

CS225 Switching Theory

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Previous Class

Minimization/ Simplification of Switching Functions

K-map (SOP)

Quine-McCluskey (Tabular) Minimization

This Class

Quine-McCluskey (Tabular) Minimization
Combinational Circuit logic design

Quine-McCluskey Tabulation Procedure

Step 1: to Obtain the Set of All Prime implicants

Example: with don't care condition

$$\text{Ex.: } f_3(v,w,x,y,z) = \sum(13,15,17,18,19,20,21,23,25, 27,29,31) + \sum_{\phi}(1,2,12,24)$$

(a)

(b)

Index Step 1

1	00001	✓
2	00010	✓
12	01100	✓
17	10001	✓
18	10010	✓
20	10100	✓
24	11000	✓
3	01101	✓
19	10011	✓
21	10101	✓
25	11001	✓
15	01111	✓
23	10111	✓
27	11011	✓
29	11101	✓
31	11111	✓

Step 2

(1,17)	0001	H
(2,18)	0010	G
(12,13)	0110	F
(17,19)	100-	✓
(17,21)	10-01	✓
(17,25)	1-001	✓
(18,19)	1001	E
(20,21)	1010	D
(24,25)	1100	C
(13,15)	011-	✓
(13,27)	-1101	✓
(19,23)	10-11	✓
(19,27)	1-101	✓
(21,23)	-101	✓
(21,29)	101-	✓
(21,27)	1-101	✓
(25,27)	110-	✓
(25,29)	11-01	✓
(15,31)	-1111	✓
(23,31)	1-111	✓
(27,31)	11-11	✓
(29,31)	111-	✓

(c)

Step 3

(17,19,21,23)	100-1	✓
(17,19,25,27)	1-0-1	✓
(17,21,25,29)	1--01	✓
(13,15,29,31)	--11-1	B
(19,23,27,31)	1--11	✓
(21,23,29,31)	1-1-1	✓
(25,27,29,31)	11--1	✓

(d)

Step 4

(17,19,21,23,25,27,29,31)	A
(17,19,25,27,21,23,29,31)	1--1
(17,21,25,27,19,23,29,31)	1--1
duplicate	1--1

$$P = \{vz, wxz, vwx'y', vw'xy', vw'x'y, v'wxy', w'x'yz', w'x'y'z'\}$$

Step 2: Find the minimal expression(s)

Don't-cares: not listed as column headings in the prime implicant chart

Example: $f_3(v,w,x,y,z) = \sum(13,15,17,18,19,20,21,23,25,27,29,31) + \sum_{\phi}(1,2,12,24)$

	✓13	✓15	✓17	18	✓19	✓20	✓21	✓23	✓25	✓27	✓29	✓31
✓A = vz		x			x		x	⊗	x	⊗	x	x
✓B = wxz	x	⊗									x	x
C = vwx'y'									x			
✓D = vw'xy'						⊗	x					
E = vw'x'y				x	x							
F = v'wxy'	x											
G = w'x'yz'				x								
H = w'x'y'z			x									

Selection of nonessential prime implicants facilitated by listing prime implicants in decreasing order of the number of minterms they cover

Essential prime implicants: A, B, and D. They cover all minterms except 18, which can be covered by E or G, giving rise to two minimal expressions

Determining the Set of All Irredundant Expressions

Deriving the minimal sum-of-products through prime implicant chart inspection: *difficult for more complex cases* Example: $f_4(v,w,x,y,z) = \sum(0,1,3,4,7,13,15,19,20,22,23,29,31)$

	0	1	3	4	7	13	15	19	20	22	23	29	31
$\checkmark A = wxz$						⊗	x					⊗	x
$B = xyz$					x		x				x		x
$\checkmark C = w'yz$			x		x			⊗			x		
$D = vw'xy$										x	x		
$E = vw'xz'$									x	x			
$F = w'xy'z'$				x					x				
$G = v'w'x'z$		x	x										
$H = v'w'y'z'$	x			x									
$I = v'w'x'y'$	x	x											

(a) Prime implicant chart.

