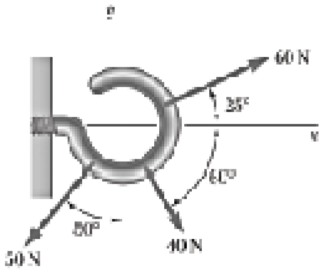
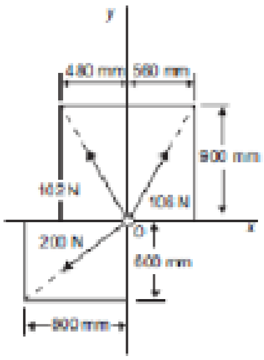


PROBLEM 2.22

Determine the x and y components of each of the forces shown.

SOLUTION

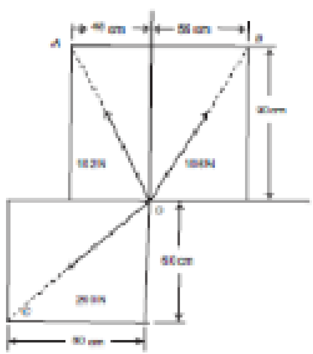
40-N Force:	$F_x = +(40\text{ N})\cos 60^\circ$	$F_x = 20.0\text{ N}$ ◀
	$F_y = -(40\text{ N})\sin 60^\circ$	$F_y = -34.6\text{ N}$ ◀
50-N Force:	$F_x = \quad (50\text{ N})\sin 50^\circ$	$F_x = \quad 38.3\text{ N}$ ◀
	$F_y = \quad (50\text{ N})\cos 50^\circ$	$F_y = \quad 32.1\text{ N}$ ◀
60-N Force:	$F_x = +(60\text{ N})\cos 25^\circ$	$F_x = 54.4\text{ N}$ ◀
	$F_y = +(60\text{ N})\sin 25^\circ$	$F_y = 25.4\text{ N}$ ◀



PROBLEM 2.24

Determine the x and y components of each of the forces shown.

SOLUTION



We compute the following distances:

$$OA = \sqrt{(48)^2 + (90)^2} = 102 \text{ cm.}$$

$$OB = \sqrt{(56)^2 + (90)^2} = 106 \text{ cm.}$$

$$OC = \sqrt{(80)^2 + (60)^2} = 100 \text{ cm.}$$

Then:

102 N Force:

$$F_x = -(102 \text{ N})\frac{48}{102}, \qquad F_x = -48.0 \text{ N} \blacktriangleleft$$

$$F_y = +(102 \text{ N})\frac{90}{102}, \qquad F_y = 90.0 \text{ N} \blacktriangleleft$$

106 N Force:

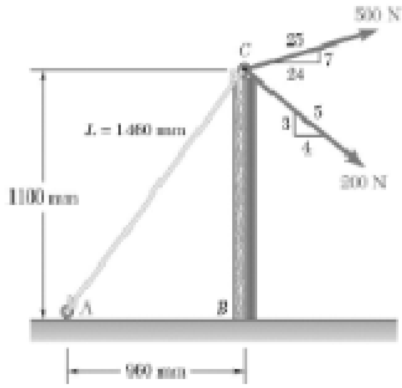
$$F_x = +(106 \text{ N})\frac{56}{106}, \qquad F_x = 56.0 \text{ N} \blacktriangleleft$$

$$F_y = +(106 \text{ N})\frac{90}{106}, \qquad F_y = 90.0 \text{ N} \blacktriangleleft$$

200 N Force:

$$F_x = -(200 \text{ N})\frac{80}{100}, \qquad F_x = -160 \text{ N} \blacktriangleleft$$

$$F_y = -(200 \text{ N})\frac{60}{100}, \qquad F_y = -120 \text{ N} \blacktriangleleft$$

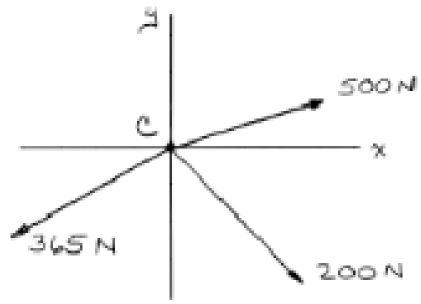


PROBLEM 2.36

Knowing that the tension in rope AC is 365 N, determine the resultant of the three forces exerted at point C of post BC.

SOLUTION

Determine force components:



Cable force AC:

$$F_x = -(365\text{ N})\frac{960}{1460} = -240\text{ N}$$

$$F_y = -(365\text{ N})\frac{1100}{1460} = -275\text{ N}$$

500-N Force:

$$F_x = (500\text{ N})\frac{24}{25} = 480\text{ N}$$

$$F_y = (500\text{ N})\frac{7}{25} = 140\text{ N}$$

200-N Force:

$$F_x = (200\text{ N})\frac{4}{5} = 160\text{ N}$$

$$F_y = -(200\text{ N})\frac{3}{5} = -120\text{ N}$$

and

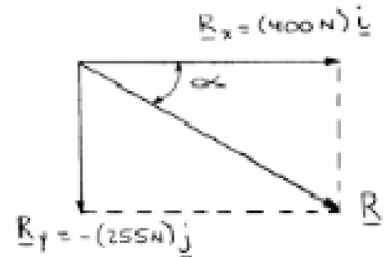
$$R_x = \Sigma F_x = -240\text{ N} + 480\text{ N} + 160\text{ N} = 400\text{ N}$$

$$R_y = \Sigma F_y = -275\text{ N} + 140\text{ N} - 120\text{ N} = -255\text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(400\text{ N})^2 + (-255\text{ N})^2}$$

$$= 474.37\text{ N}$$

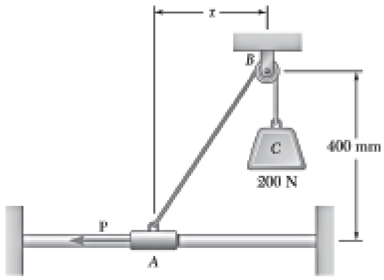


Further:

$$\tan\alpha = \frac{255}{400}$$

$$\alpha = 32.5°$$

$\mathbf{R} = 474\text{ N} \measuredangle 32.5°$ ◀

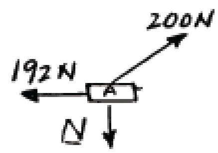


PROBLEM 2.64

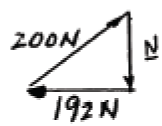
Collar A is connected as shown to a 200-N load and can slide on a frictionless horizontal rod. Determine the distance x for which the collar is in equilibrium when $P = 192$ N.

SOLUTION

Free Body: Collar A



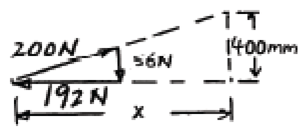
Force Triangle



$$N^2 = (200)^2 - (192)^2 = 3136$$

$$N = 56 \text{ N}$$

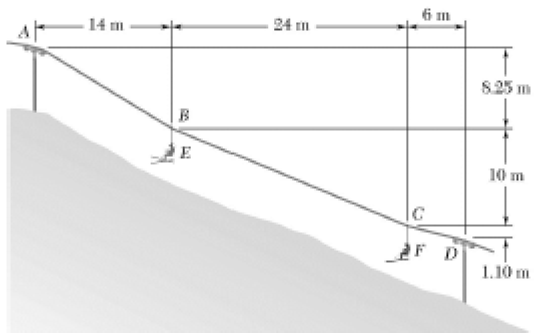
Similar Triangles



$$\frac{x}{400 \text{ mm}} = \frac{192 \text{ N}}{56 \text{ N}}$$

$$x = 1371 \text{ mm}$$

$x = 1371$ mm ◀

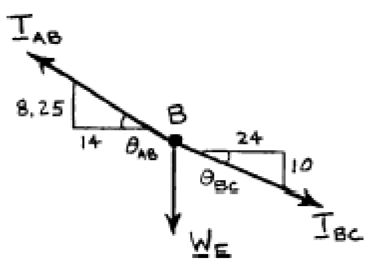


PROBLEM 2.F2

A chairlift has been stopped in the position shown. Knowing that each chair weighs 250 N and that the skier in chair E weighs 765 N, draw the free-body diagrams needed to determine the weight of the skier in chair F.

SOLUTION

Free-Body Diagram of Point B:



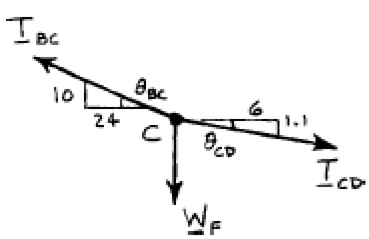
$$W_E = 250\text{ N} + 765\text{ N} = 1015\text{ N}$$

$$\theta_{AB} = \tan^{-1}\frac{8.25}{14} = 30.510^\circ$$

$$\theta_{BC} = \tan^{-1}\frac{10}{24} = 22.620^\circ$$

Use this free body to determine T_{AB} and T_{BC}.

Free-Body Diagram of Point C:



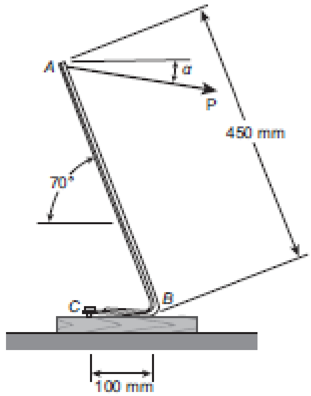
$$\theta_{CD} = \tan^{-1}\frac{1.1}{6} = 10.3889^\circ$$

Use this free body to determine T_{CD} and W_F.

Then weight of skier W_S is found by

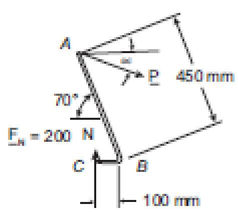
$$W_S = W_F - 250\text{ N} \blacktriangleleft$$

PROBLEM 3.8



It is known that a vertical force of 200 N is required to remove the nail at C from the board. As the nail first starts moving, determine (a) the moment about B of the force exerted on the nail, (b) the magnitude of the force **P** that creates the same moment about B if $\alpha = 10°$, (c) the smallest force **P** that creates the same moment about B.

SOLUTION

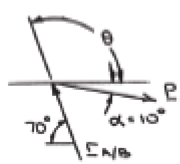


(a) We have $M_B = r_{C/B} F_N$

$$= (0.1 \text{ m})(200 \text{ N})$$

$$= 20 \text{ N} \cdot \text{m}$$

or $M_B = 20 \text{ N} \cdot \text{m}$ ◀



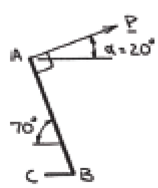
(b) By definition $M_B = r_{A/B} P \sin \theta$

$$\theta = 10° + (180° - 70°)$$

$$= 120°$$

Then $20 \text{ N} \cdot \text{m} = (0.45 \text{ m}) \times P \sin 120°$

or $P = 51.3 \text{ N}$ ◀



(c) For **P** to be minimum, it must be perpendicular to the line joining Points A and B. Thus, **P** must be directed as shown.

Thus $M_B = dP_{min}$

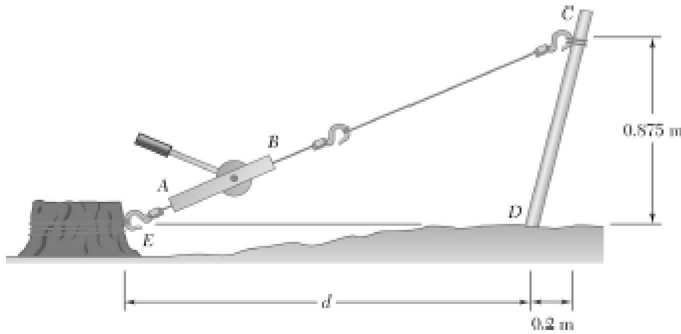
$$d = r_{A/B}$$

or $20 \text{ N·m} = (0.45)P_{min}$

or $P_{min} = 44.4 \text{ N}$ $\quad$ $\mathbf{P}_{min} = 44.4 \text{ N} \measuredangle 20°$ ◀

PROBLEM 3.11

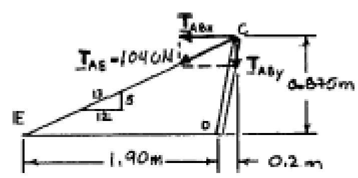
A winch puller AB is used to straighten a fence post. Knowing that the tension in cable BC is 1040 N and length d is 1.90 m, determine the moment about D of the force exerted by the cable at C by resolving that force into horizontal and vertical components applied (a) at Point C, (b) at Point E.



SOLUTION

(a) Slope of line:

$$EC = \frac{0.875 \text{ m}}{1.90 \text{ m} + 0.2 \text{ m}} = \frac{5}{12}$$



(a)

Then

$$T_{ABx} = \frac{12}{13}(T_{AB})$$

$$= \frac{12}{13}(1040 \text{ N})$$

$$= 960 \text{ N}$$

and

$$T_{ABy} = \frac{5}{13}(1040 \text{ N})$$

$$= 400 \text{ N}$$

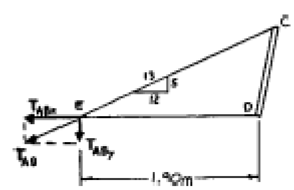
Then

$$M_D = T_{ABx}(0.875 \text{ m}) - T_{ABy}(0.2 \text{ m})$$

$$= (960 \text{ N})(0.875 \text{ m}) - (400 \text{ N})(0.2 \text{ m})$$

$$= 760 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_D = 760 \text{ N}\cdot\text{m}$ ◀



(b)

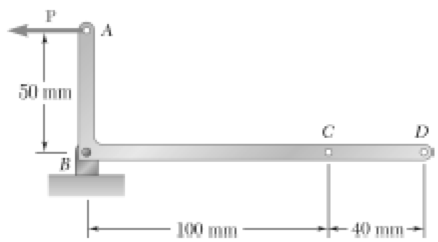
(b) We have

$$M_D = T_{ABx}(y) + T_{ABx}(x)$$

$$= (960 \text{ N})(0) + (400 \text{ N})(1.90 \text{ m})$$

$$= 760 \text{ N}\cdot\text{m}$$

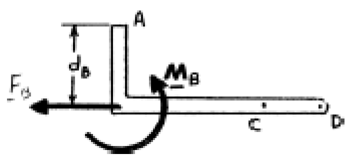
or $\mathbf{M}_D = 760 \text{ N}\cdot\text{m}$ ◀



PROBLEM 3.85

The 80-N horizontal force **P** acts on a bell crank as shown. (*a*) Replace **P** with an equivalent force-couple system at *B*. (*b*) Find the two vertical forces at *C* and *D* that are equivalent to the couple found in part *a*.

SOLUTION



(*a*) Based on $\Sigma F: \ F_B = F = 80 \text{ N}$ or $\mathbf{F}_B = 80.0 \text{ N} \longleftarrow$ ◀

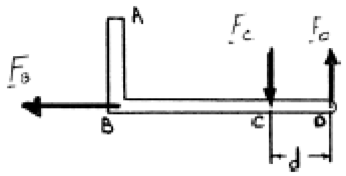
$\Sigma M: \ M_B = Fd_B$

$= 80 \text{ N } (0.05 \text{ m})$

$= 4.0000 \text{ N} \cdot \text{m}$

or $\mathbf{M}_B = 4.00 \text{ N} \cdot \text{m} \circlearrowleft$ ◀

(*b*) If the two vertical forces are to be equivalent to $\mathbf{M}_B$, they must be a couple. Further, the sense of the moment of this couple must be counterclockwise.



