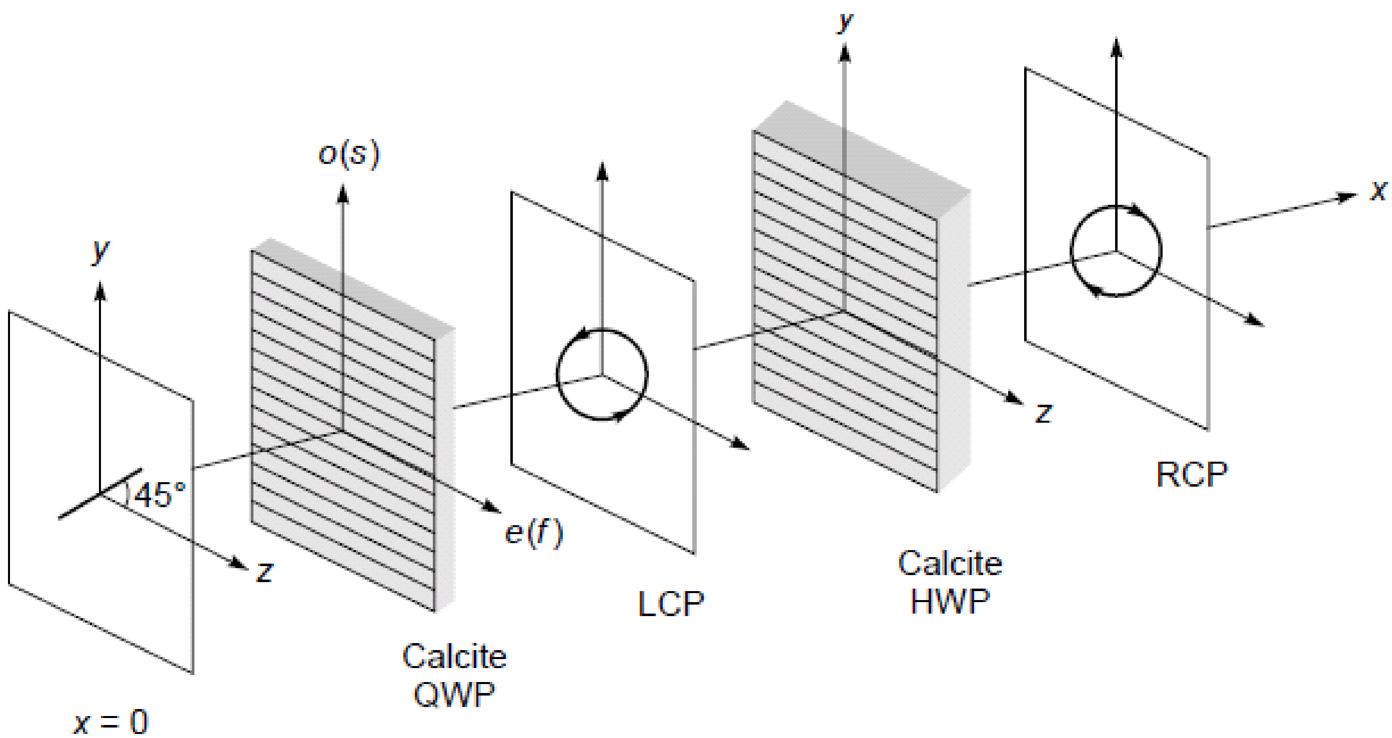
PH 201

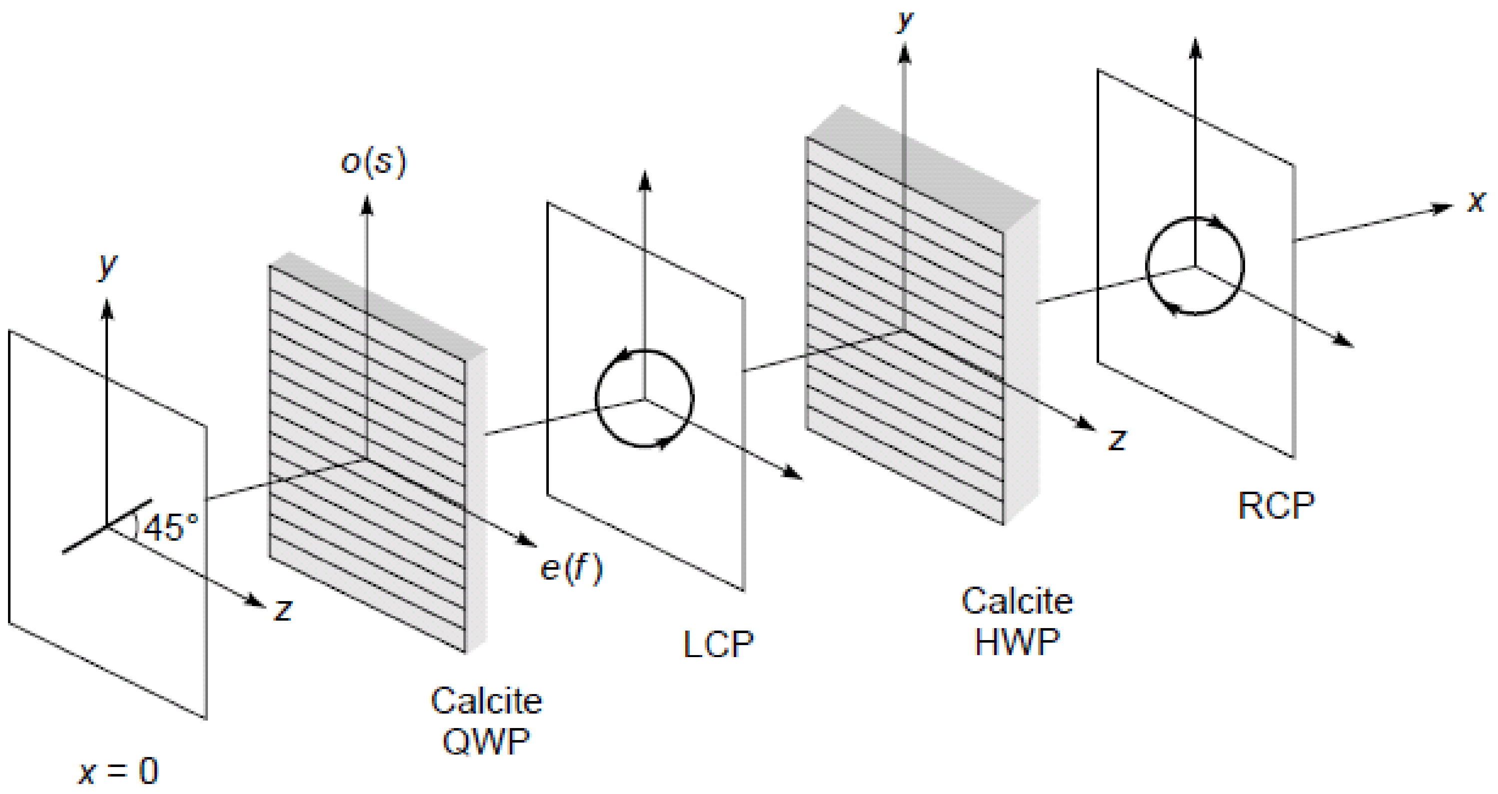
OPTICS & LASERS

Lecture_Polarization_4

Interference of Polarized Light:
Quarter Wave Plates & Half Wave Plates

- Consider normal incidence of a plane polarized beam on a calcite crystal whose optic axis is parallel to surface of crystal. Assuming z axis to be along optic axis.





A linearly polarized light making an angle 45º with *z* axis gets converted to a LCP after propagating through a calcite QWP; further, a LCP gets converted to a RCP after propagating through a calcite HWP. Optic axis in QWP & HWP is along *z* direction. [*s*-slow, *f*-fast]

- If incident beam is *y*-polarized, beam will propagate as an *o*-wave & *e*-wave will be absent.

- If incident beam is *z*-polarized, beam will propagate as an *e*-wave & *o*-wave will be absent.

- For any other state of polarization of incident beam, both extraordinary & ordinary components will be present.

- For a negative crystal such as calcite $n_e < n_o$, & *e*-wave will travel faster than *o*-wave.

- Let electric vector (of amplitude E_0) associated with incident polarized beam make an angle Φ (arbitrary angle) with z axis.

