MidTerm

CS204: Algorithms

Max Marks: 90 (excluding Q6)

Instructions: (i) This is an **open book** exam. You can refer to your textbook. But sharing of textbooks is not allowed. (ii) Please **return** the question paper at the end of the exam

1. Give asymptotic upper and lower bounds for T(n) in each of the following recurrences. Assume that T(n) is constant for n <= 2. Make your bounds as tight as possible, and justify your answers.
(a) T(n) = 2T(n/2) + n^4 **(5 pts)**
(b) T(n) = 16T(n/4) + n^2 **(5 pts)**

2. Describe an O(n) time algorithm that, given a set S of n distinct numbers and a positive integer k ≤ n, determines the k numbers in S that are closest to the median of S.
For instance, consider the following array: 4 3 6 5 100 200 300 with n=7. Let k be 2. Here the median is 6. The closest k=2 integers to the median are: 5 and 4. **(15 pts)**

3. An array A[1 . . . n] is said to have a **majority element** if more than half of its entries are the same. Given an array, the task is to design an efficient linear time algorithm to tell whether the array has a majority element, and, if so, to find that element. Explain the correctness of your algorithm (in other words, informally prove your algorithm).
Hint: Employ divide and conquer approach. Use the arguments discussed in the chip testing problem. **(20 pts)**

4. Call a sequence X [1 .. n] of numbers **oscillating** if X [i] < X [i + 1] for all even i, and X [i] > X [i + 1] for all odd i. Describe an efficient algorithm using dynamic programming to compute the length of the longest oscillating subsequence of an arbitrary array A of integers. **(15 pts)**

5. Lindonald's is considering opening a series of restaurants along Quaint Valley Highway (QVH). The n possible locations are along a straight line, and the distances of these locations from the start of QVH are, in kms and in increasing order, m1 , m2 , . . . , mn. The constraints are as follows:

- At each location, Lindonald's may open at most one restaurant. The profit from opening a restaurant at location i is pi , where pi > 0 and i = 1, 2, . . . , n.
- Any two restaurants should be at least k kms apart, where k is a positive integer.

The distances m1 , m2 , . . . , mn are represented by M array. The profits are represented by P array. Give an efficient algorithm using dynamic-programming to compute the maximum total profit subject to the given constraints. **(20 pts)**

6. **Extra Credit:** Attempt this problem only if you have time.
Given an unlimited supply of coins of denominations x1 , x2 , . . . , xn , we wish to make change for a value v; that is, we wish to find a set of coins whose total value is v. This might not be possible: for instance, if the denominations are 5 and 10 then we can make change for 15 but not for 12. Give a dynamic-programming algorithm for the following problem.

- Input: x1 , . . . , xn ; v.
- Question: Is it possible to make change for v using coins of denominations x1 , . . . , xn ?
(25 pts)

7. Class Attendance: **(10 pts)**