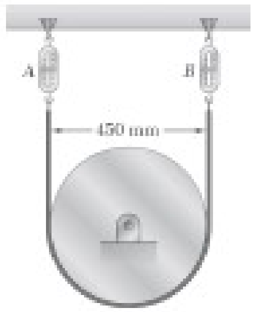
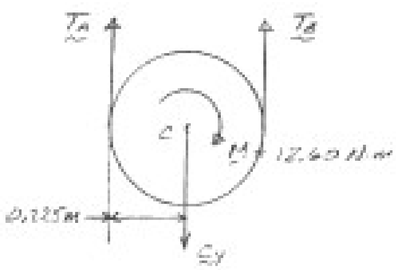
PROBLEM 8.110



The setup shown is used to measure the output of a small turbine. When the flywheel is at rest, the reading of each spring scale is 70 N. If a 12.60 N·m couple must be applied to the flywheel to keep it rotating clockwise at a constant speed, determine (*a*) the reading of each scale at that time, (*b*) the coefficient of kinetic friction. Assume that the length of the belt does not change.

SOLUTION

FBD Flywheel:



$$\circlearrowleft \Sigma M_C = 0: \quad (0.225 \text{ m})(T_B - T_A) - 12.60 \text{ N·m} = 0$$

$$T_B - T_A = 56 \text{ N}, \quad T_B = T_A + 56 \text{ N}$$

Also, since the belt doesn't change length, the additional stretch in spring B equals the decrease in stretch of spring A. Thus the increase in T_B equals the decrease in T_A.

$$\text{Thus } T_B + T_A = (70 \text{ N} + \Delta T) + (70 \text{ N} - \Delta T) = 140 \text{ N}$$

$$(T_A + 56 \text{ N}) + T_A = 140 \text{ N}, \quad T_A = 42 \text{ N}$$

$$T_B = 42 \text{ N} + 56 \text{ N} = 98 \text{ N}$$

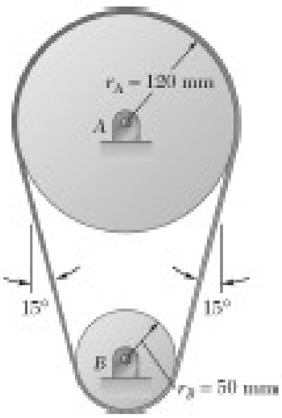
(*a*) $T_A = 42.0 \text{ N}$ ◀

$T_B = 98.0 \text{ N}$ ◀

For slip $T_B = T_A e^{\mu_k \beta}$, or $\mu_k = \dfrac{1}{\beta} \ln \dfrac{T_B}{T_A}$

$$\mu_k = \frac{1}{\pi} \ln \frac{98}{42} = 0.2697$$

(*b*) $\mu_k = 0.27$ ◀

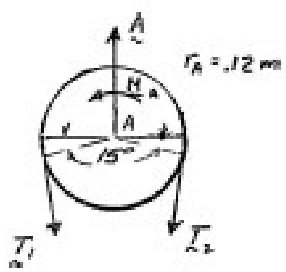


PROBLEM 8.112

A flat belt is used to transmit a couple from drum B to drum A. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum A.

SOLUTION

FBD's drums:



$$\beta_A = 180° + 30° = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

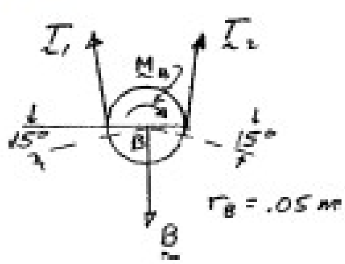
$$\beta_B = 180° - 30° = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Since $\beta_B < \beta_A$, slipping will impend first on B (friction coefficients being equal)

