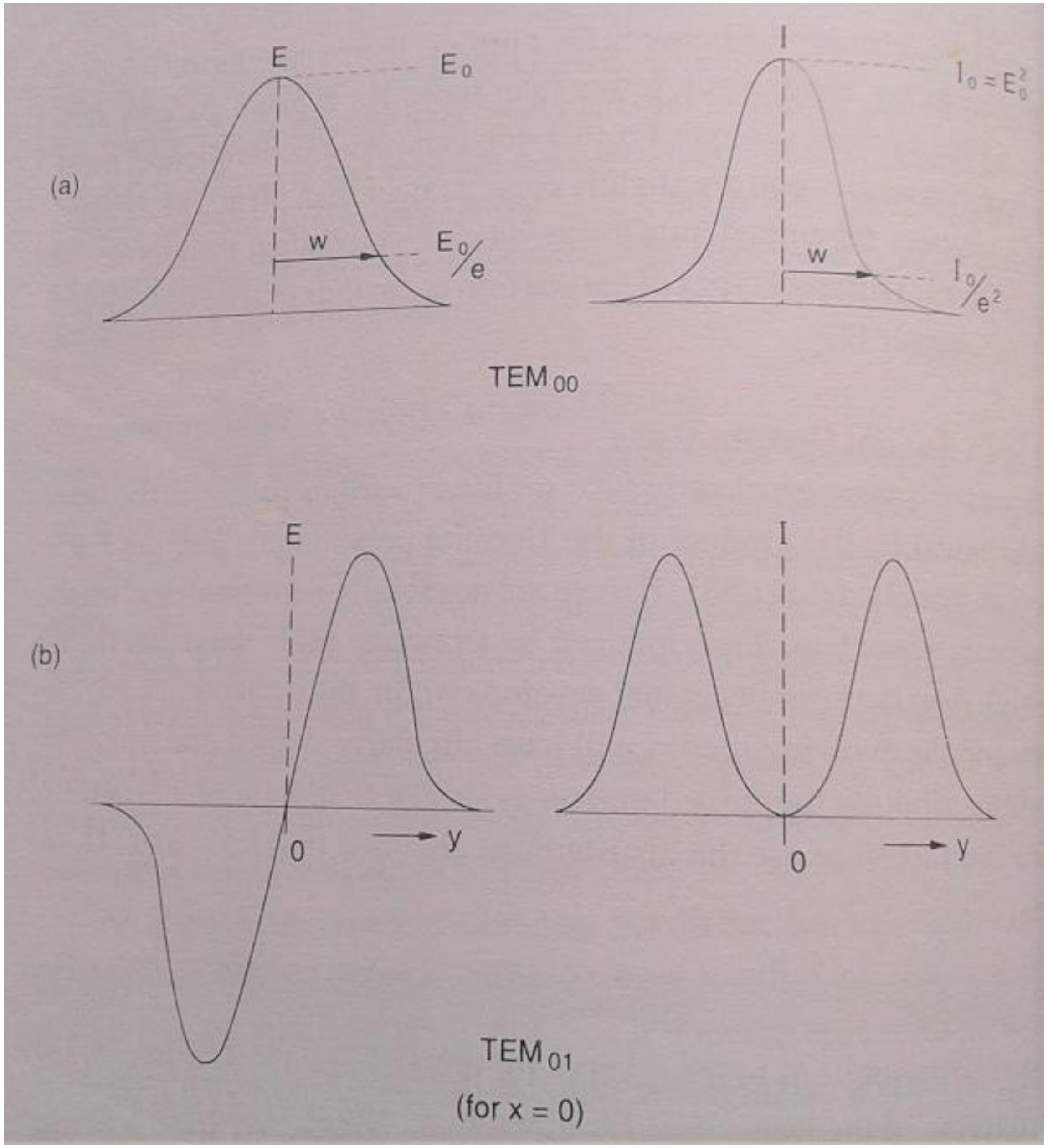
$$H_1 = \frac{2\sqrt{2}y}{\omega}$$

For TEM$_{11}$ mode, the value of *H* is same for both *x*- & *y*-directions,

$$U_{11}(x, y) = TEM_{11} = H_1\left(\frac{\sqrt{2}x}{\omega}\right)H_1\left(\frac{\sqrt{2}y}{\omega}\right)e^{-(x^2+y^2)/\omega^2}$$

$$= \frac{8xy}{\omega^2}\,e^{-(x^2+y^2)/\omega^2}$$

$$= \frac{8xy}{\omega^2}\,e^{-\rho^2/\omega^2}$$



E_0 denotes electric field at centre of mirror (ρ = 0).

