Indian Institute of Technology Patna
MA102: Mathematics II
B. Tech. I Year End Semester Examination April- 2015

Time: 3 hours **Total Marks: 50**

Note: There are **Five** questions. Attempt all the questions. Give precise and brief answer. Notations have their usual meaning. Standard formulae may be used.

Q1. a. A square matrix A is said to be idempotent if $A^2 = A$

(i). If A is idempotent matrix of order n, show that $I_n - A$ is also idempotent.

(ii). If $AB = B$ and $BA = A$. Show that A and B both are idempotent. [6]

b. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the linear mapping for which $T(1,2) = (2,3)$ and $T(0,1) = (1,4)$, where $\{(1,2),(0,1)\}$ is a basis of $\mathbb{R}^2$. Find a formula for T, i.e., find $T(a,b)$. [2]

Q2. a. Solve the differential equations
$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0.$ [3]

b. Solve the differential equation
$x^2(x\,dx + y\,dy) + y(x\,dy - y\,dx) = 0$ [4]

c. Find the Integrating Factor (I.F.) of the following differential equation
$3y\,dx - 2x\,dy + x^2y^{-1}(10y\,dx - 6x\,dy) = 0$ [3]

Q3. a. Use Undetermined Coefficients Method to solve $\frac{d^2y}{dx^2} + 4y = x^2 \sin 2x$ [4]

b. Use D-Operator Method to solve $(D^2 + 4)y = x^3 + \sin 2x$ [5]

c. Use Variation of Parameter Method to solve $\frac{d^2y}{dx^2} + a^2y = \tan ax$ [5]

Q4. a. Find the set of ordinary points, regular singular points and irregular singular points of the following differential equation
$x^2(x-2)^2y'' + (x-2)y' + 3x^2y = 0$ [3]

b. Find a **Power Series Solution** of the differential equation
$2x^2y'' - xy' + (x+1)y = 0$, about the point $x_0 = 0$ [6]

(100)

Q5. a. If J_n is Bessel's function of first kind then prove that
$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ [2]

b. If $P_n(x)$ denotes Legendre's polynomial then prove that
$P_n(-x) = (-1)^n P_n(x)$ [2]

c. Obtain the approximate solution of the following problem by using Picard's method :
$y' = 1 + ty \; ; \; y(0) = 1$ [2]

d. Solve : $(5+2x)^2 \frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 8(5+2x)^2$ [3]
