# Discrete Mathematics CS-206 Mid Semester

## Full Marks: 40 Time: 2 hours Date of Test: 18/09/2017

**Q1.**A number is said to be prime-looking if it is composite but not divisible by 2, 3 or 5. The three smallest prime-looking numbers are 49, 77 and 91. There are 168 prime numbers less than 1000. How many prime looking numbers are there less than 1000? [**Hint:** Use inclusion-exclusion principle] **[5 marks]**

**Q2.** Let p,q and r be the proposition

p: You get an A on the final exam.

q: You do every exercise in this book.

r: You get an A in the class.

Write these propositions using p, q, and r and logical connectives. **[6 marks]**

(a) You get an A in this class, but you do not do every exercise in this book.
(b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
(c) To get an A in this class, it is necessary for you to get an A on the final.
(d) You get an A on the final, but you don't do every exercise in this book is sufficient for getting an A in this class.
(e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
(f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

**Q3.** Suppose that 10 integers 1, 2, 3, …, 10 are randomly positioned around a circular wheel. Prove it using method of contradiction that the sum of some set of 3 consecutively positioned numbers is at least 17. **[5 marks]**

**Q4.** Find a formula for the following series:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}$$

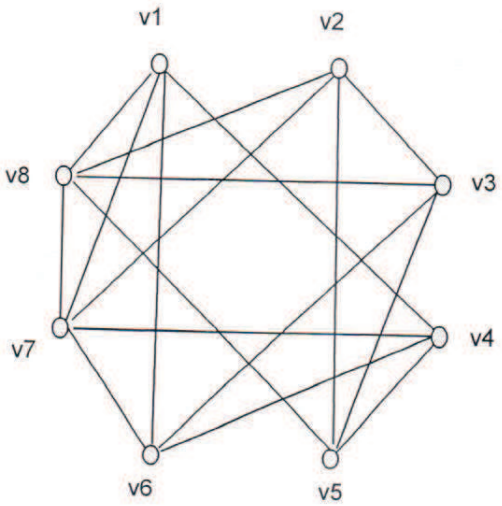
Prove the conjectured formulae by Mathematical Induction. **[5 marks]**

**Q5.**Prove the following inequality using Mathematical Induction

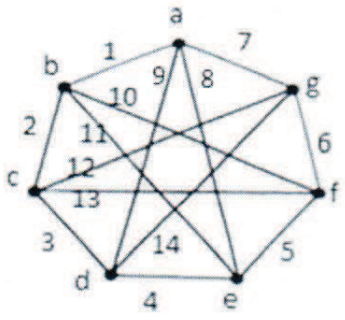
$1+\frac{1}{4}+\frac{1}{9}+\cdots+\frac{1}{n^2}<2-\frac{1}{n}$, where n is greater than 1. **[5 marks]**

**Q6.** Find two distinct set of vertices which makes the complement of the following graph Bipartite. **[4 marks]**

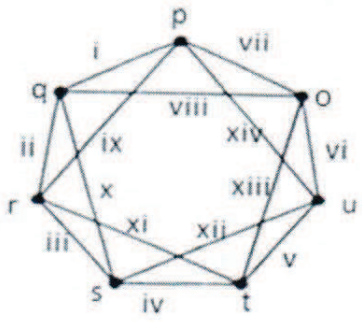
[**Hint:** Complement of a Graph $G$ is the graph $G^{\sim}$ with the same number of vertex set but whose edge set consists of the edges not present in $G$.]




**Q7.** Check whether the following two Graphs G1 and G2 are Isomorphic? Explain your answer by mapping functions: **[5 marks]**




G1




G2

**Q8.** Prove that a simple graph with n vertices and k connected components has at most $\frac{(n-k)(n-k+1)}{2}$ edges. **[5 marks]**