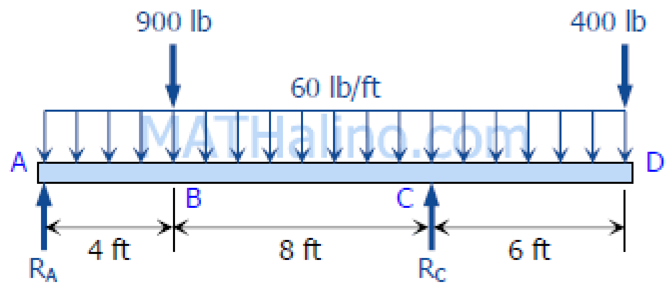
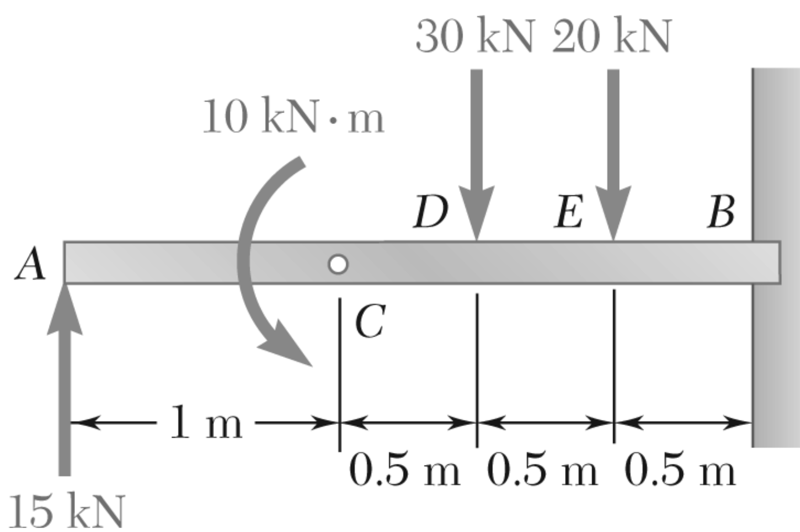
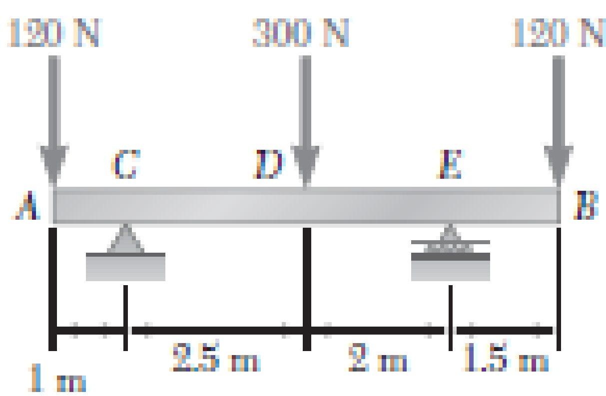


Figure P-406

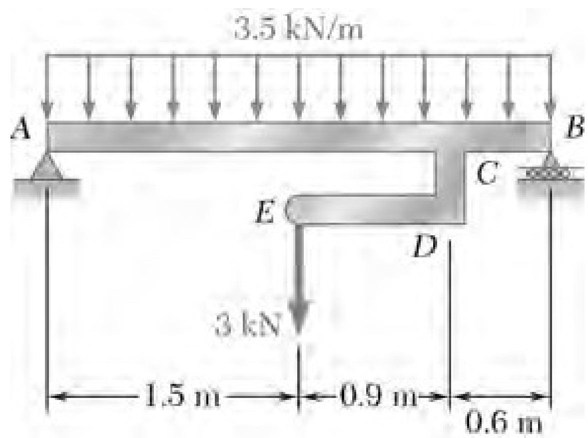
Draw shear force and bending moment diagram



