CS 225: Switching Theory

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Previous Class

Switching Algebra

This Class

- Switching Algebra
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  - Switching circuit
  - Propositional calculus

Functional Properties

Let binary constant a_i be the value of function f(x1, x2, …, xn) for the combination of variables whose decimal code is i. Thus,

$$f(x1, x2, \ldots, xn)= a_0x_1'x_2'\ldots x_n' + a_1x_1'x_2'\ldots x_n + \ldots + a_rx_1x_2\ldots x_n$$

The coefficient a_i is set to 1 (0) if the corresponding minterm is (is not) in the canonical form

Since there are 2^n coefficients, each of which can have two values, 0 and 1, there are 2^2^n possible switching functions of n variables

Example: Canonical sum-of-products form for two variables

$$f(x,y) = a_0x'y' + a_1x'y + a_2xy' + a_3xy$$

Thus 2^2^2 =16 functions corresponding to the 16 possible assignments of 0's and 1's to a_0, a_1, a_2, and a_3

List of Functions of Two Variables

a3	a2	a1	a0	f(x,y)	Name of Function	Symbol
0	0	0	0	0	Inconsistency	
0	0	0	1	$x'y'$	NOR	$x\downarrow y$
0	0	1	0	$x'y$		
0	0	1	1	x'	NOT	x'
0	1	0	0	xy'		
0	1	0	1	y'		
0	1	1	0	$x'y+xy'$	Exclusive OR	$x\oplus y$
0	1	1	1	$x'+y'$	NAND	$x\|y$

a3	a2	a1	a0	f(x,y)	Name of Function	Symbol
1	0	0	0	xy	AND	$x.y$
1	0	0	1	$xy+x'y'$	Equivalence	$x\equiv y$
1	0	1	0	y		
1	0	1	1	$x'+y$	Implication	$x\longrightarrow y$
1	1	0	0	x		
1	1	0	1	$x+y'$	Implication	$y\longrightarrow x$
1	1	1	0	$x+y$	OR	$x+y$
1	1	1	1	1	Tautology	

The Exclusive-OR Operation

Exclusive-OR: modulo-2 addition, i.e., $A \oplus B = 1$ if either A or B is 1, but not both

Commutativity: $A \oplus B = B \oplus A$

Associativity: $(A \oplus B) \oplus C = A \oplus (B \oplus C) = A \oplus B \oplus C$

Distributivity: $(AB) \oplus (AC) = A(B \oplus C)$

If $A \oplus B = C$, then

$$A \oplus C = B$$
$$B \oplus C = A$$
$$A \oplus B \oplus C = 0$$

Exclusive-OR of an even number of elements, whose value is 1, is 0
Exclusive-OR of an odd number of elements, whose value is 1, is 1

Functionally Complete Operations

Every switching function can be expressed in canonical form consisting of a finite number of switching variables, constants and operations $+, ., '$

A set of operations is functionally complete (or universal) if and only if every switching function can be expressed by operations from this set

Example: Set $\{+, ., '\}$

Set $\{+, '\}$?

Yes, since using De Morgan's theorem, $x \cdot y = (x' + y')'$.

Thus, $+$ and $'$ can replace the $.$ in any switching function

Set $\{., '\}$ Yes for similar reasons

NAND:? Yes, $\text{NAND}(x,x) = x'$ and $\text{NAND}[\text{NAND}(x,y),\text{NAND}(x,y)] = xy$

NOR:? Yes, $\text{NOR}(x,x) = x'$ and $\text{NOR}[\text{NOR}(x,y), \text{NOR}(x,y)] = x + y$

Isomorphic Systems

Isomorphism: Two algebraic systems are isomorphic if

- For every operation in one system, there exists a corresponding operation in the second system
- To each element xi in one system, there corresponds a unique element yi in the other system, and vice versa
- If each operation and element in every postulate of one system is replaced by the corresponding operation and element in the other system, then the resulting postulate is valid in the second system

Thus, two algebraic systems are isomorphic if and only if they are identical except the labels and symbols used to represent the operations and elements

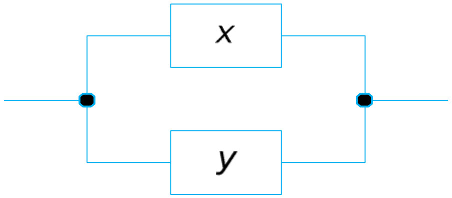
Series-parallel Switching Circuits

Gate: a two-state device capable of switching from one state, which permits flow of information, to another, which blocks it, and vice-versa

two-valued variable: denotes flowing (blocked) information

If a gate permits (blocks) the flow of information: literal associated with it takes value 1 (0)

Elementary series-parallel switching circuits



Parallel connection $x + y$

Series connection xy

Series-parallel circuits: any circuit constructed of either a series or parallel connection of two or more elementary series-parallel circuits

