Week 5 Modular Arithmetic Lecture Note

Notebook:	Computational Mathematics		
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Cornell Notes	**Topic:** Modular Arithmetic	Course: BSc Computer Science Class: Computational Mathematics[Lecture] Date: May 05, 2020

Essential Question:

What are the operations performed with congruent numbers and their application to the field of computer science?

Questions/Cues:

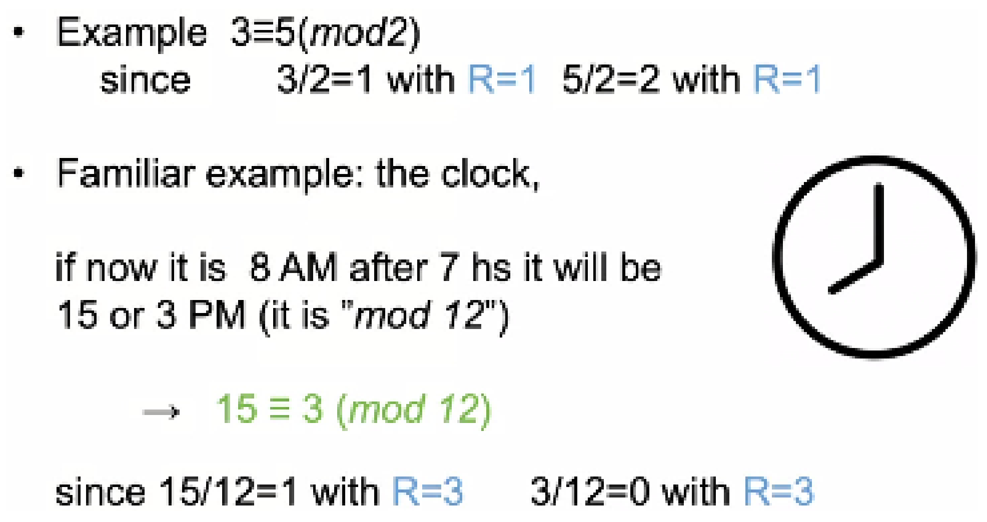
- What is modular arithmetic?
- What is general way of writing congruence between two numbers?
- What is the more formal definition of modular arithmetic?
- How do you perform modular arithmetic on negative numbers?

Notes

- Modular Arithmetic = A way to classify integers, in a sense it is an arithmetic over integers; first introduced by mathematician and physicist Carl Friedrich Gauss
  - used in comp sci when dealing with long numbers, simplifying operation with long numbers
  - At the core of modular arithmetic is the idea of congruence between integers
  - Two numbers a & b are congruent, for example "mod 2" or modulus 2 if when they are divided by 2, they have the same remainder
- Congruency (general) =

$$a \overset{congruent}{\equiv} b \, (\text{mod } k) \Leftrightarrow a = nk + R, b = mk + R$$

    - Where R is the remainder and a, b, k are generic integers



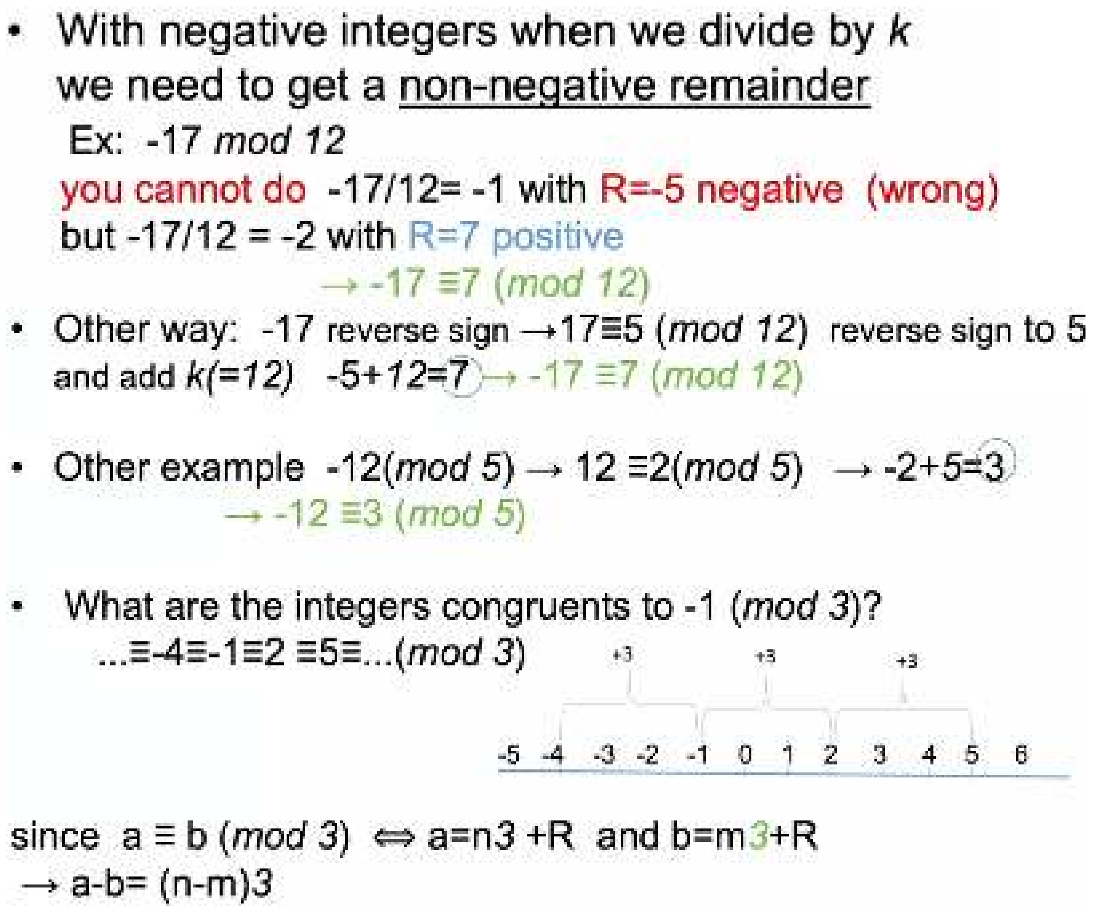
- Example $3 \equiv 5 (mod 2)$
  since $3/2=1$ with $R=1$ $5/2=2$ with $R=1$
- Familiar example: the clock,

