Engineering Optics (PH301) Mid Semester Assignment

Name of Student: M-Maheeth Reddy

Roll No : 1801CS31

Answers

1. Given,

wavelength of radiation, $\lambda = 632.8\,nm$

internode seperation of He-Ne laser, $\Delta\nu = 300\,MHz$

reflectivity of plates of interferometer, $R = 0.99$

we know,

$$c = \nu\lambda \Rightarrow \nu = \frac{c}{\lambda} = \frac{3\times10^8}{632.8\times10^{-9}}$$

$$\Rightarrow \boxed{\nu = 4.74\times10^{14}\,Hz}$$

Coefficient of Finnesse, $F = \frac{4R}{(1-R)^2} = \frac{4\times0.99}{(1-0.99)^2}$

$$\Rightarrow \boxed{F = 39600}$$

$$\text{Resolving Power} = \left|\frac{\nu}{\Delta\nu}\right| = \frac{\pi h\nu\sqrt{F}}{c}$$

$$\Rightarrow \frac{\cancel{4.74\times10^{14}}}{300\times10^6} = \frac{\pi\times h\times\cancel{4.74\times10^{14}}\times\sqrt{39600}}{3\times10^8}$$

$$\Rightarrow \boxed{h = 1.6\times10^{-3}\,m = 1.6\,mm}$$

This is minimum plate seperation required.

Free Spectral Range,

in frequency units, $(4.74 \times 10^{14}\,Hz$ to $(4.74 \times 10^{14} + 300 \times 10^{6})\,Hz)$

in wavelength units,

$$\left(\frac{c}{4.74 \times 10^{14} + 300 \times 10^{6}} ,\ 632.8\,nm \right)$$

$$= (632.79\,nm, 632.8\,nm)$$

For highest order fringe number,

$2nh\cos\theta = m\lambda$ (m is order of fringe)

if $n = 1$, $2h\cos\theta = m\lambda$

$$\Rightarrow \cos\theta = \frac{m\lambda}{2h}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{m \times 632.8 \times 10^{-9}}{2 \times 1.6 \times 10^{-3}} \right)$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{m}{5056.89} \right)$$

Hence, the maximum value 'm' can take is 5056

$\therefore$ Highest order of fringes produced is 5056

2. Refractive index of glass, $n_g = 1.5630$ (given)

(a) To make the lens non-reflecting, it must be coated with material whose refractive index is,

$$n_f = \sqrt{n_a n_g}$$

where, n_a is refractive index of surrounding medium. (air in this case)

$$\therefore \underline{n_a = 1}$$

$$n_f = \sqrt{n_a n_g} = \sqrt{1 \times 1.5630}$$

$$\Rightarrow \boxed{n_f \approx 1.250}$$

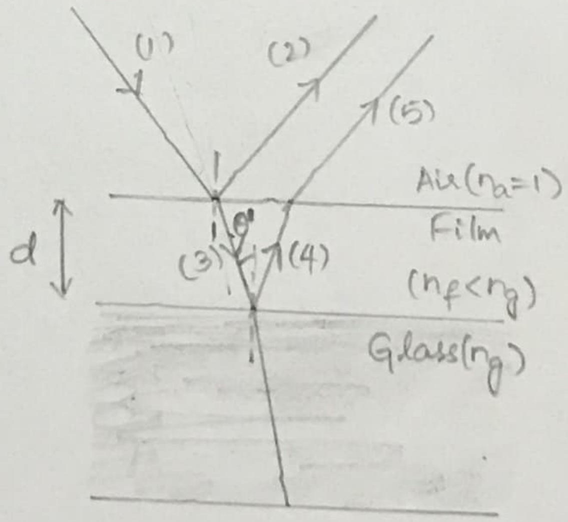
(b) Wavelength of green light, $\lambda = 550\text{nm}$

Optical path difference of rays (2) and (5),

$\Delta = 2n_f d \cos\theta'$

For normal incidence, $\cos\theta' = 1$

For zero percent reflectance, (2) and (5) must undergo destructive interference.



