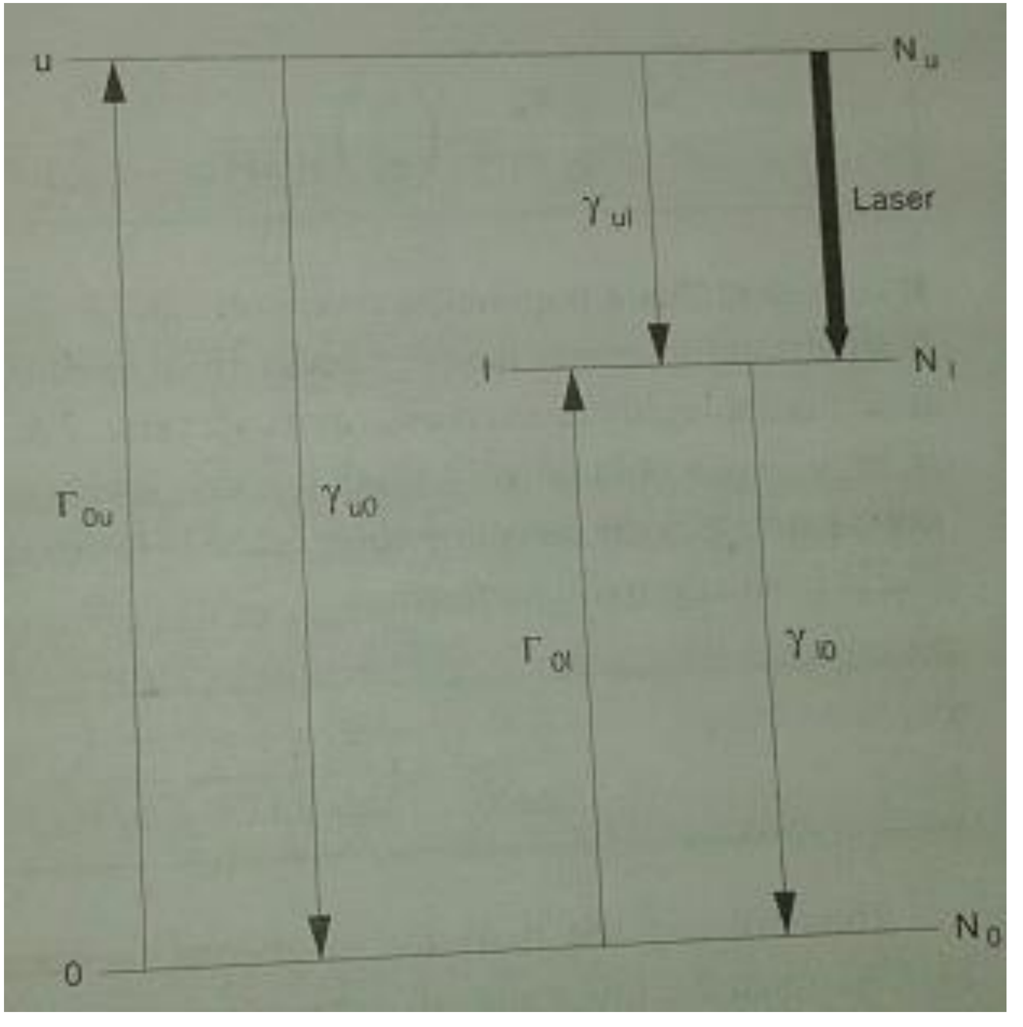
PH 201

OPTICS & LASERS

Lecture_Lasers_8

Three-level laser with Upper laser level as Highest level



Energy level diagram & relevant excitation & decay processes of an atomic three-level laser system

Consider energy level diagram with highest level as upper laser level *u*, intermediate level as lower laser level *l*, & lowest level (ground state) of that species as level 0.

Pumping process is allowed that provides flux to both levels *u* & *l* at rates Γ_{0u} & Γ_{0l} from ground state 0.

Assuming population densities N_u, N_l, & N_0, & that N_0 is nearly equal to population *N* of system before pumping of higher level is initiated.

