PH 201

OPTICS & LASERS

Lecture_Lasers_4

Conditions for Producing a Laser

Population Inversion, Gain, & Gain saturation

Absorption & Gain on a Homogeneously Broadened Radiative Transition

- **Consider a beam of light having intensity per unit frequency *I*(*v*) at frequency *v* & frequency width *Δv*, passing through a medium of thickness *L* & cross-sectional area *A*.**

$$I = I(\upsilon)\Delta\upsilon$$

I_0 = Intensity *I* just before it enters the medium

Within medium we assume there are atoms occupying at least two specific energy levels (*u* & *l*). Radiative transitions from *u* to *l* can occur over a narrow range of frequencies within emission linewidth (center frequency v_0).

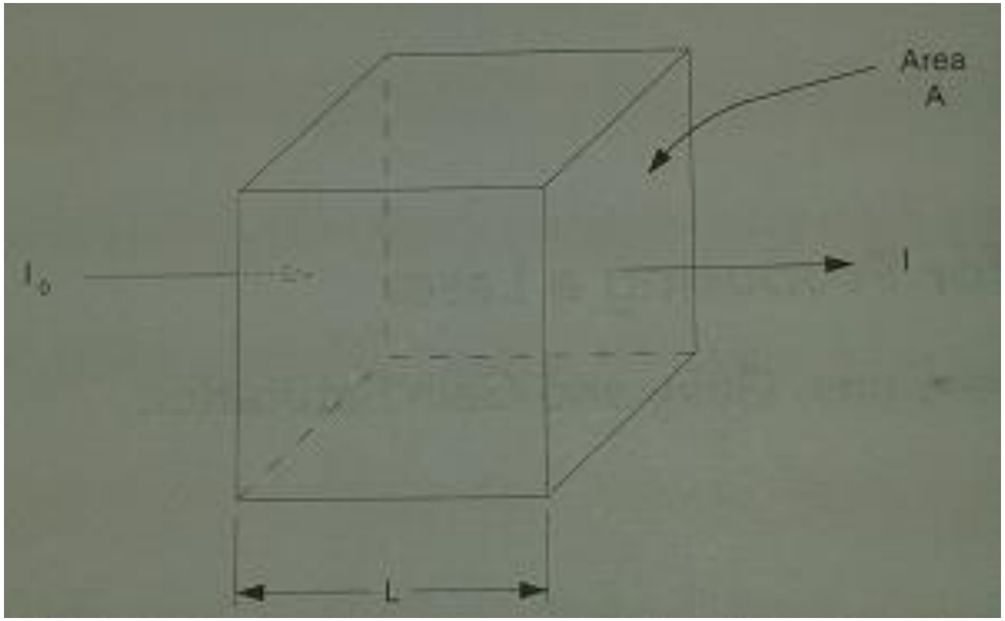
We also assume that frequency of incident beam falls somewhere within that linewidth.

$$\Delta E_{ul} = E_u - E_l = h\upsilon_{ul}$$

Determine the effect of medium on beam as beam passes through it.

How *I* is changed after it has passed through medium?

Assume that dominant broadening of transition under consideration is homogeneous, with a Lorentzian lineshape.



Light of intensity I_0 incident upon an absorptive material of length *L* & area *A*.

