Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2012-13
End Semester Examination

Maximum Marks: 50 | Total Time: 3 Hours

Note: This question paper has **TWO** pages and contains **Twelve** questions. Answer all questions.

1. Verify that the function $F_X(x) = 0,\ x < 0; \quad = \frac{3x+2}{12},\ 0 \leq x < 2; \quad = 1,\ x \geq 2;$ is the cumulative distribution function of the random variable X. Determine the probability distribution of X. Calculate the expected values of X and e^{2X}. **[0.5+0.5+1+1]**

2. The lifetime X (a nonnegative random variable) of a machine is modeled using a distribution and it is known that $P(X > x) = \frac{e^{-\frac{x}{4}}(4+x)}{4}, x \geq 0$. Given that machine has worked for more than four years, find the probability that it will work less than ten years. Also find the moment generating function of X. [1+2+2]

3. A couple with three girls has decided to keep having children until they have exactly two boys. The probability of a male birth is 0.52. Determine the probability that the couple will have at least three more girls before completing their families. Find the expected size of such a family. **[1.5 + 1.5]**

4. A manufacturer is interested in the performance of the light bulbs it produces. Light bulbs fail when they are switched on and not while they are functioning. Suppose that the probability of failure each time the light bulb is switched on is p, $0 < p < 1$ and remains unchanged for any given trial. Write the probability mass function of the random variable denoting the number of times one needs to switch on a light bulb until it actually fails. The manufacturer is interested in the probability that the light bulb functions at least 20 times before it fails. For what values of p this probability is at least 0.8. Further use definition to determine the expected value of the given random variable. [2 + 2]

5. A company is trying to determine a model for lifetime X (in years) of a particular device. It is observed that X^3 has an exponential distribution with mean 4. Determine the probability that device fails within first five years of use. Also determine the 70th percentile of X. [2 + 1]

6. (*i*) Prove the inequality *mode* < *median* < *mean* for a lognormal $LN(\mu, \sigma^2)$ distribution. [3]
(*ii*) A random variable X, denoting the losses from large fires, is known to follow a $LN(\mu, \sigma^2)$ distribution. Suppose that average loss due to fire for buildings of a particular type is Rs. 20 million and the standard deviation is Rs. 5 million. Determine

the probability that a large fire results in a losses between Rs. 25 million to Rs. 35 million. [3]

7. Suppose that the two dimensional continuous random variable (X, Y) is jointly distributed as $f_{X,Y}(x, y) = 6(1 - x - y), \quad x + y \leq 1, \ x \geq 0, \ y \geq 0; \quad = 0, \ elsewhere.$ Determine the cumulative distribution function of Z given by $Z = X + Y$ and then find the corresponding probability density function. Also compute the marginal and conditional probability density functions (for both X and Y). [2+1+2+2]

8. Consider two independent random variables X and Y distributed as gamma $G(\alpha, 1)$ and $G(\beta, 1)$ respectively. Find the probability density function of the random variable U defined by $U = \frac{\beta X}{\alpha Y}$. [3]

9. The probability mass function for a two dimensional random variable (X_1, X_2) is given by $p_{X_1,X_2}(x_1, x_2) = \frac{x_1 x_2 + 1}{13}, \quad x_1 = 1, 2, \ x_2 = 1, 2.$ Check whether X_1 and X_2 are independently distributed. Find covariance between X_1 and X_2. [1+2]

10. An assembly comprises 100 section. The length of each section (in centimeters) is a random variable with mean 10 and variance 0.9. Further more, the section are independent. The technical specification for the total length of the assembly is $500cm \pm 20cm$. Determine the approximate probability using central limit theorem that assembly fails to meet the specification. [3]

11. Suppose that accidents in a certain city involving a particular type of buses follow a Poisson process with 12 accidents per month (month is of 30 days). Find the probability that there are exactly 3 accidents in the first 15 days of a randomly chosen month of 30 days. Further, it is given that exactly 4 accidents took place in the first 18 days. Find the probability that all 4 accidents took place in the last 8 days out of these 18 days. [2+3]

12. Consider height of men and women, measured in inches, from a certain population where X denotes the height of men and Y denotes the height of women. Furthermore it is known that (X, Y) is jointly distributed as bivariate normal with $E(X) = 70, E(Y) = 68, V(X) = 4, V(Y) = 2.25, \rho = 0.6$.
(i) Determine the probability that a randomly selected woman is taller than a randomly selected man. [2]
(ii) Determine the probability that the selected woman being taller than the third quartile of all the women heights, if selected man's height is at the third quartile of all the men heights. [3]