गणित विभाग, भारतीय प्रौद्योगिकी संस्थान पटना

DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY PATNA

B.Tech - I, MA-101

End Semester Examination

November, 2012

Time: 3 Hrs | Max Marks: 50

Attempt all the questions. Write brief and precise solutions to each question.

---

(1) Check whether the following statements are true or false. Give appropriate reasons to support your answer.

(a) The differential form $2xyz\,dx + x^2 \cos y\,dy$ is not exact.

(b) The value of the integral $\int_A^B \frac{xdx + ydy}{\sqrt{x^2+y^2}}$ does not depend on the path joining A and B.

(c) If the coordinate system is transformed by the rule $x = r\cos\theta, y = r\sin\theta$ and $z = z$ the integral $\iiint_D f(x,y,z)\,dx\,dy\,dz$ becomes $\iiint_G h(r,\theta,z)\,|r|\,dr\,d\theta\,dz$.

(d) The curvature, torsion and binormal make a right handed frame of mutually orthogonal unit vectors in space.

(e) The directions in which the function $g(x,y) = (x^2/2 + y^2/2)$ shows zero change is $\mp\frac{1}{\sqrt{2}}\hat{\imath} \pm \frac{1}{\sqrt{2}}\hat{\jmath}$.

[2× 5]

(2) Find the linearization of the function $f(x,y) = e^x \cos y$ at the origin. What will be an upper bound for the error incurred in replacing f by its linearization over the region $R : |x| \le 0.01$, $|y| \le 0.01$, [1 + 2]

(3) Test the convergence and divergence of the infinite series $\frac{1}{1+x} + \frac{x}{1+x^2} + \frac{x^2}{1+x^3} + \ldots + \frac{x^{n-1}}{1+x^n} + \ldots$ where $x > 0$ (properly specify the convergence and divergence regions). [5]

(4) Define a monotone sequence. Show that the sequence $\{x_n\}$ defined as $x_1 = 1.5, x_n = \sqrt{3x_{n-1} - 2},\ n \ge 2$ converges. Find the corresponding limit. [1+3]

(5) Find a and b with $a \le b$ such that $\int_a^b (48 - 4x - x^2)^{1/3}\,dx$ has its largest value.

[3]

(6) State the increment theorem for functions of three variables. [2]

(7) Find the points of discontinuity of the function:

$$f(x, y) = \begin{cases} \dfrac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

[3]

(8) Determine $\partial w/\partial r$ if $w = x^2 + y^2 + z^2$, $x = r - s - t$, $y = r - s + t$ and $z = r + s + t$. [3]

(9) Let $a_1, a_2, \ldots, a_n$ be n positive numbers. Find the maximum of $\sum_{i=1}^{n} a_i x_i$ subject to the constraint $\sum_{i=1}^{n} x_i^2 = 1$ [3]

(10) Find the area enclosed by the cardioid $r = (1 + \cos 2\theta)$. [3]

(11) Evaluate the integral $\iint_R (x - y)^4 e^{2(x+y)}\, dx\, dy$ by applying the transformations $x = \frac{u+v}{2}$ and $y = \frac{u-v}{2}$, where the region R is the square with vertices $(1, 0), (2, 1), (1, 2)$ and $(0, 1)$. [3]

(12) Use two paths test to check whether the following function has a limit at origin:

$$f(x, y) = \begin{cases} \dfrac{2x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

[2]

(13) Find the outward flux of the field:

$$F = \left(3xy - \frac{x}{1 + y^2}\right)\hat{\imath} + (e^x + \arctan y)\hat{\jmath}$$

across the cardioid $r = a(1 + \cos\theta)$, $a > 0$. [3]

(14) Use Green's theorem to evaluate $\oint_C \{(xy + y^2)\, dx + x^2\, dy\}$, where C : The triangle bounded by $x = 0, x + y = 1, y = 0$. [3]

****