Then with F_C and F_D acting as shown,

$\Sigma M: \ M_D = F_C d$

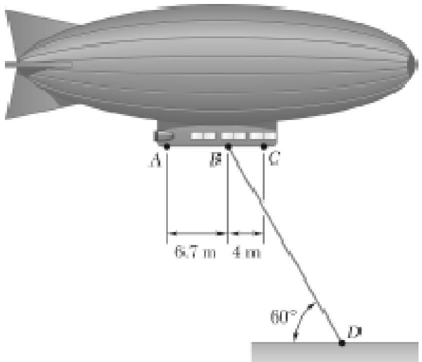
$4.0000 \text{ N} \cdot \text{m} = F_C (0.04 \text{ m})$

$F_C = 100.000 \text{ N}$ or $\mathbf{F}_C = 100.0 \text{ N} \downarrow$ ◀

$\Sigma F_y: \ 0 = F_D - F_C$

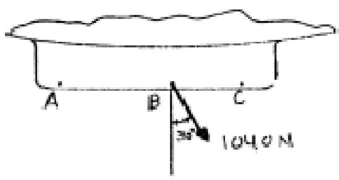
$F_D = 100.000 \text{ N}$ or $\mathbf{F}_D = 100.0 \text{ N} \uparrow$ ◀

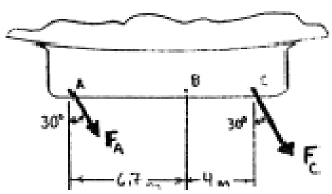
PROBLEM 3.86



A dirigible is tethered by a cable attached to its cabin at B. If the tension in the cable is 1040 N, replace the force exerted by the cable at B with an equivalent system formed by two parallel forces applied at A and C.

SOLUTION





Require the equivalent forces acting at A and C be parallel and at an angle of α with the vertical.

Then for equivalence,

$$\Sigma F_x: \quad (1040 \text{ N})\sin 30° = F_A \sin\alpha + F_B \sin\alpha \qquad (1)$$

$$\Sigma F_y: \quad -(1040 \text{ N})\cos 30° = -F_A \cos\alpha - F_B \cos\alpha \qquad (2)$$

Dividing Equation (1) by Equation (2),

$$\frac{(1040 \text{ N})\sin 30°}{-(1040 \text{ N})\cos 30°} = \frac{(F_A + F_B)\sin\alpha}{-(F_A + F_B)\cos\alpha}$$

Simplifying yields $\alpha = 30°$.

Based on

$$\Sigma M_C: \quad [(1040 \text{ N})\cos 30°](4 \text{ m}) = (F_A \cos 30°)(10.7 \text{ m})$$

$$F_A = 388.79 \text{ N}$$

or $\mathbf{F}_A = 389 \text{ N} \ \nwarrow 60.0°$ ◀

Based on

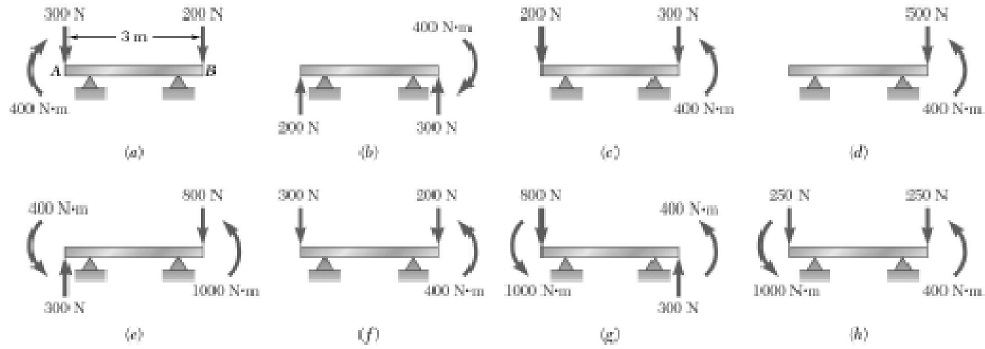
$$\Sigma M_A: \quad -[(1040 \text{ N})\cos 30°](6.7 \text{ m}) = (F_C \cos 30°)(10.7 \text{ m})$$

$$F_C = 651.21 \text{ N}$$

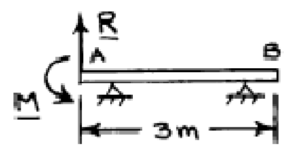
or $\mathbf{F}_C = 651 \text{ N} \ \nwarrow 60.0°$ ◀

PROBLEM 3.101

A 3-m-long beam is subjected to a variety of loadings. (*a*) Replace each loading with an equivalent force-couple system at end *A* of the beam. (*b*) Which of the loadings are equivalent?



SOLUTION



(*a*) (*a*) We have ΣF_Y: $-300\text{ N} - 200\text{ N} = R_a$

or $\mathbf{R}_a = 500\text{ N}\downarrow$ ◀

and ΣM_A: $-400\text{ N}\cdot\text{m} - (200\text{ N})(3\text{ m}) = M_a$

or $\mathbf{M}_a = 1000\text{ N}\cdot\text{m}$ ◀

(*b*) We have ΣF_Y: $200\text{ N} + 300\text{ N} = R_b$

or $\mathbf{R}_b = 500\text{ N}\uparrow$ ◀

and ΣM_A: $-400\text{ N}\cdot\text{m} + (300\text{ N})(3\text{ m}) = M_b$

or $\mathbf{M}_b = 500\text{ N}\cdot\text{m}$ ◀

(*c*) We have ΣF_Y: $-200\text{ N} - 300\text{ N} = R_c$

or $\mathbf{R}_c = 500\text{ N}\downarrow$ ◀

and ΣM_A: $400\text{ N}\cdot\text{m} - (300\text{ N})(3\text{ m}) = M_c$

or $\mathbf{M}_c = 500\text{ N}\cdot\text{m}$ ◀

PROBLEM 3.101 (Continued)

(*d*) We have ΣF_Y: $-500 \text{ N} = R_d$

or $\mathbf{R}_d = 500 \text{ N}\downarrow$ ◀

and ΣM_A: $400 \text{ N}\cdot\text{m} - (500 \text{ N})(3 \text{ m}) = M_d$

or $\mathbf{M}_d = 1100 \text{ N}\cdot\text{m}$ ↻ ◀

(*e*) We have ΣF_Y: $300 \text{ N} - 800 \text{ N} = R_e$

or $\mathbf{R}_e = 500 \text{ N}\downarrow$ ◀

and ΣM_A: $400 \text{ N}\cdot\text{m} + 1000 \text{ N}\cdot\text{m} - (800 \text{ N})(3 \text{ m}) = M_e$

or $\mathbf{M}_e = 1000 \text{ N}\cdot\text{m}$ ↻ ◀

(*f*) We have ΣF_Y: $-300 \text{ N} - 200 \text{ N} = R_f$

or $\mathbf{R}_f = 500 \text{ N}\downarrow$ ◀

and ΣM_A: $400 \text{ N}\cdot\text{m} - (200 \text{ N})(3 \text{ m}) = M_f$

or $\mathbf{M}_f = 200 \text{ N}\cdot\text{m}$ ↻ ◀

(*g*) We have ΣF_Y: $-800 \text{ N} + 300 \text{ N} = R_g$

or $\mathbf{R}_g = 500 \text{ N}\downarrow$ ◀

and ΣM_A: $1000 \text{ N}\cdot\text{m} + 400 \text{ N}\cdot\text{m} + (300 \text{ N})(3 \text{ m}) = M_g$

or $\mathbf{M}_g = 2300 \text{ N}\cdot\text{m}$ ↺ ◀

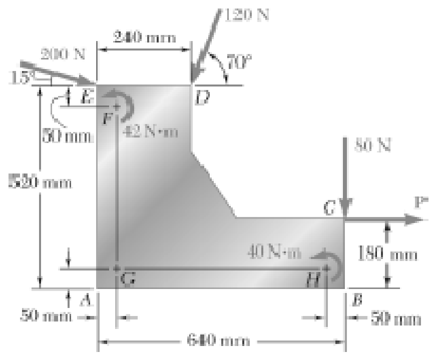
(*h*) We have ΣF_Y: $-250 \text{ N} - 250 \text{ N} = R_h$

or $\mathbf{R}_h = 500 \text{ N}\downarrow$ ◀

and ΣM_A: $1000 \text{ N}\cdot\text{m} + 400 \text{ N}\cdot\text{m} - (250 \text{ N})(3 \text{ m}) = M_h$

or $\mathbf{M}_h = 650 \text{ N}\cdot\text{m}$ ↺ ◀

(*b*) Therefore, loadings (*a*) and (*e*) are equivalent.

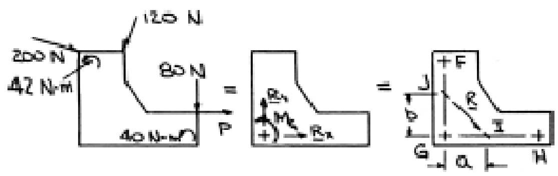


PROBLEM 3.111

A machine component is subjected to the forces and couples shown. The component is to be held in place by a single rivet that can resist a force but not a couple. For $P = 0$, determine the location of the rivet hole if it is to be located (*a*) on line *FG*, (*b*) on line *GH*.

SOLUTION

We have



First replace the applied forces and couples with an equivalent force-couple system at *G*.

Thus, ΣF_x: $200\cos 15° - 120\cos 70° + P = R_x$

or $$R_x = (152.142 + P)\text{ N}$$

ΣF_y: $-200\sin 15° - 120\sin 70° - 80 = R_y$

or $$R_y = -244.53\text{ N}$$

$$\begin{aligned}\Sigma M_G:\ & -(0.47\text{ m})(200\text{ N})\cos 15° + (0.05\text{ m})(200\text{ N})\sin 15° \\ & + (0.47\text{ m})(120\text{ N})\cos 70° - (0.19\text{ m})(120\text{ N})\sin 70° \\ & - (0.13\text{ m})(P\text{ N}) - (0.59\text{ m})(80\text{ N}) + 42\text{ N}\cdot\text{m} \\ & + 40\text{ N}\cdot\text{m} = M_G\end{aligned}$$

or $$M_G = -(55.544 + 0.13P)\text{ N}\cdot\text{m} \qquad (1)$$

Setting $P = 0$ in Eq. (1):

Now with **R** at *I*, ΣM_G: $-55.544\text{ N}\cdot\text{m} = -a(244.53\text{ N})$

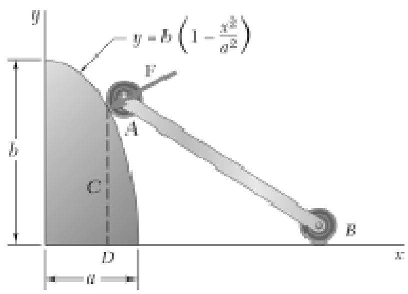
or $$a = 0.227\text{ m}$$

and with **R** at *J*, ΣM_G: $-55.544\text{ N}\cdot\text{m} = -b(152.142\text{ N})$

or $$b = 0.365\text{ m}$$

(*a*) The rivet hole is 0.365 m above *G*. ◀

(*b*) The rivet hole is 0.227 m to the right of *G*. ◀



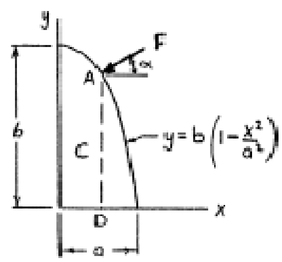
PROBLEM 3.118

As follower AB rolls along the surface of member C, it exerts a constant force $\mathbf{F}$ perpendicular to the surface. (a) Replace $\mathbf{F}$ with an equivalent force-couple system at Point D obtained by drawing the perpendicular from the point of contact to the x-axis. (b) For $a = 1$ m and $b = 2$ m, determine the value of x for which the moment of the equivalent force-couple system at D is maximum.

SOLUTION

(a) The slope of any tangent to the surface of member C is

$$\frac{dy}{dx} = \frac{d}{dx}\left[b\left(1 - \frac{x^2}{a^2}\right)\right] = \frac{-2b}{a^2}x$$



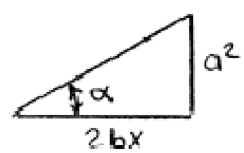
Since the force $\mathbf{F}$ is perpendicular to the surface,

$$\tan\alpha = -\left(\frac{dy}{dx}\right)^{-1} = \frac{a^2}{2b}\left(\frac{1}{x}\right)$$

For equivalence,

$$\Sigma F: \quad \mathbf{F} = \mathbf{R}$$

$$\Sigma M_D: \quad (F\cos\alpha)(y_A) = M_D$$



where

$$\cos\alpha = \frac{2bx}{\sqrt{(a^2)^2 + (2bx)^2}}$$

$$y_A = b\left(1 - \frac{x^2}{a^2}\right)$$

$$M_D = \frac{2Fb^2\left(x - \dfrac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}}$$

Therefore, the equivalent force-couple system at D is

$$\mathbf{R} = F \measuredangle \tan^{-1}\left(\frac{a^2}{2bx}\right) \blacktriangleleft$$

$$\mathbf{M} = \frac{2Fb^2\left(x - \dfrac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}} \blacktriangleleft$$

PROBLEM 3.118 (Continued)

(*b*) To maximize *M*, the value of *x* must satisfy $\frac{dM}{dx} = 0$

where for $a = 1\text{ m}, \quad b = 2\text{ m}$

$$M = \frac{8F(x - x^3)}{\sqrt{1 + 16x^2}}$$

$$\frac{dM}{dx} = 8F\frac{\sqrt{1+16x^2}(1-3x^2) - (x - x^3)\left[\frac{1}{2}(32x)(1+16x^2)^{-1/2}\right]}{(1+16x^2)} = 0$$

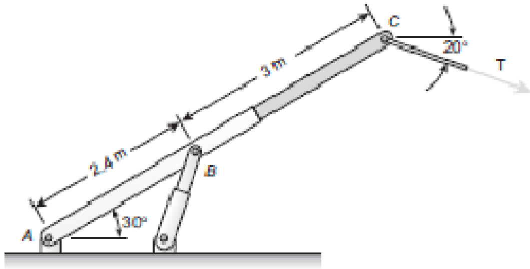
$$(1 + 16x^2)(1 - 3x^2) - 16x(x - x^3) = 0$$

or

$$32x^4 + 3x^2 - 1 = 0$$

$$x^2 = \frac{-3 \pm \sqrt{9 - 4(32)(-1)}}{2(32)} = 0.136011\text{ m}^2 \quad \text{and} \quad -0.22976\text{ m}^2$$

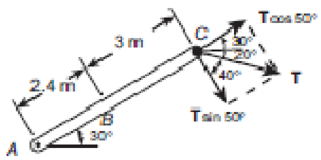
Using the positive value of x^2: $x = 0.36880\text{ m}$ or $x = 369\text{ mm}$ ◀

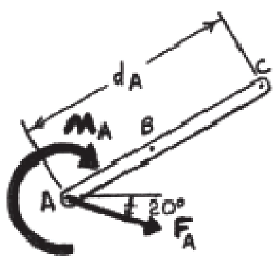


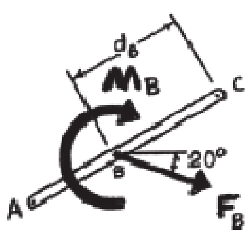
PROBLEM 3.153

The tension in the cable attached to the end C of an adjustable boom ABC is 2.24 kN. Replace the force exerted by the cable at C with an equivalent force-couple system (a) at A, (b) at B.

