

# Indian Institute of Technology Patna

## MA201: Mathematics III

### Mid Semester Exam (11-09-2012)

Time: 2hrs

Max. Marks: 30

**Note:** Answer all questions. Give precise and brief answer. Standard formulae may be used.

Q.1. Answer all parts at one place. [1+1+1+1+1+1+1]

- Find  $M$  such that  $|z^5 - 4| \leq M$  for all  $|z| \leq 1$ .
- The set  $S = \{z : 1 < |z| < 3\}$  is an Open set. (True / False).
- Find  $\oint_C \frac{\sin z}{z-2} dz$ , where  $C$  is circle of radius 1.
- Is  $f(z) = \cos(x - iy)$  analytic?
- $\text{Log}(z - i)$  is analytic at all points in  $z$ -plane except for  $x = \dots$  and  $y = \dots$ .
- Find all  $z$  such that  $e^z = -2$ .
- Find the radius of convergence for  $\sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$ .

Q.2. Find limit  $\lim_{z \rightarrow 0} \frac{\text{Re} z}{|z|}$  [3]

Q.3. Show that  $f'(z)$  exists for  $f(z) = \sin x \cosh y + i \cos x \sinh y$  [3]

Q.4 Find principle value of  $(1 - i)^{4i}$ . [3]

Q.5. Find all the roots of  $\sin z = \cosh 4$ . [4]

Q.6. Suppose that  $f(z)$  and  $\overline{f(z)}$  both are analytic in a Domain  $D$ . Hence prove that  $f(z)$  must be constant throughout  $D$ . [3]

Q.7. Given that  $u(x, y) = e^{-x}(x \sin y - y \cos y)$  is a harmonic function throughout  $xy$  plane. Find harmonic conjugate of  $u(x, y)$  and hence the corresponding analytic function. [3]

Q.8. Find the value of following integrals around the circle  $(C) |z - i| = 2$  in positive sense: [4]

a.  $\oint_C \frac{1}{z+4} dz,$

b.  $\oint_C \frac{1}{z^2+4} dz,$

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*You are expected to conduct yourself honestly and ethically, and are obliged to refrain from acts, which you know violate the academic integrity of the Institute.*

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