CS225 Switching Theory

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Previous Class

Minimization/ Simplification **of Switching Functions**

K-map (SOP)

Quine-McCluskey (Tabular) Minimization

This Class

Quine-McCluskey (Tabular) Minimization

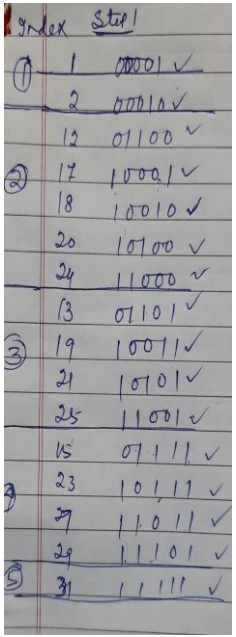
Combinational Circuit logic design

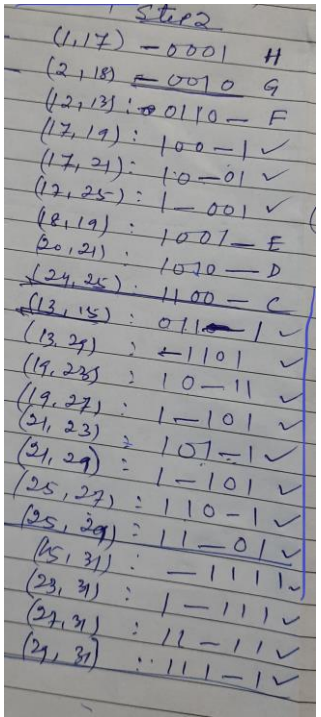
Quine-McCluskey Tabulation Procedure
Step 1: to Obtain the Set of All Prime implicants

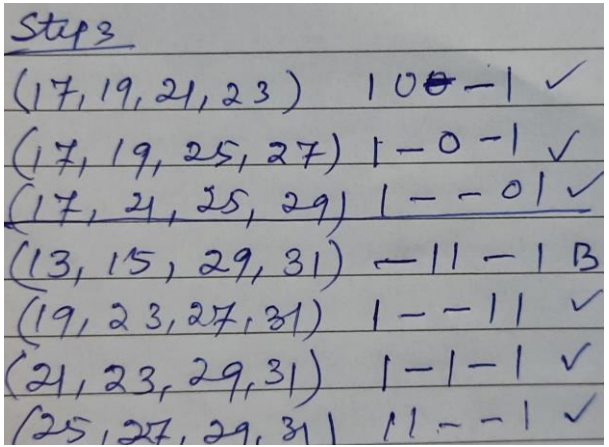
Example: with don't care condition

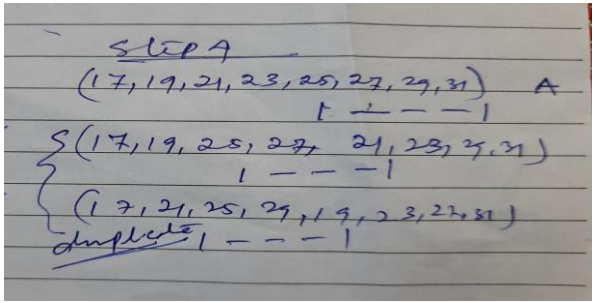
Ex.: $f_3(v,w,x,y,z) = \sum(13,15,17,18,19,20,21,23,25,27,29,31) + \sum_{\phi}(1,2,12,24)$

(a) (b) (c) (d)







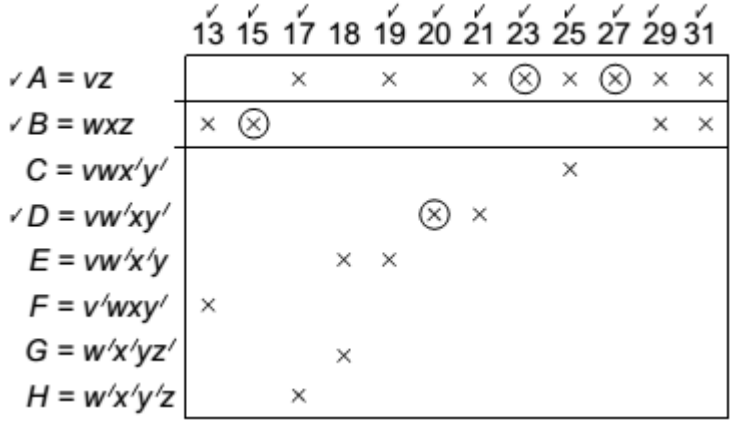


P= {vz, wxz, vwx'y', vw'xy', vw'x'y, v'wxy', w'x'yz', w'x'y'z'}

Step 2: Find the minimal expression(s)

