गणित विभाग, भारतीय प्रौद्योगिकी संस्थान पटना

DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY PATNA

B.Tech - I, MA-101

Mid Semester Examination

September, 2014

Time: 2 Hrs

Max Marks: 30

Attempt all the questions. Write brief and precise solutions to each question.

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(1) Check whether the following statements are true or false. Give appropriate reasons to support your answer.

(a) If x is a positive real number then there exists a natural number k such that $k-1 \leq x < k$.

(b) A convergent sequence of real real valued functions of a reals variable is bounded.

(c) A series of functions can be term - by - term integrated if and only if it converges uniformly.

(d) If a sequence of functions is monotone and bounded then it converges.

(e) The integral $\int_0^\infty e^{-x^2}$ is convergent.

[1× 5]

(2) (a) If x is a positive real number then show that there exists a natural number n such that $\frac{1}{2^n} < x$. [3]

(b) Show that $\lim_{n\to\infty}(\frac{2}{n!}) = 0$. [2]

(3) Using Cauchy's criterion, establish the convergence and find the limit of sequence: $x_1 = 1$, $x_2 = 1-(1/2)$, ..., $x_n = 1-(1/2)..... + (-1)^{n+1}$, ... [2]

(4) Using limit comparison test show that the series $\sum \frac{1}{n!}$ converges. [2]

(5) Suppose that $f: \mathbb{R} \to \mathbb{R}$ is continuous and that $f(x) = 0$ for every $r \in \mathbb{Q}$. Show that $f(x) = 0$ on all of $\mathbb{R}$. [2]

(6) Let $f: [0, \pi/2] \to \mathbb{R}$ be given by $f(x) = sup\{x^2, cosx\}$. Show that f has an absolute minimum in $[0, \pi/2]$. [2]

(7) Use mean value theorem to prove $|sinx - siny| \leq |x-y|$ for all $x, y \in \mathbb{R}$. [2]

(8) Using Taylor's theorem with $n = 3$, obtain approximation for $\sqrt{1.3}$. [2]

(9) State the two forms of fundamental theorem of Calculus (Riemann integration). [2]

(10) Determine whether the following integral is convergent $\int_0^1 \frac{e^{-x}}{\sqrt{x}}\,dx$. [3]

(11) Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and x-axis. [3]

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(140)