

5/11/2020

CS303 Tutorial 8

Name of Student : M. Mahesh Reddy

Roll No : 1801CS31

Ans 1:

Given: $\{a^n b^j \mid n \leq j^2\}$ is language denoted by L

To prove: L is not CFL

Proof: Assume L is CFL, then pumping lemma must hold on L .

Consider m as critical length.

Now, $a^{m^2} b^m \in L$ and $|a^{m^2} b^m| \geq m$

$\therefore \exists u, v, x, y, z$ s.t. $uvxyz = a^{m^2} b^m$

$$|vxy| \leq m$$

$$|vy| \geq 1$$

$$\forall i \geq 0, uv^i xy^i z \in L$$

Consider u, v, x, y, z as follows

Case (i):

$\overbrace{aaa \dots aaa}^{m^2 \text{ times}} \overbrace{bbb \dots bbb}^{m \text{ times}}$
 $\underbrace{\hspace{1.5cm}}_u \underbrace{\hspace{1.5cm}}_{vxy} \underbrace{\hspace{1.5cm}}_z$

Consider $\left. \begin{array}{l} v = a^{k_1} \\ y = a^{k_2} \end{array} \right\}$ s.t. $\left\{ \begin{array}{l} |vxy| \leq m \\ |vy| \geq 1 \\ 1 \leq k_1 + k_2 \leq m \\ k_1, k_2 \geq 1 \end{array} \right.$

For $i=2$, $a^{m^2+k_1+k_2}b^m \in L$, but $m^2+k_1+k_2 > m^2$

\therefore Pumping lemma doesn't hold

Case (ii):

aaa...aaabbb...bbb
 $\underbrace{\hspace{1.5cm}}_u \quad \underbrace{\hspace{1.5cm}}_{vxy} \quad \underbrace{\hspace{1.5cm}}_z$

$k_1, k_2 \geq 1 \quad v = b^{k_1} \quad y = b^{k_2} \quad k_1+k_2 \geq 1 \quad k_1+k_2+|x| \leq m$

If $i=0$, $a^{m^2}b^{m-k_1-k_2} \in L$, but $m^2 > (m-k_1-k_2)^2$

\therefore Pumping lemma doesn't hold

Case (iii):

aaa...aaabbb...bbb
 $\underbrace{\hspace{1.5cm}}_u \quad \underbrace{\hspace{1.5cm}}_{vxy} \quad \underbrace{\hspace{1.5cm}}_z$

Subcase (1): If v spans both a and b , then

if pumping is done the resultant string has a 's and b 's mixed up. So, it doesn't belong to L . Same

holds for y .

Subcase (2): $v = a^{k_1} \quad y = b^{k_2} \quad 1 \leq k_1+k_2 \leq m$
 $k_1, k_2 \geq 1$

For $i=0$, $a^{m^2-k_1}b^{m-k_2} \in L$

But, $(m-k_2)^2 \leq (m-1)^2$ ($\because k_2 \geq 1$)

$= (m^2-2m+1) < (m^2-k_1)$ ($\because k_1 < m$)

$\Rightarrow m^2-k_1 > (m-k_2)^2 \therefore$ Pumping lemma doesn't hold
 Hence L is not CFL.

Ans 2:

Given: $L = \{w: n_a(w) < n_b(w) < n_c(w)\}$ is
a language

To prove: L is not CFL

Proof:

Assume L is CFL, then Pumping Lemma must hold on it.

Consider 'm' as critical length.