SOLUTION







(a) Based on ΣF: $F_A = T = 2.24$ kN

or $\mathbf{F}_A = 2.24 \text{ kN} \measuredangle 20^\circ$ ◀

ΣM_A: $M_A = (T \sin 50^\circ)(d_A)$

$= (2.24 \text{ kN}) \sin 50^\circ (5.4 \text{ m})$

$= 9.266 \text{ kN}\cdot\text{m}$

or $\mathbf{M}_A = 9.27 \text{ kN}\cdot\text{m} \circlearrowright$ ◀

(b) Based on ΣF: $F_B = T = 2.24$ kN

or $\mathbf{F}_B = 2.24 \text{ kN} \measuredangle 20^\circ$ ◀

ΣM_B: $M_B = (T \sin 50^\circ)(d_B)$

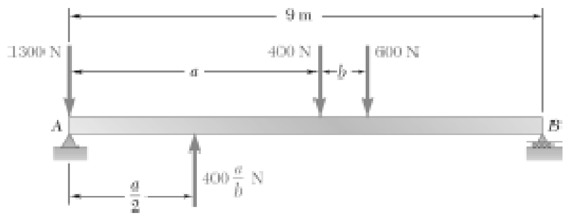
$= (2.24 \text{ kN}) \sin 50^\circ (3 \text{ m})$

$= 5.148 \text{ kN}\cdot\text{m}$

or $\mathbf{M}_B = 5.15 \text{ kN}\cdot\text{m} \circlearrowright$ ◀

PROBLEM 3.156

A beam supports three loads of given magnitude and a fourth load whose magnitude is a function of position. If $b = 1.5$ m and the loads are to be replaced with a single equivalent force, determine (*a*) the value of a so that the distance from support A to the line of action of the equivalent force is maximum, (*b*) the magnitude of the equivalent force and its point of application on the beam.



SOLUTION

For equivalence,

$$\Sigma F_y:\quad -1300 + 400\frac{a}{b} - 400 - 600 = -R$$

or
$$R = \left(2300 - 400\frac{a}{b}\right)\text{ N} \qquad (1)$$

$$\Sigma M_A:\quad \frac{a}{2}\left(400\frac{a}{b}\right) - a(400) - (a+b)(600) = -LR$$

or
$$L = \frac{1000a + 600b - 200\dfrac{a^2}{b}}{2300 - 400\dfrac{a}{b}}$$

Then with
$$b = 1.5\text{ m} \qquad L = \frac{10a + 9 - \dfrac{4}{3}a^2}{23 - \dfrac{8}{3}a} \qquad (2)$$

where a, L are in m.

(*a*) Find value of a to maximize L.

$$\frac{dL}{da} = \frac{\left(10 - \dfrac{8}{3}a\right)\left(23 - \dfrac{8}{3}a\right) - \left(10a + 9 - \dfrac{4}{3}a^2\right)\left(-\dfrac{8}{3}\right)}{\left(23 - \dfrac{8}{3}a\right)^2}$$

PROBLEM 3.156 (Continued)

or $$230 - \frac{184}{3}a - \frac{80}{3}a + \frac{64}{9}a^2 + \frac{80}{3}a + 24 - \frac{32}{9}a^2 = 0$$

or $$16a^2 - 276a + 1143 = 0$$

Then $$a = \frac{276 \pm \sqrt{(-276)^2 - 4(16)(1143)}}{2(16)}$$

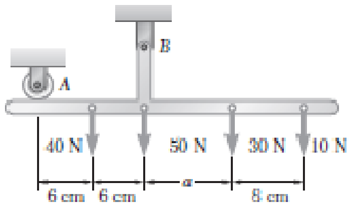
or $$a = 10.3435 \text{ m} \quad \text{and} \quad a = 6.9065 \text{ m}$$

Since $AB = 9$ m, a must be less than 9 m $\qquad a = 6.91 \text{ m}$ ◀

(*b*) Using Eq. (1), $$R = 2300 - 400\frac{6.9065}{1.5}$$ or $R = 458$ N ◀

and using Eq. (2), $$L = \frac{10(6.9065) + 9 - \frac{4}{3}(6.9065)^2}{23 - \frac{8}{3}(6.9065)} = 3.16 \text{ m}$$

R is applied 3.16 m to the right of A. ◀

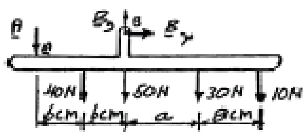


PROBLEM 4.3

A T-shaped bracket supports the four loads shown. Determine the reactions at A and B (a) if $a = 10$ cm, (b) if $a = 7$ cm.

SOLUTION

Free-Body Diagram:



$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (40 \text{ N})(6 \text{ cm}) - (30 \text{ N})a - (10 \text{ N})(a + 8 \text{ cm}) + (12 \text{ cm})A = 0$$

$$A = \frac{(40a - 160)}{12} \qquad (1)$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(40 \text{ N})(6 \text{ cm}) - (50 \text{ N})(12 \text{ cm}) - (30 \text{ N})(a + 12 \text{ cm})$$
$$-(10 \text{ N})(a + 20 \text{ cm}) + (12 \text{ cm})B_y = 0$$

$$B_y = \frac{(1400 + 40a)}{12}$$

Since $\quad B_x = 0 \quad B = \dfrac{(1400 + 40a)}{12} \qquad (2)$

(a) For $a = 10$ cm

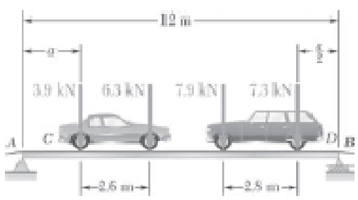
Eq. (1): $\quad A = \dfrac{(40 \times 10 - 160)}{12} = +20.0 \text{ N} \qquad \mathbf{A} = 20.0 \text{ N} \downarrow$ ◀

Eq. (2): $\quad B = \dfrac{(1400 + 40 \times 10)}{12} = +150.0 \text{ N} \qquad \mathbf{B} = 150.0 \text{ N} \uparrow$ ◀

(b) For $a = 7$ cm

Eq. (1): $\quad A = \dfrac{(40 \times 7 - 160)}{12} = +10.00 \text{ N} \qquad \mathbf{A} = 10.00 \text{ N} \downarrow$ ◀

Eq. (2): $\quad B = \dfrac{(1400 + 40 \times 7)}{12} = +140.0 \text{ N} \qquad \mathbf{B} = 140.0 \text{ N} \uparrow$ ◀

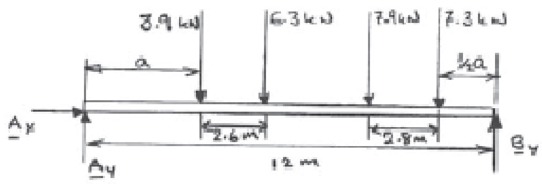


PROBLEM 4.7

When cars C and D stop on a two-lane bridge, the forces exerted by their tires on the bridge are as shown. Determine the total reactions at A and B when (*a*) $a = 2.9$ m, (*b*) $a = 8.1$ m.

SOLUTION

Free-Body Diagram:



(*a*) $a = 2.9$ m

$\xrightarrow{+} \Sigma F_x = 0$: $\quad A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$: $\quad -(12\text{ m})A_y + [(12 - 2.9)\text{m}](3.9\text{ kN}) + [(12 - 2.9 - 2.6)\text{m}](6.3\text{ kN})$

$\qquad + [(2.8 + 1.45)\text{m}](7.9\text{ kN}) + (1.45\text{ m})(7.3\text{ kN}) = 0$

or $\quad A_y = 10.0500$ kN $\qquad$ or $\mathbf{A} = 10.05$ kN $\uparrow$ ◀

$+\uparrow \Sigma F_y = 0$: $\quad 10.0500\text{ kN} - 3.9\text{ kN} - 6.3\text{ kN} - 7.9\text{ kN} - 7.3\text{ kN} + B_y = 0$

or $\quad B_y = 15.3500$ kN $\qquad$ or $\mathbf{B} = 15.35$ kN $\uparrow$ ◀

(*b*) $a = 8.1$ m

$+\circlearrowleft \Sigma M_B = 0$: $\quad -(12\text{ m})A_y + [(12 - 8.1)\text{m}](3.9\text{ kN}) + [(12 - 8.1 - 2.6)\text{m}](6.3\text{ kN})$

$\qquad + [(2.8 + 4.05)\text{m}](7.9\text{ kN}) + (4.05\text{ m})(7.3\text{ kN}) = 0$

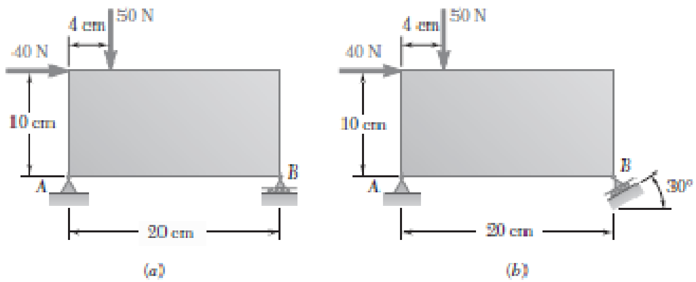
or $\quad A_y = 8.9233$ kN $\qquad$ or $\mathbf{A} = 8.92$ kN $\uparrow$ ◀

$+\uparrow \Sigma F_y = 0$: $\quad 8.9233\text{ kN} - 3.9\text{ kN} - 6.3\text{ kN} - 7.9\text{ kN} - 7.3\text{ kN} + B_y = 0$

or $\quad B_y = 16.4767$ kN $\qquad$ or $\mathbf{B} = 16.48$ kN $\uparrow$ ◀

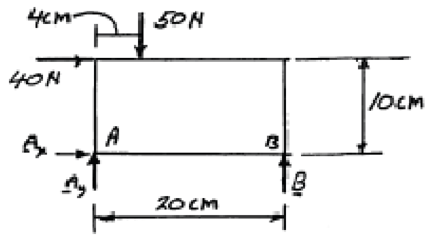
PROBLEM 4.25

For each of the plates and loadings shown, determine the reactions at A and B.



SOLUTION

(a) **Free-Body Diagram:**



$+\circlearrowleft \Sigma M_A = 0: \quad B(20 \text{ cm}) - (50 \text{ N})(4 \text{ cm}) - (40 \text{ N})(10 \text{ cm}) = 0$

$B = +30 \text{ N}$ $\qquad$ $\mathbf{B} = 30.0 \text{ N} \uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 40 \text{ N} = 0$

$A_x = -40 \text{ N}$ $\qquad$ $\mathbf{A}_x = 40.0 \text{ N} \longleftarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y + B - 50 \text{ N} = 0$

$A_y + 30 \text{ N} - 50 \text{ N} = 0$

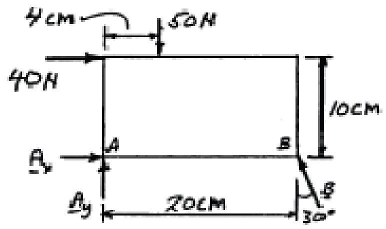
$A_y = +20 \text{ N}$ $\qquad$ $\mathbf{A}_y = 20.0 \text{ N} \uparrow$

$\alpha = 26.56^0$

$A = 44.72 \text{ N}$ $\qquad$ $\mathbf{A} = 44.7 \text{ N} \measuredangle 26.6°$ ◀

PROBLEM 4.25 (Continued)

(b) **Free-Body Diagram:**



$+\circlearrowleft \Sigma M_A = 0$: $(B\cos 30°)(20\text{ cm}) - (40\text{ N})(10\text{ cm}) - (50\text{ N})(4\text{ cm}) = 0$

$B = 34.64\text{ N}$ $\qquad$ $\mathbf{B} = 34.6\text{ N} \searrow 60.0°$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $A_x - B\sin 30° + 40\text{ N}$

$A_x - (34.64\text{ N})\sin 30° + 40\text{ N} = 0$

$A_x = -22.68\text{ N}$ $\qquad$ $\mathbf{A}_x = 22.68\text{ N} \longleftarrow$

$+\uparrow \Sigma F_y = 0$: $A_y + B\cos 30° - 50\text{ N} = 0$

$A_y + (34.64\text{ N})\cos 30° - 50\text{ N} = 0$

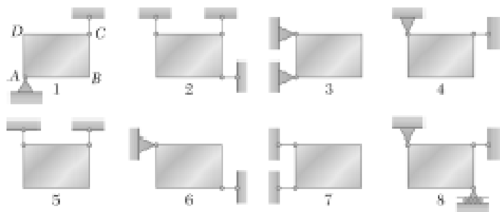
$A_y = +20\text{ N}$ $\qquad$ $\mathbf{A}_y = 20.0\text{ N} \uparrow$

$\alpha = 41.4°$

$A = 30.24\text{ N}$ $\qquad$ $\mathbf{A} = 30.2\text{ N} \searrow 41.4°$ ◀

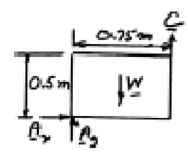
PROBLEM 4.59

Eight identical 500×750-mm rectangular plates, each of mass $m = 40$ kg, are held in a vertical plane as shown. All connections consist of frictionless pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions.



SOLUTION

1. Three non-concurrent, non-parallel reactions:



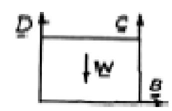
(*a*) Plate: completely constrained

(*b*) Reactions: determinate

(*c*) Equilibrium maintained

$$\mathbf{A} = \mathbf{C} = 196.2 \text{ N}\uparrow$$

2. Three non-concurrent, non-parallel reactions:



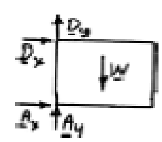
(*a*) Plate: completely constrained

(*b*) Reactions: determinate

(*c*) Equilibrium maintained

$$\mathbf{B} = 0, \quad \mathbf{C} = \mathbf{D} = 196.2 \text{ N}\uparrow$$

3. Four non-concurrent, non-parallel reactions:



(*a*) Plate: completely constrained

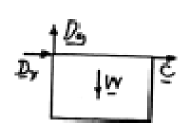
(*b*) Reactions: indeterminate

(*c*) Equilibrium maintained

$$\mathbf{A}_x = 294 \text{ N} \longrightarrow, \quad \mathbf{D}_x = 294 \text{ N} \longleftarrow$$

$$(\mathbf{A}_y + \mathbf{D}_y = 392 \text{ N}\uparrow)$$

4. Three concurrent reactions (through D):



(*a*) Plate: improperly constrained

(*b*) Reactions: indeterminate

(*c*) No equilibrium $\qquad (\Sigma M_D \neq 0)$

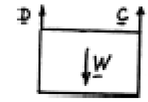
PROBLEM 4.59 (Continued)

5. Two reactions:

   (*a*) Plate: partial constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained



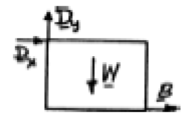
$$\mathbf{C} = \mathbf{D} = 196.2\ \text{N}\uparrow$$

6. Three non-concurrent, non-parallel reactions:

   (*a*) Plate: completely constrained

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained



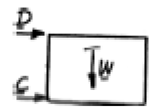
$$\mathbf{B} = 294\ \text{N}\longrightarrow, \quad \mathbf{D} = 491\ \text{N} \searrow 53.1^\circ$$

7. Two reactions:

   (*a*) Plate: improperly constrained

   (*b*) Reactions determined by dynamics

   (*c*) No equilibrium $\qquad (\Sigma F_y \neq 0)$

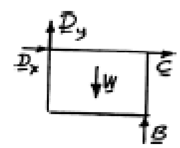


8. Four non-concurrent, non-parallel reactions:

   (*a*) Plate: completely constrained

   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained

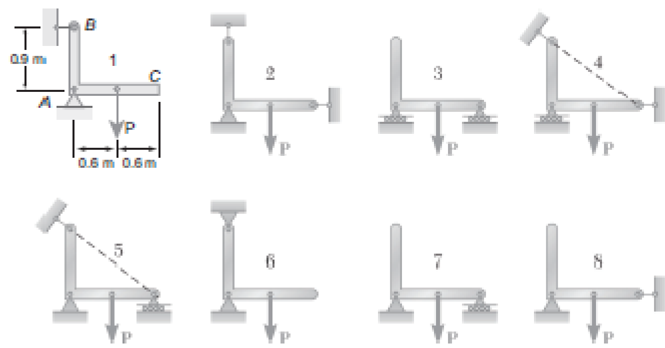


$$\mathbf{B} = \mathbf{D}_y = 196.2\ \text{N}\uparrow$$

$$(\mathbf{C} + \mathbf{D}_x = 0)$$

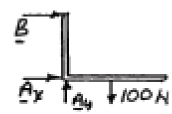
PROBLEM 4.60

The bracket *ABC* can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. For each case, answer the questions listed in Problem 4.59, and, wherever possible, compute the reactions, assuming that the magnitude of the force **P** is 100 N.