So

$$T_2 = T_{max} = T_1 e^{\mu_s \beta_B}$$

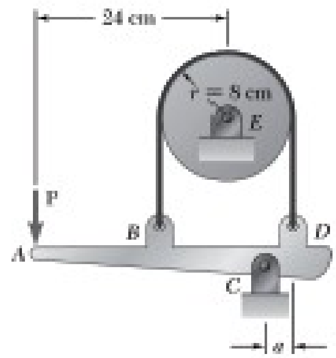
$$450 \text{ N} = T_1 e^{(0.4)5\pi/6} \quad \text{or} \quad T_1 = 157.914 \text{ N}$$



$$\circlearrowleft \Sigma M_A = 0: \quad M_A + (0.12 \text{ m})(T_1 - T_2) = 0$$

$$M_A = (0.12 \text{ m})(450 \text{ N} - 157.914 \text{ N}) = 35.05 \text{ N} \cdot \text{m}$$

$$M_A = 35.1 \text{ N} \cdot \text{m} ◀$$



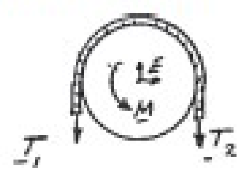
PROBLEM 8.115

The speed of the brake drum shown is controlled by a belt attached to the control bar *AD*. A force **P** of magnitude 25 N is applied to the control bar at *A*. Determine the magnitude of the couple being applied to the drum, knowing that the coefficient of kinetic friction between the belt and the drum is 0.25, that $a = 4$ cm, and that the drum is rotating at a constant speed (*a*) counterclockwise, (*b*) clockwise.

SOLUTION

(*a*) Counterclockwise rotation

Free body: Drum



$r = 8$ cm $\quad \beta = 180° = \pi$ radians

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933 T_1$$

Free body: Control bar

$$+\circlearrowleft \Sigma M_C = 0: \quad T_1(12 \text{ cm}) - T_2(4 \text{ cm}) - (25 \text{ N})(28 \text{ cm}) = 0$$

$$T_1(12) - 2.1933 T_1(4) - 700 = 0$$

$$T_1 = 216.93 \text{ N}$$

$$T_2 = 2.1933(216.93 \text{ N}) = 475.79 \text{ N}$$

Return to free body of drum

$$+\circlearrowleft \Sigma M_E = 0: \quad M + T_1(8 \text{ cm}) - T_2(8 \text{ cm}) = 0$$

$$M + (216.93 \text{ N})(8 \text{ cm}) - (475.79 \text{ N})(8 \text{ cm}) = 0$$

$$M = 2070.9 \text{ N} \cdot \text{cm}$$

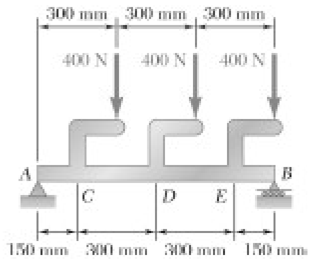
$M = 20.7$ N · m ◀

(*b*) Clockwise rotation

$r = 8$ cm $\quad \beta = \pi$ rad

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933 T_1$$

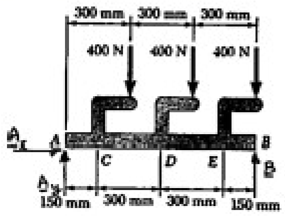


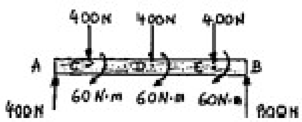
PROBLEM 7.50

Draw the shear and bending-moment diagrams for the beam AB, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

<u>Free body: Entire beam</u>





$$+\circlearrowleft \Sigma M_A = 0: \quad B(0.9\text{ m}) - (400\text{ N})(0.3\text{ m}) - (400\text{ N})(0.6\text{ m}) - (400\text{ N})(0.9\text{ m}) = 0$$

$$B = +800\text{ N} \qquad \mathbf{B} = 800\text{ N}\uparrow \triangleleft$$

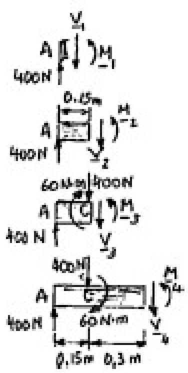
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 800\text{ N} - 3(400\text{ N}) = 0$$

$$A_y = +400\text{ N} \qquad \mathbf{A} = 400\text{ N}\uparrow \triangleleft$$

We replace the loads by equivalent force-couple systems at C, D, and E.