$$\Rightarrow \Delta = 2n_f d = (m+\tfrac{1}{2})\lambda$$

for $m=0$, $\quad 2n_f d = \frac{1}{2}\lambda \Rightarrow d = \frac{\lambda}{4n_f} = \frac{\cancel{550}^{110} \times 10^{-9}}{4 \times \cancel{1.250}_{0.25}}$

$$\Rightarrow \boxed{d = 110\text{nm}}$$

$\therefore$ Refractive index of surface coating material is 1.250 and its minimum thickness is 110nm for given wavelength of green light

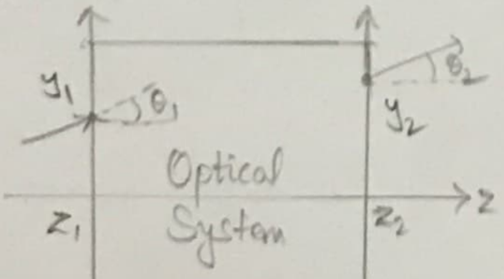
3. ## ABCD Matrix

An ABCD matrix or Ray Transfer Matrix is a 2×2 matrix used to describe the effect of an optical system on a light ray after it passes through it. This method involves the use of matrix formalism to understand the properties of rays in optical systems under paraxial conditions. We can only consider those rays whose path is completely present in a single plane containing z-axis. Such rays are called meridional rays.

Determination of position and angle of output ray in an optical system

Consider a plane containing z-axis and assume the transverse axis in this plane is y-axis. So, the plane of reference is (y, z) plane.

Now, a ray is incident on an optical system at transverse co-ordinate y_1 making an angle θ_1 as shown in figure.



It leaves the optical system making angle θ_2 at transverse co-ordinate y_2.

Generally, the incident ray is termed as input ray and emergent ray as output.

Under paraxial conditions, the y, θ of input ray and output ray are related as follows.

$$y_2 = Ay_1 + B\hat{\theta}_1$$
$$\hat{\theta}_2 = Cy_1 + D\hat{\theta}_1$$

The equivalent matrix form for above equations is

$$\begin{bmatrix} y_2 \\ \hat{\theta}_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \hat{\theta}_1 \end{bmatrix}$$

In this equation, matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called ABCD matrix or ray transfer matrix and denoted by M.

$$\boxed{M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}}$$

~~In (y,z)~~ The reduced ray-angle ($\hat{\theta}$) with respect to z-axis is related to local spatial frequency f_l as,

$$\boxed{f_l = \frac{\theta}{\lambda} = \frac{\hat{\theta}}{\lambda_0}}$$

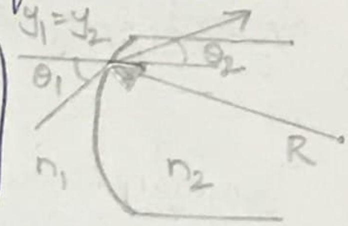
Therefore, ABCD matrix can be regarded as specifying a transformation between spatial distribution of local spatial frequency at input & corresponding distribution at output.

Ray transfer matrix when optical ray passes through a thin lens

Consider a thin lens of refractive index n_2 present in a medium of refractive index n_1

For a spherical interface, the ray transfer matrix is

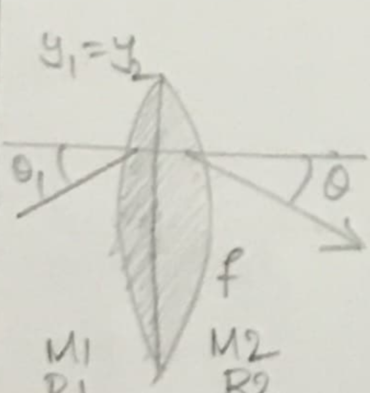
$$M = \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R} & 1 \end{bmatrix}$$



