FCS Week 4 Lecture Note

Notebook:	Fundamentals of Computer Science		
Created:	2021-04-13 10:30 AM	**Updated:**	2021-04-19 6:38 PM
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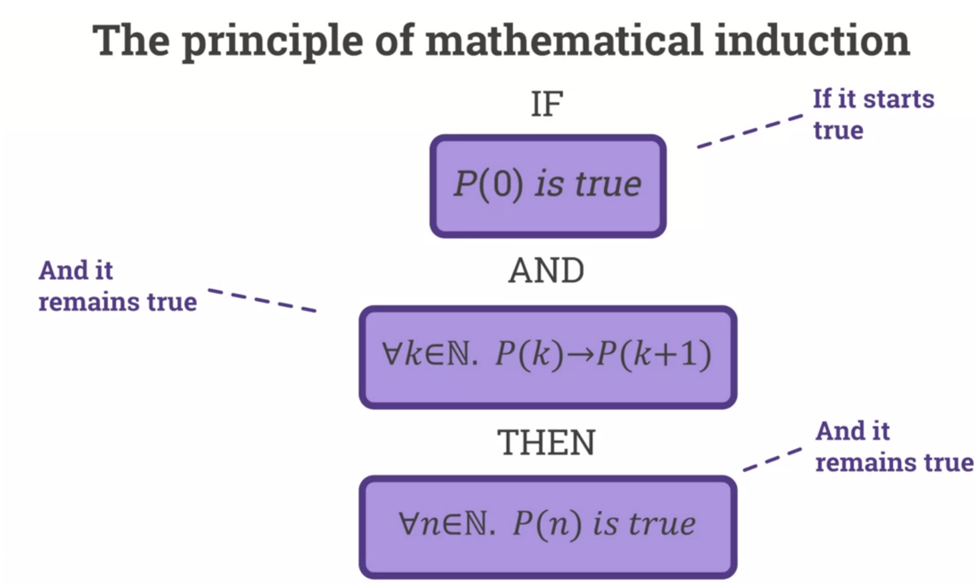
Cornell Notes	**Topic:** Proof Techniques: Part 2	Course: BSc Computer Science Class: CM1025 Fundamentals of Computer Science[Lecture] Date: April 19, 2021

Essential Question:

What is a proof?

Questions/Cues:

- What is mathematical induction?
- What are the three steps of induction?



What does it mean?

- P(0)
- $\forall k \in \mathbb{N}.\ P(k) \rightarrow P(k+1)$
- It is true at the beginning: for 0
- Since it is true for 0 then it is true for 0+1
- Since it is true for 1 then it is true for 1+1
- Since it is true for 2 then it is true for 2+1
- Since it is true for 3 then it is true for 3+1
- …

Three steps of Induction

- Prove P(0) is true
  - This step is called the Basis
- Prove If P(k) then P(k+1)
  - This step is called the inductive step
  - The assumption that P(k) is true is called inductive hypothesis
- Conclude, by induction, that P(n) is true for all n

Theorem: The sum of first n powers of 2 is $2^n - 1$

- $2^0 = 1 = 2^1 - 1$
- $2^0+2^1 = 1+2 = 3 = 2^2 - 1$
- $2^0+2^1+2^2 = 1+2+4 = 7 = 2^3 - 1$
- $2^0+2^1+2^2+2^3 = 1+2+4+8 = 15 = 2^4 - 1$
- $2^0+2^1+2^2+2^3+2^4 = 1+2+4+8+16 = 31 = 2^5 - 1$
- $2^0+2^1+2^2+2^3+2^4+2^5 = 1+2+4+8+16+32 = 63 = 2^6 - 1$

