Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2013-14
Mid Semester Examination

Maximum Marks: 30 Total Time: 2 Hours

Note: This question paper has two pages and contains ten questions. Answer all questions.

1. (i) Suppose that a fair coin is tossed twice. Assume that A denotes the event of heads on the first toss, B denotes the event of heads on the second toss and C denotes the event that exactly one head is thrown. Show that event A and C are independent. Also properly verify whether A, B and C are independent events? [1 + 1]
   (ii) Consider arbitrary events A and B. Then applying proper arguments prove that $P(A \cap B) \geq P(A) + P(B) - 1$. [1]
   (iii) Let A, B and C be independent events. Then prove that $P(A \cup B \cup C) = 1 - (1 - P(A))(1 - P(B))(1 - P(C))$. [1]

2. A signal S is sent from point A to point B and is received at B if both switches I and II are closed. It is assumed that the probabilities of I and II being closed are 0.8 and 0.6, respectively, and that $P(II \text{ is closed} | I \text{ is closed}) = P(II \text{ is closed})$. Find the probability that: (i) switch I was open given that the signal was not received at B. (ii) switch II was open given that the signal was not received at B. [1+1]

3. Drawers I and II contain black and red pencil as follows. Drawer I: b_1 black pencils and r_1 red pencils, Drawer II: b_2 black pencils and r_2 red pencils. A drawer is chosen at random and then a pencil is also chosen at random from the drawer. If it is announced that the pencil is black, find the probability that it was chosen from the drawer I. [2]

4. Let X be a random variable with PDF $f_X(x) = c(1 - x^2), \quad -1 \leq x \leq 1$ where c is a constant to be determined. Use Chebyshev inequality to find a lower bound for the probability $P(-0.9 < X < 0.9)$. Compare this bound with the corresponding exact probability. [1+1+1].

5. Suppose that a scientist has collected 6 specimens of a certain rock say, R_1 and 9 specimens of another rock say, R_2. Then a laboratory assistant selects at random 3 specimens for further analysis. Let X be the random variable denoting the number of specimens of rock R_1 selected for analysis. Write the probability distribution of X. Find the probability that at most 2 specimens of the rock R_1 are included in the analysis. Use definition to find the expected number of specimens of rock R_1 selected for analysis. [1+1+2]

6. Let PDF of a random variable X is given by $f_X(x) = \frac{2}{\pi[4+(x-1)^2]}, \quad -\infty < x < \infty$. Find x such that probability $P(X > x) = 0.25$. [2]

7. Consider a random variable X with PDF $f_X(x) = \frac{1+0.5x}{2}, \quad -1 \leq x \leq 1$ and $f_X(x) = 0$, otherwise. Find quartiles of X. [3]

8. IIT Patna has started a high $VALUE$ fellowship scheme to attract good students. A student who has four letters V, A, L and U keeps visiting to the administrative office until he receives the letter E. Assume that the probability is 0.01 of receiving the letter E on any random visit. Let X denotes the random variable the number of times student visits the office until he/she receives the letter E for the first time. Find the probability that student visits the office at least two times. Use definition to find the expected number of times the student visits the office and also properly evaluate the function $M_X(t)$. [1+1.5+1.5]

9. A company produces a particular type of microchips. There is a 0.2% chance that any given microchip of such type is defective independent of the other microchip produced. Let X denotes random variable the number of defective microchips in a shipment of 100. Find the probability that there are at least three defective microchips in the shipment. Calculate the approximate probability using Poisson approximation. [1+1]

10. (i) The members of a consulting firm rent 40%, 30% and 20% of cars from rental agencies A, B and C respectively. If 8%, 16% and 4% of cars from A, B and C agencies need tune up and if a rental car delivered to the firm does not need tune up, what is the probability that it came from C agency? [1+1]
(ii) Let random variable X follows a gamma $G(\alpha, \beta)$ distribution. Find the most probable value of X and also calculate $E(X^3)$. [1+1]