CS225 Switching Theory

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Previous Class

Minimization/ Simplification **of Switching Functions**

K-map (SOP)

This Class

Minimization/ Simplification **of Switching Functions**

Quine-McCluskey (Tabular) Minimization

Minimization using K-map

Minimal expression: covers all the 1 cells with the smallest number of cubes such that each cube is as large as possible

- A cube contained in a larger cube must never be selected
- If there is more than one way of covering the map with a minimal number of cubes, select the cover with larger cubes
- A cube contained in any combination of other cubes already selected in the cover is redundant by virtue of the consensus theorem

Rules for minimization:

1. First, cover those 1 cells by cubes that cannot be combined with other 1 cells; continue to 1 cells that have a single adjacent 1 cell (thus can form cubes of only two cells)
2. Next, combine 1 cells that yield cubes of four cells, but are not part of any cube of eight cells, and so on
3. Minimal expression: collection of cubes that are as large and as few in number as possible, such that each 1 cell is covered by at least one cube

Don't-care Combinations

Don't-care combination ϕ: combination for which the value of the function is not specified. Either

- input combinations may be invalid
- precise output value is of no importance

Since each don't-care can be specified as either 0 or 1, a function with k don't-cares corresponds to a class of 2^k distinct functions. Our aim is to choose the function with the minimal representation

- Assign 1 to some don't-cares and 0 to others in order to increase the size of the selected cubes whenever possible
- No cube containing only don't-care cells may be formed, since it is not required that the function equal 1 for these combinations

Some Definitions

- Implicant: *A* product term that has non-empty intersection with on-set *F* and does not intersect with off-set *R* .

- Prime Implicant: An implicant that is not covered by another implicant.

- Essential Prime Implicant: A prime implicant with at least one element that is not covered by one or more prime implicants

Deriving Prime Implicants and Minimal Expressions

Example: for $f(w,x,y,z) = \Sigma(0,4,5,7,8,9,13,15)$ below, set of prime implicants

$$P = \{xz,\ w'y'z',\ wx'y',\ x'y'z',\ w'xy',\ wy'z\}$$

yz \ wx	00	01	11	10
00	1	1		1
01		1	1	1
11		1	1	
10				

Essential prime implicants: covers at least one minterm not covered by any other prime implicant, e.g., *xz*

- Since all minterms must be covered, all essential prime implicants must be contained in any irredundant expression of the function

Minimal Expressions (Contd.)

Example: prime implicants of $f(w,x,y,z) = \Sigma(4,5,8,12,13,14,15)$ are all essential

yz \ wx	00	01	11	10
00		1	1	1
01		1	1	
11			1	
10			1	

Example: Cyclic prime implicant chart in which no prime implicant is essential, all prime implicants have the same size, and every 1 cell is covered by exactly two prime implicants

z \ xy	00	01	11	10
0	1	1		1
1		1	1	1

Procedure for Deriving Minimal Sum-of-products Expression

Procedure:

1. Obtain all essential prime implicants and include them in the minimal expression

2. Remove all prime implicants which are covered by the sum of some essential prime implicants

3. If the set of prime implicants derived so far covers all the minterms, it yields a unique minimal expression. Otherwise, select additional prime implicants so that the function is covered completely and the total number and size of the added prime implicants are minimal

Example: prime implicant xz is covered by the sum off our essential prime implicants, and hence xz must not be included in any irredundant expression of the function

yz \ wx	00	01	11	10
00			1	
01	1	1	1	
11		1	1	1
10		1		

Tabulation Procedure for Obtaining the Set of All Prime implicants

Systematic Quine-McCluskey tabulation procedure: for functions with a large number of variables

- Fundamental idea: repeated application of the combining theorem $Aa + Aa' = A$ on all adjacent pairs of terms yields the set of all prime implicants

Example: minimize $f_1(w,x,y,z) = \sum(0,1,8,9) = w'x'y'z' + w'x'y'z + wx'y'z' + wx'y'z$

- Combine first two and last two terms to yield

$$f_1(w,x,y,z) = w'x'y'(z' + z) + wx'y'(z' + z) = w'x'y' + wx'y'$$

- Combine this expression in turn to yield

$$f_1(w,x,y,z) = x'y'(w' + w) = x'y'$$

- Similar result can be obtained by initially combining the first and third and the second and fourth terms

Tabulation Procedure (Contd.)

Two k-variable terms can be combined into a single (k-1)-variable term if and only if they have k-1 identical literals in common and differ in only one literal

- *Using the binary representation of minterms: two minterms can be combined if their binary representations differ in only one position*

Example: $w'x'y'z$ (0001) and $wx'y'z$ (1001) can be combined into -001,

indicating w has been absorbed and the combined term is $x'y'z$

Example of Different Notations

$F(A, B, C, D) = \sum m(4,5,6,8,10,13)$

	Full variable	Cellular	1,0,-
1	$\overline{A}B\overline{C}\overline{D}$	4	0100
	$A\overline{B}\overline{C}\overline{D}$	8	1000
2	$\overline{A}B\overline{C}D$	5	0101
	$\overline{A}BC\overline{D}$	6	0110
	$A\overline{B}C\overline{D}$	10	1010
3	$AB\overline{C}D$	13	1101

Notation Forms

- Full variable form - variables and complements in algebraic form
  - hard to identify when adjacency applies
  - very easy to make mistakes
- Cellular form - terms are identified by their decimal index value
  - Easy to tell when adjacency applies; indexes must differ by power of two (one bit)
- 1,0,- form - terms are identified by their binary index value
  - Easier to translate to/from full variable form
  - Easy to identify when adjacency applies, one bit is different
  - shows variable(s) dropped when adjacency is used
- Different forms may be mixed during the minimization

Tabulation Procedure (Contd.)

Procedure:

1. *Arrange all minterms in groups, with all terms in the same group having the same number of 1's. Start with the least number of 1's (called the index) and continue with groups of increasing numbers of 1's.*

2. *Compare every term of the lowest-index group with each term in the successive group. Whenever possible, combine them using the combining theorem. Repeat by comparing each term in a group of index i with every term in the group of index i + 1. Place a check mark next to every term which has been combined with at least one term.*

3. *Compare the terms generated in step 2 in the same fashion: generate a new term by combining two terms that differ by only a single 1 and whose dashes are in the same position. Continue until no further combinations are possible. The remaining unchecked terms constitute the set of prime implicants.*

Example

Example: apply procedure to $f_2 \sum(w, x, y, z)$ = (0,1,2,5,7,8,9,10,13,15)

Step (i)

	w	x	y	z	
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
5	0	1	0	1	✓
9	1	0	0	1	✓
10	1	0	1	0	✓
7	0	1	1	1	✓
13	1	1	0	1	✓
15	1	1	1	1	✓

Step (ii)

	w	x	y	z	
0,1	0	0	0	--	✓
0,2	0	0	--	0	✓
0,8	--	0	0	0	✓
1,5	0	--	0	1	✓
1,9	--	0	0	1	✓
2,10	--	0	1	0	✓
8,9	1	0	0	--	✓
8,10	1	0	--	0	✓
5,7	0	1	--	1	✓
5,13	--	1	0	1	✓
9,13	1	--	0	1	✓
7,15	--	1	1	1	✓
13,15	1	1	--	1	✓

Step (iii)

	w	x	y	z	
0,1,8,9	--	0	0	--	A
0,2,8,10	--	0	--	0	B
1,5,9,13	--	--	0	1	C
5,7,13,15	--	1	--	1	D

P= {x′y′, x′z′, y′z, xz}

Find the K-map and match with this result