

# Binary Decision Diagrams (BDD)

# BDD History

Original idea for Binary decision Diagram due to Lee (1959) and Akers (1978)  
Critical Refinement-Ordered BDDs -due to Bryant (1986)

- Refinement impose some restrictions on structure
- Restrictions needed to make result canonical representation

## Terminology;

A BDD is a directed Acyclic graph

- **Graph**: Vertices connected by edges
- **Directed**: edges have direction (drawn them with an arrow)
- **Acyclic** : No cycles possible by following arrows in graph

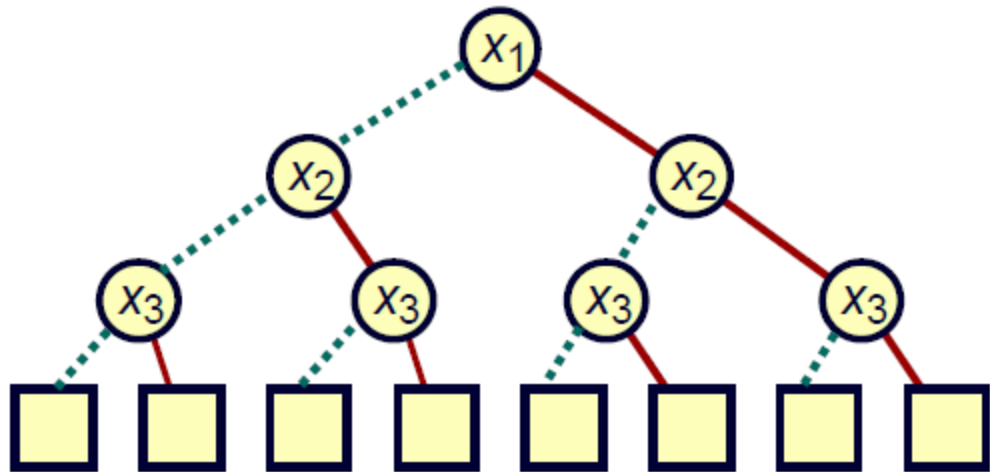
(Often see this shorted to DAG)

# BDD

## Truth Table

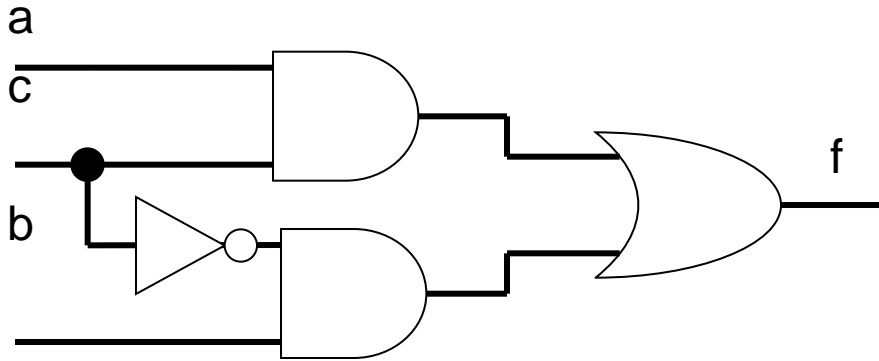
$x_1$	$x_2$	$x_3$	$f$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

