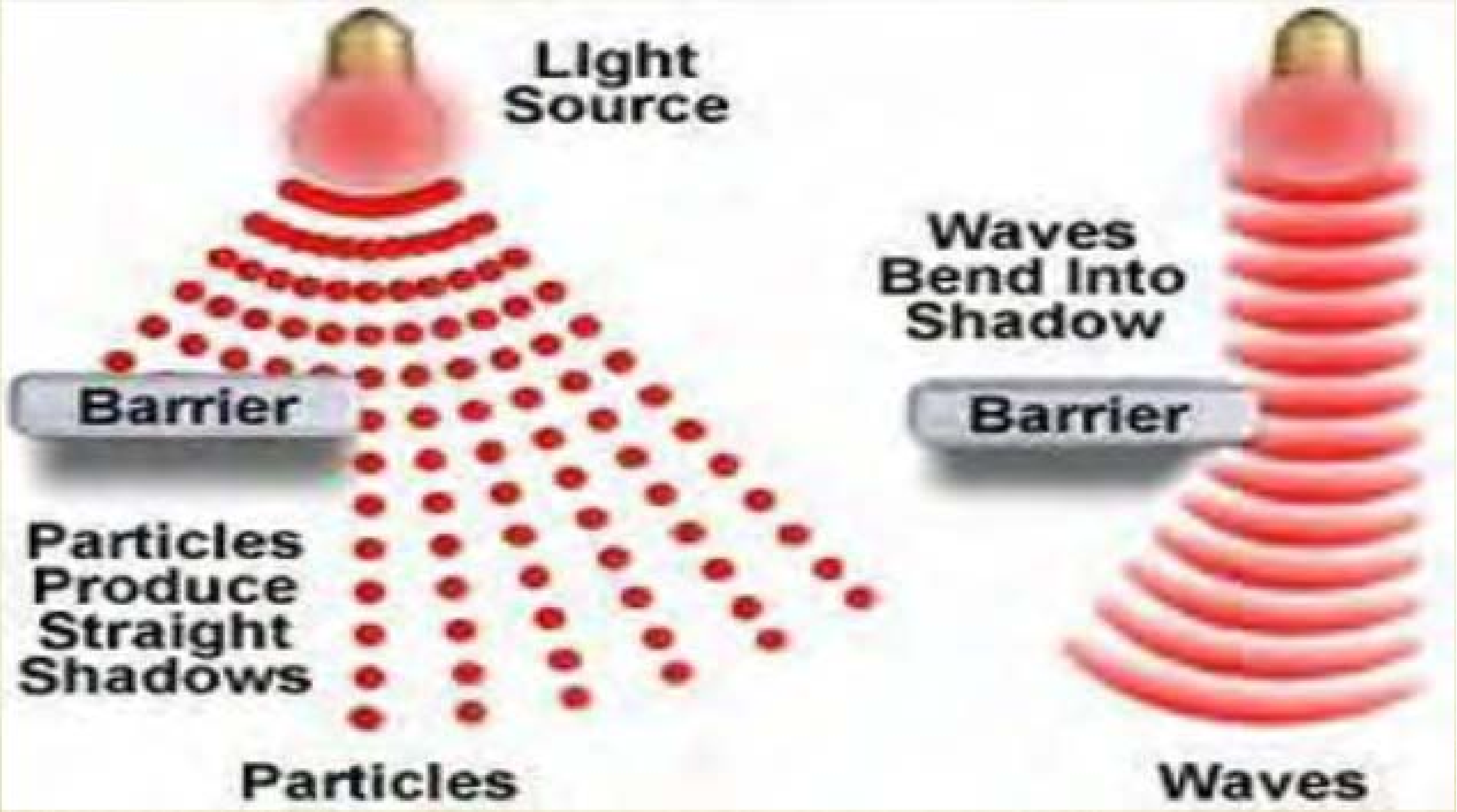
OPTICS & LASERS

PH 201

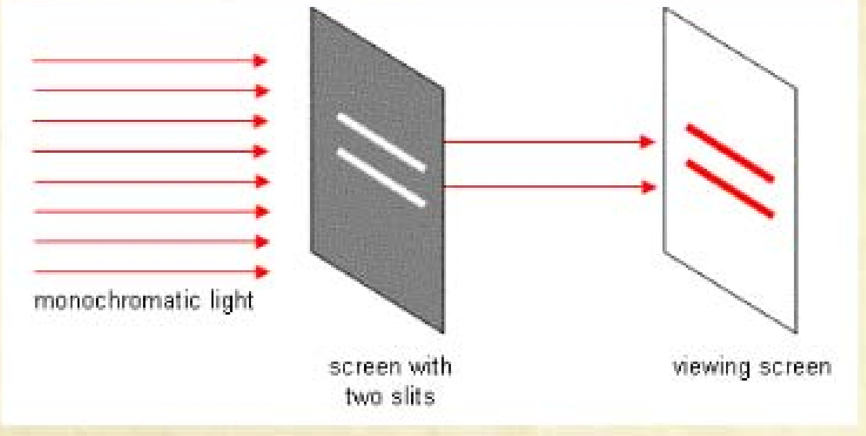
Lec_Interference

Particle vs. Wave

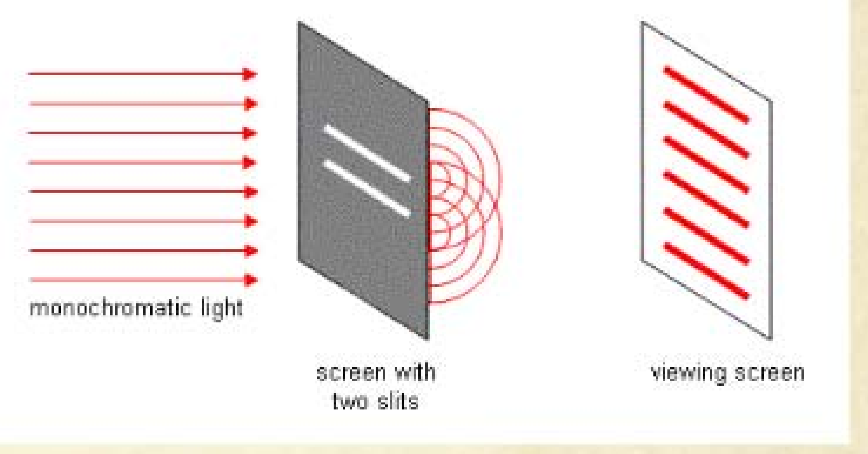
❖ **Is light a particle, a wave, or both?**



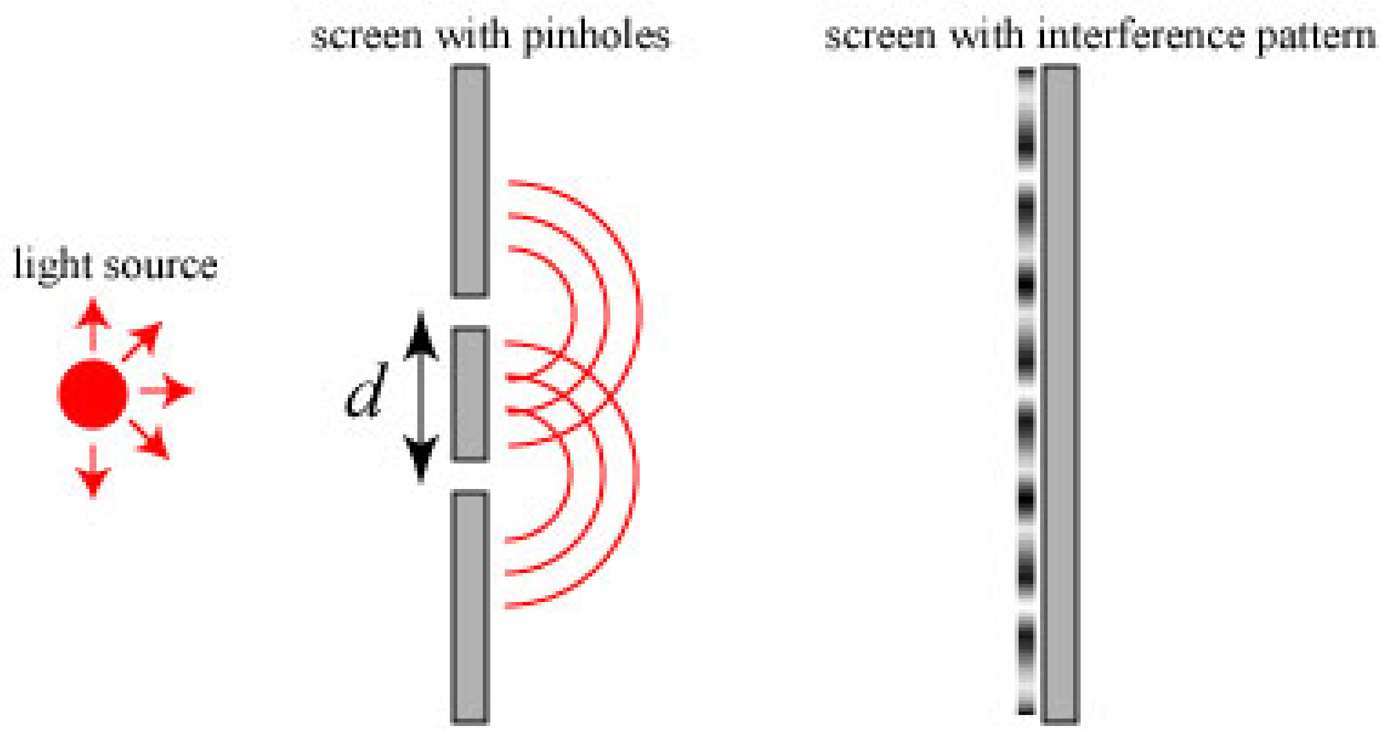
If light is a particle:



If light is a wave:

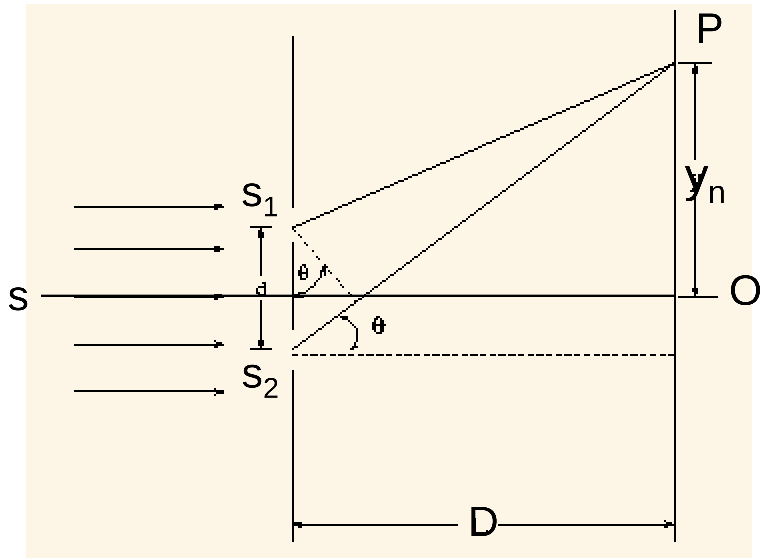


Young’s Experiment



Young's Experiment

Two light rays pass through two slits, separated by a distance *d* & strike a screen a distance *D*, from slits. Interference pattern consists of a series of dark & bright lines perpendicular to plane of Figure.



$$S_1S_2 = d, \qquad OP = y_n$$

For an arbitrary point *P* to correspond to a maximum

$$S_2P - S_1P = n\lambda; \qquad n = 0,1,2,\ldots$$

$$(S_2P)^2 - (S_1P)^2 = [D^2 + (y_n + d/2)^2] - [D^2 + (y_n - d/2)^2]$$

$$= 2y_n d$$

$$S_2P - S_1P = \frac{2y_n d}{S_2P + S_1P}$$

If y_n, d << D then negligible error will be introduced if $S_2P + S_1P \sim 2D$.

Ex. d = 0.02 cm; D = 50 cm; OP = 0.5 cm

$S_2P + S_1P$ = 100.005 cm

Error = 0.005 % Negligible

$$S_2P - S_1P = \frac{2y_n d}{2D} = \frac{y_n d}{D}$$

$$y_n = \frac{n\lambda D}{d}$$

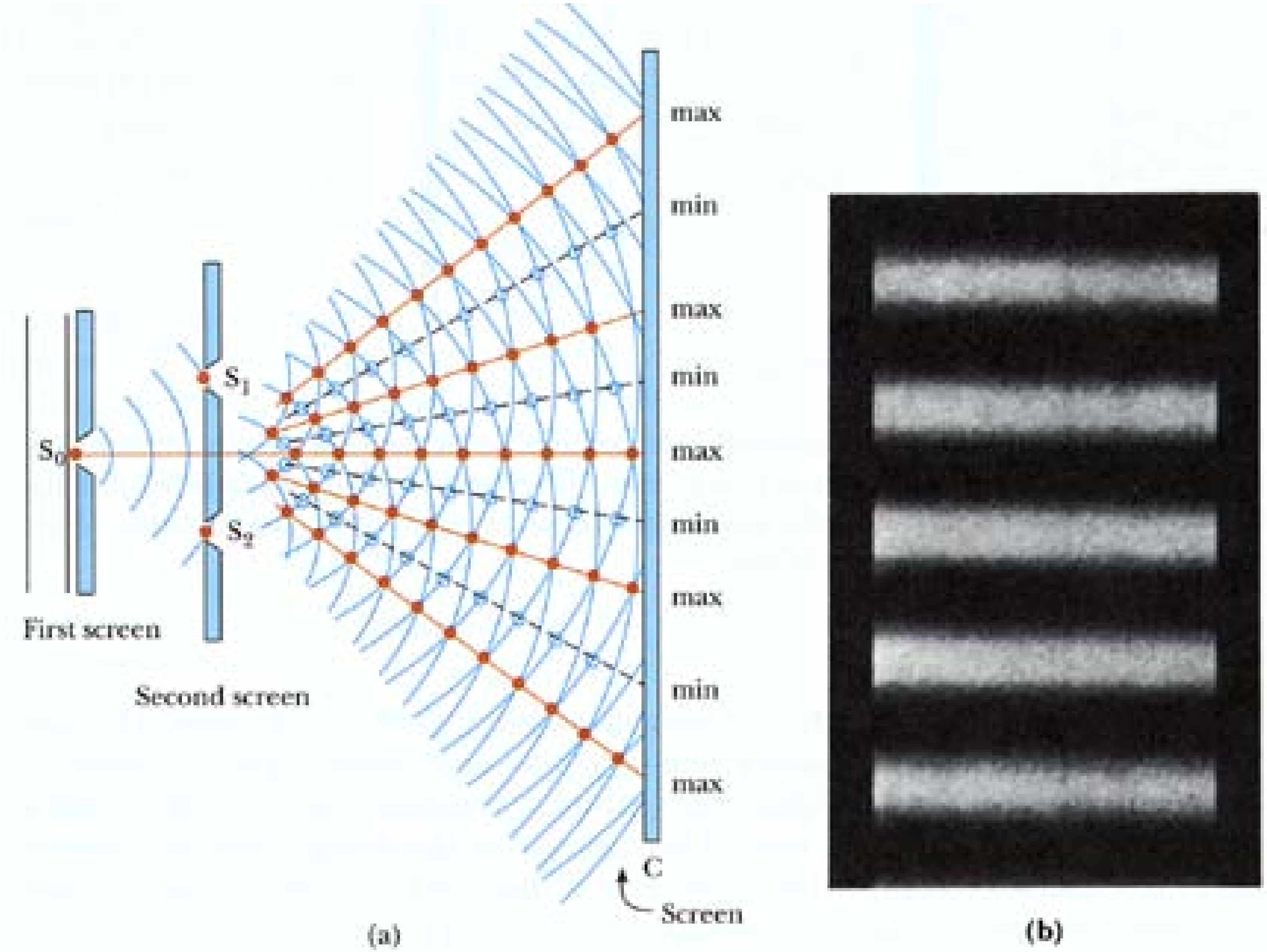
Thus dark & bright fringes are equally spaced & distance between two consecutive dark (or bright) fringes is

$$\beta = y_{n+1} - y_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

$$\beta = \frac{\lambda D}{d}$$

Expression for fringe width

Young's Double Slit Experiment



Displacement of fringes

2.30 DISPLACEMENT OF FRINGES

If a thin glass or mica strip or any other transparent plate of uniform thickness is introduced in the path of one of the two interfering beams from two coherent sources, then central bright fringe will be displaced. This displacement from C to C_0 will be towards the side of lamina. This is due to the fact that beam is retarded due to lesser velocity of light in a denser medium such as glass or mica.