<u>We consider successively the following F-B diagrams.</u>



$V_1 = +400\text{ N}$
$M_1 = 0$

$V_2 = +400\text{ N}$
$M_2 = +60\text{ N}\cdot\text{m}$

$V_3 = 0$
$M_3 = +120\text{ N}\cdot\text{m}$

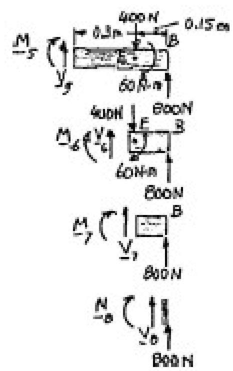
$V_4 = 0$
$M_4 = +120\text{ N}\cdot\text{m}$

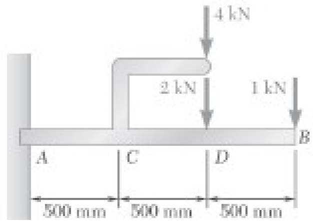
$V_5 = -400\text{ N}$
$M_5 = +180\text{ N}\cdot\text{m}$

$V_6 = -400\text{ N}$
$M_6 = +60\text{ N}\cdot\text{m}$

$V_7 = -800\text{ N}$
$M_7 = +120\text{ N}\cdot\text{m}$

$V_8 = -800\text{ N}$
$M_8 = 0$



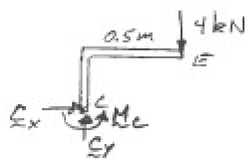


PROBLEM 7.51

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD CE:



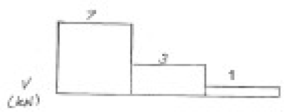
$\xrightarrow{} \Sigma F_x = 0$: $\quad \mathbf{C}_y = 0$

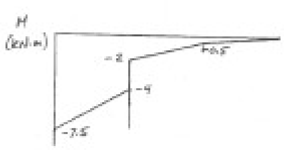
$\uparrow \Sigma F_y = 0$: $\quad C_y - 4 \text{ kN} = 0 \qquad \mathbf{C}_y = 4 \text{ kN} \uparrow$

$\circlearrowleft \Sigma M_C = 0$: $\quad M_C - (0.5 \text{ m})(4 \text{ kN}) = 0$

$$M_C = 2 \text{ kN·m}$$

Beam AB:





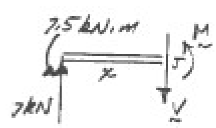
$\xrightarrow{} \Sigma F_x = 0$: $\quad A_x = 0$

$\uparrow \Sigma F_y = 0$: $\quad A_y - 4 \text{ kN} - 2 \text{ kN} - 1 \text{ kN} = 0 \qquad A_y = 7 \text{ kN} \uparrow$

$\circlearrowleft \Sigma M_A = 0$: $\quad M_A - 2 \text{ kN·m} - (0.5 \text{ m})(4 \text{ kN}) - (1 \text{ m})(2 \text{ kN})$

$$-(1.5 \text{ m})(1 \text{ kN}) = 0, \qquad \mathbf{M}_A = 7.5 \text{ kN·m} \circlearrowleft$$

Along AC:



$\uparrow \Sigma F_y = 0$: $\quad 7 \text{ kN} - V = 0$

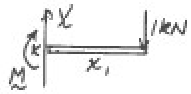
$V = 7 \text{ kN}$

$\circlearrowleft \Sigma M_J = 0$: $\quad M + 7.5 \text{ kN·m} - x(7 \text{ kN}) = 0$

$$M = (7 \text{ kN})x - 7.5 \text{ kN·m}$$

PROBLEM 7.51 CONTINUED

Along DB:



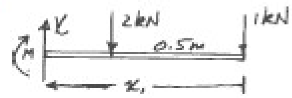
$+\uparrow \Sigma F_y = 0$: $V - 1\text{ kN} = 0$

$V = 1\text{ kN}$

$\Sigma M_K = 0$: $-M - x_1(1\text{ kN}) = 0$

$M = -(1\text{ kN})x_1$

Along CD:



$+\uparrow \Sigma F_y = 0$: $V - 2\text{ kN} - 1\text{ kN} = 0$ $V = 3\text{ kN}$

$\Sigma M_M = 0$: $M + (x_1 - 0.5\text{ m})(2\text{ kN}) + x_1(1\text{ kN}) = 0$

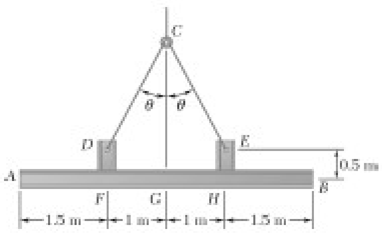
$M = 1\text{ kN}\cdot\text{m} - (3\text{ kN})x_1$

Note: M exhibits a discontinuity at C, equal to $2\text{ kN}\cdot\text{m}$, the value of M_C.

