FCS Week 2 Reading Note

Notebook:	Fundamentals of Computer Science
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Cornell Notes	**Topic:** Logic: Part 2	Course: BSc Computer Science Class: CM1025 Fundamentals of Computer Science[Reading] Date: April 14, 2021

Essential Question:

What is propositional logic?

Questions/Cues:

- What are Quantifiers?
- What is a Predicate?
- What is the universe of discourse?
- How are De Morgan's Laws applied to quantifiers?

Notes

- All people are mortal.
- Every computer is a 16-bit machine.
- No birds are black.
- Some people have blue eyes.
- There exists an even prime number.

Each contains a word indicating quantity such as *all, every, none, some,* and *one*. Such words, called **quantifiers**, give us an idea about how many objects have a certain property.

There are two different quantifiers. The first is *all*, the **universal quantifier**, denoted by $\forall$, an inverted A. You may read $\forall$ as *for all, for each,* or *for every*. The second quantifier is *some*, the **existential quantifier**, denoted by $\exists$, a backward E. You may read $\exists$ *for some, there exists a,* or *for at least one*. Note that the word *some* means *at least one*.

EXAMPLE 1.24 Let x be any apple. Then the sentence *All apples are green* can be written as *For every x, x is green*. Using the universal quantifier $\forall$, this sentence can be represented symbolically as $(\forall x)(x \text{ is green})$ or $(\forall x)P(x)$ where $P(x)$: x is green. (*Note*: x is just a dummy variable.) ■

Predicate

Here $P(x)$, called a **predicate**, states the property the object x has. Since $P(x)$ involves just one variable, it is a **unary** predicate. The set of all values x can have is called the **universe of discourse** (UD). In the above example, the UD is the set of all apples.

Note that $P(x)$ is *not* a proposition, but just an expression. However, it can be transformed into a proposition by assigning values to x. The truth value of $P(x)$ is predicated on the values assigned to x from the UD.

The variable x in the predicate $P(x)$ is a **free** variable. As x varies over the UD, the truth value of $P(x)$ can vary. On the other hand, the variable x in $(\forall x)P(x)$ is a **bound** variable, bound by the quantifier $\forall$. The proposition $(\forall x)P(x)$ has a fixed truth value.

A predicate may contain two or more variables. A predicate that contains two variables is a **binary** predicate. For instance, $P(x,y)$ is a binary predicate. If a predicate contains n variables, it is an ***n*-ary** predicate.

EXAMPLE 1.25 Rewrite the sentence *Some chalkboards are black*, symbolically.

SOLUTION:

Choose the set of all chalkboards as the UD. Let x be an arbitrary chalkboard. Then the given sentence can be written as:

There exists an x such that x is black.

Using the existential quantifier, this can be symbolized as $(\exists x)\mathrm{b}(x)$, where $\mathrm{b}(x)$: x is black. ■

The next example illustrates how to find the truth values of quantified propositions.

EXAMPLE 1.27 Rewrite each proposition symbolically, where UD = set of real numbers.

(1) For each integer x, there exists an integer y such that $x+y=0$.
(2) There exists an integer x such that $x+y=y$ for every integer y.
(3) For all integers x and y, $x \cdot y = y \cdot x$.
(4) There are integers x and y such that $x+y=5$.

SOLUTION:

(1) $(\forall x)((\exists y)(x+y=0))$, which is usually written as $(\forall x)(\exists y)(x+y=0)$.
(2) $(\exists x)(\forall y)(x+y=y)$
(3) $(\forall x)(\forall y)(x \cdot y = y \cdot x)$
(4) $(\exists x)(\exists y)(x+y=5)$ ■

The order of the variables x and y in $(\forall x)(\forall y)$ and $(\exists x)(\exists y)$ can be changed without affecting the truth values of the propositions. For instance, $(\forall x)(\forall y)(xy = yx) \equiv (\forall y)(\forall x)(xy = yx)$. Nonetheless, the order is important in $(\forall x)(\exists y)$ and $(\exists y)(\forall x)$. For example, let $P(x,y)$: $x < y$ where x and y are integers. Then $(\forall x)(\exists y)P(x,y)$ means *For every integer x, there is a suitable integer y such that* $x < y$; $y = x + 1$ is such an integer. Therefore, $(\forall x)(\exists y)P(x,y)$ is true. But $(\exists y)(\forall x)P(x,y)$ means *There exists an integer y, say, b, such that* $(\forall x)P(x,b)$; that is, *every integer x is less than b*. Clearly, it is false. Moral? The proposition $(Q_1x)(Q_2y)\mathrm{P}(x,y)$ is evaluated as $(Q_1x)[(Q_2y)\mathrm{P}(x,y)]$, where Q_1 and Q_2 are quantifiers.

De Morgan's laws

- $\sim[(\forall x)\mathrm{P}(x)] \equiv (\exists x)[\sim\mathrm{P}(x)]$
- $\sim[(\forall x)\mathrm{P}(x)] \equiv (\forall x)[\sim\mathrm{P}(x)]$

By virtue of these laws, be careful when negating quantified propositions. When you negate the universal quantifier $\forall$, it becomes the existential quantifier $\exists$; when you negate the existential quantifier, it becomes the universal quantifier. In Section 1.5, we discuss a nice application of the first law to disproving propositions.

EXAMPLE 1.29 Negate each proposition, where the UD = set of integers.

(1) $(\forall x)\ (x^2 = x)$ (2) $(\exists x)\ (|x| = x)$

SOLUTION:

- $$\sim[(\forall x)(x^2 = x)] \equiv (\exists x)[\sim(x^2 = x)] \equiv (\exists x)(x^2 \neq x).$$
- $$\sim[(\exists x)(|x| = x)] \equiv (\forall x)[\sim(|x| = x)] \equiv (\forall x)(|x| \neq x).$$

EXAMPLE 1.30 Negate each quantified proposition.

(1) Every computer is a 16-bit machine.
(2) Some girls are blondes.
(3) All chalkboards are black.
(4) No person has green eyes.

SOLUTION:
Their negations are:

(1) Some computers are not 16-bit machines.
(2) No girls are blondes.
(3) Some chalkboards are not black.
(4) Some people have green eyes.

Summary

In this week, we learned about quantifiers.