Pumping flux from level 0 to *u*: $N_0\Gamma_{0u}$

Pumping flux from level 0 to *l*: $N_0\Gamma_{0l}$

Decay rates from level *u* to *l*: $\chi_{ul}, \chi_{u0},$ & χ_{l0}

Assuming no thermal excitation, $\chi_{0u}, \chi_{0l},$ & χ_{lu} **are neglected.**

Rate Eqs for flux entering & leaving levels *u* & *l*,

$$\frac{dN_u}{dt} = N_0\Gamma_{0u} - N_u(\chi_{ul} + \chi_{u0}) = 0$$

$$\frac{dN_l}{dt} = N_0\Gamma_{0l} + N_u\chi_{ul} - N_l\chi_{l0} = 0$$

For steady state solution of N_u & N_l, we equate RHS of both Eqs to zero.

$$N_u = \frac{N_0\Gamma_{0u}}{\chi_{ul}+\chi_{u0}}$$

$$N_l = \frac{N_0[\Gamma_{0l}+\Gamma_{0u}\chi_{ul}/(\chi_{ul}+\chi_{u0})]}{\chi_{l0}}$$

In order to have gain,

$$\Delta N_{ul} = N_u - \left(\frac{g_u}{g_l}\right)N_l > 0 \qquad or \qquad \frac{g_u N_u}{g_l N_l} > 1$$

$$\frac{g_l N_u}{g_u N_l} = \left(\frac{g_l}{g_u}\right) \frac{\Gamma_{0u}\chi_{l0}}{\chi_{u0}\Gamma_{0l} + \chi_{ul}[\Gamma_{0l} + \Gamma_{0u}]} > 1$$

Considering usual situation in atomic systems,

$$\chi_{ul} = A_{ul}, \quad \chi_{u0} = A_{u0}, \quad \& \quad \chi_{l0} = A_{l0}$$

This is equivalent to saying that collisional decay processes are negligible compared to radiative decay.

$$\frac{g_l N_u}{g_u N_l} = \left(\frac{g_l}{g_u}\right) \frac{\Gamma_{0u} A_{l0}}{A_{ul}(\Gamma_{0l} + \Gamma_{0u}) + A_{u0}\Gamma_{0l}} > 1$$

Population inversion can be obtained if decay from level *l* is significantly greater than decay from level *u*, provided that pumping to level *l* is not highly favored over that to level *u*. For an atomic system in which A_{l0} is large, A_{u0} would be very small.

$$\frac{g_l N_u}{g_u N_l} \cong \left(\frac{g_l}{g_u}\right) \frac{1}{(1+\Gamma_{0l}/\Gamma_{0u})} \frac{A_{l0}}{A_{ul}} > 1$$

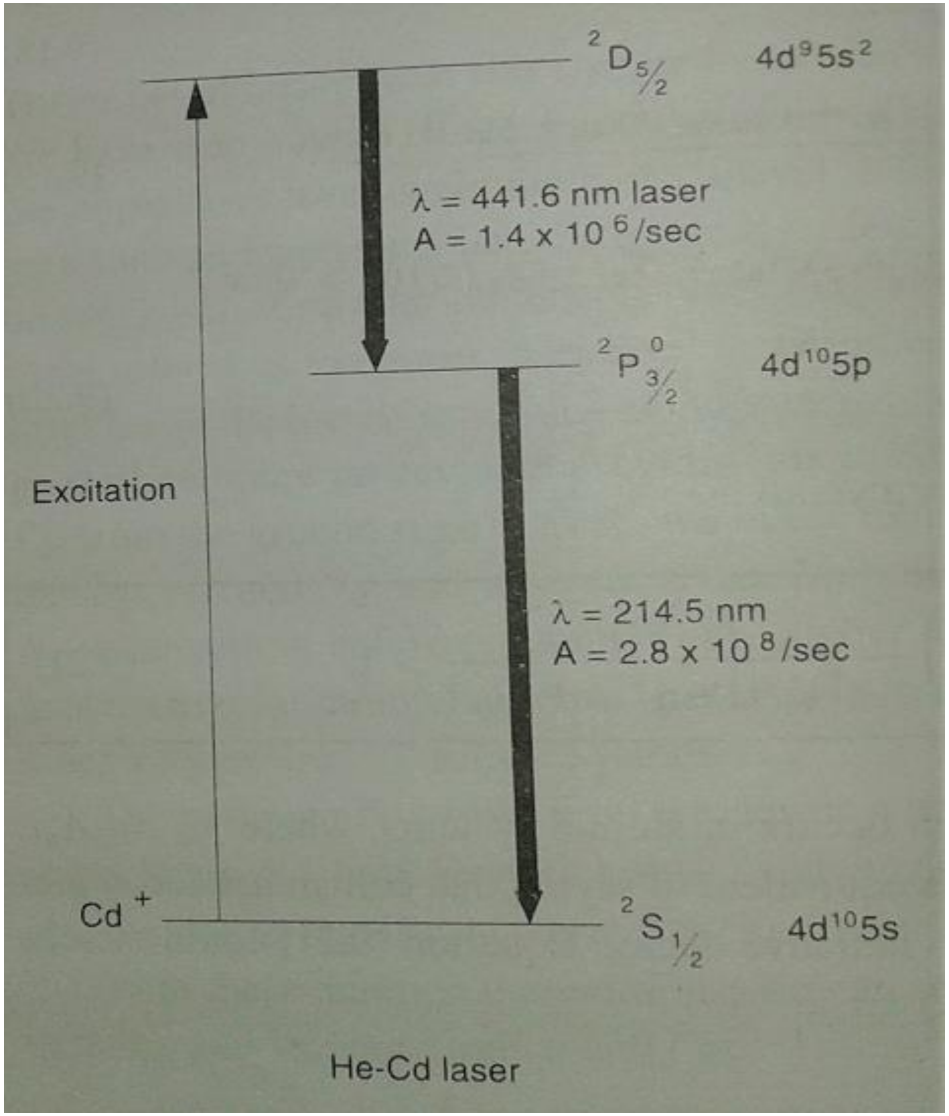
Thus, for inversions to occur, ratios of A_{l0}/A_{ul} & Γ_{0l}/Γ_{0u} must be favorable in some combination.

