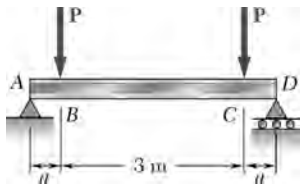
CHAPTER 8



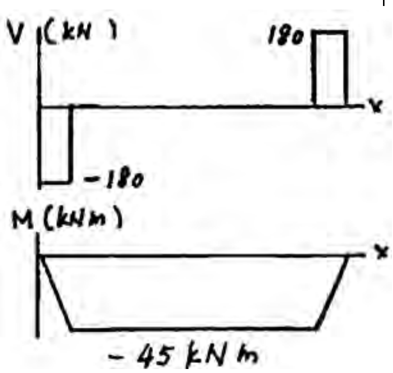
PROBLEM 8.1

A W250×101 rolled-steel beam supports a load **P** as shown. Knowing that $P = 180$ kN, $a = 0.25$ m, and $\sigma_{\text{all}} = 126$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION

$$|V|_{\max} = 180 \text{ kN}$$

$$|M|_{\max} = (180)(0.25) = 45 \text{ kNm}$$



For W250×101 rolled steel section,

$$d = 264 \text{ mm} \quad b_f = 257 \text{ mm} \quad t_f = 19.6 \text{ mm}$$

$$t_w = 11.9 \text{ mm} \quad I_x = 164\times10^6 \text{ mm}^4 \quad S_x = 1240\times10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 132 \text{ mm} \qquad y_b = c - t_f = 112.4 \text{ mm}$$

(*a*) $$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{45\times10^3}{1240\times10^{-6}}$$ $\sigma_m = 36.3$ MPa ◀

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{112.4}{132}\right)(36.3) = 30.9 \text{ MPa}$$

$$A_f = b_f t_f = 5037 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 122.2 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 615521 \text{ mm}^3$$

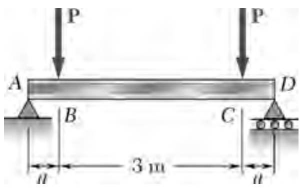
$$\tau_{xy} = \frac{|V|_{\max} Q_b}{I_x t_w} = \frac{(180\times10^3)(615521\times10^{-9})}{(164\times10^{-6})(0.0119)} = 56.77 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 58.83 \text{ MPa}$$

(*b*) $$\sigma_{\max} = \frac{\sigma_b}{2} + R = 74.29 \text{ MPa}$$ $\sigma_{\max} = 74.3$ MPa ◀

(*c*) Since $\sigma_{\max} < \sigma_{\text{all}}\,(=126 \text{ MPa})$, W250 × 101 is acceptable. ◀

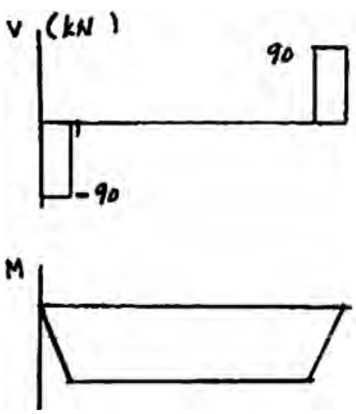
PROBLEM 8.2



Solve Prob. 8.1, assuming that $P = 90$ kN, $a = 0.5$ m.

PROBLEM 8.1 A $W250\times101$ rolled-steel beam supports a load **P** as shown. Knowing that P = 180 kN, $a = 0.25$ m, and $\sigma_{\text{all}} = 126$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION



$|V|_{\max} = 90$ kN

$|M|_{\max} = (90)(0.5) = 45$ kNm

For $W250\times101$ rolled steel section,

$$d = 264 \text{ mm} \quad b_f = 257 \text{ mm} \quad t_f = 19.6 \text{ mm}$$

$$t_w = 11.9 \text{ mm} \quad I_x = 164\times10^6 \text{ mm}^4 \quad S_x = 1240\times10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 132 \text{ mm} \qquad y_b = c - t_f = 112.4 \text{ mm}$$

(*a*) $$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{45\times10^3}{1240\times10^{-6}}$$ $\sigma_m = 36.3$ MPa ◀

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{112.4}{132}\right)(36.3) = 30.9 \text{ MPa.}$$

$$A_f = b_f t_f = 5037 \text{ mm}^2$$

$$\overline{y}_f = \frac{1}{2}(c + y_b) = 122.2 \text{ mm}$$

$$Q_b = A_f\overline{y}_f = 615521 \text{ mm}^3$$

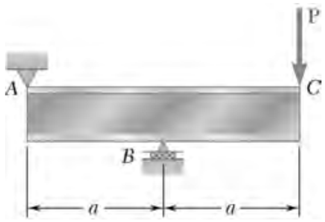
$$\tau_{xy} = \frac{|V|_{\max}Q_b}{I_x t_w} = \frac{(45\times10^3)(615521\times10^{-9})}{(164\times10^{-6})(0.119)} = 4.12 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 15.99 \text{ MPa}$$

(*b*) $$\sigma_{\max} = \frac{\sigma_b}{2} + R = 31.44 \text{ MPa}$$ $\sigma_{\max} = 31.4$ MPa ◀

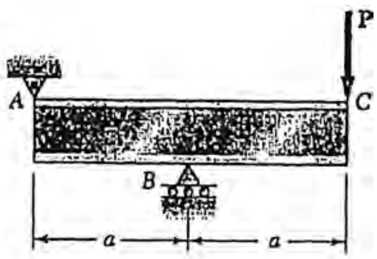
(*c*) Since $\sigma_{\max} < \sigma_{\text{all}}$ (= 126 MPa), $W250\times101$ is <u>acceptable</u>. ◀

PROBLEM 8.3



An overhanging W920×449 rolled-steel beam supports a load **P** as shown. Knowing that $P = 700$ kN, $a = 2.5$ m, and $\sigma_{\text{all}} = 100$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION



$$|V|_{\max} = 700 \text{ kN} = 700\times10^3 \text{ N}$$

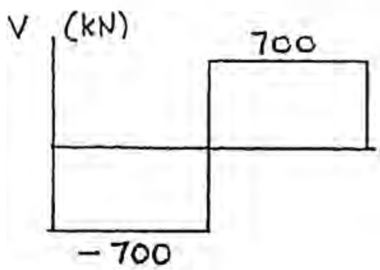
$$|M|_{\max} = (700\times10^3)(2.5) = 1.75\times10^6 \text{ N}\cdot\text{m}$$

For W920×449 rolled steel beam,

$$d = 947 \text{ mm}, \quad b_f = 424 \text{ mm}, \quad t_f = 42.7 \text{ mm},$$

$$t_w = 24.0 \text{ mm}, \quad I_x = 8780\times10^6 \text{ mm}^4, \quad S_x = 18{,}500\times10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 473.5 \text{ mm}, \quad y_b = c - t_f = 430.8 \text{ mm}$$



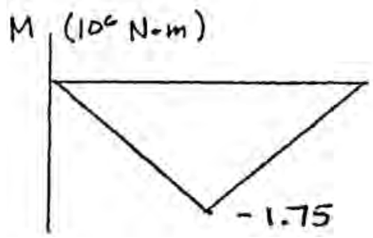
(*a*) $$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{1.75\times10^6}{18500\times10^{-6}}$$

$\sigma_m = 94.595$ MPa

$\sigma_m = 94.6$ MPa ◀

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{430.8}{473.5}(94.595) = 86.064 \text{ MPa}$$

$$A_f = b_f t_t = 18.1048\times10^3 \text{ mm}^2$$



$$\bar{y}_f = \frac{1}{2}(c + y_b) = 452.15 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 8186.1\times10^3 \text{ mm}^3 = 8186.1\times10^{-6} \text{ m}^3$$

$$\tau_{xy} = \frac{|V|_{\max} Q_b}{I_x t_w} = \frac{(700\times10^3)(8186.1\times10^{-6})}{(8780\times10^{-6})(24.0\times10^{-3})} = 27.194 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{86.064}{2}\right)^2 + 27.194^2} = 50.904 \text{ MPa}$$

(*b*) $$\sigma_{\max} = \frac{\sigma_b}{2} + R = 93.9 \text{ MPa}$$

$\sigma_{\max} = 93.9$ MPa ◀

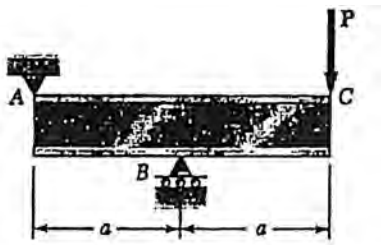
(*c*) Since $94.6 \text{ MPa} < \sigma_{\text{all}} (= 100 \text{ MPa})$, W920×449 is <u>acceptable</u>. ◀

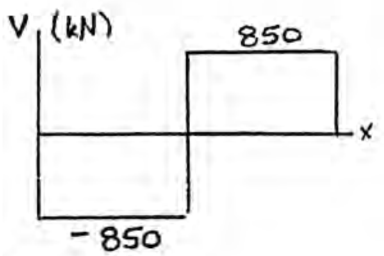
PROBLEM 8.4

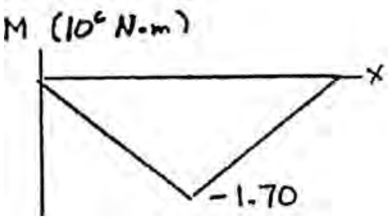
Solve Prob. 8.3, assuming that $P = 850$ kN and $a = 2.0$ m.

PROBLEM 8.3 An overhanging W920×449 rolled-steel beam supports a load **P** as shown. Knowing that $P = 700$ kN, $a = 2.5$ m, and $\sigma_{\text{all}} = 100$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION







$$|V|_{\max} = 850 \text{ kN} = 850\times10^3 \text{ N}$$

$$|M|_{\max} = (850\times10^3)(2.0) = 1.70\times10^6 \text{ N}\cdot\text{m}$$

For W920×449 rolled steel section,

$$d = 947 \text{ mm}, \quad b_f = 424 \text{ mm}, \quad t_f = 42.7 \text{ mm},$$

$$t_w = 24.0 \text{ mm}, \quad I_x = 8780\times10^6 \text{ mm}^4, \quad S_x = 18500\times10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 473.5 \text{ mm} \qquad y_b = c - t_f = 430.8 \text{ mm}$$

(*a*) $$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{1.70\times10^6}{18,500\times10^{-6}} \qquad \sigma_m = 91.892 \text{ MPa}$$

$\sigma_m = 91.9$ MPa ◀

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{430.8}{473.5}(91.892) = 83.605 \text{ MPa}$$

$$A_f = b_f t_f = 18.1048\times10^3 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 452.15 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 8186.1\times10^3 \text{ mm}^3 = 8186.1\times10^{-6} \text{ m}^3$$

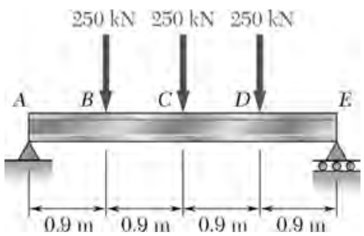
$$\tau_{xy} = \frac{|V|_{\max} Q_b}{I_x t_w} = \frac{(850\times10^3)(8186.1\times10^{-6})}{(8780\times10^{-6})(24.0\times10^{-3})} = 33.021 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{83.605}{2}\right)^2 + 33.021^2} = 53.271 \text{ MPa}$$

(*b*) $$\sigma_{\max} = \frac{\sigma_b}{2} + R = 95.1 \text{ MPa}$$

$\sigma_{\max} = 95.1$ MPa ◀

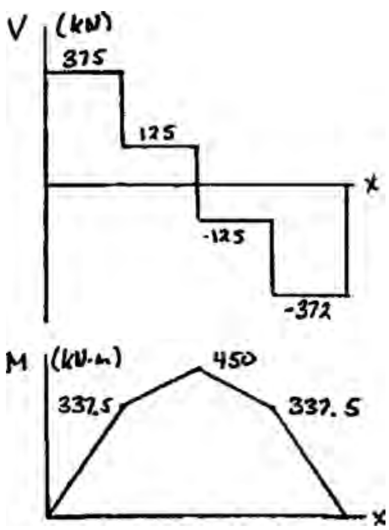
(*c*) Since 95.1 MPa < σ_{all} (= 100 MPa), W920×449 is acceptable. ◀



PROBLEM 8.5

(*a*) Knowing that $\sigma_{\text{all}} = 160$ MPa and $\tau_{\text{all}} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress $\sigma_{\max}$ at the junction of a flange and the web of the selected beam.

SOLUTION



Reactions: $R_A = 375 \text{ kN}\uparrow,\ R_E = 375 \text{ kN}\uparrow$

$|V|_{\max} = 375$ kN

$|M|_{\max} = 450$ kN·m

$|V|$ at point C: 125 kN

$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{450\times10^3}{160\times10^6} = 2.8125\times10^{-3}\ \text{m}^3$$

$$= 2812.5\times10^3\ \text{mm}^3$$

	Shape	$S_x\ (10^3\ \text{mm}^3)$
	W840×176	5890
	W760×147	4410
→	W690×125	3510
	W610×155	4220
	W530×150	3720
	W460×158	3340
	W360×216	3800

(*a*) Use W690×125. ◀

$d = 678$ mm $\quad t_f = 16.30$ mm $\quad t_w = 11.7$ mm

(*b*) $$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{450\times10^3}{3510\times10^{-6}} = 128.2\times10^6\ \text{Pa}$$ $\sigma_m = 128.2$ MPa ◀

$$\tau_m = \frac{|V|_{\max}}{A_w} = \frac{|V|_{\max}}{dt_w} = \frac{375\times10^3}{(678\times10^{-3})(11.7\times10^{-3})}$$ $\tau_m = 47.3$ MPa ◀

PROBLEM 8.5 *(Continued)*

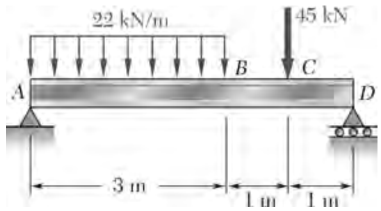
At point C, $\tau_w = \dfrac{V}{A_w} = \dfrac{125\times 10^3}{(678\times 10^{-3})(11.7\times 10^{-3})} = 15.76\times 10^6 \text{ Pa} = 15.76 \text{ MPa}$

$$c = \frac{1}{2}d = \frac{678}{2} = 339 \text{ mm} \qquad y_b = c - t_f = 339 - 16.30 = 322.7 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{322.7}{339}\right)(128.2) = 122.0 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(61.0)^2 + (15.76)^2} = 63.0 \text{ MPa}$$

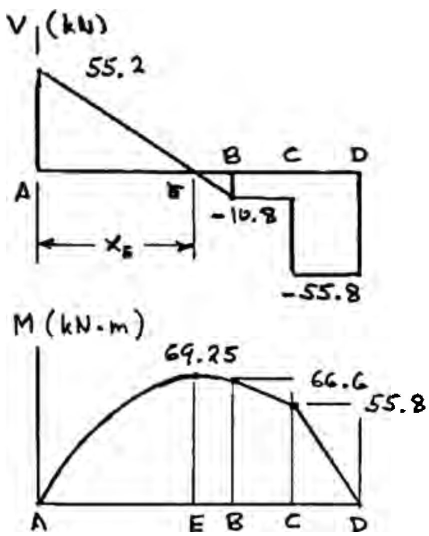
$$\sigma_{\max} = \frac{\sigma_b}{2} + R = 61.0 + 63.0 \qquad \sigma_{\max} = 124.0 \text{ MPa} ◀$$



PROBLEM 8.6

(*a*) Knowing that $\sigma_{\text{all}} = 160$ MPa and $\tau_{\text{all}} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress $\sigma_{\max}$ at the junction of a flange and the web of the selected beam.

SOLUTION



$+\circlearrowright \Sigma M_D = 0$:

$$-5R_A + (3.5)(3)(22) + (1)(45) = 0$$

$$R_A = 55.2 \text{ kN} \uparrow \quad R_D = 55.8 \text{ kN} \uparrow$$

Draw shear and bending moment diagrams.

Locate point where $V = 0$ and M_E.

$$55.2 - 22x_E = 0 \quad x_E = 2.5091 \text{ m.}$$

$$M_E = \frac{1}{2}(55.2)(2.5091) = 69.25 \text{ kN}\cdot\text{m}$$

$$S_{\min} = \frac{|M_{\max}|}{\sigma_{\text{all}}} = \frac{69.25\times10^3}{160\times10^6} = 432.8\times10^{-6} \text{ m}^3$$

$$= 432.8\times10^3 \text{ mm}^3$$

Shape	$S(10^3 \text{mm}^3)$
W410 × 38.8	637
→ W360 × 32.9	474
W310 × 38.7	549
W250 × 44.8	535
W200 × 46.1	448

(*a*) Use W360 × 32.9 ◀

For $W360\times32.9 \quad S = 474\times10^3 \text{ mm}^3 = 474\times10^{-6} \text{ m}^3$

$$A_{\text{web}} = d\,t_w = (349)(5.8) = 2024.2 \text{ mm}^2 = 2024.2\times10^{-6} \text{ m}^2$$

At point *E*: $\sigma_m = \dfrac{M_E}{S} = \dfrac{69.25\times10^3}{474\times10^{-6}}$ $\qquad \sigma_m = 146.1$ MPa ◀

PROBLEM 8.6 *(Continued)*

At point C: $\sigma_m = \dfrac{M_c}{S} = \dfrac{55.8\times10^3}{474\times10^{-6}}$ $\sigma_m = 117.7 \text{ MPa}$

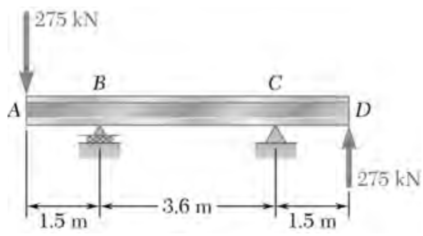
$$\tau_m = \frac{|V|}{A_{\text{web}}} = \frac{55.8\times10^3}{2024.2\times10^{-6}} \qquad \tau_m = 27.6 \text{ MPa} \blacktriangleleft$$

$$c = \frac{1}{2}d = \frac{1}{2}(349) = 174.5 \text{ mm} \qquad y_b = c - t_f = 166 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{166}{174.5}\right)(117.7) = 112.0 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = 62.4 \text{ MPa}$$

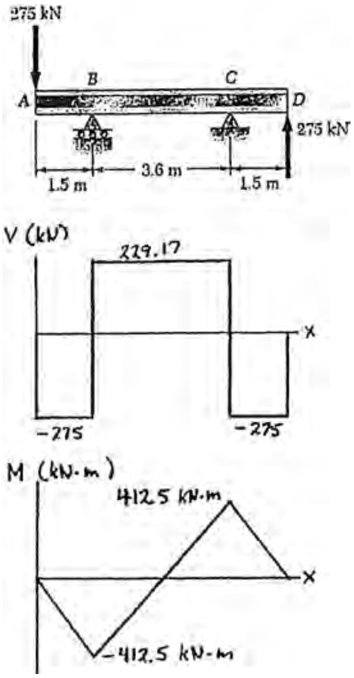
$$\sigma_{\max} = \frac{\sigma_b}{2} + R \qquad \sigma_{\max} = 118.4 \text{ MPa} \blacktriangleleft$$



PROBLEM 8.7

(*a*) Knowing that $\sigma_{all} = 160$ MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION



$$R_B = 504.17 \text{ kN} \uparrow \qquad R_C = 504.17 \downarrow$$

$$|V|_{max} = 275 \text{ kN} \qquad |M|_{max} = 412.5 \text{ kN} \cdot \text{m}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{412.5 \times 10^2}{160 \times 10^6} = 2578 \times 10^{-6} \text{ m}^3$$

$$= 2578 \times 10^3 \text{ mm}^3$$

	Shape	$S_x (10^3 \text{ mm}^3)$
	W760×147	4410
→	W690×125	3490
	W530×150	3720
	W460×158	3340
	W360×216	3800

(*a*) Use W690 × 125. ◀

$$d = 678 \text{ mm} \quad t_f = 16.3 \text{ mm} \quad t_w = 11.7 \text{ mm}$$

(*b*)
$$\sigma_m = \frac{|M|_{max}}{S} = \frac{412.5 \times 10^3}{3490 \times 10^{-6}} = 118.195 \times 10^6 \text{ Pa} \qquad \sigma_m = 118.2 \text{ MPa} ◀$$

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{dt_w} = \frac{275 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 34.667 \times 10^6 \text{ Pa} \qquad \tau_m = 34.7 \text{ MPa} ◀$$

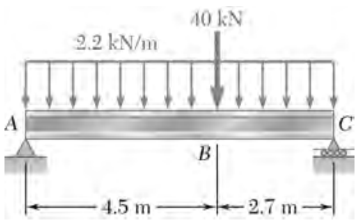
$$c = \frac{1}{2}d = \frac{67.8}{2} = 339 \text{ mm}, \quad t_f = 16.3 \text{ mm}, \quad y_b = c - t_f = 339 - 16.3 = 322.7 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{322.7}{339}\right)(118.195) = 112.512 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(56.256)^2 + (34.667)^2} = 66.080 \text{ MPa}$$

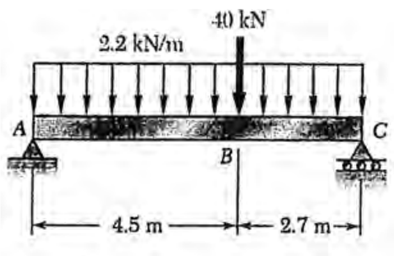
$$\sigma_{max} = \frac{\sigma_b}{2} + R = 56.256 + 66.080 \qquad \sigma_{max} = 122.3 \text{ MPa} ◀$$

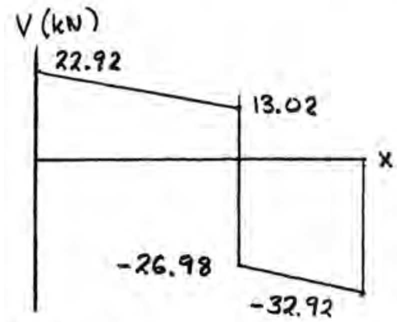
PROBLEM 8.8

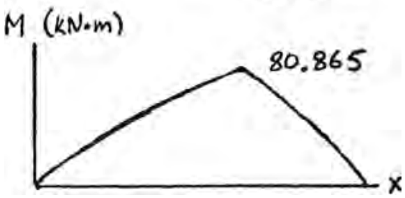


(a) Knowing that $\sigma_{\text{all}} = 160\text{ MPa}$ and $\tau_{\text{all}} = 100\text{ MPa}$, select the most economical metric wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m, τ_m, and the principal stress $\sigma_{\max}$ at the junction of a flange and the web of the selected beam.

SOLUTION







$$+\sum M_C = 0:$$

$$-7.2\,R_A + (2.2)(7.2)(3.6) + (40)(2.7) = 0$$

$$R_A = 22.92\text{ kN}$$

$$V_A = R_A = 22.92\text{ kN}$$

$$V_B^- = 22.92 - (2.2)(4.5) = 13.02\text{ kN}$$

$$V_B^+ = 13.02 - 40 = -26.98\text{ kN}$$

$$V_C = -26.98 - (2.2)(2.7) = -32.92\text{ kN}$$

$$M_A = 0$$

$$M_B = 0 + \frac{1}{2}(22.92 + 13.02)(4.5) = 80.865\text{ kN}\cdot\text{m}$$

$$M_C = 0$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{80.865\times 10^3}{165\times 10^6} = 490\times 10^{-6}\text{ m}^3$$

$$= 490\times 10^3\text{ mm}^3$$

Shape	S (10^3 mm^3)
W360 × 39	578
W310 × 38.7	547 ←
W250 × 44.8	531
W200 × 52	511

(a) Use W310 × 38.7. ◀

$d = 310$ mm $t_f = 9.65$ mm

$t_w = 5.84$ mm

PROBLEM 8.8 *(Continued)*

(*b*) $\sigma_m = \frac{M_B}{S} = \frac{80.865 \times 10^3}{547 \times 10^{-6}} = 147.834 \times 10^6 \text{ Pa}$

$$\sigma_m = 147.8 \text{ MPa} \blacktriangleleft$$

$$\tau_m = \frac{|V|_{\max}}{dt_w} = \frac{32.92 \times 10^3}{(310 \times 10^{-3})(5.84 \times 10^{-3})} = 18.1838 \times 10^6 \text{ Pa}$$

$$\tau_m = 18.18 \text{ MPa} \blacktriangleleft$$

$$c = \frac{1}{2}d = 155 \text{ mm} \qquad y_b = c - t_f = 155 - 9.65 = 145.35 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{145.35}{155}\right)(147.834) = 138.630 \text{ MPa}$$

At point *B*, $\tau_w = \frac{V}{dt_w} = \frac{(26.98 \times 10^3)}{(310 \times 10^{-3})(5.84 \times 10^{-3})} = 14.9028 \text{ MPa}$

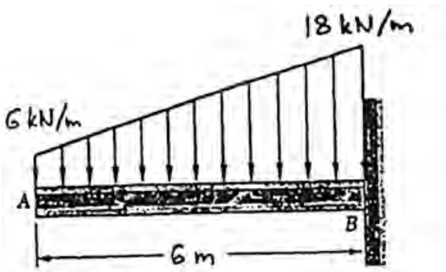
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(69.315)^2 + (14.9028)^2} = 70.899 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R = 69.315 + 70.899 \qquad \sigma_{\max} = 140.2 \text{ MPa} \blacktriangleleft$$

PROBLEM 8.9

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design (use the loading of Prob. 5.73 and selected W530 × 66 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION



From Prob. 5.73, $\sigma_{all} = 160$ MPa

$|M|_{max} = 180$ kN·m at section *B*.

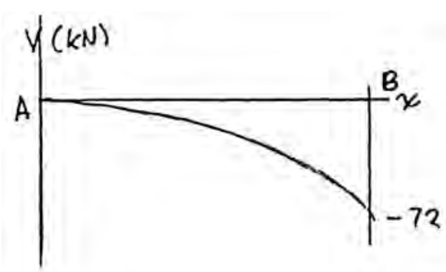
$|V| = 72$ kN at section *B*.

For W530×66 rolled steel section,

$d = 526$ mm, $\quad b_f = 165$ mm, $\quad t_f = 11.4$ mm,

$t_w = 8.89$ mm, $\quad I = 351 \times 10^6 \text{ mm}^4$, $\quad S = 1340 \times 10^3 \text{mm}^3$

$$c = \frac{1}{2}d = 263 \text{ mm}$$

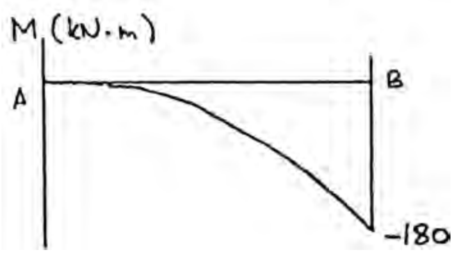


(*a*) $\quad |M|_{max} = 180 \times 10^3 \text{ N} \cdot \text{m} \qquad S = 1340 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{max}}{S} = 134.328 \times 10^6, \qquad \sigma_m = 134.3 \text{ MPa} \blacktriangleleft$$

(*b*) $\quad y_b = c - t_f = 251.6 \text{ mm} \qquad \bar{y} = \frac{1}{2}(c + y_b) = 257.3 \text{ mm}$

$$A_f = b_f t_f = 1881 \text{ mm}^2$$



At section *B*, $\qquad V = 72 \times 10^3$ N

$$\tau_b = \frac{VQ}{It} = \frac{VA_f \bar{y}}{It} = \frac{(72 \times 10^3)(1881 \times 10^{-6})(257.3 \times 10^{-3})}{(351 \times 10^{-6})(8.89 \times 10^{-3})}$$

$$= 11.1674 \times 10^6 = 11.1674 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{257.3}{263}(134.328) = 131.417 \text{ MPa}$$

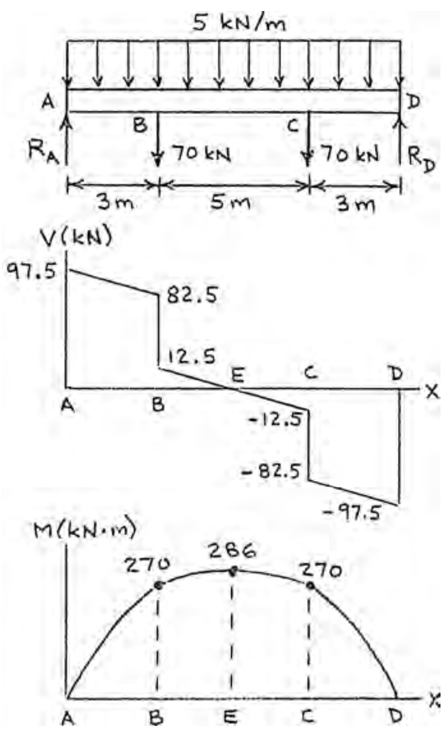
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 66.651 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = \frac{131.417}{2} + 66.651 \qquad \sigma_{max} = 132.4 \text{ MPa} \blacktriangleleft$$

PROBLEM 8.10

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{\text{all}}$. For the selected design (use the loading of Prob. 5.74 and selected W530 × 92 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

SOLUTION



Reactions: $R_A = 97.5$ kN ↑ $R_D = 97.5$ kN ↑

$|V|_{\max} = 97.5$ kN

$|M|_{\max} = 286$ kN · m

For W530 × 92 rolled steel section,

$$d = 533 \text{ mm}, \quad b_f = 209 \text{ mm}, \quad t_f = 15.6 \text{ mm},$$

$$t_w = 10.2 \text{ mm}, \quad c = \frac{1}{2}d = 266.5 \text{ mm}$$

$$I = 554 \times 10^6 \text{ mm}^4 \quad S = 2080 \times 10^3 \text{ mm}^3$$

(*a*) $$\sigma_m = \frac{|M|_{\max}}{S} = \frac{286 \times 10^3}{2080 \times 10^{-6}} = 137.5 \times 10^6 \text{ Pa}$$

$\sigma_m = 137.5$ MPa ◀

$$y_b = c - t_f = 250.9 \text{ mm}$$

$$A_f = b_f t_f = 3260.4 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 258.7 \text{ mm}$$

$$Q = A_f \bar{y} = 843.47 \times 10^3 \text{ mm}^3$$

At midspan: $V = 0 \quad \tau_b = 0$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{250.9}{266.5}(137.5) = 129.5 \text{ MPa}$$

$\sigma_{\max} = 129.5$ MPa ◀

PROBLEM 8.10 *(Continued)*

At sections B and C:

$$\sigma_m = \frac{M}{S} = \frac{270\times 10^3}{2080\times 10^{-6}} = 129.808 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{250.9}{266.5}(129.808) = 122.209 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f\bar{y}}{It_w} = \frac{(82.5\times 10^3)(3260.4\times 10^{-6})(258.7\times 10^{-3})}{(554\times 10^{-6})(10.2\times 10^{-3})}$$

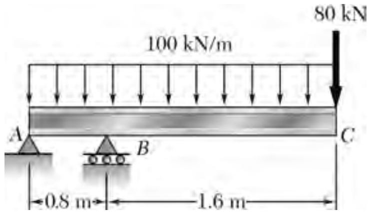
$$= 12.3143 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 62.333 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R$$

$\sigma_{\max} = 123.4 \text{ MPa}$ ◀

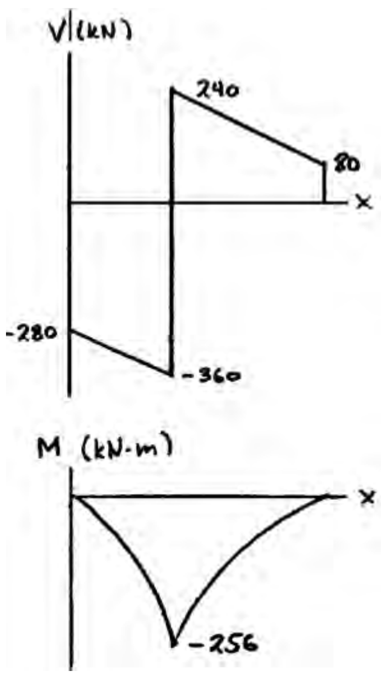
PROBLEM 8.11



Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{\text{all}}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

PROBLEM 8.12 Loading of Prob. 5.75 and selected $S20 \times 66$ shape.

