#### Summary

| Sample Mean                      | Sample Proportion       | Sample Variance                              | Sample Standard Deviation | Z-Score                   |
|----------------------------------|-------------------------|--|---------------------------|---------------------------|
| $\bar{X} = \frac{1}{n} \sum x_i$ | $\hat{p} = \frac{k}{n}$ | $S^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$ | $S = \sqrt{S^2}$          | $Z = \frac{X - Mean}{SD}$ |

### Probability

| General Addition Rule $P(E \cup F) = P(E) + P(F) - P(E \cap F)$                              | General Multiplication Rule $P(E \cap F) = P(E F)P(F)$ |
|--|--|
| Two Independent Events   | Law of Total Prob.                                     |
| $P(E \cap F) = P(E)P(F)$   | $P(E) = P(E B_1)P(B_1) + + P(E B_k)P(B_k)$             |
| Bayes' Rule  | Complement Rule  |
| $P(B_i E) = \frac{P(E B_i)P(B_i)}{P(E B_1)P(B_1) + P(E B_2)P(B_2) + \dots + P(E B_k)P(B_k)}$ | P(E') = 1 - P(E)                                       |
| Permutation  | Combination  |
| $P_k^n = \frac{n!}{(n-k)!}$  | $C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!}$           |

### Random Variables and Probability Distribution

| Expected Value (Discrete)                           | Variance (Discrete)   | Z-score                                   |
|---|---|---|
| $E[X] = \mu = \sum xp(x)$                           | $\sigma^2 = \sum (x - \mu)^2 p(x)$  | $Z = \frac{X - \mu}{\sigma}$              |
| Expected value (Binomial)                           | Variance (Binomial)   | Probability (Binomial)                    |
| $\mu = np$  | $\sigma^2 = np(1-p)$  | $p(x) = \binom{n}{k} p^k (1-p)^{n-k}$     |
| Expected value (Geometric)                          | Variance (Geometric)  | Probability (Geometric)                   |
| $\mu = \frac{1}{p}$                                 | $\sigma^2 = \frac{1-p}{p^2}$  | $p(x) = (1-p)^{x-1}p, x = 1, 2, 3, \dots$ |
| $E[y] = E[a_1x_1 + a_2x_2 + + a_nx_n]$              | $Var(y) = Var(a_1x_1 + a_2x_2 + + a_nx_n)$  | Constant, $a_o$                           |
| $\mu_y = a_1 \mu_1 + a_2 \mu_2 + \dots + a_n \mu_n$ | $\sigma_y^2 = b_1^2 \sigma_1^2 + b_2^2 \sigma_2^2 + \dots + b_n^2 \sigma_n^2; \ \sigma_y = \sqrt{Var(y)}$ | $E[a_o] = a_o; Var(a_o) = 0$              |

### Probability in density curves

| Rule      | SD within the mean | Area under the curve            |
|-----------|--------------------|---------------------------------|
| Empirical | 1                  | 68%                             |
| Empirical | 2                  | 95%                             |
| Empirical | 3                  | 99.7%                           |
| Chebyshev | k                  | $\geq (1 - \frac{1}{k^2})100\%$ |

# Normal Approximation to a Binomial Distribution

| X is Binomial      | X is Normal              |
|--------------------|--------------------------|
| P(a < X < b)       | P(a + 0.5 < X < b - 0.5) |
| $P(a \le X \le b)$ | P(a - 0.5 < X < b + 0.5) |

# Sampling Distribution

| Statistic | Mean                  | Standard Error                               | Distribution        |
|-----------|-----------------------|--|---------------------|
| $\bar{X}$ | $\mu_{\bar{x}} = \mu$ | $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ | Normal              |
| $\bar{X}$ | $\mu_{\bar{x}} = \mu$ | $\sigma_{ar{x}} = rac{S}{\sqrt{n}}$         | T with $df = n - 1$ |
| $\hat{p}$ | $\mu_{\hat{p}} = p$   | $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ | Normal              |

