Week 19 Combinatorics and Probabilities Lecture Note

Notebook:	Computational Mathematics		
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Cornell Notes	**Topic:** **Combinatorics and Probabilities**	Course: BSc Computer Science Class: Computational Mathematics[Lecture] Date: July 30, 2020

Essential Question:

What is the probability of an event and how can we use the principles of counting to evaluate such probabilities?

Questions/Cues:

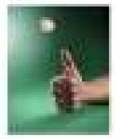
- What is the probability of an event?
- What are the properties of probability?
- What is the joint probability of two events?
- What are permutations?
- What are combinations?

Notes

Probability: definition

$$P(\text{event}) = \frac{\text{N of favourable outcomes}}{\text{N of total outcomes}}$$

Examples coin toss



What is the probability of having head (H)?

N of total outcomes 2

Favourable outcomes 1

➡ P(H)=1/2

Probability: definition

$$P(\text{event}) = \frac{\text{N of favourable outcomes}}{\text{N of total outcomes}}$$

Example: three coin tosses

What is the probability of having 2 heads and one tail (no order)

HHH
HHT
HTH
HTT
TTT
TTH
THT
THH

N total outcomes: $8=2^3= 2x2x2$
Favourable outcomes: 3

➡ P(2H1T)=3/8 ➡ P(HHT)=1/8

Probability: properties

$$P(\text{event}) = \frac{\text{N of favourable outcomes}}{\text{N of total outcomes}} \quad \Rightarrow \quad 0 \leqslant P(\text{event}) \leqslant 1$$

Probability for the occurrence of either of two incompatible events:

Given two incompatible (i.e. cannot occur simultaneously) events A and B

P(A or B)= P(A) +P(B)

Ex: one coin toss A=H B=T (incompatible)

➡ P(H or T)=1/2 +1/2=1

Probability: properties

In general for events compatible (can occur simultaneously)

P(A or B)= P(A) +P(B) –P(A and B)

Example: die roll
What's the probability of getting either 4 (A) or an even number (B)?

Favourable outcomes
A: 4 → n=1
B: 2,4,6. → n=3
A and B: 4 → n=1
Total outcomes: n=6

P(A)= 1/6 ; P(B)=3/6=1/2; P(A and B)=1/6

→P(A or B)=1/6+1/2-1/6=1/2

Joint Probability of two events

We have seen P(A or B)(= $P(A \cup B)$)
let us consider P(A and B) (also P(A,B) and $P(A \cap B)$)

Case 1) <u>Independent events:</u>
The outcome of one event does not affect the other

➡ P(A and B) =P(A) P(B)

Examples: two coin flips

P(H in flip1 and T in flip2)=P(H)P(T)=1/2x1/2=1/4

HH
HT
TH
TT
Total cases 4
Favourable cases 1 → P(A and B)=1/4

Case 2) <u>dependent events:</u>
The outcome of one event affects the other

P(A and B)=P(A)P(B|A) (=P(B)P(A|B))

P(A|B) conditional probability
probability that, given the occurrence of B, A occurs as well

Example: card deck
A= clubs B= king
A and B= king of clubs

P(A)=13/53 ; P(B)=4/53
P(A and B)=n favour./ n total =1/53 ≠ P(A)P(B)

P(B|A)=1/13
P(A|B)=1/4

P(A and B)= P(A)P(B|A) =(13/53) (1/13) =1/53
=P(B)P(A|B) =(4/53) (1/4)=1/53

Permutations (ordering important)

Let's see other situation three socks RBY from a drawer
What are the possible outcomes when you pick randomly (order important)?

RBY
RYB
YRB
YBR
BRY
BYR

Total number of outcomes: 6.
It's like the counting principle but you start from a set of 3 elements
So for the first element you have 3 possibility, the second 2, the last one 1
So it's 3x2x1

In general for the permutation n elements it's $n!=n(n-1)(n-2)....1$ $(0!=1)$

$P(n)=n!$ $P(3)=3x2x1=6$

Permutations and Combinations

You want also to calculate the permutation of only two (r) elements from a set of three (n).

RB
BR
RY
YR
BY
YB

Formula for this case is $P(r,n)=n!/(n-r)!= 3!/(3-2)!= 6/1=6$

If we don't want to count more than once the groups (couples)
that are different only for by the order
we must divide by $r!$ $(2!=2)$

$C(n,r)=\frac{n!}{(n-r)!r!} = \binom{n}{r} = 6/(1x2)=3$

These are called combinations, total n. of groups of r elements in
a set of n total elements in which order does not matter.