Now, $a^m b^{m+1} c^{m+2} \in L \Rightarrow |a^m b^{m+1} c^{m+2}| \geq m$

$\exists u, v, x, y, z \in \{a, b, c\}^*$ s.t. $uvxyz = a^m b^{m+1} c^{m+2}$,
 $|vxy| \geq m$, $|vy| \leq 1$, $uv^i xy^i z \in L \forall i \geq 0$

Consider u, v, x, y, z as follows:

Case ①:

$aa \dots aa \dots aabb \dots bb \dots bbcc \dots cc \dots cc$
 $\underbrace{\hspace{1cm}}_u \underbrace{\hspace{1cm}}_{vxy} \underbrace{\hspace{10cm}}_z$

where $v = a^{k_1}$ $y = a^{k_2}$ $k_1, k_2 \geq 1$ $k_1 + k_2 \geq 1$
 $k_1 + k_2 + |x| \leq m$

For $i=2$, $a^{m+k_1+k_2} b^{m+1} c^{m+2} \in L$

but $m+k_1+k_2 \geq m+1$ ($\because k_1+k_2 \geq 1$)

\therefore Pumping lemma doesn't hold in this case.

Case (2):

$\underbrace{aa \dots aa \dots aabb \dots}_{u} \underbrace{bb \dots}_{vxy} \underbrace{bbcc \dots cc \dots cc}_{z}$

$$v = b^{k_1}, y = b^{k_2} \quad k_1, k_2 \geq 1 \quad \begin{matrix} |vy| \geq 1 \\ 1 \leq k_1 + k_2 \leq m \\ (\because |vxy| \leq m) \end{matrix}$$

For $i=2$, $a^m b^{m+1+k_1+k_2} c^{m+2} \in L$

but, $m+1+k_1+k_2 \geq m+2 \quad (\because k_1+k_2 \geq 1)$

\therefore Pumping lemma doesn't hold in this case

Case (3):

$\underbrace{aa \dots aa \dots aabb \dots}_{u} \underbrace{bb \dots}_{vxy} \underbrace{bbcc \dots cc \dots cc}_{z}$

$$v = c^{k_1}, y = c^{k_2} \quad k_1, k_2 \geq 1 \quad |vy| \geq 1 \quad |vxy| \leq m$$

$$\Rightarrow 1 \leq k_1 + k_2 \leq m$$

For $i=0$ $a^m b^{m+1} c^{m+2-(k_1+k_2)} \in L$

but $(m+1) \geq (m+2)-(k_1+k_2) \quad [\because k_1+k_2 \geq 1]$

\therefore Pumping lemma doesn't hold in this case.

Case (4):

$\underbrace{aa \dots aa \dots aabb \dots}_{u} \underbrace{bb \dots}_{vxy} \underbrace{bbcc \dots cc \dots cc}_{z}$

Subcase (i):

Now, if either v or y contain both a and b , then, on pumping, the resultant string will have a 's & b 's mixed up. So pumping lemma won't hold.

Subcase (i):

$$\text{if } v = a^{k_1} \gamma = b^{k_2} \quad k_1, k_2 \geq 1 \quad \begin{array}{l} |vxy| \leq m \\ |vy| \geq 1 \end{array}$$
$$\Rightarrow 1 \leq k_1 + k_2 \leq m$$

Now, for $i=2$ $a^{m+k_1} b^{m+1+k_2} c^{m+2} \in L$

but $m+1+k_2 \geq m+2$ ($\because k_2 \geq 1$)

Hence pumping lemma doesn't hold

Case (5):

$aa \dots aa \dots aabb \dots bb \dots bbcc \dots cc \dots cc$
 $\underbrace{\hspace{10em}}_u \quad \underbrace{\hspace{4em}}_{vxy} \quad \underbrace{\hspace{4em}}_z$

This can be explained in a way similar to case (4)

Pumping lemma won't hold.

Case (6):

$aa \dots aa \dots aabb \dots bb \dots bbcc \dots cc \dots cc$
 $\underbrace{\hspace{2em}}_u \quad \underbrace{\hspace{12em}}_{vxy} \quad \underbrace{\hspace{4em}}_z$

This also can be explained using ~~case~~ previous cases. Pumping lemma won't hold.

\therefore The given language $L = \{w : n_a(w) < n_b(w) < n_c(w)\}$
is not context free. Hence Proved!

Ans 3:

Given: $L = \{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$ is a language?

To prove: L is not a CFL

Proof: Assume L is CFL, then pumping lemma must hold on L .

