"Page 0"

Q1(a)

32.5 Theorem.
A bounded function f on $[a,b]$ is integrable if and only if for each $\epsilon > 0$ there exists a partition P of $[a,b]$ such that

$$U(f,P) - L(f,P) < \epsilon. \qquad (1)$$

Proof
Suppose first that f is integrable and consider $\epsilon > 0$. There exist partitions P_1 and P_2 of $[a,b]$ satisfying

$$L(f,P_1) > L(f) - \frac{\epsilon}{2} \quad \text{and} \quad U(f,P_2) < U(f) + \frac{\epsilon}{2}.$$ — 1M

For $P = P_1 \cup P_2$, we apply Lemma 32.2 to obtain

$$U(f,P) - L(f,P) \leq U(f,P_2) - L(f,P_1)$$
$$< U(f) + \frac{\epsilon}{2} - \left[L(f) - \frac{\epsilon}{2}\right] = U(f) - L(f) + \epsilon.$$ — 1M

Since f is integrable, $U(f) = L(f)$, so (1) holds.

Conversely, suppose for $\epsilon > 0$ the inequality (1) holds for some partition P. Then we have

$$U(f) \leq U(f,P) = U(f,P) - L(f,P) + L(f,P)$$
$$< \epsilon + L(f,P) \leq \epsilon + L(f).$$ — 1M

Since ϵ is arbitrary, we conclude $U(f) \leq L(f)$. Hence we have $U(f) = L(f)$ by Theorem 32.4, i.e., f is integrable. ■ — 1M

Q1(b)

33.2 Theorem.
Every continuous function f on $[a,b]$ is integrable.

Proof
Again, in order to apply Theorem 32.5, consider $\epsilon > 0$. Since f is uniformly continuous on $[a,b]$ by Theorem 19.2, there exists $\delta > 0$ such that

$$x, y \in [a,b] \quad \text{and} \quad |x-y| < \delta \quad \text{imply} \quad |f(x) - f(y)| < \frac{\epsilon}{b-a}. \quad (1)$$ — 1M

Consider any partition $P = \{a = t_0 < t_1 < \cdots < t_n = b\}$ where — 1/2M

$$\max\{t_k - t_{k-1} : k = 1, 2, \ldots, n\} < \delta.$$

Since f assumes its maximum and minimum on each interval $[t_{k-1}, t_k]$ by Theorem 18.1, it follows from (1) above that — 1/2M

$$M(f, [t_{k-1}, t_k]) - m(f, [t_{k-1}, t_k]) < \frac{\epsilon}{b-a}$$ — 1/2M

for each k. Therefore we have

$$U(f,P) - L(f,P) < \sum_{k=1}^{n} \frac{\epsilon}{b-a}(t_k - t_{k-1}) = \epsilon,$$ — 1/2M

■

and Theorem 32.5 shows f is integrable.

2(a) $\vec{F} = \frac{1}{y}\hat{i} + \left(\frac{1}{z} - \frac{x}{y^2}\right)\hat{j} - \frac{y}{z^2}\hat{k} = M\hat{i} + N\hat{j} + P\hat{k}$

(i) $\left.\begin{array}{ll} \frac{\partial M}{\partial y} = -\frac{1}{y^2} & \frac{\partial N}{\partial x} = -\frac{1}{y^2} \\ \frac{\partial N}{\partial z} = -\frac{1}{z^2} & \frac{\partial P}{\partial y} = -\frac{1}{z^2} \\ \frac{\partial M}{\partial z} = 0 & \frac{\partial P}{\partial x} = 0 \end{array}\right\} \Rightarrow \vec{F}$ is conservative $\Rightarrow \vec{F} = \nabla f$

where f is a scalar function.

$\vec{F} = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$ — $\boxed{1M}$

$f_x = M, \quad f_y = N, \quad f_z = P$

$f_x = \frac{1}{y}$

$f(x,y,z) = \frac{x}{y} + h(y,z)$

$f_y = -\frac{x}{y^2} + \frac{\partial h}{\partial y}(y,z)$

$\frac{1}{z} - \cancel{\frac{x}{y^2}} = -\cancel{\frac{x}{y^2}} + \frac{\partial h}{\partial y}(y,z)$

