PH 301

ENGINEERING OPTICS

Lectures_4_Polarization-1

Syllabus

Lens systems: Basics & concepts of lens design, some lens systems.
Optical components: Reflective, refractive & diffractive systems; Mirrors, prisms, gratings, filters, polarizing components.
Interferometric systems: Two beam, multiple beam, shearing, scatter fringe & polarization interferometers.
Vision Optics: Eye & vision, colorimetry basics.
Optical sources: Incandescent, fluorescent, discharge lamps, Light emitting diode.
Optical detectors: Photographic emulsion, thermal detectors, photodiodes, photomultiplier tubes, detector arrays, Charge-coupled device, CMOS.
Optical Systems: Telescopes, microscopes (bright field, dark field, confocal, phase contrast, digital holographic), projection systems, interferometers, spectrometers.
Display devices: Cathode ray tube, Liquid crystal display, Liquid crystals on silicon, Digital light processing, Digital micro-mirror device, Gas plasma, LED display, Organic LED.
Consumer devices: Optical disc drives: CD, DVD; laser printer, photocopier, cameras, image intensifiers.

Texts

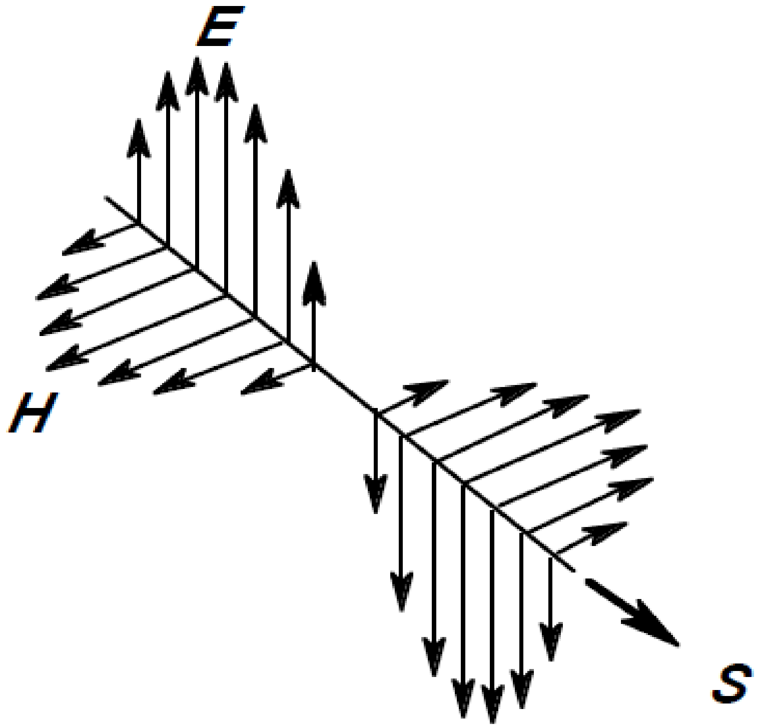
1. R. S. Longhurst, *Geometrical & Physical Optics*, 3rd ed., Orient Longman, 1988.
2. R. E. Fischer, B. Tadic-Galeb, & P. R. Yoder, *Optical System Design*, 2nd ed., SPIE, 2008
3. W. J. Smith, *Modern Optical Engineering*, 3rd ed., McGraw Hill, 2000.
4. K. Iizuka, *Engineering Optics*, Springer, 2008.
5. B. H. Walker, *Optical Engineering Fundamentals*, SPIE Press, 1995.

Engineering Optics

Engineering Optics deals with the engineering aspects of optics, & its main emphasis is on applying the knowledge of optics to the solution of engineering problems.

Electromagnetic Waves

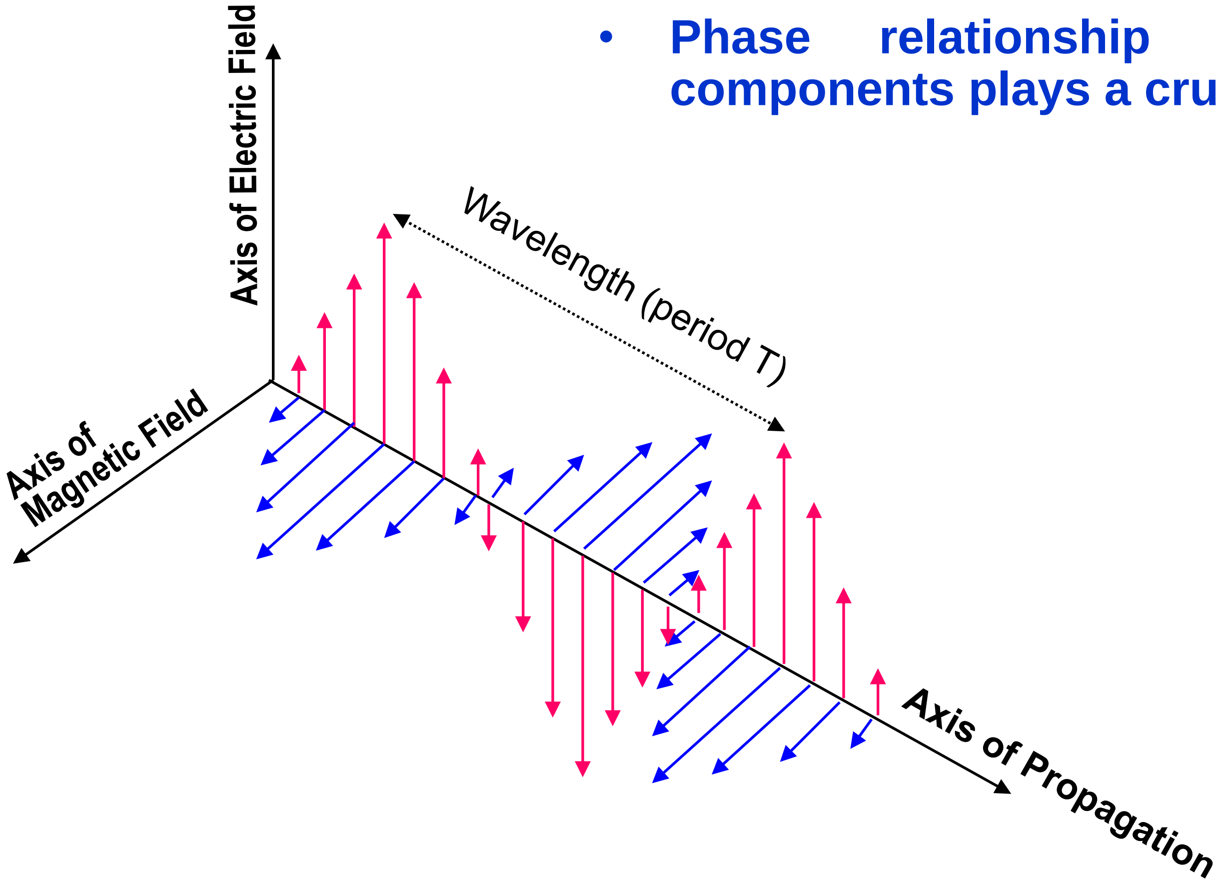
Light is an *em* wave & is produced whenever a charged particle is accelerated. In 3-D appearance of an *em* wave (if we could see it) would be two perpendicular waves, one of electric field ***E*** & one of magnetic field ***H***, in phase rippling along in a straight line.