SOLUTION

1. Three non-concurrent, non-parallel reactions



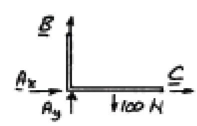
   (*a*) Bracket: complete constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

$$\mathbf{A} = 120.2\text{ N} \measuredangle 56.3°, \quad \mathbf{B} = 66.7\text{ N} \longleftarrow$$

2. Four concurrent, reactions (through *A*)

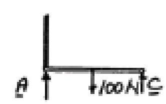


   (*a*) Bracket: improper constraint

   (*b*) Reactions: indeterminate

   (*c*) No equilibrium $(\Sigma M_A \neq 0)$

3. Two reactions



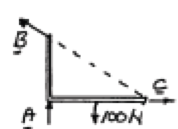
   (*a*) Bracket: partial constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

$$\mathbf{A} = 50\text{ N}\uparrow, \quad \mathbf{C} = 50\text{ N}\uparrow$$

4. Three non-concurrent, non-parallel reactions



   (*a*) Bracket: complete constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

$$\mathbf{A} = 50\text{ N}\uparrow, \quad \mathbf{B} = 83.3\text{ N} \searrow 36.9°, \quad \mathbf{C} = 66.7\text{ N} \longrightarrow$$

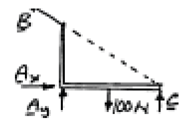
PROBLEM 4.60 (Continued)

5. Four non-concurrent, non-parallel reactions

   (*a*) Bracket: complete constraint

   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained $\quad (\Sigma M_C = 0)\ \mathbf{A}_y = 50\text{ N}\uparrow$

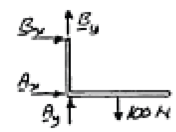


6. Four non-concurrent, non-parallel reactions

   (*a*) Bracket: complete constraint

   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained



$$\mathbf{A}_x = 66.7\text{ N}\longrightarrow,\quad \mathbf{B}_x = 66.7\text{ N}\longleftarrow$$

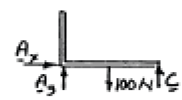
$$(\mathbf{A}_y + \mathbf{B}_y = 100\text{ N}\uparrow)$$

7. Three non-concurrent, non-parallel reactions

   (*a*) Bracket: complete constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained



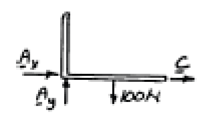
$$\mathbf{A} = \mathbf{C} = 50\text{ N}\uparrow$$

8. Three concurrent, reactions (through *A*)

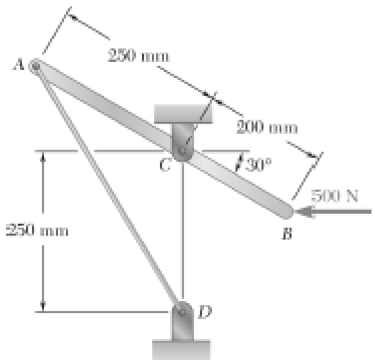
   (*a*) Bracket: improper constraint

   (*b*) Reactions: indeterminate

   (*c*) No equilibrium $\quad (\Sigma M_A \neq 0)$

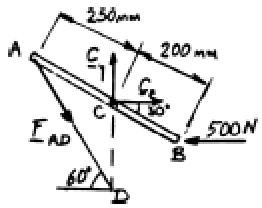


PROBLEM 4.144



A lever AB is hinged at C and attached to a control cable at A. If the lever is subjected to a 500-N horizontal force at B, determine (a) the tension in the cable, (b) the reaction at C.

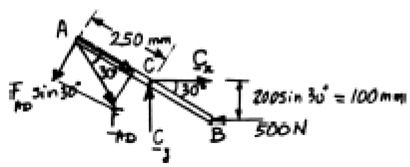
SOLUTION



Triangle ACD is isosceles with $\measuredangle C = 90° + 30° = 120°$ $\measuredangle A = \measuredangle D = \frac{1}{2}(180° - 120°) = 30°$.

Thus, DA forms angle of 60° with the horizontal axis.

(a) We resolve $\mathbf{F}_{AD}$ into components along AB and perpendicular to AB.



$$+\circlearrowleft \Sigma M_C = 0: \quad (F_{AD} \sin 30°)(250 \text{ mm}) - (500 \text{ N})(100 \text{ mm}) = 0 \qquad F_{AD} = 400 \text{ N} \blacktriangleleft$$

(b)

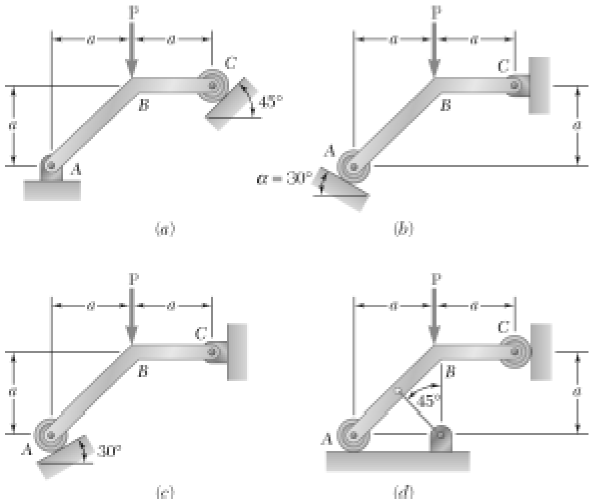
$$\xrightarrow{+} \Sigma F_x = 0: \quad -(400 \text{ N})\cos 60° + C_x - 500 \text{ N} = 0 \qquad C_x = +300 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad -(400 \text{ N})\sin 60° + C_y = 0 \qquad C_y = +346.4 \text{ N}$$

$$\mathbf{C} = 458 \text{ N} \measuredangle 49.1° \blacktriangleleft$$

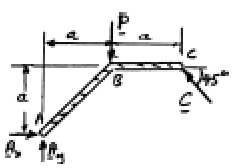
PROBLEM 4.153

A force **P** is applied to a bent rod *ABC*, which may be supported in four different ways as shown. In each case, if possible, determine the reactions at the supports.



SOLUTION

(*a*)



$$+\circlearrowleft \Sigma M_A = 0: \quad -P_a + (C\sin 45^\circ)2a + (\cos 45^\circ)a = 0$$

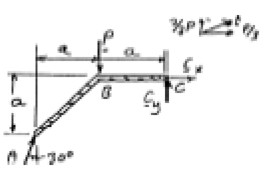
$$3\frac{C}{\sqrt{2}} = P \qquad C = \frac{\sqrt{2}}{3}P \qquad \mathbf{C} = 0.471P \measuredangle 45^\circ \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \qquad A_x = \frac{P}{3} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - P + \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \qquad A_y = \frac{2P}{3}\uparrow$$

$$\mathbf{A} = 0.745P \measuredangle 63.4^\circ \blacktriangleleft$$

(*b*)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30^\circ)2a + (A\sin 30^\circ)a = 0$$

$$A(1.732 - 0.5) = P \qquad A = 0.812P$$

$$\mathbf{A} = 0.812P \measuredangle 60.0^\circ \blacktriangleleft$$

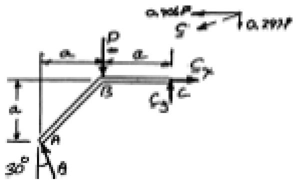
$$\xrightarrow{+} \Sigma F_x = 0: \quad (0.812P)\sin 30^\circ + C_x = 0 \qquad C_x = -0.406P$$

$$+\uparrow \Sigma F_y = 0: \quad (0.812P)\cos 30^\circ - P + C_y = 0 \qquad C_y = -0.297P$$

$$\mathbf{C} = 0.503P \measuredangle 36.2^\circ \blacktriangleleft$$

PROBLEM 4.153 (Continued)

(*c*)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30^\circ)2a + (A\sin 30^\circ)a = 0$$

$$A(1.732 + 0.5) = P \quad A = 0.448P$$

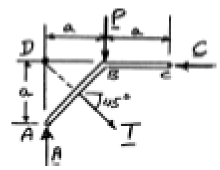
$A = 0.448P$ ⦪ 60.0° ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(0.448P)\sin 30^\circ + C_x = 0 \qquad C_x = 0.224P \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad (0.448P)\cos 30^\circ - P + C_y = 0 \quad C_y = 0.612P\uparrow$$

$\mathbf{C} = 0.652P$ ↗ 69.9° ◀

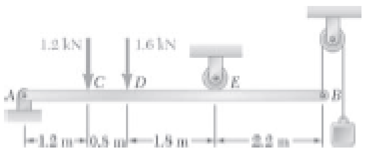
(*d*) Force **T** exerted by wire and reactions **A** and **C** all intersect at Point *D*.



$$+\circlearrowleft \Sigma M_D = 0: \quad P_a = 0$$

Equilibrium is not maintained.

Rod is improperly constrained. ◀

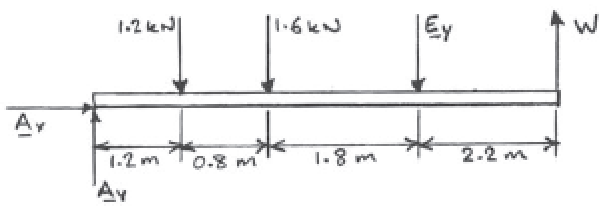


PROBLEM 4.14

For the given loading of the beam AB, determine the range of values of the mass of the crate for which the system will be in equilibrium, knowing that the maximum allowable value of the reactions at each support is 2.5 kN and that the reaction at E must be directed downward.

SOLUTION

Free-Body Diagram:



Note that $W = mg$ is the weight of the crate in the free-body diagram, and that

$0 \le E_y \le 2.5$ kN

$$\xrightarrow{+} \Sigma F_x = 0: \qquad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \qquad -(1.2 \text{ m})(1.2 \text{ kN}) - (2.0 \text{ m})(1.6 \text{ kN}) - (3.8 \text{ m})E_y + (6 \text{ m})W = 0$$

$$\text{or} \qquad 6W = 4.64 \text{ kN} + 3.8E_y \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y - 1.2 \text{ kN} - 1.6 \text{ kN} - E_y + W = 0$$

$$\text{or} \qquad A_y = 2.8 \text{ kN} + E_y - W \qquad (2)$$

Considering the smallest possible value of E_y:

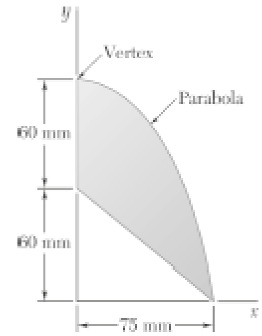
For $\quad E_y = 0,\ W = W_{min} = 0.77333$ kN

From (2) the corresponding value of A_y is:

$A_y = 2.02667 \text{ kN} \le 2.5 \text{ kN}$, which satisfies the constraint on A_y.

For the largest allowable value of E_y:

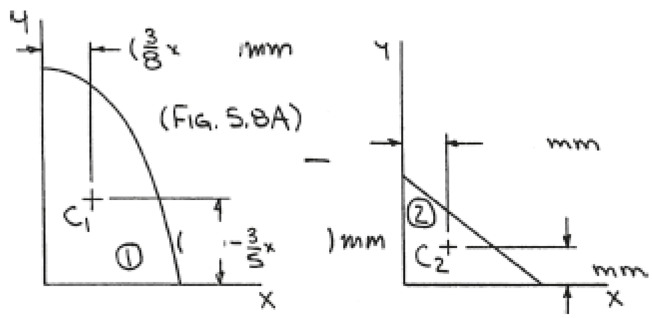
$E_y = 2.5 \text{ kN},\ W = W_{max} = 2.3567$ kN



PROBLEM 5.9

Locate the centroid of the plane area shown.

SOLUTION



	A, mm^2	$\overline{x}$, mm	$\overline{y}$, mm	$\overline{x}A$, mm^3	$\overline{y}A$, mm^3
1	$\frac{2}{3}(75)(120) = 6000$	28.125	48	168,750	288,000
2	$-\frac{1}{2}(75)(60) = -2250$	25	20	−56,250	−45,000
Σ	3750			112,500	243,000

Then

$$\overline{X}\Sigma A = \Sigma \overline{x} A$$

$$\overline{X}(3750 \text{ mm}^2) = 112{,}500 \text{ mm}^3$$

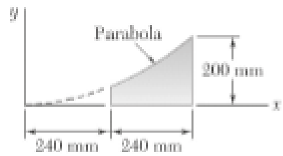
or $\overline{X} = 30.0$ mm ◀

and

$$\overline{Y}\Sigma A = \Sigma \overline{y} A$$

$$\overline{Y}(3750 \text{ mm}^2) = 243{,}000 \text{ mm}^3$$

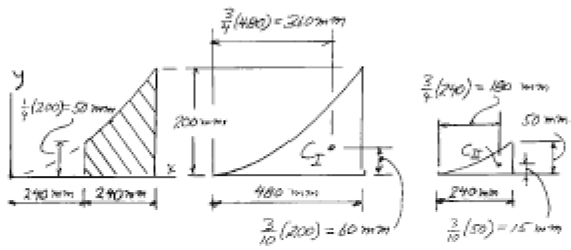
or $\overline{Y} = 64.8$ mm ◀



PROBLEM 5.12

Locate the centroid of the plane area shown.

SOLUTION



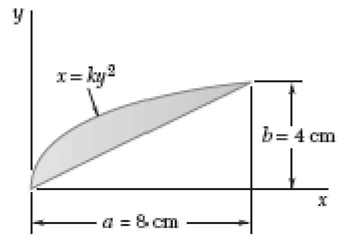
	Area mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{1}{3}(200)(480) = 32\times10^3$	360	60	11.52×10^6	1.92×10^6
2	$-\frac{1}{3}(50)(240) = 4\times10^3$	180	15	-0.72×10^6	-0.06×10^6
Σ	28×10^3			10.80×10^6	1.86×10^6

$\bar{X}\Sigma A = \Sigma \bar{x}A$: $\quad \bar{X}(28\times10^3\,\text{mm}^2) = 10.80\times10^6\,\text{mm}^3$

$\bar{X} = 385.7$ mm $\qquad \bar{X} = 386$ mm ◀

$\bar{Y}\Sigma A = \Sigma \bar{y}A$: $\quad \bar{Y}(28\times10^3\,\text{mm}^2) = 1.86\times10^6\,\text{mm}^3$

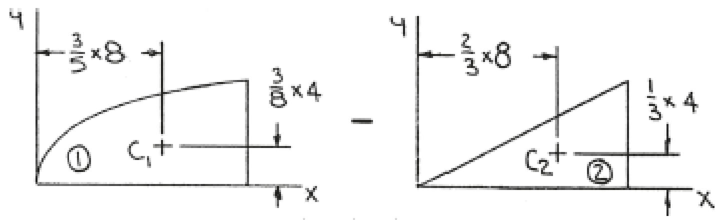
$\bar{Y} = 66.43$ mm $\qquad \bar{Y} = 66.4$ mm ◀



PROBLEM 5.14

Locate the centroid of the plane area shown.