In order to calculate the displacement, we shall find the path difference between two beams, from coherent sources S_1 and S_2, at any point P on the screen. P is at a distance y from central point C. Let t be the thickness of sheet or strip and μ the refractive index of its material.

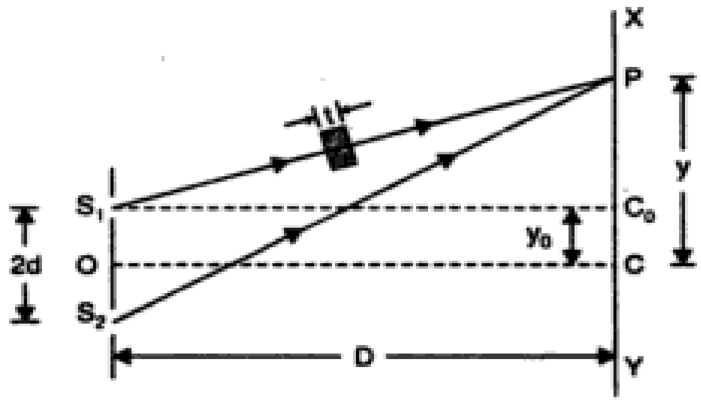


Fig. 2.27. Displacement of fringes.

Time T taken by the beam to reach from S_1 to P is given by

$$T = \frac{S_1P - t}{c} + \frac{t}{v}$$

where c is the velocity of light in air and v is the velocity of light in the medium of the plate.

or

$$T = \frac{S_1P - t}{c} + \frac{\mu t}{c} \qquad \left(\because \quad \mu = \frac{c}{v}\right)$$

$$= \frac{S_1P - t + \mu t}{c} = \frac{S_1P + (\mu - 1)t}{c}$$

Clearly, the effective path in air from S_1 to P is $S_1P + (\mu - 1)t$. So, the air path S_1P has been increased by $(\mu - 1)t$ as a result of the introduction of the plate.

The path difference between beams, reaching P, from S_1 and S_2

= path covered by beam from S_2 to P – optical path covered by beam from S_1 to P

$= S_2P - [S_1P + (\mu - 1)t] = S_2P - S_1P - (\mu - 1)t.$

But, we know that

$$S_2P - S_1P = \frac{2yd}{D}$$

Here $2d$ is the distance between two sources S_1 and S_2 and D is the distance of sources from the screen.

$\therefore$ Path difference, $\delta = \frac{2yd}{D} - (\mu - 1)t$

The Condition for Maxima

If P is the centre of the nth bright fringe, then

$$\frac{2y_n d}{D} - (\mu - 1)t = n\lambda, \text{ where } n = 0, 1, 2, \ldots\ldots \quad \text{or} \quad \frac{2y_n d}{D} = n\lambda + (\mu - 1)t$$

or

$$y_n = \frac{D}{2d}[n\lambda + (\mu - 1)t] \qquad \ldots(1)$$

Thus shift y_0 of central bright fringe, where $n = 0$, is given by $y_0 = \frac{D}{2d}(\mu - 1)t$ ...(2)

So, we conclude that the introduction of the plate in the path of one of the interfering beams displaces the entire fringe system through a distance $\frac{D}{2d}(\mu - 1)t$. *This displacement is towards the beam in the path of which the plate is introduced.*

Superposition of Waves

Resultant displacement at a particular point produced by a number of waves is vector sum of displacements produced by each disturbances.

Superposition of almost plane waves (diagonal lines) from a distant source & waves from wake of ducks.

Consider a string fixed at point A. A transverse sinusoidal wave is sent along –*x* direction. Displacement at any point on string due to this wave would be

$$y_i = a\sin\left[\frac{2\pi}{\lambda}(x+vt)+\phi\right] = a\sin\left[2\pi\left(\frac{x}{\lambda}+vt\right)\right]$$

i: Incident wave

r: Reflected wave

$$y_i\Big|_{x=0} = a\sin[2\pi vt] \qquad y_r\Big|_{x=0} = -a\sin[2\pi vt]$$

Reflected wave propagates in +*x* direction

$$y_r = +a\sin 2\pi\left(\frac{x}{\lambda}-vt\right)$$

Resultant displacement:

$$y = y_i + y_r = a\left[\sin 2\pi\left(\frac{x}{\lambda}+vt\right)+\sin 2\pi\left(\frac{x}{\lambda}-vt\right)\right]$$

$$= 2a\sin\frac{2\pi}{\lambda}x\cos 2\pi vt$$

$$y = 0, when \sin\frac{2\pi}{\lambda}x = 0 \quad at\, all\, times$$

$$At\ \ x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \ldots\ldots \qquad Nodes$$

Mid-point between two consecutive nodes, $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots\ldots$ amplitude of vibration is maximum. Displacement at these points, **antinodes**, is given by

$$y = \pm 2a \cos 2\pi \nu t$$

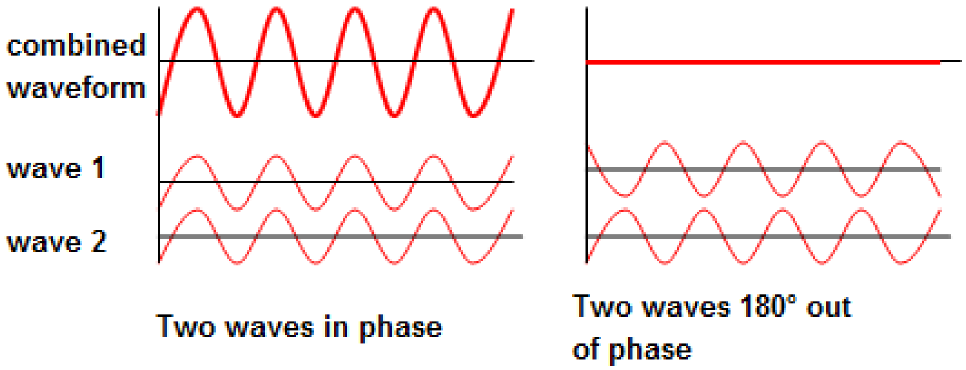
If frequency is changed, one can observe change in distance between antinodes.

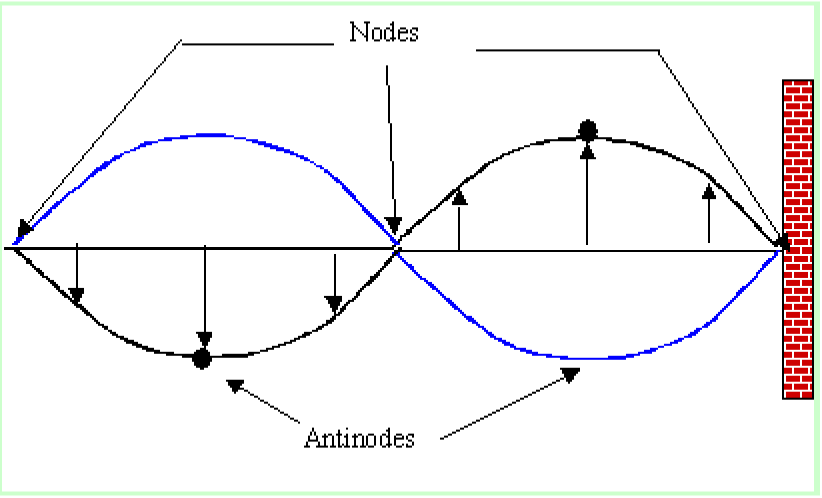
Kinetic energy density at antinodes:

Kinetic energy per unit length = $\frac{1}{2}\rho(2a)^2\omega^2\cos^2\omega t = 2\rho a^2\omega^2\cos^2\omega t$

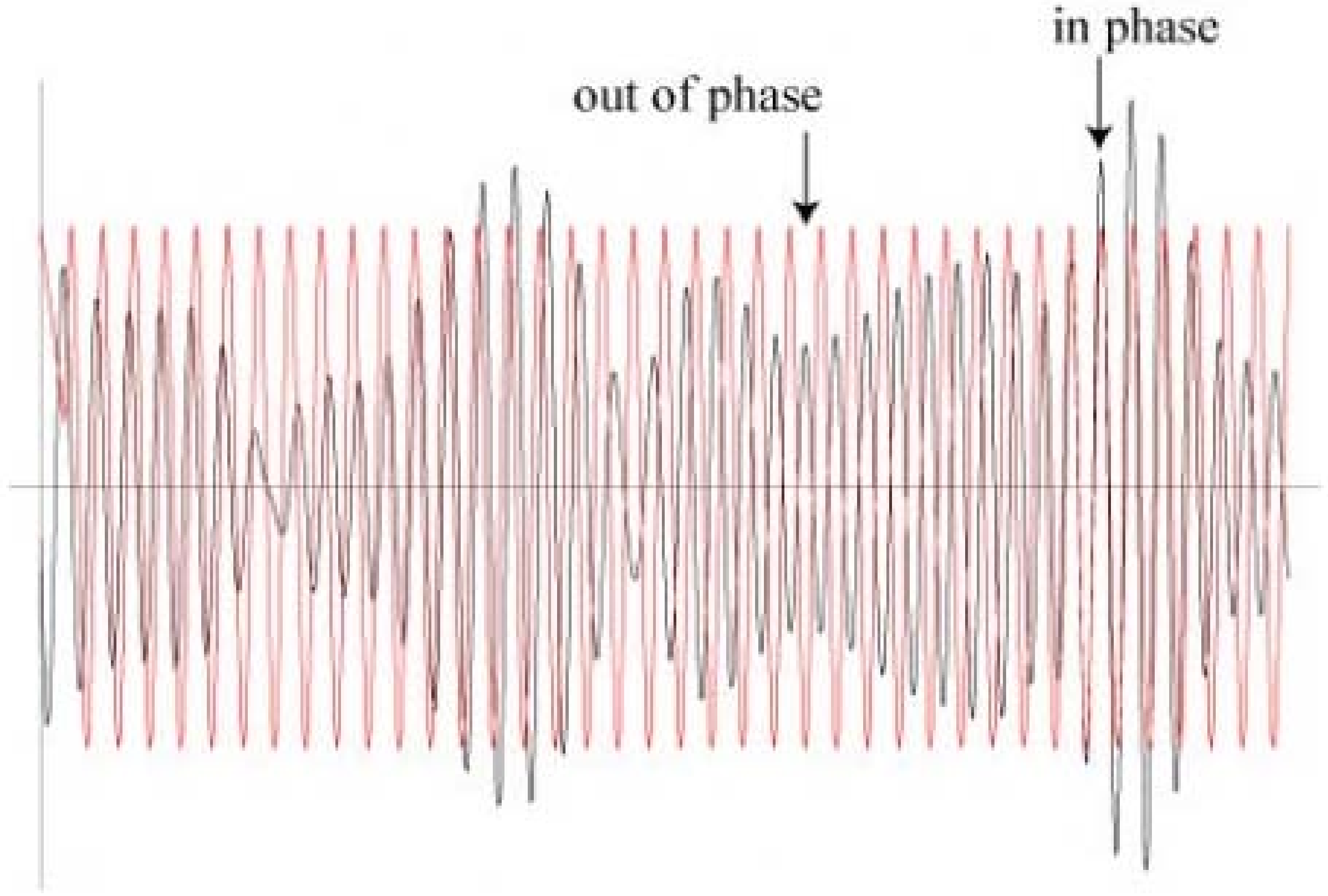
$$\omega = 2\pi\nu \rightarrow Angular\ \ frequency$$

$$\rho \rightarrow Mass\ per\ unit\ length\ of\ the\ string$$

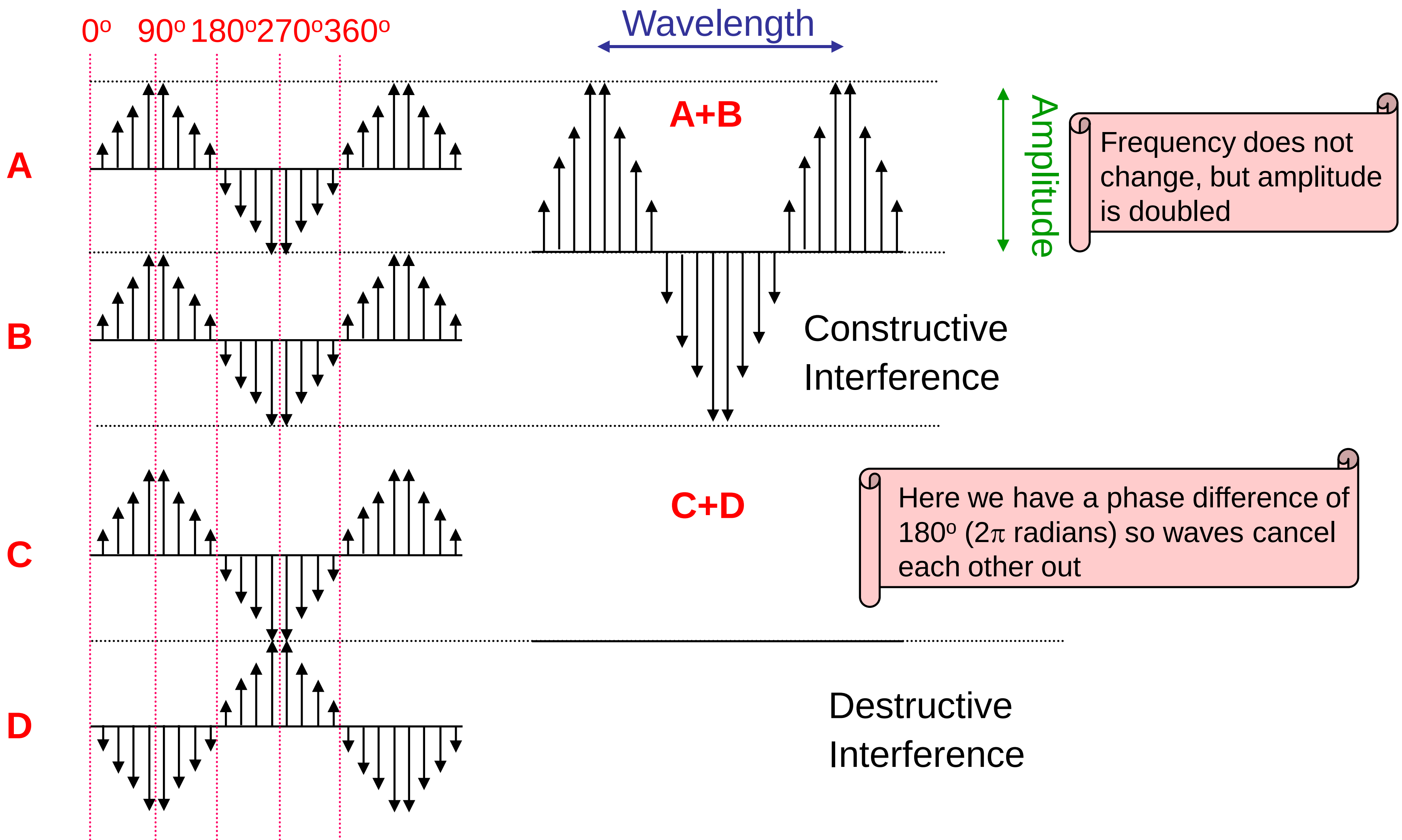




- All particles between nodes are **in phase**.
- All particles either side of a node are **in antiphase**.


in phase
out of phase

Interference in an E. M. Wave



Stationary waves on a string whose ends are fixed

Stationary waves on a string whose only one end (x = 0) is fixed, resultant displacement

$$y = 2a\sin(\frac{2\pi}{\lambda}x)\cos(2\pi\nu t)$$

If other end of string (say at x = L) is also fixed, then

$$2a\sin(\frac{2\pi}{\lambda}L)\cos(2\pi\nu t) = 0$$

This Eqn. is valid at all times.

$$\sin(\frac{2\pi}{\lambda}L) = 0 = \sin n\pi$$

$$\frac{2\pi}{\lambda}L = n\pi$$

$$\Rightarrow \lambda = \lambda_n = \frac{2L}{n}, \quad n = 1,2,3.....$$

$$Corresponding\ frequencies, \nu_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$$

If a string of length L is clamped at both ends (Sonometer wire) then it can only vibrate with certain well defined wavelengths.

$$When\ \lambda = 2L,\ n = 1$$

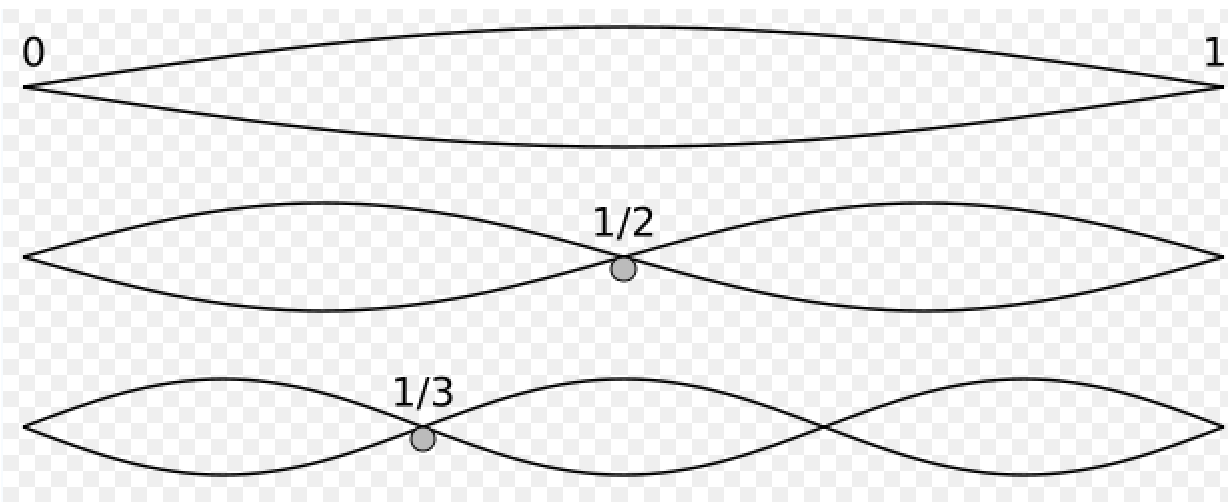
$$\Rightarrow Fundamental\ \mathrm{mod}\, e$$

$$When\ \lambda = \frac{2L}{2},\ \ n = 2 \Rightarrow First\ harmonic$$

$$When\ \lambda = \frac{2L}{3},\ \ n = 3 \Rightarrow Second\ harmonic$$

When a string is vibrating in a particular mode there is no net transfer of energy although each element of string is associated with a certain energy density. **Energy density is maximum at antinodes & minimum at nodes.**

Distance between successive antinodes & successive nodes = $\lambda/2$



$\lambda = 2L$

$\lambda = L$

$\lambda = \dfrac{2L}{3}$

Standing waves on a stretched string clamped at both ends.

Superposition of two sinusoidal waves

Consider two sinusoidal waves having same frequency at a particular point. $x_1(t)$ & $x_2(t)$ represent displacements produced by each of disturbance.

$$x_1(t) = a_1 \cos(\omega t + \theta_1)$$

$$x_2(t) = a_2 \cos(\omega t + \theta_2)$$

Resultant displacement

$$x(t) = x_1(t) + x_2(t) = a_1 \cos(\omega t + \theta_1) + a_2 \cos(wt + \theta_2)$$

$$x(t) = a\cos(\omega t + \theta)$$

Resultant disturbance is also simple harmonic in character having same frequency but different amplitude & different initial phase.

$$a\cos\theta = a_1 \cos\theta_1 + a_2 \cos\theta_2$$

$$a\sin\theta = a_1 \sin\theta_1 + a_2 \sin\theta_2$$

$$a^2 = [a_1^2 + a_2^2 + 2a_1 a_2 \cos(\theta_1 - \theta_2)]$$

$$\tan\theta = \frac{a_1 \sin\theta_1 + a_2 \sin\theta_2}{a_1 \cos\theta_1 + a_2 \cos\theta_2}$$

If we assume *a* to be always positive, then *cosθ* & *sinθ* can be determined.

$$\theta_1 \sim \theta_2 = 0,\ 2\pi,\ 4\pi \qquad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots.$$

$$\Rightarrow \quad a = a_1 + a_2 \qquad \textit{Constructive interference}$$

$$If \quad \theta_1 \sim \theta_2 = \pi, 3\pi, 5\pi \qquad x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, ...$$

$$\Rightarrow \quad a = a_1 - a_2$$

In this case, when two disturbances are not in phase, resultant amplitude is difference of two amplitudes: Destructive interference

When constructive & destructive interferences occur, there is no violation of principle of conservation of energy; energy is merely redistributed.

For n displacements:

$$x_1 = a_1 \cos(\omega t + \theta_1)$$

$$x_2 = a_2 \cos(\omega t + \theta_2)$$

$$..............................$$

$$..............................$$

$$x_n = a_n \cos(\omega t + \theta_n)$$

$$\Rightarrow x = x_1 + x_2 + ... + x_n = a\cos(\omega t + \theta)$$

$$a\cos\theta = a_1\cos\theta_1 + a_2\cos\theta_2 + ... + a_n\cos\theta_n$$

$$a\sin\theta = a_1\sin\theta_1 + a_2\sin\theta_2 + ... + a_n\sin\theta_n$$

With large no. of superposing waves (phenomenon of diffraction), graphical methods for adding displacements is very useful.

Complex representation

$$x_1 = a_1 \cos(\omega t + \theta_1)$$

$$\Rightarrow x_1 = a_1 e^{i(\omega t + \theta_1)}$$

$$x_2 = a_2 e^{i(\omega t + \theta_2)}$$

$$\Rightarrow x_1 + x_2 = (a_1 e^{i\theta_1} + a_2 e^{i\theta_2}) e^{i\omega t} = a e^{i(\omega t + \theta)}$$

If two displacements are in-phase: **Constructive interference**

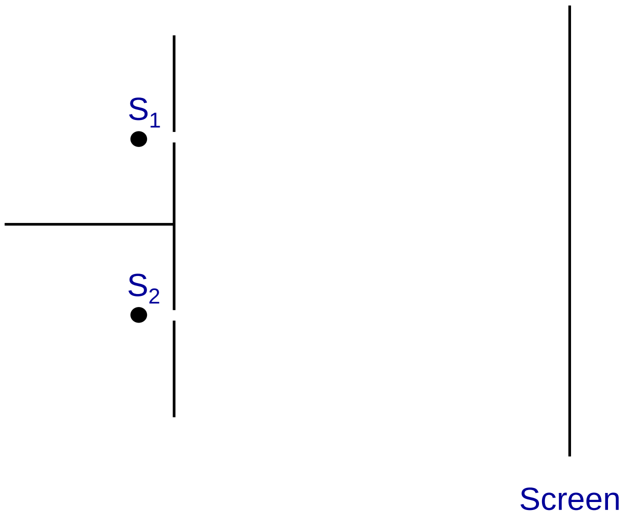
If two displacements are π out-of-phase: **Destructive interference**

Interference of Light Waves

If we use two conventional light sources, like two sodium lamps, illuminating two pin holes, **we will not observe any interference pattern** on screen.

In a conventional light source, light comes from a large no. of independent atoms; each atom emitting light for about 10^{-10} sec. i.e., light emitted by an atom is essentially **a pulse lasting for only 10^{-10} sec**.

Even if atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases.



Light sources from holes S_1 & S_2 will have a fixed phase relationship for a period of about 10^{-10} sec, **hence interference pattern will keep on changing every billionth of a second.**

Eye can notice intensity changes which last at least for a tenth of a second & hence we will observe uniform intensity over screen.

It is difficult to observe interference pattern even with two laser beams unless they are phase locked.

Thus, one tries to derive interfering waves from a single wave so that phase relationship is maintained.

Method to achieve phase relationship:

- **Division of wavefront:** A beam is allowed to fall on two closely spaced holes & two beams emanating from holes interfere.
- **Division of amplitude:** A beam is divided at two or more reflecting surfaces & reflected beams interfere.

Intensity distribution (Division of wavefront)

Let E_1 & E_2 be electric fields produced at P by S_1 & S_2. Electric fields E_1 & E_2 will have different directions & different magnitudes. If distances S_1P & S_2P are very large in comparison to distance S_1S_2, two fields will almost be in same direction.

$$E_1 = \hat{i}\, E_{01} \cos\left(\frac{2\pi}{\lambda} S_1P - \omega t\right)$$

$$E_2 = \hat{i}\, E_{02} \cos\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)$$

Resultant

$$E = E_1 + E_2 = \hat{i}\left[E_{01} \cos\left(\frac{2\pi}{\lambda} S_1P - \omega t\right) + E_{02} \cos\left(\frac{2\pi}{\lambda} S_2P - \omega t\right)\right]$$

Intensity

$$I = K\Big[E_{01}^2 \cos^2\left(\frac{2\pi}{\lambda} S_1P - \omega t\right) + E_{02}^2 \cos^2\left(\frac{2\pi}{\lambda} S_2P - \omega t\right) +$$

$$E_{01}E_{02}\{\cos[(\frac{2\pi}{\lambda} S_2P - S_1P)] + \cos[2\omega t - \frac{2\pi}{\lambda}(S_2P + S_1P)\}\Big]$$

For an optical beam frequency is very large ($\omega = 10^{15}$ sec^{-1}) & all terms depending on ωt will vary with extreme rapidity (10^{15} times in a sec); consequently any detector would record an average value of various quantities.

$$< \cos^2(\omega t - \theta) > = \frac{1}{2\tau}\int_{-\tau}^{+\tau}\frac{1+\cos[2(\omega t-\theta)]}{2}dt$$

$$= \frac{1}{2} + \frac{1}{16\pi}\frac{T}{\tau}\{[\sin 2(\omega t-\theta)]_{-\tau}^{+\tau}\} \qquad T = \frac{2\pi}{\omega}$$

$$\Rightarrow \qquad < \cos^2(\omega t - \theta) > = \frac{1}{2}$$

$$\Rightarrow \qquad I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta \qquad \delta = \frac{2\pi}{\lambda}(S_2P - S_1P)$$

Phase difference between displacements reaching point *P* from S_1 & S_2.

$$I_1 = \frac{1}{2}KE_{01}^2 \quad \Rightarrow No\ light\ from\ S_2$$

$$I_2 = \frac{1}{2}KE_{02}^2 \quad \Rightarrow No\ light\ from\ S_1$$

CASE 1

$$Max(\cos\delta) = +1, \quad Min(\cos\delta) = -1$$

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

Maximum intensity

$$\delta = 2\pi n, \quad n = 0,1,2,...$$

$$S_2P - S_1P = n\lambda$$

Minimum intensity

$$\delta = (2n+1)\pi, \quad n = 0,1,2,...$$

$$S_2P - S_1P = (n+\frac{1}{2})\lambda$$

CASE 2 If holes S_1 & S_2 are illuminated by different light sources, then δ will remain constant for about 10^{-10} sec.

$$<\cos\delta> = 0$$

$$\Rightarrow I = I_1 + I_2 \qquad \textit{Incoherent sources}$$

CASE 3 If distances S_1P & S_2P are extremely large in comparison to d.

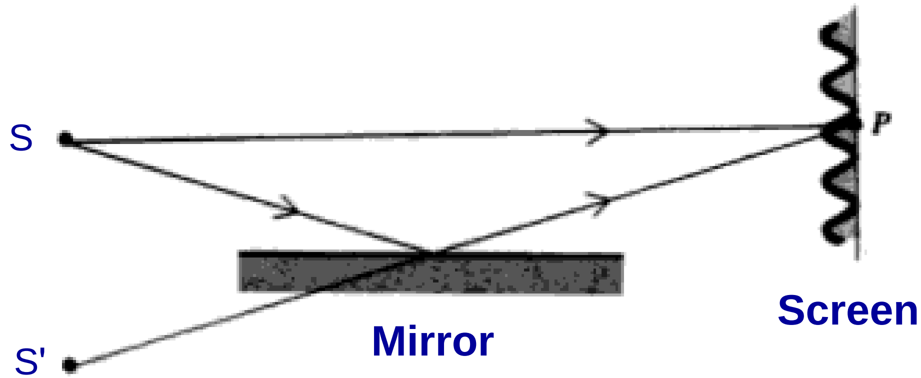
$$I_1 = I_2 = I_0 \quad (say)$$

$$\Rightarrow I = 2I_0 + 2I_0\cos\delta = 4I_0\cos^2\frac{\delta}{2}$$

Intensity distribution is $\cos^2$ pattern.

Lloyd's single mirror arrangement for producing interference fringes with a single source

Humphry Lloyd - 1834



A pinhole source S is placed near a plane mirror. Part of light that is reflected by mirror appears to come from a virtual source S'. Light directly coming from S interferes with light reflected from mirror. In calculating intensity at a point P phase change that occurs on reflection must be taken into account.

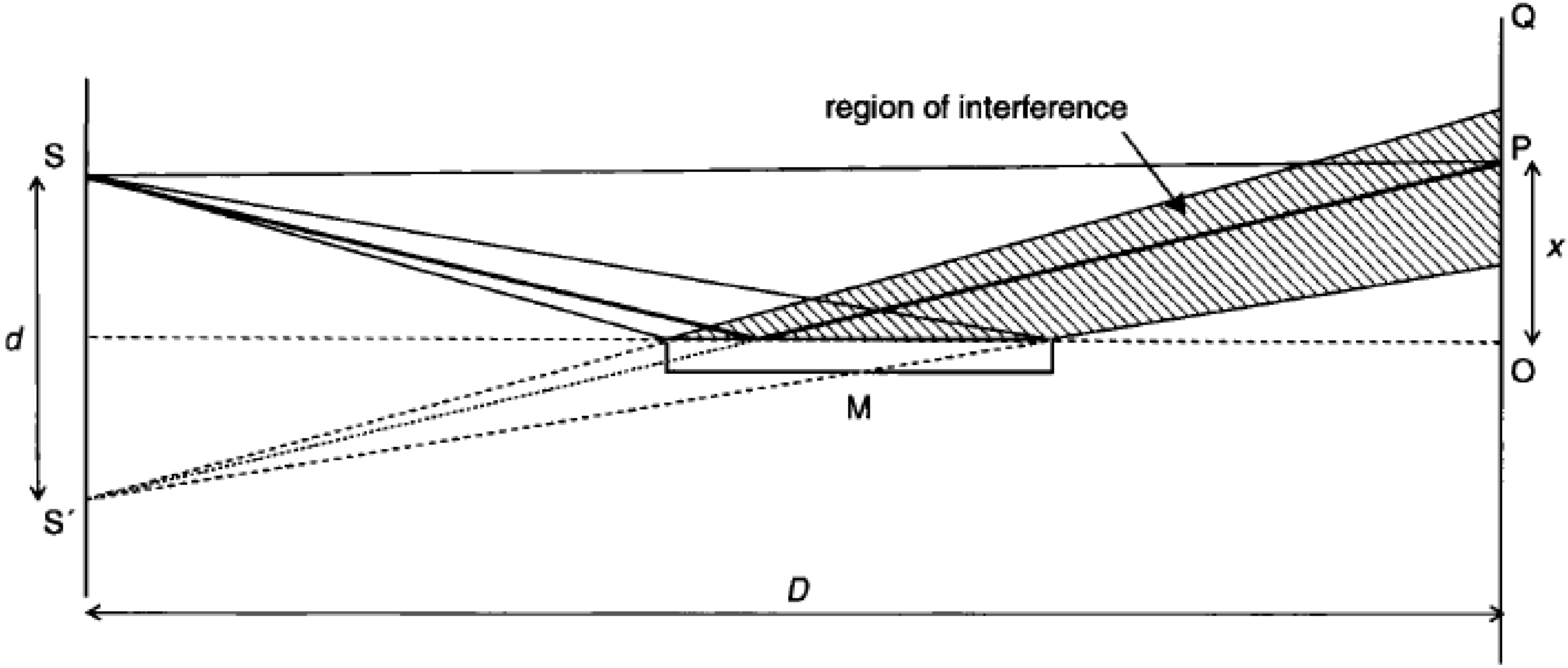
$$At\ P \qquad S'P - SP = n\lambda \qquad n = 0, 1, 2, 3, \ldots\ldots$$

$$\Rightarrow Minima\ (Destructive\ interference)$$

$$If \qquad S'P - SP = (n + \frac{1}{2})\lambda$$

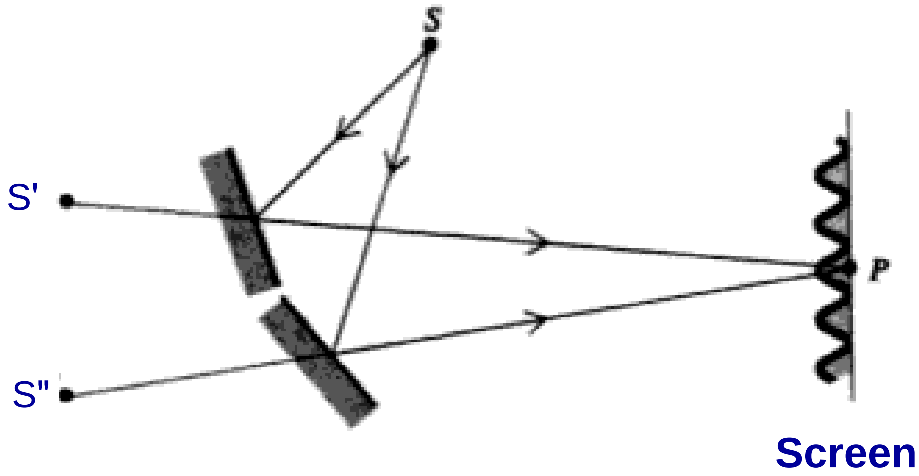
$$\Rightarrow Maxima$$

Lloyd's single mirror arrangement



Field in region of screen is equivalent to that in Young's experiment.

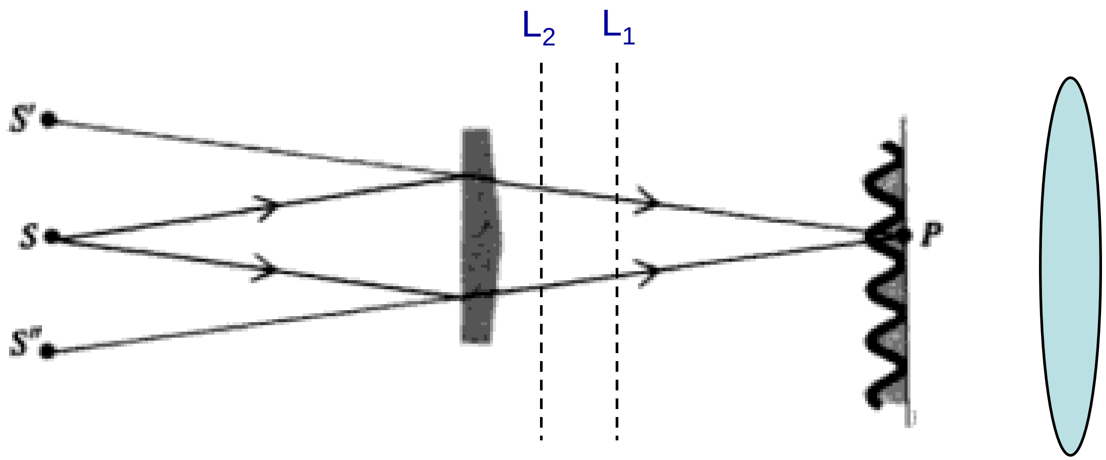
Fresnel's double mirror arrangement for producing interference fringes with a single source



It makes use of two mirrors to obtain two mutually coherent virtual sources *S*' & *S*". A portion of wavefront from *S* gets reflected from one mirror.

Another portion of wavefront from *S* gets reflected from second mirror. Both reflected wavefronts interfere to produce interference pattern.

Fresnel’s biprism arrangement for producing interference fringes with a single source



$$\lambda = \frac{\beta d}{D}$$

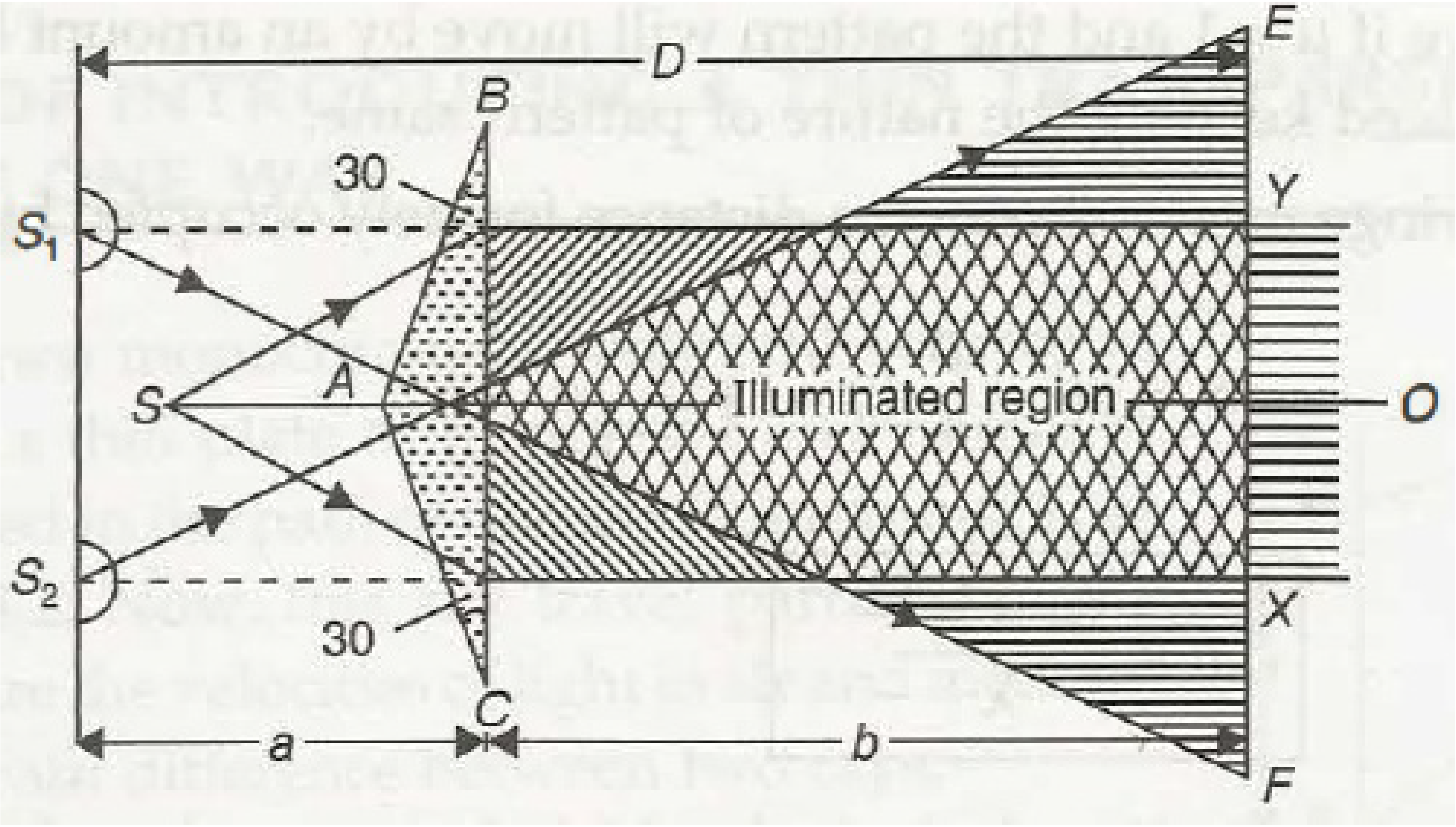
$$d = \sqrt{d_1 d_2}$$

Apex angle of prism should be nearly 180˚ in order that virtual sources are only slightly separated. Light from slit *S* gets refracted by prism & produces two virtual images *S*' & *S*''.

Biprism arrangement can be used for **determination of wavelength** of almost monochromatic light. *S*'*S*'' = *d*, *D* = *SP*. Place a convex lens between biprism & eyepiece. For a fix position of eyepiece there will be two positions of lens L_1 & L_2 where images of *S*' & *S*'' can be seen at eyepiece.

d_1 = distance between two images when lens is at L_1 & d_2 = distance between two images when lens is at L_2.

Fresnel’s biprism arrangement

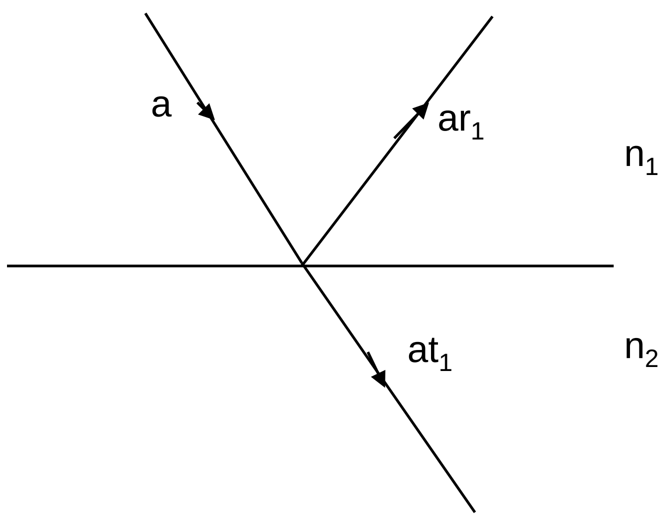


Phase change on reflection

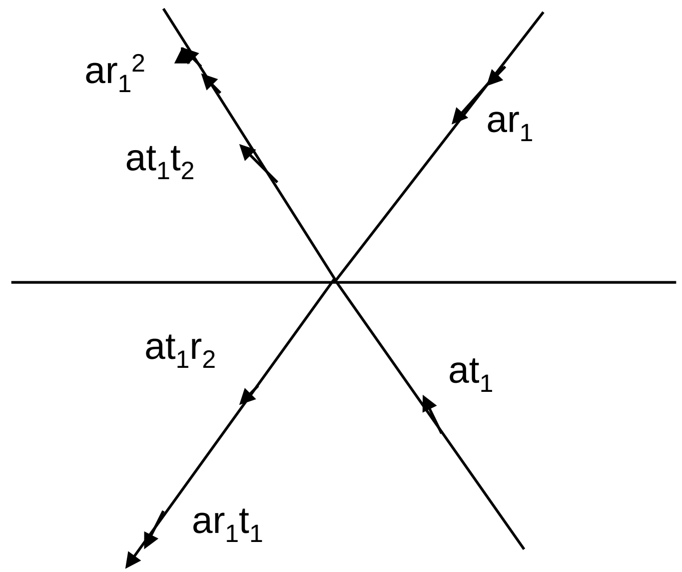
Principle of optical reversibility: Reflection of light at an interface between two media.

In absence of any absorption, a light ray that is reflected or refracted will retrace its original path if its direction is reversed.

Consider a light ray incident on an interface of two media of r.i., n_1 & n_2.



r_1 = Amplitude reflection coefficient; t_1 = Amplitude transmission coefficient; a = Amplitude of incident ray; ar_1 = Amplitude of reflected ray; at_1 = Amplitude of refracted ray. r_2 & t_2 = Amplitude reflection & transmission coefficient when a ray is incident from medium 2 to medium 1.



Reverse the rays:

Consider a light ray of amplitude at_1 incident on medium 1 & a ray of amplitude ar_1 incident on medium 2.

- Ray of amplitude at_1 will give rise to a reflected ray of amplitude at_1r_2 & a transmitted ray of amplitude at_1t_2.

- Ray of amplitude ar_1 will give rise to a ray of amplitude ar_1^2 & a refracted ray of amplitude ar_1t_1.
- According to principle of optical reversibility, two rays of amplitudes ar_1^2 & at_1t_2 must combine to give incident ray.

$$ar_1^2 + at_1t_2 = a$$
$$\Rightarrow t_1t_2 = 1 - r_1^2 \qquad \textit{Stoke's relation}$$

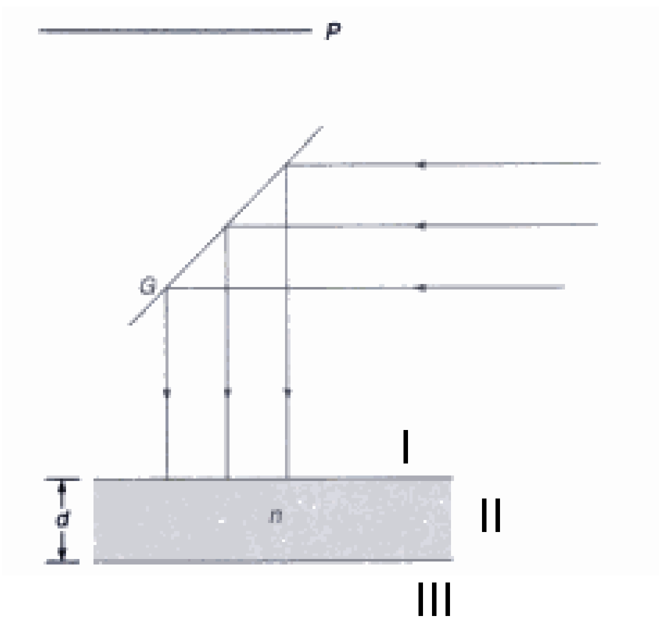
Further, the two rays of amplitudes at_1r_2 & ar_1t_1 must cancel each other,

$$at_1r_2 + ar_1t_1 = 0$$
$$\Rightarrow r_2 = -r_1 \qquad \textit{Stoke's relation}$$

- **An abrupt phase change of π occurs when light gets reflected by a denser medium.**
- **No such abrupt phase change occurs when light gets reflected by a rarer medium.**

Interference by division of amplitude

If a plane wave is incident normally on a thin film of uniform thickness *d* then waves reflected from upper surface interfere with waves reflected from lower surface. Wave reflected from lower surface of film traverses an additional optical path of *2nd*. If film is placed in air, then wave reflected from upper surface of film will undergo a sudden change in phase of *π*.

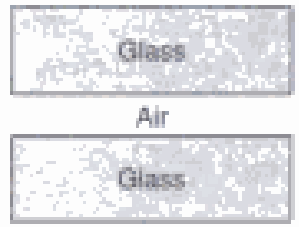


$$2nd = m\lambda \qquad \textit{Destructive interference}$$

$$= (m+\frac{1}{2})\lambda \qquad \textit{Constructive interference}$$

$$m = 0,1,2,3.....$$

Amplitudes of waves reflected from upper & lower surfaces will, in general, be slightly different, & as such interference will not be completely destructive. However, with appropriate choice of r.i. of media II & III, two amplitudes can be made very nearly equal.



Thin film of air formed between two glass plates.

For an air film between two glass plates no phase change will occur on reflection at glass-air interface, but a phase change of π will occur on reflection at air-glass interface & conditions for maxima & minima will remain same.

If medium I is crown glass, n = 1.52

medium II is oil, n = 1.60

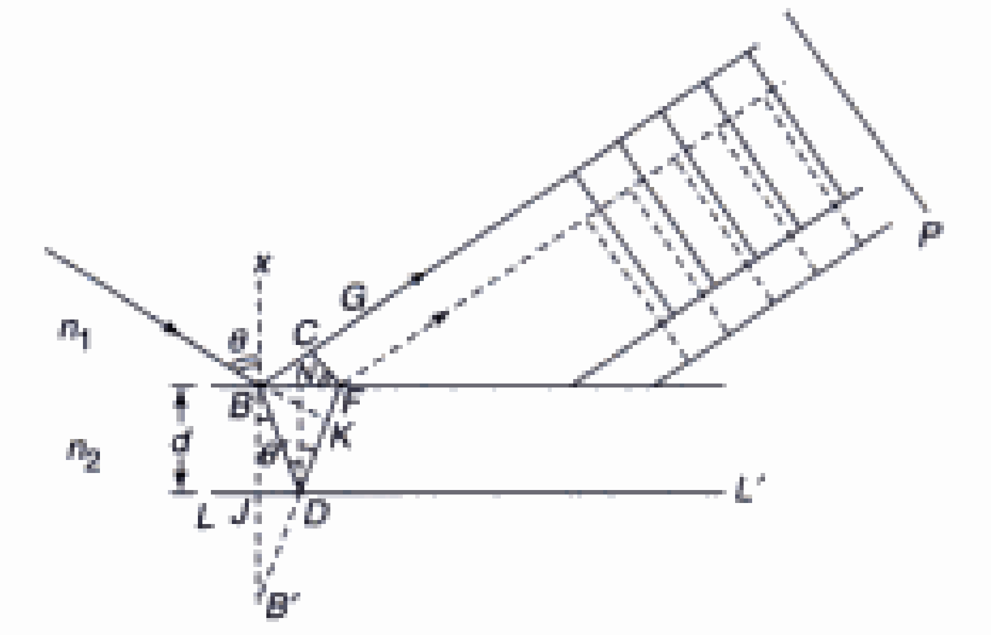
medium III is flint glass n = 1.66

Then a phase change of π will occur at both reflections & conditions for maxima & minima would be

$$2nd = (m + \frac{1}{2})\lambda \qquad\qquad Minima$$

$$= m\lambda \qquad\qquad Maxima$$

Oblique incidence of plane wave on thin film



Additional optical path, Δ

$$\Delta = n_2(BD + DF) - n_1 BC$$
$$= 2n_2 d\cos\theta'$$

For a film placed in air, a phase change of π will occur when reflection takes place at B.

$$\Delta = 2n_2 d\cos\theta' = m\lambda \qquad Minima$$
$$= (m + \frac{1}{2})\lambda \qquad Maxima$$

Cosine law: Wave reflected from lower surface of film traverses an additional optical path given by

$$\Delta = 2n_2 d\cos\theta'$$

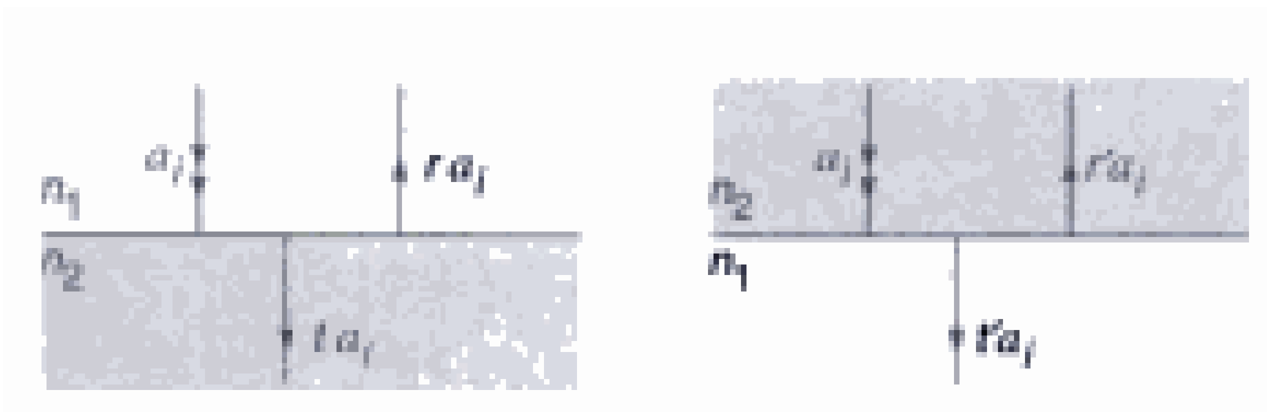
Non-reflecting films

Thin film interference phenomenon reduces reflectivity of lens surfaces.

When a light beam (propagating in a medium of *r.i.*, n_1) is incident normally on a dielectric of refractive index n_2 then amplitudes of reflected & transmitted beams are

$$a_r = \frac{n_1 - n_2}{n_1 + n_2} a_i \qquad\qquad a_t = \frac{2n_1}{n_1 + n_2} a_i$$

where a_i, a_r, & a_t are amplitudes of incident, reflected, & transmitted beams, respectively.



Amplitude reflection & transmission coefficients *r* & *t* are given by

$$r = \frac{n_1 - n_2}{n_1 + n_2} \qquad\qquad t = \frac{2n_1}{n_1 + n_2}$$

In many optical instruments (telescope) there are many interfaces & loss of intensity due to reflections can be severe.

Ex: Reflectivity of crown glass surface in air is

$$\left(\frac{n-1}{n+1}\right)^2 = \left(\frac{1.5-1}{1.5+1}\right)^2 = 0.04$$

i.e. 4% of incident light is reflected.

For a dense flint glass n = 1.67 about 6% of light is reflected. Thus, if we have a large no. of surfaces, losses at interfaces can be considerable.

To reduce losses, lens surfaces are often coated with a $\lambda/4n$ thick non-reflecting film; refractive index of film being less than that of lens.

Ex. Glass (n = 1.5) may be coated with MgF_2 film & film thickness d should be such that (considering normal incidence, $\cos\theta' = 1$, n_f = *r.i.* of film)

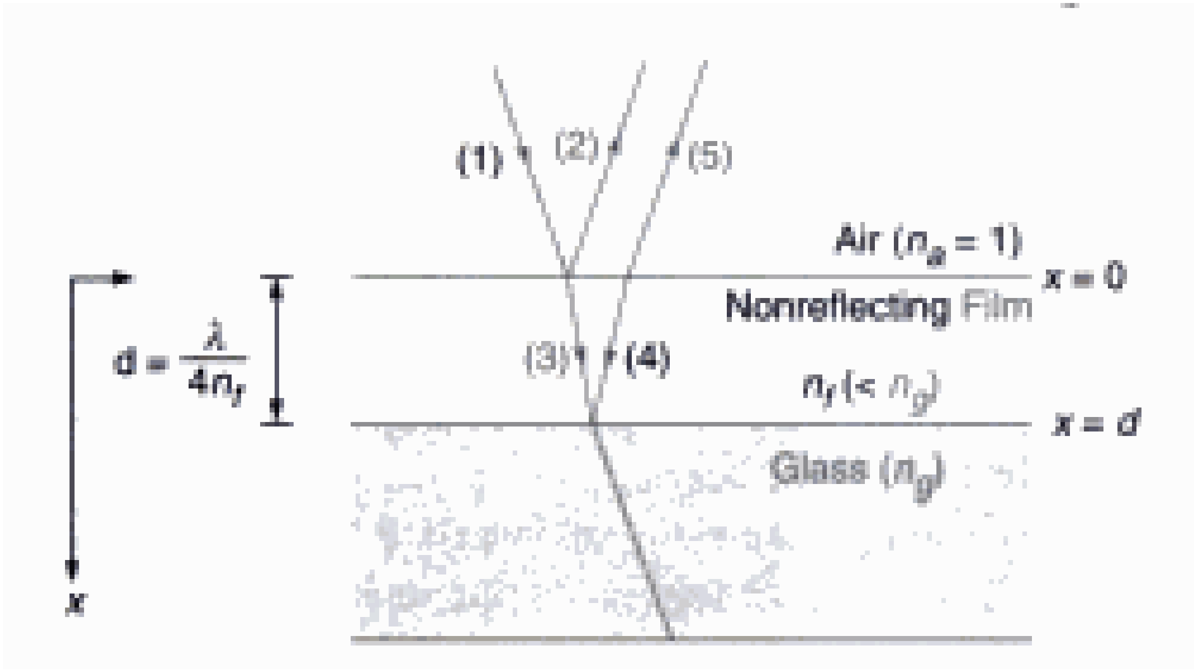
$$\Delta = 2n_2 d\cos\theta' = (m + \frac{1}{2})\lambda$$

$$2n_f d = \frac{1}{2}\lambda$$

$$d = \frac{\lambda}{4n_f}$$

For MgF_2, $n_f = 1.38$, $\lambda = 5.0 \times 10^{-5}$ cm

$$d = \frac{\lambda}{4n_f} = \frac{5.0\times 10^{-5} cm}{4\times 1.38} = 0.9\times 10^{-5} cm$$



Let n_a, n_f, & n_g be *r.i.* of air, non-reflecting film, & glass respectively. If *a* is amplitude of incident wave then amplitudes of reflected & refracted waves would be

$$-\frac{n_f - n_a}{n_f + n_a}a \qquad \frac{2n_a}{n_f + n_a}a$$

Amplitudes of waves corresponding to rays (4) & (5) would be

$$-\frac{2n_a}{n_f + n_a}\frac{n_g - n_f}{n_g + n_f}a \qquad -\frac{2n_a}{n_f + n_a}\frac{n_g - n_f}{n_g + n_f}\frac{2n_f}{n_f + n_a}a$$

For complete destructive interference, waves corresponding to rays (2) & (5) should have same amplitude.

$$-\frac{n_f - n_a}{n_f + n_a}a = -\frac{2n_a}{n_f + n_a}\frac{n_g - n_f}{n_g + n_f}\frac{2n_f}{n_f + n_a}a$$

$$or \quad \frac{n_f - n_a}{n_f + n_a} = \frac{n_g - n_f}{n_g + n_f}$$

$$\Rightarrow \quad n_f = \sqrt{n_a n_g}$$

$$\frac{4n_f n_a}{(n_f + n_a)^2} =\sim unity \quad (0.97)$$

$$for\ n_a = 1, n_f = 1.34$$

For a *λ*/4*n* thick film, reflectivity will be about $\left[\frac{n_f - n_a}{n_f + n_a} - \frac{n_g - n_f}{n_g + n_f}\right]^2$

For n_a = 1, n_f = 1.38, & n_g = 1.5, reflectivity will be about 1.3%.

In absence of film, reflectivity would have been about 4%.

Reduction of reflectivity is much more pronounced for dense flint glass. This technique of reducing reflectivity is known as **blooming**.

High reflectivity by thin film deposition

Reflectivity of glass surfaces can be increased by coating glass surface by a thin film of suitable material.

Film thickness is again $\lambda/4n_f$ where n_f represents *r.i.* of film; however, film is such that its *r.i.* is greater than that of glass; consequently an abrupt phase change of π occurs only at air-film interface & beams reflected from air-film interface & film-glass interface constructively interfere.

Ex. Consider a film of refractive index 2.37 (Zinc Sulphide) then reflectivity is about **16%.**

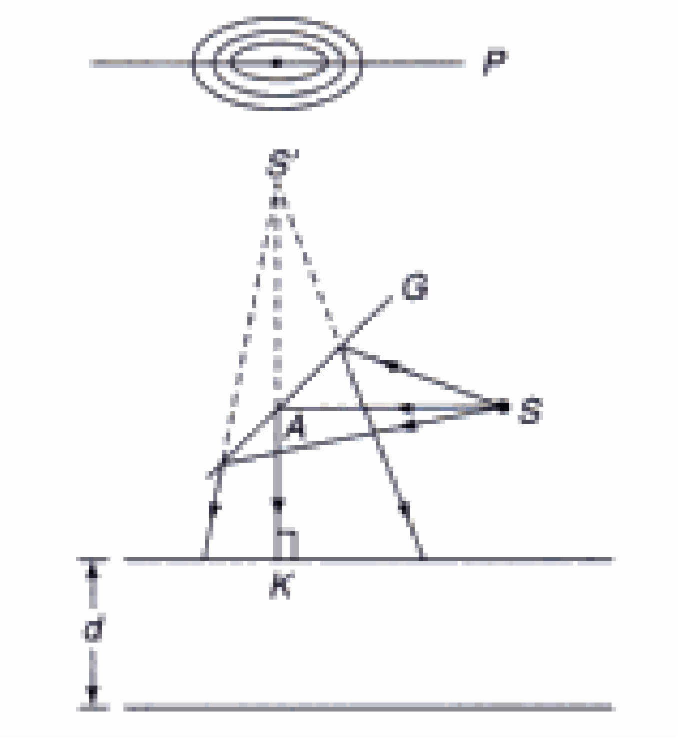
$$\frac{(2.37-1)^2}{(2.37+1)^2}$$

In presence of a glass surface of *r.i.* 1.5, reflectivity will become about **35%**

$$\left[-\frac{2.37-1}{2.37+1}-\frac{4\times1\times2.37}{(3.37)^2}\times\frac{2.37-1.5}{2.37+1.5}\right]^2$$

If difference between *r.i.* of film & glass is increased, then reflectivity will also increase.

Interference by a plane parallel film when illuminated by a point source



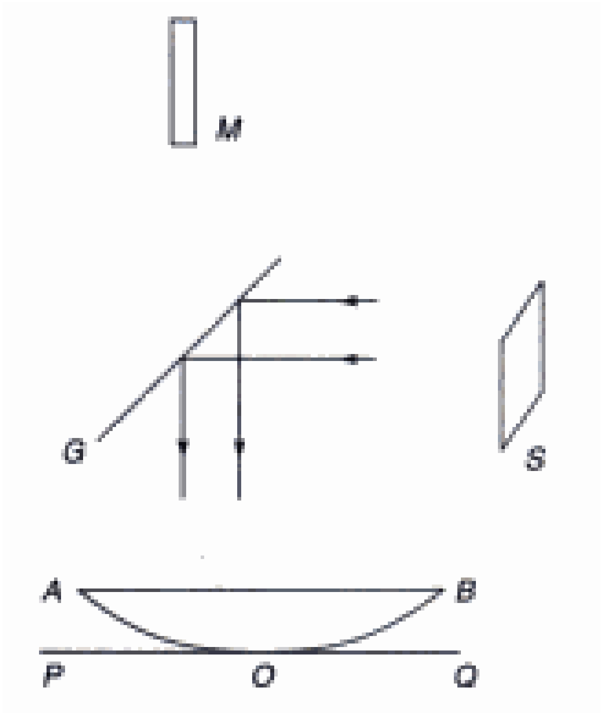
Light emanating from a point source *S* is allowed to fall on a thin film

d: thickness of thin film, *G*: partially reflecting plate, P: photographic plate.

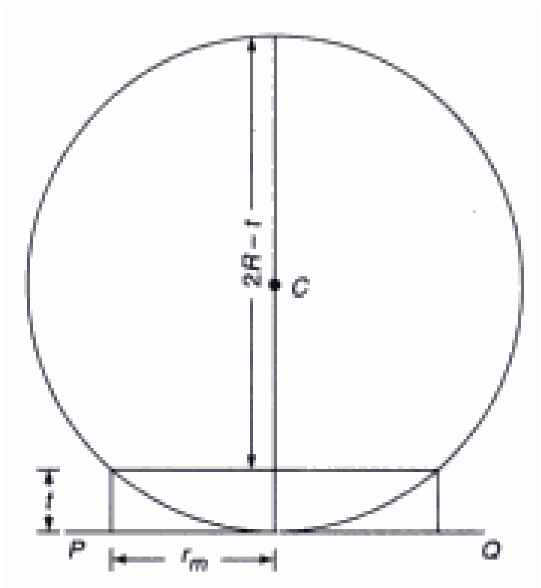
Circular fringes are obtained.

Newton's Rings

If we place a planoconvex lens on a plane glass surface, thin film of air is formed between curved surface of lens & plane glass plate. Thickness of air film is zero at point of contact & increases as one moves away from point of contact. Light reflected from surface *AOB* interferes with light reflected from surface *POQ*.



Light from an extended source *S* is allowed to fall on a thin film formed between planoconvex lens (AOB) & plane glass plate. M: traveling microscope.



r_m: radius of m^{th} dark ring; thickness of air film (where m^{th} dark ring is formed) is *t*.

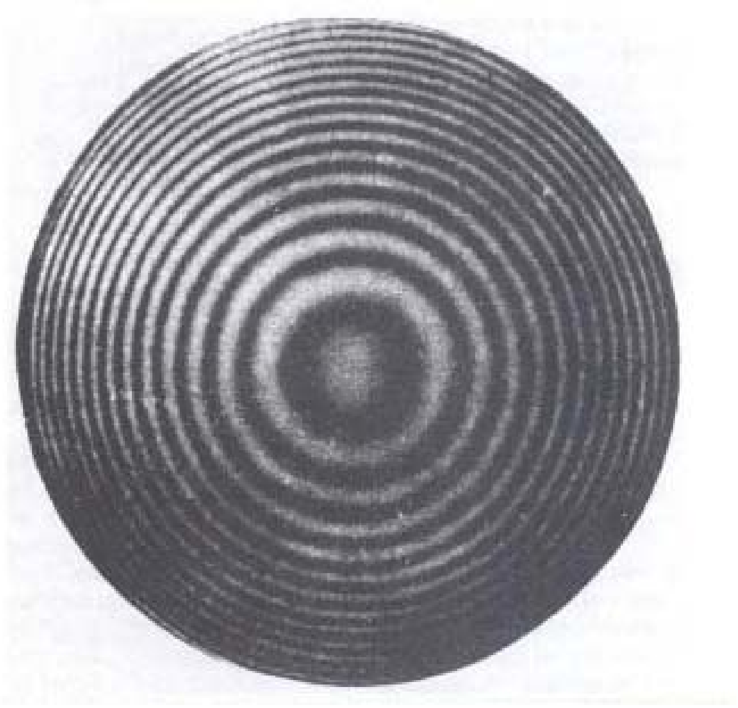
For near normal incidence optical path difference between two waves is 2*nt*, where *n* is refractive index of film & *t* is thickness of film.

$$2nt = (m + \frac{1}{2})\lambda \qquad Maxima$$

$$= m\lambda \qquad Minima$$

$$m = 0, 1, 2, ...$$

Since convex side of lens is a spherical surface, thickness of air film will be constant over a circle & we will obtain concentric dark & bright rings.



Newton's rings as observed in reflection.

Radii of various rings can easily be calculated. Thickness of air film will be constant over a circle whose centre is at point of contact *O*. Let radius of m^{th} ring be r_m & if t is thickness of air film where m^{th} dark ring appears to be formed, then

$$r_m^2 = t(2R - t)$$

where R represents radius of curvature of convex surface of lens. Say, R = 100 cm & $t \le 10^{-3}$ cm, thus we may neglect t in comparison to $2R$ to obtain

$$r_m^2 = 2Rt$$

$$\Rightarrow 2t = \frac{r_m^2}{R}$$

Condition for minima

$$2nt = m\lambda$$

$$\Rightarrow r_m^2 = m\lambda R \qquad m = 0, 1, 2, ...$$

Radii of rings vary as square root of natural numbers. Thus rings will become close to each other as radius increases. Between two dark rings there will be a bright ring whose radius will be

$$\sqrt{m + \frac{1}{2}\lambda R}$$

Condition for minima predicts that central spot should be dark, Due to presence of minute dust particles, point of contact is really not perfect & central spot may not be perfectly dark.

Ex. $\lambda = 6\times 10^{-5}\,cm \quad R = 100\,cm$

$$\Rightarrow r_m = 0.0774\sqrt{m}\ cm$$

Thus, radii of first, second, & third dark rings would be approximately 0.0774 cm, 0.110 cm, & 0.134 cm respectively. **Notice that spacing between second & third dark rings is smaller than spacing between first & second dark rings.**

While carrying out expt. one should measure radii of *m*th & (*m*+*p*)th ring & take difference in squares of radii, which is independent of *m*.

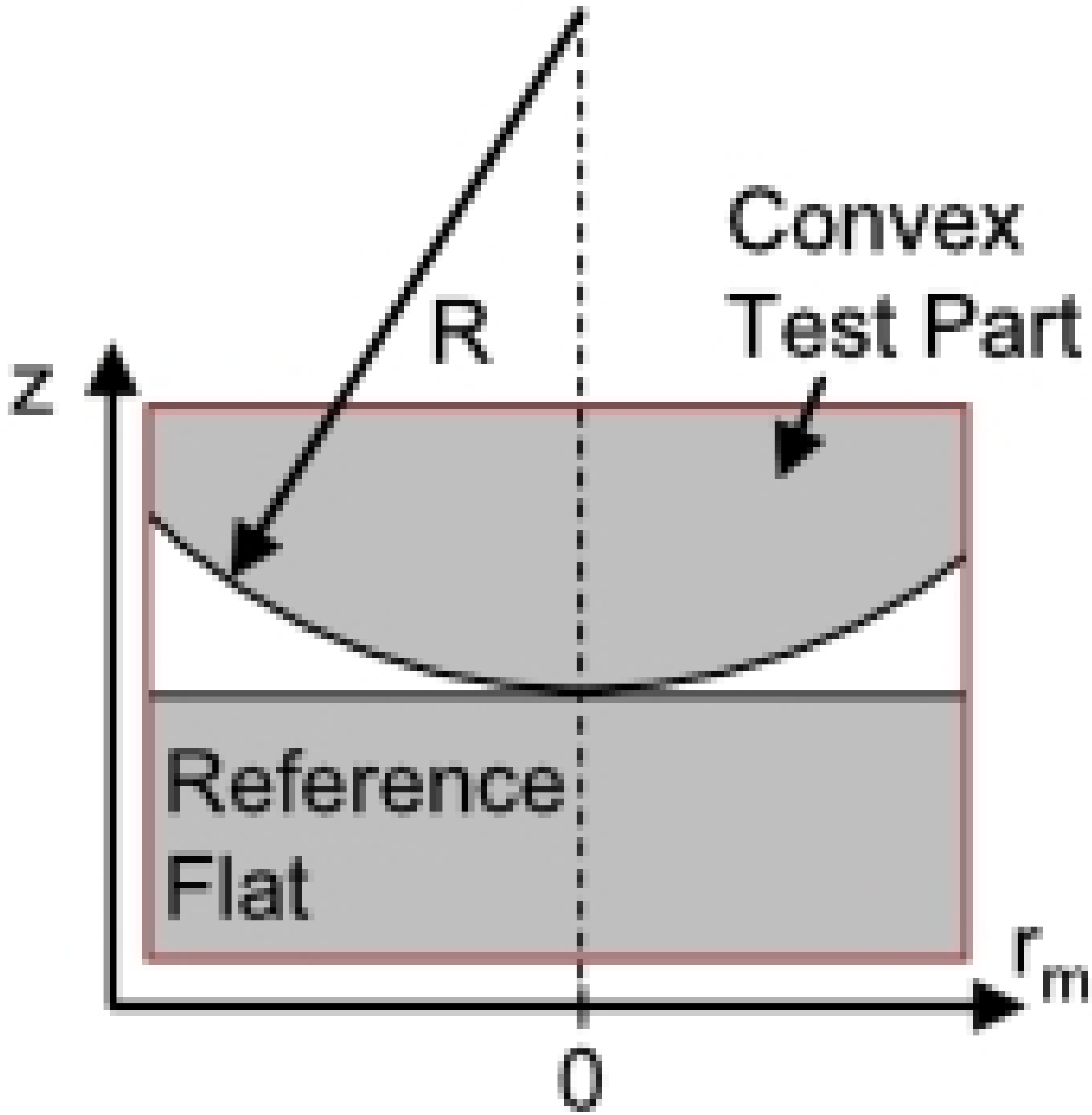
$$r_{m+p}^2 - r_m^2 = p\lambda R$$

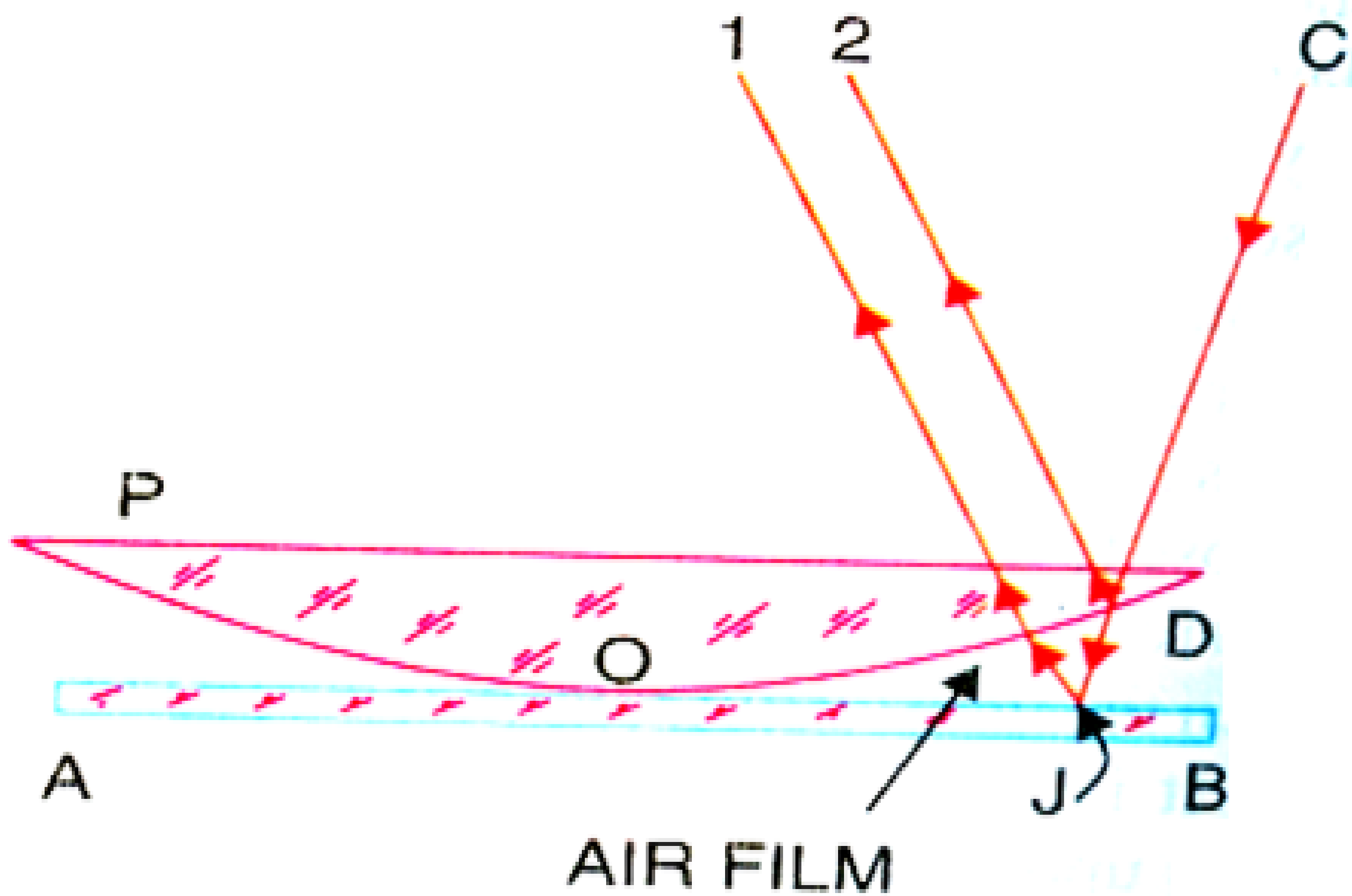
Usually, diameter can be more accurately measured & in terms of diameters, wavelength is given by:

