PH 201

OPTICS & LASERS

Lecture_Lasers_11

Ref.: William T. Silfvast, *Laser Fundamentals*, 2nd ed., Cambridge Univ. Press (2004)

Laser Cavity Modes

Properties associated with optical cavity of a laser that has mirrors located at each end of laser gain medium.

Properties – cavity modes → Output characteristics

Having mirrors at each end of cavity leads to development of both longitudinal & transverse modes superimposed upon beam.

These modes produce both longitudinal (temporal, frequency-dependent) & transverse (spatial-dependent) characteristics of beam.

Longitudinal Laser Cavity Modes

When mirrors are placed at ends of an amplifying medium, they place boundary conditions upon electromagnetic radiation field (laser beam) that develops between two mirrors.

Two mirror cavity → Fabry-Perot Resonator

$$R = r_1^2 = r_2^2$$

$$\tau = t_1 t_2 = 1 - R$$

$$\mathcal{R} = \frac{F \sin^2 \delta/2}{1 + F \sin^2 \delta/2}$$

$$T = \frac{1}{1 + F \sin^2 \delta/2}$$

$$F = \frac{4R}{(1-R)^2}$$

$$FWHM = \Delta\delta = \frac{4}{\sqrt{F}} = \frac{2(1-R)}{\sqrt{R}}$$

$$Phase\ diff = \frac{2\pi}{\lambda} \times Path\ diff$$

For bright fringe (constructive interference),

$$Phase\ diff = \frac{2\pi}{\lambda} \times Path\ diff = 2n\pi$$

$$2n\pi = \frac{2\pi}{\lambda}(2\mu d\ \cos\theta')$$

Considering refractive index, *μ* = 1 & *θ*' = 0,

$$n = \frac{2d}{\lambda}$$

***n* = integer, *λ* = wavelength, *d* = mirror separation**
Wavelength at which maxima occur,

$$\boxed{\lambda_n^{\max} = \frac{2d}{n}}$$

These maxima occur at an infinite sequence of wavelengths, decreasing in separation with increasing *n*.

In terms of frequencies,

$$\upsilon_n^{\max} = \frac{nc}{2d\mu}, \qquad \lambda\upsilon = v = c / \mu$$

Consider difference between two frequencies,

$$\upsilon_{n+1}^{\max} - \upsilon_n^{\max} = \frac{c}{2d\mu}[(n+1)-n)] = \frac{c}{2d\mu}$$

$$\boxed{\Delta\upsilon_{sep} = \frac{c}{2d\mu}}$$

Frequency difference is independent of *n*.
***dμ* = optical path length separating mirror surfaces. For *μ* = 1,**

$$\Delta\upsilon_{sep} = \frac{c}{2d}$$

These maxima (or enhancements) occur at equal frequency spacing that are independent of specific values of frequency or wavelength.

Determine the frequency difference between consecutive maxima for a Fabry-Perot etalon in which the mirrors are separated by 0.01 m and the mirrors are located in air.

For air medium $\mu = 1$,

$$\Delta \upsilon_{sep} = \frac{c}{2\mu d} = \frac{3\times 10^{8}\, m/s}{2\times 1\times 0.01m} = 1.5\times 10^{10}\ Hz$$

For separation between maxima in terms of frequency, $\Delta\upsilon_{sep} = \dfrac{c}{2\mu d}$

For mirrors of equal reflectivity,

$$\Delta\upsilon_{FWHM} = \frac{\Delta\upsilon}{F} = \left(\frac{1}{F}\right)\frac{c}{2\mu d} = \frac{c(1-R)}{2\pi\mu d\sqrt{R}}$$

For mirrors of unequal reflectivity *R_1* & *R_2*,

$$\Delta\upsilon_{FWHM} = \frac{c[1-(R_1R_2)^{1/2}]}{2\pi\mu d(R_1R_2)^{1/4}}$$

Quality factor of a cavity, which is a measure of sharpness of frequency transmission, is defined as (for mirrors of equal reflectivity),

$$Q = \frac{\upsilon_0}{\Delta\upsilon} = \frac{\upsilon_0}{\Delta\upsilon_{FWHM}} = \frac{2\pi\mu d\upsilon_0\sqrt{R}}{c(1-R)}$$

For mirrors of different reflectivities,

$$Q = \frac{2\pi\mu d\upsilon_0(R_1R_2)^{1/4}}{c[1-(R_1R_2)^{1/2}]}$$

Fabry-Perot Cavity Modes:

Considering $\mu = 1$ for simplicity,

$$\lambda_n^{\max} = \frac{2d}{n}$$

$$d = n\left(\frac{\lambda^{\max}}{2}\right)$$

This Eq. indicates that each enhancement occurs when an integral multiple of half-wavelengths fit into the cavity spacing of length *d* such that electric vector of electromagnetic wave is zero at reflecting surfaces.