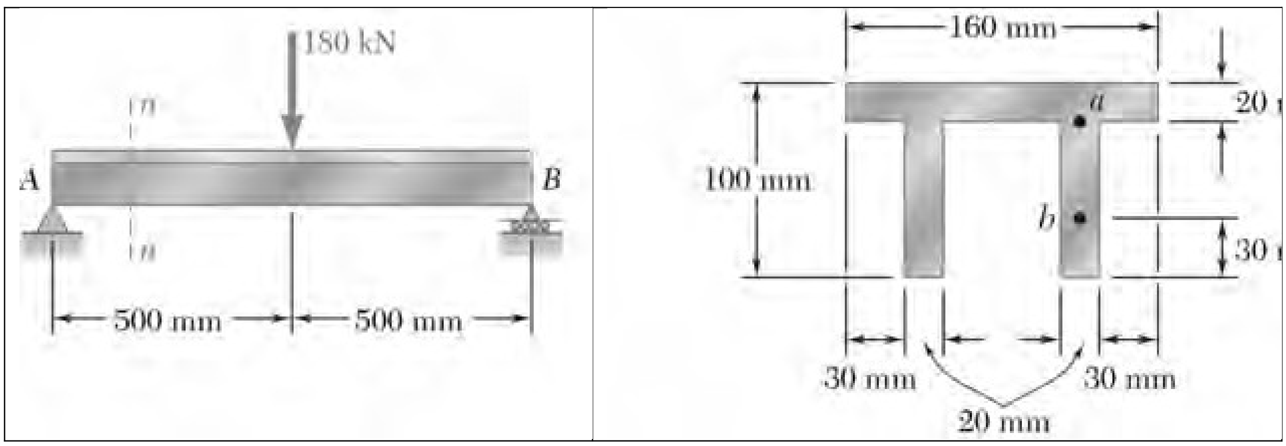
For the beam and loading shown, (a) draw the shear and bending moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.



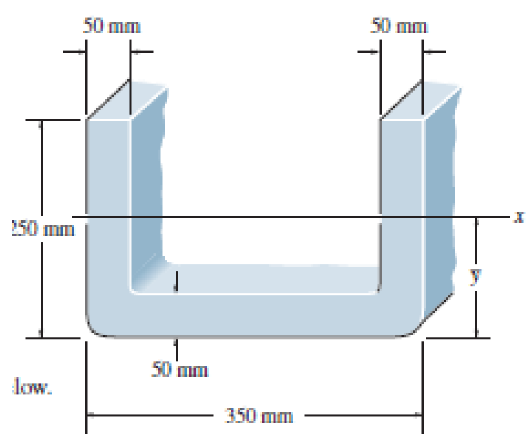
Find and construct the shear force and bending moment



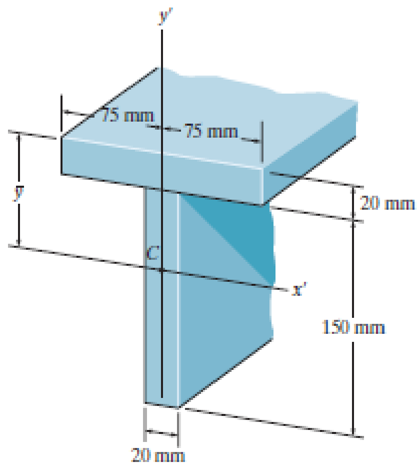
Construct the shear force and bending diagram



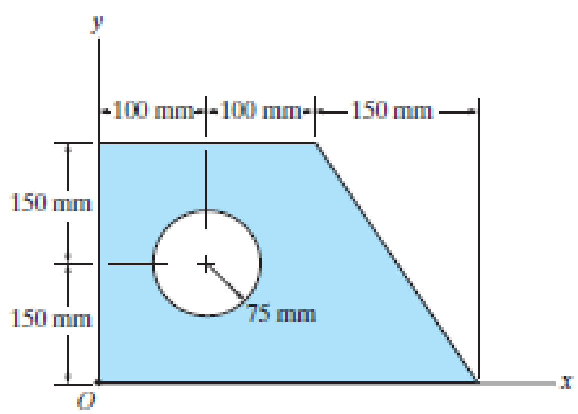
For the beam and loading shown, consider section n-n and determine the shearing stress at (*a*) point a (*b*) point *b*.



Find moment of inertia about x-x axis



Find moment of inertia about x-x- and y-y axes

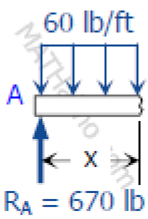


Find moment of inertia about x-x axis

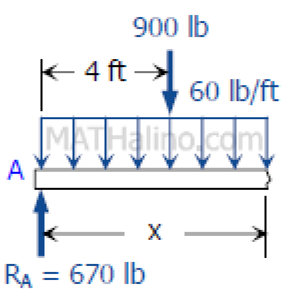
Solutions

$\Sigma M_A = 0$ ΣMA=0
$12R_C = 4(900) + 18(400) + 9[(60)(18)]$ 12RC=4(900)+18(400)+9[(60)(18)]
$R_C = 1710 \text{ lb}$ RC=1710lb