Electric field & intensity distributions at laser mirrors for (a) a TEM_{00} laser mode & (b) a TEM_{01} laser mode.

Transverse Mode Frequencies

- **Different transverse modes are modes having either same or different values of mode no. *n* (same longitudinal mode no.) but with different values on *p* &/or *q*.**

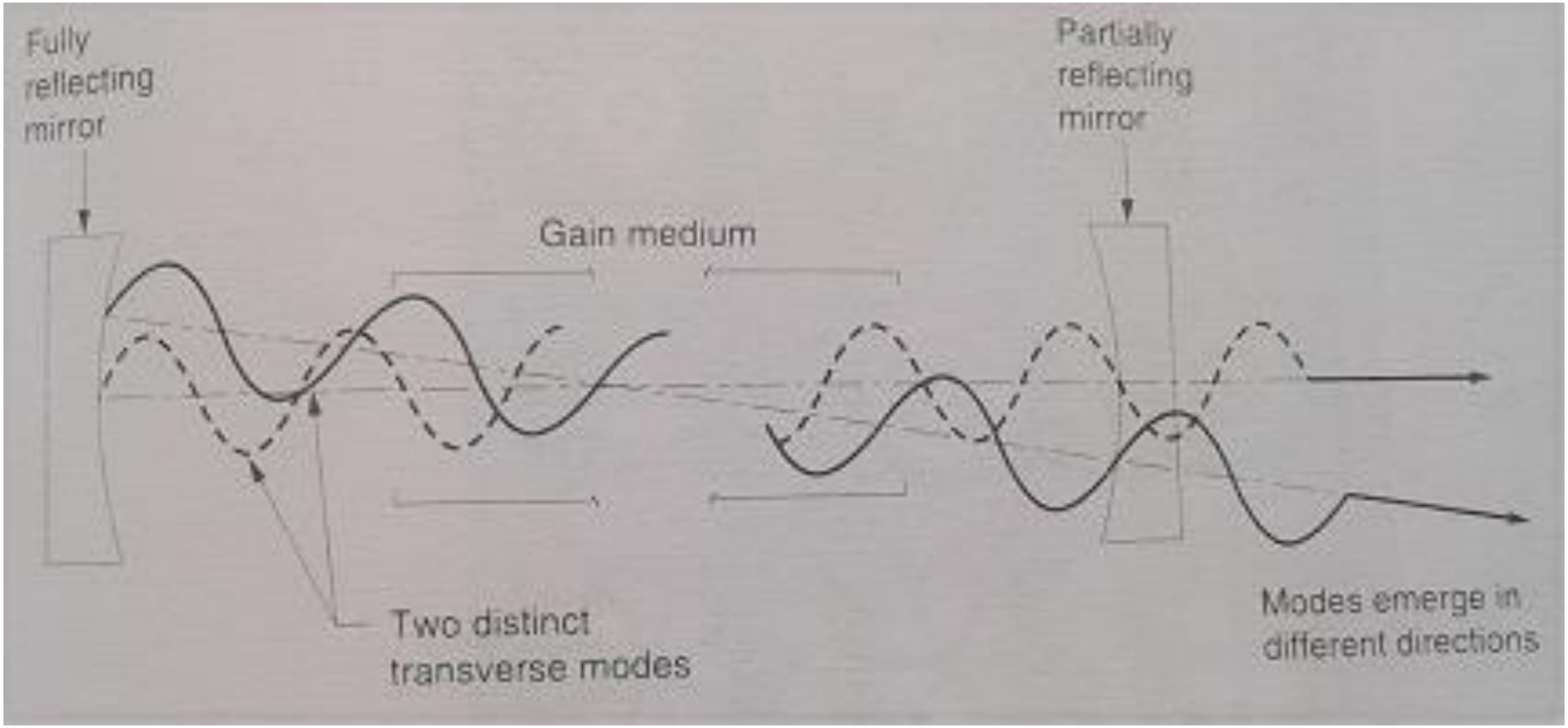
- **Transverse modes with same *n* but with different values of *p* & *q* would have slightly different optical path lengths *d* owing to slightly different angular distributions of amplitude functions within cavity.**

- **Such modes consequently have slightly different frequencies because of different value of distance *d*.**

- **Frequency difference between longitudinal modes is significantly greater than frequency difference between transverse modes.**

- **However, in a cavity with two mirrors, a standing wave pattern is always produced that leads to longitudinal modes. Hence, every transverse mode consists of one or more longitudinal modes, depending on separation *d* between mirrors.**

Ex. Two different transverse modes having different frequencies, where two transverse modes have slightly different path lengths within cavity.