Assume critical length is m

Consider a string $a^m b^m c^m \in L \Rightarrow |a^m b^m c^m| \geq m$

$\because L$ is CFL, $\exists u, v, x, y, z \in \{a, b, c\}^*$ such that

$$uvxyz = a^m b^m c^m \quad |vxy| \geq m, \quad |vy| \leq 1, \quad uv^i xy^i z \in L \quad \forall i \geq 0$$

Consider u, v, x, y, z as follows

Case ①:

$aa \dots aa \dots aabb \dots bb \dots bbcc \dots cc \dots cc$
 $\underbrace{\hspace{1cm}}_u \quad \underbrace{\hspace{1cm}}_{vxy} \quad \underbrace{\hspace{1cm}}_z$

$$v = a^{k_1}$$

$$y = a^{k_2}$$

$$k_1, k_2 \geq 1$$

$$|vxy| \leq m \text{ and } |vy| \geq 1$$

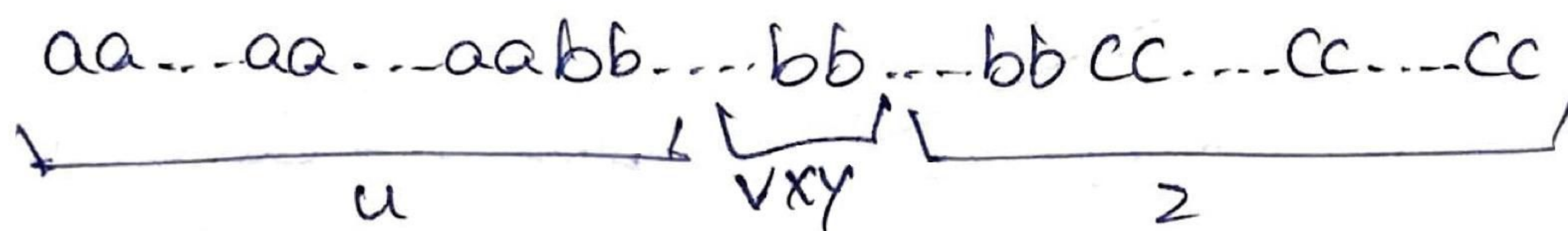
$$\Rightarrow 1 \leq k_1 + k_2 \leq m$$

For $i=2$

$$a^{m+k_1+k_2} b^m c^m \in L$$

but $m+k_1+k_2 > m$, so pumping lemma doesn't hold in this case.

Case ②:



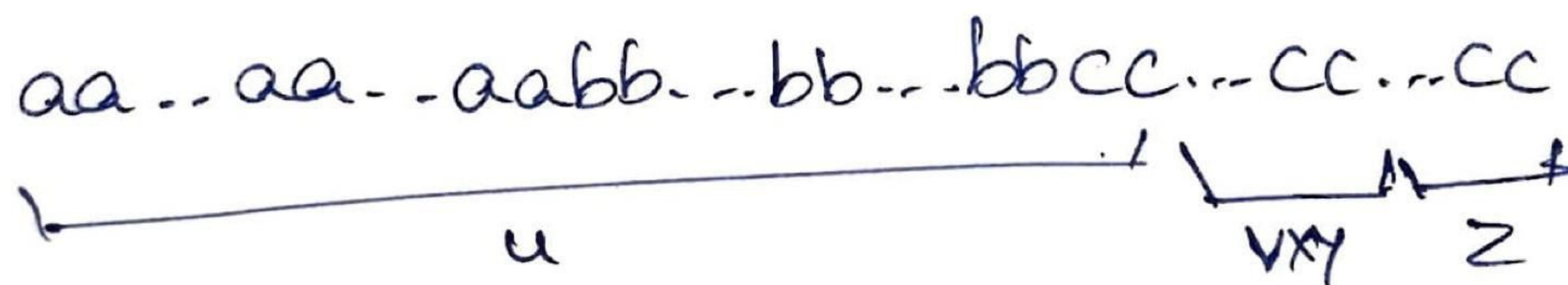
$$v = a^{k_1} \quad y = b^{k_2} \quad k_1, k_2 \geq 1 \quad |vxy| \leq m \text{ and } |vy| \geq 1$$

$$\Rightarrow 1 \leq k_1 + k_2 \leq m$$

For $i=2$, $a^m b^{m+k_1+k_2} c^m \in L$

but $m+k_1+k_2 > m$, so pumping lemma doesn't hold in this case.