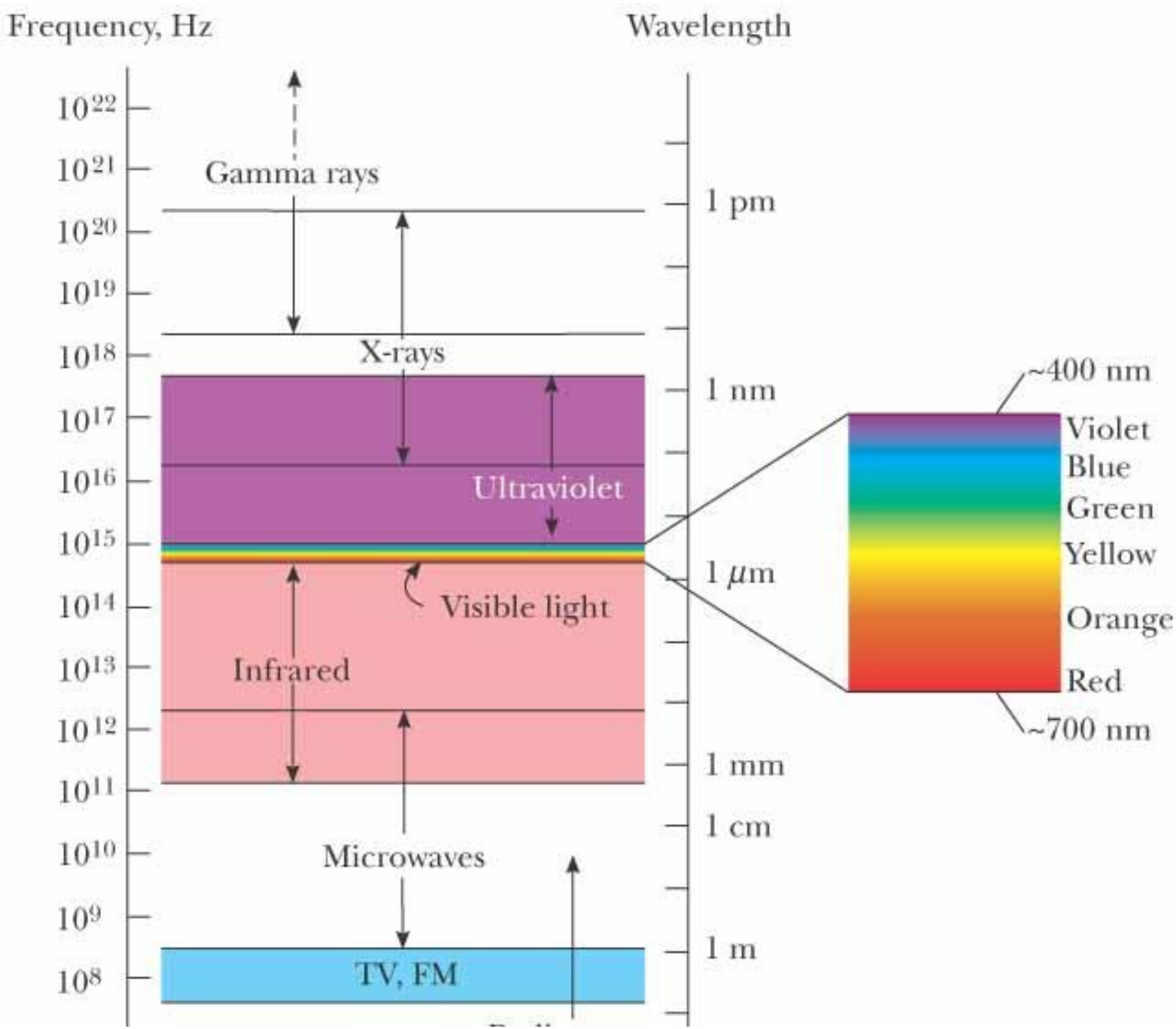


Vector *S* is pointing vector. Its magnitude is amount of energy carried by wave & its direction is direction of propagation of wave.

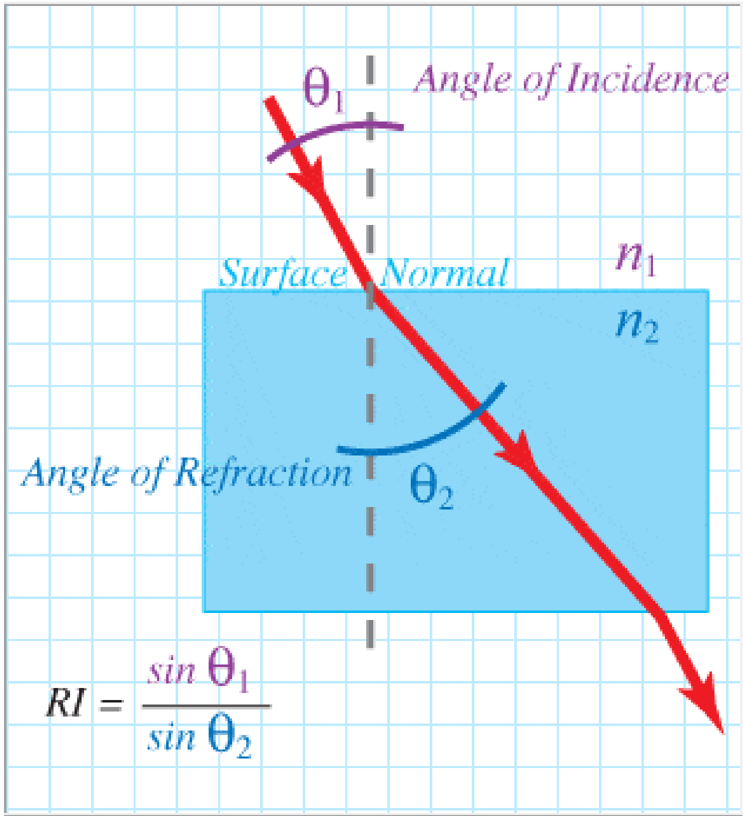
Directionality of Component

- Electric & magnetic fields are vectors - i.e. they have both magnitude & direction
- Inverse of period (wavelength) is frequency (Hz)
- Phase relationship between E & B components plays a crucial role




Frequency, Hz
Wavelength
10^{22}
10^{21}
10^{20}
10^{19}
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
Gamma rays
X-rays
Ultraviolet
Visible light
Infrared
Microwaves
TV, FM
1 pm
1 nm
1 μm
1 mm
1 cm
1 m
~400 nm
Violet
Blue
Green
Yellow
Orange
Red
~700 nm

Refractive Index



Light travels through different substances at different speeds. Generally speaking, **the denser the substance, the slower light moves**. When light enters a substance at an angle, this change in velocity causes ray to be deflected, or bent.

This bending is called **Refraction**. Amount that substance bends light is its Refraction (or Refractive) Index (RI). It's determined by using **Snell's law.**

RI for a given substance at a given temp & pressure is a constant.

RIs are used in real world to determine everything from **percentage of water in a sample of honey** to **composition & purity of gemstones**.

