Week 1 Number Bases Reading Notes

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Author:	SUKHJIT MANN		

Cornell Notes	**Topic:** Number Bases	Course: BSc Computer Science Class: Computational Mathematics[Reading] Date: April 22, 2020

Essential Question:

What is a number base and what are the different number systems available for use in computing?

Questions/Cues:

- What are the decimal numbers based upon?
- What is binary number system?
- What is octal number system?
- What is hexadecimal number system?

Notes

- Decimal numbers = based on 10 or powers of 10 and uses base 10, in which there are 10 digits 0-9 with placeholder value like "hundreds, tens and units (ones)" denoted by subscript 10

$$
\begin{aligned}
5192_{10} &= 5000 + 100 + 90 + 2 \\
&= 5(1000) + 1(100) + 9(10) + 2(1) \\
&= 5(10^3) + 1(10^2) + 9(10^1) + 2(10^0)
\end{aligned}
$$

- Binary number system = uses base 2, has only two digits 0 and 1. Numbers in binary are called binary digits or bits. Binary number system is based on powers of 2.

Converting from binary to decimal

Consider the binary number 110101_2. As the base is 2 this means that powers of 2 essentially replace powers of 10:

$$
\begin{aligned}
110101_2 &= 1(2^5) + 1(2^4) + 0(2^3) + 1(2^2) + 0(2^1) + 1(2^0) \\
&= 1(32) + 1(16) + 0(8) + 1(4) + 0(2) + 1(1) \\
&= 32 + 16 + 4 + 1 \\
&= 53_{10}
\end{aligned}
$$

Hence 110101_2 and 53_{10} are equivalent.

WORKED EXAMPLE

14.1 Convert (a) 1111_2, (b) 101010_2 to decimal.

Solution (a)

$$
\begin{aligned}
1111_2 &= 1(2^3) + 1(2^2) + 1(2^1) + 1(2^0) \\
&= 1(8) + 1(4) + 1(2) + 1(1) \\
&= 8 + 4 + 2 + 1 \\
&= 15_{10}
\end{aligned}
$$

(b)

$$
\begin{aligned}
101010_2 &= 1(2^5) + 0(2^4) + 1(2^3) + 0(2^2) + 1(2^1) + 0(2^0) \\
&= 1(32) + 0 + 1(8) + 0 + 1(2) + 0 \\
&= 32 + 8 + 2 \\
&= 42_{10}
\end{aligned}
$$

Table 14.1
Powers of 2

2^0	1	2^4	16	2^8	256
2^1	2	2^5	32	2^9	512
2^2	4	2^6	64	2^{10}	1024
2^3	8	2^7	128		

14.2 Convert 83_{10} to a binary number.

Solution We need to express 83_{10} as the sum of a set of numbers each of which is a power of 2. From Table 14.1 we see that 64 is the highest number in the table that does not exceed the given number of 83. We write

$$83 = 64 + 19$$

We now focus on the 19. From Table 14.1, 16 is the highest number that does not exceed 19. So we write

$$19 = 16 + 3$$

giving

$$83 = 64 + 16 + 3$$

We now focus on the 3 and again using Table 14.1 we may write

$$
\begin{aligned}
83 &= 64 + 16 + 2 + 1 \\
&= 2^6 + 2^4 + 2^1 + 2^0 \\
&= 1(2^6) + 0(2^5) + 1(2^4) + 0(2^3) + 0(2^2) + 1(2^1) + 1(2^0) \\
&= 1010011_2
\end{aligned}
$$

- Octal number system = use 8 as a base and there are eight digits 0-7 in use. Octal numbers use powers of 8

WORKED EXAMPLES

14.5 Convert 325_8 to a decimal number.

Solution

$$\begin{aligned} 325_8 &= 3(8^2) + 2(8^1) + 5(8^0) \\ &= 3(64) + 2(8) + 5(1) \\ &= 192 + 16 + 5 \\ &= 213_{10} \end{aligned}$$

14.6 Convert 7046_8 to a decimal number.

Solution

$$\begin{aligned} 7046_8 &= 7(8^3) + 0(8^2) + 4(8^1) + 6(8^0) \\ &= 7(512) + 0 + 4(8) + 6(1) \\ &= 3622_{10} \end{aligned}$$

Table 14.2
Powers of 8

8^0	1
8^1	8
8^2	64
8^3	512
8^4	4096
8^5	32768

WORKED EXAMPLES

14.7 Convert 1001 to an octal number.

Solution From Table 14.2 we note that the highest number that does not exceed 1001 is 512. So we write

$$1001 = 512 + 489$$

Looking at the 489, we see that 64 is the highest number that does not exceed 489. We note that

$$489 = 7(64) + 41$$

Finally, looking at 41, we note that 8 is the highest number in Table 14.2 that does not exceed 41. We note that

$$41 = 5(8) + 1$$

so we may write

$$\begin{aligned}1001 &= 512 + 489 \\ &= 512 + 7(64) + 41 \\ &= 512 + 7(64) + 5(8) + 1 \\ &= 1(8^3) + 7(8^2) + 5(8^1) + 1(8^0) \\ &= 1751_8\end{aligned}$$

As an alternative we can divide repeatedly by 8, noting the remainder:

	Remainder
$1001 \div 8 = 125$ r 1	1
$125 \div 8 = 15$ r 5	5
$15 \div 8 = 1$ r 7	7
$1 \div 8 = 0$ r 1	1

Reading up the remainder column gives the required octal number: 1751_8.

- Hexadecimal number system = uses 16 as a base, uses 16 digits 0-9, A-F(10 to 15). Hexadecimal numbers are based on powers of 16.

Table 14.3
Hexadecimal digits

Decimal	Hexadecimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

WORKED EXAMPLE

14.9 Convert the following hexadecimal numbers to decimal numbers: (a) 93A, (b) F9B3.

Solution (a) Noting that hexadecimal numbers use base 16 we have

$$\begin{aligned} 93A_{16} &= 9(16^2) + 3(16^1) + A(16^0) \\ &= 9(256) + 3(16) + 10(1) \\ &= 2362_{10} \end{aligned}$$

(b) $$\begin{aligned} F9B3_{16} &= F(16^3) + 9(16^2) + B(16^1) + 3(16^0) \\ &= 15(4096) + 9(256) + 11(16) + 3(1) \\ &= 63923_{10} \end{aligned}$$

Table 14.4
Powers of 16

16^0	1
16^1	16
16^2	256
16^3	4096
16^4	65536

WORKED EXAMPLES

14.10 Convert 14397 to a hexadecimal number.

Solution From Table 14.4 the highest number that does not exceed 14397 is 4096. We write

$$14397 = 3(4096) + 2109$$

We now focus on the 2109. From Table 14.4 the highest number that does not exceed 2109 is 256:

$$2109 = 8(256) + 61$$

Finally, $61 = 3(16) + 13$. So we have

$$\begin{aligned} 14397 &= 3(4096) + 8(256) + 3(16) + 13 \\ &= 3(16^3) + 8(16^2) + 3(16^1) + 13 \end{aligned}$$

From Table 14.3 we see that 13_{10} is D in hexadecimal so we have

$$14397_{10} = 383D_{16}$$

As with other number bases that we have studied, we can convert decimal numbers by repeated division and noting the remainder. The previous example is reworked to illustrate this.

Summary

In this week, we learned about the different number systems and how to convert between them.