① Find the limit of $f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z - i}$ at $z = i$.

Ans:

$z = i$ is the root of $g(z) = 3z^4 - 2z^3 + 8z^2 - 2z + 5 = 0$.

Therefore, $z = -i$ is the root of $g(z) = 0$.

This implies $(z+i)(z-i) = z^2 + 1$ is the factor of $g(z)$.

$$\lim_{z \to i} f(z) = \lim_{z \to i} \frac{(z^2+1)(3z^2 - 2z + 5)}{z - i}$$

$$= \lim_{z \to i} (z+i)(3z^2 - 2z + 5)$$

$$= (2i)(3i^2 - 2i + 5)$$

$$= 2i(-3 - 2i + 5)$$

$$= 2i(2 - 2i)$$

$$= 4i - 4i^2 = 4(i+1).$$