Ex. If $\Gamma_{0l}/\Gamma_{0u} = 1$ then A_{l0} must be greater than $2A_{ul}$ (assuming $g_u = g_l$).

It is most desirable to have a fast decay out of lower laser level & a higher pumping flux to upper laser level.

EXAMPLE: He-Cd LASER

He-Cd laser (441.6 nm) transition in Cd^+ ion



Energy level diagram of three-level He-Cd laser

Relevant transition probabilities are:

$$A_{ul} = 1.4 \times 10^{6} s^{-1}$$

$$A_{u0} = 0$$

$$A_{l0} = 2.8 \times 10^{8} s^{-1}$$

$$\frac{g_l N_u}{g_u N_l} = \left(\frac{4}{6}\right) \frac{200}{1 + \Gamma_{0l} / \Gamma_{0u}} > 1$$

Thus, an inversion can be obtained unless pumping flux from ground level 0 to level *l* exceeds 132 times pumping flux to level *u*, i.e., unless

$$\Gamma_{0l} > 132 \Gamma_{0u}$$

FOUR-LEVEL LASER

Consider an arrangement similar to three-level system but with level 0 added below the lower laser level *l*.

This arrangement is typical to many solid state lasers.

Level 0 is ground state, & majority of atoms are initially in that level before pumping occurs.

