FCS Week 8 Reading Note

Notebook:	Fundamentals of Computer Science		
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Cornell Notes	**Topic:** Automata Theory Part 2	Course: BSc Computer Science Class: CM1025 Fundamentals of Computer Science[Reading] Date: April 28, 2021

Essential Question:

What is an automata?

Questions/Cues:

- What are states?
- What are transitions?
- What is the formal definition of a finite automaton?
- What is the language of a automaton?
- What is a regular language?
- What are regular operations?
- What is DFA vs. NFA?
- What is meant when we say two machines are equivalent?

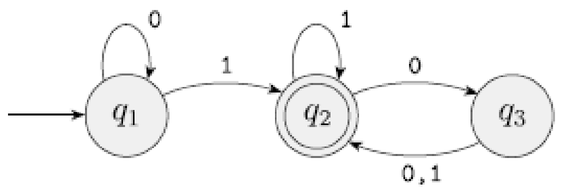


FIGURE 1.4
A finite automaton called M_1 that has three states

Figure 1.4 is called the ***state diagram*** of M_1. It has three ***states***, labeled q_1, q_2, and q_3. The ***start state***, q_1, is indicated by the arrow pointing at it from nowhere. The ***accept state***, q_2, is the one with a double circle. The arrows going from one state to another are called ***transitions***.

When this automaton receives an input string such as 1101, it processes that string and produces an output. The output is either ***accept*** or ***reject***. We will consider only this yes/no type of output for now to keep things simple. The processing begins in M_1's start state. The automaton receives the symbols from the input string one by one from left to right. After reading each symbol, M_1 moves from one state to another along the transition that has that symbol as its label. When it reads the last symbol, M_1 produces its output. The output is *accept* if M_1 is now in an accept state and *reject* if it is not.

For example, when we feed the input string 1101 into the machine M_1 in Figure 1.4, the processing proceeds as follows:

1. Start in state q_1.
2. Read 1, follow transition from q_1 to q_2.
3. Read 1, follow transition from q_2 to q_2.
4. Read 0, follow transition from q_2 to q_3.
5. Read 1, follow transition from q_3 to q_2.
6. *Accept* because M_1 is in an accept state q_2 at the end of the input.

Experimenting with this machine on a variety of input strings reveals that it accepts the strings 1, 01, 11, and 0101010101. In fact, M_1 accepts any string that ends with a 1, as it goes to its accept state q_2 whenever it reads the symbol 1. In addition, it accepts strings 100, 0100, 110000, and 0101000000, and any string that ends with an even number of 0s following the last 1. It rejects other strings, such as 0, 10, 101000. Can you describe the language consisting of all strings that M_1 accepts? We will do so shortly.

A finite automaton has several parts. It has a set of states and rules for going from one state to another, depending on the input symbol. It has an input alphabet that indicates the allowed input symbols. It has a start state and a set of accept states. The formal definition says that a finite automaton is a list of those five objects: set of states, input alphabet, rules for moving, start state, and accept states. In mathematical language, a list of five elements is often called a 5-tuple. Hence we define a finite automaton to be a 5-tuple consisting of these five parts.

We use something called a ***transition function***, frequently denoted δ, to define the rules for moving. If the finite automaton has an arrow from a state x to a state y labeled with the input symbol 1, that means that if the automaton is in state x when it reads a 1, it then moves to state y. We can indicate the same thing with the transition function by saying that $\delta(x, 1) = y$. This notation is a kind of mathematical shorthand. Putting it all together, we arrive at the formal definition of finite automata.

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta \colon Q \times \Sigma \longrightarrow Q$ is the ***transition function***,[1]
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.[2]

[1]Refer back to page 7 if you are uncertain about the meaning of $\delta \colon Q \times \Sigma \longrightarrow Q$.
[2]Accept states sometimes are called ***final states***.

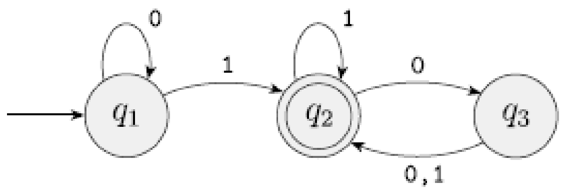


FIGURE 1.6
The finite automaton M_1

We can describe M_1 formally by writing $M_1 = (Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3\}$,
2. $\Sigma = \{0,1\}$,
3. δ is described as

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

4. q_1 is the start state, and
5. $F = \{q_2\}$.