$\frac{\partial h}{\partial y} = \frac{1}{z}$

$h(y,z) = \frac{y}{z} + g(z)$ — $\boxed{\frac{1}{2}M}$

$f(x,y,z) = \frac{x}{y} + \frac{y}{z} + g(z)$

$f_z = -\frac{y}{z^2} + g'(z)$

$-\frac{y}{z^2} = -\frac{y}{z^2} + g'(z)$

$g'(z) = 0$

$g(z) = c$ — $\boxed{\frac{1}{2}M}$

$\vec{F}\cdot d\vec{r}$
$= \vec{\nabla} f \cdot d\vec{r}$
$= df$

Potential func $f(x,y,z) = \frac{x}{y} + \frac{y}{z} + c$

(ii) $\int_{(1,1,1)}^{(2,2,2)} \left[\frac{1}{y}dx + \left(\frac{1}{z} - \frac{x}{y^2}\right)dy - \frac{y}{z^2}dz\right] = \int_{(1,1,1)}^{(2,2,2)} \vec{F}\cdot d\vec{r} = \int_{(1,1,1)}^{(2,2,2)} df$

$= f(2,2,2) - f(1,1,1)$ $\boxed{1M}$

$= \left(\frac{2}{2} + \frac{2}{2} + c\right) - \left(\frac{1}{1} + \frac{1}{1} + c\right)$

$= (2+c) - (2+c) = 0$

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2 (b) $\text{Flow} = \int_C \vec{F}\cdot\hat{T}\,ds = \int \vec{F}\cdot d\vec{r}$. $\quad C: x^2+y^2=1,\ (1,0)\text{ to }(-1,0)$

$$= \int_0^{\pi} \{(\cos t+\sin t)\hat{i}-\hat{j}\}\cdot\{-\sin t\,\hat{i}+\cos t\,\hat{j}\}\,dt \quad \text{— } \boxed{1M}$$

$x=\cos t,\ y=\sin t$

$0 \le t \le \pi$

$\vec{r}=\cos t\,\hat{i}+\sin t\,\hat{j}$

$d\vec{r}=(-\sin t\,\hat{i}+\cos t\,\hat{j})\,dt$

$$= \int_0^{\pi} (-\sin t\cos t-\sin^2 t-\cos t)\,dt$$

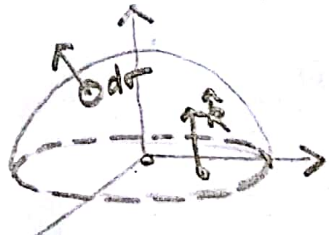
$$= \int_0^{\pi} -\frac{1}{2}\sin 2t-\frac{1}{2}(1-\cos 2t)-\cos t\ dt$$

$$= -\left[\int_0^{\pi} \frac{1}{2}\sin 2t+\frac{1}{2}-\frac{1}{2}\cos 2t+\cos t\right]dt$$

$$= -\left[\frac{\cos 2t}{-4}+\frac{1}{2}t-\frac{\sin 2t}{4}+\sin t\right]_0^{\pi} \quad \text{— } \boxed{1M}$$

$$= -\left[\frac{1-1}{-4}+\frac{1}{2}\pi-0+0\right] = -\frac{\pi}{2}.$$

2 (c) $$\iint_{\substack{x^2+y^2+z^2=1\\ z\ge 0}} x\,d\sigma = \iint_{x^2+y^2\le 1} x\,\frac{|\nabla f|}{|\nabla f\cdot\hat{k}|}\,dx\,dy \quad \text{— } \boxed{\tfrac{1}{2}M}$$



$f=x^2+y^2+z^2-1$

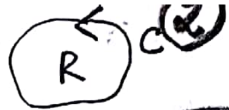
$\nabla f=2x\hat{i}+2y\hat{j}+2z\hat{k}$

$$= \iint_{x^2+y^2\le 1} x\,\frac{\sqrt{4x^2+4y^2+4z^2}}{|2z|}\,dx\,dy \quad \text{— } \boxed{\tfrac{1}{2}M}$$

$$= \iint_{x^2+y^2\le 1} \frac{x\cdot\cancel{2}\sqrt{1}}{\cancel{2}\sqrt{1-x^2-y^2}}\,dx\,dy = \iint_{x^2+y^2\le 1} \frac{x\,dx\,dy}{\sqrt{1-x^2-y^2}}$$

$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{1} \frac{r\cos\theta\ r\,dr\,d\theta}{\sqrt{1-r^2}} = \int_0^{2\pi}\cos\theta\,d\theta \int_{r=0}^{1} \frac{r^2}{\sqrt{1-r^2}}\,dr = 0$$

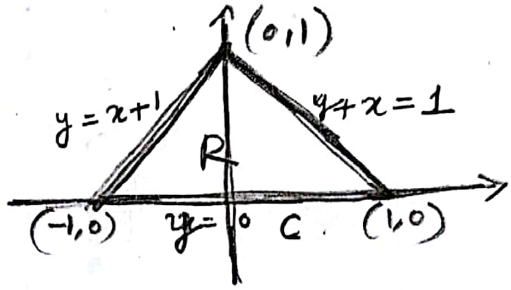
— $\boxed{1M}$

2 

3. (a) $\oint_C \vec{G}\cdot\hat{n}\,ds = \oint_C M\,dy - N\,dx = \iint_R \left(\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}\right)dx\,dy$ — $\boxed{1M}$

where R is the region enclosed by C.

