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Practical Data Science
(Optimization)

Problem 1: Applying gradient descent variations for ML objective function

As we discussed in class, machine learning objective functions are defined based on error or loss induced by train data while learning. Assume the following objective function and train data for learning.

Objective function:

$$E(w_0, w_1) = \frac{1}{n}\sum_{i=1}^{n}\left(y^{(i)} - w_0 - w_1 x_1^{(i)}\right)^2$$

Train data:

S.no	x1	y
1	10	40
2	30	60
3	40	70
4	50	90
5	60	80
6	70	70

Do the following:

a) Compute the gradient of E.

Applying batch gradient descent Algorithm

b) Write the expressions for weight updates using batch gradient descent algorithm.
c) Simulate(by hand) the batch gradient descent algorithm for 5 iterations. For each iteration, show the error value and updated weights. Assume initial values of (w0,w1) as (0,0).

Applying stochastic gradient descent Algorithm

d) Write the expressions for weight updates using stochastic gradient descent algorithm.
e) Simulate(by hand) the stochastic gradient descent algorithm for 4 iterations. For each iteration, show the error value and updated weights. Assume initial values of (w0,w1) as (0,0) and random samples picked are 3, 2, 5, 1.

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Applying stochastic minibatch gradient descent Algorithm

f) Write the expressions for weight updates using stochastic minibatch gradient descent algorithm.

g) Simulate(by hand) the stochastic mininbatch gradient descent algorithm for 3 iterations. For each iteration, show the error value and updated weights. Assume initial values of (w0,w1) as (0,0) and random batches of size 3 as follows(1,4,5), (2,1,6), (3,4,5) across iterations.

Problem 2: Find the least amount of fencing

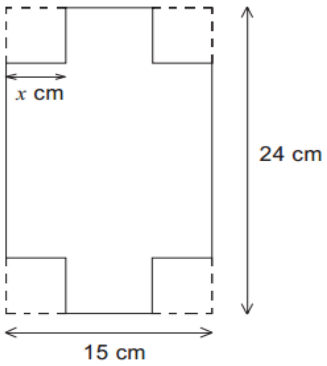
A rectangular paddock is having an area of 50 m^2. One side of the rectangle is straight wall as shown below and the remaining three sides are to be made from wire fencing. Do the following:

a. Find the expression for required fencing
b. Find the least amount of fencing required using calculus approach.

Problem 3: Design maximum volume open rectangular box

The diagram below shows a 24cm by 15cm sheet of cupboard from which a square of side x cm has been removed from each corner. The cardboard is then folded to form an open rectangular box of depth x cm and volume of v cm3. Do the following:

a. Find the expression for volume
b. Find value of x for which volume is maximum using calculus approach



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Problem 4: Design of optimal sized petrol tank

An emergency petrol tank is designed to carry 1 gallon of petrol(4546 cm^3). Its shape can be considered to be cuboid as shown below. The base of the cuboid is a rectangle with the length double the width. Do the following:

a. Find the expression for surface area of tank
b. Find the dimensions of tanks that minimizes the required surface area using calculus approach