OPTICS & LASERS

PH201

Lecture_Diffraction

Diffraction

Effects of diffraction of light ~ **Fancesco Maria Grimaldi,** who coined term diffraction.

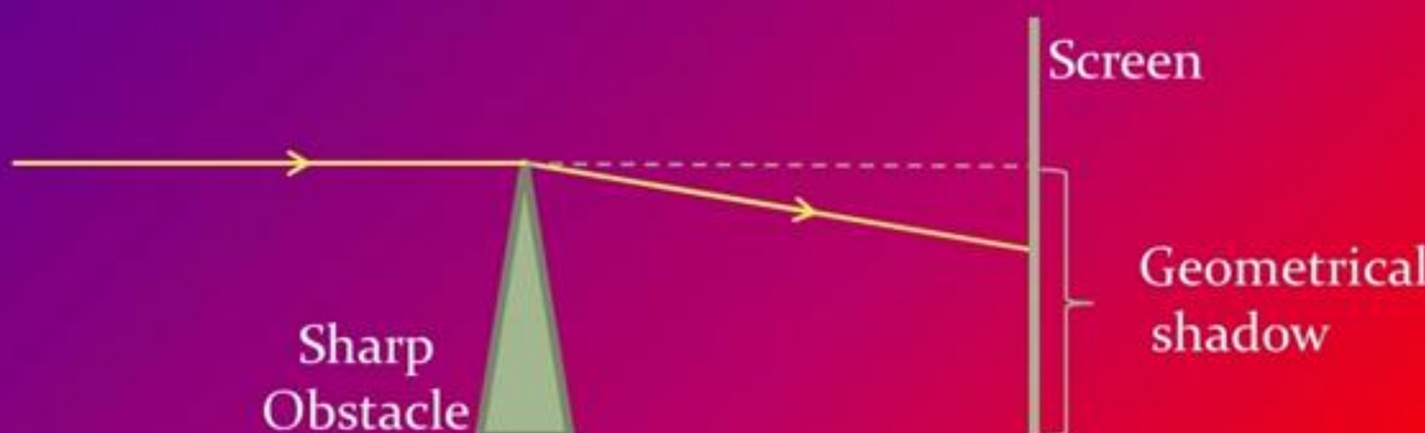
Latin ‘diffringere’ ‘to break into pieces’, referring to **light breaking up into different directions.** Results of Grimaldi’s observations were published posthumously in 1665.

Diffraction refers to various phenomena which occur when a wave encounters an obstacle. It is described as apparent **bending of waves around small obstacles** & spreading out of waves past small openings.

Diffraction occurs with all waves; sound waves, water waves, & em waves such as visible light, x-rays & radio waves. As physical objects have wave-like properties, diffraction also occurs with matter & can be studied according to the principles of quantum mechanics.

Diffraction effects are most pronounced for waves where **wavelength is on the order of the size of diffracting objects**.

Bending of light across the edges of an obstacle is called as diffraction.

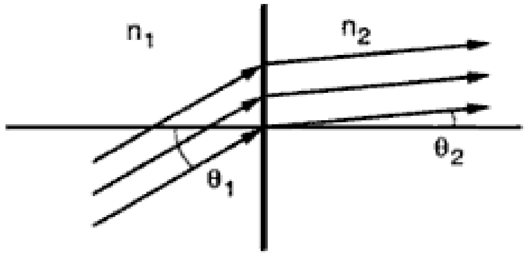


Scalar Diffraction Theory

Diffraction plays a role of utmost importance in the branches of physics & engineering that deal with wave propagation (acoustic wave & radio wave). It helps understand properties of optical imaging & data processing systems.

Refraction – Bending of light that takes place when a light wave encounters a sharp boundary between two regions having different refractive indices.

$$v_1 = c/n_1 \quad \& \quad v_2 = c/n_2,$$



c = vacuum velocity of light,
n_1 & n_2 = refractive indices of 1st & 2nd media
v_1 & v_2 = velocities of propagation in 1st & 2nd media

Incident light rays are bent at interface. Angles of incidence & refraction are related by *Snell's law*.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

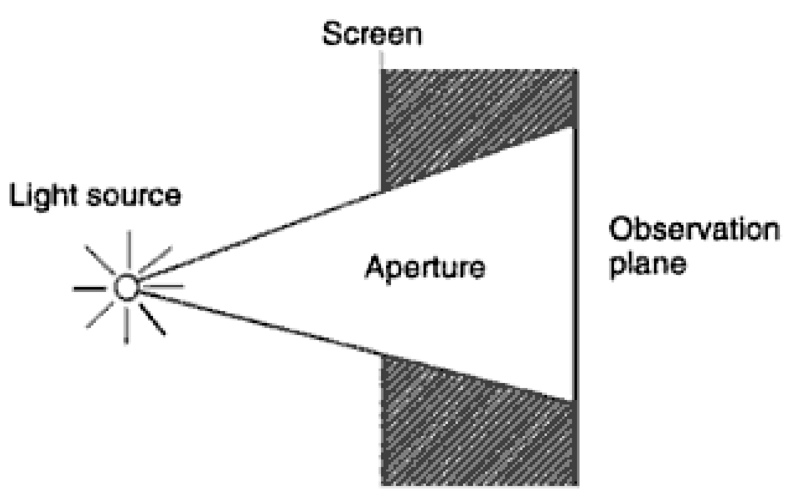
$$n_2 > n_1 \ \& \ \theta_2 < \theta_1$$

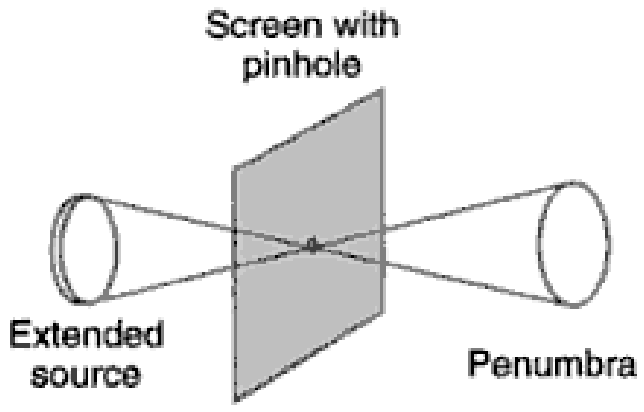
Light rays are also bent upon reflection, which can occur at a metallic or dielectric interface.

Sommerfeld: any deviation of light rays from rectilinear paths which cannot be interpreted as reflection or refraction.

Diffraction is caused by confinement of lateral extent of a wave & is most appreciable when that confinement is to sizes comparable with a wavelength of the radiation being used.

Penumbra effect: finite extent of a source causes the light transmitted by a small aperture to spread as it propagates away from that aperture. It doesn't involve any bending of the light rays.





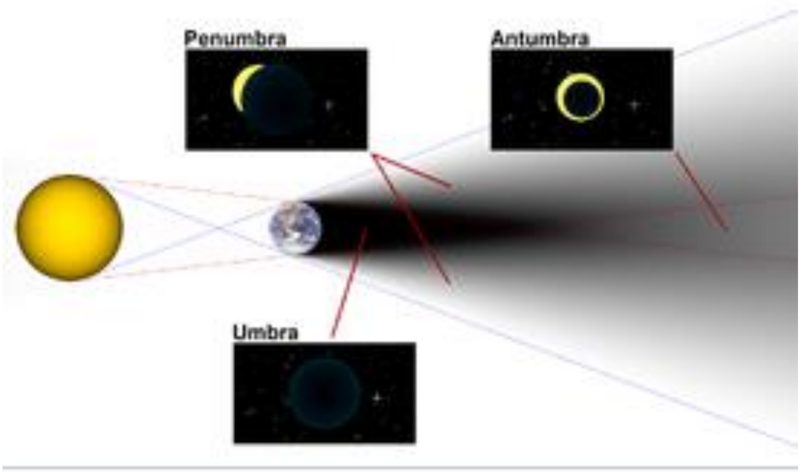
Arrangement used for observing diffraction of light

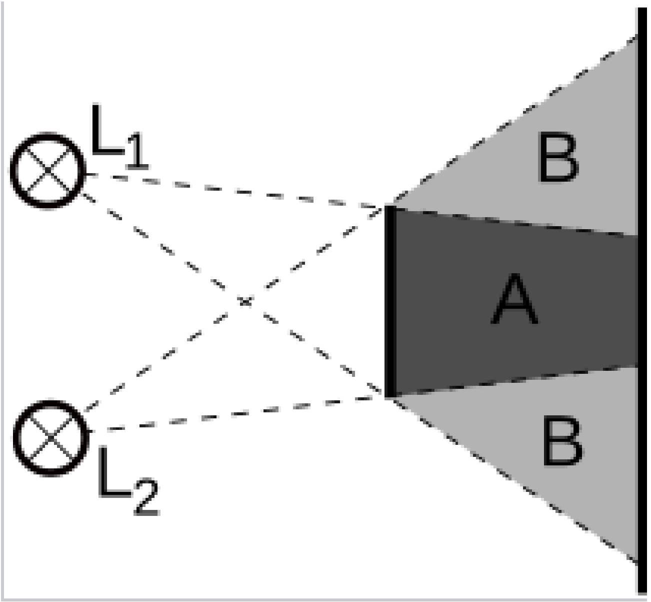
Umbra, Penumbra, Antumbra

- *Umbra*, *penumbra*, & *antumbra* are three distinct parts of a shadow, created by any light source after impinging on an opaque object.

