Discrete Mathematics CS-206 Mid Semester

Full Marks: 40 Time: 2 hours Date of Test: 25/09/2018

Q1. A number is said to be prime-looking if it is composite but not divisible by 2, 3 or 5. The three smallest prime-looking numbers are 49, 77 and 91. There are 168 prime numbers less than 1000. How many prime looking numbers are there less than 1000? [**Hint:** Use inclusion-exclusion principle] [**4 marks**]

Q2.) Choose the correct option for the following set of questions:
[1×6=6 Marks]

a.) What is the logical translation of the following statement: "None of my friends are perfect". F(x)=x is my friend. P(x)=x is perfect

i. $\exists x(F(x) \wedge \neg P(x))$
ii. $\exists x(\neg F(x) \wedge P(x))$
iii. $\exists x(\neg F(x) \wedge \neg P(x))$
iv. $\neg \exists x(F(x) \wedge P(x))$

b.) Which of the following is not equivalent to $\neg \exists x(\forall y(\alpha) \wedge \forall z(\beta))$?

i. $\forall x(\exists z(\neg \beta) \rightarrow \forall y(\alpha))$
ii. $\forall x(\forall z(\beta) \rightarrow \exists y(\neg \alpha))$
iii. $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg \beta))$
iv. $\forall x(\exists y(\neg \alpha) \rightarrow \exists z(\neg \beta))$

c.) What is the correct translation of the following statement into mathematical logic? "Some real numbers are rational"

i. $\exists x(real(x) \vee rational(x))$
ii. $\forall x(real(x) \rightarrow rational(x))$
iii. $\exists x(real(x) \wedge rational(x))$
iv. $\exists x(rational(x) \rightarrow real(x))$

d.) Which one of the following is the most appropriate logical formula to represent the statement? "Gold and silver ornaments are precious". The following notations are used: G(x): x is a gold ornament, S(x): x is a silver ornament P(x): x is precious.

i. $\forall x(P(x) \rightarrow (G(x) \wedge S(x)))$
ii. $\forall x(G(x) \wedge S(x) \rightarrow P(x))$
iii. $\exists x((G(x) \wedge S(x)) \rightarrow P(x)$
iv. $\forall x((G(x) \vee S(x)) \rightarrow P(x))$

e.) "If my computations are correct and I pay the electric bill, then I will run out of money. If I don't pay the electric bill, the power will be turned off. Therefore, if I don't run out of money and the power is still on, then my computations are incorrect." Convert this argument into logical notations using the variables c, b, r, p for propositions of computations, electric bills, out of money and the power respectively. (Where $\neg$ means NOT)

i. *if* $(c \wedge b) \rightarrow r$ *and* $\neg b \rightarrow p$, *then* $(\neg r \wedge p) \rightarrow \neg c$
ii. *if* $(c \vee b) \rightarrow r$ *and* $\neg b \rightarrow \neg p$, *then* $(r \wedge p) \rightarrow c$
iii. *if* $(c \wedge b) \rightarrow r$ *and* $\neg p \rightarrow b$, *then* $(\neg r \vee p) \rightarrow \neg c$
iv. *if* $(c \vee b) \rightarrow r$ *and* $\neg b \rightarrow \neg p$, *then* $(\neg r \wedge p) \rightarrow \neg c$

f.) Choose the correct logical formula for : "Some boys in the class are taller than all the girls".

i. $\exists x[boy(x) \rightarrow \forall y[girl(y) \wedge taller(x,y)]]$
ii. $\exists x[boy(x) \wedge \forall y[girl(y) \wedge taller(x,y)]]$
iii. $\exists x[boy(x) \rightarrow \forall y[girl(y) \rightarrow taller(x,y)]]$
iv. $\exists x[boy(x) \wedge \forall y[girl(y) \rightarrow taller(x,y)]]$

Q3. Suppose that 10 integers 1, 2, 3, …, 10 are randomly positioned around a circular wheel. Prove it using method of contradiction that the sum of some set of 3 consecutively positioned numbers is at least 17. [4 **marks**]

Q4. Let a_n be the sequence defined by $a_1 = 1$, $a_2 = 8$ and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$, Prove that

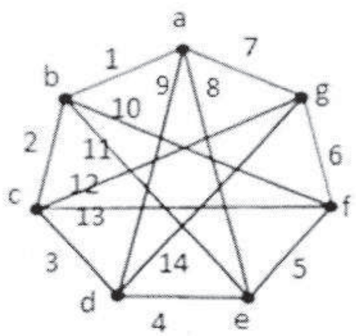
$a_n = 3.2^{n-1} + 2(-1)^n$ for all $n \in N$ **[4 marks]**

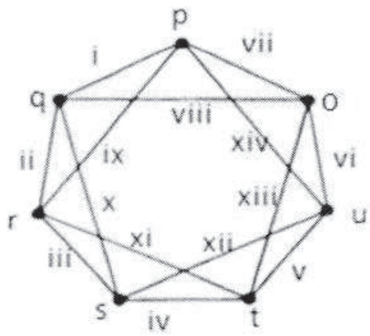
Q5. Prove the following inequality using Mathematical Induction

$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n}$, where n is greater than 1. [**5 marks**]

Q6. Consider an undirected graph G where self-loops are not allowed. The vertex set of G is {(i, j): 1 <= i <= 12, 1 <= j <= 12}. There is an edge between (a, b) and (c, d) if |a − c| <= 1 and |b − d| <= 1. The number of edges in this graph is __________. **[4 marks]**

Q7. Check whether the following two Graphs G1 and G2 are Isomorphic? Explain your answer by mapping functions and draw the corresponding adjacency matrices: **[4 marks]**





Q8. Prove that a simple graph with n vertices and k connected components has at most $\frac{(n-k)(n-k+1)}{2}$ edges. **[5 Marks]**

Q9. Find the shortest distance from A to J on the network below using Dijkstra's Algorithm **[4 Marks]**