Each of these waves that is enhanced by virtue of its exactly "fitting into the spacing *d*" or exactly "fitting into the cavity" is a standing wave known as a *mode*.

Such modes result from the interference effects that occur when light interacts with two parallel reflecting surfaces.

$$\upsilon = \frac{nc}{2d}$$

We see that there are essentially an infinite no. of frequencies that would fit within such a cavity.

If a wide range of frequencies were to be considered, the reflectivity of mirrors would have to be kept high over that entire range in order to obtain sharp discrete enhancements over that frequency range.

For visible spectral region, consider

$$\upsilon = 5 \times 10^{14} s^{-1}, \quad d = 0.01m$$

then *n* would be of order of 35,000; meaning that waves would have somewhere in range of 30,000 to 40,000 half-cycles located within cavity.

When beam is initially fed into cavity, intensity builds up within cavity until transmitted beam is equal to input beam for frequencies at which *resonances* (location of max. transmission) occur.

Longitudinal Laser Cavity Modes

- When a laser gain medium is inserted within a Fabry-Perot cavity, a similar set of enhancements or modes in form of standing wave patterns equally spaced in frequency will build up within cavity.

- When gain medium is initially turned on, amplifier begins emitting spontaneous emission over laser frequency bandwidth in all directions.

- Rays that are directed toward mirrors are reflected & return through amplifier; thus they are enhanced by gain during each transit.

- A highly directional beam eventually evolves in axial direction & reaches an intensity that exceeds I_{sat} if gain duration is sufficiently long.

Various standing waves, each of different frequency, are referred to as ***longitudinal modes*** because they are associated with longitudinal direction of electromagnetic waves within cavity.

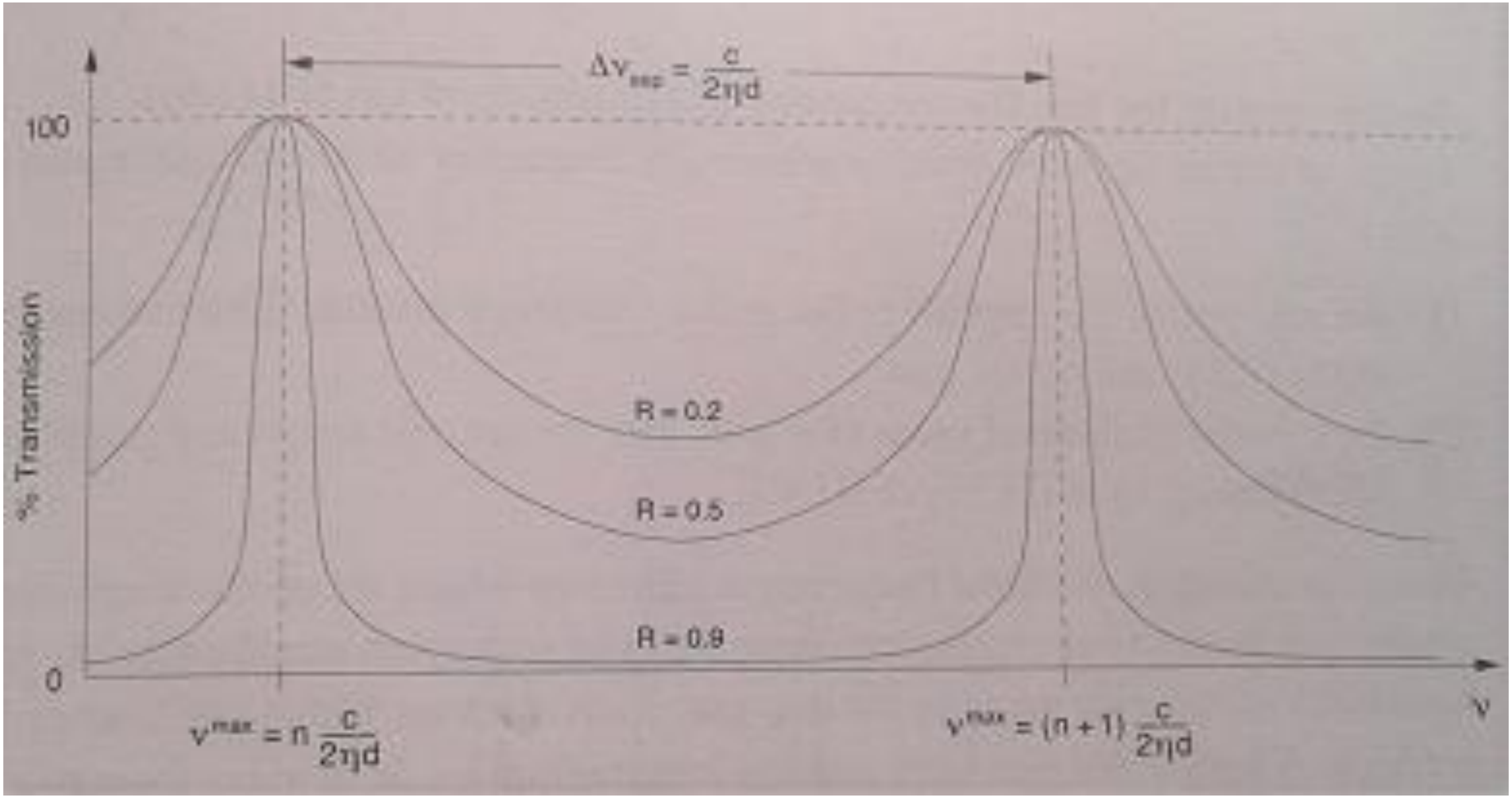
These modes occur at wavelengths or frequencies over gain bandwidth of laser beam at which electric field has an integral multiple of half-wavelengths with zero magnitude at mirror surfaces.

