Indian Institute of Technology Patna
MA-225: B.Tech. II year
Spring Semester: 2010-11
Mid Semester Examination

Maximum Marks: 30 Total Time: 2 Hours

Note: This question paper has **TWO** pages and contains **Ten** questions. Please check all pages and report the discrepancy, if any. Answer all questions.

1. Consider two urns and suppose that urn I contains 5 black and 6 white balls and urn II contains 8 black and 4 white balls. Two balls are transferred from urn II to urn I and then a ball is drawn from urn I.
   (i) Find the probability that drawn ball is white [1]
   (ii) It is known that drawn ball is white, what is the probability that at least one white ball was transferred to urn I. [2]

2. Suppose that it is suspected that a patient has one of the three diseases of type I, II and III. Probabilities that a certain target population suffering from these diseases are 0.4, 0.3 and 0.3 respectively. The patient is submitted to a test which turns out to positive in 25% of the cases of type I, 40% of type II and 60% of type III. It is given that out of three independent test taken by patient two were positive, what is the probability that patient is suffering from disease of type I, II and III respectively. [1+1+1]

3. (i) Suppose that a random variable X follows a $Weibull(\alpha, \beta)$ distribution. Using definition, find the mean and variance of X respectively. [1+1]
   (ii) A customer taking delivery of a large batch of manufactured items from a company accepts the batch if either a random sample of 6 items from the batch contains not more than one defective items, or a random sample of 6 contains two defective items. If 20% of the items in the batch are actually defective, what is the probability that the customer will reject the delivered batch? [2]

4. Let X be an random variable with density function $f(x) = 2/3,\ 0 \leq x \leq 3/2$, and $= 0$ otherwise. Find the cumulative distribution function of random variable Y where $Y = \min(X, 3/4)$ and also evaluate expected value of e^{3Y}. [1+1]

5. To test a vaccine there is a need to find patients with specific blood type which is found in 20% of the population. Consider three different random experiments, say model X the number of people sampled until we have found one with this blood type, model Y the number sampled to find four with the blood type and model Z the number with this blood type among 20 people. Find the mean and standard deviations of X, Y and Z by properly depicting the probability distribution in each case. [1+1+1]

6. Let the probability density function of a random variable X be given by $\frac{3}{\pi\{(x-4)^2+9\}}$, $-\infty < x < \infty$. Find the first, second and third quartiles of X. [3]

7. Suppose number of accidents occurring on a highway each day is a random variable having a $P(4)$ distribution.
   (i) Find the probability that four or more accidents occur today when it is known that at least two accident had already occurred today. [2]
   (ii) According to Chebyshev's theorem, there is a probability of at least 3/4 that the number of accidents on a given day will fall within what interval? [1]

8. Of 40 manufactured items 12 are defective and remaining are non defective. A sample of 6 is taken for inspection without replacement. Find the expected number of non defective items and standard deviation of the corresponding random variable using definition. [1+2]

9. (i) Suppose that $X \sim N(4, 5)$ distribution. Find the cumulative distribution function of the random variable $(1/6) - (3/4)X$ and hence properly write the probability density function of this random variable. [1+1]
   (iii) Let $X \sim G(\alpha, \beta)$ distribution. Find the value of x that maximizes this probability density function. [1]

10. Suppose that each of the 5 man at a party throws his hat into the center of the room. The hats being identical in all respect are first mixed up and then each men randomly selects a hat. Find the probability that none of the men select his own hat. [3]