भारतीय प्रौद्योगिकी संस्थान पटना

INDIAN INSTITUTE OF TECHNOLOGY PATNA

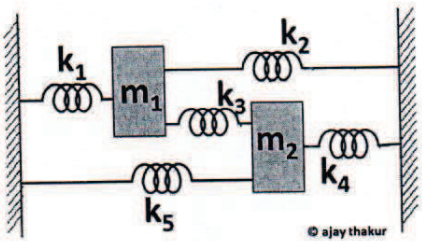
PH103 (Physics-I) Mid-Semester Examination (September 18, 2017)

[Full Marks: 30] [Time: 120 minutes]

• All the questions are compulsory. • Answers must be to the point (refrain from writing essays!. • Answers to all parts of a given question must be answered together. • Each question carries equal mark.

1. (a) Prove that in plane polar coordinates (r, θ), the velocity and acceleration vectors are given by: $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$, and $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$.
(b) For an object of mass m moving under the action of a central force $\vec{F} = F(r)\hat{r}$, show that the equation of motion can be written as: $\frac{d^2u}{d\theta^2} + u = -\frac{m}{L^2}\frac{1}{u^2}F(\frac{1}{u})$, where, $u = \frac{1}{r}$.
(c) Show that the total energy E can be written as,
$E = \frac{L^2}{2mr^4}(\frac{dr}{d\theta})^2 + \frac{L^2}{2mr^2} + V(r)$, where, $F(r)\hat{r} = -\nabla V(r)$
(d) Based on results obtained above, obtain the trajectory for a particle of mass m moving in an attractive potential $V(r) = -\frac{k}{r^2}$ for the special case when $E = 0$ and $k = \frac{L^2}{2m}$.

(a) For a coupled oscillator shown in the figure below, write down the equations of motion under the conditions of small deflections and no damping.
(b) For the special case $k_1 = k_2 = k_4 = k_5 = \frac{k}{2}$ and $k_3 = k$ (in 2 (a) above), write down the corresponding equations in a matrix form and write down the expressions for the mass and stiffness matrices $\bar{M}$ and $\bar{\mathcal{K}}$, respectively.
(c) Obtain the eigenvalues and eigenvectors (in the case 2 (b) above) and interpret the results.
(d) Show that $\Sigma_{i=0}^{2} \mid e_i >< e_i \mid = \bar{I}$, where, $\bar{I}$ is the identity matrix and $\mid e_i >$ are the eigenvectors in the 'bra-ket' notation (obtained in 2 (c) above).



3. (a) Obtain frequency of small oscillations about the equilibrium point of an object of mass m moving under the influence of a 1-d potential $V(x) = \frac{a}{x^4} - \frac{b}{x^3}$, where, $a, b > 0$.
(b) Obtain the magnitude and direction of Coriolis force acting on a sail-boat of mass 1000 kg located at the equator and moving eastward with a velocity of 10 m/s.
(c) Make a rough sketch (capturing the salient features) of amplitude versus ω/ω_0 for a driven damped harmonic oscillator for different values of damping parameters in the range of 0 and $5\omega_0$. Here, ω is the forcing frequency and ω_0 is the natural frequency of the oscillator.
(d) Illustrate the key steps involved in the application of the method of perturbation technique to solve the anharmonic oscillator problem. For this consider the case of a simple pendulum where the angle of oscillation is large enough such that the approximation $\sin(\theta) = \theta$ fails and the next term in the expansion of $\sin(\theta)$ must be considered. [Note: You have to only illustrate the steps involved along with their justification. No detailed calculation is required to be shown for this part.]

END OF EXAM PAPER