Transmission Function

Transmission function for a circuit: assumes value 1 (0) when there is (there is not) a path from one terminal of the circuit to the other through which information flows

Definition of transmission functions

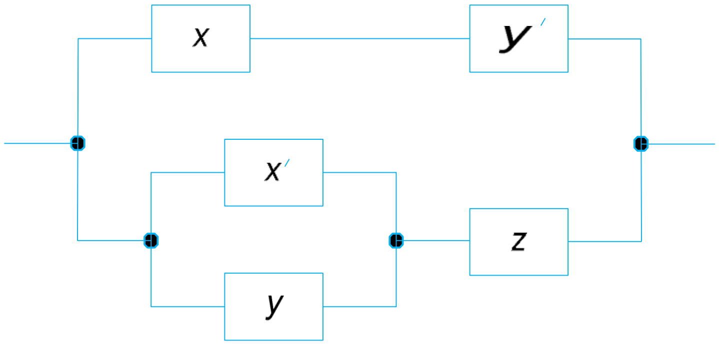
x y	x + y	xy
0 0	0	0
0 1	1	0
1 0	1	0
1 1	1	1

Analogy: OR <-> parallel; AND <-> series

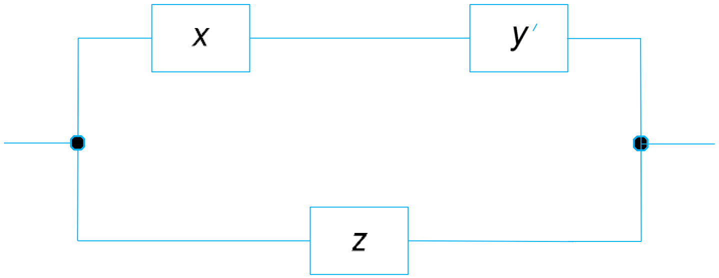
Complement of a given circuit: one that blocks all paths of information flow whenever the given circuit permits any

Thus, algebraic system for switching circuits isomorphic to switching algebra

Switching Circuit Simplification



Circuit realizing T = xy + (x′+ y)z



Simplified circuit realizing

$T = xy' + x'z + yz$

$= xy' + x'z + y'z + yz$

$= xy' + x'z + z$

$= xy' + z$

Important application of theory of switching circuits: CMOS

Propositional Calculus

Proposition: declarative statement which may be either true or false

Example: temperature is 100 degree Celsius
turtle runs faster than the hare
sum of 2 and 3 equals 5

Proposition variable: 1 (0) if proposition is true (false)

Negation p' of proposition p: 1 (0) if p is 0 (1)

Conjunction of propositions p and q is pq: true when both p and q are true and false whenever either one or both p and q are false

Disjunction of propositions p and q is p + q: true when either p or q or both are true and false whenever both p and q are false

Propositional Calculus (Contd.)

Definition of conjunction and disjunction of p and q

p q	pq	p + q
0 0	0	0
0 1	0	1
1 0	0	1
1 1	1	1

- **Analogy**: OR <-> disjunction; AND <-> conjunction
- Thus, algebraic system for switching circuits is isomorphic to propositional calculus

Propositional Calculus Example

Example: Air-conditioning of a storage warehouse to be turned on if one or more of the following three conditions occurs:

1. Weight of stored material is less than 100 kg, relative humidity is at least 60%, and temperature is above 60 degrees Celsius
2. Weight of stored material is 100 kg or more and temperature is above 60 degrees Celsius
3. Weight of the stored material is less than 100 kg and the barometer stands at 30 inches of mercury or over

Propositional Calculus Example (Contd.)

A: proposition that air-conditioning is turned on

W: weight of 100 kg or more

H: relative humidity of at least 60%

T: temperature above 60 degrees Celsius

P: barometric pressure is 30 inches of mercury or more

$$\begin{aligned} A &= W'HT + WT + W'P \\ &= HT + WT + W'P \\ &= T(H+W) + W'P \end{aligned}$$

Thus, air-conditioning is on if the temperature is above 60 degrees Celsius and either the weight is at least 100 kg or the humidity is at least 60%, or if the weight is less than 100 kg and the barometer stands at 30 inches or over

Electronic Gate Networks

Electronic gates: generally receive voltages as inputs and produce output voltages

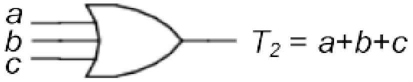
Precise values of voltages not significant: restricted to value ranges – high (value 1) and low (value 0)

Electronic gates constructed with two-state switching devices: each capable of permitting the flow of current or blocking it

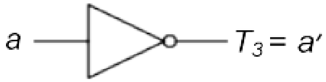
To implement arbitrary switching functions: gates must be able to implement a functionally complete set of operations

Functionally Complete Set

(a) AND gate.