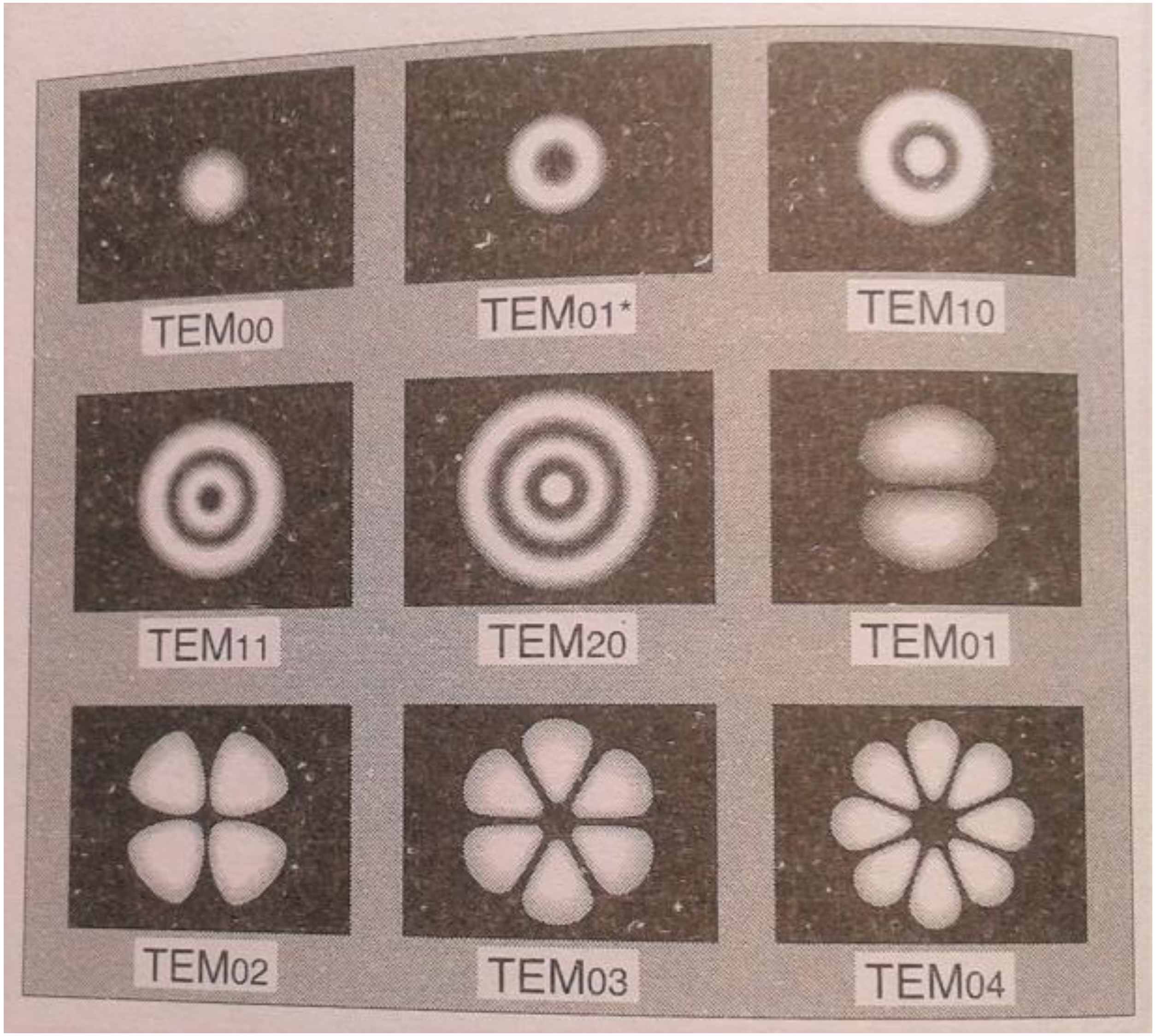


Simplified description of two distinct transverse laser modes, showing larger effective path length for an off-axis mode.

