Porting Coq Scripts to the Mathematical Components Library Version 2

Reynald Affeldt², Yves Bertot^{1,4}, Cyril Cohen^{1,4}, Pierre Roux³, Kazuhiko Sakaguchi¹, and Enrico Tassi^{1,4}

¹Inria, France

²National Institute of Advanced Industrial Science and Technology (AIST), Japan ³ONERA / DTIS, Université de Toulouse, France ⁴Université Côte d'Azur, France

Abstract

The Mathematical Components library (hereafter, MathComp) [8] provides, among others, a number of mathematical structures organized as hierarchies. Hierarchy Builder (hereafter, HB) is an extension of the Coq proof assistant to ease the development of hierarchies of structures [4]. MathComp 2 [9] is the result of the port of MathComp to HB [1].

This document is a technical report whose goal is to explain how to port MATHCOMP developments to MATHCOMP 2. It has been written by the participants of the MATHCOMP Documentation Sprint that happened from [2023-05-03] to [2023-05-10].

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1 Target Audience of this Document

Based on our experience porting several developments to MATHCOMP 2, we can distinguish three categories of users:

- 1. Mathcomp users who have not been using the Canonical command should not see much difference compared to the past version upgrades of Mathcomp. Some identifiers that are now useless have been removed but this is documented in the changelog. For example bool_eqType might need to be replaced by bool: eqType or just bool. Also, the behavior of some rewritings might have changed, requiring explicit patterns. Typically, it might happen that rewriting with associativity lemmas requires the user to indicate whether it is supposed to happen on the left or the right-hand side of an equivalence relation, so that rewrite addrA might need to be rewritten rewrite [in LHS]addrA for example. See Sect. 4.4 for a concrete example. For such users, reading this tutorial through the end might not be necessary.
- 2. The target readers are primarily MATHCOMP users who have been instantiating structures using the Canonical command.
- 3. As for the few users who have been developing their own hierarchies of structures, this tutorial might be of little help and they rather need to refer to:
 - the original paper for an extensive introduction to HB commands [4],
 - the HB development for documentation and examples [7] (start with the README),
 - various papers for more applications [1] [2, Sect. 3] [3, Sect. 4],
 - already ported developments such as odd-order, multinomials, etc.

For the sake of concreteness, we illustrate the port of COMPDECMODAL [5] in Sect. 4. Before that we review the basics of HB in Sect. 2 and review the documentation tools available for porting in Sect. 3.

2 Quick Reminder about the HB Vocabulary

The goal of this section is to briefly explain the three main commands introduced by HB: HB.mixin, HB.structure, and HB.instance. The knowledgeable reader can safely skip this section.

Let us consider the most basic scenario in generic terms. Here is the pattern to declare a structure Struct that sits at the bottom of a hierarchy. The interface of the structure goes into a mixin:

```
HB.mixin Record isStruct params carrier := {
    ... properties about the carrier ...
}
    The structure itself is declared like a sigma-type:
#[short(type=structType)]
HB.structure Definition Struct := {carrier of isStruct carrier}
```

Note that HB is using Co_Q attributes to declare the type corresponding to a structure.

Here is the pattern to declare a new structure NewStruct that extends the existing structure Struct; note the of syntax.

```
HB.mixin Record NewStruct_from_Struct params carrier
   of Struct params carrier := {
    ... more properties about the carrier ...
}
```

In the case of the extended structure, the sigma-type makes appear the dependency to the parent structure; note the & syntax.

This process results in the creation of the types structType and newStructType such that elements of the latter are also understood to be elements of the former.

Finally, the declaration of a mixin Struct is accompanied by the creation of a constructor Struct Build which is used to instantiate a structure using the command:

```
HB.instance Definition _ := Struct.Build params.
```

The command HB.instance should trigger the printing of several lines of information output such as

```
module_type__canonical__struct_Struct is defined
```

The absence of this output often indicates failure of the HB.instance command.

