Engineering Optics (PH301)
Assignment-3

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Answers

1. The emitted wavelength λ in the output spectrum of an LED is related to the photon energy E as

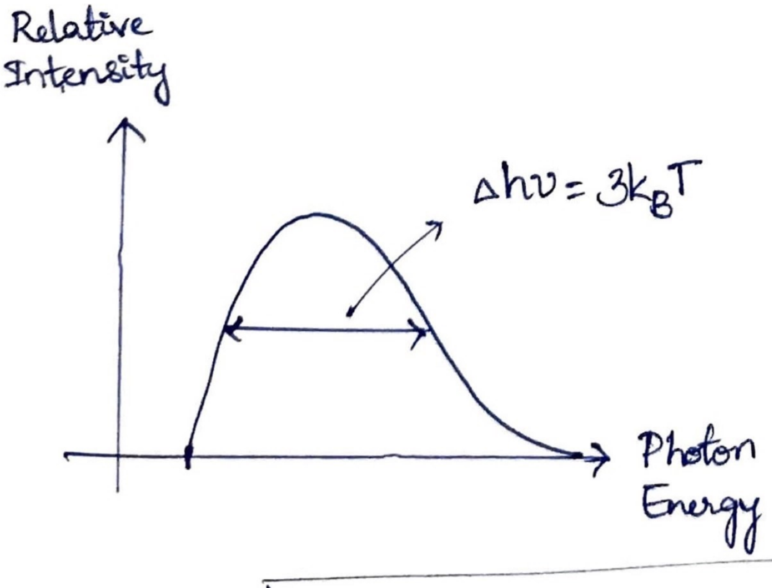
$$\boxed{\lambda = \frac{hc}{E}}$$

where h is planck's constant
c is speed of light

differentiate both sides of the equation with respect to E

$$\frac{d\lambda}{dE} = \frac{-hc}{E^2}$$

$\therefore$ Change in linewidth, $\Delta\lambda = \left|\frac{d\lambda}{dE}\right| \Delta E = \frac{hc}{E^2}(\Delta E)$



Given,
Width of relative light intensity versus photon energy spectrum is $3k_BT$.

$$\boxed{\Delta E = \Delta(h\nu) = 3k_BT}$$

Hence,
$$\Delta\lambda = \frac{hc}{E^2}(\Delta E) = \frac{hc}{E^2}(3k_BT)$$

$$= \frac{hc}{\left(\frac{hc}{\lambda}\right)^2}(3k_BT) \qquad \left[\because E = \frac{hc}{\lambda}\right]$$

$$= \frac{\cancel{hc}}{\frac{\cancel{h^2c^2}}{hc}}\lambda^2(3k_BT)$$

$$\Rightarrow \boxed{\Delta\lambda = \lambda^2 \frac{3k_BT}{hc}}$$

This is line width $\Delta\lambda$ in the output spectrum in terms of wavelength.

2. Given,

bandgap of GaAs at 300K, $E_g = 1.42\,eV$

$$\boxed{\frac{dE_g}{dT} = -4.5\times10^{-4}\,eVK^{-1}} \quad (1)$$

Change in emitted wavelength λ with temperature,

$$\boxed{\frac{d\lambda}{dT} = \frac{d\lambda}{dE_g}\times\frac{dE_g}{dT}} \quad (2) \quad \text{(chain rule for differentiation)}$$

Now, $\lambda = \frac{hc}{E_g} \Rightarrow \boxed{\frac{d\lambda}{dE_g} = \frac{-hc}{E_g^2}}$ (3)

where, h is planck's constant $= 4.135\times10^{-15}\,eVK^{-1}$
c is speed of light $= 3\times10^{8}\,ms^{-1}$

Substituting 1 & 3 in 2 $\Rightarrow$

$$\frac{d\lambda}{dT} = \frac{-hc}{E_g^2} \times -4.5 \times 10^{-4}\, eVK^{-1}$$

$$= \frac{4.135 \times 10^{-15} \times 3 \times 10^{8}}{(1.42)^2} \times 4.5 \times 10^{-4}$$

$$\Rightarrow \boxed{\frac{d\lambda}{dT} \approx 2.768 \times 10^{-10}\, mK^{-1} = 0.2768\, nm\, K^{-1}}$$