If A is the set of all strings that machine M accepts, we say that A is the ***language of machine M*** and write $L(M) = A$. We say that ***M recognizes A*** or that ***M accepts A***. Because the term *accept* has different meanings when we refer to machines accepting strings and machines accepting languages, we prefer the term *recognize* for languages in order to avoid confusion.

A machine may accept several strings, but it always recognizes only one language. If the machine accepts no strings, it still recognizes one language—namely, the empty language $\emptyset$.

In our example, let

$$A = \{w \mid w \text{ contains at least one 1 and an even number of 0s follow the last 1}\}.$$

Then $L(M_1) = A$, or equivalently, M_1 recognizes A.

EXAMPLE 1.7

Here is the state diagram of finite automaton M_2.

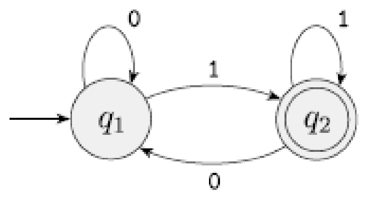


FIGURE 1.8
State diagram of the two-state finite automaton M_2

In the formal description, M_2 is $(\{q_1, q_2\}, \{0,1\}, \delta, q_1, \{q_2\})$. The transition function δ is

	0	1
q_1	q_1	q_2
q_2	q_1	q_2.

Remember that the state diagram of M_2 and the formal description of M_2 contain the same information, only in different forms. You can always go from one to the other if necessary.

A good way to begin understanding any machine is to try it on some sample input strings. When you do these "experiments" to see how the machine is working, its method of functioning often becomes apparent. On the sample string 1101, the machine M_2 starts in its start state q_1 and proceeds first to state q_2 after reading the first 1, and then to states q_2, q_1, and q_2 after reading 1, 0, and 1. The string is accepted because q_2 is an accept state. But string 110 leaves M_2 in state q_1, so it is rejected. After trying a few more examples, you would see that M_2 accepts all strings that end in a 1. Thus $L(M_2) = \{w |\, w \text{ ends in a } 1\}$.

EXAMPLE 1.9

Consider the finite automaton M_3.

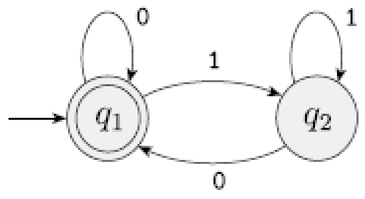


FIGURE 1.10
State diagram of the two-state finite automaton M_3

Machine M_3 is similar to M_2 except for the location of the accept state. As usual, the machine accepts all strings that leave it in an accept state when it has finished reading. Note that because the start state is also an accept state, M_3 accepts the empty string ε. As soon as a machine begins reading the empty string, it is at the end; so if the start state is an accept state, ε is accepted. In addition to the empty string, this machine accepts any string ending with a 0. Here,

$$L(M_3) = \{w |\ w \text{ is the empty string } \varepsilon \text{ or ends in a } 0\}.$$ ■

EXAMPLE 1.11

The following figure shows a five-state machine M_4.

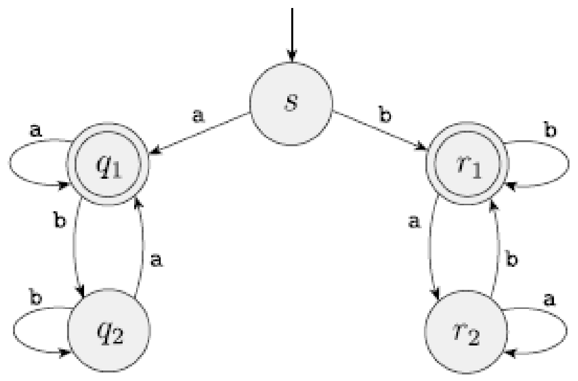


FIGURE 1.12
Finite automaton M_4

Machine M_4 has two accept states, q_1 and r_1, and operates over the alphabet $\Sigma = \{\texttt{a}, \texttt{b}\}$. Some experimentation shows that it accepts strings a, b, aa, bb, and bab, but not strings ab, ba, or bbba. This machine begins in state s, and after it reads the first symbol in the input, it goes either left into the q states or right into the r states. In both cases, it can never return to the start state (in contrast to the previous examples), as it has no way to get from any other state back to s. If the first symbol in the input string is a, then it goes left and accepts when the string ends with an a. Similarly, if the first symbol is a b, the machine goes right and accepts when the string ends in b. So M_4 accepts all strings that start and end with a or that start and end with b. In other words, M_4 accepts strings that start and end with the same symbol. ■

EXAMPLE 1.13

Figure 1.14 shows the three-state machine M_5, which has a four-symbol input alphabet, $\Sigma = \{\langle \text{RESET} \rangle, 0, 1, 2\}$. We treat $\langle \text{RESET} \rangle$ as a single symbol.

