Indian Institute of Technology, Patna
MA102, B.Tech -I year
Spring Semester: 2012-2013
(End Semester Examintaion)

Attempt all twelve problems. Worth of each problem is given. Total marks are 50.

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear mapping for which $T(1,2) = (2,3)$, $T(0,1) = (1,4)$, where $\{(1,2),(0,1)\}$ is a basis of $\mathbb{R}^2$. Find a formula for T, i.e., find $T(a,b)$. $[2\frac{1}{2}]$

2. The characteristic roots of a 3×3 matrix A are known to be in arithmetic progression. Determine them. Given that $trace(A) = 15, det(A) = 80$. $[2\frac{1}{2}]$

3. Use Gram-Schmidt process to obtain an orthogonal basis from the basis set $\{(1,0,1),(1,1,1),(1,3,4)\}$ of the Euclidean space $\mathbb{R}^3$ with standard inner product. [5]

4. If the functions $M(x,y)$ and $N(x,y)$ in the equation $M(x,y)dx + N(x,y)dy = 0$ are homogeneous functions of degree n and $Mx + Ny \neq 0$, then prove that $\frac{1}{Mx+Ny}$ is an integrating factor to make the given differential equation exact. [3]

5. Find the general solution of the following Riccati equation

$$y' = 2e^{-x}y^2 + 3y - 4e^x, \quad y = e^x \text{ is a particular solution.}$$

[3]

6. Using Picard's iteration method find the first four approximations to the solution of the following initial value problem

$$y' = x - 2y, \quad y(-1) = 1/4.$$

[4]

7. Using the method of variation of parameters, find the general solution of the differential equation

$$y'' + 6y' + 9y = \frac{e^{-3x}}{x}.$$

[5]

8. Using the method of undetermined coefficients, find the general solution of the differential equation

$$y'' + 4y' + 4y = 6e^{-2x}\cos^2 x.$$

[5]

9. Using operator method, find the general solution of the differential equation

$$(D^4 + 5D^2 + 4)y = 16\sin x + 64\cos 2x.$$

[5]

10. Find the general solution of the linear system of equations

$$\mathbf{y}' = \mathbf{A}\mathbf{y} + \mathbf{h} = \begin{pmatrix} 5 & -7 \\ 2 & -4 \end{pmatrix} y - \begin{pmatrix} 2 \\ 4 \end{pmatrix} e^t.$$

[5]

11. Find the general solution of the following equation using the power series method about the point $x_0 = 0$

$$(1 - x^2)y'' - 2xy' + 2y = 0.$$

[5]

12. Find the series solution of the following equation by the Frobenius method

$$4x^2y'' - 8xy' + 5y = 0.$$

Mention the indicial equation and the indicial roots. [5]