From the diagrams, $|V|_{max} = 7.00\text{ kN}$ along AC ◀

$|M|_{max} = 7.50\text{ kN}\cdot\text{m}$ at A ◀

PROBLEM 7.55



For the structural member of Problem 7.53, determine (*a*) the angle θ for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

PROBLEM 7.53 Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

See solution of Problem 7.50 for reduction of loading or beam *AB* to the following:

where $\qquad M_0 = (750 \text{ N}\cdot\text{m})\tan\theta \ \triangleleft$

[Equation (2)]

The largest negative bending moment occurs Just to the left of *F*:

$$+\circlearrowleft \Sigma M_1 = 0: \quad M_1 + (900 \text{ N})\left(\frac{1.5 \text{ m}}{2}\right) = 0 \qquad M_1 = -675 \text{ N}\cdot\text{m} \ \triangleleft$$

The largest positive bending moment occurs

At *G*:

$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - M_0 + (1500 \text{ N})(1.25 \text{ m} - 1 \text{ m}) = 0$$

$$M_2 = M_0 - 375 \text{ N}\cdot\text{m} \ \triangleleft$$

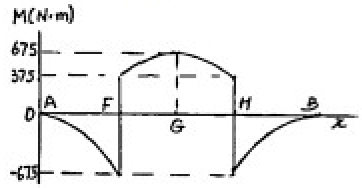
Equating M_2 and $-M_1$:

$$M_0 - 375 = +675$$

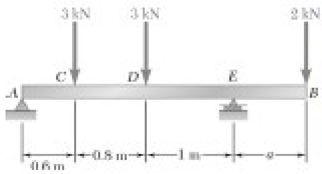
$$M_0 = 1050 \text{ N}\cdot\text{m}$$

PROBLEM 7.55 (Continued)

(*a*) From Equation (2): $\tan\theta = \dfrac{1050}{750} = 1.400$ $\theta = 54.5°$ ◀



(*b*) $|M|_{max} = 675 \text{ N} \cdot \text{m}$ ◀

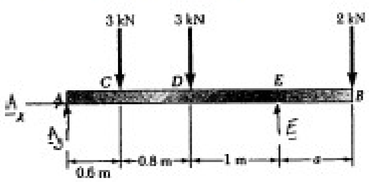


PROBLEM 7.58

For the beam and loading shown, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

SOLUTION

Free body: Entire beam

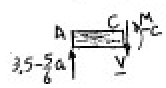


$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_E = 0: \quad -A_y(2.4) + (3)(1.8) + 3(1) - (2)a = 0$$

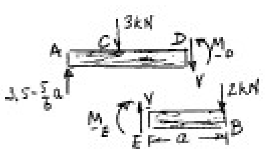
$$A_y = 3.5\text{ kN} - \frac{5}{6}a \qquad \mathbf{A} = 3.5\text{ kN} - \frac{5}{6}a \uparrow \triangleleft$$

Free body: *AC*



$$+\circlearrowleft \Sigma M_C = 0: \quad M_C - \left(3.5 - \frac{5}{6}a\right)(0.6\text{ m}) = 0, \qquad M_C = +2.1 - \frac{a}{2} \triangleleft$$

Free body: *AD*



$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - \left(3.5 - \frac{5}{6}a\right)(1.4\text{ m}) + (3\text{ kN})(0.8\text{ m}) = 0$$

$$M_D = +2.5 - \frac{7}{6}a \triangleleft$$

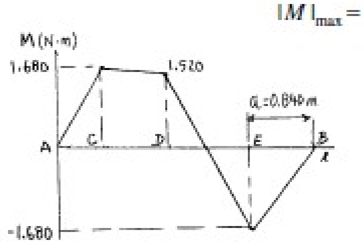
Free body: *EB*

$$+\circlearrowleft \Sigma M_E = 0: \quad -M_E - (2\text{ kN})a = 0 \qquad M_E = -2a \triangleleft$$

We shall <u>assume</u> that $M_C > M_D$ and, thus, that $M_{max} = M_C$.