Polarization

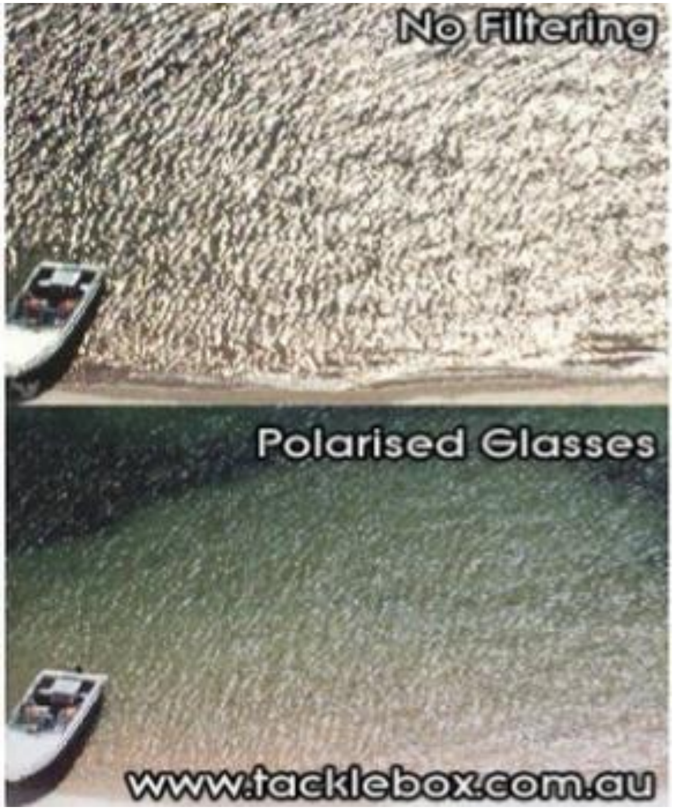
(a) (b)

If sunlight is incident on water surface at an angle close to polarizing angle, then reflected light will be almost polarized.

(a) If Polaroid allows the (almost polarized) reflected beam to pass through, we see glare from water surface.

(b) Glare can be blocked by using a vertical polarizer, & one can see inside the water.

Polarization

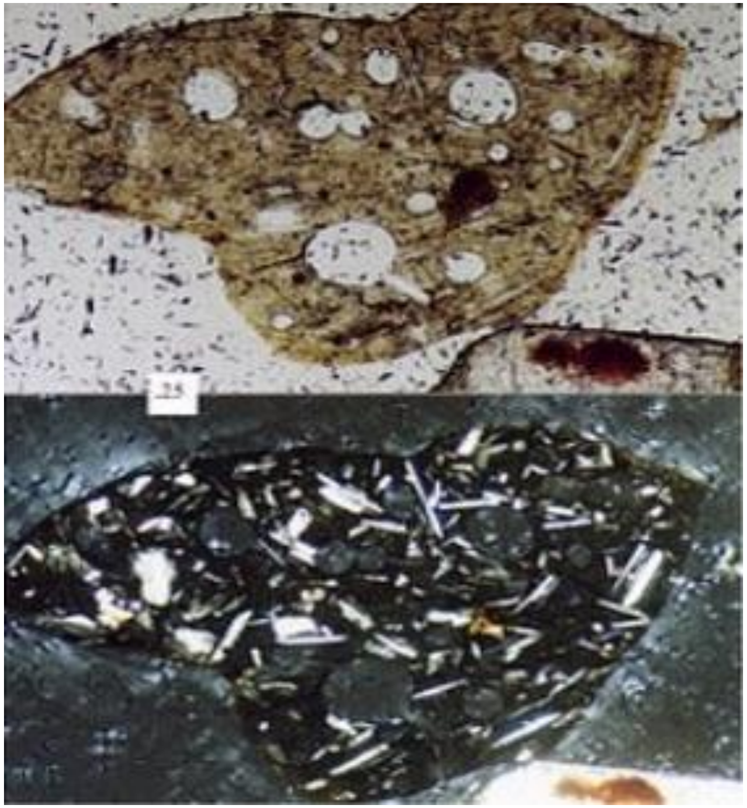


To differentiate longitudinal & transverse wave. As sun glasses to cut of unwanted reflected light best utilized by fishermen, motorist, skiers, sportsmen, etc.

Polarization

Effect of polarizing filter in sky (right picture).

Polarization



Plane polarized light

Cross-polarized light

Frequently exploited using polarization microscope for identifying minerals (geology). Photograph of a volcanic sand grain.

Polarization

Polarization is a property applying to transverse waves that specifies the geometrical orientation of the oscillations.

- In a transverse wave, direction of oscillation is transverse to the direction of motion of the wave, so oscillations can have different directions perpendicular to the wave direction.

- Ex., in a musical instrument like a guitar string. Depending on how the string is plucked, the vibrations can be in a vertical direction, horizontal direction, or at any angle perpendicular to the string.

- In contrast, in longitudinal waves, such as sound waves in a liquid or gas, displacement of particles in the oscillation is always in the direction of propagation, so these waves do not exhibit polarization.

- Transverse waves that exhibit polarization include em waves such as light & radio waves, gravitational waves, & transverse sound waves (shear waves) in solids.

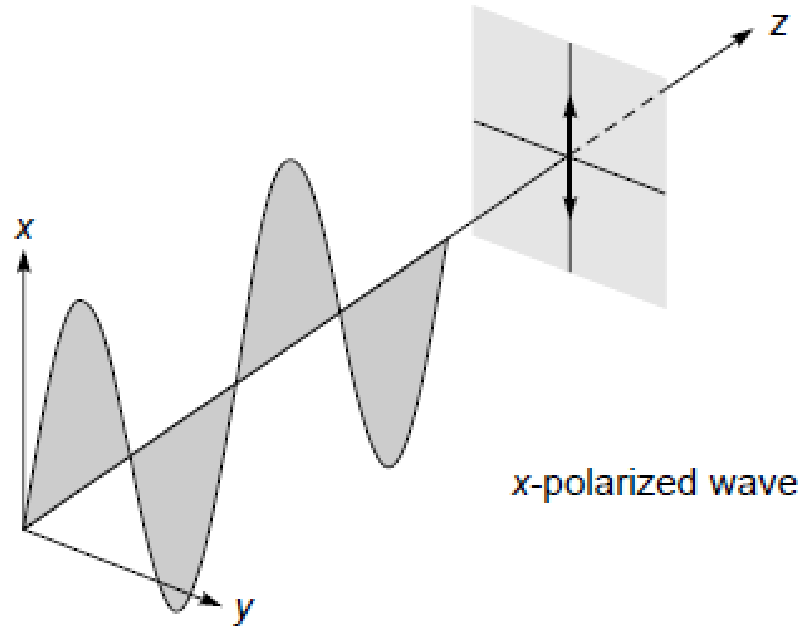
- In some types of transverse waves, the wave displacement is limited to a single direction, so these also do not exhibit polarization; for example, in surface waves in liquids (gravity waves), wave displacement of the particles is always in a vertical plane.

