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$$(a) \text{ Let } \nu \propto R^a \rho^b S^c$$

$$\Rightarrow [\nu] = [R]^a [\rho]^b [S]^c$$

$$\therefore T^{-1} = L^a (ML^{-3})^b (MT^{-2})^c$$

$$\Rightarrow \begin{cases} -2c = -1 \\ a - 3b = 0 \\ b + c = 0 \end{cases}$$

$$\text{Thus, } c = \frac{1}{2}, b = -\frac{1}{2}, a = -\frac{3}{2}.$$

$$\therefore \nu \propto \sqrt{\frac{S}{\rho R^3}}.$$

[Note: Mass of the drop $m \propto \rho R^3$.

$$\Rightarrow \nu \propto \sqrt{\frac{S}{m}}.]$$

Thus if radius is doubled, frequency is reduced by a factor of $\frac{1}{2\sqrt{2}}$.

$$(b) \quad [h] = ML^2T^{-1}$$

$$[c] = LT^{-1}$$

$$[G] = M^{-1}L^3T^{-2}$$

$$\text{Let } M_p = k_1 h^a c^b G^c$$

$$[M_p] = [h]^a [c]^b [G]^c$$

$$= [ML^2T^{-1}]^a [LT^{-1}]^b [M^{-1}L^3T^{-2}]^c$$

$$\therefore M^1 = M^{a-c} L^{2a+b+3c} T^{-a-b-2c}$$

$$\Rightarrow a-c=1, \quad 2a+b+3c=0$$

and $-a-b-2c=0$.

$$\Rightarrow a = -b-2c. \text{ Thus, } -2b-4c+b+3c=0$$

$$\therefore b = -c \Rightarrow a = -c$$

$$\therefore c = -\frac{1}{2}, \quad a = \frac{1}{2}, \quad b = \frac{1}{2}.$$

$$\therefore \boxed{M_p = k_1 \sqrt{\frac{hc}{G}}}$$

$$(c) \quad T_p = k_2 h^a c^b G^c$$

$$\Rightarrow T^1 = M^{a-c} L^{2a+b+3c} T^{-a-b-2c}$$

$$\therefore a=c, \quad b=-5c, \quad -c+5c-2c=1$$

$$\therefore c = \frac{1}{2}, \quad a = \frac{1}{2}, \quad b = -\frac{5}{2}.$$

$$\therefore \boxed{T_p = k_2 \sqrt{\frac{hG}{c^5}}}$$

$$2(a) \quad I = -G(V_2 - V_1)$$

$$\text{We have, } |V_2 - V_1| = 1 \text{ Volt.}$$

$$I = 1 \mu\text{A.}$$

$$\therefore G = \frac{1 \mu\text{A}}{1 \text{ V}} = 1 \mu\Omega^{-1} = 1 \mu\text{Siemens}$$

$$(b.) \quad \frac{dH}{dt} = -K(T_2 - T_1)$$

$$\text{We have, } |T_2 - T_1| = 100 \text{ K}$$

$$\frac{dH}{dt} = \frac{5 \text{ mJ}}{1 \text{ s}} = 5 \text{ mWatts}$$

$$\therefore K = \frac{5 \text{ mWatts}}{100 \text{ K}} = 50 \mu\text{Watts.K}^{-1}.$$

$$(c.) \quad r = 10^{-2} \text{ m} \quad \Rightarrow \quad r^4 = 10^{-8} \text{ m}^4.$$

$$l = 1 \text{ m}$$

$$\eta = 0.015 \text{ Ns/m}^2$$

$$\therefore \text{Flow conductance, } F = \frac{\pi r^4}{8\eta l}.$$

$$= \frac{3.1416 \times 10^{-8} \text{ m}^5/\text{Ns}}{8 \times 0.015 \times 1}$$

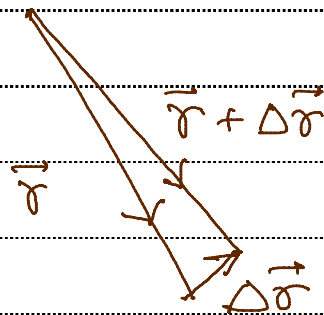
$$= 2.618 \times 10^{-7} \text{ m}^5/\text{Ns}.$$

3 (a) Simple pendulum with rigid support.

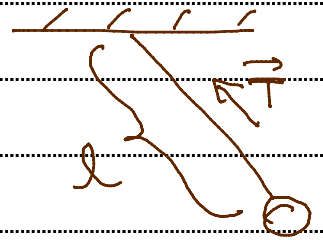
Position of bob at any time must satisfy the equation

$$|\vec{r}|^2 = l^2, \text{ independent of time.}$$

Also, tension $\vec{T} \parallel -\vec{r}$.



$$\Delta \vec{r} \perp \vec{r}.$$



$$\Rightarrow \vec{T} \cdot \Delta \vec{r} = 0.$$

\therefore The constraint is scleronomic, holonomic, bilateral & conservative in nature.

(b.) Deformable body

The constraint is given by an equation

$$|\vec{r}_i - \vec{r}_k| = f(t) \text{ (time dependent)}$$

Here $\Delta W \neq 0$.

\therefore The constraint is rheonomic, holonomic, bilateral and dissipative in nature.

3(c.) An expanding/contracting spherical container of gas

Position of gas particles at any time will satisfy

$$|\vec{r}| \leq R(t)$$

If the chamber is expanding/contracting, the kinetic energy of the bouncing particle decreases/increases at each bounce.

The constraint is therefore rheonomic, holonomic, dissipative and unilateral.

4.(a.) DOF (dumbbell in 2d) = 3.

(b.) DOF (bob of a conical pendulum) = 2.

(c.) DOF (rigid body fixed at a point) = 3.

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