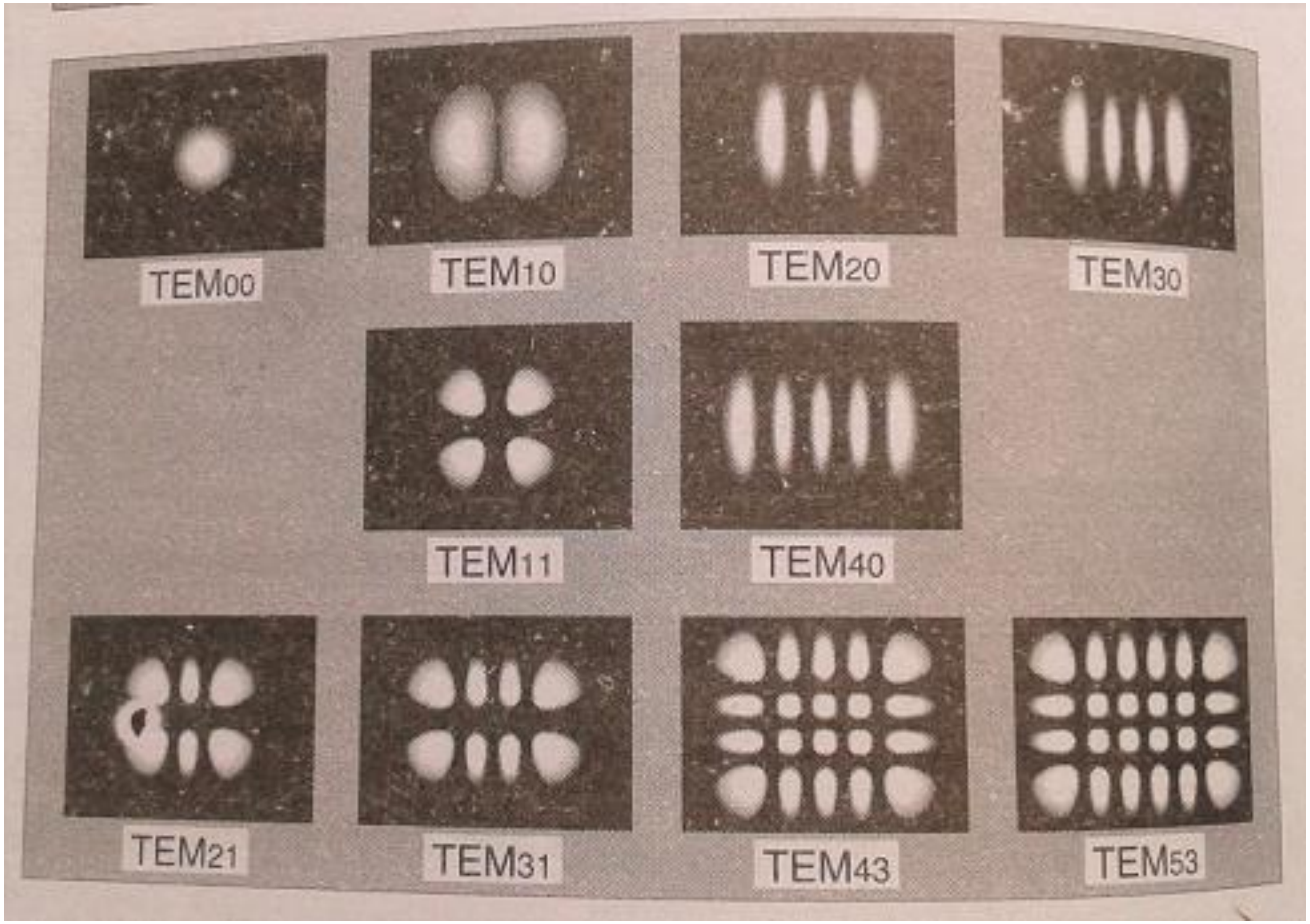
Gaussian-shaped Transverse Modes within & beyond Laser Cavity

- **TEM_{00} mode has a Gaussian shape everywhere, including between mirrors & also beyond mirrors (for part of beam that is transmitted through mirrors).**

- **Transverse modes can be described as spatial modes, whereas longitudinal modes are associated with frequencies.**



Laguerre-Gaussian mode patterns for various transverse laser modes: pure modes in circular symmetry.