Polarization

- If we move one end of a string up & down, then a transverse wave is generated.
- Each point of string executes a sinusoidal oscillation in a straight line (along x-axis), & wave is, known as a ***linearly polarized wave***. It is also known as a plane polarized wave because string is always confined to *xz* plane.
- At any instant, displacement will be a cosine curve.
- Displacement for such a wave can be written as

$$x(z,t) = a\cos(kz - \omega t + \phi_1)$$

$$y(z,t) = 0$$

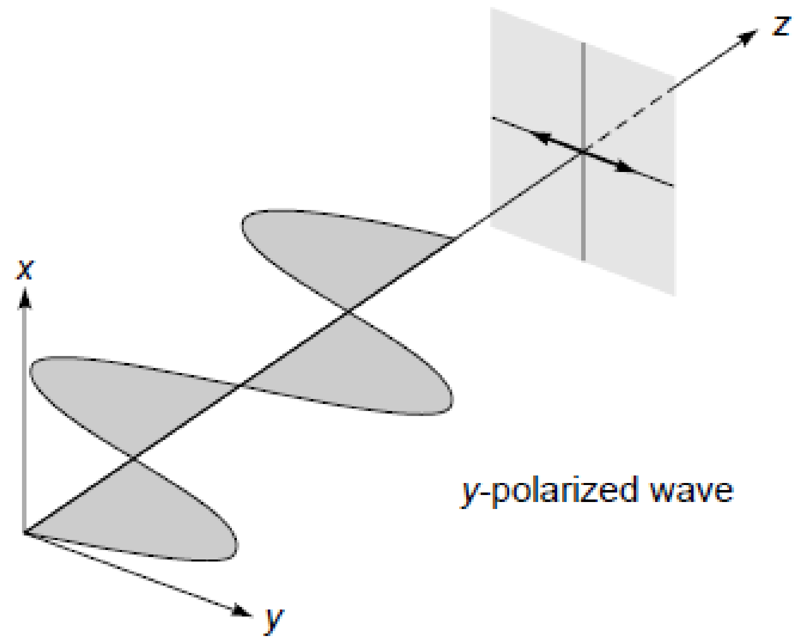


An *x*-polarized wave on a string with displacement confined to *xz* plane.

- Further, an arbitrary point $z = z_0$ will execute simple harmonic motion of amplitude a.
- String can also be made to vibrate in yz plane for which displacement is given by

$$y(z,t) = a\cos(kz - \omega t + \phi_2)$$

$$x(z,t) = 0$$



A *y*-polarized wave on a string with displacement confined to *yz* plane.

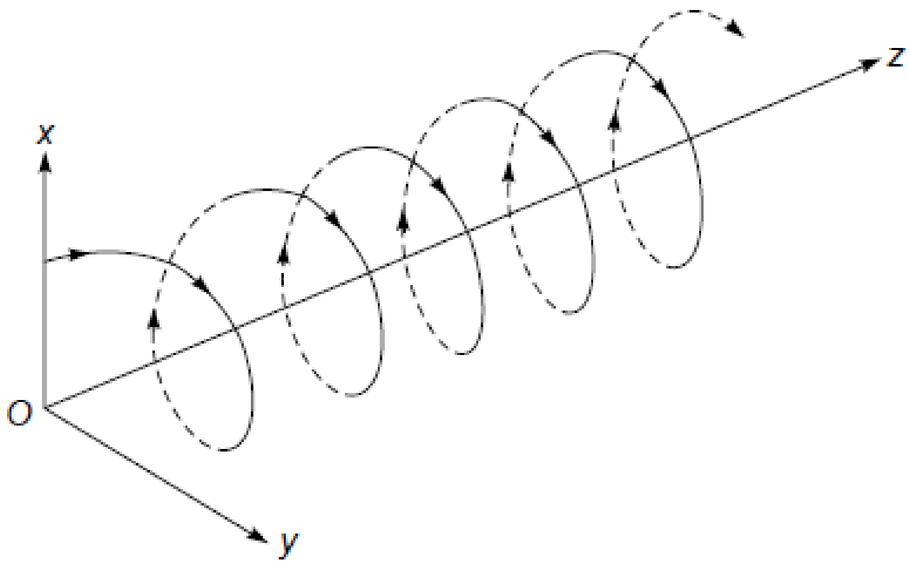
- In general, string can be made to vibrate in any plane containing z-axis.

- If one rotates the end of string on circumference of a circle, then each point of the string will move in a circular pat; such a wave is known as a **circularly polarized wave**.
- Corresponding displacement is given by

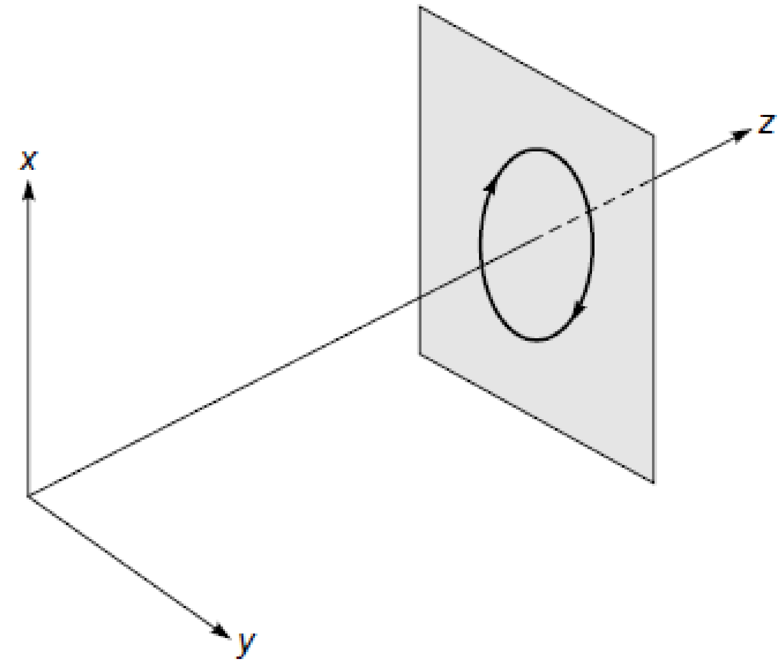
$$x(z,t) = a\cos(kz - \omega t + \phi)$$

$$y(z,t) = a\sin(kz - \omega t + \phi)$$

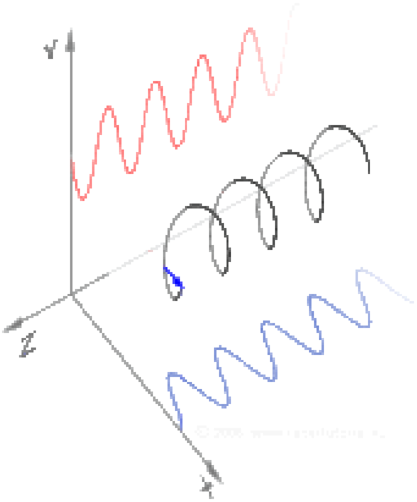
$$\Rightarrow \quad x^2 + y^2 = a^2$$



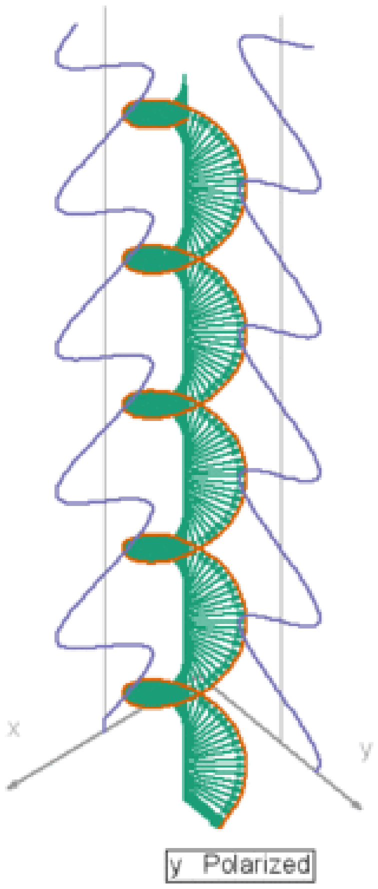
Displacement corresponding to a circularly polarized wave – all points on the string are at same distance from *z*-axis.



