Engineering Mechanics (ME102)

Lecture by

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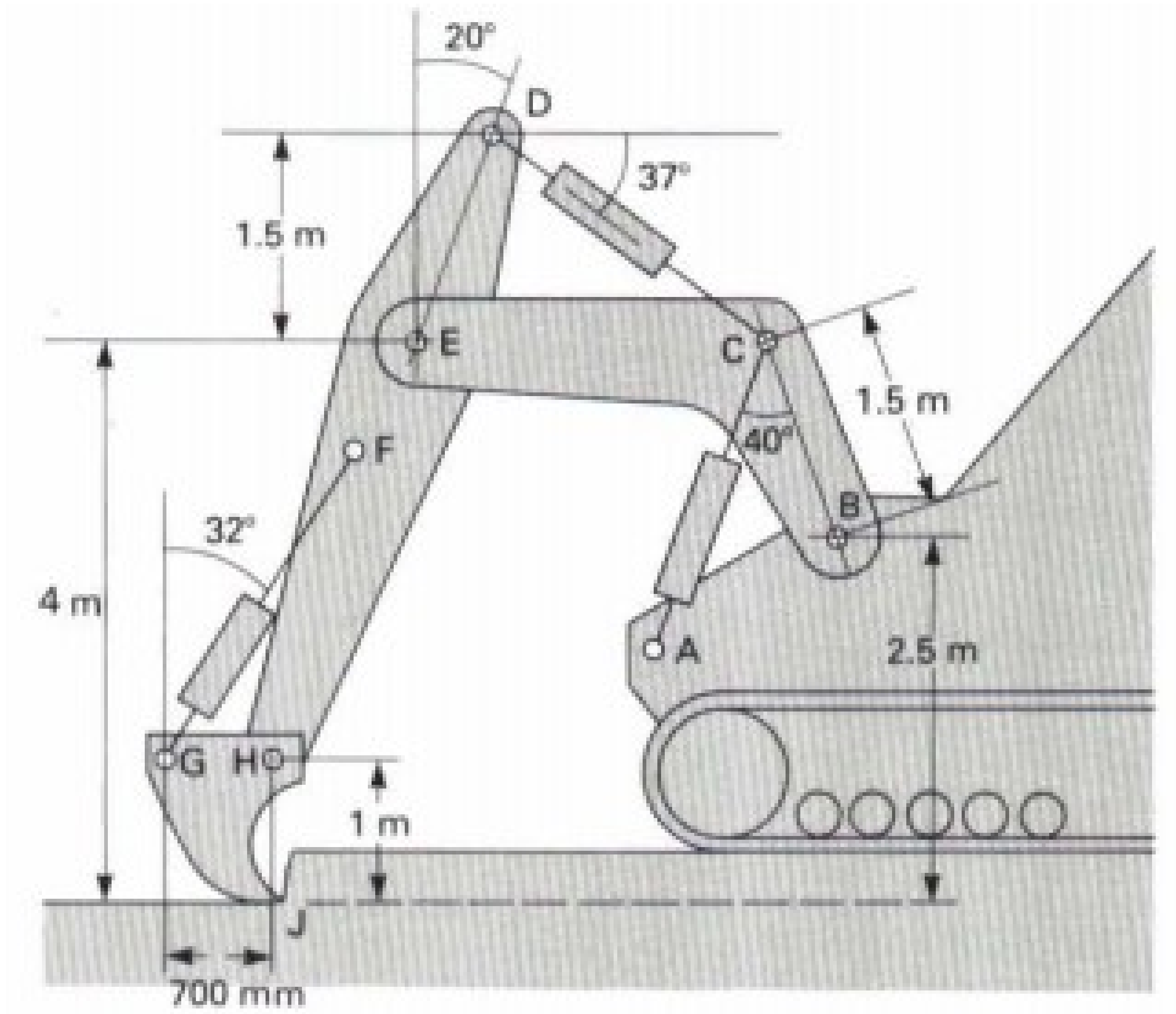
Indian Institute of Technology (IIT) Patna

Tutorial problem



Tenth Edition
in SI Units

CHAPTER

6

VECTOR MECHANICS FOR ENGINEERS:

STATICS

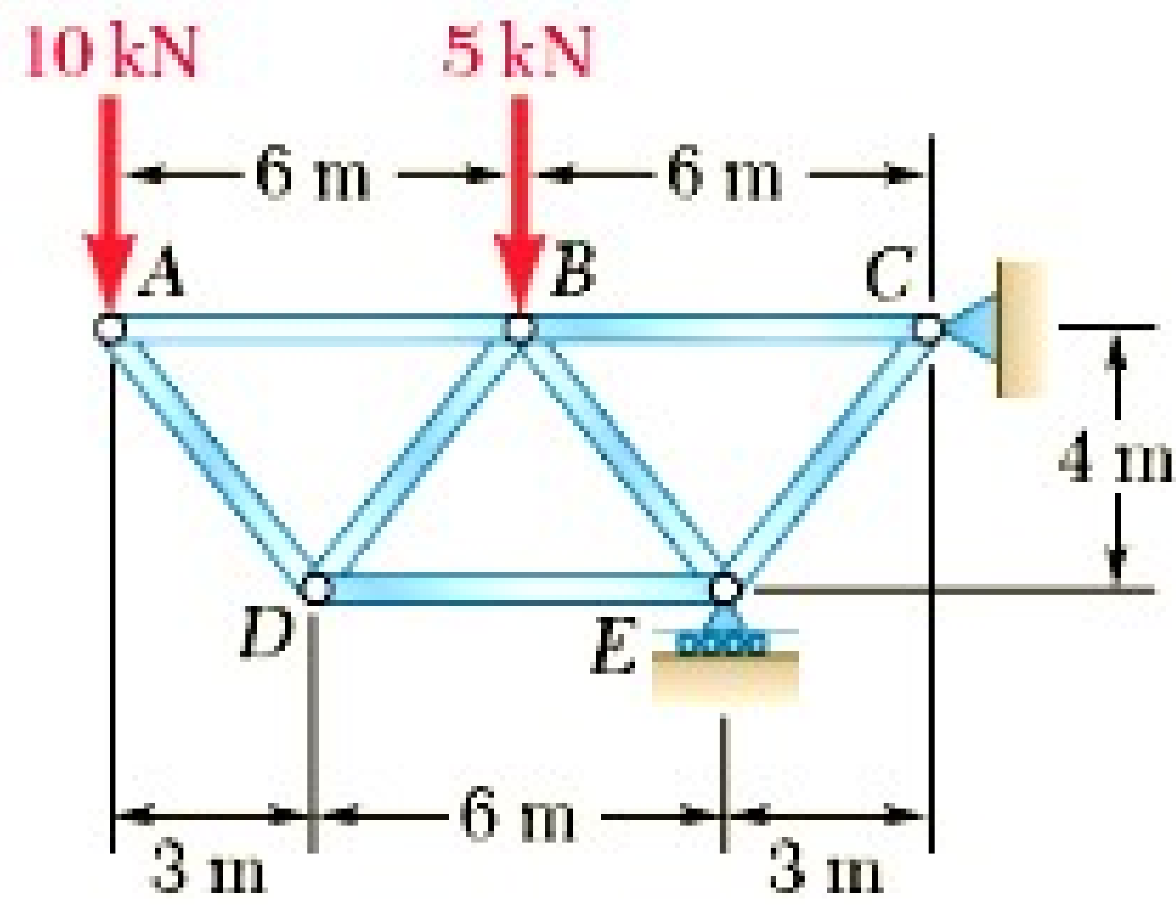
Reference for this lecture

Ferdinand P. Beer

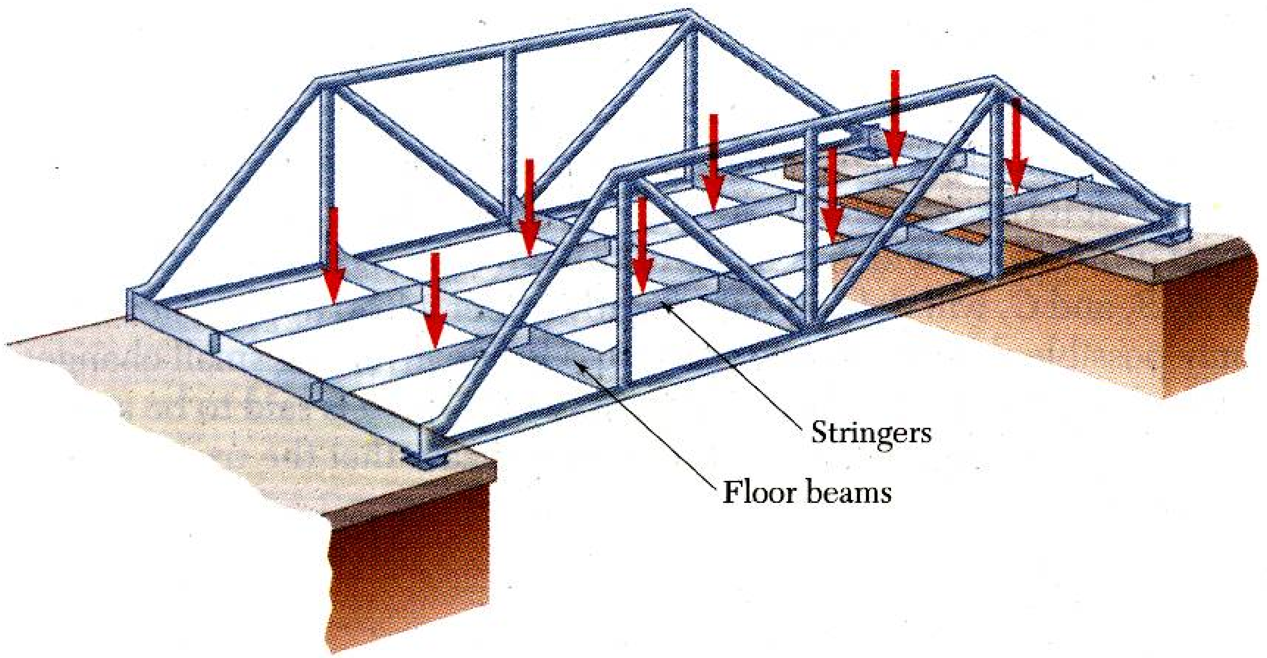
E. Russell Johnston, Jr.