#### **Parameter Estimation**

| Parameter       | Point estimator         | Confidence Interval   | Additional   |
|-----------------|-------------------------|---|--|
| p               | $\hat{p}$               | $\hat{p}\pm Z_{crit}\sqrt{rac{\hat{p}(1-\hat{p})}{n}}$   | $n = p(1-p)(\frac{1.96}{B})^2$ at 95%  |
| $\mu$           | $ar{X}$                 | $\bar{X} \pm Z_{crit} \frac{\sigma}{\sqrt{n}} \text{ or } \bar{X} \pm t_{crit} \frac{S}{\sqrt{n}}$                    | $n = (\frac{1.96\sigma}{B})^2 at 95\%$   |
| $p_1 - p_2$     | $\hat{p}_1 - \hat{p}_2$ | $(\hat{p}_1 - \hat{p}_2) \pm Z_{crit} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ |  |
| $\mu_1 - \mu_2$ | $\bar{x}_1 - \bar{x}_2$ | $(\bar{x}_1 - \bar{x}_2) \pm t_{crit} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$                                   | $df = \frac{(V_1 + V_2)^2}{\frac{V_1^2}{n_1 - 1} + \frac{V_2^2}{n_2 - 1}}; V_1 = \frac{S_1^2}{n_1} \text{ and } V_2 = \frac{S_2^2}{n_2}$ |

## Hypothesis Testing

| Testing                     | Test Statistic   | $\sigma$ | Degrees of Freedom  | Additional  |
|-----------------------------|--|----------|---|---|
| $\mu$                       | $Z = \frac{\bar{X} - \mu_o}{\sigma / \sqrt{n}}$  | known    |   |   |
| $\mu$                       | $t = \frac{\bar{X} - \mu_o}{S/\sqrt{n}}$   | unknown  | n - 1   |   |
| p                           | $Z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1 - p_o)}{n}}}$  | •        | ·   |   |
| $\mu_1 - \mu_2$             | $t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_{1o} - \mu_{2o})}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$             | unknown  | $\frac{(V_1 + V_2)^2}{\frac{V_1^2}{n_1 - 1} + \frac{V_2^2}{n_2 - 1}}$ | $V_1 = \frac{S_1^2}{n_1}$ and $V_2 = \frac{S_2^2}{n_2}$       |
| $p_1 - p_2$                 | $Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_c(1 - \hat{p}_c)}{n_1} + \frac{\hat{p}_c(1 - \hat{p}_c)}{n_2}}}$ |          |   | $\hat{p}_c = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$ |
| Goodness of Fit             | $X^2 = \sum \frac{(O-E)^2}{E}$   |          | k - 1   | $E = np_{io}$   |
| Homogeneity or Independence | $X^2 = \sum_{i} \sum_{j} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$   |          | (I - 1)(J - 1)  | $E_{ij} = \frac{n_{i.}n_{.j}}{n}$                             |
|                             | $X^2 = \sum_{i} \sum_{j} \frac{n_{ij}^2}{E_{ij}} - n$  |          | (I - 1)(J - 1)  | $E_{ij} = \frac{n_{i.}n_{.j}}{n}$                             |

# Simple Linear Regression: Inferential Methods

| $S_e$                    | $S_b$                       | $S_{xx}$               | Conf. Interval of b | Test Statistic for b  |
|--------------------------|-----------------------------|------------------------|---------------------|-----------------------|
| $\sqrt{\frac{SSE}{n-2}}$ | $\frac{S_e}{\sqrt{S_{xx}}}$ | $\sum (x - \bar{x})^2$ | $b \pm t_{crit}S_b$ | $\frac{b-eta_o}{S_b}$ |

## Summarizing Bivariate Data

| Correlation                    | y-intercept                    | slope  | Sum Sq Error                 | Sum Sq Total                  |
|--------------------------------|--------------------------------|--|------------------------------|-------------------------------|
| $r = \frac{\sum Z_x Z_y}{n-1}$ | $a = \bar{Y} - b\bar{X}$       | $b = \frac{\sum (x - \bar{X})(y - \bar{Y})}{\sum (x - \bar{X})^2}$ | $SSE = \sum (y - \hat{y})^2$ | $SSTo = \sum (y - \bar{Y})^2$ |
| Coef. Determination            | Standard Error                 |  |                              |                               |
| $r^2 = 1 - \frac{SSE}{SSTo}$   | $S_e = \sqrt{\frac{SSE}{n-2}}$ |  |                              |                               |

#### **ANOVA**

| Source    | df    | SS   | MS                | F                 |
|-----------|-------|--|-------------------|-------------------|
| Treatment | k - 1 | $\sum_{i=1}^k n_i (\bar{x}_i - \bar{x})^2$ | $\frac{SST}{k-1}$ | $\frac{MST}{MSE}$ |
| Error     | n - k | $\sum_{i=1}^{k} (n_i - 1) S_i^2$           | $\frac{SSE}{n-k}$ |                   |
| Total     | n - 1 | SST + SSE                                  |                   |                   |