	0	1	4	20	22
D					x
E				x	x
F			x	x	
G		x			
H	x		x		
I	x	x			

(b) Reduced prime implicant chart.

While every irredundant expression must contain A and C, none of them may contain B since it covers minterms already covered by A and C. The reduced chart, obtained after removing A, B, and C, has two x's in each column

Example (Contd.)

Use propositional calculus: define prime implicant function p to be 1 if each column is covered by at least one of the chosen prime implicants, and 0 if not

$$\begin{aligned} p &= (H + I)(G + I)(F + H)(E + F)(D + E) \\ &= EHI + EFI + DFI + EGH + DFGH \end{aligned}$$

At least three rows are needed to cover the reduced chart:

$E, H,$ and I , or $E, F,$ and I , and so on

Since all prime implicants in the reduced chart have the same literal count, there are four minimal sum-of-products:

$$\begin{aligned} f_4(v,w,x,y,z) &= A + C + E + H + I = wxz + w'yz + vw'xz' + v'w'y'z' + v'w'x'y' \\ f_4(v,w,x,y,z) &= A + C + E + F + I = wxz + w'yz + vw'xz' + w'xy'z' + v'w'x'y' \\ f_4(v,w,x,y,z) &= A + C + D + F + I = wxz + w'yz + vw'xy + w'xy'z' + v'w'x'y' \\ f_4(v,w,x,y,z) &= A + C + E + G + H = wxz + w'yz + vw'xz' + v'w'x'z + v'w'y'z' \end{aligned}$$

Reduction of the Chart

Aim: find just *one minimal expression* rather than all such expressions

Example: $f_5(v,w,x,y,z) = \Sigma(1,3,4,5,6,7,10,11,12,13,14,15,18,19,20,21,22,23,25,26,27)$

	1	3	4	5	6	7	10	11	12	13	14	15	18	19	20	21	22	23	25	26	27
$\checkmark A = w'x$		x	x	x	x										⊗	⊗	x	x			
$\checkmark B = v'x$		x	x	x	x				⊗	⊗	x	x									
$C = vx'y$													x	x						x	x
$D = vw'y$													x	x			x	x			
$E = wx'y$							x	x												x	x
$F = v'wy$							x	x			x	x									
$G = x'yz$	x							x						x							x
$H = w'yz$	x					x								x				x			
$I = v'yz$	x					x		x				x									
$\checkmark J = v'w'z$	⊗	x		x		x															
$\checkmark K = vwx'z$																			⊗		x

(a) Prime implicant chart.

Example (Contd.)

	10	11	18	19	26
C			x	x	x
D			x	x	
E	x	x			x
F	x	x			
G		x		x	
H				x	
I		x			

Reduced prime implicant chart

	✓10	✓11	✓18	✓19	✓26
✓C			⊗	x	x
✓E	⊗	x			x
G		x		x	

Final chart

Dominated row: row U of the chart dominates row V if U covers every column covered by V. If U does not have more literals than V then V can be deleted from the chart.

*Example: **I** is dominated by G, **D** is dominated by C and **F** is dominated by E, **H** is dominated by G, so they can be deleted*

From the final chart: $f_5(v,w,x,y,z) = A + B + J + K + C + E$

Dominating Column (Alternative choice)

	10	11	18	19	26
C			x	x	x
D			x	x	
E	x	x			x
F	x	x			
G		x		x	
H				x	
I		x			

Reduced prime implicant chart

*Dominating column: column i of the chart dominates column j if i has an x in every row in which j has an x . Hence, **dominating column i** can be deleted.*

*Example: column **11** dominates column 10. In order to cover column 10, either E or F must be selected, whereby column 11 will also automatically be covered. Similarly, since column **19** covers column 18, column 19 can be deleted.*

Final solution is still: $f_5(v,w,x,y,z) = A + B + J + K + C + E$

Quine-McCluskey Minimization (Reduction and dominance)

Aim: To find just one minimal expression

- ✓ Terms are initially listed one per line in groups
 - Each group contains terms with the same number of true and complemented variables
 - Terms are listed in numerical order within group
- ✓ Terms and implicants are identified using one of three common notations
 - full variable form
 - cellular form
 - 1,0,- form

Implication Table (1,0,-)

✓ Quine-McCluskey Method

- Tabular method to systematically find all prime implicants
- $f(A,B,C,D) = \sum m(1,2,5,6,7,9,10) + \sum d(0,13,15)$
- Part 1: Find all prime implicants
- Step 1: Fill Column 1 with active-set and DC-set minterm indices. Group by number of true variables (# of 1's).

