

Week 11 Trigonometric Functions Lecture Note

Notebook: Computational Mathematics

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Cornell Notes

Topic:
Trigonometric Functions

Course: BSc Computer Science

Class: Computational
Mathematics[Lecture]

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Essential Question:

What is the unit circle and the various trig functions associated with it?

Questions/Cues:

- What are the trig functions for a right angle triangle?
- How do we consider the trig functions for angles greater than 90 degrees?
- How do the sine and cosine change signs change across the four quadrants established by the unit circle?
- What are some special angles when mentioning the unit circle?
- What are the other trig functions and the inverse trig functions?
- What are the properties of trig functions?

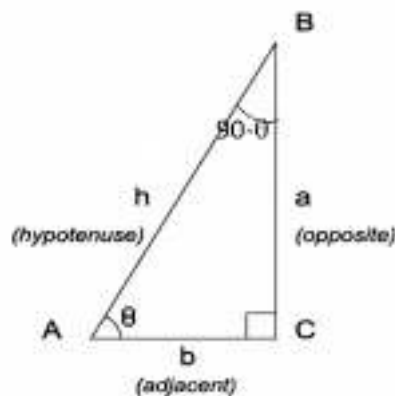
Notes

From right triangles we defined.

$$1) \sin(\theta) = a/h = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$2) \cos(\theta) = b/h = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$3) \tan(\theta) = a/b = \frac{\text{opposite}}{\text{adjacent}}$$



Limited to angles $0 \leq \theta \leq 90^\circ$

- Where the other two angles expect the 90 degree angle have to add up to 90 degrees because the sum of the all the internal angles must be equal to 180 degrees

Extension to $0 \leq \alpha \leq 360^\circ$

hypotenuse=radius=1

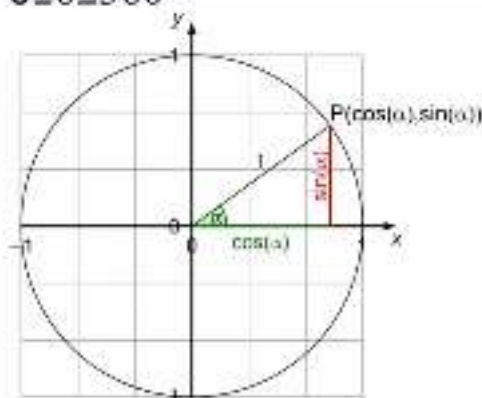
$$\sin(\alpha) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{opposite}}{1}$$

$$\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{adjacent}}{1}$$

$\cos(\alpha)$ and $\sin(\alpha)$ coincide with the x and y coordinates of the point P on the circumference

By Pythagora's theorem:

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$



hypotenuse=radius=1 let $0 \leq \alpha \leq 90^\circ$

$$\sin(180-\alpha) = \sin(\alpha)$$

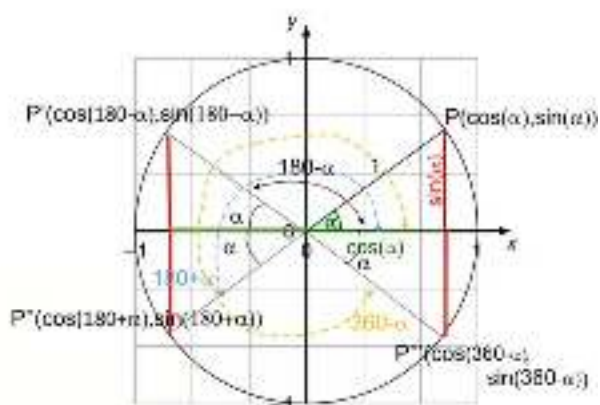
$$\cos(180-\alpha) = -\cos(\alpha)$$

$$\sin(180+\alpha) = -\sin(\alpha)$$

$$\cos(180+\alpha) = -\cos(\alpha)$$

$$\sin(360-\alpha) = -\sin(\alpha)$$

$$\cos(360-\alpha) = \cos(\alpha)$$



Special angles

hypotenuse=radius=1 let $0 \leq \alpha \leq 90^\circ$

$$\sin(0) = \sin(360) = \sin(2\pi) = 0$$

$$\cos(0) = \cos(360) = \cos(2\pi) = 1$$

$$\sin(90) = \sin(\pi/2) = 1$$

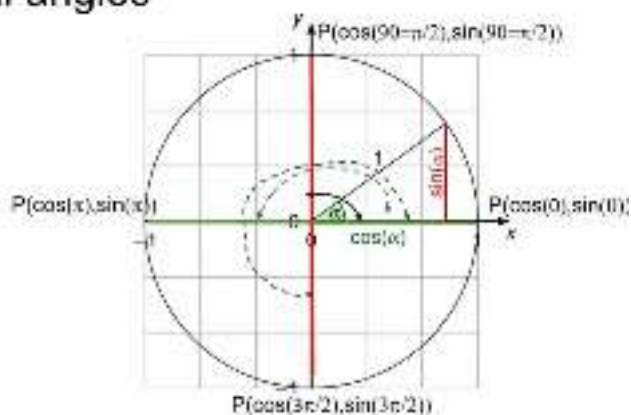
$$\cos(90) = \cos(\pi/2) = 0$$

$$\sin(180) = \sin(\pi) = 0$$

$$\cos(180) = \cos(\pi) = -1$$

$$\sin(270) = \sin(3\pi/2) = -1$$

$$\cos(270) = \cos(3\pi/2) = 0$$



in general: $\cos(2n\pi + x) = \cos(x)$

$$\sin(2n\pi + x) = \sin(x)$$

- sine and cosine both are periodic after 360 degrees or 2 pi radians, which means they repeat.

Definitions

Secant : $\sec(\theta) = 1/\cos(\theta)$

Cosecant: $\csc(\theta) = 1/\sin(\theta)$

Cotangent: $\cot(\theta) = 1/\tan(\theta)$

Inverse functions: if $y=f(x) \rightarrow x=f^{-1}(y)$ f must be bijective

ArcSin: if $\sin(\theta)=y \rightarrow \theta=\sin^{-1}(y)=\text{ArcSin}(y)$

ArcCos: if $\cos(\theta)=y \rightarrow \theta=\cos^{-1}(y)=\text{ArcCos}(y)$

ArcTan: if $\tan(\theta)=y \rightarrow \theta=\tan^{-1}(y)=\text{ArcTan}(y)$

Properties

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \quad \rightarrow \quad \cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad \rightarrow \quad \sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\sin(\alpha/2) = \sqrt{\frac{1 - \cos(\alpha)}{2}}$$

$$\cos(\alpha/2) = \sqrt{\frac{1 + \cos(\alpha)}{2}}$$

Summary

In this week, we learned about the unit circle, the trig functions for angles greater than 90 degrees, the fundamental identity of trigonometry, special trig angles, inverse trig functions and the properties of trig functions.