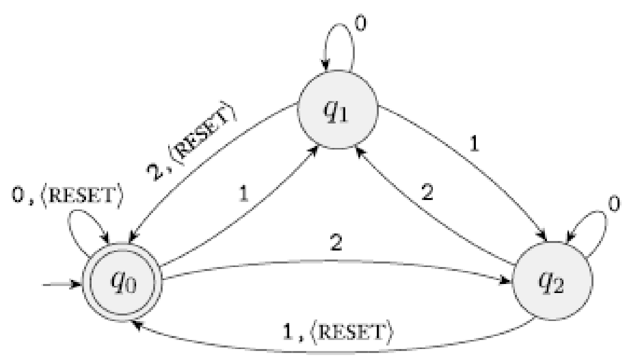


FIGURE 1.14
Finite automaton M_5

Machine M_5 keeps a running count of the sum of the numerical input symbols it reads, modulo 3. Every time it receives the $\langle \text{RESET} \rangle$ symbol, it resets the count to 0. It accepts if the sum is 0 modulo 3, or in other words, if the sum is a multiple of 3. ■

Describing a finite automaton by state diagram is not possible in some cases. That may occur when the diagram would be too big to draw or if, as in the next example, the description depends on some unspecified parameter. In these cases, we resort to a formal description to specify the machine.

EXAMPLE 1.15

Consider a generalization of Example 1.13, using the same four-symbol alphabet Σ. For each $i \geq 1$ let A_i be the language of all strings where the sum of the numbers is a multiple of i, except that the sum is reset to 0 whenever the symbol $\langle\text{RESET}\rangle$ appears. For each A_i we give a finite automaton B_i, recognizing A_i. We describe the machine B_i formally as follows: $B_i = (Q_i, \Sigma, \delta_i, q_0, \{q_0\})$, where Q_i is the set of i states $\{q_0, q_1, q_2, \ldots, q_{i-1}\}$, and we design the transition function δ_i so that for each j, if B_i is in q_j, the running sum is j, modulo i. For each q_j let

$$\begin{aligned}
&\delta_i(q_j, 0) = q_j,\\
&\delta_i(q_j, 1) = q_k, \text{where } k = j + 1 \text{ modulo } i,\\
&\delta_i(q_j, 2) = q_k, \text{where } k = j + 2 \text{ modulo } i, \text{ and}\\
&\delta_i(q_j, \langle\text{RESET}\rangle) = q_0.
\end{aligned}$$

■

FORMAL DEFINITION OF COMPUTATION

So far we have described finite automata informally, using state diagrams, and with a formal definition, as a 5-tuple. The informal description is easier to grasp at first, but the formal definition is useful for making the notion precise, resolving any ambiguities that may have occurred in the informal description. Next we do the same for a finite automaton's computation. We already have an informal idea of the way it computes, and we now formalize it mathematically.

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton and let $w = w_1 w_2 \cdots w_n$ be a string where each w_i is a member of the alphabet Σ. Then M ***accepts*** w if a sequence of states $r_0, r_1, \ldots, r_n$ in Q exists with three conditions:

1. $r_0 = q_0$,
2. $\delta(r_i, w_{i+1}) = r_{i+1}$, for $i = 0, \ldots, n-1$, and
3. $r_n \in F$.

Condition 1 says that the machine starts in the start state. Condition 2 says that the machine goes from state to state according to the transition function. Condition 3 says that the machine accepts its input if it ends up in an accept state. We say that M ***recognizes language*** A if $A = \{w \mid M \text{ accepts } w\}$.

DEFINITION 1.16

A language is called a ***regular language*** if some finite automaton recognizes it.

