CS 225: Switching Theory

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Previous Class

- Number Systems and Codes
  - Different Number systems (positional)
  - Conversion

This Class

- Number Systems and Codes
  - Binary Arithmetic
  - Codes
    - BCD, cyclic code etc.
    - Gray code
    - Parity and Error correcting code

Number Systems

Decimal Number: $123.45=1\cdot10^2 + 2\cdot10^1 + 3 \cdot10^0 +4\cdot10^{-1} +5\cdot10^{-2}$

Base b number: $N = a_{q-1}b^{q-1} + \cdots + a_0 b_0 + \cdots + a_{-p}b^{-p}$

$b > 1, 0 <= a_i <= b-1$

Integer part: $a_{q-1}a_{q-2} \cdots a_0$

Fractional part: $a_{-1}a_{-2} \cdots a_{-p}$

Most significant digit: a_{q-1}

Least significant digit: a_{-p}

Binary number ($b=2$): $1101.01 = 1\cdot2^3 + 1\cdot2^2 + 0\cdot2^1 + 1\cdot2^0 + 0\cdot2^{-1} + 1\cdot2^{-2}$

Representing number N in base b: $(N)_b$

Complement of digit a: $a' = (b-1)-a$

Decimal system: 9's complement of 3 = 9-3 = 6

Binary system: 1's complement of 1 = 1-1 = 0

Conversion (Octal)

- Octal Numbers conversions
- Binary-to-Octal conversion

1. Break the binary number into 3-bit groups
2. Replace each group with an octal equivalent

- Octal-to-decimal conversion

1. Convert the octal to groups of 3-bit binary
2. Convert the binary to decimal

- Decimal-to-Octal conversion
  - Repeated division by 8

Conversion (Hexadecimal)

- Hexadecimal Numbers conversions
- Binary-to-hexadecimal conversion
  1. Break the binary number into 4-bit groups
  2. Replace each group with the hexadecimal equivalent
- Hexadecimal-to-decimal conversion
  1. Convert the hexadecimal to groups of 4-bit binary
  2. Convert the binary to decimal
- Decimal-to-hexadecimal conversion
  - Repeated division by 16
  - Hexa decimal to Octal or vice-versa?

Ex.:

- Conversion

$(41.6875)_{10} = (?)_2$

$(101001.1011)2$

$(153.513)_{10} = (?)_8$

$(231.406517)_8$

Binary Arithmetic

Bits		Sum	Carry	Difference	Borrow	Product
a	b	a+b		a-b		a · b
0	0	0	0	0	0	0
0	1	1	0	1	1	0
1	0	1	0	1	0	0
1	1	0	1	0	0	1

Binary Addition/Subtraction

Example: Binary addition

$$\begin{array}{rl} 1111 & = \text{carries of } 1 \\ 1111.01 & = (15.25)_{10} \\ +\ \underline{0111.10} & = (\ 7.50)_{10} \\ 10110.11 & = (22.75)_{10} \end{array}$$

Example: Binary subtraction

$$\begin{array}{rl} 1 & = \text{borrows of } 1 \\ 10010.11 & = (18.75)_{10} \\ -\ \underline{01100.10} & = (12.50)_{10} \\ 00110.01 & = (\ 6.25)_{10} \end{array}$$

Answer the following

$(1001.1)_2 + (010.1)_2$ =? Show Carries and Borrows

$(100.01)_2 - (010.1)_2$ =?

$9.5+2.5 = 12.0 = (1100.0)_2$

$4.25-2.5 = 1.75 = (01.11)_2$

Binary Multiplication/Division

Example: Binary Multiplication

$$
\begin{array}{r}
11001.1 = (25.5)_{10} \\
\underline{110.1} = (\ 6.5)_{10} \\
110011 \\
000000 \\
110011 \\
\underline{110011} \\
10100101.11 = (165.75)_{10}
\end{array}
$$

Binary Multiplication/Division

Example: Binary Division

```
             10110 = quotient
       ___________
11001 | 1000100110
         11001
        _________
        00100101
            11001
         _________
           0011001
              11001
         _________
              00000   remainder
```

Signed Numbers Representation

- Three main different ways
- Sign and magnitude
- r's complement
- r-1 's complement

Signed binary number

Positive numbers can be defined with Sign bit 0

- Ex. In 8-bit representation of +9 = 00001001

- Negative numbers can be represented in three different ways:
  - Signed magnitude: -9 = 10001001
  - Signed 1's complement: -9 = 11110110
  - Signed 2's complement: -9 = 11110111

- Undesired aspect in signed binary and 1 s complement method:
- representation of 0s:

  +0 has different code from -0

Radix Complements

- Radix complement:
  - r's complement of a number N with n digits is
    $r^n - N = (r^n - 1) - N + 1.$

Ex. 10's complement of 346 is = 654 (653+1)

2's complement of 1011= 0101

NB: complement of complement of N is N

$r^n - (r^n - N) = N$

Arithmetic operations

- Addition
  - Care for overflow
- Subtraction:
  - Complements:
    - Diminished Radix (r-1)'s Complement
      - (r-1)'s complement of a number N with n digits is

        $(r^n - 1) - N$.

Ex.:9's complement of 346 is 999-346=653 (10^3 -1= 999)

- 1's complement of a binary number can be determined as just replacing 1's with 0's and vice- versa..

Subtraction using 2's complement

- M and N are two n digit numbers with radix r
- To subtract N from M
  - Compute the r's complement of N (= $r^n - N$)
  - Add M with r's complement of N

i.e. $M - N + r^n$

If M >= N result produce the carry r^n which may be discarded so resulting M –N

If M<N result does not produce carry that may be treated as . $M - N + r^n = r^n - (N - M)$ so the answer is r's complement of the result.

Overflow

- Overflow is said to occur if the result needs more than n-bits.
- If c is a carry into the sign-bit position and o is the carry from sign-bit position; then overflow is said to occur if and only if c XOR o = 1

- Thanks