SOLUTION



Dimensions in cm

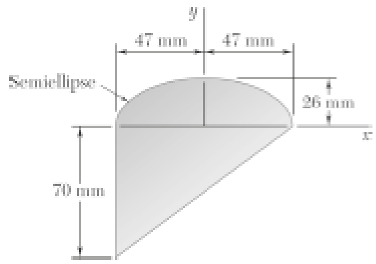
	A, cm^2	$\bar{x}$, cm	$\bar{y}$, cm	$\bar{x}A$, cm^3	$\bar{y}A$, cm^3
1	$\frac{2}{3}(4)(8) = 21.333$	4.8	1.5	102.398	32.000
2	$-\frac{1}{2}(4)(8) = -16.0000$	5.3333	1.33333	-85.333	-21.333
Σ	5.3333			17.0650	10.6670

Then $$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}(5.3333 \text{ cm}^2) = 17.0650 \text{ cm}^3$$ or $\bar{X} = 3.20$ cm ◀

and $$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}(5.3333 \text{ cm}^2) = 10.6670 \text{ cm}^3$$ or $\bar{Y} = 2.00$ cm ◀

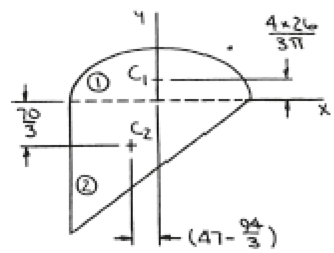


PROBLEM 5.15

Locate the centroid of the plane area shown.

SOLUTION

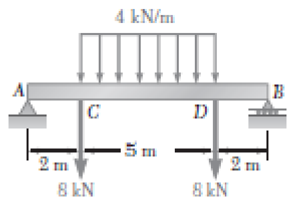
Dimensions in mm



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{\pi}{2}\times47\times26=1919.51$	0	11.0347	0	21,181
2	$\frac{1}{2}\times94\times70=3290$	−15.6667	−23.333	−51,543	−76,766
Σ	5209.5			−51,543	−55,584

Then $\quad \bar{X}=\frac{\Sigma\bar{x}A}{\Sigma A}=\frac{-51,543}{5209.5}$ $\qquad \bar{X}=-9.89$ mm ◀

$\bar{Y}=\frac{\Sigma\bar{y}A}{\Sigma A}=\frac{-55,584}{5209.5}$ $\qquad \bar{Y}=-10.67$ mm ◀



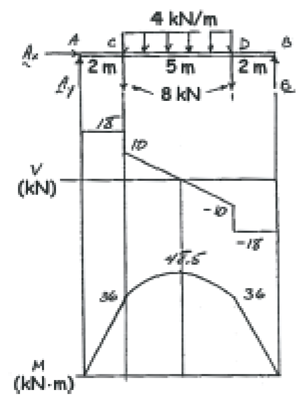
PROBLEM 7.41

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) By symmetry:

$$A_y = B = 8\text{ kN} + \frac{1}{2}(4\text{ kN})(5\text{ m}) \qquad \mathbf{A}_y = \mathbf{B} = 18\text{ kN}\uparrow$$



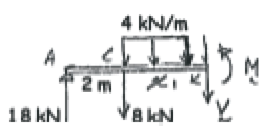
Along AC:

$$+\uparrow\Sigma F_y = 0:\ 18\text{ kN} - V = 0 \qquad V = 18\text{ kN}$$

$$\Sigma M_J = 0:\ M - x(18\text{ kN}) \qquad M = (18\text{ kN})x$$

$$M = 36\text{ kN}\cdot\text{m at } C(x = 2\text{ m})$$

Along CD:



$$+\uparrow\Sigma F_y = 0:\ 18\text{ kN} - 8\text{ kN} - (4\text{ kN/m})x_1 - V = 0$$

$$V = 10\text{ kN} - (4\text{ kN/m})x_1$$

$$V = 0 \text{ at } x_1 = 2.5\text{ m (at center)}$$

$$\Sigma M_K = 0:\quad M + \left(\frac{x_1}{2}\right)(4\text{ kN/m})x_1 + (8\text{ kN})x_1 - (2\text{ m} + x_1)(18\text{ kN}) = 0$$

$$M = 36\text{ kN}\cdot\text{m} + (10\text{ kN/m})x_1 - (2\text{ kN/m})x_1^2$$

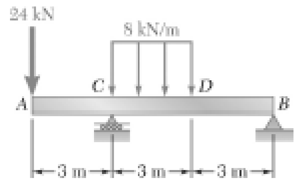
$$M = 48.5\text{ kN}\cdot\text{m at } x_1 = 2.5\text{ m}$$

Complete diagram by symmetry

(*b*) From diagrams:

$$|V|_{max} = 18.00\text{ kN on } AC \text{ and } DB \blacktriangleleft$$

$$|M|_{max} = 48.5\text{ kN}\cdot\text{m at center} \blacktriangleleft$$

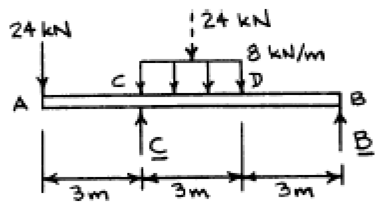


PROBLEM 7.40

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

<u>Free body: Entire beam</u>

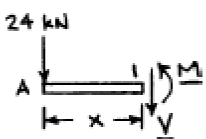


$+\circlearrowleft \Sigma M_B = 0$: $(24\text{ kN})(9\text{ m}) - C(6\text{ m}) + (24\text{ kN})(4.5\text{ m}) = 0$ $\qquad \mathbf{C} = 54\text{ kN}\uparrow$

$+\uparrow \Sigma F_y = 0$: $54 - 24 - 24 + B = 0$

$B = -6\text{ kN}$ $\qquad \mathbf{B} = 6\text{ kN}\downarrow$

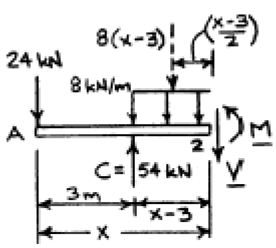
<u>From *A* to *C*:</u>



$+\uparrow \Sigma F_y = 0$: $-24 - V = 0$ $\qquad V = -24\text{ kN}$

$+\circlearrowleft \Sigma M_1 = 0$: $(24)(x) + M = 0$ $\qquad M = (-24x)\text{kN}\cdot\text{m}$

<u>From *C* to *D*:</u>



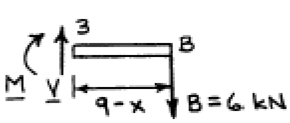
$+\uparrow \Sigma F_y = 0$: $-24 - 8(x-3) - V + 54 = 0$

$$V = (-8x + 54)\text{kN}$$

$+\circlearrowleft \Sigma M_2 = 0$: $(24)(x) + 8(x-3)\left(\frac{x-3}{2}\right) - (54)(x-3) + M = 0$

$$M = (-4x^2 + 54x - 198)\text{kN}\cdot\text{m}$$

<u>From *D* to *B*:</u>

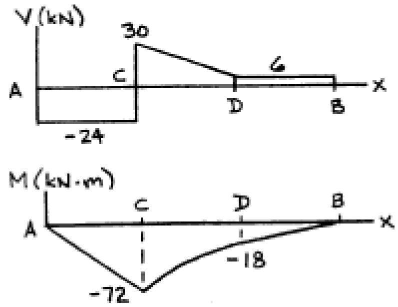


$+\uparrow \Sigma F_y = 0$: $V - 6 = 0$ $\qquad V = +6\text{ kN}$

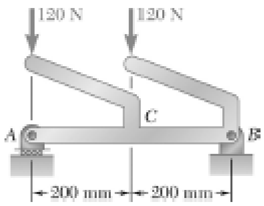
$+\circlearrowleft \Sigma M_3 = 0$: $-M - (6)(9-x) = 0$ $\qquad M = (6x - 54)\text{kN}\cdot\text{m}$

PROBLEM 7.40 (Continued)

(*b*) $|V|_{max} = 30.0 \text{ kN}$ ◀



$|M|_{max} = 72.0 \text{ kN} \cdot \text{m}$ ◀

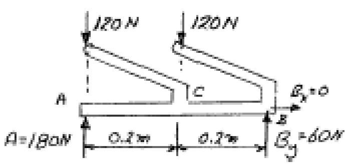


PROBLEM 7.49

Draw the shear and bending-moment diagrams for the beam AB, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

Reactions:



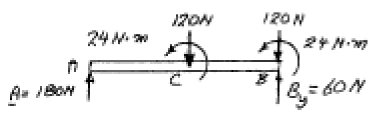
$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(0.4) - (120)(0.2) = 0$$

$$\mathbf{B}_y = 60 \text{ N}\uparrow$$

$$\Sigma F_x = 0: \qquad \mathbf{B}_x = 0$$

$$\Sigma F_y = 0: \qquad \mathbf{A} = 180 \text{ N}\uparrow$$

Equivalent loading on straight part of beam AB.

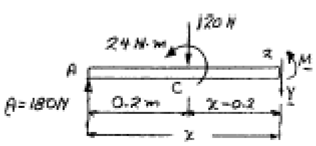


From A to C:

$$+\uparrow \Sigma F_y = 0: \quad V = +180 \text{ N}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad M = +180x$$

From C to B:

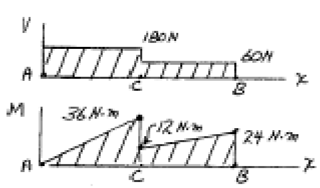


$$+\uparrow \Sigma F_y = 0: \quad 180 - 120 - V = 0$$

$$V = 60 \text{ N}$$

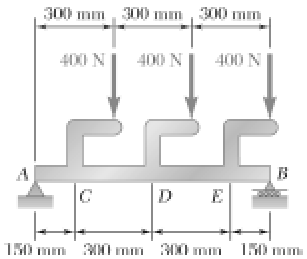
$$+\circlearrowleft \Sigma M_x = 0: \quad -(180 \text{ N})(x) + 24 \text{ N}\cdot\text{m} + (120 \text{ N})(x - 0.2) + M = 0$$

$$M = +60x$$



$$|V|_{\max} = 180.0 \text{ N} \blacktriangleleft$$

$$|M|_{\max} = 36.0 \text{ N}\cdot\text{m} \blacktriangleleft$$

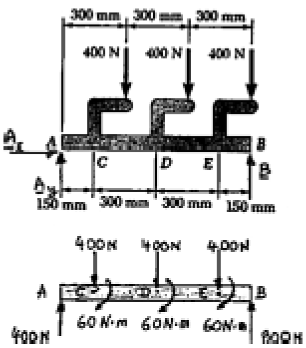


PROBLEM 7.50

Draw the shear and bending-moment diagrams for the beam AB, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_A = 0$: $B(0.9\text{ m}) - (400\text{ N})(0.3\text{ m}) - (400\text{ N})(0.6\text{ m}) - (400\text{ N})(0.9\text{ m}) = 0$

$B = +800\text{ N}$ $\qquad \mathbf{B} = 800\text{ N}\uparrow$ ◁

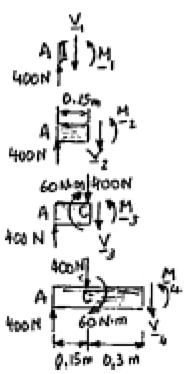
$\Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y + 800\text{ N} - 3(400\text{ N}) = 0$

$A_y = +400\text{ N}$ $\qquad \mathbf{A} = 400\text{ N}\uparrow$ ◁

We replace the loads by equivalent force-couple systems at C, D, and E.

We consider successively the following F-B diagrams.



$V_1 = +400\text{ N}$
$M_1 = 0$

$V_2 = +400\text{ N}$
$M_2 = +60\text{ N}\cdot\text{m}$

$V_3 = 0$
$M_3 = +120\text{ N}\cdot\text{m}$

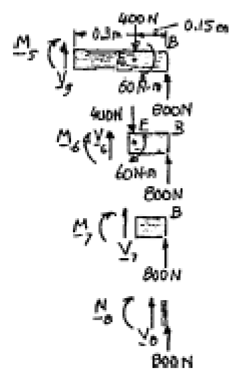
$V_4 = 0$
$M_4 = +120\text{ N}\cdot\text{m}$

$V_5 = -400\text{ N}$
$M_5 = +180\text{ N}\cdot\text{m}$

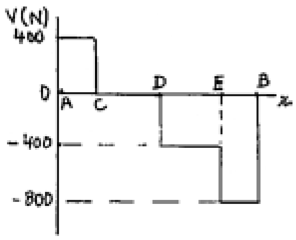
$V_6 = -400\text{ N}$
$M_6 = +60\text{ N}\cdot\text{m}$

$V_7 = -800\text{ N}$
$M_7 = +120\text{ N}\cdot\text{m}$

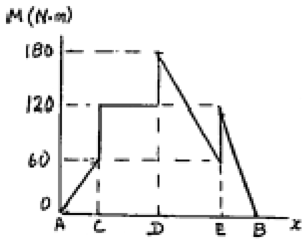
$V_8 = -800\text{ N}$
$M_8 = 0$



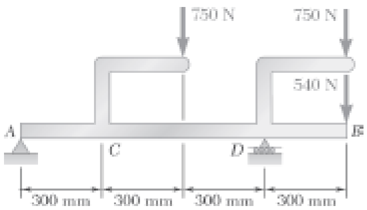
PROBLEM 7.50 (Continued)



(b) $|V|_{max} = 800\text{ N}$ ◀



$|M|_{max} = 180.0\text{ N}\cdot\text{m}$ ◀

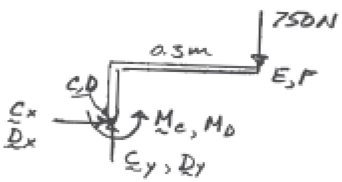


PROBLEM 7.52

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD CE or DF:



$\rightarrow \Sigma F_x = 0$: C_x, $D_x = 0$

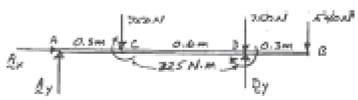
$\uparrow \Sigma F_y = 0$: $C_y - 750\text{ N} = 0$, $\quad C_y = 750\text{ N}$

$D_y = 750\text{ N}$

$\circlearrowleft \Sigma M_C = 0$: $M_C - (0.3\text{ m})(750\text{ N}) = 0$

$M_C = 225\text{ N}\cdot\text{m} = M_D$

Beam AB:



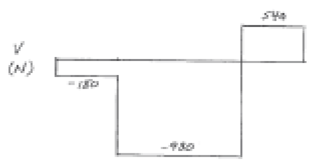
$\circlearrowleft \Sigma M_A = 0$: $(0.9\text{ m})D_y - 2(225\text{ N}\cdot\text{m}) - (0.3\text{ m})(750\text{N})$

$-(0.9\text{ m})(750\text{ N}) - (1.2\text{ m})(540\text{ N}) = 0$

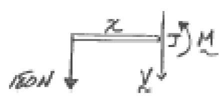
$D_y = 2220\text{ N}$

$\uparrow \Sigma F_y = 0$: $A_y - 2(750\text{ N}) - 540\text{ N} + 2220\text{ N} = 0$

$A_y = -180\text{ N} \qquad \mathbf{A}_y = 180\text{ N}\downarrow$



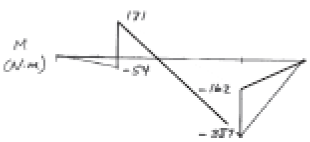
Along AC:



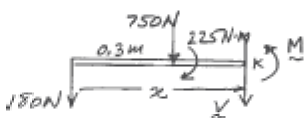
$\uparrow \Sigma F_y = 0$: $-180\text{ N} - V = 0$

$V = -180\text{ N}$

$\circlearrowleft \Sigma M_J = 0$: $M + x(180\text{ N}) = 0 \qquad M = -(180\text{ N})x$



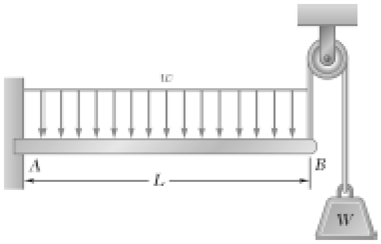
Along CD:



$\uparrow \Sigma F_y = 0$: $-180\text{ N} - 750\text{ N} - V = 0$, $\quad V = -930\text{ N}$

$\circlearrowleft \Sigma M_K = 0$: $M - 225\text{ N}\cdot\text{m} + (x - 0.3\text{ m})(750\text{ N}) + x(180\text{ N}) = 0$

$M = 450\text{ N}\cdot\text{m} - (930\text{ N})x$

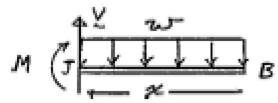


PROBLEM 7.62*

In order to reduce the bending moment in the cantilever beam AB, a cable and counterweight are permanently attached at end B. Determine the magnitude of the counterweight for which the maximum absolute value of the bending moment in the beam is as small as possible and the corresponding value of $|M|_{max}$. Consider (*a*) the case when the distributed load is permanently applied to the beam, (*b*) the more general case when the distributed load may either be applied or removed.

