1 HLW's (2017) State-Space Model (SSM) Form

HLW17 use the same standard State-Space Form (SSF) as in LW03, taking the form:

$$\begin{aligned}
\text{Measurement}: \quad & \mathbf{y}_t = \mathbf{A}\mathbf{x}_t + \mathbf{H}\boldsymbol{\xi}_t + \boldsymbol{R}^{1/2}\boldsymbol{\varepsilon}_t^{\mathbf{y}} \\
\text{State}: \quad & \boldsymbol{\xi}_t = \mathbf{F}\boldsymbol{\xi}_{t-1} + \boldsymbol{Q}^{1/2}\boldsymbol{\varepsilon}_t^{\xi}
\end{aligned}$$

where (note the different inflation specification)

$$\begin{aligned}
\mathbf{y}_t &= \begin{bmatrix} y_t & \pi_t \end{bmatrix}', \\
\mathbf{x}_t &= \begin{bmatrix} y_{t-1} & y_{t-2} & r_{t-1} & r_{t-2} & \pi_{t-1} & \pi_{t-2,4} \end{bmatrix}', \\
\boldsymbol{\xi}_t &= \begin{bmatrix} y_t^* & y_{t-1}^* & y_{t-2}^* & g_{t-1} & g_{t-2} & z_{t-1} & z_{t-2} \end{bmatrix}', \\
\mathbf{A} &= \begin{bmatrix} a_{y,1} & a_{y,2} & \frac{a_r}{2} & \frac{a_r}{2} & 0 & 0 \\ b_y & 0 & 0 & 0 & b_\pi & 1-b_\pi \end{bmatrix}, \\
\mathbf{H} &= \begin{bmatrix} 1 & -a_{y,1} & -a_{y,2} & -4\frac{a_r}{2} & -4\frac{a_r}{2} & -\frac{a_r}{2} & -\frac{a_r}{2} \\ 0 & -b_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
\end{aligned}$$

Note: This follows the notation used in the documentation file: '`HLW_Code_Guide.pdf`' included in the zip file: '`HLW_replication.zip`' that contains the replication code of HLW17 and which is available from the NYFED website at: https://www.newyorkfed.org/medialibrary/media/research/economists/williams/data/HLW_Code.zip.

Compared to LW03, the inflation dynamics are now changed and do not include oil price inflation and core import inflation anymore. Also, the constant c is now held fixed at 1, rather than estimated due to '*identification problems*', so that $r_t^* = 4g_t + z_t$. They write to this on page S63: '*In Laubach and Williams (2003), we estimated this relationship and found a coefficient of close to unity. Because this relationship is not well identified in the data, we chose to impose a coefficient of unity.*'. In the description of their SSM, the constant c has thus been removed for comparability, but in the SSM used for recovery in Section 2, it is left in the equations to make the adaption of the code easiest to the HLW23-post-COVID19 version.

In the construction of r_t^*, trend growth g_t is again annualized, but not in the state equations for g_t. That is, in the matrices above, the entries in $\mathbf{H}(1,4:5)$ corresponding to trend growth are multiplied by 4 (as well as c) in the code that performs the estimation (see `unpack.parameters.stage3.R` in `HLW_replication.zip` files, line 22, which reads):

```
H[1, 4:5] <- -parameters[3] * 2 # -a_r/2 (annualized).
```

The standard deviations of the shocks are denoted by $\begin{bmatrix} \sigma_{\tilde{y}} & \sigma_\pi & \sigma_{y^*} & \sigma_g & \sigma_z \end{bmatrix}'$ in the documentation in '`HLW_Code_Guide.pdf`'.