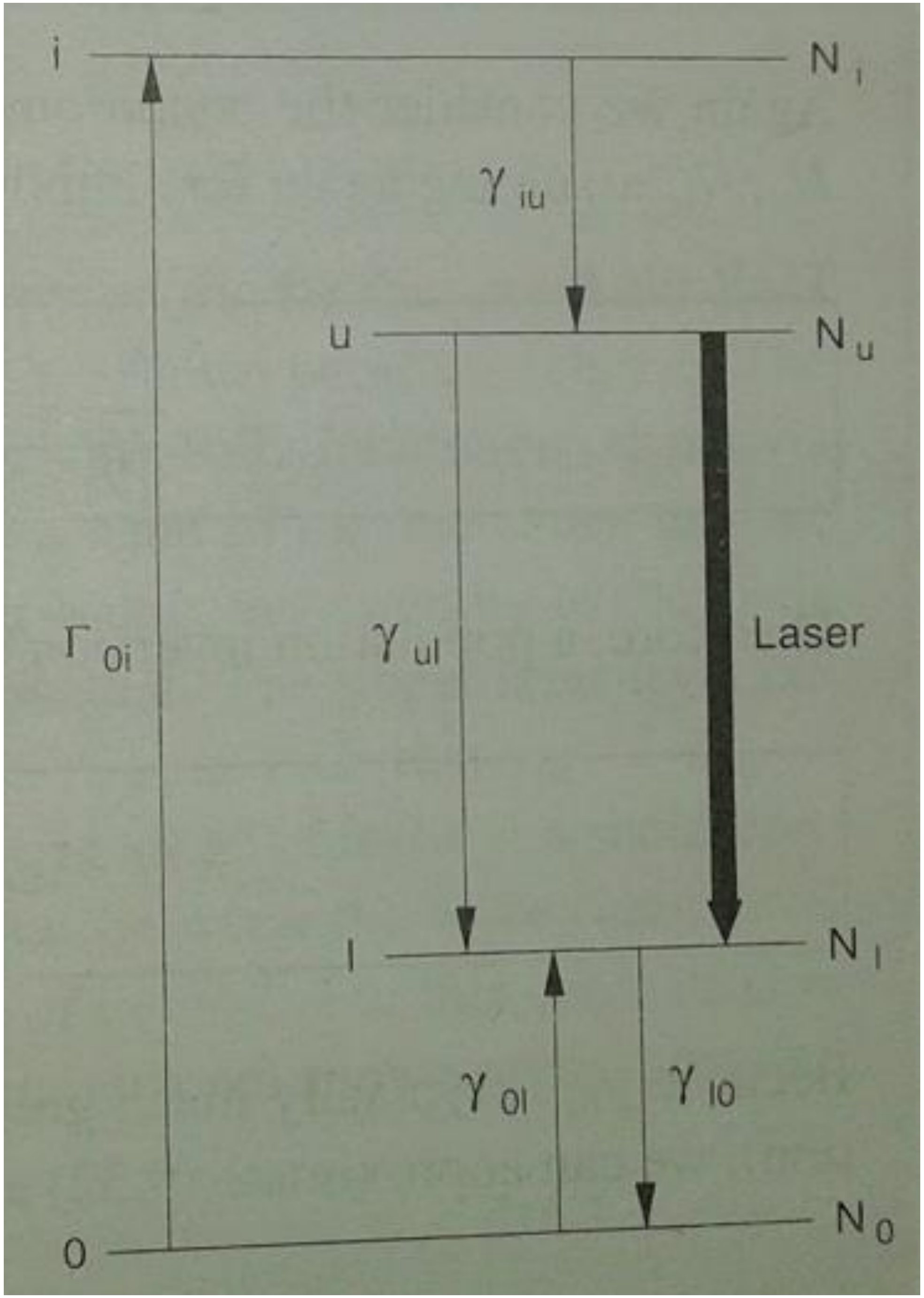
Because large energy separation upward rates are neglected.

$$\chi_{0u}, \chi_{0i}, \chi_{lu}, \chi_{li}, \,\&\, \chi_{ui}$$

Specific downward rates are also neglected since they are very small in solid state laser crystals, owing to large energy separations of specific levels.

$$\chi_{il}, \chi_{i0} \,\&\, \chi_{u0}$$

We assume that dominant decay rate from level u to l will most likely be radiative.



Energy level diagram & relevant excitation & decay processes of a four-level laser system

Rate Eqs,

$$\frac{dN_l}{dt} = \chi_{0l} N_0 - \chi_{l0} N_l + \chi_{ul} N_u = 0$$

$$\frac{dN_u}{dt} = -\chi_{ul} N_u + \chi_{iu} N_i = 0$$

$$\frac{dN_i}{dt} = \Gamma_{0i} N_0 - \chi_{iu} N_i = 0$$

Total no of laser species, (*N* is constant)

$$N_0 + N_l + N_u + N_i = N$$

By differentiation,

$$\frac{dN_0}{dt} = -\frac{dN_l}{dt} - \frac{dN_u}{dt} - \frac{dN_i}{dt}$$

We can solve for N_u & N_l,

$$N_u = \frac{\chi_{iu}\Gamma_{0i}}{\chi_{ul}\chi_{iu}} N_0 = \frac{\Gamma_{0i}}{\chi_{ul}} N_0$$

$$N_l = \left[\frac{\chi_{0l}}{\chi_{l0}} + \frac{\Gamma_{0i}}{\Gamma_{l0}}\right] N_0 = \left[\frac{(\chi_{0l} + \chi_{0i})}{\chi_{l0}}\right] N_0$$

Condition for population inversion, assuming $g_u = g_l$,

$$\frac{N_u}{N_l} = \frac{\chi_{l0}\Gamma_{0i}}{\chi_{ul}[\chi_{0l} + \Gamma_{0i}]} > 1$$

