Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2015-16 (End Semester Examination)

Maximum Marks: 50 Total Time: 3 Hours

Note: This question paper contains **Ten** questions. Answer all questions.

1. From a box containing 3 white and 2 black balls 2 balls are simultaneously drawn at random. Find the probability that they are of the same colour. **[2]**

2. The cumulative distribution function of a random variable X is given by $F_X(x) = 1-(1+x)e^{-x},\ x \geq 0$; $F_X(x) = 0$, *otherwise*. Find mean and variance of X. Name the distribution. **[1+1+1]**

3. Box 1 contains 1000 bulbs of which 10% are defective and Box 2 contains 2000 bulbs of which 5% are defective. Two bulbs are taken out simultaneously from a randomly (uniformly) selected box. Find the probability that both the bulbs are defective. Further assuming that both are defective determine the probability that they came from box 1. **[1+2]**

4. Verify that $p_X(x) = 2^{-x},\ x = 1, 2, 3, \ldots$ is a proper probability mass function. What is the probability that X is divisible by 3? Find the expected values of X and X^2. **[1+1+1+2]**

5. The two-dimensional random variable (X, Y) has the joint density function $f(x, y) = 8xy,\ 0 < x < y < 1$; $f(x, y) = 0$, *otherwise*. Determine the probability $P(X < \frac{1}{2}, Y < \frac{1}{4})$. Find both marginal density functions and also associated (both) conditional density functions. Are X and Y independent? Give reasons for your answer. **[2+3+3+1]**

6. (i) Given a two-dimensional continuous random variable $(X, Y); -\infty < X, Y < \infty$, consider a transformation $Z = X - Y$ and then find the density function of Z. **[2]**
   (ii) Now let X and Y be iid random variables distributed as one-parameter exponential distribution with mean α. Then determine the density function of Z where $Z = X - Y$. **[3]**

7. The joint density function of (X, Y) is $f(x, y) = 24xy,\ x > 0,\ y > 0, x+y \leq 1$; $f(x, y) = 0$, *otherwise*. Find the correlation coefficient between X and Y. **[9]**

8. (i) Properly define a Poisson process (write all assumptions also). **[2]**
   (ii) If customers arrive at a counter in accordance with a Poisson process with a arrival rate of 2 per minute, find the probability that a customer will arrive between 1 minute and 3 minute. **[1]**
   (iii) State central limit theorem. **[1]**

9. In an examination marks X of the students follows a normal $N(\mu, \sigma^2)$ distribution. A student is considered to have failed, secured second class, first class and distinction according to as he/she scores less than 45%, between 45% and 60%, between 60% and 75% and above 75% respectively. In a particular year 10% of the students failed in the examination and 5% of the students get distinction. Use this to find μ and σ. Find the percentage of students who have get the first class. **[1+1+2]**

10. Let X denotes the height in centimeters and Y denotes the weight in kilogram of a group of students. Assume that (X, Y) has the two-dimensional normal $BVN(185, 84, 100, 64, 0.6)$ distribution. Determine the probability $P(86.4 < Y < 95.36 \mid X = 190)$. Further derive the joint moment generating function of (X, Y). **[3+3]**