Harvard Privacy Tools Project

Mean Sensitivity Proofs

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Definition 1. *The sample mean of database X of size n is*

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

1 Neighboring Definition: Change One

1.1 ℓ_1-sensitivity

Theorem 1. *Say database X has size n and is bounded above by M and bounded below by m. Then $\bar{X}$ has ℓ_1-sensitivity bounded above by*

$$\frac{M-m}{n}.$$

Proof. Say X and X' are neighboring databases which differ at data-point x_j. Then

$$\begin{aligned}
\Delta\bar{X} &= \max_{X,X'} \left|\bar{X} - \bar{X}'\right| \\
&= \max_{X,X'} \frac{1}{n}\left|\left(\sum_{\{i\in[n]|i\neq j\}} x_i\right) + x_j - \left(\sum_{\{i\in[n]|i\neq j\}} x'_i\right) + x'_j\right| \\
&= \max_{X,X'} \frac{1}{n}\left|x_j - x'_j\right| \\
&\leq \frac{M-m}{n}.
\end{aligned}$$

□

1.2 ℓ_2-sensitivity

2 Neighboring Definition: Add/Drop One

2.1 ℓ_1-sensitivity

2.2 ℓ_2-sensitivity