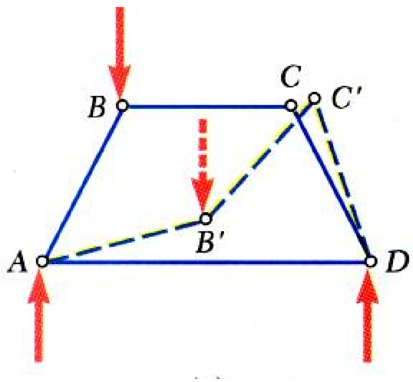
David F. Mazurek

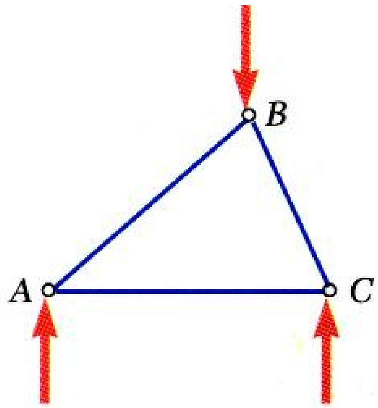
Analysis of Structures

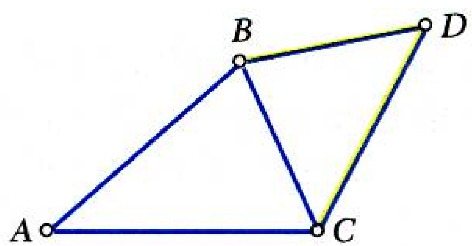

10 kN
5 kN
6 m
6 m
A
B
C
4 m
D
E
3 m
6 m
3 m

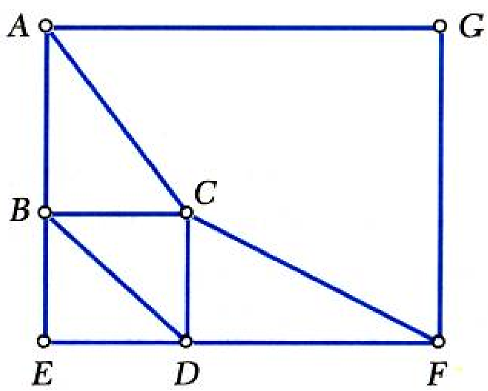
- Members of a truss are slender and not capable of supporting large lateral loads. Loads must be applied at the joints.





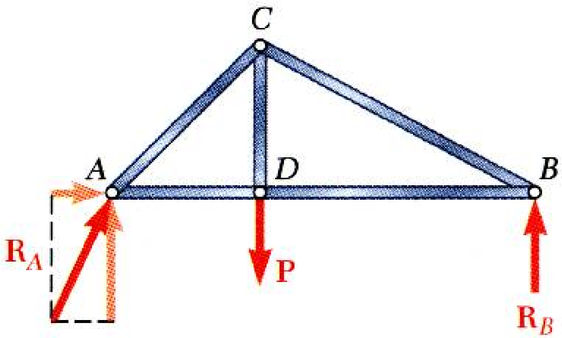


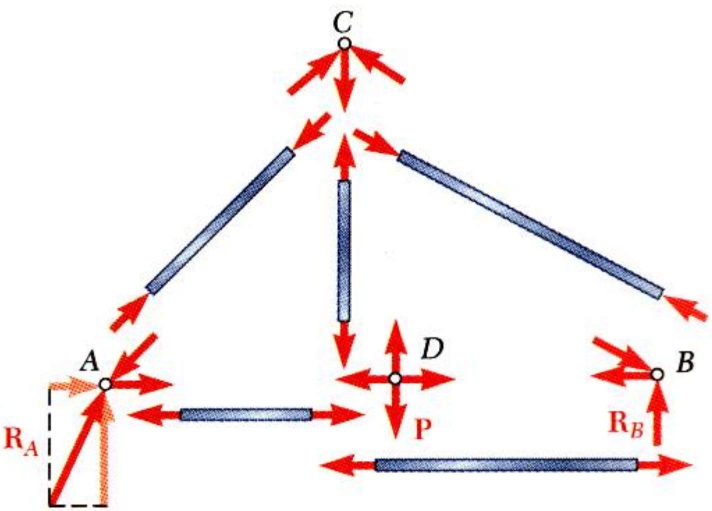




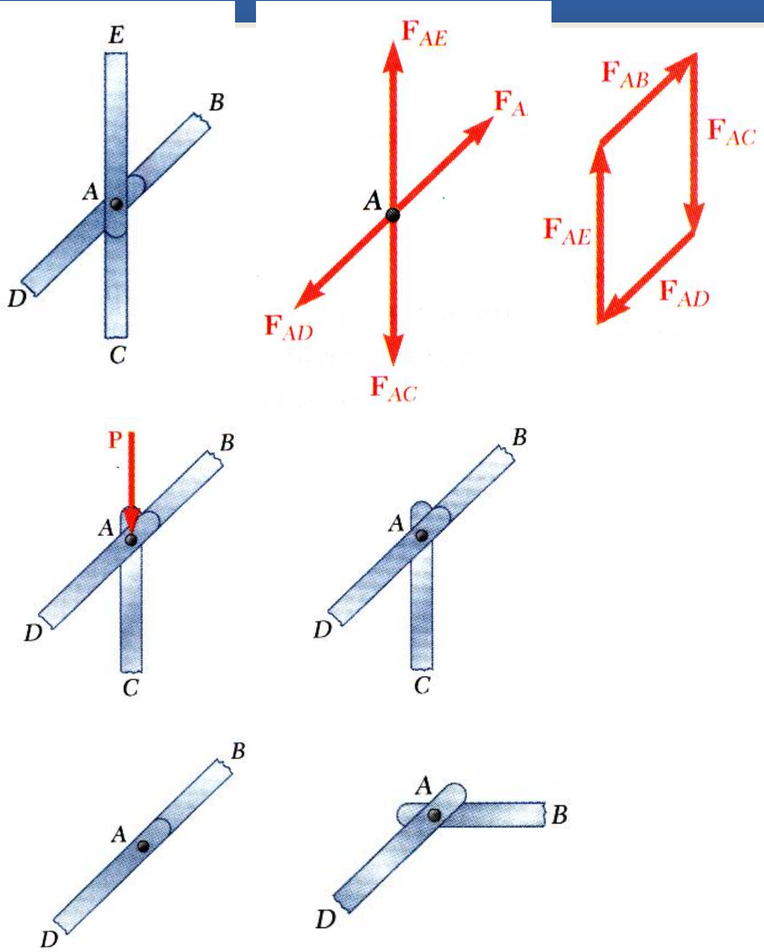
- A *rigid truss* will not collapse under the application of a load.

- A *simple truss* is constructed by successively adding two members and one connection to the basic triangular truss.