- For a point source only the umbra is cast.

- These names are most often used for *shadows* cast by celestial bodies, though they are sometimes used to describe levels of darkness.





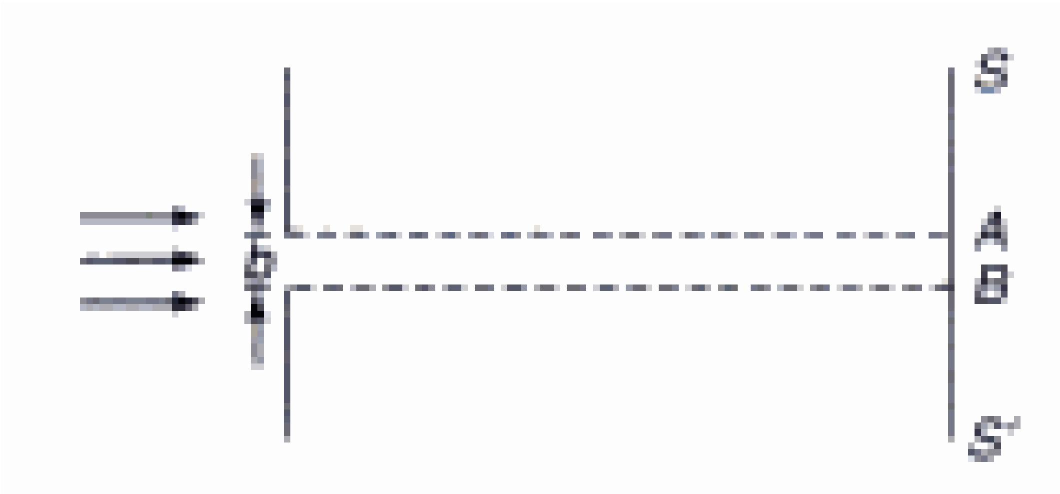
Umbra (A), Penumbra (B)

UMBRA (Latin for "shadow") is innermost & darkest part of a shadow where light source is completely blocked by occluding body. Such as an opaque object does not let light through it. An observer in the umbra experiences a total eclipse.

PENUMBRA ("almost nearly") is region in which only a portion of light source is obscured by occluding body. An observer in penumbra experiences a partial eclipse.

ANTUMBRA ("before") is region from which occluding body appears entirely contained within disc of light source. An observer in this region experiences an annular eclipse.

Umbra, Penumbra, & Antumbra formed through windows & shutters



If a plane wave is incident on an aperture then according to geometrical optics a sharp shadow will be cast in *AB* region of screen.

We can observe water wave diffraction with a ripple tank - look at the simulations of water waves below. The lines that you see in the images are called wavefronts; wavefronts connect points on the wave with the same phase.

Tracks of a *CD* act as a diffraction grating, producing a separation of colors of white light. Nominal track separation on a *CD* is 1.6 *μm*, corresponding to about 625 tracks per mm. For λ = 600 *nm*, this would give a first order diffraction maximum at about 22º.

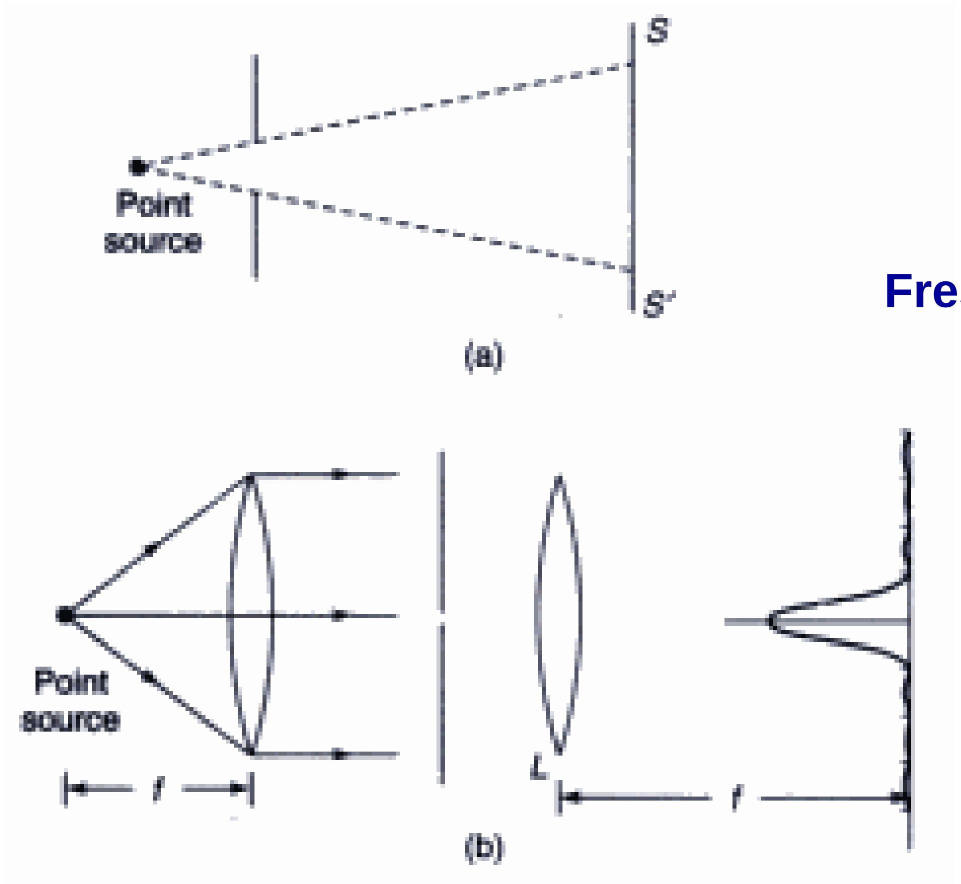
Interference corresponds to situation when we consider superposition of waves coming out from a number of point sources &

Diffraction corresponds to situation when we consider waves coming out from an area source like a circular or rectangular aperture or even a large no. of rectangular apertures (like diffraction grating).

Diffraction phenomena are usually divided into two categories:

Fresnel diffraction: Source of light & screen are at a finite distance from diffracting aperture.

Fraunhofer diffraction: Source & screen are at infinite distances from aperture. This is easily achieved by placing source on the focal plane of a convex lens & placing screen on the focal plane of another convex lens.

