
Mean Sensitivity Proofs

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Definition 1. *The sample mean of database X of size n is*

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n x_i$$

1 NEIGHBORING DEFINITION: CHANGE ONE

1.1 ℓ_1 -sensitivity

1.2 ℓ_2 -sensitivity

2 NEIGHBORING DEFINITION: ADD/DROP ONE

2.1 ℓ_1 -sensitivity

Theorem 1. *Say database X has size n and is bounded above by M and bounded below by m . Then \bar{X} has ℓ_1 -sensitivity bounded above by*

$$\frac{M - m}{n}.$$

Proof. Say X and X' are neighboring databases which differ at data-point x_j . Then

$$\begin{aligned} \Delta \bar{X} &= \max_{X, X'} |\bar{X} - \bar{X}'| \\ &= \max_{X, X'} \frac{1}{n} \left| \left(\sum_{\{i \in [n] | i \neq j\}} x_i \right) + x_j - \left(\sum_{\{i \in [n] | i \neq j\}} x'_i \right) + x'_j \right| \\ &= \max_{X, X'} \frac{1}{n} |x_j - x'_j| \\ &\leq \frac{M - m}{n}. \end{aligned}$$

□

2.2 ℓ_2 -sensitivity