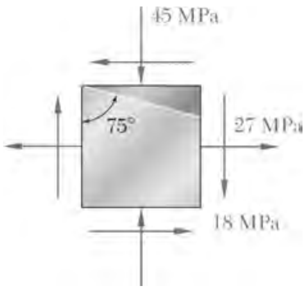
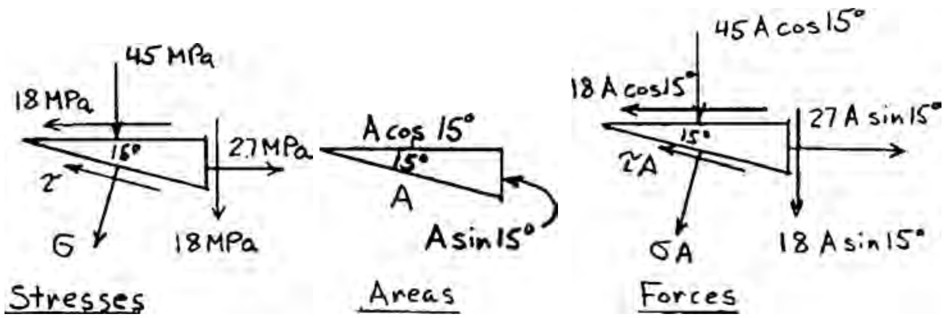
CHAPTER 7



PROBLEM 7.1

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION



$$+\swarrow \Sigma F = 0: \quad \sigma A + 18A\cos 15^\circ \sin 15^\circ + 45A\cos 15^\circ \cos 15^\circ - 27A\sin 15^\circ \sin 15^\circ + 18A\sin 15^\circ \cos 15^\circ = 0$$

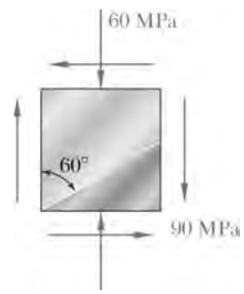
$$\sigma = -18\cos 15^\circ \sin 15^\circ - 45\cos^2 15^\circ + 27\sin^2 15^\circ - 18\sin 15^\circ \cos 15^\circ$$

$$\sigma = -49.2 \text{ MPa} \blacktriangleleft$$

$$+\nwarrow \Sigma F = 0: \quad \tau A + 18A\cos 15^\circ \cos 15^\circ - 45A\cos 15^\circ \sin 15^\circ - 27A\sin 15^\circ \cos 15^\circ - 18A\sin 15^\circ \sin 15^\circ = 0$$

$$\tau = -18(\cos^2 15^\circ - \sin^2 15^\circ) + (45 + 27)\cos 15^\circ \sin 15^\circ$$

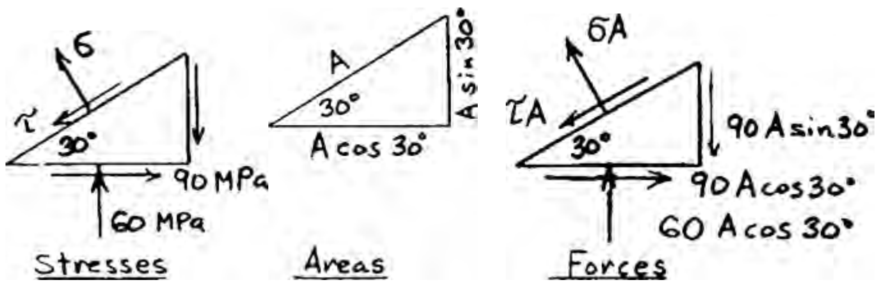
$$\tau = 2.41 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.2

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION



$+\nwarrow \Sigma F = 0$

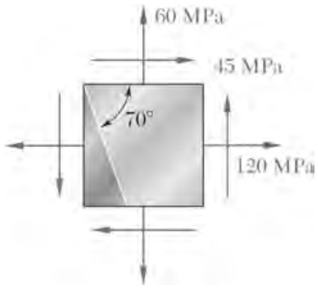
$$\sigma A - 90A\sin 30^\circ\cos 30^\circ - 90A\cos 30^\circ\sin 30^\circ + 60A\cos 30^\circ\cos 30^\circ = 0$$

$$\sigma = 180\sin 30^\circ\cos 30^\circ - 60\cos^2 30^\circ = 32.9 \text{ MPa} \quad ◀$$

$+\swarrow \Sigma F = 0$

$$\tau A + 90A\sin 30^\circ\sin 30^\circ - 90A\cos 30^\circ\cos 30^\circ - 60A\cos 30^\circ\sin 30^\circ = 0$$

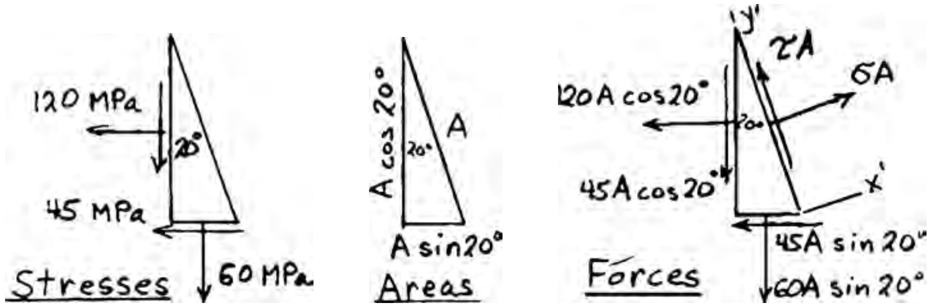
$$\tau = 90(\cos^2 30^\circ - \sin^2 30^\circ) + 60\cos 30^\circ\sin 30^\circ = 71.0 \text{ MPa} \swarrow \quad ◀$$



PROBLEM 7.3

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION



$+\nearrow\ \Sigma F = 0:$

$$\sigma A - 120A\cos 20^\circ \cos 20^\circ - 45A\cos 20^\circ \sin 20^\circ - 45A\sin 20^\circ \cos 20^\circ - 60A\sin 20^\circ \sin 20^\circ = 0$$

$$\sigma = 120\cos^2 20^\circ + 45\cos 20^\circ \sin 20^\circ + 45\sin 20^\circ \cos 20^\circ + 60\sin^2 20^\circ$$

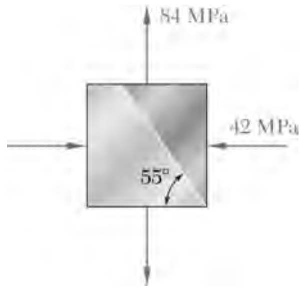
$\sigma = 14.19$ MPa ◀

$+\nwarrow\ \Sigma F = 0:$

$$\tau A + 120A\cos 20^\circ \sin 20^\circ - 45A\cos 20^\circ \cos 20^\circ + 45A\sin 20^\circ \sin 20^\circ - 60A\sin 20^\circ \cos 20^\circ = 0$$

$$\tau = -120\cos 20^\circ \sin 20^\circ + 45(\cos^2 20^\circ - \sin^2 20^\circ) + 60\sin 20^\circ \cos 20^\circ$$

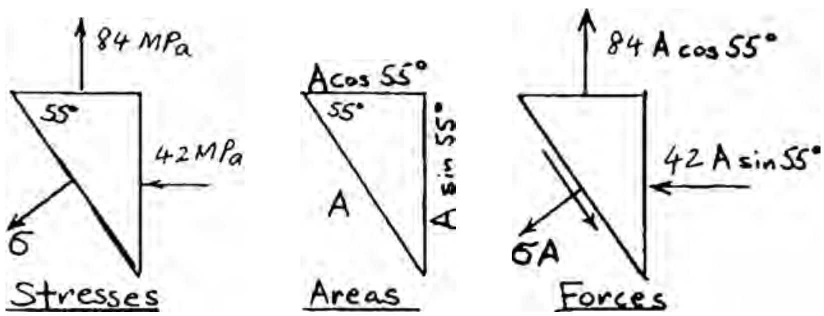
$\tau = 15.19$ MPa ◀



PROBLEM 7.4

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION



$+\swarrow \Sigma F = 0$

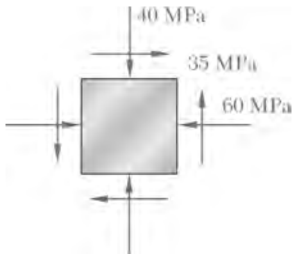
$$\sigma A - 84 A \cos 55° \cos 55° + 42 A \sin 55° \sin 55° = 0$$

$$\sigma = 84 \cos^2 55° - 42 \sin^2 55° = -0.546 \text{ MPa} \quad \blacktriangleleft$$

$+\searrow \Sigma F = 0$

$$\tau A - 84 A \cos 55° \sin 55° - 42 A \sin 55° \cos 55°$$

$$\tau = 126 \cos 55° \sin 55° = 59.2 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 7.5

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \qquad \sigma_y = -40 \text{ MPa} \qquad \tau_{xy} = 35 \text{ MPa}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(35)}{-60+40} = -3.50$$

$$2\theta_p = -74.05°$$

$$\theta_p = -37.0°, 53.0° \blacktriangleleft$$

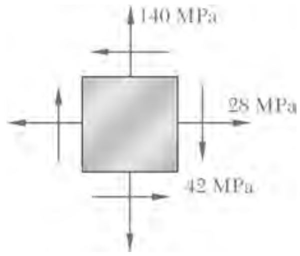
(*b*) $$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-60-40}{2} \pm \sqrt{\left(\frac{-60+40}{2}\right)^2 + (35)^2}$$

$$= -50 \pm 36.4 \text{ MPa}$$

$$\sigma_{\max} = -13.60 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -86.4 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.6

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = 28 \text{ MPa} \qquad \sigma_y = 140 \text{ MPa} \qquad \tau_{xy} = -42 \text{ MPa}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-42)}{28 - 140} = 0.750$$

$$2\theta_p = 36.87°$$

$$\theta_p = 18.43°, 108.43° \blacktriangleleft$$

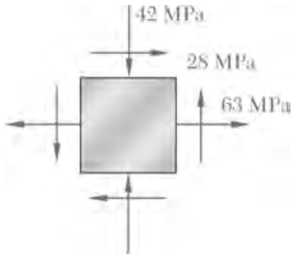
(*b*) $$\sigma_{\text{max, min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{28 + 140}{2} \pm \sqrt{\left(\frac{28 - 140}{2}\right)^2 + (-42)^2}$$

$$= 84 \pm 70$$

$$\sigma_{\text{max}} = 154 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\text{min}} = 14 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.7

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = 63 \text{ MPa} \qquad \sigma_y = -42 \text{ MPa} \qquad \tau_{xy} = 28 \text{ MPa}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(28)}{63+42} = 0.5333$$

$$2\theta_p = 28.07^\circ$$

$$\theta_p = 14.04^\circ, 104.04^\circ \blacktriangleleft$$

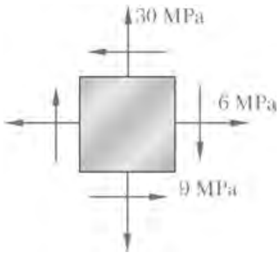
(*b*) $$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{63-42}{2} \pm \sqrt{\left(\frac{63+42}{2}\right)^2 + 28^2}$$

$$= 10.5 \pm 59.5$$

$$\sigma_{\max} = 70 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -49 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.8

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = 6 \text{ MPa} \qquad \sigma_y = 30 \text{ MPa} \qquad \tau_{xy} = -9 \text{ MPa}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-9)}{6-30} = -0.750$$

$$2\theta_p = 36.87°$$

$$\theta_p = 18.4°, 108.4° \blacktriangleleft$$

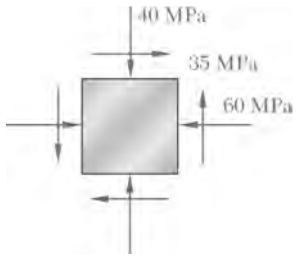
(*b*) $$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{6+30}{2} \pm \sqrt{\left(\frac{6-30}{2}\right)^2 + (-9)^2}$$

$$= 18 \pm 15$$

$$\sigma_{\max} = 33.0 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = 3.00 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.9

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \qquad \sigma_y = -40 \text{ MPa} \qquad \tau_{xy} = 35 \text{ MPa}$$

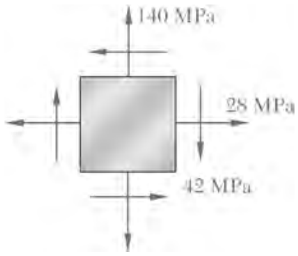
(*a*) $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{-60+40}{(2)(35)} = 0.2857$$

$$2\theta_s = 15.95° \qquad \theta_s = 8.0°, 98.0° \blacktriangleleft$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-60+40}{2}\right)^2 + (35)^2} \qquad \tau_{\max} = 36.4 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-60-40}{2} \qquad \sigma' = -50.0 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.10

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 28\text{ MPa} \qquad \sigma_y = 140\text{ MPa} \qquad \tau_{xy} = -42\text{ MPa}$$

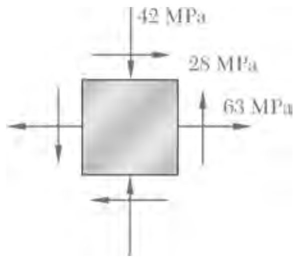
(*a*) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{28 - 140}{(2)(-42)} = -1.3333$

$2\theta_s = -53.13°$ $\qquad\theta_s = -26.57°, 63.43°$ ◀

(*b*) $\tau_{\max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{28 - 140}{2}\right)^2 + (-42)^2} = 70\text{ MPa}$ ◀

(*c*) $\sigma' = \sigma_{\text{ave}} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{28 + 140}{2} = 84\text{ MPa}$ ◀



PROBLEM 7.11

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 63 \text{ MPa} \qquad \sigma_y = -42 \text{ MPa} \qquad \tau_{xy} = 28 \text{ MPa}$$

(*a*) $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{63+42}{(2)(28)} = -1.875$$

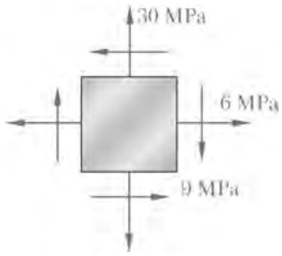
$$2\theta_s = -61.93°$$

$$\theta_s = -30.96°, 59.04° \blacktriangleleft$$

(*b*) $$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{63+42}{2}\right)^2 + (28)^2} = 59.5 \text{ MPa} \blacktriangleleft$$

(*c*) $$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{63-42}{2} = 10.5 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.12

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 6 \text{ MPa} \qquad \sigma_y = 30 \text{ MPa} \qquad \tau_{xy} = -9 \text{ MPa}$$

(*a*) $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{6 - 30}{(2)(-9)} = -1.33333$$

$$2\theta_s = -53.13^\circ \qquad \theta_s = -26.6^\circ, 63.4^\circ \blacktriangleleft$$

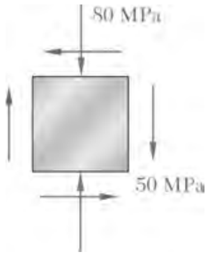
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{6 - 30}{2}\right)^2 + (-9)^2} \qquad \tau_{\max} = 15.00 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2}$$

$$= \frac{6 + 30}{2} \qquad \sigma' = 18.00 \text{ MPa} \blacktriangleleft$$

PROBLEM 7.13



For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0 \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = -50 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \qquad \frac{\sigma_x - \sigma_y}{2} = 40 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(*a*) $\underline{\theta = -25°}$ $\qquad 2\theta = -50°$

$$\sigma_{x'} = -40 + 40 \cos(-50°) - 50 \sin(-50°) \qquad \sigma_{x'} = 24.0 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -40 \sin(-50°) - 50 \cos(-50°) \qquad \tau_{x'y'} = -1.5 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = -40 - 40 \cos(-50°) + 50 \sin(-50°) \qquad \sigma_{y'} = -104.0 \text{ MPa} \blacktriangleleft$$

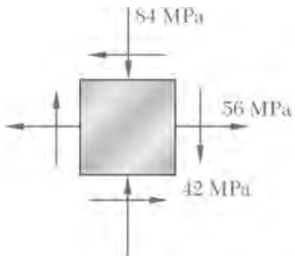
(*b*) $\underline{\theta = 10°}$ $\qquad 2\theta = 20°$

$$\sigma_{x'} = -40 + 40 \cos(20°) - 50 \sin(20°) \qquad \sigma_{x'} = -19.5 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -40 \sin(20°) - 50 \cos(20°) \qquad \tau_{x'y'} = -60.7 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = -40 - 40 \cos(20°) + 50 \sin(20°) \qquad \sigma_{y'} = -60.5 \text{ MPa} \blacktriangleleft$$

PROBLEM 7.14



For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 56 \text{ MPa} \qquad \sigma_y = -84 \text{ MPa} \qquad \tau_{xy} = -42 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = -14 \text{ MPa} \qquad \frac{\sigma_x - \sigma_y}{2} = 70 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(*a*) $\theta = -25° \qquad 2\theta = -50°$

$$\sigma_{x'} = -14 + 70\cos(-50°) - 42\sin(-50°) = 63.1 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -70\sin(-50°) - 42\cos(-50°) = 26.6 \text{ MPa} \blacktriangleleft$$

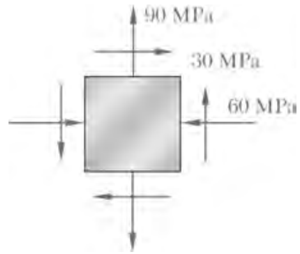
$$\sigma_{y'} = -14 - 70\cos(-50°) + 42\sin(-50°) = -91.1 \text{ MPa} \blacktriangleleft$$

(*b*) $\theta = 10° \qquad 2\theta = 20°$

$$\sigma_{x'} = -14 + 70\cos(20°) - 42\sin(20°) = 37.4 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -70\sin(20°) - 42\cos(20°) = -63.4 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = -14 - 70\cos(20°) + 42\sin(20°) = -65.4 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.15

For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \qquad \sigma_y = 90 \text{ MPa} \qquad \tau_{xy} = 30 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa} \qquad \frac{\sigma_x - \sigma_y}{2} = -75 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(*a*) $\underline{\theta = -25^\circ}$ $\quad 2\theta = -50^\circ$

$$\sigma_{x'} = 15 - 75\cos(-50^\circ) + 30\sin(-50^\circ) \qquad \sigma_{x'} = -56.2 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = +75\sin(-50^\circ) + 30\cos(-50^\circ) \qquad \tau_{x'y'} = -38.2 \text{ MPa} \blacktriangleleft$$

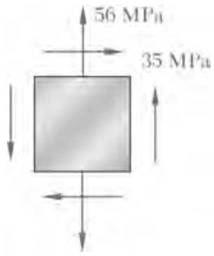
$$\sigma_{y'} = 15 + 75\cos(-50^\circ) - 30\sin(-50^\circ) \qquad \sigma_{y'} = 86.2 \text{ MPa} \blacktriangleleft$$

(*b*) $\underline{\theta = 10^\circ}$ $\quad 2\theta = 20^\circ$

$$\sigma_{x'} = 15 - 75\cos(20^\circ) + 30\sin(20^\circ) \qquad \sigma_{x'} = -45.2 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = +75\sin(20^\circ) + 30\cos(20^\circ) \qquad \tau_{x'y'} = 53.8 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = 15 + 75\cos(20^\circ) - 30\sin(20^\circ) \qquad \sigma_{y'} = 75.2 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.16

For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0 \qquad \sigma_y = 56 \text{ MPa} \qquad \tau_{xy} = 35 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = 28 \text{ MPa} \qquad \frac{\sigma_x - \sigma_y}{2} = -28 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(*a*) $\theta = -25°$ $\qquad 2\theta = -50°$

$$\sigma_{x'} = 28 - 28 \cos(-50°) + 35 \sin(-50°) = -16.8 \text{ MPa} \quad ◀$$

$$\tau_{x'y'} = 28 \sin(-50°) + 35 \cos(-50°) = 1.05 \text{ MPa} \quad ◀$$

$$\sigma_{y'} = 28 + 28 \cos(-50°) - 35 \sin(-50°) = 72.8 \text{ MPa} \quad ◀$$

(*b*) $\theta = 10°$ $\qquad 2\theta = 20°$

$$\sigma_{x'} = 28 - 28 \cos(20°) + 35 \sin(20°) = 13.7 \text{ MPa} \quad ◀$$

$$\tau_{x'y'} = 28 \sin(20°) + 35 \cos(20°) = 42.5 \text{ MPa} \quad ◀$$

$$\sigma_{y'} = 28 + 28 \cos(20°) - 35 \cos(20°) = 42.4 \text{ MPa} \quad ◀$$

PROBLEM 7.17

The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 4.2 \text{ MPa}$$

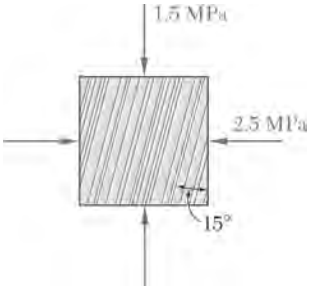
$$\theta = -15^\circ \qquad 2\theta = -30^\circ$$

(*a*) $$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -0 + 4.2\cos(-30^\circ) = 3.64 \text{ MPa} \quad \blacktriangleleft$$

(*b*) $$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + 4.2\sin(-30^\circ) = -2.1 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 7.18

The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = -2.5 \text{ MPa} \qquad \sigma_y = -1.5 \text{ MPa} \qquad \tau_{xy} = 0$$

$$\theta = -15° \qquad 2\theta = -30°$$

(*a*) $$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-2.5 - (-1.5)}{2}\sin(-30°) + 0$$

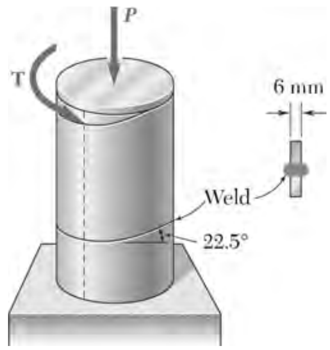
$$\tau_{x'y'} = -0.250 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-2.5 + (-1.5)}{2} + \frac{-2.5 - (-1.5)}{2}\cos(-30°) + 0$$

$$\sigma_{x'} = -2.43 \text{ MPa} \blacktriangleleft$$

PROBLEM 7.19



A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix which forms an angle of 22.5° with a plane perpendicular to the axis of the pipe. Knowing that a 160-kN axial force **P** and an 800 N · m torque **T**, each directed as shown, are applied to the pipe, determine σ and τ in directions, respectively, normal and tangential to the weld.

SOLUTION

$$d_2 = 0.3 \text{ m}, \quad c_2 = \frac{1}{2}d_2 = 0.15 \text{ m}, \quad t = 0.006 \text{ m}$$

$$c_1 = c_2 - t = 0.144$$

$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(0.15^2 - 0.144^2) = 5541.8 \times 10^{-6} \text{ m}^2$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.15^4 - 0.144^4) = 119.8 \times 10^{-6} \text{ m}^4$$

<u>Stresses</u>

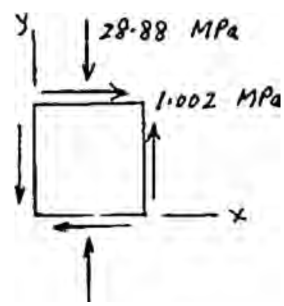
$$\sigma = -\frac{P}{A}$$

$$= -\frac{160 \times 10^3}{5541 \times 10^{-6}} = -28.88 \text{ MPa}$$

$$\tau = \frac{Tc_2}{J}$$

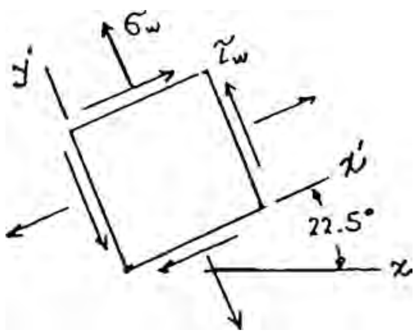
$$= \frac{(800)(0.15)}{119.8 \times 10^{-6}} = 1.002 \text{ MPa}$$

$$\sigma_x = 0, \quad \sigma_y = -28.88 \text{ MPa}, \quad \tau_{xy} = 1.002 \text{ MPa}$$



Choose the x' and y' axes respectively tangential and normal to the weld.

Then, $\sigma_w = \sigma_{y'}$ and $\tau_w = \tau_{x'y'}$ $\quad \theta = 22.5°$



$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{(-28.88)}{2} - \frac{[-(-28.88)]}{2}\cos 45° - 1.002 \sin 45°$$

$$= -25.36 \text{ MPa}$$

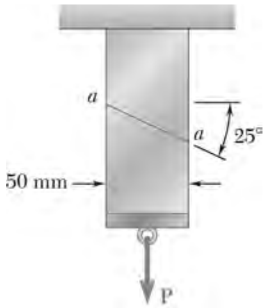
$\sigma_w = -25.4 \text{ MPa}$ ◀

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{[-(-28.88)]}{2}\sin 45° + 1.002 \cos 45°$$

$$= -9.5 \text{ MPa}$$

$\tau_w = -9.5 \text{ MPa}$ ◀

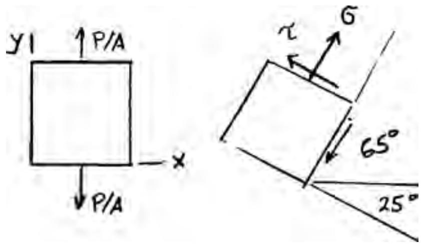


PROBLEM 7.20

Two members of uniform cross section 50×80 mm are glued together along plane *a-a* that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.

SOLUTION

For plane *a-a*, $\theta = 65°$.



$$\sigma_x = 0, \quad \tau_{xy} = 0, \quad \sigma_y = \frac{P}{A}$$

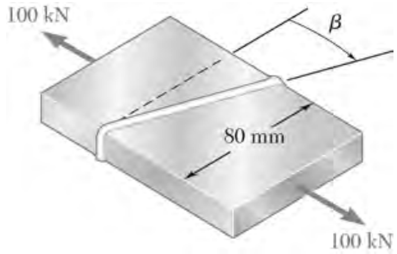
$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta = 0 + \frac{P}{A}\sin^2 65° + 0$$

$$P = \frac{A\sigma}{\sin^2 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(800\times10^{3})}{\sin^2 65°} = 3.90\times10^{3}\text{ N}$$

$$\tau = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) = \frac{P}{A}\sin 65°\cos 65° + 0$$

$$P = \frac{A\tau}{\sin 65°\cos 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(600\times10^{3})}{\sin 65°\cos 65°} = 6.27\times10^{3}\text{ N}$$

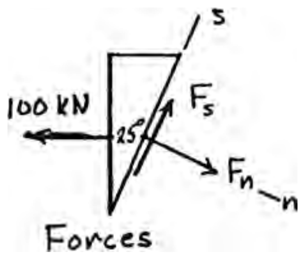
Allowable value of P is the smaller one. $P = 3.90$ kN ◀



PROBLEM 7.21

Two steel plates of uniform cross section $10 \times 80\,\text{mm}$ are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25°$, determine (*a*) the in-plane shearing stress parallel to the weld, (*b*) the normal stress perpendicular to the weld.

SOLUTION



Area of weld:

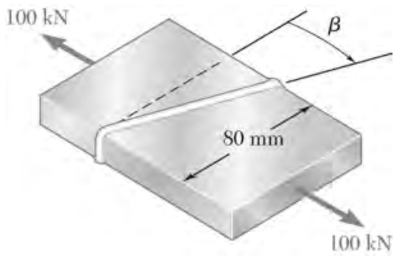
$$A_w = \frac{(10 \times 10^{-3})(80 \times 10^{-3})}{\cos 25°}$$
$$= 882.7 \times 10^{-6}\ \text{m}^2$$

(*a*) $\quad \sum F_s = 0: \quad F_s - 100 \sin 25° = 0 \qquad F_s = 42.26\ \text{kN}$

$$\tau_w = \frac{F_s}{A_w} = \frac{42.26 \times 10^3}{882.7 \times 10^{-6}} = 47.9 \times 10^6\ \text{Pa} \qquad \tau_w = 47.9\ \text{MPa} \blacktriangleleft$$

(*b*) $\quad \sum F_n = 0: \quad F_n - 100 \cos 25° = 0 \qquad F_n = 90.63\ \text{kN}$

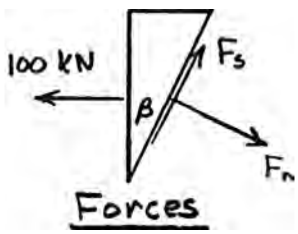
$$\sigma_w = \frac{F_n}{A_w} = \frac{90.63 \times 10^3}{882.7 \times 10^{-6}} = 102.7 \times 10^6\ \text{Pa} \qquad \sigma_w = 102.7\ \text{MPa} \blacktriangleleft$$



PROBLEM 7.22

Two steel plates of uniform cross section 10×80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (*a*) the angle β, (*b*) the corresponding normal stress perpendicular to the weld.

SOLUTION



Area of weld:

$$A_w = \frac{(10 \times 10^{-3})(80 \times 10^{-3})}{\cos \beta}$$

$$= \frac{800 \times 10^{-6}}{\cos \beta} \text{ m}^2$$

(*a*) $\sum F_s = 0$: $F_s - 100 \sin \beta = 0$ $\quad F_s = 100 \sin \beta \text{ kN} = 100 \times 10^3 \sin \beta \text{ N}$

$$\tau_w = \frac{F_s}{A_w} \quad 30 \times 10^6 = \frac{100 \times 10^3 \sin \beta}{800 \times 10^{-6} / \cos \beta} = 125 \times 10^6 \sin \beta \cos \beta$$

$$\sin \beta \cos \beta = \frac{1}{2} \sin 2\beta = \frac{30 \times 10^6}{125 \times 10^6} = 0.240$$

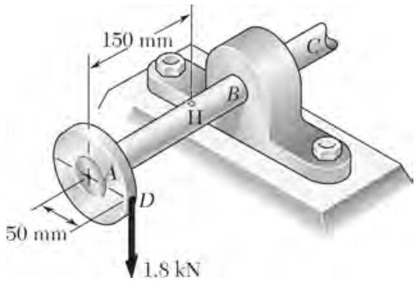
$\beta = 14.34°$ ◀

(*b*) $\sum F_n = 0$: $F_n - 100 \cos \beta = 0$ $\quad F_n = 100 \cos 14.34° = 96.88 \text{ kN}$

$$A_w = \frac{800 \times 10^{-6}}{\cos 14.34} = 825.74 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_n}{A_w} = \frac{96.88 \times 10^3}{825.74 \times 10^{-6}} = 117.3 \times 10^6 \text{ Pa}$$

$\sigma = 117.3 \text{ MPa}$ ◀

	**PROBLEM 7.23** A 1.8 kN vertical force is applied at D to a gear attached to the solid 25 mm diameter shaft AB. Determine the principal stresses and the maximum shearing stress at point H located as shown on top of the shaft.

SOLUTION

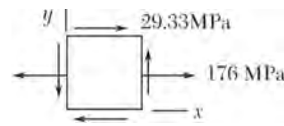
Equivalent force-couple system at center of shaft in section at point H.

$$V = 1.8 \text{ kN} \quad M = (1.8\times 10^3)(0.150) = 270 \text{ N}\cdot\text{m}$$

$$T = (1.8\times 10^3)(0.050) = 90 \text{ N}\cdot\text{m}$$

Shaft cross section $\quad d = 25 \text{ mm}, \quad c = \frac{1}{2}d = 12.5 \text{ mm}$

$$J = \frac{\pi}{2}c^4 = 38349 \text{ mm}^4 \qquad I = \frac{1}{2}J = 19174.5 \text{ mm}^4$$



Torsion: $\quad \tau = \dfrac{Tc}{J} = \dfrac{(90)(0.0125)}{38349\times 10^{-12}} = 29.33\times 10^6 \text{ Pa} = 29.33 \text{ MPa}$

Bending: $\quad \sigma = \dfrac{Mc}{I} = \dfrac{(270)(0.0125)}{19174.5\times 10^{-12}} = 176\times 10^6 \text{ Pa} = 176 \text{ MPa}$

Transverse shear: $\quad$ Stress at point H is zero.

$$\sigma_x = 176 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = 29.33 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 88 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(88)^2 + (29.33)^2}$$

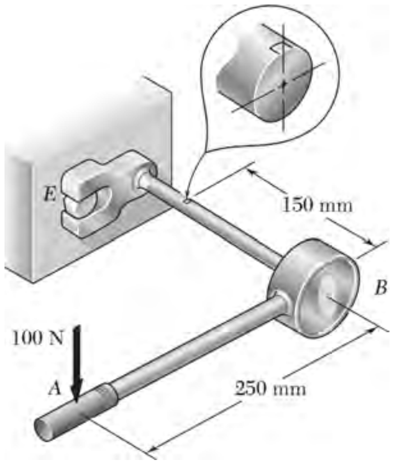
$$= 92.76 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 180.76 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R = -4.76 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{\max} = R = 92.76 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 7.24



A mechanic uses a crowfoot wrench to loosen a bolt at *E*. Knowing that the mechanic applies a vertical 100 N force at *A*, determine the principal stresses and the maximum shearing stress at point *H* located as shown on top of the 18 mm diameter shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point *H*.

$$V = 100\text{ N} \quad M = (100)(150) = 15000\text{ N}\cdot\text{mm}$$

$$T = (100)(250) = 25000\text{ N}\cdot\text{mm}$$

Shaft cross section: $d = 18\text{ mm}, \quad c = \frac{1}{2}d = 9\text{ mm}$

$$J = \frac{\pi}{2}c^4 = 10306\text{ mm}^4 \quad I = \frac{1}{2}J = 5153\text{ mm}^4$$

Torsion: $\tau = \frac{Tc}{J} = \frac{(25000)(9)}{10306} = 21.8\text{ N/mm}^2 = 21.8\text{ MPa}$

Bending: $\sigma = \frac{Mc}{I} = \frac{(15000)(9)}{5153} = 26.2\text{ N/mm}^2 = 26.2\text{ MPa}$

Transverse shear: At point *H* stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 26.2\text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = 21.8\text{ MPa}$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 13.1\text{ MPa}$$

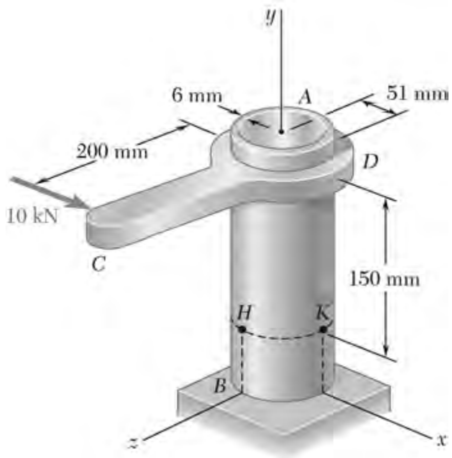
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{13.1^2 + 21.8^2} = 25.4\text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 38.5\text{ MPa} \quad ◀$$

$$\sigma_b = \sigma_{\text{ave}} - R = -12.3\text{ MPa} \quad ◀$$

$$\tau_{\max} = R = 25.4\text{ MPa} \quad ◀$$

PROBLEM 7.25



The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855 \times 10^6 \text{ mm}^4$$

$$= 4.1855 \times 10^{-6} \text{ m}^4$$

$$I = \frac{1}{2} J = 2.0927 \times 10^{-6} \text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K:

$$F_x = 10 \text{ kN}$$

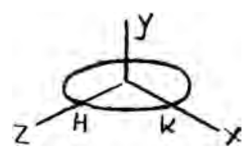
$$= 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3})$$

$$= 2000 \text{ N} \cdot \text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3})$$

$$= -1500 \text{ N} \cdot \text{m}$$



Torsion: At point K, place local x-axis in negative global z-direction.

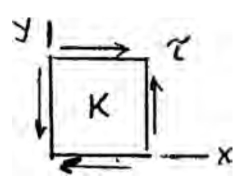
$$T = M_y = 2000 \text{ N} \cdot \text{m}$$

$$c = r_o = 51 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^6}$$

$$= 24.37 \times 10^6 \text{ Pa}$$

$$= 24.37 \text{ MPa}$$



PROBLEM 7.25 *(Continued)*

Transverse shear: Stress due to transverse shear $V = F_x$ is zero at point K.

Bending:

$$|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51\times10^{-3})}{2.0927\times10^{-6}} = 36.56\times10^{6}\,\text{Pa} = 36.56\ \text{MPa}$$

Point K lies on compression side of neutral axis:

$$\sigma_y = -36.56\ \text{MPa}$$

Total stresses at point K:

$$\sigma_x = 0, \quad \sigma_y = -36.56\ \text{MPa}, \quad \tau_{xy} = 24.37\ \text{MPa}$$

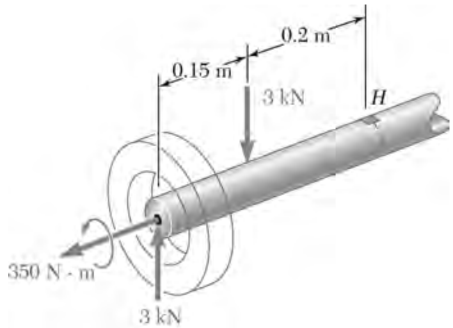
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46\ \text{MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = -18.28 + 30.46 \qquad \sigma_{\max} = 12.18\ \text{MPa} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R = -18.28 - 30.46 \qquad \sigma_{\min} = -48.7\ \text{MPa} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 30.5\ \text{MPa} \blacktriangleleft$$



PROBLEM 7.26

The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 32 mm, determine (*a*) the principal planes and principal stresses at point *H* located on top of the axle, (*b*) the maximum shearing stress at the same point.

SOLUTION

$$c = \frac{1}{2}d = \frac{1}{2}(32) = 16\text{ mm} = 16\times 10^{-3}\text{ m}$$

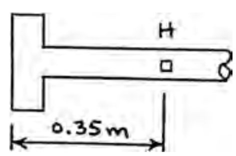
Torsion: $$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(350\text{ N}\cdot\text{m})}{\pi(16\times 10^{-3}\text{ m})^3} = 54.399\times 10^6\text{ Pa} = 54.399\text{ MPa}$$

Bending: $$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(16\times 10^{-3})^4 = 51.472\times 10^{-9}\text{ m}^4$$

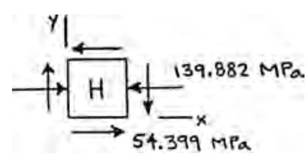
$$M = (0.15\text{ m})(3\times 10^3\text{ N}) = 450\text{ N}\cdot\text{m}$$

$$\sigma = -\frac{My}{I} = -\frac{(450)(16\times 10^{-3})}{51.472\times 10^{-9}} = -139.882\times 10^6\text{ Pa} = -139.882\text{ MPa}$$

Top view:



Stresses:



$$\sigma_x = -139.882\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -54.399\text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(-139.882 + 0) = -69.941\text{ MPa}$$

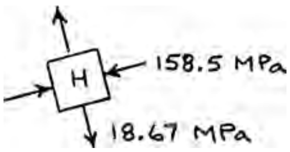
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-69.941)^2 + (-54.399)^2} = 88.606\text{ MPa}$$

(*a*) $\sigma_{\max} = \sigma_{\text{ave}} + R = -69.941 + 88.606$ $\qquad \sigma_{\max} = 18.67\text{ MPa}$ ◀

$\sigma_{\min} = \sigma_{\text{ave}} - R = -69.941 - 88.606$ $\qquad \sigma_{\min} = -158.5\text{ MPa}$ ◀

PROBLEM 7.26 *(Continued)*

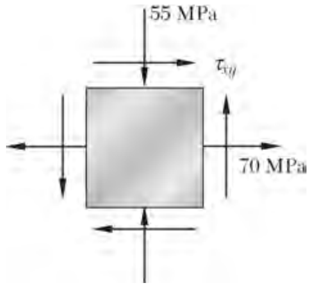
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-54.399)}{-139.882} = 0.77778 \qquad 2\theta_p = 37.88^\circ$$



$\theta_p = 18.9^\circ$ and 108.9° ◀

(*b*) $\tau_{\max} = R = 88.6\ \text{MPa}$

$\tau_{\max} = 88.6\ \text{MPa}$ ◀



PROBLEM 7.27

For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 82 MPa, (*b*) the corresponding principal stresses.

SOLUTION

$$\sigma_x = 70 \text{ MPa}, \quad \sigma_y = -55 \text{ MPa}, \quad \tau_{xy} = ?$$

$$\tau_{\max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{z}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{70-(-55)}{2}\right)^2 + \tau_{xy}^2}$$

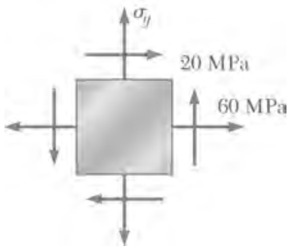
$$= \sqrt{62.5^2 + \tau_{xy}^2} = 82 \text{ MPa}$$

(*a*) $\tau_{xy} = \sqrt{82^2 - 62.5^2} = 53 \text{ MPa}$ ◀

(*b*) $\sigma_{\text{ave}} = \dfrac{1}{2}(\sigma_x + \sigma_y) = 7.5 \text{ MPa}$

$\sigma_a = \sigma_{\text{ave}} + R = 7.5 + 82 = 89.5 \text{ MPa}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R = 7.5 - 82 = -74.5 \text{ MPa}$ ◀



PROBLEM 7.28

For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.

SOLUTION

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_y = ?, \quad \tau_{xy} = 20 \text{ MPa}$$

Let

$$u = \frac{\sigma_x - \sigma_y}{2}.$$

Then

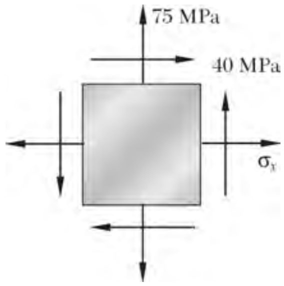
$$\sigma_y = \sigma_x - 2u$$

$$R = \sqrt{u^2 + \tau_{xy}^2} = 75 \text{ MPa}$$

$$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{75^2 - 20^2} = 72.284 \text{ MPa}$$

$$\sigma_y = \sigma_x - 2u = 60 \mp (2)(72.284) = -84.6 \text{ MPa} \quad \text{or} \quad 205 \text{ MPa}$$

Largest value of σ_y is required. $\sigma_y = 205 \text{ MPa}$ ◀



PROBLEM 7.29

Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 50 MPa.