  if now it is 8 AM after 7 hs it will be 15 or 3 PM (it is "*mod 12*")

  → $15 \equiv 3$ (*mod 12*)

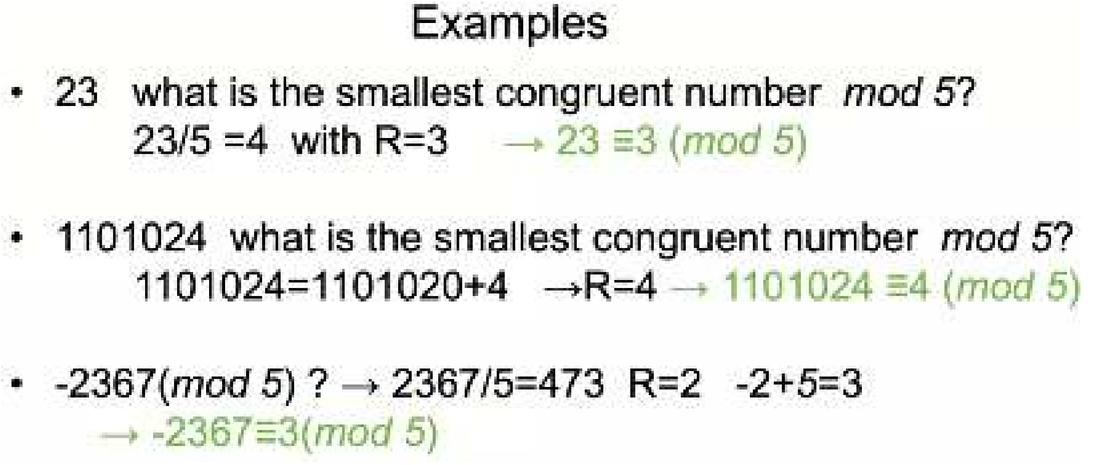
since $15/12=1$ with $R=3$ $3/12=0$ with $R=3$

- Modular Arithmetic (formal) = $\mathrm{mod}\ k$ is mapping by congruence all integers to the subset of non-negative integers smaller than k to which all other integers can be shown to be congruent to that is: $Min_k = \{0,1,2,...k-1\}$
  - The subset is referred to as the minimal subset
    - This subset is special because it is made of integers that when they are divided by the modulus k they coincide with the remainder
  - For example, modulus 12 or $\mathrm{mod}\ 12$, the minimal subset is $Min_{12} = \{0,1,2,3,4,5,6,7,8,9,10,11\}$ and no other integers can be shown to be congruent to the elements of this minimal subset. So if you have a positive integer, you just divide by k, and take the remainder & that remainder will give you the element of the minimal subset to which that number is congruent to



- With negative integers when we divide by *k* we need to get a non-negative remainder
  Ex: -17 *mod 12*
  you cannot do $-17/12=-1$ with $R=-5$ negative (wrong)
  but $-17/12=-2$ with $R=7$ positive
  → $-17 \equiv 7$ (*mod 12*)
- Other way: -17 reverse sign → $17 \equiv 5$ (*mod 12*) reverse sign to 5 and add *k(=12)* $-5+12=7$ → $-17 \equiv 7$ (*mod 12*)
- Other example -12(*mod 5*) → $12 \equiv 2$(*mod 5*) → $-2+5=3$
  → $-12 \equiv 3$ (*mod 5*)
- What are the integers congruents to -1 (*mod 3*)?
  $...\equiv -4 \equiv -1 \equiv 2 \equiv 5 \equiv ...$(*mod 3*)

since $a \equiv b$ (*mod 3*) ⟺ $a=n3+R$ and $b=m3+R$
→ $a-b=(n-m)3$



Examples

- 23 what is the smallest congruent number *mod 5*?
  $23/5 = 4$ with $R=3$ → $23 \equiv 3\ (\mathit{mod}\ 5)$
- 1101024 what is the smallest congruent number *mod 5*?
  $1101024=1101020+4$ → $R=4$ → $1101024 \equiv 4\ (\mathit{mod}\ 5)$
- $-2367(\mathit{mod}\ 5)$? → $2367/5=473$ $R=2$ $-2+5=3$
  → $-2367 \equiv 3(\mathit{mod}\ 5)$

Summary

In this week, we learned about what modular arithmetic is and how it can be performed on both positive and negative numbers.