Actual laser mode frequencies can be obtained by

$$\upsilon = n\left(\frac{c}{2\mu d}\right)$$

If a laser has a space of length (*d* – *L*) between gain medium & mirrors, & if that cavity space has a different value for index of refraction μ_c than index μ_L of laser gain medium, then a specific laser mode frequency associated with a mode no. *n* can be expressed as

$$\upsilon = n\left(\frac{c}{2}\left[\frac{1}{\mu_c(d-L)+\mu_L L}\right]\right)$$



Frequency distribution of longitudinal laser cavity modes of mode numbers *n* & *n* + 1 in which a uniform optimal medium with index of refraction *μ* exists between the two laser mirrors

Determine the mode numbers at the extreme ends of the optical frequency region for the followings: a wavelength of 700 nm in the red and a spacing of 0.35 m; and a wavelength of 400 nm in the blue and a spacing of 0.4 m.

Considering μ = 1,

$$n_{red} = \frac{2\mu d\upsilon}{c} = \frac{2\mu d}{\lambda} = \frac{2\times1\times0.35m}{700\times10^{-9}m} = 1{,}000{,}000$$

$$n_{blue} = \frac{2\mu d}{\lambda} = \frac{2\times1\times0.4m}{400\times10^{-9}m} = 2{,}000{,}000$$