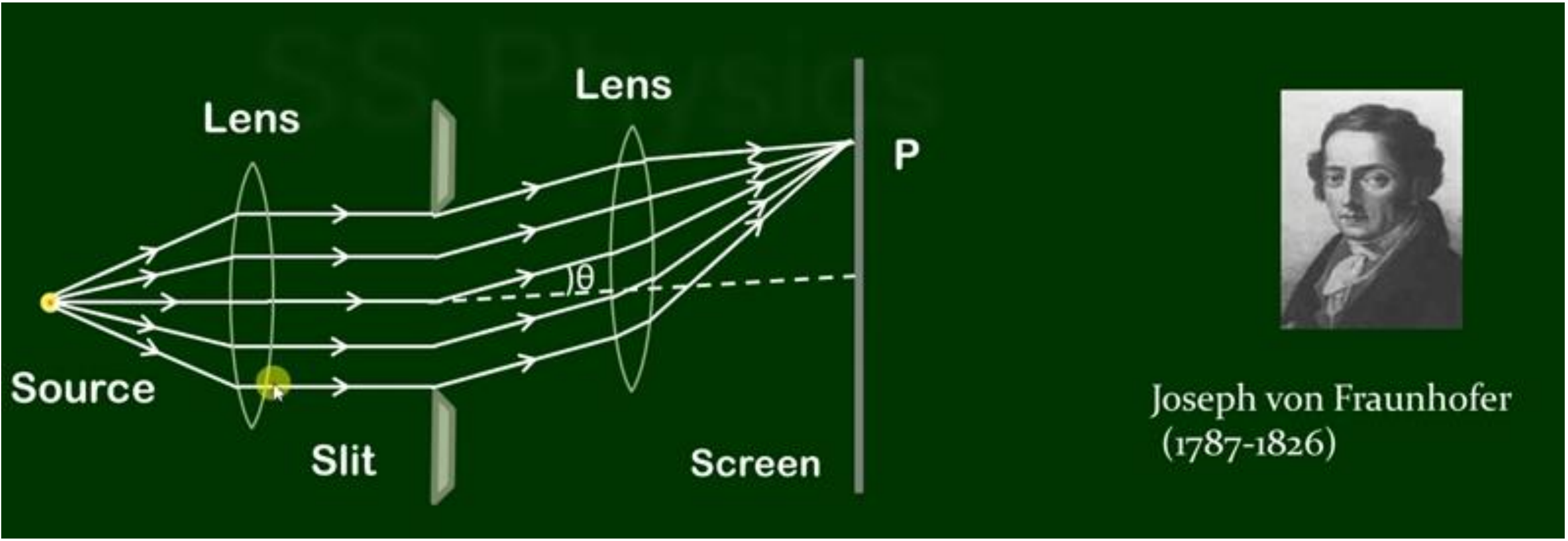


Fresnel diffraction

Fraunhofer diffraction

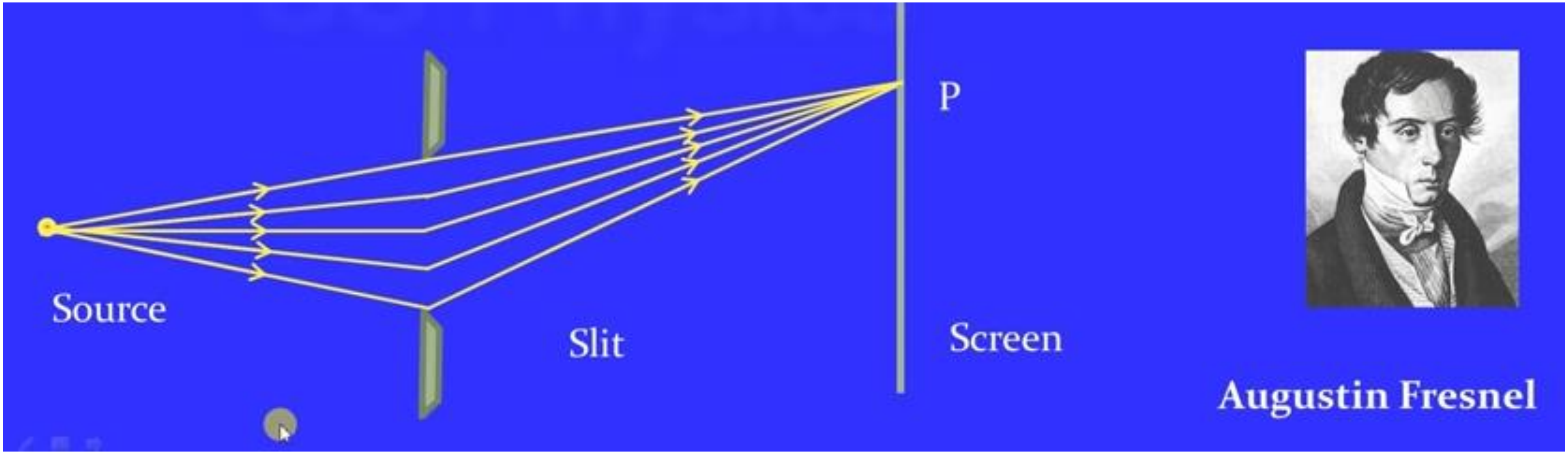
Fraunhofer Diffraction

Source of light & screen are at infinite distance from obstacle.

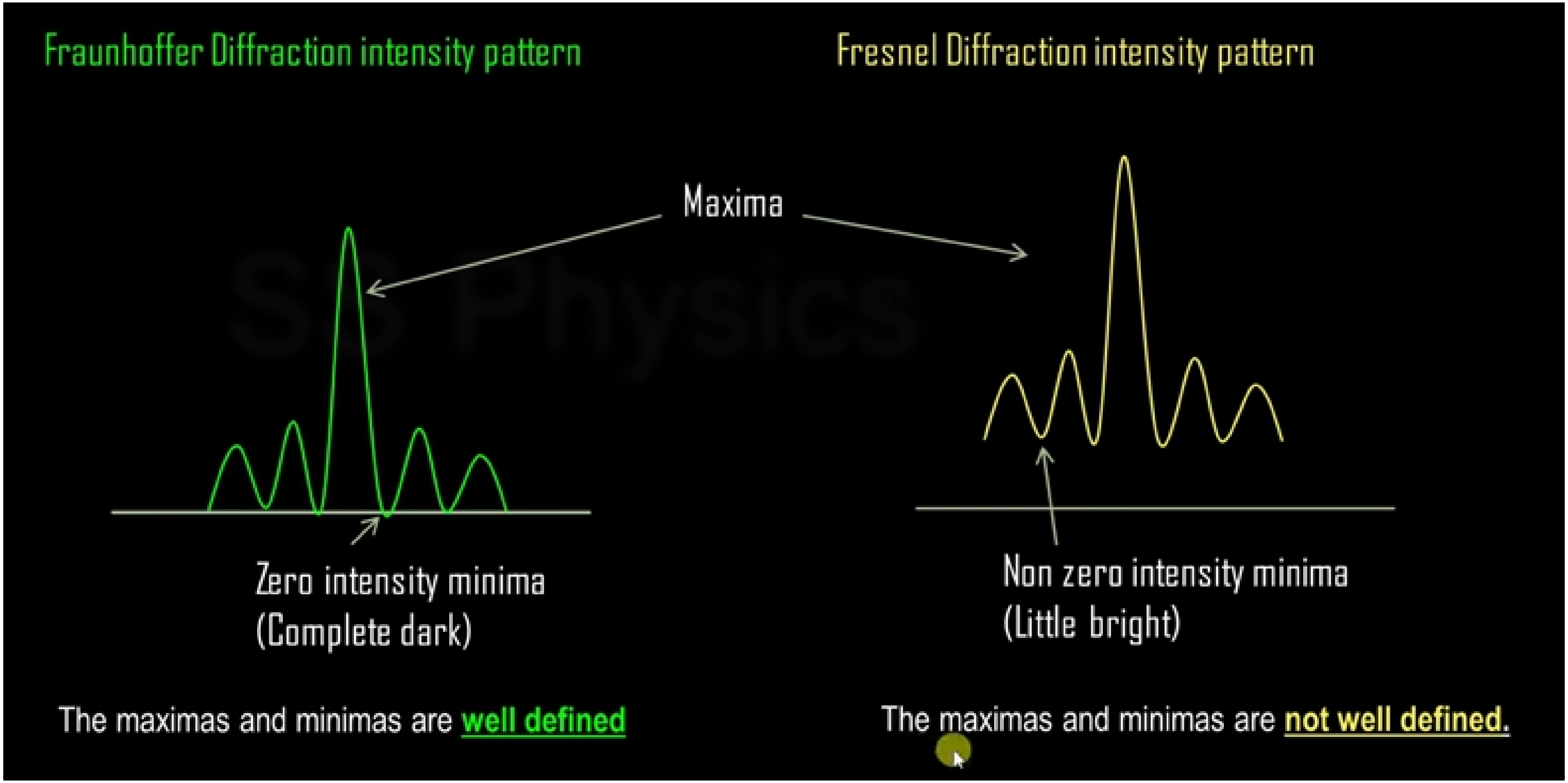


Fresnel Diffraction

Source of light & screen are at finite distance from obstacle.

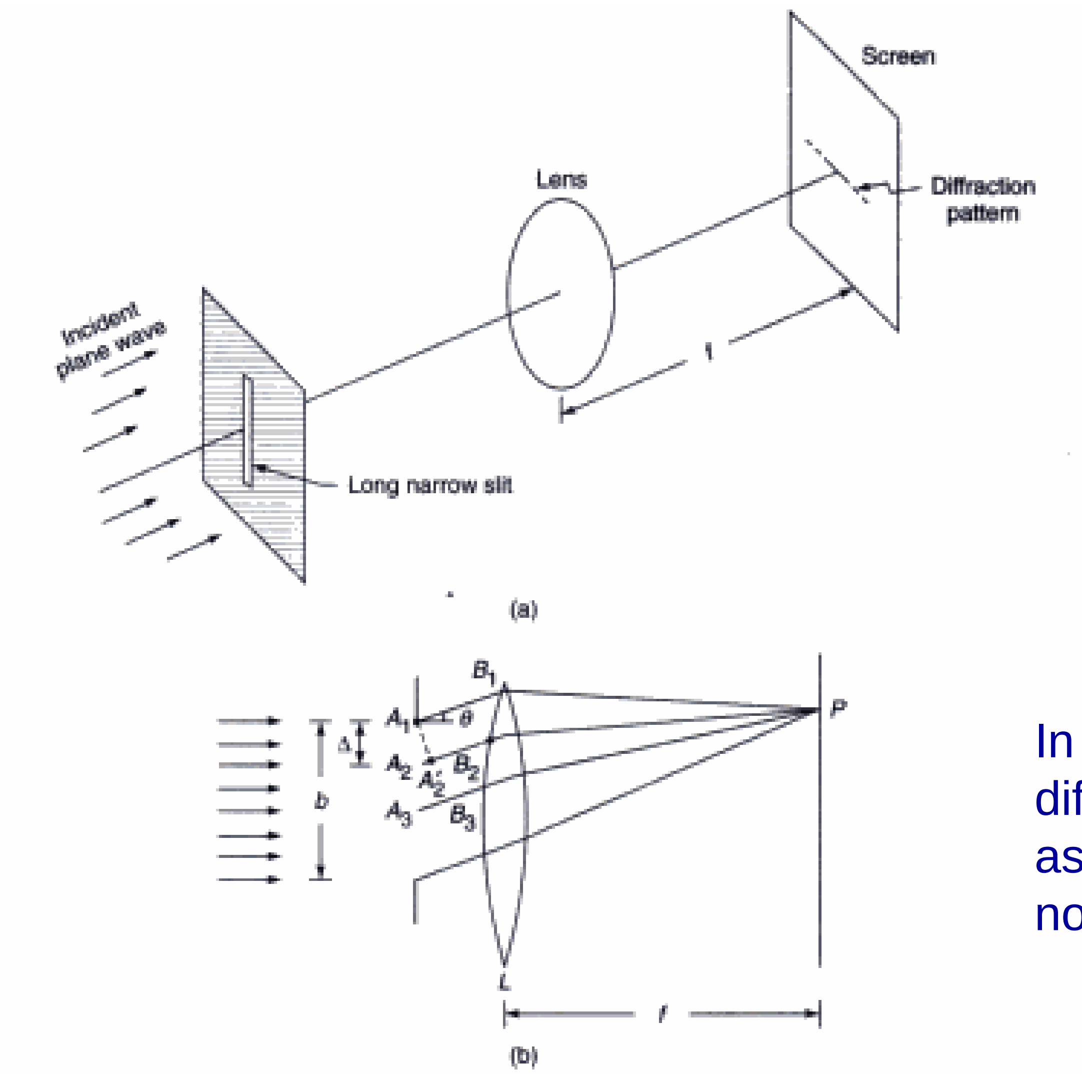


Fraunhoffer	Fresnel
Longer	Shorter
Infinite	Finite


Fraunhoffer Diffraction intensity pattern
Fresnel Diffraction intensity pattern
Maxima
Zero intensity minima
(Complete dark)
Non zero intensity minima
(Little bright)
The maximas and minimas are well defined
The maximas and minimas are not well defined.