## Decision Tree

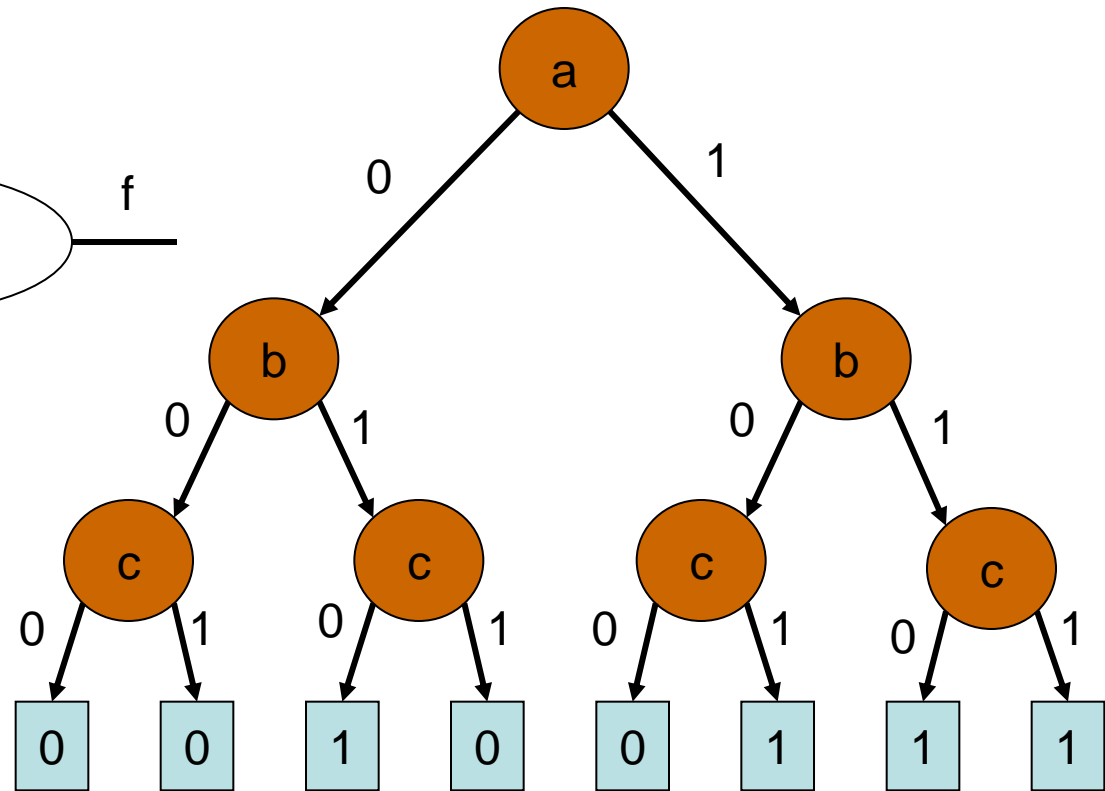


- Vertex represents a *decision*
- Follow **green** (dashed) line for value 0
- Follow **red** (solid) line for value 1
- Function value determined by leaf value.

# Binary Decision Tree

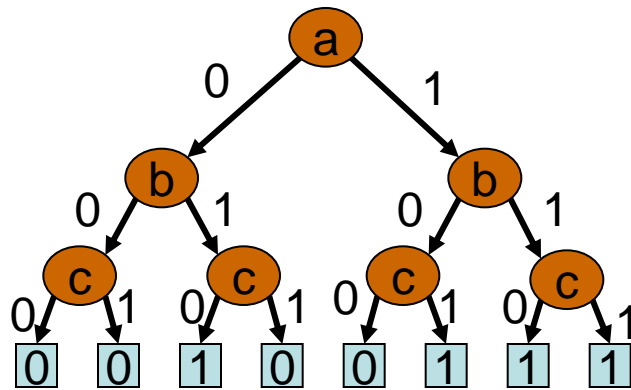
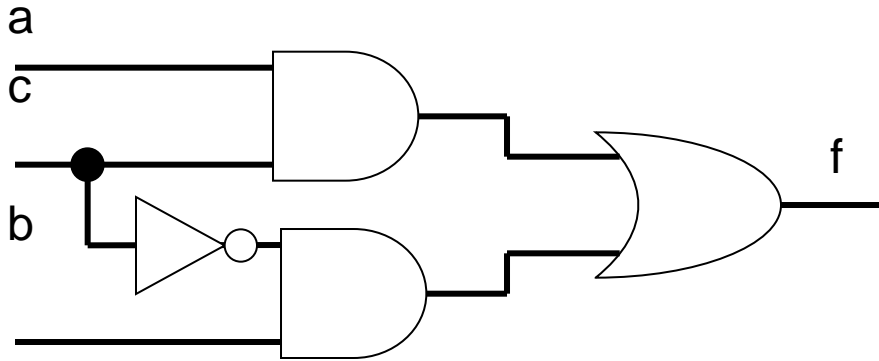