Therefore, a population inversion will occur for a pumping flux Γ_{0i},

$$\Gamma_{0i} > \frac{\chi_{0l}\chi_{ul}}{\chi_{l0} - \chi_{ul}}$$

$$\chi_{l0} >> \chi_{ul}$$

$$\Gamma_{0i} > \frac{\chi_{0l}\chi_{ul}}{\chi_{l0}} = e^{-\Delta E_{l0}/kT}\chi_{ul}$$

The lower of each pair of levels, *u* & 0 contain most of the population.

$$\frac{N_i}{N_u} \approx e^{-\Delta E_{iu}/kT}$$

$$\frac{N_l}{N_0} \approx e^{-\Delta E_{l0}/kT}$$

In case of pair of levels *i* & *u*, it is desirable to have population in level *u* since it is upper laser level.

In case of pair of levels *l* and 0, it is desirable to have population in level 0, but owing to close energy separation of *l* & 0, enough population can be in level *l* to affect N_l & thereby reduce population inversion ΔN_{ul}.

Comparing $$\Gamma_{0i} > \frac{\chi_{0l}\chi_{ul}}{\chi_{l0}} = e^{-\Delta E_{l0}/kT}\chi_{ul}$$ **Four-level**

$$\Gamma_{li} > \chi_{ul}\left(1+\frac{\chi_{il}}{\chi_{iu}}\right) \qquad \Gamma_{li} > A_{ul}$$ **Three-level**