Transverse Laser Cavity Modes

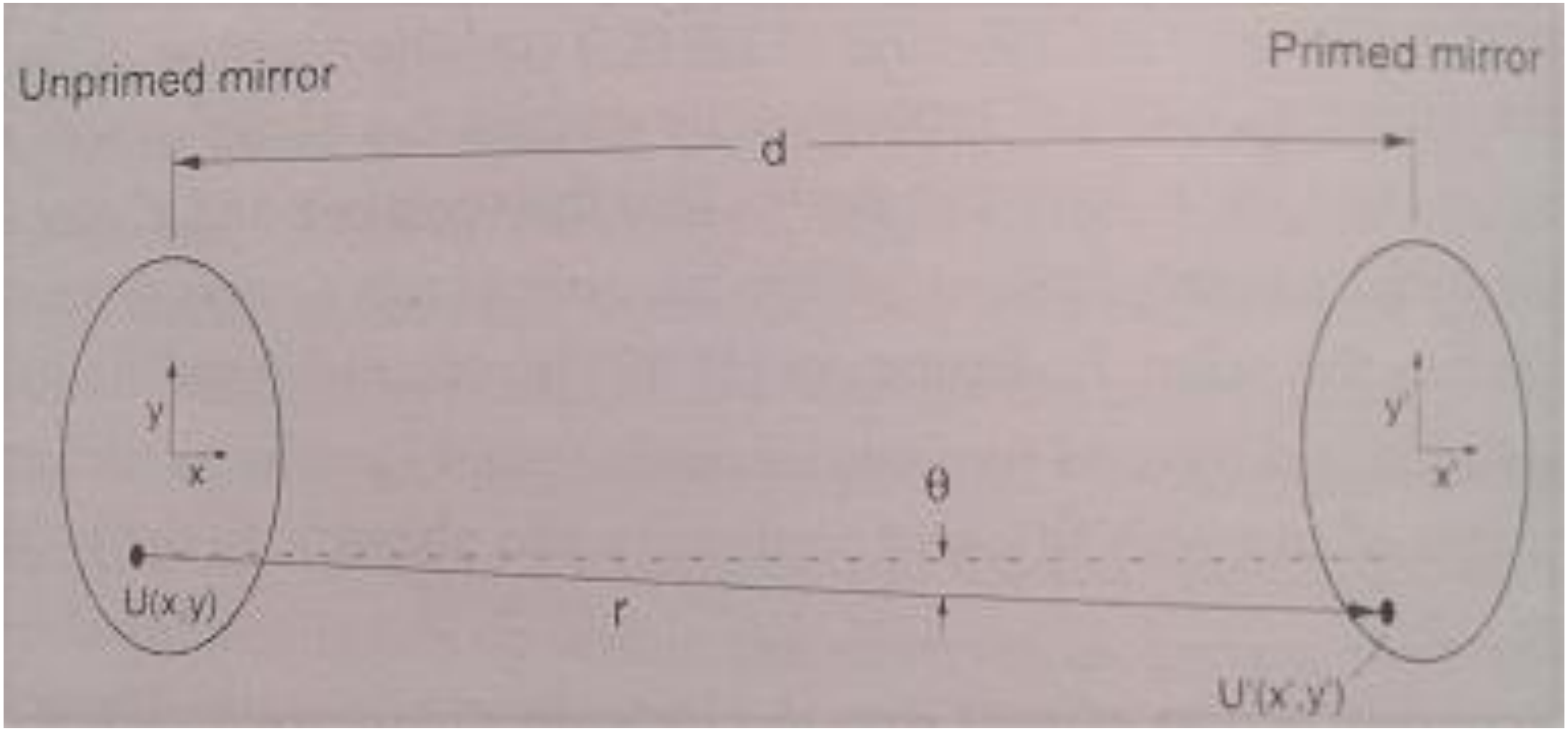
While dealing with longitudinal modes, it was considered that laser beam was similar to plane wave. However, beam is not a plane wave because a plane wave must be of infinite lateral extent, & this is not possible within a laser amplifier.

Finite lateral size of beam, due to either finite size of mirrors or some other limiting aperture within amplifier (such as amplifier diameter), will cause diffraction of beam to occur. This leads to losses within laser cavity.

Consider two modifications: Assume that laser mirrors are of finite extent & of circular shape, & also assume that source of light originates from laser amplifier region between mirrors.

Obtain expression for transverse profile of beam that builds up within cavity after having undergone many reflections as beam oscillates back & forth between mirrors.

Consider a source point function *U*(*x*,*y*) at point (*x*,*y*) on unprimed mirror, which is sum of contributions of radiation from all points leaving primed mirror that arrive at location (*x*,*y*) on unprimed mirror.



Two parallel circular mirrors considered as apertures when applying Fresnel-Kirchhoff integral formula to a laser cavity.

Source point function *U*(*x*,*y*) radiates back to primed mirror to arrive at various points (*x′*,*y′*), with an amplitude function *U′*(*x′*,*y′*) on that mirror, after having traveled a distance *r*.

