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CS303 Tutorial 7

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Ans 1:

Given: L_1 and L_2 are 2 context free languages

To prove: $L_1 \cap L_2$ need not be context free.

Proof by counter example:

Let $L_1 = \{a^n b^n c^m\}$ and $L_2 = \{a^m b^n c^n\}$ $m, n \geq 0$

$L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 0\}$

Grammar of $L_1 \cap L_2$ is

$S \rightarrow aSBC \mid aBC \mid \epsilon$

$CB \rightarrow BC$

$aB \rightarrow ab$

$bB \rightarrow bb$

$bC \rightarrow bc$

$cC \rightarrow cc$

The LHS for atleast one production is not single terminal

Hence $L_1 \cap L_2$ is not a CFL.

Hence proved.

Ans 2:

$$L_1 = \{a^n b^n c^m \mid m, n \geq 0\} \quad L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$$

$$\therefore L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 0\}$$

Grammar of $L_1 \cap L_2$ is

$$S \rightarrow aSBC \mid aBC \mid \epsilon$$

$$CB \rightarrow BC$$

$$aB \rightarrow ab$$

$$bB \rightarrow bb$$

$$bC \rightarrow bc$$

$$cC \rightarrow cc$$

\because the LHS of at least one production is not single non-terminal, this is not context free grammar.

This is a context sensitive grammar.

Ans 3:

CFL is not closed under complementation

Example:

$$L_1 = \{a^n b^n c^m \mid m, n \geq 0\}, L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$$

are context free languages.

If CFLs are closed under complementation,

\bar{L}_1 and \bar{L}_2 are also CFLs

Then $\bar{L}_1 \cup \bar{L}_2$ would also be a CFL because CFLs are closed under union.

Hence, $\overline{\bar{L}_1 \cup \bar{L}_2} = L_1 \cap L_2$ must also be CFL by our assumption.

But $L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 0\}$ is not CFL

Hence the assumption is wrong and CFLs are not closed under complementation.