Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2011-12
Mid Semester Examination

Maximum Marks: 30 Total Time: 2 Hours

Note: This question paper has **TWO** pages and contains **Ten** questions. Answer all questions.

1. (*i*) Suppose that A and B are any two events such that $A \subseteq B$ then show that $P(A) \leq P(B)$ and $P(B - A) = P(B) - P(A)$. **[0.5 +0.5]**
   (*ii*) Consider an urn with four equiprobable tickets numbered as 223, 232, 322, 333. Let A_i $(i = 1, 2, 3)$ be the events that the i^{th} digit of the number on the ticket drawn is 2. Discuss pairwise and mutual independence of considered events. **[1 +1]**
   (*iii*) Find the cumulative distribution function of the random variable X with probability density function $f_X(x) = |x|, \ -1 \leq x \leq 1$ and $f_X(x) = 0$, otherwise. [1]

2. The probability that a student will pass a mid term exam in Mathematics is 0.6 and it is 0.72 for final examination. Also probability that the student will pass the final exam given that he has passed the mid term exam is 0.90.
   (*i*) Find the probability that a student will pass both of these exams [1]
   (*ii*) A student is known to passed the final exam, what is the probability that the student also passed the mid term exam? [1]

3. Four roads leads away from a jail. A prisoner has escaped from the jail and selects a road at random. If road I is selected, the probability of escaping is $1/8$; if road II is selected the probability of success is $1/6$; if road III is selected the probability of escaping is $1/4$ and if road IV is selected the probability of success is $9/10$.
   (*i*) Find the probability that the prisoner will succeed in escaping. [1]
   (*ii*) If the prisoner succeeds, what is the probability that prisoner escaped by using road I, II, III and IV respectively ? [1+1+1+1]

4. The owner of a small hotel with eight rooms has five color television sets which he install upon request at an extra charge. If there is a fifty-fifty chance that any one of his guests wants a color television set, find the probability that on a given day when all rooms are occupied there will be more requests for television sets than there are sets. [2]

5. Write the probability mass function of a random variable X having a hypergeometric $H(M, K, n)$ distribution. Find the mean and variance of this random variable using the definition. [1+1+2]

6. Consider a probability mass function $p_X(x) = 2^{-x}, \ x = 1, 2, 3, \ldots$. Use Chebyshev inequality to find a lower bound for the probability $P(0 \leq X \leq 4)$. What is the actual probability? [1+1]

7. Suppose that out of 120 employees of a company 80 are union members while others are not. If five of the employees are to be chosen randomly to serve on a committee which administrate the pension fund. What is the probability that three of them will be union members. Also compute this probability using binomial approximation and compare it with actual probability. [1.5+1.5]

8. A manufacturer produces 3% defective items. What is the probability that at least six components are to be examined in order to get 3 defective? What is the expected number of defective items that the manufacture produces? [2+1]

9. The probability that a oil drilling company is successful is 1/13 and its success or failure is independent from one location to another. Let X denote the number of drills required to obtain the first strike. Find the cumulative distribution function of X. Use the definition to compute the moment generating function of X. [1+2]

10. (i) In a certain factory there is a small chance 1/500 that a item is defective. The items are supplied in a packet of 10. Use Poisson distribution to calculate approximate number of packets containing no defective and two defective items in a consignment of 20000 packets. [0.5+0.5]
(ii) Write the definitions of $p^{th}, 0 < p < 1$ quantile and the median of a discrete random variable. [0.5+0.5]