Indian Institute of Technology, Patna
MA101, B.Tech -I year
Autumn Semester: 2014-2015
(End Semester Examintaion)

Maximum Marks: 50 **Time: 3 Hours**

<u>**Note**</u>:

(i) This question paper has TWO pages and contain TWELVE questions. Please check all pages and report the discrepancy, if any.

(ii) Attempt all questions.

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1. If x is a positive real number then show that there exists a natural number n such that $\frac{1}{2^n} < x$. **[2]**

2. Find a point on the plane $2x + 3y - z = 5$ which is nearest to the origin. **[3]**

3. Find the slope of the curve $y = \frac{1}{x}$ at $x = a \neq 0$. Where does the slope equal -1/4? **[5]**

4. Suppose

$$f(x,y) = \begin{cases} x sin\dfrac{1}{y} + y sin\dfrac{1}{x}, & \text{for } (x,y) \neq (0,0) \\ 0, & \text{for } (x,y) = (0,0) \end{cases}$$

Then show that $\lim_{(x,y)\to(0,0)} f(x,y)$ exists. What can you say about the repeated limits? **[3]**

5. Suppose

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}, & \text{for } (x,y) \neq (0,0) \\ 0, & \text{for } (x,y) = (0,0) \end{cases}.$$

Check the differentiability of the above function at (0,0). **[3]**

6. Locate the critical points of $f(x,y) = 3x^4 + y^2 - 4x^2y$ and determine their nature. **[3]**

7. Use Lagrange Multiplier method to find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **[5]**

8. Verify Green's theorem in the plane for $\int\limits_{\Gamma}(2xy - x^2)dx + (x + y^2)dy$ where Γ is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$. [4]

9. Sketch the region bounded by the graphs of the functions $f(x) = sinx$ and $g(x) = cosx$, $\pi/4 \leq x \leq 5\pi/4$ and find the corresponding area. [1+3]

10. Consider the integral $\int_0^{\sqrt{\pi/2}}[\int_x^{\sqrt{\pi/2}}[\int_1^3 sin(y^2)dz]dy]dx$. Sketch the region over which the function $f(x, y, z) = sin(y^2)$ is being integrated. Interchange the order of integration for the variables x by y and evaluate the above integral. [1+5]

11. If $v = cos^{-1}(\frac{x+y}{\sqrt{x}+\sqrt{y}})$ then verify that $cosv$ is a homogeneous function of degree 1/2. Hence prove $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + \frac{1}{2}cotv = 0$ [2]

12. Check whether the following statements are true or false. Give appropriate reasons to support your answers.

(a) Let $f(x, y) = (x - 2)^2(y + 3)$. Then f has a local minimum at $(2, -3)$.

(b) The value of the line integral $\int_A^B \frac{xdx+ydy}{\sqrt{x^2+y^2}}$ does not depend upon the path joining A and B.

(c) $\lim_{(x,y)\to(2,1)} \frac{sin^{-1}(xy-2)}{tan^{-1}(3xy-6)} = \frac{4}{5}$.

(d) Suppose

$$f(x,y) = \begin{cases} (x^2+y^2)Sin\dfrac{1}{x^2+y^2}, & \text{for } (x,y) \neq (0,0) \\ 0, & \text{for } (x,y) = (0,0) \end{cases}$$

f has directional derivative at (0,0) in any direction.

(e) Taking $\mathbb{F} = x^2yi + xzj + 2yzk$, then $div(curlF) = 1$.

[2×5]