Fermat's Last Theorem

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Chapter 1

Applications of Jensen's inequality

In this chapter, h denotes the function $h(x) := x \log \frac{1}{x}$ for $x \in [0,1]$.

Lemma 1.1 (Concavity)**.** *h is strictly concave on $[0,1]$. NOTE: right now only concavity is established.*

Proof. Check that h' is strictly monotone decreasing. □

Lemma 1.2 (Jensen)**.** *If S is a finite set, and $\sum_{s \in S} w_s = 1$ for some non-negative w_s, and $p_s \in [0,1]$ for all $s \in S$, then*

$$\sum_{s \in S} w_s h(p_s) \leq h(\sum_{s \in S} w_s p_s).$$

Proof. Apply Jensen and Lemma **??**. □

Lemma 1.3 (Converse Jensen)**.** *If equality holds in the above lemma, then $p_s = \sum_{s \in S} w_s h(p_s)$ whenever $w_s \neq 0$.*

Proof. Need some converse form of Jensen, not sure if it is already in Mathlib. □