EXAMPLE 1.21

This example shows how to design a finite automaton E_2 to recognize the regular language of all strings that contain the string 001 as a substring. For example, 0010, 1001, 001, and 11111110011111 are all in the language, but 11 and 0000 are not. How would you recognize this language if you were pretending to be E_2? As symbols come in, you would initially skip over all 1s. If you come to a 0, then you note that you may have just seen the first of the three symbols in the pattern 001 you are seeking. If at this point you see a 1, there were too few 0s, so you go back to skipping over 1s. But if you see a 0 at that point, you should remember that you have just seen two symbols of the pattern. Now you simply need to continue scanning until you see a 1. If you find it, remember that you succeeded in finding the pattern and continue reading the input string until you get to the end.

So there are four possibilities: You

1. haven't just seen any symbols of the pattern,
2. have just seen a 0,
3. have just seen 00, or
4. have seen the entire pattern 001.

Assign the states q, q_0, q_{00}, and q_{001} to these possibilities. You can assign the transitions by observing that from q reading a 1 you stay in q, but reading a 0 you move to q_0. In q_0 reading a 1 you return to q, but reading a 0 you move to q_{00}. In q_{00} reading a 1 you move to q_{001}, but reading a 0 leaves you in q_{00}. Finally, in q_{001} reading a 0 or a 1 leaves you in q_{001}. The start state is q, and the only accept state is q_{001}, as shown in Figure 1.22.

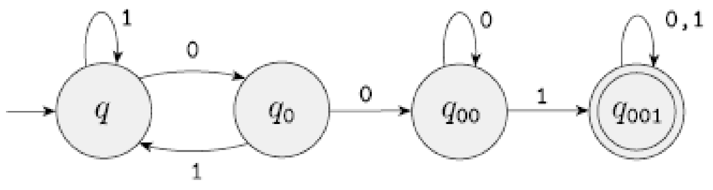


FIGURE 1.22
Accepts strings containing 001

THE REGULAR OPERATIONS

In the preceding two sections, we introduced and defined finite automata and regular languages. We now begin to investigate their properties. Doing so will help develop a toolbox of techniques for designing automata to recognize particular languages. The toolbox also will include ways of proving that certain other languages are nonregular (i.e., beyond the capability of finite automata).

In arithmetic, the basic objects are numbers and the tools are operations for manipulating them, such as $+$ and $\times$. In the theory of computation, the objects are languages and the tools include operations specifically designed for manipulating them. We define three operations on languages, called the ***regular operations***, and use them to study properties of the regular languages.

DEFINITION 1.23

Let A and B be languages. We define the regular operations ***union***, ***concatenation***, and ***star*** as follows:

- **Union**: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.
- **Concatenation**: $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$.
- **Star**: $A^* = \{x_1 x_2 \ldots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$.

You are already familiar with the union operation. It simply takes all the strings in both A and B and lumps them together into one language.

The concatenation operation is a little trickier. It attaches a string from A in front of a string from B in all possible ways to get the strings in the new language.

The star operation is a bit different from the other two because it applies to a single language rather than to two different languages. That is, the star operation is a ***unary operation*** instead of a ***binary operation***. It works by attaching any number of strings in A together to get a string in the new language. Because "any number" includes 0 as a possibility, the empty string ε is always a member of A^*, no matter what A is.

NONDETERMINISM

Nondeterminism is a useful concept that has had great impact on the theory of computation. So far in our discussion, every step of a computation follows in a unique way from the preceding step. When the machine is in a given state and reads the next input symbol, we know what the next state will be—it is determined. We call this ***deterministic*** computation. In a ***nondeterministic*** machine, several choices may exist for the next state at any point.

Nondeterminism is a generalization of determinism, so every deterministic finite automaton is automatically a nondeterministic finite automaton. As Figure 1.27 shows, nondeterministic finite automata may have additional features.

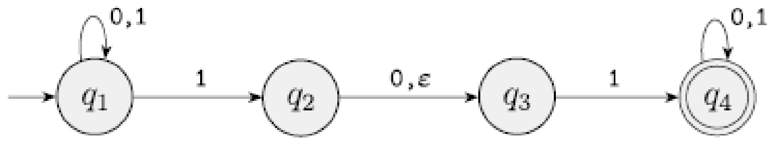


FIGURE 1.27
The nondeterministic finite automaton N_1

The difference between a deterministic finite automaton, abbreviated DFA, and a nondeterministic finite automaton, abbreviated NFA, is immediately apparent. First, every state of a DFA always has exactly one exiting transition arrow for each symbol in the alphabet. The NFA shown in Figure 1.27 violates that rule. State q_1 has one exiting arrow for 0, but it has two for 1; q_2 has one arrow for 0, but it has none for 1. In an NFA, a state may have zero, one, or many exiting arrows for each alphabet symbol.