***N*$_u$ = Total no. of atoms per unit volume in upper level**

***N*$_l$ = Total no. of atoms per unit volume in lower level**

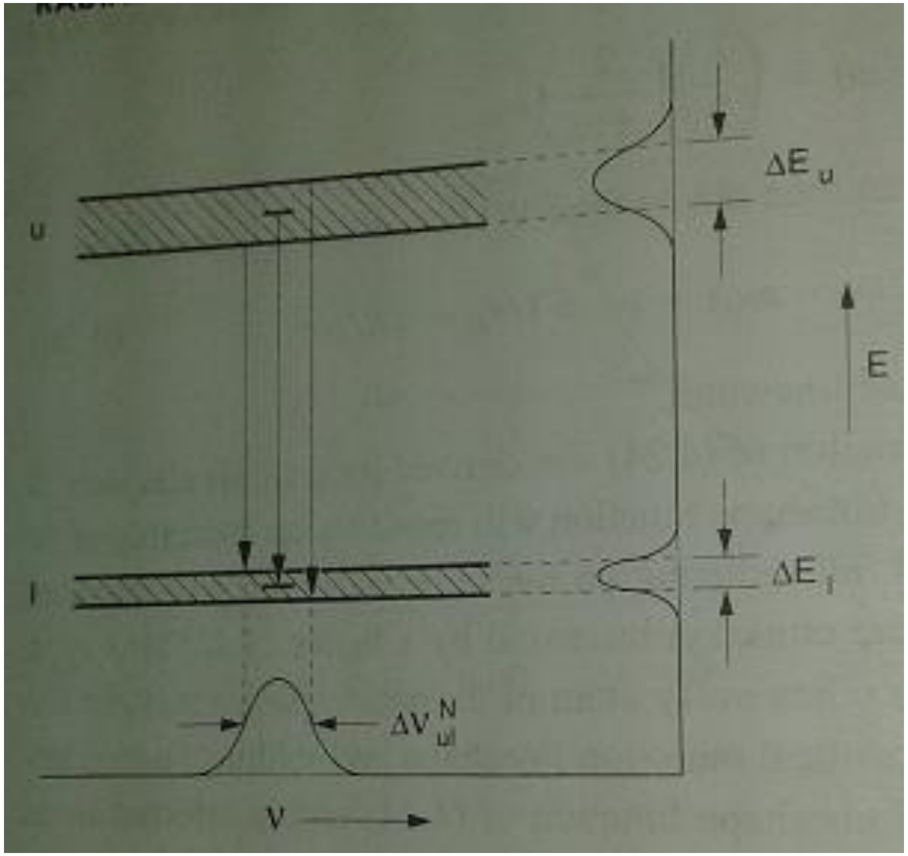
***A*$_{ul}$ = Radiative transition probability for transitions occurring between energy levels *u* & *l*.**

$$A_{ul}(\upsilon) = \frac{\chi_{ul}^{T}/4\pi^{2}}{(\upsilon-\upsilon_{0})^{2}+(\chi_{ul}^{T}/4\pi)^{2}}A_{ul}$$

χ_{ul}^{T} **= Total decay rate of upper & lower levels (minimum emission linewidth)**

$$\chi_{ul}^{T} = \chi_{u} + \chi_{l}$$

$$\chi_{ul}^{T} = 2\pi\Delta\upsilon_{ul}^{N} = \sum_{i} A_{ui} + \sum_{j} A_{lj}$$



Quantum mechanical description of natural linewidth of emission resulting from a radiating transition between two levels (*u* & *l*).

Stimulated emissions are proportional to energy density *u*(*v*) of beam at frequency *v*, as well as Einstein *B* coefficients.

No. of stimulated transitions (upward or downward) occurring between two levels per unit volume per unit time,

$$N_l B_{lu}(\upsilon)\Delta\upsilon . u(\upsilon) = N_l B_{lu}(\upsilon) I(\upsilon)\Delta\upsilon\eta / C$$

$$= N_l B_{lu}(\upsilon) I\eta / C \qquad \textbf{(upward)}$$

&

$$N_u B_{ul}(\upsilon)\Delta\upsilon . u(\upsilon) = N_u B_{ul}(\upsilon) I(\upsilon)\Delta\upsilon\eta / C$$

$$= N_u B_{ul}(\upsilon) I\eta / C \qquad \textbf{(downward)}$$

Using relation

$$u(\upsilon)\Delta\upsilon = I(\upsilon)\Delta\upsilon\eta / C = I\eta / C$$

It is also assumed that *B* coefficients per unit frequency $B_{ul}(v)$ & $B_{lu}(v)$ have same frequency dependence as $A_{ul}(v)$.

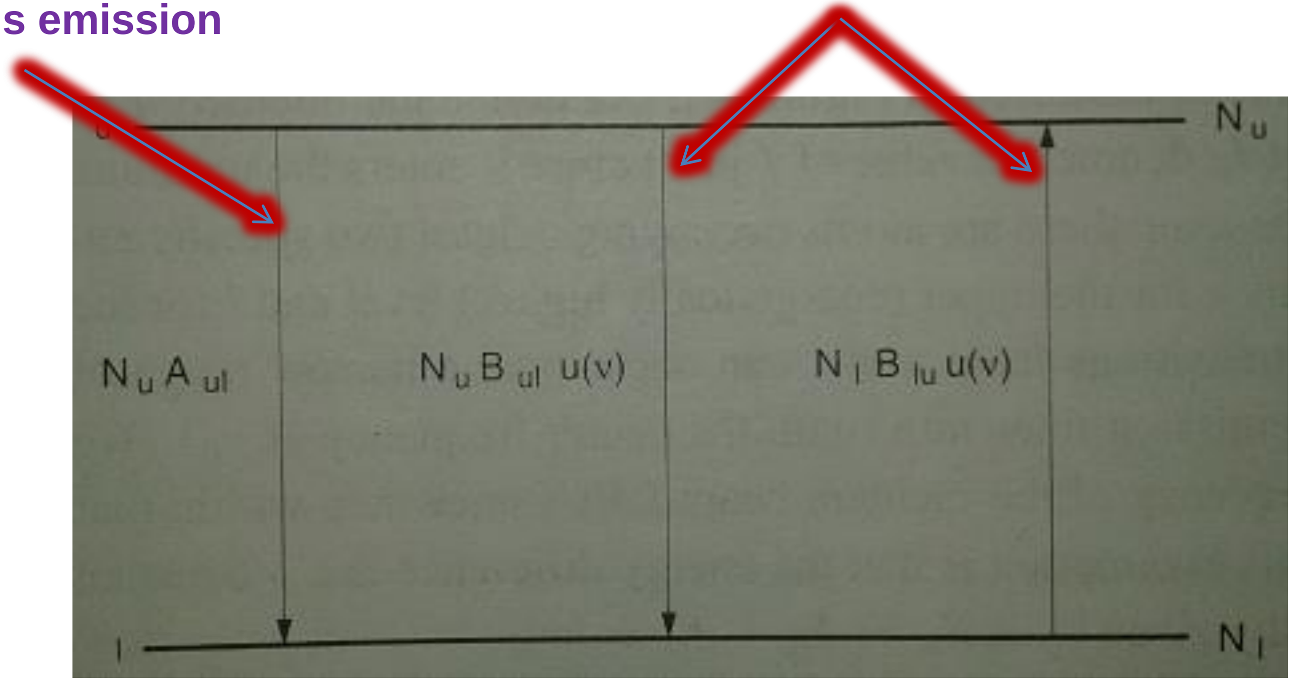
Consider three possible radiative interactions between two levels *u* & *l*.

Spontaneous emission downward from *u* to *l* at a spontaneous emission rate = A_{ul}

No. of transitions downward per unit volume per unit time at frequency *v* for transition form *u* to *l* = $N_u A_{ul}$

Stimulated emission

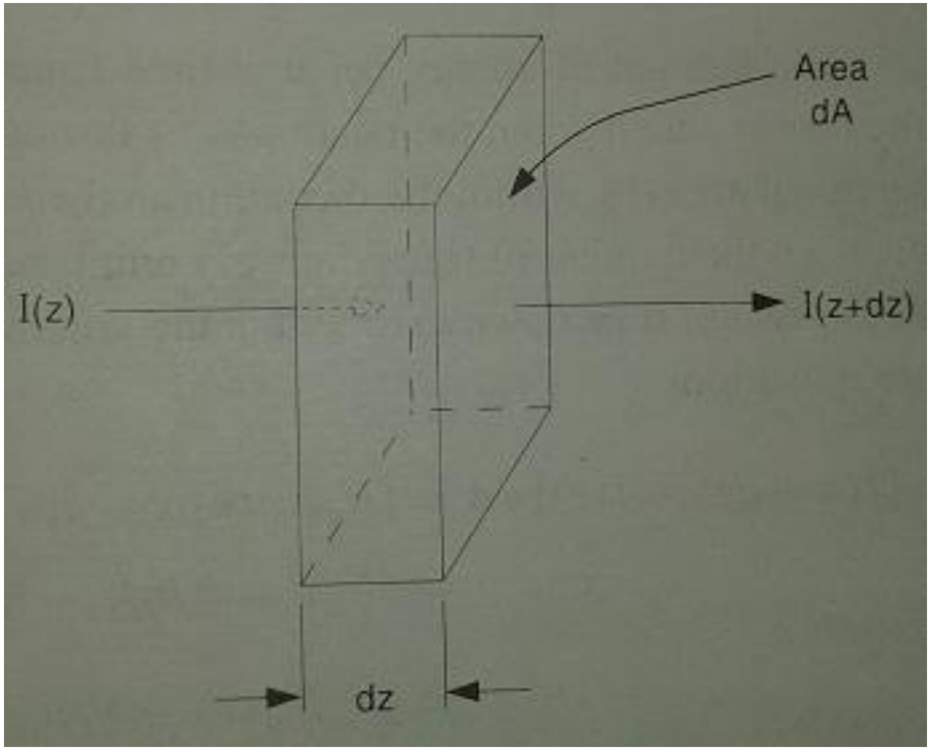
Spontaneous emission



Energy levels of a two-level system, depicting the population flux transferred between two levels via spontaneous emission, stimulated emission, & absorption.

Consider a small length *dz* of medium. In this case, we will estimate energy due to stimulated emissions.

Spontaneous emission contribution is neglected here because such emission is radiated in all directions into a solid angle 4π & thus contributes very little in direction of incident beam.



Light incident upon an incremental length *dz* of absorbing material.

Amount of energy per unit time added when beam passes through a region of length *dz* & cross-sectional area *dA* within medium can therefore be expressed as difference between no. of transitions per unit time upward & no. of transitions per unit time downward within volume, multiplied by photon energy per transition.

$$[I(z+dz)-I(z)dA=[N_u B_{ul}(\upsilon)\Delta\upsilon.u(\upsilon)-N_l B_{lu}(\upsilon)\Delta\upsilon.u(\upsilon)]h\upsilon dAdz$$

$$=[N_u B_{ul}(\upsilon)I\eta/c-N_l B_{lu}(\upsilon)I\eta/c]h\upsilon dAdz$$

$$=[N_u B_{ul}(\upsilon)-N_l B_{lu}(\upsilon)]\frac{h\upsilon I\eta}{c}dAdz$$

Energy is added to or subtracted from beam in discrete amounts *hv* as a result of the two terms within bracket.

Factor involving N_u leads to stimulated emission. Consequently, another photon is added to beam and N_u is decreased by 1 every time beam stimulates an electron from level *u* to *l*.

Hence, population within medium is transferred from level *u* to *l.* Factor containing N_l involves absorption of photons from beam with population moving from level *l* to *u*.

$$[I(z+dz)-I(z)dA=[N_u B_{ul}(\upsilon)-N_l B_{lu}(\upsilon)]\frac{h\upsilon I\eta}{c}dAdz$$

Using following relation, $dI = I(z+dz)-I(z)$

Dividing through by *dA* & *dz*,

$$\frac{dI}{dz}=[N_u B_{ul}(\upsilon)-N_l B_{lu}(\upsilon)]\frac{h\upsilon\eta}{c}I$$

It is a differential Eqn of form, $\frac{dI}{dz}=CI$

Solution of such Eqn, $I = I_0 e^{Cz}$

$$I = I_0 e^{g^H(\upsilon)z}$$

$$g^H(\upsilon)=[N_u B_{ul}(\upsilon)-N_l B_{lu}(\upsilon)]\frac{h\upsilon\eta}{c}$$

H – homogeneous broadening

Gain coefficient (dimension: 1/length)

Gain Coefficient & Stimulated Emission Cross-section for Homogeneous Broadening

$$g^{H}(\upsilon) = [N_u B_{ul}(\upsilon) - N_l B_{lu}(\upsilon)]\frac{h\upsilon\eta}{c}$$

Using following relationship of Einstein *A* & *B* coefficients,

$$\frac{A_{ul}(\upsilon)}{B_{ul}(\upsilon)} = \frac{8\pi h\eta^3\upsilon^3}{c^3} \equiv \frac{A_{ul}}{B_{ul}}$$

$$\frac{g_l B_{lu}(\upsilon)}{g_u B_{ul}(\upsilon)} = 1 \qquad or \qquad g_l B_{lu}(\upsilon) = g_u B_{ul}(\upsilon)$$

Gain coefficient can be expressed as,

$$g^{H}(\upsilon) = \left[N_u - \frac{g_u}{g_l}N_l\right]\frac{c^2}{8\pi\eta^2\upsilon^2}A_{ul}(\upsilon)$$

$$A_{ul}(\upsilon) = \frac{\chi_{ul}^{T}/4\pi^2}{(\upsilon-\upsilon_0)^2 + (\chi_{ul}^{T}/4\pi)^2}A_{ul}$$

Gain coefficient,

$$g^{H}(\upsilon)=\left[N_u-\frac{g_u}{g_l}N_l\right]\frac{c^2}{8\pi\eta^2\upsilon^2}\left[\frac{\chi_{ul}^{T}/4\pi^2}{(\upsilon-\upsilon_0)^2+(\chi_{ul}^{T}/4\pi)^2}\right]A_{ul}$$

Let us define,

$$\Delta N_{ul}=\left[N_u-\frac{g_u}{g_l}N_l\right]$$

$$\sigma_{ul}^{H}(\upsilon)=\frac{c^2}{8\pi\eta^2\upsilon^2}A_{ul}(\upsilon)=\frac{c^2}{8\pi\eta^2\upsilon^2}\left[\frac{\chi_{ul}^{T}/4\pi^2}{(\upsilon-\upsilon_0)^2+(\chi_{ul}^{T}/4\pi)^2}\right]A_{ul}$$

This term is defined as stimulated emission cross-section & it has dimension of length^2 or area.

$$\boxed{g^{H}(\upsilon)=\sigma_{ul}^{H}(\upsilon)\Delta N_{ul}}$$

Since gain coefficient is highest at center of emission line frequency ($v = v_0$).

$$g^H(\upsilon = \upsilon_0) \equiv g^H(\upsilon_0) \equiv g_0^H$$

$$g_0^H = \left[N_u - \frac{g_u}{g_l} N_l \right] \frac{c^2}{2\pi\eta^2 \upsilon_0^2 \chi_{ul}} A_{ul}$$

Stimulated emission cross-section at center frequency ($v = v_0$)

$$\sigma_{ul}^H(\upsilon_0) = \frac{c^2 A_{ul}}{2\pi\eta^2 \upsilon_0^2 \chi_{ul}} = \frac{\lambda_{ul}^2 A_{ul}}{4\pi^2 \eta^2 \Delta\upsilon_{ul}^H}$$

$$\chi_{ul} = 2\pi\Delta\upsilon_{ul}^H$$

Relationship for intensity at a specific distance *z* into medium for homogeneous broadening:

$$I = I_0 e^{g^H(\upsilon) z} = I_0 e^{\sigma_{ul}^H(\upsilon)[N_u - (g_u/g_l)N_l]z}$$

$$I = I_0 e^{\sigma_{ul}^H(\upsilon)\Delta N_{ul} z}$$

Since $g^H(\upsilon)$, $\sigma_{ul}^H(\upsilon)$**, & *z* are positive quantities, if** ΔN_{ul} **is positive**

Then beam intensity will increase exponentially with distance *z* & so provide amplification;
conversely, if ΔN_{ul} **is negative then intensity of beam will decrease leading to absorption.**

How amplification occurs in a medium with energy levels *u* & *l* & population densities N_u **&** N_l **in those levels?**

Population inversion

$$I = I_0 e^{\sigma_{ul}[N_u - (g_u/g_l)N_l]z}$$

I_0 = Intensity of beam when it enters into medium
I = Intensity at some distance z into medium

If exponent is positive, intensity of beam will increase – amplification will occur.

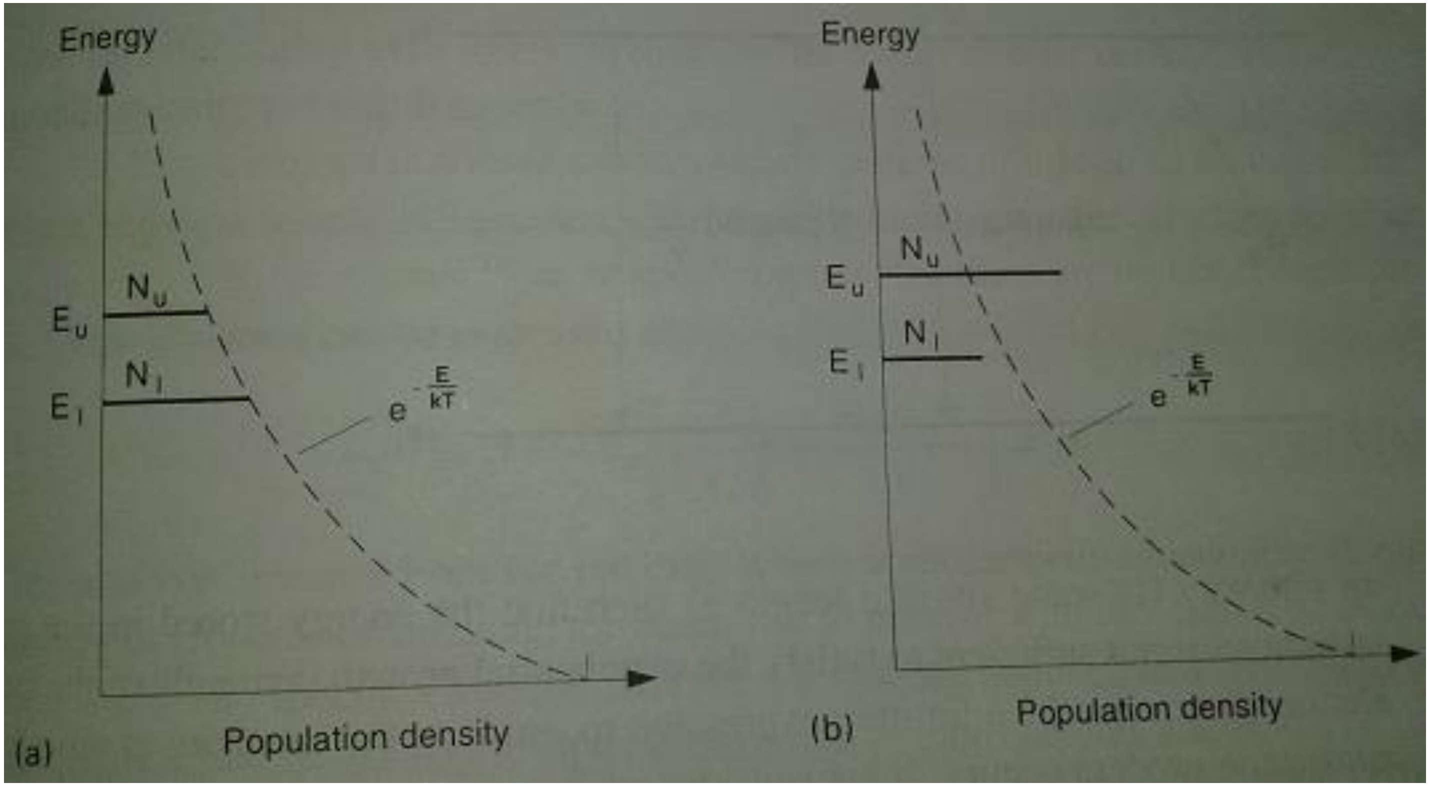
If exponent is negative, intensity of beam will decrease – absorption will occur.

Since values of σ_{ul} & z are always positive, the amplification will occur only if

$$N_u > \frac{g_u}{g_l} N_l$$

$$\frac{g_l N_u}{g_u N_l} > 1$$

The case of upper level being more populated than lower level is referred to as a population inversion. This relationship is not normal under conditions of thermal equilibrium.



Population distribution between levels *u* & *l* for conditions of (a) thermal equilibrium & (b) a population inversion.

A population inversion is a *necessary* condition for amplification or laser action to occur, but it is not a *sufficient* condition.

- **Population inversion (necessary condition)**
- **Saturation intensity (sufficient condition)**