Formal Languages and Automata Theory (CS303)

Mid-Semester Examination
Indian Institute of Technology, Patna
September, 2018
Full marks- 60, Duration- 120 min

1. Give NFAs with the specified number of states recognizing each of the following languages. In all cases, the alphabet is {0, 1}. (3 X 2) + (3 X 3)

(a) The language {w | w ends with 00} with three states.
(b) The language {0} with two states.
(c) The language 0* with one state.
(d) The language 0*1*0*0 with three states.
(e) The language {w | w contains an even number of 0s, or exactly two 1s} with six states.
(f) The language {w | w contains the substring 0101, i.e., w = x0101y for some (possibly empty) x and y} with five state.

2. Answer all the questions: 3 X 5

(a) If R_1 and R_2 are regular languages, is $\{ w \mid w \in R_1 \text{ and } w^R \in R_2 \}$ a regular language? If so, prove it; if not, give a counterexample. (w^R denotes the reversal of the string w.)
(b) Suppose that L_1 and L_2 are two languages (over the same alphabet) given to you such that both L_1 and $L_1 L_2$ are regular. Prove or disprove: L_2 must be regular too.
(c) Prove that $\{ww^R \mid w \in \Sigma^*\}$ for $\Sigma = \{0, 1\}$ is not regular using pumping lemma.

3. Consider the context-free grammar G over {a, b}, with start symbol S, and with the following productions. (2+3+5)

$S \rightarrow aaB \mid Abb$
$A \rightarrow a \mid aA$
$B \rightarrow b \mid bB$

(a) What is *L(G)*?
(b) Prove that this CFG is ambiguous.
(c) Design an unambiguous context-free grammar for *L(G)*.

4. Give context-free grammars that generate the following languages. In all parts the alphabet Σ is {0, 1}. (3+3+4)

(a) {w | w contains at least three 1s}
(b) {w | w starts and ends with the same symbol}
(c) {w | the length of w is odd and its middle symbol is a 0}

5. Answer all the questions: (5+5)

(a) Show the intersection of a context-free language C with a regular language R is always context-free.
(b) Show that the language $\{0^n1^m0^n1^m \mid n \geq 0\}$ is not context-free using pumping lemma.