Single-slit diffraction pattern

A slit is a rectangular aperture of length large compared to its breadth.



Diffraction of a plane wave incident normally on a long narrow slit of width *b*. Spreading occurs along width of the slit.

In order to calculate diffraction pattern, slit is assumed to consist of a large no. of equally spaced points.

Assume, slit consists of a large no. of equally spaced point sources & that each point on slit is a source of Huygens' secondary wavelets which interfere with wavelets emanating from other points.

Let point sources be at A_1, A_2, A_3, ….. And let distance between two consecutive points be Δ. Thus, if no. of point sources be n, then

$$b = (n-1)\Delta$$

At P, amplitudes of disturbances reaching from A_1, A_2, A_3, ….. will be very nearly same because point P is at a distance which is very large in comparison to b.

However, because of even slightly different path lengths to P, field produced by A_1 will differ in phase from field produced by A_2. If diffracted rays make an angle θ with normal to the slit then path difference would be

$$A_2 A_2' = \Delta \sin\theta \qquad \phi = \frac{2\pi}{\lambda}\Delta \sin\theta$$

If field at *P* due to disturbances emanating from A_1 is *acosωt* then field due to disturbances emanating from A_2 would be *acos*(*ωt*-*Φ*).

Resultant field

$$E = a[\cos\omega t + \cos(\omega t - \phi) + \ldots\ldots\ldots\ldots + \cos(\omega t - (n-1)\phi)]$$

$$\cos\omega t + \cos(\omega t - \phi) + \ldots\ldots\ldots\ldots + \cos(\omega t - (n-1)\phi)$$

$$= \frac{\sin n\phi/2}{\sin\phi/2}\cos[\omega t - \frac{1}{2}(n-1)\phi]$$

$$E = E_0\cos[\omega t - \frac{1}{2}(n-1)\phi]$$

$$E_0 = a\frac{\sin n\phi/2}{\sin\phi/2}$$

$$\mathit{In\ the} \lim \mathit{it} \quad n\to\infty, \quad \Delta\to 0, \quad \mathit{such\ that} \quad n\Delta\to b$$

$$\Rightarrow \frac{n\phi}{2} = \frac{\pi}{\lambda} n\Delta\sin\theta = \frac{\pi}{\lambda} b\sin\theta$$

$$E_0 = \frac{a\sin(n\phi/2)}{\phi/2} = \frac{na\sin\left(\dfrac{\pi b\sin\theta}{\lambda}\right)}{\pi b(\sin\theta)/\lambda} = A\frac{\sin\beta}{\beta}$$

$$A = na; \quad \beta = \frac{\pi b\sin\theta}{\lambda}$$

$$E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

Corresponding int *ensity*

$$I = I_0\frac{\sin^2\beta}{\beta^2} \qquad I_0 \ \mathit{represents} \ \text{int} \ \mathit{ensity\ at} \ \theta = 0$$

Positions of Maxima & Minima

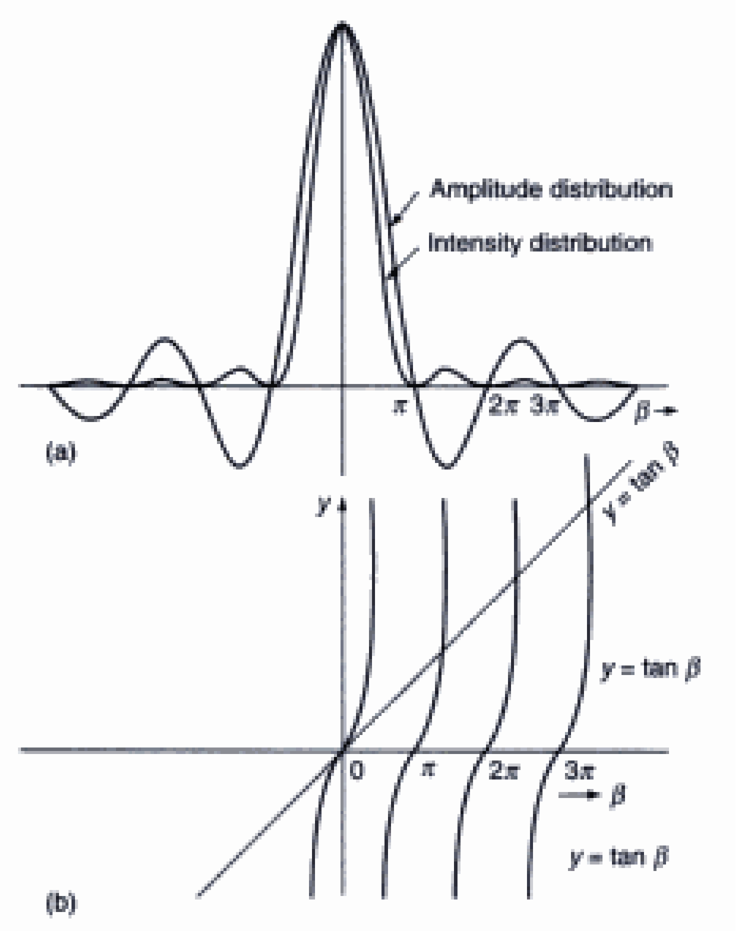
$$I = 0, \quad when \quad \beta = m\pi, \quad m \neq 0$$

$$When \quad \beta = 0, \quad \frac{\sin\beta}{\beta} = 1 \qquad \Rightarrow I = I_0$$

$$\Rightarrow b\sin\theta = m\lambda; \qquad m = \pm 1, \pm 2, \pm 3, \ldots..(Minima)$$

$$I_{\min}\ \theta = \pm\sin^{-1}(\lambda / b)$$

$$II_{\min}\ \theta = \pm\sin^{-1}(2\lambda / b)$$



Intensity distribution corresponding to single slit Fraunhofer diffraction pattern.

Graphical method for determining roots of equation tanβ = β.

To determine positions of maxima, we differentiate intensity distribution with respect to β & set it equal to zero.

$$I = I_0 \frac{\sin^2 \beta}{\beta^2}$$

$$\frac{dI}{d\beta} = I_0 \left[\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3} \right] = 0$$

$$\Rightarrow \sin\beta[\beta - \tan\beta] = 0$$

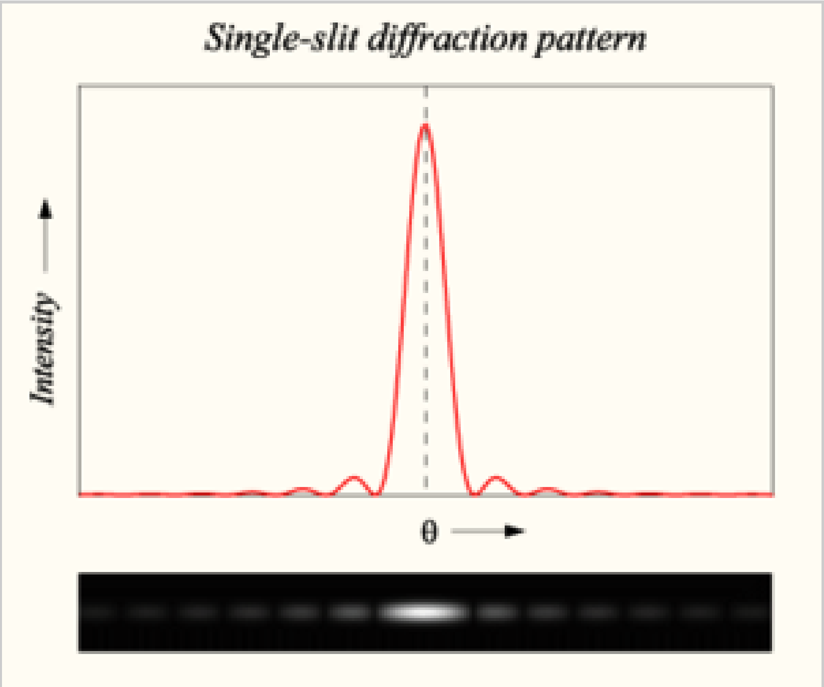
Condition $\sin\beta = 0$, or $\beta = m\pi$ ($m \neq 0$) correspond to minima. Conditions for maxima are roots of following Eqn.

