INDIAN INSTITUTE OF TECHNOLOGY PATNA

END SEMESTER EXAMINATION DATED: 26-11-2010 M.M. 50

COURSE NO.: PH 101 COURSE TITLE:PHYSICS I DURATION: 3 HOUR

Instructions:

1. Attempt all questions
2. Answer one question in one place.
3. No marks would be awarded for the question for which Instruction (2) is not followed.

1. Suppose you are in an elevator and you dropped an apple. The apple accelerated with acceleration a and strikes the floor of the elevator.
   (a) What possible explanations you can provide for such an observed motion of apple?
   (b) In what way the correct explanation could be distinguished from the set of explanations answered in part (a)? **[2+2]**

2. Show that the force acting upon a particle of mass m as observed from rotating coordinate system $\vec{F}_{rot}$ is related to the force observed from non-inertial system $\vec{F}$, through the relation: $\vec{F}_{rot} = \vec{F} - 2m\vec{\Omega}\times\vec{v_{rot}} - m\vec{\Omega}\times(\vec{\Omega}\times\vec{r})$, where $\vec{\Omega}$ is the angular velocity of rotating coordinate system and $\vec{v}_{rot}$ is the velocity of the particle as observed from rotation coordinate system. [Hint: Use the identity: $\left(\frac{d\vec{B}}{dt}\right)_{int} = \left(\frac{d\vec{B}}{dt}\right)_{rot} + \vec{\Omega}\times\vec{B}$, for any vector $\vec{B}$] **[6]**

3. The speed of a rocket (speeding away) with respect to a space station is 2.4×10^8 m/s, and observers O' and O in the rocket and the space station, respectively, synchronize their clocks in the usual fashion (i.e., $t' = t = 0$ when $x' = x = 0$). Suppose that O looks at O's clock through a telescope. What time does he see on O's clock when his own clock reads 30 s? **[4]**

4. Suppose that a particle moves relative to the primed system with the velocity u' in the x'-y' plane so that its trajectory makes an angle θ' with the x' axis. Obtain a relation for u^2 in terms of u', θ', v and c. **[6]**

5. Show that, with $u'^2 = u_x'^2 + u_y'^2$ and $u^2 = u_x^2 + u_y^2$, we can write $c^2 - u^2 = \frac{c^2\left(c^2 - u'^2\right)\left(c^2 - v^2\right)}{\left(c^2 + u_x' v\right)^2}$ **[4]**

6. Derive the relativistic acceleration transformation:
$$a_x' = \frac{a_x\left(1-\frac{v^2}{c^2}\right)^{3/2}}{\left(1-\frac{u_x v}{c^2}\right)^3}$$
   in which $a_x = du_x / dt$ and $a_x' = du_x' / dt'$. **[4]**

7. Normalize the function: $\psi(x) = e^{-|x|}\sin\alpha x$ **[3]**

8. A particle is represented (at time $t = 0$) by the wave function $\psi(x,0) = \begin{cases} \sqrt{\frac{15}{16a^5}}(a^2 - x^2), & \text{if } -a \le x \le +a \\ 0, & \text{otherwise} \end{cases}$
   (a) Evaluate the expectation value of x, p (at time $t = 0$). Can you get it from $\langle p\rangle = m\left(d\langle x\rangle / dt\right)$. If not, why?
   (b) Evaluate the expectation value of x^2 and p^2. **[3+6]**

9. In general, quantum mechanics is relevant when the de Broglie wavelength of the particle in question is greater than the characteristic size of the system d. In thermal equilibrium at temperature T, the average kinetic energy of the particle is $(3/2)kT$. The purpose of this problem is to anticipate which systems will have to be treated quantum mechanically, and which can be safely described classically. [$k = 1.38 \times 10^{-23}$ J/K]

(a) **Solids:** The lattice spacing in a typical solid is around $d = 0.3$ nm.

(*i*) Find the temperature below which the free electrons in a solid are quantum mechanical.

(*ii*) Below what temperature the nuclei of Sodium (solid form) is quantum mechanical? Given Atomic mass of Sodium is 23. **[2+2]**

(c) **Gases:** (*i*) For what temperatures are the atoms in an ideal gas at pressure P behave quantum mechanically? [Hint: Use the ideal gas law, $PV = NkT$, to deduce the inter-atomic spacing]

(*ii*) Obtain the value of temperature below which Helium behaves quantum mechanically at atmospheric pressure. Given 1 atm = 1×10^5 N/m^2.

(iii) Is hydrogen in outer space (where the inter-atomic spacing is about 1 cm and temperature is 3 K) quantum mechanical? **[3+2+1]**