Treat the left and right surfaces of the lens as seperate spherical interfaces M1 and M2 respectively. Their ray transfer matrices are

$$M_1 = \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R_1} & 1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_1}{R_2} & 1 \end{bmatrix}$$



$\therefore$ Ray transfer matrix of lens is,

$$M = M_2 M_1 = \begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R_1} & 1 \end{bmatrix}$$

$$\Rightarrow \boxed{M = \begin{bmatrix} 1 & 0 \\ -(n_2 - n_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] & 1 \end{bmatrix}}$$

We know focal length of lens is given by, $\frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ $\Rightarrow$ $\boxed{M = \begin{bmatrix} 1 & 0 \\ -\frac{n_1}{f} & 1 \end{bmatrix}}$

Ray transfer matrix of a lens system:

If there are n lenses $M_1, M_2, \ldots, M_n$ placed consecutively and light propagates through each one of them in the order $M_1, M_2, M_3, \ldots, M_n$; then,

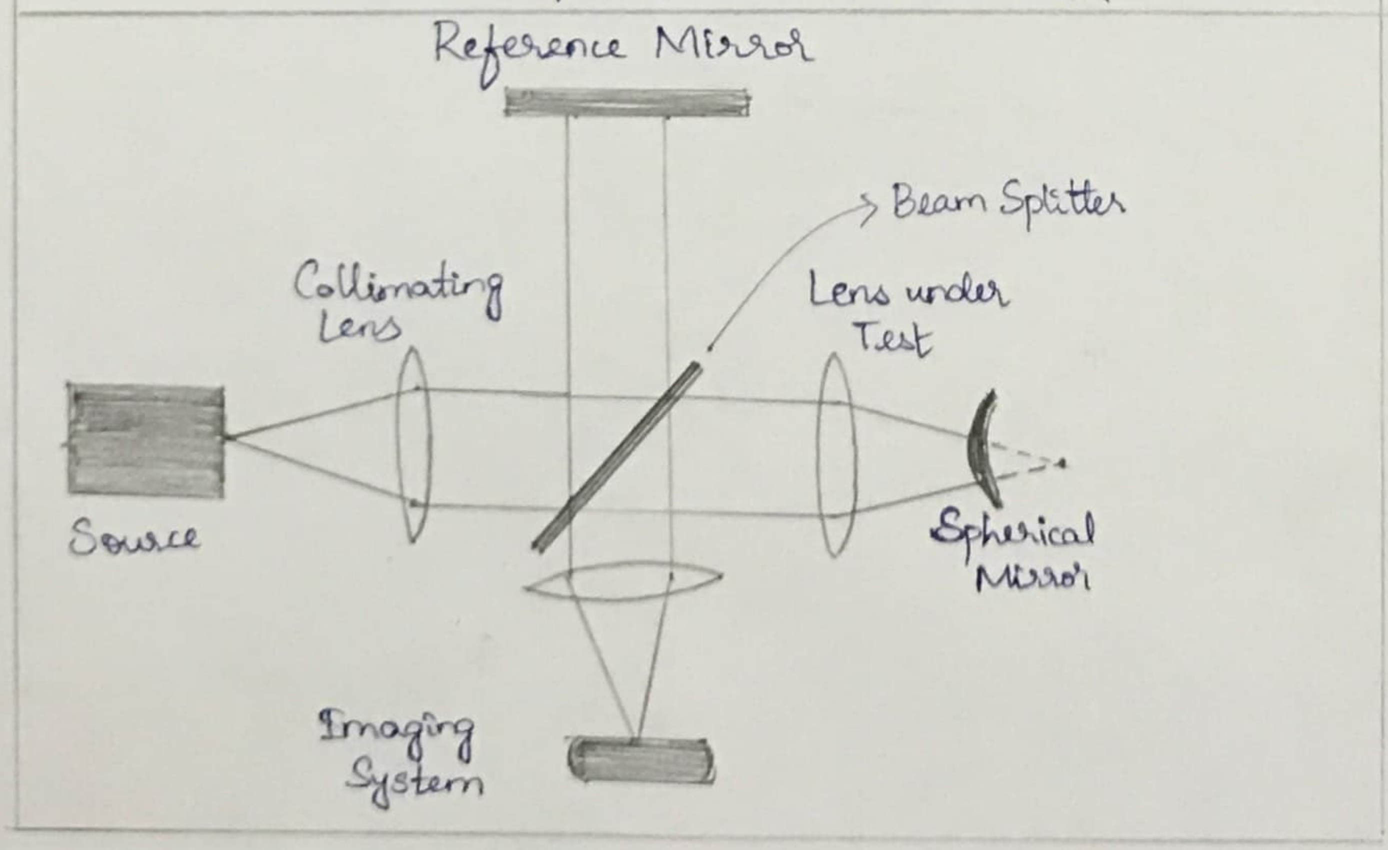
$$\boxed{M = M_n M_{n-1} \cdots M_3 M_2 M_1}$$

