

Indian Institute of Technology Patna  
Mathematics - I (MA101)  
B. Tech 1st Year (Autumn) 2017 - 18  
Mid Semester Examination

Max Time 2 Hours

Maximum Marks 30

Weightage 30%

Name and Roll Number: \_\_\_\_\_

**Note**

- (a) This question paper consists of a total 8 questions. All are compulsory.  
(b) Write your name and Roll Number on the question paper.  
(c) Notations have their usual meaning.

1. Prove that given  $x, y \in \mathbb{R}$ ,  $x > 0$ , there is a positive integer  $n$  such that [3.5M]

$$nx > y.$$

2. Let  $S$  be a bounded nonempty subset of  $\mathbb{R}$  and define  $aS = \{as : s \in S\}$  where  $0 < a \in \mathbb{R}$ . Then prove that [4M]

$$\inf(aS) = a \inf(S).$$

3. Use  $\epsilon - \delta$  definition to prove that the function  $f(x) = \frac{1}{x^2}$  is continuous on  $(0, \infty)$ . Does  $\delta$  depend on  $\epsilon$  only? [3.5M]

4. Discuss the following and justify each step. Use  $\epsilon - \delta$  definition.

(a) Is  $f(x) = ax + b$ ,  $a, b \in \mathbb{R}$  uniformly continuous on  $\mathbb{R}$ ? [2M]

(b) Is  $f(x) = \sin x \cos x$  uniformly continuous on  $\mathbb{R}$ ? [2M]

5. Discuss the convergence of the following sequences

(a)  $\left\{ \frac{\left(\frac{10}{11}\right)^n}{\left(\frac{9}{10}\right)^n + \left(\frac{11}{12}\right)^n} \right\}_{n=1}^{\infty}$ . [1.5M]

(b)  $a_1 = \sqrt{2}$ ,  $a_{n+1} = \sqrt{2a_n}$ ,  $n \in \mathbb{N}$ . [3M]

6. Find sup and inf. Justify.

(a)  $\sup \{x \in \mathbb{R} : x^2 + x + 1 > 0\}$ , [1M]

(b)  $\inf \{z = x + x^{-1} : x > 0\}$ , [1M]

(c)  $\inf \{z = 2^x + 2^{1/x} : x > 0\}$ . [1M]

7. Discuss the convergence and find out center, and interval of convergence of this power series [4M]

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3^{2n} (x-2)^n}{3n}.$$

8. Use Dirichlet Test to find the convergence of  $\sum_{n=1}^{\infty} \frac{\sin n}{n} \left(1 + \frac{1}{2} + \cdots + \frac{1}{n}\right)$ . [3.5M]