(b) $\oint_C \vec{F}\cdot\hat{n}\,ds$



$F = M\hat{i}+N\hat{j}$

$M = x+y$

$N = -(x^2+y^2)$

$$= \int_{y=0}^{1}\int_{x=y-1}^{1-y}\left(1+(-2y)\right)dx\,dy \quad — \boxed{1M}$$

$$= \int_0^1 (1-2y)(1-y-y+1)\,dy$$

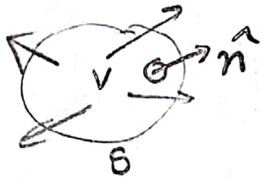
$$= \int_0^1 (1-2y)(2-2y)\,dy$$

$$= \int_0^1 (2-2y-4y+4y^2)\,dy$$

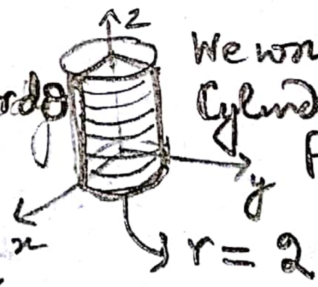
$$= \int_0^1 (2-6y+4y^2)\,dy$$

$$= 2y-6\frac{y^2}{2}+\frac{4}{3}y^3\Big|_0^1 = 2-3+\frac{4}{3} = -1+\frac{4}{3} = \frac{1}{3}. \quad — \boxed{1M}$$

(c) $\iint_S \vec{F}\cdot\hat{n}\,d\sigma = \iiint_V \vec{\nabla}\cdot\vec{F}\,dV$ — $\boxed{1M}$



(d) $$\text{Flux} = \int_{\theta=0}^{\pi/2}\int_{r=0}^{2}\int_{z=0}^{3}(12x+2y+2+0)\,dz\,r\,dr\,d\theta \quad — \boxed{1M}$$



We write in Cylindrical form.

$0\le\theta\le\pi$

First octant

$$= 2\times 3\int_0^{\pi/2}\int_{r=0}^{2}(1+y+6x)\,r\,dr\,d\theta$$

$$= 6\int_0^{\pi/2}\int_{r=0}^{r=2}(1+r\sin\theta+6r\cos\theta)\,r\,dr\,d\theta.$$

$$= 6\int_0^{\pi/2}\int_0^2 (r + r^2\sin\theta + 6r^2\cos\theta)\,dr\,d\theta$$

$$= 6\int_0^{\pi/2}\left(\frac{r^2}{2} + \frac{1}{3}r^3\sin\theta + \frac{6}{3}r^3\cos\theta\right)\Big|_0^2 d\theta$$

$$= 6\int_0^{\pi/2}\left(2 + \frac{8}{3}\sin\theta + 16\cos\theta\right)d\theta$$

$$= 6\left(2\theta + \frac{8}{3}(-\cos\theta) + 16(\sin\theta)\right)\Big|_0^{\pi/2}$$

2 $\quad = 6\left(\frac{2\pi}{2} + \frac{8}{3} + 16\right) = 6\left(\pi + \frac{56}{3}\right) = \underline{6\pi + 112}$ — $\boxed{1M}$

4.(a) $f(x,y) = \sin^{-1}(y-x)$

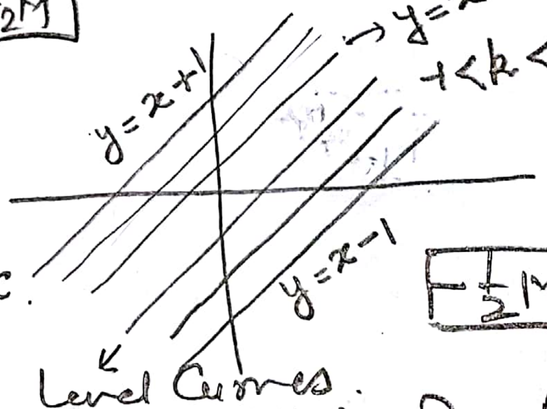
Domain $= \{(x,y) : -1 \leq y-x \leq 1\}$ — $\boxed{1/2M}$

Range $= [-\frac{\pi}{2}, \frac{\pi}{2}]$ — $\boxed{1/2M}$

Level Curves $\quad \sin^{-1}(y-x) = c$.

