Indian Institute of Technology Patna

MA102: Assignment-1, Sem-II, 2018-19

Q1. **(A)** Classify the following Differential equation: Linear/ Non-linear/ Ordinary/ Partial etc. and specify the order.
(i) $y'' + 3y' + 20y = e^x$, **(ii)** $\sqrt{1 + y'^3} = x^2$, **(iii)** $y'' + y^2 = \cos x$,
(iv) $y' + xy = \cos y'$, **(v)** $(xy')' = xy$, **(vi)** $u_x + u_y = 0$, **(vii)** $u_{xx} + u_{yy} = u_t$.

Q1. **(B)** Find the differential equation corresponding to following family of curves:
(i) $xy^2 - 1 = cy$, **(ii)** $y = ax^2 + be^{2x}$, **(iii)** $y = a\sin x + b\cos x + b$.

Q2. Graphically show the gradient filed (Slope field) for following differential equations and hence show some representative solutions.
(i) $x' = 4x$, **(ii)** $x' = -2x$, **(iii)** $x' = \frac{x}{t}$, **(iv)** $x' = t - x$,
(v) $x' = t^2 + x^2$, **(vi)** $x' = \frac{t}{x}$, **(vii)** $x' = tx$, **(viii)** $x' = x^2 - t^2$.

Q3. Let V be a linear space of all twice differentiable functions with usual operations. Show that solutions of the differential equation $y'' + \alpha y' + \beta y = 0$ form a linear space.

Q4. Consider the differential equations $y' = \alpha y, x > 0$, where α is a constant. Show that
(i) if $\phi(x)$ is any solution and $\psi(x) = \phi(x)e^{-\alpha x}$, then $\psi(x)$ is a constant;
(ii) if $\alpha < 0$, then every solution tends to zero as $x \to \infty$.

Q5. Solve the following Differential equations:
(i) $x^2y' = 3(x^2 + y^2)\arctan\frac{y}{x} + xy$, **(ii)** $y' = \sin^2(x - y + 1)$.

Q6 A. Reduce the differential equation $y' = f\Big(\dfrac{ax + by + c}{dx + ey + f}\Big)$, $ae - bd \neq 0$ to a separable variable form. What if $ae = bd$?
B. Hence find general solution of the following differential equations:
(i) $(x + 2y + 1) - (2x + y - 1)y' = 0$ **(ii)** $y' = (8x - 2y + 1)^2/(4x - y - 1)^2$

Q7. By making a substitution $v = y/x^n$ or $y = vx^n$ and choosing suitable n, show that following differential equations can be transformed into separable variables, and hence solve them:
(i) $y' = \dfrac{1 - xy^2}{2x^2y}$, **(ii)** $y' = \dfrac{2 + 3xy^2}{4x^2y}$

Q8. Show that the following equations are exact and hence find their general solution:
(i) $(\cos x\cos y - \cot x)dx - (\sin x\sin y)dy = 0$,
(ii) $2x(y + 3x - ye^{-x^2})dx + (x^2 + 3y^2 + e^{-x^2})dy = 0$.

Q9. Show that $2\sin(y^2)dx + xy\cos(y^2)dy = 0$ admits an integrating factor which is a function of x only. Hence solve the differential equation.

Q10. Show that the equation $(3y^2 - x)dx + 2y(y^2 - 3x)dy = 0$ admits an integrating factor which is a function of $(x + y^2)$. Hence solve the differential equation.

Q11. A. Consider homogeneous equation $M(x,y)dx + N(x,y)dy = 0$. If $Mx + Ny \neq 0$ then $\frac{1}{Mx+Ny}$ is an integrating factor of the differential equation. Using this solve the following equation:

$$(4y^2 + 3xy)dx - (3xy + 2x^2)dy = 0. \qquad --- (*)$$

B. Further multiply $(*)$ with $x^\alpha y^\beta$ and find α and β such that the corresponding equation becomes exact. Hence find the solution. Compare!

Q12. If an equation has an integrating factor, then it has infinitely many integrating factors.

Practice Problems: Related problems from Ross, Kreyszig & Simmons.

