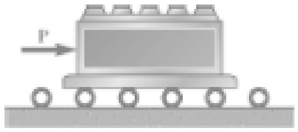
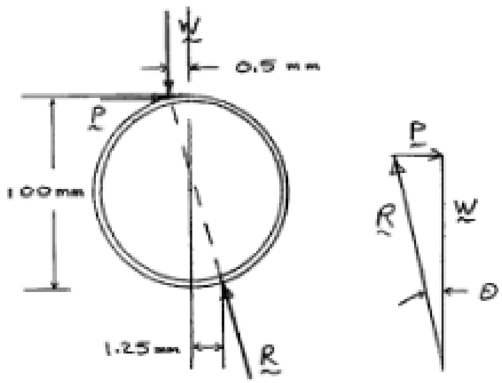
PROBLEM 8.100



A 900-kg machine base is rolled along a concrete floor using a series of steel pipes with outside diameters of 100 mm. Knowing that the coefficient of rolling resistance is 0.5 mm between the pipes and the base and 1.25 mm between the pipes and the concrete floor, determine the magnitude of the force **P** required to slowly move the base along the floor.

SOLUTION

FBD pipe:



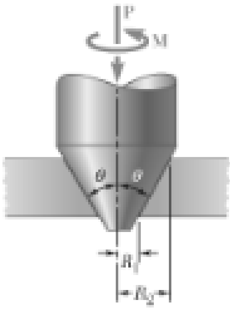
$$W = mg$$
$$= (900 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 8829.0 \text{ N}$$

$$\theta = \sin^{-1}\frac{.5 \text{ mm} + 1.25 \text{ mm}}{100 \text{ mm}}$$
$$= 1.00273°$$

$P = W\tan\theta$ for each pipe, so also for total

$$P = (8829.0 \text{ N})\tan(1.00273°)$$

$P = 154.4$ N ◀



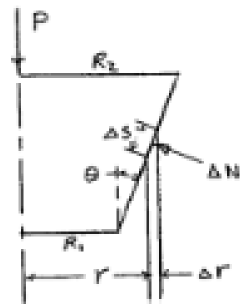
PROBLEM 8.96*

Assuming that the pressure between the surfaces of contact is uniform, show that the magnitude M of the couple required to overcome frictional resistance for the conical bearing shown is

$$M = \frac{2}{3}\frac{\mu_k P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

SOLUTION

Let normal force on ΔA be ΔN and $\dfrac{\Delta N}{\Delta A} = k$.



So $\qquad \Delta N = k\Delta A \qquad \Delta A = r\Delta s\Delta\phi \qquad \Delta s = \dfrac{\Delta r}{\sin\theta}$

where ϕ is the azimuthal angle around the symmetry axis of rotation.

$$\Delta F_y = \Delta N \sin\theta = kr\Delta r\Delta\phi$$

Total vertical force: $\qquad P = \lim_{\Delta A \to 0} \Sigma \Delta F_y$

$$P = \int_0^{2\pi}\left(\int_{R_1}^{R_2} krdr\right)d\phi = 2\pi k\int_{R_1}^{R_2} rdr$$

$$P = \pi k\left(R_2^2 - R_1^2\right) \quad \text{or} \quad k = \frac{P}{\pi\left(R_2^2 - R_1^2\right)}$$

Friction force: $\qquad \Delta F = \mu\Delta N = \mu k\Delta A$

Moment: $\qquad \Delta M = r\Delta F = r\mu kr\dfrac{\Delta r}{\sin\theta}\Delta\phi$

Total couple: $\qquad M = \lim_{\Delta A \to 0}\Sigma\Delta M = \displaystyle\int_0^{2\pi}\left(\int_{R_1}^{R_2}\frac{\mu k}{\sin\theta}r^2 dr\right)d\phi$

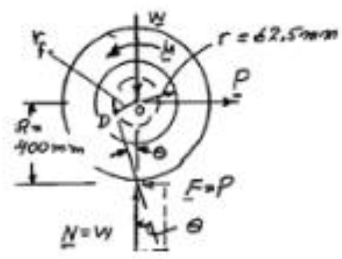
$$M = 2\pi\frac{\mu k}{\sin\theta}\int_{R_1}^{R_2} r^2 dr = \frac{2}{3}\frac{\pi\mu}{\sin\theta}\frac{P}{\pi\left(R_2^2 - R_1^2\right)}\left(R_2^3 - R_1^3\right)$$

$$M = \frac{2}{3}\frac{\mu P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \blacktriangleleft$$

PROBLEM 8.91

A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) to start the car moving, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION



$$r_f = \mu r; \quad R = 400 \text{ mm}$$

$$\sin\theta = \tan\theta = \frac{r_f}{R} = \frac{\mu r}{R}$$

$$P = W\tan\theta = W\frac{\mu r}{R}$$

$$P = W\mu\frac{62.5 \text{ mm}}{400 \text{ mm}}$$

$$= 0.15625W\mu$$

For one wheel:

$$W = \frac{1}{8}(30 \text{ mg})(9.81 \text{ m/s}^2)$$

$$= \frac{1}{8}(294.3 \text{ kN})$$

For eight wheels of railroad car:

$$\Sigma P = 8(0.15625)\frac{1}{8}(294.3 \text{ kN})\mu$$

$$= (45.984\mu) \text{ kN}$$

(*a*) To start motion:

$$\mu_s = 0.020$$

$$\Sigma P = (45.984)(0.020)$$

$$= 0.9197 \text{ kN}$$

$\Sigma P = 920$ N ◀

(*b*) To maintain motion:

$$\mu_k = 0.015$$

$$\Sigma P = (45.984)(0.015)$$

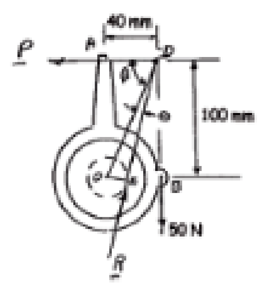
$$= 0.6897 \text{ kN}$$

$\Sigma P = 690$ N ◀

PROBLEM 8.90

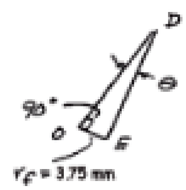
A lever AB of negligible weight is loosely fitted onto a 50-mm-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating clockwise.

SOLUTION



$$r_f = \mu_s r$$
$$= 0.15(25\text{ mm})$$
$$= 3.75\text{ mm}$$
$$\tan\beta = \frac{100\text{ mm}}{40\text{ mm}}$$
$$\beta = 68.199°$$

In ΔEOD:



$$OD = \sqrt{(40\text{ mm})^2 + (100\text{ mm})^2}$$
$$OD = 107.703\text{ mm}$$
$$\sin\theta = \frac{OE}{OD} = \frac{3.75\text{ mm}}{107.703\text{ mm}}$$
$$\theta = 1.99533°$$

Force triangle:

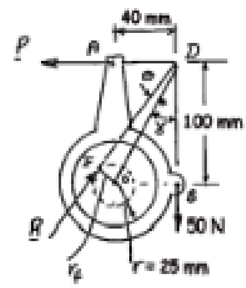
$$P = \frac{50}{\tan(\beta+\theta)} = \frac{50\text{ N}}{\tan 70.194°}$$

$\mathbf{P} = 18\text{ N} \longleftarrow$ ◀

PROBLEM 8.88

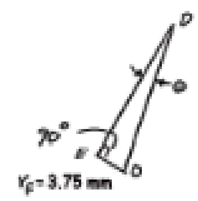
A lever AB of negligible weight is loosely fitted onto a 50-mm-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating counterclockwise.

SOLUTION



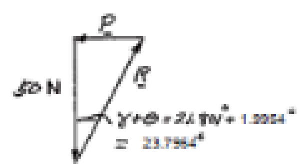
$$r_f = \mu_s r$$
$$= 0.15(25 \text{ mm})$$
$$r_f = 3.75 \text{ mm}$$
$$\tan\gamma = \frac{40 \text{ mm}}{100 \text{ mm}}$$
$$\gamma = 21.801°$$

In ΔEOD:



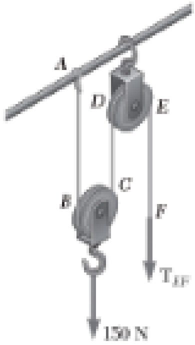
$$OD = \sqrt{(40 \text{ mm})^2 + (100 \text{ mm})^2}$$
$$= 107.7 \text{ mm}$$
$$\sin\theta = \frac{OE}{OD} = \frac{r_f}{OD}$$
$$= \frac{3.75 \text{ mm}}{107.7 \text{ mm}}$$
$$\theta = 1.9954°$$

Force triangle:



$$P = (50 \text{ N})\tan(\gamma + \theta)$$
$$= (50 \text{ N})\tan 23.79645°$$
$$= 22.05 \text{ N}$$

$\mathbf{P} = 22.0 \text{ N} \longleftarrow$ ◀



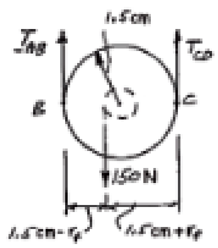
PROBLEM 8.84

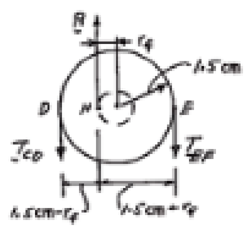
The block and tackle shown are used to lower a 150-N load. Each of the 3-cm-diameter pulleys rotates on a 0.5-cm-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly lowered.

SOLUTION

<u>For each pulley:</u>

$$r_f = r\mu_s = \left(\frac{0.5\text{ cm}}{2}\right)0.2 = 0.05\text{ cm}$$





<u>Pulley *BC*:</u>

$$+\circlearrowleft \Sigma M_B = 0: \quad T_{CD}(3\text{ cm}) - (150\text{ N})(1.5\text{ cm} - r_f) = 0$$

$$T_{CD} = \frac{(150\text{ N})(1.5\text{ cm} - 0.05\text{ cm})}{3\text{ cm}} \qquad T_{CD} = 72.5\text{ N} \blacktriangleleft$$

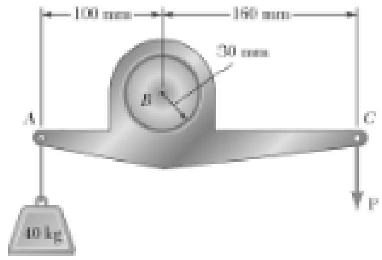
$$+\uparrow \Sigma F_y = 0: \quad T_{AB} + 72.5\text{ N} - 150\text{ N} = 0 \qquad T_{AB} = 77.5\text{ N} \blacktriangleleft$$

<u>Pulley *DE*:</u>

$$+\circlearrowleft \Sigma M_H = 0: \quad T_{CD}(1.5\text{ cm} - r_f) - T_{EF}(1.5\text{ cm} + r_f) = 0$$

$$T_{EF} = T_{CD}\frac{1.5\text{ cm} - r_f}{1.5\text{ cm} + r_f}$$

$$= (72.5\text{ N})\frac{1.5\text{ cm} - 0.05\text{ cm}}{1.5\text{ cm} + 0.05\text{ cm}} \qquad T_{EF} = 67.8\text{ N} \blacktriangleleft$$

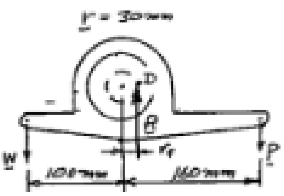


PROBLEM 8.77

A lever of negligible weight is loosely fitted onto a 30-mm-radius fixed shaft as shown. Knowing that a force **P** of magnitude 275 N will just start the lever rotating clockwise, determine (*a*) the coefficient of static friction between the shaft and the lever, (*b*) the smallest force **P** for which the lever does not start rotating counterclockwise.

SOLUTION

(*a*) <u>Impending motion</u> $\circlearrowright$



$$W = (40\text{ kg})(9.81\text{ m/s}^2) = 392.4\text{ N}$$

$$+\circlearrowright \Sigma M_D = 0: \quad P(160 - r_f) - W(100 + r_f) = 0$$

$$r_f = \frac{160P - 100W}{P + W}$$

$$r_f = \frac{(160\text{ mm})(275\text{ N}) - (100\text{ mm})(392.4\text{ N})}{275\text{ N} + 392.4\text{ N}}$$

$$r_f = 7.132\text{ mm}$$

$$r_f = r\sin\phi_s = r\mu_s$$

$$\mu_s = \frac{r_f}{r} = \frac{7.132\text{ mm}}{30\text{ mm}} = 0.2377$$

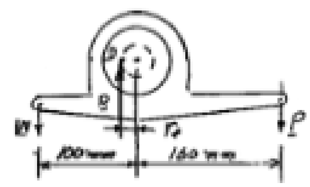
$\mu_s = 0.238$ ◀

(*b*) <u>Impending motion</u> $\circlearrowleft$

$$r_f = r\sin\phi_s = r\mu_s$$

$$= (30\text{ mm})(0.2377)$$

$$r_f = 7.132\text{ mm}$$



$$+\circlearrowleft \Sigma M_D = 0: \quad P(160 + r_f) - W(100 - r_f) = 0$$

$$P = W\frac{100 - r_f}{160 + r_f}$$

$$P = (392.4\text{ N})\frac{100\text{ mm} - 7.132\text{ mm}}{160\text{ mm} + 7.132\text{ mm}}$$

$$P = 218.04\text{ N}$$

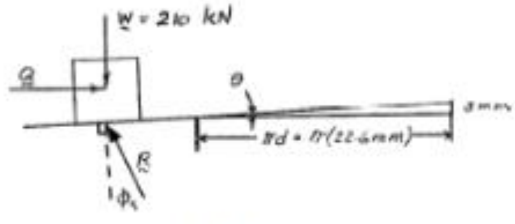
P $= 218\text{ N}\downarrow$ ◀

PROBLEM 8.71

High-strength bolts are used in the construction of many steel structures. For a 24-mm-nominal-diameter bolt the required minimum bolt tension is 210 kN. Assuming the coefficient of friction to be 0.40, determine the required couple that should be applied to the bolt and nut. The mean diameter of the thread is 22.6 mm, and the lead is 3 mm. Neglect friction between the nut and washer, and assume the bolt to be square-threaded.

SOLUTION

FBD block on incline:



$$\theta = \tan^{-1}\frac{3\text{ mm}}{(22.6\text{ mm})\pi}$$

$$= 2.4195°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.40$$

$$\phi_s = 21.801°$$

$$Q = (210\text{ kN})\tan(21.801° + 2.4195°)$$

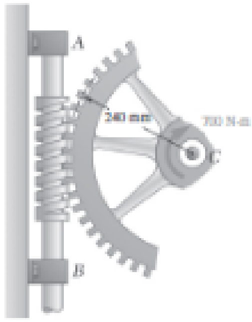
$$Q = 94.468\text{ kN}$$

$$\text{Torque} = \frac{d}{2}Q$$

$$= \frac{22.6\text{ mm}}{2}(94.468\text{ kN})$$

$$= 1067.49\text{ N}\cdot\text{m}$$

Torque = 1068 N · m ◄



PROBLEM 8.69

The square-threaded worm gear shown has a mean radius of 30 mm. and a lead of 7.5 mm. The large gear is subjected to a constant clockwise couple of 700 N.m. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

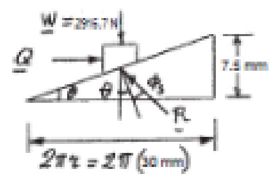
SOLUTION

Free body: Large gear

$$+\circlearrowleft \Sigma M_C = 0: \quad W(0.24 \text{ m}) - 700 \text{ N}\cdot\text{m} = 0$$

$$W = 2916.7 \text{ N}$$

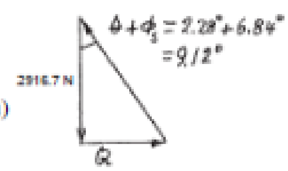
Block-and-Incline analysis of worn gear



$$\tan\theta = \frac{7.5 \text{ mm}}{2\pi(30 \text{ mm})} = 0.039789$$

$$\theta = 2.28^\circ$$

$$\mu_s = 0.12, \ \phi_s = \tan^{-1} 0.12 = 6.84^\circ$$



$$Q = (2916.7 \text{ N})\tan 9.12^\circ$$

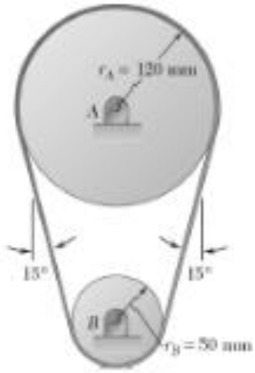
$$= 468.2 \text{ N}$$

$$\text{Torque} = Q_r = (468.2 \text{ N})(0.03 \text{ m})$$

$$= 14.05 \text{ N}\cdot\text{m}$$

Torque = 14.05 N·m ◀

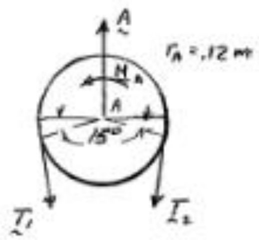
PROBLEM 8.112



A flat belt is used to transmit a couple from drum B to drum A. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum A.

SOLUTION

FBD's drums:



$$\beta_A = 180° + 30° = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

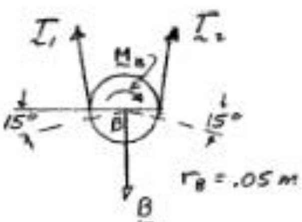
$$\beta_B = 180° - 30° = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Since $\beta_B < \beta_A$, slipping will impend first on B (friction coefficients being equal)

So

$$T_2 = T_{max} = T_1 e^{\mu_s \beta_B}$$

$$450\text{ N} = T_1 e^{(0.4)5\pi/6} \quad \text{or} \quad T_1 = 157.914\text{ N}$$

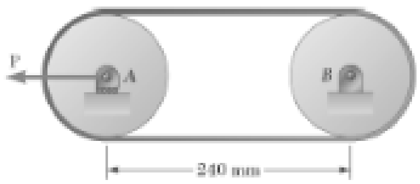


$$\circlearrowleft \Sigma M_A = 0\text{:} \quad M_A + (0.12\text{ m})(T_1 - T_2) = 0$$

$$M_A = (0.12\text{ m})(450\text{ N} - 157.914\text{ N}) = 35.05\text{ N}\cdot\text{m}$$

$$M_A = 35.1\text{ N}\cdot\text{m} \blacktriangleleft$$

PROBLEM 8.114

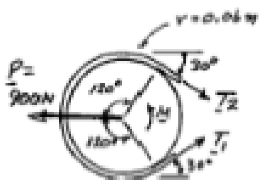


Solve Problem 8.113 assuming that the belt is looped around the pulleys in a figure eight.

PROBLEM 8.113 A flat belt is used to transmit a couple from pulley *A* to pulley *B*. The radius of each pulley is 60 mm, and a force of magnitude $P = 900$ N is applied as shown to the axle of pulley *A*. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest couple that can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

SOLUTION

Drum *A*:



$$\beta = 240° = 240° \frac{\pi}{180°} = \frac{4}{3}\pi$$

$$\sin\theta = \frac{60}{120} = \frac{1}{2}$$
$$\theta = 30°$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.35(4/3\pi)}$$
$$T_2 = 4.3322 T_1$$

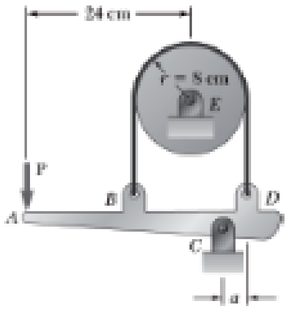
(*a*) Torque: $+\circlearrowleft \Sigma M_B = 0$: $M - (844.3\text{ N})(0.06\text{ m}) + (194.9\text{ N})(0.06\text{ m}) = 0$

$$M = 39.0\text{ N}\cdot\text{m} ◀$$

(*b*) $\xrightarrow{+} \Sigma F_x = 0$: $(T_1 + T_2)\cos 30° - 900\text{ N}$

$$(T_1 + 4.3322T_1)\cos 30° = 900$$
$$T_1 = 194.90\text{ N}$$
$$T_2 = 4.3322(194.90\text{ N}) = 844.3\text{ N}$$

$$T_{max} = 844\text{ N} ◀$$



PROBLEM 8.115

The speed of the brake drum shown is controlled by a belt attached to the control bar AD. A force $\mathbf{P}$ of magnitude 25 N is applied to the control bar at A. Determine the magnitude of the couple being applied to the drum, knowing that the coefficient of kinetic friction between the belt and the drum is 0.25, that $a = 4$ cm, and that the drum is rotating at a constant speed (a) counterclockwise, (b) clockwise.

SOLUTION

(a) Counterclockwise rotation

Free body: Drum

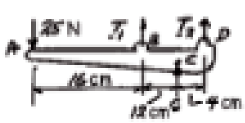
$r = 8$ cm $\beta = 180° = \pi$ radians

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933 T_1$$

Free body: Control bar

$$+\circlearrowleft \Sigma M_C = 0: \quad T_1(12 \text{ cm}) - T_2(4 \text{ cm}) - (25 \text{ N})(28 \text{ cm}) = 0$$



$$T_1(12) - 2.1933T_1(4) - 700 = 0$$

$$T_1 = 216.93 \text{ N}$$

$$T_2 = 2.1933(216.93 \text{ N}) = 475.79 \text{ N}$$

Return to free body of drum

$$+\circlearrowleft \Sigma M_E = 0: \quad M + T_1(8 \text{ cm}) - T_2(8 \text{ cm}) = 0$$

$$M + (216.93 \text{ N})(8 \text{ cm}) - (475.79 \text{ N})(8 \text{ cm}) = 0$$

$$M = 2070.9 \text{ N} \cdot \text{cm}$$

$M = 20.7$ N · m ◀

(b) Clockwise rotation

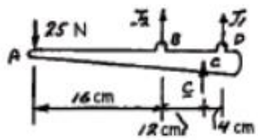
$r = 8$ cm $\beta = \pi$ rad

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933\, T_1$$

PROBLEM 8.115 (Continued)

Free body: Control rod



$$+\circlearrowleft \Sigma M_C = 0: \quad T_2(12\text{ cm}) - T_1(4\text{ cm}) - (25\text{ N})(28\text{ cm}) = 0$$

$$2.1933T_1(12) - T_1(4) - 700 = 0$$

$$T_1 = 31.363\text{ N}$$

$$T_2 = 2.1933(31.363\text{ N})$$

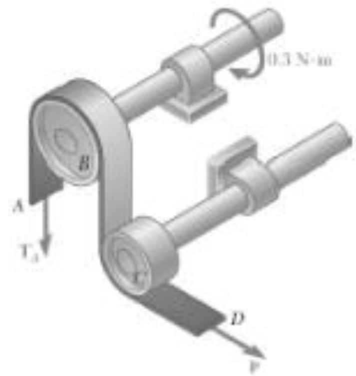
$$T_2 = 68.788\text{ N}$$

Return to free body of drum

$$+\circlearrowleft \Sigma M_E = 0: \quad M + T_1(8\text{ cm}) - T_2(8\text{ cm}) = 0$$

$$M + (31.363\text{ N})(8\text{ cm}) - (68.788\text{ N})(8\text{ cm}) = 0$$

$$M = 299.4\text{ N}\cdot\text{cm} \qquad M = 3\text{ N}\cdot\text{m} \blacktriangleleft$$

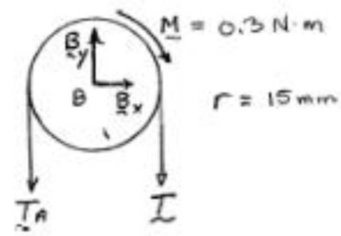


PROBLEM 8.124

A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

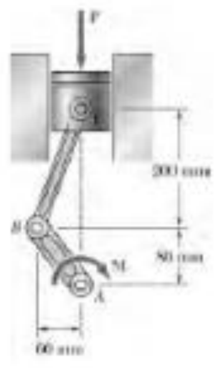
$$T_A - T = \frac{M}{r} = \frac{300 \text{ N}\cdot\text{mm}}{20 \text{ mm}} = 15.0000 \text{ N}$$

Impending slipping: $\qquad T_A = Te^{\mu_s\beta} = Te^{0.4\pi}$

So $\qquad T(e^{0.4\pi} - 1) = 15.0000 \text{ N}$

or $\qquad T = 5.9676 \text{ N}$

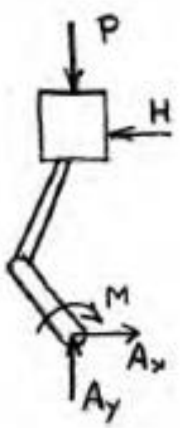
If C is free to rotate, $P = T$ $\qquad P = 5.97 \text{ N}$ ◀



PROBLEM 1.14

A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm^2 uniform cross section.

SOLUTION



Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y.

$$+\circlearrowleft \Sigma M_A = 0: \quad (0.280 \text{ m})H - 1500 \text{ N} \cdot \text{m} = 0$$

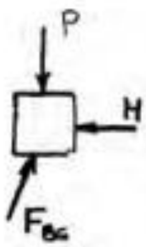
$$H = 5.3571 \times 10^3 \text{N}$$

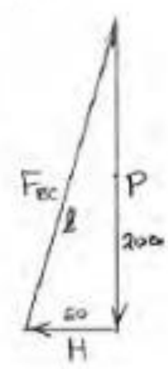
Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BE} by proportions.

$$l = \sqrt{200^2 + 60^2} = 208.81 \text{ mm}$$

$$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86 \times 10^3 \text{ N}$$

(*a*) $P = 17.86$ kN ◄





$$\frac{F_{BC}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{BC} = 18.643 \times 10^3 \text{ N}$$

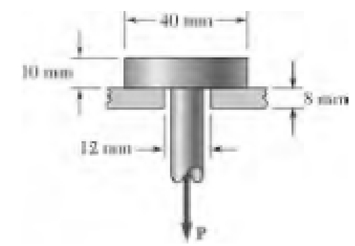
Rod BC is a compression member. Its area is

$$450 \text{ mm}^2 = 450 \times 10^{-6} \text{m}^2$$

Stress,

$$\sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-18.643 \times 10^3}{450 \times 10^{-6}} = -41.4 \times 10^6 \text{ Pa}$$

(*b*) $\sigma_{BC} = -41.4$ MPa ◄



PROBLEM 1.17

A load P is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load **P** that can be applied to the rod.

SOLUTION

For the steel rod,

$$A_1 = \pi d_1 t_1 = (\pi)(0.012)(0.010)$$

$$= 376.99 \times 10^{-6}\,\text{m}^2$$

$$\tau_1 = \frac{P}{A_1} \longrightarrow P_1 = \tau_1 A_1$$

$$P_1 = (180 \times 10^6)(376.99 \times 10^{-6}) = 67.86 \times 10^3\,\text{N}$$

For the aluminum plate,

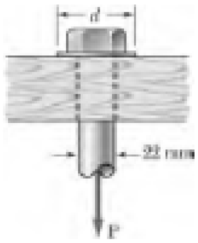
$$A_2 = \pi d_2 t_2 = (\pi)(0.040)(0.008) = 1.00531 \times 10^{-3}\,\text{m}^2$$

$$\tau_2 = \frac{P_2}{A_2} \longrightarrow P_2 = \tau_2 A_2$$

$$P_2 = (70 \times 10^6)(1.0053 \times 10^{-6}) = 70.372 \times 10^3\,\text{N}$$

The limiting value for the load P is the smaller of P_1 and P_2.

$$P = 67.86 \times 10^3\,\text{N} \qquad\qquad P = 67.9\,\text{kN} \blacktriangleleft$$



PROBLEM 1.19

The load P applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter d of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.

SOLUTION

Steel rod: $A = \frac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6}\text{m}^2$

$$\sigma = 35 \times 10^6\text{Pa}$$
$$P = \sigma A = (35 \times 10^6)(380.13 \times 10^{-6})$$
$$= 13.305 \times 10^3\text{N}$$

Washer: $\sigma_b = 5 \times 10^6\text{Pa}$

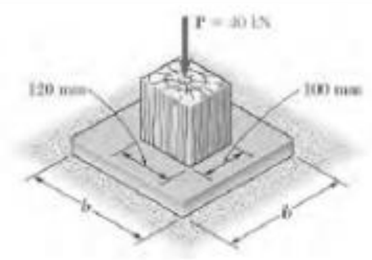
Required bearing area:

$$A_b = \frac{P}{\sigma_b} = \frac{13.305 \times 10^3}{5 \times 10^6} = 2.6609 \times 10^{-3}\text{m}^2$$

But, $A_b = \frac{\pi}{4}(d^2 - d_i^2)$

$$d^2 = d_i^2 + \frac{4A_b}{\pi}$$
$$= (0.025)^2 + \frac{(4)(2.6609 \times 10^{-3})}{\pi}$$
$$= 4.013 \times 10^{-3}\text{m}^2$$
$$d = 63.3 \times 10^{-3}\text{m}$$

$d = 63.3$ mm ◀



PROBLEM 1.22

A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

SOLUTION

(*a*) Bearing stress on concrete footing.

$$P = 40\,\text{kN} = 40 \times 10^3\,\text{N}$$

$$A = (100)(120) = 12 \times 10^3\,\text{mm}^2 = 12 \times 10^{-3}\,\text{m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.333 \times 10^6\,\text{Pa}$$

3.33 MPa ◀

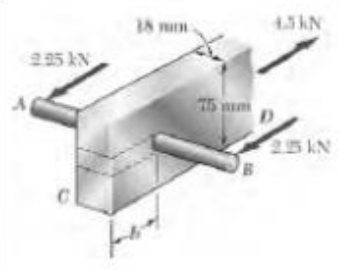
(*b*) Footing area. $P = 40 \times 10^3\,\text{N}$ $\quad \sigma = 145\,\text{kPa} = 45 \times 10^3\,\text{Pa}$

$$\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586\,\text{m}^2$$

Since the area is square, $A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525\,\text{m}$$

$b = 525$ mm ◀



PROBLEM 1.28

A 12-mm-diameter steel rod AB is fitted to a round hole near end C of the wooden member CD. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance b for which the average shearing stress is 620 kPa on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

SOLUTION

(*a*) Maximum average normal stress in the wood.

$$A_{net} = (75-12)(18) = 1.134\times 10^3\,\text{mm}^2 = 1.134\times 10^{-3}\,\text{m}^2$$

$$P = 4.50\text{ kN} = 4.50\times 10^3\text{ N}$$

$$\sigma = \frac{P}{A_{net}} = \frac{4.50\times 10^3}{1.134\times 10^{-3}} = 3.97\times 10^6\text{ Pa}$$

3.97 MPa ◀

(*b*)

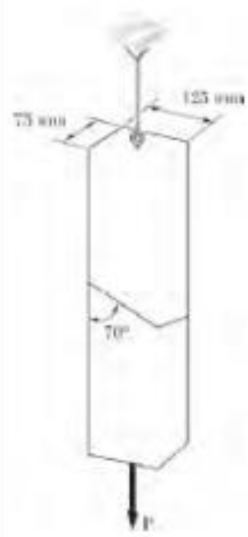
$$\tau = \frac{P}{A} = \frac{P}{2bt} \qquad b = \frac{P}{2t\tau} = \frac{4.50\times 10^3}{(2)(18\times 10^{-3})(620\times 10^3)} = 202\times 10^{-3}\text{m}$$

$b = 202$ mm ◀

(*c*)

$$\sigma_b = \frac{P}{dt} = \frac{4.50\times 10^3}{(12\times 10^{-3})(18\times 10^{-3})} = 20.8\times 10^6\text{ Pa}$$

20.8 MPa ◀



PROBLEM 1.30

Two wooden members of 75×125 mm uniform rectangular cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 500 kPa, determine (*a*) the largest load P which can be safely supported, (*b*) the corresponding shearing stress in the splice.

SOLUTION

$$A_0 = (0.075)(0.125) = 9.375\times10^{-3}\,\text{m}^2$$

$$\theta = 90° - 60° = 30° \quad \sigma = 500\times10^3\,\text{Pa}$$

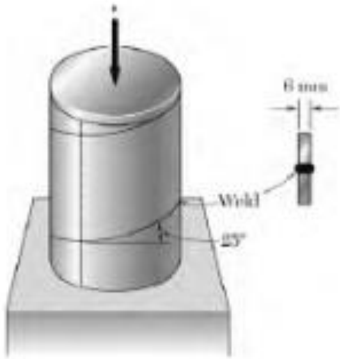
$$\sigma = \frac{P}{A_0}\cos^2\theta$$

$$P = \frac{A_0\sigma}{\cos^2\theta} = \frac{(9.375\times10^{-3})(500\times10^3)}{\cos^2 30°} = 6.25\times10^3\,\text{N}$$

(*a*) $P = 6.25\,\text{kN}$ ◄

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(6.25\times10^3)\sin 60°}{(2)(9.375\times10^{-3})} = 288.68\times10^3$$

(*b*) $\tau = 289\,\text{kPa}$ ◄



PROBLEM 1.33

A steel pipe of 300 mm outer diameter is fabricated from 6 mm thick plate by welding along a helix which forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in directions respectively normal and tangential to the weld are $\sigma = 50$ MPa and $\tau = 30$ MPa, determine the magnitude **P** of the largest axial force that can be applied to the pipe.

SOLUTION

$$d_o = 0.300 \text{ m} \qquad r_o = \frac{1}{2} d_o = 0.150 \text{ m}$$

$$r_i = r_o - t = 0.150 - 0.006 = 0.144 \text{ m}$$

$$A_0 = \pi\left(r_0^2 - r_i^2\right) = \pi(0.150^2 - 0.144^2)$$

$$= 5.54 \times 10^{-3} \text{m}^2$$

$$\theta = 25°$$

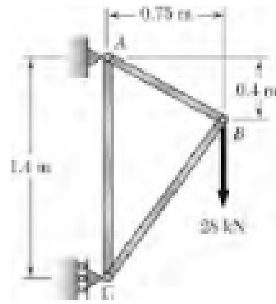
Based on $|\sigma| = 50$ MPa: $\quad \sigma = \dfrac{P}{A_0}\cos^2\theta$

$$P = \frac{A_0\sigma}{\cos^2\theta} = \frac{(5.54 \times 10^{-3})(50 \times 10^6)}{\cos^2 25°} = 337 \times 10^3 \text{ N}$$

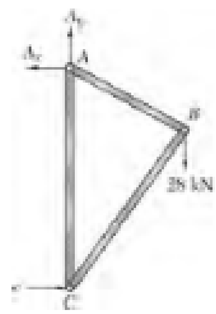
Based on $|\tau| = 30$ MPa: $\quad \tau = \dfrac{P}{2A_0}\sin 2\theta$

$$P = \frac{2A_0\tau}{\sin 2\theta} = \frac{(2)(5.54 \times 10^{-3})(30 \times 10^6)}{\sin 50°} = 434 \times 10^3 \text{ N}$$

Smaller value is the allowable value of P $\qquad \therefore P = 337 \text{ kN}$ ◄


PROBLEM 1.39 Members *AB* and *AC* of the truss shown consist of bars of square cross section made of the same alloy. It is known that a 20 mm square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If bar *AB* has a 15 mm square cross section, determine (*a*) the factor of safety for bar *AB*, (*b*) the dimensions of the cross section of bar *AC* if it is to have the same factor of safety as bar *AB*.

SOLUTION



Length of member *AB*

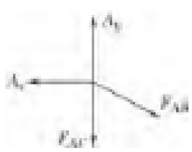
$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

Use entire truss as a free body

$$+\circlearrowleft \Sigma M_C = 0 \quad 1.4\,A_x - (0.75)(28) = 0 \qquad A_x = 15 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0 \qquad A_y - 28 = 0 \qquad A_y = 28 \text{ kN}$$

Use joint *A* as free body



$$\xrightarrow{+} \Sigma F_y = 0 \qquad \frac{0.75}{0.85} F_{AB} - A_x = 0$$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0 \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar $\qquad A = (0.020)^2 = 400 \times 10^{-6} \text{m}^2 \qquad P_U = 120 \times 10^3 \text{ N}$

For the material $\qquad \sigma_U = \dfrac{P_U}{A} = \dfrac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(*a*) For bar *AB* $\qquad \text{F.S.} = \dfrac{F_U}{F_{AB}} = \dfrac{\sigma_U A}{F_{AB}} = \dfrac{(300 \times 10^6)(0.015)^2}{17 \times 10^3} = 3.97$ ◀

(*b*) For bar *AC* $\qquad \text{F.S.} = \dfrac{F_U}{F_{AC}} = \dfrac{\sigma_U A}{F_{AC}} = \dfrac{\sigma_U a^2}{F_{AC}}$

$$a^2 = \frac{(\text{F.S.})F_{AC}}{\sigma_U} = \frac{(3.97)(20 \times 10^3)}{300 \times 10^6} = 264.7 \times 10^{-6} \text{m}^2$$

$$a = 16.27 \times 10^{-3} \text{m} \quad \text{or} \quad 16.27 \text{ mm}$$ ◀

PROBLEM 2.49

The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C.

SOLUTION

Brass core:

$$E = 105 \text{ GPa}$$
$$\alpha = 20.9\times10^{-6}/°\text{C}$$

Aluminum shell:

$$E = 70 \text{ GPa}$$
$$\alpha = 23.6\times10^{-6}/°\text{C}$$

Let L be the length of the assembly.

Free thermal expansion:

$$\Delta T = 195 - 15 = 180°\text{C}$$

Brass core: $(\delta_T)_b = L\alpha_b(\Delta T)$

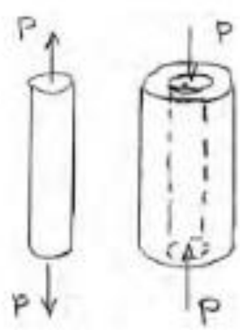
Aluminum shell: $(\delta_T) = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core: $\delta = L(\alpha_a - \alpha_b)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.

Brass core:

$$E_b = 105\times10^9 \text{ Pa}$$
$$A_b = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2$$
$$= 490.87\times10^{-6} \text{m}^2$$
$$(\delta_P)_b = \frac{PL}{E_b A_b}$$



PROBLEM 2.49 *(Continued)*

Aluminum shell:

$$E_a = 70 \times 10^9 \text{ Pa}$$

$$A_a = \frac{\pi}{4}(60^2 - 25^2)$$

$$= 2.3366 \times 10^3 \text{ mm}^2$$

$$= 2.3366 \times 10^{-3} \text{ m}^2$$

$$\delta = (\delta_P)_b + (\delta_P)_a$$

$$L(\alpha_b - \alpha_a)(\Delta T) = \frac{PL}{E_b A_b} + \frac{PL}{E_a A_a} = KPL$$

where

$$K = \frac{1}{E_b A_b} + \frac{1}{E_a A_a}$$

$$= \frac{1}{(105 \times 10^9)(490.87 \times 10^{-6})} + \frac{1}{(70 \times 10^9)(2.3366 \times 10^{-3})}$$

$$= 25.516 \times 10^{-9} \text{ N}^{-1}$$

Then

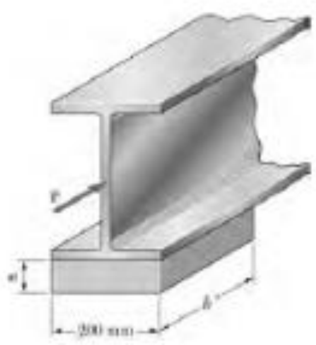
$$P = \frac{(\alpha_b - \alpha_a)(\Delta T)}{K}$$

$$= \frac{(23.6 \times 10^{-6} - 20.9 \times 10^{-6})(180)}{25.516 \times 10^{-9}}$$

$$= 19.047 \times 10^3 \text{ N}$$

Stress in aluminum:

$$\sigma_a = -\frac{P}{A_a} = -\frac{19.047 \times 10^3}{2.3366 \times 10^{-3}} = -8.15 \times 10^6 \text{ Pa} \qquad \sigma_a = -8.15 \text{ MPa} \blacktriangleleft$$



PROBLEM 2.75

An elastomeric bearing ($G = 0.9$ MPa) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22-kN lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 420 kPa, determine (*a*) the smallest allowable dimension b, (*b*) the smallest required thickness a.

SOLUTION

Shearing force: $P = 22\times 10^3\,\text{N}$

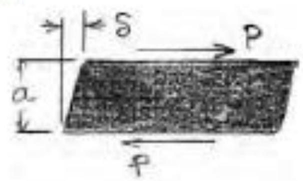
Shearing stress: $\tau = 420\times 10^3\,\text{Pa}$

$$\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau}$$

$$= \frac{22\times 10^3}{420\times 10^3} = 52.381\times 10^{-3}\,\text{m}^2$$

$$= 52.381\times 10^3\,\text{mm}^2$$

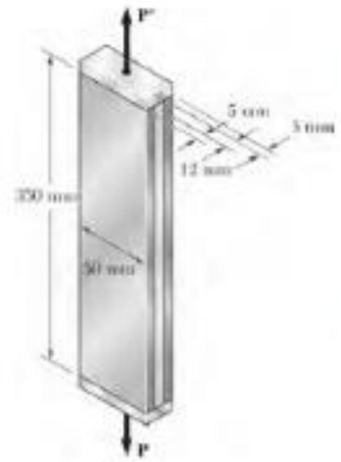
$$A = (200\text{ mm})(b)$$



(*a*) $b = \dfrac{A}{200} = \dfrac{52.381\times 10^3}{200} = 262\text{ mm}$ $\qquad b = 262\text{ mm}$ ◀

$$\gamma = \frac{\tau}{G} = \frac{420\times 10^3}{0.9\times 10^6} = 466.67\times 10^{-3}$$

(*b*) But $\gamma = \dfrac{\delta}{a} \quad \therefore \quad a = \dfrac{\delta}{\gamma} = \dfrac{10\text{ mm}}{466.67\times 10^{-3}} = 21.4\text{ mm}$ $\qquad a = 21.4\text{ mm}$ ◀



PROBLEM 2.111

Two tempered-steel bars, each 5 mm thick, are bonded to a 12 mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1$ mm and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

For the mild steel
$$A_1 = (12)(50) = 600 \text{ mm}^2$$
$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(0.350)(345\times10^6)}{200\times10^9} = 0.06\times10^{-3}\text{ m}$$

For the tempered steel
$$A_2 = 2(5)(50) = 500 \text{ mm}^2$$
$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(0.350)(690\times10^6)}{200\times10^9} = 1.2\times10^{-3}\text{ m}$$

Total area:
$$A = A_1 + A_2 = 1100 \text{ mm}^2$$

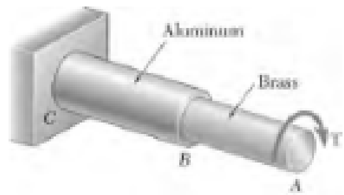
$\delta_{Y1} < \delta_m < \delta_{Y2}$. The mild steel yields. Tempered steel is elastic.

(*a*) Forces
$$P_1 = A_1\sigma_{Y1} = (600\times10^{-6})(345\times10^6) = 207\times10^3\text{ N} = 207\text{ kN}$$
$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^9)(500\times10^{-6})(1.0\times10^{-3})}{0.350} = 285.71\text{ kN}$$
$$P = P_1 + P_2 = 492.71\text{ kN} \quad ◄$$

(*b*) Stresses
$$\sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 345\text{ MPa}$$
$$\sigma_2 = \frac{P_2}{A_2} = \frac{285.71\times10^3}{500} = 571.4\text{ MPa} \quad ◄$$

Unloading
$$\delta' = \frac{PL}{EA} = \frac{(492.7\times10^3)(350)}{(1100)(2\times10^5)} = 0.783\text{ mm}$$

(*c*) Permanent set
$$\delta_p = \delta_m - \delta' = 1.0 - 0.783$$
$$= 0.217\text{ mm} \quad ◄$$



PROBLEM 3.18

The *solid* rod *BC* has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 25 MPa. Rod *AB* is *hollow* and has an outer diameter of 25 mm; it is made of a brass for which the allowable shearing stress is 50 MPa. Determine (*a*) the largest inner diameter of rod *AB* for which the factor of safety is the same for each rod, (*b*) the largest torque that can be applied at *A*.

SOLUTION

Solid rod *BC*:

$$\tau = \frac{Tc}{J} \quad J = \frac{\pi}{2}c^4$$

$$\tau_{all} = 25 \times 10^6 \text{ Pa}$$

$$c = \frac{1}{2}d = 0.015 \text{ m}$$

$$T_{all} = \frac{\pi}{2}c^3\tau_{all} = \frac{\pi}{2}(0.015)^3(25 \times 10^6) = 132.536 \text{ N} \cdot \text{m}$$

Hollow rod *AB*:

$$\tau_{all} = 50 \times 10^6 \text{ Pa}$$

$$T_{all} = 132.536 \text{ N} \cdot \text{m}$$

$$c_2 = \frac{1}{2}d_2 = \frac{1}{2}(0.025) = 0.0125 \text{ m}$$

$$c_1 = ?$$

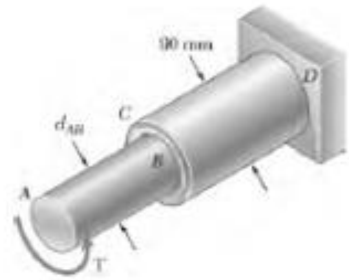
$$T_{all} = \frac{J\tau_{all}}{c_2} = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)\frac{\tau_{all}}{c_2}$$

$$c_1^4 = c_2^4 - \frac{2T_{all}c_2}{\pi\tau_{all}}$$

$$= 0.0125^4 - \frac{(2)(132.536)(0.0125)}{\pi(50 \times 10^6)} = 3.3203 \times 10^{-9} \text{ m}^4$$

(*a*) $c_1 = 7.59 \times 10^{-3} \text{ m} = 7.59 \text{ mm}$ $\qquad d_1 = 2c_1 = 15.18 \text{ mm}$ ◀

(*b*) Allowable torque. $\qquad T_{all} = 132.5 \text{ N} \cdot \text{m}$ ◀



PROBLEM 3.20

The solid rod AB has a diameter $d_{AB} = 60$ mm and is made of a steel for which the allowable shearing stress is 85 Mpa. The pipe CD, which has an outer diameter of 90 mm and a wall thickness of 6 mm, is made of an aluminum for which the allowable shearing stress is 54 MPa. Determine the largest torque **T** that can be applied at A.

SOLUTION

Rod AB:

$$\tau_{all} = 85 \times 10^6 \text{ Pa} \quad c = \frac{1}{2}d = 0.030 \text{ m}$$

$$T_{all} = \frac{J\tau_{all}}{c} = \frac{\pi}{2}c^3\tau_{all}$$

$$= \frac{\pi}{2}(0.030)^3(85 \times 10^6) = 3.605 \times 10^3 \text{ N} \cdot \text{m}$$

Pipe CD:

$$\tau_{all} = 54 \times 10^6 \text{ Pa} \quad c_2 = \frac{1}{2}d_2 = 0.045 \text{ m}$$

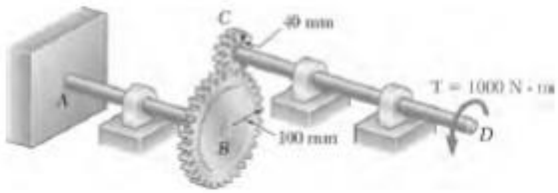
$$c_1 = c_2 - t = 0.045 - 0.006 = 0.039 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6} \text{ m}^4$$

$$T_{all} = \frac{J\tau_{all}}{c_2} = \frac{(2.8073 \times 10^{-6})(54 \times 10^6)}{0.045} = 3.369 \times 10^3 \text{ N} \cdot \text{m}$$

Allowable torque is the smaller value. $T_{all} = 3.369 \times 10^3 \text{ N} \cdot \text{m}$

3.37 kN · m ◀



PROBLEM 3.26

A torque of magnitude $T = 1000\ \text{N}\cdot\text{m}$ is applied at D as shown. Knowing that the allowable shearing stress is 60 MPa in each shaft, determine the required diameter of (*a*) shaft *AB*, (*b*) shaft *CD*.

SOLUTION

$$T_{CD} = 1000\ \text{N}\cdot\text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500\ \text{N}\cdot\text{m}$$

(*a*) Shaft *AB*:

$$\tau_{\text{all}} = 60 \times 10^6\ \text{Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(2500)}{\pi(60 \times 10^6)} = 26.526 \times 10^{-6}\ \text{m}^3$$

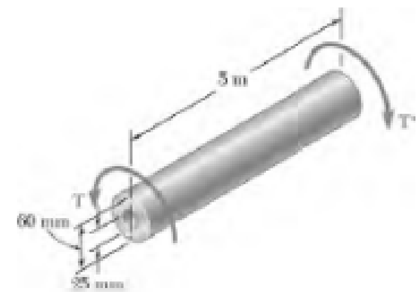
$$c = 29.82 \times 10^{-3} = 29.82\ \text{mm} \qquad d = 2c = 59.6\ \text{mm} \blacktriangleleft$$

(*b*) Shaft *CD*:

$$\tau_{\text{all}} = 60 \times 10^6\ \text{Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(1000)}{\pi(60 \times 10^6)} = 10.610 \times 10^{-6}\ \text{m}^3$$

$$c = 21.97 \times 10^{-3}\ \text{m} = 21.97\ \text{mm} \qquad d = 2c = 43.9\ \text{mm} \blacktriangleleft$$



PROBLEM 3.71

The hollow steel shaft shown ($G = 77.2$ GPa, $\tau_{all} = 50$MPa) rotates at 240 rpm. Determine (*a*) the maximum power that can be transmitted, (*b*) the corresponding angle of twist of the shaft.

SOLUTION

$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm}$$

$$c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4]$$

$$= 1.234 \times 10^6 \text{mm}^4 = 1.234 \times 10^{-6} \text{m}^4$$

$$\tau_m = 50 \times 10^6 \text{ Pa}$$

$$\tau_m = \frac{Tc}{J} \quad T = \frac{\tau_m J}{c} = \frac{(50 \times 10^6)(1.234 \times 10^{-6})}{30 \times 10^{-3}} = 2056.7 \text{ N} \cdot \text{m}$$

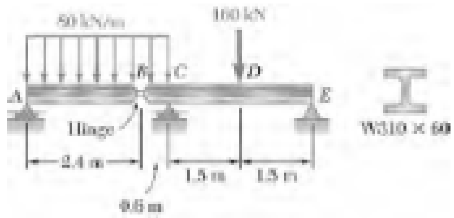
Angular speed. $f = 240 \text{ rpm} = 4 \text{ rev/sec} = 4 \text{ Hz}$

(*a*) Power being transmitted. $P = 2\pi f T = 2\pi(4)(2056.7) = 51.7 \times 10^3 \text{ W}$

$P = 51.7$ kW ◀

(*b*) Angle of twist. $\varphi = \frac{TL}{GJ} = \frac{(2056.7)(5)}{(77.2 \times 10^9)(1.234 \times 10^{-6})} = 0.1078 \text{ rad}$

$\varphi = 6.17°$ ◀

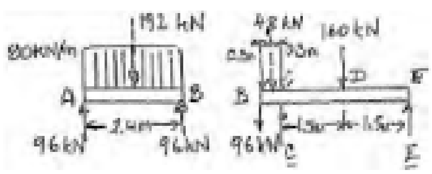


PROBLEM 5.23

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

Statics: Consider portion AB and BE separately.



Portion BE:

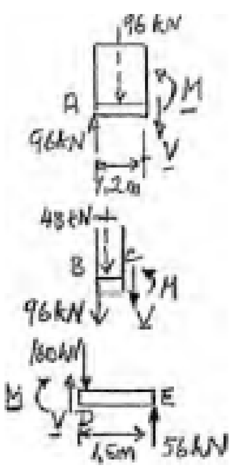
$+\circlearrowleft \Sigma M_E = 0:$

$$(96)(3.6) + (48)(3.3) - C(3) + (160)(1.5) = 0$$

$\boldsymbol{C} = 248\,\text{kN} \uparrow$

$\boldsymbol{E} = 56\,\text{kN} \uparrow$

$$M_A = M_B = M_E = 0$$



At midpoint of AB:

$\Sigma F_y = 0: \quad V = 0$

$\Sigma M = 0: \quad M = (96)(1.2) - (96)(0.6) = 57.6\,\text{kN}\cdot\text{m}$

Just to the left of C:

$\Sigma F_y = 0: \quad V = -96 - 48 = -144\,\text{kN}$

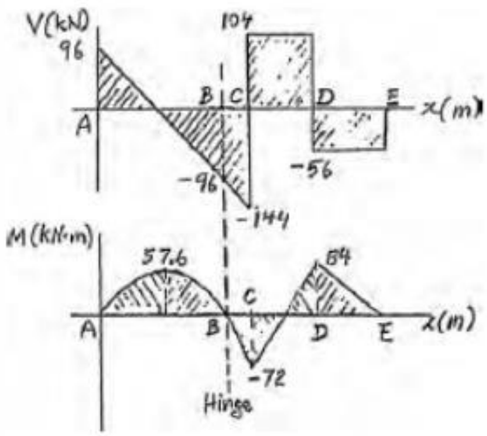
$\Sigma M_C = 0: \quad M = -(96)(0.6) - (48)(0.3) = -72\,\text{kN}$

Just to the left of D:

$\Sigma F_y = 0: \quad V = 160 - 56 = +104\,\text{kN}$

$\Sigma M_D = 0: \quad M = (56)(1.5) = +84\,\text{kN}\cdot\text{m}$

PROBLEM 5.23 *(Continued)*



From the diagram:

$$|M|_{max} = 84 \text{ kN} \cdot \text{m} = 84 \times 10^3 \text{ N} \cdot \text{m}$$

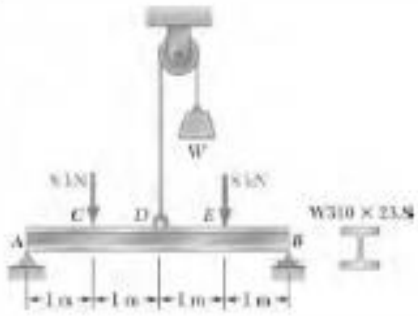
For W310 × 60 rolled steel shape,

$$S_x = 844 \times 10^3 \text{mm}^3$$
$$= 844 \times 10^{-6} \text{m}^3$$

Stress: $\sigma_m = \dfrac{|M|_{max}}{S}$

$$\sigma_m = \frac{84 \times 10^3}{844 \times 10^{-6}} = 99.5 \times 10^6 \text{ Pa}$$

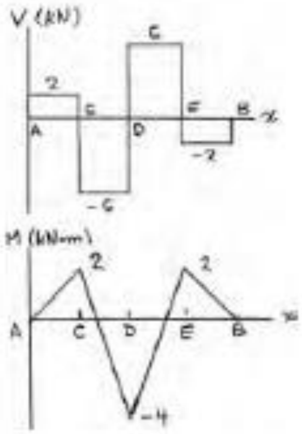
$\sigma_m = 99.5 \text{ MPa}$ ◀



PROBLEM 5.26

Knowing that $W = 12\,\text{kN}$, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.

SOLUTION



By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \; A - 8 + 12 - 8 + B = 0$$

$$A = B = 2\,\text{kN}$$

Shear:

A to C^-: $\quad V = 2\,\text{kN}$ ◄

C^+ to D^-: $\quad V = -6\,\text{kN}$ ◄

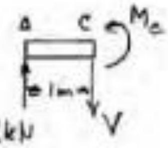
D^+ to E^-: $\quad V = 6\,\text{kN}$ ◄

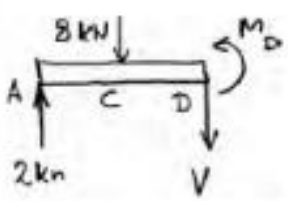
E^+ to B: $\quad V = -2\,\text{kN}$ ◄

Bending moment:

At C, $\quad +\circlearrowleft \Sigma M_C = 0: \; M_C - (1)(2) = 0$

$M_C = 2\,\text{kN}\cdot\text{m}$ ◄





At D, $\quad +\Sigma M_D = 0: \; M_D - (2)(2) + (8)(1) = 0$

$M_D - 4\,\text{kN}\cdot\text{m}$ ◄

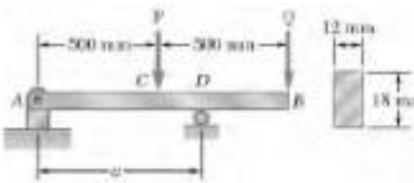
By symmetry, $\quad M = 2\,\text{kN}\cdot\text{m}$ at E. $\quad M_E = 2\,\text{kN}\cdot\text{m}$ ◄

$\max|M| = 4\,\text{kN}\cdot\text{m}$ occurs at E.

For W310 × 23.8, $\quad S_x = 280 \times 10^3\,\text{mm}^3 = 280 \times 10^{-6}\,\text{m}^3$

Normal stress: $\quad \sigma_{\max} = \dfrac{|M|_{\max}}{S_x} = \dfrac{4 \times 10^3}{280 \times 10^{-6}}$

$$= 14.29 \times 10^6\,\text{Pa} \qquad \sigma_{\max} = 14.29\,\text{MPa}$$ ◄

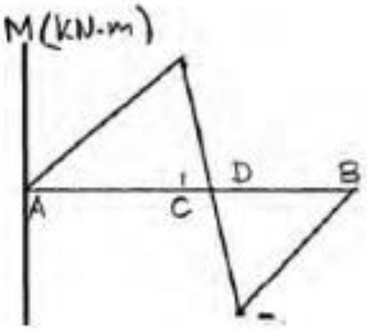


PROBLEM 5.30

Knowing that $P = Q = 480$ N, determine (*a*) the distance a for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$$P = 480 \text{ N} \qquad Q = 480 \text{ N}$$