3 Tools to Port MathComp Applications

3.1 Documentation

The following pieces of documentation are useful during the process of porting a MATHCOMP application to MATHCOMP 2:

- The changelog is the primary source of information. See CHANGELOG.md [9].
- Additionally, structures are documented in the headers of CoQ scripts according to the following format:

```
Centered Title
                                                                     *)
(* Some introductory text: what is this file about, instructions to use this *)
                                                                     *)
(* file, etc.
(*
                                                                     *)
(* Reference: bib entry if any
                                                                     *)
(* * Section Name
(*
    definition == prose explanation of the definition and its parameters
(*
     notation == prose explanation, scope information should appear nearby *)
(*
    structType == name of structures should make clear the corresponding
(*
                HB structure with the following sentence:
(*
                "The HB class is Xyz."
     shortcut := a shortcut can be explained with (pseudo-)code instead of *)
(*
(*
                                                                     *)
(* Acknowledgments: people
                                                                     *)
```

See for example the eqType structure defined in the file ssreflect/eqtype.v. See this wiki entry for more information about the documentation of scripts.

• Optionally, the user can double-check the naming of identifiers and lemmas with the naming conventions explained in CONTRIBUTING.md [9].

3.2 HB Commands Useful to Explore an Existing Hierarchy

Besides the changelog and the headers of CoQ scripts, the user can use HB commands to explore a hierarchy of mathematical structures.

3.2.1 Information about Structures with HB. about

Basic information about structures can be obtained via the command HB. about as in:

(The output message refers to a factory: this is a generalization of mixin.)

Graph of an HB Hierarchy It is also possible to explore a HB hierarchy using the command HB.graph. Inside a CoQ file:

```
HB.graph "hierarchy.dot".
```

From a terminal:

```
tred hierarchy.dot | dot -Tpng > hierarchy.png
```

For example, Fig. 1 displays the immediate vicinity of eqType.

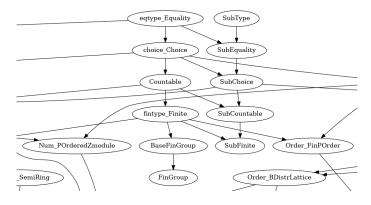


Figure 1: The vicinity of the structure eqType in MATHCOMP

3.2.2 Information about Constructors with HB.howto and HB.about

To discover constructors that build a structure, one can use the command HB.howto. For instance

```
> HB.howto eqType.   
HB: solutions (use 'HB.about F.Build' to see the arguments of each factory F):   

   - hasDecEq
```

tells us that eqType instances can be built with hasDecEq.Build. (Note that by default HB.howto may not return all the available factories; it might be necessary to increase the depth search using a natural number as in HB.howto xyzType 5.)

To learn which parameters a xyz.Build constructor is expecting, one can use the HB.about command:

The message indicates that hasDecEq.Build is expecting a type T, a predicate eq_op: rel T (implicit argument, as indicated by the square brackets) and a proof of Equality.axiom eq_op. One can thus instantiate an eqType on some type T with]

```
HB.instance Definition _ := hasDecEq.Build T proof_of_Equality_axiom.
or
HB.instance Definition _ := @hasDecEq.Build T eq_op proof_of_Equality_axiom.
```

which should output a few lines among which (recall that the absence of this output often indicate an instantiation problem)¹:

```
module_T__canonical__eqtype_Equality is defined
```

Discover Aliases and Feather Factories In addition to the structures and constructors listed by HB.about, the library defines some aliases (a.k.a. feather factories). These aliases are documented in the header comments. For instance, an eqType instance on some type T can be derived from some T' already equipped with an eqType structure, given a function $f: T \to T'$ and a proof injf: injective f:

```
HB.instance Definition _ := Equality.copy T (inj_type injf).
See eqType.v for inj_type.
```

3.2.3 Information about Instances with HB about

Instances a type is already equipped with can be listed with HB. about, for instance:

lists all the structures bool is already equipped with.

4 Porting a MathComp Development to MathComp 2

The basic strategy to port an existing Math-Comp development to Math-Comp 2 is to (1) install Math-Comp 2, (2) compile the existing Coq scripts, and (3) fix the errors one after the other. For the sake of concreteness, we explain the port of Comp-Decmodal [5]. This is a development with a moderate use of

¹We have also observed at the time of this writing that the output of the HB.instance command may not be visible by default with VSCoq.

MATHCOMP whose port involves fixing the instantiation of basic structures that most developments using MATHCOMP are likely to use.

In the following, the offending commands appear in a gray areas

```
command incompatible with MathComp 2
```

and their fixes are singly famed:

```
MathComp 2 fix for the command above
```

4.1 Import the HB Library

First thing first, any CoQ file using HB must start with:

From HB Require Import structures.