Second, in a DFA, labels on the transition arrows are symbols from the alphabet. This NFA has an arrow with the label ε. In general, an NFA may have arrows labeled with members of the alphabet or ε. Zero, one, or many arrows may exit from each state with the label ε.

How does an NFA compute? Suppose that we are running an NFA on an input string and come to a state with multiple ways to proceed. For example, say that we are in state q_1 in NFA N_1 and that the next input symbol is a 1. After reading that symbol, the machine splits into multiple copies of itself and follows *all* the possibilities in parallel. Each copy of the machine takes one of the possible ways to proceed and continues as before. If there are subsequent choices, the machine splits again. If the next input symbol doesn't appear on any of the arrows exiting the state occupied by a copy of the machine, that copy of the machine dies, along with the branch of the computation associated with it. Finally, if *any one* of these copies of the machine is in an accept state at the end of the input, the NFA accepts the input string.

If a state with an ε symbol on an exiting arrow is encountered, something similar happens. Without reading any input, the machine splits into multiple copies, one following each of the exiting ε-labeled arrows and one staying at the current state. Then the machine proceeds nondeterministically as before.

Nondeterminism may be viewed as a kind of parallel computation wherein multiple independent "processes" or "threads" can be running concurrently. When the NFA splits to follow several choices, that corresponds to a process "forking" into several children, each proceeding separately. If at least one of these processes accepts, then the entire computation accepts.

Another way to think of a nondeterministic computation is as a tree of possibilities. The root of the tree corresponds to the start of the computation. Every branching point in the tree corresponds to a point in the computation at which the machine has multiple choices. The machine accepts if at least one of the computation branches ends in an accept state, as shown in Figure 1.28.

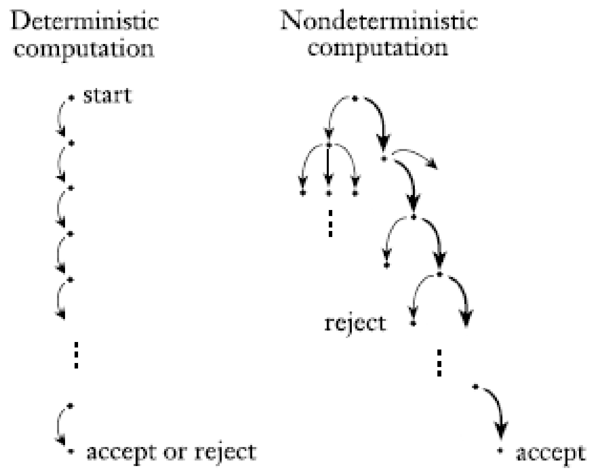


FIGURE 1.28
Deterministic and nondeterministic computations with an accepting branch

Let's consider some sample runs of the NFA N_1 shown in Figure 1.27. The computation of N_1 on input 010110 is depicted in the following figure.

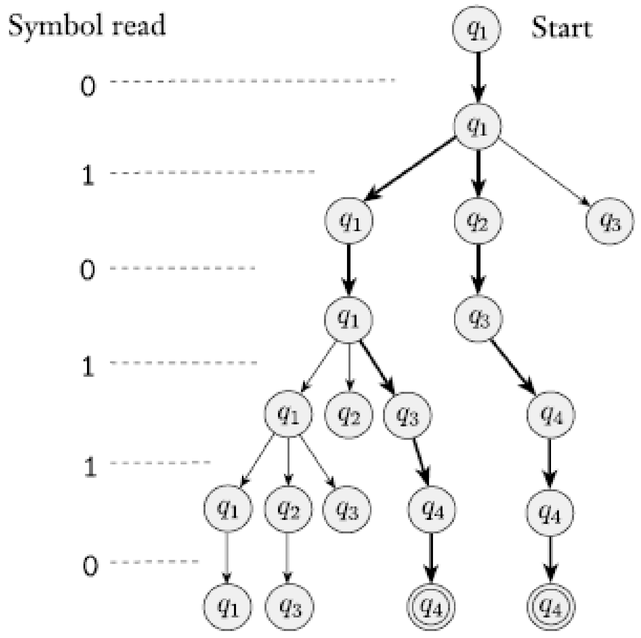


FIGURE 1.29
The computation of N_1 on input 010110

EXAMPLE 1.30

Let A be the language consisting of all strings over $\{0,1\}$ containing a 1 in the third position from the end (e.g., 000100 is in A but 0011 is not). The following four-state NFA N_2 recognizes A.

