PH 201

OPTICS & LASERS

Lecture_Lasers_3

Absorption & Stimulated Emission Coefficients

Einstein suggested concept of stimulated emission in 1917.

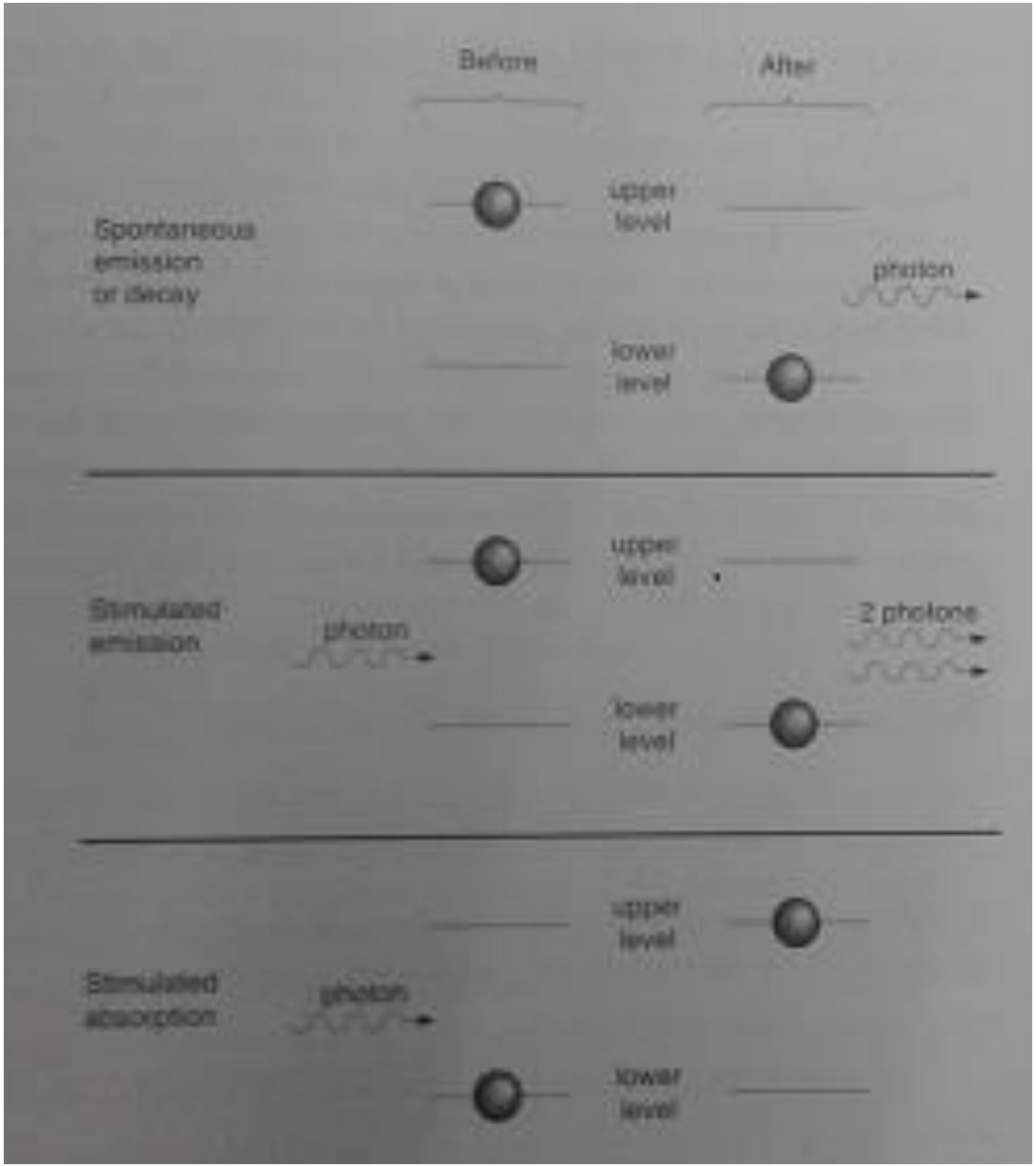
- **If a photon can stimulate an electron to move from a lower energy state *l* to higher energy state *u* by means of absorption, then a photon should also be able to stimulate an electron from same upper state u to lower state *l*.**

- **In case of absorption, photon disappears, with energy being transferred to absorbing species.**

- **In case of stimulated emission, the species would have to radiate an additional photon to conserve energy.**

  Such process must occur in order to keep population of two energy levels in thermal equilibrium.

Radiative processes producing interaction between two bound levels in a material: spontaneous emission, absorption, & stimulated emission.



Fundamental radiation processes associated with interaction of light with matter.