Don't-cares: not listed as column headings in the prime implicant chart

Example: $f_3(v,w,x,y,z) = \sum(13,15,17,18,19,20,21,23,25,27,29,31) + \sum_{\phi}(1,2,12,24)$

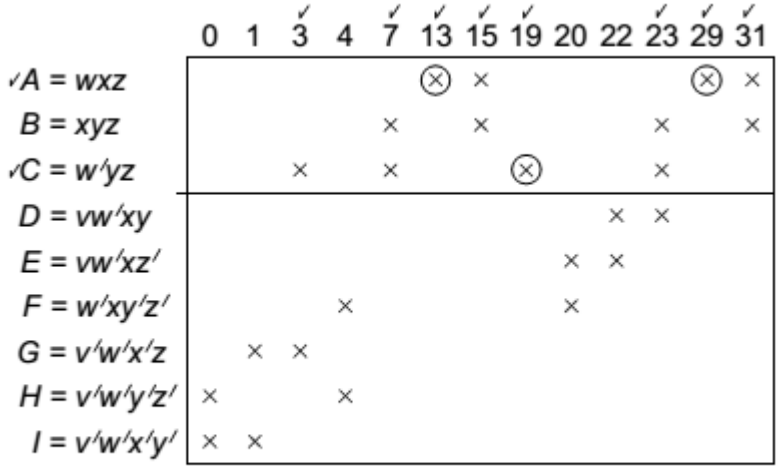


Selection of nonessential prime implicants facilitated by listing prime implicants in decreasing order of the number of minterms they cover

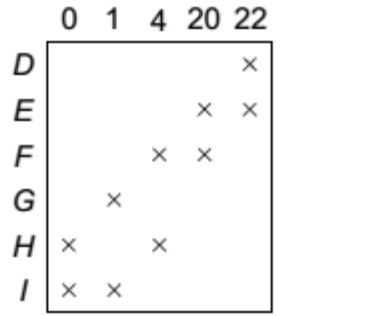
Essential prime implicants: A, B, and D. They cover all minterms except 18, which can be covered by E or G, giving rise to two minimal expressions

Determining the Set of All Irredundant Expressions

Deriving the minimal sum-of-products through prime implicant chart inspection: difficult for more complex cases *Example:* $f_4(v,w,x,y,z) = \sum(0,1,3,4,7,13,15,19,20,22,23,29,31)$



(*a*) Prime implicant chart.



(*b*) Reduced prime implicant chart.

While every irredundant expression must contain A and C, none of them may contain B since it covers minterms already covered by A and C. The reduced chart, obtained after removing A, B, and C, has two x's in each column

Example (Contd.)

Use propositional calculus: define prime implicant function p to be 1 if each

column is covered by at least one of the chosen prime implicants, and 0 if not

$$p = (H + I)(G + I)(F + H)(E + F)(D + E)$$
$$= EHI + EFI + DFI + EGH + DFGH$$

At least three rows are needed to cover the reduced chart:

E, H, and I, or E, F, and I, and so on

Since all prime implicants in the reduced chart have the same literal count, there are four minimal sum-of-products:

$$f_4(v,w,x,y,z) = A + C + E + H + I = wxz + w'yz + vw'xz' + v'w'y'z' + v'w'x'y'$$
$$f_4(v,w,x,y,z) = A + C + E + F + I = wxz + w'yz + vw'xz' + w'xy'z' + v'w'x'y'$$
$$f_4(v,w,x,y,z) = A + C + D + F + I = wxz + w'yz + vw'xy + w'xy'z' + v'w'x'y'$$
$$f_4(v,w,x,y,z) = A + C + E + G + H = wxz + w'yz + vw'xz' + v'w'x'z + v'w'y'z'$$

Reduction of the Chart

Aim: find just one minimal expression rather than all such expressions

Example: $f_5(v,w,x,y,z) = \sum(1,3,4,5,6,7,10,11,12,13,14,15,18,19,20,21,22,23,25,26,27)$

	1	3	4	5	6	7	10	11	12	13	14	15	18	19	20	21	22	23	25	26	27
✓ $A = w'x$			×	×	×	×									⊗	⊗	×	×			
✓ $B = v'x$			×	×	×	×			⊗	⊗	×	×									
$C = vx'y$													×	×						×	×
$D = vw'y$													×	×			×	×			
$E = wx'y$							×	×												×	×
$F = v'wy$							×	×			×	×									
$G = x'yz$		×						×						×							×
$H = w'yz$		×				×								×				×			
$I = v'yz$		×				×		×				×									
✓ $J = v'w'z$	⊗	×		×		×															
✓ $K = vwx'z$																			⊗		×

(*a*) Prime implicant chart.

