Notes on KMZ

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1. KF - Hamilton

The Kalman Filter/Smoother recursions following Hamilton's SSF format by HWL are:

$$\text{Measurement}: \quad \mathbf{y}_t = \mathbf{A}\mathbf{x}_t + \mathbf{H}\boldsymbol{\xi}_t + \mathbf{R}^{1/2}\varepsilon_t^{\mathbf{y}} \tag{1}$$

$$\text{State}: \quad \boldsymbol{\xi}_t = \mathbf{F}\boldsymbol{\xi}_{t-1} + \mathbf{Q}^{1/2}\varepsilon_t^{\xi} \tag{2}$$

The Kalman Filter recursions are:

$$\text{Forecasting}: \quad \hat{\mathbf{y}}_{t|t-1} = \mathbf{A}\mathbf{x}_t + \mathbf{H}\hat{\boldsymbol{\xi}}_{t|t-1} \tag{3}$$

$$\text{Forecast error}: \quad \hat{\mathbf{u}}_{t|t-1} = \mathbf{y}_t - \hat{\mathbf{y}}_{t|t-1} \tag{4}$$

$$= \mathbf{y}_t - \mathbf{A}\mathbf{x}_t - \mathbf{H}\hat{\boldsymbol{\xi}}_{t|t-1} \tag{5}$$

$$\text{(Near) Kalman Gain}: \quad \mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{H}'(\mathbf{H}\mathbf{P}_{t|t-1}\mathbf{H}' + \mathbf{R}\mathbf{R}')^{-1} \tag{6}$$

$$\text{State Filtering/Updating}: \quad \hat{\boldsymbol{\xi}}_{t|t} = \hat{\boldsymbol{\xi}}_{t|t-1} + \mathbf{K}_t\hat{\mathbf{u}}_{t|t-1} \tag{7}$$

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{K}_t\mathbf{H}\mathbf{P}_{t|t-1} \tag{8}$$

$$\text{State Forecasting}: \quad \hat{\boldsymbol{\xi}}_{t+1|t} = \mathbf{F}\hat{\boldsymbol{\xi}}_{t|t} \tag{9}$$

$$\mathbf{P}_{t+1|t} = \mathbf{F}\mathbf{P}_{t|t}\mathbf{F}' + \mathbf{Q}\mathbf{Q}' \tag{10}$$

$$\text{Kalman Gain}: \quad \mathbf{G}_t = \mathbf{F}\mathbf{K}_t \tag{11}$$

t

x

where (note that I do not transpose **A** or **H** as it is not necessary):

$$\mathbf{y}_t = \begin{bmatrix} y_t & \pi_t \end{bmatrix}',$$

$$\mathbf{x}_t = \begin{bmatrix} y_{t-1} & y_{t-2} & r_{t-1} & r_{t-2} & \pi_{t-1} & \pi_{t-2,4} \end{bmatrix}',$$

$$\boldsymbol{\xi}_t = \begin{bmatrix} y_t^* & y_{t-1}^* & y_{t-2}^* & g_{t-1} & g_{t-2} & z_{t-1} & z_{t-2} \end{bmatrix}',$$

$$
\mathbf{A} = \begin{bmatrix} a_{y,1} & a_{y,2} & \frac{a_r}{2} & \frac{a_r}{2} & 0 & 0 \\ b_y & 0 & 0 & 0 & b_\pi & 1 - b_\pi \end{bmatrix},
$$

$$
\mathbf{H} = \begin{bmatrix} 1 & -a_{y,1} & -a_{y,2} & -4\frac{a_r}{2} & -4\frac{a_r}{2} & -\frac{a_r}{2} & -\frac{a_r}{2} \\ 0 & -b_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
$$