Consider a group of atoms having electrons occupying either energy levels *u* or *l* with population densities N_u & N_l (no. of atoms per unit volume).

Assuming atoms in thermal equilibrium with each other & must therefore be related by Boltzmann distribution,

$$\frac{N_u}{N_l} = \frac{g_u}{g_l} e^{-(E_u - E_l)/kT} = \frac{g_u}{g_l} e^{-\Delta E_{ul}/kT}$$

$$\Delta E_{ul} = E_u - E_l = h\upsilon_{ul}$$

Spontaneous transition probability, rate at which spontaneous transitions occur from level *u* to level *l* (no. per unit time) = A_{ul}

No. of spontaneous transitions from *u* to *l* per unit time per unit volume = $N_u A_{ul}$

Stimulation process would be proportional to photon energy density *u*(*v*) at frequency v_{ul}

Assuming proportionality constant for such stimulated transition B, then the upward flux – the no. of stimulated upward transitions per unit volume per unit time per unit frequency = $N_l B_{lu} u(v)$

Similarly downward flux = $N_u B_{ul} u(v)$

Constants A_{ul}, B_{ul}, & B_{lu} are referred to as Einstein *A* & *B* coefficients.

For populations N_u & N_l to be in radiative thermal equilibrium & for the principle of detailed balance to apply, downward radiative flux should equal the upward radiative flux between two levels.

$$N_u A_{ul} + N_u B_{ul} u(\upsilon) = N_l B_{lu} u(\upsilon)$$

$$u(\upsilon) = \frac{N_u A_{ul}}{N_l B_{lu} - N_u B_{ul}}$$

Dividing top & bottom terms in right hand side by N_u & then for ratio N_u/N_l

$$u(\upsilon) = \frac{A_{ul}}{B_{ul}} \left(\left[\frac{g_l B_{lu}}{g_u B_{ul}} \right] e^{h\upsilon_{ul}/kT} - 1 \right)^{-1}$$

$$u(\upsilon) = \frac{A_{ul}}{B_{ul}} \left(\left[\frac{g_l B_{lu}}{g_u B_{ul}} \right] e^{h\upsilon_{ul}/kT} - 1 \right)^{-1}$$

This Eq. has a familiar form with the following Eq.

Rayleigh-Jeans law: Energy density of radiation per unit volume

$$u(\upsilon) = \frac{8\pi h \eta^3 \upsilon^3}{c^3 (e^{h\upsilon/kT} - 1)}$$

[Planck's law]

Both Eqs. Concern radiation in thermal equilibrium, if true then they must be equivalent. Equivalence follows if

$$\frac{g_l B_{lu}}{g_u B_{ul}} = 1 \qquad or \qquad g_l B_{lu} = g_u B_{ul}$$

$$\frac{A_{ul}}{B_{ul}} = \frac{8\pi h \eta^3 \upsilon^3}{c^3}$$

Relationship between stimulated emission (B_{ul}) & absorption coefficients (B_{lu}).

$$u(\upsilon)=\frac{A_{ul}}{B_{ul}}\left(\left[\frac{g_l B_{lu}}{g_u B_{ul}}\right]e^{h\upsilon_{ul}/kT}-1\right)^{-1}$$

This Eq. has a familiar form with the following Eq.

Rayleigh-Jeans law: Energy density *u*(*v*) of cavity radiation per unit volume within frequency *v* to *v* + d*v*.

$$u(\upsilon)=\frac{8\pi\eta^3\upsilon^2}{c^3}kT$$

This result suggests that there is a continuous increase in energy density with frequency for a given temp *T*.

It suggests that energy density approaches infinity as frequency is increased.

It agrees with experiments for lower frequencies but does not predict experimentally observed maximum value for a given temp at higher frequencies.

Planck's law:

Planck explored the possibility of quantizing mode energy, postulating that an oscillator of frequency *v* could have only discrete values *mhv* of energy, where *m* = 0, 1, 2, 3, …
Planck referred to this unit of energy *hv* as a quantum that could not be further divided.

$$u(\upsilon) = \frac{8\pi h \eta^3 \upsilon^3}{c^3 (e^{h\upsilon/kT} - 1)}$$

Both Eqs. Concern radiation in thermal equilibrium, if true then they must be equivalent. Equivalence follows if

$$\frac{g_l B_{lu}}{g_u B_{ul}} = 1 \qquad or \qquad g_l B_{lu} = g_u B_{ul}$$

$$\frac{A_{ul}}{B_{ul}} = \frac{8\pi h \eta^3 \upsilon^3}{c^3}$$

Relationship between stimulated emission (B_{ul}) & absorption coefficients (B_{lu}).