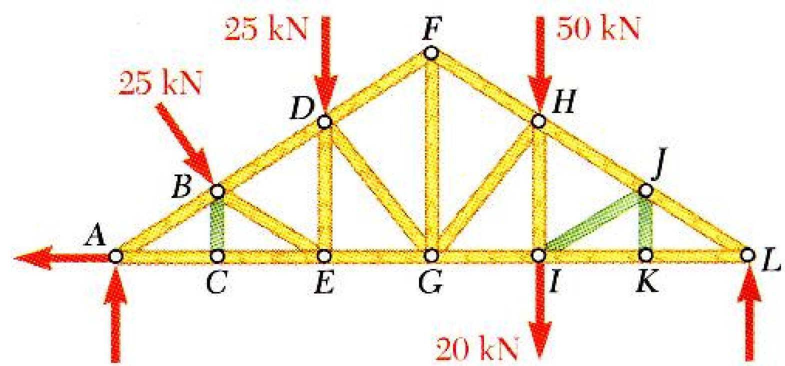




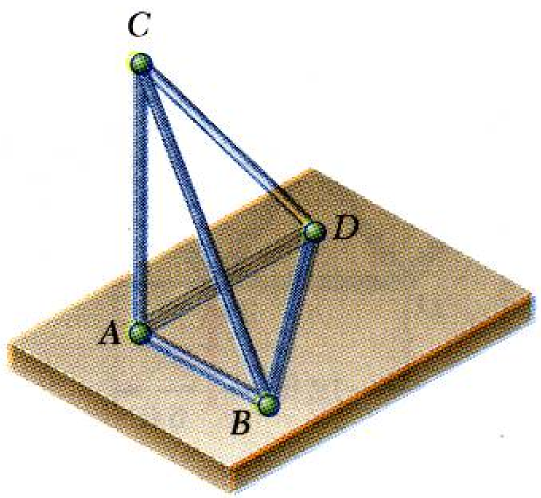
- Dismember the truss and create a free body diagram for each member and pin.
- Conditions for equilibrium for the entire truss can be used to solve for 3 support reactions.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium are used to solve for 2 unknown forces at each pin (or joint), giving a total of $2n$ solutions, where n=number of joints. Forces are found by solving for unknown forces while moving from joint to joint sequentially.

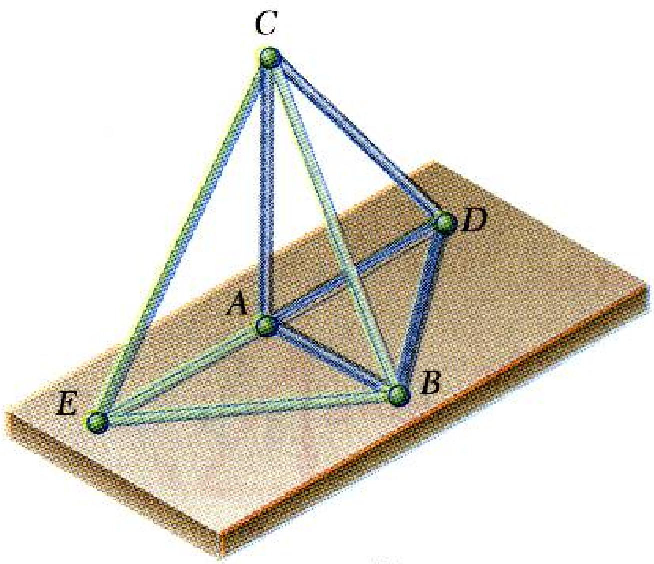


- Forces in opposite members intersecting in two straight lines at a joint are equal.
- The forces in two opposite members are equal when a load is aligned with a third member. The third member force is equal to the load (including zero load).
- The forces in two members connected at a joint are equal if the members are aligned and zero otherwise.
- Recognition of joints under special loading conditions simplifies a truss analysis.

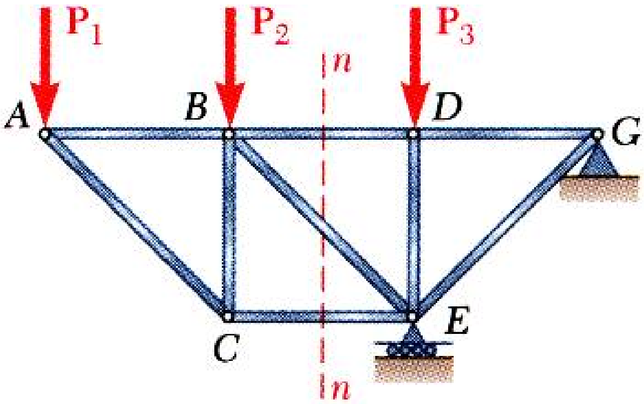


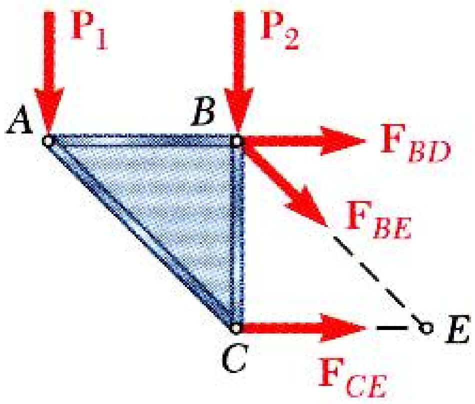
Space Truss



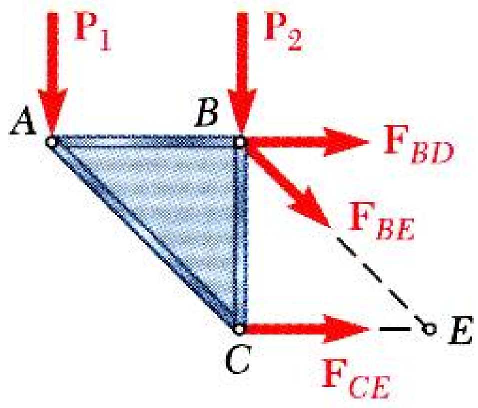


- An *elementary space truss* consists of 6 members connected at 4 joints to form a tetrahedron.
- A *simple space truss* is formed and can be extended when 3 new members and 1 joint are added at the same time.
- In a simple space truss, $m = 3n - 6$ where m is the number of members and n is the number of joints.
- Conditions of equilibrium for the joints provide $3n$ equations. For a simple truss, $3n = m + 6$ and the equations can be solved for m member forces and 6 support reactions.
- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.