SOLUTION



From Problem 5.75 $\quad \sigma_{\text{all}} = 160$ MPa

$$|M|_{\max} = 256 \text{ kN}\cdot\text{m at point } B$$

$$|V| = 360 \text{ kN at } B$$

For $S20 \times 66$ rolled steel section

$$d = 508 \text{ mm}, \qquad b_f = 159 \text{ mm}, \qquad t_f = 20.2 \text{ mm}$$

$$t_w = 12.8 \text{ mm}, \qquad I_x = 495 \times 10^6 \text{ mm}^4, \qquad S_z = 1950 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 254 \text{ mm}$$

$$(a) \quad \sigma_m = \frac{|M|_{\max}}{S_x} = \frac{256 \times 10^3}{1950 \times 10^{-6}} = 131.3 \text{ MPa} \quad \blacktriangleleft$$

$$y_b = c - t_f = 233.8$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 120.9 \text{ MPa} \qquad \frac{\sigma_b}{2} = 60.45 \text{ MPa}$$

$$A_f = b_f t_f = 3212 \text{ mm}^2$$

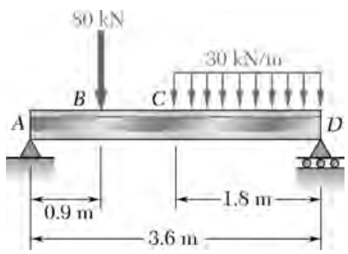
$$\overline{y} = \frac{1}{2}(c + y_b) = 243.9 \text{ mm}$$

$$Q = A_f \overline{y} = 783.4 \times 10^3 \text{ mm}^3$$

$$\tau_b = \frac{VQ}{I t_w} = \frac{(360 \times 10^3)(783.4 \times 10^{-6})}{(495 \times 10^{-6})(12.8 \times 10^{-3})} = 44.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{60.45^2 + 44.5^2} = 75.06 \text{ MPa}$$

$$(b) \quad \sigma_{\max} = \frac{\sigma_b}{2} + R = 60.45 + 75.06 = 135.5 \text{ MPa} \quad \blacktriangleleft$$

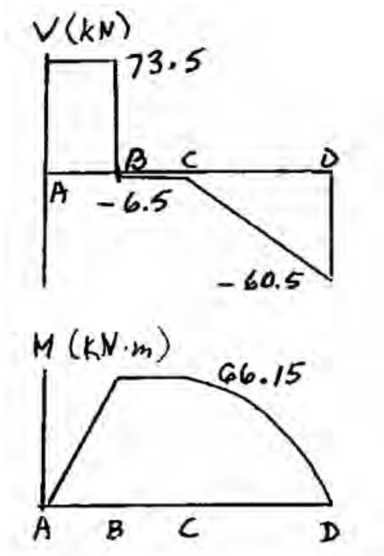


PROBLEM 8.12

Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{\text{all}}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

PROBLEM 8.11 Loading of Prob. 5.78 and selected $S310 \times 47.3$ shape.

SOLUTION



From Problem 5.78 $\sigma_{\text{all}} = 160$ MPa

$|M|_{\max} = 66.15\ \text{kN}\cdot\text{m}$ at section B.

$|V| = 73.5$ kN at B

For $S310 \times 47.3$ rolled steel section

$$d = 305\ \text{mm}, \qquad b_f = 127\ \text{mm}, \qquad t_p = 13.8\ \text{mm}$$

$$t_w = 8.9\ \text{mm}, \qquad I = 90.6\times 10^6\ \text{mm}^4, \qquad S = 593\times 10^3\ \text{mm}^3$$

$$c = \frac{1}{2}d = 152.5\ \text{mm}$$

(*a*) $$\sigma_m = \frac{|M|_{\max}}{S} = \frac{66.15\times 10^3}{593\times 10^{-6}}$$ $\sigma_m = 111.6$ MPa ◀

$$y_b = c - t_f = 138.7\ \text{mm}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 101.5\ \text{MPa}$$

$$A_f = b_f t_f = 1752.6\ \text{mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 145.6\ \text{mm}$$

$$Q = A_f \bar{y} = 255.18\times 10^3\ \text{mm}^3 = 255.18\times 10^{-6}\ \text{m}^3$$

$$\tau_b = \frac{VQ}{I\,t_w} = \frac{(73.5\times 10^3)(255.18\times 10^{-6})}{(90.6\times 10^{-6})(8.9\times 10^{-3})} = 23.26\ \text{MPa}$$

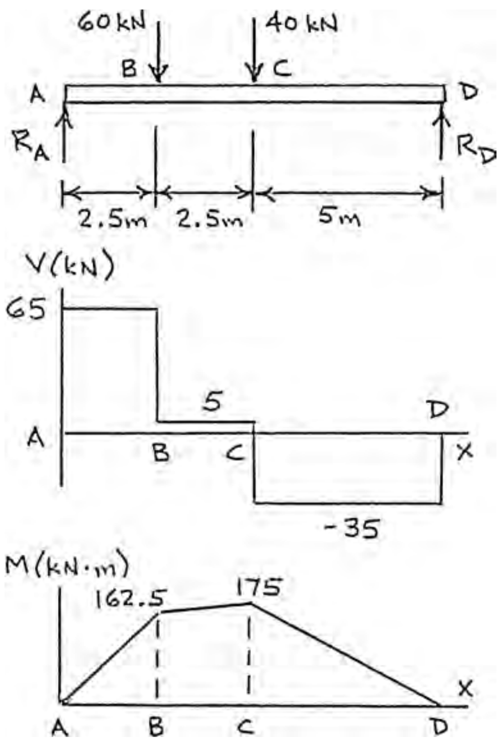
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{50.75^2 + 23.26^2} = 55.83\ \text{MPa}$$

(*b*) $$\sigma_{\max} = \frac{\sigma_b}{2} + R = 106.6\ \text{MPa}$$ $\sigma_{\max} = 106.6$ MPa ◀

PROBLEM 8.13

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{\text{all}}$. For the selected design (use the loading of Prob. 5.75 and selected S460 × 81.4 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

SOLUTION



Reactions: $R_A = 65 \text{ kN} \uparrow \qquad R_D = 35 \text{ kN} \uparrow$

$$|V|_{\max} = 65 \text{ kN}$$
$$|M|_{\max} = 175 \text{ kN} \cdot \text{m}$$

For S460×81.4 rolled steel section,

$$d = 457 \text{ mm}, \qquad b_f = 152 \text{ mm}, \qquad t_f = 17.6 \text{ mm},$$

$$t_w = 11.7 \text{ mm}, \qquad c = \frac{1}{2}d = 228.5 \text{ mm}$$

$$I = 333 \times 10^6 \text{ mm}^4 \qquad S = 1460 \times 10^3 \text{ mm}^3$$

(*a*)
$$\sigma_m = \frac{|M|_{\max}}{S} = \frac{175 \times 10^3}{1460 \times 10^{-6}} = 119.863 \times 10^6 \text{ Pa}$$

$$\sigma_m = 119.9 \text{ MPa} \blacktriangleleft$$

(*b*)
$$y_b = c - t_f = 210.9 \text{ mm}$$

$$A_f = b_f t_f = 2675.2 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 219.7 \text{ mm}$$

$$Q = A_f \bar{y} = 587.74 \times 10^3 \text{ mm}^3$$

At section *C*:
$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{210.9}{228.5}(119.863) = 110.631 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f\bar{y}}{It_w} = \frac{(35 \times 10^3)(2675.2 \times 10^{-6})(219.7 \times 10^{-3})}{(333 \times 10^{-6})(11.7 \times 10^{-3})} = 5.2799 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 55.567 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R$$

$$\sigma_{\max} = 110.9 \text{ MPa} \blacktriangleleft$$

PROBLEM 8.13 *(Continued)*

At section B: $$\sigma_m = \frac{M}{S} = \frac{162.5 \times 10^3}{1460 \times 10^{-6}} = 111.301 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{210.9}{228.5}(111.301) = 102.728 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f \bar{y}}{It_w} = \frac{(65 \times 10^3)(2675.2 \times 10^{-6})(219.7 \times 10^{-3})}{(333 \times 10^{-6})(11.7 \times 10^{-3})} = 9.8055 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 52.292 \text{ MPa}$$

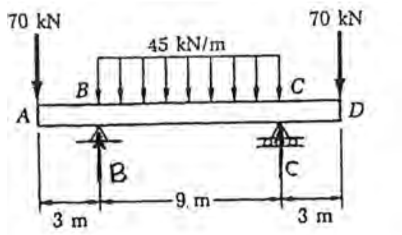
$$\sigma_{\max} = \frac{\sigma_b}{2} + R$$

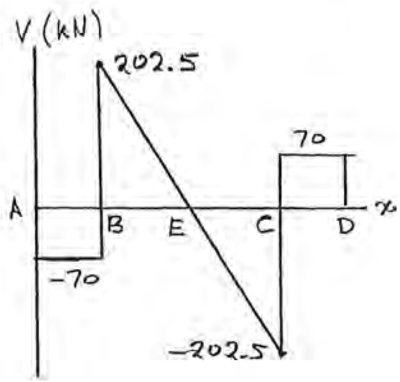
$\sigma_{\max} = 103.7 \text{ MPa}$ ◀

PROBLEM 8.14

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{\text{all}}$. For the selected design (use the loading of Prob. 5.76 and selected S510 × 98.2 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

SOLUTION





From Prob. 5.76, $\sigma_{\text{all}} = 160$ MPa

Bending moments:

$-210\ \text{kN}\cdot\text{m}$ at sections *B* and *C*.

$245.625\ \text{kN}\cdot\text{m}$ at midspan *E*.

$|V|_{\max} = 202.5$ kN at sections *B* and *C*.

For $S510\times 98.2$ rolled steel section,

$$d = 508\text{ mm},\quad b_f = 159\text{ mm},\quad t_f = 20.2\text{ mm},$$

$$t_w = 12.8\text{ mm},\quad I = 495\times 10^6\ \text{mm}^4,\quad S = 1950\times 10^3\ \text{mm}^3$$

$$c = \frac{1}{2}d = 254\text{ mm}$$

(*a*) $|M|_{\max} = 245.625\times 10^3\ \text{N}\cdot\text{m}$ $\qquad S = 1960\times 10^{-6}\ \text{m}^3$

$$\sigma_m = \frac{|M|_{\max}}{S} = \frac{245.625\times 10^3}{1950\times 10^{-6}} = 125.962\times 10^6\ \text{Pa}\qquad \sigma_m = 126.0\text{ MPa} ◀$$

(*b*) $y_b = c - t_f = 233.8$ mm, $\quad A_f = b_f t_f = 3.212\times 10^3\ \text{mm}^2$, $\quad \bar{y} = \frac{1}{2}(c + y_b) = 243.9$ mm

At midspan: $V = 0 \qquad \tau_b = 0$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{233.8}{254}(125.962) = 115.9\text{ MPa}\qquad \sigma_{\max} = 115.9\text{ MPa} ◀$$

At sections *B* and *C*:

$$\sigma_m = \frac{M}{S} = \frac{210\times 10^3}{1950\times 10^{-6}} = 107.69\text{ MPa}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{233.8}{254}(107.69) = 99.126\text{ MPa}$$

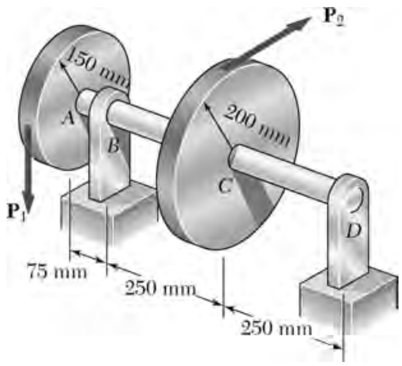
$$\tau_b = \frac{VQ}{It} = \frac{VA_f\bar{y}}{It_w} = \frac{(202.5\times 10^3)(3.212\times 10^{-3})(243.9\times 10^{-3})}{(495\times 10^{-6})(12.8\times 10^{-3})}$$

$$= 25.04\text{ MPa}$$

PROBLEM 8.14 *(Continued)*

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 55.53 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R \qquad \sigma_{\max} = 105.1 \text{ MPa} \blacktriangleleft$$



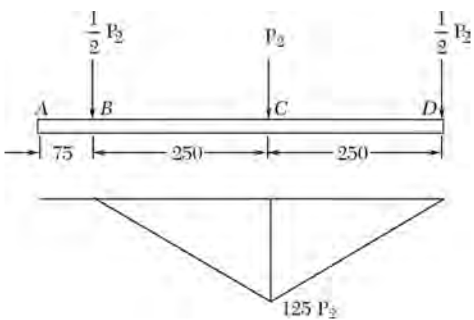
PROBLEM 8.15

The vertical force $\mathbf{P_1}$ and the horizontal force $\mathbf{P_2}$ are applied as shown to disks welded to the solid shaft *AD*. Knowing that the diameter of the shaft is 45 mm and that $\tau_{all} = 50$ MPa, determine the largest permissible magnitude of the force $\mathbf{P_2}$.

SOLUTION

Let P_2 be in kN

$$\Sigma M_{\text{shaft}} = 0 \qquad 150P_1 - 200P_2 = 0 \qquad P_1 = \frac{4}{3}P_2$$



Torque over portion *ABC* $\qquad T = 200P_2$

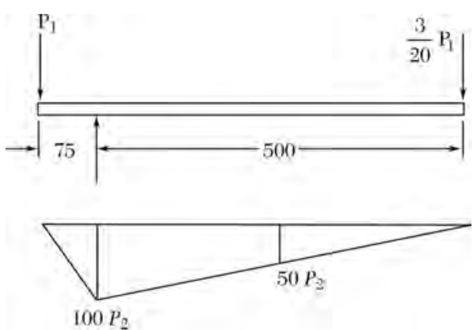
Bending in horizontal plane $\qquad M_{Cy} = 250 \cdot \frac{1}{2}P_2$

$$= 125P_2$$

Bending in vertical plane $\qquad M_{Bz} = 75P_1$

$$= 75 \cdot \frac{4}{3}P_2$$

$$= 100P_2$$



Critical point is just to the left of point *C*

$$T = 200P_2 \qquad M_y = 125P_2 \qquad M_z = 50P_2$$

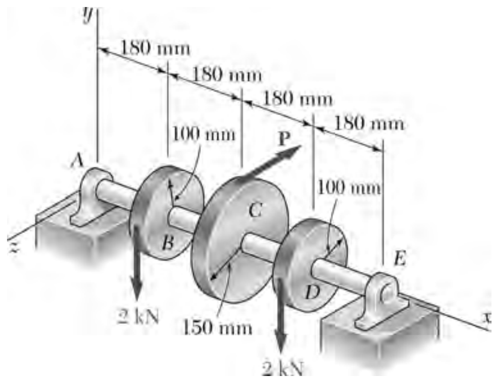
$$d = 45 \text{ mm} \qquad c = \frac{1}{2}d = 22.5 \text{ mm} \qquad J = \frac{\pi}{2}(22.5)^4$$

$$= 402.6\times10^3 \text{ mm}^4$$

$$\tau_{all} = \frac{c}{J}\sqrt{T^2 + M_y^2 + M_z^2}$$

$$50 = \frac{22.5}{402.6\times10^3}\sqrt{(200P_2)^2 + (125P_2)^2 + (50P_2)^2} = 0.01347\,P_2$$

$$P_2 = 3710 \text{ N} = 3.710 \text{ kN} \qquad \blacktriangleleft$$

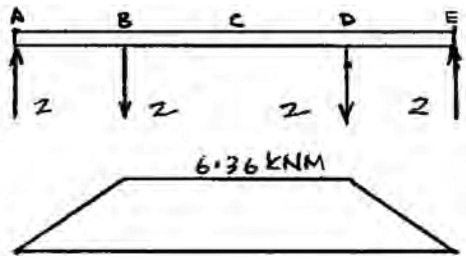


PROBLEM 8.16

The two 2-kN forces are vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{\text{all}} = 55\,\text{MPa}$, determine the smallest permissible diameter of the solid shaft AE.

SOLUTION

Forces in vertical plane



$$\Sigma M_x = 0: \quad (2)(100) - 150\text{P} + (2)(100) = 0$$

$$P = 2.67\ \text{kN}$$

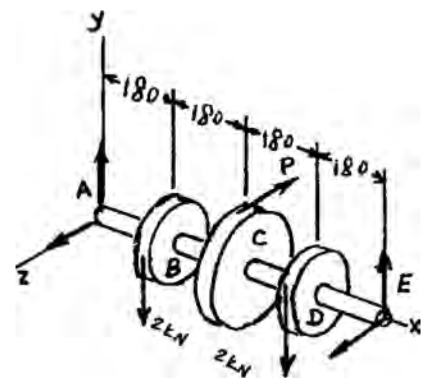
Torques:

$$AB: \quad T = 0$$

$$BC: \quad T = -(2)(0.1) = -0.2\ \text{kNm}$$

$$CD: \quad T = (2)(0.1) = 0.2\ \text{kNm}$$

$$DE: \quad T = 0$$



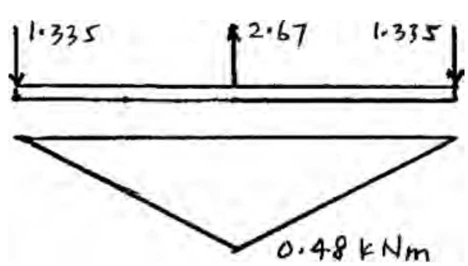
Critical sections are either side of disk C.

$$T = 0.2\ \text{kNm}$$

$$M_z = 0.36\ \text{kNm}$$

$$M_y = 0.48\ \text{kNm}$$

Forces in horizontal plane



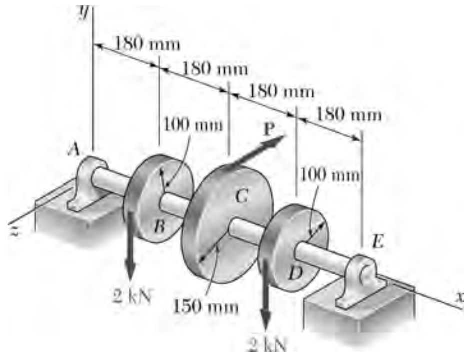
$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{\text{all}}}$$

$$= \frac{\sqrt{0.48^2 + 0.36^2 + 0.2^2}}{55 \times 10^3}$$

$$= 11.5 \times 10^{-6}\ \text{m}^3$$

$$c = 0.0194\ \text{m} \quad d = 2c = 38.8\ \text{mm}$$ ◀



PROBLEM 8.17

For the gear-and-shaft system and loading of Prob. 8.19, determine the smallest permissible diameter of shaft *AE*, knowing that the shaft is hollow and has an inner diameter that is $\frac{2}{3}$ the outer diameter.

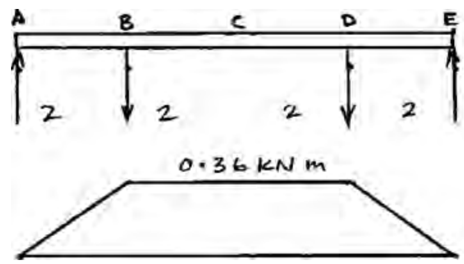
PROBLEM 8.19 The two 2-kN forces are vertical and the force **P** is parallel to the *z* axis. Knowing that $\tau_{\text{all}} = 55$ MPa, determine the smallest permissible diameter of the solid shaft *AE*.

SOLUTION

$$\Sigma M_x = 0: \quad (2)(100) - 150P + (2)(100) = 0$$

$$P = 2.67 \text{ kN}$$

Forces in vertical plane



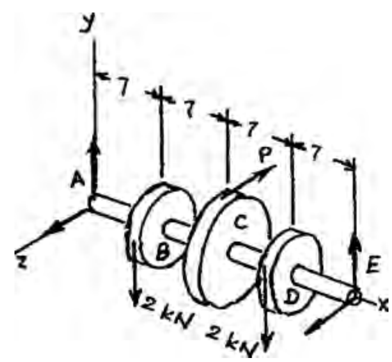
Torques:

$$AB: \quad T = 0$$

$$BC: \quad T = -(2)(0.1) = -0.2 \text{ kNm}$$

$$CD: \quad T = (2)(0.1) = 0.2 \text{ kNm}$$

$$DE: \quad T = 0$$



Critical sections are either side of disk *C*.

$$T = 0.2 \text{ kNm}$$

$$M_z = 0.36 \text{ kNm}$$

$$M_y = 0.48 \text{ kNm}$$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

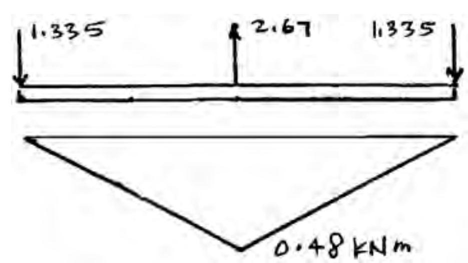
$$\frac{J}{c} = \frac{\pi}{2c}\left[c_o^4 - c_i^4\right] = \frac{\pi}{2c}\left[c^4 - \left(\frac{2}{3}c\right)^4\right]$$

$$= \frac{\pi}{2c}\frac{65c^4}{81} = \frac{65\pi}{162}c^3$$

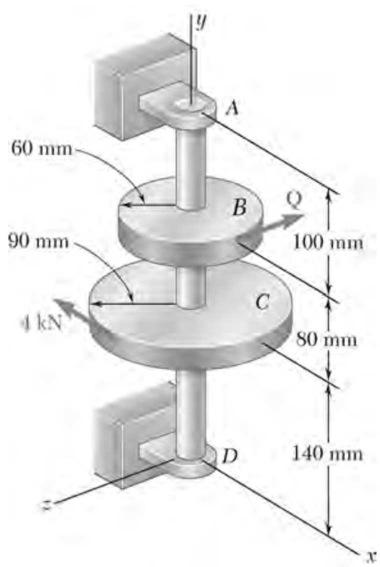
$$\frac{65\pi}{162}c^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{\text{all}}} = \frac{\sqrt{(0.48)^2 + (0.36)^2 + (0.2)^2}}{55 \times 10^3}$$

$$= 11.5 \times 10^{-6} \text{ m}^3$$

Forces in horizontal plane:



$c = 0.0209$ m $\qquad d = 2c = 41.8$ mm ◀



PROBLEM 8.18

The 4-kN force is parallel to the x axis, and the force **Q** is parallel to the z axis. The shaft AD is hollow. Knowing that the inner diameter is half the outer diameter and that $\tau_{\text{all}} = 60$ MPa, determine the smallest permissible outer diameter of the shaft.

SOLUTION

$$\Sigma M_y = 0: \quad 60\times10^{-3}\text{Q} - (90\times10^{-3})(4\times10^3) = 0$$

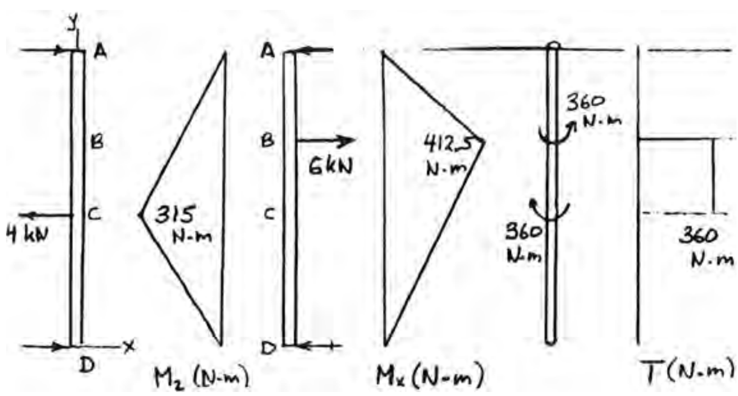
$$\text{Q} = 6\times10^3\,\text{N} = 6\text{ kN}$$

Bending moment and torque diagrams.

In xy plane: $(M_z)_{\max} = 315\text{ N}\cdot\text{m}$ at C.

In yz plane: $(M_x)_{\max} = 412.5\text{ N}\cdot\text{m}$ at B.

About z-axis: $T_{\max} = 360\text{ N}\cdot\text{m}$ between B and C.



PROBLEM 8.18 *(Continued)*

At B:

$$M_z = \left(\frac{100}{180}\right)(315) = 175 \text{ N} \cdot \text{m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{175^2 + 412.5^2 + 360^2} = 574.79 \text{ N} \cdot \text{m}$$

At C:

$$M_x = \left(\frac{140}{220}\right)(412.5) = 262.5 \text{ N} \cdot \text{m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{315^2 + 262.5^2 + 360^2} = 545.65 \text{ N} \cdot \text{m}$$

Largest value is $574.79 \text{ N} \cdot \text{m}$.

$$\tau_{\max} = \frac{\max\sqrt{M_x^2 + M_z^2 + T^2}\, c}{J}$$

$$\frac{J}{c} = \frac{\max\sqrt{M_x^2 + M_z^2 + T^2}}{\tau_{\max}} = \frac{574.79}{60 \times 10^6}$$

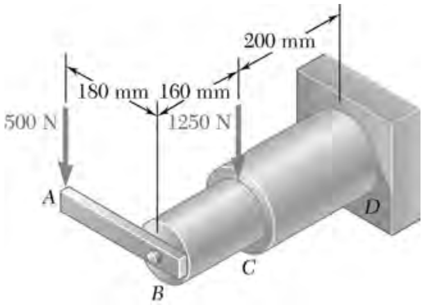
$$= 9.5798 \times 10^{-6} \text{m}^3 = 9.5798 \times 10^3 \text{mm}^3$$

$$\frac{J}{c} = \frac{\frac{\pi}{2}\left(c_o^4 - c_i^4\right)}{c_o} = \frac{\pi}{2} c_o^3 \left(1 - \frac{c_i^4}{c_o^4}\right) = \frac{\pi}{2} c_o^3 \left[1 - \left(\frac{1}{2}\right)^4\right]$$

$$= 1.47262 c_o^3$$

$$1.47262 c_o^3 = 9.5798 \times 10^3 \qquad c_o = 18.67 \text{ mm}$$

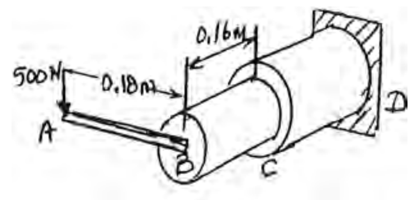
$$d_o = 2c_o \qquad\qquad d_o = 37.3 \text{ mm} \blacktriangleleft$$



PROBLEM 8.19

Neglecting the effect of fillets and of stress concentrations, determine the smallest permissible diameters of the solid rods *BC* and *CD*. Use $\tau_{\text{all}} = 60$ MPa.

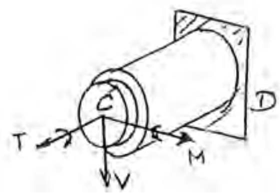
SOLUTION



$$\tau_{\text{all}} = 60 \times 10^6 \text{ Pa}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}}$$

$$c^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} \qquad d = 2c$$



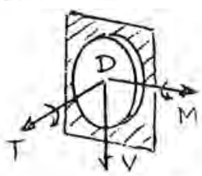
<u>Bending moments and torques</u>.

Just to the left of *C*:

$$M = (500)(0.16) = 80 \text{ N} \cdot \text{m}$$

$$T = (500)(0.18) = 90 \text{ N} \cdot \text{m}$$

$$\sqrt{M^2 + T^2} = 120.416 \text{ N} \cdot \text{m}$$

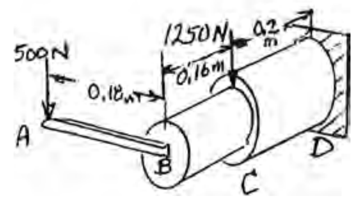


Just to the left of *D*:

$$T = 90 \text{ N} \cdot \text{m}$$

$$M = (500)(0.36) + (1250)(0.2) = 430 \text{ N} \cdot \text{m}$$

$$\sqrt{M^2 + T^2} = 439.32 \text{ N} \cdot \text{m}$$



<u>Smallest permissible diameter d_{BC}</u>.

$$c^3 = \frac{(2)(120.416)}{\pi(60 \times 10^6)} = 1.27765 \times 10^{-6} \text{m}^3$$

$$c = 0.01085 \text{ m} = 10.85 \text{ mm}$$

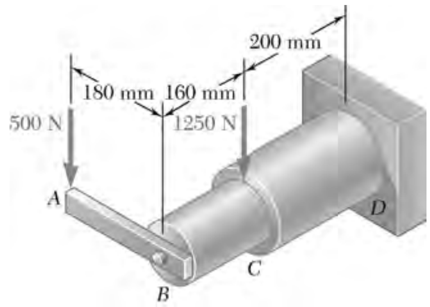
$d_{BC} = 21.7$ mm ◀

<u>Smallest permissible diameter d_{CD}</u>.

$$c^3 = \frac{(2)(439.32)}{\pi(60 \times 10^6)} = 4.6613 \times 10^{-6} \text{ m}^3$$

$$c = 0.01670 \text{ m} = 16.7 \text{ mm}$$

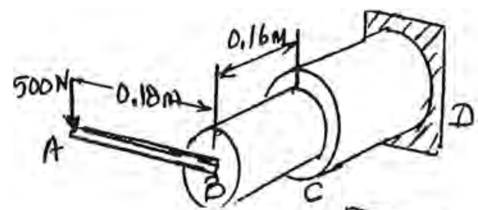
$d_{CD} = 33.4$ mm ◀



PROBLEM 8.20

Knowing that rods *BC* and *CD* are of diameter 24 mm and 36 mm, respectively, determine the maximum shearing stress in each rod. Neglect the effect of fillets and of stress concentrations.

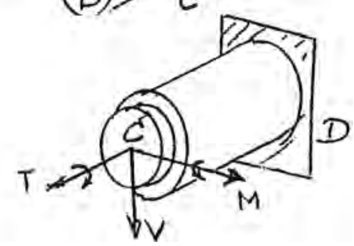
SOLUTION



Over *BC*: $c = \frac{1}{2}d = 12 \text{ mm} = 0.012 \text{ m}$

Over *CD*: $c = \frac{1}{2}d = 18 \text{ mm} = 0.018 \text{ m}$

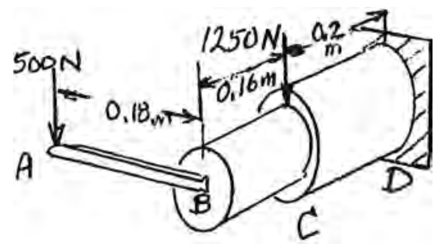
$$\tau = \frac{\sqrt{M^2+T^2}c}{J} \quad \frac{2}{\pi}\frac{\sqrt{M^2+T^2}}{c^3}$$



Bending moments and torques.

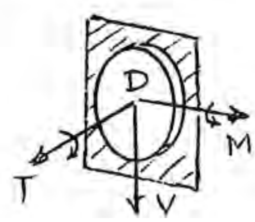
Just to the left of C:

$$M = (500)(0.16) = 80 \text{ N}\cdot\text{m}$$
$$T = (500)(0.18) = 90 \text{ N}\cdot\text{m}$$
$$\sqrt{M^2+T^2} = 120.416 \text{ N}\cdot\text{m}$$



Just to the left of *D*:

$$T = 90 \text{ N}\cdot\text{m}$$
$$M = (500)(0.36) + (1250)(0.2)$$
$$= 430 \text{ N}\cdot\text{m}$$
$$\sqrt{M^2+T^2} = 439.32 \text{ N}\cdot\text{m}$$



Maximum shearing stress in portion *BC*.

$$\tau_{\max} = \frac{(2)(120.416)}{\pi(0.012)^3} = 44.36\times10^6 \text{ Pa}$$

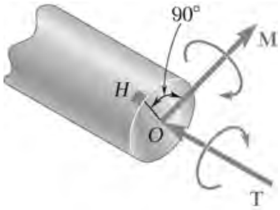
$\tau_{\max} = 44.4 \text{ MPa}$ ◀

Maximum shearing stress in portion *CD*.