1.1 SSM of HLW17

obs0 The measurement equation is:

$$\mathbf{y}_t = \mathbf{A}\mathbf{x}_t + \mathbf{H}\xi_t + \boldsymbol{R}^{1/2}\boldsymbol{\varepsilon}_t^{\mathbf{y}}$$

$$\underbrace{\begin{bmatrix} y_t \\ \pi_t \end{bmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{bmatrix} a_{y,1} & a_{y,2} & \frac{a_r}{2} & \frac{a_r}{2} & 0 & 0 \\ b_y & 0 & 0 & 0 & b_\pi & 1-b_\pi \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} y_{t-1} \\ y_{t-2} \\ r_{t-1} \\ r_{t-2} \\ \pi_{t-1} \\ \pi_{t-2,4} \end{bmatrix}}_{\mathbf{x}_t} \tag{1a}$$

$$+ \underbrace{\begin{bmatrix} 1 & -a_{y,1} & -a_{y,2} & -4\frac{a_r}{2} & -4\frac{a_r}{2} & -\frac{a_r}{2} & -\frac{a_r}{2} \\ 0 & -b_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ y_{t-2}^* \\ g_{t-1} \\ g_{t-2} \\ z_{t-1} \\ z_{t-2} \end{bmatrix}}_{\boldsymbol{\xi}_t} + \underbrace{\begin{bmatrix} \sigma_{\tilde{y}} & 0 \\ 0 & \sigma_\pi \end{bmatrix}}_{\boldsymbol{R}^{1/2}} \underbrace{\begin{bmatrix} \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^{\pi} \end{bmatrix}}_{\boldsymbol{\varepsilon}_t^{\mathbf{y}}}. \tag{1b}$$

The state equation is:

$$\boldsymbol{\xi}_t = \mathbf{F}\boldsymbol{\xi}_{t-1} + \boldsymbol{Q}^{1/2}\boldsymbol{\varepsilon}_t^{\boldsymbol{\xi}}$$

$$\underbrace{\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ y_{t-2}^* \\ g_{t-1} \\ g_{t-2} \\ z_{t-1} \\ z_{t-2} \end{bmatrix}}_{\boldsymbol{\xi}_t} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{F}} \underbrace{\begin{bmatrix} y_{t-1}^* \\ y_{t-2}^* \\ y_{t-3}^* \\ g_{t-2} \\ g_{t-3} \\ z_{t-2} \\ z_{t-3} \end{bmatrix}}_{\boldsymbol{\xi}_{t-1}} + \underbrace{\begin{bmatrix} \sigma_{y^*} & \sigma_g & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sigma_g & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_z \\ 0 & 0 & 0 \end{bmatrix}}_{\boldsymbol{Q}^{1/2}} \underbrace{\begin{bmatrix} \varepsilon_t^{y^*} \\ \varepsilon_{t-1}^{g} \\ \varepsilon_{t-1}^{z} \end{bmatrix}}_{\boldsymbol{\varepsilon}_t^{\boldsymbol{\xi}}} \tag{2}$$

state1

Expanding the relations in (1) and (2) and re-arranging yields:

$$\begin{bmatrix} y_t \\ \\ \\ \pi_t \end{bmatrix} = \begin{bmatrix} y_t^* + a_{y,1}\left(y_{t-1} - y_{t-1}^*\right) + a_{y,2}\left(y_{t-2} - y_{t-2}^*\right) \\ +\frac{1}{2}a_r\left([r_{t-1} - 4cg_{t-1} - z_{t-1}] + [r_{t-2} - 4cg_{t-2} - z_{t-2}]\right) + \sigma_{\tilde{y}}\varepsilon_t^{\tilde{y}} \\ \\ b_y\left(y_{t-1} - y_{t-1}^*\right) + b_\pi \pi_{t-1} + (1 - b_\pi)\pi_{t-2,4} + \sigma_\pi \varepsilon_t^\pi \end{bmatrix} \tag{3}$$

lwa

$$\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ y_{t-2}^* \\ g_{t-1} \\ g_{t-2} \\ z_{t-1} \\ z_{t-2} \end{bmatrix} = \begin{bmatrix} y_{t-1}^* + \overbrace{g_{t-2} + \sigma_g \varepsilon_{t-1}^g}^{g_{t-1}} + \sigma_{y^*}\varepsilon_t^{y^*} \\ y_{t-1}^* \\ y_{t-2}^* \\ g_{t-2} + \sigma_g \varepsilon_{t-1}^g \\ g_{t-2} \\ z_{t-2} + \sigma_z \varepsilon_{t-1}^z \\ z_{t-2} \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta y_t^* \\ \Delta g_{t-1} \\ \Delta z_{t-1} \end{bmatrix} = \begin{bmatrix} g_{t-1} + \sigma_{y^*}\varepsilon_t^{y^*} \\ \sigma_g \varepsilon_{t-1}^g \\ \sigma_z \varepsilon_{t-1}^z \end{bmatrix} \tag{4}$$