$$\boldsymbol{\tan\beta = \beta} \qquad \textbf{(Maxima)}$$

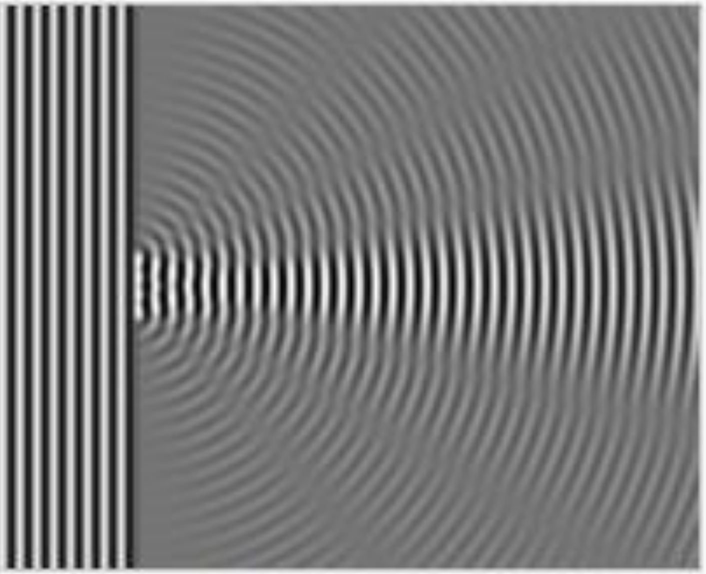
Root $\beta = 0$ corresponds to central maximum & other roots can be found by determining points of intersections of curves ***$y = \beta$*** & ***$y = \tan\beta$***. Intersections occur at $\beta = 1.43\pi$, $\beta = 2.46\pi$, etc. & are known as first maximum, second maximum, etc. Since

$$\left[\frac{\sin(1.43\pi)}{1.43\pi} \right]^2$$

is about 0.0496, intensity of 1st maximum is about 4.96% of central maximum. Similarly, intensities of 2nd & 3rd maxima are about 1.68% & 0.83% of central maximum respectively.



Graph and image of single-slit diffraction



Single-slit diffraction pattern

Two-slit Fraunhofer diffraction pattern

Fraunhofer diffraction pattern produced by two parallel slits (each of width *b*) separated by a distance *d*. **Resultant intensity is a product of single-slit diffraction pattern & interference pattern produced by two point sources separated by a distance *d***.

Assume that slits consist of a large no. of equally spaced point sources & that each point on slit is a source of Huygens' secondary wavelets. Let point sources be at A_1, A_2, A_3, ….. (1st slit) & at B_1, B_2, B_3, ……(2nd slit). Assume distance between two consecutive points in either of slits is Δ.

If diffracted rays make an angle θ with normal to plane of slits, then path difference between disturbances reaching *P* from two consecutive points in a slit will be $\boldsymbol{\Delta \sin\theta}$.

Field produced by first slit at *P*

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

Similarly, second slit will produce a field at *P*

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \phi_1) \qquad \phi_1 = \frac{2\pi}{\lambda} d\sin\theta$$

Resultant field

$$E = E_1 + E_2 = A\frac{\sin\beta}{\beta}[\cos(\omega t - \beta) + \cos(\omega t - \beta - \phi_1)]$$

This represents interference of two waves, each of amplitude $A(\sin\beta)/\beta$ & differing in phase by Φ_1.

$$E = A\frac{\sin\beta}{\beta}\cos\gamma\cos(\omega t - \frac{1}{2}\beta - \frac{1}{2}\phi_1)$$

$$\gamma = \frac{\phi_1}{2} = \frac{\pi}{\lambda} d\sin\theta$$

$$\Rightarrow I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma$$

$I_0(sin^2\beta)/\beta^2$ represents intensity distribution produced by one of the slits.

Intensity distribution is a product of two terms;

1st term $(sin^2\beta)/\beta^2$ represents diffraction pattern produced by a single slit of width b &

2nd term ($\cos^2\gamma$) represents interference pattern produced by two point sources separated by a distance d.

<u>If slit widths are very small</u> (so that there is almost no variation of $sin^2\beta/\beta^2$ term with θ) then one simply obtains Young's interference pattern.

Positions of Maxima & Minima

Intensity is zero whenever $\beta = \pi, 2\pi, 3\pi, \ldots\ldots..$ or

when $\gamma = \pi/2, 3\pi/2, 5\pi/2, \ldots\ldots$

Corresponding angles of diffraction are

$$b \sin\theta = m\lambda; \qquad (m = 1,2,3\ldots.)$$

$$d \sin\theta = (n + \frac{1}{2})\lambda; \qquad (n = 1,2,3\ldots.)$$

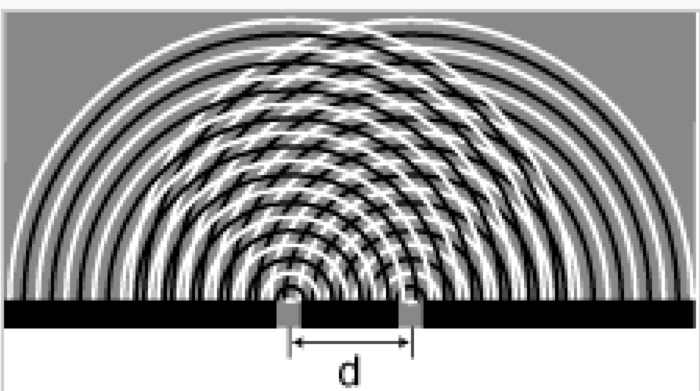
Interference maxima occur when $\gamma = 0, \pi, 2\pi, \ldots..$ or

when $d\sin\theta = 0, \lambda, 2\lambda, 3\lambda, \ldots.$

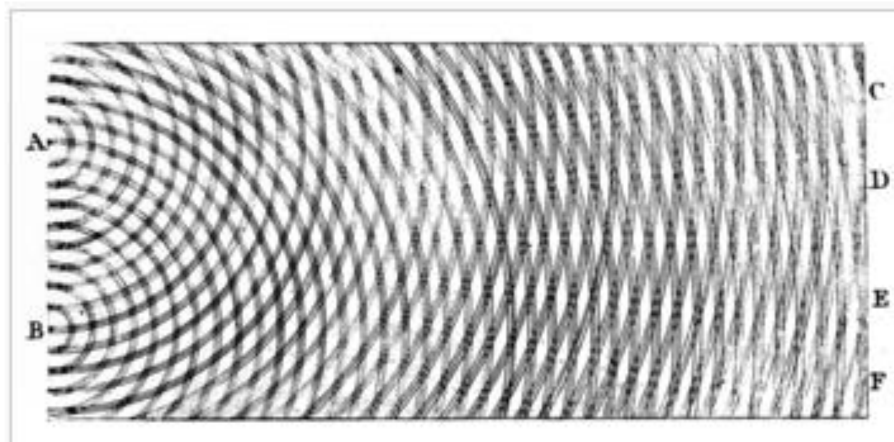
Actual positions of maxima will approximately occur at above angles provided variation of diffraction term is not too rapid. A maximum may not occur at all if θ corresponds to a diffraction minimum, i.e., if

$$b\sin\theta = \lambda, 2\lambda, 3\lambda, \ldots.$$

These are usually referred to as **missing orders**, which occur where conditions for a maximum of interference & for a minimum of diffraction are both fulfilled for same value of θ.

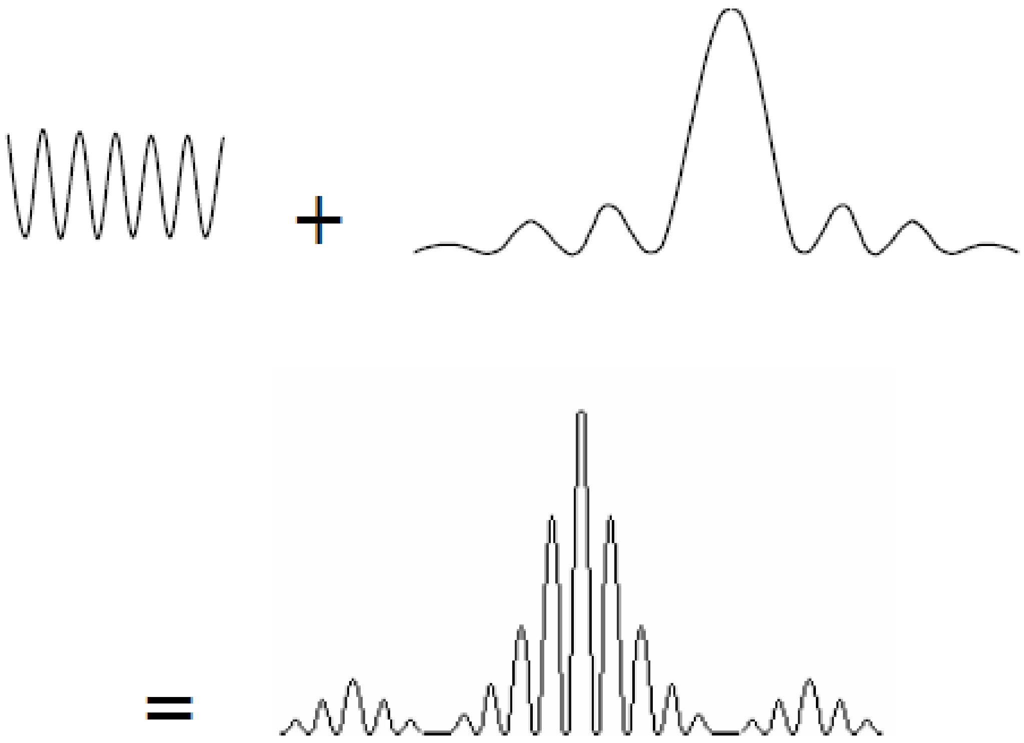


Double-slit diffraction and interference pattern



Thomas Young's sketch of two-slit diffraction, which he presented to the Royal Society in 1803

