Indian Institute of Technology Patna

MA-225: B.Tech. II year

Spring Semester: 2018-19 (End Semester Examination)

Maximum Marks: 50 Total Time: 3 Hours

Note: This question paper contains **Ten** questions. Answer all questions.

1. Suppose that length of time X (in hrs) it takes to drive to a work place every day has a cumulative distribution function given by $F_X(x) = 0,\ x < 0.25, \quad = 4(x-0.25)^2,\ 0.25 \le x \le 0.75, \quad = 1, x > 0.75$. Find the density function of Y where $Y = 3600X$. Find the expected value of Y. Also determine the moment generating function of Y. **[2+1+2]**

2. Suppose that joint density function of (X,Y) is $f_{X,Y}(x,y) = xy^2 + \frac{x^2}{8},\ 0 \le x \le 2,\ 0 \le y \le 1; \quad = 0$, elsewhere. Compute the probabilities $P(X > 1 \mid Y < 0.5)$ and $P(Y < 0.5 \mid X > 1)$. Also find marginal densities and then verify if X and Y are independent. **[1+1+1+1+1]**

3. A health-food store stocks two different brands of a certain type of grain. Let X denote the amount of brand A on hand and Y denote the amount of brand B on hand. Suppose that the joint density function of X and Y is $f_{X,Y}(x,y) = k(x^2+y^2),\ 0 < x < 1,\ 0 < y < 1; \quad = 0$, elsewhere. Find the constant k. Find the probability $P(X > 0.5, Y < 0.25)$. Find the correlation coefficient between X and Y. **[1+2+7]**

4. Suppose that joint density function of (X,Y) is $f_{X,Y}(x,y) = x + y,\ 0 < x < 1,\ 0 < y < 1; \quad = 0$, elsewhere. Then determine the density function of Z where $Z = X/Y$. Also determine the density function of U where $U = \frac{3Z}{2}$. **[2.5+2.5]**

5. Let X and Y be independent gamma variables with parameters (α, λ) and (β, λ) respectively. Find the joint density function of (U,V) where $U = X + Y$ and $V = X/(X+Y)$. Also find the marginal density functions of U and V respectively. **[2+ 1.5 + 1.5]**

6. Let (X,Y) denote the scores in two test and suppose that they have two dimensional normal distribution $BVN(82, 90, 100, 81, 0.75)$. Compute the probabilities $P(Y > 92 \mid X = 84)$, $P(X > Y)$, and $P(X + Y > 180)$ **[2+ 1.5 + 1.5]**

7. Suppose X and Y have joint density as $f_{X,Y}(x,y) = \frac{6}{5}(x+y^2),\ 0 < x < 1,\ 0 < y < 1; \quad = 0$, elsewhere. Find the conditional distribution of Y given $X = x$. Then find the conditional variance of Y given $X = 0.8$. **[2+ 3]**

8. Suppose customers arrive at a counter according to a Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of four minutes more than four customers arrive. Further using the central limit theorem compute the probability that during a three hour shift more than 530 customers arrive. **[1+2]**

9. If $X_1(t)$ and $X_2(t)$ are two independent different Poisson processes with rates λ_1 and λ_2 respectively, t is any given time interval. Then find the conditional distribution of $X_1(t)$ given $X_1(t) + X_2(t)$. **[3]**

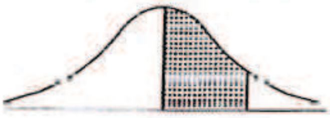
10. Write the density function of a lognormal $LN(0.5, 0.64)$ distribution. Find both mean and variance of this distribution. **[1+1.5+1.5]**

Normal probability curve is given by :

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} \quad -\infty < x < \infty$$

and standard normal probability curve is given by :

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right), -\infty < z < \infty$$



where $\quad Z = \frac{X - E(X)}{\sigma_X} \sim N(0, 1)$

The following table gives the shaded area in the diagram, *viz.*, $P(0 < Z < z)$ for different values of z.