Example (Contd.)

	10	11	18	19	26
C			×	×	×
D			×	×	
E	×	×			×
F	×	×			
G		×		×	
H				×	
I		×			

Reduced prime implicant chart

	10 ✓	11 ✓	18 ✓	19 ✓	26 ✓
✓C			⊗	×	×
✓E	⊗	×			×
G		×		×	

Final chart

Dominated row: row U of the chart dominates row V if U covers every column covered by V. If U does not have more literals than V then V can be deleted from the chart.

Example: I is dominated by G, D is dominated by C and F is dominated by E,H is dominated by G, so they can be deleted

From the final chart: $f_5(v,w,x,y,z) = A + B + J + K + C + E$

Dominating Column (Alternative choice)

	10	11	18	19	26
C			×	×	×
D			×	×	
E	×	×			×
F	×	×			
G		×		×	
H				×	
I		×			

Reduced prime implicant chart

Dominating column: column i of the chart dominates column j if i has an x in every row in which j has an x. Hence, dominating column i can be deleted.

Example: column 11 dominates column 10. In order to cover column 10, either E or F must be selected, whereby column 11 will also automatically be covered. Similarly, since column 19 covers column 18, column 19 can be deleted.

Final solution is still: $f_5(v,w,x,y,z) = A + B + J + K + C + E$

Quine-McCluskey Minimization (Reduction and dominance)
Aim: To find just one minimal expression

- ✓ Terms are initially listed one per line in groups
  - Each group contains terms with the same number of true and complemented variables
  - Terms are listed in numerical order within group
- ✓ Terms and implicants are identified using one of three common notations
  - full variable form
  - cellular form
  - 1,0,- form

Implication Table (1,0,-)

- ✓ Quine-McCluskey Method
  - Tabular method to systematically find all prime implicants
  - $f(A,B,C,D) = \Sigma\, m(1,2.5,6,7,9,10) + \Sigma\, d(0,13,15)$
  - Part 1: Find all prime implicants
  - Step 1: Fill Column 1 with active-set and DC-set minterm indices. Group by number of true variables (# of 1's).

NOTE THAT DCs ARE INCLUDED IN THIS STEP!

Implication Table		
Column I 0000 0001 0010 0101 0110 1001 1010 0111 1101 1111		

Minimization - First Pass (1,0,-)

- ✓ Quine-McCluskey Method
  - Tabular method to systematically find all prime implicants
  - $f(A,B,C,D) = \Sigma\, m(1,2.5,6,7,9,10) + \Sigma\, d(0,13,15)$
  - Part 1: Find all prime implicants
  - Step 2: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. One bit difference implies adjacent. Eliminate variable and place in next column.

    E.g., 0000 vs. 0100 yields 0-00

    0000 vs. 1000 yields -000

    When used in a combination, mark with a check. If cannot be combined, mark with a star. These are the prime implicants.

    Repeat until nothing left.

Implication Table		
Column I	Column II	
0000 ✓	000-	
	00-0	
0001 ✓		
0010 ✓	0-01	
	-001	
0101 ✓	0-10	
0110 ✓	-010	
1001 ✓		
1010 ✓	01-1	
	-101	
0111 ✓	011-	
1101 ✓	1-01	
1111 ✓	-111	
	11-1	

Minimization - Second Pass (1,0,-)

- ✓ Quine-McCluskey Method
  - Step 2 cont.: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. One bit difference implies adjacent. Eliminate variable and place in next column.