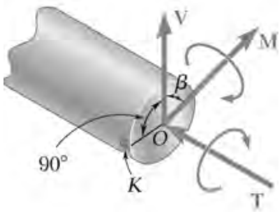
$$\tau_{\max} = \frac{(2)(439.32)}{\pi(0.018)^3} = 47.96\times10^6 \text{ Pa}$$

$\tau_{\max} = 48.0 \text{ MPa}$ ◀

PROBLEM 8.21



(*a*)



(*b*)

It was stated in Sec. 8.3 that the shearing stresses produced in a shaft by the transverse loads are usually much smaller than those produced by the torques. In the preceding problems, their effect was ignored and it was assumed that the maximum shearing stress in a given section occurred at point H (Fig. P8.21*a*) and was equal to the expression obtained in Eq. (8.5), namely,

$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2}$$

Show that the maximum shearing stress at point K (Fig. P8.21*b*), where the effect of the shear V is greatest, can be expressed as

$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\frac{2}{3}cV + T\right)^2}$$

where β is the angle between the vectors **V** and **M**. It is clear that the effect of the shear V cannot be ignored when $\tau_K \geq \tau_H$. (*Hint:* Only the component of **M** along **V** contributes to the shearing stress at K.)

SOLUTION

Shearing stress at point K.

Due to V: For a semicircle, $Q = \frac{2}{3}c^3$

For a circle cut across its diameter, $t = d = 2c$

For a circular section, $I = \frac{1}{2}J$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(V)\left(\frac{2}{3}c^3\right)}{\left(\frac{1}{2}J\right)(2c)} = \frac{2}{3}\frac{Vc^2}{J}$$

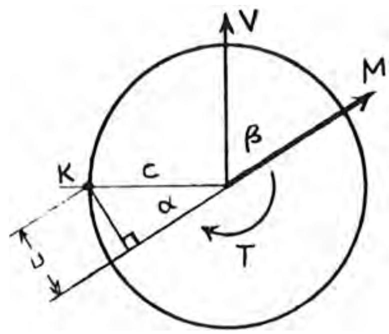
Due to T: $\tau_{xy} = \frac{Tc}{J}$

Since these shearing stresses have the same orientation,

$$\tau_{xy} = \frac{c}{J}\left(\frac{2}{3}Vc + T\right)$$

PROBLEM 8.21 *(Continued)*

Bending stress at point K: $\sigma_x = \dfrac{Mu}{I} = \dfrac{2Mu}{J}$



Cross section

where u is the distance between point K and the neutral axis,

$$u = c\ \sin\ \alpha = c\sin\left(\frac{\pi}{2} - \beta\right) = c\cos\beta$$

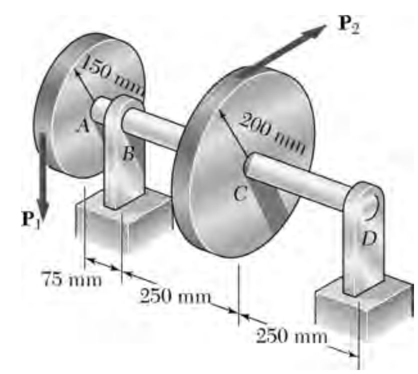
$$\sigma_x = \frac{2Mc\cos\beta}{J}$$

Using Mohr's circle,

$$\tau_K = R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{c}{J}\sqrt{(M\ \cos\beta)^2 + \left(\frac{2}{3}Vc + T\right)^2}$$

PROBLEM 8.22



Assuming that the magnitudes of the forces applied to disks A and C of Prob. 8.19 are, respectively, $P_1 = 4800$ N and $P_2 = 3600$ N, and using the expressions given in Prob. 8.21, determine the values τ_H and τ_K in a section (a) just to the left of B, (b) just to the left of C.

SOLUTION

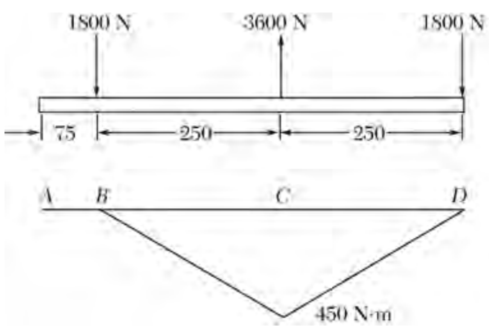
From Prob. 8.15, shaft diameter = 45 mm

$$c = \frac{1}{2}d = 22.5 \text{ mm} \quad J = \frac{\pi}{2}c^4 = 402.6\times10^3 \text{ mm}^4$$

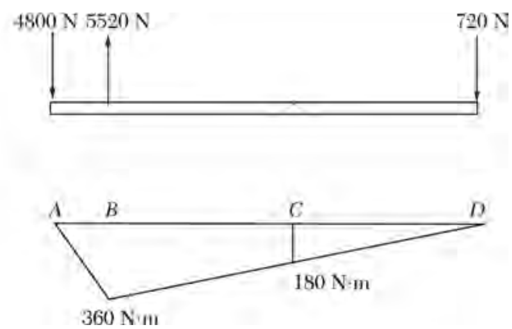
Torque over portion ABC:

$$T = (150)(4800) = (200)(3600) = 720000 \text{ N}\cdot\text{mm} = 720 \text{ N}\cdot\text{m}$$

Bending in horizontal plane

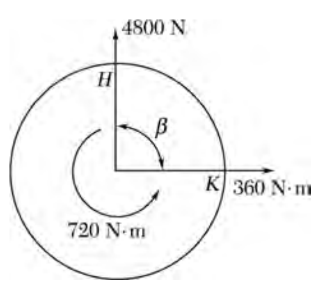


Bending in vertical plane



(a) Just to the left of point B: $V = 4800$ N $\quad M = 360$ N·m

$\beta = 90°$ $\quad T = 720$ N·m



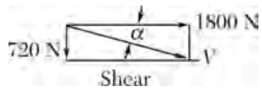
$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2} = \frac{0.0225}{402.6\times10^{-9}}\sqrt{(360)^2 + (720)^2}$$

$$= 44.98 \text{ MPa} \quad ◀$$

$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\frac{2}{3}V_C + T\right)^2} = \frac{c}{J}\left[\frac{2}{3}V_C + T\right]$$

$$= \frac{0.0225}{402.6\times10^{-9}}\left[\left(\frac{2}{3}\right)(4800)(0.0225) + 720\right] = 44.26 \text{ MPa} \quad ◀$$

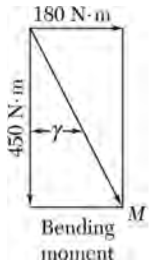
PROBLEM 8.22 *(Continued)*



(*b*) Just to the left of point *C*

$$V = \sqrt{(720)^2 + (1800)^2} = 1938.6 \text{ N}$$

$$\alpha = \tan\frac{720}{1800} = 21.80°$$

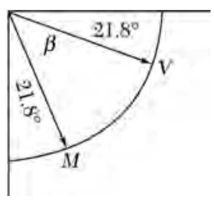


$$M = \sqrt{(180)^2 + (450)^2} = 484.66 \text{ N}\cdot\text{m}$$

$$\gamma = \tan^{-1}\frac{180}{450} = 21.80°$$

$$\beta = 90° - 21.8° - 21.8° = 46.4°$$

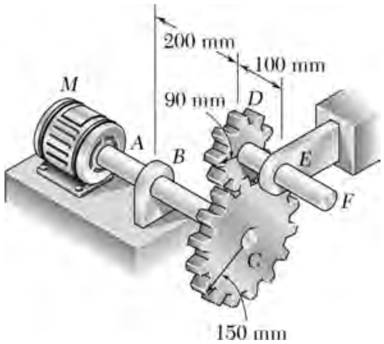
$$\tau_H = \frac{0.0225}{402.6\times10^{-9}}\sqrt{(720)^2 + (484.66)^2} = 48.50 \text{ MPa} \quad ◀$$



$$\frac{2}{3}V_C + T = \left(\frac{2}{3}\right)(1938.6)(0.0225) + 720 = 749 \text{ N}\cdot\text{m}$$

$$M\cos\beta = 484.66\cos 46.4° = 334.23 \text{ N}\cdot\text{m}$$

$$\tau_k = \frac{0.0225}{402.6\times10^{-9}}\sqrt{(334.23)^2 + (749)^2} = 45.83 \text{ MPa} \quad ◀$$



PROBLEM 8.23

The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 15 kW from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 50$ MPa, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

SOLUTION

$$240 \text{ rpm} = \frac{240}{60} = 4 \text{ Hz}$$

(*a*) Shaft *ABC*: $T = \dfrac{P}{2\pi f} = \dfrac{30}{(2\pi)(4)} = 1.194 \text{ kNm}$

Gear *C*: $F_{CD} = \dfrac{T}{r_C} = \dfrac{1.194}{0.15} = 7.96 \text{ kN}$

Bending moment at *B*: $M_B = (0.2)(7.96) = 1.592 \text{ kN}$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} = \frac{\sqrt{(1.194)^2 + (1.592)^2}}{50\times 10^3} = 39.8\times 10^{-6} \text{ m}^3$$

$c = 0.0294$ m $\qquad d = 2c = 58.8$ mm ◀

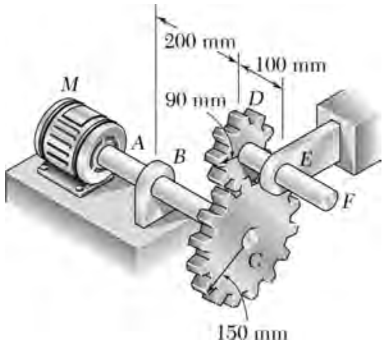
(*b*) Shaft *DEF*: $T = r_D F_{CD} = (0.09)(7.96) = 0.7164 \text{ kNm}$

Bending moment at *E*: $M_E = (0.1)(7.96) = 0.796 \text{ kNm}$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} = \frac{\sqrt{(0.796)^2 + (0.7164)^2}}{50\times 10^3} = 21.418\times 10^{-6} \text{ m}^3$$

$c = 0.0239$ m $\qquad d = 2c = 47.8$ mm ◀



PROBLEM 8.24

Solve Prob. 8.23, assuming that the motor rotates at 360 rpm.

PROBLEM 8.23 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 15 kW from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 50$ MPa, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

SOLUTION

$$360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

(*a*) Shaft *ABC*:

$$T = \frac{P}{2\pi f} = \frac{30}{(2\pi)(6)} = 0.796 \text{ kNm}$$

Gear *C*:

$$F_{CD} = \frac{T}{r_C} = \frac{0.796}{0.15} = 5.31 \text{ kN}$$

Bending moment at *B*:

$$M_B = (0.2)(5.31) = 1.062 \text{ kNm}$$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{\text{all}}} = \frac{\sqrt{1.062^2 + 0.796^2}}{50\times 10^3} = 26.544\times 10^{-6} \text{ m}^3$$

$c = 0.0257$ m $\qquad d = 2c = 51.4$ mm ◀

(*b*) Shaft *DEF*:

$$T = r_D F_{CD} = (0.09)(5.31) = 0.478 \text{ kNm}$$

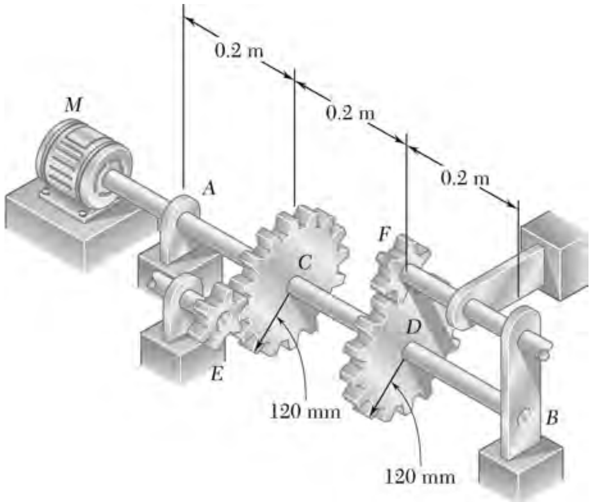
Bending moment at *E*:

$$M_E = (0.1)(5.31) = 0.531 \text{ kNm}$$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{\text{all}}} = \frac{\sqrt{0.531^2 + 0.478^2}}{50\times 10^3} = 14.29\times 10^{-6}$$

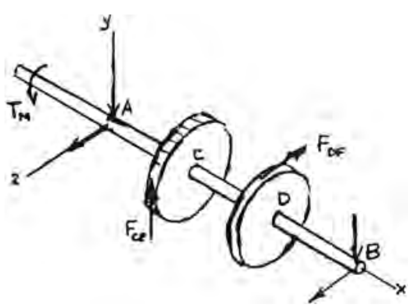
$c = 0.0209$ m $\qquad d = 2c = 41.8$ mm ◀



PROBLEM 8.25

The solid shaft *AB* rotates at 360 rpm and transmits 20 kW from the motor *M* to machine tools connected to gears *E* and *F*. Knowing that $\tau_{\text{all}} = 45\text{ MPa}$ and assuming that 10 kW is taken off at each gear, determine the smallest permissible diameter of shaft *AB*.

SOLUTION



$$f = 360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

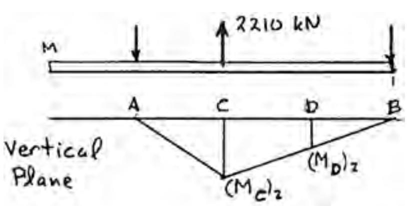
$$T_M = \frac{P_M}{2\pi f} = \frac{20\times10^3}{2\pi(6)} = 530.5 \text{ N}\cdot\text{m}$$

$$T_C = \frac{P_C}{2\pi f} = \frac{10\times10^3}{2\pi(6)} = 265.26 \text{ N}\cdot\text{m}$$

$$T_D = \frac{P_D}{2\pi f} = \frac{10\times10^3}{2\pi(6)} = 265.26 \text{ N}\cdot\text{m}$$

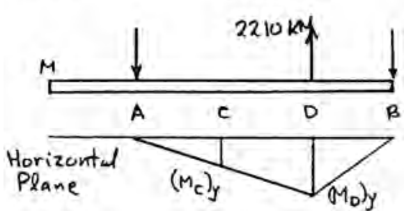
$$F_{CE} = \frac{T_C}{r_C} = \frac{265.26}{120\times10^{-3}} = 2.210\times10^3 \text{ N}$$

$$F_{DF} = \frac{T_D}{r_D} = \frac{265.26}{120\times10^{-3}} = 2.210\times10^3 \text{ N}$$



$$(M_C)_z = \frac{(0.2)(0.4)(2.210\times10^3)}{0.6} = 294.7 \text{ N}\cdot\text{m}$$

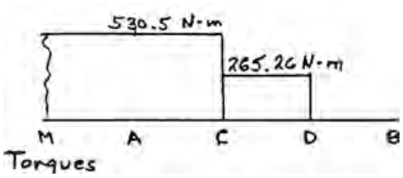
$$(M_D)_z = \frac{1}{2}(M_B)_z = 147.37 \text{ N}\cdot\text{m}$$



$$(M_C)_y = \frac{(0.4)(0.2)(2.210\times10^3)}{0.6} = 294.7 \text{ N}\cdot\text{m}$$

$$(M_D)_y = \frac{1}{2}(M_C)_y = 147.37 \text{ N}\cdot\text{m}$$

PROBLEM 8.25 *(Continued)*



Torques in shaft:

$$T_{MAC} = 530.5 \text{ N}\cdot\text{m}$$
$$T_{CD} = 265.26 \text{ N}\cdot\text{m}$$
$$T_{DB} = 0$$

Just to the left of gear *C*:

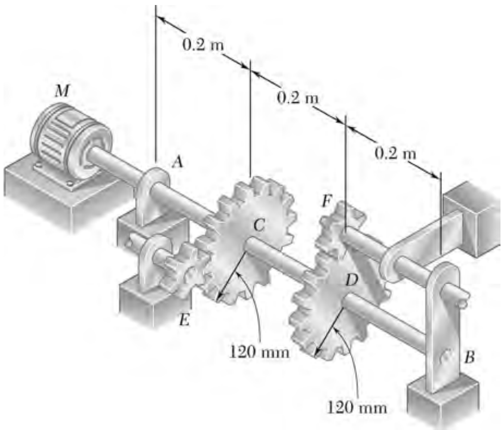
$$\max\sqrt{M_y^2 + M_z^2 + T^2} = 624.5 \text{ N}\cdot\text{m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\max\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{\max}}$$

$$= \frac{624.5}{45\times10^6} = 13.878\times10^{-6}\,\text{m}^3$$

$$c = 20.67\times10^{-3}\,\text{m} = 20.67 \text{ mm}$$

$$d = 2c \qquad d = 41.3 \text{ mm} \blacktriangleleft$$

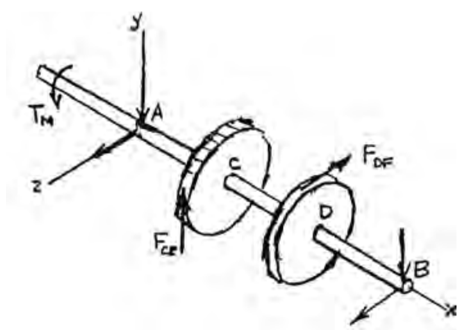


PROBLEM 8.26

Solve Prob. 8.25, assuming that the entire 20 kW is taken off at gear *E*.

PROBLEM 8.25 The solid shaft *AB* rotates at 360 rpm and transmits 20 kW from the motor *M* to machine tools connected to gears *E* and *F*. Knowing that $\tau_{\text{all}} = 45\text{ MPa}$ and assuming that 10 kW is taken off at each gear, determine the smallest permissible diameter of shaft *AB*.

SOLUTION



$$f = 360\text{ rpm} = \frac{360}{60} = 6\text{ Hz}$$

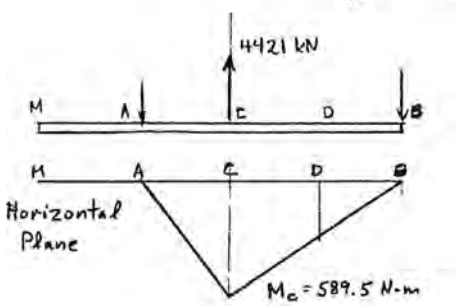
$$T_M = \frac{P_M}{2\pi f} = \frac{20\times 10^3}{2\pi(6)} = 530.5\text{ N}\cdot\text{m}$$

$$T_C = \frac{P_C}{2\pi f} = \frac{20\times 10^3}{2\pi(6)} = 530.5\text{ N}\cdot\text{m}$$

$$T_D = \frac{P_E}{2\pi f} = 0$$

$$F_{CE} = \frac{T_C}{r_C} = \frac{530.5}{120\times 10^{-3}} = 4.421\times 10^3\text{ N}$$

$$F_{DF} = \frac{T_D}{r_D} = 0$$

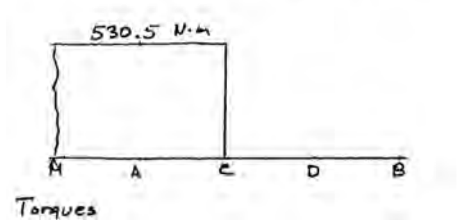


Since $F_{DF} = 0$, there is no bending in the horizontal plane.

$$M_C = \frac{(0.2)(0.4)(4.421\times 10^3)}{0.6} = 589.5\text{ N}\cdot\text{m}$$

Torques in shaft. $T_{MAC} = 530.5\text{ N}\cdot\text{m}$

$T_{CDE} = 0$



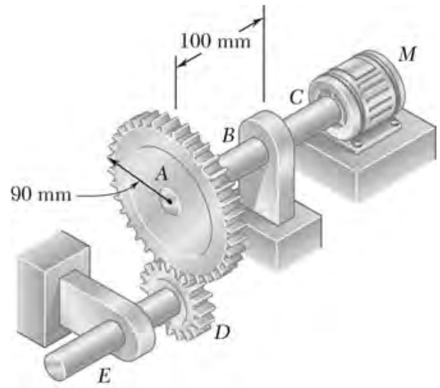
Just to the left of gear *C*:

$$\max\sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{0 + 589.5^2 + 530.5^2}$$

$$= 793.1\text{ N}\cdot\text{m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\max\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{\max}} = \frac{793.1}{45\times 10^6} = 17.624\times 10^{-6}\text{ m}^3$$

$c = 22.39\times 10^{-3} = 22.39\text{ mm}$ $\quad d = 2c$ $\qquad d = 44.8\text{ mm}$ ◀



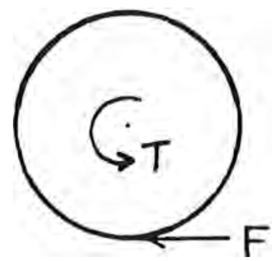
PROBLEM 8.27

The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 60$ MPa, determine the smallest permissible diameter of shaft *ABC*.

SOLUTION

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10\times 10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$



Gear *A*: $\quad Fr_A - T = 0$

$$F = \frac{T}{r_A} = \frac{397.89}{90\times 10^{-3}} = 4421 \text{ N}$$

Bending moment at *B*:

$$M_B = L_{AB}F = (100\times 10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M^2 + T^2}$$

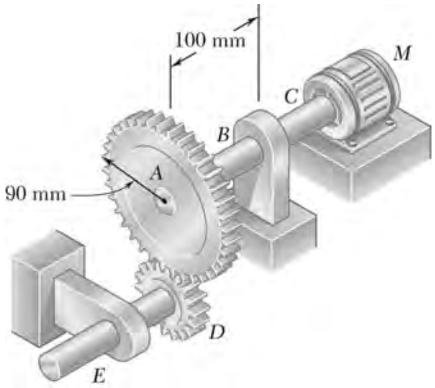
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{\text{all}}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2+T^2}}{\tau_{\text{all}}} = \frac{(2)\sqrt{442.1^2 + 397.89^2}}{\pi(60\times 10^6)} = 6.3108\times 10^{-6} \text{m}^3$$

$$c = 18.479\times 10^{-3} \text{m} \qquad d = 2c = 37.0\times 10^{-3} \text{m}$$

$d = 37.0$ mm ◀

PROBLEM 8.28



Assuming that shaft *ABC* of Prob. 8.27 is hollow and has an outer diameter of 50 mm, determine the largest permissible inner diameter of the shaft.

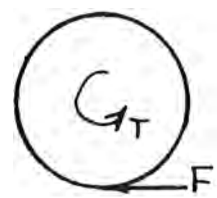
PROBLEM 8.27 The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 60$ MPa, determine the smallest permissible diameter of shaft *ABC*.

SOLUTION

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10\times10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear *A*: $Fr_A - T = 0$



$$F = \frac{T}{r_A} = \frac{397.89}{90\times10^{-3}} = 4421 \text{ N}$$

Bending moment at *B*:

$$M_B = L_{AB}F = (100\times10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$$

$$\tau_{\text{all}} = \frac{c_o}{J}\sqrt{M^2+T^2} \qquad c_o = \frac{1}{2}d_o = 25\times10^{-3}\text{m}$$

$$\frac{J}{c_o} = \frac{\pi}{2}\frac{(c_o^4 - c_i^4)}{c_o} = \frac{\sqrt{M^2+T^2}}{\tau_{\text{all}}}$$

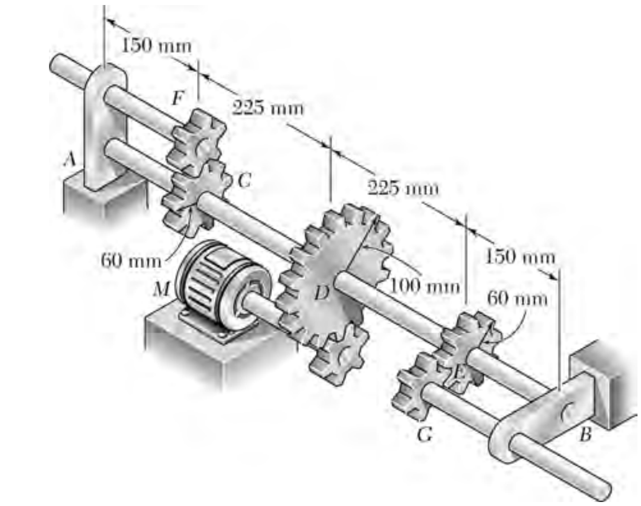
$$c_i^4 = c_o^4 - \frac{2c_o\sqrt{M^2+T^2}}{\pi\tau_{\text{all}}}$$

$$= (25\times10^{-3})^4 - \frac{(2)(25\times10^{-3})\sqrt{442.1^2+397.89^2}}{\pi(60\times10^6)}$$

$$= 390.625\times10^{-9} - 157.772\times10^{-9} = 232.85\times10^{-9}$$

$$c_i = 21.967\times10^{-3}\text{m} \qquad d_i = 2c_i = 43.93\times10^{-3}\text{m}$$

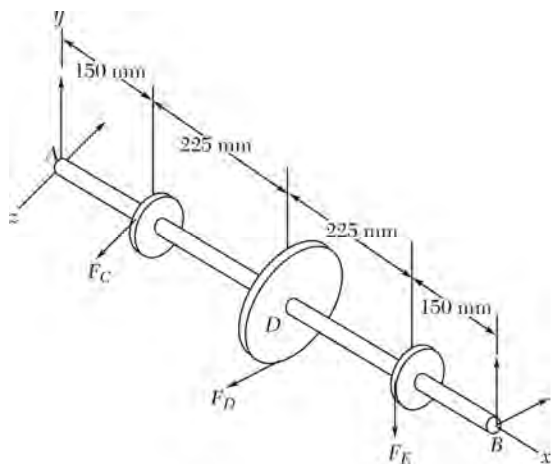
$d = 43.9$ mm ◀



PROBLEM 8.29

The solid shaft *AB* rotates at 450 rpm and transmits 20 kW from the motor *M* to machine tools connected to gears *F* and *G*. Knowing that $\tau_{\text{all}} = 55\text{ MPa}$ and assuming that 8 kW is taken off at gear *F* and 12 kW is taken off at gear *G*, determine the smallest permissible diameter of shaft *AB*.

SOLUTION



$$f = \frac{450}{60} = 7.5\text{ Hz}$$

Torque applied at *D*:

$$T_D = \frac{P}{2\pi f} = \frac{20\times 10^3}{2\pi(7.5)} = 424.41\text{ N}\cdot\text{m}$$

Torques on gears *C* and *E*:

$$T_C = \frac{8}{20}T_D = 169.76\text{ N}\cdot\text{m}$$

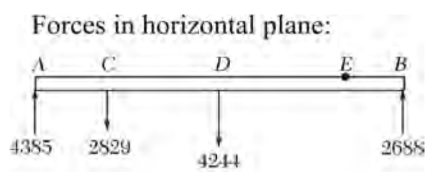
$$T_E = \frac{12}{20}T_D = 254.65\text{ N}\cdot\text{m}$$

Forces on gears

$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100\times 10^{-3}} = 4244\text{ N}$$

$$F_C = \frac{T_C}{r_C} = \frac{169.76}{60\times 10^{-3}} = 2829\text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{254.65}{60\times 10^{-3}} = 4244\text{ N}$$



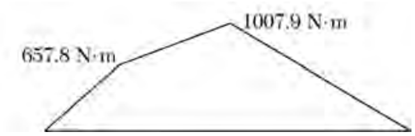
Torques in various parts

$$AC:\quad T = 0$$
$$CD:\quad T = 169.76\text{ N}\cdot\text{m}$$
$$DE:\quad T = 254.65\text{ N}\cdot\text{m}$$
$$EB:\quad T = 0$$

PROBLEM 8.29 *(Continued)*

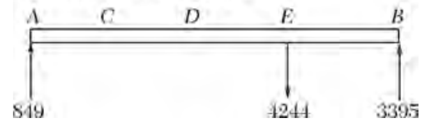
Critical point lies just the right of D:

$$T = 254.65 \text{ N}\cdot\text{m}$$
$$M_y = 1007.9 \text{ N}\cdot\text{m}$$
$$M_z = 318.3 \text{ N}\cdot\text{m}$$



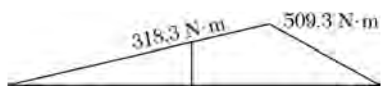
$$\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max} = 1087.2 \text{ N}\cdot\text{m}$$

Forces in vertical plane:



$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}$$

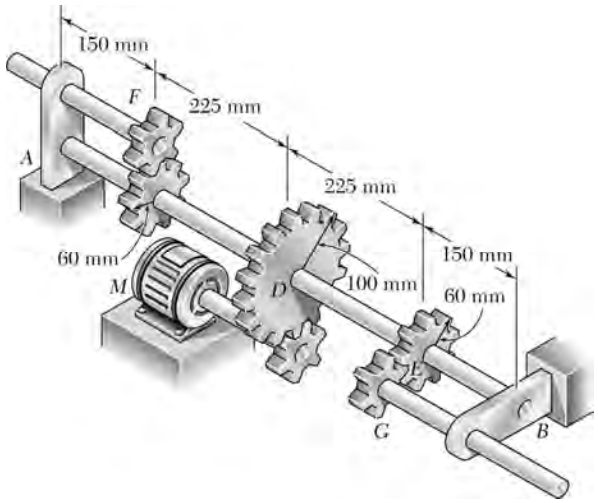
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}}{\tau_{\text{all}}} = \frac{1087.2}{55\times 10^6}$$



$$= 19.767\times 10^{-3} \text{ m}$$

$$c = 23.26\times 10^{-3} \text{ m}$$

$$d = 2c = 46.5\times 10^{-3} \text{ m} = 46.5 \text{ m} \quad \blacktriangleleft$$

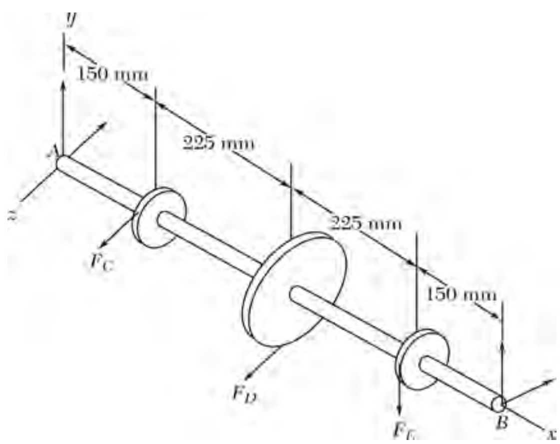


PROBLEM 8.30

Solve Prob. 8.29, assuming that the entire 20 kW is taken off at gear *G*.

PROBLEM 8.29 The solid shaft *AB* rotates at 450 rpm and transmits 20 kW from the motor *M* to machine tools connected to gears *F* and *G*. Knowing that $\tau_{\text{all}} = 55\ \text{MPa}$ and assuming 8 kW is taken off at gear *F* and 12 kW is taken off at gear *G*, determine the smallest permissible diameter of shaft *AB*.

