PH 201

OPTICS & LASERS

Lecture_Lasers_7

Requirements for Obtaining Population Inversions

Population inversion between upper & lower laser levels is essential in order to have amplification in a medium.

How inversions are created even though they seem to defy concept of thermal equilibrium?

It is not possible to have an inversion between two levels that are the only two levels of system.

Inversions & Two-Level Systems

Consider a hypothetical atom having just two levels *u* & *l* with an energy difference,

$$\Delta E_{ul} = E_u - E_l = h\upsilon_{ul}$$

Assume v_{ul} is in or near visible spectral region so that thermal excitation of upper level *u* from lower level *l* via collisions with other particles is negligible at room temp.

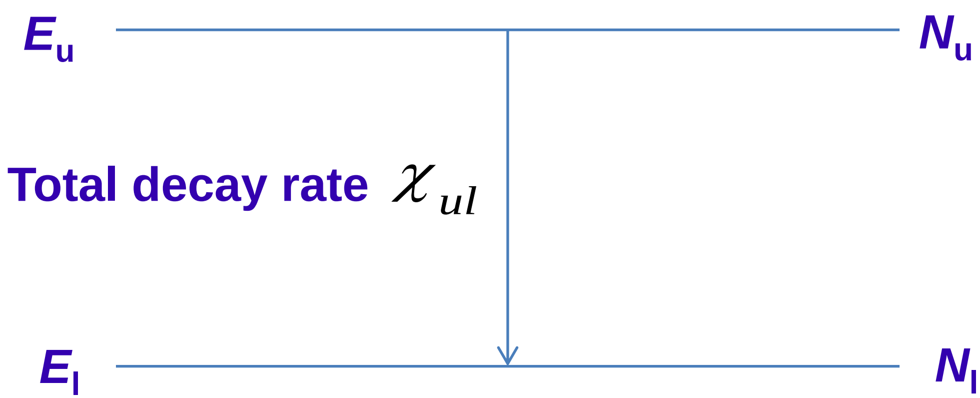
Whether or not an inversion could be created between these two levels?

Assume a cell of dimension *L* containing atoms of total density *N* at room temp, & that cell is square-shaped with optical-quality walls so that a beam could be transmitted through cell without being distorted by cell walls.

Total decay rate from upper to lower level is γ_{ul}, & radiative decay from level *u* to *l* occurs at a rate A_{ul}.

Thus, $\chi_{ul} > A_{ul}$

depending upon whether collisions increase decay rate from level *u* to level *l* above rate of radiative decay.



Atoms in cell are at room temp so that nearly all atoms would be in lower level (Boltzmann distribution). It means collisional or thermalizing excitation from *l* to *u* can be neglected.

Thus, initially $N_l \cong N$

***N* = Total population density of combined two levels**

We will attempt to pump atoms from *l* to *u* by shining light of intensity I_0 & frequency ν_{ul} into cell & will then examine intensity *I* emerging from opposite side after passing through cell.

In this case, light would be absorbed by atoms in level *l*, & for every absorbed photon, an atom would be promoted to level *u*.

$$I = I_0 e^{\sigma_{ul} \Delta N_{ul} z} = I_0 e^{\sigma_{ul} (N_u - N_l) L} \qquad \frac{g_u}{g_l} = 1$$

$$I = I_0 e^{-\sigma_{ul} N L} \qquad N_l \cong N$$

As soon as population is pumped up to level *u*, N_u becomes greater than zero.

$$N_u + N_l = N \qquad or \qquad N_u = N - N_l$$

$$I = I_0 e^{\sigma_{ul} [1 - 2(N_l / N)] N L}$$

As population leaves level *l*, ratio N_l/N begins to drop from a value of unity, & when it reaches 0.5, no more energy will be absorbed because value of exponent will be reduced to zero.

If no more energy can be absorbed, then there is no mechanism to increase population in level *u*.

Also, if $N_l/N = 0.5$ then $N_u = N_l$

& since no further absorption can occur, N_u never exceed N_l.

Radiative decay will continually reduce population in level *u*, which will also increase population in level *l* & therefore lead to more absorption. But N_l/N will never decrease to a value lower than 0.5, so there will never be a population inversion.

