Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2017-18 (End Semester Examination)

Maximum Marks: 50 Total Time: 3 Hours

Note: This question paper contains **Ten** questions. Answer all questions.

1. Consider the density function $f_X(x) = cx(1+x),\ 0 < x < 1; = 0,$ otherwise. Compute the constant c and then determine the probability $P(X > 0.5 \mid 0.4 < X < 0.75)$. Also derive the moment generating function. [1+2+2]

2. The response time X on an online computer terminal follows gamma distribution with mean value 4 minutes and variance 8 minutes^2. Write the probability density function of X. Then determine the probability $P(X > 1)$. Also compute the mode value of X. [1+2+1]

3. Suppose that joint density function of (X, Y) is $f_{X,Y}(x, y) = \frac{3}{2},\ x^2 \le y \le 1,\ 0 \le x \le 1$. Compute the probabilities $P(0.5 \le X \le 1, 0.5 \le Y \le 1)$ and $P(0 \le X \le 1)$. Are X and Y independent random variables? [1.5+1.5+1]

4. Let X and Y respectively denote height in centimeters and weight in kilograms of a college students. Assume that (X, Y) follows a bivariate normal $BVN(185, 84, 100, 64, 0.6)$. Find c such that $P(X > c) = 0.45$. Find the conditional probability $(86.4 < Y < 95.36 \mid X = 190)$. Determine the covariance between X and Y. [2+3+2]

5. Let random variable Y is distributed as Poisson $P(\lambda)$ and that the conditional distribution of X given $Y = y$ is binomial $B(y, p)$ distribution. Then show that marginal PMF of X is Poisson $P(\lambda p)$. [5]

6. Let X and Y denote life times of two electronic devices with joint density function given by $f_{X,Y}(x, y) = 12e^{-(3x+4y)},\ x > 0,\ y > 0$ and zero elsewhere. Let $U = X + Y$ and $V = X - Y$. Find the joint density function of (U, V). Also evaluate marginal densities of U and V. [2+2+2]

7. Suppose that joint density function of (X, Y) is $f_{X,Y}(x, y) = \frac{1}{40},\ 0 \le x \le 10,\ 10 - x \le y \le 14 - x$ and zero elsewhere. Compute the marginal density of X and the probability $P(\sqrt{3} < X \le 7)$. Find the conditional density function of Y given x and evaluate the variance $V(Y \mid x)$. [2+2+2+2]

8. Let the magnitude of earthquakes recorded in a region of India can be modeled using a one-parameter exponential distribution with mean 2.4 (measured on the Richter Scale). Find the probability that an earthquake striking this region will exceed 3.5 on the Richter Scale. Also compute the probability it will fall between 1.5 and 2.5 on the Richter Scale. Further compute the median magnitude of such earthquakes. [1+1+1]

9. Let customers visiting a restaurant have to wait for service and assume that the wait time X (in minutes) follows a lognormal $LN(3.5, 0.36)$ distribution. Compute the probability that a customer will have to wait more than 20 minutes for service. Also determine the average wait time. [2+2]

10. (*i*) Properly define a Poisson process by stating all the assumptions. [2]
(*ii*) Suppose that an average of 30 customers per hour arrive at a shop in accordance with a Poisson process. What is the probability that in any 5 minutes duration at least 3 customers arrive. [2]