SOLUTION

$$\sigma_x = ?, \quad \sigma_y = 75 \text{ MPa}, \quad \tau_{xy} = 40 \text{ MPa}$$

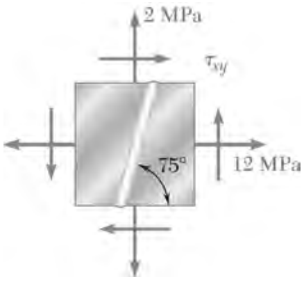
Let $U = \dfrac{\sigma_x - \sigma_y}{2} \qquad \sigma_x = \sigma_y + 2U$

$$R = \sqrt{U^2 + \tau_{xy}^2} = \tau_{\max} = 50 \text{ MPa}$$

$$U = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{50^2 - 40^2} = \pm 30 \text{ MPa}$$

$$\sigma_x = \sigma_y + 2U = 75 \pm (2)(30) = 135 \text{ MPa}, 15 \text{ MPa}$$

Allowable range $\qquad 15 \text{ MPa} \le \sigma_x \le 135 \text{ MPa}$ ◀



PROBLEM 7.30

For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

SOLUTION

$$\sigma_x = 12 \text{ MPa}, \quad \sigma_y = 2 \text{ MPa}, \quad \tau_{xy} = ?$$

Since $\tau_{x'y'} = 0$, x'-direction is a principal direction.

$$\theta_p = -15°$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

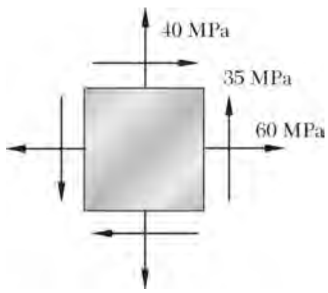
(*a*) $\tau_{xy} = \frac{1}{2}(\sigma_x - \sigma_y) \tan 2\theta_p = \frac{1}{2}(12 - 2) \tan(-30°)$ $\qquad \tau_{xy} = -2.89 \text{ MPa}$ ◀

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5^2 + 2.89^2} = 5.7735 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 7 \text{ MPa}$$

(*b*) $\sigma_a = \sigma_{\text{ave}} + R = 7 + 5.7735$ $\qquad \sigma_a = 12.77 \text{ MPa}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R = 7 - 5.7735$ $\qquad \sigma_b = 1.226 \text{ MPa}$ ◀



PROBLEM 7.31

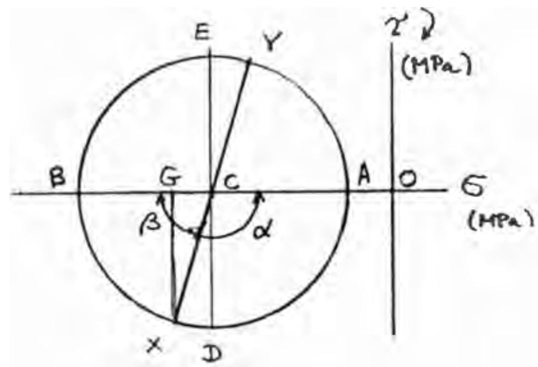
Solve Probs. 7.5 and 7.9, using Mohr's circle.

PROBLEM 7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

SOLUTION

$$\sigma_x = -60 \text{ MPa},$$
$$\sigma_y = -40 \text{ MPa},$$
$$\tau_{xy} = 35 \text{ MPa}$$
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -50 \text{ MPa}$$



<u>Plotted points for Mohr's circle</u>.

$$X: (\sigma_x, -\tau_{xy}) = (-60 \text{ MPa}, -35 \text{ MPa})$$
$$Y: (\sigma_y, \tau_{xy}) = (-40 \text{ MPa}, 35 \text{ MPa})$$
$$C: (\sigma_{ave}, 0) = (-50 \text{ MPa}, 0)$$

(*a*) $\tan \beta = \dfrac{GX}{CG} = \dfrac{35}{10} = 3.500$

$\beta = 74.05°$

$\theta_B = -\dfrac{1}{2}\beta = -37.03°$ ◀

$\alpha = 180° - \beta = 105.95°$

$\theta_A = \dfrac{1}{2}\alpha = 52.97°$ ◀

$R = \sqrt{\overline{CG}^2 + \overline{GX}^2} = \sqrt{10^2 + 35^2} = 36.4 \text{ MPa}$

(*b*) $\sigma_{min} = \sigma_{ave} - R = -50 - 36.4$ $\qquad \sigma_{min} = -86.4 \text{ MPa}$ ◀

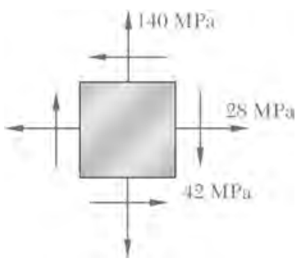
$\sigma_{max} = \sigma_{ave} + R = -50 + 36.4$ $\qquad \sigma_{max} = -13.6 \text{ MPa}$ ◀

(*a*′) $\theta_D = \theta_B + 45° = 7.97°$ $\qquad \theta_d = 8.0°$ ◀

$\theta_E = \theta_A + 45° = 97.97°$ $\qquad \theta_e = 98.0°$ ◀

$\tau_{max} = R = 36.4 \text{ MPa}$ $\qquad \tau_{max} = 36.4 \text{ MPa}$ ◀

(*b*′) $\sigma' = \sigma_{ave} = -50 \text{ MPa}$ $\qquad \sigma' = -50.0 \text{ MPa}$ ◀



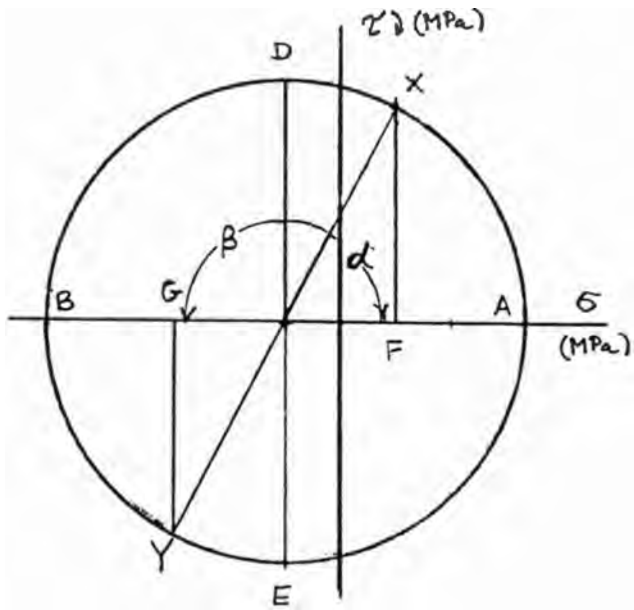
PROBLEM 7.32

Solve Probs. 7.6 and 7.10, using Mohr's circle.

PROBLEM 7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

SOLUTION



$$\sigma_x = 28 \text{ MPa},$$
$$\sigma_y = -140 \text{ MPa},$$
$$\tau_{xy} = -42 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -56 \text{ MPa}$$

<u>Plotted points for Mohr's circle</u>.

$$X: (\sigma_x, -\tau_{xy}) = (28 \text{ MPa}, 42 \text{ MPa})$$
$$Y: (\sigma_y, \tau_{xy}) = (-140 \text{ MPa}, -42 \text{ MPa})$$
$$C: (\sigma_{\text{ave}}, 0) = (-56 \text{ MPa}, 0)$$

(*a*) $\tan\alpha = \dfrac{FX}{CF} = \dfrac{42}{84} = 0.5$

$$\alpha = 26.57°$$
$$\theta_a = -\frac{1}{2}\alpha = -13.29° \quad ◀$$
$$\beta = 180° - \alpha = 153.43°$$
$$\theta_b = \frac{1}{2}\beta = 76.72° \quad ◀$$
$$R = \sqrt{(CF)^2 + (FX)^2} = \sqrt{(84)^2 + (42)^2} = 93.9 \text{ MPa}$$

(*b*) $\sigma_a = \sigma_{\max} = \sigma_{\text{ave}} + R = -56 + 93.9$ $\qquad \sigma_{\max} = 37.9 \text{ MPa}$ ◀

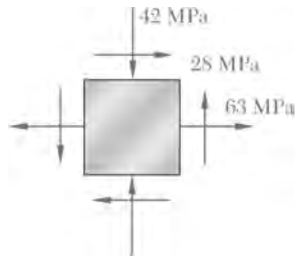
$\sigma_{\min} = \sigma_{\min} = \sigma_{\text{ave}} - R = -56 - 93.9$ $\qquad \sigma_{\min} = -149.9 \text{ MPa}$ ◀

(*a′*) $\theta_d = \theta_a + 45° = 31.71°$ $\qquad \theta_d = 31.7°$ ◀

$\theta_e = \theta_b + 45° = 121.72°$ $\qquad \theta_e = 121.7°$ ◀

$\tau_{\max} = R$ $\qquad \tau_{\max} = 93.9 \text{ MPa}$ ◀

(*b′*) $\sigma' = \sigma_{\text{ave}}$ $\qquad \sigma' = -56 \text{ MPa}$ ◀



PROBLEM 7.33

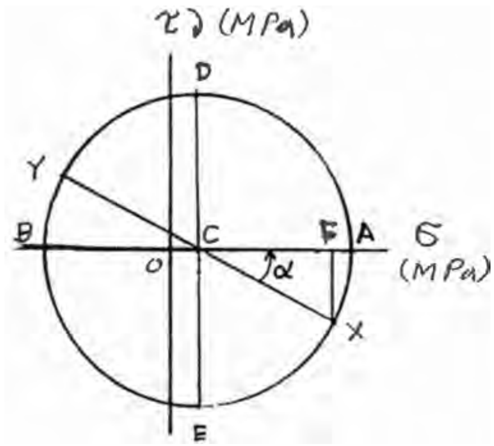
Solve Prob. 7.11, using Mohr's circle.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 63 \text{ MPa}, \quad \sigma_y = -42 \text{ MPa}, \quad \tau_{xy} = 28 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 10.5 \text{ MPa}$$



Points

$$X: \ (\sigma_x, -\tau_{xy}) = (63 \text{ MPa}, 28 \text{ MPa})$$

$$Y: \ (\sigma_y, \tau_{xy}) = (-42 \text{ MPa}, 28 \text{ MPa})$$

$$C: \ (\sigma_{\text{ave}}, 0) = (10.5 \text{ MPa}, 0)$$

$$\tan \alpha = \frac{FX}{CF} = \frac{28}{52.5} = 0.5333$$

$$\alpha = 28.07°$$

$$\theta_a = \frac{1}{2}\alpha = 14.04°$$

$$\theta_d = \theta_a + 45° = 59.04° \quad ◀$$

$$\theta_e = \theta_a - 45° = -30.96° \quad ◀$$

$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{52.5^2 + 28^2} = 59.5 \text{ MPa}$$

$$\tau_{\max} = R = 59.5 \text{ MPa} \quad ◀$$

$$\sigma' = \sigma_{\text{ave}} = 10.5 \text{ MPa} \quad ◀$$

PROBLEM 7.34

Solve Prob. 7.12, using Mohr's circle.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 8 \text{ MPa}, \quad \sigma_y = 30 \text{ MPa}, \quad \tau_{xy} = -9 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(6 + 36) = 18 \text{ MPa}$$

Plotted points for Mohr's circle.

$$X: (\sigma_x, -\tau_{xy}) = (6 \text{ MPa}, 9 \text{ MPa})$$

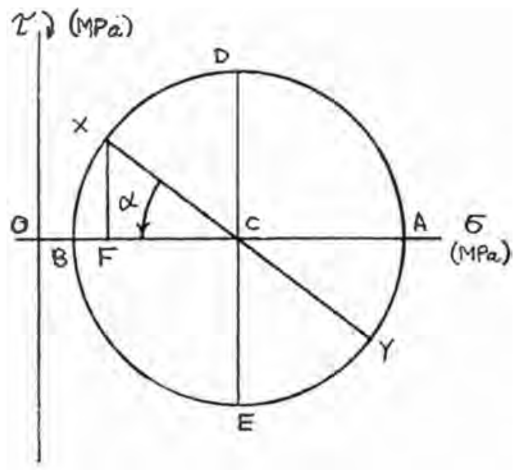
$$Y: (\sigma_y, \tau_{xy}) = (30 \text{ MPa}, -9 \text{ MPa})$$

$$C: (\sigma_{\text{ave}}, 0) = (18 \text{ MPa}, 0)$$

$$\tan \alpha = \frac{FX}{FC} = \frac{9}{12} = 0.75$$

$$\alpha = 36.87°$$

$$\theta_b = \frac{1}{2}\alpha = 18.43°$$



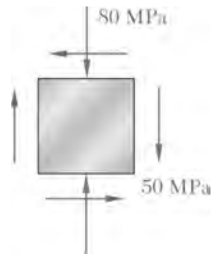
(*a*) $\theta_d = \theta_b - 45°$ $\qquad \theta_d = -26.6°$ ◀

$\theta_e = \theta_b + 45°$ $\qquad \theta_e = 63.4°$ ◀

$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{12^2 + 9^2} = 15 \text{ MPa}$$

$\tau_{\text{max (in-plane)}} = R$ $\qquad \tau_{\text{max (in-plane)}} = 15.00 \text{ MPa}$ ◀

(*b*) $\sigma' = \sigma_{\text{ave}}$ $\qquad \sigma' = 18.00 \text{ MPa}$ ◀



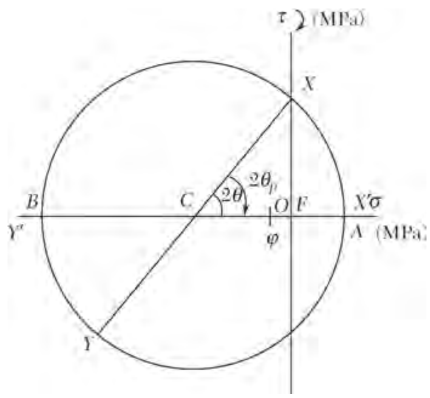
PROBLEM 7.35

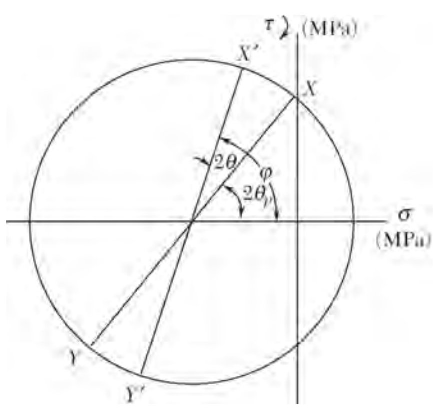
Solve Prob. 7.13, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0,$$
$$\sigma_y = -80 \text{ MPa},$$
$$\tau_{xy} = -50 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$$





<u>Plotted points for Mohr's circle</u>.

$$X: (0,\ 50 \text{ MPa})$$
$$Y: (-80 \text{ MPa},\ -50 \text{ MPa})$$
$$C: (-40 \text{ MPa},\ 0)$$

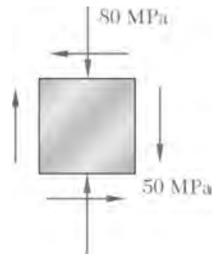
$$\tan 2\theta_p = \frac{FX}{CF} = \frac{50}{40} = 1.25$$
$$2\theta_p = 51.34°$$
$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{40^2 + 50^2} = 64.03 \text{ MPa}$$

(*a*) $\underline{\theta = 25°}$. $\quad 2\theta = 50°$

$$\varphi = 51.34° - 50° = 1.34°$$
$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi = 24.0 \text{ MPa} \quad ◀$$
$$\tau_{x'y'} = -R\sin\varphi = -1.5 \text{ MPa} \quad ◀$$
$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi = -104.0 \text{ MPa} \quad ◀$$

(*b*) $\underline{\theta = 10°}$. $\quad 2\theta = 20°$

$$\varphi = 51.34° + 20° = 71.34°$$
$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi = -19.5 \text{ MPa} \quad ◀$$
$$\tau_{x'y'} = -R\sin\varphi = -60.7 \text{ MPa} \quad ◀$$
$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi = -60.5 \text{ MPa} \quad ◀$$



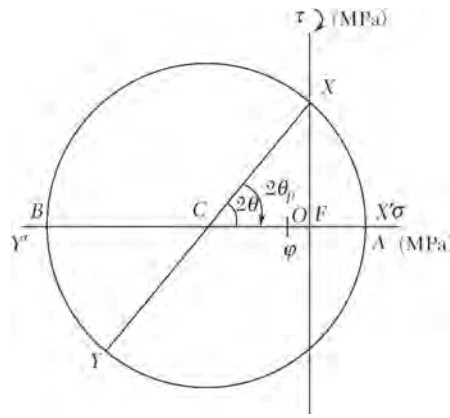
PROBLEM 7.36

Solve Prob. 7.14, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

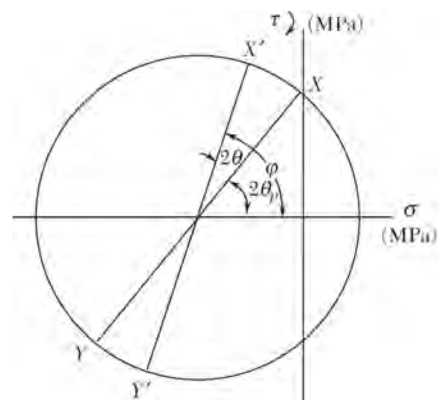
$$\sigma_x = 0$$
$$\sigma_y = -80 \text{ MPa}$$
$$\tau_{xy} = -50 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$$



Points

X: (0, 50 MPa)
Y: (–80 MPa, –50 MPa)
C: (–40 MPa, 0)

$$\tan 2\theta_p = \frac{FX}{FC} = \frac{50}{40} = 1.25$$
$$2\theta_p = 51.34°$$
$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{40^2 + 50^2} = 64.03 \text{ MPa}$$

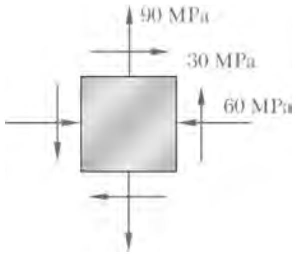


(*a*) $\theta = 25°$ ↻ $\qquad 2\theta = 50°$ ↻

$$\varphi = 51.34° - 50° = 1.34° \circlearrowleft$$
$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi = 24.0 \text{ MPa}$$ ◄
$$\tau_{x'y'} = -R\sin\varphi = -1.5 \text{ MPa}$$ ◄
$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi = -104.0 \text{ MPa}$$ ◄

(*b*) $\theta = 10°$ ↺ $\qquad 2\theta = 20°$ ↺

$$\varphi = 51.34° + 20° = 71.34°$$
$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi = -19.5 \text{ MPa}$$ ◄
$$\tau_{x'y'} = +R\sin\varphi = -60.7 \text{ MPa}$$ ◄
$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi = -60.5 \text{ MPa}$$ ◄



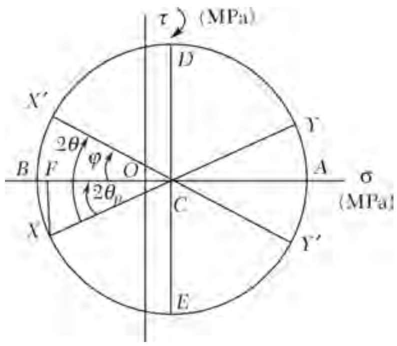
PROBLEM 7.37

Solve Prob. 7.15, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = -60 \text{ MPa},$$
$$\sigma_y = 90 \text{ MPa},$$
$$\tau_{xy} = 30 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa}$$



<u>Plotted points for Mohr's circle</u>.

$$X: \ (-60 \text{ MPa}, -30 \text{ MPa})$$
$$Y: \ (90 \text{ MPa}, 30 \text{ MPa})$$
$$C: \ (15 \text{ MPa}, 0)$$

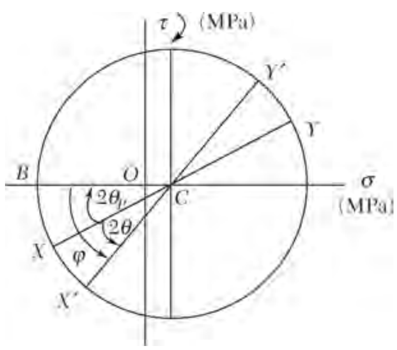
$$\tan 2\theta_p = \frac{FX}{FC} = \frac{30}{75} = 0.4$$
$$2\theta_p = 21.80°$$
$$\theta_p = 10.90°$$

$$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{75^2 + 30^2} = 80.78 \text{ MPa}$$

(*a*) $\underline{\theta = 25°}$. $\quad 2\theta = 50°$

$$\varphi = 2\theta - 2\theta_p = 50° - 21.80° = 28.20°$$

$$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi = -56.2 \text{ MPa} \quad ◀$$
$$\tau_{x'y'} = -R\sin\varphi = -38.2 \text{ MPa} \quad ◀$$
$$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi = 86.2 \text{ MPa} \quad ◀$$

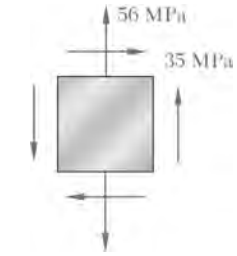


(*b*) $\underline{\theta = 10°}$. $\quad 2\theta = 20°$

$$\varphi = 2\theta_p + 2\theta = 21.80° + 20° = 41.80°$$

$$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi = -45.2 \text{ MPa} \quad ◀$$
$$\tau_{x'y'} = R\sin\varphi = 53.8 \text{ MPa} \quad ◀$$
$$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi = 75.2 \text{ MPa} \quad ◀$$

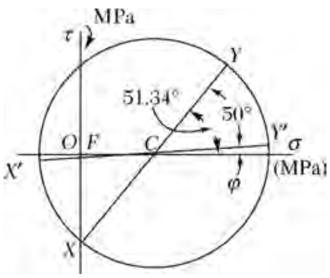
PROBLEM 7.38



Solve Prob. 7.16, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

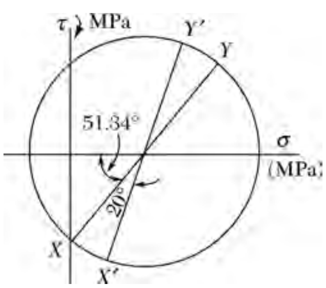


$$\sigma_x = 0$$
$$\sigma_y = 56 \text{ MPa}$$
$$\tau_{xy} = 35 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 28 \text{ MPa}$$

Points:

$$X: \ (0, -35 \text{ MPa})$$
$$Y: \ (56 \text{ MPa}, 35 \text{ MPa})$$
$$C: \ (28 \text{ MPa}, 0)$$

$$\tan 2\theta_p = \frac{FX}{FC} = \frac{35}{28} = 1.25$$
$$2\theta_p = 51.34°$$



$$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{28^2 + 35^2}$$
$$= 44.8 \text{ MPa}$$

(*a*) $\underline{\theta = 25°}$ ↻ $\qquad 2\theta = 50°$ ↻

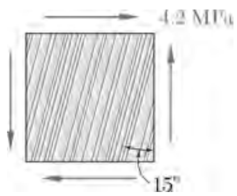
$$\varphi = 51.34° - 50° = 1.34°$$

$$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi = -16.8 \text{ MPa} \quad ◀$$
$$\tau_{x'y'} = R\sin\varphi = 1.05 \text{ MPa} \quad ◀$$
$$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi = 72.8 \text{ MPa} \quad ◀$$

(*b*) $\underline{\theta = 10°}$ ↺ $\qquad 2\theta = 20°$ ↺

$$\varphi = 51.34° + 20° = 71.34°$$

$$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi = 13.7 \text{ MPa} \quad ◀$$
$$\tau_{x'y'} = R\sin\varphi = 42.3 \text{ MPa} \quad ◀$$
$$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi = 42.4 \text{ MPa} \quad ◀$$



PROBLEM 7.39

Solve Prob. 7.17, using Mohr's circle.

PROBLEM 7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = \sigma_y = 0$$

$$\tau_{xy} = 2.8 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Points

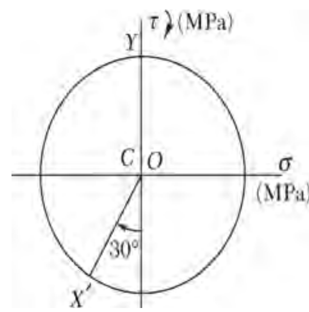
$$X: (\sigma_x, -\tau_{xy}) = (0, -4.2 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (0, 4.2 \text{ MPa})$$

$$C: (\sigma_{\text{ave}}, 0) = (0, 0)$$

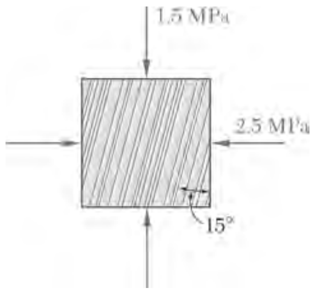
$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

$$\overline{CX} = R = 4.2 \text{ MPa}$$



(*a*) $\tau_{x'y'} = R\cos 30^\circ = 600\cos 30^\circ = 3.6 \text{ MPa}$ ◀

(*b*) $\sigma_{x'} = \sigma_{\text{ave}} - R\sin 30^\circ = -600\sin 30^\circ = -2.1 \text{ MPa}$ ◀



PROBLEM 7.40

Solve Prob. 7.18, using Mohr's circle.

PROBLEM 7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = -2.5 \text{ MPa}$$

$$\sigma_y = -1.5 \text{ MPa}$$

$$\tau_{xy} = 0$$

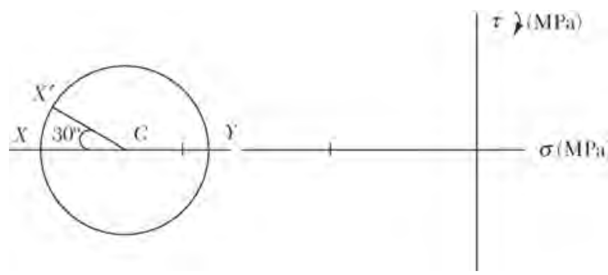
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -2.0 \text{ MPa}$$

<u>Plotted points for Mohr's circle</u>.

$$X: \ (\sigma_x, -\tau_{xy}) = (-2.5 \text{ MPa}, 0)$$

$$Y: \ (\sigma_y, \tau_{xy}) = (-1.5 \text{ MPa}, 0)$$

$$C: \ (\sigma_{\text{ave}}, 0) = (-2.0 \text{ MPa}, 0)$$

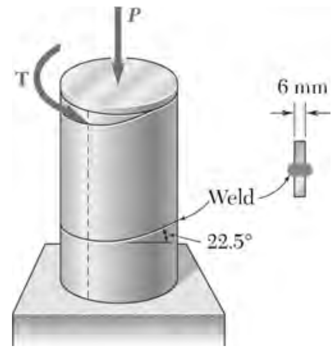


<u>$\theta = -15°$</u>. $\quad 2\theta = -30°$

$$\overline{CX} = 0.5 \text{ MPa} \quad R = 0.5 \text{ MPa}$$

(*a*) $\quad \tau_{x'y'} = -\overline{CX'}\sin 30° = -R\sin 30° = -0.5\sin 30°$ $\quad \tau_{x'y'} = -0.25 \text{ MPa}$ ◀

(*b*) $\quad \sigma_{x'} = \sigma_{\text{ave}} - \overline{CX'}\sin 30° = -2.0 - 0.5\cos 30°$ $\quad \sigma_{x'} = -2.43 \text{ MPa}$ ◀

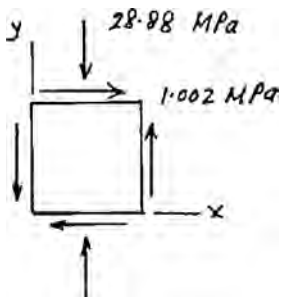


PROBLEM 7.41

Solve Prob. 7.19, using Mohr's circle.

PROBLEM 7.20 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix which forms an angle of 22.5° with a plane perpendicular to the axis of the pipe. Knowing that a 160-kN axial force **P** and an 800 N · m torque **T**, each directed as shown, are applied to the pipe, determine σ and τ in directions, respectively, normal and tangential to the weld.

SOLUTION



$$d_2 = 0.3 \text{ m}, c_2 = \frac{1}{2}d_2 = 0.15 \text{ m}, t = 0.006 \text{ m}$$

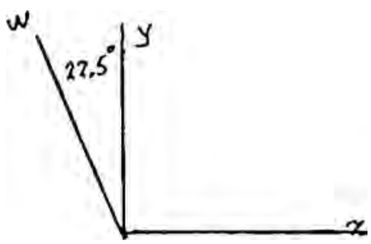
$$c_1 = c_2 - t = 0.144 \text{ m}$$

$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(0.15^2 - 0.144^2) = 5541\times10^{-6} \text{ m}^2$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.15^4 - 0.144^4) = 119.8\times10^{-6} \text{ m}^4$$

<u>Stresses</u>

$$\sigma = -\frac{P}{A} = -\frac{160\times10^3}{5541\times10^{-6}} = -28.88 \text{ MPa}$$



$$\tau = \frac{Tc_2}{J} = \frac{(800)(0.15)}{119.8\times10^{-6}} = 1.002 \text{ MPa}$$

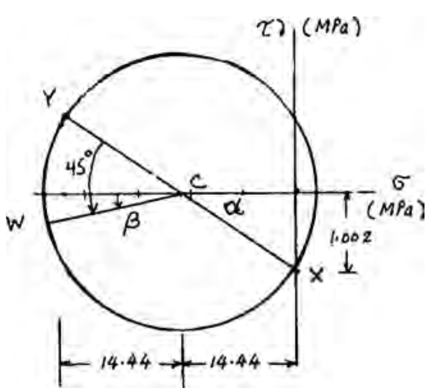
$$\sigma_x = 0, \sigma_y = -28.88 \text{ MPa}, \tau_{xy} = 1.002 \text{ MPa}$$

Draw the Mohr's circle.

$$X: \ (0, -1.002 \text{ MPa})$$

$$Y: \ (-28.88 \text{ MPa}, 1.002 \text{ MPa})$$

$$C: \ (-14.44, 0)$$



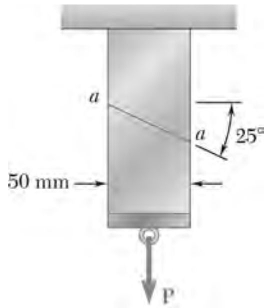
$$\tan\alpha = \frac{1.002}{14.44} = 0.06939 \quad \alpha = 4.0°$$

$$\beta = (2)(22.5°) - \alpha = 41°$$

$$R = \sqrt{(14.44)^2 + (1.002)^2} = 14.47 \text{ MPa}$$

$$\sigma_w = -14.44 - 14.47 \cos 41° \qquad \sigma_w = -25.4 \text{ MPa} \blacktriangleleft$$

$$\tau_w = -14.47 \sin 41° \qquad \tau_w = -9.5 \text{ MPa} \blacktriangleleft$$

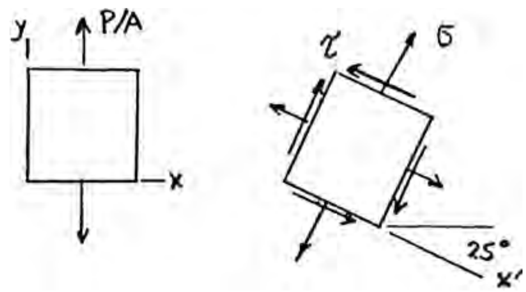


PROBLEM 7.42

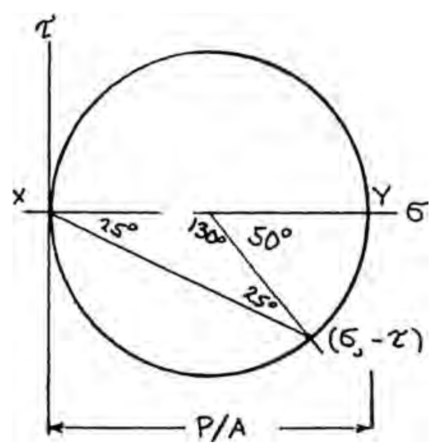
Solve Prob. 7.20, using Mohr's circle.

PROBLEM 7.20 Two members of uniform cross section 50×80 mm are glued together along plane *a-a* that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.

SOLUTION

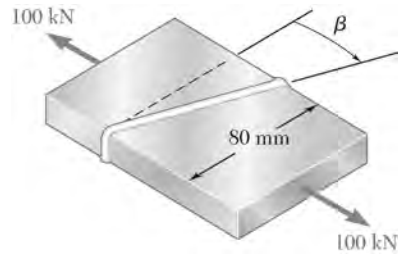


$$\sigma_x = 0$$
$$\tau_{xy} = 0$$
$$\sigma_y = P/A$$
$$A = (50 \times 10^{-3})(80 \times 10^{-3})$$
$$= 4 \times 10^{-3}\,\text{m}^2$$
$$\sigma = \frac{P}{2A}(1 + \cos 50°)$$
$$P = \frac{2A\sigma}{1 + \cos 50°}$$
$$P \leq \frac{(2)(4 \times 10^{-3})(800 \times 10^3)}{1 + \cos 50°}$$
$$P \leq 3.90 \times 10^3\,\text{N}$$



$$\tau = \frac{P}{2A}\sin 50° \quad P = \frac{2A\tau}{\sin 50°} \leq \frac{(2)(4 \times 10^{-3})(600 \times 10^3)}{\sin 50°} = 6.27 \times 10^3\,\text{N}$$

Choosing the smaller value, $P = 3.90$ kN ◀

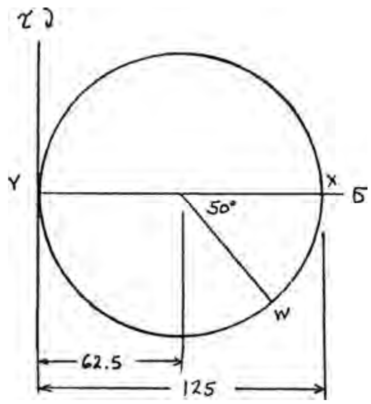


PROBLEM 7.43

Solve Prob. 7.21, using Mohr's circle.

PROBLEM 7.21 Two steel plates of uniform cross section 10×80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25°$, determine (*a*) the in-plane shearing stress parallel to the weld, (*b*) the normal stress perpendicular to the weld.

SOLUTION



$$\sigma_x = \frac{P}{A} = \frac{100 \times 10^3}{(10 \times 10^{-3})(80 \times 10^{-3})} = 125 \times 10^6 \text{ Pa} = 125 \text{ MPa}$$

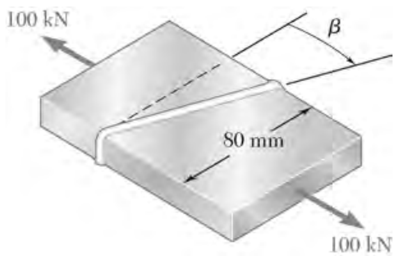
$$\sigma_y = 0 \qquad \tau_{xy} = 0$$

From Mohr's circle:

(*a*) $\tau_w = 62.5 \sin 50°$ $\qquad \tau_w = 47.9 \text{ MPa}$ ◀

(*b*) $\sigma_w = 62.5 + 62.5 \cos 50°$

$\sigma_w = 102.7 \text{ MPa}$ ◀

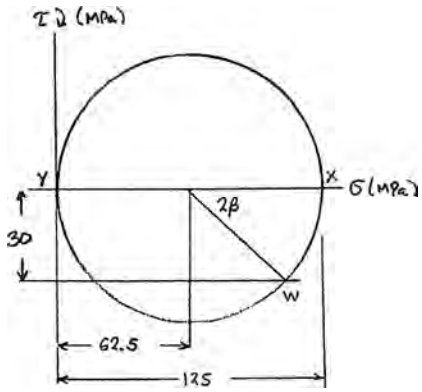


PROBLEM 7.44

Solve Prob. 7.22, using Mohr's circle.

PROBLEM 7.22 Two steel plates of uniform cross section 10×80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (*a*) the angle β, (*b*) the corresponding normal stress perpendicular to the weld.

SOLUTION



$$\sigma_x = \frac{P}{A} = \frac{100 \times 10^3}{(10 \times 10^{-3})(80 \times 10^{-3})} = 125 \times 10^6 \text{ Pa} = 125 \text{ MPa}$$

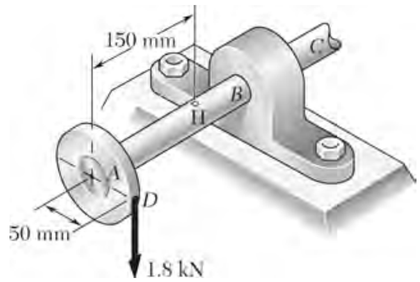
$$\sigma_y = 0 \qquad \tau_{xy} = 0$$

From Mohr's circle:

(*a*) $\quad \sin 2\beta = \dfrac{30}{62.5} = 0.48 \qquad\qquad \beta = 14.3° \blacktriangleleft$

(*b*) $\quad \sigma = 62.5 + 62.5 \cos 2\beta$

$$\sigma = 117.3 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.45

Solve Prob. 7.23, using Mohr's circle.

PROBLEM 7.23 A 1.8 kN vertical force is applied at *D* to a gear attached to the solid one-inch diameter shaft *AB*. Determine the principal stresses and the maximum shearing stress at point *H* located as shown on top of the shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point *H*.

$$V = 1.8 \text{ kN} \qquad M = (1.8\times 10^3)(0.150) = 270 \text{ N}\cdot\text{m}$$

$$T = (1.8\times 10^3)(0.050) = 90 \text{ N}\cdot\text{m}$$

Shaft cross section $\quad d = 25 \text{ mm} \qquad c = \frac{1}{2}d = 12.5 \text{ mm}$

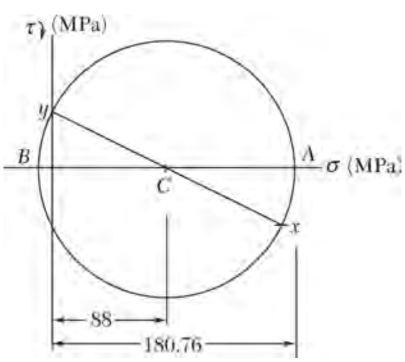
$$J = \frac{\pi}{2}c^4 = 38349 \text{ mm}^4 \qquad I = \frac{1}{2}J = 19174.5 \text{ mm}^4$$

Torsion: $\quad \tau = \frac{Tc}{J} = \frac{(90)(0.0125)}{38394\times 10^{-12}} = 29.33 \text{ MPa}$

Bending: $\quad \sigma = \frac{Mc}{I} = \frac{(270)(0.0125)}{19174.5\times 10^{-12}} = 176 \text{ MPa}$

Transverse Shear: Stress at point *H* is zero.

Resultant Stress: $\quad \sigma_x = 176 \text{ MPa}, \qquad \sigma_y = 0, \qquad \tau_{xy} = 29.33 \text{ MPa}$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 88 \text{ MPa}$$

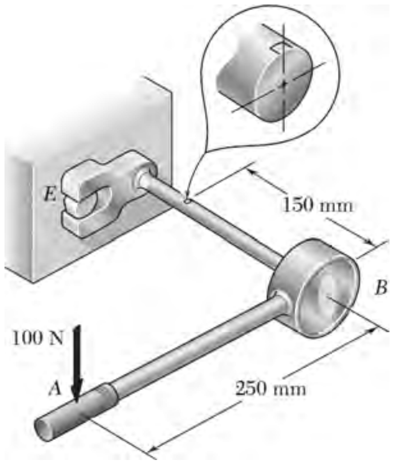
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(88)^2 + (29.33)^2} = 92.76 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 180.76 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R = -4.76 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{\max} = R = 92.76 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 7.46

Solve Prob. 7.24, using Mohr's circle.

PROBLEM 7.24 A mechanic uses a crowfoot wrench to loosen a bolt at *E*. Knowing that the mechanic applies a vertical 100 N force at *A*, determine the principal stresses and the maximum shearing stress at point *H* located as shown on top of the 18 mm diameter shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point *H*.

$$V = 100 \text{ N} \qquad M = (100)(150) = 15000 \text{ N} \cdot \text{mm}$$

$$T = (100)(250) = 25000 \text{ N} \cdot \text{mm}$$

Shaft cross section: $d = 18 \text{ mm} \qquad c = \frac{1}{2}d = 9 \text{ mm}$

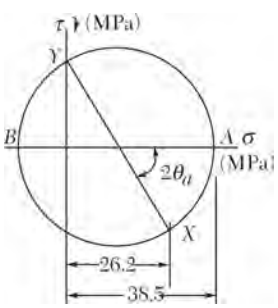
$$J = \frac{\pi}{2}c^4 = 10306 \text{ mm}^4 \qquad I = \frac{1}{2}J = 5153 \text{ mm}^4$$

Torsion: $\tau = \frac{Tc}{J} = \frac{(25000)(9)}{10306} = 21.8 \text{ N/mm}^2 = 21.8 \text{ MPa}$

Bending: $\sigma = \frac{Mc}{J} = \frac{(15000)(9)}{5153} = 26.2 \text{ N/mm}^2 = 26.2 \text{ MPa}$

Transverse Shear: At point *H* stress due to transverse shear is zero.

Resultant Stresses: $\sigma_x = 26.2 \text{ MPa}, \qquad \sigma_y = 0, \qquad \tau_{xy} = 21.8 \text{ MPa}$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 13.1 \text{ MPa}$$

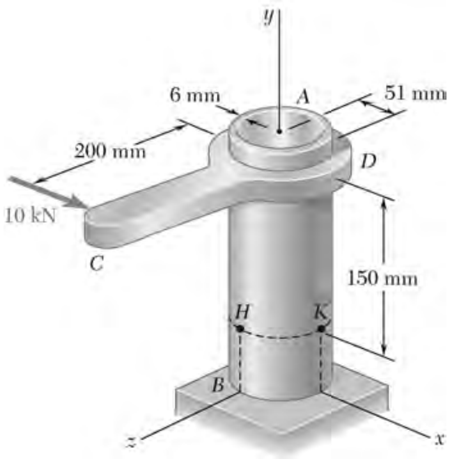
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

$$= \sqrt{13.1^2 + 21.8^2} = 25.4 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 38.5 \text{ MPa} \quad ◀$$

$$\sigma_b = \sigma_{\text{ave}} - R = -12.3 \text{ MPa} \quad ◀$$

$$\tau_{\max} = R = 25.4 \text{ MPa}$$



PROBLEM 7.47

Solve Prob. 7.25, using Mohr's circle.

PROBLEM 7.25 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51\,\text{mm} \qquad r_i = r_o - t = 45\,\text{mm}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855 \times 10^6\,\text{mm}^4 = 4.1855 \times 10^{-6}\,\text{m}^4$$

$$I = \frac{1}{2}J = 2.0927 \times 10^{-6}\text{m}^4$$

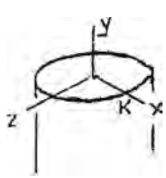
Force-couple system at center of tube in the plane containing points *H* and *K*:

$$F_x = 10 \times 10^3\,\text{N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000\,\text{N} \cdot \text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500\,\text{N} \cdot \text{m}$$

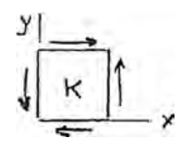
Torsion:



$$T = M_y = 2000\,\text{N} \cdot \text{m}$$

$$c = r_o = 51 \times 10^{-3}\,\text{m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37\,\text{MPa}$$



Note that the local *x*-axis is taken along a negative global *z*-direction.

Transverse shear: Stress due to $V = F_x$ is zero at point *K*.