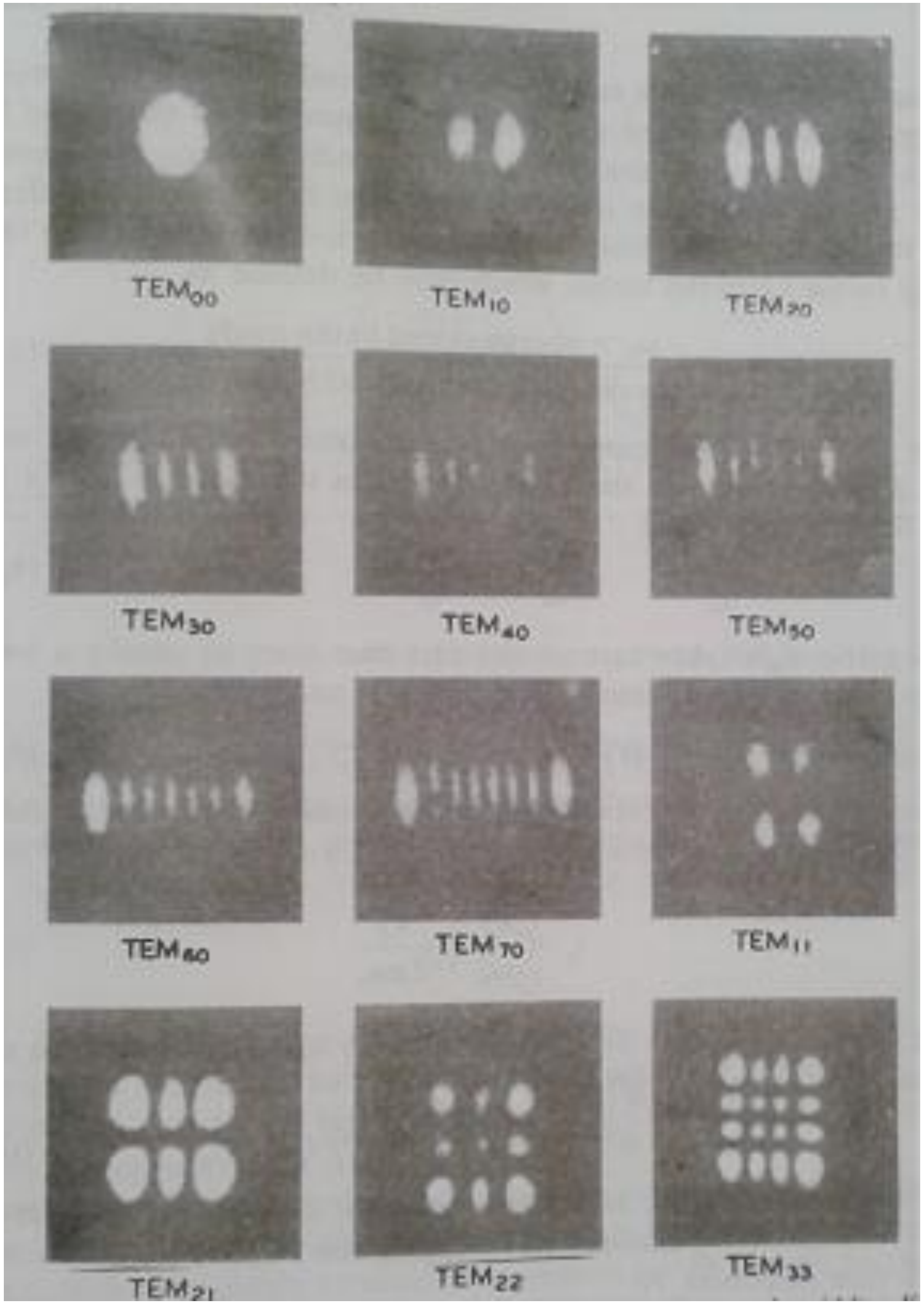


Mode patterns have a definite preferred orientation rather than a cylindrical symmetry.

Radially nonsymmetric loss within cavity

Hermite-Gaussian mode patterns for various transverse laser modes: pure modes in *x-y* symmetry.

Lower order modes of a stable resonator cavity.



