Binary:

2^0	1	2^4	16	2^8	256
2^1	2	2^5	32	2^9	512
2^2	4	2^6	64	2^{10}	1024
2^3	8	2^7	128		

Octal:

8^0	1
8^1	8
8^2	64
8^3	512
8^4	4096
8^5	32768

Hexadecimal:

Decimal	Hexadecimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

16^0	1
16^1	16
16^2	256
16^3	4096
16^4	65536

Non-integers

↳ binary to decimal

$1101.101 = 2^3+2^2+2^0+2^{-1}+2^{-3} = 13.625_{10}$

↳ decimal to binary

17.375

10001_2

0.375×2=0.75=0+0.75
0.75×2= 1.5 = 1+0.5
0.5 ×2= 1.0 = 1+0
0 → STOP ➡ $0.375_{10}=0.011_2$

10001.011_2

Decimal to Octal	Remainder
$3726 \div 8 = 465$ r 6	6
$465 \div 8 = 58$ r 1	1
$58 \div 8 = 7$ r 2	2
$7 \div 8 = 0$ r 7	7

read ↑ 7216

Operations:

$101 + 111 = 1100$

$110 - 101 = 001$

1100×1111

$1100 + 1100 + 1100 + 1100 = 10110100$

$11100110_2 \div 110_2 = ?$

110 | 11100110=
-110 → 1
10
-110 won't go → 0
100
-110 won't go → 0
1001
-110 → 1
0111
-110 → 1
Remainder→ 10 → 0

➡ $11100110_2 \div 110_2 = 100110_2$
With remainder 10
in decimal 230/6=38.333

Sequence $a: N \to X$

convergent, if $\lim_{n\to\infty} a_n = L < \infty$

else → divergent (goes to ∞ or oscillates)

defined
↳ explicit ex: $a_n = 2n+1$
↳ by recursion

↳ **Arithmetic** ex: (common difference) $a_n = d + a_{n-1} = a_0 + (n-1)d$

Series: $S_n = \sum_{i=0}^{n} a_i = \frac{n}{2}(2a_0 + (n-1)d)$

↳ **Geometric** $a_n = r \times a_{n-1} = a_0 r^{n-1}$ (r: common ratio)

$S_n = \sum_{i=0}^{n} a_i = \frac{a_0(1-r^n)}{1-r}$

Congruence

a and b are congruent mod k: $a \equiv b \pmod k$ → maps integers to $Min_k = \{0,1,\dots,k-1\}$

if they have the same remainder when divided by k

$a \bmod k = b \bmod k$

$a \overset{+}{\underset{\times}{}} b \bmod k = (a \bmod k \overset{+}{\underset{\times}{}} b \bmod k) \bmod k$

for negatives: $-17 \bmod 12 = 7$ (not -5)

for division ⇒ Multiplicative Inverse: $(m \times m^{-1}) \bmod k = 1$

ex: $\frac{1}{2} \bmod 7 = ?$ $= 2^{-1} \bmod 7$

try (2×0), (2×1), ⋮

$(2\times 4) \bmod 7 = 1$

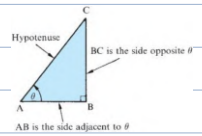
$2^{-1} \bmod 7 = 4$

$\frac{6}{2} \bmod 7 = ?$

$= (6 \times 2^{-1}) \bmod 7 = (6\times 4) \bmod 7 = 24 \bmod 7 = 3$

Trigonometry

Degree	Radian
180°	π



$\sin\theta = \frac{\text{side opposite to } \theta}{\text{hypotenuse}} = \frac{BC}{AC}$

$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{AB}{AC}$

$\tan\theta = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta} = \frac{BC}{AB}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

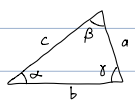
$\sin^2\theta + \cos^2\theta = 1$

$\cos(0°) = \sin(90°) = 1$

$\sin(0°) = \cos(90°) = 0$

$h^2 = a^2 + b^2$

for general triangle



sine rule:
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

cosine rule:
$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$

$\alpha + \beta + \gamma = 180°$

Functions

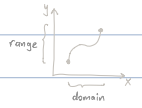
$f(x) = y$

x: independent

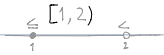
y: dependent

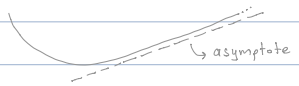
domain: possible values of x

range: " " " y



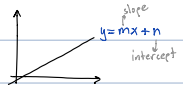
$[1, 2)$





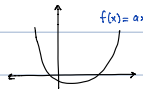
linear function

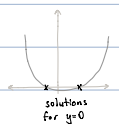
$y = mx + n$ (m: slope, n: intercept)



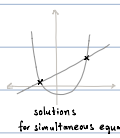
quadratic function

$f(x) = ax^2 + bx + c$





solutions for $y = 0$



solutions for simultaneous equations

$x^2 + px + q = 0$

$x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$

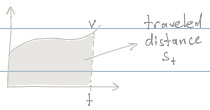
$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Motion

Velocity $v = \frac{\Delta s}{\Delta t}$

Acceleration $a = \frac{\Delta v}{\Delta t}$



$v_t = v_0 + at$

$s_t = v_0 t + \frac{1}{2} a t^2$

$s_t = \frac{1}{2} (v_0 + v_t) t$

$v_t^2 = v_0^2 + 2 a s_t$