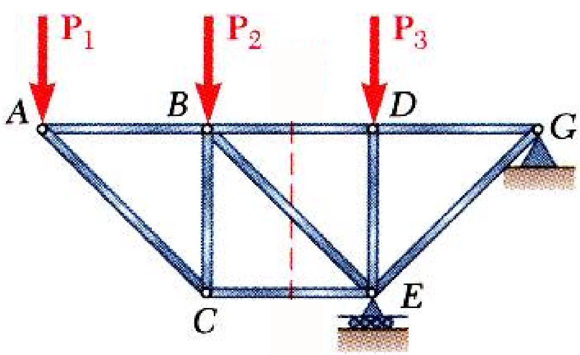




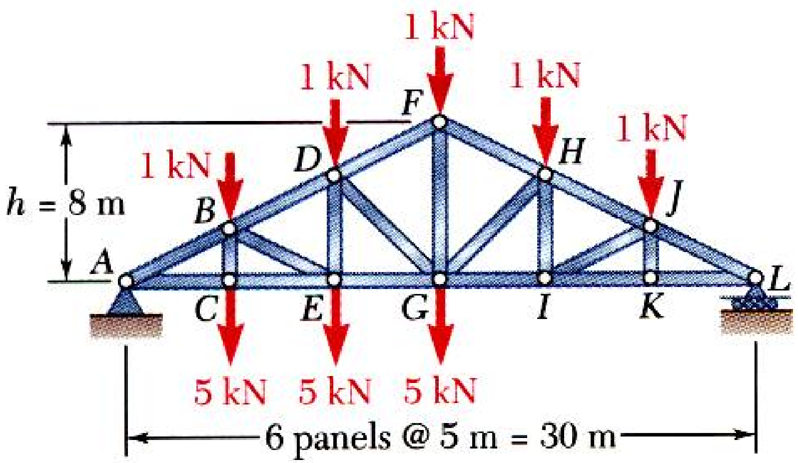
- When the force in only one member or the forces in a very few members are desired, the *method of sections* works well.
- To determine the force in member *BD,* form a *section* by “cutting” the truss at *n-n* and create a free body diagram for the left side.
- A FBD could have been created for the right side, but why is this a less desirable choice? Think and discuss.
- Notice that the exposed internal forces are all *assumed* to be in tension.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including $\mathbf{F}_{BD}$.



- Using the left-side FBD, write <u>one</u> equilibrium equation that can be solved to find $\mathbf{F}_{BD}$. Check your equation with a neighbor; resolve any differences between your answers if you can.



- Assume that the initial section cut was made using line *k*-*k*. Why would this be a poor choice? Think, then discuss with a neighbor.
- Notice that *any* cut may be chosen, so long as the cut creates a separated section.
- So, for example, this cut with line *p*-*p* is acceptable.

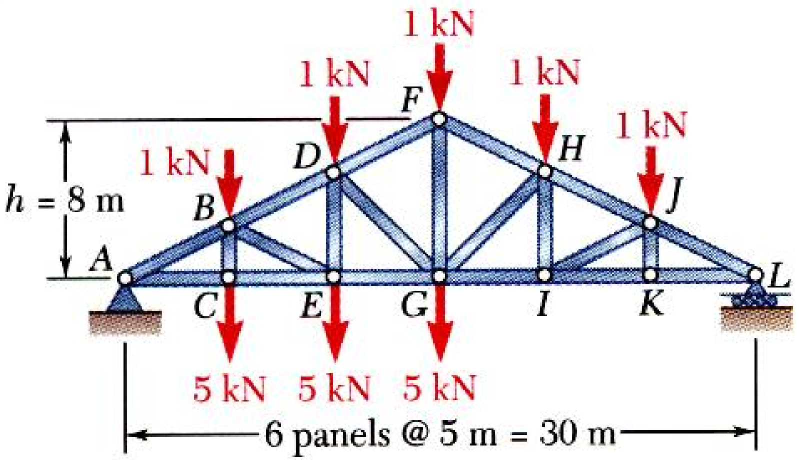


Determine the force in members *FH*, *GH*, and *GI*.

<u>SOLUTION</u>:

- List the steps for solving this problem. Discuss your list with a neighbor.

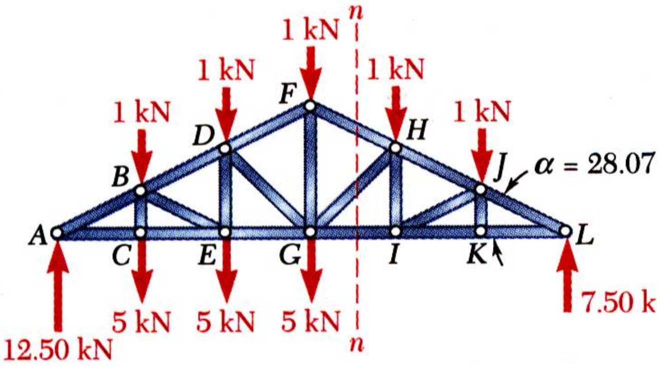
1. Draw the FBD for the entire truss. Apply the equilibrium conditions and solve for the reactions at *A* and *L*.
2. Make a cut through members *FH*, *GH*, and *GI* and take the right-hand section as a free body (the left side would also be good).
3. Apply the conditions for static equilibrium to determine the desired member forces.