$y - x = \sin c$

$y - x = k \quad k = \sin c$.



$\boxed{\frac{1}{2}M}$

Level Curves.

$\boxed{1/2M}$ — Domain is closed as it contains all of its Boundary Point.

$\boxed{1/2M}$ — Set of Boundary Points $= \{(x,y) : y = x+1, \; y = x-1\}$

$\boxed{1/2M}$ — Domain is UNBOUNDED.

4(b) $\quad f_{xy}(0,0) = \lim_{k\to 0}\frac{f_x(0,k) - f_x(0,0)}{k}$ — $\boxed{1M}$

$f_x(0,k) = ? \quad f_x(0,0) = ?$

$$f_x(0,k) = \lim_{h\to 0} \frac{f(h,k)-f(0,k)}{h} = \lim_{h\to 0} \frac{\frac{\cancel{h}k(h^2-k^2)}{h^2+k^2} - 0}{\cancel{h}}$$

$$= -k \quad \boxed{\tfrac{1}{2}M}$$

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0 \quad \boxed{\tfrac{1}{2}M}$$

$$f_{xy}(0,0) = \lim_{k\to 0} \frac{-k-0}{k} = -1.$$

$$f_{yx}(0,0) = \lim_{h\to 0} \frac{f_y(h,0)-f_y(0,0)}{h} \quad - \boxed{1M}$$

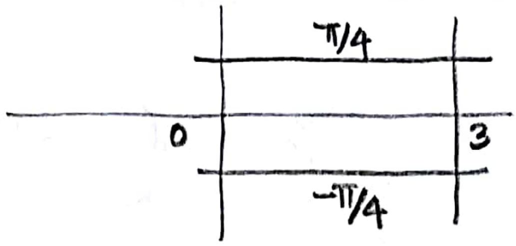
$$f_y(h,0) = \lim_{k\to 0} \frac{f(h,k)-f(h,0)}{k}$$

$$= \lim_{k\to 0} \frac{\frac{h\cancel{k}(h^2-k^2)}{h^2+k^2} - 0}{\cancel{k}} = h \quad - \boxed{\tfrac{1}{2}M}$$

$$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k)-f(0,0)}{k} = \lim_{k\to 0} \frac{0-0}{k} = 0 \quad - \boxed{\tfrac{1}{2}M}$$

$$f_{yx}(0,0) = \lim_{h\to 0} \frac{h-0}{h} = 1$$

5 a) $f(x,y)=(4x-x^2)\cos y$



$f_x=(4-2x)\cos y \quad =0$

$f_y=(4x-x^2)(-\sin y)=0$

$\cos y \neq 0$ in $[-\pi/4, \pi/4]$ so $x=2$

$x(4-x)\sin y=0$

$x=0.$ or $x=4$ or $\sin y=0$

$x=0.$ or $x=4$ or $y=0$. Therefore the critical point $(2,0)$.

Max is 4 at $(2,0)$

min is 0 at along the line $x=0$, $-\pi/4 \le y \le \pi/4$.

$\boxed{1/2 M}$

$f_{xx}=-2\cos y$, $f_{yy}=-(4x-x^2)\cos y$, $f_{xy}=-(4-2x)\sin y$

at $(2,0)$ $\quad f_{xx}=-2$, $f_{yy}=-4$, $f_{xy}=0$.

$\left.\begin{array}{l}(-2)(-4)-0^2=8>0\\ f_{xx}=-2<0\end{array}\right]$ $(2,0)$ is point of local maximum.

$\boxed{1M}$

along $x=0$, $f(0,y)=0$ — $\boxed{\frac{1}{2}M}$

$x=3$, $\quad f(3,y)=3\cos y$, $\quad f'(3,y)=-3\sin y=0 \Rightarrow y=0$

$f''(3,y)=-3\cos y<0$ at $y=0$

local maxima at $(3,0)$ — $\boxed{1M}$

at $y=\pm\pi/4$ $\quad \left.\begin{array}{l} f(x,\frac{\pi}{4})=\frac{1}{\sqrt{2}}(4x-x^2)\\ f(x,-\frac{\pi}{4})=\frac{1}{\sqrt{2}}(4x-x^2)\end{array}\right\}$ $\quad f'(x,\pm\frac{\pi}{4})=\frac{4-2x}{\sqrt{2}}=0$

