Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2014-15
Mid Semester Examination

Maximum Marks: 30 — Total Time: 2 Hours

Note: This question paper has two pages and contains eight questions. Answer all questions.

1. Suppose that A, B and C are events such that probabilities, $P(A) = 0.55, P(B) = 0.6, P(C) = 0.45, P(A \cap B) = 0.25, P(A \cap C) = 0.2, P(B^c \cap C) = 0.15$ and $P(A \cap B \cap C) = 0.10$ are known. Then using the laws of probability determine $P(A^c \cap B \cap C^c)$ and $P(A \cup B^c \cup C)$. **[1 + 1]**

2. A fair coin is tossed three times. Let A denotes the event at least one of the first two tosses is a head, B denotes the event same result on tosses 1 and 3, C denotes the event no heads, D denotes the event same result on tosses 1 and 2. Among these events which pairs are independent? Which pairs are mutually exclusive? **[3]**

3. At IIT Patna suppose 2000 students are enrolled in a course. The grades are given by a team of teaching instructors. However a sample of the papers are examined for grading consistency by a professor. It is known that 1% of all papers are improperly graded. The professor selects 10 papers at random from the 2000 submitted and examines them for grading inconsistencies. Let X denotes a hypergeometric random variable the number of papers in the sample that are improperly graded. Write probability mass function of X. Determine the probability that at most one student is improperly graded. Use definition to find the expected number of students who are improperly graded. Compute the above probability using binomial approximation. **[1 + 2 + 2 + 1]**

4. An immunologist is studying blood disorders exhibited by people with rare blood types. It is estimated that 10% of a target population has the type of blood being investigated. People whose blood type is unknown are tested until 100 people with the desired blood type are found. Let X denotes the random variable the number of people tested who do not have the desired rare blood type. Write probability distribution of X. Then use the definition to compute the moment generating function of X. Compute variance of X using the definition. **[1 + 2 + 2]**

5. (i) State and prove Markov inequality. Explain why it is useful for computing tail probabilities? **[2 + 1]**
   (ii) The average amount of time that it takes to change a tire at a certain garage is 10 minutes. Use Markov inequality to give an estimate for the probability that it will take more than 25 minutes to change the next tire. **[1]**

6. Clearly mentioning all the steps, show that mean, median and mode of a normal $N(1.5, 2)$ distribution are the same. What is that value. **[3+0.5]**

(55)

7. IIT system has observed that 20% of its incoming freshmen are unqualified and drop out within first 6 months. To better predict a student success the institute has decided to administer a test to all freshmen when they first enroll. It is observed that 85% of qualified students pass the test and 80% of unqualified students fail the test. Let Q represent the event that a student is qualified and P represent the event a student passes the test. Determine the probabilities $P(Q^c \mid P)$ and $P(Q \mid P^c)$. Do you think this is a good test? Explain.

[1 + 1 + 1]

8. (*i*) Write probability density function of a $Cauchy(4, 9)$ distribution. Determine its CDF.

(*ii*) Show that **[0.5 + 1]**

$$f_X(x;\theta) = \begin{cases} \theta^2\, x\, e^{-\theta x}, & x > 0, \\ 0, & elsewhere, \end{cases}$$

is proper probability density function where $\theta > 0$ is a parameter of the distribution? Compute the probability $P(X \geq 1)$.

[1 + 1]