- Such a beam can be assumed to be a superposition of two linearly polarized beams (vibrating in phase), polarized along y & z directions with amplitudes; $E_0\sin\Phi$ & $E_0\cos\Phi$, respectively.

- z component (whose amplitude is $E_0\cos\Phi$) passes through as an *e*-beam propagating with wave velocity c/n_e.

- y component (whose amplitude is $E_0\sin\Phi$) passes through as an *o*-beam propagating with wave velocity c/n_o.

- Since $n_e \neq n_o$, two beams will propagate with different velocities; as such, when they come out of crystal, they will not be in phase. Consequently, emergent beam (superposition of these two beams) will be, in general, **elliptically polarized**.

- Let plane x = 0 represent surface of crystal on which beam is incident. y & z components of incident beam can be written as

$$E_y = E_0 \sin\phi \cos(kx - \omega t)$$

$$E_z = E_0 \cos\phi \cos(kx - \omega t)$$

- where k (= ω/c) represents free space wave number. Thus, at x = 0,

$$E_y(x=0) = E_0 \sin\phi \cos\omega t$$

$$E_z(x=0) = E_0 \cos\phi \cos\omega t$$

- Inside crystal, two components will be given by

$$E_y = E_0 \sin\phi \cos(n_0 kx - \omega t) \qquad o - wave$$

$$E_z = E_0 \cos\phi \cos(n_e kx - \omega t) \qquad e - wave$$

- If thickness of crystal is d, then at emerging surface, we have

$$E_y = E_0 \sin\phi \cos(\omega t - \theta_o)$$

$$E_z = E_0 \cos\phi \cos(\omega t - \theta_e)$$

$$\theta_o = n_o kd, \quad \theta_e = n_e kd$$

- By appropriately choosing the instant t = 0, components may be rewritten as

$$E_y = E_0 \sin\phi \cos(\omega t - \theta)$$

$$E_z = E_0 \cos\phi \cos\omega t$$

- where $$\theta = \theta_o - \theta_e = kd(n_o - n_e) = \frac{\omega}{c}(n_o - n_e)d$$

  represents phase difference between *o*- & *e*-beams.

- If thickness *d* of crystal is such that $\theta = 2\pi, 4\pi, \ldots.$ the emergent wave will have same state of polarization as incident beam.

- If thickness *d* of crystal is such that $\theta = \pi/2$, the crystal is said to be a **quarter wave plate** (QWP) – a phase difference of $\pi/2$ implies a path difference of $\lambda/4$.

- If thickness *d* of crystal is such that $\theta = \pi$, the crystal is said to be a **half wave plate** (HWP) – a phase difference of π implies a path difference of $\lambda/2$.

Example:

- Consider $\Phi = \pi/4$ & $\theta = \pi/2$, i.e., y & z components of incident wave have equal amplitudes, & crystal introduces a phase difference of $\pi/2$. For emergent wave we have

$$E_y = \frac{E_0}{\sqrt{2}}\sin\omega t \qquad\qquad E_z = \frac{E_0}{\sqrt{2}}\cos\omega t$$

