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INDIAN INSTITUTE OF TECHNOLOGY PATNA
END SEMESTER EXAM Nov.2010 - MA 201

Time : 3 Hrs **Max Marks : 50**

ATTEMPT ALL THE QUESTIONS. PLEASE GIVE APPROPRIATE REASONS AND EXPLANATION FOR EACH ANSWER.

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(1) Apply the residue technique to evaluate $\int_0^\infty \frac{x \sin x}{x^4+1}\, dx$. [4]

(2) Show that the function $e^x[(x^2-y^2)\cos y - 2xy \sin y]$, (x, y real) is harmonic. Find the corresponding conjugate harmonic function. Finally, construct the corresponding analytic function in terms of $z(=x+iy)$. **[2+3+1]**

(3) (i) Find the surface orthogonal to the system $z = cxy(x^2+y^2)$ and which passes through the hyperbola $x^2-y^2=4$, $z=0$, c being constant. (ii) Solve the Cauchy problem : $4\sqrt{x}\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$, $z(2,y) = y^2$. [2+2]

(4) Consider the PDE $a^2 u_{xx} + 2u_{xy} + b^2 u_{yy} + 2u_x + 3u_y = 0$. Find the values of a and b such that the PDE is (i) elliptic, (ii) parabolic. Put $a=1$ and $b=2$ transform the resulting PDE to canonical form. [2+2]

(5) Use Parseval's formula to determine the sum of series $\sum_{n=1}^{\infty}\frac{1}{n^4}$ and determine the value of $\int_{-\pi}^{\pi}\cos^8 x\, dx$. [3+3]

(6) Show that $ln|2\sin\frac{x}{2}| = -\sum_{n=1}^{\infty}\frac{\cos nx}{n}$ for $x \neq 2m\pi$ $m \in \mathbb{Z}$, where ln denotes natural logarithm. [4]

(7) (i) Let $f(x)$ denote a function whose Fourier sine transform $F_s(d)$ is $2\frac{d}{d^2+1}$. Determine the function $f(x)$. (ii) Using a suitable Fourier transform show that $\int_0^\infty \frac{d\alpha}{(a^2+\alpha^2)(b^2+\alpha^2)} = \frac{\pi}{2ab(a+b)}$, where $a, b > 0$. [3+3]

(8) Use Duhamel's principle to solve the heat flow problem described by the equation :

$$u_t(x,t) = c\,u_{xx}(x,t) + f(x,t)$$
$$u(x,t) = 0 \text{ at } t = 0$$
$$-\infty < x < \infty \text{ and } t > 0$$

[5]

(9) Let a solid rod be placed along the real line with its one end at origin and other at infinity. Determine the temperature distribution in the rod when the end at origin is maintained at zero temperature and the initial temperature distribution is given by function $f(x)$. [5]

(10) (i) Determine whether following PDEs are compatible pairs : (a) $xp = yq$ and $z(xp+yq) = 2xy$, (b) $p^2+q^2 = 7$ and $(p^2+q^2)x = p$. (ii) Find the complete integral of $z = px + qy - \cos(pq)$. [3+3]
