MA101, Real Analysis
Series and Power Series

1. Discuss the convergence and divergence of the following series:

(a) $\sum\left[\sqrt{n+1}-\sqrt{n}\right]$.

(b) $\sum(1/2)^n\left(50+\frac{2}{n}\right)$.

(c) $\sum\frac{100^n}{n!}$.

(d) $\sum\frac{1}{\sqrt{n!}}$.

(e) $\sum\frac{n^3}{3^n}$.

(f) $\sum\frac{(-1)^n n!}{2^n}$.

(g) $\sum_2^\infty\frac{1}{\sqrt{n}\log n}$.

(h) $\sum_2^\infty\frac{\log n}{n}$.

(i) $\sum_4^\infty\frac{1}{n\log n\log\log n}$.

(j) $\sum_2^\infty\frac{\log n}{n^2}$.

(k) $\sum_2^\infty\frac{1}{n(\log n)^p}$.

(l) $\sum_2^\infty(-1)^n\left(\frac{\ln n}{(\log n)^2}\right)^n$.

(m) $\sum_1^\infty(-1)^n\operatorname{sech} n$.

(n) $\sum_2^\infty(-1)^n\left(\frac{\tan^{-1}n}{1+n^2}\right)$.

2. Let $\sum_{n=1}^\infty a_n$ be a convergent series of positive terms. What can be said about the convergence of

$$\sum_{n=1}^\infty\frac{a_1+a_2+\cdots+a_n}{n}.$$

3. Let p_n be a sequence of all consecutive prime numbers. Study convergence of $\sum_{n=1}^\infty\frac{1}{p_n}$.

4. Decide whether the series

$$\sum_{n=1}^\infty\frac{(-1)^{[\ln n]}}{n}$$

is absolutely convergent, conditional convergent or divergent.

5. For a sequence $\{a_n\}$ tends to zero and for a,b,c such that $a+b+c\neq 0$, prove that the series $\sum_{n=1}^\infty a_n$ and $\sum_{n=1}^\infty(aa_n+ba_{n+1}+ca_{n+2})$ either both converge or both diverge.

6. Apply Dirichlet's test and study the convergence of the series where $a\in\mathbb{R}$

(a)
$$\sum_{n=1}^\infty\frac{\sin(na)\sin(n^2a)}{n},$$

(b)
$$\sum_{n=1}^\infty\frac{\sin(na)\cos(n^2a)}{n}.$$

7. In the following exercises (a) find the series' radius and interval of convergence. For what values of x does the series converge (b) absolutely, (c) conditionally?

(a) $\sum_0^\infty(x+5)^n$.

(b) $\sum_0^\infty(-2)^n(n+1)(x-1)^n$.

(c) $\sum_2^\infty\frac{x^n}{n\ln n}$.

(d) $\sum_1^\infty\frac{(4x-5)^{2n+1}}{n^{3/2}}$.

(e) $\sum_1^\infty\frac{\left(x-\sqrt{2}\right)^{2n+1}}{2^n}$.