It is impossible to have a population inversion between levels *u* & *l* for a system with only those two energy levels.

Radiative Decay Rates – Radiative versus Collisional

Properties of radiative decay (spontaneous emission) & collisional (vibronic) decay from excited energy levels:

Radiative decay rate from any level *u* to a lower level *l*, $Rate \propto \upsilon_{ul}^{2}$

$$\Delta E_{ul} = E_u - E_l = h\upsilon_{ul}$$

$$A_{ul} \propto \Delta E_{ul}^{2}$$

Radiative transition probability from an excited level *u* increases very rapidly as energy separation between levels *u* & *l* increases.

Collisional decay rate constant associated with electron collisions transferring population from *u* to *l*,

$$k_{ul} = 7.8\,\frac{\lambda_{ul}^{3} A_{ul}}{T^{1/2}}$$

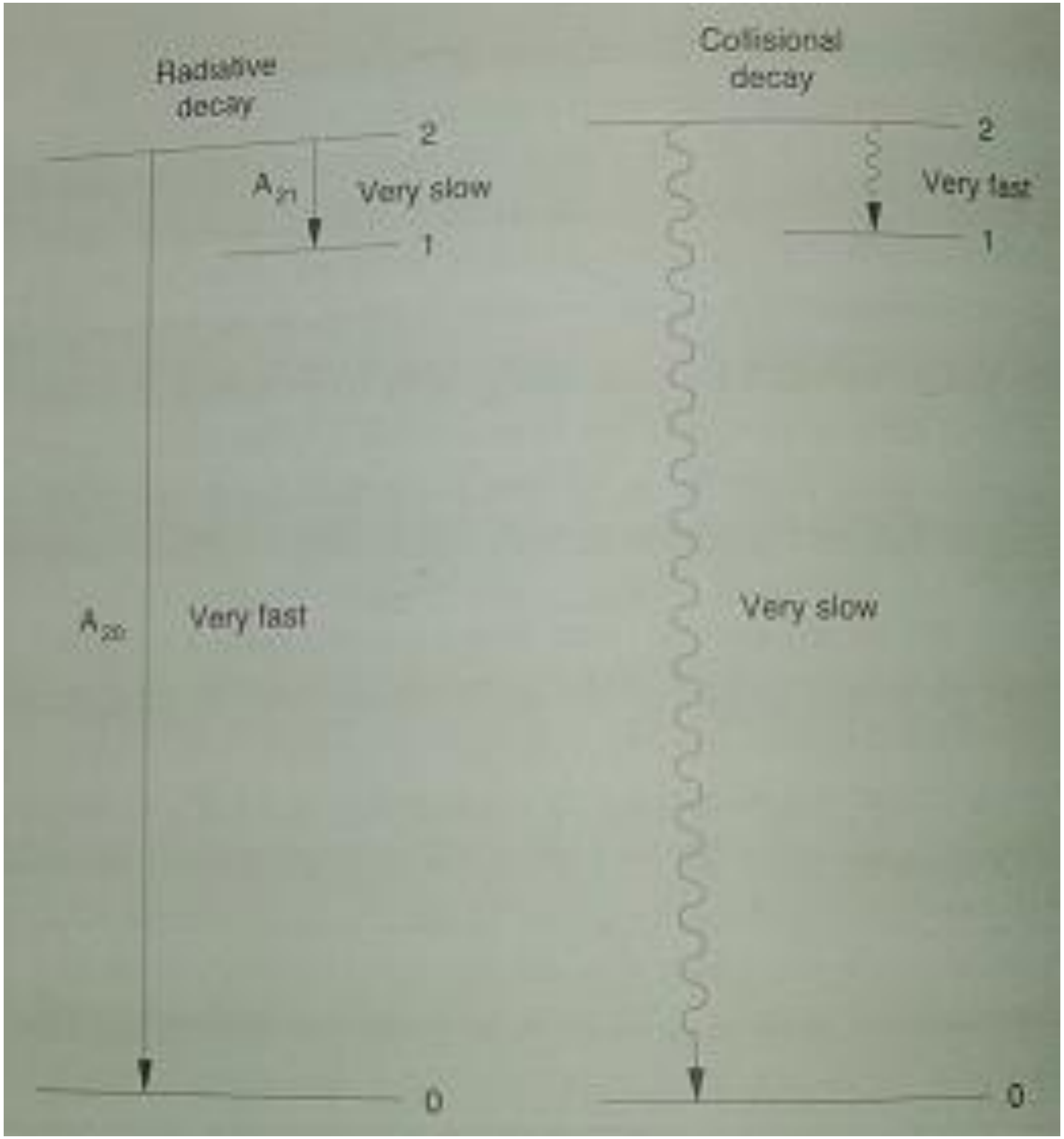
$$\lambda_{ul}^{3} = 1/(h\upsilon_{ul})^{3} = 1/\Delta E_{ul}^{3}$$