$x=2$

$f''(x,\pm\pi/4)=-\frac{2}{\sqrt{2}}<0$

local maxima at $(2,\pm\frac{\pi}{4})$ — $\boxed{1M}$

At corner pts $\rightarrow f(0,\pm\frac{\pi}{4})=0$, $\quad f(3,\pm\frac{\pi}{4})=(12-9)\frac{1}{\sqrt{2}}=\frac{3}{\sqrt{2}}=1.5\sqrt{2}=1.5\times1.414$

$=1.0721$

$f(2,0)=(8-4)1=4$

Max at $(2,0)$, Value 0

Min at 0, $-\frac{\pi}{4}\le y\le\frac{\pi}{4}$ value is zero

$f(0,y)=0$

$f(3,0)=(12-9)=3$

$f(2,\pm\pi/4)=(8-4)\frac{1}{\sqrt{2}}=\frac{4}{\sqrt{2}}=2\sqrt{2}=2\times1.414=2.828$

5(b)

For max, min $\nabla f = \lambda \nabla g_1 + \mu \nabla g_2$ $\quad y\hat{i} + x\hat{j} + 2z\hat{k} = \lambda(-\hat{i}+\hat{j}) + \mu(2x\hat{i} + 2y\hat{j} + 2z\hat{k})$

$g_1 = 0$

$g_2 = 0$

$y = -\lambda + 2\mu x$

$x = \lambda + 2\mu y$

$2z = 2\mu z$

$y = x$

$x^2 + y^2 + z^2 = 4$

$\Rightarrow$ at $y = x$, we get

$x = -\lambda + 2\mu x$

$x = \lambda + 2\mu x$

$2z = 2\mu z$

$2x^2 + z^2 = 4$

$\boxed{1M}$

$z(1-\mu) = 0$

$z = 0$, or $\mu = 1$ $\quad \boxed{1M}$

For $z = 0$, $2x^2 = 4$, $x = \pm\sqrt{2}$, $y = x = \pm\sqrt{2}$ so pts $(\pm\sqrt{2}, \pm\sqrt{2}, 0)$

For $\mu = 1$, $\quad x = \lambda + 2x$

$x = -\lambda$, $\quad x = -\lambda + 2x$

$x = \lambda$

$\Rightarrow x = \lambda$ and $x = -\lambda \Rightarrow x = 0 \ (\lambda = 0)$ $\quad \boxed{1M}$