For change in temperature of $10^\circ C$,

$\Delta T = 10^\circ C = 10K$,

$$\Rightarrow \Delta\lambda = \left(\frac{d\lambda}{dT}\right)\Delta T = 0.2768 \times 10\ nm$$

$$\Rightarrow \boxed{\Delta\lambda = 2.768\, nm}$$

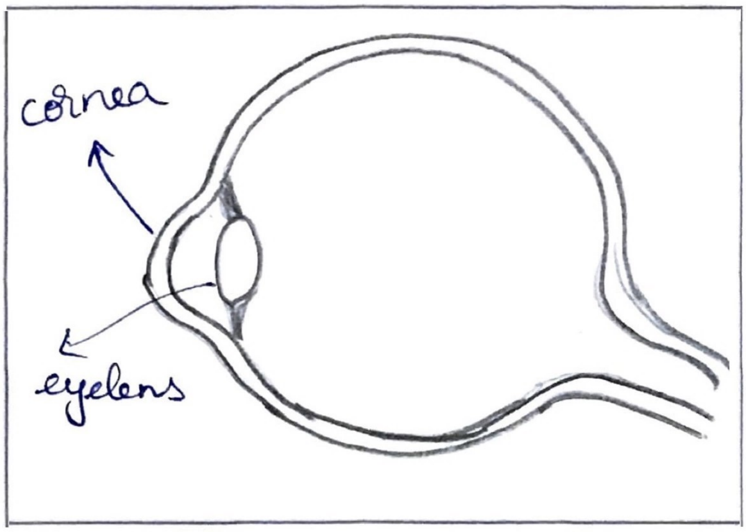
$\therefore$ For change in temperature of $10^\circ C$, the emitted wavelength increases by 2.768 nm.

3. LED's generally emit visible, UV, IR rays with very high brightness. They are made using direct bandgap semiconductors like GaAs, GaAsP, GaP.

Silicon and Germanium are indirect bandgap semiconductors with reference to visible light. Their p-n junctions emit energy in the form of heat, instead of light. Therefore they are very inefficient. Hence, Silicon (Si) or Germanium (Ge) diodes are not used for LED material.

4. Focal length of cornea = 2.3 cm
Focal length of eyelens = 6.4 cm

The cornea and eye lens are closely spaced as shown.



For closely spaced lenses of focal lengths f_1, f_2, the net focal length is given by

$$\boxed{\frac{1}{f_{net}} = \frac{1}{f_1} + \frac{1}{f_2}}$$

Here, $f_1 = 2.3$ cm
$f_2 = 6.4$ cm

∴ Focal length of eye is found by

$$\frac{1}{f_{eye}} = \frac{1}{2.3} + \frac{1}{6.4} \Rightarrow f_{eye} = \frac{2.3 \times 6.4}{2.3 + 6.4}$$

$$\Rightarrow \boxed{f_{eye} = \frac{14.72}{8.7}\text{ cm} \approx 1.69\text{cm} = 0.0169\text{m}}$$

∴ Optical Power of eye is $\frac{1}{f_{eye}} = \frac{1}{0.0169} \simeq 59.17D$

Hence, net focal length of eye is 1.69cm and optical power is 59.17D

5. Yes, a virtual image can be photographed. It is evident from the recent trend of people posting pictures on social media by taking selfies with their new mobile phones while standing in front of a mirror.

A virtual image is formed when rays of light appear to converge/diverge from a location behind a mirror or lens system but don't actually do so.

When these rays pass through a camera, they actually converge onto the film or chip inside the camera and form a real image. In this manner, a virtual image can be photographed. Hence, photograph is taken.

6. Resolution limit of a microscope objective is given by

$$\boxed{R = 0.61 \frac{\lambda}{NA}}$$

where λ is wavelength of light ($= 550\,nm$)

NA is numerical aperture

a) for $\boxed{NA = 0.25}$, $R = 0.61 \times \frac{550 \times 10^{-9}}{0.25}$

$\Rightarrow \boxed{R = 1.342 \times 10^{-6} m}$

b) for $\boxed{NA = 0.80}$, $R = 0.61 \times \frac{550 \times 10^{-9}}{0.80}$

$\Rightarrow \boxed{R = 4.194 \times 10^{-7} m}$

c) for $\boxed{NA = 1.2}$, $R = 0.61 \times \frac{550 \times 10^{-9}}{1.2}$

$\Rightarrow \boxed{R = 2.796 \times 10^{-7} m}$

Hence, resolution limit of microscope objective is

$1.342 \times 10^{-6} m$ for $NA = 0.25$ (numerical aperture)

$4.194 \times 10^{-7} m$ for $NA = 0.80$

$2.796 \times 10^{-7} m$ for $NA = 1.2$