Relationship between k_{ul} & ΔE_{ul},

$$k_{ul} \propto \frac{1}{\Delta E_{ul}}$$

Collisional rate downward from level *u* is inversely proportional to energy separation ΔE_{ul} between *u* & *l*.

For rapid decay, radiative transitions like to have large energy separations between levels whereas collisional transitions downward prefer a close energy separation.



Comparison of radiative decay rates & collisional decay rates to nearby energy levels & to distant energy levels.

Steady-State Inversions in Three- & Four-Level Systems

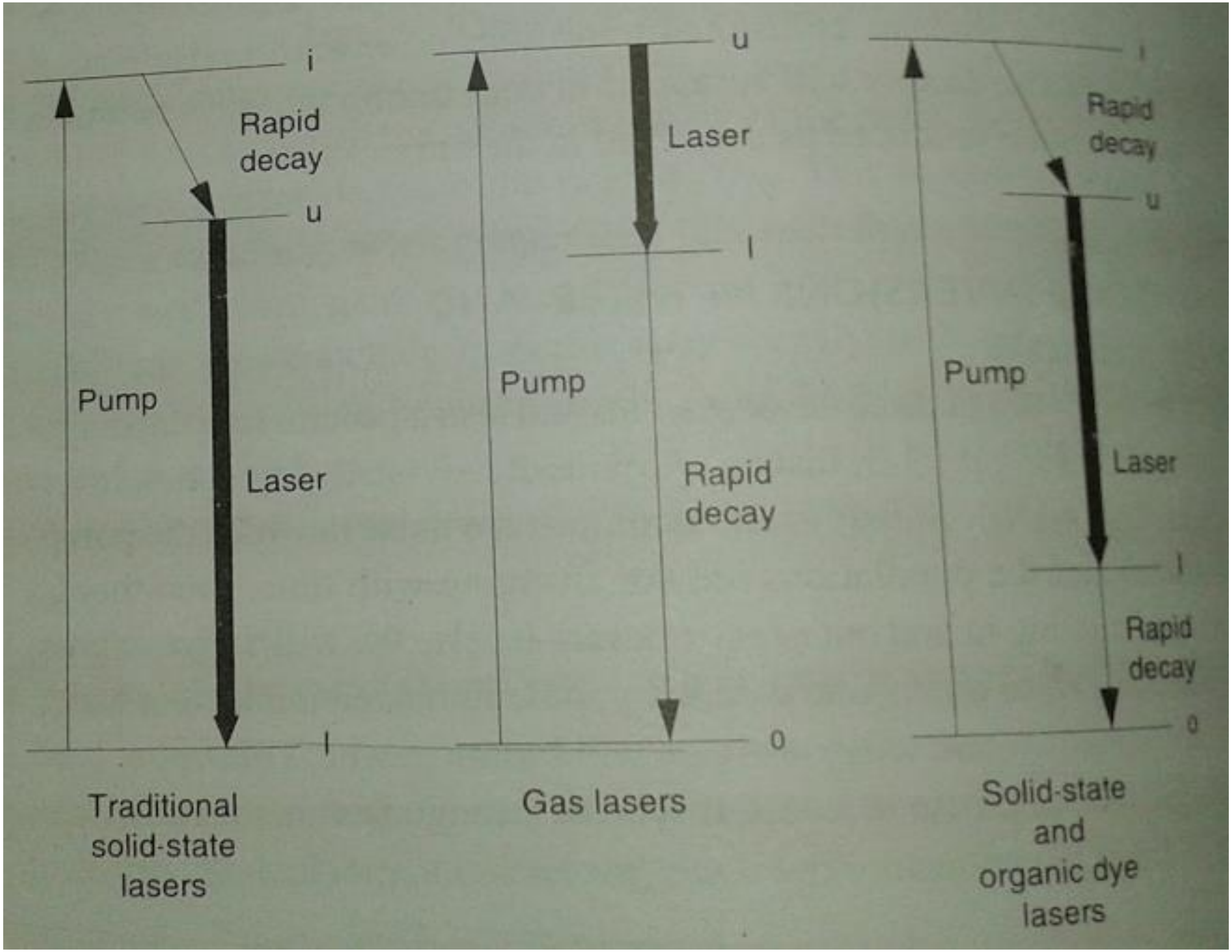
Steady-state conditions are those in which pumping flux is constant & populations are not changing with time, even though there is population moving in & out of relevant levels.