$\Rightarrow$ so $2 \times 0 + z^2 = 4$ $\quad$ as $y = x = 0$

$z = \pm 2$

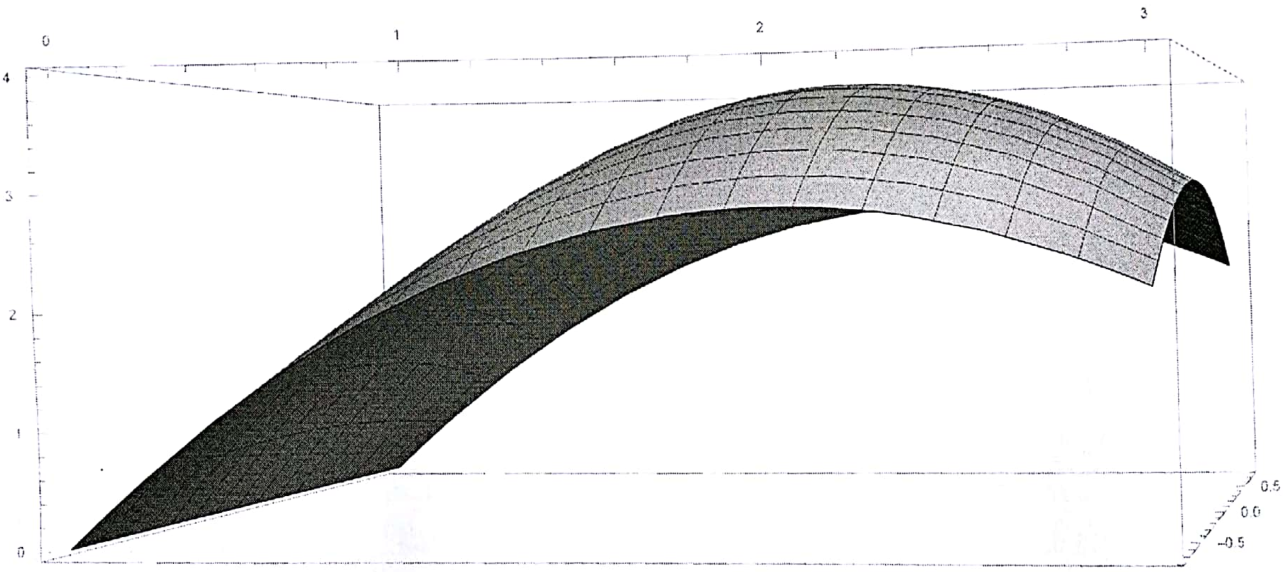
So the point is $(0, 0, \pm 2)$

$f(0,0,\pm 2) = 0 + 2^2 = 4$

$f(\pm\sqrt{2}, \pm\sqrt{2}, 0) = (\pm\sqrt{2})(\pm\sqrt{2}) + 0 = 2$

Max at $(0, 0, \pm 2)$ value $= 4$

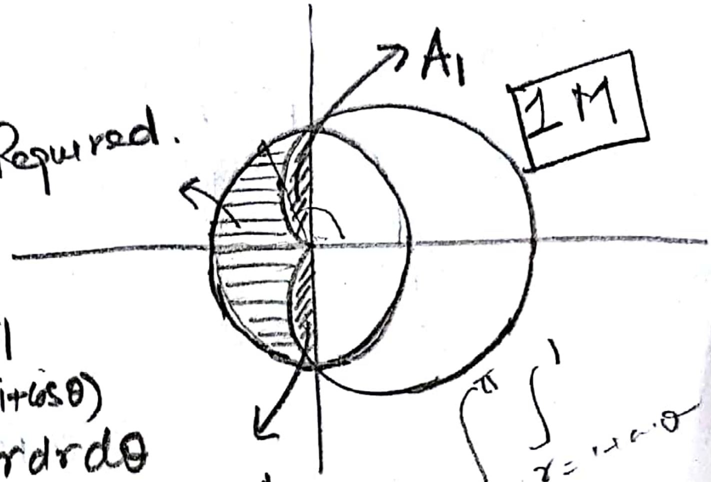
Min at $(\pm\sqrt{2}, \pm\sqrt{2}, 0)$ value $= 2$.



Question 5(a)

7(b):-

Required $= \frac{1}{2}$ (Area of Circle) $-(A_1+A_2)$ Required.



$\boxed{1M}$

$= \frac{1}{2}\times\pi\times 1^2 - 2A_1$

$\int_{\pi/2}^{\pi}\int_{r=1+\cos\theta}^{1}$

$= \frac{1}{2}\pi - 2\times\int_{\frac{\pi}{2}}^{\pi}\int_{0}^{(1+\cos\theta)} r\,dr\,d\theta$ — $\boxed{1M}$

$A_1 = A_2$ (due to Symmetry)

$= \frac{1}{2}\pi - 2\times\frac{1}{2}\int_{\pi/2}^{\pi}(1+\cos\theta)^2\,d\theta$

$= \frac{\pi}{2} - \int_{\frac{\pi}{2}}^{\pi}(1+\cos^2\theta+2\cos\theta)\,d\theta$

$\boxed{= 2-\frac{\pi}{4}}$

$\boxed{= \frac{\pi}{2}-\left(-2+\frac{3\pi}{4}\right)}$

$= \frac{\pi}{2} - \left[\left(\pi-\frac{\pi}{2}\right)+2\left(\sin\pi-\sin\frac{\pi}{2}\right)\right] - \int_{\frac{\pi}{2}}^{\pi}\cos^2\theta\,d\theta$

$= \cancel{\frac{\pi}{2}} - \cancel{\frac{\pi}{2}} - 2(-1) - \int_{\pi/2}^{\pi}\frac{1+\cos 2\theta}{2}\,d\theta$