which represents a circularly polarized wave because

$$E_y^2 + E_z^2 = \frac{E_0^2}{\sqrt{2}}$$

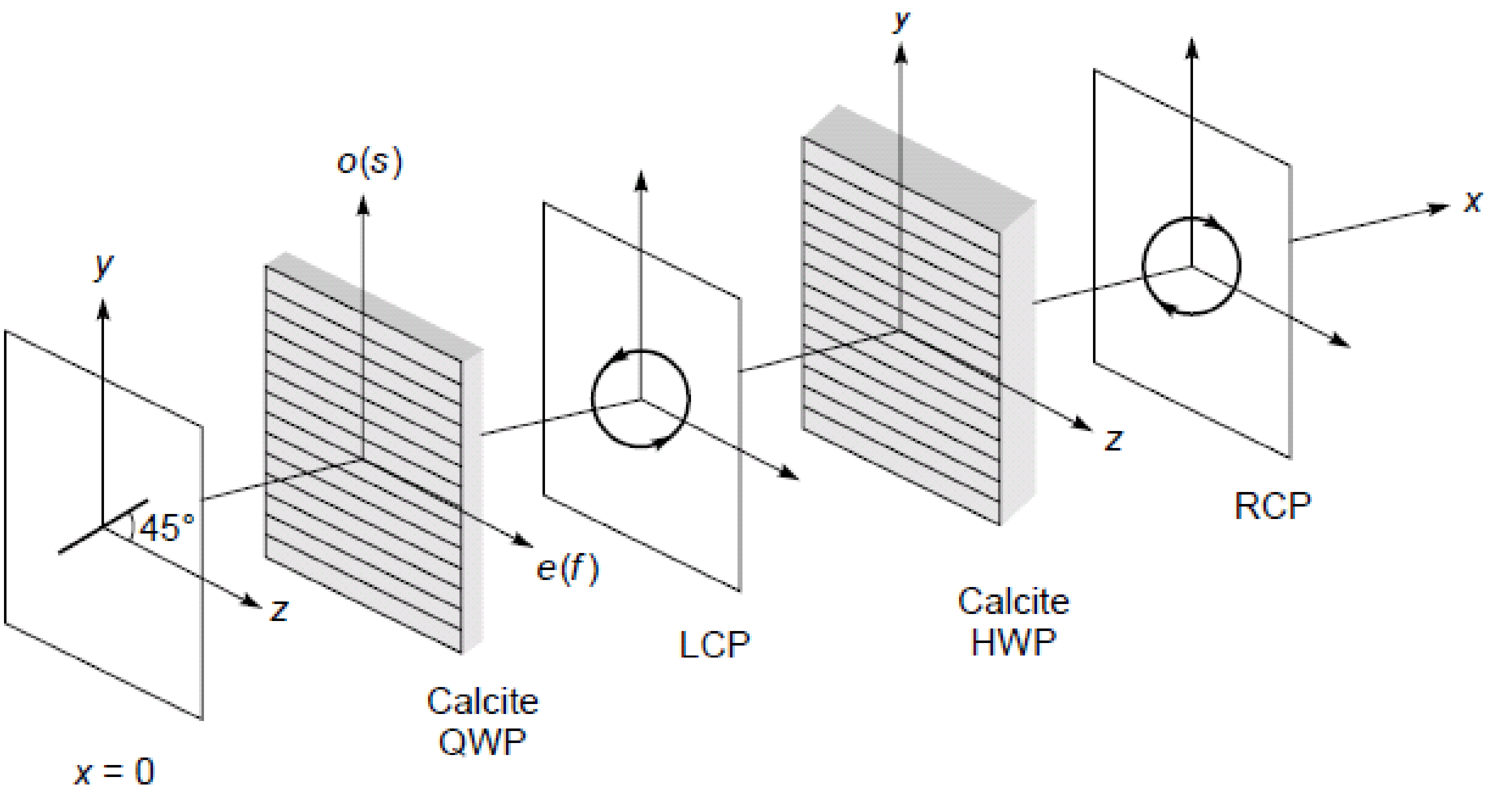
- To determine direction of rotation of electric vector, we note that at $t = 0$

$$E_y = 0 \qquad\qquad E_z = \frac{E_0}{\sqrt{2}}$$

& at $t = \Delta t$,

$$E_y \approx \frac{E_0}{\sqrt{2}}\omega\Delta t \qquad\qquad E_z \approx \frac{E_0}{\sqrt{2}}$$

These Eqs. show that as time increases, electric vector rotates in counterclockwise direction & hence beam is left circularly polarized.



A linearly polarized light making an angle 45º with *z* axis gets converted to a LCP after propagating through a calcite QWP; further, a LCP gets converted to a RCP after propagating through a calcite HWP. Optic axis in QWP & HWP is along *z* direction. [*s*-slow, *f*-fast]

- To introduce a phase difference of $\pi/2$, the thickness of crystal should have the value given by

$$d = \frac{c}{\omega(n_o - n_e)} \frac{\pi}{2} = \frac{1}{4} \frac{\lambda_0}{n_o - n_e}$$

where λ_0 is free space wavelength.

- For Calcite, n_0 = 1.65836, n_e = 1.48641 which correspond to λ_0 = 5893 Å at 18ºC. Substituting these values, we obtain

$$d = \frac{5893 \times 10^{-8}}{4 \times 0.17195} cm \approx 0.000857\ mm$$

- Thus a Calcite QWP (at λ_0 = 5893 Å) will have a thickness of 0.000857 mm & will have its optic axis parallel to surface; such a QWP will introduce a phase difference of $\pi/2$ between *o*- & *e*-components at λ_0 = 5893 Å.