$\Sigma M_C = 0$ ΣMC=0
$12R_A + 6(400) = 8(900) + 3[60(18)]$ 12RA+6(400)=8(900)+3[60(18)]
$R_A = 670 \text{ lb}$ RA=670lb

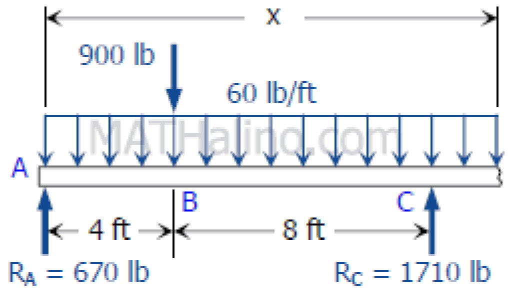


Segment AB:
$V_{AB} = 670 - 60x \text{ lb}$ VAB=670−60xlb
$M_{AB} = 670x - 60x(x/2)$ MAB=670x−60x(x/2)
$M_{AB} = 670x - 30x^2 \text{ lb}\cdot\text{ft}$ MAB=670x−30x2lb·ft



Segment BC:
$V_{BC} = 670 - 900 - 60x$ VBC=670−900−60x
$V_{BC} = -230 - 60x \text{ lb}$ VBC=−230−60xlb
$M_{BC} = 670x - 900(x-4) - 60x(x/2)$ MBC=670x−900(x−4)−60x(x/2)
$M_{BC} = 3600 - 230x - 30x^2 \text{ lb}\cdot\text{ft}$ MBC=3600−230x−30x2lb·ft

Segment CD:
$V_{CD} = 670 + 1710 - 900 - 60x$ VCD=670+1710−900−60x
$V_{CD} = 1480 - 60x \text{ lb}$ VCD=1480−60xlb

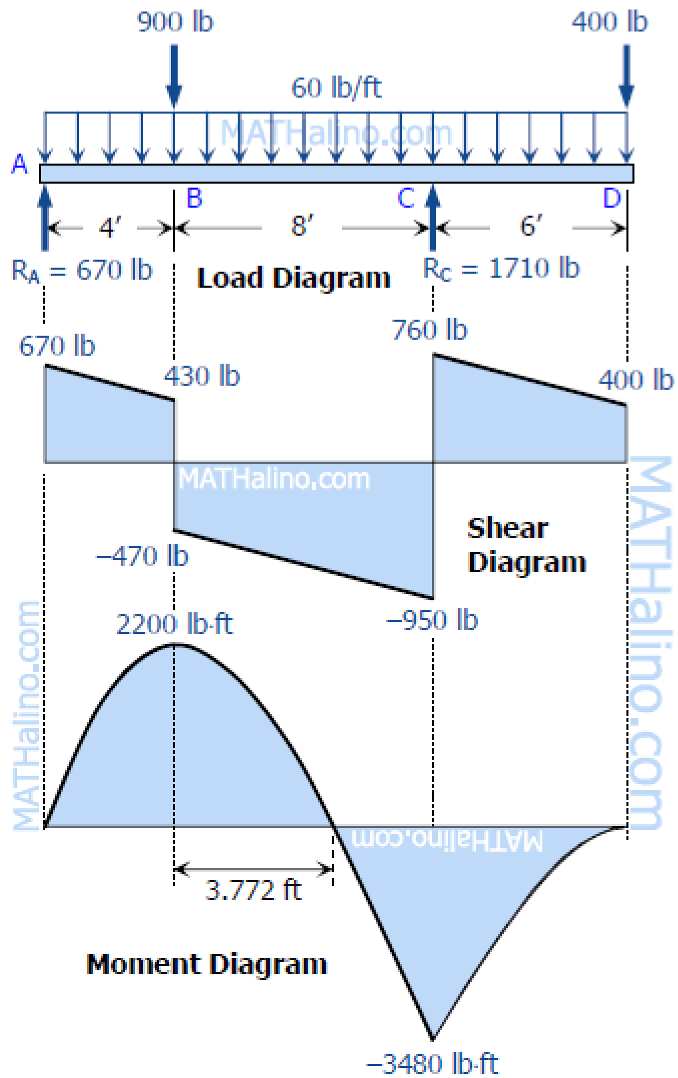


$M_{CD} = 670x + 1710(x - 12) - 900(x - 4) - 60x(x/2)$

$M_{CD} = -16920 + 1480x - 30x^2 \text{ lb}\cdot\text{ft}$

To draw the Shear Diagram:

1. V_{AB} = 670 - 60x for segment AB is linear; at x = 0, V_{AB}= 670 lb; at x = 4 ft, V_{AB} = 430 lb.
2. For segment BC, V_{BC} = -230 - 60x is also linear; at x= 4 ft, V_{BC} = -470 lb, at x = 12 ft, V_{BC} = -950 lb.
3. V_{CD} = 1480 - 60x for segment CD is again linear; at x = 12, V_{CD} = 760 lb; at x = 18 ft, V_{CD} = 400 lb.