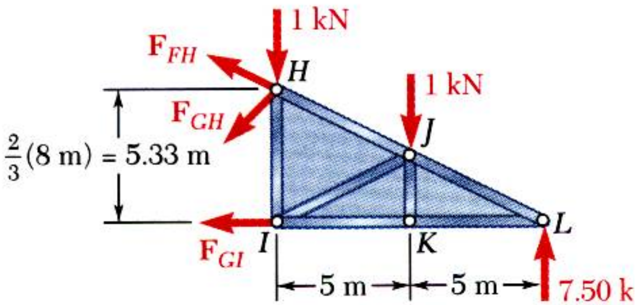
SOLUTION:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at A and L.

$$\begin{aligned}
\sum M_A &= 0 = -(5\text{ m})(6\text{ kN}) - (10\text{ m})(6\text{ kN}) - (15\text{ m})(6\text{ kN}) \\
&\quad - (20\text{ m})(1\text{ kN}) - (25\text{ m})(1\text{ kN}) + (25\text{ m})L \\
L &= 7.5\text{ kN} \\
\sum F_y &= 0 = -20\text{ kN} + L + A_y \\
A_y &= 12.5\text{ kN} \\
\sum F_x &= 0 = A_x
\end{aligned}$$



- Make a cut through members *FH*, *GH*, and *GI* and take the right-hand section as a free body. Draw this FBD.



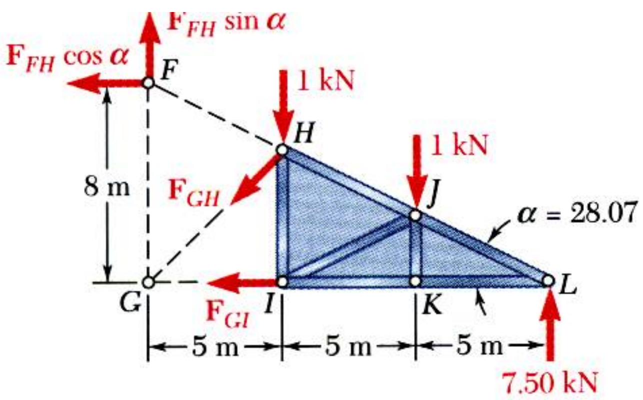
- What is the one equilibrium equation that could be solved to find $\mathbf{F}_{GI}$? Confirm your answer with a neighbor.
- Sum of the moments about point *H*:

$$\sum M_H = 0$$

$$(7.50\,\text{kN})(10\,\text{m}) - (1\,\text{kN})(5\,\text{m}) - F_{GI}(5.33\,\text{m}) = 0$$

$$F_{GI} = +13.13\,\text{kN}$$

$$F_{GI} = 13.13\,\text{kN}\ \ T$$



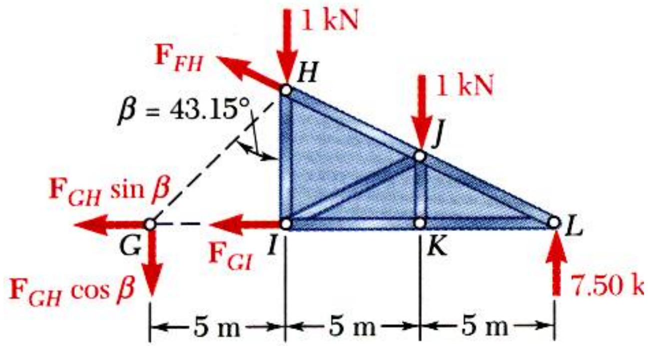
- $\mathbf{F}_{FH}$ is shown as its components. What one equilibrium equation will determine $\mathbf{F}_{FH}$?

$$\tan\alpha = \frac{FG}{GL} = \frac{8\text{ m}}{15\text{ m}} = 0.5333 \qquad \alpha = 28.07^\circ$$

$$\sum M_G = 0$$

$$(7.5\text{ kN})(15\text{ m}) - (1\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) + (F_{FH}\cos\alpha)(8\text{ m}) = 0$$

$$F_{FH} = -13.81\text{ kN} \qquad F_{FH} = 13.81\text{ kN } C$$



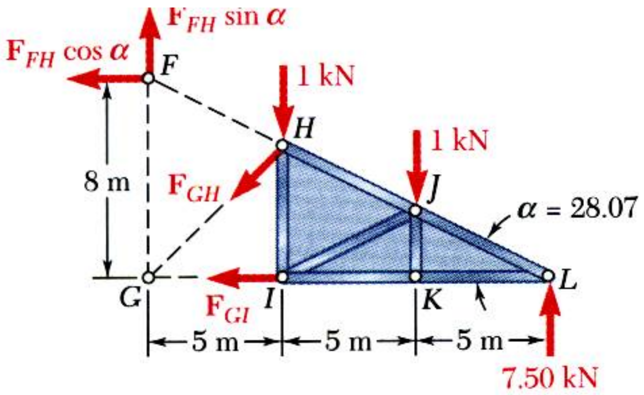
- There are many options for finding $\mathbf{F}_{GH}$ at this point (e.g., $\Sigma F_x=0$, $\Sigma F_y=0$). Here is one more:

$$\tan\beta = \frac{GI}{HI} = \frac{5\text{ m}}{\frac{2}{3}(8\text{ m})} = 0.9375 \qquad \beta = 43.15^\circ$$

$$\sum M_L = 0$$

$$(1\text{ kN})(10\text{ m}) + (1\text{ kN})(5\text{ m}) + (F_{GH}\cos\beta)(15\text{ m}) = 0$$

$$F_{GH} = -1.371\text{ kN} \qquad F_{GH} = 1.371\text{ kN } C$$



- $\mathbf{F}_{FH}$ is shown as its components. What one equilibrium equation will determine $\mathbf{F}_{FH}$?