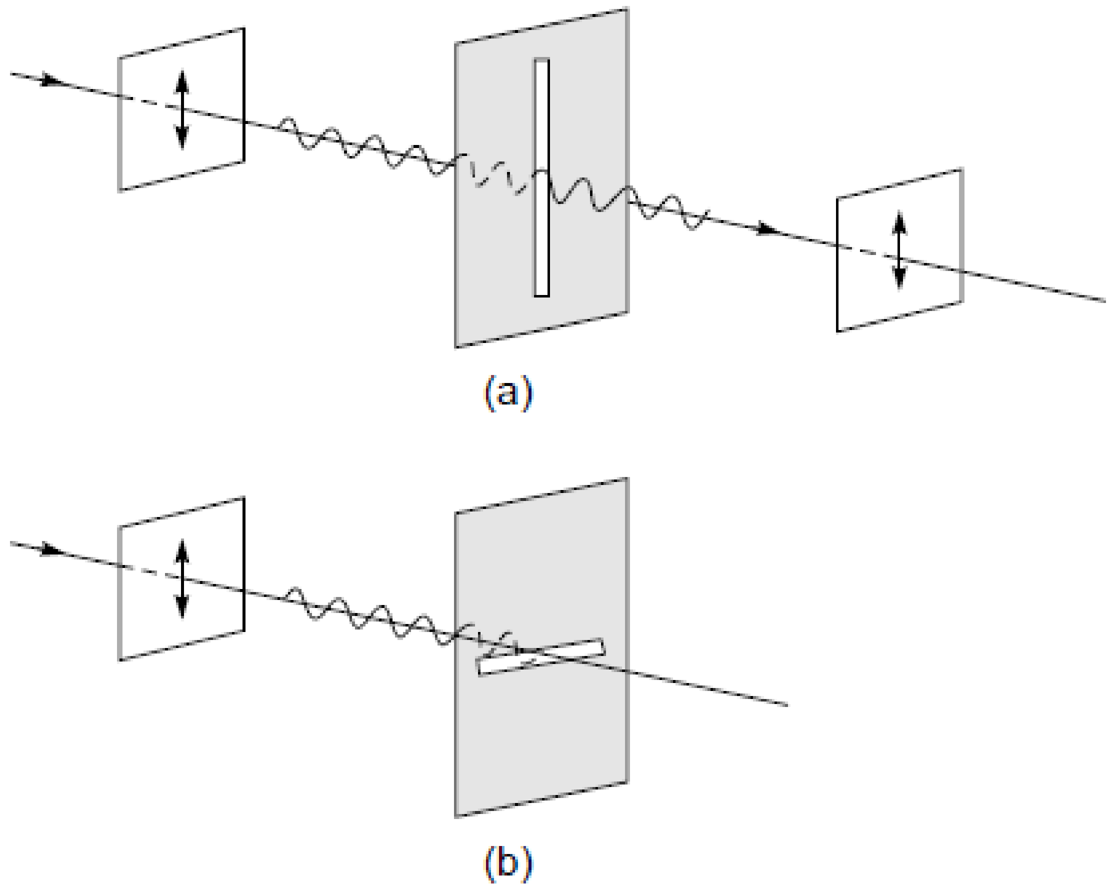
Each point on the string rotates on the circumference of the circle.



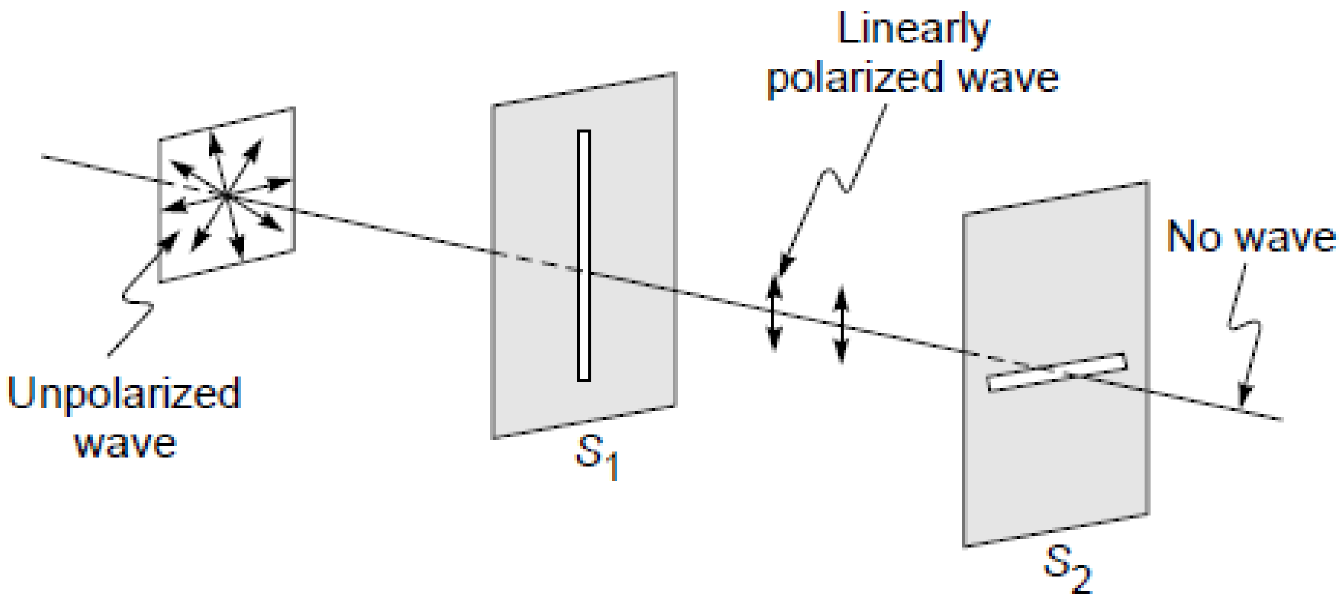
Circularly polarized wave as a sum of two linearly polarized components 90° out of phase.



Four different polarization states & two orthogonal projections.



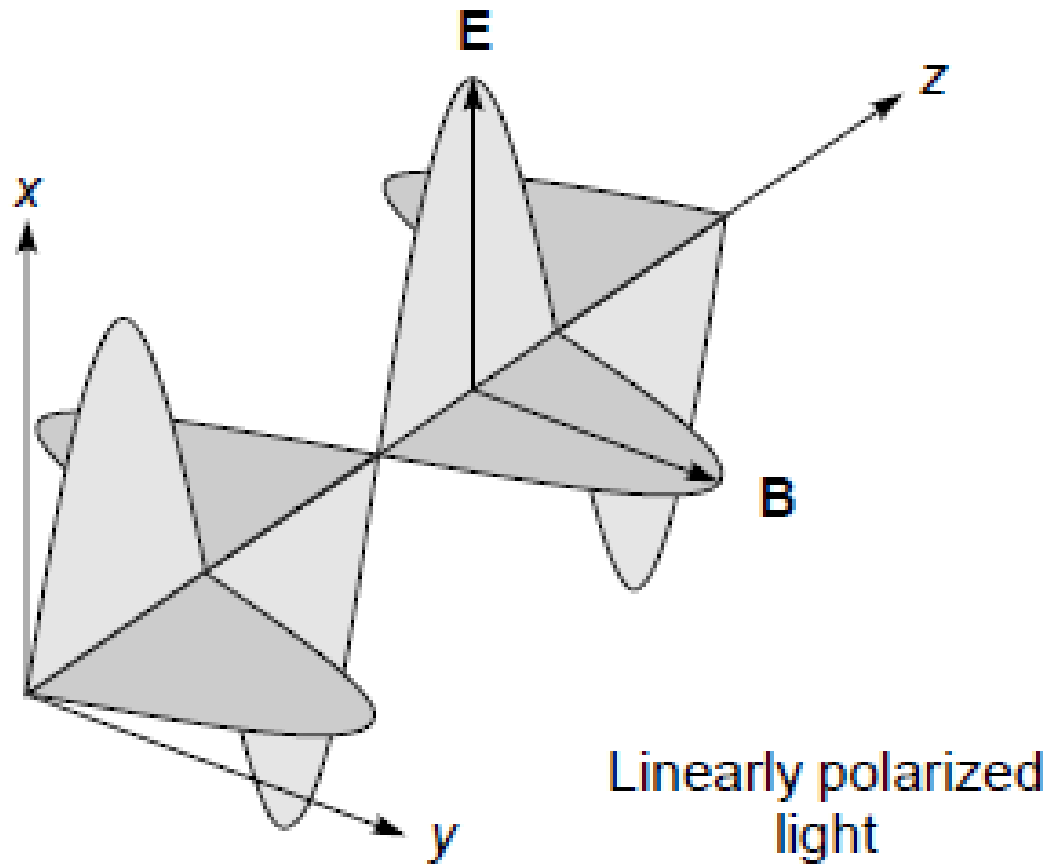
If a linearly polarized transverse wave (propagating on a string) is incident on a long narrow slit, then slit will allow only component of displacement, which is along length of slit, to pass through.