To draw the Moment Diagram:

1. $M_{AB} = 670x - 30x^2$ for segment AB is a second degree curve; at $x = 0$, $M_{AB} = 0$; at $x = 4$ ft, $M_{AB} = 2200$ lb·ft.
2. For BC, $M_{BC} = 3600 - 230x - 30x^2$, is a second degree curve; at $x = 4$ ft, $M_{BC} = 2200$ lb·ft, at $x = 12$ ft, $M_{BC} = -3480$ lb·ft; When $M_{BC} = 0$, $3600 - 230x - 30x^2 = 0$, $x = -15.439$ ft and 7.772 ft. Take $x = 7.772$ ft, thus, the moment is zero at 3.772 ft from B.
3. For segment CD, $M_{CD} = -16920 + 1480x - 30x^2$ is a second degree curve; at $x = 12$ ft, $M_{CD} = -3480$ lb·ft; at $x = 18$ ft, $M_{CD} = 0$.

SOLUTION

(*a*) Just to the right of *A*:

$$+\!\uparrow \Sigma F_y = 0 \quad V_1 = +15\text{ kN} \quad M_1 = 0$$

Just to the left of *C*:

$$V_2 = +15\text{ kN} \quad M_2 = +15\text{ kN}\cdot\text{m}$$

Just to the right of *C*:

$$V_3 = +15\text{ kN} \quad M_3 = +5\text{ kN}\cdot\text{m}$$

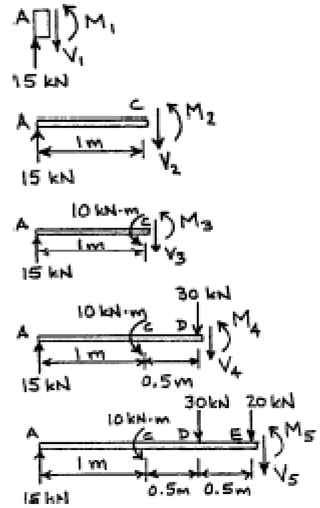
Just to the right of *D*:

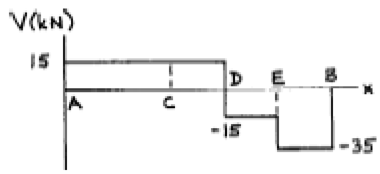
$$V_4 = -15\text{ kN} \quad M_4 = +12.5\text{ kN}\cdot\text{m}$$

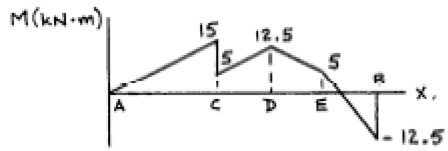
Just to the right of *E*:

$$V_5 = -35\text{ kN} \quad M_5 = +5\text{ kN}\cdot\text{m}$$

At *B*: $M_B = -12.5\text{ kN}\cdot\text{m}$



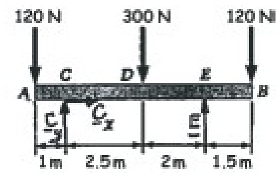




(*b*) $|V|_{max} = 35.0\text{ kN}$ $|M|_{max} = 12.50\text{ kN}\cdot\text{m}$ ◀

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_C = 0: \quad (120\text{ N})(1\text{ m}) - (300\text{ N})(2.5\text{ m}) + E(4.5\text{ m}) - (120\text{ N})(6\text{ m}) = 0$$

$$E = +300\text{ N} \qquad \mathbf{E} = 300\text{ N}\uparrow \ \triangleleft$$

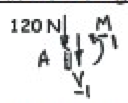
$$\Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 300\text{ N} - 120\text{ N} - 300\text{ N} - 120\text{ N} = 0$$

$$C_y = +240\text{ N} \qquad \mathbf{C} = 240\text{ N}\uparrow \ \triangleleft$$

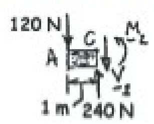
(*a*) Shear and bending moment

Just to the right of *A*:



$$+\uparrow \Sigma F_y = 0: \quad -120\text{ N} - V_1 = 0 \qquad V_1 = -120\text{ N}, \ M_1 = 0 \ \triangleleft$$

