2.2 More about Functions

Notebook:	Discrete Mathematics [CM1020]		
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Cornell Notes	**Topic:** 2.2 More about functions	Course: BSc Computer Science Class: Discrete Mathematics-Lecture Date: October 28, 2019

Essential Question:

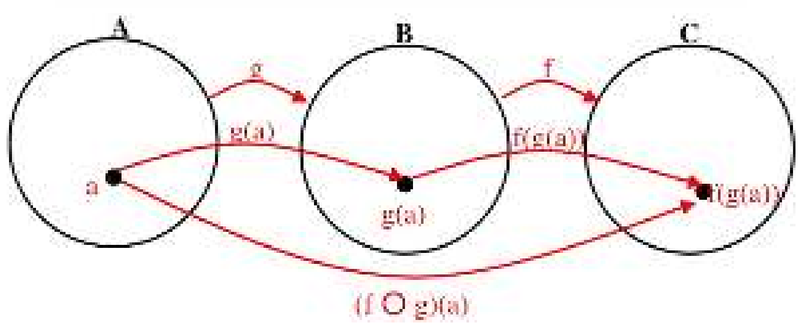
What is function composition, also what does it mean for a function to be bijective(invertible)? Alongside this, what are logarithmic, floor and ceiling functions?

Questions/Cues:

- What is function composition?
- What does it mean when a function is bijective or invertible?
- What is the identity function in terms of function composition?
- What can be said about the graphs of f and its inverse?
- What is logarithmic function and what is its inverse?
- What is the laws of logarithms?
- What is the graph of the logarithmic functions and what are some of it's properties?
- What is the floor function and its respective graph?
- What is the ceiling function and its respective graph?

Notes

- Given 2 functions f and g, $(f\ o\ g)(x) = f(g(x))$



- Function composition is not commutative! $f\,o\,g \neq g\,o\,f$
  - If we change order of f and g, we get different function
- Bijective or Invertible = if and only if it's both injective and surjective
  - Injective = one-to-one, for every x there is unique image in the co-domain
  - Surjective = onto, unique pre-image for each element in co-domain, and range = co-domain
- Inverse function = Let $f\colon A \to B$, if f is bijective (invertible), then inverse function f^{-1} exists and defined $f^{-1}\colon B \to A$
- $(f\,o\,f^{-1})(x) = (f^{-1}o\,f)(x) = x$

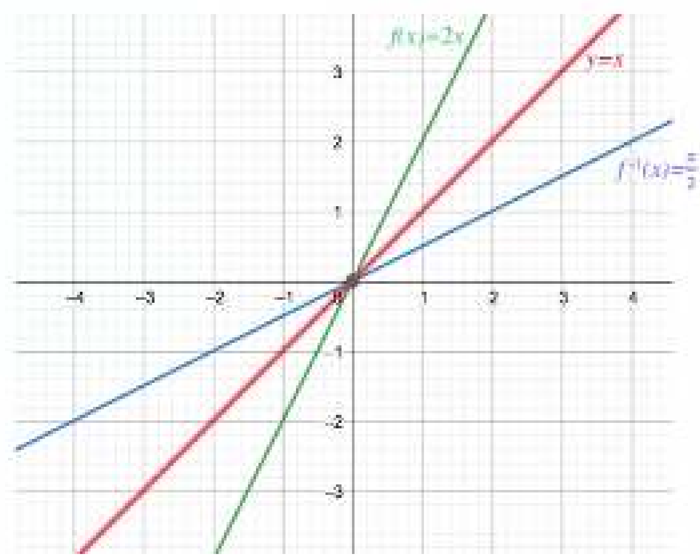
$$f\colon \mathrm{R} \to R \quad \textit{with } f(x)=2x$$

$$f^{-1}\colon \mathrm{R} \to R \quad \textit{with } f^{-1}(x)=\frac{x}{2}$$

$$(f \circ f^{-1})(\mathrm{x}) = f(f^{-1}(\mathrm{x})) = f\left(\frac{x}{2}\right) = 2\,\frac{x}{2} = x$$

$$(f^{-1} of)(\mathrm{x}) = f^{-1}(\mathrm{f(x)}) = f^{-1}(2x) = \frac{2x}{2} = x$$

The curves of f and f^{-1} *are symmetric with respect to the straight line* $y = x$.