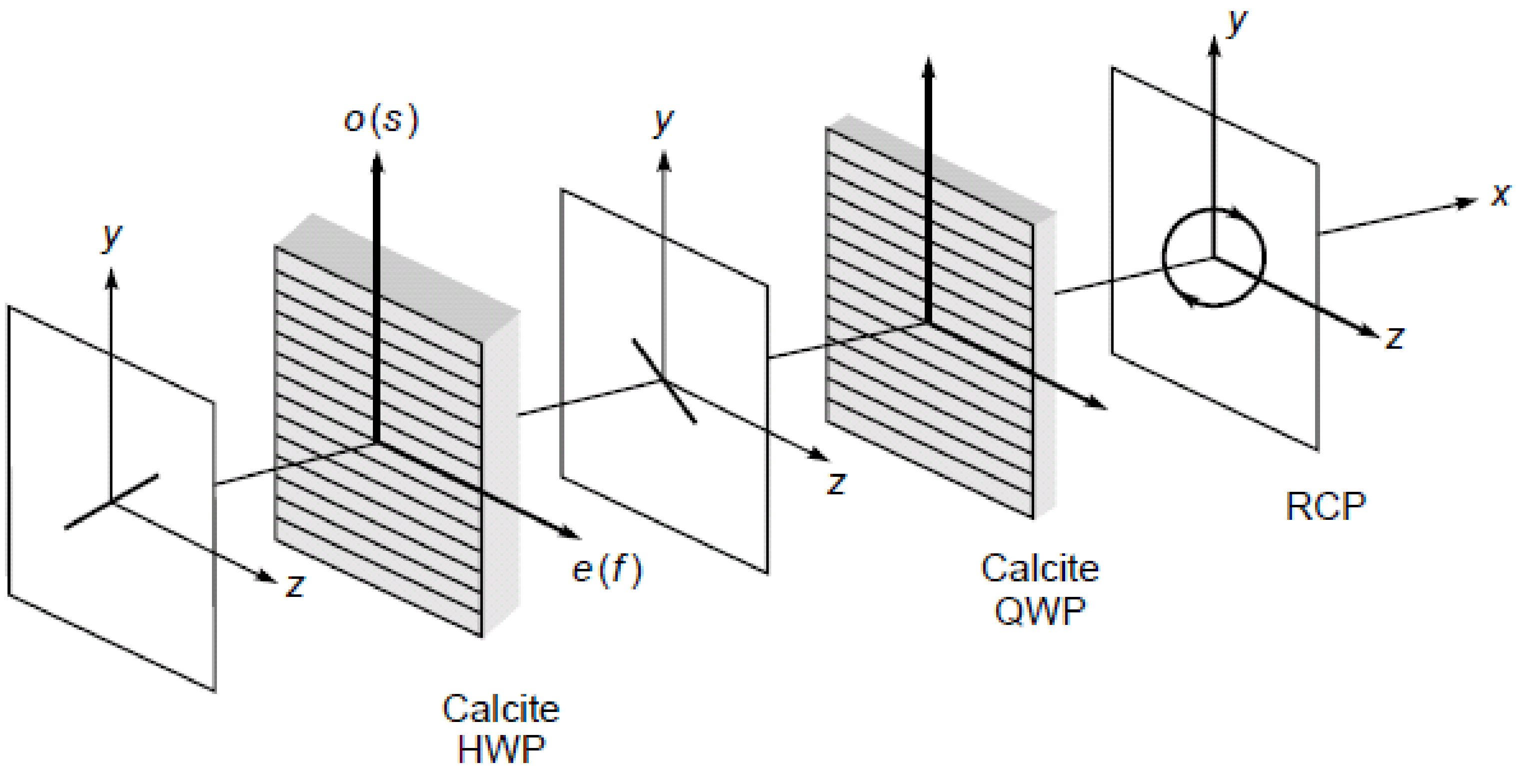
- If thickness is an odd multiple of above quantity, i.e., if

$$d = (2m+1)\frac{1}{4}\frac{\lambda_0}{n_o - n_e} \qquad m = 0,1,2....$$

then in Ex. considered above (i.e. when $\Phi = \pi/4$) it can be shown that emergent beam will be **left circularly polarized** for m = 0, 2, 4, … & **right circularly polarized** for m = 1, 3, 5, ….

y-polarized *o*-wave in Calcite has a smaller wave velocity (= c/n_o), & hence it is referred to as a slow wave *o*(*s*).