lwb

and with $r_t^* = 4g_t + z_t$, we get:

$$\begin{aligned} \Delta r_t^* &= 4\overbrace{\Delta g_t}^{\sigma_g \varepsilon_t^g} + \overbrace{\Delta z_t}^{\sigma_z \varepsilon_t^z} \\ &= 4\sigma_g \varepsilon_t^g + \sigma_z \varepsilon_t^z. \end{aligned} \tag{5}$$

drstar

2 Shock recovery SSM

2.1 SSM with lagged states

SSM Kurz's (2018) SSM has the following general from:

$$\text{Measurement}: \quad Z_t = D_1 X_t + D_2 X_{t-1} + R\varepsilon_t \tag{6a}$$

ssm1

$$\text{State}: \quad X_t = AX_{t-1} + C\varepsilon_t, \tag{6b}$$

ssm2

where $\varepsilon_t \sim MN(0, I_m)$, D_1, D_2, A, R are C are conformable system matrices, Z_t the observed variable and X_t the latent state variable.

2.2 HLW17 equations

Following the same format as for LW03, HLW17's SSM equations in (3) and (4) (with original labelling of the shocks) gives the following SSM equations:

LW03

$$
\begin{aligned}
y_t &= y_t^* + \sum_{i=1}^{2} a_{y,i}\left(y_{t-i} - y_{t-i}^*\right) + \tfrac{1}{2}a_r \sum_{i=1}^{2}\left(r_{t-i} - r_{t-i}^*\right) + \sigma_{\tilde{y}}\varepsilon_t^{\tilde{y}} && \text{(7a)} \quad \texttt{LW03a} \\
\pi_t &= b_y\left(y_{t-1} - y_{t-1}^*\right) + b_\pi \pi_{t-1} + (1-b_\pi)\pi_{t-2,4} + \sigma_\pi \varepsilon_t^\pi && \text{(7b)} \\
\Delta z_t &= \sigma_z \varepsilon_t^z && \text{(7c)} \quad \texttt{LW03c} \\
\Delta y_t^* &= g_{t-1} + \sigma_{y^*}\varepsilon_t^{y^*} && \text{(7d)} \quad \texttt{LW03d} \\
\Delta g_t &= \sigma_g \varepsilon_t^g && \text{(7e)} \quad \texttt{LW03e}
\end{aligned}
$$

with

$$
\Delta r_t^* = 4\sigma_g \varepsilon_t^g + \sigma_z \varepsilon_t^z. \tag{7f}
$$
LW03f

where the numbered shock to named shock mapping is:

$$
\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix} = \begin{bmatrix} \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^{\pi} \\ \varepsilon_t^{z} \\ \varepsilon_t^{y^*} \\ \varepsilon_t^{g} \end{bmatrix}. \tag{8}
$$

2.3 HLW17 SSM for shock recovery

To assess recovery, re-write the model in '*shock recovery*' form. That is, collect all observables in Z_t, and all shocks (and other state variables) in state vector X_t. The relevant equations
ssm0 from (7) (incorporating (7f)) for the '*shock recovery*' SSM are (note the inclusion of c in (9e)):

$$
\begin{aligned}
\text{Measurement}: \quad Z_{1t} &= y_t^* - a_{y,1}y_{t-1}^* - a_{y,2}y_{t-2}^* - \tfrac{1}{2}a_r\left(r_{t-1}^* + r_{t-2}^*\right) + \sigma_{\tilde{y}}\varepsilon_t^{\tilde{y}} && \text{(9a)} \\
Z_{2t} &= -b_y y_{t-1}^* + \sigma_\pi \varepsilon_t^\pi && \text{(9b)} \\
\text{State}: \quad \Delta y_t^* &= g_{t-1} + \sigma_{y^*}\varepsilon_t^{y^*} && \text{(9c)} \\
\Delta g_t &= \sigma_g \varepsilon_t^g && \text{(9d)} \\
\Delta r_t^* &= 4c\sigma_g \varepsilon_t^g + \sigma_z \varepsilon_t^z, && \text{(9e)} \quad \texttt{drstar2}
\end{aligned}
$$

