Indian Institute of Technology Patna
MA102: Mathematics II
End Semester Exam (28-4-2018)

Time: 3hrs | Max. Marks: 50

<u>Note:</u> There are total 9 questions. Answer all questions. Give precise and brief answer. Standard formulae may be used. **DO NOT WRITE ANYTHING ON THE QUESTION PAPER. Write your Roll number at the end. CALCULATOR or any other electronic gadget is NOT allowed.** Notations are standard and are same as used in class.

Que 1. Answer all parts of this question at one place.

(a.) The condition $M_y = N_x$ for the differential equation $Mdx + Ndy = 0$ to be exact is: Necessary/ Sufficient/ Necessary & Sufficient (choose correct answer). [1]

(b.) Solve $x \cos xy' + y(x \sin x + \cos x) = 1$. [1]

(c.) The wronskian of $y_1 = x^3$ and $y_2 = x^2|x|$ vanishes identically on interval $[-1, 1]$. Are y_1 and y_2 LD? Justify your answer. [1]

(d.) Solve $y' = \ln(x + y - 2)$. [1]

(e.) Is $x = 1$ a regular singular point of equation $(1-x)^2y'' + xy' + y = 0$. Justify your answer. [1]

(f.) Find the Laplace transform of $f(t) = \int_0^t e^{3x}dx$. [1]

(g.) True/False (Justify your answer): If system $Ax = b$ has infinitely many solutions, then columns of A are linearly dependent. [1]

(h.) True/False (Justify your answer): A linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$ that transform $\begin{bmatrix}1\\2\end{bmatrix}$ to $\begin{bmatrix}7\\3\end{bmatrix}$ and $\begin{bmatrix}3\\4\end{bmatrix}$ to $\begin{bmatrix}-1\\1\end{bmatrix}$ will also transform $\begin{bmatrix}5\\8\end{bmatrix}$ to $\begin{bmatrix}13\\7\end{bmatrix}$. [1]

(i.) True/False (Justify your answer): Assume that A and B are matrices such that AB is defined and AB has a column that has all its entries equal to zero. Then one of the columns of B also has all its entries equal to zero. [1]

(j.) Find a 3×3 permutation matrix P such that for any $3 \times n$ matrix A the matrix PA is A with its last two rows exchanged. [1]

Que 2. a) Solve the differential equation $2\sin(y^2)dx + xy\cos(y^2)dy = 0$ by converting it into an exact equation. [2]

b) Solve the equation: $x^3y' - x^2y = -y^4 \cos x$. [2]

Que 3. a) Solve the differential equation (for constant p, q): [1.5]

$$y'' + 2py' + (p^2 + q^2)y = 0.$$

b) Given that $y_1 = e^{2x}$ is a solution of the differential equation

$$(x + 2)y'' - (2x + 5)y' + 2y = 0.$$

Find the other LI solution y_2 and hence find the general solution of the differential equation. [1.5]

c) Solve the equation: $x^2y'' + 7xy' + 9y = 0$ and find general solution by finding two LI solutions. [2]

Que 4. If $y_1(x)$ and $y_2(x)$ are any two solutions of the differential equation $y'' + P(x)y' + Q(x)y = 0$ on interval I (here P, Q are continuous functions on interval I) then show the wronskian $W(y_1, y_2) = y_1y_2' - y_1'y_2$ is either identically zero or never zero. [2]

Que 5. a) Solve the following differential equation: [2]

$$y'''' - 2y''' + 5y'' - 8y' + 4y = 0.$$

b) Using variation of parameters, find the **general** solution of the ODE: [4]

$$x^2y'' - 2x(x + 1)y' + 2(x + 1)y = x^3.$$

Que 6. a) Use Euler method to solve the IVP: $y' = x^2 + y, y(0) = 0$ for $x \in [0, 1]$ choosing $h = 0.2$. [2]

a) Use Picards iteration to find successive approximations for $y' = x^2y$, $y(0) = 1$. Find upto 4 terms. [2]

Que 7. Solve the following system of first order linear equations:

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Find two LI solutions and hence find the general solution. [4]

Que 8. Find the series solutions of Legendre Equation: $(1-x^2)y''-2xy'+p(p+1)y = 0$. Show that the solutions obtained are LI. Also, show that these LI solutions will reduce to polynomials for non-negative integer values of p. [3+1+1]

Que 9. a) Let $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ and $\sin(A) = B \times C \times E$. Then find matrices B, C, and E. [5]

b) Prove: for $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$, the Gauss-Seidel method diverges while the Jacobi method converges. [5]

ROLL NUMBER: