

Indian Institute of Technology Patna
MA-201: B.Tech. II year (Date of Exam: 15.09.2015)

Maximum Marks: 30

Total Time: 2 Hours

Note: Answer all **Fourteen** questions.

1. By one example, show that $\text{Arg}(z_1 z_2) \neq \text{Arg}(z_1) + \text{Arg}(z_2)$. [1]
2. Find the roots of the equation $z^2 + 2z + (1 - i) = 0$. Write the answer in $x + iy$ form. [2]
3. Find $\lim_{z \rightarrow 0} \frac{xy^3}{x^2 + 2y^6} + i \frac{x^3 y}{5x^6 + y^2}$. [2]
4. Define the harmonic conjugate of any function $u(x, y)$. Determine it if the function $u(x, y) = 2x - x^3 + 3xy^2$. [1 + 2]
5. Let the function $f(z) = u(x, y) + iv(x, y)$ be analytic in some complex domain. Show that the functions $U(x, y) = e^{u(x, y)} \cos v(x, y)$, $V(x, y) = e^{u(x, y)} \sin v(x, y)$ are harmonic there. [2]
6. With proper argument show that the function $f(z) = 2z^2 - 3 - ze^z + e^{-z}$ is entire. [2]
7. Find all values of z such that $e^{2z-1} = 1$. [1]
8. Show that $-i \sinh iz = \sin z$ and $\cosh iz = \cos z$. Determine $|\sin z|^2$ in terms of real sine and real sine hyperbolic functions and show that $\sin z$, $z \in \mathbb{C}$ is an unbounded function of $z = (x, y)$ on the complex plane \mathbb{C} . [3]
9. Find all the roots of the equation $\sin z = 2$. [1+2]
10. Determine the value of the contour integral $\int_C \frac{1}{z^2(z^2+1)} dz$ where C is given as $|z - i| = \frac{3}{2}$. [3]
11. Find an upper bound for the absolute of the integral $\left| \int_C \frac{z+4}{(z^3-1)} dz \right|$ where C is the arc of circle $|z| = 0.5$ from $z = 0.5$ to $z = 0.5i$ lying in first quadrant. [2]
12. Let $f(t) = u(t) + iv(t)$ be a complex valued function continuous on the interval $a \leq t \leq b$ then show that $\int_{-b}^{-a} f(-t) dt = \int_a^b f(\tau) d\tau$. [1]
13. Let $f(z) = u(x, y) + iv(x, y)$ be some complex function. Prove: if $f'(z_0)$ exists then CR equations are satisfied at $z_0 = (x_0, y_0)$. Does converse hold? [2+1]
14. Let $f(z) = u(x, y) + iv(x, y)$ be analytic and real valued function in a domain D . Prove that $f(z)$ is constant throughout D . [2]