

2.1 Introduction to Functions-Reading

Notebook: Discrete Mathematics [CM1020]

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Cornell Notes	Topic: 2.1 Introduction to functions-Reading	Course: BSc Computer Science
		Class: Discrete Mathematics-Reading
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Essential Question:		
What is a function and what are its properties?		
Questions/Cues:		
<ul style="list-style-type: none">• What is a function?• What is the domain, co-domain, range, image and pre-image of a function?• How do we the sum and multiplication of two functions?• How is an image of a subset of a set defined?• What is a one-to-one function?• What is an increasing or decreasing function?• What is an onto function?• What is a bijective function?• What is the inverse of a function?		
Notes		
<ul style="list-style-type: none">• Function = let A and B be nonempty set, function f from A to B is assignment of exactly 1 element of B to each element of A; write $f(a) = b$ if b is unique element of B assigned by function f to element a of A.<ul style="list-style-type: none">◦ If f is function from A to B, write $f:A \rightarrow B$◦ Functions sometimes also called mappings or transformations• If function from A to B, A is domain of f and B is co-domain of f; if $f(a) = b$, b is image of a and a is pre-image of b. Range or image of f is set of all images of elements of A<ul style="list-style-type: none">◦ If function f from A to B, say f maps A to B◦ 2 functions equal if have same domain, co-domain and map each elem. of common domain to same elem. in common co-domain◦ function called real-valued if co-domain is set of real numbers; called integer-valued if co-domain is set of integers◦ Two real-val or int-valued functions with same domain can added, well as multiplied• Let f_1 and f_2 be function A to $\mathbf{R} = f_1 + f_2$ and $f_1 f_2$ also functions from A to \mathbf{R} defined for all $x \in A$ by:<ul style="list-style-type: none">◦ $(f_1 + f_2)(x) = f_1(x) + f_2(x)$		

- $(f_1 f_2)(x) = f_1(x) f_2(x)$
- For function A to B , image of subset of A = image of S under function f is subset of B that consists of images of elements of S
 - image of S , $f(S)$
 - $f(S) = \{ t \mid \exists s \in S (t = f(s)) \}$
 - shorthand, $\{ f(s) \mid s \in S \}$ to denote set
 - *** $f(S)$ denotes set, not value of the function f for the set S
- one-to-one or an injection of function f = if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in domain of f
 - function said to be injective if it's one-to-one
 - function f is one-to-one if and only if $f(a) \neq f(b)$ whenever $a \neq b$
- increasing function = function f whose domain and codomain are subsets of set of real \mathbb{R} , called increasing if $f(x) \leq f(y)$, strictly increasing if $f(x) < f(y)$, whenever $x < y$ and x and y in the domain of f
- decreasing function = f called decreasing if $f(x) \geq f(y)$, strictly decreasing if $f(x) > f(y)$, whenever $x < y$ and x and y in domain of f .
 - strictly = strict inequality
- onto or surjective function = function f from A to B is onto, if and only if for every element $b \in B$ there an element $a \in A$ with $f(a) = b$
 - function is onto if also range and co-domain are equal
- bijection = function f is one-to-one correspondence or bijection, if both one-to-one and onto; said to be bijective
 - called invertible if inverse is definable, not invertible b/c no one-to-one correspondence and no inverse exists

Suppose that $f : A \rightarrow B$.

To show that f is injective Show that if $f(x) = f(y)$ for arbitrary $x, y \in A$ with $x \neq y$, then $x = y$.

To show that f is not injective Find particular elements $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$.

To show that f is surjective Consider an arbitrary element $y \in B$ and find an element $x \in A$ such that $f(x) = y$.

To show that f is not surjective Find a particular $y \in B$ such that $f(x) \neq y$ for all $x \in A$.

Let f be a one-to-one correspondence from the set A to the set B . The inverse function of f is the function that assigns to an element b belonging to B the unique element a in A such that $f(a) = b$. The inverse function of f is denoted by f^{-1} . Hence, $f^{-1}(b) = a$ when $f(a) = b$.

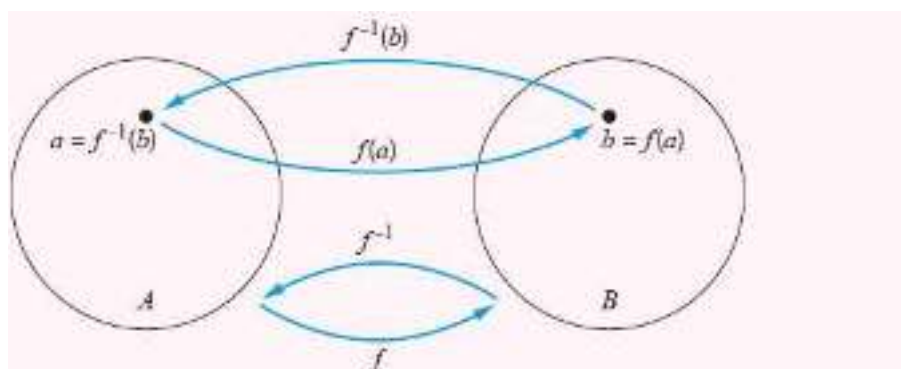


FIGURE 6 The Function f^{-1} Is the Inverse of Function f .

Summary

In this week, we learned what is function is, with what the domain, co-domain, and range of a function represent. Also we explored injective, surjective and bijective functions. Lastly, we looked at the invertibility of a function.