Indian Institute of Technology Patna
Mathematics - I (MA101)
B. Tech 1st Year (Autumn) 2016 - 17
Mid Semester Examination

Max Time 2 Hours | Maximum Marks 60 | Weightage 30%

Name and ID:____________________________

Note

(a) This question paper consists of a total 10 questions. All are compulsory.
(b) Write your name and id on the question paper.
(c) $\mathbb{R}$ is set of reals, $\mathbb{N}$ is set of naturals.
(d) Other notations have their usual meaning.

1. Find sup and inf of the set
$$A = \left\{ \frac{m}{n} + \frac{4n}{m} : m, n \in \mathbb{N} \right\}.$$
Justify your answer. [6]

2. Let S be a bounded nonempty subset of $\mathbb{R}$ and define $a + S = \{a + x : x \in S\}$ for some $a \in \mathbb{R}$. Then prove that
$$\sup(a + S) = a + \sup(S).$$
[6]

3. Use $\epsilon - \delta$ definition of continuity to prove that $f(x) = x^2 + 20$ is continuous on $\mathbb{R}$. [6]

4. Use $\epsilon - \delta$ definition to check whether function $f(x) = \frac{1}{x^4}$ is uniformly continuous on $[1, \infty)$? [6]

5. Let f be continuously differentiable on $[a, b]$ and twice differentiable on (a, b) and suppose that $f(a) = 0$, $f'(a) = 0$ and $f(b) = 0$. Prove that there is $x_1 \in (a, b)$ such that $f''(x_1) = 0$. [6]

6. Lat $a > 0$, $x_1 > 0$, and
$$x_{n+1} = \frac{1}{3}\left(2x_n + \frac{a}{x_n^2}\right).$$
Show that the sequence $\{x_n\}$ is monotone and converges. Justify. [8]

7. Let $\alpha > 0, \beta > 0$ and
$$a_n = \frac{(1+\alpha)^n + n^\beta}{(1+\alpha)^{n+1}}.$$
Does the sequence $\{a_n\}$ converge or diverge? If it converges find its limit. [4]

8. Two sequences $\{x_n\}$ and $\{y_n\}$ are defined by $x_{n+1} = \frac{1}{2}(x_n + y_n)$ and $y_{n+1} = \sqrt{x_n y_n}$ for $n \geq 1$ and $x_1 > 0, y_1 > 0$. Prove that both the sequences $\{x_n\}$ and $\{y_n\}$ converge and they converge to the same limit. [8]

9. Let $y \in (0, 1)$. Discuss the convergence of $\sum_{n=1}^{\infty} [(n+1)y^n + \sin n]$. [4]

10. Determine all values of y for which
$$\sum_{n=1}^{\infty} \frac{(y-1)^{2n}}{3^n n^2}$$
converges. [6]

End