with the observables Z_t in the measurement equations defined as:

$$
\begin{aligned}
Z_{1t} &= y_t - \sum_{i=1}^{2} a_{y,i}y_{t-i} - \tfrac{1}{2}a_r \sum_{i=1}^{2} r_{t-i} \\
Z_{2t} &= \pi_t - b_\pi \pi_{t-1} - (1-b_\pi)\pi_{t-2,4} - b_y y_{t-1}.
\end{aligned}
$$

KOSSM The '*shock recovery*' SSF corresponding to (9) is then:

$$\text{Measurement :}\quad Z_t = D_1 X_t + D_2 X_{t-1} + R\varepsilon_t$$

$$\underbrace{\begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix}}_{Z_t} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \sigma_{\tilde{y}} & 0 & 0 & 0 & 0 \\ 0 & -b_y & 0 & 0 & 0 & 0 & \sigma_\pi & 0 & 0 & 0 \end{bmatrix}}_{D_1} \underbrace{\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ g_t \\ r_t^* \\ r_{t-1}^* \\ \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^{\pi} \\ \varepsilon_t^{z} \\ \varepsilon_t^{y^*} \\ \varepsilon_t^{g} \end{bmatrix}}_{X_t} \tag{10a}$$

$$+ \underbrace{\begin{bmatrix} -a_{y,1} & -a_{y,2} & 0 & -\frac{a_r}{2} & -\frac{a_r}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{D_2} \underbrace{\begin{bmatrix} y_{t-1}^* \\ y_{t-2}^* \\ g_{t-1} \\ r_{t-1}^* \\ r_{t-2}^* \\ \varepsilon_{t-1}^{\tilde{y}} \\ \varepsilon_{t-1}^{\pi} \\ \varepsilon_{t-1}^{z} \\ \varepsilon_{t-1}^{y^*} \\ \varepsilon_{t-1}^{g} \end{bmatrix}}_{X_{t-1}} + \underbrace{\mathbf{0}_{2\times 5}}_{R} \underbrace{\begin{bmatrix} \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^{\pi} \\ \varepsilon_t^{z} \\ \varepsilon_t^{y^*} \\ \varepsilon_t^{g} \end{bmatrix}}_{\varepsilon_t} \tag{10b}$$

$$\text{State :}\quad X_t = AX_{t-1} + C\varepsilon_t,$$

$$\underbrace{\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ g_t \\ r_t^* \\ r_{t-1}^* \\ \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^{\pi} \\ \varepsilon_t^{z} \\ \varepsilon_t^{y^*} \\ \varepsilon_t^{g} \end{bmatrix}}_{X_t} = \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y_{t-1}^* \\ y_{t-2}^* \\ g_{t-1} \\ r_{t-1}^* \\ r_{t-2}^* \\ \varepsilon_{t-1}^{\tilde{y}} \\ \varepsilon_{t-1}^{\pi} \\ \varepsilon_{t-1}^{z} \\ \varepsilon_{t-1}^{y^*} \\ \varepsilon_{t-1}^{g} \end{bmatrix}}_{X_{t-1}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \sigma_{y^*} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_g \\ 0 & 0 & \sigma_z & 0 & 4c\sigma_g \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^{\pi} \\ \varepsilon_t^{z} \\ \varepsilon_t^{y^*} \\ \varepsilon_t^{g} \end{bmatrix}}_{\varepsilon_t} \tag{10c}$$