SOLUTION



$$f = \frac{450}{60} = 7.5\ \text{Hz}$$

Torque applied at *D*:

$$T_D = \frac{P}{2\pi f} = \frac{20\times 10^3}{(2\pi)(7.5)} = 424.41\ \text{N}\cdot\text{m}$$

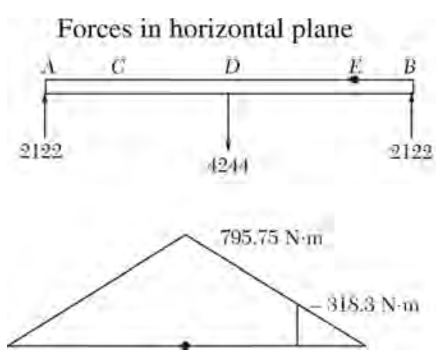
Torque on gears *E*:

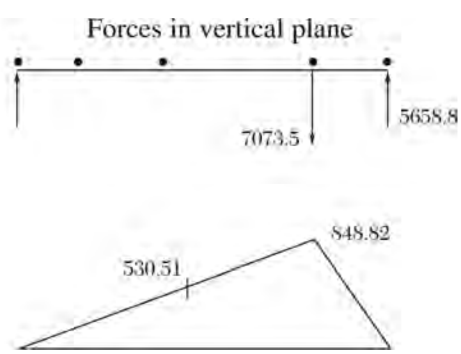
$$T_E = T_D = 424.41\ \text{N}\cdot\text{m}$$

Forces on gears *D* and *E*:

$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100\times 10^{-3}} = 4244\ \text{N}$$

$$F_E = \frac{T_E}{r_D} = \frac{424.41}{60\times 10^{-3}} = 7073.5\ \text{N}$$





PROBLEM 8.30 *(Continued)*

Bending moments

$$M_D = \sqrt{530.51^2 + 795.75^2} = 956.4 \text{ N}\cdot\text{m}$$

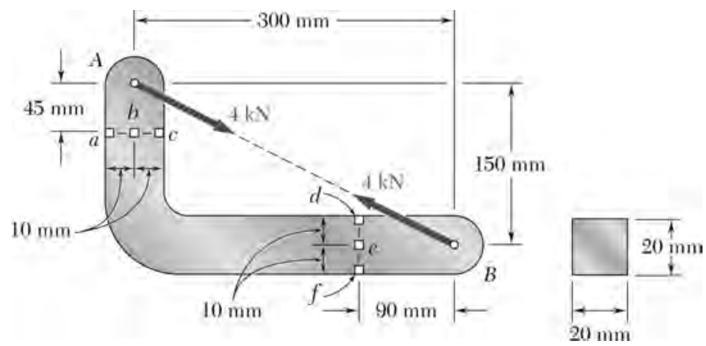
$$M_E = \sqrt{848.82^2 + 318.3^2} = 906.5 \text{ N}\cdot\text{m}$$

$$\left(\sqrt{M^2 + T^2}\right)_{\max} = \sqrt{956.4^2 + 424.41^2} = 1046.3 \text{ N}\cdot\text{m}$$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M^2 + T^2}\right)_{\max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} = \frac{1046.3}{55\times 10^6} = 19.024\times 10^{-6} \text{ m}^3$$

$$c = 22.96\times 10^{-3} \text{ m} \quad d = 2c = 45.9\times 10^{-3} \text{ m} = 45.9 \text{ mm} \quad \blacktriangleleft$$



PROBLEM 8.31

Two 4-kN forces are applied to an L-shaped machine element *AB* as shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

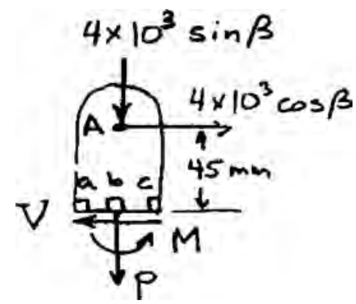
SOLUTION

Let β be the slope angle of line *AB*.

$$\tan\beta = \frac{150}{300} \qquad \beta = 26.565°$$

Draw free body sketch of the portion of the machine element lying above section *abc*.

$$P = -(4\times10^3)\sin\beta$$
$$= -1.78885\times10^3 \text{ N}$$
$$V = (4\times10^3)\cos\beta = 3.5777\times10^3 \text{ N}$$
$$M = (45\times10^{-3})(4\times10^3)\cos\beta = 160.997 \text{ N}\cdot\text{m}$$



Section properties:

$$A = (20)(20) = 400 \text{ mm}^2 = 400\times10^{-6} \text{ m}^2$$
$$I = \frac{1}{12}(20)(20)^3 = 13.3333\times10^3 \text{ mm}^4$$
$$= 13.3333\times10^{-9} \text{ m}^4$$

(*a*) Point *a*:

$$\sigma = \frac{P}{A} - \frac{Mx}{I} = -\frac{-1.78885\times10^3}{400\times10^{-6}} - \frac{(160.997)(-10\times10^{-3})}{13.3333\times10^{-9}}$$

$= 116.3$ MPa ◀

$\tau = 0$ ◀

(*b*) Point *b*:

$$\sigma = \frac{P}{A} = -\frac{1.78885\times10^3}{400\times10^{-6}}$$

$= -4.47$ MPa ◀

$$Q = (20)(10)(5) = 1000 \text{ mm}^3 = 10^{-6} \text{ m}^3$$
$$\tau = \frac{VQ}{It} = \frac{(3.5777)(10^{-6})}{(13.3333\times10^{-6})(20\times10^{-3})}$$

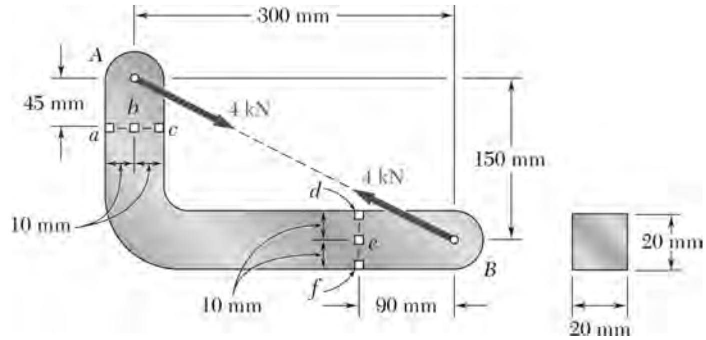
$= 13.42$ MPa ◀

(*c*) Point *c*:

$$\sigma = \frac{P}{A} - \frac{Mx}{I} = -\frac{1.78885\times10^3}{400\times10^{-6}} - \frac{(160.997)(10\times10^{-3})}{13.3333\times10^{-9}}$$

$= -125.2$ MPa ◀

$\tau = 0$ ◀



PROBLEM 8.32

Two 4-kN forces are applied to an L-shaped machine element *AB* as shown. Determine the normal and shearing stresses at (*a*) point *d*, (*b*) point *e*, (*c*) point *f*.

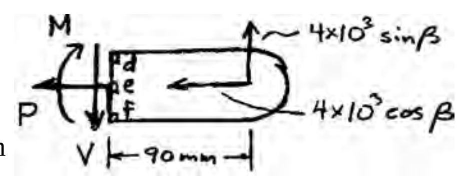
SOLUTION

Let β be the slope angle of line *AB*.

$$\tan\beta = \frac{150}{300} \qquad \beta = 26.565^\circ$$

Draw free body sketch of the portion of the machine element lying to the right of section *def*.

$$P = -(4\times10^3)\cos\beta$$
$$= -3.5777\times10^3\ \text{N}$$
$$V = (4\times10^3)\sin\beta = 1.78885\times10^3\ \text{N}$$
$$M = (90\times10^{-3})(4\times10^{-3})\sin\beta = 160.997\ \text{N}\cdot\text{m}$$



Section properties:

$$A = (20)(20) = 400\ \text{mm}^2 = 400\times10^{-6}\ \text{m}^3$$
$$I = \frac{1}{12}(20)(20)^3 = 13.3333\times10^3\ \text{mm}^4$$
$$= 13.3333\times10^{-9}\ \text{m}^4$$

(*a*) Point *d*:

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{-3.5777\times10^3}{400\times10^{-6}} - \frac{(160.997)(10\times10^{-3})}{13.3333\times10^{-9}}$$

$= -129.7$ MPa ◀

$\tau = 0$ ◀

(*b*) Point *e*:

$$\sigma = \frac{P}{A} = \frac{-3.5777\times10^3}{400\times10^{-6}}$$ $= -8.94$ MPa ◀

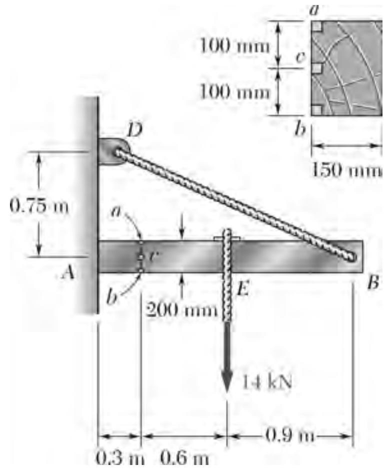
$$Q = (20)(10)(5) = 1000\ \text{mm}^3 = 10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(1.78885\times10^3)(10^{-6})}{(13.3333\times10^{-9})(20\times10^{-3})}$$ $= 6.71$ MPa ◀

(*c*) Point *f*:

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{-3.5777\times10^3}{400\times10^{-6}} - \frac{(160.997)(-10\times10^{-3})}{13.3333\times10^{-9}}$$ $= 111.8$ MPa ◀

$\tau = 0$ ◀

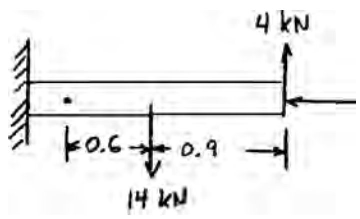


PROBLEM 8.33

The cantilever beam AB has a rectangular cross section of 150×200 mm. Knowing that the tension in cable BD is 10.4 kN and neglecting the weight of the beam, determine the normal and shearing stresses at the three points indicated.

SOLUTION

$$\overline{DB} = \sqrt{0.75^2 + 1.8^2} = 1.95 \text{ m}$$

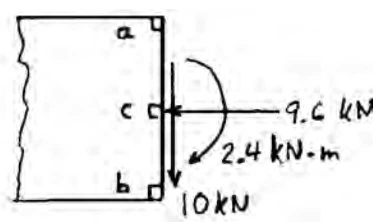


Vertical component of T_{DB}:

$$\left(\frac{0.75}{1.95}\right)(10.4) = 4 \text{ kN}$$

Horizontal component of T_{DB}: $\left(\frac{1.8}{1.95}\right)(10.4) = 9.6 \text{ kN}$

At section containing points a, b, and c:



$$P = -9.6 \text{ kN} \qquad V = 14 - 4 = 10 \text{ kN}$$

$$M = (1.5)(4) - (0.6)(14) = -2.4 \text{ kN} \cdot \text{m}$$

Section properties: $A = (0.150)(0.200) = 0.030 \text{ m}^2$

$$I = \frac{1}{12}(0.150)(0.200)^3 = 100 \times 10^{-6} \text{ m}^2$$

$$c = 0.100 \text{ m}$$

At point a: $\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} + \frac{(2.4 \times 10^3)(0.100)}{100 \times 10^{-6}} = 2.08 \text{ MPa}$ ◀

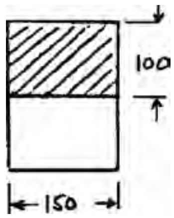
$\tau_{xy} = 0$ ◀

At point b: $\sigma_x = -\frac{P}{A} + \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} - \frac{(2.4 \times 10^3)(0.100)}{100 \times 10^{-6}} = -2.72 \text{ MPa}$ ◀

$\tau_{xy} = 0$ ◀

PROBLEM 8.33 *(Continued)*

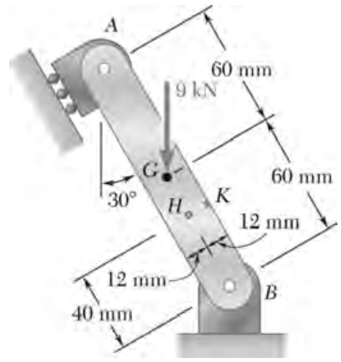
At point *c*:



$$\sigma_x = -\frac{P}{A} = -\frac{9.6\times 10^3}{0.030} = -0.320 \text{ MPa} \blacktriangleleft$$

$$Q = (150)(100)(50) = 750\times 10^3 \text{ mm}^3 = 750\times 10^{-6} \text{ m}^3$$

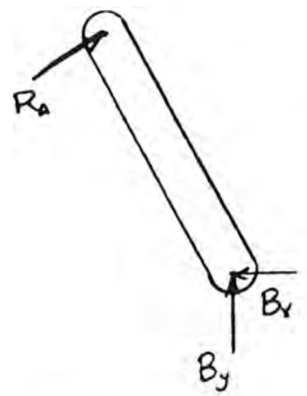
$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(10\times 10^3)(750\times 10^{-6})}{(100\times 10^{-6})(0.150)} = -0.500 \text{ MPa} \blacktriangleleft$$



PROBLEM 8.34

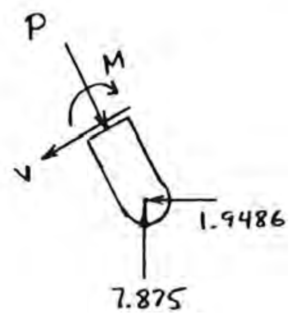
Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (9)(60\sin 30°) - 120R_A = 0$$
$$R_A = 2.25 \text{ kN}$$
$$\xrightarrow{+} \Sigma F_x = 0: \quad 2.25\cos 30° - B_x = 0$$
$$B_x = 1.9486 \text{ kN} \leftarrow$$
$$+\uparrow \Sigma F_y = 0: \quad 2.25\sin 30° - 9 + B_y = 0$$
$$B_y = 7.875 \text{ kN} \uparrow$$

At the section containing points *H* and *K*,

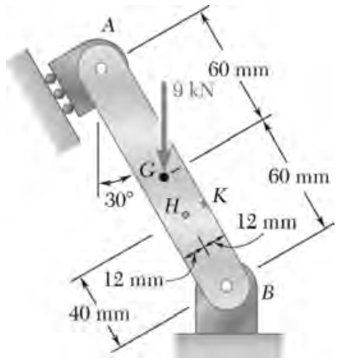


$$P = 7.875\cos 30° + 1.9486\sin 30° = 7.794 \text{ kN}$$
$$V = 7.875\sin 30° - 1.9486\cos 30° = 2.25 \text{ kN}$$
$$M = (7.875\times10^3)(40\times10^{-3}\sin 30°)$$
$$- (1.9486\times10^3)(40\times10^{-3}\cos 30°)$$
$$= 90 \text{ N}\cdot\text{m}$$
$$A = 10\times24 = 240 \text{ mm}^2 = 240\times10^{-6}\text{m}^2$$
$$I = \frac{1}{12}(10)(24)^3 = 11.52\times10^3\text{mm}^4 = 11.52\times10^{-9}\text{m}^4$$

(*a*) At point *H*,
$$\sigma_x = -\frac{P}{A} = -\frac{7.794\times10^3}{240\times10^{-6}}$$
$$\sigma_x = -32.5 \text{ MPa} \blacktriangleleft$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25\times10^3}{240\times10^{-6}}$$
$$\tau_{xy} = 14.06 \text{ MPa} \blacktriangleleft$$

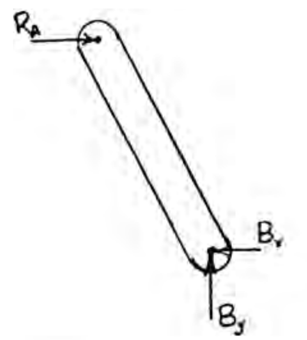
(*b*) At point *K*,
$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{7.794\times10^3}{240\times10^{-6}} - \frac{(90)(12\times10^{-3})}{11.52\times10^{-9}}$$
$$\sigma_x = -126.2 \text{ MPa} \blacktriangleleft$$
$$\tau_{xy} = 0 \blacktriangleleft$$



PROBLEM 8.35

Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point H, (*b*) point K.

SOLUTION



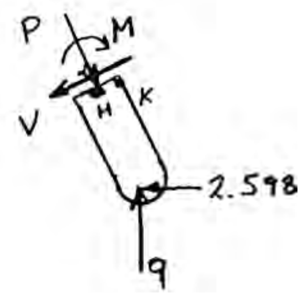
$$+\circlearrowleft \Sigma M_B = 0: \quad (120\cos 30°)R_A - (60\sin 30°)(9) = 0$$

$$R_A = 2.598 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 9 = 0 \qquad B_y = 9 \text{ kN} \uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 2.598 - B_x = 0 \quad B_x = 2.598 \text{ kN} \leftarrow$$

At the section containing points H and K,



$$P = 9 \cos 30° + 2.598 \sin 30° = 9.093 \text{ kN}$$

$$V = 9 \sin 30° - 2.598 \cos 30° = 2.25 \text{ kN}$$

$$M = (9\times10^3)(40\times10^{-3}\sin 30°)$$
$$- (2.598\times10^3)(40\times10^{-3}\cos 30°)$$
$$= 90 \text{ N}\cdot\text{m}$$

$$A = 10 \times 240 = 240 \text{ mm}^2 = 240\times10^{-6}\,\text{m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52\times10^3\,\text{mm}^4 = 11.52\times10^{-9}\,\text{m}^4$$

(*a*) <u>At point *H*</u>:

$$\sigma_x = -\frac{P}{A} = -\frac{9.093\times10^3}{240\times10^{-6}} \qquad \sigma = -37.9 \text{ MPa} \blacktriangleleft$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25\times10^3}{240\times10^{-6}} \qquad \tau = 14.06 \text{ MPa} \blacktriangleleft$$

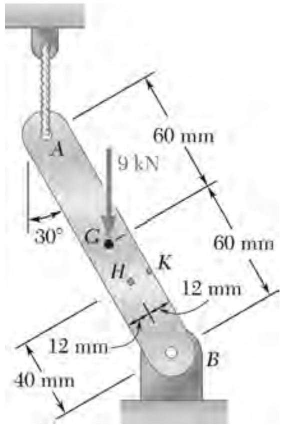
(*b*) <u>At point *K*</u>:

$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I}$$

$$= -\frac{9.093\times10^3}{240\times10^{-6}} - \frac{(90)(12\times10^{-3})}{11.52\times10^{-9}}$$

$$\sigma = -131.6 \text{ MPa} \blacktriangleleft$$

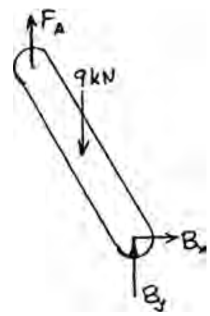
$$\tau = 0 \blacktriangleleft$$



PROBLEM 8.36

Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION

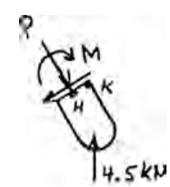


$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(120 \sin 30°) - 9(60 \sin 30°) = 0$$

$$B_y = 4.5 \text{ kN}$$

At the section containing points *H* and *K*,



$$P = 4.5 \cos 30° = 3.897 \text{ kN}$$

$$V = 4.5 \sin 30° = 2.25 \text{ kN}$$

$$M = (4.5\times 10^3)(40\times 10^{-3} \sin 30°)$$

$$= 90 \text{ N}\cdot\text{m}$$

$$A = 10\times 24 = 240 \text{ mm}^2 = 240\times 10^{-6}\,\text{m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52\times 10^3\,\text{mm}^4$$

$$= 11.52\times 10^{-9}\,\text{m}^4$$

(*a*) <u>At point *H*,</u>

$$\sigma_x = -\frac{P}{A} = -\frac{3.897\times 10^3}{240\times 10^{-6}}$$

$\sigma = -16.24$ MPa ◀

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25\times 10^3}{240\times 10^{-6}}$$

$\tau = 14.06$ MPa ◀

(*b*) <u>At point *K*,</u>

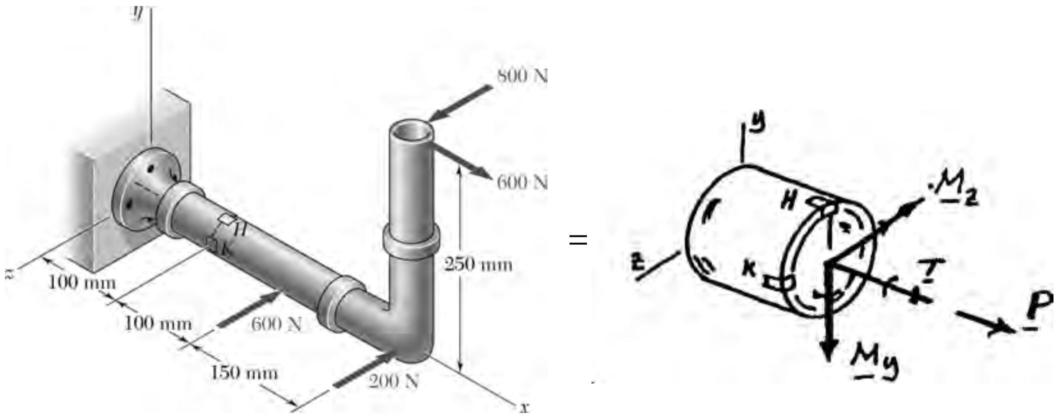
$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{3.897\times 10^3}{240\times 10^{-6}} - \frac{(90)(12\times 10^{-3})}{11.52\times 10^{-9}}$$

$\sigma = 110.0$ MPa ◀

$\tau = 0$ ◀

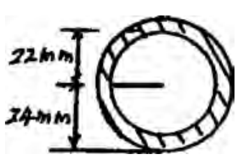
PROBLEM 8.37

Several forces are applied to the pipe assembly shown. Knowing that the pipe has inner and outer diameters equal to 40 mm and 48 mm, respectively, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.



SOLUTION

Section properties:



$$P = 600 \text{ N}$$
$$T = (800 \text{ N})(0.25 \text{ m}) = 200 \text{ Nm}$$
$$M_z = (600 \text{ N})(0.25 \text{ m}) = 150 \text{ Nm}$$
$$M_y = (800 \text{ N} - 200 \text{ N})(0.25 \text{ m}) - (600)(0.1 \text{ m}) = 90 \text{ Nm}$$
$$V_z = 800 - 600 - 200 = 0$$
$$V_y = 0$$
$$A = \pi(24^2 - 22^2) = 289 \text{ mm}^2$$
$$I = \frac{\pi}{4}(24^4 - 22^4) = 76592 \text{ mm}^4$$
$$J = 2I = 153184 \text{ mm}^4$$

(*a*) Point *H*:

$$\sigma_H = \frac{P}{A} + \frac{M_z c}{I} = \frac{600}{289\times10^{-6}} + \frac{(150)(0.024)}{76592\times10^{-12}} = 49.078 \text{ MPa}$$

$\sigma_H = 49.1 \text{ MPa}$ ◀

$$\tau_N = \frac{Tc}{J} = \frac{(200)(0.024)}{153184\times10^{-12}} = 31.33 \text{ MPa}$$

$\tau_N = 31.3 \text{ MPa}$ ◀

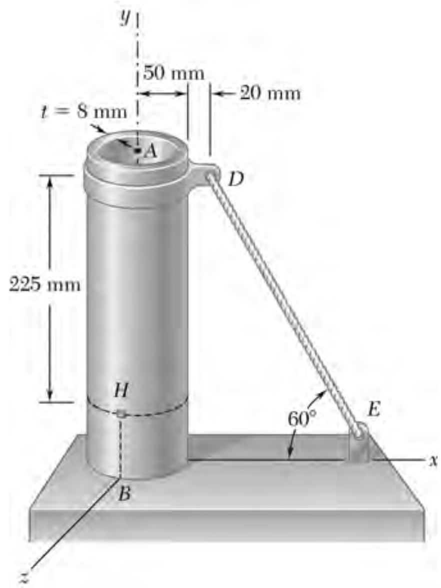
(*b*) Point *K*:

$$\sigma_K = \frac{P}{A} + \frac{M_y c}{I} = \frac{600}{289\times10^{-6}} - \frac{(90)(0.024)}{76592\times10^{-12}} = -26.125 \text{ MPa}$$

$\sigma_K = -26.1 \text{ MPa}$ ◀

$$\tau_K = \frac{Tc}{J} = \text{same as for } \tau_N$$

$\tau_K = 31.03 \text{ MPa}$ ◀



PROBLEM 8.38

The steel pipe *AB* has a 100-mm outer diameter and an 8-mm wall thickness. Knowing that the tension in the cable is 40 kN, determine the normal and shearing stresses at point *H*.

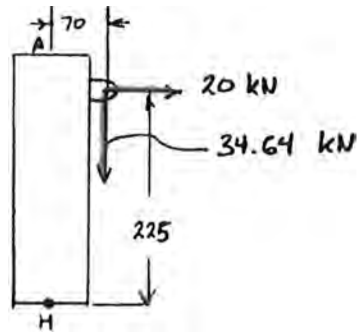
SOLUTION

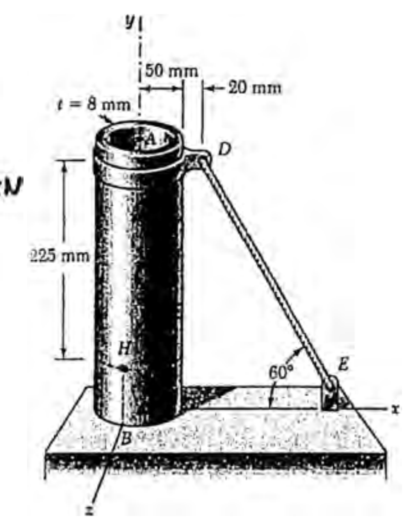
Vertical force:

$40\cos 30° = 34.64\ \text{kN}$

Horizontal force:

$40\sin 30° = 20\ \text{kN}$





Point *H* lies on neutral axis of bending.

Section properties:

$$d_o = 100\ \text{mm}, \qquad c_o = \frac{1}{2}d_o = 50\ \text{mm}, \qquad c_i = c_o - t = 42\ \text{mm},$$

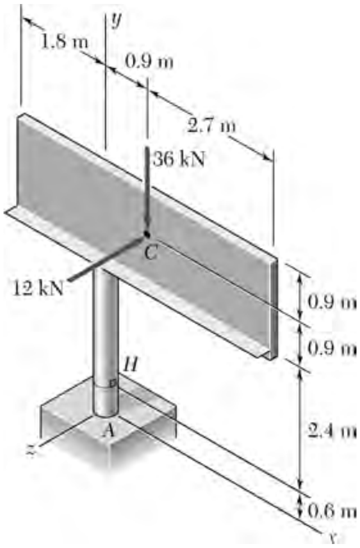
$$A = \pi(c_o^2 - c_i^2) = 2.312\times 10^3\ \text{mm}^2 = 2.312\times 10^{-3}\ \text{m}^2$$

$$\sigma = -\frac{P}{A} = -\frac{34.64\times 10^3}{2.312\times 10^{-6}}$$

$\sigma = -14.98\ \text{MPa}$ ◀

For thin pipe, $\tau = 2\dfrac{V}{A} = \dfrac{(2)(20\times 10^3)}{2.314\times 10^{-3}}$

$\tau = 17.29\ \text{MPa}$ ◀



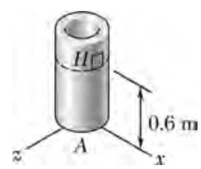
PROBLEM 8.39

The billboard shown weighs 36 kN and is supported by a structural tube that has a 375 mm outer diameter and a 12 mm wall thickness. At a time when the resultant of the wind pressure is 12 kN located at the center *C* of the billboard, determine the normal and shearing stresses at point *H*.

SOLUTION

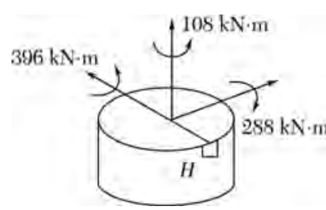
At section containing point *H*

$$P = 36 \text{ kN (compression)}$$
$$T = (12)(0.9) = 10.8 \text{ kN}\cdot\text{m}$$
$$M_x = -(-3.3)(12) = -39.6 \text{ kN}\cdot\text{m}$$
$$M_z = -(0.9)(36) = -32.4 \text{ kN}\cdot\text{m}$$
$$V = 12 \text{ kN}$$



Section properties

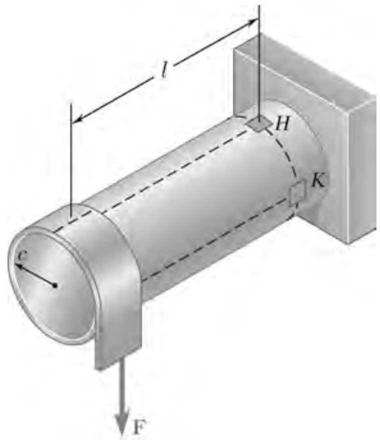
$$d_o = 375 \text{ mm} \quad c_o = \frac{1}{2}d_o = 187.5 \text{ mm} \quad c_i = c_o - t = 175.5 \text{ mm}$$



$$A = \pi\left(c_o^2 - c_i^2\right) = 13684.8 \text{ mm}^2$$
$$I = \frac{\pi}{4}\left(c_o^4 - c_i^4\right) = 225.65\times10^6 \text{ mm}^4$$
$$J = 2I = 451.30\times10^6 \text{ mm}^4$$
$$Q = \frac{2}{3}\left(c_o^3 - c_i^3\right) = 790.9\times10^3 \text{ mm}^3$$

$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{36\times10^3}{13684.8\times10^{-6}} - \frac{(32.4\times10^3)(0.1875)}{225.65\times10^{-6}} \quad ◀$$
$$= -2.63 - 26.92 = -29.55 \text{ MPa}$$

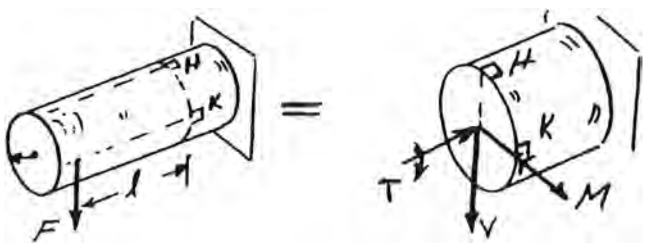
$$\tau = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(10.8\times10^3)(0.1875)}{451.30\times10^{-6}} + \frac{(12\times10^3)(790.9\times10^{-6})}{(225.65\times10^{-6})(0.024)}$$
$$= 4.48 + 1.75 = 6.23 \text{ MPa} \quad ◀$$



PROBLEM 8.40

A thin strap is wrapped around a solid rod of radius $c = 20$ mm as shown. Knowing that $l = 100$ mm and $F = 5$ kN, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

SOLUTION



At the section containing points *H* and *K*,

$$T = Fc \qquad M = Fl \qquad V = F$$

$$J = \frac{\pi}{2}c^4 \qquad I = \frac{\pi}{4}c^4$$

Point *H*:

$$\sigma = \frac{Mc}{I} = \frac{Flc}{\frac{\pi}{4}c^4} \qquad \sigma = \frac{4Fl}{\pi c^2}$$

$$\tau = \frac{Tc}{J} = \frac{Fc^2}{\frac{\pi}{4}c^4} \qquad \tau = \frac{2F}{\pi c^2}$$

Point *K*: Point *K* lies on the neutral axis. $\sigma = 0$

Due to torque:

$$\tau = \frac{Tc}{J} = \frac{2F}{\pi c^2}$$

Due to shear: For a semicircle, $Q = \frac{2}{3}c^3, \quad t = d = 2c$

$$\tau = \frac{VQ}{It} = \frac{F\,\frac{2}{3}c^3}{\frac{\pi}{4}c^4(2c)} = \frac{4F}{3\pi c^2}$$

Combined.

$$\tau = \frac{2F}{\pi c^2} + \frac{4F}{3\pi c^2} \qquad \tau = \frac{10F}{3\pi c^2}$$

PROBLEM 8.40 *(Continued)*

Data: $F = 5\text{ kN} = 5\times 10^3\text{ N}, \quad l = 100\text{ mm} = 0.100\text{ m}$

$c = 20\text{ mm} = 0.020\text{ m}$

(*a*) Point *H*:

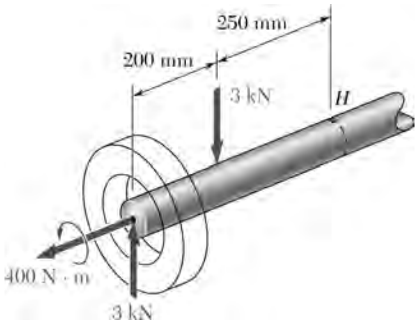
$$\sigma = \frac{(4)(5\times 10^3)(0.100)}{\pi(0.020)^3} \qquad \sigma = 79.6\text{ MPa} \blacktriangleleft$$

$$\tau = \frac{(2)(5\times 10^3)}{\pi(0.020)^2} \qquad \tau = 7.96\text{ MPa} \blacktriangleleft$$

(*b*) Point *K*:

$$\sigma = 0 \blacktriangleleft$$

$$\tau = \frac{(10)(5\times 10^3)}{3\pi(0.020)^2} \qquad \tau = 13.26\text{ MPa} \blacktriangleleft$$

	**PROBLEM 8.41** The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter of the axle is 36 mm, determine the normal and shearing stress at point *H* located on the top of the axle.

