Indian Institute of Technology Patna
MA201: Mathematics III
Mid Semester Exam (17-09-2013)

Time: 2hrs — Max. Marks: 30

Note: Answer all questions. Give precise and brief answer. Standard formulae may be used.

Q.1. Answer all parts at one place. [1x8]

a. Sketch the set $S = \{z \in \mathbb{C} : |Rez| > 2\}$. Is S a *Domain*.

b. For a non-zero z and $-\pi < Argz \leq \pi$, show that $|z-1| \leq ||z|-1| + |z||Argz|$.

c. Determine the *domain of definition* of the function $f(z) = \dfrac{e^z}{z\cos z}$.

d. A necessary condition for complex valued function $f(z) = u(x,y) + iv(x,y)$ to be differentiable at a point z_0 is $\dfrac{\partial f}{\partial \bar{z}}(z_0) = 0$. *Justify.*

e. Find the limit $\lim\limits_{z \to i} \dfrac{z^4 - 1}{z - i}$

f. Find $M > 0$ such that $\left| \int\limits_C z^2 dz \right| \leq M$, where C is circle of radius 1.

g. $Log(z-i)$ is analytic at all points in $z-$plane except for $x \cdots$ and $y \cdots$.

h. Find the principal value of $(-i)^i$.

Q.2. Discuss the continuity of the function $f(z) = \begin{cases} \frac{x^2+iy^2}{|z|^2}, & z \neq 0; \\ 0, & z = 0. \end{cases}$ at $z = 0$. [2]

Q.3. *State C-R equations* for an analytic function at a point.
Given that $f(z) = \sin x \cosh y + i \cos x \sinh y$ is analytic for all $z \in \mathbb{C}$, verify that C-R equations are satisfied. [3]

Q.4. Is the function $u(x,y) = e^{-x}\cos y + xy$, a *harmonic function* throughout xy plane? If yes, then find harmonic conjugate of $u(x,y)$ and hence the corresponding analytic function. [3]

Q.5. Find all z such that $\cos z = \cosh 2$. [3]

Q.6. Prove following inequality: [4]

(*i*) $|\sinh y| \leq |\sin z| \leq \cosh y$,

(*ii*) $|\sinh y| \leq |\cos z| \leq \cosh y$.

P.T.O

Q.7. Let $\phi(z) = U(x,y)+iV(x,y)$ be defined over a simply connected domain $\mathcal{D} \subset \mathbb{C}$ such that at each point in $\mathcal{D}$, functions $U(x,y)$ & $V(x,y)$ satisfy C-R equations and U_x, U_y, V_x, V_y exists & are continuous. Let C be a closed curve in $\mathcal{D}$, then, giving proper justification, evaluate $\int_C e^{\phi(z)}dz$. [3]

Q.8. Evaluate $\int_{C_1} \overline{z}dz$ and $\int_{C_2} \overline{z}dz$, where C_1 is semi circular path from 1 to -1 and C_2 is polygonal path from 1 to -1, respectively, shown in Figure-1. [4]

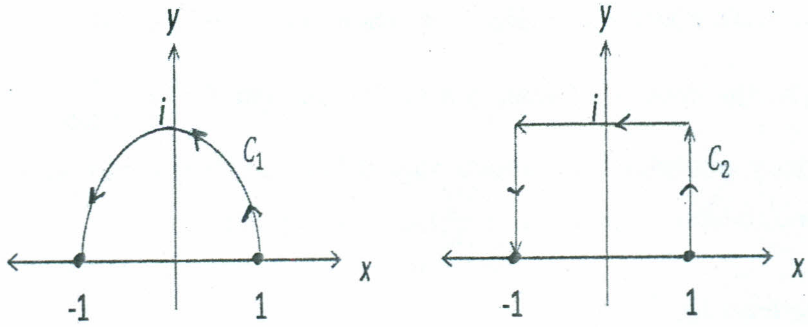

Figure 1: Two contours C_1 and C_2 joining -1 and 1

Good Luck