    E.g., 0000 vs. 0100 yields 0-00

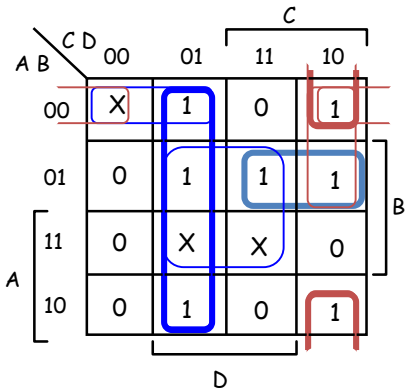
    00-0 vs. 10-0 yields -0-0

    When used in a combination, mark with a check. If cannot be combined, mark with a star. **THESE ARE THE PRIME IMPLICANTS.**

    Repeat until nothing left.
- ✓ The set of ✶ constitutes the Complete Sum $\sum_c$

Implication Table		
Column I	Column II	Column III
0000 ✓	000- *	--01 *
	00-0 *	- 1-1 *
0001 ✓		
0010 ✓	0-01 ✓	
	-001 ✓	
0101 ✓	0-10 *	
0110 ✓	-010 *	
1001 ✓		
1010 ✓	01-1 ✓	
	-101 ✓	
0111 ✓	011- *	
1101 ✓	1-01 ✓	
1111 ✓	-111 ✓	
	11-1 ✓	

Prime Implicants



Prime Implicants:

$000- = \overline{A}\,\overline{B}\,\overline{C}$

$00-0 = \overline{A}\,\overline{B}\,\overline{D}$

$0-10 = \overline{A}\,C\,\overline{D}$

$-010 = \overline{B}\,C\,\overline{D}$

$011- = \overline{A}\,B\,C$

$-1-1 = B\,D$

$--01 = \overline{C}\,D$

Stage 2: find smallest set of prime implicants that cover the active-set

Note that essential prime implicants must be in final expression

Coverage Table

Coverage Chart

		1	2	5	6	7	9	10
0,1	000-	X						
0,2	00-0		X					
2,6	0-10		X		X			
2,10	-010		X					X
6,7	011-				X	X		
1,5,9,13	--01	X		X			X	
5,7,13,15	-1-1			X		X		

NOTE: DON'T INCLUDE DCs IN COVERAGE TABLE; THEY DON'T HAVE COVERED BY THE FINAL LOGIC EXPRESSION!

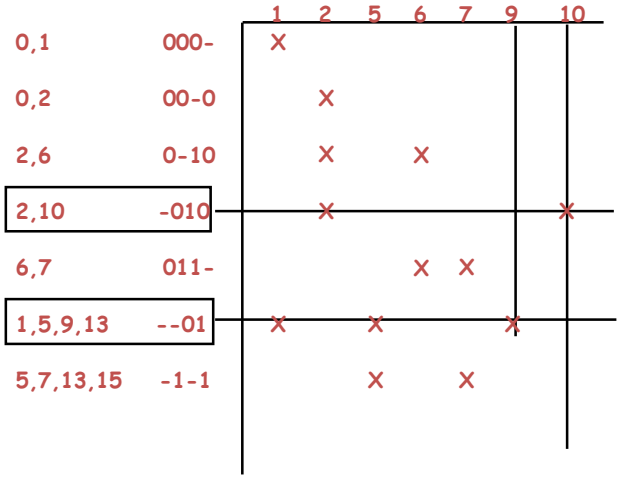
rows = prime implicants
columns = ON-set elements (minterms)

place an "X" if ON-set element is covered by the prime implicant

Coverage Table (cont.)

Coverage Chart

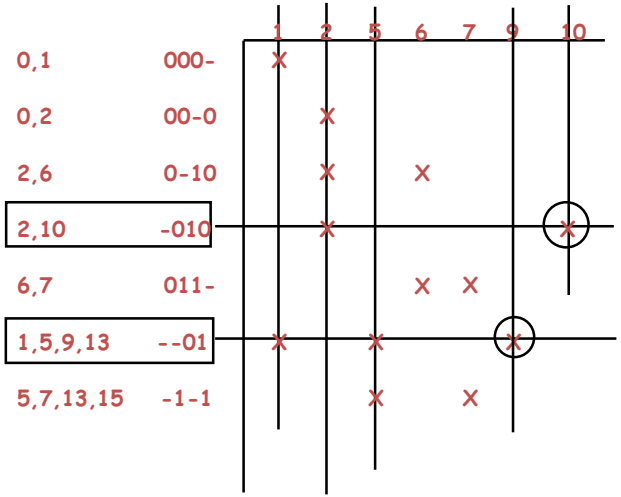
		1	2	5	6	7	9	10
0,1	000-	X						
0,2	00-0		X					
2,6	0-10		X		X			
2,10	-010		X					X
6,7	011-				X	X		
1,5,9,13	--01	X		X			X	
5,7,13,15	-1-1			X		X		



