भारतीय प्रौद्योगिकी संस्थान पटना
INDIAN INSTITUTE OF TECHNOLOGY PATNA

PH101 (Physics-I) **End-Semester Examination (November 19, 2015)**

[Full Marks: 50] **[Time: 180 minutes]**

• All the questions are compulsory. • Answers must be to the point (refrain from writing essays!). • Answers to all parts of a given question must be written together. • Marks for the questions are given in bold within square brackets.

1. For a certain driven-damped oscillator in one-dimension, $\frac{d^2x}{dt^2} + 3\,\frac{dx}{dt} + 2\,x = 10\,cost$; At $t = 0$, the particle is at rest at the origin. Obtain the following:
(a) Discuss an example (physical situation) where such an equation may arise. Explain the significance of each term with reference to the above example.**[2.5]**
(b) Using 'irreducible' (or another suitable approach), obtain the most general solution of the equations of motion.**[2.5]**
(c) For the given initial conditions, write down the expressions for the driven response and the transient response.**[2.5]**
(d) Graphically illustrate the results obtained in (c) above.**[2.5]**

2. Two bobs of masses m each are hung from a flat roof using strings of length l. These masses are further attached by a spring of stiffness κ such that the two strings are hung from the roof separated by a distance equal to the equilibrium length of the spring.
(a) Write down the equations of motion for the coupled pendulum described above.**[2.5]**
(b) Express the equations in a matrix notation. Write down the corresponding $\bar{M}$ and $\bar{K}$ matrices.**[2.5]**
(c) Obtain the normal mode angular frequencies and the corresponding normal modes. Physically interpret the results.**[2.5]**

3. (a) Estimate the minimum intensity of audibility in air in Watt-cm^{-2} for a note of 1000 Hz. Assume, density of air $= 0.0013\,\mathrm{g/cc}$, velocity of sound $= 340\,\mathrm{m/s}$ and amplitude of vibration $= 0.1\,\mathring{A}$. What about notes at 20 Hz and 20 kHz? What can you say about human ear as a detector for sound wave based on above?**[2.5]**
(b) Using the *principle of optical reversibility*, derive Stoke's relations.**[2.5]**
(c) A biprism is placed 10 cm away from a slit illuminated by sodium light ($\lambda = 5890\,\mathring{A}$). The width of the fringes obtained on a screen placed at a distance of 90 cm from the biprism is 1 mm. Obtain the distance between the two resulting coherent sources?**[2.5]**
(d) In a double-slit interference experiment, one of the slits is covered by a thin crown glass cover-slip (refractive index 1.52). Due to the introduction of the crown glass cover-slip the central fringe gets shifted by 0.25 cm. Determine the thickness of the cover-slip. The distance between the two slits is 0.1 cm and the screen is kept 1 m away.**[2.5]**

4. (a) Explain using suitable $V(x)$, the difference between bound states and scattering states and write down the conditions for their existence for the Coulomb potential (where the potential goes to zero at infinity).**[2.5]**
(b) What is a Dirac delta 'function'? Write down its properties? Explain using a suitable representation for the Dirac delta 'function'.**[2.5]**
(c) Show that the one-dimensional attractive Dirac delta function potential given by $V(x) = -\alpha\delta(x)$ has a unique bound state. Obtain the corresponding wave function and the energy eigenvalue.**[2.5]**
(d) Write down the wave functions for the scattering states for the potential in (c) above and determine the reflection and transmission coefficients.**[2.5]**
(e) Obtain the M-matrix (transfer matrix) for the attractive Dirac delta function potential in (c) above.**[2.5]**

5. (a) Schrödinger equation has a remarkable property that it automatically preserves the normalization of the wave function. i.e., $\frac{d}{dt}\int_{-\infty}^{\infty} \mid \Psi(x,t) \mid^2 = 0$. Prove it.**[2.5]**
(b) The solution for the time independent Schrödinger equation in one dimension for a given potential is given by a wave function $\psi(x) = x\,exp(-\alpha x)$ for $x > 0$ and zero elsewhere. Normalize the wave function and obtain $< x >$ and $< x^2 >$.**[2.5]**
(c) For the wave function given in (b) above, find the corresponding wave function in the momentum space and obtain $< p_x >$ and $< p_x^2 >$.**[2.5]**
(d) A one-dimensional Dirac comb potential is represented by $V(x) = \Sigma_n V_n \delta(x - na)$. Can you suggest a physical