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MA - 225: B.Tech. II year
Tutorial Sheet-4

1. (i) Comment on 'Variance of a $B(n,p)$ distribution can be more than its mean'. False
   (ii) Let the RV X has a $B(6, 0.5)$ distribution. Check which outcome is most likely. $P(X=3)$
   (iii) Find the k^{th} factorial moment of a $B(n,p)$ and $P(\lambda)$ distribution. $\frac{n!}{(n-k)!} p^k$, λ^k
   * (iv) Ten percent of the articles produced by a certain machine are defective. If 10 independent articles fabricated by this machine are taken at random what is the probability that exactly two of them are defective? (0.194) ⟶ Why only binomial, Why not Hyper Geometric ?
   (v) We are interested in the proportion p of defective in a batch of manufactured articles. We draw at random with replacement a sample of 20 articles from the batch. Let X be the number of defective articles in the sample. Write the PMF of X. For $p = 0.25$ find $P(X = 10), P(X \geq 2)$ and compare them with approximate probability obtained through Poisson approximation 0.01, 0.975 ↓ 0.018, 0.9595
   (vi) A device is made up of five independent components and it will operate if at least 4 of its 5 components are active. Each components operates with probability 0.95. Find the probability that a device taken at random operates (0.997) 0.977
   (vii) During a war 1 ship out of 9 was sunk on an average in making a certain voyage. Find the probability that exactly 3 out of a convoy of 6 ships would arrive safely 0.0001
   (viii) In the long run 3 vessels out of every 100 are sunk. If 10 vessels are out, what is the probability that exactly 6, at least 6 will arrive safely 1.35×10^{-7}, 1.38×10^{-7}
   (ix) Find the probability $P(X \geq 1 \mid X \leq 1)$ where $X \sim P(5)$. (5/6)

2. An unprepared student appears a true false examination consisting of 10 questions and randomly guesses answers. (i) Find the probability that he guesses correctly at least five times (319/512)
   (ii) Find the probability that he guesses correctly nine times 5/512
   (iii) Find the smallest n so that probability of guessing at least n correct answers is less than 1/2. (6)

3. A owner of a hotel with five room is considering buying TV sets to rent to room occupants. He expects that about half of his customers would be willing to rent sets and finally he buy three sets. Assume that rooms are full all times: (i) What fraction of the evening will there be more requests than TV sets (ii) Find the probability that a customer who requests a TV set will get it (iii) If owner's cost per set per day is C what rent R must be charged in order to break even (neither gain nor loss) in the long run.

4. Consider a binomial distribution with 4 independent trials where it is known that probabilities of 1 and 2 successes are 2/3 and 1/3 respectively. Find the parameter p of the distribution. Also find the mean and variance of the distribution. (1/4, 1, 3/4)

5. A coin is biased so that a head is thrice as likely to appear as a tail. Suppose that the coin is tossed 5 times, find the probability of getting (i) at least 3 heads, (ii) at most 3 heads, (iii) exactly 3 tails. (459/512, 47/128, 45/512)

6. A fair die is thrown, and an outcome of 4 or 5 is considered to be a successes. If the die is thrown 9 times and X denotes the number of successes, find (i) mean and variance of X (ii) $P(X = 2)$ (iii) $P(X \leq 2)$ (iv) $P(X \geq 2)$. (3, 2, 0.234, 0.378, 0.857)

7. The overall percentage of failures in a certain examination is 40. What is the probability that out of a group of 6 candidates at least 4 passed the examinations? (1701/3125)

8. A certain company produces bulbs of which 10% are defective. What is the probability of getting exactly 3 defective in a sample of 10 bulbs selected at random. Compute the probability using Poisson approximations and also compare your result with that obtained from binomial distribution. (0.061 by approximation, 0.057)

9. Derive a recurrence formula formula to find the the k^{th} central moments of a $B(n,p)$ distribution. Also discuss the case of $P(\lambda)$ distribution.

10. A machine normally makes items of which 4% are defective. Every hour the producer draws a sample of size 10 for inspection. If the sample contains no defective items he does not stop the machine. What is the probability that the machine will not stopped when it has started producing items of which 10% are defective. (0.9^{10})

11. One per thousand of population is subject to certain kinds of accident each year. Given that an insurance company has insured 5000 persons from the population, find the probability that at most 2 persons will incur this accident. ($15.5e^{-5}$) $18.5e^{-5}$

12. In a certain factory producing razor blades, there is 1% for any blades to be defective. The blades are in packets of 10. In a consignment of 1000 packets, calculate the approximate number of packets containing (i) no defective blades (ii) one defective blades (iii) at most two defective blades (iv) at least two defective blades. (905, 90, 1000, 5)

13. A certain airline company having observed that 5% of the persons making reservations on a flight do not show up for the flight, sells 100 seats on a plane that has 95 seats. What is the probability that there will a seat available for every person who shows up for the flight? 0.564

14. A pair of die is rolled 50 times. Find the probability of getting a double six at least three times. 0.1617

15. Suppose number of accidents occurring on a highway each day is a random variable having a $P(3)$ distribution.
(i) Find the probability that three or more accidents occur today. (0.5768)
(ii) Repeat part (i) under the assumption that at least one accident had already occurred today. (0.607)

16. Suppose that an airplane engine will fail, when in flight, with probability $1 - p$ independently from engine to engine. Suppose that airplane will make a successful flight if at least 50% of engines remain operative. For what values of p is a four engine plane preferable to a two engine plane? (probability at least 2/3)

17. Suppose that a trainee soldier shoots a target according to a geometric distribution. If the probability that a target is shot in any one trial is 0.8, find the probability that odd number of trials are needed. What is the probability that even number of trials will be needed. (0.833 for odd number of trials)

18. In a company 3% defective components are produced. Find the probability that at least 6 components are to be examined in order to get 3 defective. (0.999)

19. A marksman is firing bullets at a target and the probability of hitting the target at any trial is 0.7. Find the probability that his seventh shot is his forth hit. (0.129)

20. (i) Probability that a certain person will die from a certain respiratory infection is 0.002. Find the probability that fewer than 5 of the next 2000 so infected will die. What are the mean and variance in this case. (0.6288, 4, 4)

(ii) According to Chebyshev's theorem, there is a probability of at least 3/4 that the number of persons to die among 2000 persons infected will fall within what interval? (interval is (0,8))

21. An electronic firm claims that the portions of defective units of a certain process is 5%. A buyer has a standard procedure of inspecting 15 units selected randomly from a large lot. On a Particular occasion the buyer found 5 items defective. What is the probability of this occurrence, given that the claim of 5% defective is correct? What would be your reaction if you were the buyer? (0.000562)

22. Imperfections in computer circuits boards and computer chips lend themselves to statistical treatment. For a particular type of board the probability of diode failure is 0.03. Suppose a circuit board contains 200 diodes. What is the mean number of failures among the diodes? What is the variance? The board will work if there are no defective diodes. What is the probability that a board will work? (6, 5.82, 0.0025 by approximation, 0.0023 using binomial)