Case ③:



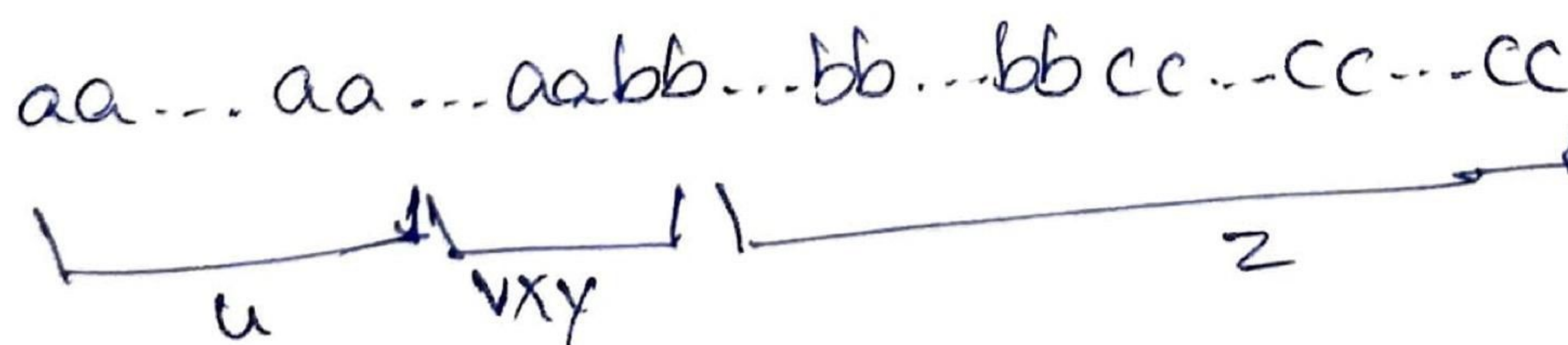
$$v = c^{k_1} \quad y = c^{k_2} \quad k_1, k_2 \geq 1 \quad |vxy| \leq m \text{ \& } |vy| \geq 1$$

$$\Rightarrow 1 \leq k_1 + k_2 \leq m$$

For $i=0$, $a^m b^m c^{m-k_1-k_2} \in L$

but $m-k_1-k_2 < m$, so pumping lemma doesn't hold in this case.

Case ④:



Subcase (i): if either v or y contains both a & b , then the string obtained by pumping will have a & b mixed up. So, pumping lemma won't hold.

Subcase (ii)

$$v = a^{k_1} \quad y = b^{k_2}$$

$$k_1, k_2 \geq 1 \quad |vxy| \leq m \\ |vy| \geq 1$$

$$\text{for } i=2, \quad a^{m+k_1} b^{m+k_2} c^m \in L$$

but $m+k_2 > m$, so pumping lemma doesn't hold.

Case ⑤:

$$\begin{array}{c} aa \dots aa \dots aa bb \dots bb \dots bbcc \dots cc \dots cc \\ \underbrace{\hspace{10em}}_u \quad \underbrace{\hspace{2em}}_{vxy} \quad \underbrace{\hspace{2em}}_z \end{array}$$

Pumping lemma won't hold in this case and can be explained similarly as case ④.

Case ⑥:

$$\begin{array}{c} aa \dots aa \dots aa bb \dots bb \dots bbcc \dots cc \dots cc \\ \underbrace{\hspace{3em}}_u \quad \underbrace{\hspace{15em}}_{vxy} \quad \underbrace{\hspace{2em}}_z \end{array}$$

Pumping lemma won't hold in this case and can be explained similarly as previous cases.

\therefore The given language $L = \{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$ is not a CFL. Hence Proved!