Indian Institute of Technology Patna
Department of Mathematics
MA - 225: B.Tech. II year

Tutorial Sheet-3

1. A continuous random variable has the probability density function (PDF) defined as $f_X(x) = cxe^{-x/2}, \ x \geq 0$ and $f_X(x) = 0, \ x < 0$.
(*i*) Find the constant c (*ii*) What is the CDF of X (*iii*) Find the mean, variance and standard deviation of X (*iv*) Where is the median of X located

2. In a dart game the player wins at a circular target having a radius of 25 centimeters. Let X be the distance (in centimeters) between the dart's impact point and the center of the target. Suppose that $P(X \leq x) = c\pi x^2, \ 0 \leq x \leq 25$ and $= 1, \ x > 25$, where c is a constant. Evaluate (*i*) the constant c (*ii*) the PDF of X (*iii*) the mean of X (*iv*) the probability $P(X \leq 10 \mid X \geq 5)$ (*iv*) It costs 1\$ to throw a dart and the player wins 10\$ $if \ X \leq r$, 1\$ $if \ r < X \leq 2r$, 0\$ $if \ 2r < X \leq 25$. For what values of r is the average gain of the player equal to 0.25\$.

3. Show that the function defined as
$$\frac{9(3+2x)}{x^2(3+x)^2} I_{(3,\infty)}(x) + \frac{x(6+x)}{3(3+x)^2} I_{(0,3]}(x)$$
is a probability density function (PDF).

4. Does the function $\theta^2 x e^{-\theta x}, \ x > 0$, and $= 0, \ x \leq 0, \ \theta > 0$ defines a probability density function? If yes, find the corresponding distribution function and also evaluate $P(X \geq 1)$.

5. Are the following functions distribution functions. If so, find the corresponding PDF/PMF.
(*i*) $F(x) = 0, \ x \leq 0, = x/2, \ 0 \leq x < 1, = 1/2, \ 1 \leq x < 2, \ = x/4, \ 2 \leq x < 4, \ = 1, \ x \geq 4$.
(*ii*) $F(x) = 0, \ x < -\theta, = \frac{1}{2}(x/\theta + 1), \ |x| \leq \theta, \ = 1, x > \theta$
(*iii*) $F(x) = 0, \ x < 1, = \frac{(x-1)^2}{8}, \ 1 \leq x < 3, = 1, \ x \geq 3$.

6. Consider the following probability mass function:
$P(X = 0) = 0, P(X = 1) = k, P(X = 2) = 2k = P(X = 3), P(X = 4) = 3k, P(X = 5) = k^2, P(X = 6) = 2k^2, P(X = 7) = 7k^2 + k$. Find (*i*) k (ii) Evaluate $P(X < 6), P(X \geq 6), P(0 < X < 5)$ (iii) the corresponding distribution function.

7. Two fair dice are thrown. If X is the sum of numbers showing up, prove that $P(|X - 7| \geq 4) \leq \frac{35}{36}$. Compare it with actual probability.

8. Let X be continuous random variable with PDF $0.5e^{-0.5x} I_{(0,\infty)}(x)$. Show that $P(|X - 2| \geq 2) \leq \frac{1}{2}$. Find the actual probability.
Continue

9. Let X be an random variable (RV) such that $E|X| < \infty$. Show that $E|X - c|$ is minimized if we choose c to be the median of the distribution of X.

10. Let X be an RV with pdf $f(x) = \frac{\Gamma(m)}{\Gamma(1/2)\Gamma(m-\frac{1}{2})(1+x^2)^m}, \quad -\infty < x < \infty, \ m \geq 1$. Evaluate EX^{2r} whenever it exists.

11. Let $f(x)$ be the density function of the RV X. Suppose that X has symmetric distribution about a. Show that the mean of X is a itself.

12. Show that the first k noncentral moments determine the first k central moments and conversely, that the first k central moments determine the first k noncentral moments.

13. Let X be a RV with Distribution Function $F(x) = 1 - 0.8e^{-x}, \ x \geq 0$ and $F(x) = 0, \ x < 0$. Find EX.

14. Let $f_X(x) = \frac{1}{2}\left[1 - \frac{|x-3|}{2}\right] I_{(1,5)}(x)$. Check that $f_X(x)$ is a PDF. Find mean, median, variance and p^{th} quantile of X.

15. Let $f_X(x) = \frac{k}{\beta}\left[1 - \frac{(x-\alpha)^2}{\beta^2}\right] I_{(\alpha-\beta,\alpha+\beta)}(x)$ where $-\infty < \alpha < \infty$, $\beta > 0$. Find the value of k so that $f_X(x)$ is a PDF. Then find mean, median, variance and p^{th} quantile of X. Also evaluate $E|X - \alpha|$.

16. Let X be an RV with density function $f(x) = 1/2, \ -1 \le x \le 1$, and $= 0$ otherwise. Find the distribution function of $\max(X, 0)$.

17. Let X be an RV with PMF $P(X = -2) = P(X = 0) = 1/4, P(X = 1) = 1/3, P(X = 2) = 1/6$. Find median of X. Also find quantile of order 0.2.

18. Find the moment generating function for the density function $\frac{1}{2a}e^{-\frac{|x-\mu|}{a}}, \ -\infty < x < \infty, a > 0, \ -\infty < \mu < \infty$. First check whether or not it is a density function. Also find the mean deviation about mean.

19. A coin is tossed until a head appears. Find the expected number of tosses required to obtain the first head.

20. A man with n keys wants to open his door and tries the keys independently and at random. Find the mean and variance of the number of trials required to open the door, (i) if unsuccessful keys are not eliminated from further selection, (ii) if they are eliminated.

$f_X(x) = \frac{k}{\ }$