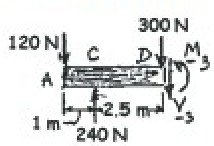
Just to the right of *C*:



$$+\uparrow \Sigma F_y = 0: \quad 240\text{ N} - 120\text{ N} - V_2 = 0 \qquad V_2 = +120\text{ N} \ \triangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: \quad M_2 + (120\text{ N})(1\text{ m}) = 0 \qquad M_2 = -120\text{ N}\cdot\text{m} \ \triangleleft$$

Just to the right of *D*:



$$+\uparrow \Sigma F_y = 0: \quad 240 - 120 - 300 - V_3 = 0 \qquad V_3 = -180\text{ N} \ \triangleleft$$

$$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 + (120)(3.5) - (240)(2.5) = 0 \qquad M_3 = +180\text{ N}\cdot\text{m} \ \triangleleft$$

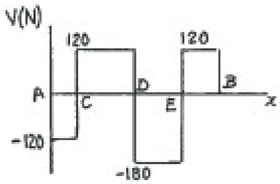
Just to the right of E:

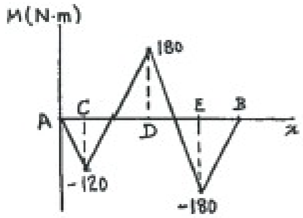
$+\Sigma F_y = 0$: $V_4 - 120\text{ N} = 0$ $\qquad V_4 = +120\text{ N}$ ◁

$+\circlearrowleft \Sigma M_4 = 0$: $-M_4 - (120\text{ N})(1.5\text{ m}) = 0$ $\qquad M_4 = -180\text{ N}\cdot\text{m}$ ◁

At B: $\qquad V_B = M_B = 0$ ◁

(*b*) $\qquad |V|_{max} = 180.0\text{ N}$ ◀





$|M|_{max} = 180\text{ N}\cdot\text{m}$ ◀

SOLUTION

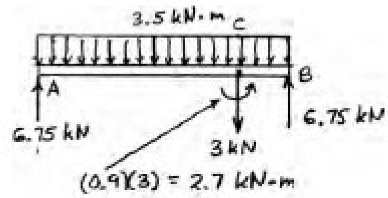
Reaction at A:

$$+\circlearrowleft \Sigma M_B = 0: \quad -3.0A + (1.5)(3.0)(3.5) + (1.5)(3) = 0$$

$$A = 6.75 \text{ kN} \uparrow$$

Reaction at B: $\quad B = 6.75 \text{ kN} \uparrow$

Beam ACB and loading: (See sketch.)



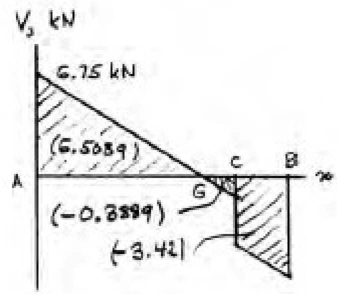
Areas of load diagram:

A to C: $\quad (2.4)(3.5) = 8.4 \text{ kN}$

C to B: $\quad (0.6)(3.5) = 2.1 \text{ kN}$

Shear diagram:

$$V_A = 6.75 \text{ kN}$$
$$V_{C^-} = 6.75 - 8.4 = -1.65 \text{ kN}$$
$$V_{C^+} = -1.65 - 3 = -4.65 \text{ kN}$$
$$V_B = -4.65 - 2.1 = -6.75 \text{ kN}$$



Over A to C, $\quad V = 6.75 - 3.5x$

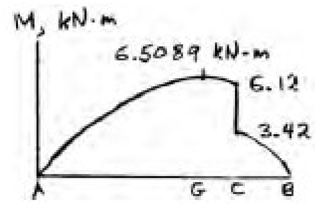
At G, $\quad V = 6.75 - 3.5x_G = 0 \quad x_G = 1.9286 \text{ m}$

Areas of shear diagram:

A to G: $\quad \frac{1}{2}(1.9286)(6.75) = 6.5089 \text{ kN} \cdot \text{m}$

G to C: $\quad \frac{1}{2}(0.4714)(-1.65) = -0.3889 \text{ kN} \cdot \text{m}$

C to B: $\quad \frac{1}{2}(0.6)(-4.65 - 6.75) = -3.42 \text{ kN} \cdot \text{m}$



PROBLEM 5.44 *(Continued)*

Bending moments:

$$M_A = 0$$
$$M_G = 0 + 6.5089 = 6.5089 \text{ kN} \cdot \text{m}$$
$$M_{C^-} = 6.5089 - 0.3889 = 6.12 \text{ kN} \cdot \text{m}$$
$$M_{C^+} = 6.12 - 2.7 = 3.42 \text{ kN} \cdot \text{m}$$
$$M_B = 3.42 - 3.42 = 0$$

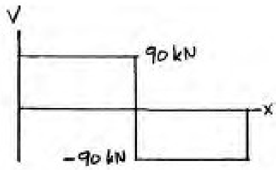
(a) $\quad |V|_{max} = 6.75 \text{ kN}$ ◀

(b) $\quad |M|_{max} = 6.51 \text{ kN} \cdot \text{m}$ ◀

SOLUTION

Draw the shear diagram. $|V|_{max} = 90$ kN

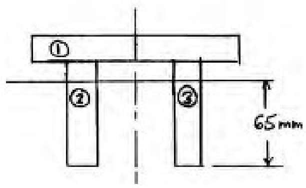
Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3\,\text{mm}^3)$	$d(\text{mm})$	$Ad^2(10^6\text{mm}^4)$	$\bar{I}(10^6\text{mm}^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	−25	1.000	0.8533
③	1600	40	64	−25	1.000	0.8533
Σ	6400		416		4.000	1.8133



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416\times 10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = (4.000 + 1.8133)\times 10^6\text{mm}^4$$

$$= 5.8133\times 10^6\text{mm}^4 = 5.8133\times 10^{-6}\text{m}^4$$

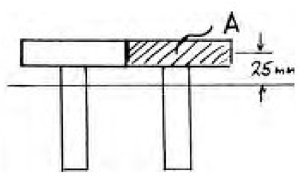


(*a*) $A = (80)(20) = 1600 \text{ mm}^2$

$\bar{y} = 25$ mm

$$Q_a = A\bar{y} = 40\times 10^3\text{mm}^3 = 40\times 10^{-6}\text{m}^3$$

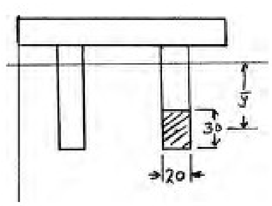
$$\tau_a = \frac{VQ_a}{It} = \frac{(90\times 10^3)(40\times 10^{-6})}{(5.8133\times 10^{-6})(20\times 10^{-3})} = 31.0\times 10^6\text{Pa}$$



$\tau_a = 31.0$ MPa ◀

(*b*) $A = (30)(20) = 600 \text{ mm}^2 \quad \bar{y} = 65 - 15 = 50$ mm

$$Q_b = A\bar{y} = 30\times 10^3\text{mm}^3 = 30\times 10^{-6}\text{m}^3$$

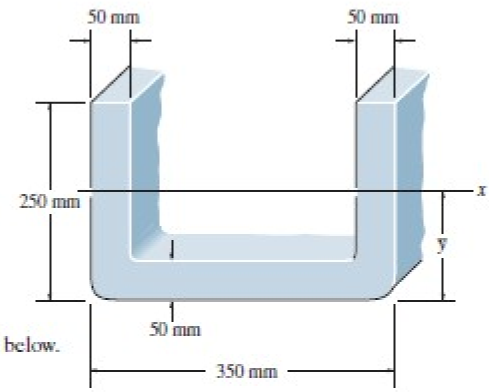


$$\tau_b = \frac{VQ_b}{It} = \frac{(90\times 10^3)(30\times 10^{-6})}{(5.8133\times 10^{-6})(20\times 10^{-3})} = 23.2\times 10^6\text{Pa}$$

$\tau_b = 23.2$ MPa ◀

***10–28.**

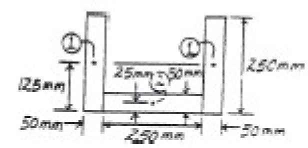
Determine the location $\bar{y}$ of the centroid of the channel's cross-sectional area and then calculate the moment of inertia of the area about this axis.



SOLUTION

Centroid: The area of each segment and its respective centroid are tabulated below.