*Graph representation  
of a Boolean function.*

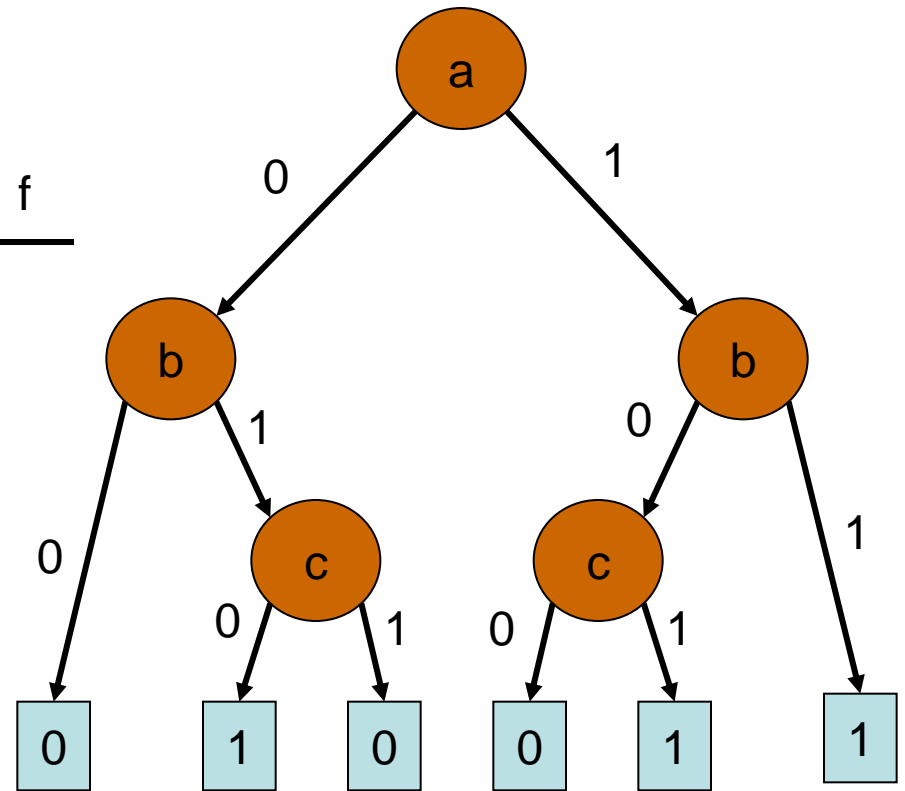


*Leaf nodes*

# Ordered Binary Decision Diagram (OBDD)