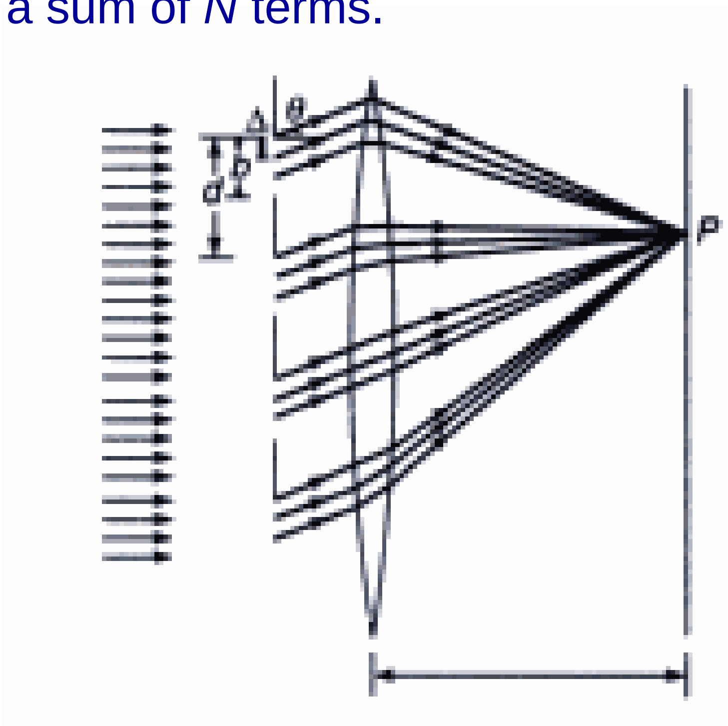
If one or other of two slits is covered, we obtain exactly same single-slit pattern in same position, while if both slits are uncovered, pattern, instead of being a single-slit one with twice intensity, breaks up into narrow maxima & minima called **interference fringes**. Intensity at maximum of these fringes is 4 times intensity of either single-slit pattern at that point, while it is zero at minima.



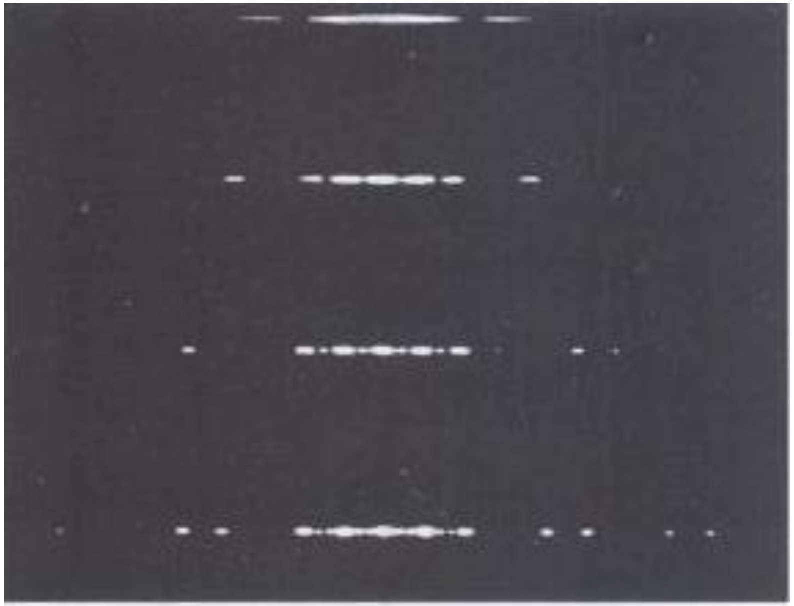
Double-slit pattern can be said to be due to **combination of interference & diffraction**. Interference of beams from two slits produces narrow maxima & minima, & diffraction modulates intensities of these interference fringes.

N-slit Fraunhofer diffraction pattern

Consider diffraction pattern produced by *N* parallel slits, each of width *b*; distance between two consecutive slits is *d*. Assume each slit consists of *n* equally spaced point sources with spacing *Δ*. Field at point *P* will essentially be a sum of *N* terms.



Fraunhofer diffraction of a plane wave incident normally on a multiple slit.



Multiple-slit Fraunhofer diffraction pattern corresponding to b = 0.0044 cm, d = 0.0132 cm & $\lambda = 6.328 \times 10^{-5}$ cm. No. of slits are 1, 2, 3, & 4 respectively.

$$E = A\frac{\sin\beta}{\beta}\cos(\omega t-\beta) + A\frac{\sin\beta}{\beta}\cos(\omega t-\beta-\phi_1)$$

$$+ ....+ A\frac{\sin\beta}{\beta}\cos(\omega t-\beta-(N-1)\phi_1)$$

1st term represents amplitude produced by 1st slit, 2nd term by 2nd slit, etc. Rewriting above Eq.

$$E = A\frac{\sin\beta}{\beta}[\cos(\omega t-\beta)+\cos(\omega t-\beta-\phi_1)+....+\cos(\omega t-\beta-(N-1)\phi_1)]$$

$$= A\frac{\sin\beta}{\beta}\frac{\sin N\gamma}{\sin\gamma}\cos(\omega t-\beta-\frac{1}{2}(N-1)\phi_1)$$

$$\gamma = \frac{\phi_1}{2} = \frac{\pi}{\lambda}d\sin\theta$$

Corresponding intensity distribution $$I = I_0\frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma}$$

$I_0(\sin^2\beta)/\beta^2$ represents intensity distribution produced by a single slit. Intensity distribution is a product of two terms; **1st term ($\sin^2\beta$)/β^2 represents diffraction pattern produced by a single slit** & **2nd term ($\sin^2 N\gamma/\sin^2\gamma$) represents interference pattern produced by *N* equally spaced point sources.**

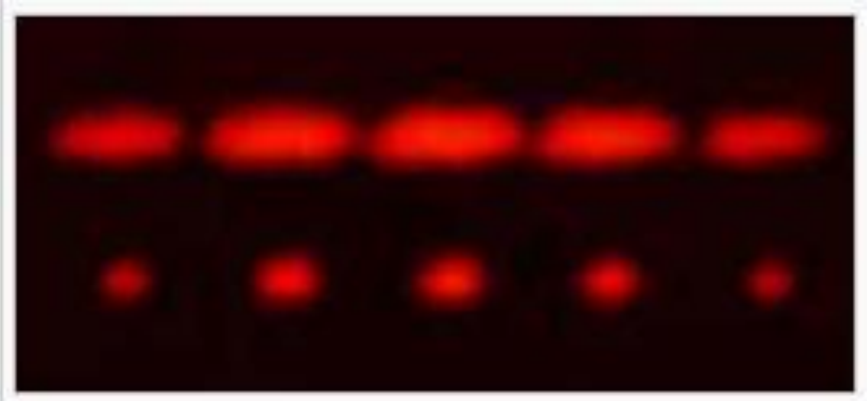
For $N = 1$, intensity distribution reduces to single slit diffraction pattern

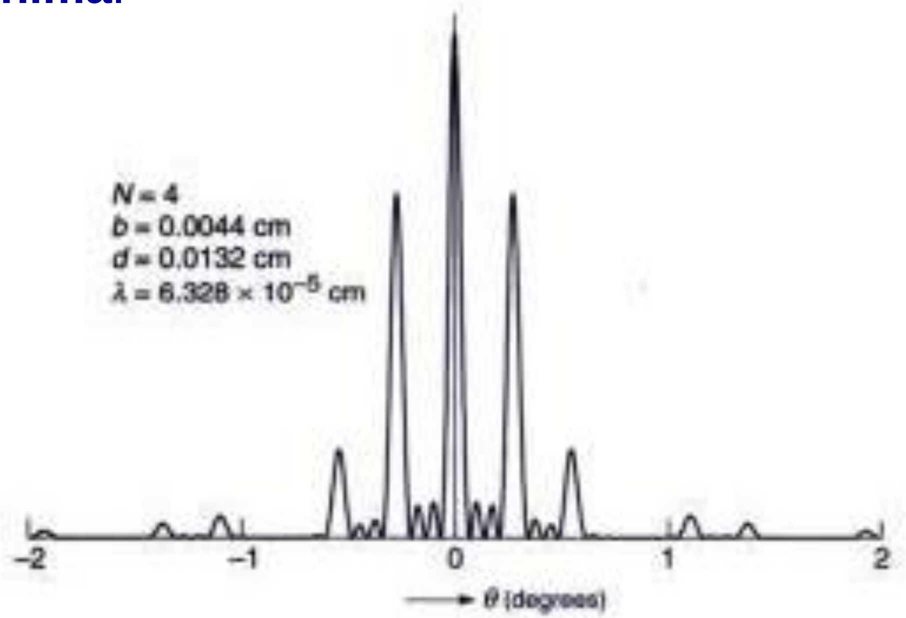
For $N = 2$, intensity distribution reduces to double slit diffraction pattern.