TABLE OF AREAS

↓Z→	0	1	2	3	4	5	6	7	8	9
·0	·0000	·0040	·0080	·0120	·0160	·0199	·0239	·0279	·0319	·0359
·1	·0398	·0438	·0478	·0517	·0557	·0596	·0636	·0675	·0714	·0759
·2	·0793	·0832	·0871	·0910	·0948	·0987	·1026	·1064	·1103	·1141
·3	·1179	·1217	·1255	·1293	·1331	·1368	·1406	·1443	·1480	·1517
·4	·1554	·1591	·1628	·1664	·1700	·1736	·1772	·1808	·1844	.1879
·5	·1915	·1950	·1985	·2019	·2054	·2088	·2123	·2157	·2190	·2224
·6	·2257	·2291	·2324	·2357	·2389	·2422	·2454	·2486	·2517	·2549
·7	·2580	·2611	·2642	·2673	·2703	·2734	·2764	·2794	·2823	·2852
·8	·2881	·2910	·2939	·2967	·2995	·3023	·3051	·3078	·3106	·3133
·9	·3159	·3186	·3212	·3238	·3264	·3289	·3315	·3340	·3365	·3389
1·0	·3413	·3438	·3461	·3485	·3508	·3531	·3554	·3577	·3599	·3621
1·1	·3643	·3655	·3686	·3708	·3729	·3749	·3770	·3790	·3810	·3830
1·2	·3849	·3869	·3888	·3907	·3925	·3944	·3962	·3980	·3997	·4015
1·3	·4032	·4049	·4066	·4082	·4099	·4115	·4131	·4147	·4162	·4177
1·4	·4192	·4207	·4222	·4236	·4251	·4265	·4279	·4292	·4306	·4319
1·5	·4332	·4345	·4357	·4370	·4382	·4394	·4406	·4418	·4429	·4441
1·6	·4452	·4463	·4474	·4484	·4495	·4505	·4515	·4525	·4535	·4545
1·7	·4554	·4564	·4573	·4582	·4591	·4599	·4608	·4616	·4625	·4633
1·8	·4641	·4649	·4656	·4664	·4671	·4678	·4686	·4693	·4699	·4706
1·9	·4713	·4719	·4726	·4732	·4738	·4744	·4750	·4756	·4761	·4767
2·0	·4772	·4778	·4783	·4788	·4793	·4798	·4803	·4808	·4812	·4817
2·1	·4821	·4826	·4830	·4834	·4838	·4842	·4846	·4850	·4854	·4857
2·2	·4861	·4864	·4868	·4871	·4875	·4678	·4881	·4884	·4887	·4890
2·3	·4893	·4896	·4898	·4901	·4904	·4906	·4909	·4911	·4913	·4916
2·4	·4918	·4920	·4922	·4925	·4927	·4929	·4931	·4932	·4934	·4936
2·5	·4938	·4940	·4941	·4943	·4945	·4946	·4948	·4959	·4951	·4952
2·6	·4953	·4955	·4956	·4957	·4959	·4960	·4961	·4962	·4963	·4964
2·7	·4965	·4966	·4967	·4968	·4969	·4970	·4971	·4972	·4973	·4974
2·8	·4974	·4975	·4976	·4977	·4977	·4978	·4979	·4979	·4980	·4981
2·9	·4981	·4982	·4982	·4983	·4984	·4984	·4985	·4985	·4986	·4986
3·0	·4987	·4987	·4987	·4988	·4988	·4989	·4989	·4989	·4990	·4990
3·1	·4990	·4991	·4991	·4991	·4992	·4992	·4992	·4992	·4993	·4993
3·2	·4993	·4993	·4994	·4994	·4994	·4994	·4994	·4995	·4995	·4995
3·3	·4995	·4995	·4995	·4996	·4996	·4996	·4996	·4996	·4996	·4997
3·4	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4998
3·5	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998
3·6	·4998	·4998	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999
3·7	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999
3·9	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000