2.3.1 Correlation between true change in natural rate and estimate

The correlation between the true and estimated Δr_t^* from the SSM can be constructed from the relation:

$$\rho = 0.5\frac{\text{Var}(\Delta r_t^*) + \text{Var}(E_T\Delta r_t^*) - \phi}{\sigma(\Delta r_t^*)\sigma(E_T\Delta r_t^*)}, \tag{11}$$

where $\text{Var}(\Delta r_t^*) = 4^2c^2\sigma_5^2 + \sigma_3^2$, $\sigma(\Delta r_t^*) = \sqrt{\text{Var}(\Delta r_t^*)}$, and $\text{Var}(E_T\Delta r_t^*)$ can be computed from simulating from the true model, applying the Kalman Filter and Smoother to get $E_T\Delta r_t^*$ and then computing the sample variance of $E_T\Delta r_t^*$ as an estimate of $\text{Var}(E_T\Delta r_t^*)$.

To obtain ϕ, add Δr_t^* to the state-vector X_t and augment the remaining matrices to be conformable. The required ϕ term is then the entry of $\text{diag}(P_{t|T}^*)$ that corresponds to Δr_t^*, which will be the very last element.

The augmented SSF of (10) is then:

$$\text{Measurement}: \quad Z_t = D_1X_t + D_2X_{t-1} + R\varepsilon_t$$

$$\underbrace{\begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix}}_{Z_t} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \sigma_{\tilde{y}} & 0 & 0 & 0 & 0 & 0 \\ 0 & -b_y & 0 & 0 & 0 & 0 & \sigma_\pi & 0 & 0 & 0 & 0 \end{bmatrix}}_{D_1} \underbrace{\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ g_t \\ r_t^* \\ r_{t-1}^* \\ \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^\pi \\ \varepsilon_t^z \\ \varepsilon_t^{y^*} \\ \varepsilon_t^g \\ \Delta r_t^* \end{bmatrix}}_{X_t} \tag{12}$$

$$+ \underbrace{\begin{bmatrix} -a_{y,1} & -a_{y,2} & 0 & -\frac{a_r}{2} & -\frac{a_r}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{D_2} \underbrace{\begin{bmatrix} y_{t-1}^* \\ y_{t-2}^* \\ g_{t-1} \\ r_{t-1}^* \\ r_{t-2}^* \\ \varepsilon_{t-1}^{\tilde{y}} \\ \varepsilon_{t-1}^\pi \\ \varepsilon_{t-1}^z \\ \varepsilon_{t-1}^{y^*} \\ \varepsilon_{t-1}^g \\ \Delta r_{t-1}^* \end{bmatrix}}_{X_{t-1}} + \underbrace{\mathbf{0}_{2\times 5}}_{R} \underbrace{\begin{bmatrix} \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^\pi \\ \varepsilon_t^z \\ \varepsilon_t^{y^*} \\ \varepsilon_t^g \end{bmatrix}}_{\varepsilon_t} \tag{13}$$

$$\text{State}: \quad X_t = AX_{t-1} + C\varepsilon_t,$$

$$
\underbrace{\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ g_t \\ r_t^* \\ r_{t-1}^* \\ \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^{\pi} \\ \varepsilon_t^{z} \\ \varepsilon_t^{y^*} \\ \varepsilon_t^{g} \\ \Delta r_t^* \end{bmatrix}}_{X_t} = \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y_{t-1}^* \\ y_{t-2}^* \\ g_{t-1} \\ r_{t-1}^* \\ r_{t-2}^* \\ \varepsilon_{t-1}^{\tilde{y}} \\ \varepsilon_{t-1}^{\pi} \\ \varepsilon_{t-1}^{z} \\ \varepsilon_{t-1}^{y^*} \\ \varepsilon_{t-1}^{g} \\ \Delta r_{t-1}^* \end{bmatrix}}_{X_{t-1}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \sigma_{y^*} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_g \\ 0 & 0 & \sigma_z & 0 & 4c\sigma_g \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \sigma_z & 0 & 4c\sigma_g \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^{\pi} \\ \varepsilon_t^{z} \\ \varepsilon_t^{y^*} \\ \varepsilon_t^{g} \end{bmatrix}}_{\varepsilon_t} \tag{14}
$$