# 2.2 More about Functions-Reading

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| **Cornell Notes** | **Topic:** 2.2 More about functions-Reading | Course: BSc Computer Science<br>Class: Discrete Mathematics-Reading<br>Date: October 29, 2019 |
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**Essential Question:**

What is function composition? Alongside this, what are the floor, ceiling and partial functions ?

**Questions/Cues:**

- What is function composition?
- What is the identify function in terms of function composition?
- What is the graph of a function?
- What are floor and ceiling functions?
- What is a partial function?

Notes

- Let $g: A \rightarrow B$ and $f: B \rightarrow C$, composition of f and g denoted for all $a \in A$ by $f\ o\ g$ is: $(f\ o\ g)(a) = f(g(a))$
    - In this case $f\ o\ g$ cannot be unless range of g is subset of domain f
    - composition of functions not commutative
- Identity function :

$$(f^{-1} o\, f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a$$
$$(f\, o\, f^{-1})(b) = f(f^{-1}(b)) = f(a) = b$$

- That is, $(f^{-1})^{-1} = f$

Let $f$ be a function from the set $A$ to the set $B$. The *graph* of the function $f$ is the set of ordered pairs $\{(a, b) \mid a \in A \text{ and } f(a) = b\}$.

The *floor function* assigns to the real number $x$ the largest integer that is less than or equal to $x$. The value of the floor function at $x$ is denoted by $\lfloor x \rfloor$. The *ceiling function* assigns to the real number $x$ the smallest integer that is greater than or equal to $x$. The value of the ceiling function at $x$ is denoted by $\lceil x \rceil$.

- Floor function often also called greatest integer function, denoted by [ x ]

**TABLE 1 Useful Properties of the Floor and Ceiling Functions.**
($n$ is an integer, $x$ is a real number)

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| (1a) | $\lfloor x \rfloor = n$ if and only if $n \le x < n+1$ |
| (1b) | $\lceil x \rceil = n$ if and only if $n-1 < x \le n$ |
| (1c) | $\lfloor x \rfloor = n$ if and only if $x-1 < n \le x$ |
| (1d) | $\lceil x \rceil = n$ if and only if $x \le n < x+1$ |
| (2) | $x - 1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x+1$ |
| (3a) | $\lfloor -x \rfloor = -\lceil x \rceil$ |
| (3b) | $\lceil -x \rceil = -\lfloor x \rfloor$ |
| (4a) | $\lfloor x+n \rfloor = \lfloor x \rfloor + n$ |
| (4b) | $\lceil x+n \rceil = \lceil x \rceil + n$ |

A *partial function* $f$ from a set $A$ to a set $B$ is an assignment to each element $a$ in a subset of $A$, called the *domain of definition* of $f$, of a unique element $b$ in $B$. The sets $A$ and $B$ are called the *domain* and *codomain* of $f$, respectively. We say that $f$ is *undefined* for elements in $A$ that are not in the domain of definition of $f$. When the domain of definition of $f$ equals $A$, we say that $f$ is a *total function*.

- We write $f: A \to B$, denoting f is partial function from A to B, same notation used for functions, context is different; determines whether f is a partial or total function

EXAMPLE 32 The function $f : \mathbf{Z} \to \mathbf{R}$ where $f(n) = \sqrt{n}$ is a partial function from $\mathbf{Z}$ to $\mathbf{R}$ where the domain of definition is the set of nonnegative integers. Note that $f$ is undefined for negative integers. ◄

## Summary

In this week, we learned what function composition is and what is means for a function to be partial. Also, we looked at the floor and ceiling functions.