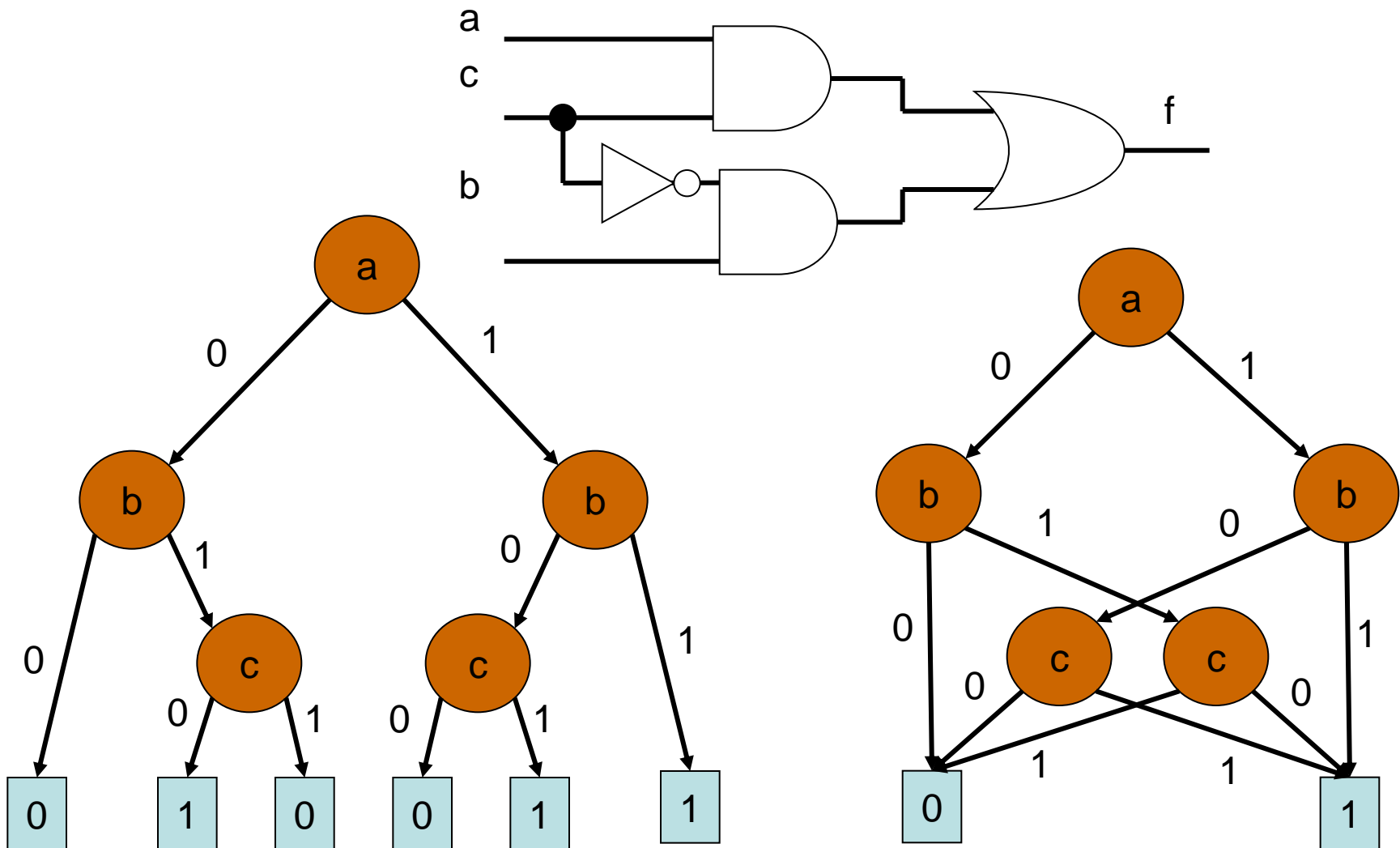


Tree

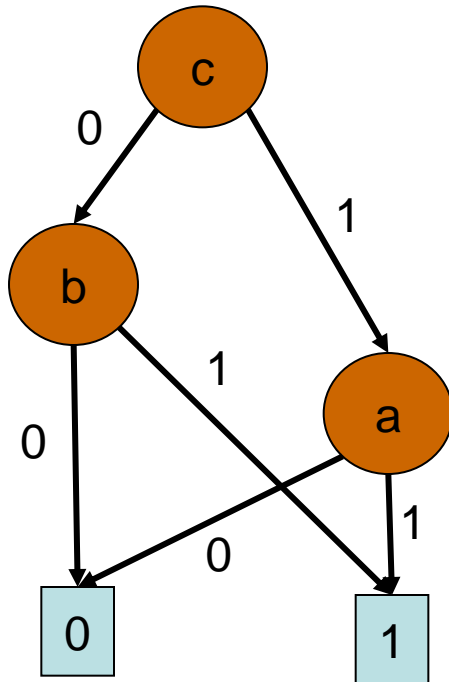
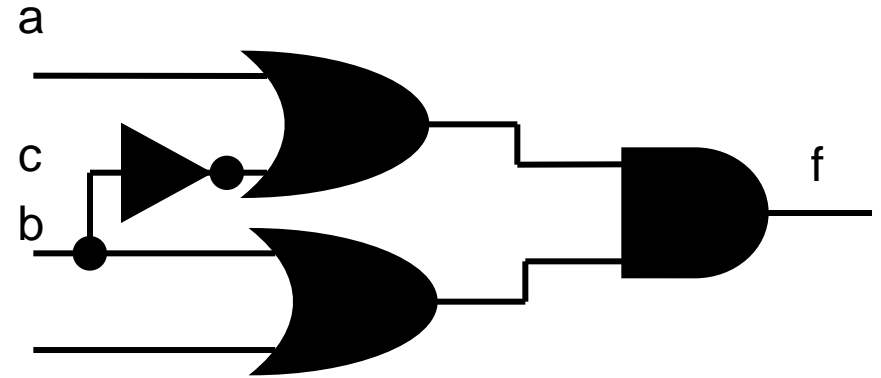
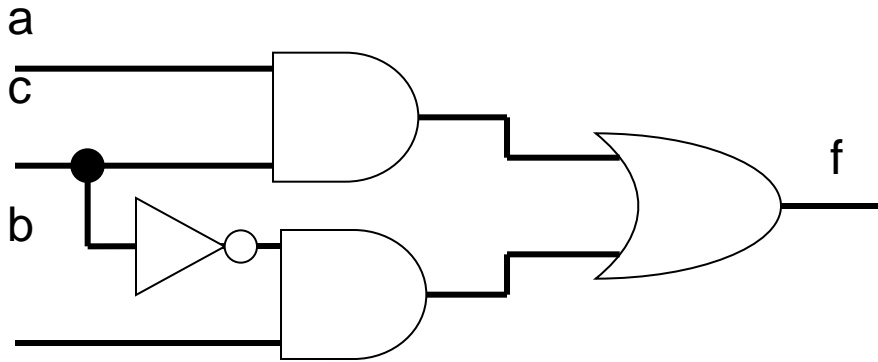