Bending:

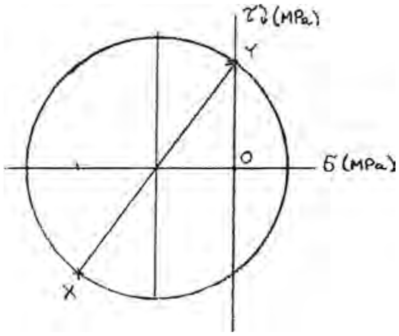
$$|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56\,\text{MPa}$$

Point *K* lies on compression side of neutral axis. $\sigma_y = -36.56\,\text{MPa}$

PROBLEM 7.47 *(Continued)*

Total stresses at point K: $\sigma_x = 0, \quad \sigma_y = -36.56 \text{ MPa}, \quad \tau_{xy} = 24.37 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$$

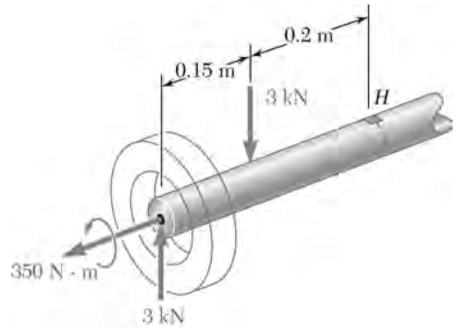
$$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46$$

$$\sigma_{max} = 12.18 \text{ MPa} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46$$

$$\sigma_{min} = -48.74 \text{ MPa} \blacktriangleleft$$

$$\tau_{max} = R \qquad \tau_{max} = 30.46 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.48

Solve Prob. 7.26, using Mohr's circle.

PROBLEM 7.26 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 32 mm, determine (*a*) the principal planes and principal stresses at point *H* located on top of the axle, (*b*) the maximum shearing stress at the same point.

SOLUTION

$$c = \frac{1}{2}d = \frac{1}{2}(32) = 16\text{ mm} = 16 \times 10^{-3}\text{ m}$$

Torsion:

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau = \frac{2(350\text{ N}\cdot\text{m})}{\pi(16\times 10^{-3}\text{m})^3} = 54.399\times 10^6\text{ Pa} = 54.399\text{ MPa}$$

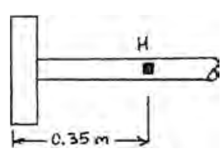
Bending:

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(16\times 10^{-3})^4 = 51.472\times 10^{-9}\text{m}^4$$

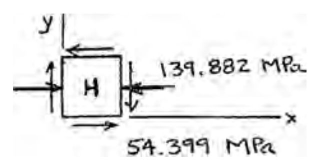
$$M = (0.15\text{ m})(3\times 10^3\text{ N}) = 450\text{ N}\cdot\text{m}$$

$$\sigma = -\frac{My}{I} = -\frac{(450)(16\times 10^{-3})}{51.472\times 10^{-9}} = -139.882\times 10^6\text{ Pa} = -139.882\text{ MPa}$$

Top view

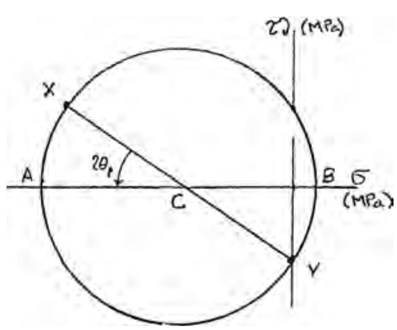


Stresses



$$\sigma_x = -139.882\text{ MPa}, \qquad \sigma_y = 0, \qquad \tau_{xy} = -54.399\text{ MPa}$$

Plotted points: $X: (-139.882, 54.399)$; $Y: (0, -54.399)$; $C: (-69.941, 0)$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -69.941\text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-139.882}{2}\right)^2 + (54.399)^2} = 88.606\text{ MPa}$$

PROBLEM 7.48 *(Continued)*

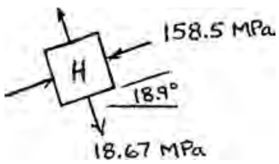
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-54.399)}{-139.882}$$

$$= 0.77778$$

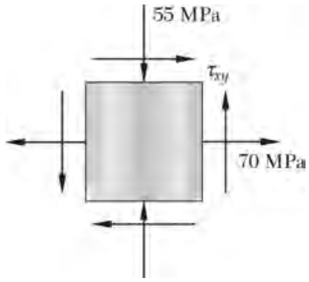
(*a*) $\theta_a = 18.9°$, $\theta_b = 108.9°$ ◀

$\sigma_a = \sigma_{\text{ave}} - R = -69.941 - 88.606$ $\sigma_a = -158.5 \text{ MPa}$ ◀

$\sigma_b = \sigma_{\text{ave}} + R = -69.941 + 88.606$ $\sigma_b = 18.67 \text{ MPa}$ ◀

(*b*) 

$\tau_{\max} = R$ $\tau_{\max} = 88.6 \text{ MPa}$ ◀



PROBLEM 7.49

Solve Prob. 7.27, using Mohr's circle.

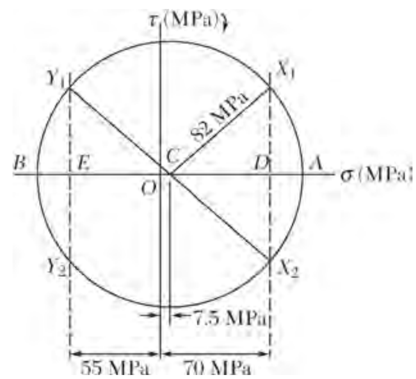
PROBLEM 7.27 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 82 MPa, (*b*) the corresponding principal stresses.

SOLUTION

The center of the Mohr's circle lies at point C with coordinates

$$\left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{70 - 55}{2}, 0\right) = (7.5 \text{ MPa}, 0).$$

The radius of the circle is $\tau_{\max}\,(\text{in-plane}) = 82$ MPa



The stress point $(\sigma_x, -\tau_{xy})$ lie along the line X_1X_2 of the Mohr's circle diagram. The extreme points with $R \le 82$ MPa are X_1 and X_2.

(*a*) The largest allowable value of τ_{xy} is obtained from triangle CDX,

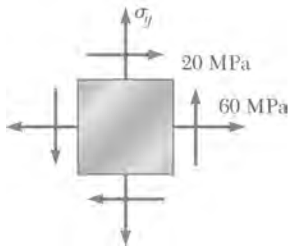
$$\overline{DX}_1^2 = \overline{DX}_2^2 = \sqrt{\overline{CX}_1^2 - \overline{CD}^2}$$

$$\tau_{xy}^2 = \sqrt{82^2 - 62.5^2} = 53 \text{ MPa}$$

(*b*) The principal stresses are

$$\sigma_a = 7.5 + 82 = 89.5 \text{ MPa}$$

$$\sigma_b = 7.5 - 82 = -74.5 \text{ MPa} \quad ◀$$



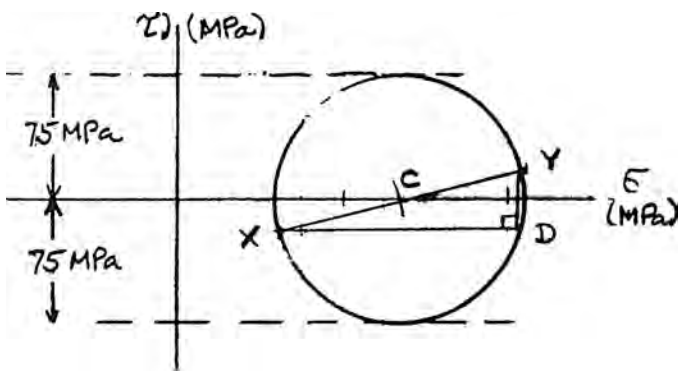
PROBLEM 7.50

Solve Prob. 7.28, using Mohr's circle.

PROBLEM 7.28 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.

SOLUTION

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_y = ?, \quad \tau_{xy} = 20 \text{ MPa}$$



Given:

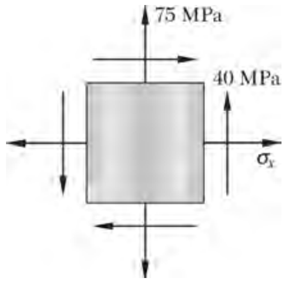
$$\tau_{\max} = R = 75 \text{ MPa}$$

$$\overline{XY} = 2R = 150 \text{ MPa}$$

$$\overline{DY} = (2)(\tau_{xy}) = 40 \text{ MPa}$$

$$\overline{XD} = \sqrt{\overline{XY}^2 - \overline{DY}^2} = \sqrt{150^2 - 40^2} = 144.6 \text{ MPa}$$

$$\sigma_y = \sigma_x + \overline{XD} = 60 + 144.6 \qquad \sigma_y = 205 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.51

Solve Prob. 7.29, using Mohr's circle.

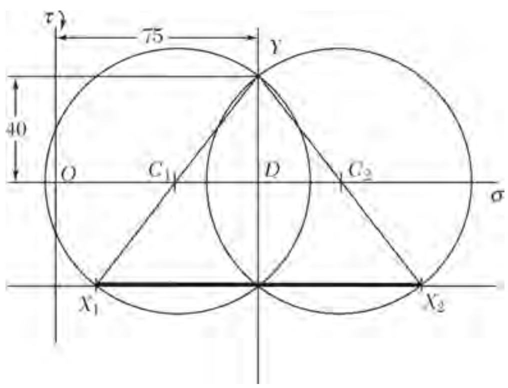
PROBLEM 7.29 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 50 MPa.

SOLUTION

For the Mohr's circle, point Y lies at (75 MPa, 40 MPa). The radius of limiting circles is $R = 50$ MPa.

Let C_1 be the location of the left most limiting circle and C_2 be that of the right most one.

$$\overline{C_1Y} = 50 \text{ MPa}$$
$$\overline{C_2Y} = 50 \text{ MPa}$$



Noting right triangles C_1DY and C_2DY

$$\overline{C_1D}^2 + \overline{DY}^2 = \overline{C_1Y}^2 \qquad \overline{C_1D}^2 = 50^2 - 40^2 \qquad \overline{C_1D} = 30$$

Coordinates of point C_1 are $(0,\ 75 - 30) = (0,\ 45 \text{ MPa})$.

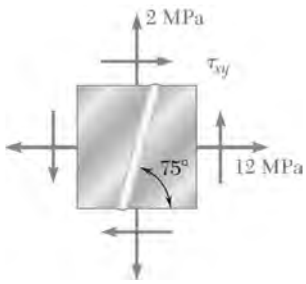
Likewise, coordinates of point C_2 are $= (0,\ 75 + 30) = (0,\ 105 \text{ MPa})$

Coordinates of point X_1 $(45 - 30,\ -40) = (15 \text{ MPa},\ -40 \text{ MPa})$

Coordinates of point X_2 $(105 + 30,\ -40) = (135 \text{ MPa},\ -40 \text{ MPa})$

The point $(\sigma_x, -\tau_{xy})$ must lie on the line X_1X_2

Thus $15 \text{ MPa} \le \sigma_x \le 135 \text{ MPa}$



PROBLEM 7.52

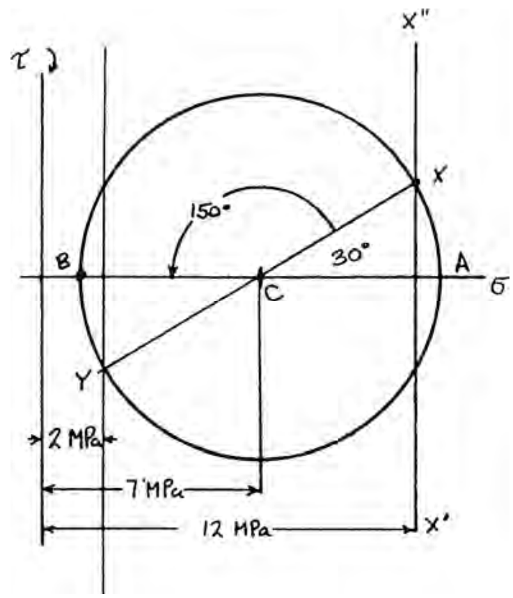
Solve Prob. 7.30, using Mohr's circle.

PROBLEM 7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

SOLUTION

Point X of Mohr's circle must lie on $X'X''$ so that $\sigma_x = 12$ MPa. Likewise, point Y lies on line $Y'Y''$ so that $\sigma_y = 2$ MPa. The coordinates of C are

$$\frac{2+12}{2}, \quad 0 = (7 \text{ MPa}, 0).$$



Counterclockwise rotation through 150° brings line CX to CB, where $\tau = 0$.

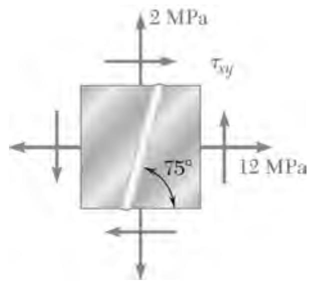
$$R = \frac{\sigma_x - \sigma_y}{2}\sec 30° = \frac{12-2}{2}\sec 30° = 5.77 \text{ MPa}$$

(*a*) $\tau_{xy} = -\dfrac{\sigma_x - \sigma_y}{2}\tan 30°$

$= -\dfrac{12-2}{2}\tan 30°$ $\qquad \tau_{xy} = -2.89 \text{ MPa}$ ◀

(*b*) $\sigma_a = \sigma_{\text{ave}} + R = 7 + 5.77$ $\qquad \sigma_a = 12.77 \text{ MPa}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R = 7 - 5.77$ $\qquad \sigma_b = 1.23 \text{ MPa}$ ◀

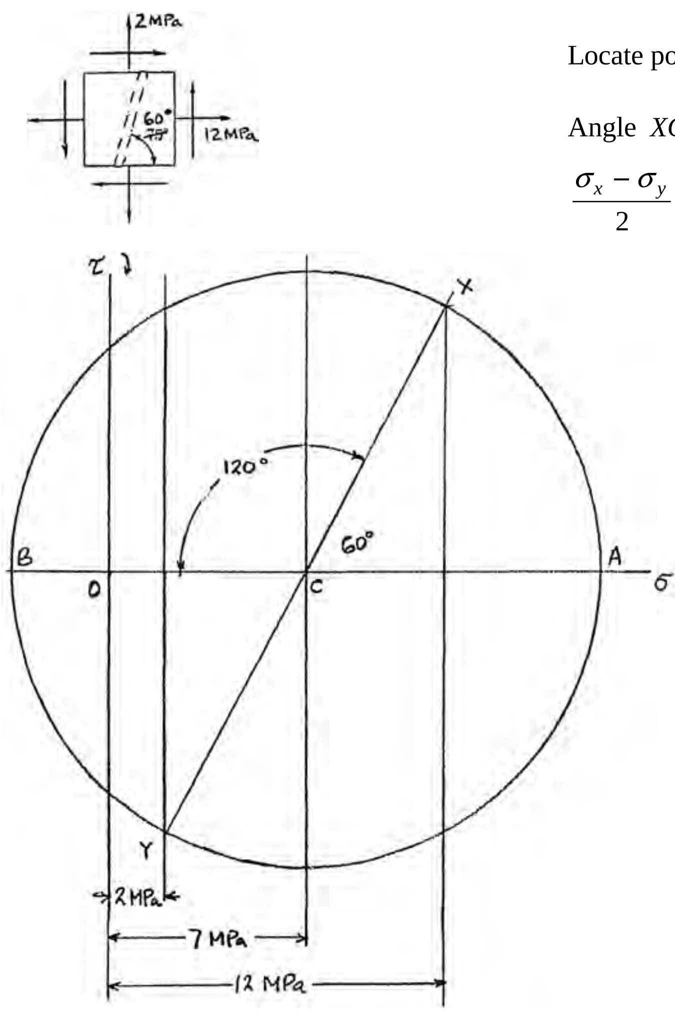


PROBLEM 7.53

Solve Problem 7.30, using Mohr's circle and assuming that the weld forms an angle of 60° with the horizontal.

PROBLEM 7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

SOLUTION



Locate point C at $\sigma = \dfrac{12+2}{2} = 7$ MPa with $\tau = 0$.

Angle $XCB = 120°$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{12-2}{2} = 5 \text{ MPa}$$

$$R = 5\sec 60° = 10 \text{ MPa}$$

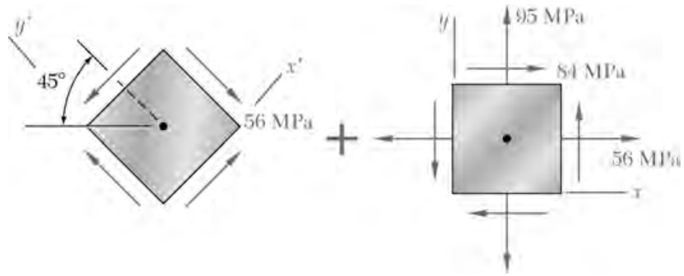
$$\tau_{xy} = -5\tan 60°$$

$\tau_{xy} = -8.66$ MPa ◀

$\sigma_a = \sigma_{ave} + R = 7 + 10$ $\sigma_a = 17$ MPa ◀

$\sigma_b = \sigma_{ave} - R = 7 - 10$ $\sigma_b = -3$ MPa ◀

PROBLEM 7.54



Determine the principal planes and the principal stress for the state of plane stress resulting from the superposition of the two states of stress shown.

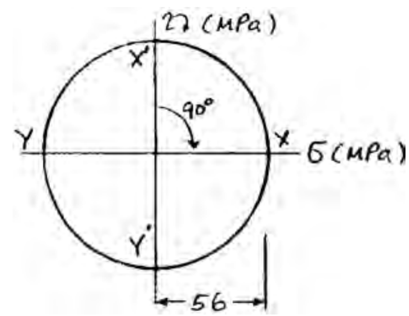
SOLUTION

Mohr's circle for 1st stress state.

$$\sigma_x = 56 \text{ MPa}$$

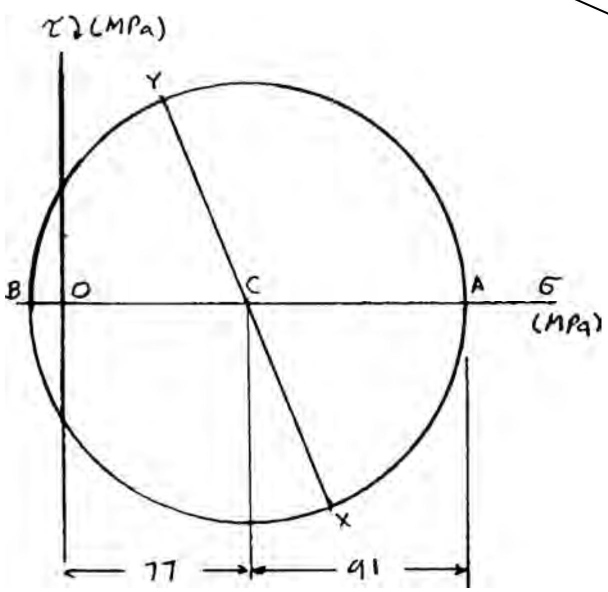
$$\sigma_y = -56 \text{ MPa}$$

$$\tau_{xy} = 0$$



Resultant stresses

$$\left.\begin{aligned} \sigma_x &= 56 + 56 = 112 \text{ MPa} \\ \sigma_y &= -56 + 98 = 42 \text{ MPa} \\ \tau_{xy} &= 84 + 0 = 84 \text{ MPa} \end{aligned}\right\}$$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 77 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(84)}{70} = 2.4$$

$$2\theta_p = 67.38°$$

$\theta_a = 33.69°$ ◀

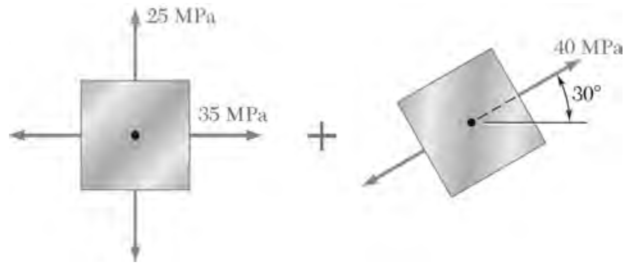
$\theta_b = 123.69°$ ◀

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{35^2 - 84^2} = 91 \text{ MPa}$$

$\sigma_a = \sigma_{\text{ave}} + R = 168 \text{ MPa}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R = -14 \text{ MPa}$ ◀

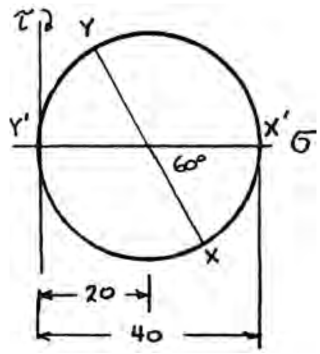


PROBLEM 7.55

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

Mohr's circle for 2nd stress state:



$$\sigma_x = 20 + 20\cos 60°$$
$$= 30 \text{ MPa}$$
$$\sigma_y = 20 - 20\cos 60°$$
$$= 10 \text{ MPa}$$
$$\tau_{xy} = 20\sin 60°$$
$$= 17.32 \text{ MPa}$$

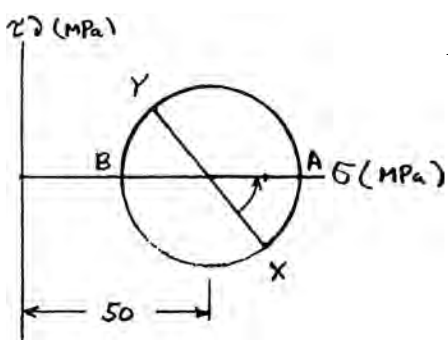
Resultant stresses:

$$\sigma_x = 35 + 30 = 65 \text{ MPa}$$

$$\sigma_y = 25 + 10 = 35 \text{ MPa}$$

$$\tau_{xy} = 0 + 17.32 = 17.32 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(65 + 35) = 50 \text{ MPa}$$



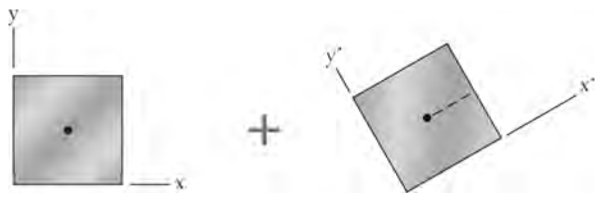
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(17.32)}{65 - 35} = 1.1547$$

$$2\theta_p = 49.11°, \qquad \theta_a = 24.6°, \quad \theta_b = 114.6° \blacktriangleleft$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 22.91 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 72.91 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = 27.09 \text{ MPa} \blacktriangleleft$$

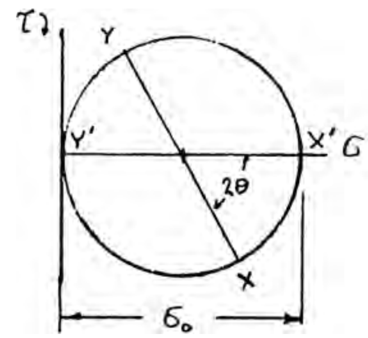


PROBLEM 7.56

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

Mohr's circle for 2nd stress state:



$$\sigma_x = \frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\sigma_y = \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\tau_{xy} = \frac{1}{2}\sigma_0 \sin 2\theta$$

Resultant stresses:

$$\sigma_x = \sigma_0 + \frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta = \frac{3}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta$$

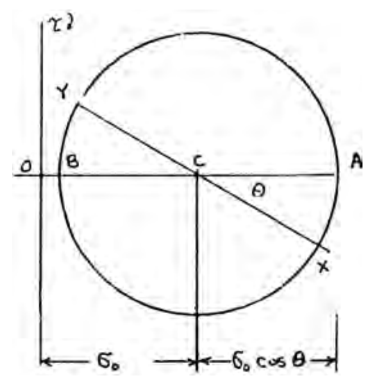
$$\sigma_y = 0 + \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta = \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\tau_{xy} = 0 + \frac{1}{2}\sigma_0 \sin 2\theta = \frac{1}{2}\sigma_0 \sin 2\theta$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sigma_0 \sin 2\theta}{\sigma_0 + \sigma_0 \cos 2\theta}$$

$$= \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$$



$$\theta_p = \frac{1}{2}\theta \blacktriangleleft$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta\right)^2 + \left(\frac{1}{2}\sigma_0 \sin 2\theta\right)^2}$$

$$= \frac{1}{2}\sigma_0\sqrt{1 + 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta} = \frac{\sqrt{2}}{2}\sigma_0\sqrt{1 + \cos 2\theta} = \sigma_0\,|\cos\theta|$$

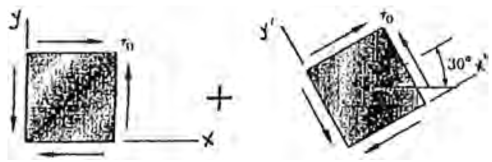
$\sigma_a = \sigma_{\text{ave}} + R$ $\qquad$ $\sigma_a = \sigma_0 + \sigma_0 \cos\theta \blacktriangleleft$

$\sigma_b = \sigma_{\text{ave}} - R$ $\qquad$ $\sigma_b = \sigma_0 - \sigma_0 \cos\theta \blacktriangleleft$

PROBLEM 7.57

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

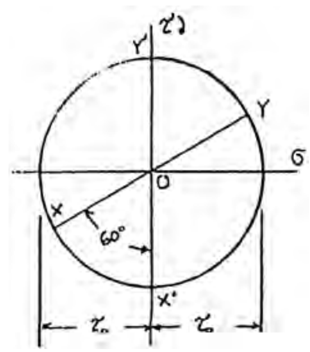


Mohr's circle for 2nd state of stress:

$$\sigma_{x'} = 0$$

$$\sigma_{y'} = 0$$

$$\tau_{x'y'} = \tau_0$$



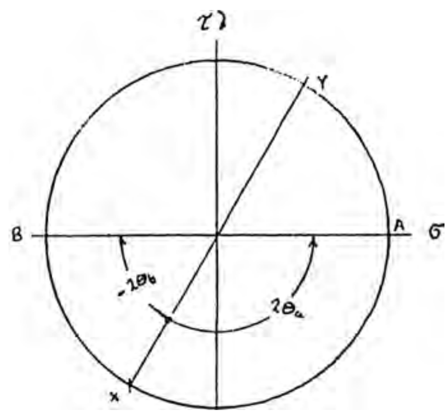
$$\sigma_x = -\tau_0 \sin 60° = -\frac{\sqrt{3}}{2}\tau_0 \qquad \sigma_y = \tau_0 \sin 60° = \frac{\sqrt{3}}{2}\tau_0$$

$$\tau_{xy} = \tau_0 \cos 60° = \frac{1}{2}\tau_0$$

Resultant stresses:

$$\sigma_x = 0 - \frac{\sqrt{3}}{2}\tau_0 = -\frac{\sqrt{3}}{2}\tau_0 \qquad \sigma_y = 0 + \frac{\sqrt{3}}{2}\tau_0 = \frac{\sqrt{3}}{2}\tau_0$$

$$\tau_{xy} = \tau_0 + \frac{1}{2}\tau_0 = \frac{3}{2}\tau_0$$

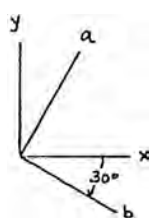


$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sqrt{3}}{2}\tau_0\right)^2 + \left(\frac{3}{2}\tau_0\right)^2} = \sqrt{3}\,\tau_0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)\left(\frac{3}{2}\right)}{-\sqrt{3}} = -\sqrt{3}$$

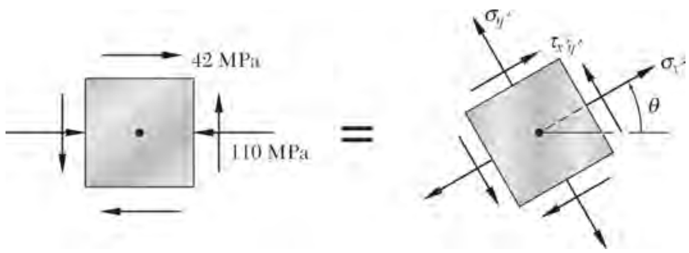
$$2\theta_p = -60°$$



$$\theta_b = -30° \qquad \theta_a = 60° \blacktriangleleft$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = \sqrt{3}\,\tau_0 \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -\sqrt{3}\,\tau_0 \blacktriangleleft$$



PROBLEM 7.58

For the state of stress shown, determine the range of values of θ for which the magnitude of the shearing stress $\tau_{x'y'}$ is equal to or less than 56 MPa.

SOLUTION

$$\sigma_x = -110 \text{ MPa}, \quad \sigma_y = 0$$

$$\tau_{xy} = 42 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -55 \text{ MPa}$$

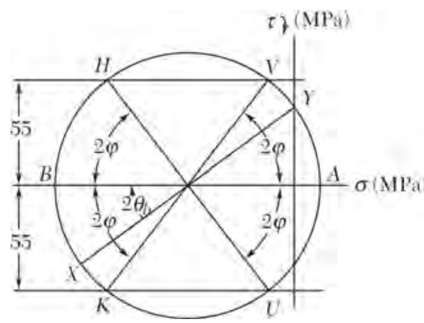
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-55)^2 + (42)^2} = 69.2 \text{ MPa}$$

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(42)}{-110} = -0.7636$$

$$2\theta_P = -37.36°$$

$$\theta_b = -18.68°$$



$|\tau_{x'y'}| \le 56$ MPa for states of stress corresponding to arcs *HBK* and *UAV* of Mohr's circle. The angle φ is calculated from

$$R \sin 2\varphi = 55 \qquad \sin 2\varphi = \frac{55}{69.2} = 0.7948$$

$$2\varphi = 52.64° \qquad \varphi = 26.32°$$

$$\theta_H = \theta_b - \varphi = -18.68 - 26.32 = -45°$$

$$\theta_K = \theta_b + \varphi = -18.68 + 26.32 = 7.64°$$

$$\theta_U = \theta_H + 90° = 45°$$

$$\theta_V = \theta_K + 90° = 97.64°$$

Permissible range of θ $\qquad \theta_H \le \theta \le \theta_K$

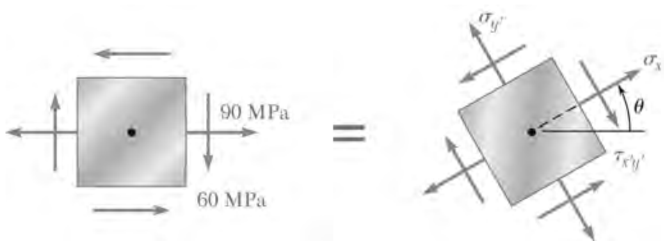
$$-45° \le \theta \le 7.64°$$

$$\theta_U \le \theta \le \theta_V \quad ◀$$

$$45° \le \theta \le 97.64° \quad ◀$$

Also, $\qquad 135° \le \theta \le 187.64°$

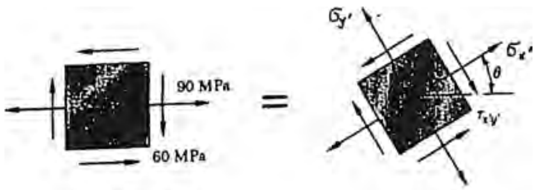
$$225° \le \theta \le 277.64°$$



PROBLEM 7.59

For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 50 MPa.

SOLUTION



$$\sigma_x = 90 \text{ MPa}, \quad \sigma_y = 0$$

$$\tau_{xy} = -60 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 45 \text{ MPa}$$

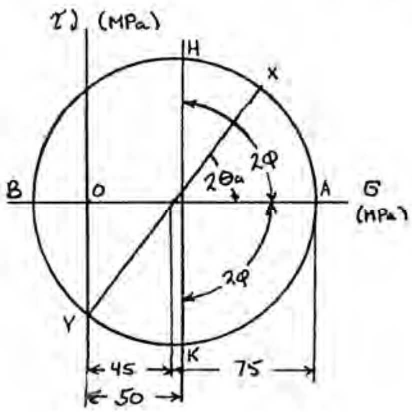
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{90} = -\frac{4}{3}$$

$$2\theta_p = -53.13°$$

$$\theta_a = -26.565°$$



$\sigma_{x'} \leq 50$ MPa for states of stress corresponding to the arc *HBK* of Mohr's circle. From the circle,

$$R\cos 2\varphi = 50 - 45 = 5 \text{ MPa}$$

$$\cos 2\varphi = \frac{5}{75} = 0.066667$$

$$2\varphi = 86.177° \qquad \varphi = 43.089°$$

$$\theta_h = \theta_a + \varphi = -26.565° + 43.089° = 16.524°$$

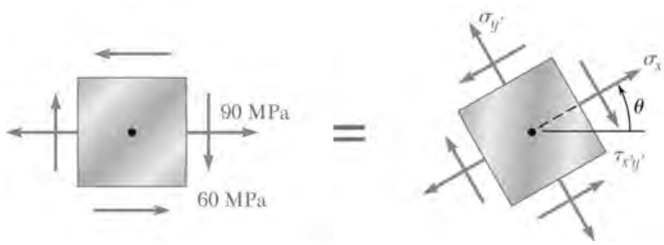
$$2\theta_k = 2\theta_h + 360° - 4\varphi = 32.524° + 360° - 172.355° = 220.169°$$

$$\theta_k = 110.085°$$

Permissible range of θ: $\theta_h \leq \theta \leq \theta_k$

$16.524° \leq \theta \leq 110.085°$ ◀

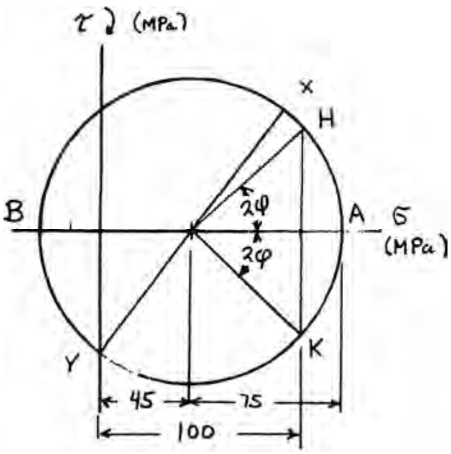
Also, $196.524° \leq \theta \leq 290.085°$



PROBLEM 7.60

For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 100 MPa.

SOLUTION



$$\sigma_x = 90\text{ MPa}, \quad \sigma_y = 0$$

$$\tau_{xy} = -60\text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 45\text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{45^2 + 60^2} = 75\text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{90} = -\frac{4}{3}$$

$$2\theta_p = -53.13°$$

$$\theta_a = -26.565°$$

$\sigma_{x'} \le 100$ MPa for states of stress corresponding to arc HBK of Mohr's circle. From the circle,

$$R\cos 2\varphi = 100 - 45 = 55\text{ MPa}$$

$$\cos 2\varphi = \frac{55}{75} = 0.73333$$

$$2\varphi = 42.833° \qquad \varphi = 21.417°$$

$$\theta_h = \theta_a + \varphi = -26.565° + 21.417° = -5.15°$$

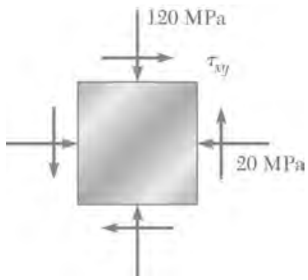
$$2\theta_k = 2\theta_h + 360° - 4\varphi = -10.297° + 360° - 85.666° = 264.037°$$

$$\theta_k = 132.02°$$

Permissible range of θ is $\qquad \theta_h \le \theta \le \theta_k$

$$-5.15° \le \theta \le 132.02° \quad ◀$$

Also, $\qquad 174.85° \le \theta \le 312.02°$



PROBLEM 7.61

For the element shown, determine the range of values of τ_{xy} for which the maximum tensile stress is equal to or less than 60 MPa.

SOLUTION

$$\sigma_x = -20 \text{ MPa} \quad \sigma_y = -120 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -70 \text{ MPa}$$

Set

$$\sigma_{\max} = 60 \text{ MPa} = \sigma_{\text{ave}} + R$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 130 \text{ MPa}$$

But

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

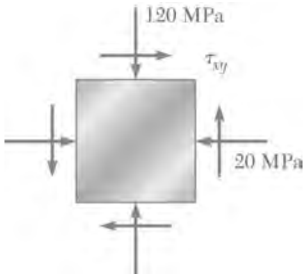
$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_x}{2}\right)^2}$$

$$= \sqrt{130^2 - 50^2}$$

$$= 120 \text{ MPa}$$

Range of τ_{xy} :

$$-120 \text{ MPa} \le \tau_{xy} \le 120 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.62

For the element shown, determine the range of values of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 150 MPa.

SOLUTION

$$\sigma_x = -20\,\text{MPa} \quad \sigma_y = -120\ \text{MPa}$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = 50\ \text{MPa}$$

Set $$\tau_{\max\,(\text{in-plane})} = R = 150\ \text{MPa}$$

But $$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

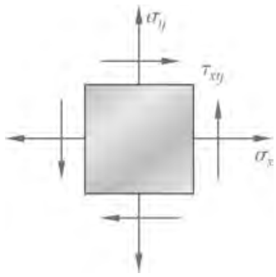
$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

$$= \sqrt{150^2 - 50^2}$$

$$= 141.4\ \text{MPa}$$

Range of τ_{xy}: $$-141.4\ \text{MPa} \le \tau_{xy} \le 141.4\ \text{MPa}$$ ◀

PROBLEM 7.63



For the state of stress shown it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 98$ MPa, $\sigma_y = 63$ MPa, and $\sigma_{\min} = 35$ MPa. Determine (*a*) the orientation of the principal planes, (*b*) the principal stress $\sigma_{\max}$, (*c*) the maximum in-plane shearing stress.

SOLUTION

$$\sigma_x = 98 \text{ MPa}, \quad \sigma_y = 63 \text{ MPa}, \quad \sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 80.5 \text{ MPa}$$

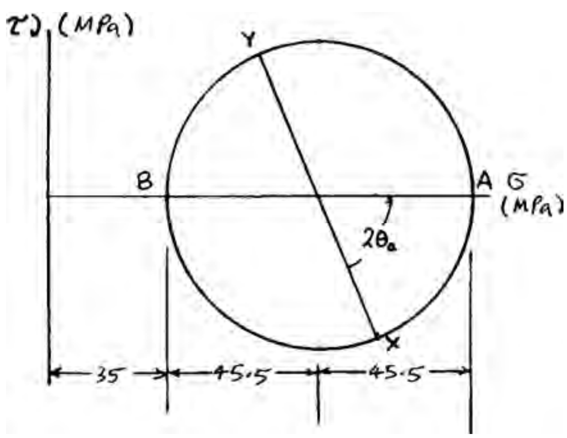
$$\sigma_{\min} = \sigma_{\text{ave}} - R \quad \therefore \quad R = \sigma_{\text{ave}} - \sigma_{\min}$$

$$= 80.5 - 35 = 45.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \pm\sqrt{45.5^2 - 17.5^2} = \pm 42 \text{ MPa}$$

But it is given that τ_{xy} is positive, thus $\tau_{xy} = +42$ MPa



(*a*) $\tan 2\theta_P = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$$= \frac{(2)(6)}{5} = 2.4$$

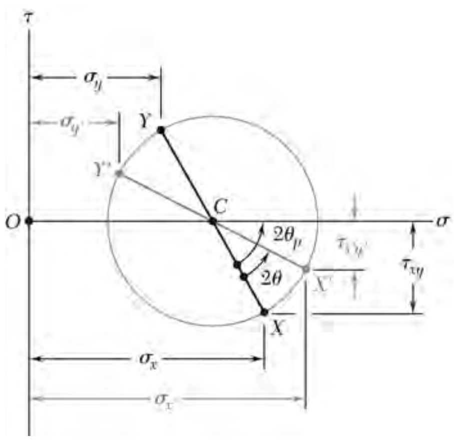
$$2\theta_P = 67.38°$$

$\theta_a = 33.69°$ ◀

$\theta_b = 123.69°$ ◀

(*b*) $\sigma_{\max} = \sigma_{\text{ave}} + R = 126$ MPa ◀

(*c*) $\tau_{\max(\text{in-plane})} = R = 45.5$ MPa ◀



PROBLEM 7.64

The Mohr's circle shown corresponds to the state of stress given in Fig. 7.5*a* and *b*. Noting that $\sigma_{x'} = OC + (CX')\cos(2\theta_p - 2\theta)$ and that $\tau_{x'y'} = (CX')\sin(2\theta_p - 2\theta)$, derive the expressions for $\sigma_{x'}$ and $\tau_{x'y'}$ given in Eqs. (7.5) and (7.6), respectively. [*Hint:* Use $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and $\cos(A+B) = \cos A \cos B - \sin A \sin B$.]

SOLUTION

$$\overline{OC} = \frac{1}{2}(\sigma_x + \sigma_y) \qquad \overline{CX'} = \overline{CX}$$

$$\overline{CX'} \cos 2\theta_p = \overline{CX} \cos 2\theta_p = \frac{\sigma_x - \sigma_y}{2}$$

$$\overline{CX'} \sin 2\theta_p = \overline{CX} \sin 2\theta_p = \tau_{xy}$$

$$\begin{aligned}
\sigma_{x'} &= \overline{OC} + \overline{CX'} \cos(2\theta_p - 2\theta) \\
&= \overline{OC} + \overline{CX'}(\cos 2\theta_p \cos 2\theta + \sin 2\theta_p \sin 2\theta) \\
&= \overline{OC} + \overline{CX'} \cos 2\theta_p \cos 2\theta + \overline{CX'} \sin 2\theta_p \sin 2\theta \\
&= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \quad \blacktriangleleft
\end{aligned}$$

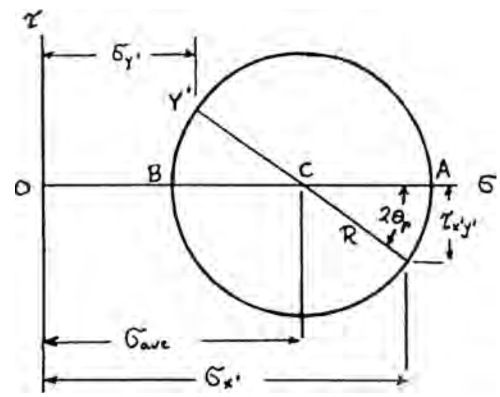
$$\begin{aligned}
\tau_{x'y'} &= \overline{CX'} \sin(2\theta_p - 2\theta) = \overline{CX'}(\sin 2\theta_p \cos 2\theta - \cos 2\theta_p \sin 2\theta) \\
&= \overline{CX'} \sin 2\theta_p \cos 2\theta - \overline{CX'} \cos 2\theta_p \sin 2\theta \\
&= \tau_{xy} \cos 2\theta - \frac{\sigma_x - \sigma_y}{2} \sin 2\theta \quad \blacktriangleleft
\end{aligned}$$

PROBLEM 7.65

(a) Prove that the expression $\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$, where $\sigma_{x'}, \sigma_{y'}$, and $\tau_{x'y'}$ are components of the stress along the rectangular axes x' and y', is independent of the orientation of these axes. Also, show that the given expression represents the square of the tangent drawn from the origin of the coordinates to Mohr's circle. (b) Using the invariance property established in part a, express the shearing stress τ_{xy} in terms of σ_x, σ_y, and the principal stresses $\sigma_{\max}$ and $\sigma_{\min}$.