$$N_u > \left(\frac{g_u}{g_l}\right) N_l$$

Derive expressions for ratio N_u/N_l to determine where a population inversion is obtained.

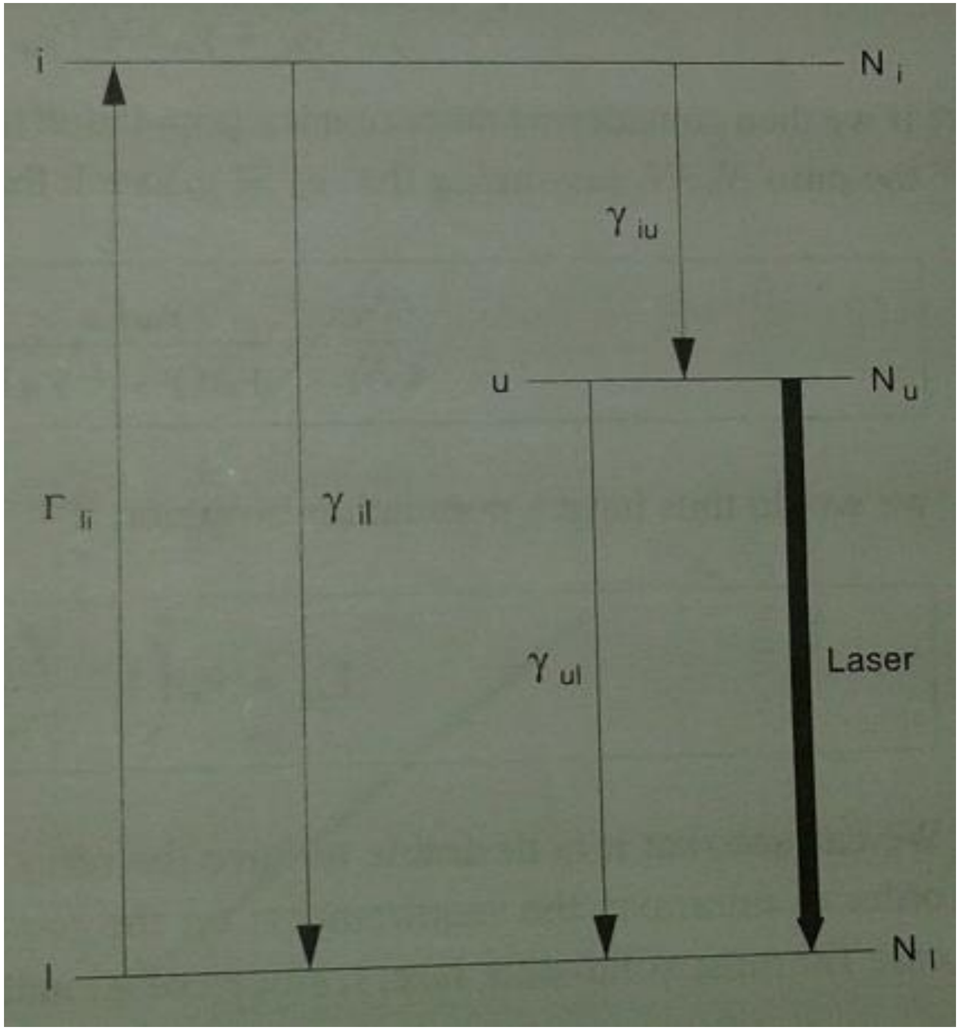
Consider following cases:

- **Upper level is intermediate of three levels – Ruby laser**
- **Upper of three levels is upper laser level – Gas lasers**



Energy-level arrangements & pumping & decay processes for three- & four-level lasers

Three-level laser with intermediate level as Upper laser level



Energy levels & relevant excitation & decay processes for three-level laser

We have levels: *i*, *u*, *l* with populations N_i, N_u, N_l & $E_i > E_u > E_l$.

Assume that laser transition occurs from level *u* to level *l*.

We also assume that gain medium is in thermal equilibrium before we begin pumping from level *l* to level *i* at a rate of Γ_{li}.

Decay from level *i* to level *u* – χ_{iu}

Decay from level *i* to level *l* - χ_{il}

Decay from level *u* to level *l* - χ_{ul}

We assume no thermalizing excitation from ground state level *l* to levels *u* & *i* since it is assumed that energies of *u* & *i* are sufficiently high that such processes are very small.

$$N_u \chi_{ul} = N_l \chi_{lu}$$

Considering Boltzmann distribution,

$$\frac{N_u}{N_l} = e^{-\Delta E_{ul} / kT}$$

& $g_u = g_l$

$$\frac{\chi_{lu}}{\chi_{ul}} = e^{-(E_u - E_l)/kT}$$

At room temp, χ_{lu} & χ_{lu}

are very small for any value of *E*$_u$ associated with visible & near infrared lasers.

Rate equations,

$$\frac{dN_l}{dt} = -\Gamma_{li} N_l + \chi_{ul} N_u + \chi_{il} N_i = 0$$

$$\frac{dN_u}{dt} = -\chi_{ul} N_u + \chi_{iu} N_i = 0$$

$$\frac{dN_i}{dt} = +\Gamma_{li} N_l - (\chi_{il} + \chi_{iu}) N_i = 0$$

Considering steady-state solutions, they have been equated to zero.

Γ – Externally applied pumping or excitation rate

χ – Excitation or decay process inherently associated with medium

χ_{lu}, χ_{li}, & χ_{ui} – Neglected because energy separations are much greater than kT

Solutions for populations of upper & lower laser levels N_u & N_l in terms of total population N.

$$N_i + N_u + N_l = N$$

$$N_l = \frac{\chi_{ul}(\chi_{iu} + \chi_{il})}{\chi_{ul}(\chi_{il} + \chi_{iu}) + (\chi_{ul} + \chi_{iu})\Gamma_{li}} N$$

$$N_u = \frac{\chi_{iu}\Gamma_{li}}{\chi_{ul}(\chi_{il} + \chi_{iu}) + (\chi_{ul} + \chi_{iu})\Gamma_{li}} N$$

If we then consider whether or not a population inversion can occur, we investigate ratio N_u/N_l (assuming $g_u = g_l$).

$$\frac{N_u}{N_l} = \frac{\chi_{iu}\Gamma_{li}}{\chi_{ul}(\chi_{il} + \chi_{iu})} > 1$$

We would thus have a population inversion if

$$\Gamma_{li} > \chi_{ul}\left(1 + \frac{\chi_{il}}{\chi_{iu}}\right)$$

It is desirable to have ratio $\frac{\chi_{il}}{\chi_{iu}}$ be as small as possible in order to minimize requirements on pumping rate.

This is the case with most solid state lasers, since collisional depopulation occurs much more frequently between nearby energy levels than between levels separated by a large energy.