is the overall ray transfer matrix for the lens system.

4. Twyman – Green Interferometer

Twyman – Green Interferometer is a variant of Michelson Interferometer. It was invented by Frank Twyman and Arthur Green in 1916 and was originally intended for testing prisms and microscope objectives.

TWYMAN–GREEN INTERFEROMETER



Working :

1. The interferometer consists of a point-source of light, a Laser. Light from the laser is expanded and collimated into a parallel beam.
2. A convex spherical mirror is positioned so that its center of curvature coincides with focus of the lens being tested. An imaging system records the emergent beam for analysis.
3. The TGI consists of a rotatable mirror unlike Michelson Interferometer. Rotation of the mirror results in straight fringes appearing in the interference pattern, a fringing which is used to test quality of optical components by observing changes in fringe pattern. when component is placed in one arm of TGI.
4. A test beam is incident on unknown test part and also returns to beam splitter.
5. Beams are both split a second time, creating two complementary interferograms. One is projected towards point source, while more useful interferogram is relayed by an imaging lens to observation plane.

Lens aberration can be identified as one of the following by looking at the fringe pattern.

1. Spherical Aberration
2. Coma
3. Astigmatism
4. Curvature of Field
5. Distortion

Uses of Twyman-Green Interferometer.

It is used for testing:

1. Prisms
2. Microscope Objectives
3. Microspheres
4. Photographic Lenses

Differences with Michelson Interferometer

1. Mirror in TGI is rotatable, but in Michelson Interferometer. it is fixed.
2. TGI consists of laser which is a point-source of light. In Michelson Interferometer, light source is usually an extended source but it can also be point source.

Differences with Wavefront Shearing Interferometer

In TGI, wavefront under test is compared to a reference wavefront, which is generated by a dividing mirror and comparision mirror, which are highly corrected over an aperture equal to that of optical system under test.

This is highly difficult while testing very large aperture astronomical ~~telesp~~ telescope objectives.

In wavefront shearing interferometer, this difficulty is overcome. When one part of a wavefront is under test, a difficult part of the same wavefront acts as reference wavefront.

5.

Lens Design

Optical lens design is the process of designing a lens to meet a set of performance requirements and constraints, including cost and manufacturing limitations.

The parameters involved in lens design are:

1. Surface profile types (spherical, aspheric, holographic, diffractive etc)
2. Radius of Curvature
3. Distance to next surface
4. Material Type
5. Optionally tilt and decenter.

The process of designing a lens is computationally intensive and requires ray tracing or other techniques to model how lens affects light that passes through it.

Depending on the use case, a lens design must satisfy certain requirements. They are classified as follows:

1. Performance Requirements
   i) Optical performance (image quality) : Choice is application specific
   ii) Encircled energy.

iii) Modulation transmission function

iv) Strehl ratio

v) Ghost reflection ratio

vi) Pupil performance (size, location & aberration control)

② Physical requirements

i) Weight
iii) Dynamic Volume
ii) Static Volume
iv) Centre of gravity
v) Overall configuration requirements

③ Environmental requirements :

i) Ranges for temperature
ii) Pressure
iii) Vibration
iv) Electromagnetic Shielding.

