
Count Sensitivity Proofs

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Definition 1. Let \mathcal{X} be the universe of possible rows (individuals) and let $I : \mathcal{X} \rightarrow \{0, 1\}$ be a predicate on rows. Let $x \in \mathcal{X}^n$ be a dataset. Then a count over x is defined as

$$q(x) = \sum_{i=1}^n I(x_i).$$

Definition 2. Let q_1, \dots, q_k be a series of counts with predicates I_1, \dots, I_k . These counts are disjoint for every row in the database, only one of them evaluates to 1. In other words, they are disjoint if $\forall x_i \in \mathcal{X}$,

$$\sum_{j=1}^k I_j(x_i) = 1.$$

1 NEIGHBORING DEFINITION: CHANGE ONE

1.1 ℓ_1 -sensitivity

Theorem 1. A single count query has sensitivity 1. A series of k disjoint counts has sensitivity 2.

Proof. Let q be a count query with predicate I , and let x' be equal to x with point x_i changed to x'_i . Then $I(x_i)$ will change by at most 1, and since no other x_j with $i \neq j$ is changed, this term in q is the only thing affected. Thus, the sensitivity of a single query is bounded by 1.

Consider k disjoint count on the same databases x and x' . Since they are disjoint, only one of the k counts is influenced by x_i , and only one of the counts is affected by x'_i . Since each query can be affected by at most 1 by a single data point by the above logic, in total there will be a change of at most 2. \square

1.2 ℓ_2 -sensitivity

Theorem 2. A single count query has sensitivity 1. A series of k disjoint counts has sensitivity 4.

Proof. From the proof of Theorem 1, the difference between counts on two neighboring databases is at most 1. Squaring this gives the same value. Similarly, the difference between the results of k disjoint counts on two neighboring databases is 2, and squaring this gives 4. \square

2 NEIGHBORING DEFINITION: ADD/DROP ONE

2.1 ℓ_1 -sensitivity

2.2 ℓ_2 -sensitivity