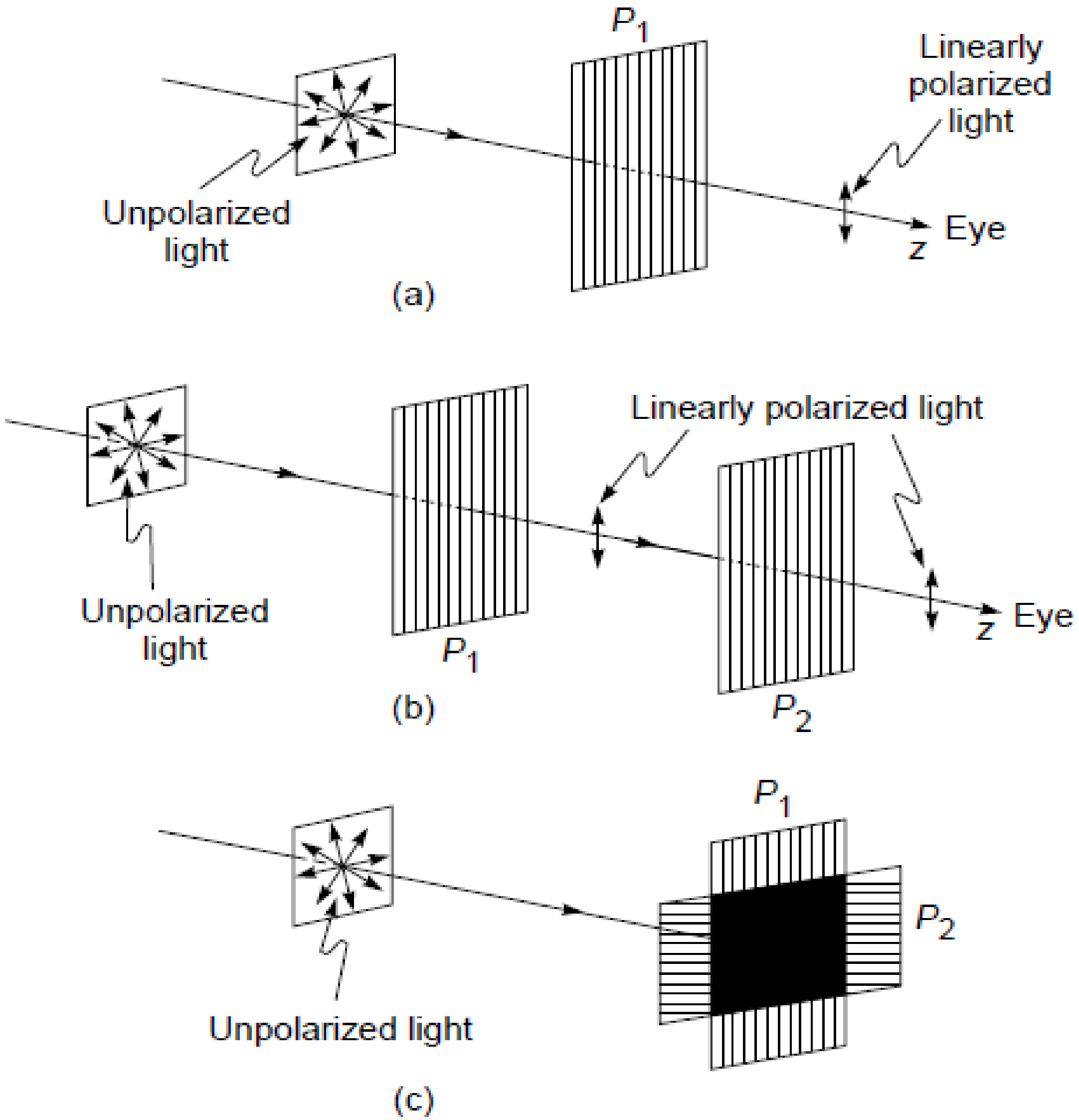
If an unpolarized wave propagating on a string is incident on a long narrow slit S_1, then transmitted beam is linearly polarized & its amplitude does not depend on orientation of S_1. If this polarized wave is allowed to pass through another slit S_2, then intensity of emerging wave depends on relative orientation of S_2 with respect to S_1.

EM theory

$$B_0 = \frac{1}{v} E_0$$



An x-polarized electromagnetic wave propagating in *z* direction.
Direction of propagation is along the vector E × B.



If an ordinary light beam is allowed to fall on a Polaroid, then emerging beam will be linearly polarized; & if we place another Polaroid P_2, then intensity of transmitted light will depend on relative orientation of P_2 with respect to P_1.

Superposition of two disturbances

- Propagation of two linearly polarized *em* waves (both propagating along *z* axis) with their electric vectors oscillating along *x* axis.

$$E_1 = \hat{x} a_1 \cos(kz - \omega t + \theta_1)$$
$$E_2 = \hat{x} a_2 \cos(kz - \omega t + \theta_2)$$

a_1 & a_2: Amplitudes, $\hat{x}$: Unit vector along *x* axis, θ_1 & θ_2: Phase const.

Resultant:

$$E = E_1 + E_2$$
$$\Rightarrow \quad E = \hat{x} a \cos(kz - \omega t + \theta)$$
$$where \quad a = [a_1^2 + a_2^2 + 2a_1 a_2 \cos(\theta_1 - \theta_2)]^{1/2}$$

- Resultant is a linearly polarized wave with its electric vector oscillating along the same axis.

- **Superposition of two linearly polarized em waves (both propagating along *z* axis) but with their electric vectors along two mutually perpendicular directions.**

$$E_1 = \hat{x} a_1 \cos(kz - \omega t)$$
$$E_2 = \hat{y} a_2 \cos(kz - \omega t + \theta)$$

For $\theta = n\pi$, resultant will also be a linearly polarized wave with its electric vector oscillating along a direction making a certain angle with *x* axis; this angle will depend on relative values of a_1 & a_2.