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MA102: Assignment-2, Sem-II, 2018-19,

Q1. Reduce the following differential equations into linear form and solve:
(i) $y^2y' + y^3/x = \sin x$, **(ii)** $y'\sin y + x\cos y = x$, **(iii)** $y' = y(xy^3 - 1)$,
(iv) $(e^y - 2xy)y' = y^2$, **(v)** $y - xy' = y'y^2e^y$,

Q2. Solve the following linear first order differential equations using the method of variation of parameters:
(i) $xy' - 2y = x^4$ **(ii)** $y' + (\cos x)y = \sin x \cos x$

Q3. If $y_1(x)$ and $y_2(x)$ are any two solutions of the differential equation $y'' + P(x)y' + Q(x)y = 0$ on I (here P, Q are continuous functions on interval I) then show that $y(x) = c_1y_1(x) + c_2y_2(x)$ is a solution of the differential equation.

Q4. If $y_1(x)$ and $y_2(x)$ are any two solutions of the differential equation $y'' + P(x)y' + Q(x)y = 0$ on I (here P, Q are continuous functions on interval I) then the wronskian $W(y_1, y_2) = y_1y_2' - y_1'y_2$ is either identically zero or never zero.

Q5. If $y_1(x)$ and $y_2(x)$ are any two solutions of the differential equation $y''+P(x)y'+Q(x)y = 0$ on I (here P, Q are continuous functions on interval I) then the solutions $y_1(x)$, $y_2(x)$ are LD on I iff wronskian $W(y_1, y_2)$ is identically zero.

Q6. (a) Find the values of m such that $y = e^{mx}$ is a solution of
(i) $y'' + 3y' + 2y = 0$, **(ii)** $y'' - 4y' + 4y = 0$, **(iii)** $y''' - 2y'' - y' + 2y = 0$.

(b) Find the values of m such that $y = x^m (x > 0)$ is a solution of
(i) $x^2y'' - 4xy' + 4y = 0$, **(ii)** $x^2y'' - 3xy' - 5y = 0$.

Q7. Use Picard's method of successive approximation to solve the following initial value problems and compare these results with the exact solutions:
(i) $y' = 2\sqrt{x},\ y(0) = 1$ **(ii)** $y' + xy = x,\ y(0) = 0$
(iii) $y' = 2\sqrt{y}/3,\ y(0) = 0.$

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MA102: Assignment-3, Sem-II, 2018-19

Q1. Solve the following differential equations:
(i) $y'' - 4y' + 3y = 0,$ **(ii)** $y'' + 2y' + (\omega^2 + 1)y = 0,\ \ \omega$ is real,
(iii) $4y'' - 12y' + 9y = 0.$

Q2. Find general solution of the following differential equations given a known solution y_1:
(i) $x(1-x)y'' + 2(1-2x)y' - 2y = 0, \quad y_1 = 1/x,$
(ii) $(1-x^2)y'' - 2xy' + 2y = 0, \quad y_1 = x.$

Q3. Solve the following differential equations:
(i) $y''' - 8y = 0,$ **(ii)** $y^{(4)} + y = 0,$ **(iii)** $y''' - 3y' - 2y = 0,$
(iv) $y''' - 6y'' + 11y' - 6y = 0.$

Q4. Solve the following Cauchy-Euler equations:
(i) $x^2y'' + 2xy' - 12y = 0,$ **(ii)** $x^2y'' + xy' + y = 0,$ **(iii)** $x^2y'' - xy' + y = 0.$

Hint with details for Q4: Assume that $y = x^m$ is a solution of the equation $Ly = 0$. Now find out corresponding polynomial $p(m)$ such that $L(x^m) = p(m)x^m$. Now for distinct roots of $p(m) = 0$ one will get x^{m_1}, x^{m_2} as LI Solution. Similarly for complex case $m = a \pm ib$, we have solution as $x^{a+ib} = x^a.x^{ib} = x^a.e^{ib\ln x} = x^a(\cos b\ln x + i\sin b\ln x)$ will give $x^a\cos b\ln x, x^a\sin b\ln x$ as LI solution. Further for repeated case $m = m_1, m_1$, one solution will be x^{m_1} and another LI Solution can be found using method of reduction of order.

Q5. Solve the following equations using Method of variation of parameter:
(i) $y'' + 2y' + 5y = e^{-x}\ sec2x,$ **(ii)** $(x^2-1)y'' - 2xy' + 2y = (x^2-1)^2,$
(iii) $x^2y'' - 2xy' + 2y = xe^{-x}.$

Practice Problems: Related problems from Ross, Kreyszig & Simmons.