SOLUTION

(a) From Mohr's circle,

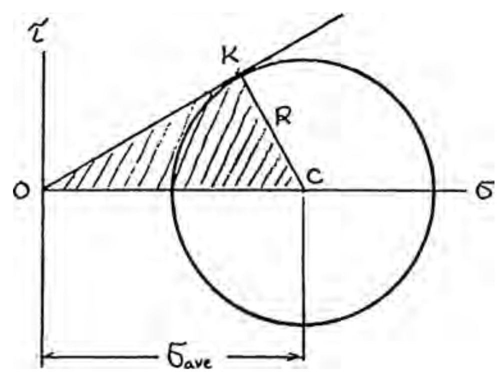


$$\tau_{x'y'} = R\sin 2\theta_p$$
$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos 2\theta_p$$
$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos 2\theta_p$$

$$\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$$
$$= \sigma_{\text{ave}}^2 - R^2\cos^2 2\theta_p - R^2\sin^2 2\theta_p$$
$$= \sigma_{\text{ave}}^2 - R^2; \text{ independent of } \theta_p.$$

Draw line $\overline{OK}$ from origin tangent to the circle at K. Triangle OCK is a right triangle.

$$\overline{OC}^2 = \overline{OK}^2 + \overline{CK}^2$$
$$\overline{OK}^2 = \overline{OC}^2 - \overline{CK}^2$$
$$= \sigma_{\text{ave}}^2 - R^2$$
$$= \sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$$



(b) Applying above to σ_x, σ_y, and τ_{xy}, and to σ_a, σ_b,

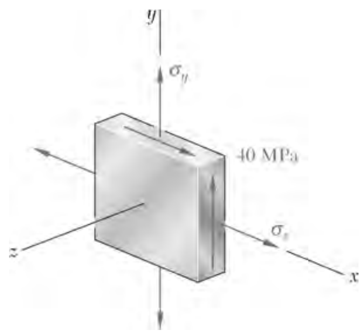
$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_a\sigma_b - \tau_{ab}^2 = \sigma_{\text{ave}}^2 - R^2$$

But $\quad \tau_{ab} = 0, \quad \sigma_a = \sigma_{\max}, \quad \sigma_b = \sigma_{\min}$

$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_{\max}\sigma_{\min}$$
$$\tau_{xy}^2 = \sigma_x\sigma_y - \sigma_{\max}\sigma_{\min}$$
$$\tau_{xy} = \pm\sqrt{\sigma_x\sigma_y - \sigma_{\max}\sigma_{\min}} \quad ◀$$

The sign cannot be determined from above equation.

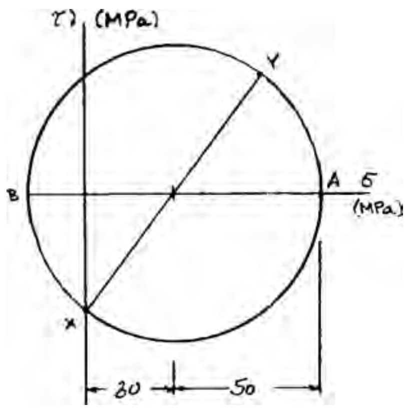
PROBLEM 7.66



For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 0$ and $\sigma_y = 60$ MPa, (*b*) $\sigma_x = 105$ MPa and $\sigma_y = 45$ MPa. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(*a*) $\sigma_x = 0, \quad \sigma_y = 60 \text{ MPa}, \quad \tau_{xy} = 40 \text{ MPa}$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 30 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-30)^2 + 40^2} = 50 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 80 \text{ MPa} \quad \text{(max)}$$

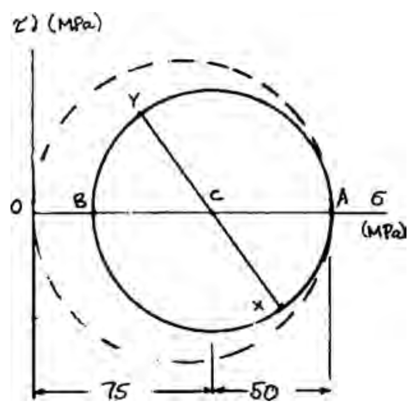
$$\sigma_b = \sigma_{\text{ave}} - R = -20 \text{ MPa} \quad \text{(min)}$$

$$\sigma_c = 0$$

$$\sigma_{\max} = 80 \text{ MPa} \qquad \sigma_{\min} = -20 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 50 \text{ MPa} \quad ◀$$

(*b*) $\sigma_x = 105 \text{ MPa}, \quad \sigma_y = 45 \text{ MPa} \quad \tau_{xy} = 40 \text{ MPa}$



$$\sigma_{\text{ave}} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-30)^2 + 40^2} = 50 \text{ MPa}$$

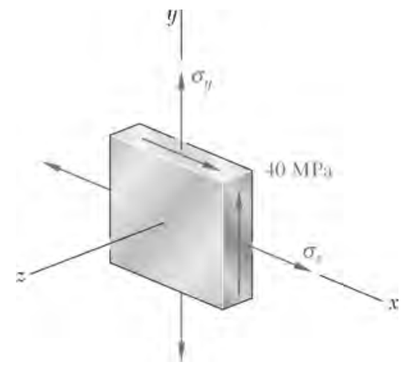
$$\sigma_a = \sigma_{\text{ave}} + R = 125 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 25 \text{ MPa}$$

$$\sigma_c = 0 \qquad \text{(min)}$$

$$\sigma_{\max} = 125 \text{ MPa}, \sigma_{\min} = 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 62.5 \text{ MPa} \quad ◀$$

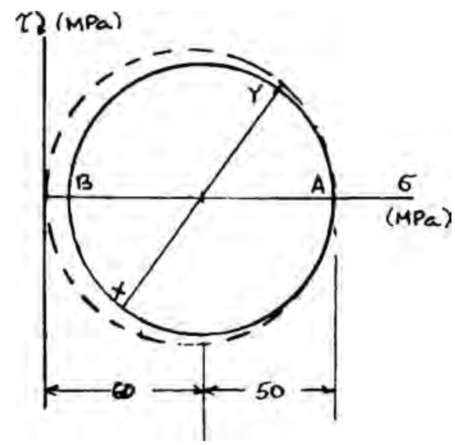


PROBLEM 7.67

For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 30$ MPa and $\sigma_y = 90$ MPa, (*b*) $\sigma_x = 70$ MPa and $\sigma_y = 10$ MPa. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(*a*) $\sigma_x = 30 \text{ MPa} \quad \sigma_y = 90 \text{ MPa} \quad \tau_{xy} = 40 \text{ MPa}$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 60 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + 40^2} = 50 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 60 + 50 = 110 \text{ MPa} \quad \text{(max)}$$

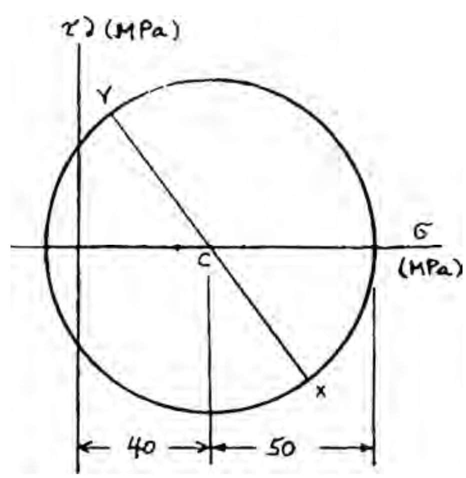
$$\sigma_b = \sigma_{\text{ave}} - R = 60 - 50 = 10 \text{ MPa}$$

$$\sigma_c = 0 \quad \text{(min)}$$

$$\tau_{\text{max (in-plane)}} = R = 50 \text{ MPa}$$

$$\tau_{\text{max}} = \frac{1}{2}(\sigma_{\text{max}} - \sigma_{\text{min}}) = 55 \text{ MPa} \quad ◀$$

(*b*) $\sigma_x = 70 \text{ MPa} \quad \sigma_y = 10 \text{ MPa} \quad \tau_{xy} = 40 \text{ MPa}$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 40 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + 40^2} = 50 \text{ MPa}$$

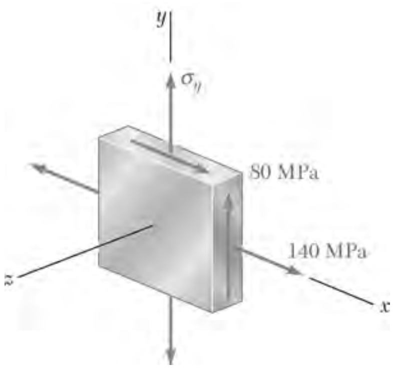
$$\sigma_a = \sigma_{\text{ave}} + R = 90 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -10 \text{ MPa} \quad \text{(min)}$$

$$\sigma_c = 0$$

$$\sigma_{\text{max}} = 90 \text{ MPa} \quad \sigma_{\text{min}} = -10 \text{ MPa}$$

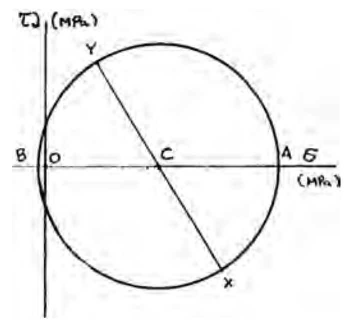
$$\tau_{\text{max}} = \frac{1}{2}(\sigma_{\text{max}} - \sigma_{\text{min}}) = 50 \text{ MPa} \quad ◀$$



PROBLEM 7.68

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 40$ MPa, (*b*) $\sigma_y = 120$ MPa. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION



(*a*) $\sigma_x = 140 \text{ MPa}, \quad \sigma_y = 40 \text{ MPa}, \quad \tau_{xy} = 80 \text{ MPa}$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 90 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{50^2 + 80^2} = 94.34 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 184.34 \text{ MPa} \quad \text{(max)}$$

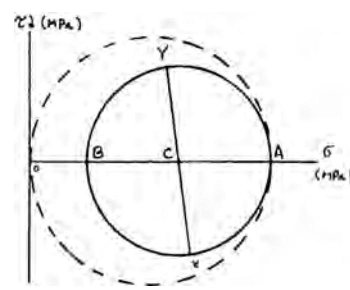
$$\sigma_b = \sigma_{\text{ave}} - R = -4.34 \text{ MPa} \quad \text{(min)}$$

$$\sigma_c = 0$$

$$\tau_{\max(\text{in-plane})} = \frac{1}{2}(\sigma_a - \sigma_b) = R = 94.34 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = \frac{1}{2}(\sigma_a - \sigma_b) = 94.3 \text{ MPa}$$

$\tau_{\max} = 94.3 \text{ MPa}$ ◀



(*b*) $\sigma_x = 140 \text{ MPa}, \quad \sigma_y = 120 \text{ MPa}, \quad \tau_{xy} = 80 \text{ MPa}$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 130 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{10^2 + 80^2} = 80.62 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 210.62 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 49.38 \text{ MPa}$$

$$\sigma_c = 0 \quad \text{(min)}$$

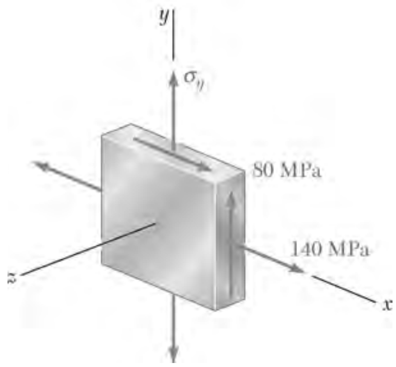
$$\sigma_{\max} = \sigma_a = 210.62 \text{ MPa} \quad \sigma_{\min} = \sigma_c = 0$$

$$\tau_{\max(\text{in-plane})} = R = 86.62 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 105.3 \text{ MPa}$$

$\tau_{\max} = 105.3 \text{ MPa}$ ◀

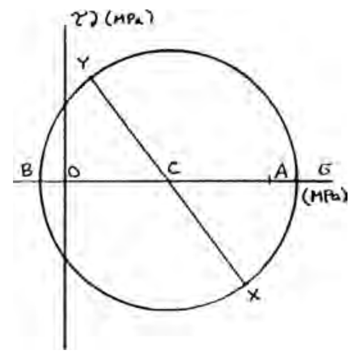
PROBLEM 7.69



For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 20$ MPa, (*b*) $\sigma_y = 140$ MPa. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(*a*) $\sigma_x = 140 \text{ MPa}, \quad \sigma_y = 20 \text{ MPa}, \quad \tau_{xy} = 80 \text{ MPa}$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 80 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{60^2 + 80^2} = 100 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 80 + 100 = 180 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 80 - 100 = -20 \text{ MPa} \quad \text{(min)}$$

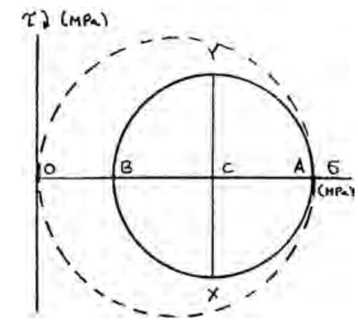
$$\sigma_c = 0$$

$$\tau_{\max(\text{in-plane})} = \frac{1}{2}(\sigma_a - \sigma_b) = 100 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 100 \text{ MPa}$$

$\tau_{\max} = 100$ MPa ◀

(*b*) $\sigma_x = 140 \text{ MPa}, \quad \sigma_y = 140 \text{ MPa}, \quad \tau_{xy} = 80 \text{ MPa}$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 140 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + 80^2} = 80 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 220 \text{ MPa} \quad \text{(max)}$$

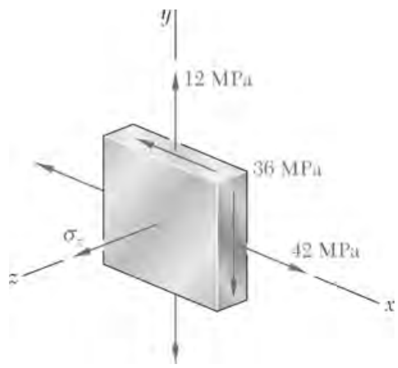
$$\sigma_b = \sigma_{\text{ave}} - R = 60 \text{ MPa}$$

$$\sigma_c = 0 \quad \text{(min)}$$

$$\tau_{\max\,(\text{in-plane})} = \frac{1}{2}(\sigma_a - \sigma_b) = 80 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 110 \text{ MPa}$$

$\tau_{\max} = 110$ MPa ◀



PROBLEM 7.70

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = +24$ MPa, (*b*) $\sigma_z = -24$ MPa, (*c*) $\sigma_z = 0$.

SOLUTION

$$\sigma_x = 42 \text{ MPa}, \qquad \sigma_y = 12 \text{ MPa}, \qquad \tau_{xy} = -36 \text{ MPa}$$

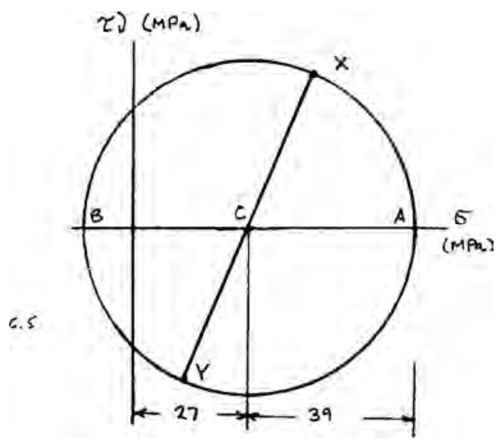
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 2.7 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(15)^2 + (-36)^2} = 39 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 66 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -12 \text{ MPa}$$



(*a*) $\underline{\sigma_z = +24 \text{ MPa}}$ $\quad \sigma_a = 66 \text{ MPa} \quad \sigma_b = -12 \text{ MPa}$

$$\sigma_{\max} = 66 \text{ MPa} \quad \sigma_{\min} = -12 \text{ MPa} \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 39 \text{ MPa} \quad \blacktriangleleft$$

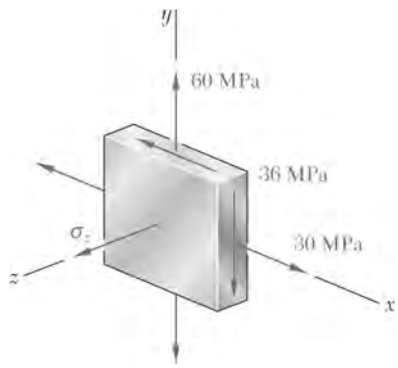
(*b*) $\underline{\sigma_z = -24 \text{ MPa}}$ $\quad \sigma_a = 66 \text{ MPa} \quad \sigma_b = -12 \text{ MPa}$

$$\sigma_{\max} = 66 \text{ MPa} \quad \sigma_{\min} = -24 \text{ MPa} \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 45 \text{ MPa} \quad \blacktriangleleft$$

(*c*) $\underline{\sigma_z = 0}$ $\quad \sigma_a = 66 \text{ MPa} \quad \sigma_b = -12 \text{ MPa}$

$$\sigma_{\max} = 66 \text{ MPa} \quad \sigma_{\min} = -12 \text{ MPa} \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 39 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 7.71



For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = +24$ MPa, (*b*) $\sigma_z = -24$ MPa, (*c*) $\sigma_z = 0$.

SOLUTION

$$\sigma_x = 30 \text{ MPa}, \qquad \sigma_y = 60 \text{ MPa}, \qquad \tau_{xy} = -36 \text{ MPa}$$

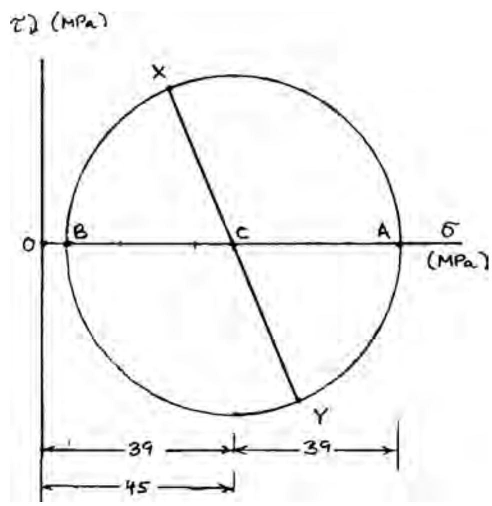
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(15) + (-36)^2} = 39 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 84 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 6 \text{ MPa}$$



(*a*) $\underline{\sigma_z = +24 \text{ MPa}}$ $\quad \sigma_a = 84 \text{ MPa} \quad \sigma_b = 6 \text{ MPa}$

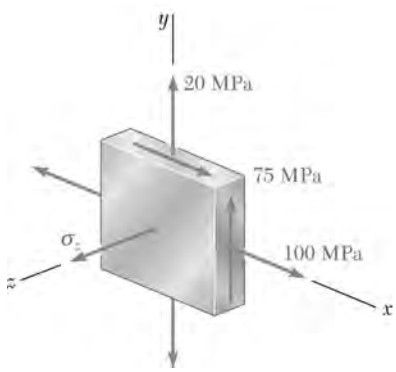
$$\sigma_{\max} = 84 \text{ MPa} \quad \sigma_{\min} = 6 \text{ MPa} \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 39 \text{ MPa} \quad \blacktriangleleft$$

(*b*) $\underline{\sigma_z = -24 \text{ MPa}}$ $\quad \sigma_a = 84 \text{ MPa} \quad \sigma_b = 6 \text{ MPa}$

$$\sigma_{\max} = 84 \text{ MPa} \quad \sigma_{\min} = -24 \text{ MPa} \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 54 \text{ MPa} \quad \blacktriangleleft$$

(*c*) $\underline{\sigma_z = 0}$ $\quad \sigma_a = 84 \text{ MPa} \quad \sigma_b = 6 \text{ MPa}$

$$\sigma_{\max} = 84 \text{ MPa} \quad \sigma_{\min} = 0 \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 42 \text{ MPa} \quad \blacktriangleleft$$

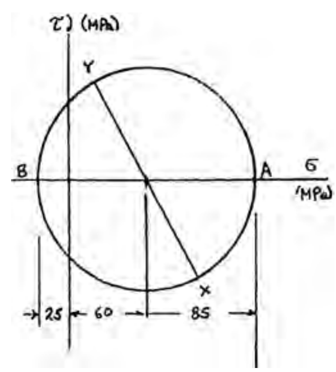


PROBLEM 7.72

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +45$ MPa, (*c*) $\sigma_z = -45$ MPa.

SOLUTION

$\sigma_x = 100$ MPa, $\quad \sigma_y = 20$ MPa, $\quad \tau_{xy} = 75$ MPa



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 60 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -25 \text{ MPa}$$

(*a*) $\quad \sigma_z = 0, \quad \sigma_a = 145 \text{ MPa}, \quad \sigma_b = -25 \text{ MPa}$

$$\sigma_{\max} = 145 \text{ MPa}, \quad \sigma_{\min} = -25 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 85 \text{ MPa} \blacktriangleleft$$

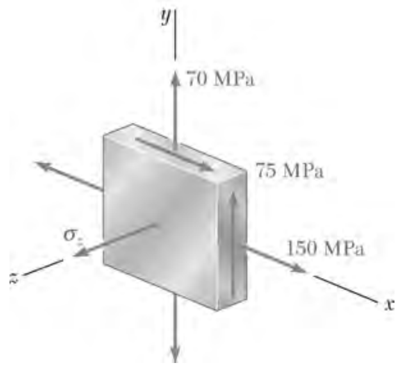
(*b*) $\quad \sigma_z = +45 \text{ MPa}, \quad \sigma_a = 145 \text{ MPa}, \quad \sigma_b = -25 \text{ MPa}$

$$\sigma_{\max} = 145 \text{ MPa}, \quad \sigma_{\min} = -25 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 85 \text{ MPa} \blacktriangleleft$$

(*c*) $\quad \sigma_z = -45 \text{ MPa}, \quad \sigma_a = 145 \text{ MPa}, \quad \sigma_b = -25 \text{ MPa}$

$$\sigma_{\max} = 145 \text{ MPa}, \quad \sigma_{\min} = -45 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 95 \text{ MPa} \blacktriangleleft$$

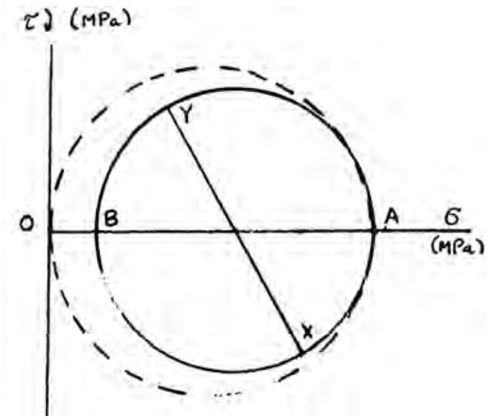
PROBLEM 7.73



For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +45$ MPa, (*c*) $\sigma_z = -45$ MPa.

SOLUTION

$$\sigma_x = 150 \text{ MPa}, \qquad \sigma_y = 70 \text{ MPa}, \qquad \tau_{xy} = 75 \text{ MPa}$$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 110 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 195 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 25 \text{ MPa}$$

(*a*) $\sigma_z = 0, \quad \sigma_a = 195 \text{ MPa}, \quad \sigma_b = 25 \text{ MPa}$

$$\sigma_{\max} = 195 \text{ MPa}, \quad \sigma_{\min} = 0, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$\tau_{\max} = 97.5 \text{ MPa}$ ◀

(*b*) $\sigma_z = +45 \text{ MPa}, \quad \sigma_a = 195 \text{ MPa}, \quad \sigma_b = 25 \text{ MPa}$

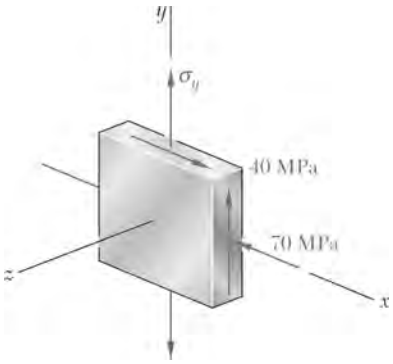
$$\sigma_{\max} = 195 \text{ MPa}, \quad \sigma_{\min} = 25 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$\tau_{\max} = 85 \text{ MPa}$ ◀

(*c*) $\sigma_z = -45 \text{ MPa}, \quad \sigma_a = 195 \text{ MPa}, \quad \sigma_b = 25 \text{ MPa}$

$$\sigma_{\max} = 195 \text{ MPa}, \quad \sigma_{\min} = -45 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$\tau_{\max} = 120 \text{ MPa}$ ◀



PROBLEM 7.74

For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 75 MPa.

SOLUTION

$$\sigma_x = -70 \text{ MPa}, \quad \tau_{xy} = 40 \text{ MPa}$$

Let $$U = \frac{\sigma_y - \sigma_x}{2} \qquad \sigma_y = 2U + \sigma_x$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + U$$

$$R = \sqrt{U^2 + \tau_{xy}^2}, \qquad U = \pm\sqrt{R^2 - \tau_{xy}^2}$$

Case (1) $$\tau_{\max} = R = 75 \text{ MPa}, \qquad U = \pm\sqrt{75^2 - 40^2} = \pm 63.44 \text{ MPa}$$

(1a) $$U = +63.44 \text{ MPa}, \quad \sigma_y = 2U + \sigma_x = 56.88 \text{ MPa}$$ ◀

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.56 \text{ MPa},$$

$$\sigma_a = \sigma_{\text{ave}} + R = 68.44 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = -81.56 \text{ MPa}$$

$$\sigma_c = 0 \quad \sigma_{\max} = 68.44 \text{ MPa}, \quad \sigma_{\min} = -81.56 \text{ MPa} \quad \tau_{\max} = 75 \text{ MPa}$$

(1b) $$U = -63.44 \text{ MPa} \quad \sigma_y = 2U + \sigma_x = -196.88 \text{ MPa} \quad \text{(reject)}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -133.44 \text{ MPa} \quad \sigma_a = \sigma_{\text{ave}} + R = -58.44 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -208.44 \text{ MPa}, \quad \sigma_c = 0, \quad \sigma_{\max} = 0$$

$$\sigma_{\min} = -208.44 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 104.22 \text{ MPa} \neq 75 \text{ MPa}$$

PROBLEM 7.74 *(Continued)*

Case (2) Assume $\sigma_{\max} = 0, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 75 \text{ MPa}$

$$\sigma_{\min} = -150 \text{ MPa} = \sigma_b$$

$$\sigma_b = \sigma_{\text{ave}} - R = \sigma_x + U - \sqrt{U^2 + \tau_{xy}^2}$$

$$\sqrt{U^2 + \tau_{xy}^2} = -\sigma_x + U - \sigma_b$$

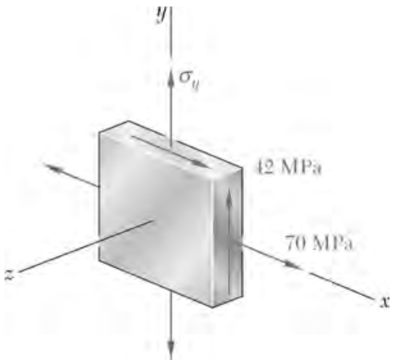
$$\cancel{U^2} + \tau_{xy}^2 = (\sigma_x - \sigma_b)^2 + 2(\sigma_x - \sigma_b)U + \cancel{U^2}$$

$$2U = \frac{\tau_{xy}^2 - (\sigma_x - \sigma_b)^2}{\sigma_x - \sigma_b} = \frac{(40)^2 - (-70 + 150)^2}{-70 + 150} = -160 \text{ MPa}$$

$$U = -30 \text{ MPa} \quad \sigma_y = 2U + \sigma_x = -130 \text{ MPa} \quad ◀$$

$$R = \sqrt{U^2 + \tau_{xy}^2} = 50 \text{ MPa}$$

$$\sigma_a = \sigma_b + 2R = -150 + 100 = -50 \text{ MPa} \qquad \text{O.K.}$$



PROBLEM 7.75

For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 52 MPa.

SOLUTION

$$\sigma_x = 70 \text{ MPa}, \quad \tau_{xy} = 42 \text{ MPa}, \quad \tau_{\max} = 52 \text{ MPa}$$

$$\text{Let} \qquad U = \frac{\sigma_y - \sigma_z}{2} \qquad \sigma_y = 2U + \sigma_x$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + U$$

$$R = \sqrt{U^2 + \tau_{xy}^2} \qquad U = \pm\sqrt{R^2 - \tau_{xy}^2}$$

Case (1) $\qquad \tau_{\max} = R = 52 \text{ MPa}, \qquad U = \pm 30.6 \text{ MPa}$

(1a) $\qquad U = +30.6 \text{ MPa} \quad \sigma_y = 2U + \sigma_x = 131.2 \text{ MPa}$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 100.6 \text{ MPa},$$

$$\sigma_a = \sigma_{\text{ave}} + R = 152.6 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = 48.6 \text{ MPa}$$

$$\sigma_{\max} = 152.6 \text{ MPa}, \quad \sigma_{\min} = 0, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 76.3 \text{ MPa} \neq 52 \text{ MPa. Hence rejected.}$$

(1b) $\qquad U = -30.6 \text{ MPa} \quad \sigma_y = 2U + \sigma_x = 8.8 \text{ MPa}$ ◀

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 39.4 \text{ MPa}, \quad \sigma_a = \sigma_{\text{ave}} + R = 91.4 \text{ MPa},$$

$$\sigma_b = \sigma_{\text{ave}} - R = -12.6 \text{ MPa}$$

$$\sigma_{\max} = 91.4 \text{ MPa}, \quad \sigma_{\min} = -12.6 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 52 \text{ MPa} \quad \text{O.K.}$$

PROBLEM 7.75 *(Continued)*

Case (2) Assume $\sigma_{\text{min}} = 0 \quad \sigma_{\text{max}} = 2\tau_{\text{max}} = 104 \text{ MPa} = \sigma_a$

$$\sigma_a = \sigma_{\text{ave}} + R = \sigma_x + U + \sqrt{U^2 + \tau_{xy}^2}$$

$$\sigma_a - \sigma_x - U = \sqrt{U^2 + \tau_{xy}^2}$$

$$(\sigma_a - \sigma_x - U)^2 = U^2 + \tau_{xy}^2$$

$$(\sigma_a - \sigma_x)^2 - 2(\sigma_a - \sigma_x)U + \cancel{U^2} = \cancel{U^2} + \tau_{xy}^2$$

$$2U = \frac{(\sigma_a - \sigma_x)^2 - \tau_{xy}^2}{\sigma_a - \sigma_x} = \frac{(104 - 70)^2 - 42^2}{104 - 70} = -17.88 \text{ MPa}$$

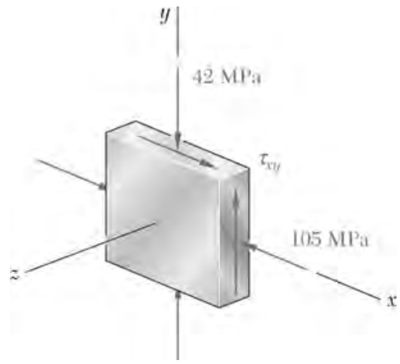
$$U = -8.94 \text{ MPa} \qquad \sigma_y = 2U + \sigma_x = 52.12 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 61.06 \text{ MPa} \quad R = \sqrt{U^2 + \tau_{xy}^2} = 42.94 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 104 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 18.12 \text{ MPa}$$

$$\sigma_{\text{max}} = 104 \text{ MPa}, \quad \sigma_{\text{min}} = 0 \quad \tau_{\text{max}} = 52 \text{ MPa} \qquad \text{O.K.} \quad \blacktriangleleft$$



PROBLEM 7.76

For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 63 MPa, (*b*) 84 MPa.

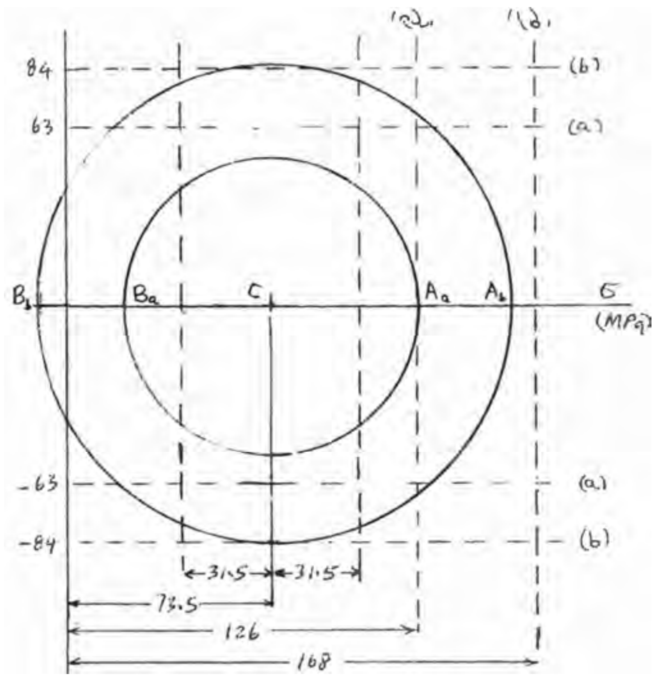
SOLUTION

$$\sigma_x = 105 \text{ MPa} \quad \sigma_y = 42 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 73.5 \text{ MPa}$$

$$U = \frac{\sigma_x - \sigma_y}{2} = 31.5 \text{ MPa}$$

τ (MPa)



(*a*) For $\tau_{\max} = 63$ MPa

Center of Mohr's circle lies at point *C*.
Lines marked (*a*) show the limits on $\tau_{\max}$.
Limit on $\sigma_{\max}$ is $\sigma_{\max} = 2\tau_{\max} = 126$ MPa.
For the Mohr's circle $\sigma_a = \sigma_{\max}$ corresponds to point A_a.

$$R = \sigma_a - \sigma_{\text{ave}} = 126 - 73.5 = 52.5 \text{ MPa}$$

$$R = \sqrt{U^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - U^2} = \pm\sqrt{52.5^2 - 31.5^2} = \pm 42 \text{ MPa} \quad ◀$$

(*b*) For $\tau_{\max} = 84$ MPa

Center of Mohr's circle lies at point *C*.

$$R = 84 \text{ MPa}$$

$$\tau_{xy} = \pm\sqrt{R^2 - U^2} = \pm 78.7 \text{ MPa} \quad ◀$$

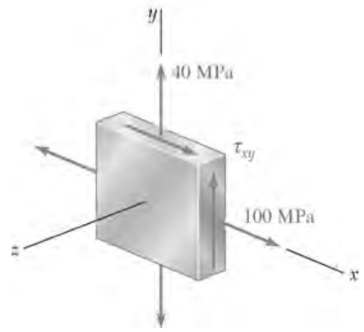
Checking

$$\sigma_a = 73.5 + 84 = 157.5 \text{ MPa}$$

$$\sigma_b = 73.5 - 84 = -10.5 \text{ MPa}$$

$$\sigma_c = 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 84 \text{ MPa} \qquad \text{O.K.}$$



PROBLEM 7.77

For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 60 MPa, (*b*) 78 MPa.

SOLUTION

$$\sigma_x = 100 \text{ MPa}, \quad \sigma_y = 40 \text{ MPa}, \quad \sigma_z = 0$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 70 \text{ MPa}$$

(*a*) $\underline{\tau_{\max} = 60 \text{ MPa.}}$

If σ_z is $\sigma_{\min}$, then $\sigma_{\max} = \sigma_{\min} + 2\tau_{\max}$.

$$\sigma_{\max} = 0 + (2)(60) = 120 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 120 - 70 = 50 \text{ MPa}$$

$$\sigma_b = \sigma_{\max} - 2R = 20 \text{ MPa} \; > 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2} = 50 \text{ MPa}$$

$$\tau_{xy} = \sqrt{50^2 - 30^2} \qquad \tau_{xy} = 40 \text{ MPa} \blacktriangleleft$$

(*b*) $\underline{\tau_{\max} = 78 \text{ MPa.}}$

If σ_z is $\sigma_{\min}$, then $\sigma_{\max} = \sigma_{\min} + 2\tau_{\max} = 0 + (2)(78) = 156 \text{ MPa.}$

$$\sigma_{\max} = \sigma_{\text{ave}} + R$$

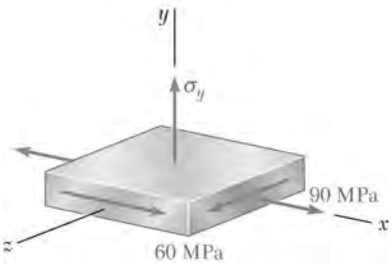
$$R = \sigma_{\max} - \sigma_{\text{ave}} = 156 - 70 = 86 \text{ MPa} > \tau_{\max} = 78 \text{ MPa}$$

Set

$$R = \tau_{\max} = 78 \text{ MPa.} \quad \sigma_{\min} = \sigma_{\text{ave}} - R = -8 \text{ MPa} < 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{78^2 - 30^2} \qquad \tau_{xy} = 72 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.78

For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 80 MPa.

SOLUTION

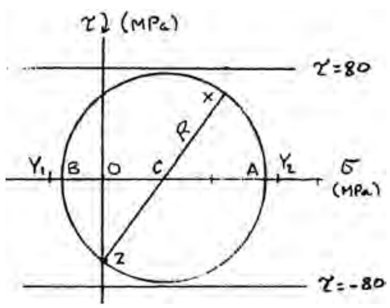
$$\sigma_x = 90 \text{ MPa}, \quad \sigma_z = 0, \quad \tau_{xz} = 60 \text{ MPa}$$

Mohr's circle of stresses in *zx*-plane:

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_z) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{zx}^2} = \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 120 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = -30 \text{ MPa}$$



Assume $\sigma_{\max} = \sigma_a = 120 \text{ MPa}.$

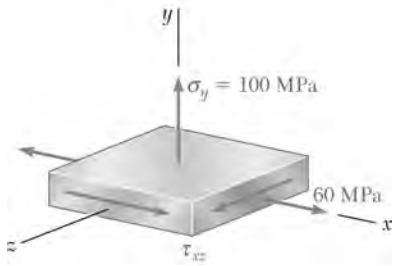
$$\sigma_y = \sigma_{\min} = \sigma_{\max} - 2\tau_{\max}$$
$$= 120 - (2)(80)$$

$\sigma_y = -40 \text{ MPa}$ ◀

Assume $\sigma_{\min} = \sigma_b = -30 \text{ MPa}.$

$$\sigma_y = \sigma_{\max} = \sigma_{\min} + 2\tau_{\max}$$
$$= -30 + (2)(8)$$

$\sigma_y = 130 \text{ MPa}$ ◀



PROBLEM 7.79

For the state of stress shown, determine the range of values of τ_{xz} for which the maximum shearing stress is equal to or less than 60 MPa.

SOLUTION

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_z = 0, \quad \sigma_y = 100 \text{ MPa}$$

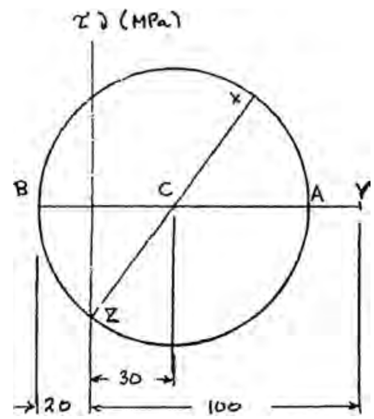
For Mohr's circle of stresses in *zx*-plane

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_z) = 30 \text{ MPa}$$

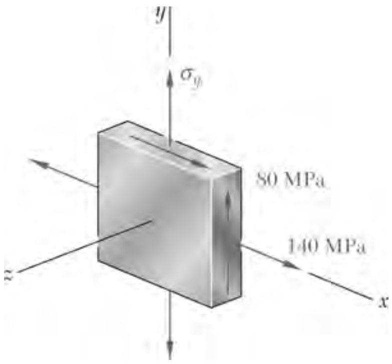
$$u = \frac{\sigma_x - \sigma_z}{2} = 30 \text{ MPa}$$

Assume $\quad \sigma_{\max} = \sigma_y = 100 \text{ MPa}$

$$\sigma_{\min} = \sigma_b = \sigma_{\max} - 2\tau_{\max}$$
$$= 100 - (2)(60) = -20 \text{ MPa}$$
$$R = \sigma_{\text{ave}} - \sigma_b$$
$$= 30 - (-20) = 50 \text{ MPa}$$
$$\sigma_a = \sigma_{\text{ave}} + R$$
$$= 30 + 50 = 80 \text{ MPa} < \sigma_y$$
$$R = \sqrt{u^2 + \tau_{xz}^2}$$
$$\tau_{xz} = \pm\sqrt{R^2 - u^2}$$
$$= \pm\sqrt{50^2 - 30^2} = \pm 40 \text{ MPa}$$



$-40 \text{ MPa} \leq \tau_{xz} \leq 40 \text{ MPa}$ ◀



PROBLEM 7.80*

For the state of stress of Prob. 7.69, determine (*a*) the value of σ_y for which the maximum shearing stress is as small as possible, (*b*) the corresponding value of the shearing stress.

SOLUTION

Let $$u = \frac{\sigma_x - \sigma_y}{2} \qquad \sigma_y = \sigma_x - 2u$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x - u$$

$$R = \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{\text{ave}} - R = \sigma_x - u - \sqrt{u^2 + \tau_{xy}^2}$$

Assume $\tau_{\max}$ is the in-plane shearing stress. $\tau_{\max} = R$

Then $\tau_{\max\,(\text{in-plane})}$ is minimum if $u = 0$.

$$\sigma_y = \sigma_x - 2u = \sigma_x = 140 \text{ MPa}, \qquad \sigma_{\text{ave}} = \sigma_x - u = 140 \text{ MPa}$$

$$R = |\tau_{xy}| = 80 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 140 + 80 = 220 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 140 - 80 = 60 \text{ MPa}$$

$$\sigma_{\max} = 220 \text{ MPa}, \quad \sigma_{\min} = 0, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 110 \text{ MPa}$$

Assumption is incorrect.

Assume $$\sigma_{\max} = \sigma_a = \sigma_{\text{ave}} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_{\min} = 0 \qquad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = \frac{1}{2}\sigma_a$$

$$\frac{d\sigma_a}{du} = -1 + \frac{u}{\sqrt{u^2 + \tau_{xy}^2}} \neq 0 \qquad \text{(no minimum)}$$

PROBLEM 7.80* *(Continued)*

Optimum value for u occurs when $\tau_{\text{max (out-of-plane)}} = \tau_{\text{max (in-plane)}}$

$$\frac{1}{2}(\sigma_a + R) = R \quad \text{or} \quad \sigma_a = R \quad \text{or} \quad \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

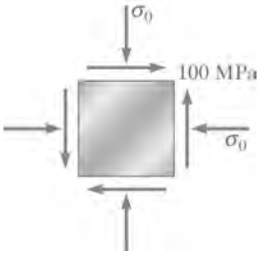
$$(\sigma_x - u)^2 = \sigma_x^2 - 2u\sigma_x + \cancel{u}^2 = \cancel{u}^2 + \tau_{xy}^2$$

$$2u = \frac{\sigma_x^2 - \tau_{xy}^2}{\sigma_x} = \frac{140^2 - 80^2}{140} = 94.3 \text{ MPa} \qquad u = 47.14 \text{ MPa}$$

(*a*) $\sigma_y = \sigma_x - 2u = 140 - 94.3$ $\qquad \sigma_y = 45.7 \text{ MPa}$ ◀

(*b*) $R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{\max} = 92.9 \text{ MPa}$ $\qquad \tau_{\max} = 92.9 \text{ MPa}$ ◀

PROBLEM 7.81



The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_{\text{ave}} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(*a*) $\underline{\sigma_0 = 200 \text{ MPa}}$ $\quad \sigma_{\text{ave}} = -200$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -100 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -300 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 264.56 \text{ MPa} < 325 \text{ MPa}$$ (No yielding)

$$F.S. = \frac{325}{264.56}$$ $F.S. = 1.228$ ◀

(*b*) $\underline{\sigma_0 = 240 \text{ MPa}}$ $\quad \sigma_{\text{ave}} = -240$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -140 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -340 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 295.97 \text{ MPa} < 325 \text{ MPa}$$ (No yielding)

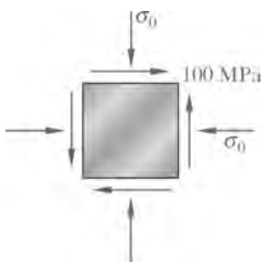
$$F.S. = \frac{325}{295.97}$$ $F.S. = 1.098$ ◀

(*c*) $\underline{\sigma_0 = 280 \text{ MPa}}$ $\quad \sigma_{\text{ave}} = -280$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -180 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -380 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 329.24 \text{ MPa} > 325 \text{ MPa}$$ (Yielding occurs) ◀

PROBLEM 7.82



Solve Prob. 7.81, using the maximum-shearing-stress criterion.

