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Department of Mathematics
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Tutorial Sheet-2

1. Let A and B be two events. Then verify that following statements are equivalent.
   (i) Events A and B are independent. (ii) Events A^c and B are independent. (iii) Events A and B^c are independent. (iv) Events A^c and B^c are independent.

2. A box contains three white balls w_1, w_2, w_3 and two red balls r_1 and r_2. We remove at random two balls in succession. What is the probability that the first removed ball is white and the second is red?

3. A box contains white and black balls. When two balls are drawn without replacement, suppose the probability that both are white is $1/3$. (i) Find the smallest number of balls in the box. (ii) How small can the total number of ball be if black balls are even in number?

4. Suppose box 1 contains a white balls and b black balls, and box 2 contains c white balls and d black balls. One ball of unknown color is transferred from the first box into the second one and then a ball is drawn from the later. What is the probability that it will be a white ball?

5. A certain test for a particular cancer is known to be 95% accurate. A person submits to the test and results are positive. Suppose that the person comes from a population of 100,000 where 2000 people suffer from that disease. What can be concluded about the probability that the person under test has that particular cancer?

6. Trains X and Y arrives at station at random between 8 AM and 8:20 AM. Train X stops for four minutes and Y stops for five minutes. Assuming that the trains arrive independently of each other, determine the probabilities: (i) That train X arrives before train Y. (ii) That trains meets at the station. (iii) Assuming that the trains met, determine the probability that train X arrived before train Y.

7. A biased coin is tossed till a head appears for the first time. What is the probability that the number of required tosses is odd?

8. In examining a past records of a corporation's account balances, an auditor finds that 15% of them have contained errors. Of those balances in error, 60% were regarded as unusual values based on historical figures. Of all the account balances, 20% were unusual values. If the figure for a particular balance appears unusual on this basis, what is the probability that it is in error?

9. A stock market analyst examined the prospects of the share of a large number of corporations. When the performance of these stocks was investigated one year later, it turned out that 25% performed much better than the market average, 25% much worse and the remaining 50% about the same as the average. Forty percent of the stocks that turned out to do much better than the market were rated 'good buy' by the analyst, as were 20% of those that did about as well the market and 10% of those that did much worse. What is the probability that a stock rated a 'good buy' by the analyst performed much better than the market average?

10. Suppose that each of N men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. What is the probability that: (i) none of the men selects his own hat (ii) exactly k of the men select their own hats?

11. Ten married couple are seated at random at a round table, compute the probability that no wife sits next to her husband.

12. A pair of dice is rolled until a sum of either 5 or 7 appears. Find the probability that a 5 occurs first.

$1 - P[A_1 \cap A_2 \cdots \cap A_n]$

13. There are n socks, 3 of which are red, in a drawer. What is the value of n if when 2 of the socks are chosen randomly, the probability that they both are red is $1/2$?

14. A box contain n identical balls numbered 1 through n. Suppose k balls are drawn in succession. (i) What is the probability that m is the largest number drawn? (ii) What is the probability that the largest number drawn is less than or equal to m?

15. Let E and F be mutually exclusive events in the sample space of an experiment. Suppose experiment is repeated until either E or F occurs. Show that the probability of the event E occurs before event F is $P(E)/(P(E)+P(F))$.

16. Two persons go target shooting together. Both shoot at a target at the same time. Suppose first person hits the target with probability 0.7 and second, independently, hits the target with probability 0.4. (i) Given that exactly one shot hit the target, what is the probability that it was the person shot? (ii) Given that the target is hit, what is the probability that second one hit?

17. A box contains m white and n black balls.Suppose k balls are drawn. Find the probability of drawing at least one white ball?

18. Three prisoners are informed by their jailer that one of them has been chosen at random to be executed, and the other two are to be freed. Prisoner A asks the jailer to tell him privately which of his fellow prisoners will be set free, claiming that there would be no harm in divulging this information since he already knows at least one will go free. The jailer refuses to answer this question pointing out that if A knew which of his fellows were set to be free then his own probability of being executed would rise from $1/3$ to $1/2$, since he would then be one of two prisoners. What do you think of jailer's reasoning?

19. In the New York State lottery , six numbers are drawn from the sequence of numbers 1 through 51. What is the probability that the six numbers drawn will have (i)) all one digit numbers? (ii) two one-digit and four two-digit numbers?

20. Box 1 contains 1 white and 999 red balls. Box 2 contains 999 white and 1 red balls.A ball is drawn from a randomly selected box. If the ball is red what is the probability that it came from box 1?

21. Box 1 contains 1000 bulbs of which 10 are defective. Box 2 contains 2000 bulbs of which 5 are defective.Two bulbs are drawn from a randomly selected box. (i) Find the probability that both bulbs are defective? (ii) Assuming that both are defective, find the probability that they came from box 1?

22. A train and a bus arrive at the station at random between 9 A.M. and 10A.M. The train stops for 10 minutes and the bus for x minutes. Find x so that the probability that the bus and the train will meet equals 0.5.

23. We have two coins. The first is fair and the second two headed. We pick one of the coins at random, we toss it twice and heads shows both times. Finds the probability that the coin picked is fair.