Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2014-15
End Semester Examination

Maximum Marks: 50 Total Time: 3 Hours

Note: This question paper has **ONE** page and contains **Ten** questions. Answer all questions.

1. Suppose that the probability that in a pair of twins both are boys is 0.3 and that both are girls is 0.26. It is given that the probability of the first child being a boy is 0.52. What is the probability that in a randomly selected twins the second baby is a girl child given that the first one is a girl? Also determine the probability the first twin is boy and the second one is a girl. **[1.5 + 1.5]**

2. Let X follows a binomial $B(n,p)$ distribution. Assume that as $n \to \infty$, $p \to 0$ then $np \to \lambda$. Under this assumption show that the given binomial distribution approaches to a Poisson distribution with parameter λ. **[2]**

3. Let X be a random variable with pdf $f_X(x) = ax^2, \quad -1 \leq x \leq 0$, $f_X(x) = b(1-x^2), \quad 0 \leq x \leq 1$, $f_X(x) = 0$, *elsewhere*. Given that mean of this distribution is zero, evaluate the constants a and b. Determine the median of this distribution. **[1+1+1]**

4. Incoming telephone calls to an operator are assumed to be a Poisson process with rate 30 calls per hour. Find the density function of the length of the time for 10 calls to be received. Find the mean of this distribution. **[3 + 1]**

5. If X and Y are RVs with the joint pdf $f_{X,Y}(x,y) = \frac{x^2+y^2}{4\pi}e^{-\frac{x^2+y^2}{2}}, -\infty < x < \infty, -\infty < y < \infty$. Verify whether X and Y are independent. Find covariance between X and Y. Define correlation coefficient between two random variables. **[2+2+1]**

6. Let X have the exponential distribution with mean 1. Consider the transformation $Y = \theta X^{\frac{1}{m}}$, $\theta > 0$, $m > 0$. Find pdf of Y. Express mean and variance of Y in terms of θ and m. Find the 0.9th quantile of Y with $m = 3, \theta = 5$. **[3+2+2+1]**

7. Suppose that X and Y have gamma $G(\alpha, 1)$ and $G(\beta, 1)$ distributions respectively and that they are independent. Let $U = X+Y$, $V = \frac{X}{X+Y}$. Show that U and V are independent (evaluate joint pdf). Find the corresponding marginal probability density functions. Further let $\alpha = 1$ and then find the cdf of $Z = \frac{X}{Y}$ and use it to determine the probability $P(Z \leq 10)$ for $\beta = 10$.**[3+2+2+1]**

8. Let $X_1, X_2, \ldots, X_k$ are independent random variables distributed as gamma $G(r_i, \lambda)$, $i = 1, 2, \ldots k$ distribution. Use mgf technique to find the distribution of the sum of these random variables. Determine the mean and the variance of this distribution using the definition. **[3+1+2]**

9. Let (X,Y) be the heights in centimeters of randomly selected husband-wife pairs. Suppose (X,Y) has the bivariate $BVN(178, 165, 7, 6, 0.4)$ normal distribution. Find the probability that the wife is taller than 170 cm given that her husband has height 171 cm. Further determine the probability that total height of a randomly selected husband-wife pair is less than 320 cm. **[3 + 3]**

10. Given the joint pdf of X and Y as $f_{X,Y}(x,y) = 0.5xe^{-y}$, $0 < x < 2$, $y > 0$; $f_{X,Y}(x,y) = 0$, *elsewhere*, determine the joint pdf of (U,V) where $U = X+Y$, $V = Y$. Also evaluate the marginal distribution of U. **[3 + 2]**

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