4.2 Instantiation of Structures with MathComp 2

From the viewpoint of the MATHCOMP user, the main change is the way mathematical structures are now instantiated. Most Canonical (or Canonical Structure) commands are replaced by HB.instance (see Sect. 2) and there are small changes to MATHCOMP notations such [subType ...], etc.

Regarding COMPDECMODAL, the first offending set of commands is the following (file fset.v):

```
Section FinSets.
Variable T : choiceType.
...
Canonical Structure fset_subType := [subType for elements by fset_type_rect].
Canonical Structure fset_eqType := EqType _ [eqMixin of fset_type by <:].
Canonical Structure fset_predType := PredType (fun (X : fset_type) x => nosimpl x \in elements X)
Canonical Structure fset_choiceType := Eval hnf in ChoiceType _ [choiceMixin of fset_type by <:].
End FinSets.

Canonical Structure fset_countType (T : countType) :=
    Eval hnf in CountType _ [countMixin of fset_type T by <:].
Canonical Structure fset_subCountType (T : countType) :=
    Eval hnf in [subCountType of fset_type T].</pre>
```

Let us consider compilation errors in order:

```
> Canonical Structure fset_subType := [subType for elements by fset_type_rect].
Error: Syntax error: [reduce] expected after ':=' (in [def_body]).
```

This error is due to a change of notation that is documented in the changelog. Search for the string, say, "[subType" in CHANGELOG.md:

```
- in `eqtype.v`
...
+ notation `[subType for v by rec]`, use `[isSub for v by rec]`
...
```

The fix is therefore the following:

```
> HB.instance Definition _ := [isSub for elements by fset_type_rect].
HB_unnamed_factory_3 is defined
fset_fset_type__canonical__eqtype_SubType is defined
```

Note that the instance need not be named and better not be since it is the job of HB to figure out instances automatically. It is important to check that HB displays more than one message as a response to HB.instance, otherwise this might indicate a failed instantiation.

Next compilation error:

```
> Canonical Structure fset_eqType := EqType _ [eqMixin of fset_type by <:].
Error: The reference EqType was not found in the current environment.</pre>
```

This error is primarily due to the remove of the EqType constructor [6, Sect. 2.1]. In fact, most xyzType constructors from MATHCOMP should not be necessary anymore. See the changelog. Similarly to the [subType for _ by _] notation above, the [eqMixin of _ by <:] has changed:

```
- in `eqtype.v`
...
+ notation `[eqMixin of T by <:]`, use `[Equality of T by <:]`
...</pre>
```

The fix is therefore:

```
> HB.instance Definition _ := [Equality of fset_type by <:].
HB_unnamed_factory_8 is defined
eqtype_Equality__to__eqtype_hasDecEq is defined
HB_unnamed_mixin_10 is defined
fset_fset_type__canonical__eqtype_Equality is defined
fset_fset_type__canonical__eqtype_SubEquality is defined
```

The next two compilation errors are similarly due to the removal of choiceType and CountType, and to the change of the notations [choiceMixin of _ by <:] and [countMixin of _ by <:]:

The fix can again be inferred from the changelog:

```
> HB.instance Definition _ := [Choice of fset_type by <:].

HB_unnamed_factory_11 is defined

choice_Choice_to__choice_hasChoice is defined

HB_unnamed_mixin_14 is defined

fset_fset_type__canonical__choice_Choice is defined

fset_fset_type__canonical__choice_SubChoice is defined

> HB.instance Definition _ (T : countType) := [Countable of fset_type T by <:].

T is declared

HB_unnamed_factory_30 is defined

choice_Countable__to__choice_hasChoice is defined

choice_Countable__to__etype_hasDecEq is defined

choice_Countable__to__etype_hasDecEq is defined

HB_unnamed_mixin_34 is defined

fset_fset_type__canonical__choice_Countable is defined

fset_fset_type__canonical__choice_SubCountable is defined
```

Note that, although HB does provide an #[hnf] attribute, it should not be necessary in general. Finally, the last Canonical command causes a deprecation warning that needs to be addressed:

```
> Canonical Structure fset_subCountType (T : countType) :=
> Eval hnf in [subCountType of fset_type T].
Warning: Notation "[ subCountType of _ ]" is deprecated since mathcomp 2.0.0.
Use SubCountable.clone instead.
[deprecated-notation,deprecated]
fset_subCountType is defined
```

In fact, going back one step, it can be observed by the output the HB.instance commands that the instantiation of the Countable structure already triggered the instantiation of the SubCountable structure, rendering the last Canonical command harmful. It therefore needs to be deleted.