SOLUTION

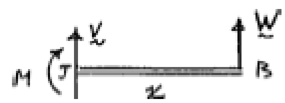
M due to distributed load:



$$\circlearrowleft \Sigma M_J = 0: \quad -M - \frac{x}{2}wx = 0$$

$$M = -\frac{1}{2}wx^2$$

M due to counter weight:



$$\circlearrowleft \Sigma M_J = 0: \quad -M + xw = 0$$

$$M = wx$$

(*a*) **Both applied:**

$$M = W_x - \frac{w}{2}x^2$$

$$\frac{dM}{dx} = W - wx = 0 \text{ at } x = \frac{W}{w}$$

And here $M = \dfrac{W^2}{2w} > 0$ so M_{max}; M_{min} must be at $x = L$

So $M_{min} = WL - \dfrac{1}{2}wL^2$. For minimum $|M|_{max}$ set $M_{max} = -M_{min}$,

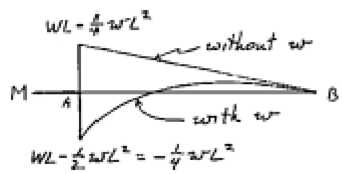
so $$\frac{W^2}{2w} = -WL + \frac{1}{2}wL^2 \quad \text{or} \quad W^2 + 2wLW - w^2L^2 = 0$$

$$W = -wL \pm \sqrt{2w^2L^2} \text{ (need+)} \qquad W = (\sqrt{2} - 1)wL = 0.414\, wL \blacktriangleleft$$

PROBLEM 7.62* (Continued)

(*b*) **w may be removed**

$$M_{max} = \frac{W^2}{2w} = \frac{(\sqrt{2}-1)^2}{2}wL^2 \qquad M_{max} = 0.0858\,wL^2 \blacktriangleleft$$



Without w, $\quad M = Wx$

$$M_{max} = WL \text{ at } A$$

With w (see Part *a*) $\quad M = Wx - \frac{w}{2}x^2$

$$M_{max} = \frac{W^2}{2w} \text{ at } x = \frac{W}{w}$$

$$M_{min} = WL - \frac{1}{2}wL^2 \text{ at } x = L$$

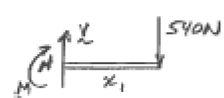
For minimum M_{max}, set $M_{max}(\text{no } w) = -M_{min}(\text{with } w)$

$$WL = -WL + \frac{1}{2}wL^2 \rightarrow W = \frac{1}{4}wL \rightarrow \qquad M_{max} = \frac{1}{4}wL^2 \blacktriangleleft$$

With $\qquad W = \frac{1}{4}wL \blacktriangleleft$

PROBLEM 7.52 CONTINUED

Along DB:



$\uparrow \Sigma F_y = 0: \quad V - 540 \text{ N} = 0$

$$V = 540 \text{ N}$$

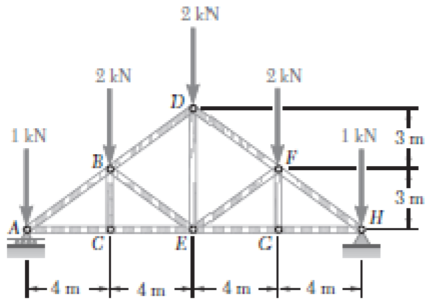
$\circlearrowleft \Sigma M_N = 0: \quad M + x_1(540 \text{ N}) = 0 \qquad M = -(540 \text{ N})x_1$

Note: The discontinuities in M, at C and D, equal 225 N·m, M_C and M_D

From the diagrams $\qquad |V|_{max} = 930$ N along $CD \blacktriangleleft$

$$|M|_{max} = 387 \text{ N·m at } D \blacktriangleleft$$

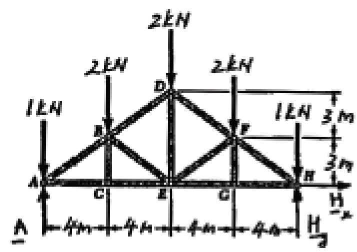
PROBLEM 6.10



Determine the force in each member of the Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



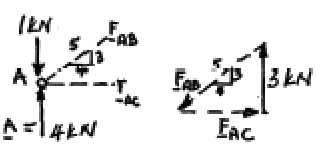
$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

Because of the symmetry of the truss and loading:

$$A = H_y = \frac{1}{2} \text{ Total load}$$

$$\mathbf{A} = \mathbf{H}_y = 4 \text{ kN}\uparrow$$

Free body: Joint *A*:



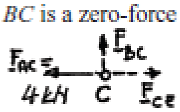
$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{3 \text{ kN}}{3}$$

$\mathbf{F}_{AB} = 5 \text{ kN } C$ ◀

$\mathbf{F}_{AC} = 4 \text{ kN } T$ ◀

Free body: Joint *C*:

BC is a zero-force member

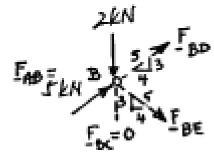


$\mathbf{F}_{BC} = 0$

$\mathbf{F}_{CE} = 4 \text{ kN } T$ ◀

PROBLEM 6.10 (Continued)

Free body: Joint *B*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{BD} + \frac{4}{5}F_{BE} + \frac{4}{5}(5\text{ kN}) = 0$$

or $$F_{BD} + F_{BE} = -5\text{ kN} \qquad (1)$$

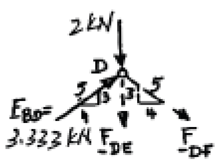
$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - \frac{3}{5}F_{BE} + \frac{3}{5}(5\text{ kN}) - 2\text{ kN} = 0$$

or $$F_{BD} - F_{BE} = -1.667\text{ kN} \qquad (2)$$

Add Eqs. (1) and (2): $2F_{BD} = -6.667\text{ kN}$ $\qquad F_{BD} = 3.333\text{ kN}\quad C$ ◀

Subtract (2) from (1): $2F_{BE} = -3.333\text{ kN}$ $\qquad F_{BE} = 1.667\text{ kN}\quad C$ ◀

Free Body: Joint *D*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}(3.333\text{ kN}) + \frac{4}{5}F_{DF} = 0$$

$F_{DF} = -3.333\text{ kN}$ $\qquad F_{DF} = 3.333\text{ kN}\quad C$ ◀

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}(3.333\text{ kN}) - \frac{3}{5}(-3.333\text{ kN}) - 2\text{ kN} - F_{DE} = 0$$

$F_{DE} = +2\text{ kN}$ $\qquad F_{DE} = 2\text{ kN}\quad T$ ◀

Because of the symmetry of the truss and loading, we deduce that

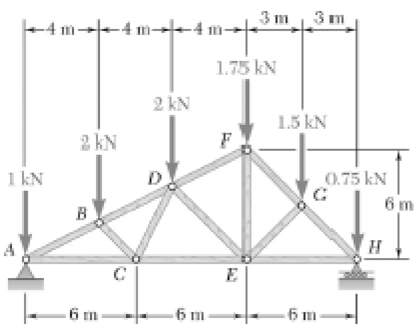
$F_{EF} = F_{BE}$ $\qquad F_{EF} = 1.667\text{ kN}\quad C$ ◀

$F_{EG} = F_{CE}$ $\qquad F_{EG} = 4\text{ kN}\quad T$ ◀

$F_{FG} = F_{BC}$ $\qquad F_{FG} = 0$ ◀

$F_{FH} = F_{AB}$ $\qquad F_{FH} = 5\text{ kN}\quad C$ ◀

$F_{GH} = F_{AC}$ $\qquad F_{GH} = 4\text{ kN}\quad T$ ◀

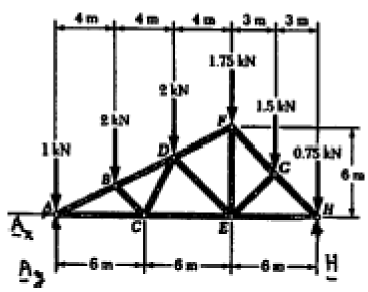


PROBLEM 6.13

Using the method of joints, determine the force in each member of the double-pitch roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



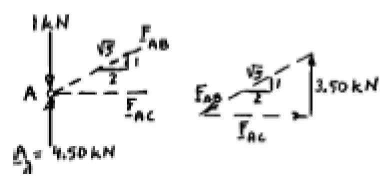
$$+\circlearrowleft \Sigma M_A = 0: \quad H(18\text{ m}) - (2\text{ kN})(4\text{ m}) - (2\text{ kN})(8\text{ m}) - (1.75\text{ kN})(12\text{ m})$$
$$- (1.5\text{ kN})(15\text{ m}) - (0.75\text{ kN})(18\text{ m}) = 0$$

$\mathbf{H} = 4.50\text{ kN}\uparrow$

$$\Sigma F_x = 0: \quad A_x = 0$$
$$\Sigma F_y = 0: \quad A_y + H - 9 = 0$$
$$A_y = 9 - 4.50$$

$\mathbf{A}_y = 4.50\text{ kN}\uparrow$

Free body: Joint A:



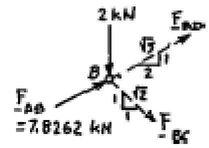
$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{3.50\text{ kN}}{1}$$

$$F_{AB} = 7.8262\text{ kN} \quad C$$

$F_{AB} = 7.83\text{ kN} \quad C$ ◀

$F_{AC} = 7.00\text{ kN} \quad T$ ◀

PROBLEM 6.13 (Continued)



Free body: Joint B:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}}(7.8262 \text{ kN}) + \frac{1}{\sqrt{2}} F_{BC} = 0$$

or $$F_{BD} + 0.79057 F_{BC} = -7.8262 \text{ kN} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{5}}(7.8262 \text{ kN}) - \frac{1}{\sqrt{2}} F_{BC} - 2 \text{ kN} = 0$$

or $$F_{BD} - 1.58114 F_{BC} = -3.3541 \qquad (2)$$

Multiply Eq. (1) by 2 and add Eq. (2):

$$3F_{BD} = -19.0065$$

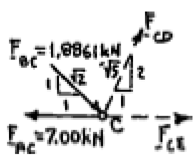
$$F_{BD} = -6.3355 \text{ kN} \qquad F_{BD} = 6.34 \text{ kN} \quad C \blacktriangleleft$$

Subtract Eq. (2) from Eq. (1):

$$2.37111 F_{BC} = -4.4721$$

$$F_{BC} = -1.8861 \text{ kN} \qquad F_{BC} = 1.886 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint C:

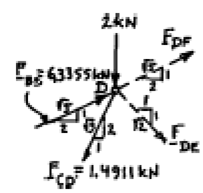


$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{5}} F_{CD} - \frac{1}{\sqrt{2}}(1.8861 \text{ kN}) = 0$$

$$F_{CD} = +1.4911 \text{ kN} \qquad F_{CD} = 1.491 \text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} - 7.00 \text{ kN} + \frac{1}{\sqrt{2}}(1.8861 \text{ kN}) + \frac{1}{\sqrt{5}}(1.4911 \text{ kN}) = 0$$

$$F_{CE} = +5.000 \text{ kN} \qquad F_{CE} = 5.00 \text{ kN} \quad T \blacktriangleleft$$



Free body: Joint D:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{DF} + \frac{1}{\sqrt{2}} F_{DE} + \frac{2}{\sqrt{5}}(6.3355 \text{ kN}) - \frac{1}{\sqrt{5}}(1.4911 \text{ kN}) = 0$$

or $$F_{DF} + 0.79057 F_{DE} = -5.5900 \text{ kN} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{2}} F_{DE} + \frac{1}{\sqrt{5}}(6.3355 \text{ kN}) - \frac{2}{\sqrt{5}}(1.4911 \text{ kN}) - 2 \text{ kN} = 0$$

or $$F_{DF} - 0.79057 F_{DE} = -1.1188 \text{ kN} \qquad (2)$$

Add Eqs. (1) and (2): $$2F_{DF} = -6.7088 \text{ kN}$$

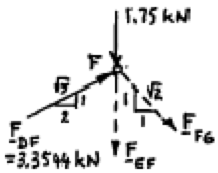
$$F_{DF} = -3.3544 \text{ kN} \qquad F_{DF} = 3.35 \text{ kN} \quad C \blacktriangleleft$$

Subtract Eq. (2) from Eq. (1): $$1.58114 F_{DE} = -4.4712 \text{ kN}$$

$$F_{DE} = -2.8278 \text{ kN} \qquad F_{DE} = 2.83 \text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.13 (Continued)

Free body: Joint F:



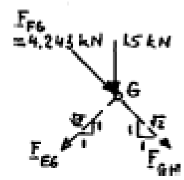
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{2}} F_{FG} + \frac{2}{\sqrt{5}}(3.3544 \text{ kN}) = 0$$

$$F_{FG} = -4.243 \text{ kN} \qquad F_{FG} = 4.24 \text{ kN} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{EF} - 1.75 \text{ kN} + \frac{1}{\sqrt{5}}(3.3544 \text{ kN}) - \frac{1}{\sqrt{2}}(-4.243 \text{ kN}) = 0$$

$$F_{EF} = 2.750 \text{ kN} \qquad F_{EF} = 2.75 \text{ kN} \quad T \blacktriangleleft$$

Free body: Joint G:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} + \frac{1}{\sqrt{2}}(4.243 \text{ kN}) = 0$$

or

$$F_{GH} - F_{EG} = -4.243 \text{ kN} \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} - \frac{1}{\sqrt{2}}(4.243 \text{ kN}) - 1.5 \text{ kN} = 0$$

or

$$F_{GH} + F_{EG} = -6.364 \text{ kN} \tag{2}$$

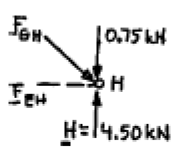
Add Eqs. (1) and (2): $\quad 2F_{GH} = -10.607$

$$F_{GH} = -5.303 \qquad F_{GH} = 5.30 \text{ kN} \quad C \blacktriangleleft$$

Subtract Eq. (1) from Eq. (2): $\quad 2F_{EG} = -2.121 \text{ kN}$

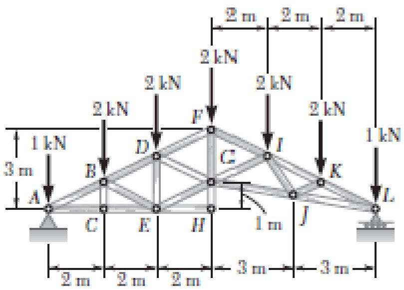
$$F_{EG} = -1.0605 \text{ kN} \qquad F_{EG} = 1.061 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint H:



$$\frac{F_{EH}}{1} = \frac{3.75 \text{ kN}}{1} \qquad F_{EH} = 3.75 \text{ kN} \quad T \blacktriangleleft$$

