Indian Institute of Technology Patna
MA201- (Partial Differential Equation) July-November 2019

<u>**Tutorial - 2**</u> **Instructor: Dr. Pratibhamoy Das**

Note that Nonlinear=Semilinear+ Quasilinear + Fully Nonlinear.
Classification for first order PDEs:

1. **Linear PDE :** A PDE is said to be linear if it is of the form

$$P(x,y)u_x + Q(x,y)u_y = R(x,y)u + S(x,y)$$

2. **Semi-linear PDE :** It is said to be semi-linear if it is of the form

$$P(x,y)u_x + Q(x,y)u_y = R(x,y,u)$$

3. **Quasi-linear PDE :** It is said to be quasi-linear if it is of the form

$$P(x,y,u)u_x + Q(x,y,u)u_y = R(x,y,u)$$

4. **Fully Non-linear PDE :** A PDE is said to be non-linear if it does not fall under any one of the above three categories.

Questions:

1. Classify the following PDEs (Linear/Semilinear/Quasilinear/Fully Nonlinear):

$$(i) \quad yu_x - xu_y = xyu + x, \quad (ii) \quad (1+u^2)u_{xx} - 2u_x u_y u_{xy} + (1+u_x^2)u_{yy} = 0,$$

$$(iii) \quad xu_{xx} + uu_x + u^2 u_y = u^4, \quad (iv) \quad uu_x + u_y^2 = 1.$$

2. Find a partial differential equation (of least order) by eliminating the arbitrary function f from the following expressions:

$$(i) \quad u = e^{ay} f(x+by) \ , \qquad (ii) \quad f(u - xy, x^2 + y^2) = 0.$$

3. Find a partial differential equation (of least order) which describes all planes which are at a constant distance k from the origin.

4. Find a partial differential equation which arises from the following surfaces:
   $(i) \quad \log u = a \log x + \sqrt{1-a^2}\log y + b\ , \quad (ii) \quad f(x^2+y^2, x^2-u^2) = 0.$

5. Find the solution of the following Cauchy problems:

$$(i) \quad u_x + u_y = 2, \quad u(x,0) = x^2\ , \quad (ii) \quad 5u_x + 2u_y = 0, \quad u(x,0) = \sin x.$$

6. Show that the Cauchy problem $u_x + u_y = 1, \quad u(x,x) = x$ has infinitely many solutions.

7. Find a function $u(x,y)$ that solves the Cauchy problem

$$x^2 u_x + y^2 u_y = u^2, \quad u(x, 2x) = x^2, \quad x \in \mathbb{R}.$$

   Is the solution defined for all x and y?

8. Find the surface which is orthogonal to the one parameter family $u = cxy(x^2+y^2)$ and passes through the hyperbola $x^2 - y^2 = a^2, u = 0$.

9. Find the general solution of the following PDEs ($p = u_x,\ q = u_y$):

   (i) $(y+u)p+(x+u)q = x+y$, (ii) $u(xp-yq) = y^2-x^2$, (iii) $y^2p-xyq = x(u-2y)$,

   (iv) $x(y^2+u)p - y(x^2+u)q = (x^2-y^2)u$, (v) $x^2p + yq + u^2 = 0$,

   (vi) $(p-q)u = u^2 + (x+y)$, (vii) $x^2(y-u)p + y^2(u-x)q = u^2(x-y)$,

   (viii) $\frac{y^2u}{x}p + xuq = y^2$.