To sum up, here follows the complete fix:

```
Section FinSets.

Variable T : choiceType.

...

HB.instance Definition _ := [isSub for elements by fset_type_rect].

HB.instance Definition _ := [Equality of fset_type by <:].

Canonical Structure fset_predType := PredType (fun (X : fset_type) x => nosimpl x \in elements X).

HB.instance Definition _ := [Choice of fset_type by <:].

End FinSets.

HB.instance Definition _ (T : countType) := [Countable of fset_type T by <:].
```

In fact, we can go one step further. Instead of instantiating the Equality structure at line 5 and then the Choice structure at line 7, one can start by instantiating the Choice structure and get the Equality structure automatically, so that a shorter fix would be:

```
Section FinSets.
Variable T : choiceType.
...
HB.instance Definition _ := [isSub for elements by fset_type_rect].
HB.instance Definition _ := [Choice of fset_type by <:].
Canonical Structure fset_predType := PredType (fun (X : fset_type) x => nosimpl x \in elements X).
End FinSets.

HB.instance Definition _ (T : countType) := [Countable of fset_type T by <:].</pre>
```

This small example already illustrates the advantage of porting to MATHCOMP 2.

4.3 Finitely Iterated Operators

The next series of compilation errors that occur when porting COMPDECMODAL is about finitely iterated operators, which are likely to be used by most MATHCOMP developments:

```
Canonical Structure fsetU_law (T : choiceType) :=
  Monoid.Law (@fsetUA T) (@fsetUU T) (@fsetUU T).
Canonical Structure fsetU_comlaw (T : choiceType) :=
  Monoid.ComLaw (@fsetUC T).
```

The first compilation error indicates a change in the signature of a constructor:

```
> Canonical Structure fsetU_law (T : choiceType) :=
> Monoid.Law (@fsetUA T) (@fsetUU T) (@fsetUU T).
Error:
```

```
In environment
T : choiceType
The term "fsetUA (T:=T)" has type "forall X Y Z : \{fset T\}, X `|` (Y `|` Z) = X `|` Y `|` Z"
while it is expected to have type "Type"
We can use HB about to discover the relevant structure (see Sect. 3.2.1):
> HB.about Monoid.Law.
HB: Monoid.Law.type is a structure (from "./ssreflect/bigop.v", line 415)
We can now use HB.howto to discover how to build this structure (see Sect. 3.2.2):
> HB.howto Monoid.Law.type
HB: solutions (use 'HB.about F.Build' to see the arguments of each factory F):
    - Monoid.isLaw
    - SemiGroup.isLaw; Monoid.isMonoidLaw
Finally, we can use HB about to learn about the parameters of a constructot (see Sect. 3.2.2):
> HB.about Monoid.isLaw.Build
HB: arguments: Monoid.isLaw.Build T idm op opA op1m opm1
    - T : Type
    - idm : T
    - op : T \rightarrow T
    - opA : associative op
    - op1m : left_id idm op
   - opm1 : right_id idm op
```

We have now enough information to fix the compilation error:

```
HB.instance Definition _ (T : choiceType) :=
Monoid.isLaw.Build {fset T} fset0 fsetU (@fsetUA T) (@fsetOU T) (@fsetUO T).
```

Note that we have made explicit the key (the operator fsetU) in the instantiation. Though not strictly necessary, this is good practice to document it on this occasion.

Next compilation error:

```
> Canonical Structure fsetU_comlaw (T : choiceType) :=
> Monoid.ComLaw (@fsetUC T).
Error:
In environment
T : choiceType
The term "fsetUC (T:=T)" has type "forall X Y : {fset T}, X `|` Y = Y `|` X"
   while it is expected to have type "Type".
```

This is similar to above: use HB.about to learn about Monoid.ComLaw and use HB.howto to inquire about its construction.

```
> HB.howto Monoid.ComLaw.type.

HB: solutions (use 'HB.about F.Build' to see the arguments of each factory F):

- Monoid.isComLaw

- SemiGroup.isComLaw; Monoid.isMonoidLaw

- SemiGroup.isCommutativeLaw; Monoid.isLaw

- SemiGroup.isLaw; SemiGroup.isCommutativeLaw; Monoid.isMonoidLaw
```

