Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2019-20 (End Semester Assignment)

Maximum Marks: 30
Note: Answer all six questions.

1. Let independent random variables X and Y be distributed as one-parameter exponential random variable with same mean 2. Find the expectation $E[\min(X, Y)]$. **[5]**

2. Let X and Y be independent and identically distributed normal variables with mean zero and variance one. Consider functions $U = X + Y$ and $V = X - Y$. Find joint probability density of (U, V). Also evaluate marginal densities of U and V, respectively. **[3+1+1]**

3. Suppose random variables X and Y have joint density $f_{X,Y}(x, y) = 6x,\ x > 0,\ y > 0,\ 0 < x + y \leq 1$; $f_{X,Y}(x, y) = 0,$ elsewhere. Find correlation coefficient between X and Y. **[5]**

4. Consider the function $f_{X,Y}(x, y) = (6/7)\Big(x^2 + (xy/2)\Big),\ 0 \leq x \leq 1,\ 0 \leq y \leq 2$; $f_{X,Y}(x, y) = 0,$ elsewhere. Evaluate probabilities $P(X > Y)$ and $P(Y > 0.5 \mid X < 0.5)$. **[2+3]**

5. Suppose random variables X and Y have joint density $f_{X,Y}(x, y) = 1,\ 0 \leq x \leq 2,\ 0 \leq y \leq 1,\ 2y \leq x$; $f_{X,Y}(x, y) = 0,$ elsewhere. Find probability density function of $Z = X + Y$. **[5]**

6. Describe a Poisson process. **[5]**