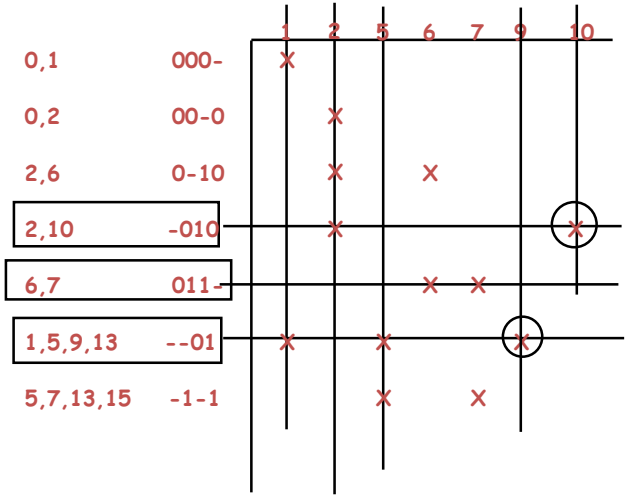
rows = prime implicants
columns = ON-set elements
place an "X" if ON-set element is covered by the prime implicant

If column has a single X, than the implicant associated with the row is essential. It must appear in minimum cover

Coverage Table (cont.)



Eliminate all columns covered by essential primes



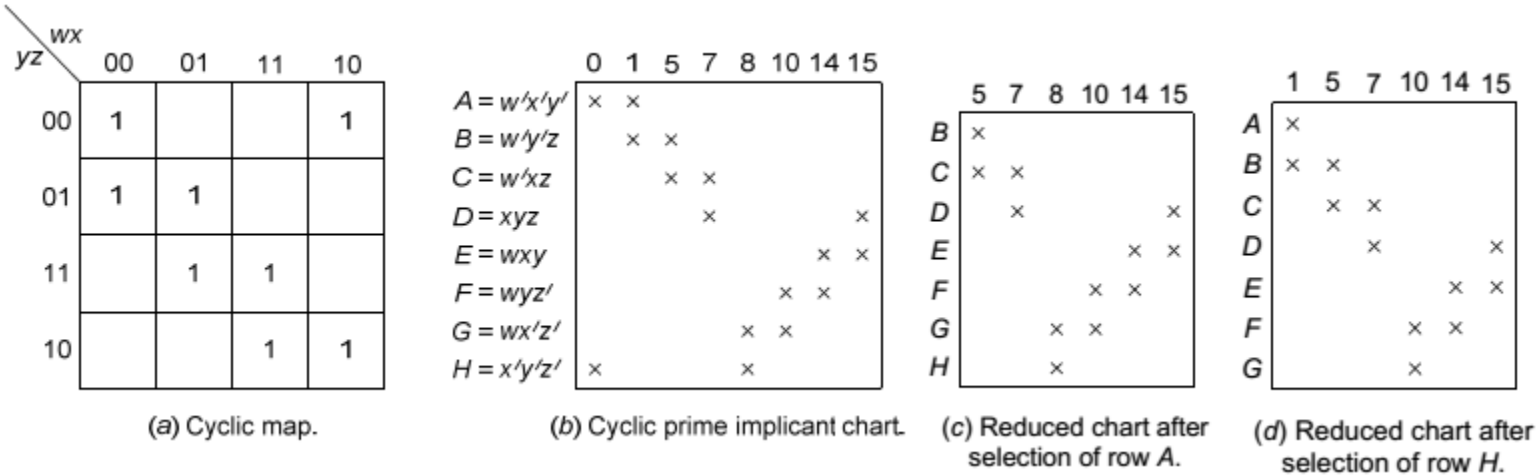
Find minimum set of rows that cover the remaining columns

$$F = \bar{B}C\bar{D} + \bar{A}BC + \bar{C}D$$

Branching Method

When chart has no essential prime implicant, dominated row or dominating column: use branching method

Example: $f_6(w,x,y,z) = (0,1,5,7,8,10,14,15)$



(*a*) Cyclic map. (*b*) Cyclic prime implicant chart. (*c*) Reduced chart after selection of row *A*. (*d*) Reduced chart after selection of row *H*.

To cover column 0:either A or H has to be selected
If A is arbitrarily chosen: C(G) dominates B(H): $f_6(w,x,y,z) = A+C+G+E$
If H is arbitrarily chosen: B(F) dominates A(G): $f_6(w,x,y,z) = H+B+D+F$

Thanks