PROBLEM 7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_{ave} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(*a*) $\underline{\sigma_0 = 200 \text{ MPa},}$ $\quad \sigma_{ave} = -200$ MPa

$$\sigma_a = \sigma_{ave} + R = -100 \text{ MPa} \qquad \sigma_b = \sigma_{ave} - R = -300 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -300 \text{ MPa}$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 300 \text{ MPa} < 325 \text{ MPa}$$ (No yielding)

$$F.S. = \frac{325}{300}$$ $F.S. = 1.083$ ◀

(*b*) $\underline{\sigma_0 = 240 \text{ MPa},}$ $\quad \sigma_{ave} = -240$ MPa

$$\sigma_a = \sigma_{ave} + R = -140 \text{ MPa}, \qquad \sigma_b = \sigma_{ave} - R = -340 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -340 \text{ MPa}$$

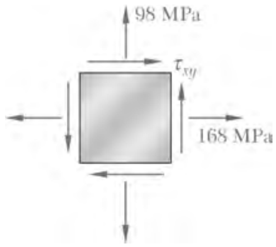
$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 340 \text{ MPa} > 325 \text{ MPa}$$ (Yielding occurs) ◀

(*c*) $\underline{\sigma_0 = 280 \text{ MPa},}$ $\quad \sigma_{ave} = -280$ MPa

$$\sigma_a = \sigma_{ave} + R = -180 \text{ MPa}, \qquad \sigma_b = \sigma_{ave} - R = -380 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -380 \text{ MPa}$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 380 \text{ MPa} > 325 \text{ MPa}$$ (Yielding occurs) ◀



PROBLEM 7.83

The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 210$ MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\tau_{xy} = 42$ MPa, (*b*) $\tau_{xy} = 84$ MPa, (*c*) $\tau_{xy} = 98$ MPa. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_x = 168 \text{ MPa} \quad \sigma_y = 98 \text{ MPa} \quad \sigma_z = 0$$

For stresses in *xy*-plane

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 133 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = 35 \text{ MPa}$$

(*a*) $\tau_{xy} = 6$ MPa $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (42)^2} = 54.7$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = 187.7 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = 78.3 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 163.3 \text{ MPa} < 210 \text{ MPa}$$ (No yielding)

$$F.S. = \frac{210}{163.3} = 1.286$$ ◀

(*b*) $\tau_{xy} = 84$ MPa $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (84)^2} = 91$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = 224 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = 42 \text{ MPa}$$

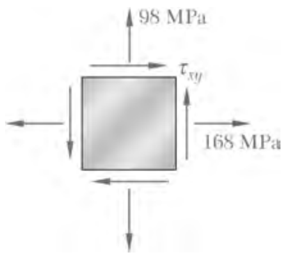
$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 206.2 \text{ MPa} < 210 \text{ MPa}$$ (No yielding)

$$F.S. = \frac{210}{206.2} = 1.018$$ ◀

(*c*) $\tau_{xy} = 98$ MPa $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (98)^2} = 104.1$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = 237.1 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = 28.9 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 224 \text{ MPa} > 210 \text{ MPa}$$ (Yielding occurs)



PROBLEM 7.84

Solve Prob. 7.83, using the maximum-shearing-stress criterion.

PROBLEM 7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 210$ MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\tau_{xy} = 42$ MPa, (*b*) $\tau_{xy} = 84$ MPa, (*c*) $\tau_{xy} = 98$ MPa. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_x = 168 \text{ MPa} \quad \sigma_y = 98 \text{ MPa} \quad \sigma_z = 0$$

For stresses in *xy*-plane $\quad \sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 133 \text{ MPa}$

$$\frac{\sigma_x - \sigma_y}{2} = 35 \text{ MPa}$$

(*a*) $\quad \tau_{xy} = 42 \text{ MPa} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 54.7 \text{ MPa}$

$$\sigma_a = \sigma_{\text{ave}} + R = 187.7 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = 78.3 \text{ MPa}$$

$$\sigma_{\max} = 187.7 \text{ MPa}, \quad \sigma_{\min} = 0$$

$$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 187.7 \text{ MPa} < 210 \text{ MPa}$$ (No yielding)

$$F.S. = \frac{210}{187.7} = 1.119$$ ◀

(*b*) $\quad \tau_{xy} = 84 \text{ MPa} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 91 \text{ MPa}$

$$\sigma_a = \sigma_{\text{ave}} + R = 224 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = 42 \text{ MPa}$$

$$\sigma_{\max} = 224 \text{ MPa} \quad \sigma_{\min} = 0$$

$$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 224 \text{ MPa} > 210 \text{ MPa}$$ (Yielding occurs)

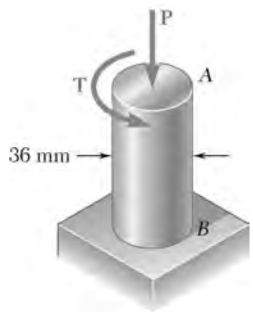
(*c*) $\quad \tau_{xy} = 98 \text{ MPa} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 104.1 \text{ MPa}$

$$\sigma_a = \sigma_{\text{ave}} + R = 237.1 \text{ MPa} \quad \sigma_b = \sigma_{\text{ave}} - R = 28.9 \text{ MPa}$$

$$\sigma_{\max} = 237.1 \text{ MPa} \quad \sigma_{\min} = 0$$

$$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 237.1 > 210 \text{ MPa}$$ (Yielding occurs)

PROBLEM 7.85



The 36-mm-diameter shaft is made of a grade of steel with a 250-MPa tensile yield stress. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 200$ kN.

SOLUTION

$$P = 200 \text{ kN} = 200 \times 10^3 \text{ N} \qquad c = \frac{1}{2}d = 18 \text{ mm} = 18 \times 10^{-3}\text{m}$$

$$A = \pi c^2 = \pi(18 \times 10^{-3})^2 = 1.01788 \times 10^{-3}\text{m}^2$$

$$\sigma_y = -\frac{P}{A} = -\frac{200 \times 10^3}{1.01788 \times 10^{-3}} = 196.488 \times 10^6 \text{ Pa}$$

$$= 196.488 \text{ MPa}$$

$$\sigma_x = 0 \qquad \sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_y = 98.244 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(98.244)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R \quad \text{(positive)}$$

$$\sigma_b = \sigma_{\text{ave}} - R \quad \text{(negative)}$$

$$|\sigma_a - \sigma_b| = 2R \qquad |\sigma_a - \sigma_b| > |\sigma_a| \qquad |\sigma_a - \sigma_b| > |\sigma_b|$$

<u>Maximum shear stress criterion</u> under the above conditions:

$$|\sigma_a - \sigma_b| = 2R = \sigma_Y = 250 \text{ MPa} \qquad R = 125 \text{ MPa}$$

Equating expressions for R,

$$125 = \sqrt{(98.244)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{(125)^2 - (98.244)^2} = 77.286 \text{ MPa} = 77.286 \times 10^6\text{Pa}$$

<u>Torsion</u>:

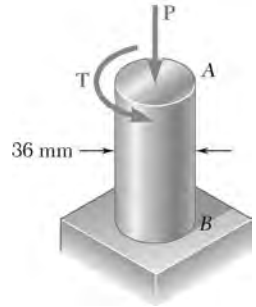
$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(18 \times 10^{-3})^4 = 164.896 \times 10^{-9}\text{m}^4$$

$$\tau_{xy} = \frac{Tc}{J}$$

$$T = \frac{J\tau_{xy}}{c} = \frac{(164.846 \times 10^{-9})(77.286 \times 10^6)}{18 \times 10^{-3}}$$

$$= 708 \text{ N} \cdot \text{m}$$

$T = 708$ N · m ◀



PROBLEM 7.86

Solve Prob. 7.85, using the maximum-distortion-energy criterion.

PROBLEM 7.85 The 36-mm-diameter shaft is made of a grade of steel with a 250-MPa tensile yield stress. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 200$ kN.

SOLUTION

$$P = 200 \text{ kN} = 200\times10^3 \text{ N} \qquad c = \frac{1}{2}d = 18 \text{ mm} = 18\times10^{-3} \text{ m}$$

$$A = \pi c^2 = \pi(18\times10^{-3})^2 = 1.01788\times10^{-3} \text{ m}^2$$

$$\sigma_y = -\frac{P}{A} = -\frac{200\times10^3}{1.01788\times10^{-3}} = 196.488\times10^6 \text{ Pa}$$

$$= 196.448 \text{ MPa}$$

$$\sigma_x = 0 \qquad \sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_y = 98.244 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(98.244)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_b = \sigma_{\text{ave}} - R$$

<u>Distortion energy criterion</u>:

$$\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = \sigma_Y^2$$

$$(\sigma_{\text{ave}} + R)^2 + (\sigma_{\text{ave}} - R)^2 - (\sigma_{\text{ave}} + R)(\sigma_{\text{ave}} - R) = \sigma_Y^2$$

$$\sigma_{\text{ave}}^2 + 3R^2 = \sigma_Y^{-2}$$

$$(98.244)^2 + (3)[(98.244)^2 + \tau_{xy}^2] = (250)^2$$

$$\tau_{xy} = \pm 89.242 \text{ MPa}$$

<u>Torsion</u>:

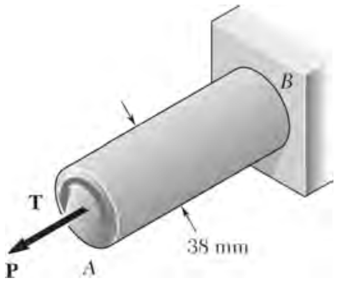
$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(18\times10^{-3})^4 = 164.846\times10^{-9} \text{ m}^4$$

$$\tau_{xy} = \frac{Tc}{J}$$

$$T = \frac{J\tau_{xy}}{c} = \frac{(164.846\times10^{-9})(89.242\times10^6)}{18\times10^{-3}}$$

$$= 818 \text{ N}\cdot\text{m}$$

$T = 818$ N·m ◀



PROBLEM 7.87

The 38 mm diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 240$ kN.

SOLUTION

$$P = 240\times10^3 \text{ N}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(38)^2 = 1.1341\times10^3 \text{ mm}^2 = 1.1341\times10^{-3} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{240\times10^3}{1.1341\times10^{-3}} = 211.6\times10^6 \text{ Pa} = 211.6 \text{ MPa}$$

$$\sigma_y = 0$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$2\tau_{\max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_y$$

$$4\tau_{xy}^2 = \sigma_y^2 - \sigma_x^2$$

$$\tau_{xy} = \frac{1}{2}\sqrt{\sigma_y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{250^2 - 211.6^2}$$

$$= 65.568\times10^6 \text{ Pa} = 66.568 \text{ MPa}$$

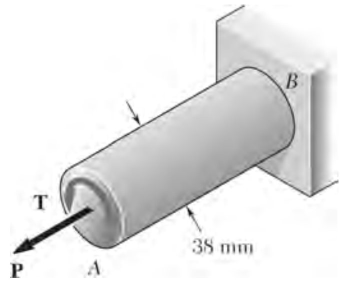
From torsion $\qquad \tau_{xy} = \dfrac{Tc}{J} \quad T = \dfrac{J\tau_{xy}}{c}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{38}{2}\right)^4 = 204.71\times10^3 \text{ mm}^4 = 204.71\times10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}d = 19\times10^{-3} \text{ m}$$

$$T = \frac{(204.71\times10^{-9})(65.668\times10^6)}{19\times10^{-3}} = 717 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

PROBLEM 7.88



Solve Prob. 7.87, using the maximum-distortion-energy criterion.

PROBLEM 7.87 The 38 mm diameter shaft *AB* is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 240$ kN.

SOLUTION

$$P = 240 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(38)^2 = 1.1341 \times 10^3 \text{ mm}^2 = 1.1341 \times 10^{-3} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.1341 \times 10^{-3}} = 211.6 \times 10^6 \text{ Pa} = 211.6 \text{ MPa}$$

$$\sigma_y = 0$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R = \frac{1}{2}\sigma_x + \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{\text{ave}} - R = \frac{1}{2}\sigma_x - \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\begin{aligned}\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b &= \frac{1}{4}\sigma_x^2 + \sigma_x\sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ &\quad + \frac{1}{4}\sigma_x^2 - \sigma_x\sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ &\quad - \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ &= \sigma_x^2 + 3\tau_{xy}^2 = \sigma_y^2\end{aligned}$$

$$\tau_{xy}^2 = \frac{1}{3}\left(\sigma_y^2 - \sigma_x^2\right)$$

$$\tau_{xy} = \frac{1}{\sqrt{3}}\sqrt{250^2 - 211.6^2} = 76.867 \times 10^6 \text{ Pa} = 76.867 \text{ MPa}$$

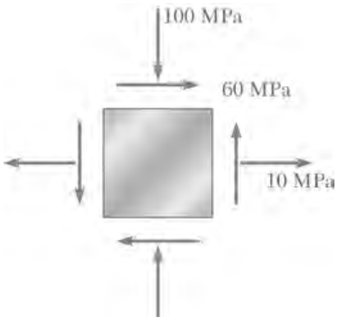
PROBLEM 7.88 *(Continued)*

From torsion $\tau_{xy} = \frac{Tc}{J} \quad T = \frac{J\tau_{xy}}{c}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{38}{2}\right)^4 = 204.71\times10^3 \text{ mm}^4 = 204.71\times10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}d = 19\times10^{-3} \text{ m}$$

$$T = \frac{(204.71\times10^{-9})(76.876\times10^6)}{19\times10^{-3}} = 828 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$



PROBLEM 7.89

The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 80$ MPa and $\sigma_{UC} = 200$ MPa and using Mohr's criterion, determine whether rupture of the casting will occur.

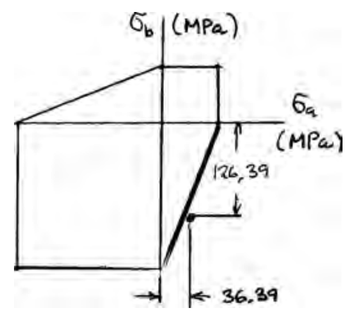
SOLUTION

$$\sigma_x = 10 \text{ MPa},$$
$$\sigma_y = -100 \text{ MPa},$$
$$\tau_{xy} = 60 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{10 - 100}{2} = -45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(55)^2 + (60)^2} = 81.39 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = -45 + 81.39 = 36.39 \text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -45 - 81.39 = -126.39 \text{ MPa}$$

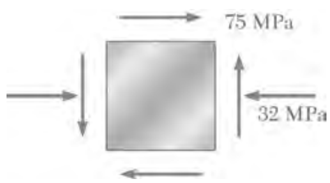


Equation of 4th quadrant of boundary:

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{36.39}{80} - \frac{(-126.39)}{200} = 1.087 > 1$$

Rupture will occur. ◀

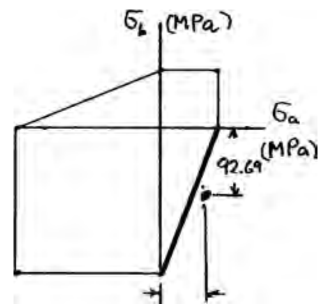


PROBLEM 7.90

The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 80$ MPa and $\sigma_{UC} = 200$ MPa and using Mohr's criterion, determine whether rupture of the casting will occur.

SOLUTION

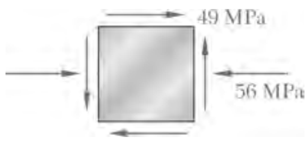
$$\sigma_x = -32 \text{ MPa},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = 75 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -16 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(16)^2 + (75)^2} = 76.69 \text{ MPa}$$
$$\sigma_a = \sigma_{\text{ave}} + R = -16 + 76.69 = 60.69 \text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -16 - 76.69 = -92.69 \text{ MPa}$$



Equation of 4th quadrant of boundary:

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{60.69}{80} - \frac{(-92.69)}{200} = 1.222 > 1$$

Rupture will occur. ◀

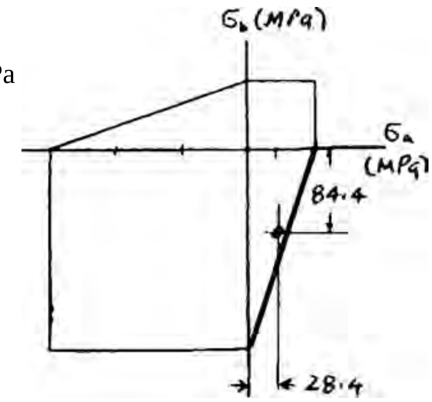


PROBLEM 7.91

The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 210$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.

SOLUTION

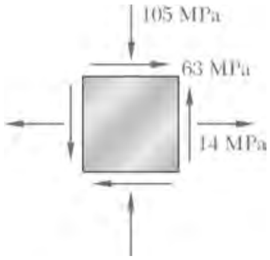
$$\sigma_x = -56 \text{ MPa},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = 49 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -28 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{28^2 + 49^2} = 56.4 \text{ MPa}$$
$$\sigma_a = \sigma_{\text{ave}} + R = -28 + 56.4 = 28.4 \text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -28 - 56.4 = -84.4 \text{ MPa}$$



Equation of 4th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{28.4}{70} - \frac{(-84.4)}{210} = 0.808 < 1$$

No rupture. ◀



PROBLEM 7.92

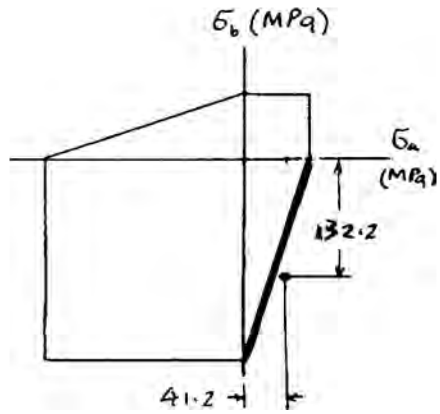
The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used σ_{UT} = 70 MPa and σ_{UC} = 210 MPa and using Mohr's criterion, determine whether rupture of the component will occur.

SOLUTION

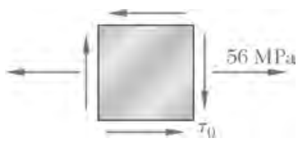
$$\sigma_x = 14 \text{ MPa}$$
$$\sigma_y = -105 \text{ MPa}$$
$$\tau_{xy} = 63 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -45.5 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{59.5^2 + 63^2} = 86.7 \text{ MPa}$$
$$\sigma_a = \sigma_{\text{ave}} + R = 41.2 \text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -132.2 \text{ MPa}$$

Equation of 4th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{41.2}{70} - \frac{(-132.2)}{210} = 1.218 > 1$$



Rupture will occur. ◀

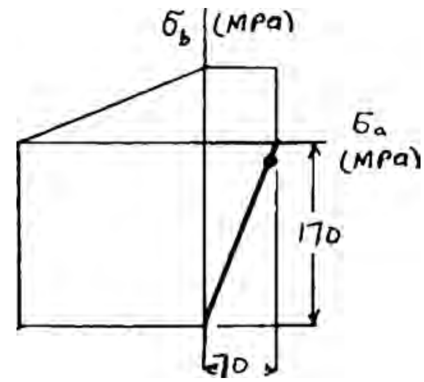


PROBLEM 7.93

The state of plane stress shown will occur at a critical point in an aluminum casting that is made of an alloy for which $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 170$ MPa. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.

SOLUTION

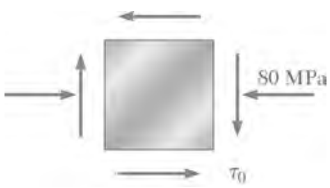
$$\sigma_x = 56 \text{ MPa},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = \tau_0$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 28 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{28^2 + \tau_0^2},$$
$$\tau_0 = \pm\sqrt{R^2 - 4^2}$$
$$\sigma_a = \sigma_{\text{ave}} + R = (28 + R) \text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = (28 - R) \text{ MPa}$$



Since $|\sigma_{\text{ave}}| < R,$ stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{28 + R}{70} - \frac{28 - R}{170} = 1$$
$$\left(\frac{1}{70} + \frac{1}{170}\right)R = 1 - \frac{28}{70} + \frac{28}{170}$$
$$R = 37.9 \text{ MPa} \qquad \tau_0 = \pm\sqrt{37.9^2 - 28^2} = \pm 25.5 \text{ MPa} \quad \blacktriangleleft$$

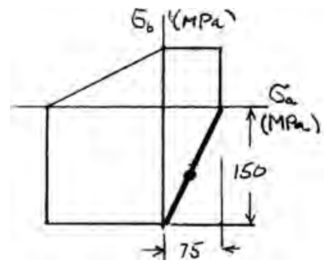
PROBLEM 7.94



The state of plane stress shown will occur at a critical point in a pipe made of an aluminum alloy for which $\sigma_{UT} = 75$ MPa and $\sigma_{UC} = 150$ MPa. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.

SOLUTION

$$\sigma_x = -80 \text{ MPa},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = -\tau_0$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -40 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + \tau_0^2} \text{ MPa}$$
$$\sigma_a = \sigma_{\text{ave}} + R$$
$$\sigma_b = \sigma_{\text{ave}} - R$$
$$\tau_0 = \pm\sqrt{R^2 - 40^2}$$

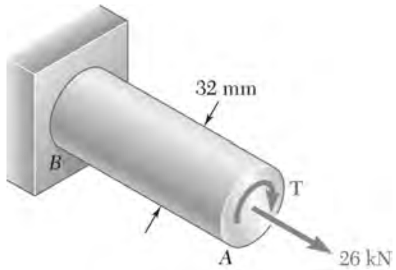


Since $|\sigma_{\text{ave}}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{-40 + R}{75} - \frac{-40 - R}{150} = 1$$
$$\frac{R}{75} + \frac{R}{150} = 1 + \frac{40}{75} - \frac{40}{150} = 1.2667$$
$$R = 63.33 \text{ MPa}, \qquad \tau_0 = \pm\sqrt{63.33^2 - 40^2}$$

$\tau_0 = \pm 8.49$ MPa ◀

PROBLEM 7.95

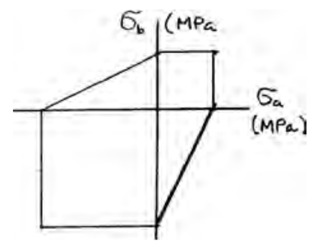


The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 60$ MPa and $\sigma_{UC} = 120$ MPa. Using Mohr's criterion, determine the magnitude of the torque **T** for which failure should be expected.

SOLUTION

$$P = 26\times 10^3\,\text{N} \qquad A = \frac{\pi}{4}(32)^2 = 804.25\ \text{mm}^2 = 804.25\times 10^{-6}\,\text{m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{26\times 10^3}{804.25\times 10^{-6}} = 32.328\times 10^6\,\text{Pa} = 32.328\ \text{MPa}$$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(32.328 + 0) = 16.164\ \text{MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{1}{2}(32.328 - 0) = 16.164\ \text{MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 16.164 + R\ \text{MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 16.164 - R\ \text{MPa}$$

Since $|\sigma_{\text{ave}}| < R,$ stress point lies in the 4th quadrant. Equation of the 4th quadrant is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1 \qquad \frac{16.164 + R}{60} - \frac{16.164 - R}{120} = 1$$

$$\left(\frac{1}{60} + \frac{1}{120}\right)R = 1 - \frac{16.164}{60} + \frac{16.164}{120} \qquad R = 34.612\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right) + \tau_{xy}^2} \qquad \tau_{xy} = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{34.612^2 - 16.164^2} = 30.606\ \text{MPa}$$

$$= 30.606\times 10^6\,\text{Pa}$$

For torsion, $\quad \tau_{xy} = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3} \qquad$ where $c = \dfrac{1}{2}d = 16\ \text{mm} = 16\times 10^{-3}\ \text{m}$

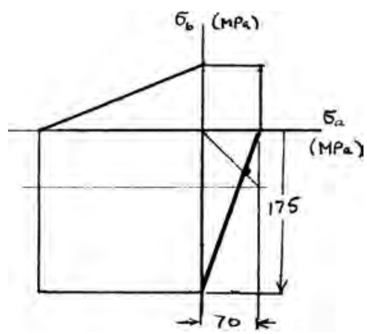
$$T = \frac{\pi}{2}c^3\tau_{xy} = \frac{\pi}{2}(16\times 10^{-3})^3(30.606\times 10^6) \qquad T = 196.9\ \text{N}\cdot\text{m} \ ◀$$

PROBLEM 7.96

The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 175$ MPa. Knowing that the magnitude T of the applied torques is slowly increased and using Mohr's criterion, determine the shearing stress τ_0 that should be expected at rupture.

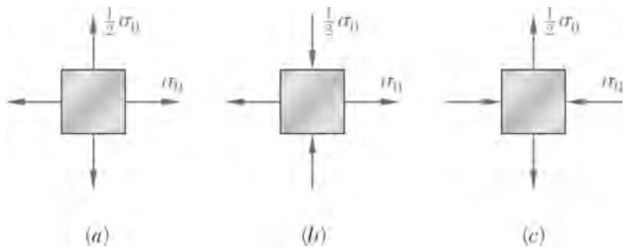
SOLUTION

$$\sigma_x = 0$$
$$\sigma_y = 0$$
$$\tau_{xy} = -\tau_0$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + \tau_{xy}^2} = |\tau_{xy}|$$
$$\sigma_a = \sigma_{\text{ave}} + R = R$$
$$\sigma_b = \sigma_{\text{ave}} - R = -R$$



Since $|\sigma_{\text{ave}}| < R,$ stress point lies in 4th quadrant. Equation of boundary of 4th quadrant is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{R}{70} - \frac{-R}{175} = 1$$
$$\left(\frac{1}{70} + \frac{1}{175}\right)R = 1$$
$$R = 50 \text{ MPa}$$
$$\tau_0 = R \qquad\qquad \tau_0 = 50.0 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.97

A machine component is made of a grade of cast iron for which $\sigma_{UT} = 56$ MPa and $\sigma_{UC} = 140$ MPa. For each of the states of stress shown, and using Mohr's criterion, determine the normal stress σ_0 at which rupture of the component should be expected.

SOLUTION

(*a*) $\sigma_a = \sigma_0,$

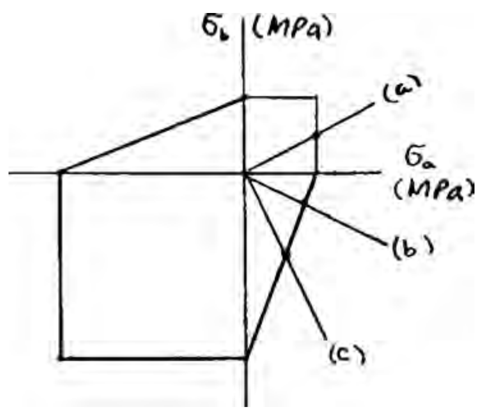
$$\sigma_b = \frac{1}{2}\sigma_0$$

Stress point lies in 1st quadrant.

$$\sigma_a = \sigma_0 = \sigma_{UT} = 56 \text{ MPa} \quad \blacktriangleleft$$

(*b*) $\sigma_a = \sigma_0,$

$$\sigma_b = -\frac{1}{2}\sigma_0$$



Stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{\sigma_0}{56} - \frac{-\frac{1}{2}\sigma_0}{140} = 1 \qquad \sigma_0 = 46.7 \text{ MPa} \quad \blacktriangleleft$$

(*c*) $\sigma_a = \frac{1}{2}\sigma_0, \quad \sigma_b = -\sigma_0,$ 4th quadrant

$$\frac{\frac{1}{2}\sigma_0}{56} - \frac{-\sigma_0}{140} = 1 \qquad \sigma_0 = 62.2 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 7.98

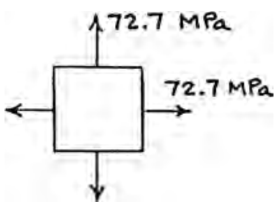
A spherical gas container made of steel has a 5-m outer diameter and a wall thickness of 6 mm. Knowing that the internal pressure is 350 kPa, determine the maximum normal stress and the maximum shearing stress in the container.

SOLUTION

$$d = 5\text{ m} \quad t = 6\text{ mm} = 0.006\text{ m}, \quad r = \frac{d}{2} - t = 2.494\text{ m}$$

$$\sigma = \frac{pr}{2t} = \frac{(350 \times 10^3\text{ Pa})(2.494\text{ m})}{2(0.006\text{ m})} = 72.742 \times 10^6\text{ Pa}$$

$\sigma = 72.7\text{ MPa}$ ◀



$$\sigma_{max} = 72.742\text{ MPa}$$

$$\sigma_{min} \approx 0 \quad (\text{Neglecting small radial stress})$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$

$\tau_{max} = 36.4\text{ MPa}$ ◀

PROBLEM 7.99

The maximum gage pressure is known to be 8 MPa in a spherical steel pressure vessel having a 250-mm outer diameter and a 6-mm wall thickness. Knowing that the ultimate stress in the steel used is $\sigma_U = 400$ MPa, determine the factor of safety with respect to tensile failure.

SOLUTION

$$r = \frac{d}{2} - t = \frac{250}{2} - 6 = 119 \text{ mm} = 119 \times 10^{-3} \text{ m}, \quad t = 6 \times 10^{-3} \text{ m}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(8 \times 10^6 \text{ Pa})(119 \times 10^{-3} \text{ m})}{2(6 \times 10^{-3} \text{ m})} = 79.333 \times 10^6 \text{ Pa}$$

$$F.S. = \frac{\sigma_U}{\sigma_{\max}} = \frac{400 \times 10^6}{79.333 \times 10^6}$$

$F.S. = 5.04$ ◀

PROBLEM 7.100

A basketball has a 300-mm outer diameter and a 3-mm wall thickness. Determine the normal stress in the wall when the basketball is inflated to a 120-kPa gage pressure.

SOLUTION

$$r = \frac{1}{2}d - t = 147 \text{ mm} = 147 \times 10^{-3} \text{ m} \qquad p = 120 \times 10^{3} \text{ Pa}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(120 \times 10^{3})(147 \times 10^{-3})}{(2)(3 \times 10^{-3})} = 2.94 \times 10^{4} \text{ Pa} \qquad \sigma = 2.94 \text{ MPa} \blacktriangleleft$$

PROBLEM 7.101

A spherical pressure vessel of 900-mm outer diameter is to be fabricated from a steel having an ultimate stress $\sigma_U = 400$ MPa. Knowing that a factor of safety of 4.0 is desired and that the gage pressure can reach 3.5 MPa, determine the smallest wall thickness that should be used.

SOLUTION

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{400}{4.0} = 100 \text{ MPa} \qquad r = \frac{d}{2} - t = (0.45 - t) \text{ m}$$

$$\sigma_{\text{all}} = \frac{pr}{2t} \qquad 2\sigma_{\text{all}} t = pr$$

$$2(100)\,t = 3.5(0.45 - t)$$

$$203.5\,t = 1.575$$

$$t = 7.74 \times 10^{-3} \text{ m} \qquad t_{\text{min}} = 7.74 \text{ mm} \blacktriangleleft$$

PROBLEM 7.102

A spherical pressure vessel is 3 m in diameter and has a wall thickness of 12 mm. Knowing that for the steel used $\sigma_{\text{all}} = 80$ MPa, $E = 200$ GPa and $v = 0.29$, determine (*a*) the allowable gage pressure, (*b*) the corresponding increase in the diameter of the vessel.

SOLUTION

$$r = \frac{1}{2}d - t = \frac{1}{2}(3000) - 12 = 1488 \text{ mm}$$

$$\sigma_1 = \sigma_2 = \sigma_{\text{all}} = 8 \text{ MPa}$$

(*a*) $\sigma_1 = \sigma_2 = \dfrac{pr}{2t} \qquad p = \dfrac{2t\sigma_1}{r} = \dfrac{(2)(12)(80)}{1488} = 1.290 \text{ MPa}$ ◀

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - v\sigma_2) = \frac{1-v}{E}\sigma_1 = \frac{1-0.29}{200\times 10^9}(8\times 10^6) = 28.4\mu$$

(*b*) $\Delta d = d\varepsilon_1 = (3000)(28.4\times 10^{-6}) = 85.2\times 10^{-3} \text{ mm} = 0.0852 \text{ mm}$ ◀

PROBLEM 7.103

A spherical gas container having an outer diameter of 5 m and a wall thickness of 22 mm is made of steel for which $E = 200$ GPa and $\nu = 0.29$. Knowing that the gage pressure in the container is increased from zero to 1.7 MPa, determine (*a*) the maximum normal stress in the container, (*b*) the corresponding increase in the diameter of the container.

SOLUTION

$$r = \frac{d}{2} - t = \frac{5}{2} - 22\times10^{-3} = 2.478\text{ m},\quad t = 22\times10^{-3}\text{ m}$$

(*a*) $$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(1.7\times10^6\text{ Pa})(2.478\text{ m})}{2(22\times10^{-3}\text{ m})} = 95.741\times10^6\text{ Pa}$$

$$\sigma_{\max} = 95.7\text{ MPa} \blacktriangleleft$$

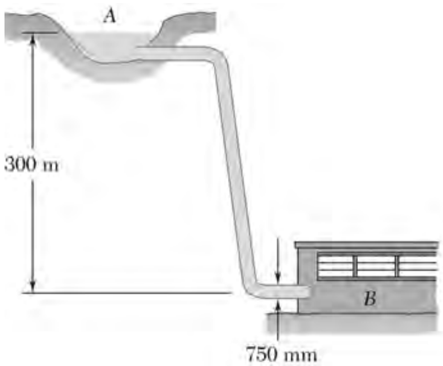
(*b*) $$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1$$

$$= \frac{(1-0.29)(95.741\times10^6\text{ Pa})}{200\times10^9\text{ Pa}} = 339.88\times10^{-6}$$

$$\Delta d = d\varepsilon_1 = (5\times10^3\text{ mm})(339.88\times10^{-6})$$

$$\Delta d = 1.699\text{ mm} \blacktriangleleft$$

PROBLEM 7.104



A steel penstock has a 750-mm outer diameter, a 12-mm wall thickness, and connects a reservoir at A with a generating station at B. Knowing that the density of water is $1000\ \text{kg/m}^3$, determine the maximum normal stress and the maximum shearing stress in the penstock under static conditions.

SOLUTION

$$r = \frac{1}{2}d - t = \frac{1}{2}(750) - 12 = 363\ \text{mm} = 363\times 10^{-3}\text{m}$$

$$t = 12\ \text{mm} = 12\times\ 10^{-3}\text{m}$$

$$p = \rho g h = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(300\ \text{m})$$

$$= 2.943\times 10^6\ \text{Pa}$$

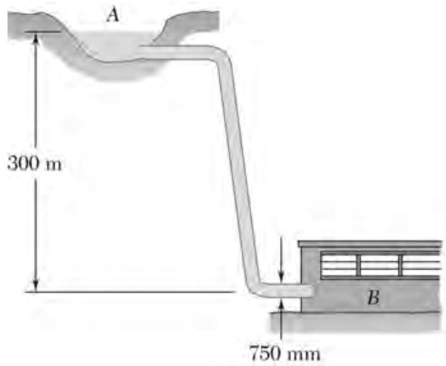
$$\sigma_1 = \frac{pr}{t} = \frac{(2.943\times 10^6)(363\times 10^{-3})}{12\times 10^{-3}} = 89.0\times 10^6\ \text{Pa}$$

$$\sigma_{\max} = \sigma_1 \qquad \sigma_{\max} = 89.0\ \text{MPa}\ \blacktriangleleft$$

$$\sigma_{\min} = -p \approx 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 44.5\ \text{MPa}\ \blacktriangleleft$$

PROBLEM 7.105



A steel penstock has a 750-mm outer diameter and connects a reservoir at *A* with a generating station at *B*. Knowing that the density of water is $1000\ \text{kg/m}^3$ and that the allowable normal stress in the steel is 85 MPa, determine the smallest thickness that can be used for the penstock.

SOLUTION

$$p = \rho g h = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(300\ \text{m})$$

$$= 2.943 \times 10^6\ \text{Pa}$$

$$\sigma_1 = 85\ \text{MPa} = 85 \times 10^6\ \text{Pa}$$

$$r = \frac{1}{2}d - t = \frac{1}{2}(750 \times 10^{-3}) - t = 0.375 - t$$

$$\sigma_1 = \frac{pr}{t}$$

$$85 \times 10^6 = \frac{(2.943 \times 10^6)(0.375 - t)}{t}$$

$$(87.943 \times 10^6)t = 1.103625 \times 10^6 \qquad t = 12.549 \times 10^{-3}\ \text{m}$$

$$t = 12.55\ \text{mm} \blacktriangleleft$$

PROBLEM 7.106

The bulk storage tank shown in Photo 7.3 has an outer diameter of 3.3 m and a wall thickness of 18 mm. At a time when the internal pressure of the tank is 1.5 MPa, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$$r = \frac{d}{2} - t = \frac{3.3}{2} - 18 \times 10^{-3} = 1.632 \text{ m}, \qquad t = 18 \times 10^{-3} \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.5 \times 10^6 \text{ Pa})(1.632 \text{ m})}{18 \times 10^{-3} \text{ m}} = 136 \times 10^6 \text{ Pa}$$

$$\sigma_{\max} = \sigma_1 = 136 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 136.0 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -p \approx 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 68 \times 10^6 \text{ Pa} \qquad \tau_{\max} = 68.0 \text{ MPa} \blacktriangleleft$$

PROBLEM 7.107

Determine the largest internal pressure that can be applied to a cylindrical tank of 1.75-m outer diameter and 16-mm wall thickness if the ultimate normal stress of the steel used is 450 MPa and a factor of safety of 5.0 is desired.

SOLUTION

$$\sigma_1 = \frac{\sigma_U}{F.S.} = \frac{450 \text{ MPa}}{5} = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{1.75}{2} - 16 \times 10^{-3} = 0.859 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} \quad p = \frac{t\sigma_1}{r} = \frac{(16 \times 10^{-3})(90 \times 10^6)}{0.859} = 1.676 \times 10^6 \text{ Pa} \quad \sigma_1 = 1.676 \text{ MPa} \blacktriangleleft$$

PROBLEM 7.108

A cylindrical storage tank contains liquefied propane under a pressure of 1.5 MPa at a temperature of 38°C. Knowing that the tank has an outer diameter of 320 mm and a wall thickness of 3 mm, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$$r = \frac{d}{2} - t = \frac{320}{2} - 3 = 157 \text{ mm} = 157 \times 10^{-3} \text{ m}$$

$$t = 3 \times 10^{-3} \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.5 \times 10^6 \text{ Pa})(157 \times 10^{-3} \text{m})}{3 \times 10^{-3} \text{ m}} = 78.5 \times 10^6 \text{ Pa}$$

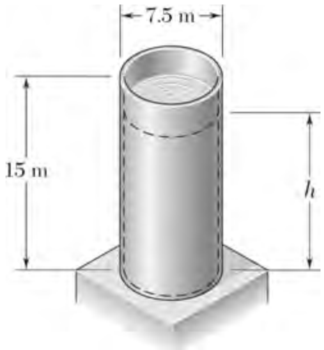
$$\sigma_{\max} = \sigma_1 = 78.5 \times 10^6 \text{ Pa}$$

$\sigma_{\max} = 78.5 \text{ MPa}$ ◀

$$\sigma_{\min} = -p \approx 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 39.25 \times 10^6 \text{ Pa}$$

$\tau_{\max} = 39.3 \text{ MPa}$ ◀



PROBLEM 7.109

The unpressurized cylindrical storage tank shown has a 5-mm wall thickness and is made of a steel having a 420-MPa ultimate strength in tension. Determine the maximum height h to which it can be filled with water if a factor of safety of 4.0 is desired. (Density of water = 9810 N/m^3.)

SOLUTION

$$d_0 = 7500 \text{ mm}$$

$$r = \frac{1}{2}d - t = 3750 - 5 = 3745 \text{ mm}$$

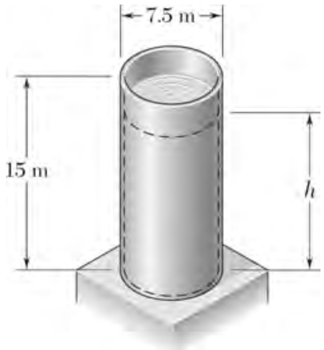
$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{420}{4.0} = 105 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{pr}{t}$$

$$p = \frac{t\sigma_{\text{all}}}{r} = \frac{(5)(105)}{3745} = 140.2 \text{ kPa}$$

But $p = \gamma h$

$$h = \frac{p}{\gamma g} = \frac{140100}{(1000)(9.81)} = 14.3 \text{ m} \quad ◀$$



PROBLEM 7.110

For the storage tank of Prob. 7.109, determine the maximum normal stress and the maximum sharing stress in the cylindrical wall when the tank is filled to capacity ($h = 14.4$ m).

PROBLEM 7.109 The unpressurized cylindrical storage tank shown has a 5-mm wall thickness and is made of steel having a 420-MPa ultimate strength in tension. Determine the maximum height h to which it can be filled with water if a factor of safety of 4.0 is desired. (Specific weight of water = 9810 N/m^3.)

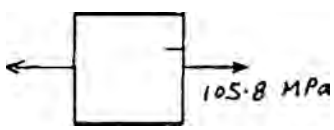
SOLUTION

$$d_0 = 7.5 \text{ m}$$

$$t = 0.005 \text{ m}$$

$$r = \frac{1}{2}d - t = 3.745 \text{ m}$$

$$p = \gamma h = (9810)(14.4) = 141264 \text{ Pa}$$



$$\sigma_1 = \frac{pr}{t} = \frac{(141264)(3.745)}{0.005} = 105.81 \text{ MPa}$$

$$\sigma_{\text{max}} = \sigma_1 \qquad \sigma_{\text{max}} = 105.8 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\text{min}} \approx 0 \quad \tau_{\text{max}} = \frac{1}{2}(\sigma_{\text{max}} - \sigma_{\text{min}}) \qquad \tau_{\text{max}} = 52.9 \text{ MPa} \blacktriangleleft$$

PROBLEM 7.111

A standard-weight steel pipe of 300-mm nominal diameter carries water under a pressure of 2.8 MPa. (*a*) Knowing that the outside diameter is 320 mm and the wall thickness is 10 mm, determine the maximum tensile stress in the pipe. (*b*) Solve part *a*, assuming an extra-strong pipe is used, of 320 mm outside diameter and 12-mm wall thickness.