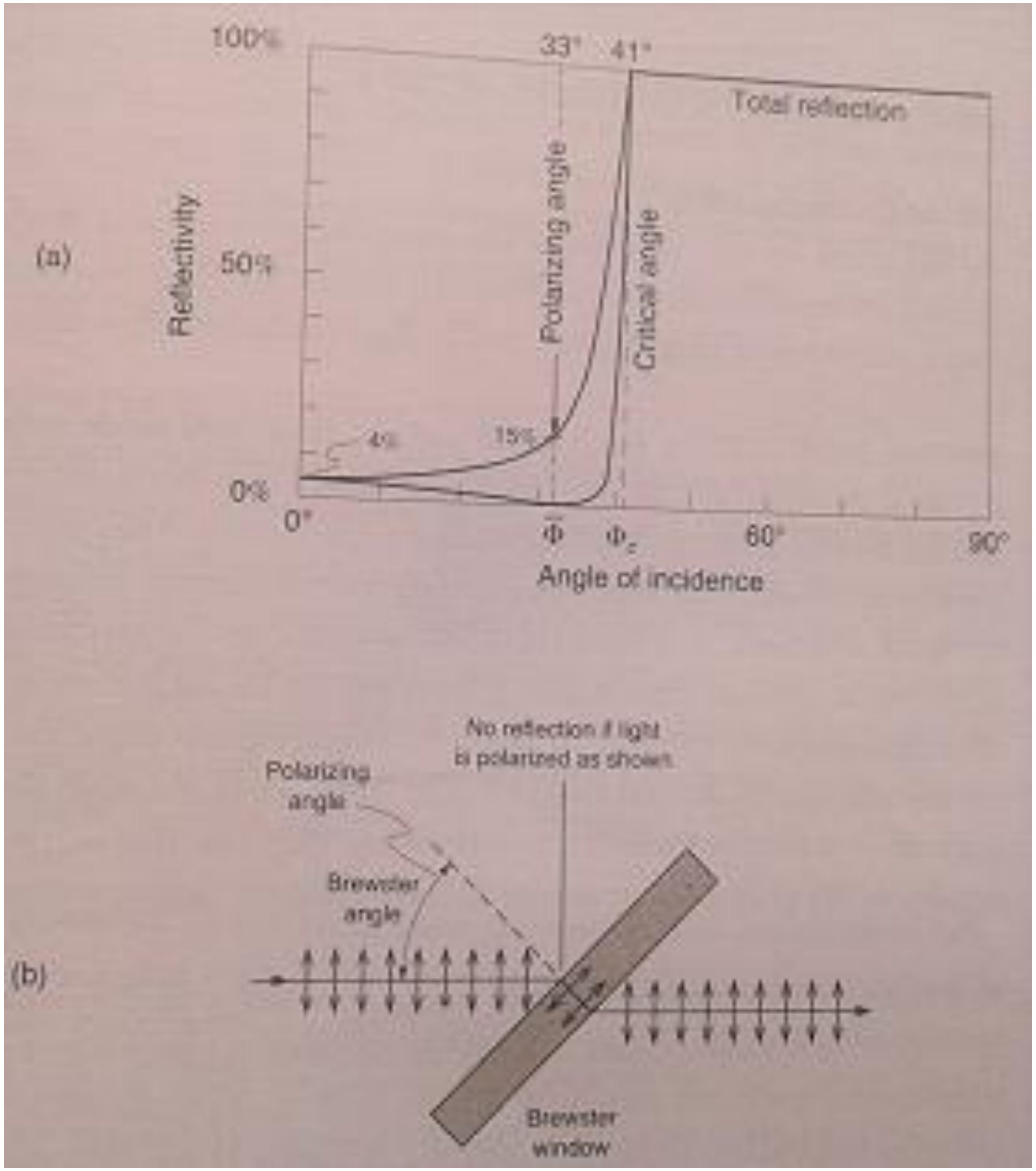
Brewster Angle Windows within Laser Cavity

- **Brewster's law:** $$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

- **Brewster angle window arrangement is used to minimize the losses resulting when laser beam passes from amplifier to region between gain medium & laser mirrors.**

- **It is useful when laser mirrors are mounted external to laser gain medium rather than directly at ends of gain medium, or at ends of a gas laser discharge tube in case of some gas lasers.**

- **At Brewster's angle there is no loss for component of laser beam polarized in transverse direction that lies in a plane normal to plane of window & perpendicular to direction of propagation of beam (vertical arrows).**



(a) Reflective intensity versus angle for light reflected from a dielectric interface; (b) Brewster angle laser window providing very low reflection loss for light polarized in plane of figure.