OBDD

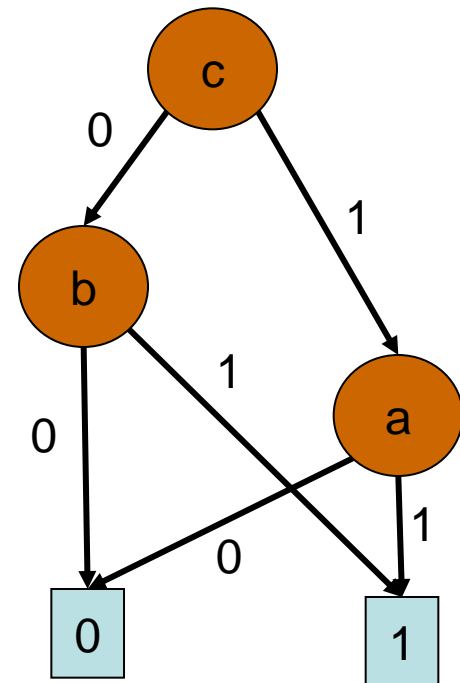
# Reduction: OBDD to ROBDD



# ROBDDs

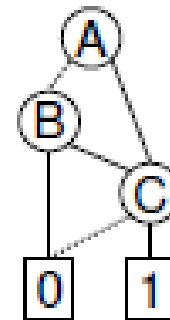
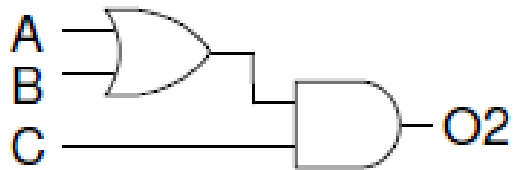
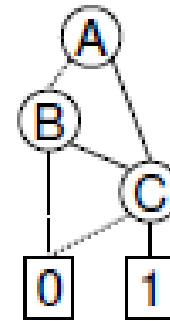
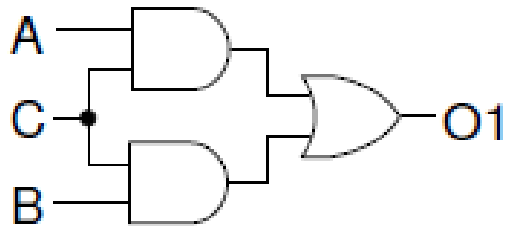


Isomorphic  
graphs



Functions are equal iff ROBDDs identical

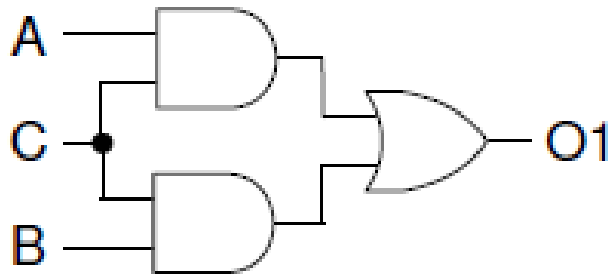
Solution using ROBDD



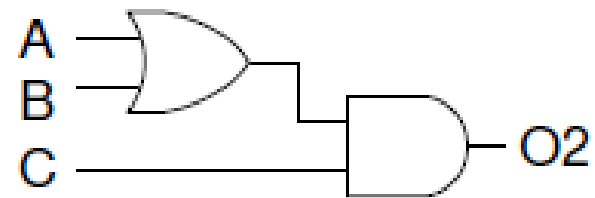


## Equivalence Checking Example

**Circuit 1**



**Circuit 2**



## Equivalence Checking Example

