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Indian Institute of Technology Patna
MA 101: B.Tech. I year
Autumn Semester: 2012-13

Mid Semester Examination

Maximum Marks: 30 Total Time: 2 Hours

Note: This question paper contains **Twelve** questions. Answer all questions.

1. State the principle of mathematical induction for natural numbers and then using it prove the inequality $1 + 3^n a > (1+a)^n$ with $0 < a < 1$ for $n = 1, 2, \ldots$. [1+1]

2. Consider a nonempty set A of real numbers which is bounded above. A set B is defined as $\{-x : x \in A\}$. Prove that $\inf B = -\sup(A)$. [2]

3. Use the definition to show that the sequence $\{\frac{2n}{n+4\sqrt{n}}\}$ converges to 2. [2]

4. Consider a sequence of real numbers $\{x_n\}$ defined as $x_1 = 1, x_2 = 2$ and $x_n = \frac{1}{2}(x_{n-1} + x_{n-2})$ for $n > 2$. Show that $\{x_n\}$ is a bounded sequence and nonmonotone. Further by providing proper arguments prove that it is a Cauchy sequence. [1.5+.5+1.5]

5. Prove that a sequence $\{x_n\}$ converges to a number l if and only if for any given $\epsilon > 0$ all but a finite number of terms of $\{x_n\}$ lie in the interval $(l-\epsilon, l+\epsilon)$. [2]

6. Apply ratio test to make inference about convergence and divergence of the infinite series $x^2 + \frac{2^2}{3\cdot4}x^4 + \frac{2^2\cdot4^2}{3\cdot4\cdot5\cdot6}x^6 + \frac{2^2\cdot4^2\cdot6^2}{3\cdot4\cdot5\cdot6\cdot7\cdot8}x^8 + \cdots$, $(x \in R)$. The case where this test fails apply Raabe's test to check its convergence and divergence. For what values of x the given series converges and for what values of x it diverges? [4 + 1]

7. Use $(\epsilon - \delta)$ definition to evaluate the limit of the function $f(x) = \frac{x+5}{2x+3}$ as $x \to -1$. [2]

8. Find the interval in which the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$ converges. Here x is a real number. [2]

9. Use $(\epsilon - \delta)$ definition to evaluate limits $\lim_{x\to0^+} f(x)$ and $\lim_{x\to0^-} f(x)$ where
$$f(x) = \begin{cases} 2x+3, & x \geq 0 \\ 2x-3 & x < 0 \end{cases}$$
Is $f(x)$ continuous at $x = 0$?(support your answer with proper justification) [1+1+0.5]

10. Show that if $x > 0$, then $1 + \frac{x}{2} - \frac{x^2}{8} \leq \sqrt{1+x} \leq 1 + \frac{x}{2}$ [2]

11. Define uniform continuity. Show that the function $f(x) = \frac{1}{x^2}$, $x \geq 1.5$ is uniformly continuous [1+2]

12. Suppose that the function $f : [0,4] \to R$ is defined as $f(x) = x^2$. Use Riemann integrability criteria to show that $\int_0^4 x^2 dx = \frac{64}{3}$. [2]