SOLUTION

The bending moment causing normal stress at point *H* is

$$M = (0.2)(3) = 0.6 \text{ kNm}$$

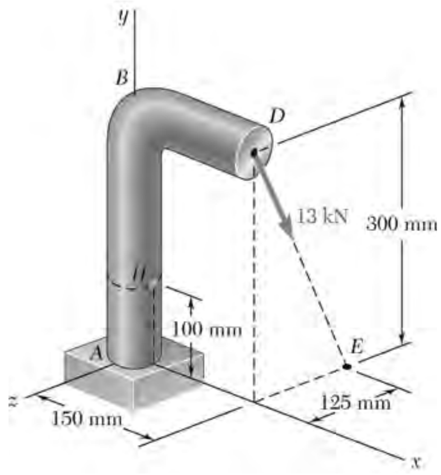
$$C = \frac{1}{2}d = 18 \text{ mm}$$

$$I = \frac{\pi}{4}C^4 = 82448 \text{ mm}^4, \ J = 2I = 164896 \text{ mm}^4$$

Normal stress at *H*: $\sigma_H = -\frac{Mc}{I} = -\frac{(0.6\times 10^6)(18)}{82448} = -131 \text{ MPa}$ ◀

At the section containing point *H*, $V = 0, \ T = 400 \text{ Nm}$

$$\tau_H = \frac{Tc}{J} = \frac{(400000)(18)}{164896} = 43.7 \text{ MPa}$$ ◀



PROBLEM 8.42

A 13-kN force is applied as shown to the 60-mm-diameter cast-iron post *ABD*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.

SOLUTION

$$DE = \sqrt{125^2 + 300^2} = 325 \text{ mm}$$

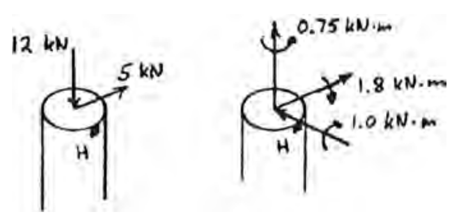
At point *D*, $F_x = 0$

$$F_y = -\left(\frac{300}{325}\right)(13) = -12 \text{ kN}$$

$$F_z = -\left(\frac{125}{300}\right)(13) = -5 \text{ kN}$$

Moment of equivalent force-couple system at *C*, the centroid of the section containing point *H*:

$$\vec{\boldsymbol{M}} = \begin{vmatrix} \vec{\boldsymbol{i}} & \vec{\boldsymbol{j}} & \vec{\boldsymbol{k}} \\ 0.150 & 0.200 & 0 \\ 0 & -12 & -5 \end{vmatrix} = -1.00\vec{\boldsymbol{i}} + 0.75\vec{\boldsymbol{j}} - 1.8\vec{\boldsymbol{k}} \text{ kN} \cdot \text{m}$$



Section properties:

$$d = 60 \text{ mm} \quad c = \frac{1}{2}d = 30 \text{ mm}$$

$$A = \pi c^2 = 2.8274 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{4}c^4 = 636.17 \times 10^3 \text{ mm}^4$$

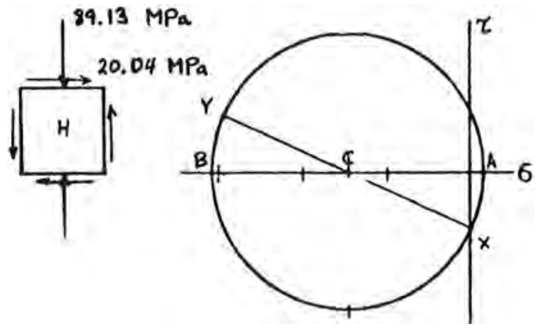
$$J = 2I = 1.2723 \times 10^6 \text{ mm}^4$$

For a semicircle, $Q = \frac{2}{3}c^3 = 18.00 \times 10^3 \text{ mm}^3$

PROBLEM 8.42 *(Continued)*

At point H, $$\sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{12\times 10^3}{2.8274\times 10^{-3}} - \frac{(1.8\times 10^3)(30\times 10^{-3})}{636.17\times 10^{-9}} = -89.13 \text{ MPa}$$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(0.75\times 10^3)(30\times 10^{-3})}{1.2723\times 10^{-6}} + \frac{(5\times 10^3)(18.00\times 10^{-6})}{(636.17\times 10^{-9})(60\times 10^{-3})} = 20.04 \text{ MPa}$$



(*a*) $$\sigma_{\text{ave}} = \frac{\sigma_H}{2} = -44.565 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 48.863 \text{ MPa}$$

$\sigma_a = \sigma_{\text{ave}} + R$ $\qquad\sigma_a = 4.3 \text{ MPa}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R$ $\qquad\sigma_b = -93.4 \text{ MPa}$ ◀

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = 0.4497$$

$\theta_a = 12.1°, \quad \theta_b = 102.1°$ ◀

(*b*) $\tau_{\max} = R = 48.9 \text{ MPa}$ ◀

PROBLEM 8.43

A 10-kN force and a 1.4-kN · m couple are applied at the top of the 65-mm diameter brass post shown. Determine the principal stresses and maximum shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION

At the section containing points *H* and *K*,

$$V = 10 \text{ kN} = 10\times10^3 \text{ N}$$

$$M = (10\times10^3)(240\times10^{-3}) = 2.4\times10^3 \text{ N}\cdot\text{m}$$

$$T = 1.4\times10^3 \text{ N}\cdot\text{m}$$

$$c = \frac{1}{2}d = 32.5 \text{ mm} = 0.0325 \text{ m}$$

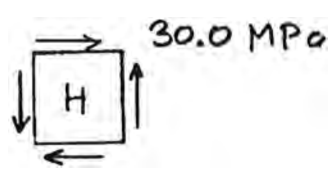
$$J = \frac{\pi}{2}c^4 = 1.75248\times10^{-6} \text{ m}^4$$

$$I = \frac{1}{2}J = 0.87624\times10^{-6} \text{ m}^4$$

For a semicircle, $$Q = \frac{2}{3}c^3 = \frac{2}{3}(0.0325)^3 = 22.885\times10^{-6} \text{ m}^3$$

(*a*) Stresses at point *H*. *H* lies on the neutral axis: $\sigma = 0$

Due to torque: $$\tau = \frac{Tc}{J} = \frac{(1.4\times10^3)(0.0325)}{1.75248\times10^{-6}} = 25.963 \text{ MPa}$$



Due to shear: $$\tau = \frac{VQ}{It} = \frac{(10\times10^3)(22.885\times10^{-6})}{(0.87624\times10^{-6})(0.065)} = 4.018 \text{ MPa}$$

Total at *H*: $\tau = 30.0 \text{ MPa}$

$$\sigma_{\text{ave}} = 0, \quad R = 30.0 \text{ MPa}$$

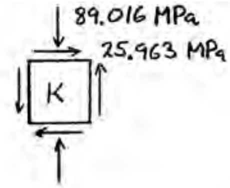
$\sigma_{\max} = \sigma_{\text{ave}} + R$ $\quad\quad$ $\sigma_{\max} = 30.0 \text{ MPa}$ ◀

$\sigma_{\min} = \sigma_{\text{ave}} - R$ $\quad\quad$ $\sigma_{\min} = -30.0 \text{ MPa}$ ◀

$\tau_{\max} = R$ $\quad\quad$ $\tau_{\max} = 30.0 \text{ MPa}$ ◀

PROBLEM 8.43 *(Continued)*

(*b*) Stresses at point K.

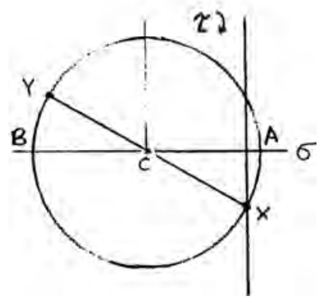


Due to shear: $\tau = 0$

Due to torque: $\tau = \dfrac{Tc}{J} = 25.963 \text{ MPa}$

Due to bending: $\sigma = -\dfrac{Mc}{I} = -\dfrac{(2.4\times10^3)(0.0325)}{(0.87624\times10^{-6})}$

$= -89.016 \text{ MPa}$



$$\sigma_{\text{ave}} = \frac{-89.016}{2} = -44.508 \text{ MPa}$$

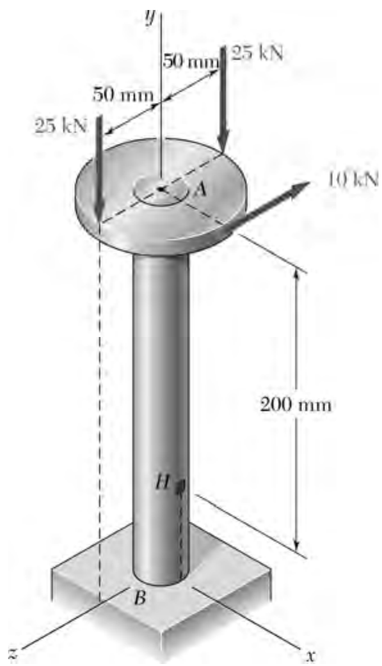
$$R = \sqrt{\left(\frac{89.016}{2}\right)^2 + (25.963)^2}$$

$$= 51.527 \text{ MPa}$$

$\sigma_{\max} = \sigma_{\text{ave}} + R$ $\qquad\sigma_{\max} = 7.02 \text{ MPa}$ ◀

$\sigma_{\min} = \sigma_{\text{ave}} - R$ $\qquad\sigma_{\min} = -96.0 \text{ MPa}$ ◀

$\tau_{\max} = R$ $\qquad\tau_{\max} = 51.5 \text{ MPa}$ ◀

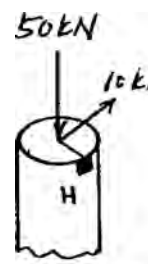


PROBLEM 8.44

Three forces are applied to a 100-mm diameter plate that is attached to the solid 45-mm diameter shaft *AB*. At point *H*, determine (*a*) the principal stresses and principal plates, (*b*) the maximum shearing stress.

SOLUTION

At the section containing points *H*,



$$P = 50 \text{ kN} \quad \text{(compression)}$$

$$V = 10 \text{ kN}$$

$$T = (10)(50) = 500 \text{ kNmm}$$

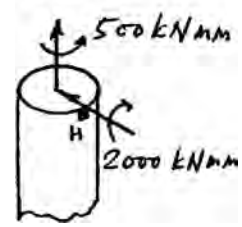
$$M = (10)(200) = 2000 \text{ kNmm}$$

$$d = 45 \text{ mm} \quad C = \frac{1}{2}d = 22.5 \text{ mm}$$

$$A = \pi C^2 = 1590 \text{ mm}^2$$

$$I = \frac{\pi}{4}C^4 = 0.2013\times 10^6 \text{ mm}^4$$

$$J = 2I = 0.4026\times 10^6 \text{ mm}^4$$



For a semicircle,

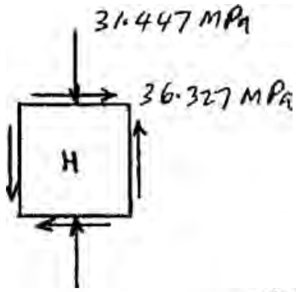
$$Q = \frac{2}{3}c^3 = 7.594\times 10^3 \text{ mm}^3$$

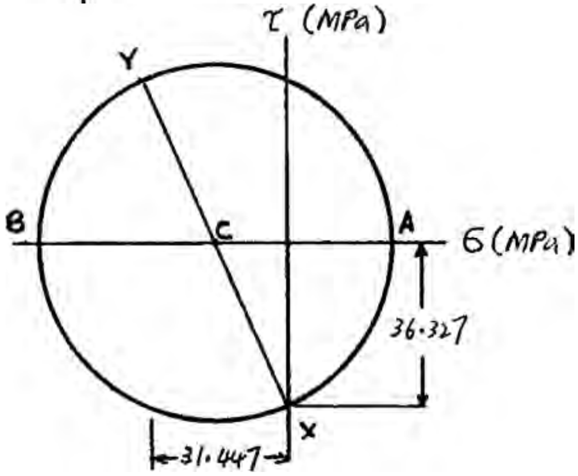
Point *H* lies on neutral axis of bending.

$$\sigma_H = \frac{P}{A} = -\frac{50000}{1590} = -31.447 \text{ MPa}$$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(5\times 10^5)(22.5)}{0.4026\times 10^6} + \frac{(10\times 10^3)(7.594\times 10^3)}{(0.2013\times 10^6)(45)} = 36.327 \text{ MPa}$$

PROBLEM 8.44 *(Continued)*





Use Mohr's circle.

$$\sigma_c = \frac{1}{2}(-31.447) = -15.7235 \text{ MPa}$$

$$R = \sqrt{\left(\frac{15.7235}{2}\right)^2 + 36.327^2} = 37.168 \text{ MPa}$$

(*a*) $\sigma_a = \sigma_c + R = 21.4 \text{ MPa}$

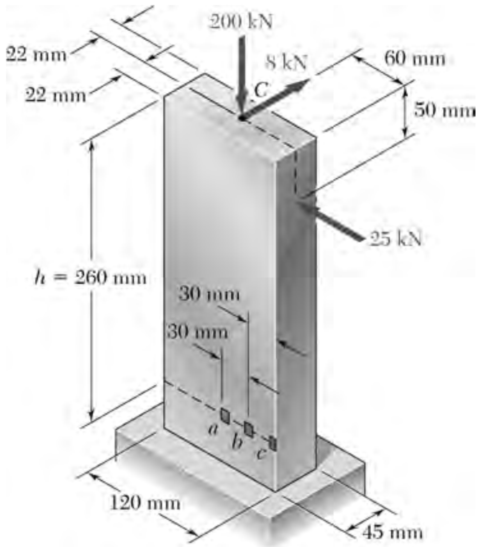
$\sigma_b = \sigma_c - R = -52.9 \text{ MPa}$ ◀

$$\tan 2\theta_p = \frac{(2)(36.327)}{31.447} = 2.3164$$

$\theta_a = 33.3°$ ◀

$\theta_b = 123.3°$ ◀

(*b*) $\tau_{\max} = R = 37.2 \text{ MPa}$ ◀



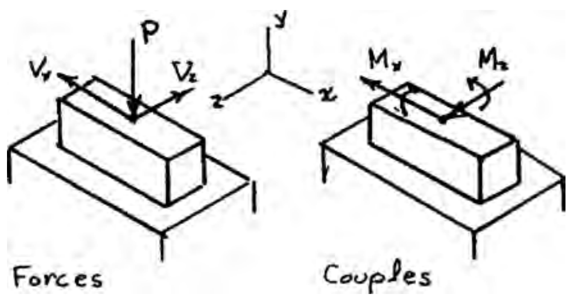
PROBLEM 8.45

Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Calculate forces and couples at section containing points *a*, *b*, and *c*.

$$h = 260 \text{ mm}$$



$$P = 200 \text{ N}, \quad V_x = 25 \text{ kN}, \quad V_z = 8 \text{ kN}$$

$$M_z = (260 - 50)(25) = 5250 \text{ kN}\cdot\text{mm}$$

$$M_x = (260)(8) = 2080 \text{ kN}\cdot\text{mm}$$

Section properties.

$$A = (120)(44) = 5280 \text{ mm}^2$$

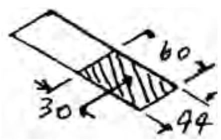
$$I_x = \frac{1}{12}(120)(44)^3 = 851840$$

$$I_z = \frac{1}{12}(44)(120)^3 = 6.336\times 10^6 \text{ mm}^4$$

Stresses.

$$\sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \qquad \tau = \frac{V_x Q}{I_z t}$$

PROBLEM 8.45 *(Continued)*

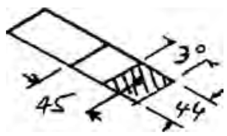


(*a*) Point *a*:

$$x = 0, \quad z = 22 \text{ mm}, \quad Q = (44)(60)\left(\frac{60}{2}\right) = 79200 \text{ mm}^3$$

$$\sigma = -\frac{200\times10^3}{5280} + 0 + \frac{(2.08\times10^6)(22)}{0.85184\times10^6} = 15.8 \text{ MPa} \quad ◀$$

$$\tau = \frac{(25\times10^3)(79200)}{(6.336\times10^6)(44)} = 7.1 \text{ MPa} \quad ◀$$



(*b*) Point *b*:

$$x = 30 \text{ mm}, \quad z = 22 \text{ mm}, \quad Q = (44)(30)(45) = 59400 \text{ mm}^3$$

$$\sigma = -\frac{200\times10^3}{5280} + \frac{(5.25\times10^6)(30)}{6.336\times10^6} + \frac{(208\times10^6)(22)}{0.85184\times10^6} = 40.7 \text{ MPa} \quad ◀$$

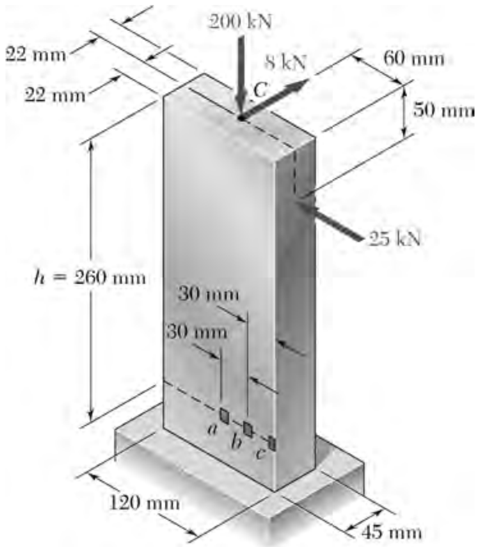
$$\tau = \frac{(25\times10^3)(59400)}{[(6.336\times10^6)(44)]} = 5.3 \text{ MPa} \quad ◀$$

(*c*) Point *c*:

$$x = 60 \text{ mm}, \quad z = 22 \text{ mm}, \quad Q = 0$$

$$\sigma = -\frac{200\times10^3}{5280} + \frac{(5.25\times10^6)(60)}{6.336\times10^6} + \frac{(2.08\times10^6)(22)}{0.85184\times10^6} = 65.6 \text{ MPa} \quad ◀$$

$$\tau = 0 \quad ◀$$



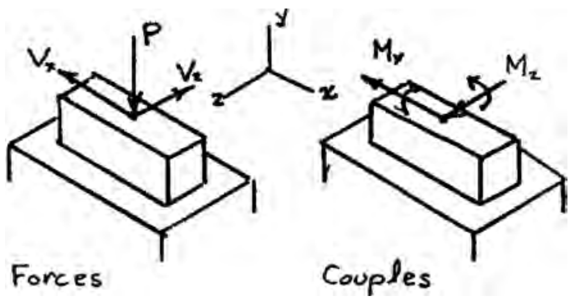
PROBLEM 8.46

Solve Prob. 8.45, assuming that $h = 300$ mm

PROBLEM 8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Calculate forces and couples at section containing points *a*, *b*, and *c*. $h = 300$ mm



$$P = 200 \text{ kN}, \quad V_x = 25 \text{ kN}, \quad V_z = 8 \text{ kN}$$

$$M_z = (300 - 50)(25) = 6250 \text{ kN}\cdot\text{mm}$$

$$M_x = (300)(8) = 2400 \text{ kN}\cdot\text{mm}$$

Section properties. $A = (120)(44) = 5280 \text{ mm}^2$

$$I_x = \frac{1}{12}(120)(44)^3 = 851840 \text{ mm}^4$$

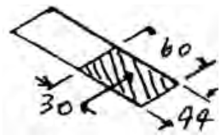
$$I_z = \frac{1}{12}(44)(120)^3 = 6.336\times 10^6 \text{ mm}^4$$

Stresses. $\sigma = -\dfrac{P}{A} + \dfrac{M_z x}{I_z} + \dfrac{M_x z}{I_x}$

$$\tau = \frac{V_x Q}{I_z t}$$

PROBLEM 8.46 *(Continued)*

(*a*) Point *a*:

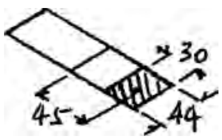


$x = 0, \quad z = 22 \text{ mm}, \quad Q = (44)(60)(30) = 79200 \text{ mm}^3$

$$\sigma = -\frac{200\times10^3}{5280} + 0 + \frac{(2.4\times10^6)(22)}{0.85184\times10^6} = 24.1 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = \frac{(25\times10^3)(79200)}{(6.336\times10^6)(44)} = 7.1 \text{ MPa} \quad \blacktriangleleft$$

(*b*) Point *b*:



$x = 30 \text{ mm}, \quad z = 22 \text{ mm}, \quad Q = (30)(44)(45) = 59400 \text{ mm}^3$

$$\sigma = -\frac{200\times10^3}{5280} + \frac{(6.25\times10^6)(30)}{10.336\times10^6} + \frac{(2.4\times10^6)(22)}{0.85184\times10^6} = 53.7 \text{ MPa} \quad \blacktriangleleft$$

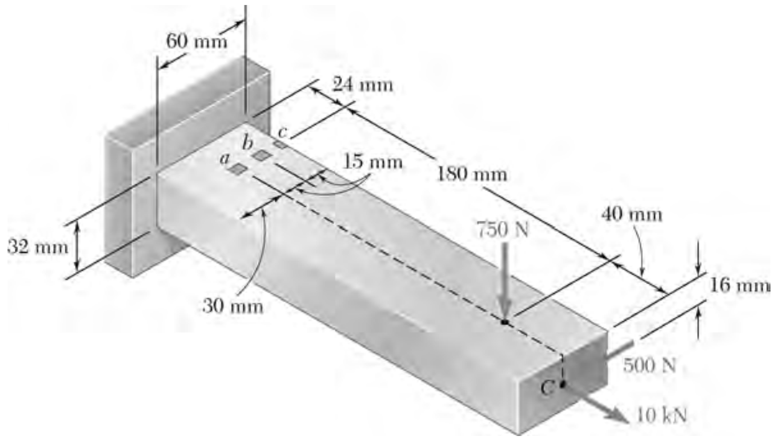
$$\tau = \frac{(25\times10^3)(59400)}{(6.336\times10^6)(44)} = 5.3 \text{ MPa} \quad \blacktriangleleft$$

(*c*) Point *c*:

$x = 60 \text{ mm}, \quad z = 22 \text{ mm}, \quad Q = 0$

$$\sigma = -\frac{200\times10^3}{5280} + \frac{(6.25\times10^6)(60)}{6.336\times10^6} + \frac{(2.4\times10^6)(22)}{0.85184\times10^6} = 83.3 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = 0 \quad \blacktriangleleft$$



PROBLEM 8.47

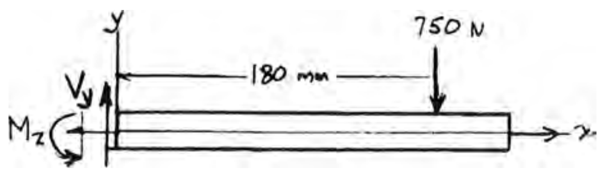
Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

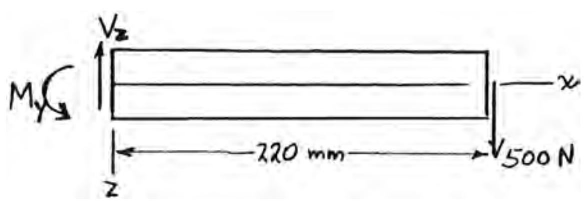
$$A = (60)(32) = 1920 \text{ mm}^2$$
$$= 1920 \times 10^{-6} \text{ m}^2$$
$$I_z = \frac{1}{12}(60)(32)^3 = 163.84 \times 10^3 \text{ mm}^4$$
$$= 163.84 \times 10^{-9} \text{ m}^4$$
$$I_y = \frac{1}{12}(32)(60)^3$$
$$= 579 \times 10^3 \text{ mm}^4$$
$$= 576 \times 10^{-9} \text{ m}^4$$

At the section containing points *a*, *b*, and *c*,

$$P = 10 \text{ kN}$$
$$V_y = 750 \text{ N},$$
$$V_z = 500 \text{ N}$$
$$M_z = (180 \times 10^{-3})(750)$$
$$= 135 \text{ N} \cdot \text{m}$$
$$M_y = (220 \times 10^{-3})(500)$$
$$= 110 \text{ N} \cdot \text{m}$$
$$T = 0$$
$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$
$$\tau = \frac{V_z Q}{I_y t}$$



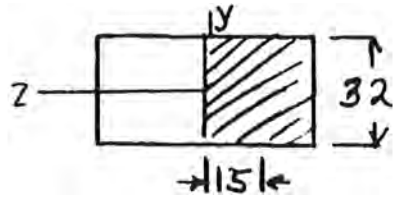
Side View



Top View

PROBLEM 8.47 *(Continued)*

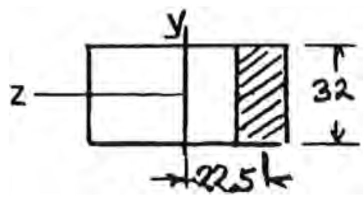
(*a*) Point *a*: $y = 16\text{ mm}, \quad z = 0, \quad Q = A\bar{z} = (32)(30)(15) = 14.4\times10^3\text{ mm}^3$



$$\sigma = \frac{10\times10^3}{1920\times10^{-6}} + \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - 0 \qquad \sigma = 18.39\text{ MPa} \blacktriangleleft$$

$$\tau = \frac{(500)(14.4\times10^{-6})}{(576\times10^{-9})(32\times10^{-3})} \qquad \tau = 0.391\text{ MPa} \blacktriangleleft$$

(*b*) Point *b*: $y = 16\text{ mm}, \quad z = -15\text{ mm}, \quad Q = A\bar{z} = (32)(15)(22.5) = 10.8\times10^3\text{ mm}^3$



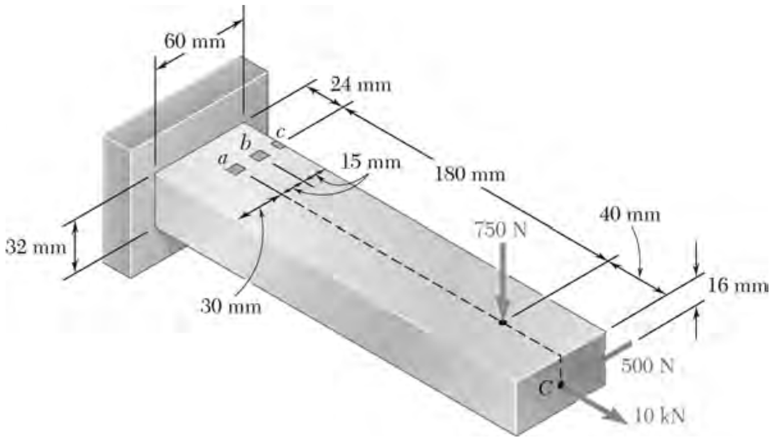
$$\sigma = \frac{10\times10^3}{1920\times10^{-6}} + \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-15\times10^{-3})}{576\times10^{-9}} \qquad \sigma = 21.3\text{ MPa} \blacktriangleleft$$

$$\tau = \frac{(500)(10.8\times10^{-6})}{(576\times10^{-9})(32\times10^{-3})} \qquad \tau = 0.293\text{ MPa} \blacktriangleleft$$

(*c*) Point *c*: $y = 16\text{ mm}, \quad z = -30\text{ mm}, \quad Q = 0$

$$\sigma = \frac{10\times10^3}{1920\times10^{-6}} + \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-30\times10^{-6})}{576\times10^{-9}} \qquad \sigma = 24.1\text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$



PROBLEM 8.48

Solve Prob. 8.47, assuming that the 750-N force is directed vertically upward.

PROBLEM 8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

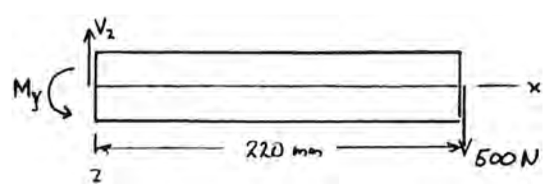
SOLUTION

$$A = (60)(32) = 1920\text{ mm}^2$$
$$= 1920\times10^{-6}\text{ m}^2$$
$$I_z = \frac{1}{12}(60)(32)^3$$
$$= 163.84\times10^3\text{ mm}^4$$
$$= 163.84\times10^{-9}\text{ m}^4$$
$$I_y = \frac{1}{12}(32)(60)^3$$
$$= 576\times10^3\text{ mm}^4$$
$$= 576\times10^{-9}\text{ m}^4$$

At the section containing points *a*, *b*, and *c*,

$$P = 10\text{ kN} \qquad T = 0$$
$$V_y = 750\text{ N}$$
$$V_z = 500\text{ N}$$
$$M_z = (180\times10^{-3})(750)$$
$$= 135\text{ N}\cdot\text{m}$$
$$M_y = (220\times10^{-3})(500)$$
$$= 110\text{ N}\cdot\text{m}$$
$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$
$$\tau = \frac{V_z Q}{I_y t}$$

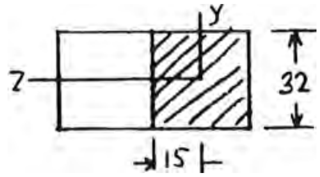
Side View



Top View

PROBLEM 8.48 *(Continued)*

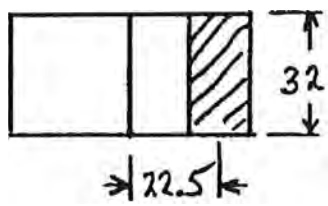
(*a*) Point *a*: $y = 16\text{ mm},\quad z = 0,\quad Q = A\bar{z} = (32)(30)(15) = 14.4\times10^3\,\text{mm}^3$



$$\sigma = \frac{10\times10^3}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} = 0 \qquad \sigma = -7.98\text{ MPa} \blacktriangleleft$$

$$\tau = \frac{(500)(14.4\times10^{-6})}{(163.84\times10^{-9})(32\times10^{-3})} \qquad \tau = 0.391\text{ MPa} \blacktriangleleft$$

(*b*) Point *b*: $y = 16\text{ mm},\quad z = -15\text{ mm},\quad Q = A\bar{z} = (32)(15)(22.5) = 10.8\times10^3\,\text{mm}^3$



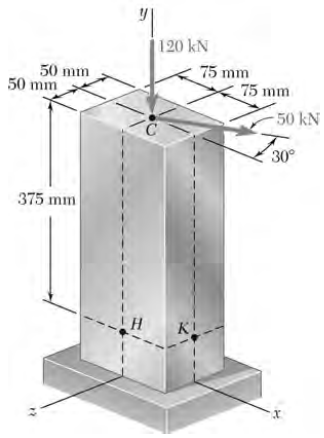
$$\sigma = \frac{10\times10^3}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-15\times10^{-3})}{576\times10^{-9}} \qquad \sigma = -5.11\text{ MPa} \blacktriangleleft$$

$$\tau = \frac{(500)(10.8\times10^{-6})}{(163.84\times10^{-9})(32\times10^{-9})} \qquad \tau = 0.293\text{ MPa} \blacktriangleleft$$

(*c*) Point *c*: $y = 16\text{ mm},\quad z = -30\text{ mm},\quad Q = 0$

$$\sigma = \frac{10\times10^3}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-30\times10^{-3})}{576\times10^{-9}} \qquad \sigma = -2.25\text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$



PROBLEM 8.49

For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

SOLUTION

Components of force at point *C*:

$$F_x = 50 \cos 30° = 43.301 \text{ kN}$$

$$F_z = -50 \sin 30° = -25 \text{ kN}$$

$$F_y = -120 \text{ kN}$$

Section forces and couples at the section containing points *H* and *K*:

$$P = 120 \text{ kN} \qquad \text{(compression)}$$

$$V_x = 43.301 \text{ kN}$$

$$V_z = -25 \text{ kN}$$

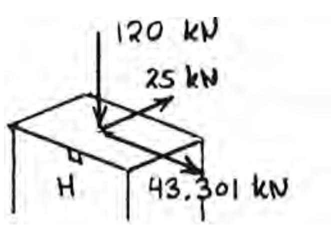
$$M_x = -(25)(0.375) = -9.375 \text{ kN} \cdot \text{m}$$

$$M_y = 0$$

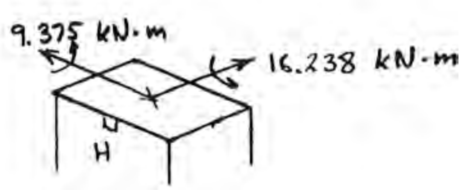
$$M_z = -(43.301)(0.375) = -16.238 \text{ kN} \cdot \text{m}$$

$$A = (100)(150) = 15 \times 10^3 \text{mm}^2 = 15 \times 10^{-3} \text{m}^2$$

$$I_x = \frac{1}{12}(150)(100)^3 = 12.5 \times 10^6 \text{mm}^4 = 12.5 \times 10^{-6} \text{m}^4$$



Forces



Couples

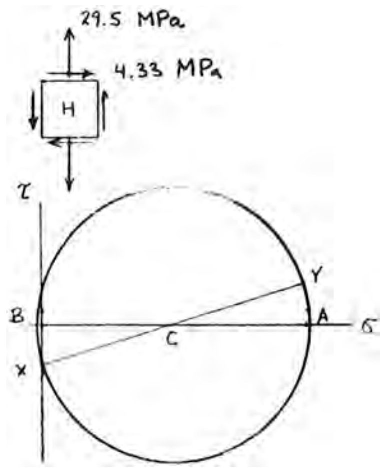
PROBLEM 8.49 *(Continued)*

Stresses at point H:

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} = -\frac{(120\times10^3)}{15\times10^{-3}} - \frac{(-9.375\times10^3)(50\times10^{-3})}{12.5\times10^{-6}} = 29.5 \text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{V_x}{A} = \frac{3}{2}\frac{43.301\times10^3}{15\times10^{-3}} = 4.33 \text{ MPa}$$

Use Mohr's circle.