- Logarithmic function = with base b, b > 0 and b $\neq$ 1 is defined:

$$log_b\, x = y \quad \text{if and only if} \quad x = b^y$$

$log_b\, x$ is the inverse function of the exponential function b^x

$$\log_b m * n = \log_b m + \log_b n$$

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

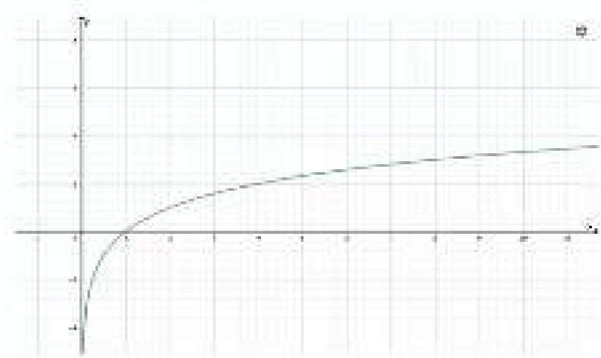
$$\log_b m^n = \text{n}\log_b m$$

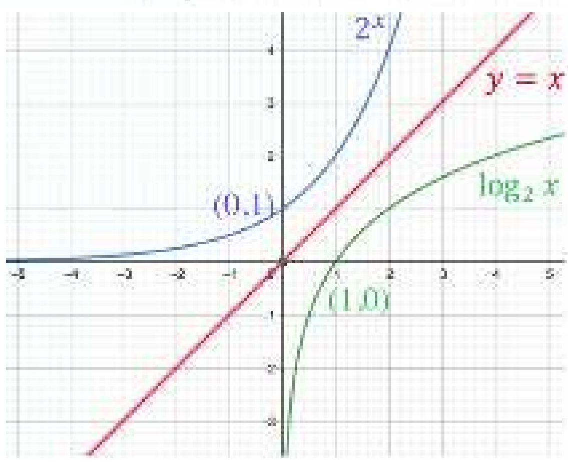
$$\log_b 1 = 0$$

$$\log_b b = 1$$

Consider **f(x) = $\log_2$ x**. We will create a table of values for **x** and f(x) and then sketch a graph of **f**.

x	1/8	1/4	1/2	1	2	4	8
f(x)=$\log_2$ x	-3	-2	-1	0	1	2	3





Log properties:

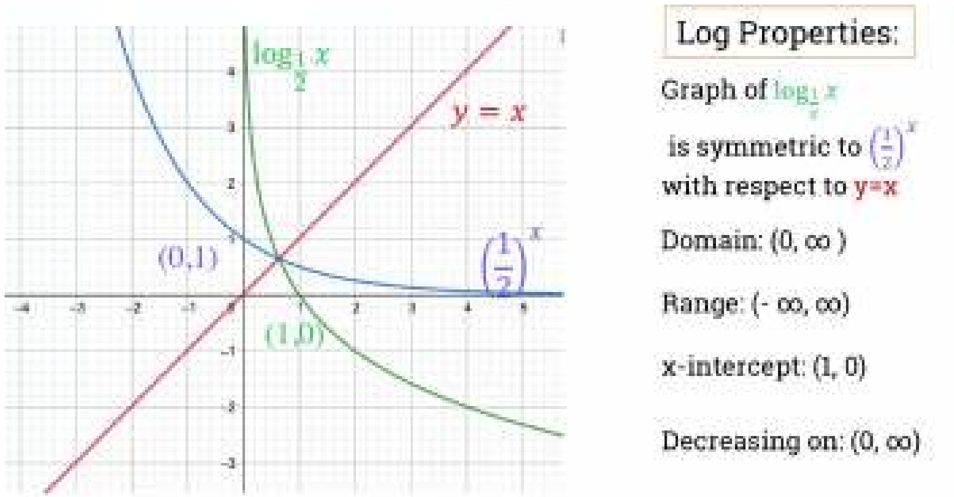
Graph of $\log_2 x$ is symmetric to 2^x with respect to y=x

Domain: (0, ∞)

Range: (- ∞, ∞)