We set $M_{max} = |M_{min}|$ or $M_C = |M_E| = 2.1 - \dfrac{a}{2} = 2a$ $\qquad a = 0.840\text{ m} \triangleleft$

$$|M|_{max} = M_C = |M_E| = 2a = 2(0.840) \qquad |M|_{max} = 1.680\text{ N}\cdot\text{m} \triangleleft$$



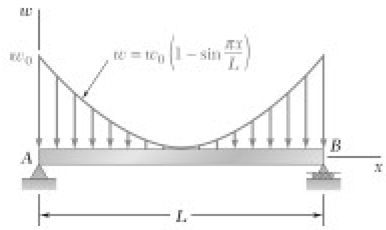
We must check our assumption.

$$M_D = 2.5 - \frac{7}{6}(0.840) = 1.520\text{ N}\cdot\text{m}$$

Thus, $M_C > M_D$, O.K.

The answers are (*a*) $a = 0.840\text{ m}$ ◀

(*b*) $|M|_{max} = 1.680\text{ N}\cdot\text{m}$ ◀



PROBLEM 7.88

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

SOLUTION

(*a*) <u>Reactions at supports</u>: $A = B = \frac{1}{2}W$, where $\frac{W}{L}$ = Total load

$$W = \int_0^L w\,dx = w_0 \int_0^L \left(1 - \sin\frac{\pi x}{L}\right)dx$$

$$= w_0\left[x + \frac{L}{x}\cos\frac{\pi x}{L}\right]_0^L$$

$$= w_0 L\left(1 - \frac{2}{\pi}\right)$$

Thus $$V_A = A = \frac{1}{2}W = \frac{1}{2}w_0 L\left(1 - \frac{2}{\pi}\right)$$

$$M_A = 0 \qquad (1)$$

<u>Load</u>: $$w(x) = w_0\left(1 - \sin\frac{\pi x}{L}\right)$$

<u>Shear</u>: From Eq. (7.2):

$$V(x) - V_A = -\int_0^x w(x)\,dx$$

$$= -w_0 \int_0^x \left(1 - \sin\frac{\pi x}{L}\right)dx$$

Integrating and recalling first of Eqs. (1),

$$V(x) - \frac{1}{2}w_0 L\left(1 - \frac{2}{\pi}\right) = -w_0\left[x + \frac{L}{\pi}\cos\frac{\pi x}{L}\right]_0^x$$

$$V(x) = \frac{1}{2}w_0 L\left(1 - \frac{2}{\pi}\right) - w_0\left(2 + \frac{L}{\pi}\cos\frac{\pi x}{L}\right) + w_0\frac{L}{\pi}$$

$$V(x) = w_0\left(\frac{L}{2} - x - \frac{L}{\pi}\cos\frac{\pi x}{L}\right) \qquad (2) ◀$$

PROBLEM 7.88 (Continued)

Bending moment: From Eq. (7.4) and recalling that $M_A = 0$.

$$M(x) - M_A = \int_0^x V(x)dx$$

$$= w_0\left[\frac{L}{2}x - \frac{1}{2}x^2 - \left(\frac{L}{\pi}\right)^2 \sin\frac{\pi x}{L}\right]_0^x$$

$$M(x) = \frac{1}{2}w_0\left(Lx - x^2 - \frac{2L^2}{\pi^2}\sin\frac{\pi x}{L}\right) \qquad (3) ◀$$

(*b*) Maximum bending moment

$$\frac{dM}{dx} = V = 0.$$

This occurs at $x = \frac{L}{2}$ as we may check from (2):

$$V\left(\frac{L}{2}\right) = w_0\left(\frac{L}{2} - \frac{L}{2} - \frac{L}{\pi}\cos\frac{\pi}{2}\right) = 0$$

From (3):

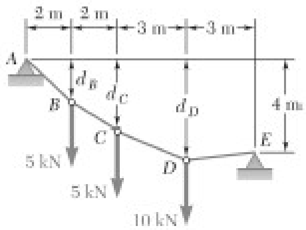
$$M\left(\frac{L}{2}\right) = \frac{1}{2}w_0\left(\frac{L^2}{2} - \frac{L^2}{4} - \frac{2L^2}{\pi^2}\sin\frac{\pi}{2}\right)$$

$$= \frac{1}{8}w_0L^2\left(1 - \frac{8}{\pi^2}\right)$$

$$= 0.0237w_0L^2$$

$$M_{\max} = 0.0237w_0L^2, \quad \text{at} \quad x = \frac{L}{2} \qquad ◀$$

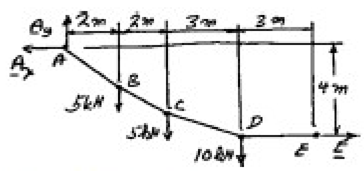
PROBLEM 7.98



Determine (*a*) distance d_C for which portion *DE* of the cable is horizontal, (*b*) the corresponding reactions at *A* and *E*.