$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H = 14.75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{29.5}{2}\right)^2 + 4.33^2} = 15.37 \text{ MPa}$$

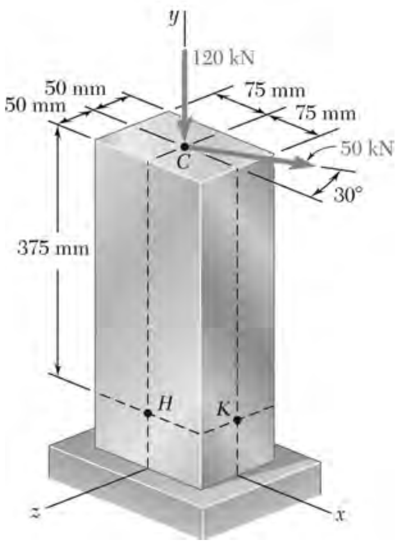
$\sigma_a = \sigma_{\text{ave}} + R$ $\qquad$ $\sigma_a = 30.1 \text{ MPa}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R$ $\qquad$ $\sigma_b = -0.62 \text{ MPa}$ ◀

$$\tan 2\theta_p = \frac{2\tau_H}{-\sigma_H} = -0.2936$$

$\theta_a = -8.2°$ $\quad$ $\theta_b = 81.8°$ ◀

$\tau_{\max} = R$ $\qquad$ $\tau_{\max} = 15.37 \text{ MPa}$ ◀



PROBLEM 8.50

For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point K.

SOLUTION

Components of force at point C:

$$F_x = 50\cos 30° = 43.301 \text{ kN}$$

$$F_z = -50\sin 30° = -25 \text{ kN} \qquad F_y = -120 \text{ kN}$$

Section forces and couples at the section containing points H and K:

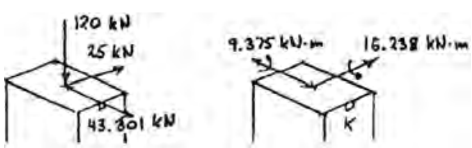
$$P = 120 \text{ kN (compression)}$$

$$V_x = 43.301 \text{ kN}, \qquad V_z = -25 \text{ kN}$$

$$M_x = -(25)(0.375) = -9.375 \text{ kN} \cdot \text{m}$$

$$M_y = 0$$

$$M_z = -(43.301)(0.375) = -16.238 \text{ kN} \cdot \text{m}$$



$$A = (100)(150) = 15 \times 10^3 \text{ mm}^2$$

$$= 15 \times 10^{-3} \text{ m}^2$$

$$I_z = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$

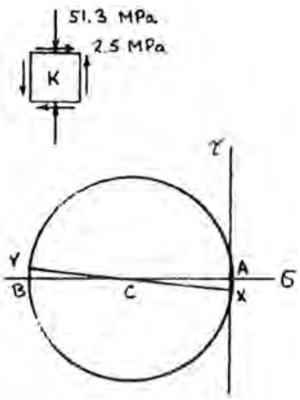
$$= 28.125 \times 10^{-6} \text{ m}^4$$

PROBLEM 8.50 *(Continued)*

Stresses at point K:

$$\sigma_K = -\frac{P}{A} + \frac{M_z x}{I_z} = -\frac{120 \times 10^3}{15 \times 10^{-3}} + \frac{(-16.238 \times 10^3)(75 \times 10^{-3})}{28.125 \times 10^{-6}} = -51.3 \text{ MPa}$$

$$\tau_K = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{25 \times 10^3}{15 \times 10^{-3}} = 2.5 \text{ MPa}$$



Use Mohr's circle.

$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_K = -25.65 \text{ MPa}$$

$$R = \sqrt{\left(\frac{51.3}{2}\right)^2 + (2.5)^2} = 25.77 \text{ MPa}$$

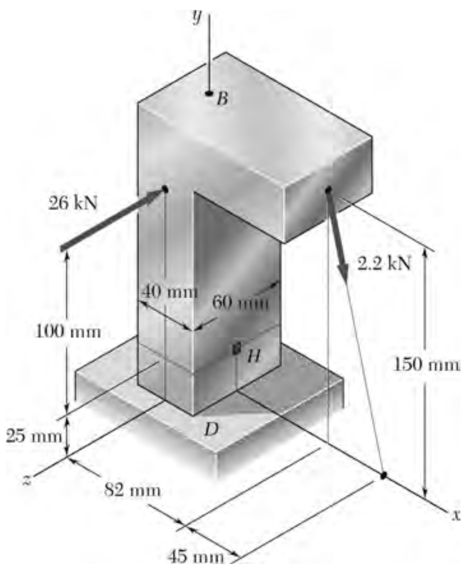
$\sigma_a = \sigma_{\text{ave}} + R$ $\qquad\qquad$ $\sigma_a = 0.12 \text{ MPa}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R$ $\qquad\qquad$ $\sigma_b = -51.4 \text{ MPa}$ ◀

$$\tan 2\theta_p = \frac{2\tau_K}{-\sigma_K} = 0.09747$$

$\theta_a = 2.8^\circ \qquad \theta_b = 92.8^\circ$ ◀

$\tau_{\max} = R$ $\qquad\qquad$ $\tau_{\max} = 25.8 \text{ MPa}$ ◀



PROBLEM 8.51

Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 40×60 mm, determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

SOLUTION

Components of 2.2 kN force:

$$F_x = \frac{(2.2\times 10^3)(45)}{156.6} = 632.2 \text{ N}$$

$$F_y = -\frac{(2.2\times 10^3)(150)}{156.6} = -2107.3 \text{ N}$$

Moment arm of 2.2 kN force:

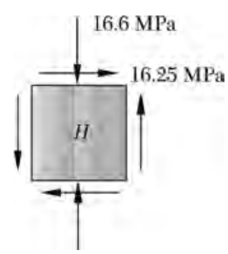
$$\vec{r} = 82\vec{i} + (150-25)\vec{j}$$

Moment of 2.2 kN force:

$$\overrightarrow{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 82 & 125 & 0 \\ 632.2 & -21073 & 0 \end{vmatrix} = -251823.6\vec{k} \text{ N}\cdot\text{mm} = -251.8\vec{k} \text{ N}\cdot\text{m}$$

At the section containing point *H*, $\quad P = -2107.3 \text{ N} \qquad V_x = 632.2 \text{ N}$

$$V_z = -26 \text{ kN}, \qquad M_z = -251.8 \text{ N}\cdot\text{m}, \qquad M_x = -(0.100)(26) = -2.6 \text{ kN}\cdot\text{m}$$



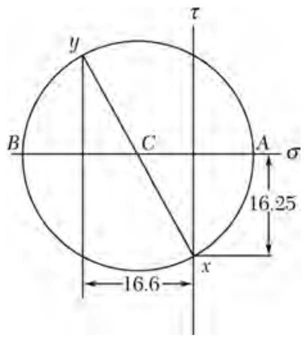
$$A = (40)(60) = 2400 \text{ mm}^2 \qquad I_z = \frac{1}{12}(60)(40)^3 = 32\times 10^4 \text{ mm}^4$$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{2107.3}{2400\times 10^{-6}} + \frac{(-251.8)(0.020)}{32\times 10^{-8}} = -16.6 \text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{-26\times 10^3}{2400\times 10^{-6}} = 16.25 \text{ MPa}$$

$$\sigma_c = -\frac{16.6}{2} = -8.3 \text{ MPa}$$

PROBLEM 8.51 *(Continued)*



$$R = \sqrt{\left(\frac{16.6}{2}\right)^2 + (16.25)^2} = 18.24 \text{ MPa}$$

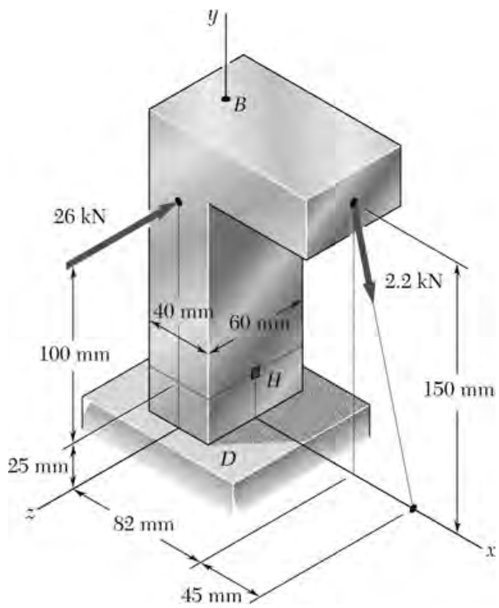
$$\sigma_a = \sigma_c + R = 9.94 \text{ MPa}$$ ◀

$$\sigma_b = \sigma_c - R = -26.54 \text{ MPa}$$ ◀

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(16.25)}{16.6} = 1.958$$

$$\theta_a = 31.47°, \quad \theta_b = 121.47°$$

$$\tau_{\max} = R = 18.24 \text{ MPa}$$ ◀



PROBLEM 8.52

Solve Prob. 8.51, assuming that the magnitude of the 26 kN force is reduced to 6.5 kN.

PROBLEM 8.51 Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 38×60 mm, determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

SOLUTION

Components of 2.2 kN force:

$$F_x = \frac{(2.2\times 10^3)(45)}{156.6} = 632.2 \text{ N}$$

$$F_y = \frac{(2.2\times 10^3)(150)}{156.6} = -2107.3 \text{ N}$$

Moment arm of 2.2 kN force:

$$\vec{r} = 82\vec{i} + (150-25)\vec{j}$$

$$\overline{\vec{M}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 82 & 125 & 0 \\ 632.2 & -21073 & 0 \end{vmatrix} = -251823.6\vec{k} \text{ kN}\cdot\text{mm} = 251.8\vec{k} \text{ N}\cdot\text{m}$$

At the section containing point *H*, $P = -2107.3 \text{ N}$ $\quad V_x = 632.2 \text{ N}$

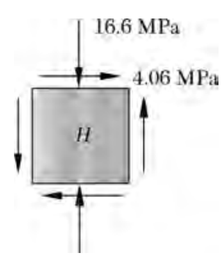
$$V_z = -6500 \text{ N}, \quad M_z = -251.8 \text{ N}\cdot\text{m}, \quad M_x = -(0.100)(6500) = 650 \text{ N}\cdot\text{m}$$

$$A = (40)(60) = 2400 \text{ mm}^2 \qquad I_z = \frac{1}{12}(60)(40)^3 = 32\times 10^4 \text{ mm}^4$$

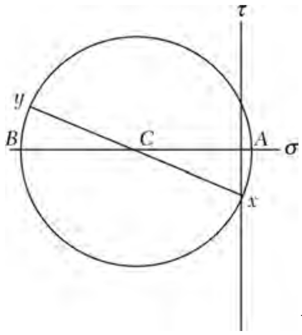
$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{2107.3}{2400\times 10^{-6}} + \frac{(-251.8)(0.020)}{32\times 10^{-8}} = -16.6 \text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{6500}{2400\times 10^{-6}} = 4.06 \text{ MPa}$$

$$\sigma_c = \frac{1}{2}\sigma_H = -8.3 \text{ MPa}$$



PROBLEM 8.52 *(Continued)*



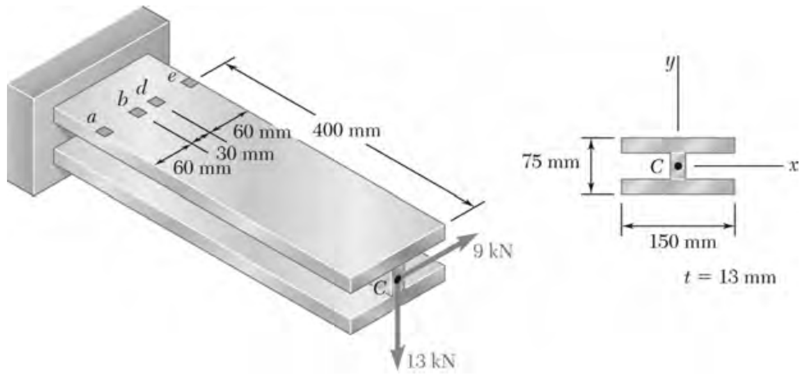
$$R = \sqrt{\left(\frac{16.6}{2}\right)^2 + (4.06)^2} = 9.24 \text{ MPa}$$

$$\sigma_a = \sigma_c + R = 0.94 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_c - R = -17.54 \text{ MPa} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(4.06)}{16.6} = 0.4891$$

$$\theta_a = 13.03^\circ \qquad \theta_b = 103.03^\circ \qquad \tau_{\max} = R = 9.24 \text{ MPa} \blacktriangleleft$$



PROBLEM 8.53

Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *a* and *b*.

SOLUTION

Equivalent force-couple system at section containing points *a* and *b*.

$$F_x = 9 \text{ kN}, \quad F_y = -13 \text{ kN}, \quad F_z = 0$$

$$M_x = (0.400)(13\times10^3) = 5200 \text{ N}\cdot\text{m}$$

$$M_y = (0.400)(9\times10^3) = 3600 \text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75-26)$$

$$= 4537 \text{ mm}^2$$

$$= 4537\times10^{-6}\,\text{m}^2$$

$$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \frac{1}{12}(13)(75-26)^3$$

$$= 3.9303\times10^6\,\text{mm}^4$$

$$= 3.9303\times10^{-6}\,\text{m}^4$$

$$I_y = 2\cdot\frac{1}{12}(13)(150)^3 + \frac{1}{12}(75-26)(13)^3$$

$$= 7.3215\times10^6\,\text{mm}^4 = 7.3215\times10^{-6}\,\text{mm}^4$$

For point *a*,

$$Q_x = 0, \quad Q_y = 0$$

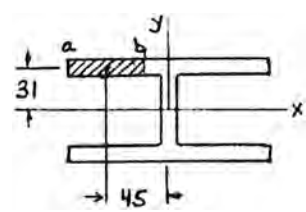
For point *b*,

$$A^* = (60)(13) = 780 \text{ mm}^2$$

$$\bar{x} = -45 \text{ mm} \quad \bar{y} = 31 \text{ mm}$$

$$Q_x = A^*\bar{y} = 24.18\times10^3 \text{ mm}^3 = 24.18\times10^{-6} \text{ m}^3$$

$$Q_y = A^*\bar{x} = -35.1\times10^{-3} \text{ mm}^3 = -35.1\times10^{-6} \text{ m}^3$$



PROBLEM 8.53 *(Continued)*

Direction of shearing stress for horizontal and for vertical components of shear.

At point *a*:

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-75\times10^{-3})}{7.3215\times10^{-6}} \qquad \sigma = 86.5 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

At point *b*:

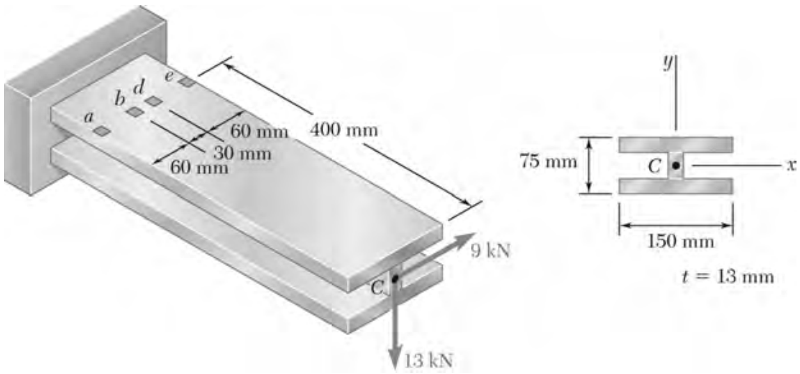
$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-15\times10^{-3})}{7.3215\times10^{-6}} \qquad \sigma = 57.0 \text{ MPa} \blacktriangleleft$$

$$\tau = \frac{|V_x||Q_y|}{I_y t} + \frac{|V_y||Q_x|}{I_x t}$$

$$= \frac{(9\times10^3)(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})} + \frac{(13\times10^3)(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})}$$

$$= 3.32 \text{ MPa} + 6.15 \text{ MPa} \qquad \tau = 9.47 \text{ MPa} \blacktriangleleft$$



PROBLEM 8.54

Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *d* and *e*.

SOLUTION

Equivalent force-couple system at section containing points *d* and *e*.

$$F_x = 9 \text{ kN}, \quad F_y = -13 \text{ kN}, \quad F_z = 0$$

$$M_x = (0.400)(13\times 10^3) = 5200 \text{ N}\cdot\text{m}$$

$$M_y = (0.400)(9\times 10^3) = 3600 \text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75-26)$$

$$= 4537 \text{ mm}^2$$

$$= 4537\times 10^{-6} \text{ m}^2$$

$$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \frac{1}{12}(13)(75-26)^3$$

$$= 3.9303\times 10^6 \text{ mm}^4$$

$$= 3.9303\times 10^{-6} \text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(13)(150)^3\right] + \frac{1}{12}(75-26)(13)^3$$

$$= 7.3215\times 10^6 \text{ mm}^4 = 7.3215\times 10^{-6} \text{ m}^4$$

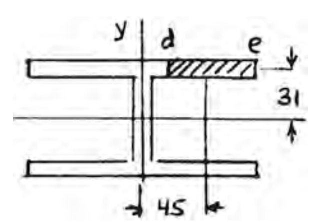
For point *d*,

$$A^* = (60)(13) = 780 \text{ mm}^2$$

$$\bar{x} = 45 \text{ mm} \quad \bar{y} = 31 \text{ mm}$$

$$Q_x = A^*\bar{y} = 24.18\times 10^3 \text{ mm}^3 = 24.18\times 10^{-6} \text{ m}^3$$

$$Q_y = A^*\bar{x} = 35.1\times 10^3 \text{ mm}^3 = 35.1\times 10^{-6} \text{ m}^3$$



For point *e*, $Q_x = 0, \quad Q_y = 0$

PROBLEM 8.54 *(Continued)*

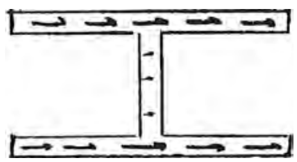
<u>At point *d*:</u>

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(15\times10^{-3})}{7.3215\times10^{-6}} \qquad \sigma = 42.2 \text{ MPa} \blacktriangleleft$$

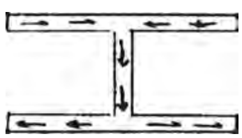
Due to V_x:

$$\tau = \frac{|V_x||Q_y|}{I_y t} = \frac{(9000)(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})}$$
$$= 3.32 \text{ MPa} \rightarrow$$



Due to V_y:

$$\tau = \frac{|V_y||Q_x|}{I_x\ t} = \frac{(13000)(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})} = 6.15 \text{ MPa} \leftarrow$$



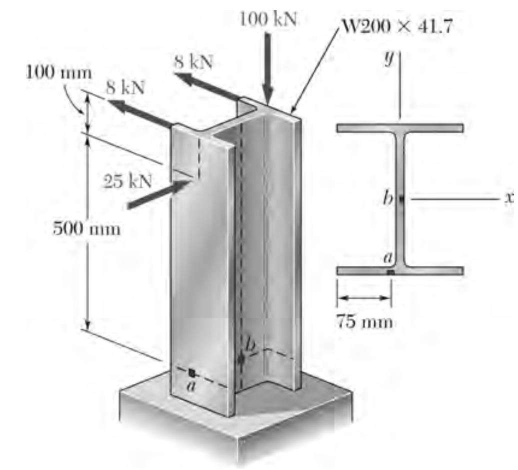
By superposition, the net value is $\qquad \tau = 2.83 \text{ MPa} \blacktriangleleft$

<u>At point *e*:</u>

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(75\times10^{-3})}{7.3215\times10^{-6}}$$

$$\sigma = 12.74 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

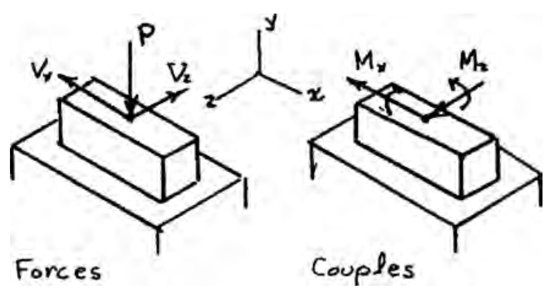


PROBLEM 8.55

Four forces are applied to a W200×41.7 rolled beam as shown. Determine the principal stresses and maximum shearing stress at point *a*.

SOLUTION

Calculate forces and couples at section containing point *a*.



Section properties.

$$A = 5310 \text{ mm}^2 \quad d = 205 \text{ mm}$$
$$b_f = 166 \text{ mm} \quad \tau_f = 11.8 \text{ mm}$$
$$t_w = 7.2 \text{ mm}$$

$$P = 100 \text{ kN}, \quad V_x = -16 \text{ kN}, \quad V_y = 25 \text{ kW}$$
$$M_x = -(500\times10^{-3})(25\times10^{3}) - \left(\frac{205}{2}\times10^{-3}\right)(100\times10^{3})$$
$$= -122.75\times10^{3} \text{ N}\cdot\text{m}$$
$$M_y = (500+100)(10^{-3})(16\times10^{3})$$
$$= 9.6\times10^{3} \text{ N}\cdot\text{m}$$

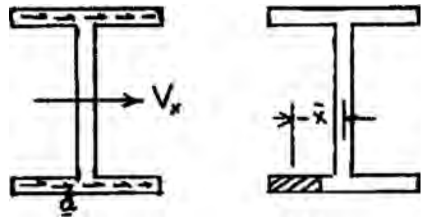
$$I_x = 40.9\times10^{6} \text{ mm}^4 = 40.9\times10^{-6} \text{ m}^4, \quad I_y = 9.01\times10^{6} \text{ mm}^4 = 9.01\times10^{-6} \text{ m}^4$$

Point *a*: $x_a = -\dfrac{166}{2} + 75 = -8 \text{ mm}, \quad y_a = -\dfrac{205}{2} = -102.5 \text{ mm}$

$$\sigma_a = -\frac{P}{A} + \frac{M_x y_a}{I_x} + \frac{M_y x_a}{I_y}$$
$$= -\frac{100\times10^{3}}{5310\times10^{-6}} + \frac{(-22.75\times10^{3})(-102.5\times10^{-3})}{40.9\times10^{-6}} + \frac{(9.6\times10^{3})(-8\times10^{-3})}{9.01\times10^{-6}}$$
$$= -18.83\times10^{6} + 57.01\times10^{6} - 8.52\times10^{6} = 29.66 \text{ MPa}$$

PROBLEM 8.55 *(Continued)*

Searing stress at point *a* due to V_x:



$$A = (75)(11.8) = 885 \text{ mm}^2$$

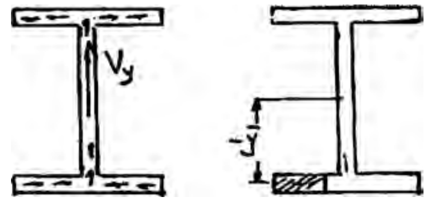
$$\bar{x} = -\frac{166}{2} + \frac{75}{2} = -45.5 \text{ mm}$$

$$Q = -A\bar{x} = 40.2675\times 10^3 \text{ mm}^2$$

$$\tau_{xz} = \frac{V_x Q}{I_y t} = \frac{(-16\times 10^3)(40.2675\times 10^{-6})}{(9.01\times 10^{-6})(11.8\times 10^{-3})}$$

$$= -6.060 \text{ MPa}$$

Shearing stress at point *a* due to V_y



$$A = (75)(11.8) = 885 \text{ mm}^2$$

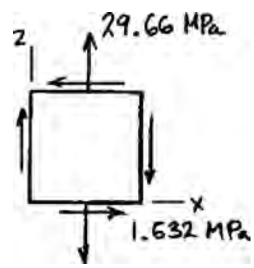
$$\bar{y} = -\frac{205}{2} + \frac{11.8}{2} = -96.6 \text{ mm}$$

$$Q = -A\bar{y} = -85.491\times 10^3 \text{ mm}^2$$

$$\tau_{xz} = -\frac{V_y Q}{I_x t} = -\frac{(25\times 10^3)(-85.491\times 10^{-6})}{(40.9\times 10^{-6})(11.8\times 10^{-3})}$$

$$= 4.428 \text{ MPa}$$

Combined shearing stress: $\tau_a = -6.060 + 4.428 = -1.632$ MPa



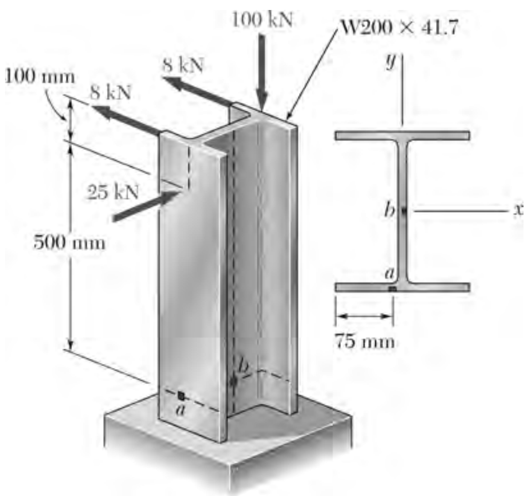
$$\sigma_{\text{ave}} = \frac{29.66 + 0}{2} = 14.83 \text{ MPa}$$

$$R = \sqrt{\left(\frac{29.66 - 0}{2}\right)^2 + (1.632)^2} = 14.92 \text{ MPa}$$

$\sigma_{\max} = \sigma_{\text{ave}} + R$ $\qquad$ $\sigma_{\max} = 29.8$ MPa ◀

$\sigma_{\min} = \sigma_{\text{ave}} - R$ $\qquad$ $\sigma_{\min} = -0.09$ MPa ◀

$\tau_{\max} = R$ $\qquad$ $\sigma_{\max} = 14.92$ MPa ◀

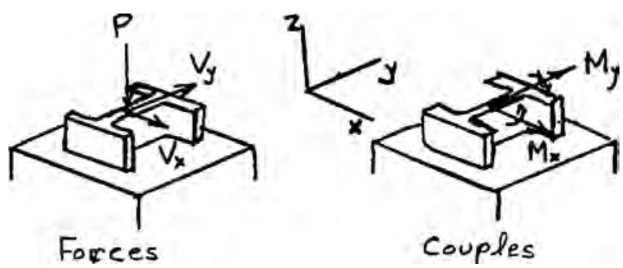


PROBLEM 8.56

Four forces are applied to a W200×41.7 rolled beam as shown. Determine the principal stresses and maximum shearing stress at point *b*.

SOLUTION

Calculate forces and couples at section containing point *b*.



Section properties.

$$A = 5310 \text{ mm}^4 \quad d = 205 \text{ mm}$$
$$b_f = 166 \text{ mm} \quad \tau_f = 11.8 \text{ mm}$$
$$t_w = 7.2 \text{ mm}$$

$$P = 100 \text{ kN}, \quad V_x = -16 \text{ kN}, \quad V_y = 25 \text{ kN}$$
$$M_x = -(500\times10^{-3})(25\times10^3) - \left(\frac{205}{2}\times10^{-3}\right)(100\times10^3)$$
$$= -22.75\times10^3 \text{ N}\cdot\text{m}$$
$$M_y = (500+100)(10^{-3})(16\times10^3)$$
$$= 9.6\times10^3 \text{ N}\cdot\text{m}$$

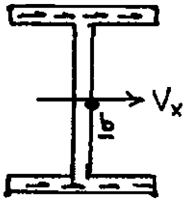
$$I_x = 40.9\times10^6 \text{ mm}^4 = 40.9\times10^{-6} \text{ m}^4, \quad I_y = 9.01\times10^6 \text{ mm}^4 = 9.01\times10^{-6} \text{ m}^4$$

Point *b*: $x_b = 0, \quad y_b = 0$

$$\sigma_b = -\frac{P}{A} + \frac{M_x y_b}{I_x} + \frac{M_y x_b}{I_y} = -\frac{100\times10^3}{5310\times10^{-6}} = -18.83 \text{ MPa}$$

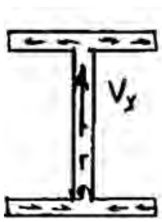
PROBLEM 8.56 *(Continued)*

Shearing stress at point b due to V_x.



$$\tau_{xz} = 0$$

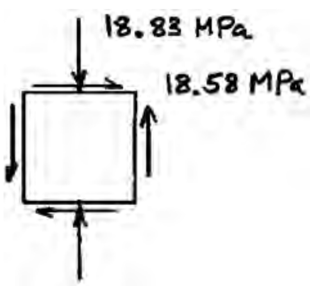
Shearing stress at point b due to V_y.



	$A\,(\text{mm}^2)$	$\bar{y}$ (mm)	$A\bar{y}$ (10^3 mm^3)
①	1958.8	96.6	189.220
②	653.04	45.35	29.615
Σ			218.835

$$Q = \Sigma A\bar{y} = 218.835\times10^3 \text{ mm}^3 = 218.835\times10^{-6} \text{ m}^3$$

$$\tau_b = \frac{V_y Q}{I_x I_w} = \frac{(25\times10^3)(218.835\times10^{-6})}{(40.9\times10^{-6})(7.2\times10^{-3})} = 18.58 \text{ MPa}$$



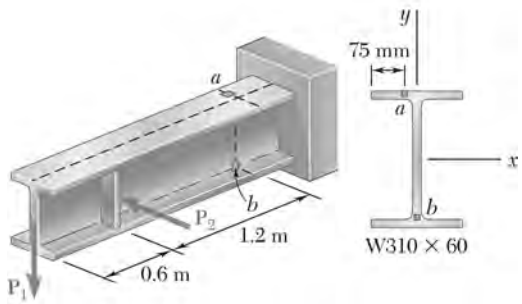
$$\sigma_{\text{ave}} = \frac{-18.83+0}{2} = -9.415 \text{ MPa}$$

$$R = \sqrt{\left(\frac{-18.83-0}{2}\right)^2 + (18.58)^2} = 20.829 \text{ MPa}$$

$\sigma_{\text{max}} = \sigma_{\text{ave}} + R$ $\qquad\qquad$ $\sigma_{\text{max}} = 11.41 \text{ MPa}$ ◀

$\sigma_{\text{min}} = \sigma_{\text{ave}} - R$ $\qquad\qquad$ $\sigma_{\text{min}} = -30.2 \text{ MPa}$ ◀

$\tau_{\text{max}} = R$ $\qquad\qquad$ $\sigma_{\text{max}} = 20.8 \text{ MPa}$ ◀



PROBLEM 8.57

Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $\mathbf{P}_1 = 25\text{ kN}$ and $\mathbf{P}_2 = 24\text{ kN}$, determine the principal stresses and the maximum shearing stress at point *a*.