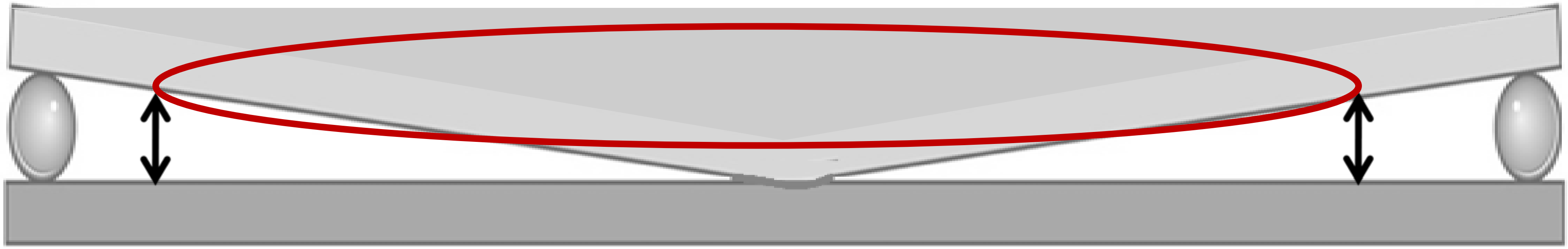
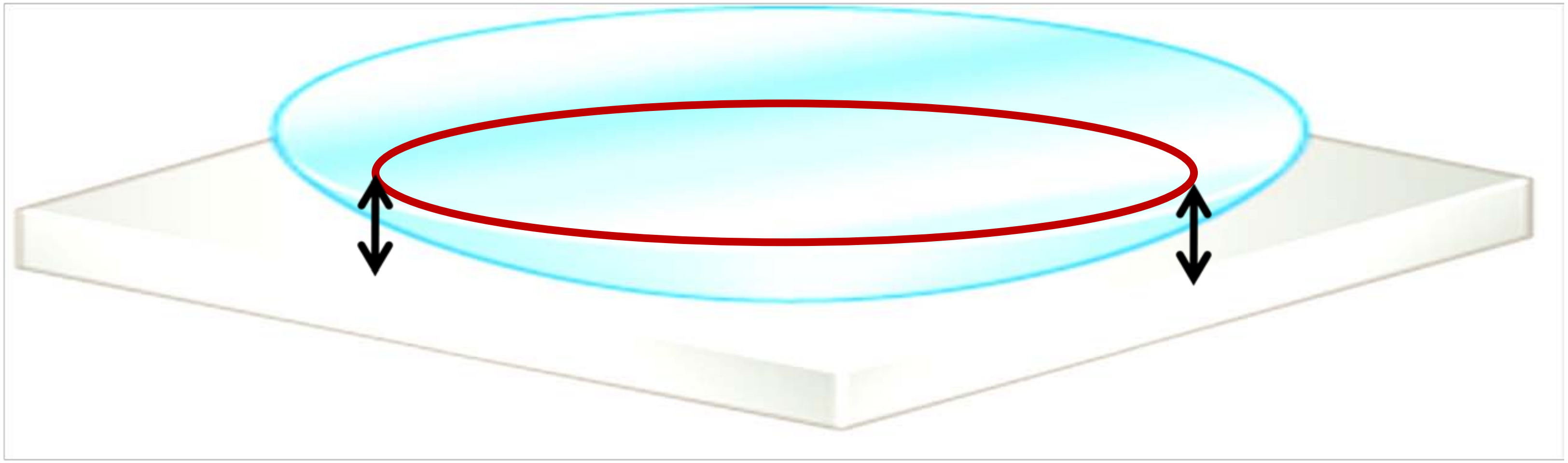
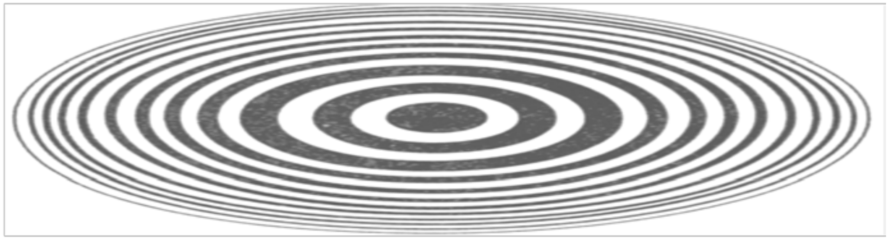
$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}$$

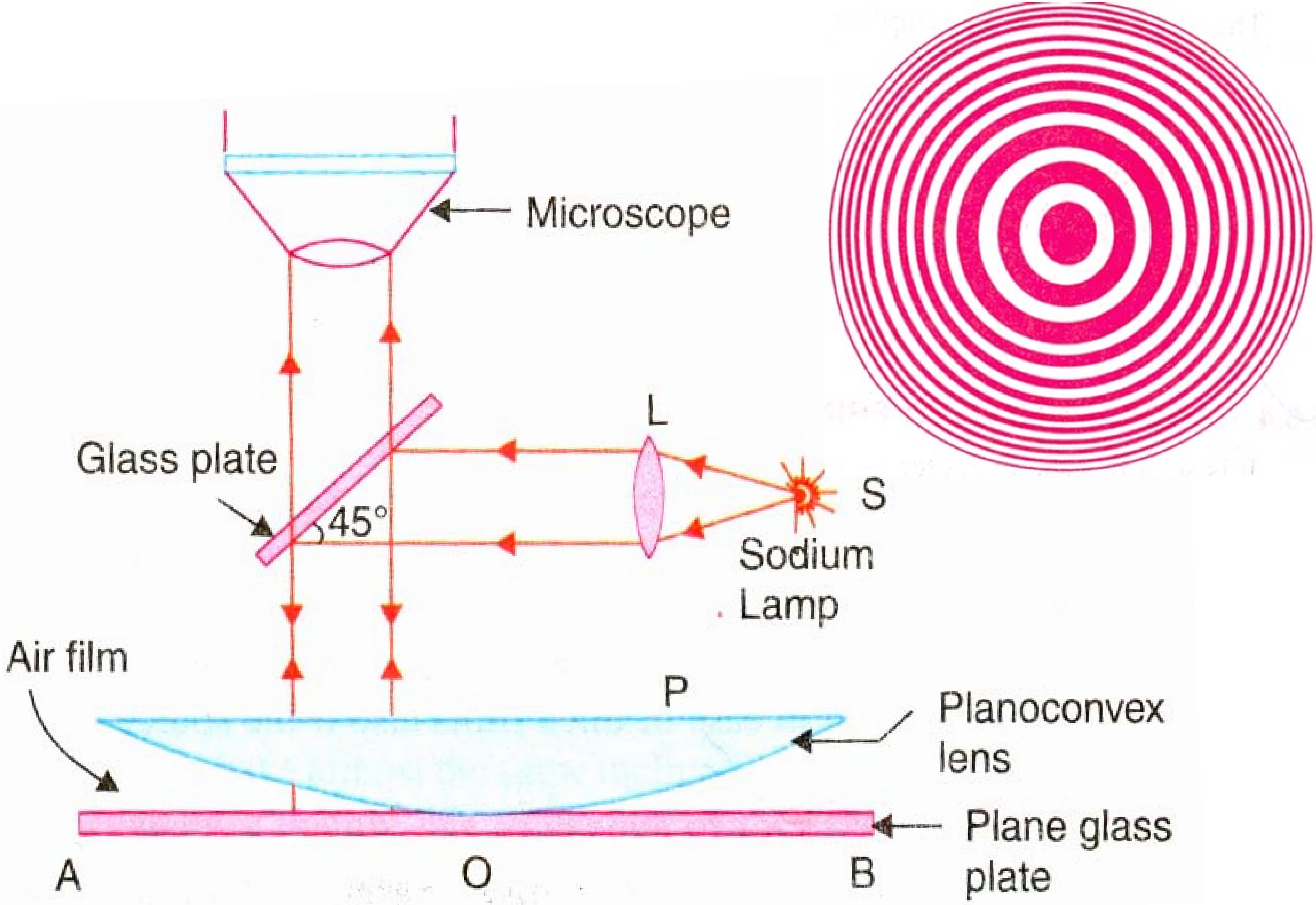
If a liquid of *r.i. n* is introduced between lens & glass plate, radii of rings would be given by:

$$r_m = \left(\frac{m\lambda R}{n}\right)^{1/2}$$


R
Convex
Test Part
z
Reference
Flat
r_m
0


1
2
C
P
D
O
A
J
B
AIR FILM


Microscope
L
Glass plate
45°
S
Sodium
Lamp
Air film
P
Planoconvex
lens
Plane glass
plate
A
O
B

Refractive Index of a Liquid

Liquid whose refractive index is to be determined is filled between lens & glass plate. Now air film is replaced by liquid. Condition for interference then (for darkness)

For normal incidence

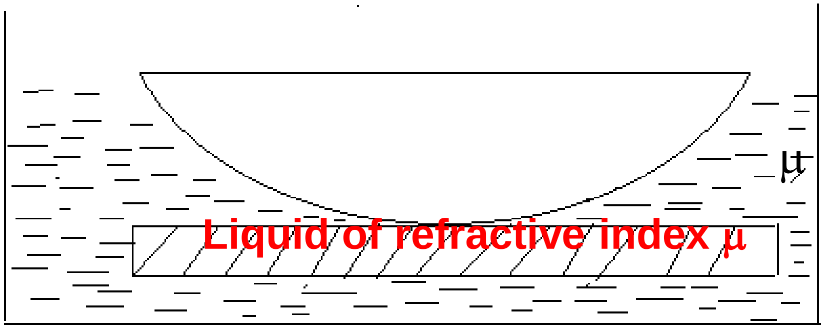
$$2\mu t \cos r = m\lambda$$

Therefore

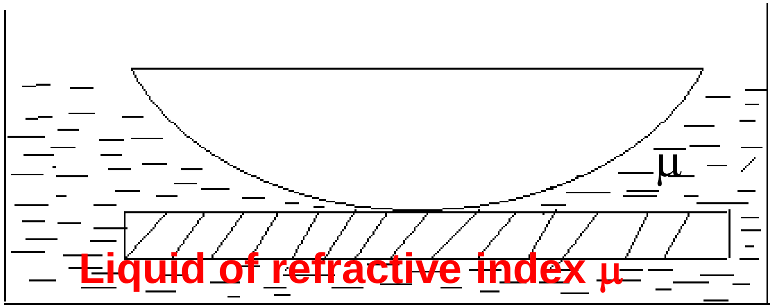
$$\cos r = 1$$

$$2\mu t = m\lambda$$

$$t = \frac{r^2}{2R}$$



$$2\mu \frac{r^2}{2R} = m\lambda$$



$$r^2 = \frac{m\lambda R}{\mu}$$

Therefore

$$D^2 = \frac{4m\lambda R}{\mu}$$

Following the above relation the diameter of mth dark ring is

$$[D_m^2]_{liq} = \frac{4m\lambda R}{\mu}$$

Similarly diameter of (m+p)th ring is given by

$$\left[D_{m+p}^2\right]_{liq} = \frac{4(m+p)\lambda R}{\mu}$$

On subtracting

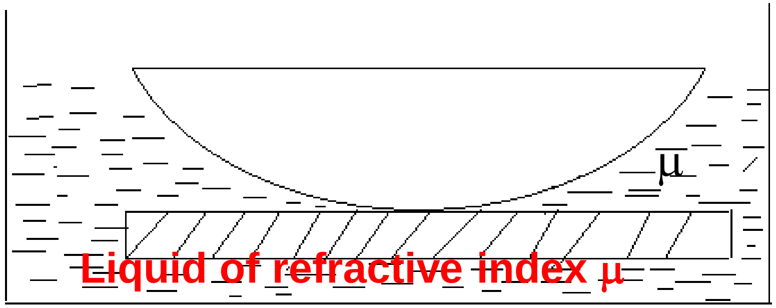
$$\left[D_{m+p}^{2}\right]_{liq} - \left[D_{m}^{2}\right]_{liq} = \frac{4p\lambda R}{\mu}$$

For air

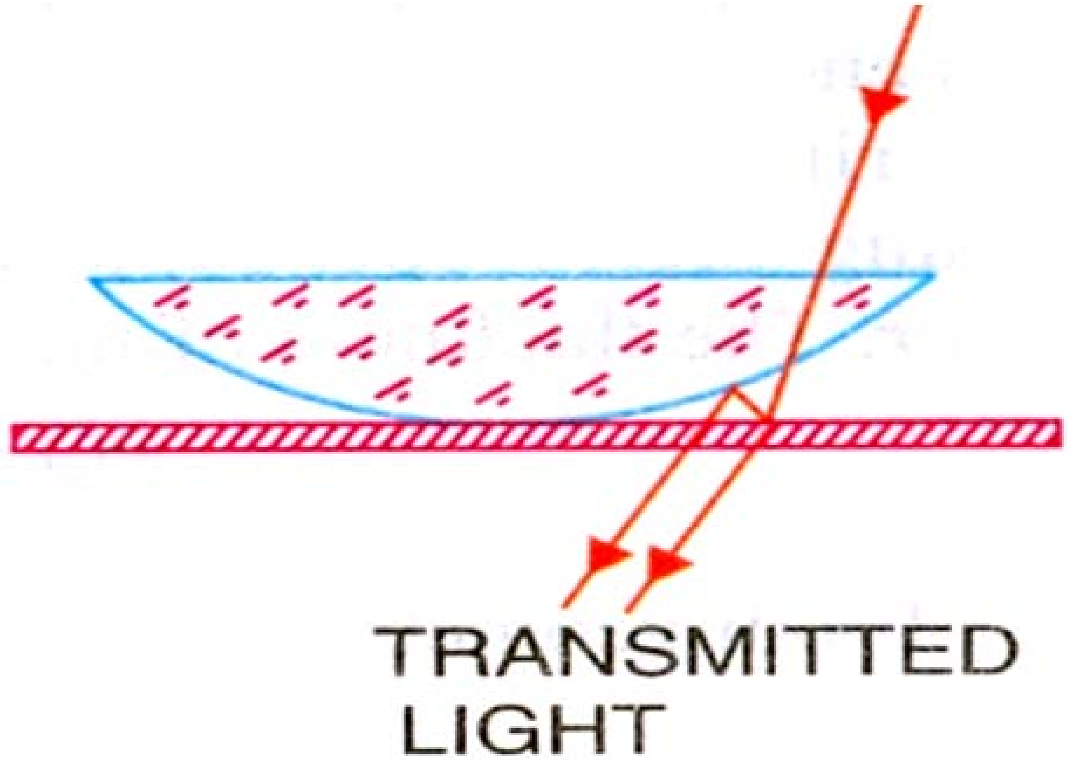
$$\left[D_{m+p}^{2}\right]_{air} - \left[D_{m}^{2}\right]_{air} = 4p\lambda R$$

Therefore

$$\boxed{\mu = \frac{\left[D_{m+p}^{2}\right]_{air} - \left[D_{m}^{2}\right]_{air}}{\left[D_{m+p}^{2}\right]_{liq} - \left[D_{m}^{2}\right]_{liq}}}$$



Newton's rings in transmitted light



Condition for maxima (brightness) is

$$2\mu t \cos r = m\lambda$$

Michelson Interferometer

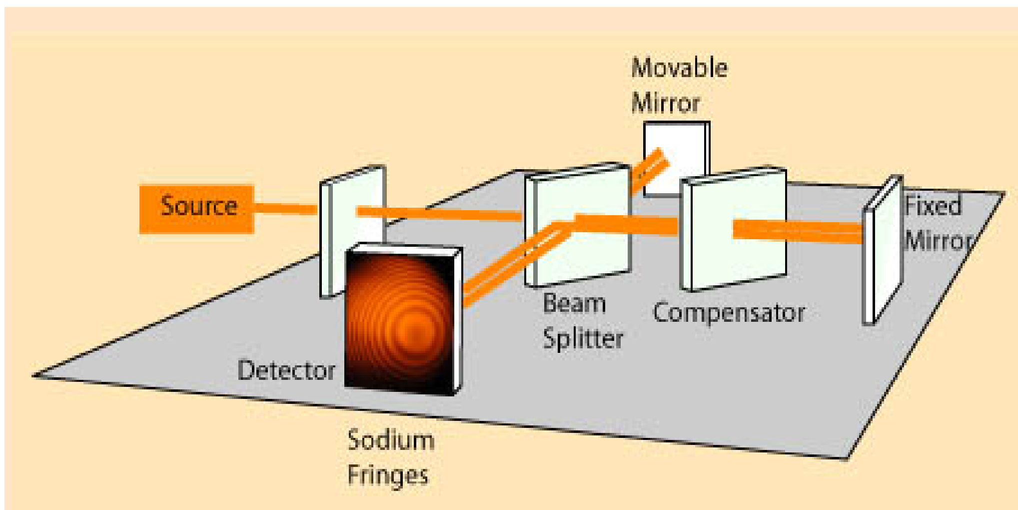
Michelson interferometer produces interference fringes by splitting a beam of monochromatic light so that one beam strikes a fixed mirror & other a movable mirror. When reflected beams are brought back together, interference pattern results.