Segment	A (mm^2)	$\tilde{y}$ (mm)	$\tilde{y}A$ (mm^3)
1	100(250)	125	3.125(10^6)
2	250(50)	25	0.3125(10^6)
Σ	37.5(10^3)		3.4375(10^6)

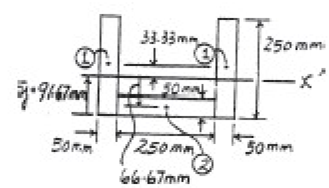


Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{3.4375(10^6)}{37.5(10^3)} = 91.67 \text{ mm} = 91.7 \text{ mm} \qquad \textbf{Ans.}$$

Moment of Inertia: The moment of inertia about the x' axis for each segment can be determined using the parallel-axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

Segment	A_i (mm^2)	$(d_y)_i$ (mm)	$(\bar{I}_{x'})_i$ (mm^4)	$(Ad_y^2)_i$ (mm^4)	$(I_{x'})_i$ (mm^4)
1	100(250)	33.33	$\frac{1}{12}(100)(250^3)$	27.778(10^6)	157.99(10^6)
2	250(50)	66.67	$\frac{1}{12}(250)(50^3)$	55.556(10^6)	58.16(10^6)

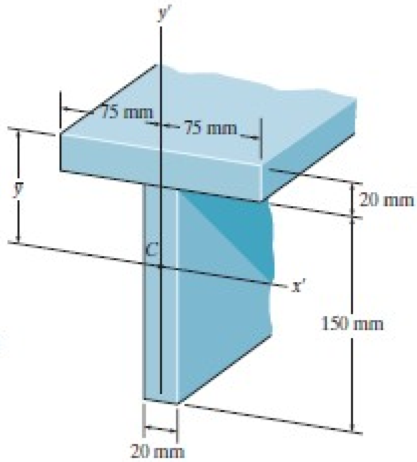


Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 216.15(10^6) \text{ mm}^4 = 216(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

10–29.

Determine $\bar{y}$, which locates the centroidal axis x' for the cross-sectional area of the T-beam, and then find the moments of inertia $I_{x'}$ and $I_{y'}$.



SOLUTION

Centroid. Referring to Fig. *a*, the areas of the segments and their respective centroids are tabulated below.

Segment	$A(\text{mm}^2)$	$\tilde{y}(\text{mm})$	$\tilde{y}A(\text{mm}^3)$
1	150(20)	10	$30(10^3)$
2	20(150)	95	$285(10^3)$
Σ	$6(10^3)$		$315(10^3)$

Thus, $\bar{y} = \dfrac{\Sigma\tilde{y}^2 A}{\Sigma A} = \dfrac{315(10^3)}{6(10^3)} = 52.5\text{ mm}$ **Ans.**

Moment of Inertia. The moment of inertia about the x' axis for each segment can be determined using the parallel axis theorem, $I_{x'} = \bar{I}_{x'} + Ad_y^2$. Referring to Fig. *b*,

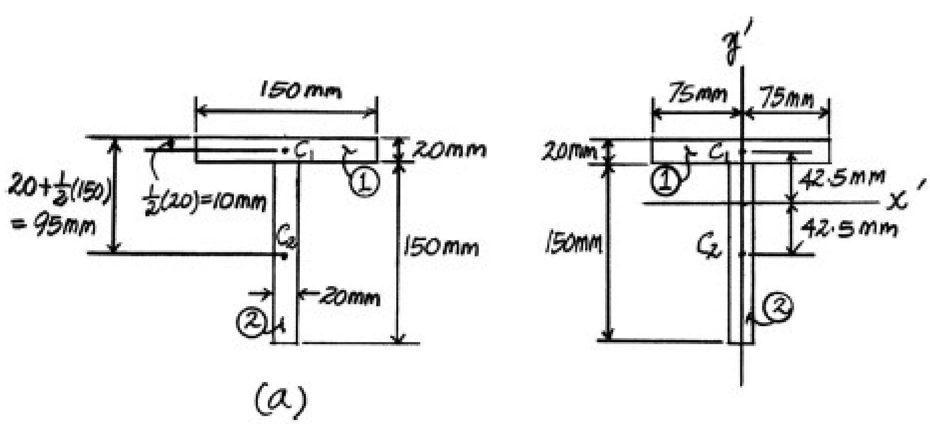
Segment	$A_i(\text{mm}^2)$	$(d_y)_i(\text{mm})$	$(\bar{I}_{x'})_i(\text{mm}^4)$	$(Ad_y^2)_i(\text{mm}^4)$	$(\bar{I}_{x'})_i(\text{mm}^4)$
1	150(20)	42.5	$\frac{1}{12}(150)(20^3)$	$5.41875(10^6)$	$5.51875(10^6)$
2	20(150)	42.5	$\frac{1}{12}(20)(150^3)$	$5.41875(10^6)$	$11.04375(10^6)$