NOTE THAT DCs ARE INCLUDED IN THIS STEP!

Implication Table		
Column I		
0000		
0001		
0010		
0101		
0110		
1001		
1010		
0111		
1101		
1111		

Minimization - First Pass (1,0,-)

✓ Quine-McCluskey Method

- Tabular method to systematically find all prime implicants
- $f(A,B,C,D) = \sum m(1,2,5,6,7,9,10) + \sum d(0,13,15)$
- Part 1: Find all prime implicants
- Step 2: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. One bit difference implies adjacent. Eliminate variable and place in next column.

E.g., 0000 vs. 0100 yields 0-00

0000 vs. 1000 yields -000

When used in a combination, mark with a check. If cannot be combined, mark with a star. These are the prime implicants.

Repeat until nothing left.

Implication Table		
Column I	Column II	
0000 ✓	000-	
	00-0	
0001 ✓		
0010 ✓	0-01	
	-001	
0101 ✓	0-10	
0110 ✓	-010	
1001 ✓		
1010 ✓	01-1	
	-101	
0111 ✓	011-	
1101 ✓	1-01	
1111 ✓	-111	
	11-1	

Minimization - Second Pass (1,0,-)

✓ Quine-McCluskey Method

- Step 2 cont.: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. One bit difference implies adjacent. Eliminate variable and place in next column.

E.g., 0000 vs. 0100 yields 0-00

00-0 vs. 10-0 yields -0-0

When used in a combination, mark with a check. If cannot be combined, mark with a star.

THESE ARE THE PRIME IMPLICANTS.

Repeat until nothing left.

- ✓ The set of ★ constitutes the **Complete Sum** \sum_c

Implication Table		
Column I	Column II	Column III
0000 ✓	000- *	--01 *
	00-0 *	- 1-1 *
0001 ✓		
0010 ✓	0-01 ✓	
	-001 ✓	
0101 ✓	0-10 *	
0110 ✓	-010 *	
1001 ✓		
1010 ✓	01-1 ✓	
	-101 ✓	
0111 ✓	011- *	
1101 ✓	1-01 ✓	
1111 ✓	-111 ✓	
	11-1 ✓	

Prime Implicants

		C D			
		00	01	11	10
A B	00	X	1	0	1
	01	0	1	1	1
	11	0	X	X	0
	10	0	1	0	1

Diagram illustrating a 4-variable Karnaugh map for variables A, B, C, and D. The map shows the following values:

- Row 00 (A=0, B=0): (0,0)=X, (0,1)=1, (0,3)=0, (0,2)=1
- Row 01 (A=0, B=1): (0,0)=0, (0,1)=1, (0,3)=1, (0,2)=1
- Row 11 (A=1, B=1): (0,0)=0, (0,1)=X, (0,3)=X, (0,2)=0
- Row 10 (A=1, B=0): (0,0)=0, (0,1)=1, (0,3)=0, (0,2)=1

Groupings (Prime Implicants) are indicated by colored boxes:

- Blue boxes: (0,1), (0,3), (1,1), (1,3) forming $\bar{A} B \bar{C}$; (0,1), (1,1) forming $\bar{A} C \bar{D}$; (0,1), (1,1) forming $\bar{A} B C$; (0,1), (1,1) forming $\bar{C} D$.
- Red boxes: (0,0), (0,2), (1,0), (1,2) forming $\bar{A} \bar{B} \bar{D}$; (0,2), (1,2) forming $\bar{B} C \bar{D}$; (0,2), (1,2) forming $B D$.