SOLUTION

(*a*) $d_0 = 0.32\text{ m} \quad t = 0.01\text{ m} \quad r = \frac{1}{2}d_0 - t = 0.15\text{ m}$

$$\sigma = \frac{pr}{t} = \frac{(2.8)(0.15)}{0.01} = 42\text{ MPa}$$

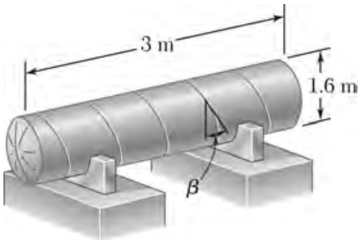
$\sigma = 42\text{ MPa}$ ◀

(*b*) $d_0 = 0.32\text{ m} \quad t = 0.012\text{ m} \quad r = \frac{1}{2}d_0 - t = 0.148\text{ m}$

$$\sigma = \frac{pr}{t} = \frac{(2.8)(0.148)}{0.012} = 34.5\text{ MPa}$$

$\sigma = 34.5\text{ MPa}$ ◀

PROBLEM 7.112



The pressure tank shown has an 8-mm wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 600 kPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

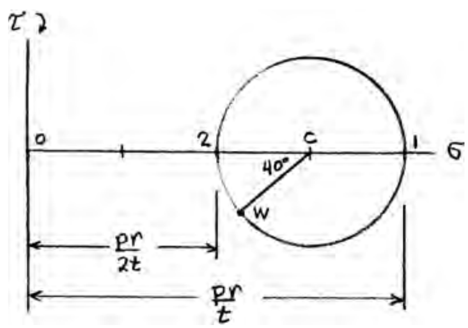
$$d = 1.6 \text{ m} \quad t = 8\times10^{-3}\text{m} \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(600\times10^3)(0.792)}{8\times10^{-3}} = 59.4\times10^6\,\text{Pa}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(600\times10^3)(0.792)}{(2)(8\times10^{-3})} = 29.7\times10^6\,\text{Pa}$$

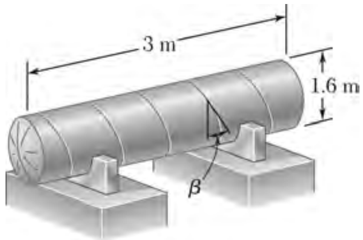
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = 44.56\times10^6\,\text{Pa}$$

$$R = \frac{1}{2}(\sigma_1 - \sigma_2) = 14.85\times10^6\,\text{Pa}$$



(*a*) $\sigma_w = \sigma_{\text{ave}} - R\cos 40° = 33.17\times10^6$ Pa $\qquad \sigma_w = 33.2$ MPa ◀

(*b*) $\tau_w = R\sin 40° = 9.55\times10^6\,\text{Pa}$ $\qquad \tau_w = 9.55$ MPa ◀



PROBLEM 7.113

For the tank of Prob. 7.112, determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 120 MPa and the allowable shearing stress parallel to the weld is 80 MPa.

PROBLEM 7.112 The pressure tank shown has a 8-mm wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 600 kPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$d = 1.6 \text{ m} \quad t = 8\times10^{-3}\text{ m} \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

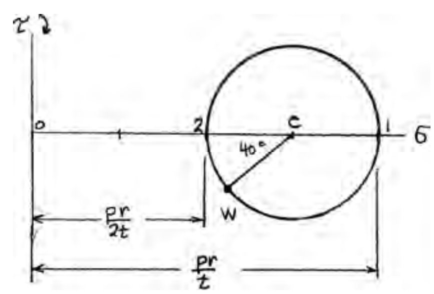
$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{\text{ave}} - R\cos 40°$$

$$= \left(\frac{3}{4} - \frac{1}{4}\cos 40°\right)\frac{pr}{t} = 0.5585\frac{pr}{t}$$

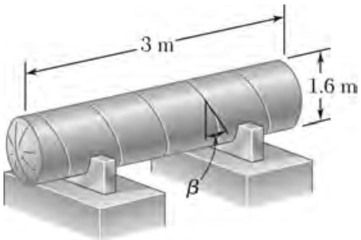


$$p = \frac{\sigma_w t}{0.5585r} = \frac{(120\times10^6)(8\times10^{-3})}{(0.5585)(0.792)} = 2.17\times10^6\text{ Pa} = 2.17\text{ MPa}$$

$$\tau_w = R\sin 40° = \left(\frac{1}{4}\sin 40°\right)\frac{pr}{t} = 0.1607\frac{pr}{t}$$

$$p = \frac{\tau_w t}{0.1607r} = \frac{(80\times10^6)(8\times10^{-3})}{(0.1607)(0.792)} = 5.03\times10^6\text{ Pa} = 5.03\text{ MPa}$$

The largest allowable pressure is the smaller value. $p = 2.17$ MPa ◀



PROBLEM 7.114

For the tank of Prob. 7.112, determine the range of values of β that can be used if the shearing stress parallel to the weld is not to exceed 12 MPa when the gage pressure is 600 kPa.

PROBLEM 7.112 The pressure tank shown has a 8-mm wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 600 kPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$d = 1.6 \text{ m} \quad t = 8\times10^{-3}\text{mm} \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

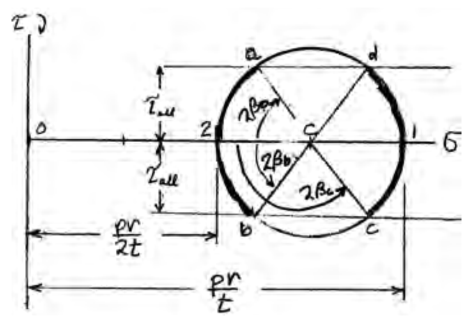
$$\sigma_1 = \frac{pr}{t} = \frac{(600\times10^3)(0.792)}{8\times10^{-3}}$$

$$= 59.4\times10^6\,\text{Pa} = 59.4 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 29.7 \text{ MPa}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 14.85 \text{ MPa}$$

$$\tau_w = R|\sin 2\beta|$$



$$|\sin\, 2\beta_a| = \frac{\tau_N}{R} = \frac{12}{14.85} = 0.80808$$

$$2\beta_a = -53.91° \qquad \beta_a = +27.0°$$

$$2\beta_b = +53.91° \qquad \beta_b = 27.0°$$

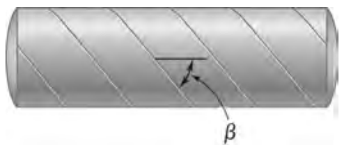
$$2\beta_c = 180° - 53.91° = +126.09° \qquad \beta_c = 63.0°$$

$$2\beta_d = 180° + 53.91° = +233.91° \qquad \beta_d = 117.0°$$

Let the total range of values for β be $-180° < \beta \le 180°$

<u>Safe ranges for β:</u> $-22.0° \le \beta \le 27.0°$ ◀

and $63.0° \le \beta \le 117.0°$ ◀



PROBLEM 7.115

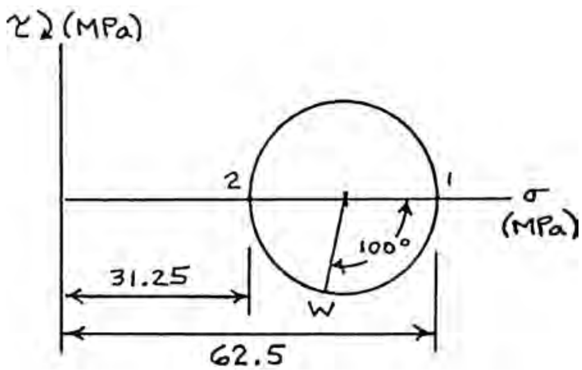
The steel pressure tank shown has a 750-mm inner diameter and a 9-mm wall thickness. Knowing that the butt-welded seams form an angle $\beta = 50°$ with the longitudinal axis of the tank and that the gage pressure in the tank is 1.5 MPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$r = \frac{d}{2} = 375\text{ mm} = 0.375\text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.5\times 10^6\text{ Pa}\times 0.375\text{ m})}{0.009\text{ m}} = 62.5\times 10^6\text{ Pa} = 62.5\text{ MPa}$$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 31.25\text{ MPa} \qquad 2\beta = 100°$$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = 46.875\text{ MPa}$$

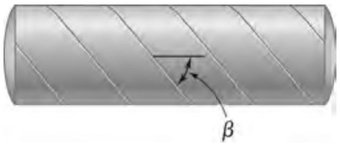
$$R = \frac{\sigma_1 - \sigma_2}{2} = 15.625\text{ MPa}$$

(*a*) $\sigma_w = \sigma_{\text{ave}} + R\cos 100°$

$$\sigma_w = 44.2\text{ MPa} \blacktriangleleft$$

(*b*) $\tau_w = R\sin 100°$

$$\tau_w = 15.39\text{ MPa} \blacktriangleleft$$



PROBLEM 7.116

The pressurized tank shown was fabricated by welding strips of plate along a helix forming an angle β with a transverse plane. Determine the largest value of β that can be used if the normal stress perpendicular to the weld is not to be larger than 85 percent of the maximum stress in the tank.

SOLUTION

$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

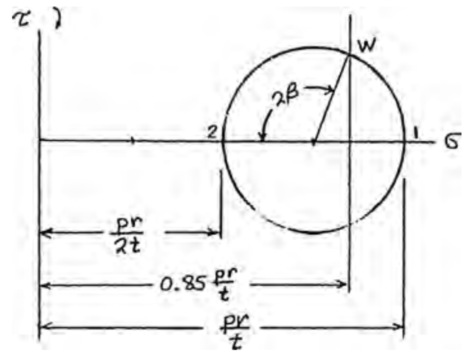
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{\text{ave}} - R\cos 2\beta$$

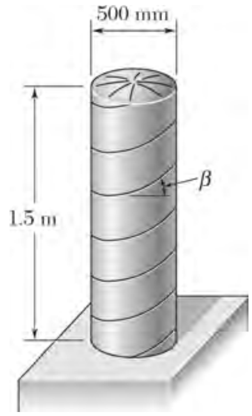
$$0.85\frac{pr}{t} = \left(\frac{3}{4} - \frac{1}{4}\cos 2\beta\right)\frac{pr}{t}$$

$$\cos 2\beta = -4\left(0.85 - \frac{3}{4}\right) = -0.4$$

$$2\beta = 113.6^\circ$$



$\beta = 56.8^\circ$ ◀



PROBLEM 7.117

The cylindrical portion of the compressed air tank shown is fabricated of 6 mm thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Knowing that the allowable stress normal to the weld is 75 MPa, determine the largest gage pressure that can be used in the tank.

SOLUTION

$$r = \frac{1}{2}d - t = \frac{1}{2}(500) - 6 = 244 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{1}{2}\frac{pr}{t}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

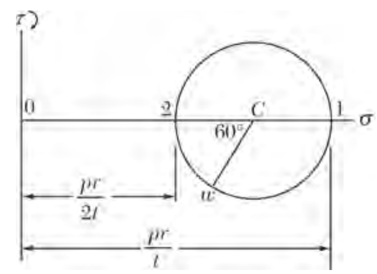
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{\text{ave}} - R\cos 60°$$

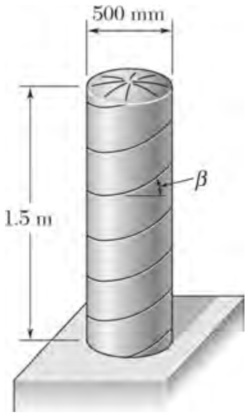
$$= \frac{5}{8}\frac{pr}{t}$$

$$p = \frac{8}{5}\frac{\sigma_w t}{r}$$

$$p = \frac{8}{5}\frac{(75)(6)}{244} = 2.95 \text{ MPa} \quad ◄$$



PROBLEM 7.118



The cylindrical portion of the compressed air tank shown is fabricated of 6 mm thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Determine the gage pressure that will cause a shearing stress parallel to the weld of 30 MPa.

SOLUTION

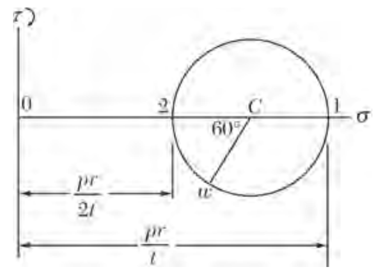
$$r = \frac{1}{2}d - t = \frac{1}{2}(500) - 6 = 244 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t}, \quad \sigma_2 = \frac{1}{2}\frac{pr}{t}$$

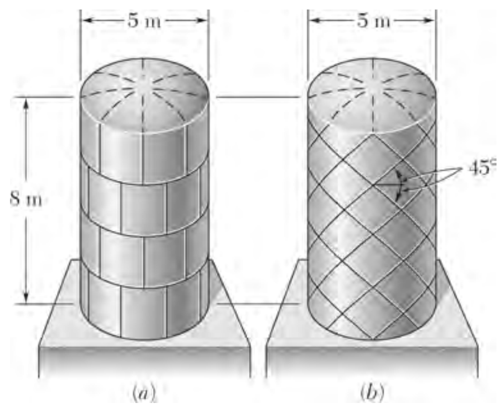
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\tau_w = R \sin 60°$$

$$= \frac{\sqrt{3}}{8}\frac{pr}{t}$$



$$p = \frac{8}{\sqrt{3}}\frac{\tau_w t}{R} \quad p = \frac{8}{\sqrt{3}}\frac{(30)(6)}{244} = 3.41 \text{ MPa}$$ ◀



PROBLEM 7.119

Square plates, each of 16-mm thickness, can be bent and welded together in either of the two ways shown to form the cylindrical portion of a compressed air tank. Knowing that the allowable normal stress perpendicular to the weld is 65 MPa, determine the largest allowable gage pressure in each case.

SOLUTION

$$r = \frac{1}{2}d - t = \frac{1}{2}(5) - 16\times 10^{-3} = 2.484 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$

(*a*) $$\sigma_1 = 65 \text{ MPa} = 65\times 10^6 \text{ Pa}$$

$$p = \frac{\sigma_1 t}{r} = \frac{(65\times 10^6)(16\times 10^{-3})}{2.484} = 419\times 10^3 \text{ Pa}$$

$p = 419$ kPa ◀

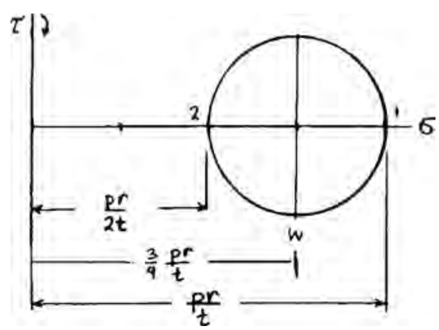
(*b*) $$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\beta = \pm 45°$$

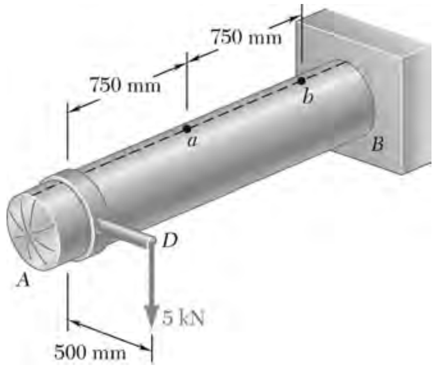
$$\sigma_w = \sigma_{\text{ave}} + R\cos\beta$$

$$= \frac{3}{4}\frac{pr}{t}$$



$$p = \frac{4\sigma_w t}{3r} = \frac{(4)(65\times 10^6)(16\times 10^{-3})}{(3)(2.484)} = 558\times 10^3 \text{ Pa}$$

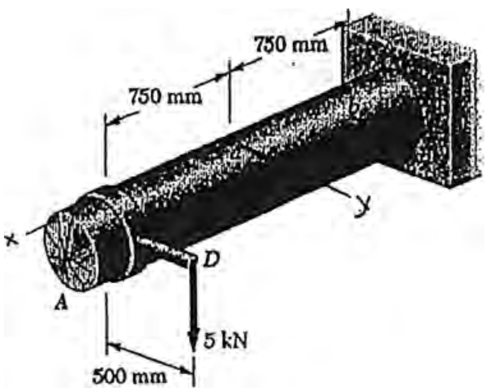
$p = 558$ kPa ◀



PROBLEM 7.120

The compressed-air tank *AB* has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure inside the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point *a* on the top of the tank.

SOLUTION



Internal pressure:

$$r = \frac{1}{2}d = 225\,\text{mm} \quad t = 6\,\text{mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45\,\text{MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 22.5\,\text{MPa}$$

Torsion: $c_1 = 225$ mm, $\quad c_2 = 225 + 6 = 231\,\text{mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 446.9 \times 10^6\,\text{mm}^4 = 446.9 \times 10^{-6}\,\text{m}^4$$

$$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500\ \text{N} \cdot \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6\ \text{Pa} = 1.292\ \text{MPa}$$

Transverse shear: $\tau = 0$ at point *a*.

Bending: $I = \frac{1}{2}J = 223.45 \times 10^{-6}\,\text{m}^4, \quad c = 231 \times 10^{-3}\text{m}$

At point *a*, $M = (5 \times 10^3)(750 \times 10^{-3}) = 3750\ \text{N} \cdot \text{m}$

$$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.88\ \text{MPa}$$

Total stresses (MPa).

Longitudinal: $\sigma_x = 22.5 + 3.88 = 26.38\,\text{MPa}$

Circumferential: $\sigma_y = 45\,\text{MPa}$

Shear: $\tau_{xy} = 1.292\ \text{MPa}$

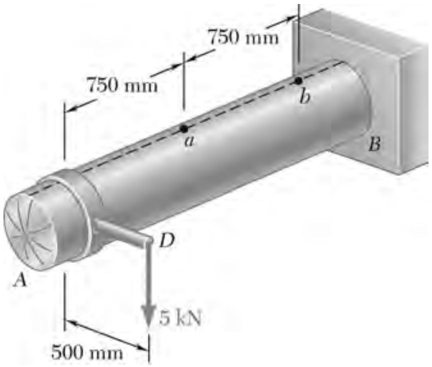
PROBLEM 7.120 *(Continued)*

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 35.69 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 9.40 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = 45.1 \text{ MPa}$$ $\sigma_{\max} = 45.1 \text{ MPa}$ ◀

$$\tau_{\max(\text{in-plane})} = R = 9.40 \text{ MPa}$$ $\tau_{\max(\text{in-plane})} = 9.40 \text{ MPa}$ ◀

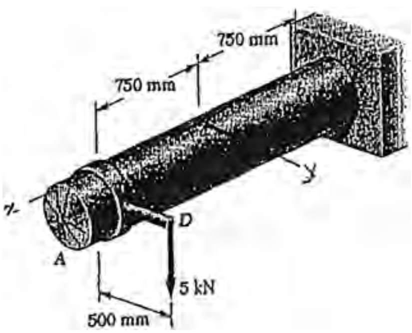


PROBLEM 7.121

For the compressed-air tank and loading of Prob. 7.120, determine the maximum normal stress and the maximum in-plane shearing stress at point b on the top of the tank.

PROBLEM 7.120 The compressed-air tank AB has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure inside the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point a on the top of the tank.

SOLUTION



Internal pressure:

$$r = \frac{1}{2}d = 225\text{ mm} \quad t = 6\text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45\text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 22.5\text{ MPa}$$

Torsion: $c_1 = 225\text{ mm}, \quad c_2 = 225 + 6 = 231\text{ mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 446.9\times 10^6\text{mm}^4 = 446.9\times 10^{-6}\text{ m}^4$$

$$T = (5\times 10^3)(500\times 10^{-3}) = 2500\text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(2500)(231\times 10^{-3})}{446.9\times 10^{-6}} = 1.292\times 10^6\text{ Pa} = 1.292\text{ MPa}$$

Transverse shear: $\tau = 0$ at point b.

Bending: $I = \frac{1}{2}J = 223.45\times 10^{-6}\text{ m}^4, \quad c = 231\times 10^{-3}\text{ m}$

At point b, $M = (5\times 10^3)(2\times 750\times 10^{-3}) = 7500\text{ N}\cdot\text{m}$

$$\sigma = \frac{Mc}{I} = \frac{(7500)(231\times 10^{-3})}{223.45\times 10^{-6}} = 7.75\text{ MPa}$$

Total stresses (MPa).

Longitudinal: $\sigma_x = 22.5 + 7.75 = 30.25\text{ MPa}$

Circumferential: $\sigma_y = 45\text{ MPa}$

Shear: $\tau_{xy} = 1.292\text{ MPa}$

PROBLEM 7.121 *(Continued)*

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 37.625 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.487 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = 45.1 \text{ MPa} \qquad \sigma_{\max} = 45.1 \text{ MPa} \blacktriangleleft$$

$$\tau_{\max\,(\text{in-plane})} = R = 7.49 \text{ MPa} \qquad \tau_{\max\,(\text{in-plane})} = 7.49 \text{ MPa} \blacktriangleleft$$

PROBLEM 7.122

A torque of magnitude $T = 12\ \text{kN}\cdot\text{m}$ is applied to the end of a tank containing compressed air under a pressure of 8 MPa. Knowing that the tank has a 180-mm inner diameter and a 12-mm wall thickness, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$$d = 180\ \text{mm} \quad r = \frac{1}{2}d = 90\ \text{mm} \quad t = 12\ \text{mm}$$

Torsion:

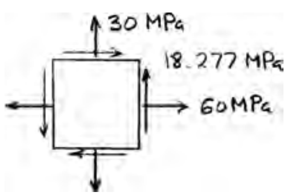
$$c_1 = 90\ \text{mm} \quad c_2 = 90 + 12 = 102\ \text{mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 66.968\times 10^6\text{mm}^4 = 66.968\times 10^{-6}\text{m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(12\times 10^3)(102\times 10^{-3})}{66.968\times 10^{-6}} = 18.277\ \text{MPa}$$

Pressure:

$$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60\ \text{MPa} \quad \sigma_2 = \frac{pr}{2t} = 30\ \text{MPa}$$



Summary of stresses:

$$\sigma_x = 60\ \text{MPa}, \quad \sigma_y = 30\ \text{MPa}, \ \tau_{xy} = 18.277\ \text{MPa}$$

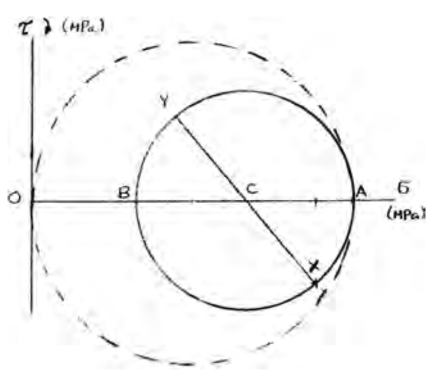
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 45\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.64\ \text{MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 68.64\ \text{MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 21.36\ \text{MPa}$$

$$\sigma_c \approx 0$$



$$\sigma_{\max} = 68.6\ \text{MPa} \ ◀$$

$$\sigma_{\min} = 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 34.3\ \text{MPa} \ ◀$$

PROBLEM 7.123

The tank shown has a 180-mm inner diameter and a 12-mm wall thickness. Knowing that the tank contains compressed air under a pressure of 8 MPa, determine the magnitude T of the applied torque for which the maximum normal stress is 75 MPa.

SOLUTION

$$r = \frac{1}{2}d = \left(\frac{1}{2}\right)(180) = 90 \text{ mm} \qquad t = 12 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 30 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_y) = 45 \text{ MPa}$$

$$\sigma_{\max} = 75 \text{ MPa}$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 30 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{xy}^2} = \sqrt{15^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{R^2 - 15^2} = \sqrt{30^2 - 15^2} = 25.98 \text{ MPa}$$

$$= 25.98 \times 10^6 \text{ Pa}$$

Torsion:

$$c_1 = 90 \text{ mm}$$

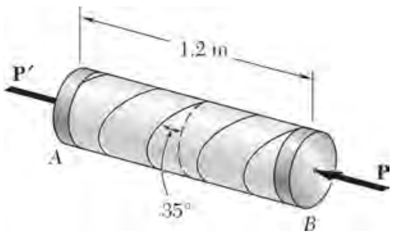
$$c_2 = 90 + 12 = 102 \text{ mm}$$

$$J = \frac{T}{2}\left(c_2^4 - c_1^4\right) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$$

$$\tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c} = \frac{(66.968 \times 10^{-6})(25.98 \times 10^6)}{102 \times 10^{-3}} = 17.06 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 17.06 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROBLEM 7.124



A pressure vessel of 250 mm inside diameter and 6 mm wall thickness is fabricated from a 1.2 m section of spirally welded pipe *AB* and is with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 45 kN centric axial forces **P** and **P′** are applied to the end plates. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$r = \frac{1}{2}d = 125 \text{ mm} \qquad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(2)(125)}{(2)(6)} = 20.83 \text{ MPa}$$

$$r_0 = r + t = 125 + 6 = 131 \text{ mm}$$

$$A = \pi\left(r_0^2 - r^2\right) = 4.825\times10^3 \text{ mm}^2 = 4.825\times10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} = -\frac{45\times10^3}{4.825\times10^{-3}} = -9.326\times10^6 \text{ Pa} = -9.326 \text{ MPa}$$

Total stresses: Longitudinal $\sigma_x = 20.83 - 9.326 = 11.504$ MPa

Circumferential $\sigma_y = 41.67$ MPa

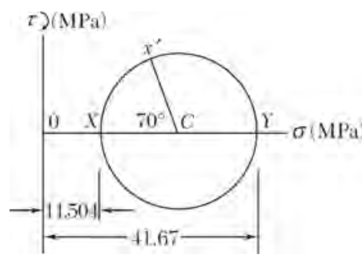
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 26.585 \text{ MPa}$$

$$R = \frac{\sigma_x - \sigma_y}{2} = 15.081$$

(*a*) $\sigma_{x'} = \sigma_{\text{ave}} + R \cos 70°$

$= 26.585 - 15.081 \cos 70°$

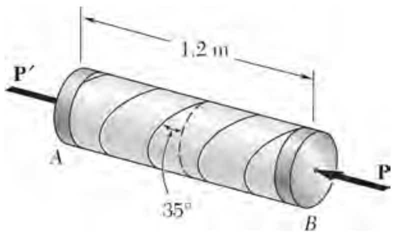
$= 21.4$ MPa ◀



(*b*) $\tau_{x'y'} = R \sin 70° = 15.081 \sin 70°$

$= 14.17$ MPa ◀

PROBLEM 7.125



Solve Prob. 7.124, assuming that the magnitude P of the two forces is increased to 120 kN.

PROBLEM 7.124 A pressure vessel of 250 mm inside diameter and 6 mm wall thickness is fabricated from a 1.2 m section of spirally welded pipe AB and is with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 45 kN centric axial forces **P** and **P′** are applied to the end plates. Determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

SOLUTION

$$r = \frac{1}{2}d = 125 \text{ mm} \qquad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 20.833 \text{ MPa}$$

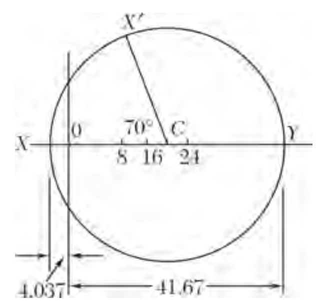
$$r_0 = r + t = 125 + 6 = 131 \text{ mm}$$

$$A = \pi\left(r_0^2 - r^2\right) = 4.825\times10^3 \text{ mm}^2 = 4.825\times10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} = -\frac{120\times10^3}{4.825\times10^{-3}} = -24.870\times10^6 \text{ Pa} = -24.870 \text{ MPa}$$

Total stresses: Longitudinal $\qquad \sigma_x = 20.833 - 24.870 = -4.037$ MPa

Circumferential $\qquad \sigma_y = 41.67$ MPa



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 18.815 \text{ MPa}$$

$$R = \left|\frac{\sigma_x - \sigma_y}{2}\right| = 22.852 \text{ MPa}$$

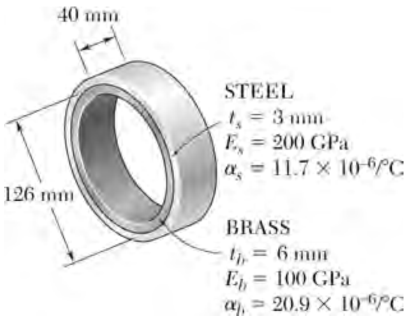
(a) $\quad \sigma_{x'} = \sigma_{\text{ave}} - R\cos 70°$

$= 18.815 - 22.852\cos 70°$

$= 11.00$ MPa ◀

(b) $\quad \tau_{x'y'} = R\sin 70° = 22.852\sin 70°$

$= 21.5$ MPa ◀



PROBLEM 7.126

A brass ring of 126-mm outer diameter and 6-mm thickness fits exactly inside a steel ring of 126-mm inner diameter and 3-mm thickness when the temperature of both rings is 10°C. Knowing that the temperature of both rings is then raised to 52°C, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.

SOLUTION

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p, $\sigma_s = \dfrac{pr}{t_s}$ (1)

Corresponding strain $\varepsilon_{sp} = \dfrac{\sigma_s}{E_s} = \dfrac{pr}{E_s t_s}$

Strain due to temperature change $\varepsilon_{sT} = \alpha_s \Delta T$

Total strain $\varepsilon_s = \dfrac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r\,\varepsilon_s = 2\pi r\left(\frac{pr}{E_s t_s} + \alpha_s \Delta T\right)$$

Brass ring: External pressure p, $\sigma_b = -\dfrac{pr}{t_b}$

Corresponding strains $\varepsilon_{bp} = -\dfrac{pr}{E_b t_b}, \quad \varepsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r\left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T\right)$$

Equating ΔL_s to ΔL_b $\dfrac{pr}{E_s t_s} + \alpha_s \Delta T = -\dfrac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b}\right) p = (\alpha_b - \alpha_s)\Delta T \qquad (2)$$

PROBLEM 7.126 *(Continued)*

Data:
$$\Delta T = 52\ °\mathrm{C} - 10\ °\mathrm{C} = 42\ °\mathrm{C}$$
$$r = \frac{1}{2}d = \frac{1}{2}(126) = 63 \text{ mm}$$

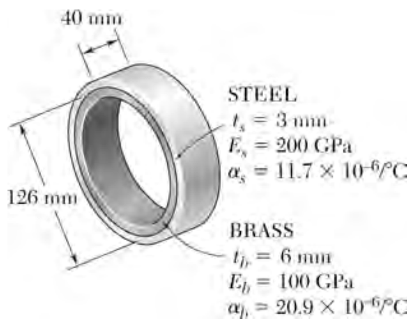
From Eq. (2)
$$\left[\frac{63\times10^{-3}}{(200\times10^{9})(0.003)} + \frac{63\times10^{-3}}{(100\times10^{9})(0.006)}\right] p = (20.9-11.7)(10^{-6})(38)$$
$$2.1\times10^{-10}\, p = 3.496\times10^{-4}$$
$$p = 1.67 \text{ MPa}$$

From Eq. (1)
$$\sigma_s = \frac{pr}{t_s} = \frac{(1.67)(63)}{3} = 35.1 \text{ MPa}$$

(*a*) $\sigma_s = 35.1$ MPa ◀

(*b*) $p = 1.67$ MPa ◀

PROBLEM 7.127



Solve Prob. 7.126, assuming that the brass ring is 3 mm thick and the steel ring is 6 mm thick.

PROBLEM 7.126 A brass ring of 126-mm outer diameter and 6-mm thickness fits exactly inside a steel ring of 126-mm inner diameter and 3-mm thickness when the temperature of both rings is 10 °C. Knowing that the temperature of both rings is then raised to 52°C, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.

SOLUTION

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p,
$$\sigma_s = \frac{pr}{t_s} \tag{1}$$

Corresponding strain
$$\varepsilon_{sp} = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$$

Strain due to temperature change
$$\varepsilon_{sT} = \alpha_s \Delta T$$

Total strain
$$\varepsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$$

Change in length of circumference

$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r\left(\frac{pr}{E_s t_s} + \alpha_s \Delta T\right)$$

Brass ring: External pressure p,
$$\sigma_b = -\frac{pr}{t_b}$$

Corresponding strains
$$\varepsilon_{bp} = -\frac{pr}{E_b t_b}, \quad \varepsilon_{bT} = \alpha_b \Delta T$$

Change in length of circumference

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r\left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T\right)$$

Equating ΔL_s to ΔL_b
$$\frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b}\right) p = (\alpha_b - \alpha_s)\ \Delta T \tag{2}$$

PROBLEM 7.127 *(Continued)*

Data: $\Delta T = 52\,°\text{C} - 10\,°\text{C} = 42\,°\text{C}$

$$r = \frac{1}{2}d = \frac{1}{2}(126) = 63 \text{ mm}$$

From Eq. (2)

$$\left[\frac{63\times10^{-3}}{(200\times10^{9})(0.006)} + \frac{63\times10^{-3}}{(100\times10^{9})(0.003)}\right] p = (20.9-11.7)(10^{-6})(38)$$

$$2.625\times10^{-10}\, p = 3.496\times10^{-4}$$

$$p = 1.33 \text{ MPa}$$

From Eq. (1)

$$\sigma_s = \frac{pr}{t_s} = \frac{(1.33)(63)}{6} = 13.3 \text{ MPa}$$

(*a*) $\sigma_s = 14 \text{ MPa}$ ◀

(*b*) $p = 1.33 \text{ MPa}$ ◀

PROBLEM 7.128

For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -720\mu, \quad \varepsilon_y = 0, \quad \gamma_{xy} = +300\mu, \quad \theta = -30^\circ$$

SOLUTION

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -360\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -360\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-360 - 360\cos(-60^\circ) + \frac{300}{2}\sin(-60^\circ)\right\}\mu = -670\mu \quad \blacktriangleleft$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-360 - (-360)\cos(-60^\circ) - \frac{300}{2}\sin(-60^\circ)\right\}\mu = -50\mu \quad \blacktriangleleft$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \{-(-720 - 0)\sin(-60^\circ) + 300\cos(-60^\circ)\}\mu = -474\mu \quad \blacktriangleleft$$

PROBLEM 7.129

For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = 0 \quad \varepsilon_y = +320\mu \quad \gamma_{xy} = -100\mu \quad \theta = 30^\circ$$

SOLUTION

$$\frac{\varepsilon_x + \varepsilon_y}{2} = 160\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -160\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{160 - 160\cos 60^\circ - \frac{100}{2}\sin 60^\circ\right\}\mu = +36.7\mu \quad ◀$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{160 + 160\cos 60^\circ + \frac{100}{2}\sin 60^\circ\right\}\mu = +283\mu \quad ◀$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \{-(0 - 320)\sin 60^\circ - 100\cos 60^\circ\}\mu = +227\mu \quad ◀$$

PROBLEM 7.130

For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -800\mu, \quad \varepsilon_y = +450\mu, \quad \gamma_{xy} = +200\mu, \quad \theta = -25°$$

SOLUTION

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -175\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -625\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-175 - 625\cos(-50°) + \frac{200}{2}\sin(-50°)\right\}\mu = -653\mu \quad \blacktriangleleft$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-175 + 625\cos(-50°) - \frac{200}{2}\sin(-50°)\right\}\mu = +303\mu \quad \blacktriangleleft$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \{-(-800 - 450)\sin(-50°) + 200\cos(-50°)\}\mu = -829\mu \quad \blacktriangleleft$$

PROBLEM 7.131

For the given state of plane strain, use the method of Sec 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -500\mu, \quad \varepsilon_y = +250\mu, \quad \gamma_{xy} = 0, \quad \theta = 15°$$

SOLUTION

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -125\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -375\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \{-125 - 375\cos 30° + 0\}\mu = -450\mu \quad ◀$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \{-125 + 375\cos 30° - 0\}\mu = +200\mu \quad ◀$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

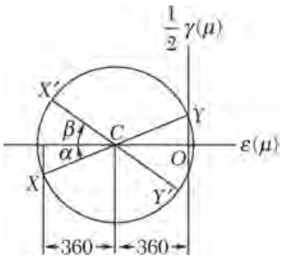
$$= \{-(-500 - 250)\sin 30° + 0\}\mu = +375\mu \quad ◀$$

PROBLEM 7.132

For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$$\varepsilon_x = -720\mu, \quad \varepsilon_y = 0, \quad \gamma_{xy} = +300\mu, \quad \theta = -30°$$

SOLUTION



Plotted points

$X: \ (-720\mu, -150\mu)$

$Y: \ (0, 150\mu)$

$C: \ (-360\mu, 0)$

$$\tan\alpha = \frac{150\mu}{360\mu}$$

$$\alpha = 22.62°$$

$$R = \sqrt{(360\mu)^2 + (150\mu)^2} = 390\mu$$

$$\beta = 2\theta - \alpha = 60° - 22.62° = 37.38°$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} - R\cos\beta = -360\mu - 390\mu\cos 37.38° = -670\mu \quad ◀$$

$$\varepsilon_{y'} = \varepsilon_{\text{ave}} + R\cos\beta = -360\mu + 390\mu\cos 37.38° = -50\mu \quad ◀$$

$$\frac{\gamma_{x'y'}}{2} = -R\sin\beta = -390\mu\sin 37.38° \qquad \gamma_{x'y'} = -474\text{ m} \quad ◀$$

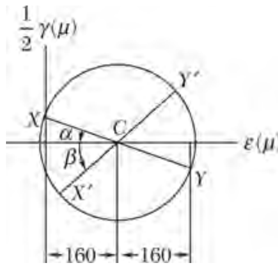
PROBLEM 7.133

For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$$\varepsilon_x = 0 \quad \varepsilon_y = +320\mu \quad \gamma_{xy} = -100\mu \quad \theta = 30°$$

SOLUTION

Plotted points



$$X: \ (0, 50\mu)$$
$$Y: \ (320\mu, -50\mu)$$
$$C: \ (160\mu, 0)$$

$$\tan\ \alpha = \frac{50}{160} \quad \alpha = 17.35°$$

$$R = \sqrt{(160\mu)^2 + (50\mu)^2} = 167.63\mu$$

$$\beta = 2\theta - \alpha = 60° - 17.35° = 42.65°$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} - R\cos\beta = 160\mu - 167.63\mu\cos 42.65° = -36.7\mu \ ◀$$

$$\varepsilon_{y'} = \varepsilon_{\text{ave}} + R\cos\ \beta = 160\mu + 167.63\mu\cos 42.65° = 283\mu \ ◀$$

$$\frac{\gamma_{x'y'}}{2} = R\ \sin\ \beta = 167.63\mu\ \sin\ 42.65°$$

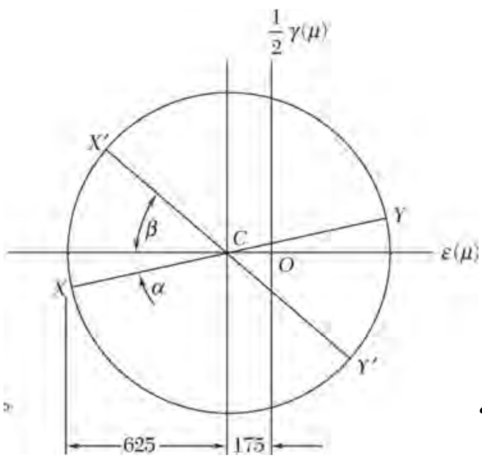
$$\gamma_{x'y'} = 227\mu \ ◀$$

PROBLEM 7.134

For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$$\varepsilon_x = -800\mu \quad \varepsilon_y = 450\mu \quad \gamma_{xy} = +200\mu \quad \theta = -25°$$

SOLUTION



Plotted points:

$X: (-800\mu,\ -100\mu)$

$Y: (+450\mu,\ +100\mu)$

$C: (-175\mu,\ 0)$

$$\tan\alpha = \frac{100}{625} \qquad \alpha = 9.09°$$

$$R = \sqrt{(625\mu)^2 + (100\mu)^2} = 632.95\mu$$

$$\beta = 2\theta - \alpha = 50° - 9.09° = 40.91°$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} - R\cos\beta = -175\mu - 632.95\mu\cos 40.91°$$

$$= -653\mu \quad ◀$$

$$\varepsilon_{y'} = \varepsilon_{\text{ave}} + R\cos\beta = -175$$

$$= +303\mu \quad ◀$$

$$\frac{\gamma_{x'y'}}{2} = -R\sin\beta = -632.95\mu\sin 40.91° \qquad \gamma_{x'y'} = -829\mu \quad ◀$$

PROBLEM 7.135

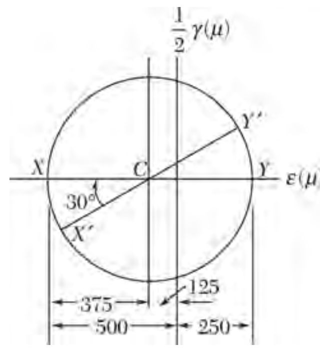
For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$$\varepsilon_x = -500\mu, \quad \varepsilon_y = +250\mu, \quad \gamma_{xy} = 0, \quad \theta = 15°$$

SOLUTION

Plotted points

$$X: \ (-500\mu, \ 0)$$
$$Y: \ (+250\mu, \ 0)$$
$$C: \ (-125\mu, \ 0)$$



$$R = 375\mu$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} - R\cos 2\theta = -125 - 375\cos 30°$$
$$= -450\mu \quad \blacktriangleleft$$

$$\varepsilon_{y'} = \varepsilon_{\text{ave}} + R\cos 2\theta = -125 + 375\cos 30°$$
$$= 200\mu \quad \blacktriangleleft$$

$$\frac{1}{2}\gamma_{xy} = R\sin 2\theta = 375\sin 30°$$
$$\gamma_{x'y'} = 375\mu \quad \blacktriangleleft$$

PROBLEM 7.136

The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$.)

$$\varepsilon_x = -260\mu, \quad \varepsilon_y = -60\mu, \quad \gamma_{xy} = +480\mu$$

SOLUTION

For Mohr's circle of strain, plot points:

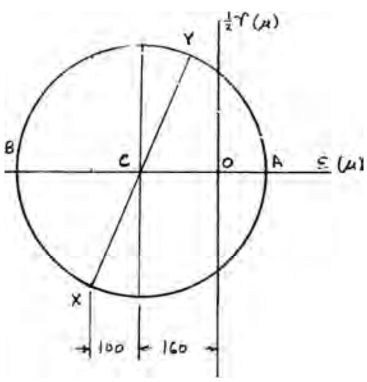
$$X: \ (-260\mu, -240\mu)$$
$$Y: \ (-60\mu, 240\mu)$$
$$C: \ (-160\mu, 0)$$

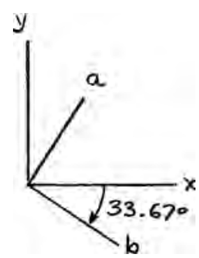
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{480}{-260+60} = -2.4$$

$$2\theta_p = -67.38^\circ \qquad \theta_b = -33.67^\circ \blacktriangleleft$$

$$\theta_a = 56.31^\circ \blacktriangleleft$$

$$R = \sqrt{(100\mu)^2 + (240\mu)^2}$$
$$R = 260\mu$$





(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = -160\mu + 260\mu$ $\qquad \varepsilon_a = 100\mu \blacktriangleleft$

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -160\mu - 260\mu$ $\qquad \varepsilon_b = -420\mu \blacktriangleleft$

(*b*) $\frac{1}{2}\gamma_{\max(\text{in-plane})} = R \quad \gamma_{\max(\text{in-plane})} = 2R$ $\qquad \gamma_{\max\,(\text{in-plane})} = 520\mu \blacktriangleleft$