Thus

$$I_{x'} = \Sigma(I_{x'})_i = 16.5625(10^6)\text{ mm}^4 = 16.6(10^6)\text{ mm}^4$$ **Ans.**

Since the y' axis passes through the centroids of segments 1 and 2,

$$I_{y'} = \frac{1}{12}(20)(150^3) + \frac{1}{12}(150)(20^3)$$
$$= 5.725(10^6)\text{ mm}^4$$ **Ans.**

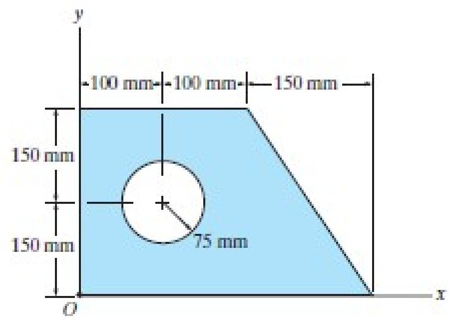


(a)

Ans:
$\bar{y} = 52.5$ mm
$I_{x'} = 16.6(10^6)\text{ mm}^4$
$I_{y'} = 5.725(10^6)\text{ mm}^4$

***10–32.**

Determine the moment of inertia I_x of the shaded area about the x axis.



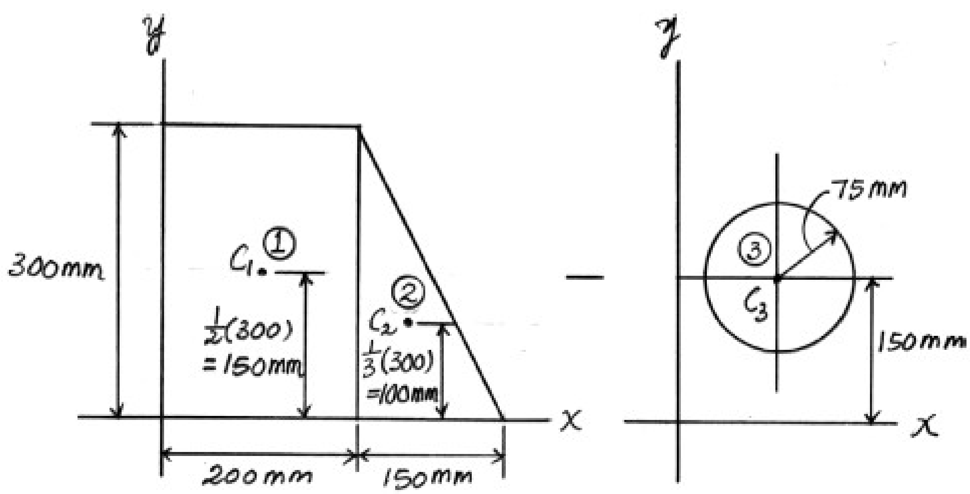
SOLUTION

Moment of Inertia. The moment of inertia about the x axis for each segment can be determined using the parallel axis theorem, $I_x = \bar{I}_{x'} + Ad^2_y$. Referring to Fig. *a*

Segment	$A_i(\text{mm}^2)$	$(d_y)_i(\text{mm})$	$(\bar{I}_{x'})_i(\text{mm}^4)$	$(Ad_y)^2_i(\text{mm}^4)$	$(I_x)_i(\text{mm}^4)$
1	$200(300)$	150	$\frac{1}{12}(200)(300^3)$	$1.35(10^9)$	$1.80(10^9)$
2	$\frac{1}{2}(150)(300)$	100	$\frac{1}{36}(150)(300^3)$	$0.225(10^9)$	$0.3375(10^9)$
3	$-\pi(75^2)$	150	$-\frac{\pi(75^4)}{4}$	$-0.3976(10^9)$	$-0.4225(10^9)$

Thus,

$$I_x = \Sigma(I_x)_i = 1.715(10^9)\ \text{mm}^4 = 1.72(10^9)\ \text{mm}^4 \qquad \textbf{Ans.}$$



(a)

Ans:
$I_x = 1.72(10^9)\ \text{mm}^4$