भारतीय प्रौद्योगिकी संस्थान पटना

INDIAN INSTITUTE OF TECHNOLOGY PATNA

PH101 (Physics-I) Mid-Semester Examination (September 15, 2014)

[Full Marks: 50] [Time: 120 minutes]

- All the questions are compulsory.
- Answers must be to the point. Refrain from writing essays!.
- Answers to all parts of a given question must be answered together.
- Marks for the questions are given in bold within square brackets.

1. Fill in the blanks using most suitable words/short phrases/mathematical expressions (wherever applicable):
(a) In the case of a mass moving under a central force, ________ are conserved.
(b) Elemental volume dV in cylindrical polar coordinate system is given by ________.
(c) For a conservative force $\vec{F}$, ________.
(d) The number of degrees of freedom for a rigid body is ________.
(e) Coriolis force acts on an object ________.
(f) For a Foucault's pendulum, the plane of ________ in the northern hemisphere and ________ in the southern hemisphere.
[6]

2. An object of mass m is projected vertically upwards with initial speed u. In absence of any air drag, the maximum height reached by the object is $\frac{u^2}{2g}$, where g is the acceleration due to gravity.
(a) In the presence of a linear air drag proportional to mass and velocity of the object (with κ being the proportionality constant, *i.e.*, the drag coefficient), show that the maximum height z_{max} reached by the object is given by $\frac{u}{\kappa} - \frac{g}{\kappa^2} ln(1 + \frac{\kappa u}{g})$.
(b) For small values of κ, obtain an approximate expression for z_{max} in (a) above [truncate the series to terms proportional to κ^2 in the final result].
(c) For $u = 20m/s$ and $g = 10m/s^2$, obtain the percentage change in the maximum height reached by the object in the presence of air drag than when it is absent. Assume $\kappa = 0.001$ **[7]**

3. An F16 fighter aircraft can can accelerate uniformly to 400 m/s in 10 sec. Its maximum braking rate cannot exceed $5g$. Obtain the minimum time required to travel 40 km, assuming it begins and ends at rest [assume $g = 10m/s^2$]. **[5]**

4. A particle of mass m which moves in a plane is acted upon by a force $\vec{F_\theta} = m\dot{r}\dot{\theta}\hat{\theta}$. Show that $\dot{r} = \sqrt{Alnr + B}$, where A and B are constants which can be determined from initial conditions of motion. **[5]**

5. Set up the differential equation for a simple harmonic oscillator in the presence of a damping force proportional to velocity. Illustrate graphically the difference between underdamped, overdamped and critically damped motions. **[5]**

6. A chain of mass M and length L is suspended vertically with its lowest end touching a scale. The chain is released and falls onto the scale. What is the reading of the scale when a length of chain, x, has fallen? **[5]**

7. For a given rigid body, moments of inertia about the principal axes are I_1, I_2 and I_3, such that $I_1 > I_2 > I_3$. Using Euler's equations, show that the rotational motion is unstable about the 2-axis. **[6]**

8. A mass m moves in a central force field with the potential $V(r) = -\frac{\alpha}{r}$ $(\alpha > 0)$. A quantity $\vec{W}$ called the Laplace-Runge-Lenz vector is constructed such that $\vec{W} = \frac{1}{m\alpha}(\vec{L} \times \vec{p}) - \hat{r}$.
(a) Show that $\vec{W}$ is a conserved quantity.
(b) Provide a suitable geometrical interpretation of $\vec{W}$. **[6]**

9. A surveillance boat races across the ocean at the equator at a speed of 300 km/h. Obtain the fractional change in gravity experienced by the driver of the surveillance boat, when the boat heads: (a) westward, (b) southward. **[5]**

END OF EXAM PAPER

(157)