Theorem: The sum of first n powers of 2 is $2^n - 1$

- Let P(n) be $2^0+2^1+...+2^{n-1} = 2^n-1$
- Prove that P(n) is true for all n
- Basis: Prove P(1) is true. P(1): $2^0=1=2^1-1$
- Inductive Step: Prove $P(k) \rightarrow P(k+1)$
  - Assume P(k) is true, $2^0+2^1+...+2^{k-1} = 2^k-1$
  - Prove P(k+1) is true P(k+1): $2^0+2^1+...+2^{k-1}+2^k = 2^{k+1}-1$

$$\text{LHS} = \underbrace{2^0+2^1+...+2^{k-1}}_{=\,2^k-1} + 2^k$$

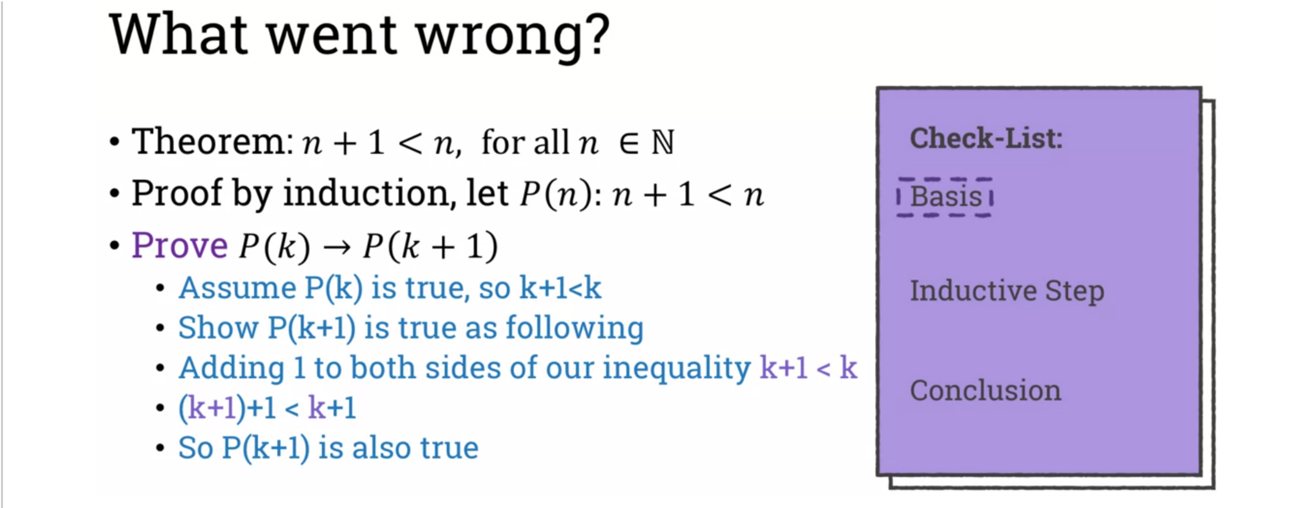
Inductive hypothesis $= 2^k-1 \quad +2^k$

Theorem: The sum of first n powers of 2 is $2^n - 1$

- Let P(n): $2^0+2^1+...+2^{n-1} = 2^n-1$
- Inductive Step: Prove $P(k) \rightarrow P(k+1)$
  - Assume P(k) is true, $2^0+2^1+...+2^{k-1} = 2^k-1$
  - LHS = $2^0+2^1+...+2^{k-1}+2^k$
    $= 2^k-1+2^k = 2^k + 2^k - 1 = 2^{k+1}-1$ = RHS
  - So P(k+1) is true
- Therefore P(n) is true for all n.

Theorem: $n < 3^n$, for all n, natural numbers

- Let P(n): $n < 3^n$
- Prove by induction that P(n) is true for all n
- Basis: Prove P(1) is true. $1 < 3^1$
- Inductive Step: Prove $P(k) \rightarrow P(k+1)$
  - Assume $P(k)$ is true: $k < 3^k$
  - Prove $P(k+1)$ is true $\equiv k+1 < 3^{(k+1)}$
  - $k+1 < 3^k + 1 < 3^k + 3^k + 3^k = 3.3^k = 3^{(k+1)}$
  - $k+1 < 3^{(k+1)}$ so P(k+1) is true



Summary

In this week, we learned about mathematical induction.