Pumping requirements of four-level system are significantly reduced by factor

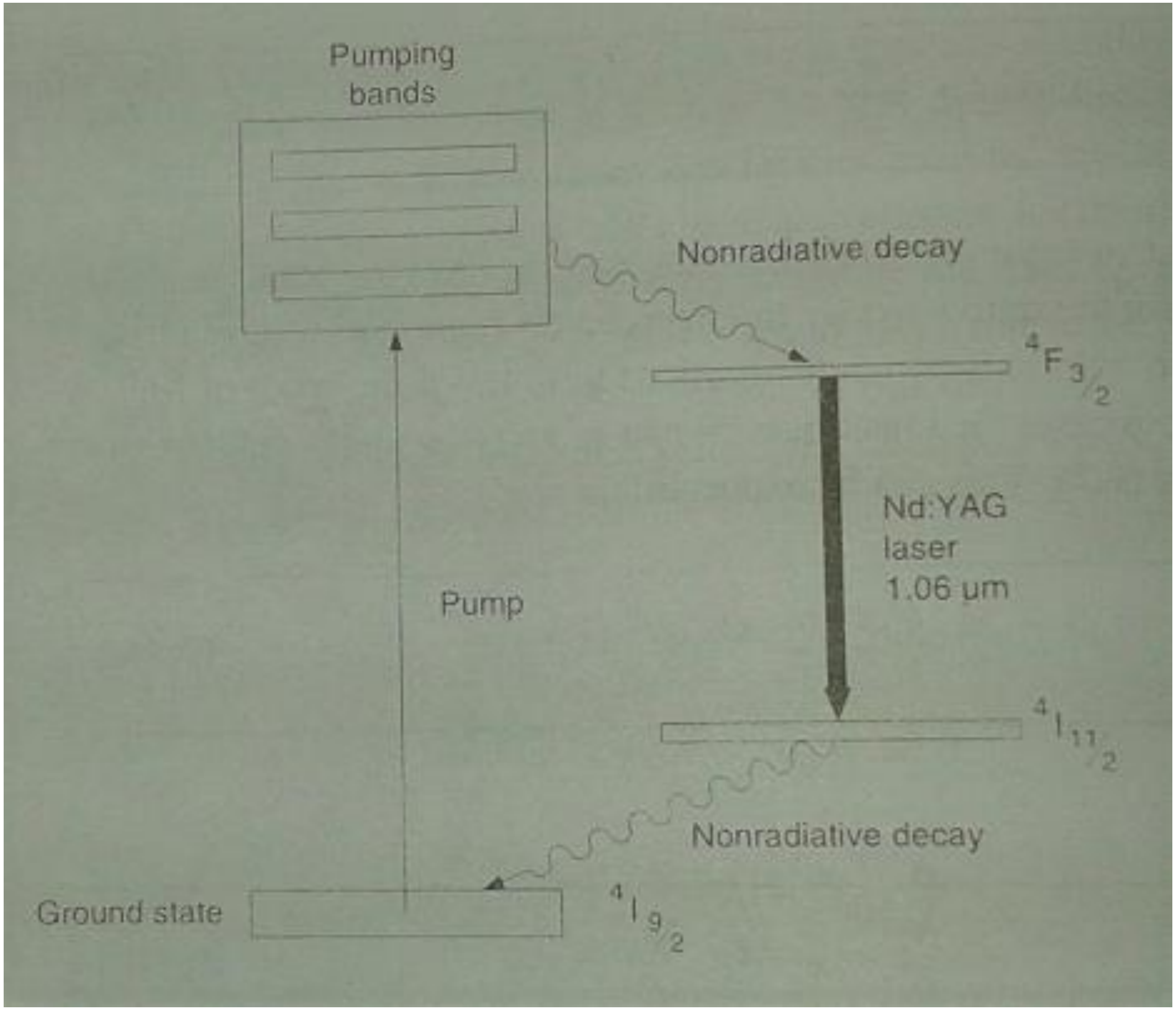
$$e^{-\Delta E_{l0}/kT}$$

compared to three-level system.

Since ΔE_{lo} is typically much greater than kT at or near room temp & thus for most solid state lasers,

$$e^{-\Delta E_{l0}/kT} << 1$$

EXAMPLE: Nd:YAG LASER



Energy level diagram of a Nd:YAG four-level laser

Excitation occurs via optical pumping from $^4I_{9/2}$ ground state to a band of excited states that we will refer to as level *i*.

Nonradiative decay occurs very rapidly to $^4F_{3/2}$ upper laser level *u* with $\gamma_{iu} \approx 10^{12}$ to 10^{14} per second.

Upper laser level decays primarily radiatively & has a lifetime of 230 μs such that

$$\chi_{ul} \cong A_{ul} = 1/\tau_u = 1/(2.3\times10^{-4}\,s) = 4.35\times10^{3}\,s^{-1}$$

For this case, ΔE_{lo} = 0.25 eV & we assume laser crystal is at room temp (*T* = 300 K),

$$\Gamma_{0i} > e^{-\Delta E_{l0}/kT}\chi_{ul} = e^{-0.25/(8.6\times10^{-5}\times300)}(4.35\times10^{3}\,s^{-1})$$

$$= e^{-9.7}(4.35\times10^{3}\,s^{-1}) \cong 0.265$$

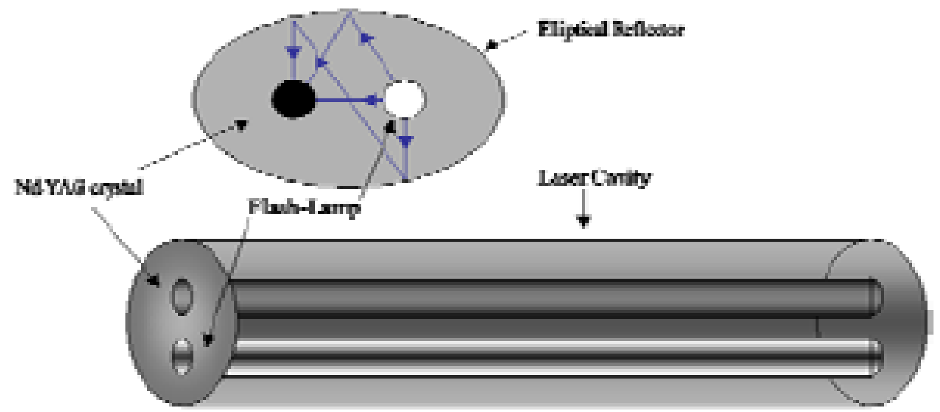
Comparing this pumping rate to that obtained for Ruby laser, we find that pumping rate is reduced by a factor of 333/0.265 = 1257.

Thus, even though transition probability of Nd:YAG is significantly higher than that of Ruby laser & would therefore increase the pumping threshold, reduction due to exponential factor far overweighs this increase.

Hence we have a much lower threshold pumping rate for a four level system than for a three level system.

Solid State Lasers

Nd:YAG Laser:
$\lambda = 1.064\ \mu m$



• YAG = Yttrium-Aluminium-Garnet ($Y_3Al_5O_{12}$), it is transparent and colourless.

•Nd:YAG Laser is doped with about 1% Nd3+ ions into the YAG crystal. The crystal color then changed to a light blue color.