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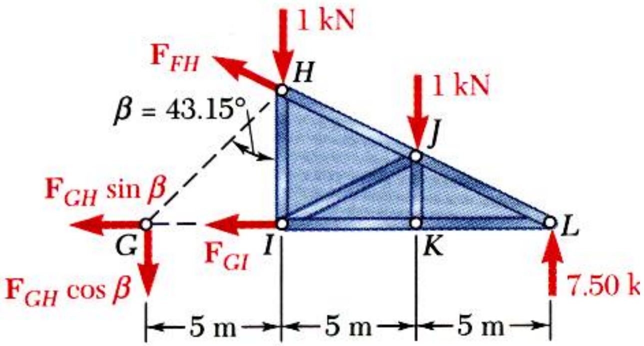
$$\sum M_G = 0$$

$$(7.5\text{ kN})(15\text{ m}) - (1\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) + (F_{FH}\cos\alpha)(8\text{ m}) = 0$$

$$F_{FH} = -13.81\text{ kN} \qquad F_{FH} = 13.81\text{ kN } C$$



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$$(1\text{ kN})(10\text{ m}) + (1\text{ kN})(5\text{ m}) + (F_{GH}\cos\beta)(15\text{ m}) = 0$$

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Method of Section: Truss Analysis

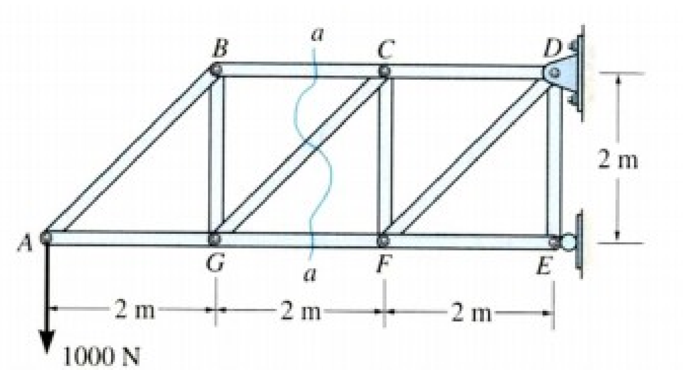
http://site.iugaza.edu.ps/malqedra/files/Lecture-6.2.pdf

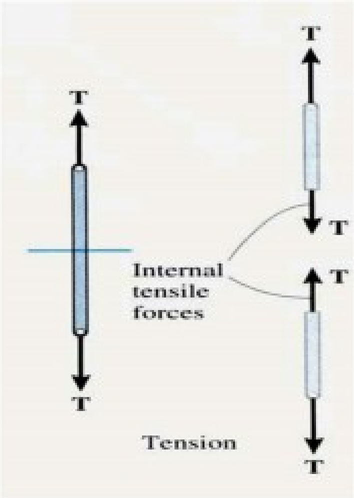
Long trusses are often used to construct bridges.

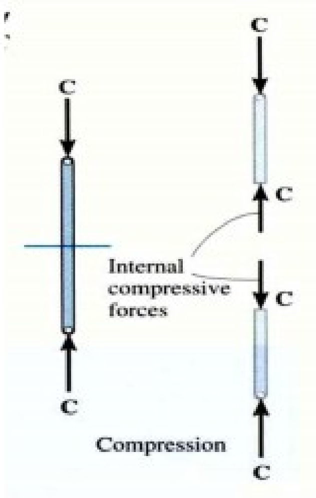
The method of joints requires that many joints be analyzed before we can determine the forces in the middle part of the truss.

Is there another method to determine these forces directly?

6.4 THE METHOD OF SECTIONS



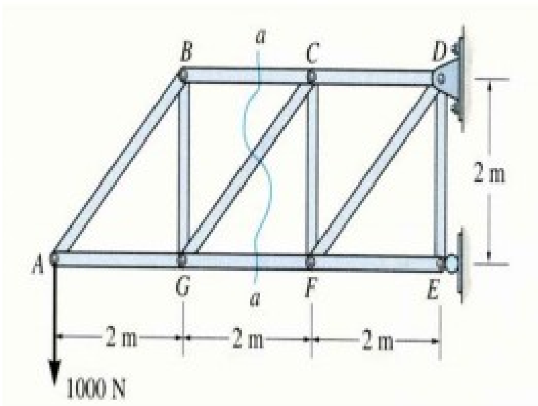


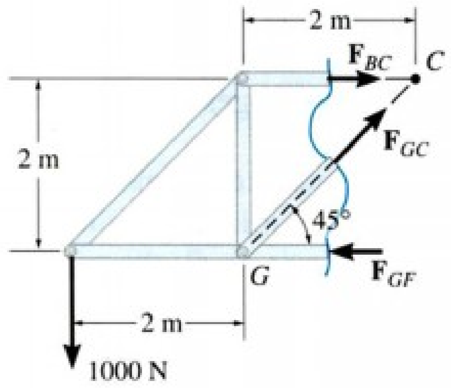


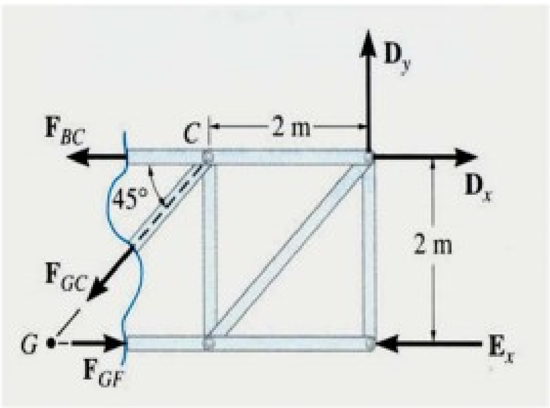
In the method of sections, a truss is divided into two parts by taking an imaginary "cut" (shown here as a-a) through the truss.