Design constraints include realistic lens element centre and edge thicknesses, minimum & maximum airspaces between lenses, maximum constraints on entrance & exit angles, physically realizable glass index of refraction & dispersion properties.

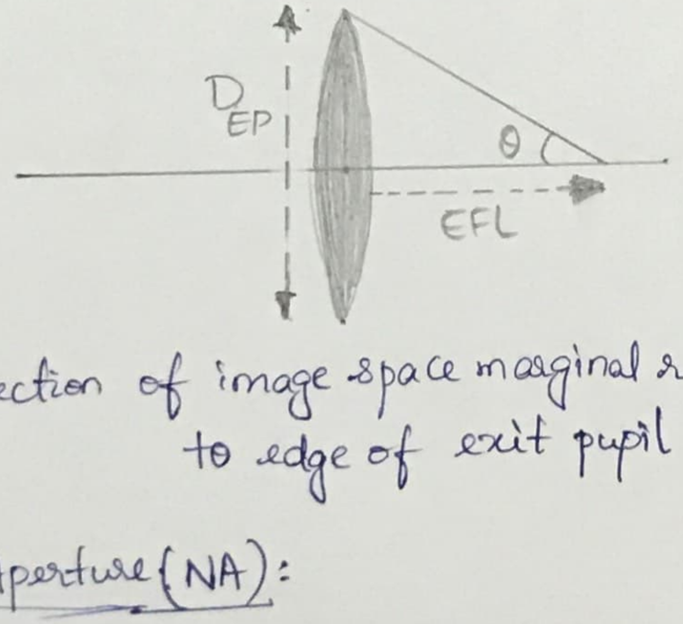
f/# Lens Iris/Aperture Setting:

Defined as the ratio of effective focal length of lens to effective aperture diameter.

f/# setting controls the following lens' parameters:

$$f/\# = \frac{EFL}{D_{EP}}$$

1. Overall light throughput
2. Depth of field
3. Ability to produce contrast at a given resolution

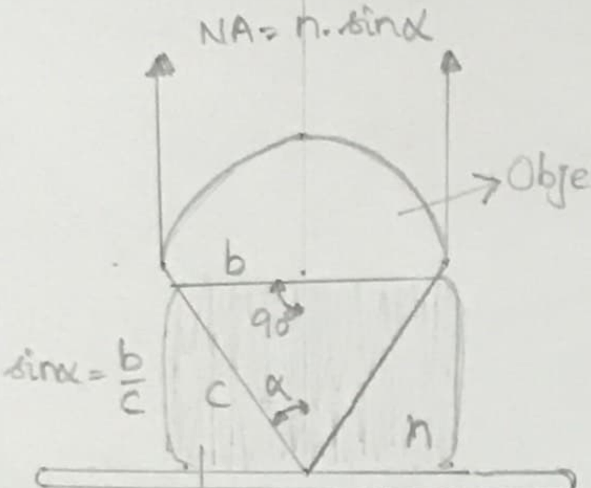


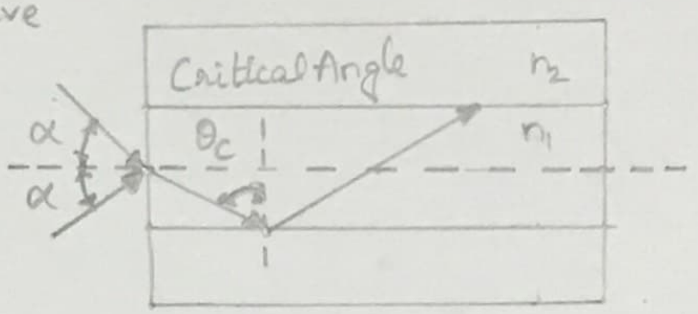
Projection of image space marginal ray angle to edge of exit pupil

Numerical Aperture (NA):