Prime Implicants:

$$000 - = \bar{A} \bar{B} \bar{C}$$

$$00 - 0 = \bar{A} \bar{B} \bar{D}$$

$$0 - 10 = \bar{A} C \bar{D}$$

$$- 010 = \bar{B} C \bar{D}$$

$$011 - = \bar{A} B C$$

$$- 1 - 1 = B D$$

$$-- 01 = \bar{C} D$$

Stage 2: find smallest set of prime implicants that cover the active-set
 Note that essential prime implicants must be in final expression

Coverage Table

Coverage Chart

		1	2	5	6	7	9	10
0,1	000-	X						
0,2	00-0		X					
2,6	0-10		X		X			
2,10	-010		X					X
6,7	011-				X	X		
1,5,9,13	--01	X		X			X	
5,7,13,15	-1-1			X		X		

**NOTE: DON'T INCLUDE DCs IN
COVERAGE TABLE; THEY DON'T
HAVE COVERED BY THE FINAL
LOGIC EXPRESSION!**

rows = prime implicants

columns = ON-set elements (minterms)

place an "X" if ON-set element is covered by the prime implicant

Coverage Table (cont.)

Coverage Chart

		1	2	5	6	7	9	10
0,1	000-	X						
0,2	00-0		X					
2,6	0-10		X		X			
2,10	-010		X					X
6,7	011-				X	X		
1,5,9,13	--01	X		X			X	
5,7,13,15	-1-1			X		X		

rows = prime implicants
columns = ON-set elements
place an "X" if ON-set element is
covered by the prime implicant

		1	2	5	6	7	9	10
0,1	000-	X						
0,2	00-0		X					
2,6	0-10		X		X			
2,10	-010		X					X
6,7	011-				X	X		
1,5,9,13	--01	X		X			X	
5,7,13,15	-1-1			X		X		

If column has a single X, then the
implicant associated with the row
is essential. It must appear in
minimum cover

Coverage Table (cont.)

		1	2	5	6	7	9	10
0,1	000-	X						
0,2	00-0		X					
2,6	0-10		X		X			
2,10	-010		X					X
6,7	011-				X	X		
1,5,9,13	--01	X		X			X	
5,7,13,15	-1-1			X		X		

Eliminate all columns covered by essential primes

		1	2	5	6	7	9	10
0,1	000-	X						
0,2	00-0		X					
2,6	0-10		X		X			
2,10	-010		X					X
6,7	011-				X	X		
1,5,9,13	--01	X		X			X	
5,7,13,15	-1-1			X		X		

Find minimum set of rows that cover the remaining columns

$$F = \bar{B}\bar{C}\bar{D} + \bar{A}BC + \bar{C}D$$

Branching Method

When chart has no essential prime implicant, dominated row or dominating column: use branching method

Example: $f_6(w,x,y,z) = (0,1,5,7,8,10,14,15)$

yz \ wx	00	01	11	10
00	1			1
01	1	1		
11		1	1	
10			1	1

(a) Cyclic map.

	0	1	5	7	8	10	14	15
A = $w'x'y'$	x	x						
B = $w'y/z$		x	x					
C = $w'xz$			x	x				
D = xyz				x				x
E = wxy							x	x
F = wyz'						x	x	
G = $wx'z'$					x	x		
H = $x'y/z'$	x				x			

(b) Cyclic prime implicant chart.

	5	7	8	10	14	15
B	x					
C	x	x				
D		x				x
E					x	x
F			x	x		
G			x	x		
H			x			

(c) Reduced chart after selection of row A.

	1	5	7	10	14	15
A	x					
B	x	x				
C		x	x			
D			x			x
E					x	x
F				x	x	
G				x		

(d) Reduced chart after selection of row H.

To cover column 0: either A or H has to be selected

If A is arbitrarily chosen: C(G) dominates B(H): $f_6(w,x,y,z) = A+C+G+E$

If H is arbitrarily chosen: B(F) dominates A(G): $f_6(w,x,y,z) = H+B+D+F$

Thanks