SOLUTION

Free body: Entire cable



(*b*) $+\uparrow \Sigma F_y = 0: \quad A_y - 5\text{ kN} - 5\text{ kN} - 10\text{ kN} = 0$ $\qquad \mathbf{A}_y = 20\text{ kN}\uparrow$

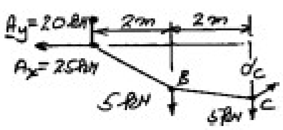
$+\circlearrowleft \Sigma M_A = 0: \quad E(4\text{ m}) - (5\text{ kN})(2\text{ m}) - (5\text{ kN})(4\text{ m}) - (10\text{ kN})(7\text{ m}) = 0$

$E = +25\text{ kN}$ $\qquad \mathbf{E} = 25.0\text{ kN} \longrightarrow$ ◀

$\Sigma F_x = 0: \quad -A_x + 25\text{ kN} = 0$ $\qquad \mathbf{A}_x = 25\text{ kN} \longleftarrow$

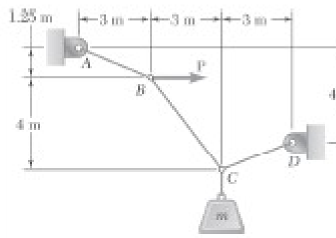
$\mathbf{A} = 32.0\text{ kN} \searrow 38.7^\circ$ ◀

(*a*) Free body: Portion *ABC*



$+\circlearrowleft \Sigma M_C = 0: \quad (25\text{ kN})d_C - (20\text{ kN})(4\text{ m}) + (5\text{ kN})(2\text{ m}) = 0$

$25d_C - 70 = 0$ $\qquad d_C = 2.80\text{ m}$ ◀

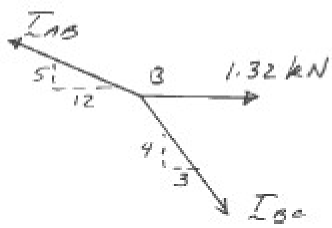


PROBLEM 7.103

A force **P** applied at *B* and a block attached at *C* maintain cable *ABCD* in the position shown. Knowing that the force **P** has a magnitude of 1.32 kN, determine (*a*) the reaction at *A*, (*b*) the required mass *m* of the block, (*c*) the tension in each portion of the cable.

SOLUTION

FBD B:



$\rightarrow \Sigma F_x = 0: \quad -\frac{12}{13}T_{AB} + \frac{3}{5}T_{BC} + 1.32 \text{ kN} = 0$

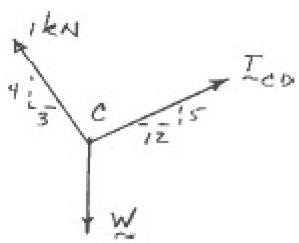
$\uparrow \Sigma F_y = 0: \quad \frac{5}{13}T_{AB} - \frac{4}{5}T_{BC} = 0$

Solving: $T_{AB} = 2.08$ kN, $T_{BC} = 1$ kN

By inspection of *A*,

(*a*) $\mathbf{A} = 2.08 \text{ kN} \searrow 22.6°$ ◀

FBD C:



$\rightarrow \Sigma F_x = 0: \quad \frac{12}{13}T_{CD} - \frac{3}{5}(1 \text{ kN}) = 0, \qquad T_{CD} = 0.65 \text{ kN}$

$\uparrow \Sigma F_y = 0: \quad \frac{4}{5}(1 \text{ kN}) + \frac{5}{13}(0.65 \text{ kN}) - W = 0$

