

## Week 7 Angles, Triangles, Trigonometry Lecture Note

Notebook: Computational Mathematics

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### Cornell Notes

Angles, triangles,  
trigonometry

Course: BSc Computer Science

Class: Computational  
Mathematics[Lecture]

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### Essential Question:

What are angles and what is trigonometry and how are these related to the study of triangles?

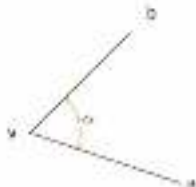
### Questions/Cues:

- What is an angle?
- What are some special types of angles, how do we measure radians and how do we convert between radians/degrees?
- What are some properties of triangles?
- What are similar triangles?
- What are properties of right triangles?

### Notes

#### What is an angle?

- It is a measure of the separation of two rays emanating from a vertex  $v$



- It is measured in degrees (sexagesimal) or radians

- 1 Degree is  $1/180$  of a flat angle 

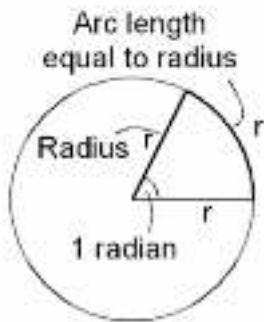
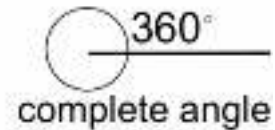
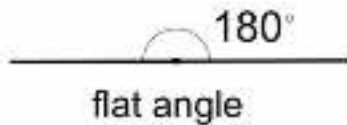
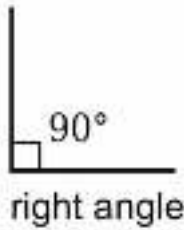
- 1 min is  $1/60$  of a degree, 1 sec is  $1/60$  of min

- ex.  $35^\circ 23' 12''$

- A flat angle is just the separation between two rays that emanate from the vertex and they depart in opposite direction. A flat angle corresponds to  $180^\circ$

- When working with degrees, we are working in base 60

## Types of angle



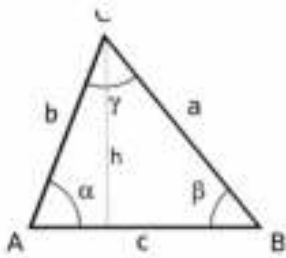
$$r \rightarrow 1 \text{ radian} \Rightarrow (\text{circ.}) 2\pi r \rightarrow 2\pi \text{ radians}$$

$$360^\circ = 2\pi \text{ radians}$$

$$\text{radians} = \text{degrees} \times \pi / 180^\circ$$

- The complete angle is the angle that is formed by two rays emanating from a vertex when they coincide
- Angles are periodic, in degree they periodical of  $360^\circ$

## Triangles: properties

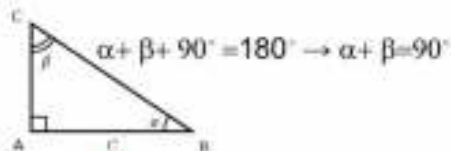


Triangle: Right trian.

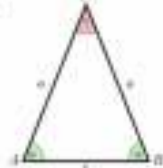
$$\alpha + \beta + \gamma = 180^\circ$$

$$S = c \times h / 2$$

$$P = a + b + c$$



Isosceles



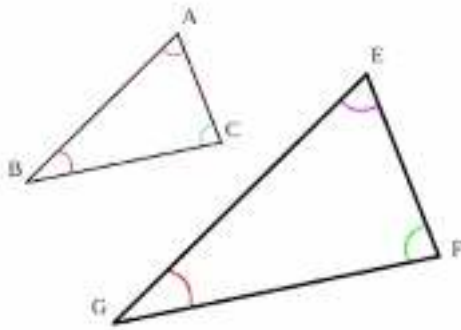
Equilateral



$$3\alpha = 180^\circ \rightarrow \alpha = 60^\circ$$

- In an isosceles triangle, two sides are the same length and the angles adjacent to the non-equal side are equal
- Similar Triangles = have the same angles but they have sides which are rescaled by the same rescaling factor; they have the same angles and proportional sides. This means the ratio between each side of one triangle and the corresponding side of the similar triangle are the same

# Similar Triangles

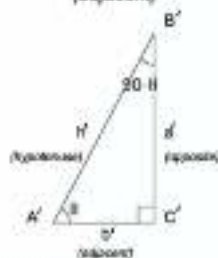
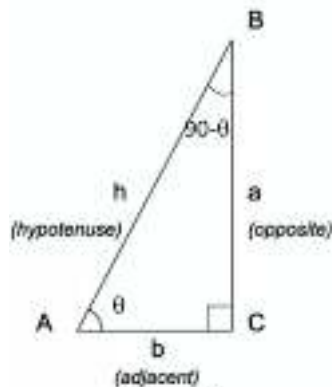


Similar Triangles  
rescale one (zoom in or out) and  
will coincide with the other

Same angles, proportional sides:

$$\rightarrow AB/EG = AC/EF = BC/GF$$

## Right Triangles: properties



$$h/h' = b/b' \rightarrow b/h = b'/h' = \cos(\theta) = \sin(90-\theta)$$

$$h/h' = a/a' \rightarrow a/h = a'/h' = \sin(\theta) = \cos(90-\theta)$$

$$a/a' = b/b' \rightarrow a/b = a'/b' = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$$

$$1) \sin(\theta) = a/h = \frac{\text{opposite}}{\text{hypotenuse}} \rightarrow a = h \sin(\theta)$$

$$2) \cos(\theta) = b/h = \frac{\text{adjacent}}{\text{hypotenuse}} \rightarrow b = h \cos(\theta)$$

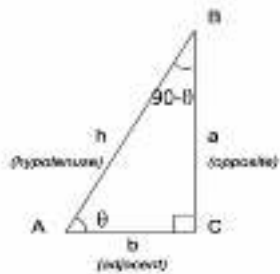
$$3) \tan(\theta) = a/b = \frac{\text{opposite}}{\text{adjacent}} \rightarrow a = b \tan(\theta)$$

$$a^2 + b^2 = h^2 \text{ Pythagora's theorem}$$

From 1) and 2) it follows

$$h^2 \sin^2(\theta) + h^2 \cos^2(\theta) = h^2$$

$$\rightarrow \sin^2(\theta) + \cos^2(\theta) = 1$$



$$\rightarrow a = h \sin(\theta)$$

$$\rightarrow b = h \cos(\theta) = h \sin(90^\circ - \theta)$$

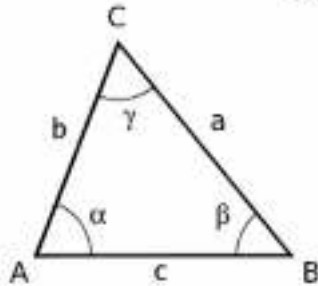
what if  $\theta \rightarrow 0$ ?  $h \rightarrow b$  and  $a \rightarrow 0$

$$\rightarrow \cos(0^\circ) = \sin(90^\circ) = 1, \quad \sin(0^\circ) = \cos(90^\circ) = 0$$

$$\rightarrow \frac{a}{\sin(\theta)} = \frac{b}{\sin(90^\circ - \theta)} = \frac{h}{\sin(90^\circ)}$$

*Sine rule*

## General triangle



$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

generalized *Pithagora's th.*

$$\rightarrow a^2 = b^2 + c^2 - 2bc \cos(\alpha)$$

(a.k.a. cosine rule)

## Summary

In this week, we learned about what an angle is. Alongside this, we looked at properties of triangles, Pythagoras theorem, the laws of sines and cosines