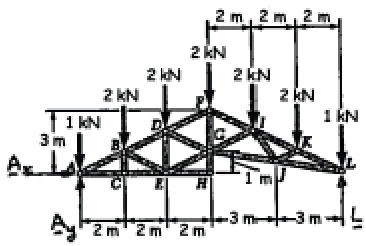
We can also write $\quad \dfrac{F_{GH}}{\sqrt{2}} = \dfrac{3.75 \text{ kN}}{1} \qquad F_{GH} = 5.30 \text{ kN} \quad C \quad \text{(Checks)}$



PROBLEM 6.15

Determine the force in each of the members located to the left of line *FGH* for the studio roof truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss $\quad \Sigma F_x = 0: \quad \mathbf{A}_x = 0$

Because of symmetry of loading:

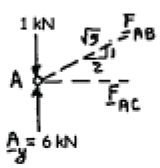
$$A_y = L = \frac{1}{2} \text{ Total load}$$

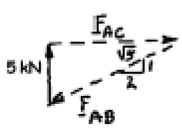
$$\mathbf{A}_y = \mathbf{L} = 6 \text{ kN} \uparrow$$

Zero-Force Members. Examining joints C and H, we conclude that BC, EH, and GH are zero-force members. Thus

$$\mathbf{F}_{BC} = \mathbf{F}_{EH} = 0 \quad ◀$$

Also, $\quad F_{CE} = F_{AC} \quad (1)$





Free body: Joint A $\quad \dfrac{F_{AB}}{\sqrt{5}} = \dfrac{F_{AC}}{2} = \dfrac{5 \text{ kN}}{1}$

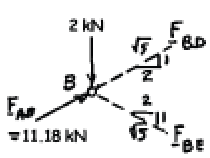
$$F_{AB} = 11.18 \text{ kN} \quad C$$

$$F_{AB} = 11.2 \text{ kN} \quad C \quad ◀$$

$$F_{AC} = 10 \text{ kN} \quad T \quad ◀$$

From Eq. (1): $\quad F_{CE} = 10 \text{ kN} \quad T \quad ◀$

Free body: Joint B



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}} F_{BE} + \frac{2}{\sqrt{5}}(11.18 \text{ kN}) = 0$$

or $\quad F_{BD} + F_{BE} = -11.18 \text{ kN} \quad (2)$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{5}} F_{BE} + \frac{1}{\sqrt{5}}(11.18 \text{ kN}) - 2 \text{ kN} = 0$$

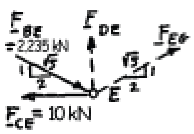
or $\quad F_{BD} - F_{BE} = -6.71 \text{ kN} \quad (3)$

PROBLEM 6.15 (Continued)

Add (2) and (3): $2F_{BD} = -17.89\text{ kN}$ $\qquad F_{BD} = 8.95\text{ kN}\ C$ ◀

Subtract (3) from (2): $2F_{BE} = -4.47\text{ kN}$ $\qquad F_{BE} = 2.24\text{ kN}\ C$ ◀

<u>Free body: Joint *E*</u>



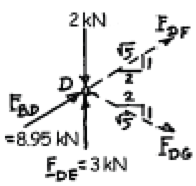
$$\xrightarrow{+} \Sigma F_x = 0:\quad \frac{2}{\sqrt{5}}F_{EG} + \frac{2}{\sqrt{5}}(2.235\text{ kN}) - 10\text{ kN} = 0$$

$F_{EG} = 8.95\text{ kN}\ T$ ◀

$$+\!\uparrow \Sigma F_y = 0:\quad F_{DE} + \frac{1}{\sqrt{5}}(8.95\text{ kN}) - \frac{1}{\sqrt{5}}(2.235\text{ kN}) = 0$$

$F_{DE} = -3\text{ kN}$ $\qquad F_{DE} = 3\text{ kN}\quad C$ ◀

<u>Free body: Joint *D*</u>



$$\xrightarrow{+} \Sigma F_x = 0:\quad \frac{2}{\sqrt{5}}F_{DF} + \frac{2}{\sqrt{5}}F_{DG} + \frac{2}{\sqrt{5}}(8.95\text{ kN}) = 0$$

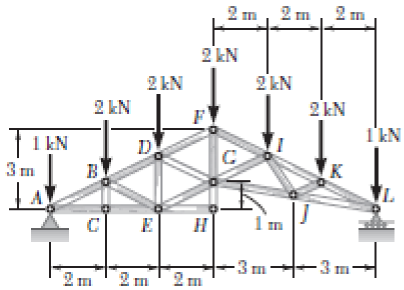
or $\qquad F_{DF} + F_{DG} = -8.95\text{ kN} \qquad (4)$

$$+\!\uparrow \Sigma F_y = 0:\quad \frac{1}{\sqrt{5}}F_{DF} - \frac{1}{\sqrt{5}}F_{DG} + \frac{1}{\sqrt{5}}(8.95\text{ kN}) + 3\text{ kN} - 2\text{ kN} = 0$$

or $\qquad F_{DF} - F_{DG} = -11.18\text{ kN} \qquad (5)$

Add (4) and (5): $2F_{DF} = -20.13\text{ kN}$ $\qquad F_{DF} = 10.07\text{ kN}\ C$ ◀

Subtract (5) from (4): $2F_{DG} = 2.23\text{ kN}$ $\qquad F_{DG} = 1.12\text{ kN}\ T$ ◀



PROBLEM 6.16

Determine the force in member FG and in each of the members located to the right of FG for the studio roof truss shown. State whether each member is in tension or compression.

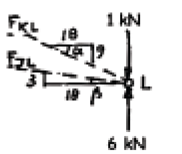
SOLUTION

Reaction at L: Because of the symmetry of the loading,

$$L = \frac{1}{2}\text{Total load,} \quad \mathbf{L} = 6\text{ kN} \uparrow$$

(See free body diagram to the left for more details)

Free body: Joint L



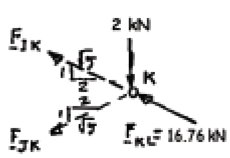
$$\alpha = \tan^{-1}\frac{3}{6} = 26.57°$$

$$\beta = \tan^{-1}\frac{1}{6} = 9.46°$$

$$\frac{F_{JL}}{\sin 63.43°} = \frac{F_{KL}}{\sin 99.46°} = \frac{5\text{ kN}}{\sin 17.11°} \qquad F_{JL} = 15.2\text{ kN} \quad T ◀$$

$$F_{KL} = 16.76\text{ kN} \quad C \qquad F_{KL} = 16.76\text{ kN} \quad C ◀$$

Free body: Joint K



$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{5}}F_{IK} - \frac{2}{\sqrt{5}}F_{JK} - \frac{2}{\sqrt{5}}(16.76\text{ kN}) = 0$$

or: $$F_{IK} + F_{JK} = -16.76\text{ kN} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{IK} - \frac{1}{\sqrt{5}}F_{JK} + \frac{1}{\sqrt{5}}(16.76\text{ kN}) - 2\text{ kN} = 0$$

or: $$F_{IK} - F_{JK} = -12.29\text{ kN} \qquad (2)$$

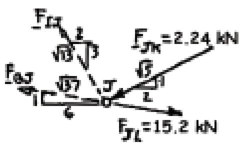
Add (1) and (2): $$2F_{IK} = -29.05$$

$$F_{IK} = -14.53\text{ kN} \qquad F_{IK} = 14.53\text{ kN} \quad C ◀$$

Subtract (2) from (1): $$2F_{JK} = -4.47$$

$$F_{JK} = -2.24\text{ kN} \qquad F_{JK} = 2.24\text{ kN} \quad C ◀$$

PROBLEM 6.16 (Continued)



Free body: Joint J

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{13}}F_{IJ} - \frac{6}{\sqrt{37}}F_{GJ} + \frac{6}{\sqrt{37}}(15.2) - \frac{2}{\sqrt{5}}(2.24) = 0 \qquad (3)$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{3}{\sqrt{13}}F_{IJ} + \frac{1}{\sqrt{37}}F_{GJ} - \frac{1}{\sqrt{37}}(15.2) - \frac{1}{\sqrt{5}}(2.24) = 0 \qquad (4)$$

Multiply (4) by 6 and add to (3):

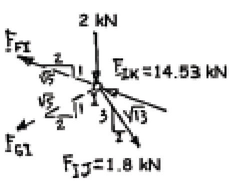
$$\frac{16}{\sqrt{13}}F_{IJ} - \frac{8}{\sqrt{5}}(2.24) = 0$$

$F_{IJ} = 1.8 \text{ kN}$ $\qquad$ $F_{IJ} = 1.8 \text{ kN } T$ ◀

Multiply (3) by 3, (4) by 2, and add:

$$-\frac{16}{\sqrt{37}}(F_{GJ} - 15.2) - \frac{8}{\sqrt{5}}(2.24) = 0$$

$F_{GJ} = 12.15 \text{ kN}$ $\qquad$ $F_{GJ} = 12.15 \text{ kN } T$ ◀



Free body: Joint I

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{5}}F_{FI} - \frac{2}{\sqrt{5}}F_{GI} - \frac{2}{\sqrt{5}}(14.53 \text{ kN}) + \frac{2}{\sqrt{13}}(1.8 \text{ kN}) = 0$$

or $\qquad F_{FI} + F_{GI} = -13.4 \text{ kN} \qquad (5)$

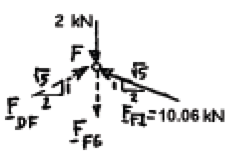
$$+\!\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{FI} - \frac{1}{\sqrt{5}}F_{GI} + \frac{1}{\sqrt{5}}(14.53) - \frac{3}{\sqrt{13}}(1.8 \text{ kN}) - 2 \text{ kN} = 0$$

or $\qquad F_{FI} - F_{GI} = -6.71 \text{ kN} \qquad (6)$

Add (5) and (6): $\qquad 2F_{FI} = -20.11$

$F_{FI} = -10.06 \text{ kN}$ $\qquad$ $F_{FI} = 10.06 \text{ kN } C$ ◀

Subtract (6) from (5): $\qquad 2F_{GI} = -6.69 \text{ kN}$ $\qquad$ $F_{GI} = 3.35 \text{ kN } C$ ◀

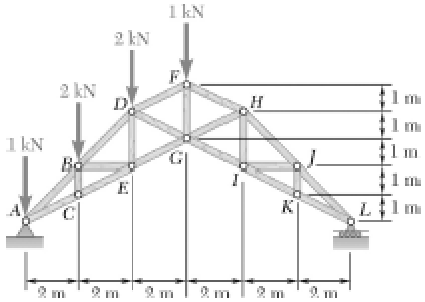


Free body: Joint F

From $\qquad \Sigma F_x = 0: \quad F_{DF} = F_{FI} = 10.06 \text{ kN } C$

$$+\!\uparrow \Sigma F_y = 0: \; -F_{FG} - 2 \text{ kN} + 2\left(\frac{1}{\sqrt{5}}10.06\right) = 0$$

$F_{FG} = +7.0 \text{ kN}$ $\qquad$ $F_{FG} = 7.0 \text{ kN } T$ ◀

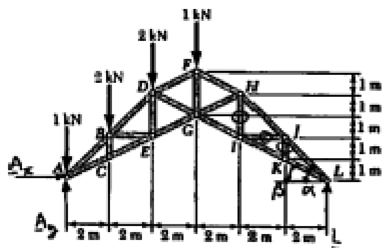


PROBLEM 6.18

Determine the force in member FG and in each of the members located to the right of FG for the scissors roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



$$+\circlearrowleft \Sigma M_A = 0: \quad L(12 \text{ m}) - (2 \text{ kN})(2 \text{ m}) - (2 \text{ kN})(4 \text{ m}) - (1 \text{ kN})(6 \text{ m}) = 0$$

$$\mathbf{L} = 1.500 \text{ kN} \uparrow$$

Angles:

$$\tan\alpha = 1 \qquad \alpha = 45^\circ$$

$$\tan\beta = \frac{1}{2} \qquad \beta = 26.57^\circ$$

Zero-force members:

Examining successively joints K, J, and I, we note that the following members to the right of FG are zero-force members: JK, IJ, and HI.

Thus, $F_{HI} = F_{IJ} = F_{JK} = 0$ ◀

We also note that

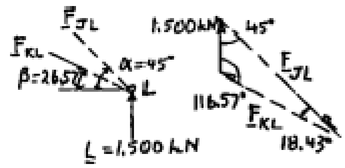
$$F_{GI} = F_{IK} = F_{KL} \tag{1}$$

and

$$F_{HJ} = F_{JL} \tag{2}$$

PROBLEM 6.18 (Continued)

Free body: Joint L:



$$\frac{F_{JL}}{\sin 116.57^\circ} = \frac{F_{KL}}{\sin 45^\circ} = \frac{1.500 \text{ kN}}{\sin 18.43^\circ}$$

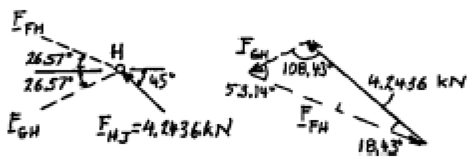
$F_{JL} = 4.2436$ kN $\qquad F_{JL} = 4.24$ kN C ◀

$F_{KL} = 3.35$ kN T ◀

From Eq. (1): $F_{GI} = F_{IK} = F_{KL}$ $\qquad F_{GI} = F_{IK} = 3.35$ kN T ◀

From Eq. (2): $F_{HJ} = F_{JL} = 4.2436$ kN $\qquad F_{HJ} = 4.24$ kN C ◀

Free body: Joint H:

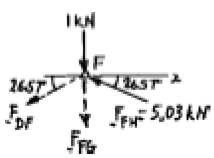


$$\frac{F_{FH}}{\sin 108.43^\circ} = \frac{F_{GH}}{\sin 18.43^\circ} = \frac{4.2436}{\sin 53.14^\circ}$$

$F_{FH} = 5.03$ kN C ◀

$F_{GH} = 1.677$ kN T ◀

Free body: Joint F:

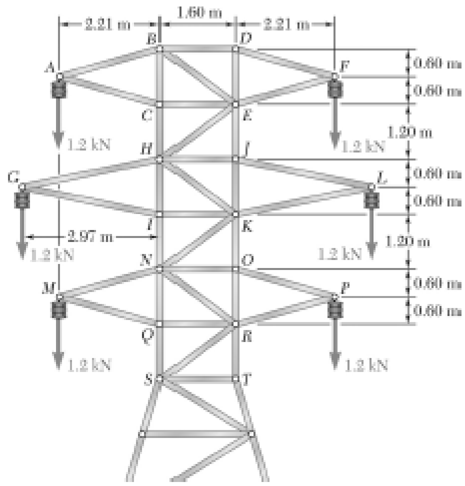


$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{DF} \cos 26.57^\circ - (5.03 \text{ kN}) \cos 26.57^\circ = 0$$

$$F_{DF} = -5.03 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{FG} - 1 \text{ kN} + (5.03 \text{ kN}) \sin 26.57^\circ - (-5.03 \text{ kN}) \sin 26.57^\circ = 0$$

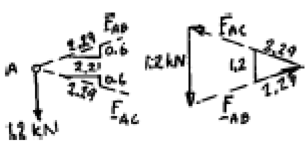
$F_{FG} = 3.500$ kN $\qquad F_{FG} = 3.50$ kN T ◀



