Indian Institute of Technology Patna

Department of Computer Science and Engineering

End Semester Examination, Autumn 2017-18

Discrete Mathematics (CS-206)

Full Marks: 60 **Date:** 20/11/2017 **Time:** 3 hours

Instructions:

1. Write your name, roll number in the answer sheet.
2. Marks for every question is shown with the question.
3. Write your roll number in every extra sheet you take.
4. Explanation for each answer is mandatory. No marks will be given for answers without proper explanation.

Q1. Prove by Mathematical Induction clearly indicating each steps **[4+6 marks]**

a) $\frac{1}{1*5}+\frac{1}{5*9}+\frac{1}{9*13}+....+\frac{1}{(4n-3)*(4n+1)}=\frac{n}{4n+1}$

b) $f_{n-1}*f_{n+1} = (f_n)^2+(-1)^n$, where f_n refers to the Fibonacci sequence.

Q2. Solve the following questions with proper explanation. **[2+4+4 marks]**

a) For two sets X = { 1, 1, A}, Y = {8, 9, B} $f(x) = x^3 + 9$ is a bijection from X to Y. What's the value of A+B?

b) Let f(x) and g(x) be functions. Prove, using contradiction method, that if f(g(x)) is one-to-one, then g(x) is one-to-one. That is, suppose that g(x) were not one-to-one and derive that f(g(x)) cannot be one-to-one.

c) Let X, Y be two sets where X = {1, 2, 3, 4, 5, 6} and Y = {a, b, c, d}. F denote the set of all possible functions defined from X to Y. Let g be randomly chosen from F. What is the probability of g being onto?

Q3. Solve the following questions with proper explanation. **[3+2+5 marks]**

a) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

b) Prove that if n is a positive integer, then n is even if and only if $7n+4$ is even.

c) In a room there are only two types of people, namely type A and type B. Type A always tells the truth and type B always lies. You give a fair coin to a person, without knowing which type he is from and tell her/him to toss it and hide the result till you ask for it. Upon asking, the person replies the following:

"The result of the toss is head if and only if I am lying."

What is the result of the toss? Justify using Propositional Logic.

Q4. Solve the following questions with proper explanation. **[3+3+4 marks]**

a) If G is a group in which $(ab)^i = a^i b^i$ for three consecutive integers i for all a,b $\in$ G. Show that G is abelian.

b) Let *(Z, *)* be an algebraic structure where Z is the set of integers and the operation $*$ is defined by $n * m = max(n, m)$. Check whether *(Z, *)* is a monoid?

c) $(G, +)$ and (H, *) are two groups that are defined over the field of real numbers and are related with via function $f : G \to H$, such that $f(x) = e^x$. Prove that G and H are isomorphic to each other.

Q5. Solve the following questions related to poset and Hasse diagram.

[2+2+4+2 marks]

a) Find the number of edges formed by the Hasse diagram for the relation $R = \{(a, b) : a \subseteq b\}$ created on the power set $A = \{1, 2, 3\}$.

b) Find the number of edges formed by the Hasse diagram for the relation $S = \{(a, b) : a \text{ divides } b\}$ created on the set $B = \{2, 3, 6, 12, 15, 48, 120, 240\}$.

c) A total order $\leq$ on a set ***S*** is a partial order along with one more condition: $\forall\ a, b \in S, a \leq b \text{ or } b \leq a$. A partial order is defined on a set $S = \{x, a_1, a_2, a_3, \cdots, a_n, y\}$ such that $x \leq a_i, \forall i$ and $a_i \leq y, \forall i$, where $n \geq 1$. What is the number of total orders on the set S(in terms of n) which contain the partial order $<$?

d) Let there be two partial ordering; $S1 = [\{1, 3, 6, 9, 12\}, divides]$ and $S2 = [\{1, 5, 25, 125\}, divides]$. Check whether S1 and S2 is/are lattice/s or not.

Q6. Solve the following questions with proper explanation. **[6+4 marks]**

a) Show that isomorphism of simple graphs is an equivalence relation.

b) Check whether the following graph is planar or not. Justify your claim.