SOLUTION

At the section containing points *a* and *b*,

$$M_x = (1.8)(25) = 45\text{ kN}\cdot\text{m}$$

$$M_y = -(1.2)(24) = -28.8\text{ kN}\cdot\text{m}$$

$$V_x = -24\text{ kN} \qquad V_y = -25\text{ kN}$$

For W310 × 60 rolled steel section,

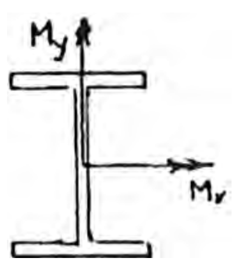
$$d = 302\text{ mm}, \quad b_f = 203\text{ mm}, \quad t_f = 13.1\text{ mm}, \quad t_w = 7.49\text{ mm}$$

$$I_x = 128\times 10^6\text{ mm}^4 = 128\times 10^{-6}\text{ m}^4, \quad I_y = 18.4\times 10^6\text{ mm}^4 = 18.4\times 10^{-6}\text{ m}^4$$

Normal stress at point *a*:

$$x = -\frac{b_f}{2} + 75 = -26.5\text{ mm}$$

$$y = \frac{1}{2}d = 151\text{ mm}$$

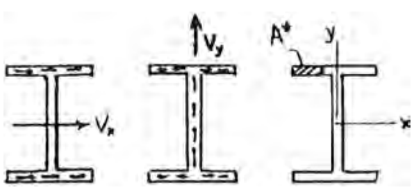


$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45\times 10^3)(151\times 10^{-3})}{128\times 10^{-6}} - \frac{(-28.8\times 10^3)(-26.5\times 10^{-3})}{18.4\times 10^{-6}}$$

$$= 53.086\text{ MPa} - 41.478\text{ MPa} = 11.608\text{ MPa}$$

Shearing stress at point *a*:

$$\tau_{xz} = -\frac{V_x A^* \bar{x}}{I_y t_f} - \frac{V_y A^* \bar{y}}{I_x t_f}$$



$$A^* = (75\times 10^{-3})(13.1\times 10^{-3}) = 982.5\times 10^{-6}\text{ m}^2$$

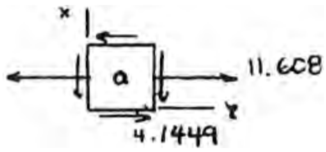
$$\bar{x} = -\frac{b_f}{2} + \frac{75}{2} = -64\text{ mm}$$

$$\bar{y} = \frac{d}{2} - \frac{t_f}{2} = 144.45\text{ mm}$$

$$\tau_{xz} = -\frac{(-24\times 10^3)(982.5\times 10^{-6})(-64\times 10^{-3})}{(18.4\times 10^{-6})(13.1\times 10^{-3})} - \frac{(-25\times 10^3)(982.5\times 10^{-6})(144.45\times 10^{-3})}{(128\times 10^{-6})(13.1\times 10^{-3})}$$

$$= -6.2609\text{ MPa} + 2.1160\text{ MPa} = -4.1449\text{ MPa}$$

PROBLEM 8.57 *(Continued)*



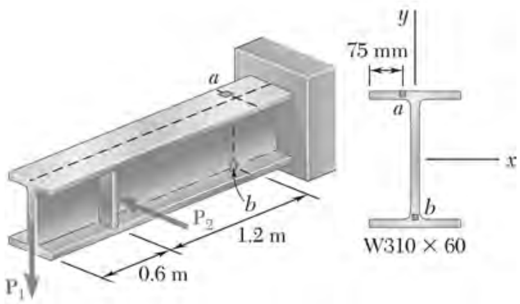
$$\sigma_{\text{ave}} = \frac{11.608}{2} = 5.804 \text{ MPa}$$

$$R = \sqrt{\left(\frac{11.608}{2}\right)^2 + (4.1449)^2} = 7.1321 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 12.94 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -1.33 \text{ MPa} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 7.13 \text{ MPa} \blacktriangleleft$$



PROBLEM 8.58

Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $P_1 = 25$ kN and $P_2 = 24$ kN, determine the principal stresses and the maximum shearing stress at point *b*.

SOLUTION

At the section containing points *a* and *b*,

$$M_x = (1.8)(25) = 45 \text{ kN}\cdot\text{m}$$

$$M_y = -(1.2)(24) = -28.8 \text{ kN}\cdot\text{m}$$

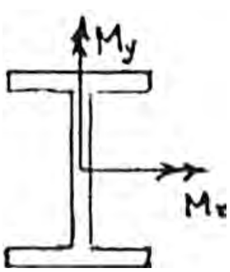
$$V_x = -24 \text{ kN}, \qquad V_y = -25 \text{ kN}$$

For W310 × 60 rolled steel section,

$$d = 302 \text{ mm}, \qquad b_f = 203 \text{ mm}, \qquad t_f = 13.1 \text{ mm}, \qquad t_w = 7.49 \text{ mm}$$

$$I_x = 128\times 10^6 \text{ mm}^4 = 128\times 10^{-6} \text{ m}^4, \qquad I_y = 18.4\times 10^6 \text{ mm}^4 = 18.4\times 10^{-6} \text{ m}^4$$

Normal stress at point *b*: $\qquad x \approx 0, \qquad y = -\frac{1}{2}d + t_f = -137.9$ mm

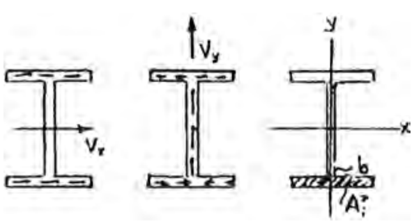


$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45\times 10^3)(-137.9\times 10^{-3})}{128\times 10^{-6}} - 0$$

$$= -48.480 \text{ MPa}$$

Shearing stress at point *b*:

$$\tau_{yz} = -\frac{V_y A^* \bar{y}}{I_x t_w}$$



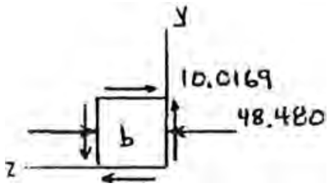
$$A^* = A_f = b_f t_f = 2659.3 \text{ mm}^2$$

$$= 2659.3\times 10^{-6} \text{ m}^2$$

$$\bar{x} = 0, \qquad \bar{y} = -\frac{1}{2}d + \frac{1}{2}t_f = -144.45 \text{ mm}$$

$$\tau_{yz} = -\frac{(-25\times 10^3)(2659.3\times 10^{-6})(-144.45\times 10^{-3})}{(128\times 10^{-6})(7.49\times 10^{-3})} = -10.0169 \text{ MPa}$$

PROBLEM 8.58 *(Continued)*



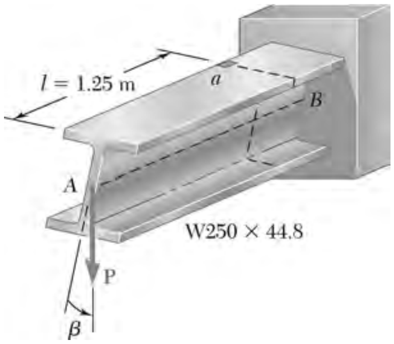
$$\sigma_{\text{ave}} = -\frac{48.480}{2} = -24.240 \text{ MPa}$$

$$R = \sqrt{\left(\frac{48.48}{2}\right)^2 + (10.0169)^2} = 26.228 \text{ MPa}$$

$\sigma_{\max} = \sigma_{\text{ave}} + R$ $\qquad \sigma_{\max} = 1.99 \text{ MPa}$ ◀

$\sigma_{\min} = \sigma_{\text{ave}} - R$ $\qquad \sigma_{\min} = -50.5 \text{ MPa}$ ◀

$\tau_{\max} = R$ $\qquad \tau_{\max} = 26.2 \text{ MPa}$ ◀



PROBLEM 8.59

A vertical force **P** is applied at the center of the free end of cantilever beam *AB*. (*a*) If the beam is installed with the web vertical $(\beta = 0)$ and with its longitudinal axis *AB* horizontal, determine the magnitude of the force **P** for which the normal stress at point *a* is +120 MPa. (*b*) Solve part *a*, assuming that the beam is installed with $\beta = 3°$.

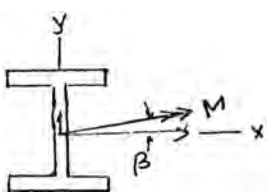
SOLUTION

For W250 × 44.8 rolled steel section,

$$S_x = 531 \times 10^3 \text{ mm}^3 = 531 \times 10^{-6} \text{ m}^3$$

$$S_y = 94.2 \times 10^3 \text{ mm}^3 = 94.2 \times 10^{-6} \text{ m}^3$$

At the section containing point *a*,



$$M_x = Pl\cos\beta, \qquad M_y = Pl\sin\beta$$

Stress at *a*:

$$\sigma = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{Pl\cos\beta}{S_x} + \frac{Pl\sin\beta}{S_y}$$

<u>Allowable load</u>. $$P_{\text{all}} = \frac{\sigma_{\text{all}}}{l}\left[\frac{\cos\beta}{S_x} + \frac{\sin\beta}{S_y}\right]^{-1}$$

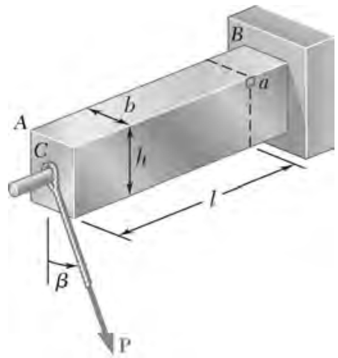
(*a*) $\underline{\beta = 0}$: $$P_{\text{all}} = \frac{120 \times 10^6}{1.25}\left[\frac{1}{531 \times 10^{-6}} + 0\right]^{-1} = 51.0 \times 10^3 \text{ N}$$

$P_{\text{all}} = 51.0$ kN ◀

(*b*) $\underline{\beta = 3°}$: $$P_{\text{all}} = \frac{120 \times 10^6}{1.25}\left[\frac{\cos 3°}{531 \times 10^{-6}} + \frac{\sin 3°}{94.2 \times 10^{-6}}\right]^{-1} = 39.4 \times 10^3 \text{ N}$$

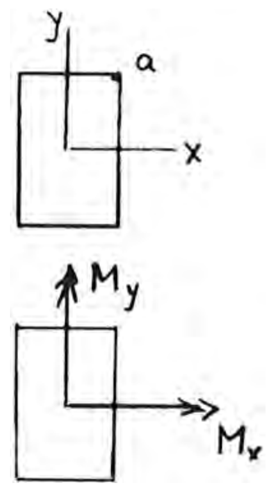
$P_{\text{all}} = 39.4$ kN ◀

PROBLEM 8.60



A force **P** is applied to a cantilever beam by means of a cable attached to a bolt located at the center of the free end of the beam. Knowing that **P** acts in a direction perpendicular to the longitudinal axis of the beam, determine (*a*) the normal stress at point *a* in terms of *P*, *b*, *h*, *l*, and β, (*b*) the values of β for which the normal stress at *a* is zero.

SOLUTION



$$I_x = \frac{1}{12}bh^3 \quad I_y = \frac{1}{12}hb^3$$

$$\sigma = \frac{M_x(h/2)}{I_x} - \frac{M_y(b/2)}{I_y}$$

$$= \frac{6M_x}{bh^2} - \frac{6M_y}{hb^2}$$

$$\boldsymbol{P} = P\sin\beta\,\boldsymbol{i} - P\cos\beta\,\boldsymbol{j} \quad \boldsymbol{r} = l\,\boldsymbol{k}$$

$$\boldsymbol{M} = \boldsymbol{r}\times\boldsymbol{P} = l\,\boldsymbol{k}\times(P\sin\beta\boldsymbol{i} - P\cos\beta\,\boldsymbol{j})$$

$$= Pl\cos\beta\,\boldsymbol{i} + Pl\sin\beta\,\boldsymbol{j}$$

$$M_x = Pl\cos\beta \qquad M_y = Pl\sin\beta$$

(*a*) $$\sigma = \frac{6\,Pl\cos\beta}{bh^2} - \frac{6\,Pl\sin\beta}{hb^2} = \frac{6\,Pl}{bh}\left[\frac{\cos\beta}{h} - \frac{\sin\beta}{b}\right]$$ ◀

(*b*) $$\sigma = 0 \qquad \frac{\cos\beta}{h} - \frac{\sin\beta}{b} = 0 \qquad \tan\beta = \frac{b}{h}$$

$$\beta = \tan^{-1}\left(\frac{b}{h}\right)$$ ◀

PROBLEM 8.61*

A 5-kN force **P** is applied to a wire that is wrapped around bar *AB* as shown. Knowing that the cross section of the bar is a square of side $d = 40$ mm, determine the principal stresses and the maximum shearing stress at point *a*.

SOLUTION

Bending: Point *a* lies on the neutral axis.

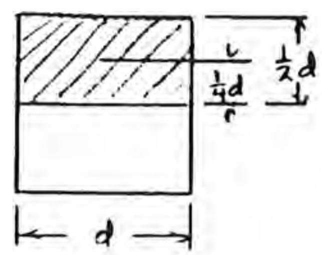
$$\sigma = 0$$

Torsion: $\tau = \dfrac{T}{c_1 ab^2}$ where $a = b = d$

and $c_1 = 0.208$ for a square section.

Since $T = \dfrac{Pd}{2}, \quad \tau_T = \dfrac{P}{0.416\, d^2} = 2.404\dfrac{P}{d^2}.$

Transverse shear:



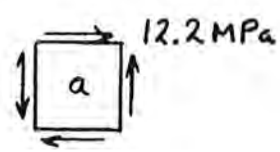
$$V = P \qquad I = \frac{1}{12}d^4$$

$$A = \frac{1}{2}d^2 \qquad \bar{y} = \frac{1}{4}d \qquad Q = A\bar{y} = \frac{1}{8}d^3$$

$$t = d$$

$$\tau_V = \frac{VQ}{It} = 1.5\frac{P}{d^2}$$

By superposition,



$$\tau = \tau_T + \tau_V = 3.904\frac{P}{d^2}$$

$$\tau = \frac{(3.904)(5\times 10^3)}{(40\times 10^{-3})^2}$$

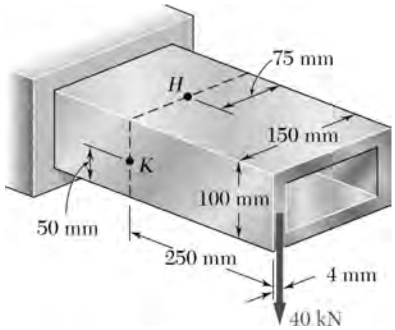
$$= 12.2\times 10^6 \text{Pa} \qquad 12.2 \text{ MPa.}$$

By Mohr's circle,

$\sigma_{max} = 12.2$ MPa ◀

$\sigma_{min} = -12.2$ MPa ◀

$\tau_{max} = 12.2$ MPa ◀



PROBLEM 8.62*

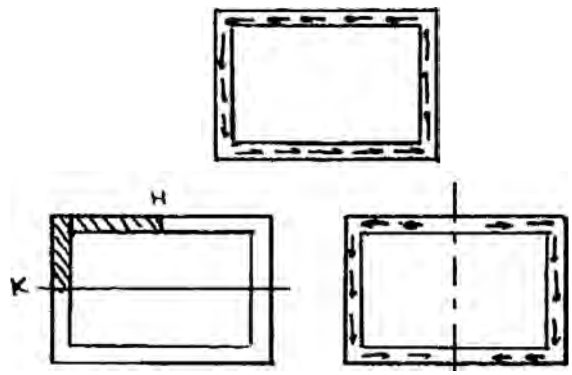
Knowing that the structural tube shown has a uniform wall thickness of 8 mm, determine the principal stresses, principal planes, and maximum shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION

At the section containing points *H* and *K*,

$$V = 40 \text{ kN} \quad M = (40)(0.25) = 10 \text{ kNm}$$

$$T = (40)(0.075 - 0.004) = 2.84 \text{ kNm}$$



Torsion:

$$a = (142)(92) = 13064 \text{ mm}^2$$

$$\tau = \frac{T}{2ta} = \frac{(2.84\times10^6)}{(2)(8)(13064)} = 13.59 \text{ MPa}$$

Transverse shear:

$$Q_H = 0$$

$$Q_K = (75)(50)(25) - (67)(42)(21) = 34656 \text{ mm}^3$$

$$I = \frac{1}{12}(150)(100)^3 - \frac{1}{12}(134)(841)^3 = 5.88\times10^6 \text{ mm}^4$$

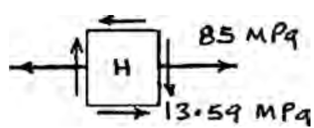
$$\tau_H = 0 \qquad \tau_K = \frac{VQ_a}{It} = \frac{(40\times10^3)(34656)}{(5.88\times10^6)(8)} = 29.47 \text{ MPa}$$

Bending:

$$\sigma_H = \frac{Mc}{I} = \frac{(10\times10^6)(50)}{5.88\times10^6} = 85 \text{ MPa}, \qquad \sigma_K = 0$$

(*a*) Point *H*:

$$\sigma_c = \frac{85}{2} = 42.5 \text{ MPa}$$



$$R = \sqrt{\left(\frac{85}{2}\right)^2 + (13.59)^2} = 44.62 \text{ MPa}$$

$$\sigma_{\max} = \sigma_c + R = 87.1 \text{ MPa} \quad ◀$$

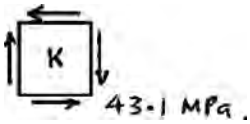
$$\sigma_{\min} = \sigma_c - R = -2.1 \text{ MPa} \quad ◀$$

$$\tan 2\theta_p = \frac{2\tau}{\sigma} = -0.32 \qquad \theta_p = -8.9°,\ 81.1° \quad ◀$$

$$\tau_{\max} = R = 44.6 \text{ MPa} \quad ◀$$

PROBLEM 8.62* *(Continued)*

(*b*) Point *K*: $\sigma = 0 \qquad \tau = 13.59 + 29.47 = 43.06 \text{ MPa}$ ◀

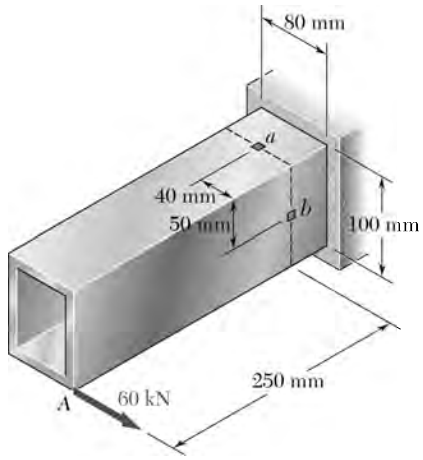


$$\sigma_{\max} = 43.1 \text{ MPa}$$ ◀

$$\sigma_{\min} = -43.1 \text{ MPa}$$ ◀

$$\theta_P = \pm 45°$$ ◀

$$\tau_{\max} = 43.1 \text{ MPa}$$ ◀



PROBLEM 8.63*

The structural tube shown has a uniform wall thickness of 8 mm. Knowing that the 60-kN load is applied 4 mm above the base of the tube, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

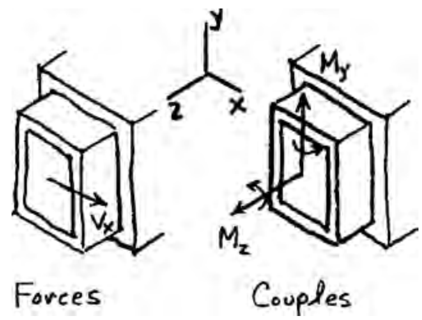
SOLUTION

Calculate forces and couples at section containing points *a* and *b*.

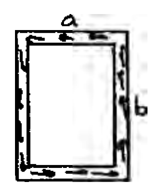
$$V_x = 60 \text{ kN}$$

$$M_z = (50 - 4)(60) = 2760 \text{ kN} \cdot \text{mm}$$

$$M_y = (60)(250) = 15000 \text{ kN} \cdot \text{mm}$$



Shearing stresses due to torque $T = M_z$.



$$a = [80 - (2)(4)][100 - (2)(4)] = 6624 \text{ mm}^2$$

$$q = \frac{M_z}{2a} = \frac{2760 \times 10^3}{(2)(6624)} = 208.3 \text{ N/mm}$$

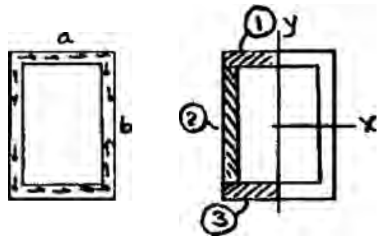
At point *a*. $t = 8$ mm $\quad \tau_a = \dfrac{q}{t} = \dfrac{208.3}{8} = 26.04 \text{ MPa} \longleftarrow$

At point *b*. $t = 8$ mm $\quad \tau_b = \dfrac{q}{t} = \dfrac{208.3}{8} = 26.04 \text{ MPa} \uparrow$

PROBLEM 8.63* *(Continued)*

Shearing stresses due to V_x.

At point *a:*



Part	$A(\text{mm}^2)$	$\bar{x}(\text{mm})$	$A\bar{x}(\text{mm}^3)$
①	320	–20	–6400
②	672	36	24192
③	320	–20	–6400
Σ			–36992

$$Q = |\Sigma A\bar{x}| = 36992 \text{ mm}^3$$

$$t = (2)(8) = 16 \text{ mm}$$

$$I_y = \frac{1}{12}(100)(80)^3 - \frac{1}{12}(84)(64)^3 = 2.4317 \times 10^6 \text{ mm}^4$$

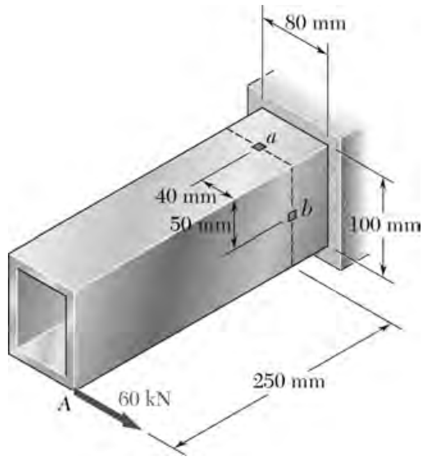
$$\tau = \frac{V_x Q}{I_y t} = \frac{(60000)(36992)}{(2.4317\times10^6)(16)} = 57.05 \text{ MPa}$$

At point *b*: $\tau_b = 0$

Combined shearing stresses.

At point *a*. $\tau_a = 26.04 \leftarrow +57.05 \rightarrow = 31 \text{ MPa} \rightarrow$ ◀

At point *b*. $\tau_b = 26.04 \uparrow + 0 \qquad = 26 \text{ MPa} \uparrow$ ◀



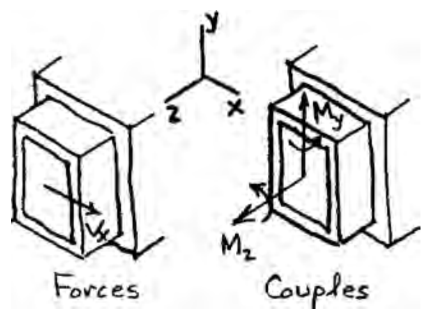
PROBLEM 8.64*

For the tube and loading of Prob. 8.63, determine the principal stresses and the maximum shearing stress at point *b*.

PROBLEM 8.63* The structural tube shown has a uniform wall thickness of 8 mm. Knowing that the 60-kN load is applied 4 mm above the base of the tube, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

Calculate forces and couples at section containing point *b*.



$$V_x = 60 \text{ kN}$$

$$M_z = (50-4)(60) = 2760 \text{ kN}\cdot\text{mm}$$

$$M_y = (60)(250) = 15000 \text{ kN}\cdot\text{mm}$$

$$I_y = \frac{1}{12}(100)(80)^3 - \frac{1}{12}(84)(64)^3 = 2.4317\times 10^6 \text{ mm}^4$$

$$\sigma_b = -\frac{M_y x_b}{I_y} = -\frac{(15\times 10^6)(40)}{2.4317\times 10^6} = -246.7 \text{ MPa}$$

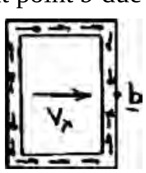
Shearing stress at point *b* due to torque M_z.

$$a = [80-(2)(4)][100-(2)(4)] = 6624 \text{ mm}^2$$

$$q = \frac{M_z}{2a} = \frac{2760\times 10^3}{(2)(6624)} = 208.3 \text{ N/mm}$$

$$\tau = \frac{q}{t} = \frac{208.3}{8} = 26.04 \text{ MPa}$$

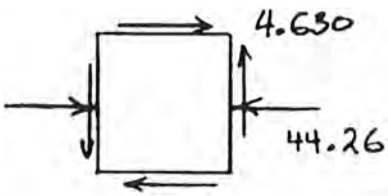
Shearing at point *b* due to V_x.



$$\tau = 0$$

PROBLEM 8.64* *(Continued)*

Calculation of principal stresses and maximum shearing stress.



$$\sigma_{\text{ave}} = \frac{-246.7 + 0}{2} = -123.35 \text{ MPa}$$

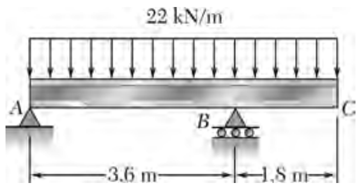
$$R = \sqrt{\left(\frac{-246.7 - 0}{2}\right)^2 + (26.04)^2} = 126.07 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = 2.72 \text{ MPa} \quad ◀$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R = -249.4 \text{ MPa} \quad ◀$$

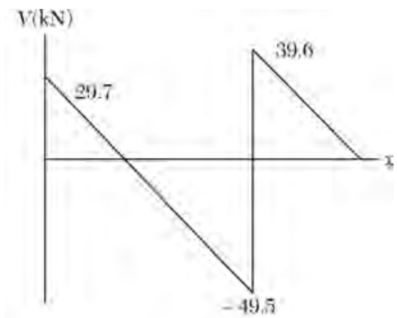
$$\tau_{\max} = R = 126 \text{ MPa} \quad ◀$$

PROBLEM 8.65



(*a*) Knowing that $\sigma_{all} = 165$ MPa and $\tau_{all} = 100$ MPa, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0 \quad -3.6R_A + (22)(5.4)(0.9) = 0 \quad R_A = 29.7 \text{ kN}\uparrow$$

$$+\circlearrowleft \Sigma M_A = 0 \quad 3.6R_B - (22)(5.4)(2.7) = 0 \quad R_B = 89.1 \text{ kN}\uparrow$$

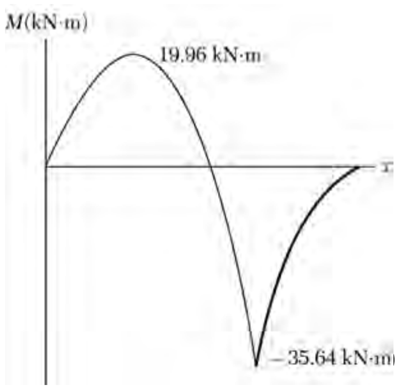
$$|V|_{max} = 49.5 \text{ kN}$$

$$|M|_{max} = 35.64 \text{ kN}\cdot\text{m}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{35.64\times10^3}{165\times10^6} = 216\times10^{-6} \text{ m}^3$$

$$= 216\times10^3 \text{ mm}^3$$

Shape	$S_x(10^3 \text{ mm}^3)$
W 310 × 23.8	280
→ W 250 × 22.3	228
W 200 × 26.6	249
W150 × 29.8	219



(*a*) Use W 250 × 22.3 ◀

$$d = 254 \text{ mm}$$

$$t_f = 6.9 \text{ mm}$$

$$t_w = 5.8 \text{ mm}$$

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{35.64\times10^3}{228\times10^{-6}} = 156.31 \text{ MPa} \quad ◀$$

$$\tau_m = \frac{|V|_{max}}{dt_w} = \frac{49.5\times10^3}{(0.254)(0.0058)} = 33.6 \text{ MPa} \quad ◀$$

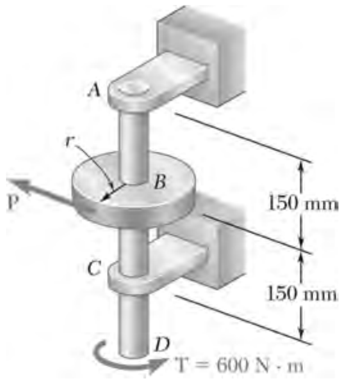
PROBLEM 8.65 *(Continued)*

$$c = \frac{1}{2}d = \frac{254}{2} = 127 \text{ mm} \quad y_b = c - t_f = 127 - 6.9 = 120.1 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{120.1}{127}\right)(156.3) = 147.8 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(73.9)^2 + (33.6)^2} = 81.18 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R = \frac{147.8}{2} + 81.18 = 155.08 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 8.66

Determine the smallest allowable diameter of the solid shaft *ABCD*, knowing that $\tau_{\text{all}} = 60$ MPa and that the radius of disk *B* is $r = 80$ mm.

SOLUTION

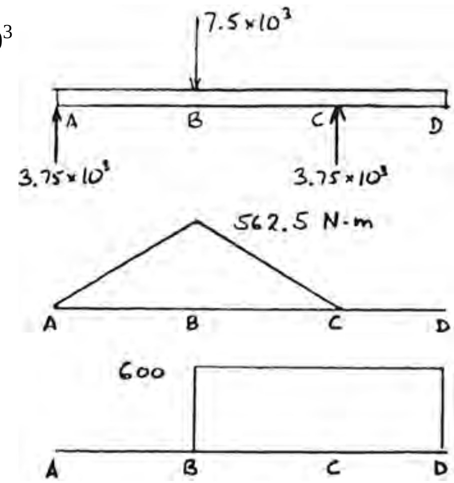
$$\Sigma M_{\text{axis}} = 0: \quad T - Pr = 0 \quad P = \frac{T}{r} = \frac{600}{80 \times 10^{-3}} = 7.5 \times 10^3$$

$$R_A = R_C = \frac{1}{2}P$$

$$= 3.75 \times 10^3 \text{N}$$

$$M_B = (3.75 \times 10^3)(150 \times 10^{-3})$$

$$= 562.5 \text{ N} \cdot \text{m}$$



Bending moment: (See sketch).

Torque: (See sketch).