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-260-60)$$
$$= 160\mu$$

$$\varepsilon_{\max} = 160\mu \quad \varepsilon_{\min} = -420\mu$$

(*c*) $\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 160\mu + 420\mu$ $\qquad \gamma_{\max} = 580\mu \blacktriangleleft$

PROBLEM 7.137

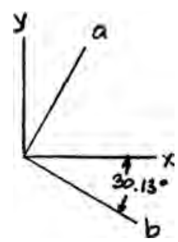
The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $v = \frac{1}{3}$.)

$$\varepsilon_x = -600\mu, \quad \varepsilon_y = -400\mu, \quad \gamma_{xy} = +350\mu$$

SOLUTION

Plotted points for Mohr's circle:

$$X: \ (-600\mu, -175\mu)$$
$$Y: \ (-400\mu, +175\mu)$$
$$C: \ (-500\mu, 0)$$



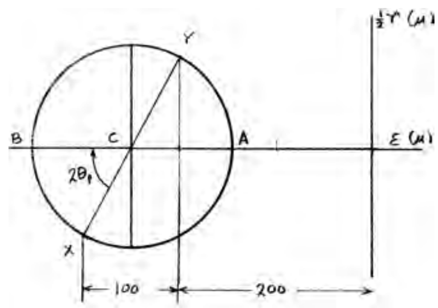
$$\tan 2\theta_p = -\frac{175}{100}$$

$$2\theta_p = -60.26°$$

$\theta_b = -30.13°$ ◀

$\theta_a = 59.87°$ ◀

$$R = \sqrt{(100\mu)^2 + (175\mu)^2}$$
$$= 201.6\mu$$



(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = -500\mu + 201.6\mu$ $\qquad \varepsilon_a = -298\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -500\mu - 201.6\mu$ $\qquad \varepsilon_b = -702\mu$ ◀

(*b*) $\gamma_{\max\,(\text{in-plane})} = 2R$ $\qquad \gamma_{\max\,(\text{in-plane})} = 403\mu$ ◀

$$\varepsilon_c = -\frac{v}{1-v}(\varepsilon_a + \varepsilon_b) = -\frac{v}{1-v}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-600\mu - 400\mu)$$

$\varepsilon_c = 500\mu$ ◀

$$\varepsilon_{\max} = 500\mu \quad \varepsilon_{\min} = -702\mu$$

(*c*) $\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 500\mu + 702\mu$ $\qquad \gamma_{\max} = 1202\mu$ ◀

PROBLEM 7.138

The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $v = \frac{1}{3}$.)

$$\varepsilon_x = +160\mu, \quad \varepsilon_y = -480\mu, \quad \gamma_{xy} = -600\mu$$

SOLUTION

(*a*) For Mohr's circle of strain, plot points:

$$X: \ (160\mu, \ 300\mu)$$
$$Y: \ (-480\mu, -300\mu)$$
$$C: \ (-160\mu, \ 0)$$

(*a*) $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{-300}{320} = -0.9375$

$$2\theta_p = -43.15° \quad \theta_p = -21.58° \quad \text{and} \quad -21.58 + 90 = 68.42°$$

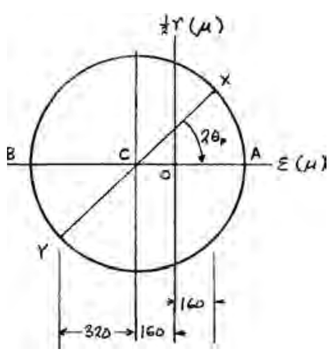
$\theta_a = -21.58°$ ◀

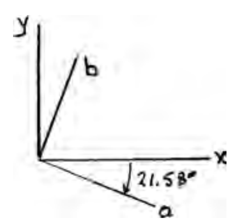
$\theta_b = 68.42°$ ◀

$$R = \sqrt{(320\mu)^2 + (300\mu)^2} = 438.6\mu$$

$\varepsilon_a = \varepsilon_{\text{ave}} + R = -160\mu + 438.6\mu$ $\qquad \varepsilon_a = +278.6\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -160\mu - 438.6\mu$ $\qquad \varepsilon_b = -598.6\mu$ ◀





(*b*) $\dfrac{1}{2}\gamma_{(\text{max, in-plane})} = R \quad \gamma_{(\text{max, in-plane})} = 2R$ $\qquad \gamma_{(\text{max, in-plane})} = 877\mu$ ◀

(*c*) $\varepsilon_c = -\dfrac{v}{1-v}(\varepsilon_a + \varepsilon_b) = -\dfrac{v}{1-v}(\varepsilon_x + \varepsilon_y) = -\dfrac{1/3}{2/3}(160\mu - 480\mu)$ $\qquad \varepsilon_c = 160.0\mu$ ◀

$$\varepsilon_{\max} = 278.6\mu \quad \varepsilon_{\min} = -598.6\mu$$

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 278.6\mu + 598.6\mu$ $\qquad \gamma_{\max} = 877\mu$ ◀

PROBLEM 7.139

The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $v = \frac{1}{3}$)

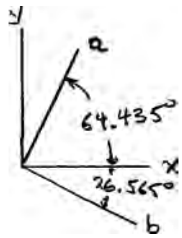
$$\varepsilon_x = +30\mu, \quad \varepsilon_y = +570\mu, \quad \gamma_{xy} = +720\mu$$

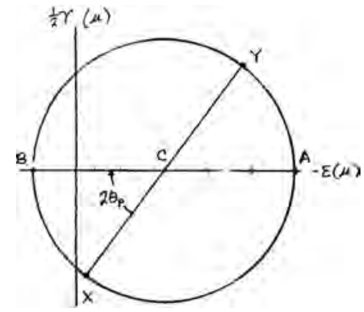
SOLUTION

Plotted points for Mohr's circle:

$$X: \ (30\mu, -360\mu)$$
$$Y: \ (570\mu, +360\mu)$$
$$C: \ (300\mu, 0)$$

$$\tan 2\theta_p = \frac{-360}{270} = -1.3333$$
$$2\theta_p = -53.13°$$





(*a*) $\theta_b = -26.565°$ ◀

$\theta_a = 64.435°$ ◀

$$R = \sqrt{(270\mu)^2 + (360\mu)^2} = 450\mu$$

$\varepsilon_a = \varepsilon_{\text{ave}} + R = 300\mu + 450\mu$ $\qquad \varepsilon_a = 750\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = 300\mu - 450\mu$ $\qquad \varepsilon_b = -150\mu$ ◀

(*b*) $\gamma_{\text{max (in-plane)}} = 2R$ $\qquad \gamma_{\text{max (in-plane)}} = 900\mu$ ◀

$\varepsilon_c = -\dfrac{v}{1-v}(\varepsilon_a + \varepsilon_b) = -\dfrac{1/3}{2/3}(750\mu - 150\mu)$ $\qquad \varepsilon_c = -300\mu$ ◀

$$\varepsilon_{\max} = \varepsilon_a = 750\mu, \quad \varepsilon_{\min} = \varepsilon_c = -300\mu$$

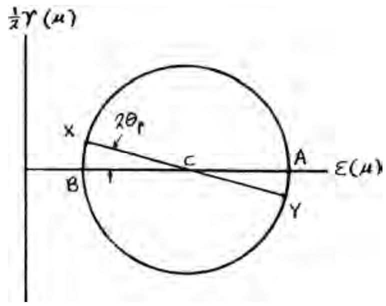
(*c*) $\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 750\mu - (-300\mu)$ $\qquad \gamma_{\max} = 1050\mu$ ◀

PROBLEM 7.140

For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = +60\mu, \quad \varepsilon_y = +240\mu, \quad \gamma_{xy} = -50\mu$$

SOLUTION



Plotted points:

$$X: \ (60\mu, 25\mu)$$
$$Y: \ (240\mu, -25\mu)$$
$$C: \ (150\mu, 0)$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-50}{60 - 240} = 0.277778$$

$$2\theta_p = 15.52° \qquad \theta_a = 97.76° \blacktriangleleft$$

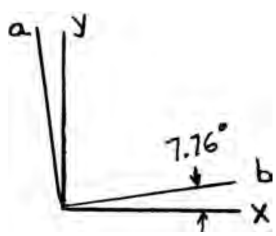
$$\theta_b = 7.76° \blacktriangleleft$$

$$R = \sqrt{(90\mu)^2 + (25\mu)^2} = 93.4\mu$$

(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 150\mu + 93.4\mu$ $\qquad \varepsilon_a = 243.4\mu \blacktriangleleft$

$\varepsilon_b = \varepsilon_{\text{ave}} - R = 150\mu - 93.4\mu$ $\qquad \varepsilon_b = 56.6\mu \blacktriangleleft$

(*b*) $\gamma_{\text{max (in-plane)}} = 2R$ $\qquad \gamma_{\text{max (in-plane)}} = 186.8\mu \blacktriangleleft$



(*c*) $\varepsilon_c = 0, \quad \varepsilon_{\max} = 243.4\mu, \quad \varepsilon_{\min} = 0$

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min}$ $\qquad \gamma_{\max} = 243.4 \blacktriangleleft$

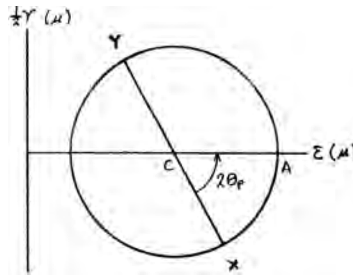
PROBLEM 7.141

For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = +400\mu, \quad \varepsilon_y = +200\mu, \quad \gamma_{xy} = 375\mu$$

SOLUTION

Plotted points for Mohr's circle:



$$X: \ (+400\mu, -187.5\mu)$$
$$Y: \ (+200\mu, +187.5\mu)$$
$$C: \ (+300\mu, 0)$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{375}{400-200} = 1.875$$

$$2\theta_p = 61.93^\circ \qquad \theta_a = 30.96^\circ \blacktriangleleft$$

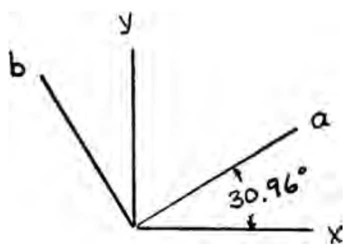
$$\theta_b = 120.96^\circ \blacktriangleleft$$

$$R = \sqrt{(100\mu)^2 + (187.5\mu)^2} = 212.5\mu$$

(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 300\mu + 212.5\mu$ $\qquad \varepsilon_a = 512.5\mu \blacktriangleleft$

$\varepsilon_b = \varepsilon_{\text{ave}} - R = 300\mu - 212.5\mu$ $\qquad \varepsilon_b = 87.5\mu \blacktriangleleft$

(*b*) $\gamma_{\text{max (in-plane)}} = 2R$ $\qquad \gamma_{\text{max (in-plane)}} = 425\mu \blacktriangleleft$



(*c*) $\varepsilon_c = 0 \quad \varepsilon_{\max} = 512.5\mu \quad \varepsilon_{\min} = 0$

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min}$ $\qquad \gamma_{\max} = 512.5\mu \blacktriangleleft$

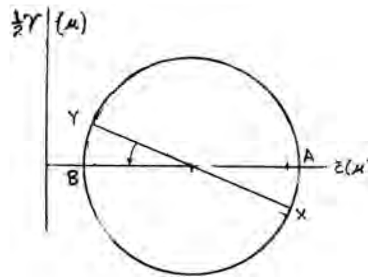
PROBLEM 7.142

For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = +300\mu, \quad \varepsilon_y = +60\mu, \quad \gamma_{xy} = +100\mu$$

SOLUTION

$$X: \ (300\mu, -50\mu)$$
$$Y: \ (60\mu, 50\mu)$$
$$C: \ (180\mu, 0)$$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{100}{300-60}$$

$$2\theta_p = 22.62°$$

$\theta_a = 11.31°$ ◀

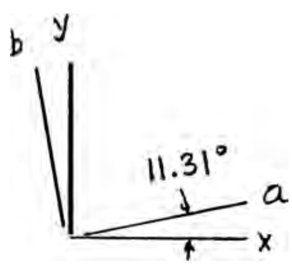
$\theta_b = 101.31°$ ◀

$$R = \sqrt{(120\mu)^2 + (50\mu)^2} = 130\mu$$

(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 180\mu + 130\mu$ $\qquad \varepsilon_a = 310\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = 180\mu - 130\mu$ $\qquad \varepsilon_b = 50\mu$ ◀

(*b*) $\gamma_{\max\text{ (in-plane)}} = 2R$ $\qquad \gamma_{\max\text{ (in-plane)}} = 260\mu$ ◀



(*c*) $\varepsilon_c = 0, \quad \varepsilon_{\max} = 310\mu, \quad \varepsilon_{\min} = 0$

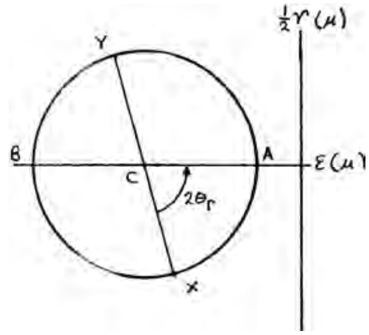
$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min}$ $\qquad \gamma_{\max} = 310\mu$ ◀

PROBLEM 7.143

For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = -180\mu, \quad \varepsilon_y = -260\mu, \quad \gamma_{xy} = +315\mu$$

SOLUTION



Plotted points for Mohr's circle:

$$X: \ (-180\mu, -157.5\mu)$$
$$Y: \ (-260\mu, +157.5\mu)$$
$$C: \ (-220\mu, 0)$$

(*a*) $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{315}{80} = 3.9375$

$2\theta_p = 75.75°$ $\qquad \theta_a = 37.87°$ ◀

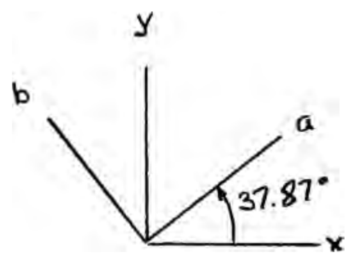
$\theta_b = 127.87°$ ◀

$$R = \sqrt{(40\mu)^2 + (157.5\mu)^2} = 162.5\mu$$

$\varepsilon_a = \varepsilon_{\text{ave}} + R = -220\mu + 162.5\mu$ $\qquad \varepsilon_a = -57.5\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -220\mu - 162.5$ $\qquad \varepsilon_b = -382.5\mu$ ◀

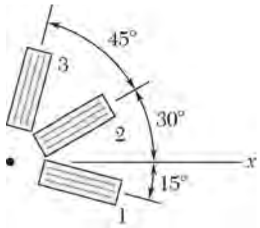
(*b*) $\gamma_{\text{max (in-plane)}} = 2R = 325\mu$ ◀



(*c*) $\varepsilon_c = 0, \quad \varepsilon_{\max} = 0, \quad \varepsilon_{\min} = -382.5\mu$

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 0 + 382.5\mu$ $\qquad \gamma_{\max} = 382.5\mu$ ◀

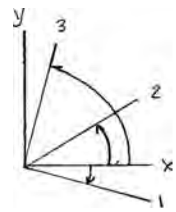
PROBLEM 7.144



Determine the strain ε_x knowing that the following strains have been determined by use of the rosette shown:

$$\varepsilon_1 = +480\mu \quad \varepsilon_2 = -120\mu \quad \varepsilon_3 = +80\mu$$

SOLUTION



$$\theta_1 = -15°$$
$$\theta_2 = 30°$$
$$\theta_3 = 75°$$

$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1\cos\theta_1 = \varepsilon_1$$

$$0.9330\varepsilon_x + 0.06699\varepsilon_y - 0.25\gamma_{xy} = 480\mu \qquad (1)$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2\cos\theta_2 = \varepsilon_2$$

$$0.75\varepsilon_x + 0.25\varepsilon_y + 0.4330\gamma_{xy} = -120\mu \qquad (2)$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3\cos\theta_3 = \varepsilon_3$$

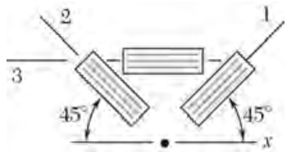
$$0.06699\varepsilon_x + 0.9330\varepsilon_y + 0.25\gamma_{xy} = 80\mu \qquad (3)$$

Solving (1), (2), and (3) simultaneously,

$$\varepsilon_x = 253\mu, \qquad \varepsilon_y = 307\mu, \qquad \gamma_{xy} = -893\mu$$

$$\varepsilon_x = 253\mu \blacktriangleleft$$

PROBLEM 7.145



Determine the largest in-plane normal strain, knowing that the following strains have been obtained by the use of the rosette shown:

$$\varepsilon_1 = -50 \times 10^{-6} \text{in./in.}$$
$$\varepsilon_2 = +360 \times 10^{-6} \text{in./in.}$$
$$\varepsilon_3 = +315 \times 10^{-6} \text{in./in.}$$

SOLUTION

$$\theta_1 = 45°, \quad \theta_2 = -45°, \quad \theta_3 = 0$$

$$\varepsilon_x \cos^2 \theta_1 + \varepsilon_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 = \varepsilon_1$$

$$0.5\varepsilon_x + 0.5\varepsilon_y + 0.5\gamma_{xy} = -50 \times 10^{-6} \quad (1)$$

$$\varepsilon_x \cos^2 \theta_2 + \varepsilon_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 = \varepsilon_2$$

$$0.5\varepsilon_x + 0.5\varepsilon_y - 0.5\gamma_{xy} = 360 \times 10^{-6} \quad (2)$$

$$\varepsilon_x \cos^2 \theta_3 + \varepsilon_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 = \varepsilon_3$$

$$\varepsilon_x + 0 + 0 = 315 \times 10^{-6} \quad (3)$$

From (3), $\varepsilon_x = 315 \times 10^{-6}$ mm/mm

Eq. (1) – Eq. (2): $\gamma_{xy} = -50 \times 10^{-6} - 360 \times 10^{-6} = -410 \times 10^{-6}$ mm/mm

Eq. (1) + Eq. (2): $\varepsilon_x + \varepsilon_y = \varepsilon_1 + \varepsilon_2$

$$\varepsilon_y = \varepsilon_1 + \varepsilon_2 - \varepsilon_x = -50 \times 10^{-6} + 360 \times 10^{-6} - 315 \times 10^{-6}$$

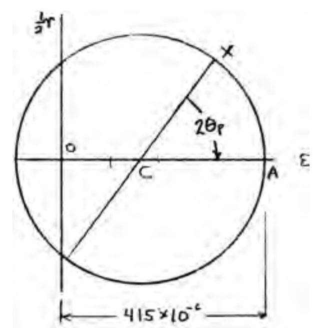
$$= -5 \times 10^{-6} \text{ mm/mm}$$

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 155 \times 10^{-6} \text{ mm/mm}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \sqrt{\left(\frac{315 \times 10^{-6} + 5 \times 10^{-6}}{2}\right)^2 + \left(\frac{-410 \times 10^{-6}}{2}\right)^2}$$

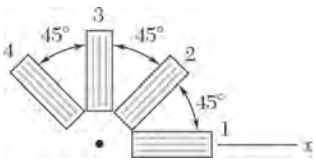
$$= 260 \times 10^{-6} \text{mm/mm}$$



$$\varepsilon_{\max} = \varepsilon_{\text{ave}} + R = 155 \times 10^{-6} + 260 \times 10^{-6}$$

$\varepsilon_{\max} = 415 \times 10^{-6}$ mm/mm ◀

PROBLEM 7.146



The rosette shown has been used to determine the following strains at a point on the surface of a crane hook:

$$\varepsilon_1 = +420\mu \quad \varepsilon_2 = -45\mu \quad \varepsilon_4 = +165\mu$$

(*a*) What should be the reading of gage 3? (*b*) Determine the principal strains and the maximum in-plane shearing strain.

SOLUTION

(*a*) Gages 2 and 4 are 90° apart

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_2 + \varepsilon_4)$$

$$\varepsilon_{\text{ave}} = \frac{1}{2}(-45\mu + 165\mu) = 60\mu$$

Gages 1 and 3 are also 90° apart

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

$$\varepsilon_3 = 2\varepsilon_{\text{ave}} - \varepsilon_1 = (2)(60\mu) - 420\mu = -300\mu \quad \blacktriangleleft$$

(*b*) $\varepsilon_x = \varepsilon_1 = 420\mu \quad \varepsilon_y = \varepsilon_3 = -300\mu$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_x - \varepsilon_1 = (2)(-45\mu) - 420\mu + 300\mu$$

$$= -210\mu$$

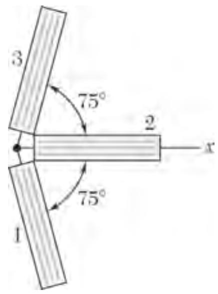
$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{420\mu + 300\mu}{2}\right)^2 + \left(\frac{-210\mu}{2}\right)^2}$$

$$= 375\mu$$

$$\varepsilon_a = \varepsilon_{\text{ave}} + R = 60\mu + 375\mu = 435\mu \quad \blacktriangleleft$$

$$\varepsilon_b = \varepsilon_{\text{ave}} - R = 60\mu - 375\mu = -315\mu \quad \blacktriangleleft$$

$$\gamma_{\text{max (in-plane)}} = 2R = 750\mu \quad \blacktriangleleft$$



PROBLEM 7.147

The strains determined by the use of a rosette attached as shown to the surface of a machine element are

$$\varepsilon_1 = -93.1\times10^{-6}\text{ mm/mm} \quad \varepsilon_2 = +385\times10^{-6}\text{ mm/mm}$$
$$\varepsilon_3 = +210\times10^{-6}\text{ mm/mm}$$

Determine (*a*) the orientation and magnitude of the principal strains in the plane of the rosette, (*b*) the maximum in-plane shearing stress.

SOLUTION

Use $\varepsilon_{x'} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$

where $\theta = -75°$ for gage 1,

$\theta = 0$ for gage 2,

and $\theta = +75°$ for gage 3.

$$\varepsilon_1 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(-150°) + \frac{\gamma_{xy}}{2}\sin(-150°) \tag{1}$$

$$\varepsilon_2 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 0 + \frac{\gamma_{xy}}{2}\sin 0 \tag{2}$$

$$\varepsilon_3 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(150°) + \frac{\gamma_{xy}}{2}\sin(150°) \tag{3}$$

From Eq. (2), $\varepsilon_x = \varepsilon_z = 385\times10^{-6}$ mm/mm

Adding Eq's. (1) and (3),

$$\begin{aligned}\varepsilon_1 + \varepsilon_3 &= (\varepsilon_x + \varepsilon_y) + (\varepsilon_x - \varepsilon_y)\cos 150° \\ &= \varepsilon_x(1 + \cos 150°) + \varepsilon_y(1 - \cos 150°)\end{aligned}$$

$$\begin{aligned}\varepsilon_y &= \frac{\varepsilon_1 + \varepsilon_3 - \varepsilon_x(1 + \cos 150°)}{(1 - \cos 150°)} \\ &= \frac{-93.1\times10^{-6} + 210\times10^{-6} - 385\times10^{-6}(1 + \cos 150°)}{1 - \cos 150°} \\ &= 35.0\times10^{-6}\text{ mm/mm}\end{aligned}$$

PROBLEM 7.147 *(Continued)*

Subtracting Eq. (1) from Eq. (3),

$$\varepsilon_3 - \varepsilon_1 = \gamma_{xy} \sin 150°$$

$$\gamma_{xy} = \frac{\varepsilon_3 - \varepsilon_1}{\sin 150°} = \frac{210\times10^{-6} - (-93.1\times10^{-6})}{\sin 150°} = 606.2\times10^{-6} \text{ mm/mm}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{606.2\times10^{-6}}{385\times10^{-6} - 35.0\times10^{-6}} = 1.732$$

(*a*) $\theta_a = 30.0°, \theta_b = 120.0°$ ◀

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = \frac{1}{2}(385\times10^{-6} + 35.0\times10^{-6})$$

$$= 210\times10^{-6} \text{ mm/mm}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

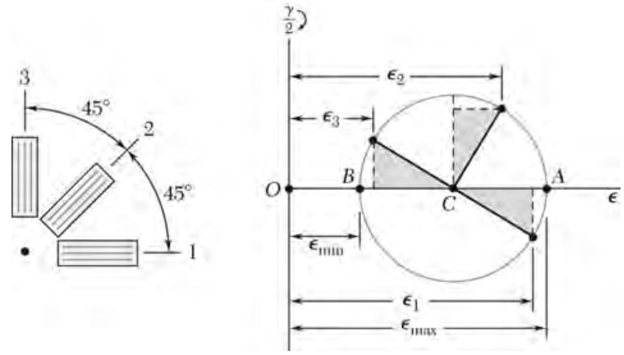
$$= \sqrt{\left(\frac{385\times10^{-6} - 35.0\times10^{-6}}{2}\right)^2 + \left(\frac{606.2}{2}\right)^2} = 350.0\times10^{-6}$$

$$\varepsilon_a = \varepsilon_{\text{ave}} + R = 210\times10^{-6} + 350.0\times10^{-6} = 560\times10^{-6} \text{ mm/mm}$$ ◀

$$\varepsilon_b = \varepsilon_{\text{ave}} - R = 210\times10^{-6} - 350.0\times10^{-6} = -140.0\times10^{-6} \text{ mm/mm}$$ ◀

(*b*) $\dfrac{\gamma_{\text{max (in-plane)}}}{2} = R = 350.0\times10^{-6}$ mm/mm

$\gamma_{\text{max (in-plane)}} = 700\times10^{-6}$ mm/mm ◀



PROBLEM 7.148

Using a 45° rosette, the strains ε_1, ε_2, and ε_3 have been determined at a given point. Using Mohr's circle, show that the principal strains are:

$$\varepsilon_{\max,\min} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3) \pm \frac{1}{\sqrt{2}}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{\frac{1}{2}}$$

(*Hint:* The shaded triangles are congruent.)

SOLUTION

Since gage directions 1 and 3 are 90° apart,

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

Let $\quad \text{u} = \varepsilon_1 - \varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_1 - \varepsilon_3).$

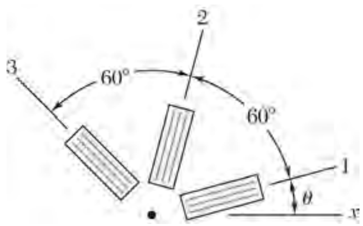
$$\text{v} = \varepsilon_2 - \varepsilon_{\text{ave}} = \varepsilon_2 - \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

$$\begin{aligned}
R^2 &= \text{u}^2 + \text{v}^2 \\
&= \frac{1}{4}(\varepsilon_1 - \varepsilon_3)^2 + \varepsilon_2^2 - \varepsilon_2(\varepsilon_1 + \varepsilon_3) + \frac{1}{4}(\varepsilon_1 + \varepsilon_3)^2 \\
&= \frac{1}{4}\varepsilon_1^2 - \cancel{\frac{1}{2}\varepsilon_1\varepsilon_3} + \frac{1}{4}\varepsilon_3^2 + \varepsilon_2^2 - \varepsilon_2\varepsilon_1 - \varepsilon_2\varepsilon_3 + \frac{1}{4}\varepsilon_1^2 + \cancel{\frac{1}{2}\varepsilon_1\varepsilon_3} + \frac{1}{4}\varepsilon_3^2 \\
&= \frac{1}{2}\varepsilon_1^2 - \varepsilon_2\varepsilon_1 + \varepsilon_2^2 - \varepsilon_2\varepsilon_3 + \frac{1}{2}\varepsilon_3^2 \\
&= \frac{1}{2}(\varepsilon_1 - \varepsilon_2)^2 + \frac{1}{2}(\varepsilon_2 - \varepsilon_3)^2
\end{aligned}$$

$$R = \frac{1}{\sqrt{2}}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2}$$

$$\varepsilon_{\max,\,\min} = \varepsilon_{\text{ave}} \pm R$$

gives the required formula.



PROBLEM 7.149

Show that the sum of the three strain measurements made with a 60° rosette is independent of the orientation of the rosette and equal to

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{\text{avg}}$$

where ε_{avg} is the abscissa of the center of the corresponding Mohr's circle.

SOLUTION

$$\varepsilon_1 = \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (1)$$

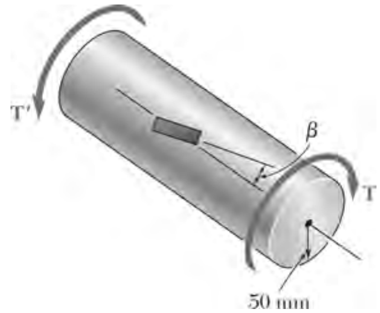
$$\begin{aligned}
\varepsilon_2 &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 120°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 120°) \\
&= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 120°\cos 2\theta - \sin 120°\sin 2\theta) \\
&\quad + \frac{\gamma_{xy}}{2}(\cos 120°\sin 2\theta + \sin 120°\cos 2\theta) \\
&= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta\right) \\
&\quad + \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta + \frac{\sqrt{3}}{2}\cos 2\theta\right) \qquad (2)
\end{aligned}$$

$$\begin{aligned}
\varepsilon_3 &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 240°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 240°) \\
&= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 240°\cos 2\theta - \sin 240°\sin 2\theta) \\
&\quad + \frac{\gamma_{xy}}{2}(\cos 240°\sin 2\theta + \sin 240°\cos 2\theta) \\
&= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta + \frac{\sqrt{3}}{2}\sin 2\theta\right) \\
&\quad + \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta - \frac{\sqrt{3}}{2}\cos 2\theta\right) \qquad (3)
\end{aligned}$$

Adding (1), (2), and (3),

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{\text{ave}} + 0 + 0$$

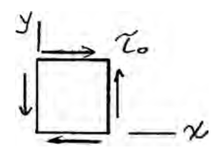
$$3\varepsilon_{\text{ave}} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad ◀$$



PROBLEM 7.150

A single gage is cemented to a solid 100-mm-diameter steel shaft at an angle $\beta = 25°$ with a line parallel to the axis of the shaft. Knowing that $G = 79$ GPa, determine the torque **T** indicated by a gage reading of 300×10^{-6} mm/mm.

SOLUTION



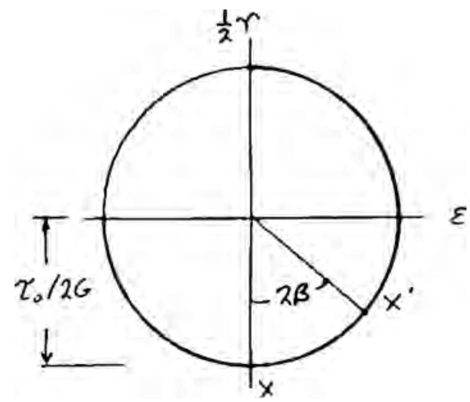
For torsion, $\sigma_x = \sigma_y = 0, \quad \tau = \tau_0$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y) = 0$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x) = 0$$

$$\gamma_{xy} = \frac{\tau_0}{G} \quad \frac{1}{2}\gamma_{xy} = \frac{\tau_0}{2G}$$

Draw the Mohr's circle for strain



$$R = \frac{\tau_0}{2G}$$

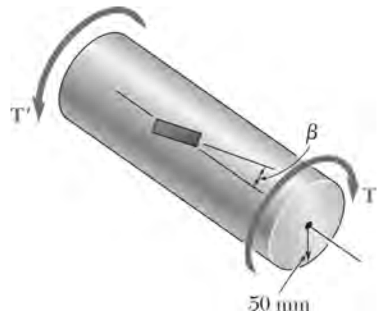
$$\varepsilon_{x'} = R \sin 2\beta = \frac{\tau_0}{2G} \sin 2\beta$$

But $\tau_0 = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3} = \dfrac{2G\,\varepsilon_{x'}}{\sin 2\beta}$

$$T = \frac{\pi c^3 G \varepsilon_{x'}}{\sin 2\beta}$$

$$= \frac{\pi (0.05)^3 (79 \times 10^9)(300 \times 10^{-6})}{\sin 50°}$$

$$= 12.2 \text{ kNm}$$ ◀



PROBLEM 7.151

Solve Prob. 7.150, assuming that the gage forms an angle $\beta = 35°$ with a line parallel to the axis of the shaft.

PROBLEM 7.150 A single gage is cemented to a solid 100-mm-diameter steel shaft at an angle $\beta = 25°$ with a line parallel to the axis of the shaft. Knowing that $G = 79$ GPa, determine the torque **T** indicated by a gage reading of 300×10^{-6} mm/mm.

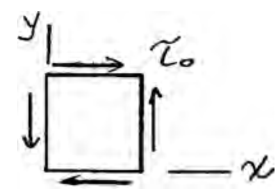
SOLUTION

For torsion, $\quad \sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = \tau_0$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y) = 0$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x) = 0$$

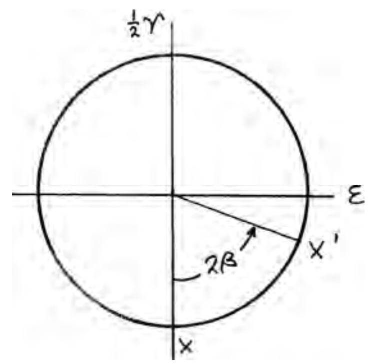
$$\gamma_{xy} = \frac{\tau_0}{G} \quad \frac{1}{2}\gamma_{xy} = \frac{\tau_0}{2G}$$



Draw Mohr's circle for strain

$$R = \frac{\tau_0}{2G}$$

$$\varepsilon_{x'} = R \sin 2\beta = \frac{\tau_0}{2G}\sin 2\beta$$

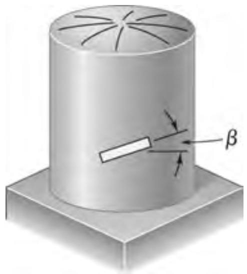


But

$$\tau_0 = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2G\varepsilon_{x'}}{\sin 2\beta}$$

$$T = \frac{\pi c^3 G\varepsilon_{x'}}{\sin 2\beta} = \frac{\pi(0.05)^3(79\times 10^9)(300\times 10^{-6})}{\sin 70°}$$

$$= 9.9 \text{ kNm} \quad ◀$$



PROBLEM 7.152

A single strain gage forming an angle $\beta = 18°$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200$ GPa and $\nu = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280μ.

SOLUTION

$$\sigma_x = \sigma_1 = \frac{pr}{t}$$

$$\sigma_y = \frac{1}{2}\sigma_x, \quad \sigma_z \approx 0$$

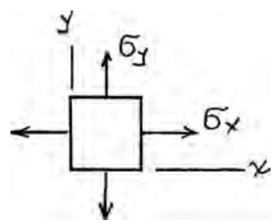
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \left(1 - \frac{\nu}{2}\right)\frac{\sigma_x}{E}$$

$$= 0.85\frac{\sigma_x}{E}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \left(\frac{1}{2} - \nu\right)\frac{\sigma_x}{E}$$

$$= 0.20\frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



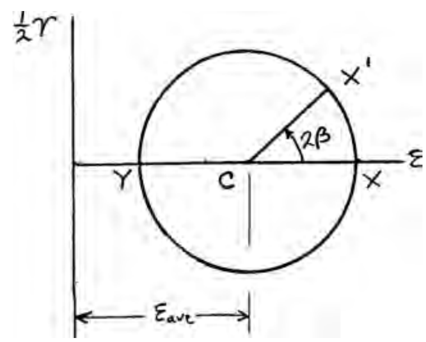
Draw Mohr's circle for strain.

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\frac{\sigma_x}{E}$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos 2\beta = (0.525 + 0.325\cos 2\beta)\frac{\sigma_x}{E}$$

$$p = \frac{t\sigma_x}{r} = \frac{tE\varepsilon_{x'}}{r(0.525 + 0.325\cos 2\beta)}$$

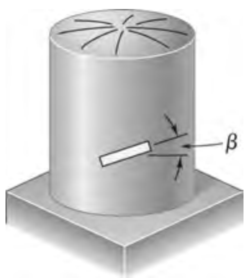


<u>Data</u>: $r = \frac{1}{2}d = \frac{1}{2}(600) = 300 \text{ mm} = 0.300 \text{ m}$

$$t = 6\times10^{-3}\text{mm} \quad E = 200\times10^9\,\text{Pa}, \quad \varepsilon_{x'} = 280\times10^{-6} \quad \beta = 18°$$

$$p = \frac{(6\times10^{-3})(200\times10^9)(280\times10^{-6})}{(0.300)(0.525 + 0.325\cos 36°)} = 1.421\times10^6\,\text{Pa}$$

$p = 1.421$ MPa ◀



PROBLEM 7.153

Solve Prob. 7.152, assuming that the gage forms an angle $\beta = 35°$ with a horizontal plane.

PROBLEM 7.152 A single strain gage forming an angle $\beta = 18°$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200\,\text{GPa}$ and $v = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280μ.

SOLUTION

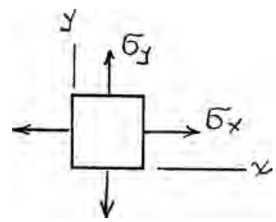
$$\sigma_x = \sigma_1 = \frac{pr}{t}$$

$$\sigma_y = \frac{1}{2}\sigma_x, \quad \sigma_z \approx 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = \left(1 - \frac{v}{2}\right)\frac{\sigma_x}{E} = 0.85\frac{\sigma_x}{E}$$

$$\varepsilon_y = \frac{1}{E}(-v\sigma_x + \sigma_y - v\sigma_z) = \left(\frac{1}{2} - v\right)\frac{\sigma_x}{E} = 0.20\frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



Draw Mohr's circle for strain.

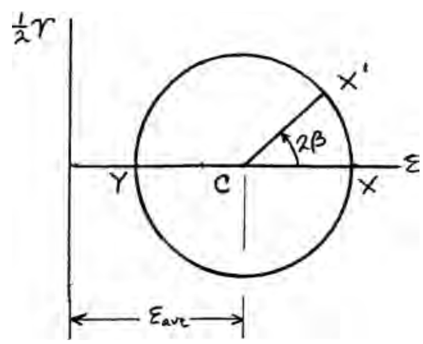
$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\frac{\sigma_x}{E}$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos 2\beta$$

$$= (0.525 + 0.325\cos 2\beta)\frac{\sigma_x}{E}$$

$$p = \frac{t\sigma_x}{r} = \frac{tE\varepsilon_{x'}}{r(0.525 + 0.325\cos 2\beta)}$$



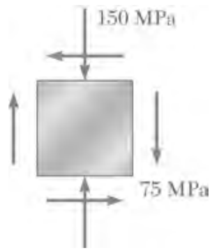
Data: $r = \frac{1}{2}d = \frac{1}{2}(600) = 300\text{ mm} = 0.300\text{ m}$

$$t = 6\times10^{-3}\,\text{m} \quad E = 200\times10^9\,\text{Pa}, \quad \varepsilon_{x'} = 280\times10^{-6} \quad \beta = 35°$$

$$p = \frac{(6\times10^{-3})(200\times10^9)(280\times10^{-6})}{(0.300)(0.525 + 0.325\cos 70°)} = 1.761\times10^6\text{ Pa}$$

$p = 1.761\text{ MPa}$ ◀

PROBLEM 7.154



The given state of plane stress is known to exist on the surface of a machine component. Knowing that $E = 200\text{ GPa}$ and $G = 77.2\text{ GPa}$, determine the direction and magnitude of the three principal strains (*a*) by determining the corresponding state of strain [use Eq. (2.43) and Eq. (2.38)] and then using Mohr's circle for strain, (*b*) by using Mohr's circle for stress to determine the principal planes and principal stresses and then determining the corresponding strains.

SOLUTION

(*a*) $\sigma_x = 0, \quad \sigma_y = -150\times10^6\text{ Pa}, \quad \tau_{xy} = -75\times10^6\text{ Pa}$

$$E = 200\times10^9\text{ Pa} \quad G = 77\times10^9\text{ Pa}$$

$$G = \frac{E}{2(1+v)} \quad v = \frac{E}{2G} - 1 = 0.2987$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y) = \frac{1}{200\times10^9}[0 + (0.2987)(150\times10^6)]$$
$$= 224\mu$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x) = \frac{1}{200\times10^9}[(-150\times10^6) - 0]$$
$$= -750\mu$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-75\times10^6}{77\times10^9} = -974\mu$$

$$\frac{\gamma_{xy}}{2} = -487.0\mu$$

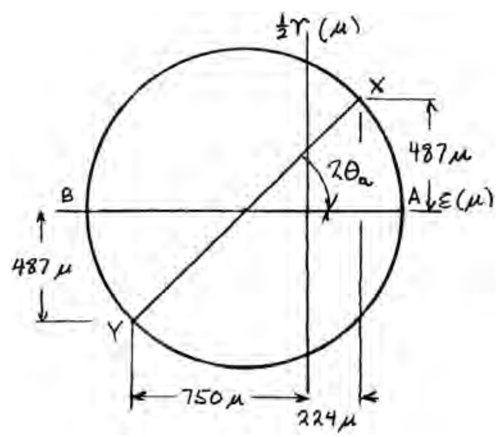
$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = -263\mu$$

$$\varepsilon_x - \varepsilon_y = 974\mu$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-974}{974} = -1.000$$

$$2\theta_a = -45.0°$$

$\theta_a = -22.5°$ ◀



$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 689\mu$$

$$\varepsilon_a = \varepsilon_{\text{ave}} + R$$

$\varepsilon_a = 426\mu$ ◀

$$\varepsilon_b = \varepsilon_{\text{ave}} - R$$

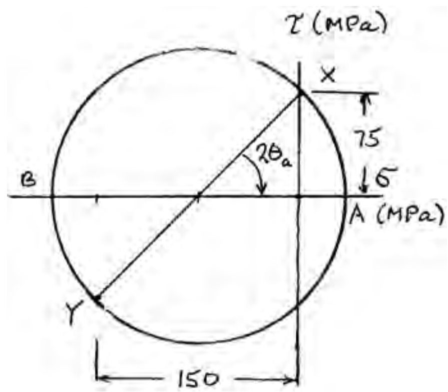
$\varepsilon_b = -952\mu$ ◀

$$\varepsilon_c = -\frac{v}{E}(\sigma_x + \sigma_y) = -\frac{(0.2987)(0 - 150\times10^6)}{200\times10^9}$$

$\varepsilon_c = -224\mu$ ◀

PROBLEM 7.154 *(Continued)*

(*b*) $$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -75 \text{ MPa}$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0+150}{2}\right)^2 + 75^2}$$

$$= 106.07 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 31.07 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -181.07 \text{ MPa}$$

$$\varepsilon_a = \frac{1}{E}(\sigma_a - v\sigma_b)$$

$$= \frac{1}{200\times 10^9}[31.07\times 10^6 - (0.2987)(-181.07\times 10^6)]$$

$$= 426\times 10^{-6}$$

$\varepsilon_a = 426\mu$ ◀

$$\tan 2\theta_a = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.000$$

$2\theta_a = -45°$

$\theta_a = -22.5°$ ◀

PROBLEM 7.155

The following state of strain has been determined on the surface of a cast-iron machine part:

$$\varepsilon_x = -720\mu \quad \varepsilon_y = -400\mu \quad \gamma_{xy} = +660\mu$$

Knowing that $E = 69\,\text{GPa}$ and $G = 28\,\text{GPa}$, determine the principal planes and principal stresses (*a*) by determining the corresponding state of plane stress [use Eq. (2.36), Eq. (2.43), and the first two equations of Prob. 2.72] and then using Mohr's circle for stress, (*b*) by using Mohr's circle for strain to determine the orientation and magnitude of the principal strains and then determining the corresponding stresses.