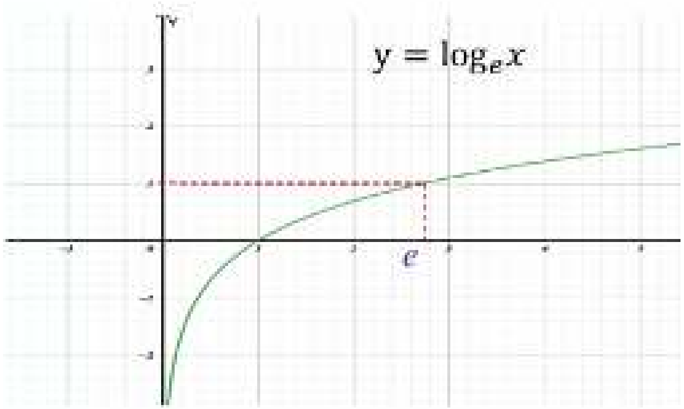
x-intercept: (1, 0)

Increasing on: (0, ∞)

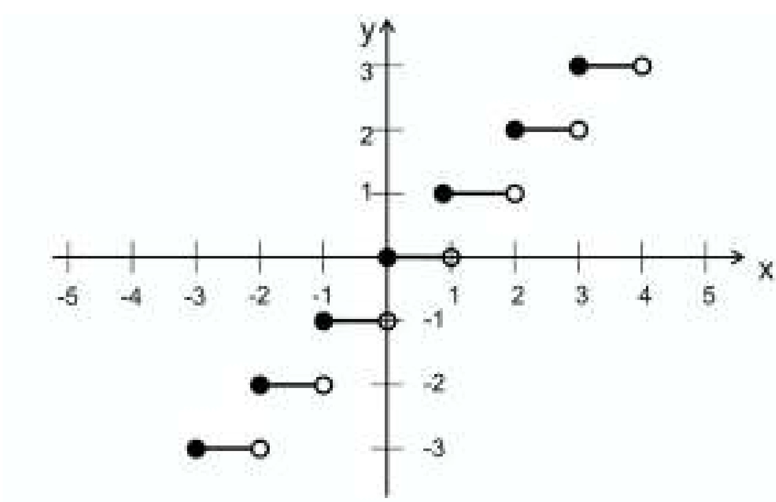


$$\ln x = \log_e x \quad \text{where } e = 2.71828$$

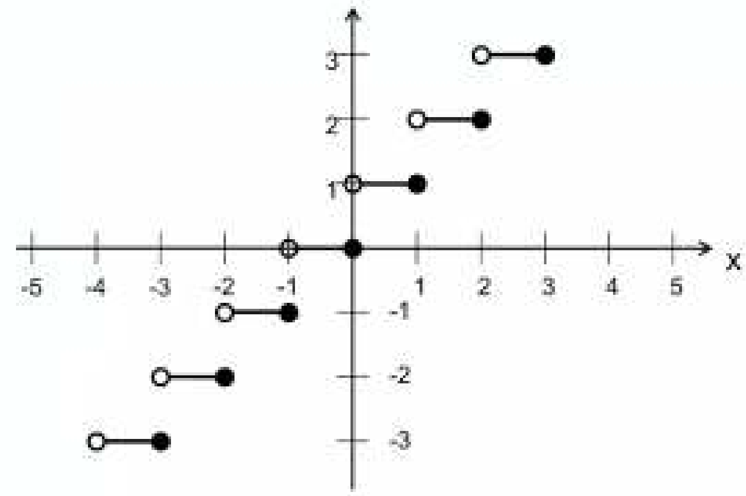
$$\ln e = \log_e e = 1$$



- Floor function = function $R \rightarrow Z$ takes real number x as input, returns largest integer that is less than or equal to x, denoted floor(x) = ⌊ x ⌋
  - Floor of any integer is itself



- Ceiling function = function $R \rightarrow Z$ takes real x as input, returns smallest integer that is greater than or equal to x, denoted ceiling(x) = ⌈ x ⌉
  - Ceiling of any integer is itself



Let n be an integer and x a real number. Show that:

$$\lfloor x+n \rfloor = \lfloor x \rfloor + n$$

Proof:

- Let $m = \lfloor x \rfloor$
- hence, $\mathbf{m \le x < m+1}$ (by definition)
- $\mathbf{m+n \le x+n < m+n+1}$
- this implies that $\mathbf{\lfloor x+n \rfloor = m + n}$ (by definition)
- hence, $\mathbf{\lfloor x+n \rfloor = \lfloor x \rfloor + n}$

Summary

In this week, we learned what function composition is and what is means for a function to be bijective (invertible). Also, we looked at the logarithmic, floor and ceiling functions.