Also called cone angle of lens, defined as sine of the marginal ray angle in image space. It is the light gathering ability of an objective or condenser.

$$NA = \frac{1}{2(f/\#)}$$





$NA = \sin\alpha = \sqrt{n_1^2 - n_2^2}$

Full acceptance angle $= 2\alpha$

Numerical Aperture

Modulation Transfer Function (MTF)

1. It is a measure to compare performance of optical systems
2. Components of MTF are i) Resolution
   ii) Contrast

Resolution

i) Imaging system's ability to distinguish object-detail

ii) Expressed in terms of line-pairs per millimeter (a line pair is sequence of one black line and one white line)

Contrast/ Modulation

i) Degree of difference between elements that form an image.

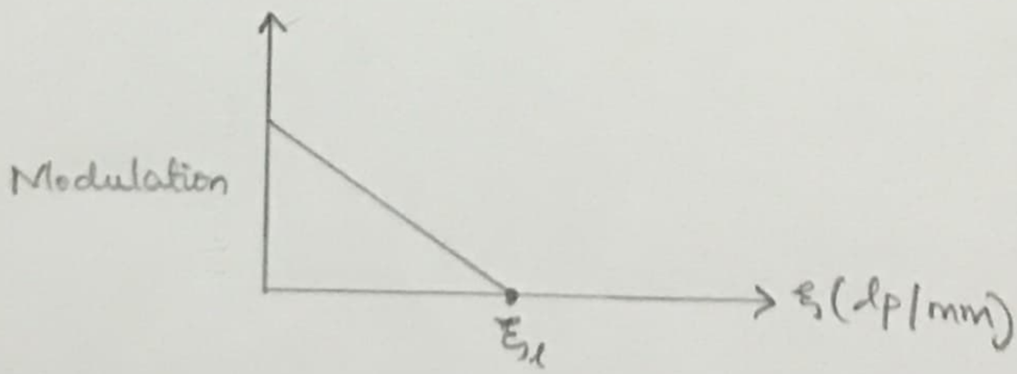
ii) Higher contrast gives different feel to an image than Lower Contrast.

iii) As spatial frequency of lines increases, contrast of image decreases. This effect is always present when working with imaging lenses of the same resolution. For the image to appear defined, black must be truly black and white truly white, with minimal amount of grayscale in between.

iv) In imaging applications, imaging lens, camera sensor and illumination play key roles in determining resulting image contrast.

MTF of a lens is its ability to transfer contrast at a particular resolution from the object to the image.

MTF is a way to incorporate resolution & contrast into a single specification. It is a function of spatial resolution, ξ_1, which refers to smallest line-pair the system can resolve.



MTF for an aberration-free lens with rectangular aperture.

Aberration Handling in Lens Design

The inability of a lens to form a perfect image is ~~called~~ caused by lens aberrations. This occurs due to some of the approximations taken during lens design. Various aberrations are listed below along with the method of dealing them.

Spherical Aberration: Rays which pass through outer zones of the lens do not meet the axis at the same point as paraxial rays and form a spot on the screen. In almost all applications where spherical aberration is present, the overall image quality is best when lens is focused close to the point of minimum size.

Coma: Linear magnification of a small object, located on the axis of the instrument, is different when different zones of lenses are used to form the image. This is minimized by arranging the lenses in a symmetric fashion so as to reduce the amount of coma.

Field Curvature: A plane object that doesn't result in a plane image of the focal plane is said to have curvature. It can only be minimized by balancing it with astigmatism.

Astigmatism:

A thin pencil of light which converges to two focal lines instead of one focal point is said to be astigmatic. A lens can't be made free of astigmatism, but image quality can be improved by balancing astigmatism and field curvature.

Distortion:

Image is distorted when object, which is supposed to be a plane figure at right angles to the axis of the lens, gives rise to an image which is not similar to itself geometrically. With the exception of certain metrological systems where critical measurements are taken from the image, distortion errors in 5 to 10 percent region usually are deemed acceptable.