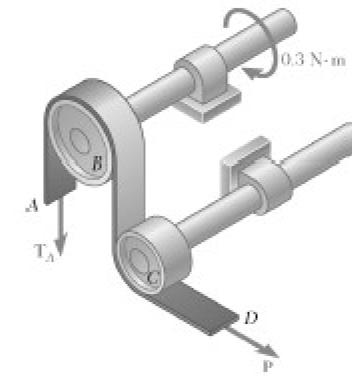
$W = 1.05$ kN

(*b*) $m = \frac{W}{g} = \frac{1050 \text{ N}}{9.81 \text{ m/s}^2} = 107.03 \text{ kg}$ $\qquad m = 107.0 \text{ kg}$ ◀

(*c*) From above $\qquad T_{AB} = 2.08$ kN ◀

$T_{BC} = 1.000$ kN ◀

$T_{CD} = 650$ N ◀



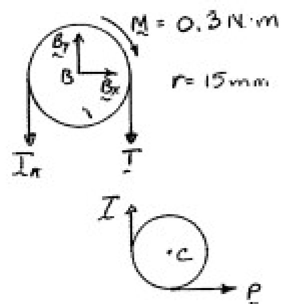
PROBLEM 8.125

Solve Problem 8.124 assuming that the idler drum C is frozen and cannot rotate.

PROBLEM 8.124 A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

$$T_A - T = \frac{M}{r} = 300\ \text{N}\cdot\text{mm} = 15.0000\ \text{N}$$

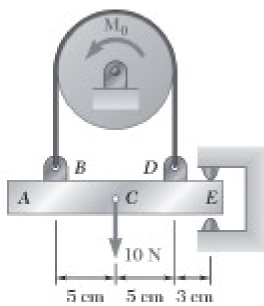
Impending slipping: $\quad T_A = Te^{\mu_s \beta} = Te^{0.4\pi}$

So $\quad (e^{0.4\pi} - 1)T = 15.000\ \text{N}$

or $\quad T = 5.9676\ \text{N}$

If C is fixed, the tape must slip $\circlearrowleft$

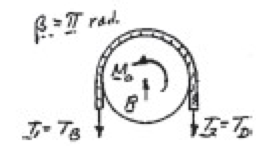
So $\quad P = Te^{\mu_k \beta_C} = (5.9676\ \text{N})e^{0.3\pi/2} = 9.5600\ \text{N} \qquad P = 9.56\ \text{N}$ ◀



PROBLEM 8.128

The 10-N bar AE is suspended by a cable that passes over a 5-cm-radius drum. Vertical motion of end E of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (*a*) the largest counterclockwise couple $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force exerted on end E of the bar.

SOLUTION



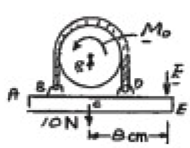
Drum: Slipping impends

$$\mu_s = 0.30$$

$$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{T_D}{T_B} = e^{0.30\pi} = 2.5663$$

$$T_D = 2.5663T_B$$

(*a*) Free-body: Drum and bar



From (b), $E = 3.7834$ N

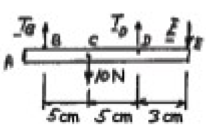
$$+\circlearrowleft \Sigma M_C = 0: \quad M_0 - E(8\text{ cm}) = 0$$

$$M_0 = (3.7834\text{ N})(8\text{ cm})$$

$$= 30.27\text{ N}\cdot\text{cm}$$

$\mathbf{M}_0 = 0.3\text{ N}\cdot\text{m}$ ◀

(*b*) Bar AE:



$$+\uparrow \Sigma F_y = 0: \quad T_B + T_D - E - 10\text{ N} = 0$$

$$T_B + 2.5663T_B - E - 10\text{ N} = 0$$

$$3.5663T_B - E - 10\text{ N} = 0$$

$$E = 3.5663T_B - 10\text{ N} \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad E(3\text{ cm}) - (10\text{ N})(5\text{ cm}) + T_B(10\text{ cm}) = 0$$

$$(3.5663T_B - 10\text{ N})(3\text{ cm}) - 50\text{ N}\cdot\text{cm} + T_B(10\text{ cm}) = 0$$

$$20.699T_B = 80 \quad T_B = 3.8649\text{ N}$$

Eq. (1):

$$E = 3.5663(3.8649\text{ N}) - 10\text{ N}$$

$$E = +3.7834\text{ N}$$

$\mathbf{E} = 3.8\text{ N} \downarrow$ ◀