Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2012-13
Mid Semester Examination

Maximum Marks: 30 Total Time: 2 Hours

Note: This question paper has two pages and contains eight questions. Answer all questions.

1. (*i*) Let A and B be any two events. Then show that $P(A^c \cap B) = P(B) - P(A \cap B)$ and $P(A \cap B^c) = P(A) - P(A \cap B)$. [0.5 +0.5]

   (*ii*) It is known that IIT Patna buys three newspapers A, B and C every day. A survey suggest that 80% read A, 75% read B, 80% read C, 80% read both A and B, 85% read both A and C, 90% read both B and C and 95% read at least one of the papers.
   (*a*) Find what percentage read none of the papers? [1]
   (*b*) What percentage read exactly one of the three papers? [3]

2. (*i*) Define independent events. Consider any four events A, B, C and D and then write all the conditions under which these events become independent. [1 + 2]

   (*ii*) An urn contains 6 black and 7 white balls. Two balls are drawn at random from the urn without replacement. Let $B_i, i = 1, 2,$ denotes the event that ith draw yields a black ball. Find probabilities $P(B_1)$ and $P(B_2)$. Also use the definition to verify whether the given events B_1 and B_2 are independent or not. [3]

3. Five percent of the patients suffering from a certain disease are selected to undergo a new treatment that is believed to increase the recovery rate from 30% to 50%. A person is randomly selected from these patients after the completion of the treatment and found to have recovered. What is the probability that the patient received the new treatment? [3]

4. Let X be a continuous random variable with probability density function $f_X = cx,\ 0 \leq x \leq 1, \quad = c,\ 1 \leq x \leq 2, \quad = -c(x-3), 2 \leq x \leq 3, \quad = 0,$ *otherwise*. Find the constant c and then find the corresponding cumulative distribution function. Finally, compute the probability $P(X < 1.5 \mid 1 < X < 2)$. [1+1+1]

5. (*i*) State and prove the Chebyshev inequality (give proper justification for your steps in the proof). [0.5+1]

(*ii*) Let random variable X be distributed with probability density function $f_X(x) = 2, \; 0 < x < 0.5, \quad = 0, \; otherwise.$ Find a lower bound for the probability $P(|X - \frac{1}{4}| < \frac{1}{\sqrt{3}})$. Compare your bound with the actual probability. **[1.5]**

6. An airport has eight radar in operation and each radar has a 0.8 probability of detecting an arriving plane. It is assumed that radars operate independent of each other. Write probability mass function of a random variable X which denotes the event: 'number of radars that detect the airplane'. Then
(*a*) Compute the probability that an arriving plane will be detected by at most one radar. [1]
(*b*) It is known that at least six radars detected an plane, find the probability that eight radars detected this airplane. [2]

7. A man shoots a target until he has hit it three times. Let probability that a given shot hits the target is equal to 0.7. Let X denotes the random variable that the number of times man shoot the target in order to get the desire number of hit. What is the probability that the man shoot the target at least four times. Use the definition to compute the variance of the given random variable. **[1.5 + 2.5]**

8. A merchant receives a batch of 100 electrical devices. To save time, she decides to use the following sampling plan: she take two devices at random and without replacement and she decides to accept the whole batch if devices selected are nondefective. Let X be the random variable denoting the number of defective devices in the sample. Then
(*a*) Properly write the probability distribution of X. [1]
(*b*) If the batch contains exactly two defective devices, calculate the probability that it is accepted. [1]
(*c*) Approximate the probability computed in part (b) with the help of a binomial distribution. [1]