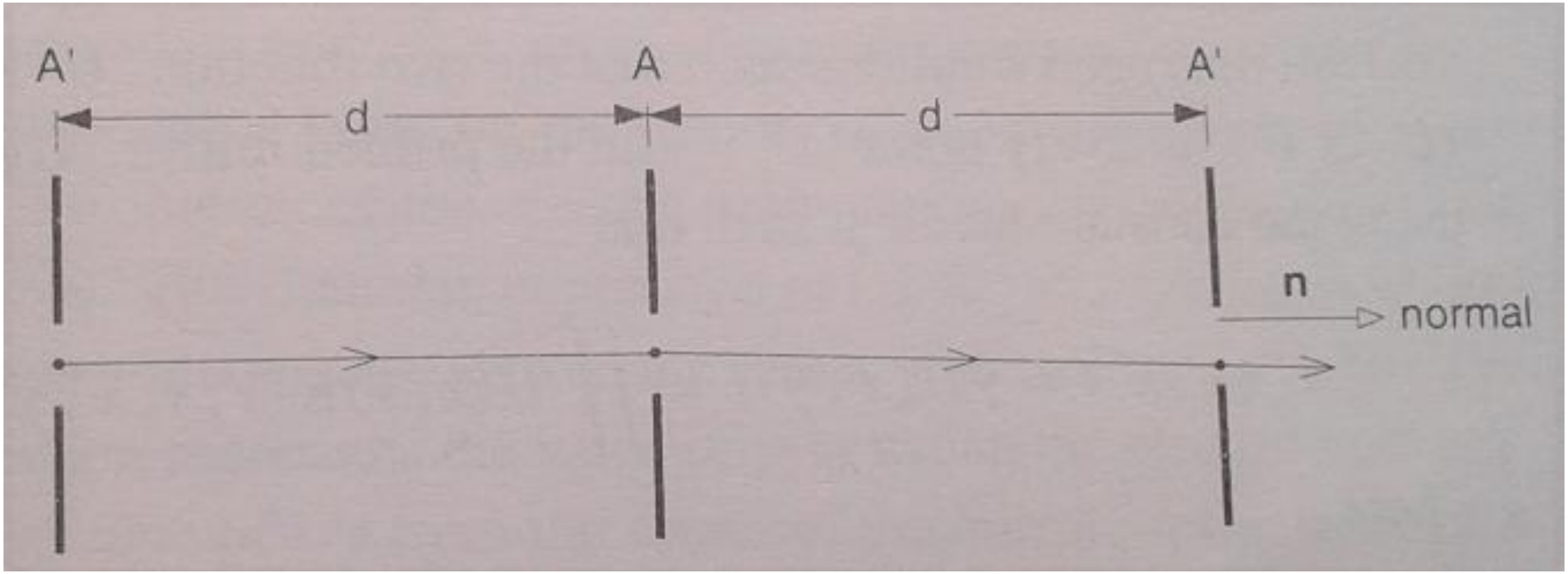
$$r = \{(d^2 + (x' - x)^2 + (y' - y)^2\}^{1/2}$$

***θ* is defined as angle between *d* & *r*.**

$$\cos\theta = \frac{d}{r}$$

We wish to determine the distribution of radiation *U′*(*x′*,*y′*) on primed mirror as a result of radiation of distribution *U*(*x*,*y*) at unprimed mirror.

Picture the application of Fresnel-Kirchhoff integral formula if two mirrors are transposed into a series of successive apertures.



Equivalent aperture description of a two mirror reflective laser cavity.

$$U'(x', y') = -\frac{ik}{4\pi}\iint_A U(x,y)\frac{e^{ikr}}{r}(\cos\theta+1)\,dxdy$$

Considering possibility of a decrease in amplitude at all values of *x′* & *y′* by a constant factor of *γ* (eigenvalue), which is determined by diffraction. This diffraction loss.

$$U'(x', y') = \gamma U'(x', y') = \iint_{\Sigma} U(x,y) K(x,y,x',y') dxdy$$

$$where \quad K(x, y, x', y') = \frac{e^{ikr}}{r}(\cos\theta + 1) \qquad \textbf{(A)}$$

There are an infinite no. of solutions U_n & γ_n; each set is associated with a specific value of *n*, where *n* can take on values of *n* = 1, 2, 3, ….

These solutions correspond to normal modes of this resonator. They are referred to as *transverse modes* because they represent amplitude distribution of electromagnetic field in directions transverse to propagation of laser beam within resonator.

We describe γ_n by allowing both amplitude & phase factor as,

$$\gamma_n = |\gamma_n| e^{i\phi_n}$$

$|\gamma_n|$ denotes ratio of amplitudes of distributions after two successive passes from one mirror to next

ϕ_n phase shift

Energy loss per transit, $$\frac{energy\,loss}{transit} = 1 - |\gamma_n|^2$$

Solution of (A) can be obtained by making approximation for its kernel:

$$K(x, y, x', y') = Ce^{-ik_1(xx'+yy')}$$

C & k_1 are constants.

$$\gamma U(x', y') = C \iint_{\Sigma} U(x,y) e^{ik_1(xx'+yy')} dxdy$$

It can be seen that *U*(*x*,*y*) is its own Fourier transform. Simplest solution of such a function is Gaussian function,

$$U(x, y) = e^{-\rho^2/\omega^2} = e^{-(x^2+y^2)/\omega^2}$$

ω **= scaling factor &** ***ρ*** **is radial distance to any location on mirror from central point on mirror.**

Function *U*(*x*,*y*) gives variation of distribution of electric field amplitude over mirror at various locations (*x*,*y*).

Gaussian expression is a symmetrical distribution that decreases with distance *ρ* from center of mirror (*ρ* = 0).

ω **represents value of *ρ* at which electric field amplitude decreases to a value of 1/*e* from centre of mirror, or a value of *ρ* at which intensity of beam decreases to** $1/e^2$ **= 0.86 of value at centre of mirror.**