As N becomes very large, function $\sin^2 N\gamma / \sin^2\gamma$ would become very sharply peaked at $\gamma = 0, \pi, 2\pi, \ldots\ldots$ Between two peaks, function vanishes when

$$\gamma = \frac{p\pi}{N}; \quad p = \pm 1, \pm 2, \ldots. \quad but \quad p \neq 0, \pm N, \pm 2N$$

which are referred to as **secondary minima**.


2-slit (top) and 5-slit diffraction of red laser light



Intensity distribution corresponding to four-slit Fraunhofer diffraction pattern.

Positions of Maxima & Minima

When N is very large, one obtains intense maxima at $\gamma = m\pi$, i.e., when **$d\sin\theta = m\lambda$ (m = 0,1,2,…)**

$$\underset{\gamma \to m\pi}{Lt} \frac{\sin N\gamma}{\sin\gamma} = \underset{\gamma \to m\pi}{Lt} \frac{N\cos N\gamma}{\cos\gamma} = \pm N$$

Resultant amplitude & corresponding intensity distributions are given by

$$E = N\frac{A\sin\beta}{\beta}$$

$$I = N^2 I_0 \frac{\sin^2\beta}{\beta^2}$$

$$where \quad \beta = \frac{\pi b\sin\theta}{\lambda} = \frac{\pi b}{\lambda}\frac{m\lambda}{d} = \frac{\pi b m}{d}$$

Such maxima are known as **principal maxima**. Physically, at these maxima, fields produced by each slit are in phase &, therefore, they add up & resultant field is N times field produced by each slit. There will only be a finite no. of principal maxima.

Minima: Intensity is zero when either ***b*sin*θ* = *nλ*, *n* = 1, 2, 3,….** or ***Nγ* = *pπ*, *p* ≠ *N*, 2*N*, ….**

Between two principal maxima we have (*N*-1) minima. Between two such consecutive minima intensity has to have a maximum; these maxima are known as **secondary maxima**.

- A particular principal maximum may be absent if it corresponds to angle which also determines min. of single-slit diffraction pattern.

$$d\sin\theta = m\lambda \qquad \& \, b\sin\theta = \lambda, 2\lambda, 3\lambda, ...$$

are satisfied simultaneously & is usually referred to as a **missing order**. Even if 2nd Eq. does not hold exactly (i.e., if *b*sin*θ* is close to an integral multiple of *λ*), intensity of corresponding principal max. will be very weak.

- In addition to minima predicted by *Nγ* = *pπ*, we will also have diffraction minima; however, when *N* is very large, no. of such minima will be very small.

Width of principal maxima

*m*th order principal maximum occurs at

$$d\sin\theta_m = m\lambda, \qquad m = 0, 1, 2, ...$$

Angles of diffraction corresponding to $N\gamma = p\pi$ are

$$d\sin\theta = \frac{\lambda}{N}, \frac{2\lambda}{N}, \ldots.\frac{(N-1)\lambda}{N}, \frac{(N+1)\lambda}{N}, \frac{(N+2)\lambda}{N},$$
$$\ldots..\frac{(2N-1)\lambda}{N}, \frac{(2N+1)\lambda}{N}, \frac{(2N+2)\lambda}{N}, \ldots..$$

If $\theta_m + \Delta\theta_{1m}$ & $\theta_m - \Delta\theta_{2m}$ represent angles of diffraction corresponding to first minimum on either side of principal maximum, then $(\Delta\theta_{1m} + \Delta\theta_{2m})/2$ is known as **angular half width** of m^{th} order principal maximum.

For a large value of N, $\Delta\theta_{1m} = \Delta\theta_{2m}$ which we write as $\Delta\theta_m$.

$$d\sin(\theta_m \pm \Delta\theta_m) = m\lambda \pm \frac{\lambda}{N}$$

$$\sin(\theta_m \pm \Delta\theta_m) = \sin\theta_m \cos\Delta\theta_m \pm \cos\theta_m \sin\Delta\theta_m$$
$$= \sin\theta_m \pm \Delta\theta_m \cos\theta_m$$

$$\Delta\theta_m = \frac{\lambda}{Nd\cos\theta_m}$$

Principal maximum becomes sharper as *N* increases.

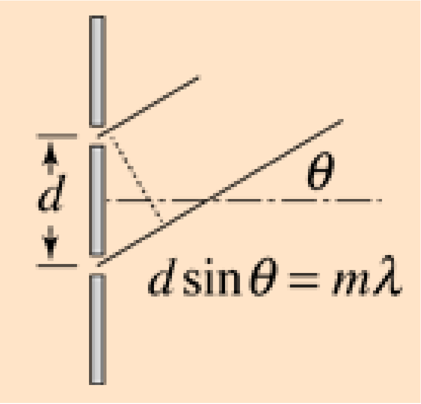
Diffraction grating

An arrangement which essentially consists of a large no. of equidistant slits is known as a **diffraction grating**; corresponding diffraction pattern is known as **grating spectrum**.

$$\Delta\theta_m = \frac{\lambda}{Nd\cos\theta_m}$$

For narrow principal maxima, a large value of *N* is required. A good quality grating, therefore, requires a large no. of slits (typically about 15,000 per inch).

This is achieved by ruling grooves with a diamond point on an optically transparent sheet of material; grooves act as opaque spaces. After each groove is ruled, machine lifts diamond point & moves sheet forward for ruling of next groove. For a good quality grating lines should be as equally spaced as possible.