- **To find state of polarization of resultant field, we consider time variation of resultant electric field at an arbitrary plane perpendicular to *z* axis, which can be assumed to be $z = 0$.**

- **If E_x & E_y represent *x* & *y* component of resultant field *E* (= E_1 + E_2), then**

$$E_x = a_1 \cos \omega t$$
$$E_y = a_2 \cos(\omega t - \theta)$$

For $\theta = n\pi$,

$$E_x = a_1 \cos\omega t$$

$$E_y = (-1)^n a_2 \cos\omega t$$

$$\Rightarrow \quad \frac{E_y}{E_x} = \pm\frac{a_2}{a_1} \qquad (independent\ of\ t)$$

where upper & lower signs correspond to *n* even & *n* odd, respectively. In E_xE_y plane, this Eq. represents a straight line; angle Φ that this line makes with E_x axis depends on ratio a_2/a_1.

$$\phi = \tan^{-1}\left(\pm\frac{a_2}{a_1}\right)$$

Condition $\theta = n\pi$ implies that the two vibrations are either in phase (n = 0, 2, 4, ….) or out of phase (n = 1, 3, 5, ….).

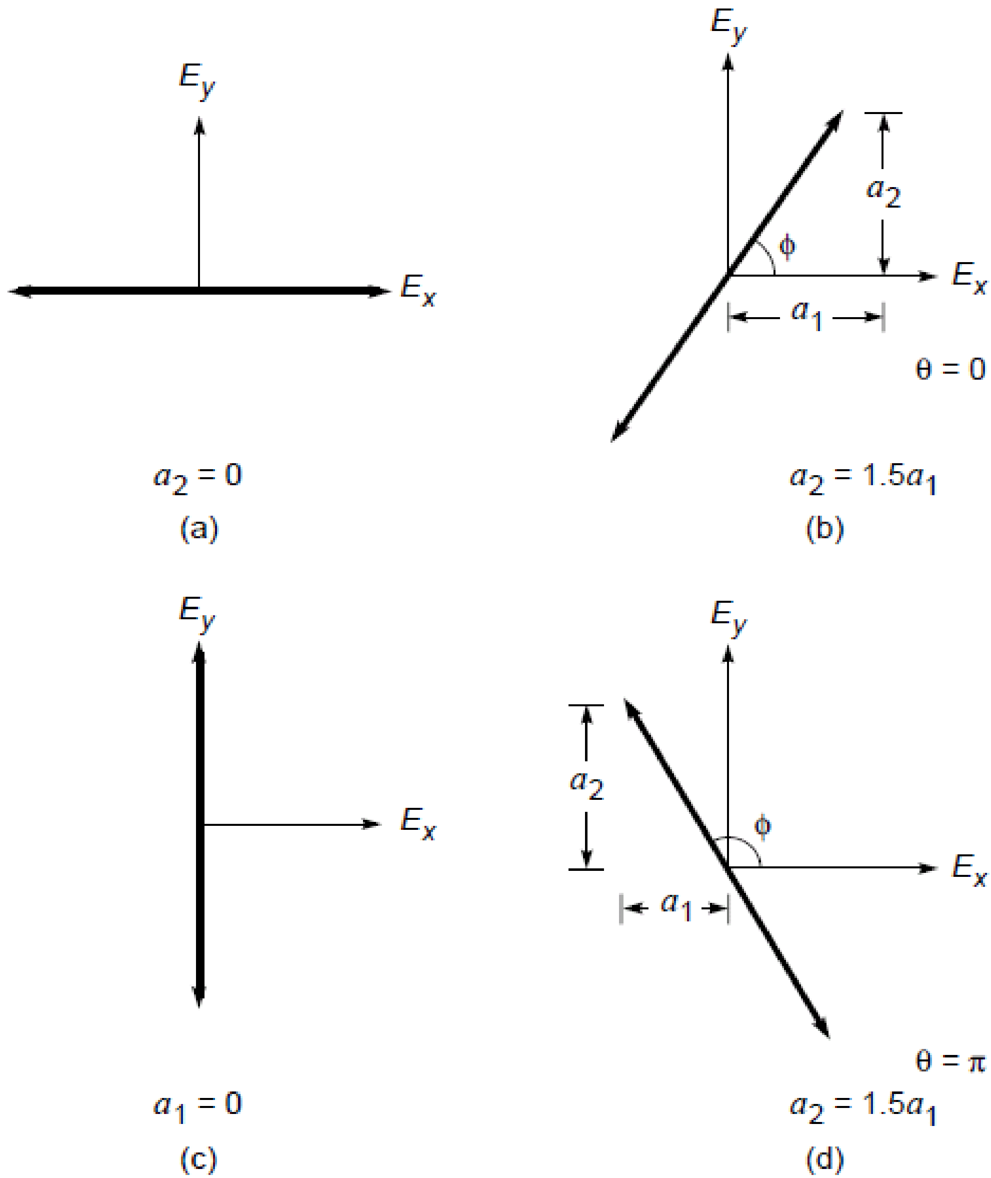
For $\theta \neq n\pi$ (n = 0, 1, 2, ….), resultant electric vector does not oscillate along a straight line.

Ex. Consider $\theta = \pi/2$ with $a_1 = a_2$. Thus,

$$E_x = a_1 \cos \omega t$$

$$E_y = a_1 \sin \omega t$$

- If we plot time variation of resultant electric vector we find that tip of electric vector rotates on circumference of a circle (of radius a_1) in counterclockwise direction, & propagation is in +z direction which is coming out of page. Such a wave is known as a right circularly polarized wave.



Superposition of two linearly polarized waves with their electric fields oscillating in phase. Resultant is again a linearly polarized wave with its electric vector oscillating in a direction making an angle Φ with x axis.