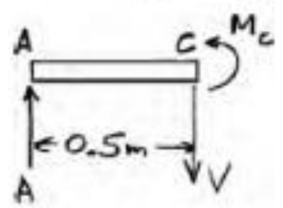


Reaction at A: $\quad +\circlearrowleft \Sigma M_D = 0: \; -Aa + 480(a - 0.5) - 480(1 - a) = 0$

$$A = \left(960 - \frac{720}{a}\right) \text{N}$$

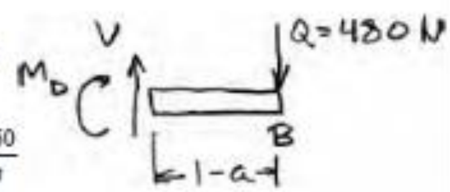
Bending moment at C: $\quad +\circlearrowleft \Sigma M_C = 0: \; -0.5A + M_C = 0$

$$M_C = 0.5A = \left(480 - \frac{360}{a}\right) \text{N} \cdot \text{m}$$



Bending moment at D: $\quad +\circlearrowleft \Sigma M_D = 0: \; -M_D - 480(1 - a) = 0$

$$M_D = -480(1 - a) \text{ N} \cdot \text{m}$$



(*a*) Equate: $\quad -M_D = M_C \qquad 480(1 - a) = 480 - \dfrac{360}{a}$

$$a = 0.86603 \text{ m}$$

$a = 866$ mm ◄

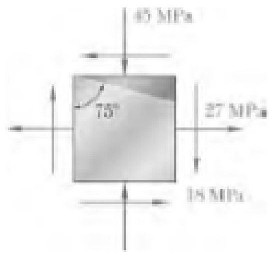
$$A = 128.62 \text{ N} \quad M_C = 64.31 \text{ N} \cdot \text{m} \qquad M_D = -64.31 \text{ N} \cdot \text{m}$$

(*b*) For rectangular section, $S = \dfrac{1}{6}bh^2$

$$S = \frac{1}{6}(12)(18)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{m}^3$$

$$\sigma_{max} = \frac{|M|_{max}}{S} = \frac{64.31}{6.48 \times 10^{-9}} = 99.2 \times 10^6 \text{ Pa}$$

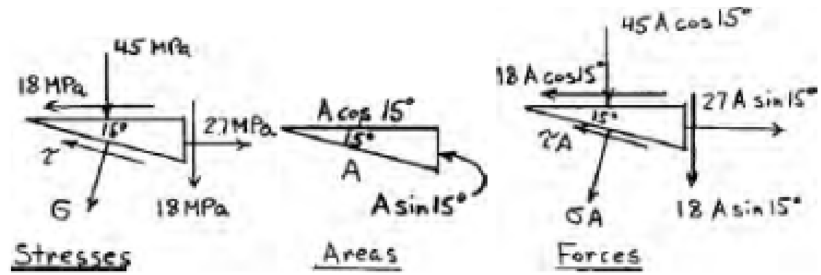
$\sigma_{max} = 99.2$ MPa ◄



PROBLEM 7.1

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION



$+\nearrow \Sigma F = 0$: $\sigma A + 18A\cos 15^\circ \sin 15^\circ + 45A\cos 15^\circ \cos 15^\circ - 27A\sin 15^\circ \sin 15^\circ + 18A\sin 15^\circ \cos 15^\circ = 0$

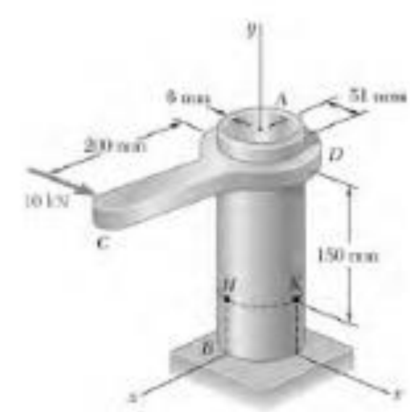
$$\sigma = -18\cos 15^\circ \sin 15^\circ - 45\cos^2 15^\circ + 27\sin^2 15^\circ - 18\sin 15^\circ \cos 15^\circ$$

$\sigma = -49.2$ MPa ◄

$+\nwarrow \Sigma F = 0$: $\tau A + 18A\cos 15^\circ \cos 15^\circ - 45A\cos 15^\circ \sin 15^\circ - 27A\sin 15^\circ \cos 15^\circ - 18A\sin 15^\circ \sin 15^\circ = 0$

$$\tau = -18(\cos^2 15^\circ - \sin^2 15^\circ) + (45+27)\cos 15^\circ \sin 15^\circ$$

$\tau = 2.41$ MPa ◄



PROBLEM 7.25

The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51\text{ mm} \qquad r_i = r_o - t = 45\text{ mm}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855\times 10^6\text{ mm}^4$$

$$= 4.1855\times 10^{-6}\text{ m}^4$$

$$I = \frac{1}{2}J = 2.0927\times 10^{-6}\text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K:

$$F_x = 10\text{ kN}$$

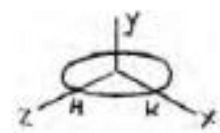
$$= 10\times 10^3\text{ N}$$

$$M_y = (10\times 10^3)(200\times 10^{-3})$$

$$= 2000\text{ N}\cdot\text{m}$$

$$M_z = -(10\times 10^3)(150\times 10^{-3})$$

$$= -1500\text{ N}\cdot\text{m}$$



Torsion: At point K, place local x-axis in negative global z-direction.

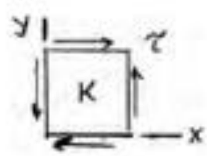
$$T = M_y = 2000\text{ N}\cdot\text{m}$$

$$c = r_o = 51\times 10^{-3}\text{ m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51\times 10^{-3})}{4.1855\times 10^{-6}}$$

$$= 24.37\times 10^6\text{ Pa}$$

$$= 24.37\text{ MPa}$$



Transverse shear: Stress due to transverse shear $V = F_x$ is zero at point K.

Bending:

$$|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51\times10^{-3})}{2.0927\times10^{-6}} = 36.56\times10^{6}\,\text{Pa} = 36.56\text{ MPa}$$

Point K lies on compression side of neutral axis:

$$\sigma_y = -36.56\text{ MPa}$$

Total stresses at point K:

$$\sigma_x = 0, \quad \sigma_y = -36.56\text{ MPa}, \quad \tau_{xy} = 24.37\text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28\text{ MPa}$$

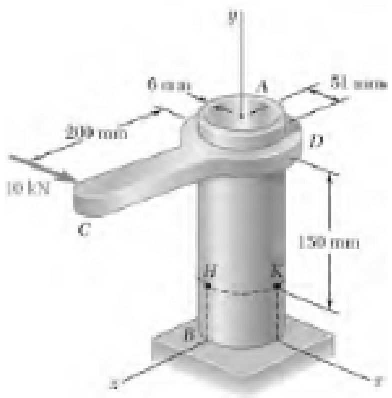
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46\text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46 \qquad \sigma_{max} = 12.18\text{ MPa} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46 \qquad \sigma_{min} = -48.7\text{ MPa} \blacktriangleleft$$

$$\tau_{max} = R \qquad \tau_{max} = 30.5\text{ MPa} \blacktriangleleft$$

PROBLEM 7.47



Solve Prob. 7.25, using Mohr's circle.

PROBLEM 7.25 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51\text{ mm} \qquad r_i = r_o - t = 45\text{ mm}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855 \times 10^6\text{ mm}^4 = 4.1855 \times 10^{-6}\text{ m}^4$$

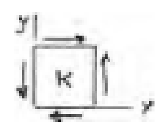
$$I = \frac{1}{2}J = 2.0927 \times 10^{-6}\text{m}^4$$

Force-couple system at center of tube in the plane containing points *H* and *K*:

$$F_x = 10 \times 10^3\text{ N}$$
$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000\text{ N}\cdot\text{m}$$
$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500\text{ N}\cdot\text{m}$$

Torsion:

$$T = M_y = 2000\text{ N}\cdot\text{m}$$
$$c = r_o = 51 \times 10^{-3}\text{ m}$$
$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37\text{ MPa}$$



Note that the local x-axis is taken along a negative global z-direction.

Transverse shear: Stress due to $V = F_x$ is zero at point *K*.

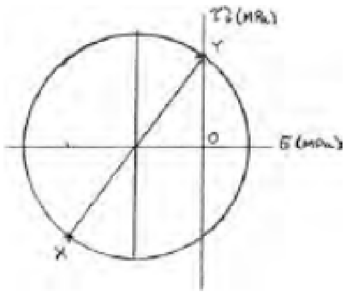
Bending: $$\left|\sigma_y\right| = \frac{\left|M_z\right|c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56\text{ MPa}$$

Point *K* lies on compression side of neutral axis. $\sigma_y = -36.56\text{ MPa}$

PROBLEM 7.47 *(Continued)*

Total stresses at point K: $\sigma_x = 0, \quad \sigma_y = -36.56 \text{ MPa}, \quad \tau_{xy} = 24.37 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46$$

$\sigma_{max} = 12.18 \text{ MPa}$ ◀

$$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46$$

$\sigma_{min} = -48.74 \text{ MPa}$ ◀

$\tau_{max} = R$ $\qquad$ $\tau_{max} = 30.46 \text{ MPa}$ ◀