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Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2015-16
Mid Semester Examination

Maximum Marks: 30 Total Time: 2 Hours

Note: Answer all questions. You can use scientific calculator.

1. Students in an institute subscribe to three news magazines A, B and C with respective proportions being 20%, 15% and 10%. For both A and B it is 5%, for A and C it is 4%, for B and C it is 3% and for all three A, B and C the proportion is 2%. One student is chosen at random, then determine the probability that he/she subscribes to none of the news magazines. [2]

2. A signal is sent from point A to point B and is received at B if both switches I and II are closed. It is assumed that the probabilities I and II being closed are 0.8 and 0.6 respectively and that $P(II \text{ is closed} \mid I \text{ is closed}) = P(II \text{ is closed})$. Determine the probability that switch II was open given that the signal was not received at B. [3]

3. Suppose that events A_1, A_2 and B_1 are independent, the events A_1, A_2 and B_2 are independent and it is known that $B_1 \cap B_2 = \phi$. Then show that events $A_1, A_2, B_1 \cup B_2$ are independent. [4]

4. The lifetime in hours of electric tubes is a random variable X with PDF $f_X(x) = \lambda^2 x e^{-\lambda x}$, $x > 0$, $\lambda > 0$. Determine the probability $P(X > x)$ and compute the variance of X. Find the mode of X. [2 + 2 + 1]

5. In an undergraduate class of 80 students 10 of the students are actually graduate students. If 5 students are chosen at random and let X denotes the number of undergraduate students in this sample. Write the probability mass function of X and then calculate $E(X(X-1))$. Determine the probability that at least 3 undergraduate students are included in the selected sample. [1 + 2 + 2]

6. A target is either hit or missed. Let the probability of hitting the target is 1/5 and this probability remains unchanged from one trial to the other. Assume that 10 shots are fired. Let X denotes the number of times the target is hit. Write the appropriate probability distribution of X. Determine the probability of the target being hit at least twice given that at least one hit is already scored. [1 + 2]

7. In a Bernoulli experiment it is known that probability that an item produced by a certain machine will be defective is 0.01. By applying Poisson approximation determine the probability that a random sample of 100 items will contain at least three defective. Find the moment generating function for defective items. [1 + 2]

8. Suppose that life of a machine in hours is a continuous random variable with PDF given by $f_X(x) = \frac{k}{x^2}$, $x \geq k$ where k is a known constant. Determine the probability a machine will last less than u hours given that it is functioning after v, $(v < u)$ hours. Note that u and v are known constants and both are greater than k. [2]

9. State and prove the memoryless property of a geometric distribution. [1+2]