❖ **Tip of resultant electric vector should lie on circumference of a circle.**

$$E_x^2 + E_y^2 = a_1^2 \qquad (independent\ of\ t)$$

For $\theta = 3\pi/2$,

$$E_x = a_1 \cos\omega t$$

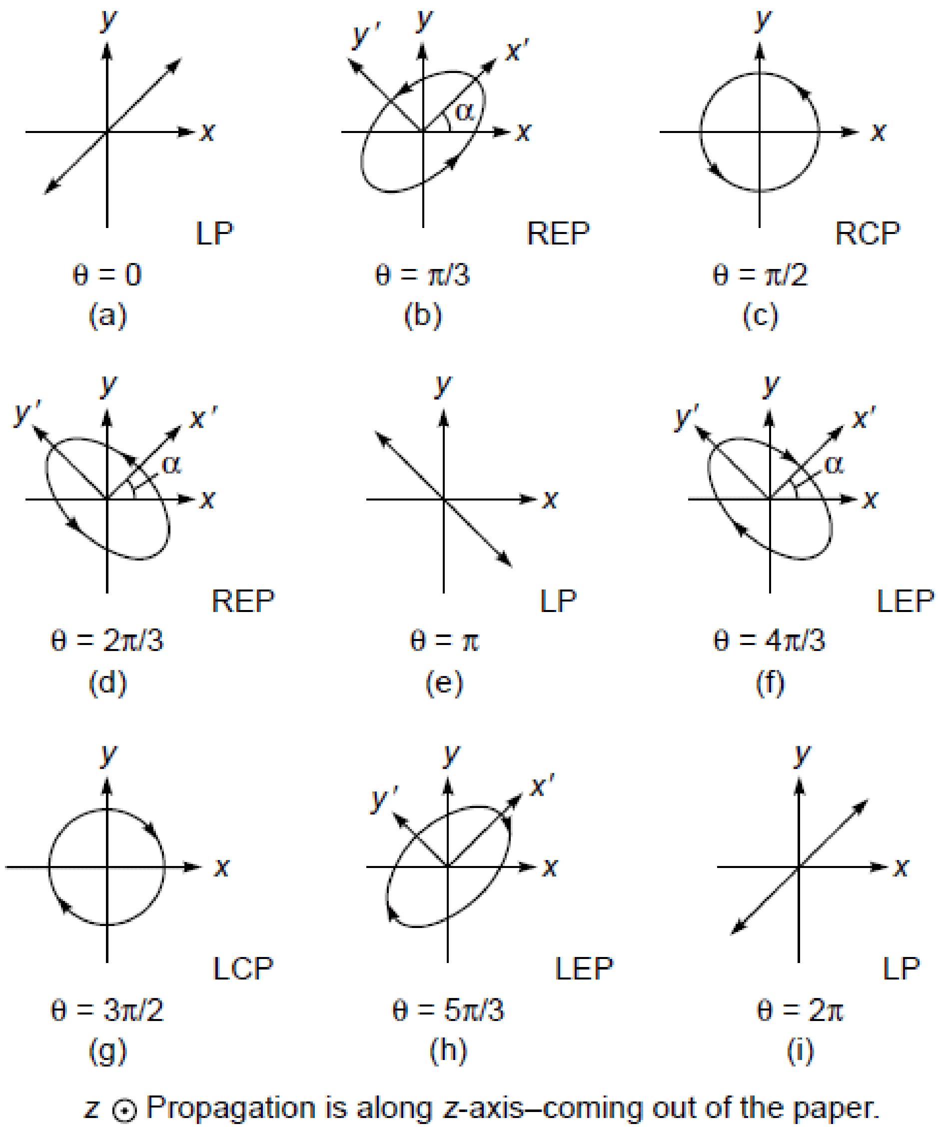
$$E_y = -a_1 \sin\omega t$$

which would also represent a circularly polarized wave; however, the electric vector will rotate in clockwise direction. Such a wave is known as a left circularly polarized wave.

❖ **For $\theta \neq m\pi/2$ (m = 0, 1, 2, …), the tip of electric vector rotates on circumference of an ellipse.**

❖ **This ellipse will degenerate into a straight line or a circle when θ becomes an even or an odd multiple of $\pi/2$.**

❖ **When $a_1 \neq a_2$, one obtains an elliptically polarized wave which degenerates into a straight line for $\theta = 0, \pi, 2\pi$, … etc.**



Propagation is out of the page.

States of polarization for various values of θ corresponding to $a_1 = a_2$. Ex. (c) & (g) correspond to right circularly & left circularly polarized light, respectively; similarly, (b) & (d) correspond to right elliptically polarized light, & (f) & (h) correspond to left elliptically polarized light.

Mathematical Analysis

$$E_x = a_1 \cos \omega t$$

$$E_y = a_2 \cos(\omega t - \theta)$$

❖ **Assume that major axis of ellipse is along x' or y' axes & that x' axis makes an angle *α* with *x* axis; i.e.,**

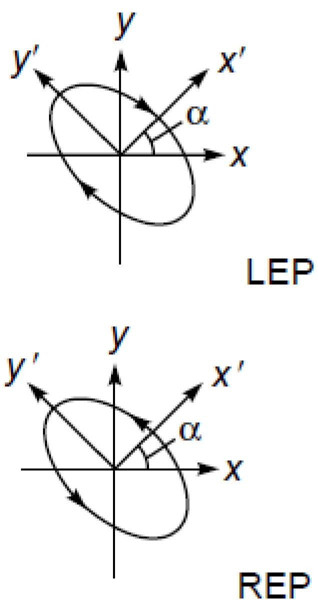
$$E'_x = E_1 \cos(\omega t - \phi)$$

$$\frac{E'_x}{E_1} = \cos(\omega t - \phi)$$

$$E'_y = E_2 \sin(\omega t - \phi)$$

$$\frac{E'_y}{E_2} = \sin(\omega t - \phi)$$

$$\Rightarrow \left(\frac{E'_x}{E_1}\right)^2 + \left(\frac{E'_y}{E_2}\right)^2 = 1$$



which represents equation of an ellipse.

- **For rotated coordinates,**

$$E_x = E'_x \cos\alpha - E'_y \sin\alpha$$
$$E_y = E'_x \sin\alpha - E'_y \cos\alpha$$

If we multiply 1st Eq. by *cosα* & 2nd Eq. by *sinα* & add,

$$E'_x = E_x \cos\alpha + E_y \sin\alpha$$

Similarly,

$$E'_y = -E_x \sin\alpha + E_y \cos\alpha$$

Substituting above Eqs., we get

$$E_1 \cos(\omega t - \phi) = a_1 \cos\omega t \cos\alpha + a_2 \cos(\omega t - \theta) \sin\alpha$$
$$E_2 \sin(\omega t - \phi) = -a_1 \cos\omega t \sin\alpha + a_2 \cos(\omega t - \theta) \cos\alpha$$

These Eqs. have to be valid at all times; thus we equate coefficients of *cosωt* & *sinωt* on both sides of Eq.

$$E_1 \cos\phi = a_1 \cos\alpha + a_2 \cos\theta \sin\alpha$$
$$E_1 \sin\phi = a_2 \sin\theta \sin\alpha$$

& $$-E_2 \sin\phi = -a_1 \sin\alpha + a_2 \cos\theta \cos\alpha$$
$$E_2 \cos\phi = a_2 \sin\theta \cos\alpha$$

If we square the four equations & add, we get

$$E_1^2 + E_2^2 = a_1^2 + a_2^2$$

Further,

$$\frac{E_2}{E_1} = \frac{a_2 \sin\theta \cos\alpha}{a_1 \cos\alpha + a_2 \cos\theta \sin\alpha} = \frac{a_1 \sin\alpha - a_2 \cos\theta \cos\alpha}{a_2 \sin\theta \sin\alpha}$$

$$\Rightarrow \quad a_2^2 \sin^2\theta \sin\alpha \cos\alpha = a_1^2 \sin\alpha \cos\alpha - a_2^2 \cos^2\theta \sin\alpha \cos\alpha$$
$$- a_1 a_2 \cos\theta (\cos^2\alpha - \sin^2\alpha)$$

With simple manipulations,

$$\tan 2\alpha = \frac{2a_1 a_2 \cos\theta}{a_1^2 - a_2^2}$$

Examples. For

$$a_1 = a_2 \qquad 2\alpha = \frac{\pi}{2} \quad \Rightarrow \quad \alpha = \frac{\pi}{4}$$

implying that major (or minor) axis of ellipse makes 45° with *x* axis. Further,

$$\frac{E_2}{E_1} = \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$$

Thus, for $a_1 = a_2$ & for

$$\theta = \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$$

$$\frac{E_2}{E_1} = +0.577, 1, 1.732, -1.732, -1, -0.577$$

which correspond to REP, RCP, REP, LEP, LCP, & LEP.

For $\theta = 4\pi/3$,

$$E'_x = E_1 \cos(\omega t - \phi)$$
$$E'_y = -1.732 E_1 \sin(\omega t - \phi)$$

Thus major axis of ellipse is along y' axis. To determine the state of polarization, we may choose $t = 0$ at the instant so that Φ may be assumed to be zero:

$$E'_x = E_1 \cos \omega t$$
$$E'_y = -1.732 E_1 \sin \omega t$$

Thus at

$$t = 0 \qquad E'_x = E_1 \qquad E'_y = 0$$

$$t = \frac{\pi}{2\omega} \qquad E'_x = 0 \qquad E'_y = -1.732 E_1$$

$$t = \frac{\pi}{\omega} \qquad E'_x = -E_1 \qquad E'_y = 0$$

etc., & the electric vector will rotate in clockwise direction.