11 SOCKS $\quad \begin{matrix} R & G & B \\ 2 & 4 & 5 \end{matrix}$

1) $P(R)$? $\quad N_{TOT} = 11$, $N(R) = 2$ $\Rightarrow P(R) = \frac{2}{11} \approx 18\%$

2) $P(RR)$

RR
RR
RB
BR
BB
BB

$$N_{TOT} = P_2^{11} = \frac{11!}{(11-2)!} = \frac{\cancel{9!} \times 10 \times 11}{\cancel{9!}} = 110$$

$$N_{TOT} = 11 \times 10 = 110$$

$$N(RR) = 2 = P_2^2 = \frac{2!}{(2-2)!} = 2 \Rightarrow P(RR) = \frac{2}{110} \approx 1.8\%$$

3) $P(GGG)$?

$$N_{TOT} = C_3^{11} = \frac{11!}{(11-3)!\,3!} = \frac{11!}{8!\,3!} =$$

$$= \frac{\cancel{8!}\,\cancel{9}^{3} \times \cancel{10}^{5} \times 11}{\cancel{8!}\,\cancel{2} \times \cancel{3}} = 165$$

$$N(GGG) = C_3^4 = \frac{4!}{(4-3)!\,3!} = 4$$

$$P(GGG) = \frac{4}{165} = 2.4\%$$

4) $P(XXX)$ $\quad \begin{matrix} R & G & B \\ 2 & 4 & 5 \end{matrix}$

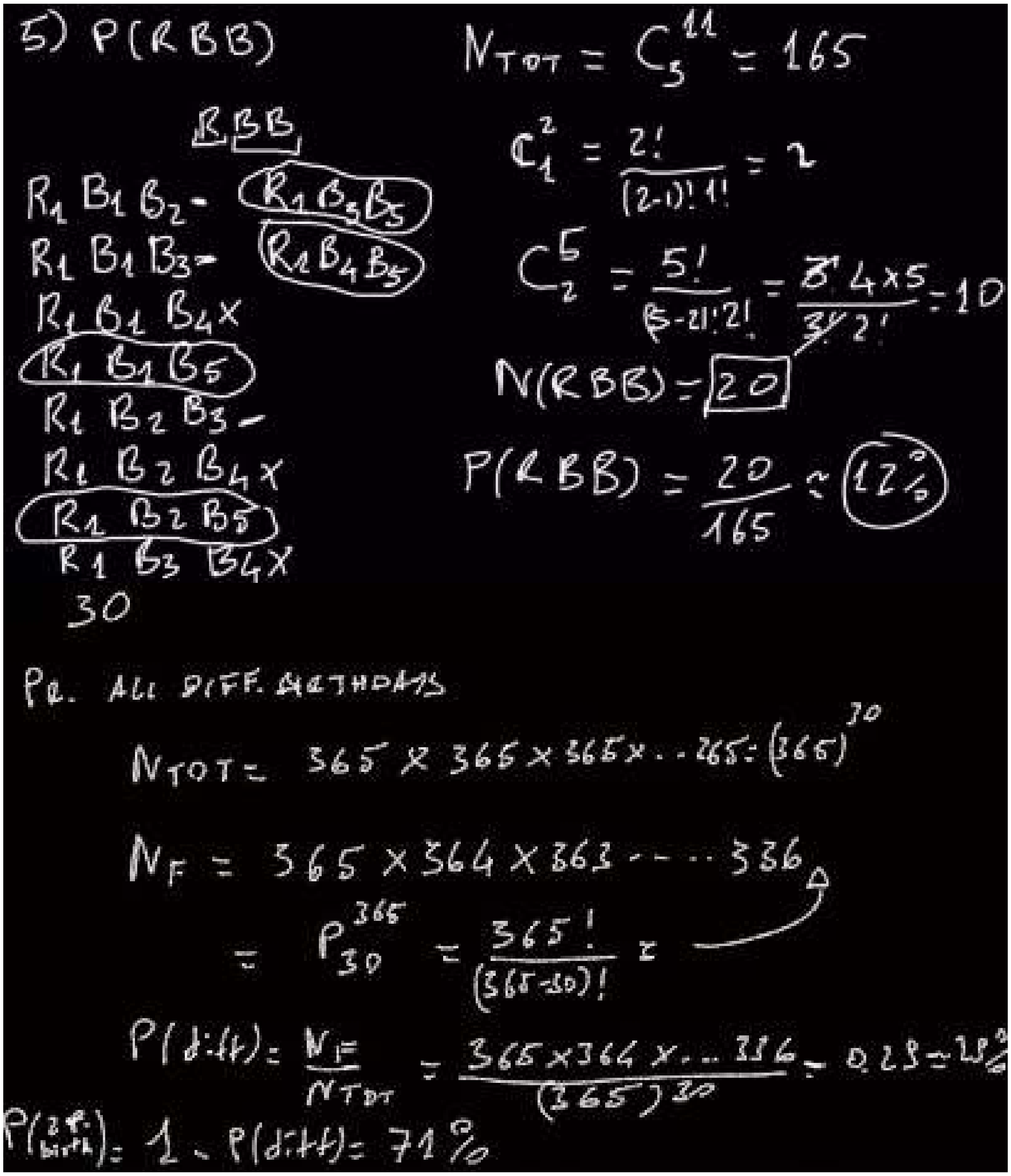
$$P(XXX) = P(GGG) + \underline{P(BBB)}$$

$$N_{TOT} = C_3^{11} = 165$$

$$N(BBB) = C_3^5 = \frac{5!}{(5-3)!\,3!} = \frac{4 \times 5}{2!} = 10$$

$$P(BBB) = 10/165 = 6\%$$

$$P(XXX) = P(GGG) + P(BBB) = (2.4 + 6)\% = 8.4\%$$



Summary

In this week, we learned about what the probability of an event is, the properties of probability, the joint probability of two events, what permutations are and what combinations are.