Relationship with spontaneous emission coefficient (A_{ul}).

$$B_{ul} = \frac{c^3}{8\pi h \eta^3 \upsilon^3} A_{ul}$$

Ratio of stimulated to spontaneous emission rates from level *u*.

$$\frac{B_{ul} u(\upsilon)}{A_{ul}} = \frac{1}{e^{h\upsilon_{ul}/kT} - 1}$$

Thus, stimulated emission plays a significant role only for temps in which *kT* is of, or greater than, the order of photon energy hv_{ul}.

Ratio is unity when

$$\frac{h\upsilon_{ul}}{kT} = \ln 2 = 0.693$$

For visible transitions, in green portion of spectrum (photons of order of 2.5 eV), such a relationship would be achieved for a temp of 33,500 K. Thus, in visible spectrum, dominance of stimulated emission over spontaneous emission normally happens only in stars, in high-temp & density laboratory plasmas such as laser produced plasma or in lasers.

In low-pressure plasmas the radiation can readily escape, so there is no opportunity for radiation density to build up to a value where stimulated decay rate is comparable to radiative decay rate.

In lasers, the ratio can be significantly greater than unity.

- **A He-Ne laser operating at 632.8 nm has an output power 1.0 mW with 1.0 mm beam diameter. The beam passes through a mirror that has 99% reflectivity and 1% transmission at the laser wavelength. For this laser calculate the ratio**

$$\frac{B_{ul}u(\upsilon)}{A_{ul}}$$

What is the effective blackbody temp of the laser beam as it emerges from the laser output mirror?

Assume beam diameter is 1.0 mm inside the laser cavity & that power is uniform over beam cross-section. Also, laser linewidth is approximately one tenth of Doppler width for transition.

Laser frequency

$$\upsilon = \frac{c}{\lambda} = \frac{3\times10^{8}\,m/s}{6.328\times10^{-7}\,m} = 4.74\times10^{14}\,Hz$$

$$\frac{A_{ul}}{B_{ul}} = \frac{8\pi h\eta^3\upsilon^3}{c^3} = \frac{(8\pi)(6.63\times10^{-34}\,J-s)(1)^3(4.74\times10^{14}\,Hz)^3}{(3\times10^8\,m/s)^3}$$

$$= 6.57\times10^{-14}\,J-s/m^3$$

$$\frac{B_{ul}}{A_{ul}} = 1.52\times10^{13}\,m^3/J-s$$

Energy density *u*(*v*) is related to intensity per unit frequency. Intensity *I*(*v*) can be obtained by dividing laser beam power in cavity by beam cross-sectional area & frequency width of beam.

Power of beam within cavity traveling toward output mirror must be 100 mW & that reflected would be 99 mW (1 mW passes through mirror). Thus, total power in cavity is 199 mW.

Doppler width of He-Ne 632.8 nm transition is 1.5 × 10^9 Hz.

$$u(\upsilon) = \frac{I(\upsilon)}{c} = \frac{[(199\times1.0mW)/(\pi.(5\times10^{-4}\,m)^2)]/(0.1)(1.5\times10^9\,Hz)}{3\times10^8\,m/s}$$

$$= 5.63\times10^{-12}\,J-s/m^3$$

$$\frac{B_{ul}u(\upsilon)}{A_{ul}} = (1.52\times10^{13}\, m^3 / J-s)(5.63\times10^{-12}\, J-s/m^3)$$

$$= 85.6$$

The stimulated emission rate is therefore almost 86 times the spontaneous emission rate or transitions from upper to lower laser level at 632.8 nm.

$$\frac{B_{ul}u(\upsilon)}{A_{ul}} = \frac{1}{e^{h\upsilon_{ul}/kT}-1} = 85.6$$

$$e^{h\upsilon_{ul}/kT} - 1 = 1/85.6 = 0.0117$$

$$e^{h\upsilon_{ul}/kT} = \ln(1.0117) = 1.16\times10^{-2}$$

$$T = \frac{h\upsilon_{ul}}{(1.16\times10^{-2})k} = \frac{(6.63\times10^{-34}\, J-s)(4.74\times10^{14}\, Hz)}{(1.16\times10^{-2})(1.38\times10^{-23}\, J/K)}$$

$$= 1.96\times10^{6}\, K = 19{,}60{,}000 K$$

Radiation intensity inside laser cavity has a value equivalent to that of a nearly 20,00,000 K blackbody – if we consider only radiation emitted from blackbody in frequency (or wavelength interval) over which laser operates.

Einstein *A* & *B* coefficients are associated with interaction of radiation with two specific energy levels, where radiation has exact frequency corresponding to energy separation between two levels.

How gain (amplification) & absorption of radiation can occur in a medium containing populations in those two levels?