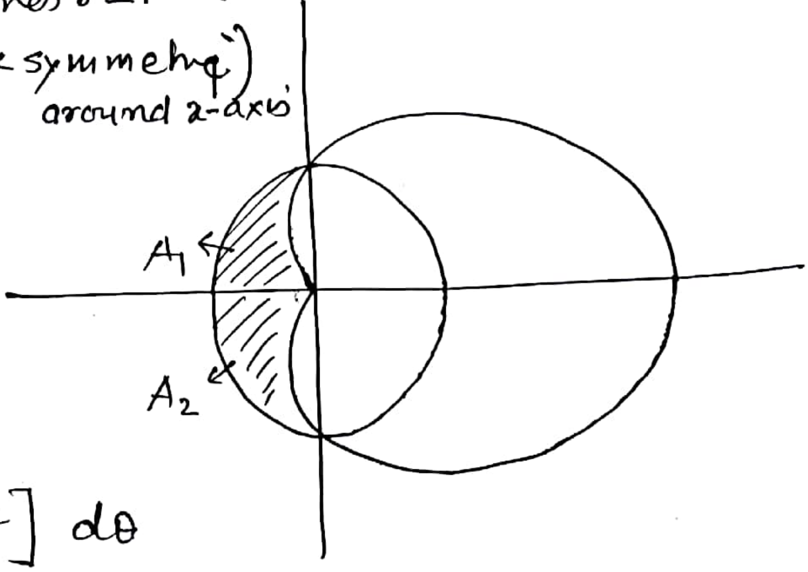
$= 2 - \frac{1}{2}\left[\left(\pi-\frac{\pi}{2}\right)+\frac{\sin 2\theta}{2}\Big|_{\pi/2}^{\pi}\right]$

$= 2-\frac{1}{2}\left[\frac{\pi}{2}+0\right] = \underline{2-\frac{\pi}{4}}$ — $\boxed{1M}$

(7b) Curves $r = 1$ & $r = 1+\cos\theta$

$A_1 = A_2$ (Both are symmetric around x-axis)

Required Area $= 2A_1$



$$= 2\int_{\frac{\pi}{2}}^{\pi}\int_{1+\cos\theta}^{1} r\,dr\,d\theta$$

$$= 2\,\frac{1}{2}\int_{\frac{\pi}{2}}^{\pi}\left[1-(1+\cos\theta)^2\right]d\theta$$

$$= \int_{\frac{\pi}{2}}^{\pi} -\cos^2\theta - 2\cos\theta\,d\theta = \int_{\frac{\pi}{2}}^{\pi}\left(-\frac{1-\cos 2\theta}{2} - 2\cos\theta\right)d\theta$$

$$= \int_{\frac{\pi}{2}}^{\pi}\left(-\frac{1}{2} - \frac{\cos 2\theta}{2} - 2\cos\theta\right)d\theta$$

$$= -\frac{1}{2}\theta - \frac{\sin 2\theta}{4} - 2\sin\theta\Big|_{\pi/2}^{\pi}$$

$$= -\frac{1}{2}\frac{\pi}{2} - 0 - 2(-1)$$

$$= 2 - \frac{\pi}{4}.$$

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6(a) $\nabla f = 4x\hat{i} + 2y\hat{j}$

∇f at $(-1,1) = -4\hat{i} + 2\hat{j}$ — $\boxed{\tfrac{1}{2}M}$

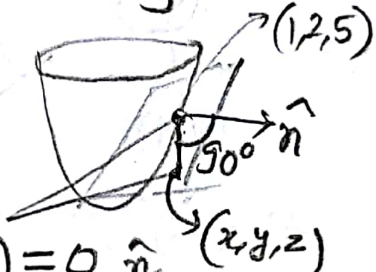
Unit vector $\hat{u} = \dfrac{3\hat{i} - 4\hat{j}}{\sqrt{9+16}} = \dfrac{3\hat{i} - 4\hat{j}}{5}$ — $\boxed{1M}$

Directional Derivative

Directional Derivative $\nabla f\big|_{at(-1,1)} \cdot \hat{u} = (-4\hat{i} + 2\hat{j}) \cdot \left(\dfrac{3\hat{i} - 4\hat{j}}{5}\right) = \dfrac{-12-8}{5} = -4$ — $\boxed{\tfrac{1}{2}M}$

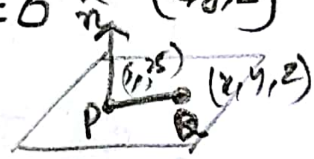
6(b) $\nabla f = -2x\hat{i} - 2y\hat{j} + \hat{k}$ — $f = z - x^2 - y^2$

∇f at $(1,2,5) = -2\hat{i} - 4\hat{j} + \hat{k}$ $\boxed{1M}$



Eq^n of Tangent Plane: $\vec{PQ} \cdot \hat{n} = 0$

$(x-1)\hat{i} + (y-2)\hat{j} + (z-5)\hat{k} \cdot (-2\hat{i} - 4\hat{j} + \hat{k}) = 0$



$-2(x-1) - 4(y-2) + (z-5) = 0$

$2x - 2 + 4y - 8 - z + 5 = 0$

$\boxed{2x + 4y - z = 5}$ — $\boxed{1M}$

6(c) $0 \leq \left|\dfrac{x^3 y}{x^2+y^2}\right| = \left| xy \left(\dfrac{x^2}{x^2+y^2}\right)\right| \leq |xy|$ — $\boxed{1M}$