If distance *d* is such that

$$\frac{2d}{c} << \tau_c$$

$\Rightarrow$

then a definite phase relationship exists between two beams & well-defined interference fringes are observed.



If distance *d* is such that

$$\frac{2d}{c} >> \tau_c$$

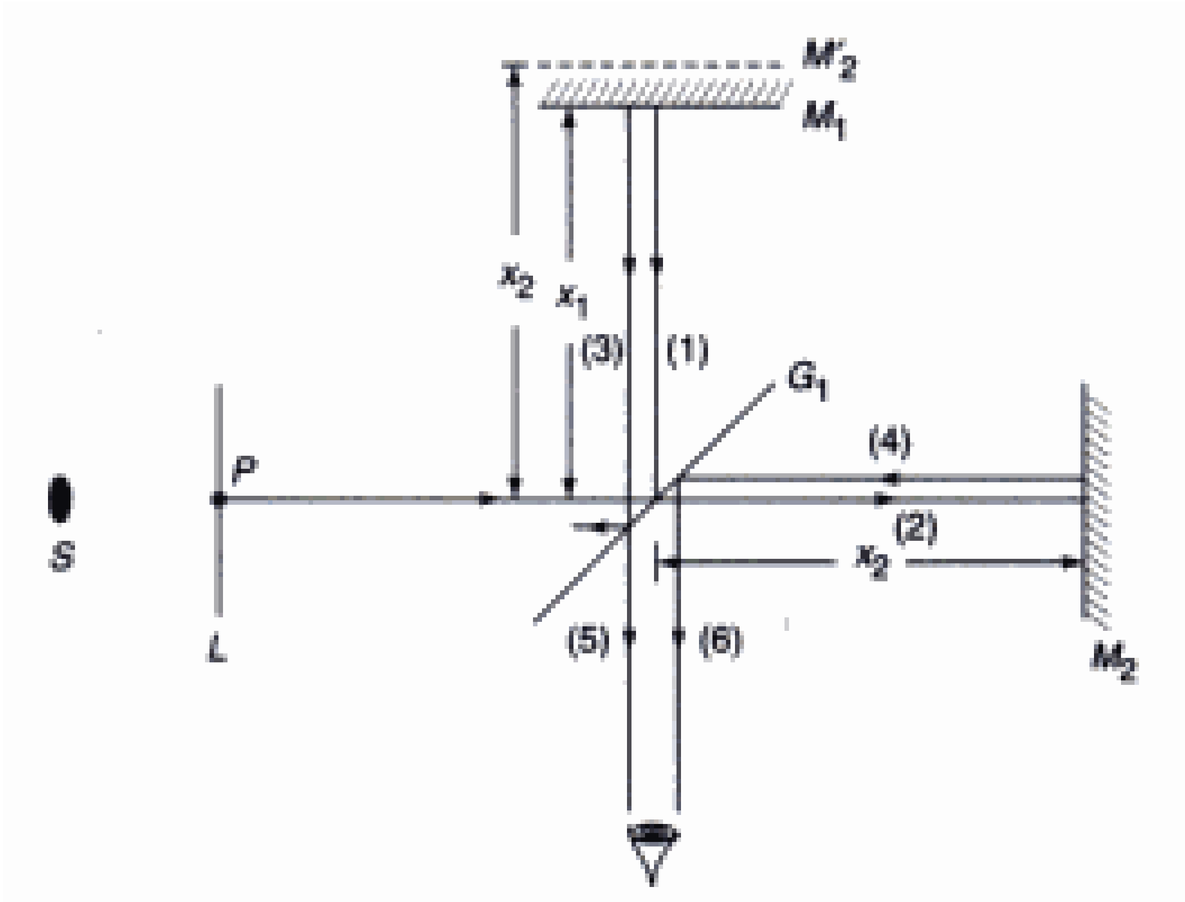
$\Rightarrow$

then there is no definite phase relationship between two beams & no interference pattern is observed.

There is no definite distance at which interference pattern disappears; as distance increases, contrast of fringes becomes gradually poorer & eventually fringe system disappears.

Michelson (1852-1931)

Michelson Interferometer



Waves emanating from *P* get partially reflected & partially transmitted by G_1, & two resulting beams interfere. Reflected wave (1) undergoes a further reflection at M_1 & this reflected wave gets (partially) transmitted through G_1.Transmitted wave (2) gets reflected by M_2 & gets (partially) reflected by G_1 & results in wave (6). Waves (5) & (6) interfere.

If x_1 & x_2 are distances of M_1 & M_2 from plate G_1, then to eye, waves emanating from *P* will appear to get reflected by two parallel mirrors [M_1 & M'_2] separated by a distance ($x_1 \sim x_2$).

If we have a camera focused for infinity, then on focal plane we will obtain circular fringes; each circle corresponding to a definite value of θ. Beam reflected from M_2 will undergo an abrupt phase change of π & since extra path that one of beams traverse will be $2(x_1 \sim x_2)$.

Condition for a dark ring:

$$2d\cos\theta = m\lambda \qquad m = 0,1,2,....$$

$$d = x_1 - x_2$$

Condition for a bright ring:

$$2d\cos\theta = (m + \frac{1}{2})\lambda$$

Ex. Angles at which dark rings will occur, if

$$\lambda = 6\times10^{-5}cm, \quad d = 0.3\,mm$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{m}{1000}\right) = 0^\circ, 2.56^\circ, 3.62^\circ, 4.44^\circ, ...$$

$$corresponding\ to\ m = 1000, 999, 998, ...$$

Central dark ring corresponds to *m* = 1000, first dark ring corresponds to *m* = 999, etc. If we now reduce separation between two mirrors so that *d* = 0.15 mm, angles at which dark rings will occur

$$\theta = \cos^{-1}\left(\frac{m}{500}\right) = 0^\circ, 3.62^\circ, 5.13^\circ, ...$$

$$corresponding\ to\ m = 500, 499, 498, ...$$

As we start reducing value of *d*, fringes will appear to collapse at centre & fringes become less closely placed. Thus, **as *d* decreases, fringe pattern tends to collapse towards centre.**

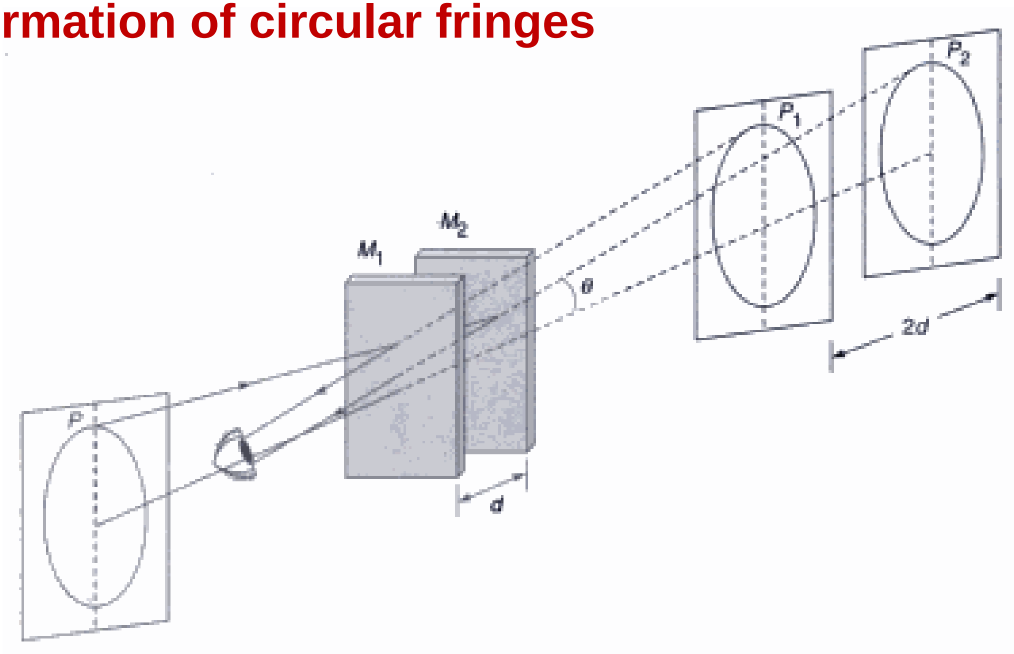
If N fringes collapse to centre as mirror M_1 moves by a distance d_0, then we must have

$$2d = m\lambda$$

$$2(d - d_0) = (m - N)\lambda$$

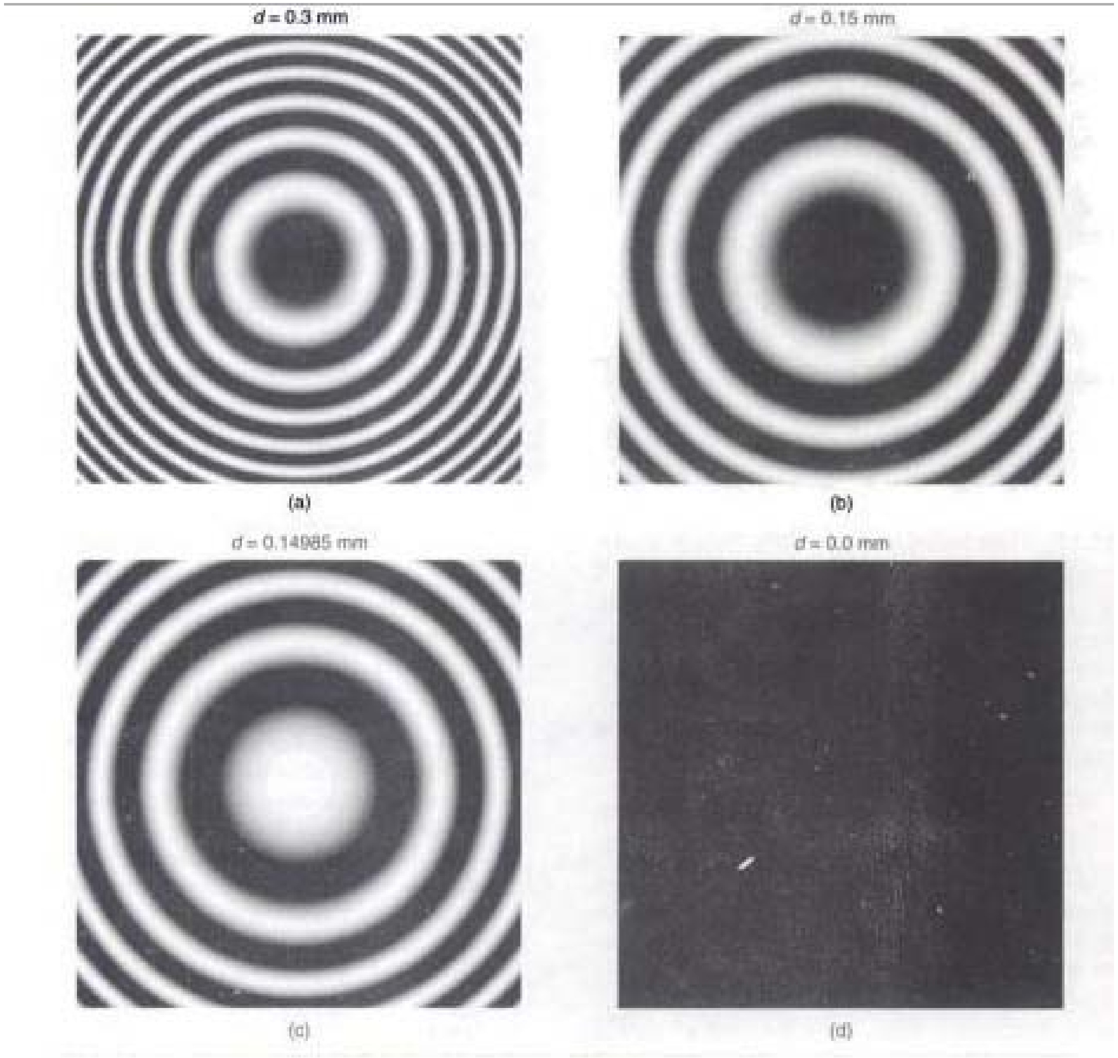
Taking $\theta = 0$, because we are looking at central fringe.

Schematic of formation of circular fringes

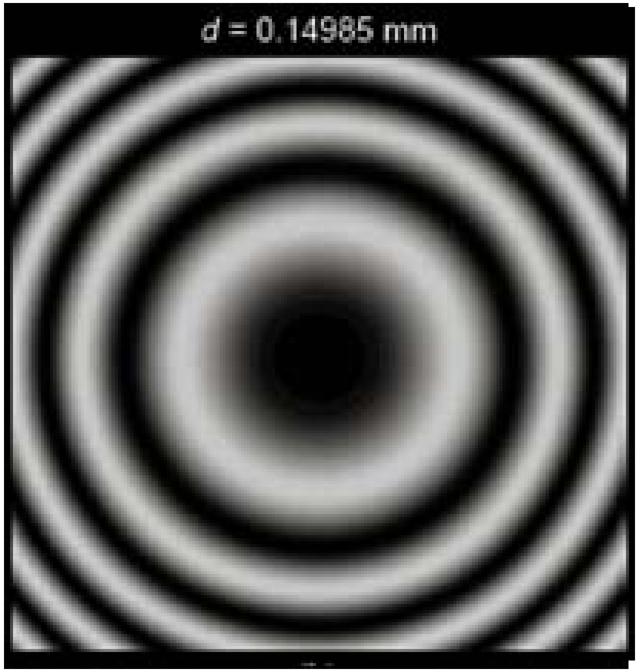


$$Thus \quad \boxed{\lambda = \frac{2d_0}{N}}$$

This provides us with a method for measurement of wavelength.



Computer generated interference pattern produced by Michelson interferometer



Now, if '*d' is slightly* decreased, from 0.15 to 0.14985 mm, then bright central spot corresponding to m = 500 would disappear & central fringe will become *dark.*

Thus, *as d decreases, the fringe patterns tends to collapse* toward the center. Conversely, if *d is increased, the* fringe pattern will expand.