Indian Institute of Technology Patna
Department of Mathematics
MA - 225: B.Tech. II year

Tutorial Sheet-2

1. Let A and B be two events. Then verify that following statements are equivalent.
(i) Events A and B are independent. (ii) Events A^c and B are independent. (iii) Events A and B^c are independent. (iv) Events A^c and B^c are independent.

2. Consider events A and B such that $P(A) = p_1 > 0$ and $P(B) = p_2 > 0$ and $p_1 + p_2 > 1$. Show that $P(B \mid A) \geq 1 - \left[\frac{1-p_2}{p_1}\right]$.

3. Consider a random experiment consisting of three identical coins one of which is fair and other two are biased with probabilities 1/4 and 3/4 respectively for turning up head. One Coin is taken up at random and tossed twice. If a head appears both times, show that the probability that the fair coin was chosen is 2/7.

4. A box contains three white balls w_1, w_2, w_3 and two red balls r_1 and r_2. We remove at random two balls in succession. What is the probability that the first removed ball is white and the second is red? (3/10)

5. Rain is forecast half the time in a certain region during a given time period. We estimate that the weather forecasts are accurate two times out of three. Mr. X goes out every day and he really fears being caught in the rain without an umbrella. Consequently, he always carries his umbrella if rain is forecast. Moreover, he even carries his umbrella one time out of three if rain is not forecast. Calculate the probability that it is raining and Mr. X does not have his umbrella. (1/9)

6. A commuter has two vehicles, one being a compact car and the other one a minivan. Three times out of four, he uses the compact car to go to work and the remainder of the time he uses the minivan. When he uses the compact car (respectively, the minivan), he gets home before 5:30pm 75% (resp. 60%) of the time. Calculate the probability that
(a) he gets home before 5:30pm on a given day (b) he used the compact car if he did not get home before 5:30pm (c) he uses the minivan and he gets home after 5:30pm (d) he gets home before 5:30pm on two (independent) consecutive days and he does not use the same vehicle on these two days. (0.712, 0.652, 0.1, 0.169)

7. Five percent of the patients suffering from a certain disease are selected to undergo a new treatment that is believed to increase the recovery rate from 30% to 50%. A person is randomly selected from these patients after the completion of the treatment and found to have recovered. What is the probability that the patient received the new treatment? (0.08)

8. Consider all families with exactly two children. Also let each child has a 50-50 chance of being a boy. Let the events be: A_1 = both male and female child are represented among the children
A_2 = at most one child is a girl
(a) Are A_1 and A_2^c incompatible events (b) Are A_1 and A_2^c independent events (c) We also suppose that the probability that the third child of an arbitrary family is a boy is equal to 11/20 if the first two children are boys, to 2/5 if the first two children are girls, and to 1/2 in the other cases. Knowing that the third child of a given family is a boy what is the probability that the first two are also boys? (Y, N, 0.282)

9. Suppose box 1 contains a white balls and b black balls, and box 2 contains c white balls and d black balls. One ball of unknown color is transferred from the first box into the second one and then a ball is drawn from the later. What is the probability that it will be a white ball?

10. A certain test for a particular cancer is known to be 95% accurate. A person submits to the test and results are positive. Suppose that the person comes from a population of 100,000 where 2000 people suffer from that disease. What can be concluded about the probability that the person under test has that particular cancer? (0.278)

11. Four roads leads away from a jail. A prisoner has escaped from the jail and selects a road at random. If road I is selected, the probability of escaping is 1/8; if road II is selected the probability of success is 1/6; if road III is selected the probability of escaping is 1/4 and if road IV is selected the probability of success is 9/10. (i) Find the probability that the prisoner will succeed in escaping. (ii) If the prisoner succeeds, what is the probability that prisoner escaped by using road IV? By using road I? ((i) 173/480 (ii) 108/173, 15/173)

12. A biased coin is tossed till a head appears for the first time (assume that p and q $(p + q = 1)$ are the probability of getting head and tail respectively in a single trial). What is the probability that the number of required tosses is odd? (1/(2-p))

13. In examining a past records of a corporation's account balances, an auditor finds that 15% of them have contained errors. Of those balances in error, 60% were regarded as unusual values based on historical figures. Of all the account balances, 20% were unusual values. If the figure for a particular balance appears unusual on this basis, what is the probability that it is in error? (0.45)

14. A stock market analyst examined the prospects of the share of a large number of corporations. When the performance of these stocks was investigated one year later, it turned out that 25% performed much better than the market average, 25% much worse and the remaining 50% about the same as the average. Forty percent of the stocks that turned out to do much better than the market were rated 'good buy' by the analyst, as were 20% of those that did about as well the market and 10% of those that did much worse. What is the probability that a stock rated a 'good buy' by the analyst performed much better than the market average? (0.444)

15. A pair of dice is rolled until a sum of either 5 or 7 appears. Find the probability that a 5 occurs first. (2/5)

16. There are n socks, 3 of which are red, in a drawer. What is the value of n if when 2 of the socks are chosen randomly, the probability that they both are red is 1/2? (4)

17. Let E and F be mutually exclusive events in the sample space of an experiment. Suppose experiment is repeated until either E or F occurs. Show that the probability of the event E occurs before event F is $P(E)/(P(E) + P(F))$.

18. Box 1 contains 1 white and 999 red balls. Box 2 contains 999 white and 1 red balls. A ball is drawn from a randomly selected box. If the ball is red what is the probability that it came from box 1? (0.999)

19. Box 1 contains 1000 bulbs of which 10% are defective. Box 2 contains 2000 bulbs of which 5% are defective. Two bulbs are drawn from a randomly selected box. (i) Find the probability that both bulbs are defective? (ii) Assuming that both are defective, find the probability that they came from box 1? (0.006, 0.8)

20. We have two coins. The first is fair and the second two headed. We pick one of the coins at random, we toss it twice and heads shows both times. Finds the probability that the coin picked is fair. (1/5)