(29)

INDIAN INSTITUTE OF TECHNOLOGY PATNA

MSE, PH 101 (PHYSICS I), Sept. 2012

Time: 2 hrs, Full Marks: 50

(Answer all questions. You should answer according to the marks allotted for each question)

1. a) If the cosine of the angles between a vector and the x, y, z axes are α, β, γ, respectively. Prove that $\alpha^2 + \beta^2 + \gamma^2 = 1$. [2]

b) Write down the three most important properties of a conservative force. [3]

c) A rocket of mass M moves in space with a velocity **v**. Consider the rocket at time t. Between t & $t+\Delta t$ a mass of fuel Δm is burned and expelled as gas with velocity **u** relative to the rocket. **u** is independent of the velocity of the rocket. Derive the expression of the external force. [4]

2. a) For a fixed axis rotation of a rigid body, show that the velocity is always perpendicular to the radius vector. [2]

b) Find the position of center of mass of a uniform solid hemisphere of radius R and mass M. [3]

c) An automobile enters a turn whose radius is R. The road is banked at angle θ, and the coefficient of friction between wheels and road is μ. Find the maximum and minimum speeds for the car to stay on the road without skidding sideways. [4]

3. a) Prove that the force is everywhere perpendicular to the constant energy surfaces and points from higher to lower potential energy. [2]

b) For a potential energy function: $U = \epsilon\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right]$

i) What are the radius at the potential minimum and the depth of the potential well? [2]

ii) Find the frequency of small oscillations about equilibrium for two identical atoms of mass m bound to each other inside the potential. [2]

4. a) Prove that, motion of a body under central force field always lies in a plane. Also show that the area swept out by the body per unit time is constant. [2+3]

b) Derive the angular momentum of a system of particle involving both translation and rotation. [2]

c) For a general rotation with an angular velocity w, around an arbitrary axis, prove that the velocity, $\mathbf{v} = w \times \mathbf{r}$, where **r** is the radius vector from the origin of the coordinate system. [2]

5. a) Explain the theory & stability of a Gyroscope ? [3]

b) A gyroscope wheel is at one end of an axle of length l. The other end of the axle is suspended from a string of length L. The wheel is set into motion so that it executes uniform precession in the horizontal plane. The wheel has mass M and moment of inertia about its center of mass I_0. Its spin angular velocity is w_s . Neglect the mass of the shaft and of the string. Find the angle that the string makes with the vertical. Assume the angle is small. [4]

6. a) Write down the expression of angular momentum for the general rotation of a rigid body and define inertia tensor. What do you mean by principal moment of inertia? [3+2]

b) A particle of mass m is located at $x = 2$, $y = 0$, $z = 3$.

i) Find the inertia tensor relative to the origin. ii) The particle undergoes pure rotation about the z axis through a small angle α. Show that its moments of inertia are unchanged to first order in α if $\alpha << 1$. [2+3]