Similarly, *e*-wave is fast wave (in Calcite).



If linearly polarized light beam making an angle 45º with *z* axis is incident on a HWP, the plane of polarization gets rotated by 90º; this beam gets converted to a RCP after propagating through a calcite QWP. Optic axis in HWP & QWP is along *z* direction as shown by lines parallel to *z* axis. [*s*-slow, *f*-fast]

❖ Consider the case when linearly polarized beam (with when $\Phi = \pi/4$) is incident on a HWP so that $\theta = \pi$; i.e., y & z components of incident wave have equal amplitudes, & crystal introduces a phase difference of π.

❖ Thus, for emergent beam we have

$$E_y = -\frac{E_0}{\sqrt{2}}\cos\omega t \qquad\qquad E_z = \frac{E_0}{\sqrt{2}}\cos\omega t$$

which represents a linearly polarized wave with the direction of polarization making an angle of 135º with z axis.

❖ If we now pass this beam through a calcite QWP, the emergent beam will be right circularly polarized.

❖ If a left circularly polarized beam is incident normally on a calcite HWP, the emergent beam will be right circularly polarized.

- Thus, for a HWP, thickness (for a negative crystal) is given by

$$d = (2m+1)\frac{\lambda_0}{2(n_o - n_e)}$$

- If crystal thickness is such that if $\theta \neq \pi/2$, π, $3\pi/2$, 2π, ….. the emergent beam will be elliptically polarized.

- For a positive crystal (such as quartz), $n_e > n_o$

$$E_y = E_0 \sin\phi \cos(\omega t + \theta')$$

$$E_z = E_0 \cos\phi \cos\omega t$$

where $$\theta' = \frac{\omega}{c} d(n_e - n_o)$$

For a QWP,

$$d = (2m+1)\frac{1}{4}\frac{\lambda_0}{n_e - n_o} \qquad m = 0,1,2....$$

Analysis of Polarized Light

Different states of polarization:

- Linearly polarized
- Circularly polarized
- Elliptically polarized
- Unpolarized
- Mixture of linearly polarized & unpolarized
- Mixture of circularly polarized & unpolarized
- Mixture of elliptically polarized & unpolarized

To naked eyes, all states of polarization appear to be same.

Procedure for determining state of polarization of a light beam

- **If we introduce a Polaroid in path of beam & rotate it about direction of propagation, then one of the following three possibilities can occur:**

  1. **If there is complete extinction at two positions of polarizer, then beam is linearly polarized.**

Procedure for determining state of polarization of a light beam

2. **If there is no variation of intensity, then beam is unpolarized or circularly polarized or a mixture of unpolarized & circularly polarized light.**

   We now put a quarter wave plate on the path of beam followed by rotating Polaroid.

   If there is no variation of intensity, then incident beam is unpolarized.

   If there is complete extinction at two positions, then beam is circularly polarized (this is so because a quarter wave plate will transform a circularly polarized light into a linearly polarized light).

   If there is a variation of intensity (without complete extinction), then beam is a mixture of unpolarized & circularly polarized light.

Procedure for determining state of polarization of a light beam

3. **If there is a variation of intensity (without complete extinction), then beam is elliptically polarized or a mixture of linearly polarized & unpolarized or a mixture of elliptically polarized & unpolarized light.**

   We now put a quarter wave plate in front of Polaroid with its optic axis parallel to pass axis of Polaroid at position of maximum intensity.

   Elliptically Polarized light will transform to a linearly polarized light.

   Thus, if one obtains two positions of Polaroid where complete extinction occurs, then original beam is elliptically polarized.

   If complete extinction does not occur & position of maximum intensity occurs at same orientation as before, the beam is a mixture of unpolarized & linearly polarized light.

   Finally, if position of maximum intensity occurs at a different orientation of Polaroid, the beam is a mixture of elliptically polarized & unpolarized light.