Note that since the key fsetU has already been equipped with a Monoid.Law structure, we can add this key as an additional parameter to HB.howto to restrict the search:

We can then check the parameters of SemiGroup.isCommutativeLaw.Build to come up with the following fix:

```
HB.instance Definition _ (T : choiceType) :=
SemiGroup.isCommutativeLaw.Build _ fsetU (@fsetUC T).
```

To sum up, we fix the compilation errors about finitely iterated operators using:

```
HB.instance Definition _ (T : choiceType) :=
SemiGroup.isCommutativeLaw.Build _ fsetU (@fsetUC T).
HB.instance Definition _ (T : choiceType) :=
Monoid.isLaw.Build {fset T} fsetO fsetU (@fsetUA T) (@fsetOU T) (@fsetUO T).
```

In fact, we could have proceeded with only one instantiation since we can get the Monoid.Law structure from the instantiation of the Monoid.ComLaw structure, so that a better fix would be:

```
HB.instance Definition _ (T : choiceType) :=

Monoid.isComLaw.Build _ _ fsetU (@fsetUA T) (@fsetUC T) (@fsetOU T).
```

4.4 Other Compilation Errors

Since other compilation errors are similar to the one already explained above, we go faster about them.

Instantiation of an Equality Structure Next failure in the file K/K_def.v:

```
Definition form_eqMixin := EqMixin (compareP eq_form_dec).

Canonical Structure form_eqType := Eval hnf in @EqType form form_eqMixin.
```

The failure is primarily caused by the removal of EqMixin. The changelog suggests to use the constructor hasDecEq.Build whose parameters can be double-checked with HB.howto leading to the following fix:

```
HB.instance Definition _ := hasDecEq.Build form (compareP eq_form_dec).
```

Instantiation of a Countable Structure Next failure in the file K/K_def.v:

```
Definition form_countMixin := PcanCountMixin formChoice.pickleP.

Definition form_choiceMixin := CountChoiceMixin form_countMixin.

Canonical Structure form_ChoiceType := Eval hnf in ChoiceType form form_choiceMixin.

Canonical Structure form_CountType := Eval hnf in CountType form form_countMixin.
```

We learn that PcanCountMixin is deprecated and that CountChoiceMixin is no more. The changelog suggests to use PCanIsCountable instead of PcanCountMixin. According to Locate, PCanIsCountable is located in the file ssreflect/choice.v and it has the following type:

```
PCanIsCountable :
forall [T : countType] [sT : Type] [f : sT -> T] [f' : T -> option sT], pcancel f f' ->
  isCountable.axioms_ sT
```

The fix in this case is therefore:

```
HB.instance Definition _ : isCountable form := PCanIsCountable formChoice.pickleP.
```

Note that we have added a type information to make the key (here, form) explicit as recommended when performing instantiations. This fix is sufficient because it generates the Choice instance along with the Countable instance.

More Instantiations of Equality Structures The next compilation errors in the files Kstar_def.v, gen_def.v, and CTL_def.v are handled as already explained above.

Failing Rewriting The next compilation "error" is in the file CTL/demo.v. It is actually a tactic whose execution has been substantially slowed down by the upgrade to MATHCOMP 2:

```
move => [p' y]. rewrite /MRel /Mstate (negbTE (root_internal _)) [_ && _]/= orbF.
```

The offending rewrite is the one with orbF. It used to apply by default to the left-hand side of an <-> equivalence but now the user is required to indicate the rewrite location with a pattern to recover a reasonable execution time:

```
move => [p' y]. rewrite /MRel /Mstate (negbTE (root_internal _)) [_ && _]/=.
rewrite [in X in X <-> _]orbF.
```

This kind of slow down is however rather exceptional.

There are a few more compilation errors but they are similar to the ones we explained so far.

5 Conclusion

This document illustrated the port of a typical MATHCOMP development to MATHCOMP 2. We reviewed the available tools (documentation and HB tactics) and went through a number of concrete sample errors and warnings that we explained and corrected.

The example of porting COMPDECMODAL demonstrated that the use of HB improves the CoQ scripts in terms of readability and even allows for improvements. The complete fix can be found online, it required the edition of 10 files, 35 insertions, and 67 deletions, which arguably represents a moderate amount of work.

The migration process to MATHCOMP 2 will surely generate more questions that the community is ready to answer via the math-comp streams on https://coq.zulipchat.com.

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