Critical section lies at point *B*.

$$M = 562.5 \text{ N} \cdot \text{m}, \quad T = 600 \text{ N} \cdot \text{m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2 + T^2}\right)_{\max}}{\tau_{\text{all}}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} = \frac{2}{\pi}\frac{\sqrt{(562.5)^2 + (600)^2}}{60 \times 10^6}$$

$$= 8.726 \times 10^{-6} \text{m}^3$$

$$c = 20.58 \times 10^{-3} \text{m} \qquad d = 2c = 41.2 \times 10^{-3} \text{m}$$

$d = 41.2$ mm ◀

PROBLEM 8.67

Using the notation of Sec. 8.3 and neglecting the effect of shearing stresses caused by transverse loads, show that the maximum normal stress in a circular shaft can be expressed as follows:

$$\sigma_{\max} = \frac{c}{J}\left[\left(M_y^2 + M_z^2\right)^{1/2} + \left(M_y^2 + M_z^2 + T^2\right)^{1/2}\right]_{\max}$$

SOLUTION

Maximum bending stress:

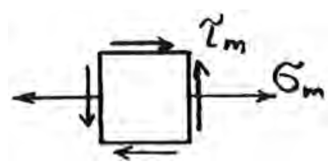
$$\sigma_m = \frac{|M|c}{I} = \frac{\sqrt{M_y^2 + M_z^2}\,c}{I}$$

Maximum torsional stress:

$$\tau_m = \frac{Tc}{J}$$

$$\frac{\sigma_m}{2} = \frac{\sqrt{M_y^2 + M_z^2}\,c}{2I} = \frac{c}{J}\sqrt{M_y^2 + M_z^2}$$

Using Mohr's circle,

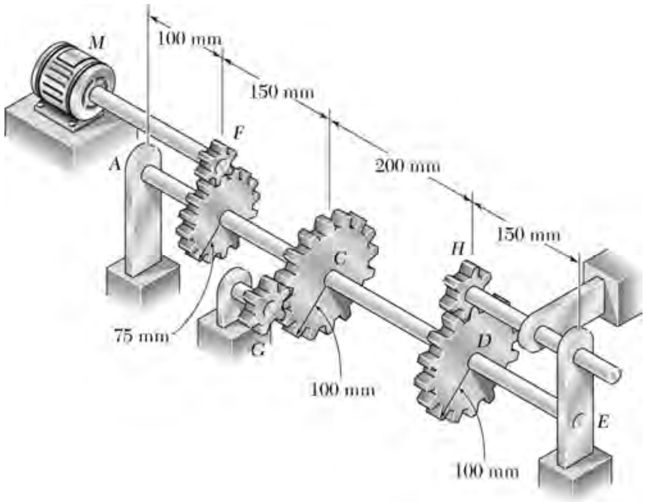


$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2} = \sqrt{\frac{c^2}{J^2}\left(M_y^2 + M_z^2\right) + \frac{T^2c^2}{J^2}}$$

$$= \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\sigma_{\max} = \frac{\sigma_m}{2} + R = \frac{c}{J}\sqrt{M_y^2 + M_z^2} + \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$= \frac{c}{J}\left[\left(M_y^2 + M_z^2\right)^{1/2} + \left(M_y^2 + M_z^2 + T^2\right)^{1/2}\right] \quad ◀$$



PROBLEM 8.68

The solid shaft *AE* rotates at 600 rpm and transmits 45 kW from the motor *M* to machine tools connected to gears *G* and *H*. Knowing that τ_{all} = 55 MPa and that 30 kW is taken off at gear *G* and 15 kW is taken off at gear *H*, determine the smallest permissible diameter of shaft *AE*.

SOLUTION

$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

Torque on gear B:

$$T_B = \frac{P}{2\pi f} = \frac{(45000)}{2\pi(10)}$$
$$= 716.2 \text{ Nm}$$

Torques on gears C and D:

$$T_C = \frac{30}{45} T_B = 477.5 \text{ Nm}$$

$$T_D = \frac{15}{45} T_B = 238.7 \text{ Nm}$$

Shaft torques:

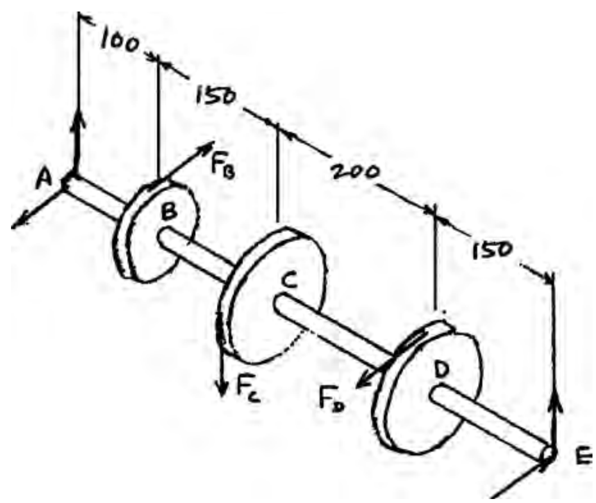
$$AB: \quad T_{AB} = 0$$
$$BC: \quad T_{BC} = 716.2 \text{ Nm}$$
$$CD: \quad T_{CD} = 238.7 \text{ Nm}$$
$$DE: \quad T_{DE} = 0$$

Gear forces:

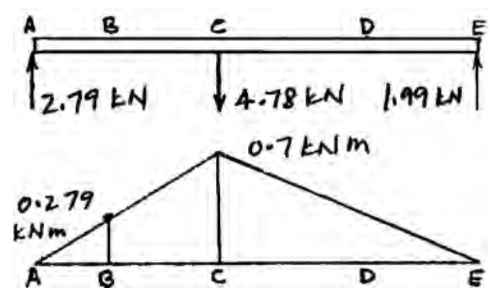
$$F_B = \frac{T_B}{r_B} = \frac{716.2}{0.075} = 9.55 \text{ kN}$$

$$F_C = \frac{T_C}{r_C} = \frac{477.5}{0.1} = 4.78 \text{ kN}$$

$$F_D = \frac{T_D}{r_D} = \frac{238.7}{0.1} = 2.39 \text{ kN}$$

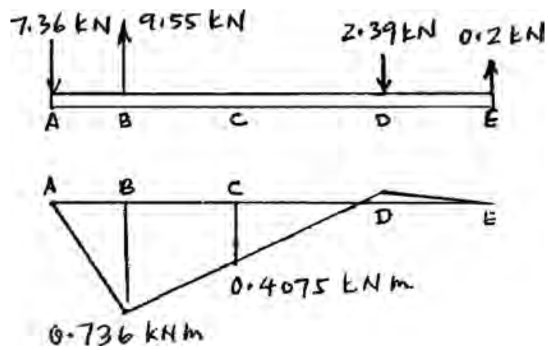


Forces in vertical plane



PROBLEM 8.68 *(Continued)*

Forces in horizontal plane



At B^+, $\sqrt{M_z^2 + M_y^2 + T^2}$

$$= \sqrt{0.279^2 + 0.736^2 + 0.7162^2}$$

$$= 1.064 \text{ kNm}$$

At C^-, $\sqrt{M_z^2 + M_y^2 + T^2}$

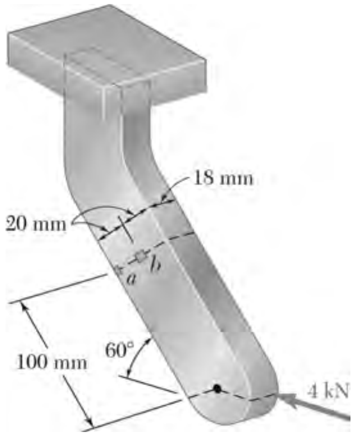
$$= \sqrt{0.7^2 + 0.4075^2 + 0.7162^2}$$

$$= 1.081 \text{ kNm} \qquad \text{(maximum)}$$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{\text{max}}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{\text{all}}} = \frac{1.081}{55 \times 10^3} = 19.655 \times 10^{-6} \text{ m}^3$$

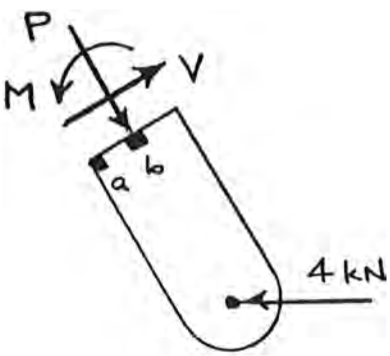
$c = 0.0232$ m $\qquad d = 2c = 46.4$ mm ◀



PROBLEM 8.69

For the bracket and loading shown, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.

SOLUTION



Draw free body diagram of portion below section *ab*.

From statics:

$$P = 4000\cos 60^\circ = 2000 \text{ N}$$

$$V = 4000\sin 60^\circ = 3464.1 \text{ N}$$

$$M = (0.1)(4000)\sin 60^\circ = 346.41 \text{ N}\cdot\text{m}$$

<u>Section properties</u>:

$$A = (0.018)(0.040) = 720\times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(0.018)(0.040)^3 = 96\times 10^{-9} \text{ m}^4$$

(*a*) <u>Point *a*</u>: $\sigma = -\dfrac{P}{A} - \dfrac{Mc}{I} = -\dfrac{2000}{720\times 10^{-6}} - \dfrac{(346.41)(0.02)}{96\times 10^{-9}}$

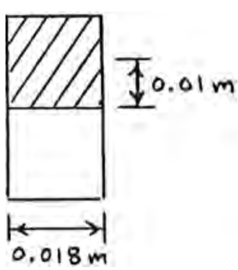
$$= -74.9\times 10^6 \text{ Pa}$$

$\sigma = -74.9$ MPa ◀

$\tau = 0$ ◀

(*b*) <u>Point *b*</u>: $\sigma = -\dfrac{P}{A} = -\dfrac{2000}{720\times 10^{-6}}$

$\sigma = -2.78$ MPa ◀

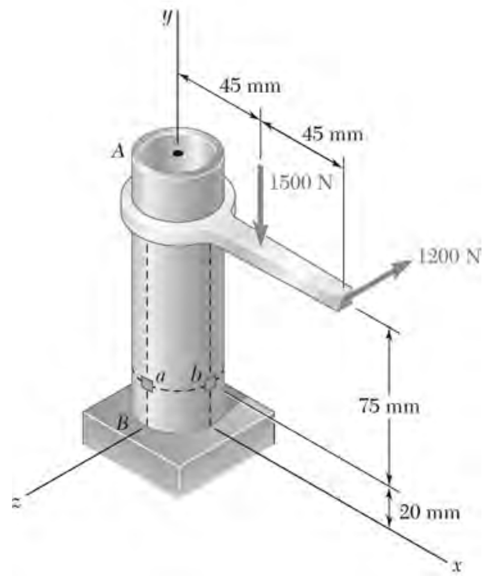


$$A = (0.018)(0.02) = 360\times 10^{-6} \text{ m}^3 \qquad \bar{y} = 0.01 \text{ m}$$

$$Q = A\bar{y} = 3.6\times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(3464.1)(3.6\times 10^{-6})}{(96\times 10^{-9})(0.018)} = 7.22\times 10^6 \text{ Pa}$$

$\tau = 7.22$ MPa ◀



PROBLEM 8.70

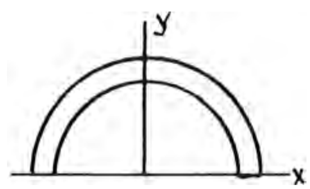
Two forces are applied to the pipe *AB* as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$c_0 = \frac{d_o}{2} = 21 \text{ mm}, \quad c_i = \frac{d_i}{2} = 17.5 \text{ mm} \quad A = \pi\left(c_o^2 - c_i^2\right) = 423.33 \text{ mm}^2$$

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = 158.166 \times 10^3 \text{mm}^4 \quad I = \frac{1}{2}J = 79.083 \times 10^3 \text{mm}^4$$

For semicircle with semicircular cutout,



$$Q = \frac{2}{3}\left(c_o^3 - c_i^3\right) = 2.6011 \times 10^3 \text{mm}^3$$

At the section containing points *a* and *b*,

$$P = -1500 \text{ N} \quad V_z = -1200 \text{ N} \quad V_x = 0$$

$$M_z = -(45 \times 10^{-3})(1500) = -67.5 \text{ N} \cdot \text{m}$$

$$M_x = -(75 \times 10^{-3})(1200) = -90 \text{ N} \cdot \text{m}$$

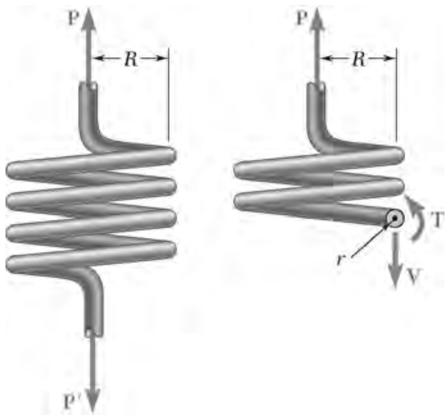
$$T = (90 \times 10^{-3})(1200) = 108 \text{ N} \cdot \text{m}$$

(*a*) $$\sigma = \frac{P}{A} - \frac{M_x c}{I} = \frac{-1500}{423.33 \times 10^{-6}} - \frac{(-90)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$$ $\sigma = 20.4 \text{ MPa}$ ◀

$$\tau = \frac{Tc}{J} + \frac{V_x Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + 0$$ $\tau = 14.34 \text{ MPa}$ ◀

(*b*) $$\sigma = \frac{P}{A} + \frac{M_z c}{I} = \frac{-1500}{423.33 \times 10^{-6}} + \frac{(-67.5)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$$ $\sigma = -21.5 \text{ MPa}$ ◀

$$\tau = \frac{Tc}{J} + \frac{|V_z| Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + \frac{(1200)(2.6011 \times 10^{-6})}{(79.083 \times 10^{-9})(7 \times 10^{-3})}$$ $\tau = 19.98 \text{ MPa}$ ◀



PROBLEM 8.71

A close-coiled spring is made of a circular wire of radius r that is formed into a helix of radius R. Determine the maximum shearing stress produced by the two equal and opposite forces **P** and **P′**. (*Hint:* First determine the shear **V** and the torque **T** in a transverse cross section.)

SOLUTION

$$+\uparrow \Sigma F_y = 0: \qquad P - V = 0 \qquad V = P$$

$$+\circlearrowleft \Sigma M_C = 0: \qquad T - PR = 0 \qquad T = PR$$

Shearing stress due to T.

$$\tau_T = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2PR}{\pi r^3}$$

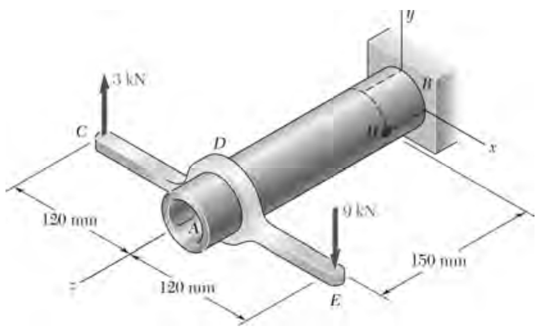
Shearing stress due to V.

For semicircle, $$Q = \frac{2}{3}r^3, \quad t = d = 2r$$

For solid circular section, $$I = \frac{1}{2}J = \frac{\pi}{4}r^3$$

$$\tau_V = \frac{VQ}{It} = \frac{V\left(\frac{2}{3}r^3\right)}{\frac{\pi}{4}r^4(2r)} = \frac{4V}{3\pi r^2} = \frac{4P}{3\pi r^2}$$

By superposition, $$\tau_{\max} = \tau_T + \tau_V$$ $$\tau_{\max} = P(2R + 4r/3)/\pi r^3 \blacktriangleleft$$



PROBLEM 8.72

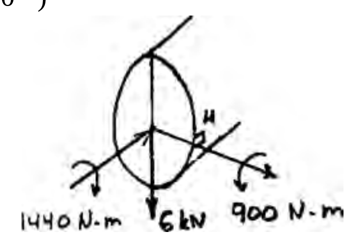
The steel pipe *AB* has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that the arm *CDE* is rigidly attached to the pipe, determine the principal stresses, principal planes, and the maximum shearing stress at point *H*.

SOLUTION

Replace the forces at *C* and *E* by an equivalent force-couple system at *D*.

$$F_D = 9 - 3 = 6\ \text{kN}\downarrow$$

$$T_D = (9\times10^3)(120\times10^{-3}) + (3\times10^3)(120\times10^{-3})$$
$$= 1440\ \text{N}\cdot\text{m}$$



At the section containing point *H*, $P = 0,\quad V = 6\ \text{kN},\quad T = 1440\ \text{N}\cdot\text{m}$

$$M = (6\times10^3)(150\times10^{-3}) = 900\ \text{N}\cdot\text{m}$$

Section properties: $d_o = 72\ \text{mm} \quad c_o = \frac{1}{2}d_o = 36\ \text{mm} \quad C_i = C_o - t = 31\ \text{mm}$

$$A = \pi\left(C_o^2 - C_i^2\right) = 1.0524\times10^3\ \text{mm}^2 = 1.0524\times10^{-3}\ \text{m}^2$$

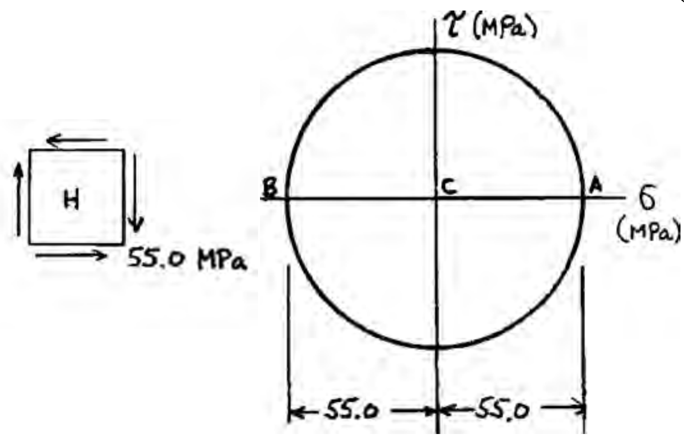
$$I = \frac{\pi}{4}\left(C_o^4 - C_i^4\right) = 593.84\times10^{-3}\ \text{mm}^4 = 593.84\times10^{-9}\ \text{m}^4$$

$$J = 2I = 1.877\times10^{-6}\ \text{m}^4$$

For half-pipe, $Q = \frac{2}{3}\left(C_o^3 - C_i^3\right) = 11.243\times10^3\ \text{mm}^3 = 11.243\times10^{-6}\ \text{m}^3$

<u>At point *H*</u>. Point *H* lies on the neutral axis of bending. $\sigma_H = 0.$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(1440)(36\times10^{-3})}{1.1877\times10^{-6}} + \frac{(6\times10^3)(11.243\times10^{-6})}{(593.84\times10^{-9})(10\times10^{-3})} = 55.0\ \text{MPa}$$



Use Mohr's circle. $\sigma_c = 0$

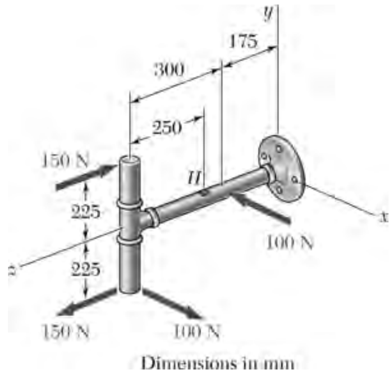
$R = 55.0\ \text{MPa}$

$\sigma_a = \sigma_c + R \qquad \sigma_a = 55.0\ \text{MPa}$ ◀

$\sigma_b = \sigma_c - R \qquad \sigma_b = -55.0\ \text{MPa}$ ◀

$\theta_c = -45°, \qquad \theta_b = +45°$ ◀

$\tau_{\max} = R \qquad \tau_{\max} = 55.0\ \text{MPa}$ ◀



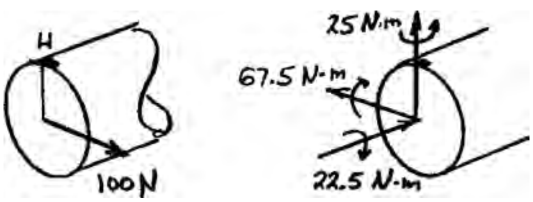
PROBLEM 8.73

Several forces are applied to the pipe assembly shown. Knowing that each section of pipe has inner and outer diameters equal to 36 and 42 mm, respectively, determine the normal and shearing stresses at point *H* located at the top of the outer surface of the pipe.

SOLUTION

At the section containing points *H*,

$$P = 0, \quad V_x = 100 \text{ N}, \quad V_y = 0$$



$$M_x = -(0.450)(150) = -67.5 \text{ N}\cdot\text{m}$$
$$M_y = (0.250)(100) = 25 \text{ N}\cdot\text{m}$$
$$M_z = -(0.225)(100) = -22.5 \text{ N}\cdot\text{m}$$
$$d_o = 42 \text{ mm} \quad d_i = 32 \text{ mm}$$
$$c_o = 21 \text{ mm} \quad c_i = 18 \text{ mm}$$
$$t = c_o - c_i = 3 \text{ mm}$$

$$A = \pi\left(C_o^2 - C_i^2\right) = 367.57 \text{ mm}^2 = 367.57\times10^{-6} \text{ m}^2$$

$$I = \frac{\pi}{4}\left(C_o^4 - C_i^4\right) = 70.30\times10^3 \text{ mm}^4 = 70.30\times10^{-9} \text{ m}^4, \; J = 2I = 140.59\times10^{-9} \text{ m}^4$$

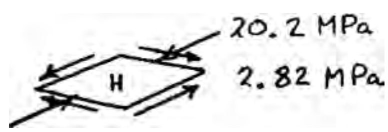
For half-pipe, $$Q = \frac{2}{3}\left(C_o^3 - C_i^3\right) = 2.286\times10^3 \text{ mm}^3 = 2.286\times10^{-6} \text{ m}^3$$

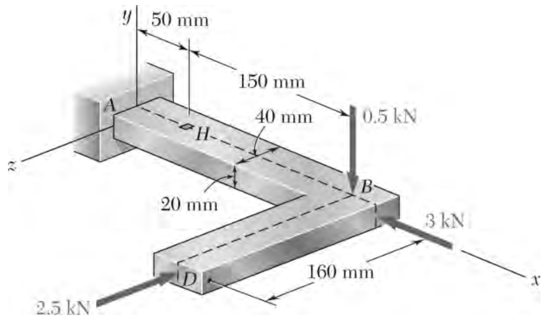
$$\sigma_H = \frac{M_x y}{I_x} = \frac{(-67.5)(21\times10^{-3})}{70.30\times10^{-9}} \qquad \sigma = -20.2 \text{ MPa} \blacktriangleleft$$

Due to torque: $$(\tau_H)_T = \frac{Tc}{J} = \frac{(22.5)(21\times10^{-3})}{140.59\times10^{-9}} = 3.36 \text{ MPa}$$

Due to shear: $$(\tau_H)_V = \frac{VQ}{It} = \frac{(100)(2.286\times10^{-6})}{(70.30\times10^{-9})(6\times10^{-3})} = 0.54 \text{ MPa}$$

Net: $$\tau_H = 3.36 - 0.54 \qquad \tau = 2.82 \text{ MPa} \blacktriangleleft$$





PROBLEM 8.74

Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a 20×40-mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.

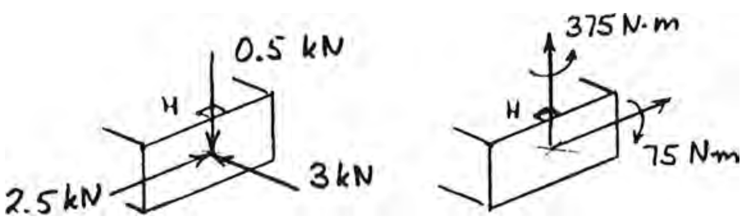
SOLUTION

Equivalent force-couple system at section containing point *H*:

$$F_x = -3\text{ kN}, \quad F_y = -0.5\text{ kN}, \quad F_z = -2.5\text{ kN}$$

$$M_x = 0, \quad M_y = (0.150)(2500) = 375\text{ N}\cdot\text{m}$$

$$M_z = -(0.150)(500) = -75\text{ N}\cdot\text{m}$$



$$A = (20)(40) = 800\text{ mm}^2$$

$$= 800 \times 10^{-6}\text{m}^2$$

$$I_z = \frac{1}{12}(40)(20)^3$$

$$= 26.667 \times 10^3\text{mm}^4$$

$$= 26.667 \times 10^{-9}\text{m}^4$$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z}$$

$$= \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}}$$

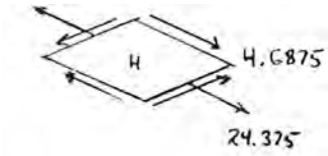
$$= 24.375\text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_z|}{A} = \frac{3}{2} \cdot \frac{2500}{800 \times 10^{-6}}$$

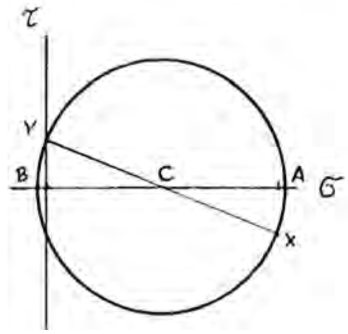
$$= 4.6875\text{ MPa}$$

PROBLEM 8.74 *(Continued)*

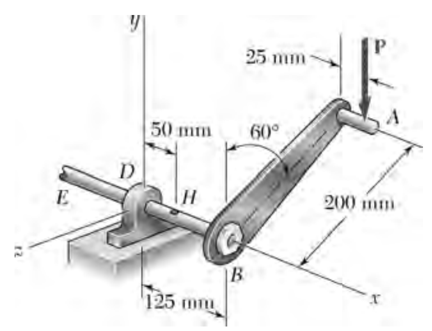
Use Mohr's circle.



$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H$$
$$= 12.1875 \text{ MPa}$$
$$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (4.6875)^2}$$
$$= 13.0579 \text{ MPa}$$



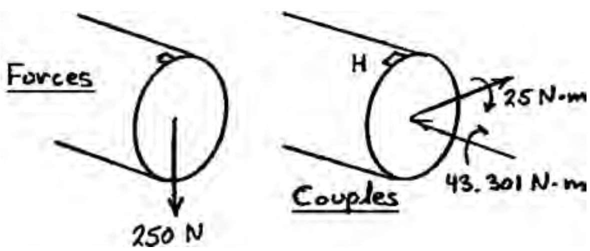
$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 25.2 \text{ MPa} \blacktriangleleft$$
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -0.87 \text{ MPa} \blacktriangleleft$$
$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(4.6875)}{24.375} = 0.3846$$
$$\theta_a = 10.5^\circ, \quad \theta_b = 100.5^\circ$$
$$\tau_{\max} = R \qquad \tau_{\max} = 13.06 \text{ MPa} \blacktriangleleft$$

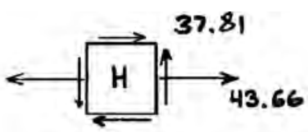


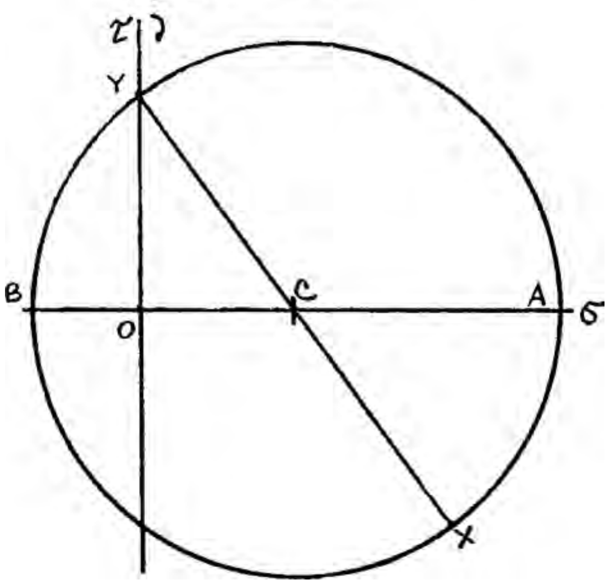
PROBLEM 8.75

A vertical force **P** of magnitude 250 N is applied to the crank at point *A*. Knowing that the shaft *BDE* has a diameter of 18 mm, determine the principal stresses and the maximum shearing stress at point *H* located at the top of the shaft, 50 mm to the right of support *D*.

SOLUTION







Force-couple system at the centroid of the section containing point *H*:

$$F_x = 0, \qquad V_y = -250 \text{ N}, \qquad V_z = 0$$

$$M_z = -(125 - 50 + 25)(10^{-3})(250) = -25 \text{ N} \cdot \text{m}$$

$$M_x = -(200 \sin 60°)(10^{-3})(250) = -43.301 \text{ N} \cdot \text{m}$$

$$d = 18 \text{ mm} \qquad c = \frac{1}{2}d = 9 \text{ mm}$$

$$I = \frac{\pi}{4}c^4 = 5.153 \times 10^3 \text{ mm}^4 = 5.153 \times 10^{-9} \text{ m}^4$$

$$J = 2I = 10.306 \times 10^{-9} \text{ m}^4$$

At point *H*,

$$\sigma_H = -\frac{M_z y}{I_x} = -\frac{(-25)(9 \times 10^{-3})}{5.153 \times 10^{-9}} = 43.66 \text{ MPa}$$

$$\tau_H = \frac{Tc}{J} = \frac{(43.301)(9 \times 10^{-3})}{10.306 \times 10^{-3}} = 37.81 \text{ MPa}$$

Use Mohr's circle,

$$\sigma_C = \frac{1}{2}\sigma_H = 21.83 \text{ MPa}$$

$$R = \sqrt{\left(\frac{43.66}{2}\right)^2 + (37.81)^2} = 43.66 \text{ MPa}$$

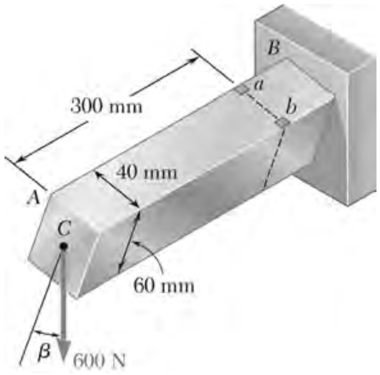
$\sigma_a = \sigma_c + R$ $\qquad\qquad$ $\sigma_{\max} = 65.5 \text{ MPa}$ ◀

$\sigma_b = \sigma_c - R$ $\qquad\qquad$ $\sigma_{\min} = -21.8 \text{ MPa}$ ◀

$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{75.62}{43.66} = 1.7320$$

$$\theta_a = 30.0°, \qquad \theta_b = 120°$$

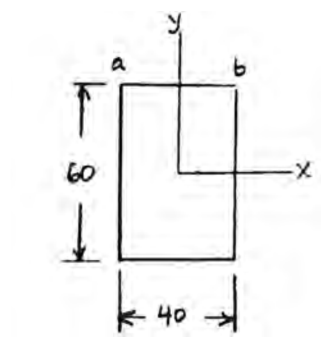
$\tau_{\max} = R$ $\qquad\qquad$ $\tau_{\max} = 43.7 \text{ MPa}$ ◀



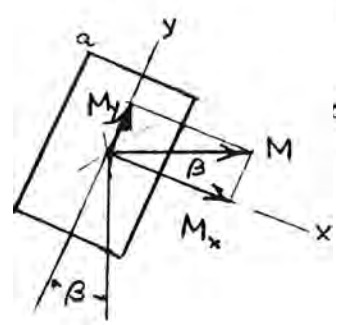
PROBLEM 8.76

The cantilever beam AB will be installed so that the 60-mm side forms an angle β between 0 and 90° with the vertical. Knowing that the 600-N vertical force is applied at the center of the free end of the beam, determine the normal stress at point a when (a) $\beta = 0$, (b) $\beta = 90°$. (c) Also, determine the value of β for which the normal stress at point a is a maximum and the corresponding value of that stress.

SOLUTION



$$S_x = \frac{1}{6}(40)(60)^2 = 24\times10^3\,\text{mm}^3$$
$$= 24\times10^{-6}\,\text{m}^3$$
$$S_y = \frac{1}{6}(60)(40)^2 = 16\times10^3\,\text{mm}^3$$
$$= 16\times10^{-3}\,\text{m}^3$$
$$M = Pl = (600)(300\times10^{-3}) = 180\ \text{N}\cdot\text{m}$$
$$M_x = M\cos\beta = 180\cos\beta$$
$$M_y = M\sin\beta = 180\sin\beta$$



$$\sigma_a = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{180\cos\beta}{24\times10^{-6}} + \frac{180\sin\beta}{16\times10^{-6}}$$
$$= (7.5\times10^6)\left(\cos\beta + \frac{3}{2}\sin\beta\right)\text{Pa}$$
$$= 7.5\left(\cos\beta + \frac{3}{2}\sin\beta\right)\text{MPa}$$

(a) $\underline{\beta = 0.}$ $\quad \sigma_a = 7.50$ MPa $\quad\quad \sigma = 7.50$ MPa ◀

(b) $\underline{\beta = 90°.}$ $\quad \sigma_a = 11.25$ MPa $\quad\quad \sigma = 11.25$ MPa ◀

(c) Maximum.

$$\frac{d\sigma_a}{d\beta} = 7.5\left(-\sin\beta + \frac{3}{2}\cos\beta\right) = 0$$

$$\sin\beta = \frac{3}{2}\cos\beta \qquad \tan\beta = \frac{3}{2}$$

$\beta = 56.3°$ ◀

$$\sigma_a = 7.5\left(\cos 56.3° + \frac{3}{2}\sin 56.3°\right)$$

$\sigma = 13.52$ MPa ◀