Mid Term Examination (16 Sep, 2014): MA201

Time: 2 Hrs $\hfill$ Maximum Marks: 30

Answer all **12** questions. Answers to all the subparts of a question must appear together. Give precise and brief answer. You are free to use any standard formula. Symbols/notations carry their usual meanings as per the Lecture's discussion.

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1. Show that $\sqrt{2}|z| \geq |Re(z| + |Img(z)|$, $\forall z \in \mathbb{C}$. [2]

2. Verify that the roots of the polynomial $z^2+(1+i)z+5i$ are not in the form of complex conjugate pair. Write one sufficient condition under which $\overline{z}$ is always a root of any polynomial $p(z)$ if z is a root of this polynomial. [2]

3. Find the harmonic conjugate of the function $u(x,y) = y^3 - 3x^2y$. Write the corresponding function $f(z)$ in terms of z. [2+1]

4. Show that $2\pi i$ is a period of $\sinh z$. Find one solution of $\cos z = 2$ (write answer in $x+iy$ form). Show that all values of $(i)^{-2i}$ are real. [1+1.5+1.5]

5. Without evaluating the integral, show that $|\oint_C (e^z - \bar{z})\, dz| \leq 60$, where C is the boundary of the triangle with vertices at the points 0, $3i$ and -4, oriented in the counterclockwise (positive) direction. [2]

6. Show that $f'(z)$ does not exist at any point if $f(z) = e^x e^{-iy}$. [1]

7. Let $f(z)$ be analytic and real-valued in a domain D. Then show that it must be a constant function throughout D. [1]

8. State the Cauchy integral and the Morera theorem. Also show that $\oint_{|z|=1} e^{1/z^2} dz = 0$. Verify if $f(z) = e^{1/z^2}$ entire? **[1+1.5+0.5]**

9. Determine the nature of all possible singular points for the following functions. In case of pole, find the corresponding order also. (i) $f(z) = \dfrac{1}{z(e^z-1)}$ (ii) $f(z) = cosec(\dfrac{\pi}{z})$. **[1.5+2.5]**

10. By finding the Laurent series expansion of $f(z) = \dfrac{5z-2}{z(z-1)}$ in the domain $0 < |z-1| < 1$, calculate the residue of $f(z)$ at $z=1$. Also, without finding the Laurent series, calculate the residue of $f(z) = \dfrac{z}{z^4+4}$ at $z = 1+i$. [2+1]

11. Evaluate $\oint_{|z|=2} \dfrac{z}{(9-z^2)(z+i)^2}\, dz$ (given contour is positively oriented). [2]

12. Evaluate $\oint_{|z|=3} \dfrac{z+1}{z^2-2z}\, dz$ (given contour is positively oriented). [3]