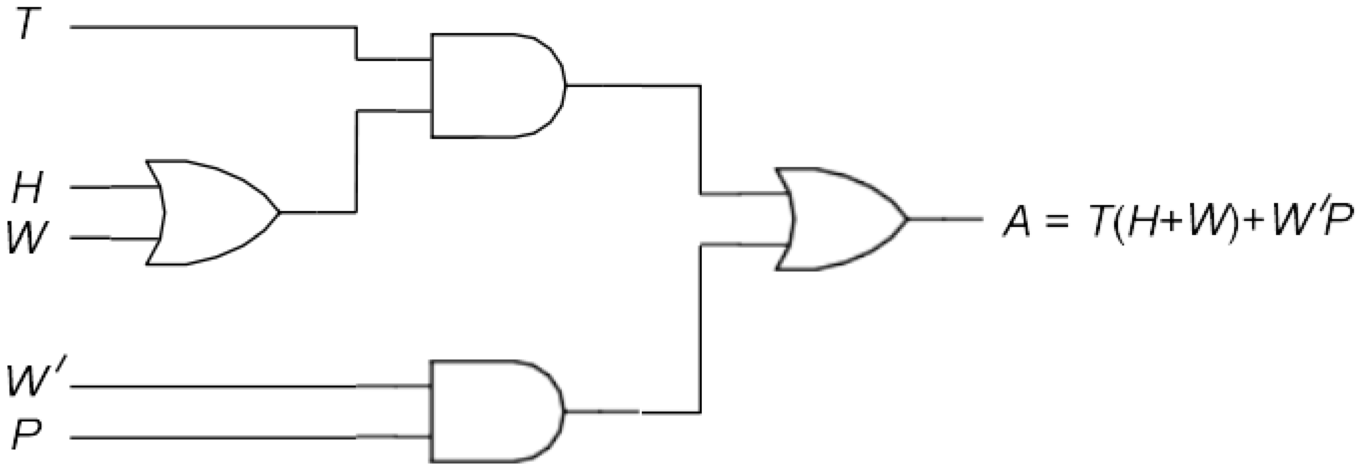


(b) OR gate.



(c) NOT gate.

Gate Network for Air-conditioning Function



Boolean Algebra

Boolean algebra B: A set of elements a, b, c, …, and binary operations + and . that satisfy the idempotent, commutative, absorption, and associative laws, and are mutually distributive

Complement a′ of any element a in B is unique, i.e., there exists only element a′ such that a + a′ = 1 and a . a′ = 0

- Suppose there exists element a which has two complements, b1 and b2, i.e., a + b1 = 1, a . b1 = 0, a + b2 = 1, a . b2 = 0
- Then b1 = b1 . 1 = b1 . (a + b2) = b1 . a + b1 . b2 = 0 + b1 . b2 = b1 . b2
- Similar arguments show b2 = b1 . b2.
- Thus, b1 = b2, proving the uniqueness of the complement

Complements of elements 0 and 1: since by definition 0 + 0′ = 1, 0′ = 1. Similarly, 1′ = 0

De Morgan's Theorem

Prove De Morgan's theorem for two variables:

$$(a + b)' = a' \,.\, b'$$
$$(a \,.\, b)' = a' + b'$$

- We have to show that $(a + b)(a' \,.\, b') = 0$ and $(a + b) + a' \,.\, b' = 1$
- Applying the distributive law:
  - $(a + b) + a'b' = (a + b + a')(a + b + b') = (b + 1)(a + 1) = 1$
- Dual property proved similarly

Definition of a Boolean algebra isomorphic to switching algebra

+	**0**	**1**	**.**	**0**	**1**	
0	0	1	0	0	0	$0' = 1$
1	1	1	1	0	1	$1' = 0$

De Morgan's Theorem

Example of Boolean algebra:

+	0 1 a b	.	0 1 a b	
0	0 1 a b	0	0 0 0 0	$0' = 1$
1	1 1 1 1	1	0 1 a b	$1' = 0$
a	a 1 a 1	a	0 a a 0	$a' = b$
b	b 1 1 b	b	0 b 0 b	$b' = a$

Boolean Algebra Operator Precedence

- Evaluate the following Boolean equations, assuming a=1, b=1, c=0, d=1.
  - Q1. $F = (ac)'$.
    - Answer: we first evaluate what is inside the parentheses, then we NOT the result, yielding $(1*0)' = (0)' = 0' = 1$.
  - Q2. $F = ab + c$.
    - Answer: the problem is identical to the previous problem, using the shorthand notation for *.
  - Q3. $F = ab'$.
    - Answer: we first evaluate b' because NOT has precedence over AND, resulting in $F = 1 * (1') = 1 * (0) = 1 * 0 = 0$.

a

Boolean algebra precedence, highest precedence first.

Symbol	Name	Description
( )	Parentheses	Evaluate expressions nested in parentheses first
’	NOT	Evaluate from left to right
*	AND	Evaluate from left to right
+	OR	Evaluate from left to right