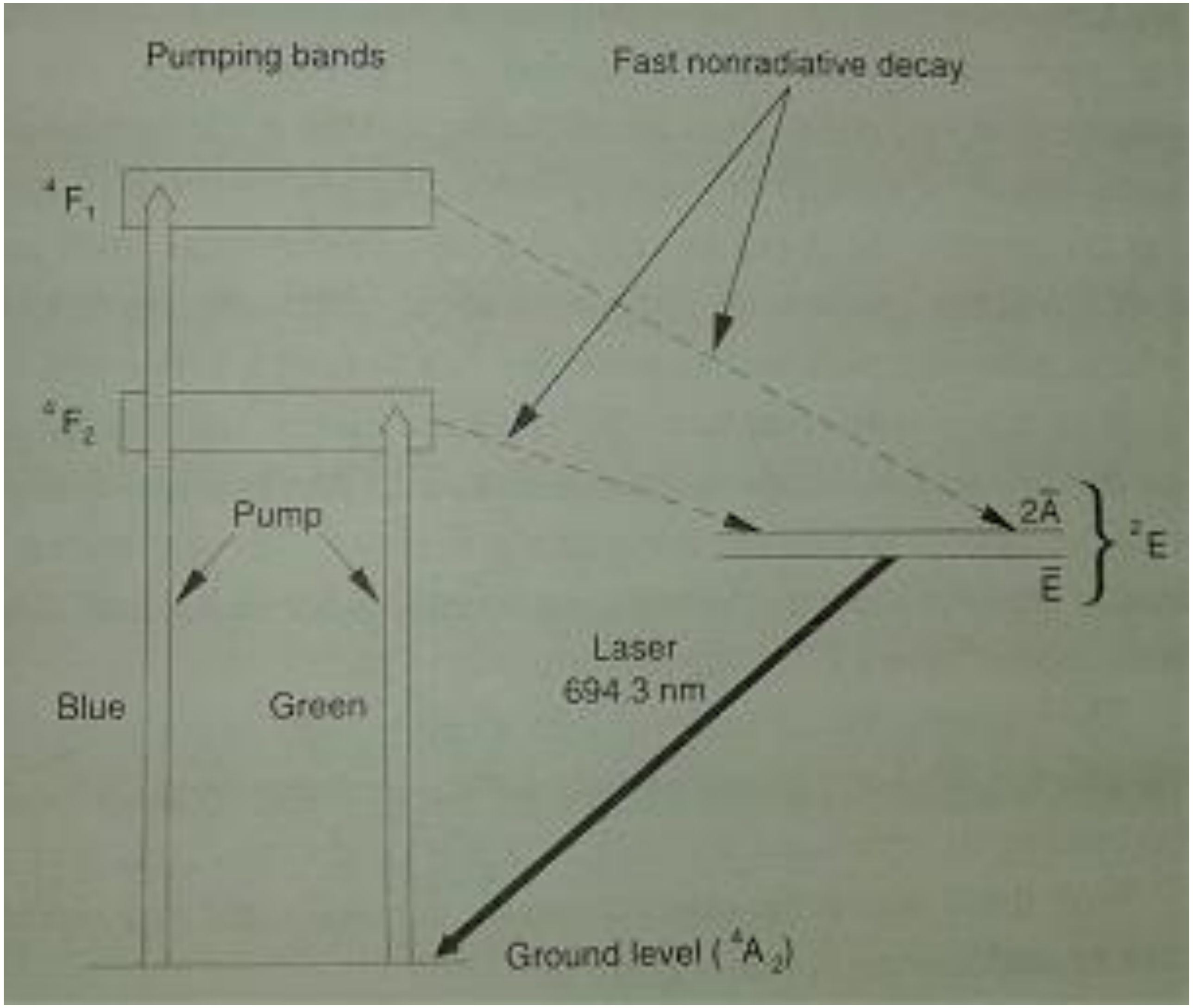
If $\frac{\chi_{il}}{\chi_{iu}}$ is small & if decay from level *u* to *l* is primarily by radiative decay at a rate A_{ul}, then $\chi_{ul} \cong A_{ul}$

Inversion is produced if $\Gamma_{li} > A_{ul}$

Hence for this type of three-level laser, if pumping rate exceeds radiative rate then population will be transferred to level *i* & then to level *u* by rapid collisional decay, which eventually produces an inversion between level *u* & level *l*.

Thus, for extremely high pumping rates, most of population will reside in level *u* & thereby allow production of large gains.

