2.1 Introduction to Functions

Notebook: Discrete Mathematics [CM1020]

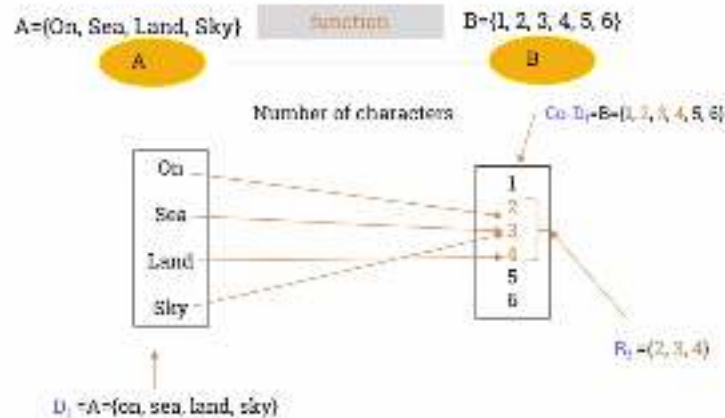**Created:** 2019-10-07 2:31 PM **Updated:** 2019-10-21 7:08 PM

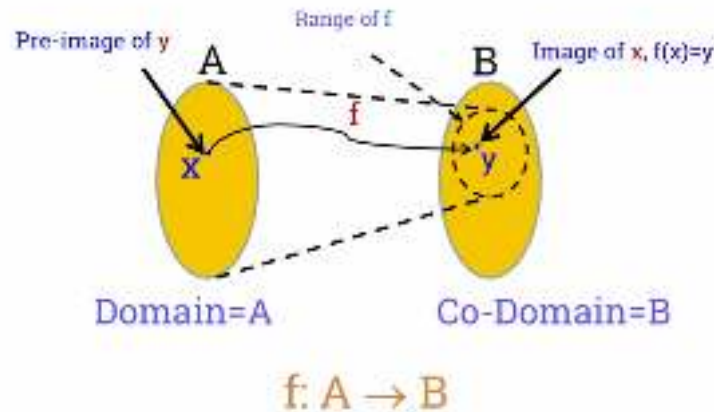**Author:** SUKHJIT MANN

Tags: Domain, Exponential, Injective, Linear, Quadratic, Range, Surjective

Cornell Notes	Topic: 2.1 Introduction to functions	Course: BSc Computer Science
		Class: Discrete Mathematics-Lecture
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Essential Question:

What is a function and what are its properties and how do we graph it?

Questions/Cues:

- What is a function?
- What is the formal definition of a function?
- What is the domain, co-domain and range of a function?
- What is a linear function?
- What is a Quadratic function?
- What is an Exponential function?
- What is an Injective function?
- What is an Surjective function?

Notes

- Function = A rule that relates how one quantity depends on other quantities, a relation between set of inputs and set of outputs, in which each input maps to exactly one output
    - function maps an element of set to an element in another set
- Formal Def of Function = function *f* from set *A* to set *B* is an assignment of exactly 1 element of *B* to each element of *A*
    - $f : A \longrightarrow B$
    - $x \in A : x \to f(x) = y \, (y \in B)$
- $x \in A \to f(x) = y \in B$; f : A → B
    1. A is set of inputs called domain of f, $D_f = A$
    2. B is set of containing outputs called co-domain of f, $co - D_f = B$
    3. set of all outputs called range of f, R_f
    4. Y called image of x, x called pre-image of y, $f(x) = y$

Domain, co-domain and range

A={On, Sea, Land, Sky}   function   B={1, 2, 3, 4, 5, 6}

A   B

Number of characters   Co-D$_j$-B={1, 2, 3, 4, 5, 6}

On
Sea
Land
Sky

1
2
3
4
5
6

R$_j$=(2, 3, 4)

D$_j$ =A={on, sea, land, sky}

Visualization

Range of f

Pre-image of y   A   B   Image of x, f(x)=y

f

X   y

Domain=A   Co-Domain=B

$$f: A \rightarrow B$$

- Linear function = of the form $f(x) = ax + b$
  - straight line function
  - passes through point (0, b)
  - a is gradient or slope
    - if a is positive, increasing function, $x_1 \leq x_2 \, then \, f(x_1) \leq f(x_2)$
    - if a is negative, then decreasing function,
      $x_1 \leq x_2 \, then \, f(x_1) \geq f(x_2)$
- Quadratic function = of the form $f(x) = ax^2 + bx + c$
  - where a, b, c are number with $a \neq 0$
- Exponential function = of the form $f(x) = b^x \, where \, b > 0 \, and \, b \neq 1$
  - variable b called the base of function
  - if b > 1, then exponential growth
  - if 0 < b < 1 , then exponential decay
    - (0,1) is common point for both graphs and x-axis is the horizontal asymptote for both graphs
    - Domain is set of all real numbers and range is > 0

$$b^x b^y = b^{x+y}$$

$$b^{-x} = \frac{1}{b^x}$$

$$\frac{b^x}{b^y} = b^{x-y}$$

$$(b^x)^y = b^{xy}$$

$$(ab)^x = a^x b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

- Injective function (one to one) = any two different inputs lead to two different outputs
  - $for\ all\ a, b \in A, if\ a \neq b\ then\ f(a) \neq f(b)$
  - Equivalently, $for\ all\ a, b \in A, if\ f(a) = f(b)\ then\ a = b$
- Surjective function (onto) = every element of the co-domain of f, B has at least one pre-image in domain of f, A
  - $for\ all\ y \in B\ there\ exists\ x \in A\ such\ that\ y = f(x)$
  - Equivalently, $D_f = R_f$

Summary

In this week, we learned what is function is, with what the domain, co-domain, and range of a function represent. Also we explored the equations and graphs of three common function, Linear, Quadratic and Exponential. Lastly, we looked at the injective and surjective functions.