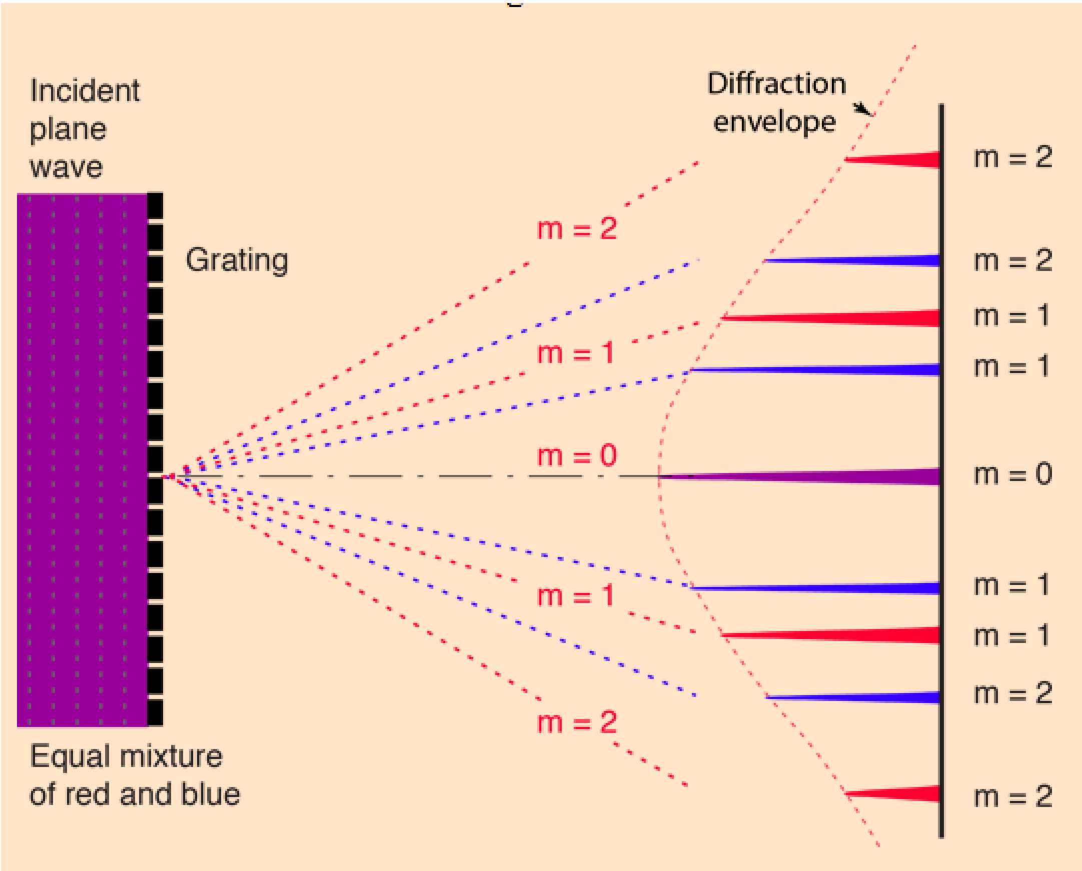


Grating equation

$$d\sin\theta = m\lambda$$

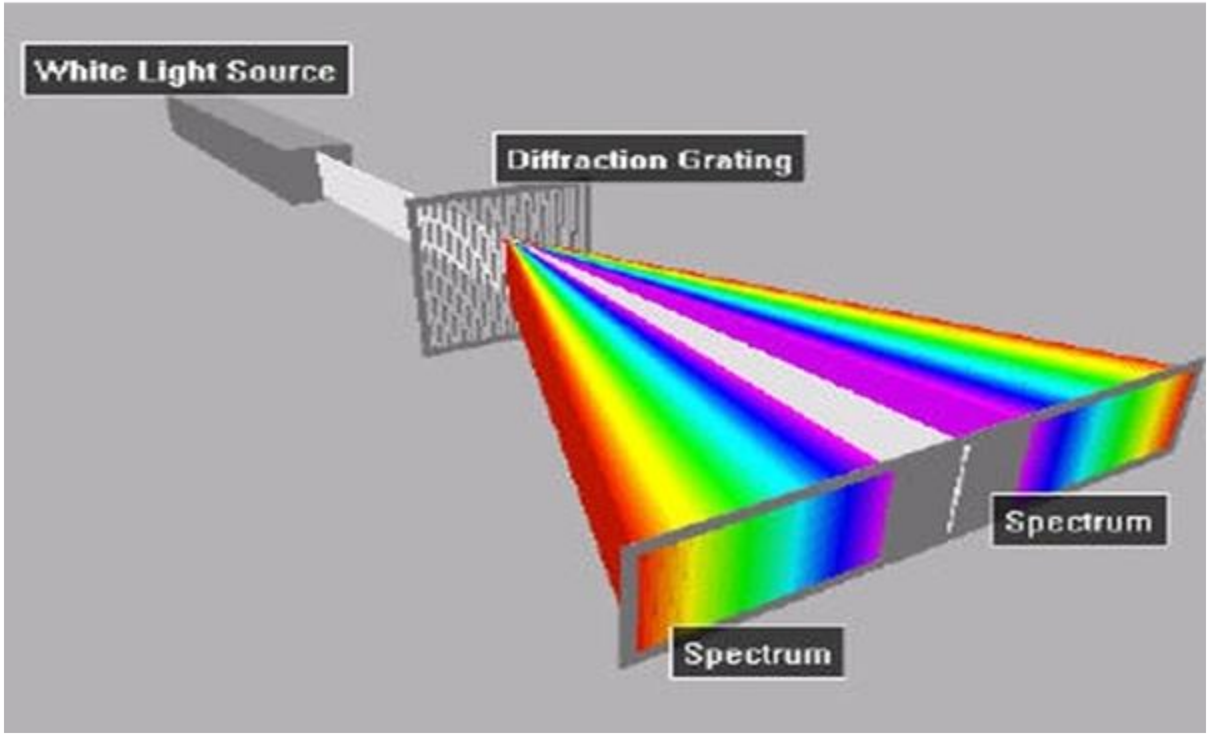
Dependence of angle of diffraction θ on λ.

Diffraction grating



A diffraction grating is a tool of choice for separating colors in incident light.

Diffraction Grating

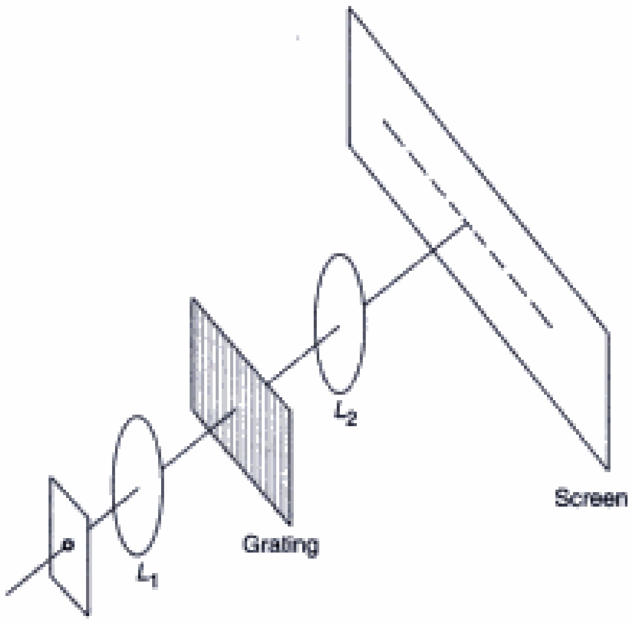


Differentiating grating equation

$$\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}$$

• Assuming θ to be very small (i.e., cosθ = 1) we see that $\Delta\theta$ is directly proportional to order of spectrum (m) for a given $\Delta\lambda$, so that for a given m, $\Delta\theta/\Delta\lambda$ is a constant. Such a spectrum is known as a **normal spectrum**.

• Δθ is inversely proportional to d, & therefore smaller grating element, larger will be angular dispersion.



Fraunhofer diffraction of a plane wave incident normally on a grating.

Resolving power of a grating

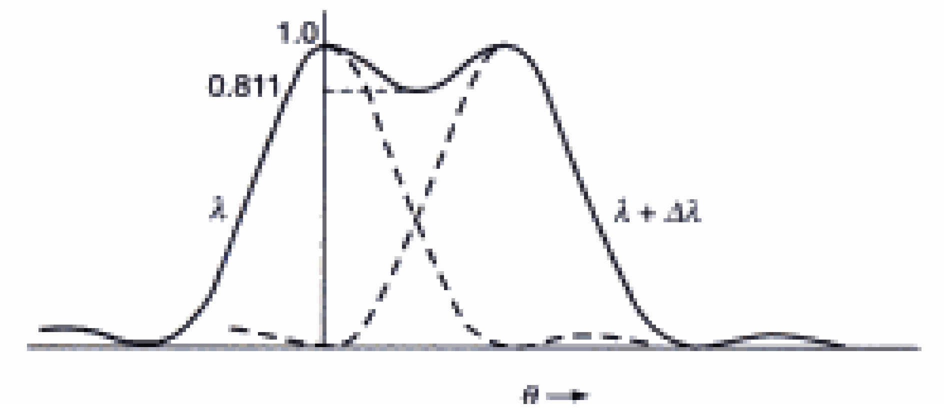
Resolving power of a grating refers to power of distinguishing two nearby spectral lines & is defined by (Chromatic resolving power)

$$R = \frac{\lambda}{\Delta\lambda}$$

$\Delta\lambda$ = separation of two wavelengths which grating can just resolve; **the smaller the value of $\Delta\lambda$, the larger the resolving power**.

Rayleigh criterion

If principal maximum corresponding to wavelength *λ + Δλ* falls on first minimum (on either side of principal maximum) of wavelength *λ*, then two wavelengths *λ* & *λ + Δλ* are said to be just **resolved**.



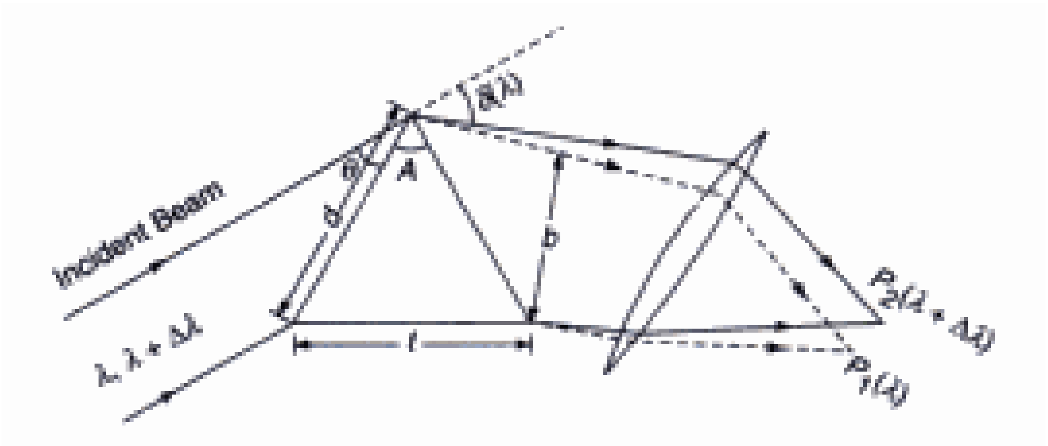
If this common diffraction angle is represented by *θ* & if we are looking at *m*th order spectrum, then two wavelengths *λ* & *λ + Δλ* will be just resolved if following two eqs. are simultaneously satisfied:

$$d\sin\theta = m(\lambda + \Delta\lambda) \qquad \&$$

$$d\sin\theta = m\lambda + \frac{\lambda}{N}$$

$$\Rightarrow R = \frac{\lambda}{\Delta\lambda} = mN$$

Resolving power depends on total no. of lines in grating. Further, resolving power is proportional to order of the spectrum.



Resolving power of a prism

$$n(\lambda)=\frac{\sin\dfrac{A+\delta(\lambda)}{2}}{\sin\dfrac{A}{2}}$$

$$\frac{dn}{d\lambda}=\frac{1}{\sin\dfrac{A}{2}}\cos\left[\frac{A+\delta(\lambda)}{2}\right]\frac{1}{2}\frac{d\delta}{d\lambda}$$

Resolving power, R

$$R=\frac{\lambda}{\Delta\lambda}=t\frac{dn}{d\lambda}$$

Cauchy formula: wavelength dependence of the refractive index

$$n=A+\frac{B}{\lambda^2}+\frac{C}{\lambda^4}+\ldots\ldots$$

Negative sign implies that the refractive index decreases with increase in wavelength.

$$\frac{dn}{d\lambda}=-\left[\frac{2B}{\lambda^3}+\frac{4C}{\lambda^5}+\ldots.\right]$$

No one has ever been able to define the difference between interference & diffraction satisfactorily. It is just a question of usage, & there is no specific, important physical difference between them. The best we can do is, roughly speaking, is to say that when there are only a few sources, say two, interfering, then the result is usually called interference, but if there is a large no. of them, it seems that the word diffraction is more often used.

Richard Feynman