$\lim\limits_{(x,y)\to(0,0)} |xy| = \lim\limits_{r\to 0} |r^2 \sin\theta \cos\theta| = 0$ $\{|\sin\theta\cos\theta| < 1$

$\lim\limits_{(x,y)\to(0,0)} 0 = \lim\limits_{(x,y)\to(0,0)} |xy| = 0$ — $\boxed{\tfrac{1}{2}M}$

$\Rightarrow \lim\limits_{(x,y)\to(0,0)} \left|\dfrac{x^3 y}{x^2+y^2}\right| = 0 \Rightarrow \lim\limits_{(x,y)\to(0,0)} \dfrac{x^3 y}{x^2+y^2} = 0$ — $\boxed{\tfrac{1}{2}M}$

6(a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y}{x^4+y^4}$

along $y = mx$. — $\boxed{1M}$

$$\lim_{x\to 0} \frac{x^3 mx}{x^4+m^4x^4} = \lim_{x\to 0} \frac{m}{1+m^4} = \frac{m}{1+m^4} \text{ — } \boxed{1M}$$

so limit is path dependant hence limit does not exist.

OR $x = r\cos\theta$, $y = r\sin\theta$

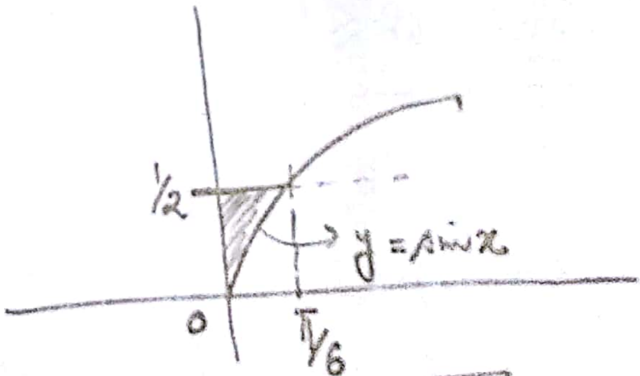
$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^4} = \lim_{r\to 0} \frac{\cancel{r^4}\cos^3\theta\sin\theta}{\cancel{r^4}(\cos^4\theta+\sin^4\theta)} = \lim_{r\to 0} \frac{\cos^3\theta\sin\theta}{\cos^4\theta+\sin^4\theta} \text{ — } \boxed{1M}$$

Along $\theta = \frac{\pi}{4}$ $\quad \lim\limits_{r\to 0} \dfrac{\cos^3\frac{\pi}{4}\sin\frac{\pi}{4}}{\cos^4\frac{\pi}{4}+\sin^4\frac{\pi}{4}} = \dfrac{\left(\frac{1}{\sqrt2}\right)^3\frac{1}{\sqrt2}}{\frac14+\frac14} = \dfrac{\frac14}{\frac12} = \dfrac12$

$\theta = \frac{\pi}{2}$ $\quad \lim\limits_{r\to 0} \dfrac{\cos^3\frac{\pi}{2}\sin\frac{\pi}{2}}{\cos^4\frac{\pi}{2}+\sin^4\frac{\pi}{2}} = 0$ — $\boxed{1M}$

So along two paths two different limits hence limit does not exist.

7(a) $\displaystyle\int_0^{\pi/6}\int_{\sin x}^{1/2} xy^2\,dy\,dx$



— $\boxed{1M}$

$$= \int_{y=0}^{1/2}\int_{x=0}^{\sin^{-1}y} xy^2\,dx\,dy \longleftarrow \boxed{1M}$$

(7c) $x^2+4y^2+9z^2=1.$

$$V=\iiint\limits_{x^2+4y^2+9z^2\leq 1} dz\,dx\,dy.$$

$x=u,\quad 2y=v,\quad 3z=w$ — $\boxed{1\text{ M}}$

$$J=\begin{vmatrix}1 & 0 & 0\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{3}\end{vmatrix}=\frac{1}{6} \text{ — } \boxed{1\text{M}}$$

$$V=\iiint\limits_{u^2+v^2+w^2\leq 1}\frac{1}{6}\,du\,dv\,dw=\frac{1}{6}\iiint\limits_{u^2+v^2+w^2\leq 1} du\,dv\,dw$$

$$V=\frac{1}{\cancel{6}_{3}}\,\frac{\cancel{4}^{2}}{3}\pi 1^3=\frac{2\pi}{9} \text{ — } \boxed{1\text{M}}$$