EXAMPLE: RUBY LASER



Cr:Al_2O_3

Energy level diagram of three-level Ruby laser

- **Ruby laser** consists of transition metal chromium in the form of Cr^{3+} ions doped in a sapphire (Al_2O_3) host crystal – $Cr{:}Al_2O_3$ – at a concentration of order of 1%.

- Pumping is achieved by absorption of pump light on transitions from ground level to $^4F_1$ excited level in blue spectral region & to $^4F_2$ excited level in green.

- Both these transitions have a broad range absorption spectrum, allowing a broad-spectrum pumping lamp to be used with reasonable efficiency.

- Population pumped to these excited states rapidly decays to ^2E state, where a population inversion builds up on Ê upper laser level with respect to ground state $^4A_2$, & laser action occurs on radiative transition between those two states.

- **Broad excited level – level *i* :** $\chi_{iu} >> \chi_{il}$ **since collisional relaxation or phonon relaxation occurs preferentially to energetically nearby upper laser level rather than to more distant ground state.**

- **Also, decay from upper level is primarily due to radiative decay, since dopant ion in this state is shielded from surrounding medium & thereby protected from detrimental collisions.**

$$\chi_{ul} \cong A_{ul} \cong 1/\tau_u$$

$$For\ Ruby \quad \tau_u = 3ms, \quad 1/\tau_u = 3.33\times10^2 s^{-1}$$

$$\Gamma_{li} > A_{ul} = 1/\tau_u$$

Threshold pumping condition

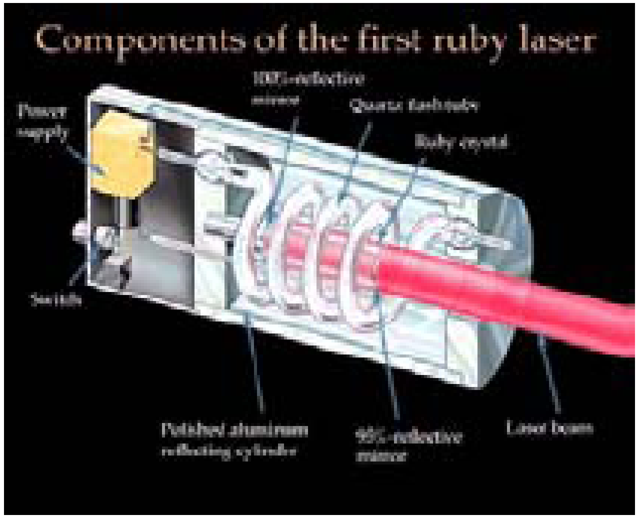
- **Pumping flux (photons per cubic meter per second) =** $N_l\Gamma_{li}$

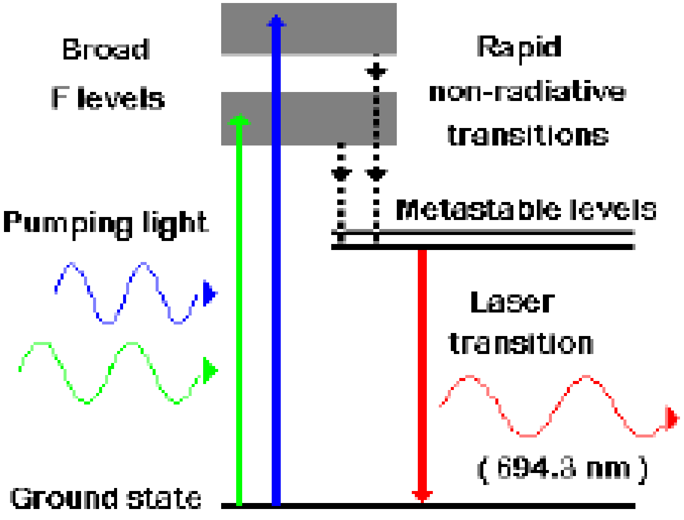
This is necessary to reach a population inversion.

Solid State Lasers

Ruby Laser:
$\lambda = 694.3$ nm

- First laser invented (1960).
- Ruby: Al_2O_3 in which some of the Al atoms have been replaced with Cr.
- Cr gives its characteristic red color and is responsible for the lasing behavior of the crystal.
- Cr atoms absorb green and blue light and emit or reflect only red light.





Energy levels of chromium ions in ruby