Since truss members are subjected to only tensile or compressive forces along their length, the internal forces at the cut member will also be either tensile or compressive with the same magnitude. This result is based on the equilibrium principle and Newton's third law.

STEPS FOR ANALYSIS

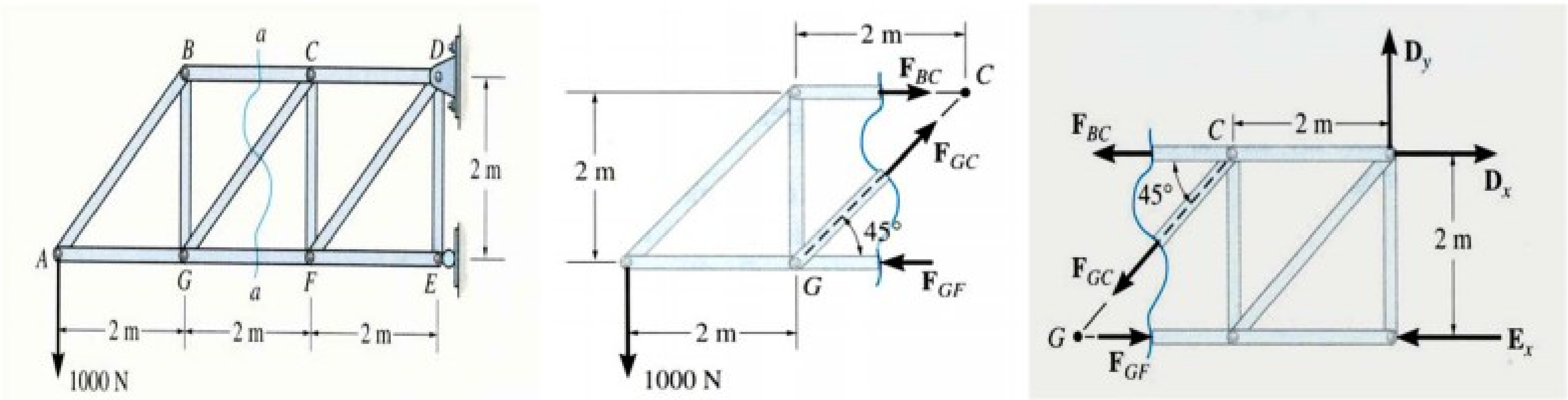






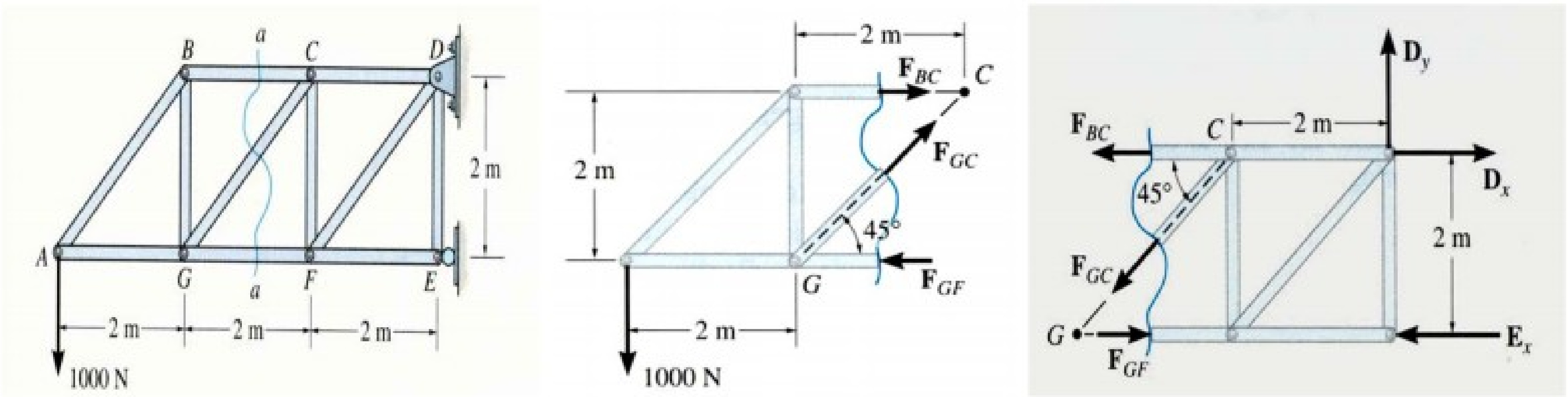
1. Decide how you need to "cut" the truss. This is based on: a) where you need to determine forces, and, b) where the total number of unknowns does not exceed three (in general).
2. Decide which side of the cut truss will be easier to work with (minimize the number of reactions you have to find).
3. If required, determine the necessary support reactions by drawing the FBD of the entire truss and applying the E-of-E.

STEPS FOR ANALYSIS (continued)



4. Draw the FBD of the selected part of the cut truss. We need to indicate the unknown forces at the cut members. Initially we may assume all the members are in tension, as we did when using the method of joints. Upon solving, if the answer is positive, the member is in tension as per our assumption. If the answer is negative, the member must be in compression. (Please note that you can also assume forces to be either tension or compression by inspection as was done in the figures above.)

STEPS FOR ANALYSIS (continued)



5. Apply the equations of equilibrium (E-of-E) to the selected cut section of the truss to solve for the unknown member forces. Please note that in most cases it is possible to write one equation to solve for one unknown directly.