SOLUTION

The 3rd principal stress is $\sigma_z = 0.$

$$G = \frac{E}{2(1+v)} \quad v = \frac{E}{2G} - 1 = \frac{69}{56} - 1 = 0.2321$$

$$\frac{E}{1-v^2} = \frac{69}{1-(0.232)^2} = 72.93 \text{ GPa}$$

(*a*) $$\sigma_x = \frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_y)$$

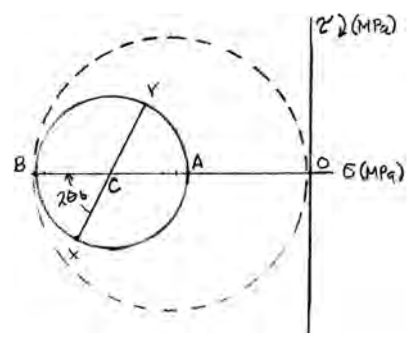
$$= (72.93\times10^9)[-720\times10^{-6} + (0.232)(-400\times10^{-6})]$$

$$= -59.28 \text{ MPa}$$

$$\sigma_y = \frac{E}{1-v^2}(\varepsilon_y + v\varepsilon_x)$$

$$= (72.93\times10^9)[-400\times10^{-6} + (0.2321)(-720\times10^{-6})]$$

$$= -41.36 \text{ MPa}$$



$$\tau_{xy} = G\gamma_{xy} = (28\times10^9)(660\times10^{-6})$$

$$= 18.48 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -50.32 \text{ MPa}$$

$$\tan 2\theta_b = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -2.0625$$

$$2\theta_b = -64.1°, \quad \theta_b = -32.1°, \quad \theta_a = 57.9°$$

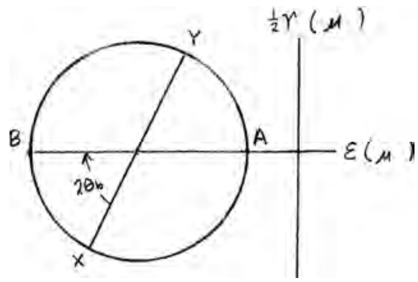
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 20.54 \text{ MPa}$$

$\sigma_a = \sigma_{\text{ave}} + R$ $\qquad\qquad$ $\sigma_a = -29.8 \text{ MPa}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R$ $\qquad\qquad$ $\sigma_b = -70.9 \text{ MPa}$ ◀

PROBLEM 7.155 *(Continued)*

(*b*) $\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = -560\mu$



$$\tan 2\theta_b = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -2.0625$$

$$2\theta_b = -64.1°, \quad \theta_b = -32.1°, \quad \theta_a = 57.9°$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 366.74\mu$$

$$\varepsilon_a = \varepsilon_{\text{ave}} + R = -193.26\mu$$

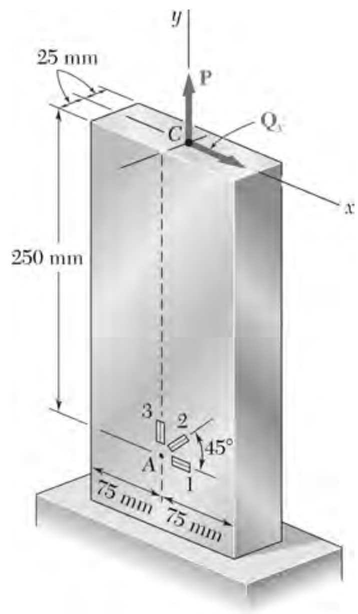
$$\varepsilon_b = \varepsilon_{\text{ave}} - R = -926.74\mu$$

$$\sigma_a = \frac{E}{1-v^2}(\varepsilon_a + v\varepsilon_b)$$

$\sigma_a = -29.8$ MPa ◀

$$\sigma_b = \frac{E}{1-v^2}(\varepsilon_b + v\varepsilon_a)$$

$\sigma_b = -70.9$ MPa ◀



PROBLEM 7.156

A centric axial force **P** and a horizontal force **Q** are both applied at point *C* of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point *A* indicates the following strains:

$$\varepsilon_1 = -75\times10^{-6}\ \text{mm/mm} \quad \varepsilon_2 = +300\times10^{-6}\ \text{mm/mm}$$
$$\varepsilon_3 = +250\times10^{-6}\ \text{mm/mm}$$

Knowing that $E = 200$ GPa and $v = 0.30$, determine the magnitudes of **P** and **Q**.

SOLUTION

$$\varepsilon_x = \varepsilon_1 = -75\times10^{-6} \quad \varepsilon_y = \varepsilon_3 = 250\times10^{-6}$$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 425\times10^{-6}$$

$$\sigma_x = \frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_y) = \frac{200\times10^9}{1-0.3^2}[-75+(0.3)(250)](10^{-6}) = 0$$

$$\sigma_y = \frac{E}{1-v^2}(\varepsilon_y + v\varepsilon_x) = \frac{200\times10^9}{1-(0.3)^2}[250+(0.3)(-75)](10^{-6}) = 50\ \text{MPa}$$

$$\frac{P}{A} = \sigma_y \quad P = A\sigma_y = (50)(150)(50^3)$$

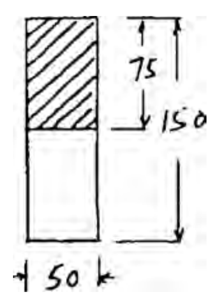
$$= 375\ \text{kN} \quad ◀$$

$$G = \frac{E}{2(1+v)} = \frac{200\times10^9}{(2)(1.3)} = 76.92\ \text{GPa}$$

$$\tau_{xy} = G\gamma_{xy} = (76.92\times10^9)(425)(10^{-6}) = 32.69\ \text{MPa}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(50)(150)^3 = 14062500\ \text{mm}^4$$

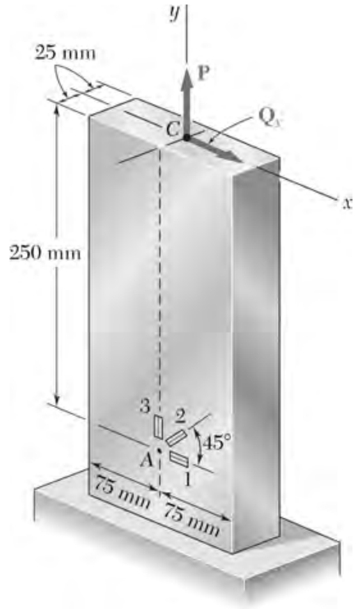
$$Q = A\bar{y} = (75)(50)\left(\frac{75}{2}\right) = 140625\ \text{mm}^3 \qquad t = 50\ \text{mm}$$



$$\tau_{xy} = \frac{VQ}{It}$$

$$V = \frac{It\tau_{xy}}{Q} = \frac{(14062500)(50)(32.69)}{140625} = 163.45\,\text{kN}$$

$$Q_x = V = 163.5\ \text{kN} \quad ◀$$



PROBLEM 7.157

Solve Prob. 7.156, assuming that the rosette at point A indicates the following strains:

$$\varepsilon_1 = -60\times 10^{-6}\ \text{mm/mm} \quad \varepsilon_2 = +410\times 10^{-6}\ \text{mm/mm}$$
$$\varepsilon_3 = +200\times 10^{-6}\ \text{mm/mm}$$

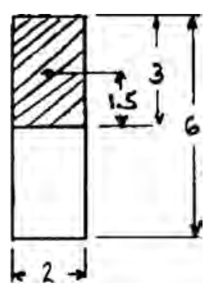
PROBLEM 7.156 A centric axial force **P** and a horizontal force **Q** are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:

$$\varepsilon_1 = -75\times 10^{-6}\ \text{mm/mm} \quad \varepsilon_2 = +310\times 10^{-6}\ \text{mm/mm}$$
$$\varepsilon_3 = +250\times 10^{-6}\ \text{mm/mm}$$

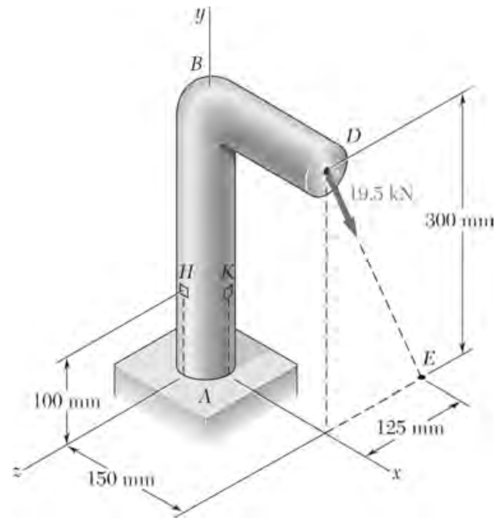
Knowing that $E = 200$ GPa and $\nu = 0.30$, determine the magnitudes of **P** and **Q**.

SOLUTION

$$\varepsilon_x = \varepsilon_1 = -60\times 10^{-6}$$
$$\varepsilon_y = \varepsilon_3 = 200\times 10^{-6}$$
$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 680\times 10^{-6}$$
$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{200\times 10^9}{1-(0.3)^2}[-60+(0.3)(200)](10^{-6}) = 0$$
$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{200\times 10^9}{1-(0.3)^2}[200+(0.3)(-60)](10^{-6}) = 40\ \text{MPa}$$
$$\frac{P}{A} = \sigma_y \quad P = A\sigma_y = (50)(150)(40) = 300\ \text{kN} \quad \blacktriangleleft$$
$$G = \frac{E}{2(1+\nu)} = \frac{200\times 10^9}{(2)(1.3)} = 76.92\ \text{GPa}$$
$$\tau_{xy} = G\gamma_{xy} = (76.92\times 10^9)(680)(10^{-6}) = 52.306\ \text{MPa}$$
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(50)(150)^3 = 14062500\ \text{mm}^4$$
$$Q = A\bar{y} = (75)(50)\left(\frac{75}{2}\right) = 140625\ \text{mm}^3 \quad t = 50\ \text{mm}$$



$$\tau_{xy} = \frac{VQ}{It}$$
$$V = \frac{It\,\tau_{xy}}{Q} = \frac{(14062500)(50)(52.306)}{140625} = 261.53\ \text{kN}$$
$$Q_x = V = 261.5\ \text{kN} \quad \blacktriangleleft$$



PROBLEM 7.158

A 19.5-kN force is applied at point D of the cast-iron post shown. Knowing that the post has a diameter of 60 mm, determine the principal stresses and the maximum shearing stress at point H.

SOLUTION

$$l_{DE} = \sqrt{(300)^2 + (125)^2} = 325 \text{ mm}$$

Resolve the 19.5 kN force F at point D into x, y, and z components.

$$F_x = 0$$

$$F_y = -\frac{300}{325}(19.5) = -18 \text{ kN} = -18\times 10^3 \text{ N}$$

$$F_z = -\frac{125}{325}(19.5) = -7.5 \text{ kN} = -7.5\times 10^3 \text{ N}$$

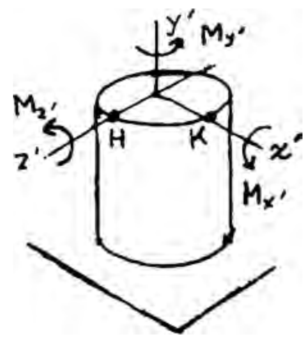
Determine the force-couple system at the point on the y-axis where it intersects the plane containing elements H and K.

$$F_{x'} = 0, \quad F_{y'} = -18\times 10^3 \text{ N}, \quad F_{z'} = -7.5\times 10^3 \text{ N}$$

$$M_{x'} = -(7.5 \text{ kN})(300 \text{ mm} - 100 \text{ mm}) = -1.5\times 10^3 \text{ N}\cdot\text{m}$$

$$M_{y'} = (7.5 \text{ kN})(150 \text{ mm}) = 1.125\times 10^3 \text{ N}\cdot\text{m}$$

$$M_{z'} = -(18 \text{ kN})(150 \text{ mm}) = -2.7\times 10^3 \text{ N}\cdot\text{m}$$



PROBLEM 7.158 *(Continued)*

Properties of section. (Circle) $c = \frac{1}{2}d = 30 \text{ mm} = 30\times10^{-3} \text{ m}$

$$A = \pi c^2 = \pi(30)^2 = 2.8274\times10^3 \text{ mm}^2 = 2.8274\times10^{-3} \text{ m}^4$$

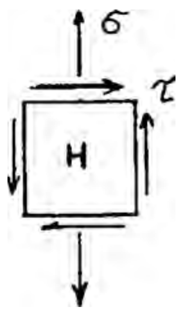
$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(30)^4 = 636.17\times10^3 \text{ mm}^4 = 636.17\times10^{-9} \text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30)^4 = 1.27235\times10^6 \text{ mm}^4 = 1.27235\times10^{-6} \text{ m}^4$$

(Semicircle) $Q = \frac{2}{3}c^3 = \frac{2}{3}(30)^3 = 18\times10^3 \text{ mm}^3 = 18\times10^{-6} \text{ m}^3$

$$t = d = 60 \text{ mm} = 60\times10^{-3} \text{ m}$$

Stresses at H.



$$\sigma = \frac{Fy}{A} - \frac{M_x c}{I} = \frac{-18\times10^3}{2.8274\times10^{-3}} - \frac{(-1.5\times10^3)(30\times10^{-3})}{636.17\times10^{-9}}$$

$$= 64.370\times10^6 \text{ Pa} = 64.37 \text{ MPa}$$

$$\tau = \frac{F_x Q}{It} + \frac{M_y c}{j} = 0 + \frac{(1.125\times10^3)(30\times10^{-3})}{1.27235\times10^{-6}}$$

$$= 26.526\times10^6 \text{ Pa} = 26.526 \text{ MPa}$$

Principal stresses. $\sigma_{\text{max,min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}.$

$$\sigma_{\text{max,min}} = \frac{0 + 64.37}{2} + \sqrt{\left(\frac{0 - 64.37}{2}\right)^2 + (26.526)^2}$$

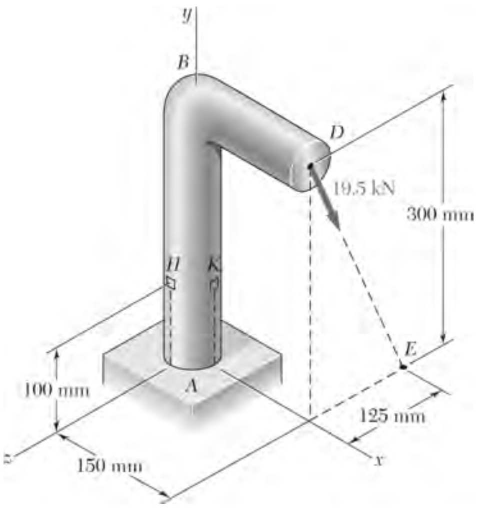
$$= -32.185 \pm 41.707 \text{ MPa}$$

$\sigma_{\text{max}} = 73.9 \text{ MPa}$

$\sigma_{\text{min}} = -9.52 \text{ MPa}$ ◀

Maximum shearing stresses. $\tau_{\text{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\tau_{\text{max}} = 41.7 \text{ MPa}$ ◀



PROBLEM 7.159

A 19.5-kN force is applied at point D of the cast-iron post shown. Knowing that the post has a diameter of 60 mm, determine the principal stresses and the maximum shearing stress at point K.

SOLUTION

$$l_{DE} = \sqrt{(300)^2 + (125)^2} = 325 \text{ mm}$$

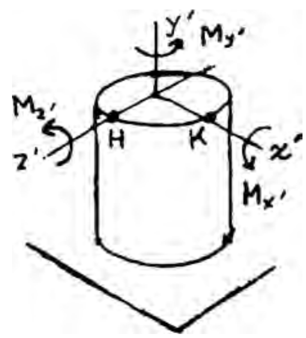
Resolve the 19.5 kN force F at point D into x, y, and z components.

$$F_x = 0$$

$$F_y = -\frac{300}{325}(19.5) = -18 \text{ kN} = -18\times10^3 \text{ N}$$

$$F_z = -\frac{125}{325}(19.5) = -7.5 \text{ kN} = -7.5\times10^3 \text{ N}$$

Determine the force-couple system at the point on the y-axis where it intersects the plane containing elements H and K.

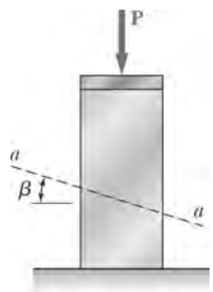


$$F_{x'} = 0, \quad F_{y'} = -18\times10^3 \text{ N}, \quad F_{z'} = -7.5\times10^3 \text{ N}$$

$$M_{x'} = -(7.5 \text{ kN})(300 \text{ mm} - 100 \text{ mm}) = -1.5\times10^3 \text{ N}\cdot\text{m}$$

$$M_{y'} = (7.5 \text{ kN})(150 \text{ mm}) = 1.125\times10^3 \text{ N}\cdot\text{m}$$

$$M_{z'} = -(18 \text{ kN})(150 \text{ mm}) = -2.7\times10^3 \text{ N}\cdot\text{m}$$

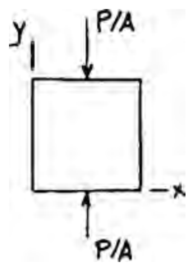


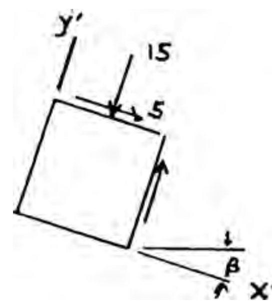
PROBLEM 7.160

The centric force **P** is applied to a short post as shown. Knowing that the stresses on plane *a-a* are $\sigma = -105$ MPa and $\tau = 35$ MPa, determine (*a*) the angle β that plane *a-a* forms with the horizontal, (*b*) the maximum compressive stress in the post.

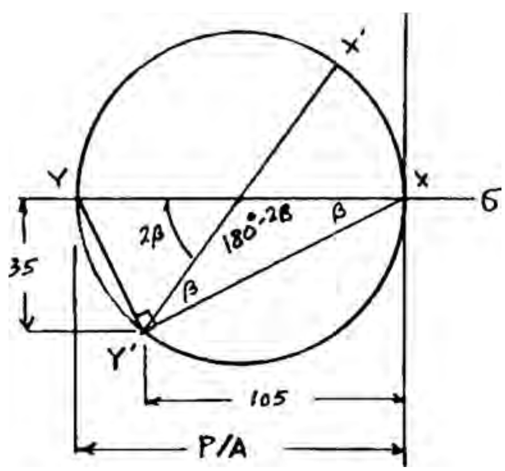
SOLUTION

$$\sigma_x = 0$$
$$\tau_{xy} = 0$$
$$\sigma_y = -P/A$$





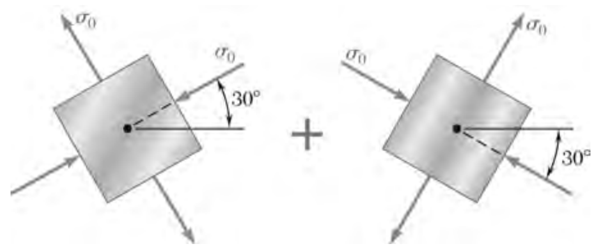
(*a*) From the Mohr's circle,



$$\tan\beta = \frac{35}{105} = 0.3333 \qquad \beta = 18.4° \blacktriangleleft$$

$$-\sigma = \frac{P}{2A} + \frac{P}{2A}\cos 2\beta$$

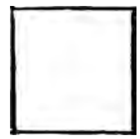
(*b*) $$\frac{P}{A} = \frac{2(-\sigma)}{1+\cos 2\beta} = \frac{(2)(105)}{1+\cos 2\beta} \qquad \frac{P}{A} = 116.6 \text{ MPa} \blacktriangleleft$$



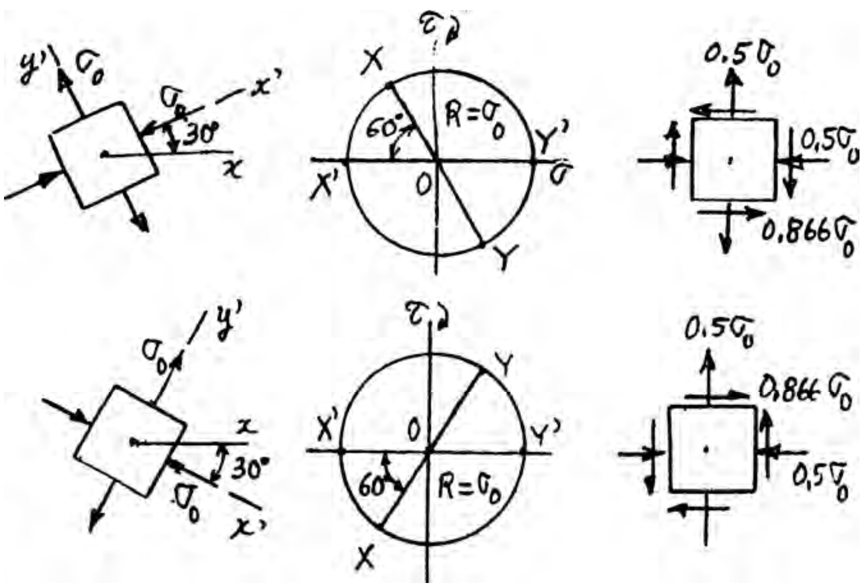
PROBLEM 7.161

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

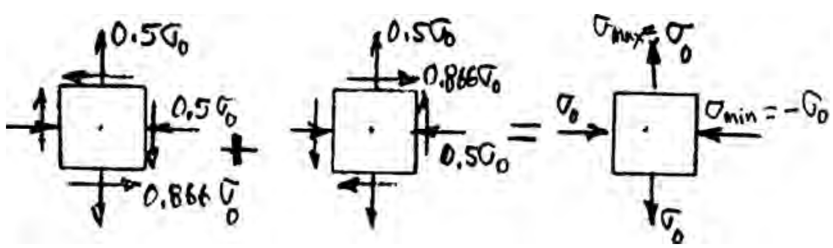
SOLUTION



Express each state of stress in terms of components acting on the element shown above.



Add like components of the two states of stress.

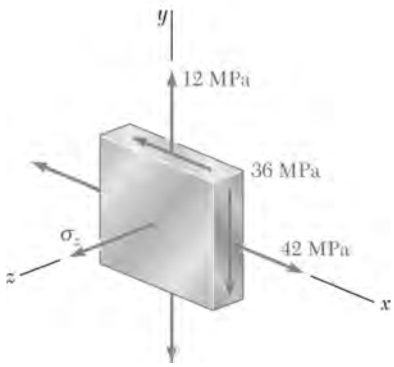


$\theta_p = 0$ and $90°$ ◀

$\sigma_{max} = \sigma_0$ ◀

$\sigma_{min} = -\sigma_0$ ◀

PROBLEM 7.162



For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = +24$ MPa, (*b*) $\sigma_z = -24$ MPa, (*c*) $\sigma_z = 0$.

SOLUTION

$$\sigma_x = 42 \text{ MPa}, \quad \sigma_y = 12 \text{ MPa}, \quad \tau_{xy} = -36 \text{ MPa}$$

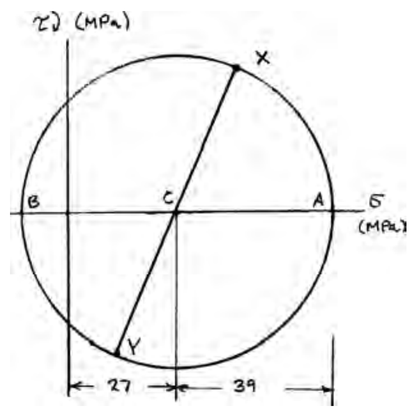
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 27 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(15)^2 + (-36)^2} = 39 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 66 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -12 \text{ MPa}$$



(*a*) $\underline{\sigma_z = +24 \text{ MPa}}$ $\quad \sigma_a = 66$ MPa $\quad \sigma_b = -12$ MPa

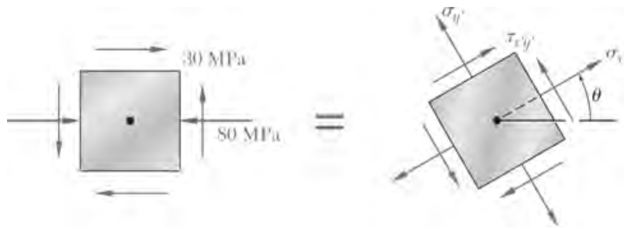
$\sigma_{\max} = 66$ MPa $\quad \sigma_{\min} = -12$ MPa $\quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 39$ MPa ◀

(*b*) $\underline{\sigma_z = -24 \text{ MPa}}$ $\quad \sigma_a = 66$ MPa $\quad \sigma_b = -12$ MPa

$\sigma_{\max} = 66$ MPa $\quad \sigma_{\min} = -24$ MPa $\quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 45$ MPa ◀

(*c*) $\underline{\sigma_z = 0}$ $\quad \sigma_a = 66$ MPa $\quad \sigma_b = -12$ MPa

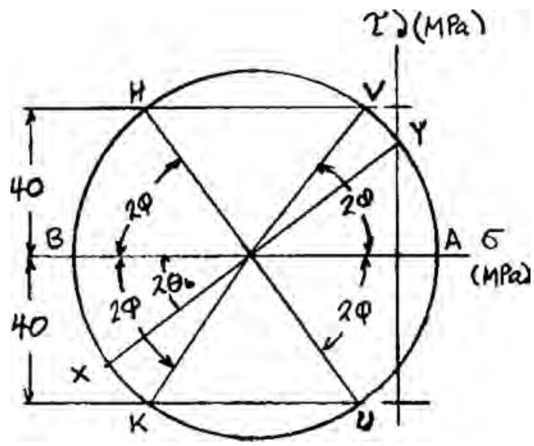
$\sigma_{\max} = 66$ MPa $\quad \sigma_{\min} = -12$ MPa $\quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 39$ MPa ◀



PROBLEM 7.163

For the state of stress shown, determine the range of values of θ for which the magnitude of the shearing stress $\tau_{x'y'}$ is equal to or less than 40 MPa.

SOLUTION



$$\sigma_x = -80 \text{ MPa}, \quad \sigma_y = 0$$

$$\tau_{xy} = 30 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -40 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-40)^2 + 30^2} = 50 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(30)}{-80} = -0.75$$

$$2\theta_p = -36.870°$$

$$\theta_b = -18.435°$$

$|\tau_{x'y'}| \leq 40$ MPa for states of stress corresponding to arcs HBK and UAV of Mohr's circle. The angle φ is calculated from

$$R \sin 2\varphi = 40$$

$$\sin 2\varphi = \frac{40}{50} = 0.8$$

$$2\varphi = 53.130° \qquad \varphi = 26.565°$$

$$\theta_N = \theta_b - \varphi = -18.435° - 26.565° = -45°$$

$$\theta_k = \theta_b + \varphi = -18.435° + 26.565° = 8.13°$$

$$\theta_U = \theta_H + 90° = 45°$$

$$\theta_v = \theta_k + 90° = 98.13°$$

Permissible ranges of θ: $\quad \theta_H \leq \theta \leq \theta_k$

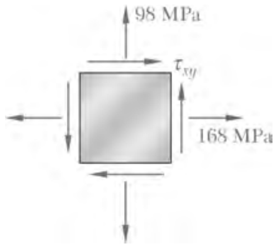
$$-45° \leq \theta \leq 8.13° \quad ◀$$

$$\theta_U \leq \theta \leq \theta_v$$

$$45° \leq \theta \leq 98.13° \quad ◀$$

Also, $\quad 135° \leq \theta \leq 188.13°$

$$225° \leq \theta \leq 278.13°$$



PROBLEM 7.164

The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 210$ MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\tau_{xy} = 42$ MPa, (*b*) $\tau_{xy} = 84$ MPa, (*c*) $\tau_{xy} = 98$ MPa. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_x = 168 \text{ MPa} \quad \sigma_y = 98 \text{ MPa} \quad \sigma_z = 0$$

For stresses in *xy*-plane $$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 133 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = 35 \text{ MPa}$$

(*a*) $\tau_{xy} = 6$ MPa $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (42)^2} = 54.7$ MPa

$\sigma_a = \sigma_{\text{ave}} + R = 187.7$ MPa, $\quad \sigma_b = \sigma_{\text{ave}} - R = 78.3$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 163.3 \text{ MPa} < 210 \text{ MPa}$ (No yielding)

$F.S. = \frac{210}{163.3} = 1.286$ ◀

(*b*) $\tau_{xy} = 84$ MPa $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (84)^2} = 91$ MPa

$\sigma_a = \sigma_{\text{ave}} + R = 224$ MPa, $\quad \sigma_b = \sigma_{\text{ave}} - R = 42$ MPa

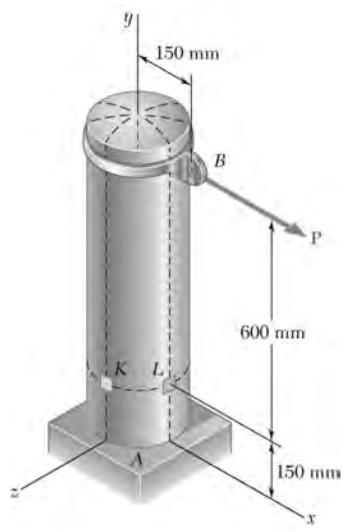
$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 206.2 \text{ MPa} < 210 \text{ MPa}$ (No yielding)

$F.S. = \frac{210}{206.2} = 1.018$ ◀

(*c*) $\tau_{xy} = 98$ MPa $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (98)^2} = 104.1$ MPa

$\sigma_a = \sigma_{\text{ave}} + R = 237.1$ MPa, $\quad \sigma_b = \sigma_{\text{ave}} - R = 28.9$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 224 \text{ MPa} > 210 \text{ MPa}$ (Yielding occurs)

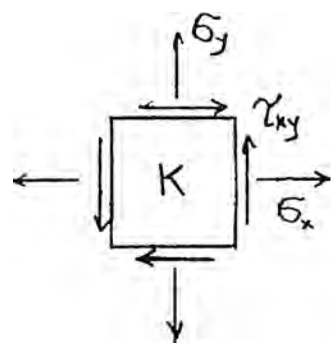


PROBLEM 7.165

The compressed-air tank *AB* has a 250-mm outside diameter and an 8-mm wall thickness. It is fitted with a collar by which a 40-kN force **P** is applied at *B* in the horizontal direction. Knowing that the gage pressure inside the tank is 5 MPa, determine the maximum normal stress and the maximum shearing stress at point *K*.

SOLUTION

Consider element at point *K*.



Stresses due to internal pressure:

$$p = 5\text{ MP} = 5\times 10^6\text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{250}{2} - 8 = 117\text{ mm}$$

$$\sigma_x = \frac{pr}{t} = \frac{(5\times 10^6)(117\times 10^{-3})}{(8\times 10^{-3})} = 73.125\text{ MPa}$$

$$\sigma_y = \frac{pr}{2t} = \frac{(5\times 10^6)(117\times 10^{-3})}{(2)(8\times 10^{-3})} = 36.563\text{ MPa}$$

Stress due to bending moment: Point *K* is on the neutral axis.

$$\sigma_y = 0$$

Stress due to transverse shear:

$$V = P = 40\times 10^3\text{ N}$$

$$c_2 = \frac{1}{2}d = 125\text{ mm}$$

$$c_1 = c_2 - t = 117\text{ mm}$$

$$Q = \frac{2}{3}\left(c_2^3 - c_1^3\right) = \frac{2}{3}(125^3 - 117^3)$$

$$= 234.34\times 10^3\text{ mm}^3 = 234.34\times 10^{-6}\text{ m}^3$$

$$I = \frac{\pi}{4}\left(c_2^4 - c_1^4\right) = \frac{\pi}{4}(125^4 - 117^4)$$

$$= 44.573\times 10^6\text{ mm}^4 = 44.573\times 10^{-6}\text{ m}^4$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{PQ}{I(2t)} = \frac{(40\times 10^3)(234.34\times 10^{-6})}{(44.573\times 10^{-6})(16\times 10^{-3})}$$

$$= 13.144\times 10^6\text{ Pa} = 13.144\text{ MPa}$$

PROBLEM 7.165 *(Continued)*

Total stresses: $\sigma_x = 73.125 \text{ MPa}, \quad \sigma_y = 36.563 \text{ MPa}, \quad \tau_{xy} = 13.144 \text{ MPa}$

Mohr's circle: $\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 54.844 \text{ MPa}$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(18.281)^2 + (13.144)^2} = 22.516 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 77.360 \text{ MPa}$$

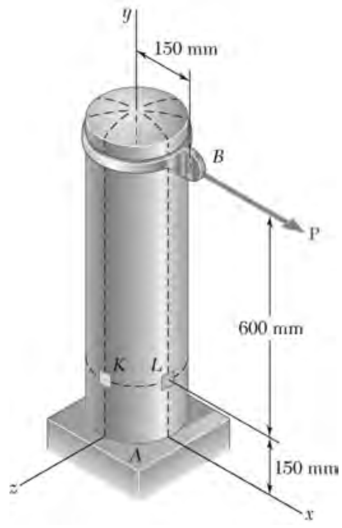
$$\sigma_b = \sigma_{\text{ave}} - R = 32.328 \text{ MPa}$$

Principal stresses: $\sigma_a = 77.4 \text{ MPa}, \quad \sigma_b = 32.3 \text{ MPa}$ ◀

The 3rd principal stress is the radial stress. $\sigma_z \approx 0$ ◀

$$\sigma_{\max} = 77.4 \text{ MPa}, \quad \sigma_{\min} = 0$$

Maximum shearing stress: $\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$ $\tau_{\max} = 38.7 \text{ MPa}$ ◀



PROBLEM 7.166

In Prob. 7.165, determine the maximum normal stress and the maximum shearing stress at point *L*.

PROBLEM 7.165 The compressed-air tank *AB* has a 250-mm outside diameter and an 8-mm wall thickness. It is fitted with a collar by which a 40-kN force **P** is applied at *B* in the horizontal direction. Knowing that the gage pressure inside the tank is 5 MPa, determine the maximum normal stress and the maximum shearing stress at point *K*.

SOLUTION

Consider element at point *L*.

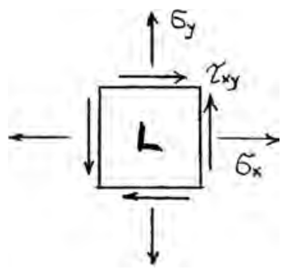
Stresses due to internal pressure:

$$p = 5 \text{ MPa} = 5\times10^6 \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{250}{2} - 8 = 117 \text{ mm}$$

$$\sigma_x = \frac{pr}{t} = \frac{(5\times10^6)(117\times10^{-3})}{8\times10^{-3}} = 73.125 \text{ MPa}$$

$$\sigma_y = \frac{pr}{2t} = \frac{(5\times10^3)(117\times10^{-3})}{(2)(8\times10^{-3})} = 36.563 \text{ MPa}$$



Stress due to bending moment: $M = (40 \text{ kN})(600 \text{ mm}) = 24000 \text{ N}\cdot\text{m}$

$$c_2 = \frac{1}{2}d = 125 \text{ mm}$$

$$c_1 = c_2 - t = 125 - 8 = 117 \text{ mm}$$

$$I = \frac{\pi}{4}\left(c_2^4 - c_1^4\right) = \frac{\pi}{4}(125^4 - 117^4)$$

$$= 44.573\times10^6 \text{ mm}^4 = 44.573\times10^{-6} \text{m}^4$$

$$\sigma_y = -\frac{Mc}{I} = -\frac{(24000)(125\times10^{-3})}{44.573\times10^{-6}} = -67.305 \text{ MPa}$$

PROBLEM 7.166 *(Continued)*

Stress due to transverse shear: Point L lies in a plane of symmetry.

$$\tau_{xy} = 0$$

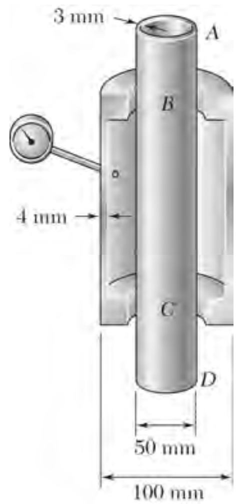
Total stresses: $\sigma_x = 73.125$ MPa, $\sigma_y = -30.742$ MPa, $\tau_{xy} = 0$

Principal stresses: Since $\tau_{xy} = 0$, σ_x and σ_y are principal stresses. The 3rd principal stress is in the radial direction, $\sigma_z \approx 0$.

$$\sigma_{\max} = 73.125 \text{ MPa}, \quad \sigma_{\min} = 0, \quad \sigma_a = 73.1 \text{ MPa}, \quad \sigma_b = -30.7 \text{ MPa}, \quad \sigma_z = 0$$

Maximum stress: $\sigma_{\max} = 73.1$ MPa ◀

Maximum shearing stress: $\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$ $\tau_{\max} = 51.9$ MPa ◀



PROBLEM 7.167

The brass pipe *AD* is fitted with a jacket used to apply a hydrostatic pressure of 3.5 MPa to portion *BC* of the pipe. Knowing that the pressure inside the pipe is 0.7 MPa, determine the maximum normal stress in the pipe.

SOLUTION

Let r be the mean radius of the pipe.

$$r = \frac{1}{2}(50 - 3) = 23.5\text{ mm}$$

Do not distinguish among inner, mean, and outer radius.

For pipe *AB*, $t = 3\text{ mm}$

Longitudinal stress $\sigma_1 = \dfrac{P_i r}{2t}$

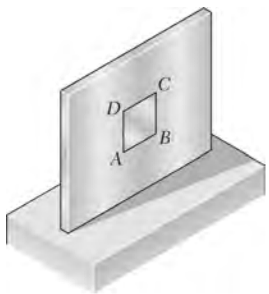
$$\sigma_1 = \frac{(0.7)(23.5)}{(2)(3)} \qquad \sigma_1 = 2.74\text{ MPa (tension)} \blacktriangleleft$$

Hoop stress $\sigma_2 = \dfrac{P_i r}{t} - \dfrac{p_o r}{t}$

$$\sigma_2 = \frac{(0.7)(23.5)}{3} - \frac{(3.5)(23.5)}{3}$$

$$\sigma_2 = -21.9\text{ MPa} \qquad \sigma_2 = 21.9\text{ MPa (compression)} \blacktriangleleft$$

PROBLEM 7.168

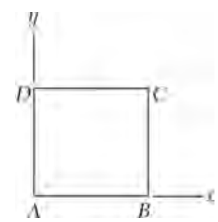


A square *ABCD* of 60 mm side is scribed on the surface of a thin plate while the plate is unloaded. After the plate is loaded, the lengths of sides *AB* and *AD* are observed to have increased, respectively, by 13.5×10^{-3} mm and 22.5×10^{-3} mm, while the angle *DAB* is observed to have decreased by 360×10^{-6} rad. Knowing that $v=\dfrac{1}{3}$, determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain.

SOLUTION

$$\varepsilon_x=\frac{\Delta l_x}{\Delta x}=\frac{\Delta\overline{AB}}{\overline{AB}}=\frac{540\times10^{-6}}{2.4}=225\times10^{-6}$$

$$\varepsilon_y=\frac{\Delta l_y}{\Delta y}=\frac{\Delta\overline{AD}}{\overline{AD}}=\frac{900\times10^{-6}}{2.4}=375\times10^{-6}$$



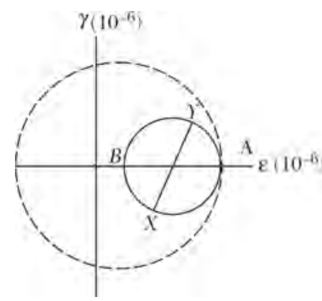
$$\gamma_{xy}=\text{decrease in right angle } DAB=360\times10^{-6}\text{ rad}=360\times10^{-6}$$

$$\varepsilon_{\text{ave}}=\frac{1}{2}(\varepsilon_x+\varepsilon_y)=300\times10^{-6}$$

$$\tan 2\theta_p=\frac{\gamma_{xy}}{\varepsilon_x-\varepsilon_y}=\frac{360}{225-375}=-2.4$$

$$2\theta_p=-67.38^\circ\qquad \theta_p=-33.7^\circ$$

$$R=\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}=195\times10^{-6}$$



$$\varepsilon_a=\varepsilon_{\text{ave}}+R=495\times10^{-6}$$ ◀

$$\varepsilon_b=\varepsilon_{\text{ave}}-R=105\times10^{-6}$$ ◀

(*b*) $$\gamma_{\max(\text{in-plane})}=\varepsilon_a-\varepsilon_b=390\times10^{-6}$$ ◀

$$\varepsilon_c=-\frac{v}{1-v}(\varepsilon_a+\varepsilon_b)$$

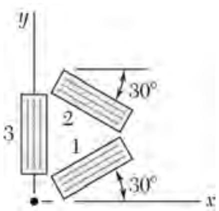
$$=-\frac{(1/3)}{(2/3)}(495\times10^{-6}+105\times10^{-6})=-300\times10^{-6}$$

$$\varepsilon_{\max}=495\times10^{-6}\qquad \varepsilon_{\min}=-300\times10^{-6}$$

(*c*) $$\gamma_{\max}=\varepsilon_{\max}-\varepsilon_{\min}=795\times10^{-6}$$ ◀

$$\left.\begin{aligned}\varepsilon_{\text{ave}}&=\frac{1}{2}(\varepsilon_{\max}+\varepsilon_{\min})=97.5\times10^{-6}\\ R&=\frac{1}{2}\gamma_{\max}=397.5\times10^{-6}\end{aligned}\right\}\text{For dotted Mohr's circle}$$

PROBLEM 7.169

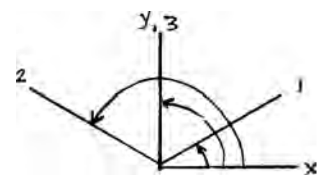


The strains determined by the use of the rosette shown during the test of a machine element are

$$\varepsilon_1 = +600\mu \quad \varepsilon_2 = +450\mu \quad \varepsilon_3 = -75\mu$$

Determine (*a*) the in-plane principal strains, (*b*) the in-plane maximum shearing strain.

SOLUTION



$$\theta_1 = 30°$$
$$\theta_2 = 150°$$
$$\theta_3 = 90°$$

$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1 = \varepsilon_1$$

$$0.75\varepsilon_x + 0.25\varepsilon_y + 0.43301\gamma_{xy} = 600\mu \qquad (1)$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2 = \varepsilon_2$$

$$0.75\varepsilon_x + 0.25\varepsilon_y - 0.43301\gamma_{xy} = 450\mu \qquad (2)$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3 = \varepsilon_3$$

$$0 \;+\; \varepsilon_y \;+\; 0 \;= -75\mu \qquad (3)$$

Solving (1), (2), and (3) simultaneously,

$$\varepsilon_x = 725\mu, \quad \varepsilon_y = -75\mu, \quad \gamma_{xy} = 173.21\mu$$

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 325\mu$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{725+75}{2}\right)^2 + \left(\frac{173.21}{2}\right)^2} = 409.3\mu$$

(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 734\mu$ $\qquad \varepsilon_a = 734\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -84.3\mu$ $\qquad \varepsilon_b = -84.3\mu$ ◀

(*b*) $\gamma_{\text{max (in-plane)}} = 2R = 819\mu$ $\qquad \gamma_{\text{max (in-plane)}} = 819\mu$ ◀