PROBLEM 6.23

The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

SOLUTION

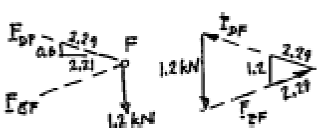


Free body: Joint *A*:

$$\frac{F_{AB}}{2.29} = \frac{F_{AC}}{2.29} = \frac{1.2 \text{ kN}}{1.2}$$

$F_{AB} = 2.29 \text{ kN} \quad T$ ◀

$F_{AC} = 2.29 \text{ kN} \quad C$ ◀

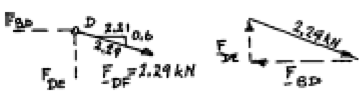


Free body: Joint *F*:

$$\frac{F_{DF}}{2.29} = \frac{F_{EF}}{2.29} = \frac{1.2 \text{ kN}}{2.1}$$

$F_{DF} = 2.29 \text{ kN} \quad T$ ◀

$F_{EF} = 2.29 \text{ kN} \quad C$ ◀



Free body: Joint *D*:

$$\frac{F_{BD}}{2.21} = \frac{F_{DE}}{0.6} = \frac{2.29 \text{ kN}}{2.29}$$

$F_{BD} = 2.21 \text{ kN} \quad T$ ◀

$F_{DE} = 0.600 \text{ kN} \quad C$ ◀

Free body: Joint *B*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5} F_{BE} + 2.21 \text{ kN} - \frac{2.21}{2.29}(2.29 \text{ kN}) = 0$$

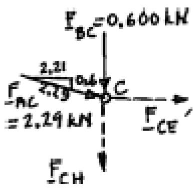
$F_{BE} = 0$ ◀

$$+\uparrow \Sigma F_y = 0: \quad -F_{BC} - \frac{3}{5}(0) - \frac{0.6}{2.29}(2.29 \text{ kN}) = 0$$

$F_{BC} = -0.600 \text{ kN}$ $\qquad F_{BC} = 0.600 \text{ kN} \quad C$ ◀

PROBLEM 6.23 (Continued)

Free body: Joint C:



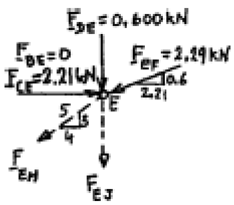
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} + \frac{2.21}{2.29}(2.29 \text{ kN}) = 0$$

$F_{CE} = -2.21 \text{ kN}$ $\qquad$ $F_{CE} = 2.21 \text{ kN} \quad C$ ◀

$$+\uparrow \Sigma F_y = 0: \quad -F_{CH} - 0.600 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) = 0$$

$F_{CH} = -1.200 \text{ kN}$ $\qquad$ $F_{CH} = 1.200 \text{ kN} \quad C$ ◀

Free body: Joint E:

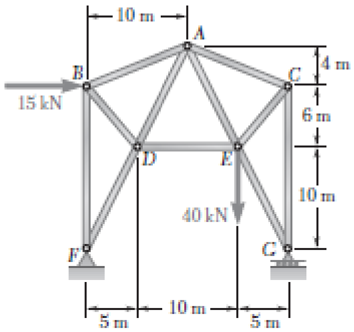


$$\xrightarrow{+} \Sigma F_x = 0: \quad 2.21 \text{ kN} - \frac{2.21}{2.29}(2.29 \text{ kN}) - \frac{4}{5}F_{EH} = 0$$

$F_{EH} = 0$ ◀

$$+\uparrow \Sigma F_y = 0: \quad -F_{EJ} - 0.600 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) - 0 = 0$$

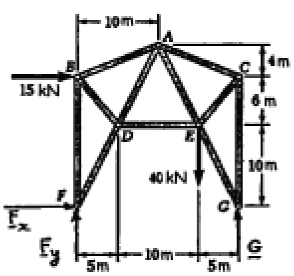
$F_{EJ} = -1.200 \text{ kN}$ $\qquad$ $F_{EJ} = 1.200 \text{ kN} \quad C$ ◀



PROBLEM 6.27

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss

$$+\circlearrowleft \Sigma M_F = 0: \quad G(20\text{ m}) - (15\text{ kN})(16\text{ m}) - (40\text{ kN})(15\text{ m}) = 0$$

$$\mathbf{G} = 42\text{ kN} \uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_x + 15\text{ kN} = 0$$

$$\mathbf{F}_x = 15\text{ kN} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad F_y - 40\text{ kN} + 42\text{ kN} = 0$$

$$\mathbf{F}_y = 2\text{ kN} \downarrow$$

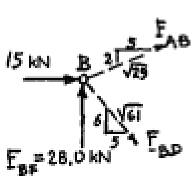
Free body: Joint F

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - 15\text{ kN} = 0$$

$$F_{DF} = 33.54\text{ kN} \qquad F_{DF} = 33.5\text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{BF} - 2\text{ kN} + \frac{2}{\sqrt{5}}(33.54\text{ kN}) = 0$$

$$F_{BF} = -28.00\text{ kN} \qquad F_{BF} = 28.0\text{ kN} \quad C \blacktriangleleft$$



Free body: Joint B

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{5}{\sqrt{29}} F_{AB} + \frac{5}{\sqrt{61}} F_{BD} + 15\text{ kN} = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{29}} F_{AB} - \frac{6}{\sqrt{61}} F_{BD} + 28\text{ kN} = 0 \qquad (2)$$

PROBLEM 6.27 (Continued)

Multiply (1) by 6, (2) by 5, and add:

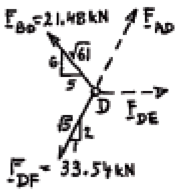
$$\frac{40}{\sqrt{29}}F_{AB} + 230 \text{ kN} = 0$$

$$F_{AB} = -30.96 \text{ kN} \qquad F_{AB} = 31.0 \text{ kN} \quad C \blacktriangleleft$$

Multiply (1) by 2, (2) by –5, and add:

$$\frac{40}{\sqrt{61}}F_{BD} - 110 \text{ kN} = 0$$

$$F_{BD} = 21.48 \text{ kN} \qquad F_{BD} = 21.5 \text{ kN} \quad T \blacktriangleleft$$



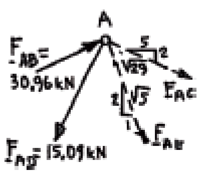
Free body: joint D

$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{5}}F_{AD} - \frac{2}{\sqrt{5}}(33.54) + \frac{6}{\sqrt{61}}(21.48) = 0$$

$$F_{AD} = 15.09 \text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{DE} + \frac{1}{\sqrt{5}}(15.09 - 33.54) - \frac{5}{\sqrt{61}}(21.48) = 0$$

$$F_{DE} = 22.0 \text{ kN} \quad T \blacktriangleleft$$



Free body: joint A

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{5}{\sqrt{29}}F_{AC} + \frac{1}{\sqrt{5}}F_{AE} + \frac{5}{\sqrt{29}}(30.96) - \frac{1}{\sqrt{5}}(15.09) = 0 \qquad (3)$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{2}{\sqrt{29}}F_{AC} - \frac{2}{\sqrt{5}}F_{AE} + \frac{2}{\sqrt{29}}(30.96) - \frac{2}{\sqrt{5}}(15.09) = 0 \qquad (4)$$

Multiply (3) by 2 and add (4):

$$\frac{8}{\sqrt{29}}F_{AC} + \frac{12}{\sqrt{29}}(30.96) - \frac{4}{\sqrt{5}}(15.09) = 0$$

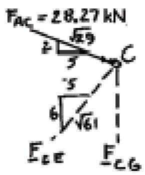
$$F_{AC} = -28.27 \text{ kN}, \qquad F_{AC} = 28.3 \text{ kN} \quad C \blacktriangleleft$$

Multiply (3) by 2, (4) by 5 and add:

$$-\frac{8}{\sqrt{5}}F_{AE} + \frac{20}{\sqrt{29}}(30.96) - \frac{12}{\sqrt{5}}(15.09) = 0$$

$$F_{AE} = 9.50 \text{ kN} \quad T \blacktriangleleft$$

PROBLEM 6.27 (Continued)



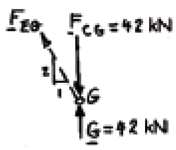
Free body: Joint C

From force triangle

$$\frac{F_{CE}}{\sqrt{61}} = \frac{F_{CG}}{8} = \frac{28.27 \text{ kN}}{\sqrt{29}}$$

$F_{CE} = 41.0 \text{ kN} \quad T$ ◀

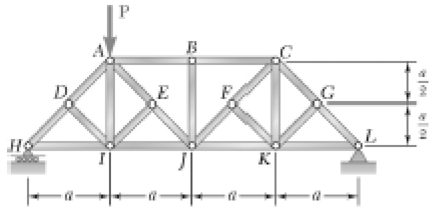
$F_{CG} = 42.0 \text{ kN} \quad C$ ◀



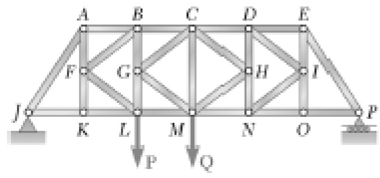
Free body: Joint G

$\xrightarrow{+} \Sigma F_x = 0:$ $\quad F_{EG} = 0$ ◀

$+\uparrow \Sigma F_y = 0: \quad 42 \text{ kN} - 42 \text{ kN} = 0$ (Checks)



(a)

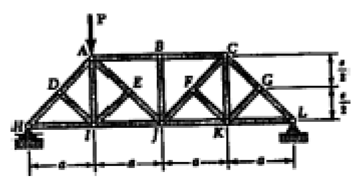


(b)

PROBLEM 6.31

For the given loading, determine the zero-force members in each of the two trusses shown.

SOLUTION



(a)

Truss (a):

FB: Joint B: $F_{BJ} = 0$

FB: Joint D: $F_{DI} = 0$

FB: Joint E: $F_{EI} = 0$

FB: Joint I: $F_{AI} = 0$

FB: Joint F: $F_{FK} = 0$

FB: Joint G: $F_{GK} = 0$

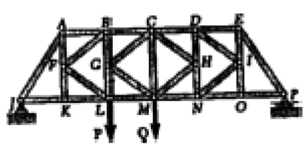
FB: Joint K: $F_{CK} = 0$

The zero-force members, therefore, are $AI, BJ, CK, DI, EI, FK, GK$ ◀

Truss (b):

FB: Joint K: $F_{FK} = 0$

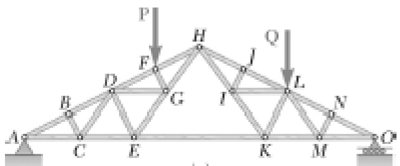
FB: Joint O: $F_{IO} = 0$

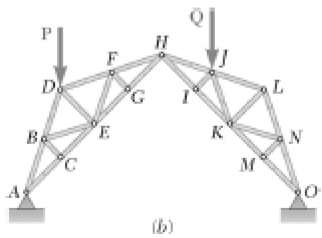


(b)

The zero-force members, therefore, are FK and IO ◀

All other members are either in tension or compression.





PROBLEM 6.32

For the given loading, determine the zero-force members in each of the two trusses shown.

SOLUTION

Truss (a):

FB: Joint B: $F_{BC} = 0$

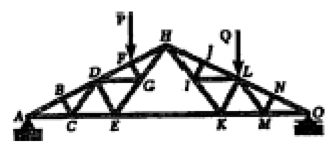
FB: Joint C: $F_{CD} = 0$

FB: Joint J: $F_{IJ} = 0$

FB: Joint I: $F_{IL} = 0$

FB: Joint N: $F_{MN} = 0$

FB: Joint M: $F_{LM} = 0$



(a)

The zero-force members, therefore, are BC, CD, IJ, IL, LM, MN ◀

Truss (b):

FB: Joint C: $F_{BC} = 0$

FB: Joint B: $F_{BE} = 0$

FB: Joint G: $F_{FG} = 0$

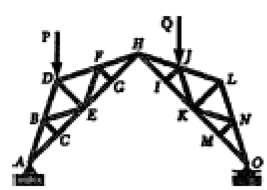
FB: Joint F: $F_{EF} = 0$

FB: Joint E: $F_{DE} = 0$

FB: Joint I: $F_{IJ} = 0$

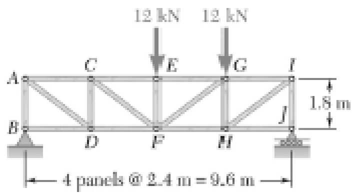
FB: Joint M: $F_{MN} = 0$

FB: Joint N: $F_{KN} = 0$



(b)

The zero-force members, therefore, are $BC, BE, DE, EF, FG, IJ, KN, MN$ ◀

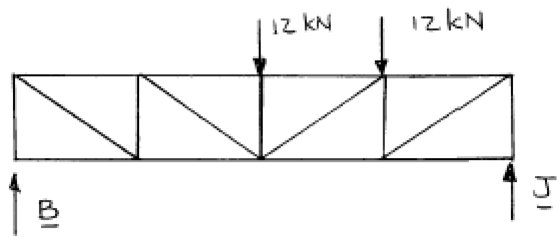


PROBLEM 6.43

Determine the force in members CD and DF of the truss shown.

SOLUTION

Reactions:



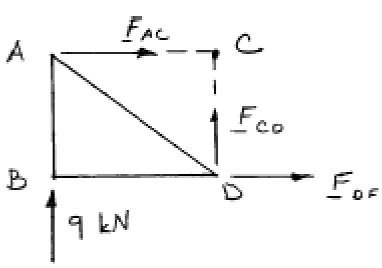
$$+\circlearrowleft \Sigma M_J = 0: \quad (12 \text{ kN})(4.8 \text{ m}) + (12 \text{ kN})(2.4 \text{ m}) - B(9.6 \text{ m}) = 0$$

$$B = 9.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad 9.00 \text{ kN} - 12.00 \text{ kN} - 12.00 \text{ kN} + J = 0$$

$$J = 15.00 \text{ kN}$$

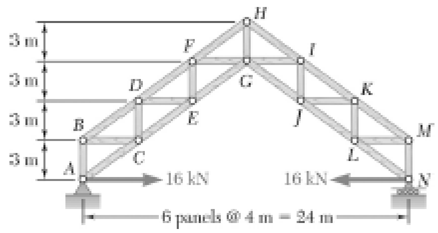
Member CD:



$$+\uparrow \Sigma F_y = 0: \quad 9.00 \text{ kN} + F_{CD} = 0 \qquad F_{CD} = 9.00 \text{ kN} \quad C \blacktriangleleft$$

Member DF:

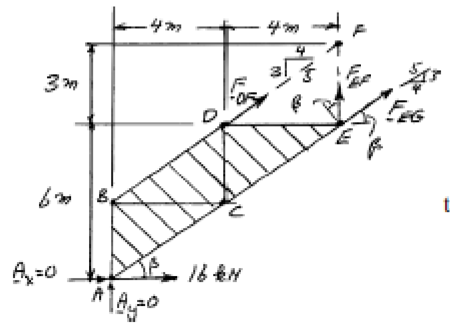
$$+\circlearrowleft \Sigma M_C = 0: \quad F_{DF}(1.8 \text{ m}) - (9.00 \text{ kN})(2.4 \text{ m}) = 0 \qquad F_{DF} = 12.00 \text{ kN} \quad T \blacktriangleleft$$



PROBLEM 6.47

Determine the force in members DF, EF, and EG of the truss shown.

SOLUTION



$$\tan \beta = \frac{3}{4}$$

Reactions:

$$\mathbf{A} = \mathbf{N} = 0$$

Member DF: $+\circlearrowleft \Sigma M_E = 0: \quad +(16\text{ kN})(6\text{ m}) - \frac{3}{5}F_{DF}(4\text{ m}) = 0$

$F_{DF} = +40\text{ kN}$ $\qquad F_{DF} = 40.0\text{ kN}\quad T$ ◀

Member EF: $+\searrow \Sigma F = 0: \quad (16\text{ kN})\sin\beta - F_{EF}\cos\beta = 0$

$F_{EF} = 16\tan\beta = 16(0.75) = 12\text{ kN}$ $\qquad F_{EF} = 12.00\text{ kN}\quad T$ ◀

Member EG: $+\circlearrowleft \Sigma M_F = 0: \quad (16\text{ kN})(9\text{ m}) + \frac{4}{5}F_{EG}(3\text{ m}) = 0$

$F_{EG} = -60\text{ kN}$ $\qquad F_{EG} = 60.0\text{ kN}\quad C$ ◀