FCS Week 10 Lecture Note

Notebook:	Fundamentals of Computer Science
Created:	2021-04-13 10:30 AM
Updated:	2021-05-01 12:18 PM
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Cornell Notes	**Topic:** Regular Languages: Part 2	Course: BSc Computer Science Class: CM1025 Fundamentals of Computer Science[Lecture] Date: May 01, 2021

Essential Question:

What is a regular expression and/or language?

Questions/Cues:

- What are closure properties?
- What are non-regular languages?
- What is pumping lemma?
- How do we prove a language is non-regular using the pumping lemma?

What we know

- A language is regular if it can be accepted by a finite automaton
- A language is regular if it can be accepted by a regular expression
- Every finite language is regular

Another way?

- Break it into smaller regular languages
- Then build it up again using the closure properties.

Closure Properties

- Theorem: If L_1 and L_2 are regular languages on alphabet Σ, then the following languages are also regular:
  - $U - L_1$. This means $\Sigma^* - L_1$ or the complement of L_1
  - $L_1 \cup L_2$. This means the union of L_1 and L_2
  - $L_1 \cap L_2$. This means the intersection of L_1 and L_2
  - $L_1 L_2$. This means the product of L_1 and L_2
  - ${L_1}^*$. This means the Kleene star of L_1.

Examples of non-regular language

- $L = \{a^n b^n | n \in \mathbb{N}\}$
- $L = \{xx | x \in \{\mathrm{a, b}\}^* \}$
- $L = \{a^{n!} | n \in \mathbb{N}\}$
- $L = \{xx^R | x \in \{\mathrm{a, b}\}^* \}$
- $L = \{a^{n^2} | n \in \mathbb{N}\}$
- $L = \{a^n | n \in \mathbb{N}, n \text{ is a prime number}\}$

Using closure properties – intersection

- Prove $L = \{x \in \{a,b\}^* | \#a \text{ in } x = \#b\ in\ x\}$ is not regular
- $L = \{ab, aabb, abab, abba, baab, \dots\}$
- Proof: Let's assume L is regular
- We know $L' = \{x\ \in a^*b^*\}$ is regular
- We know if L and L' are regular so is $L \cap L'$
- $L \cap L' = \{a^n b^n | n\ \in \mathbb{N}\}$
- Contradiction!

Not regular

Using closure properties – complement

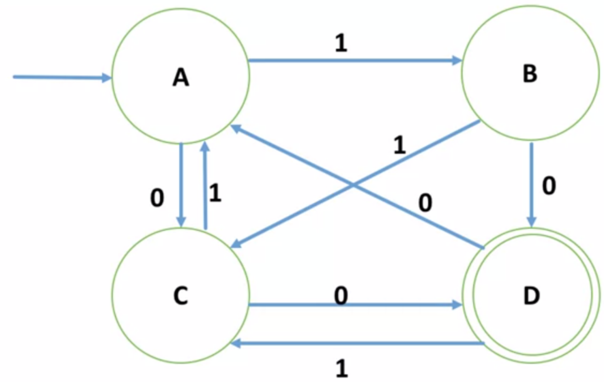
- Prove \$L = \{a^i b^j \mid i, j\ \in \mathbb{N}, i \neq j\}$ is not regular
- $L = \{abb, abbb, abbbb, aab, aabbb, \dots\}$
- Proof: Let us assume L is regular so is $\neg L$
- We know $\neg L = \{\ a^n b^n\} \cup$ non-bitonic is regular
- We know $L' = \{x\ \in a^*b^*\}$ is regular
- We know if $\neg L$ and L' are regular so is $\neg \mathrm{L} \cap \mathrm{L}'$.
- $\neg L \cap L' = \{a^n b^n | n\ \in \mathbb{N}\}$
- Contradiction!

Not regular

Proving a language is not regular

- 11
- 00
- 101
- No repeating states

- 1011- **A**BDC**A**
- 110010- **AB**CD**AB**D
- Repeating states



If length of the input $\geq$ number of states, then there are repeating states

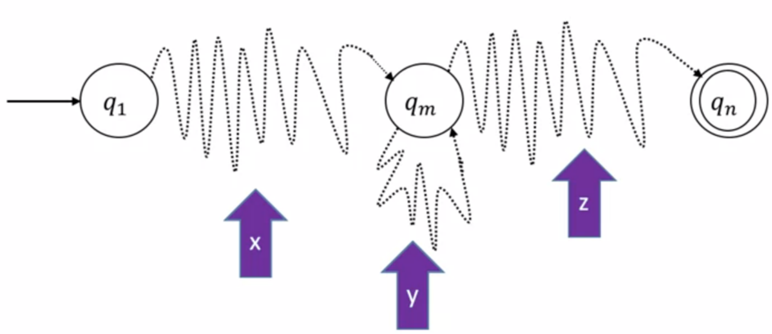
Pumping Lemma

If L is a regular language, then there is a number $\boldsymbol{p}$ (the pumping length) where, if **s is any string in L of length at least p**, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

- *for each* $i \geq 0,\ xy^iz \in L$
- $|y| > 0$ *and*
- $|xy| \leq \boldsymbol{p}$.

What does it mean?

- If the language is finite, it is regular
- We choose $\boldsymbol{p}$ to be the number of states in the FA representing L
- If $|s| \geq \boldsymbol{p}$, s must have a repeated state **(Pigeonhole Principle)**



- Note if all s's in the language have a limited length then the language is finite

Prove $L = \{a^n b^n | n \in \mathbb{N}\}$ is not regular

- Assume ***L* is regular**. Let **p** be the pumping length.
- Let $s = a^p b^p$, $|s| > p$
- Pumping Lemma: $s = xyz$
- For any i, $xy^i z \in L$. Let us try ***i=2***
- Cases:
  1) y is only *a's*. xyyz will have more *a's* than *b's*.
  2) y is only *b's*. xyyz will have more *b's* than *a's*.
  3) y has *a's* and *b's*. xyyz will have *a's* and *b's* jumbled up.

Contradiction! *L* is not regular

Example: $L = \{xx | x \in \{a, b\}^*\}$

- Assume ***L* is regular**. Let **p** be the pumping length.
- Let $s = a^p b a^p b$, $|s| > p$
- Pumping Lemma: $s = xyz$
- For any i, $xy^i z \in L$. Let's try ***i=2***
- The third condition: $|xy| \leq p$
- So $y = a^q, q \leq p$
- $xyyz = a^{p+q} b a^p b \notin L$
- $xyyz \notin L$

Contradiction! *L* is not regular

Summary

In this week, we learned about the pumping lemma.