Indian Institute of Technology Patna
End semester examination
(November 24, 2010)

MA-101 (Mathematics I) **M.M. 50 Time: 3Hrs.**

Note: This question paper has **TWO** pages and contains **FIFTEEN** questions. Please check all the pages and inform discrepancy, if any. Answer **ALL** the questions. Marks against each question are indicated.

1. Introducing polar coordinates $x = r\cos\theta$ and $y = r\sin\theta$ changes $f(x,y)$ to $g(r,\theta)$. Find the value of $\partial^2 g/\partial\theta^2$ at the point $(r,\theta) = (2, \pi/2)$, given that

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial y^2} = 1 \quad \text{at that point.}$$ [3]

2. Change the order of the following double integral:

$$\int_0^2 \int_{\sqrt{2x-x^2}}^{\sqrt{2x}} f(x,y)\,dy\,dx.$$ [4]

3. Use Lagrange Multiplier method to find the point closest to the origin on the curve of intersection of the plan $x + y + z = 1$ and the cone $z^2 = 2x^2 + 2y^2$. [4]

4. Find the absolute maxima and minima of the function

$f(x,y) = (4x - x^2)\cos y$, on the rectangular region $1 \le x \le 3, -\pi/4 \le y \le \pi/4$. [5]

5. Use an appropriate transformation to find the integral

$$\int_0^{2/3}\int_y^{2-2y} (x+2y)e^{(y-x)}\,dx\,dy$$ [5]

6. Find a nonzero h for which

$$F(x,y) = h(x)[x\sin y + y\cos y]\hat{i} + h(x)[x\cos y - y\sin y]\hat{j}$$

is conservative. [3]

7. State Stokes' theorem and verify it for the vector function $F(x,y,z) = 2z\,\hat{i} + 3x\,\hat{j} + 5y\,\hat{k}$ taking σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \ge 0$ with upward orientation, and C to be positively oriented circle $x^2 + y^2 = 4$ that forms boundary of σ in the xy plane. [1+5]

8. A heat seeking particle is located at the point (2,3) on a flat metal plate whose temperature at a point *(x, y)* is $T(x,y)=10-8x^2-2y^2$ Find an equation of the trajectory of the particle if it moves continuously in the direction of maximum temperature increase. [3]

9. Let $f(x,y)=\frac{y}{|y|}\sqrt{x^2+y^2}$, $y\neq 0$ and $f(x,y)=0, y=0$. Show that f has all directional derivatives at (0,0) but f is not differentiable at (0,0). [4]

10. Find the volume bounded by the surfaces $z=x^2+3y^2$ and $z=8-x^2-y^2$. [3]

11. Using the $\delta-\varepsilon$ definition show that $f(x)=x$ is continuous on [0, 1].

True or False: $f:[a,b]\to R$ is well defined and continuous function $\Rightarrow f([a,b])=[f(a),f(b)]$ (Hint: Think by geometry). In case of False, write the correct relation between $f([a,b])$ and $[f(a),f(b)]$ in above statement. [2]

12. Compute upper integral and lower integral for the function $f:[0,1]\to R$ where $f(x)=1$ for $x\in[0,1]\cap Q$ and $f(x)=0$ otherwise (Q is the set of rational numbers). Further, conclude whether this function is Riemann integrable or not. [2]

13. Discuss the convergence of the improper integral $\int_1^\infty \frac{dx}{\sqrt{x^3+1}}$. [2]

14. Find all points of intersection of the polar curves $r=1$ and $r^2=2\sin 2\theta$. [2]

15. Check for uniform convergence: $f_n(x)=\frac{x}{1+nx}$: $x\in[0,4]$, $n\in N$. [2]

ALL THE BEST