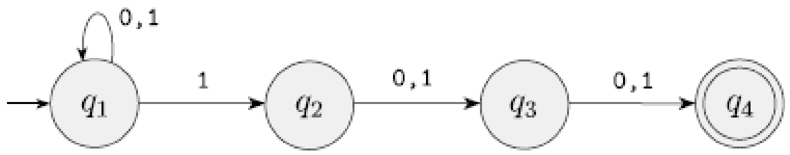


FIGURE 1.31
The NFA N_2 recognizing A

One good way to view the computation of this NFA is to say that it stays in the start state q_1 until it "guesses" that it is three places from the end. At that point, if the input symbol is a 1, it branches to state q_2 and uses q_3 and q_4 to "check" on whether its guess was correct.

As mentioned, every NFA can be converted into an equivalent DFA; but sometimes that DFA may have many more states. The smallest DFA for A contains eight states. Furthermore, understanding the functioning of the NFA is much easier, as you may see by examining the following figure for the DFA.

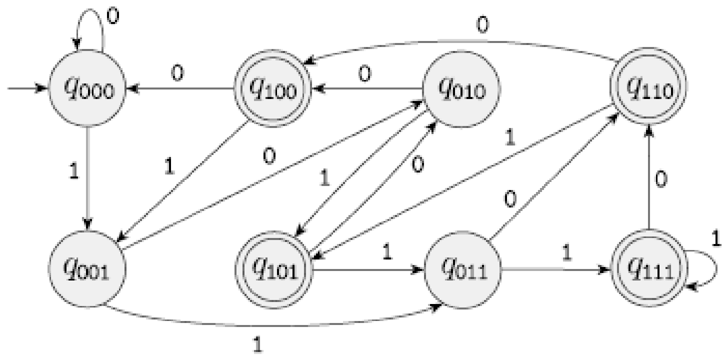


FIGURE 1.32
A DFA recognizing A

Suppose that we added ε to the labels on the arrows going from q_2 to q_3 and from q_3 to q_4 in machine N_2 in Figure 1.31. So both arrows would then have the label $0, 1, \varepsilon$ instead of just $0, 1$. What language would N_2 recognize with this modification? Try modifying the DFA in Figure 1.32 to recognize that language.

EXAMPLE 1.33

The following NFA N_3 has an input alphabet $\{0\}$ consisting of a single symbol. An alphabet containing only one symbol is called a ***unary alphabet.***

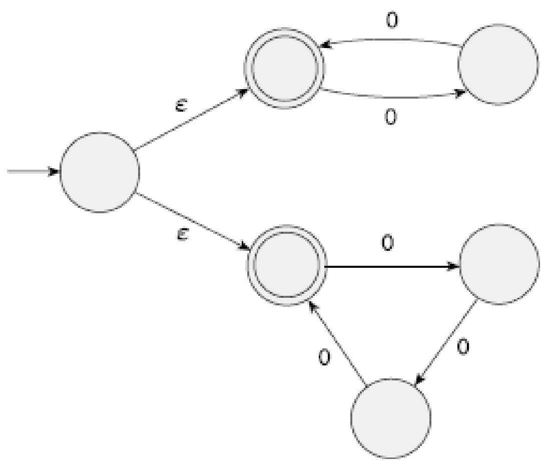


FIGURE 1.34
The NFA N_3

This machine demonstrates the convenience of having ε arrows. It accepts all strings of the form 0^k where k is a multiple of 2 or 3. (Remember that the superscript denotes repetition, not numerical exponentiation.) For example, N_3 accepts the strings ε, 00, 000, 0000, and 000000, but not 0 or 00000.

Think of the machine operating by initially guessing whether to test for a multiple of 2 or a multiple of 3 by branching into either the top loop or the bottom loop and then checking whether its guess was correct. Of course, we could replace this machine by one that doesn't have ε arrows or even any nondeterminism at all, but the machine shown is the easiest one to understand for this language. ■

EXAMPLE 1.35

We give another example of an NFA in Figure 1.36. Practice with it to satisfy yourself that it accepts the strings ε, a, baba, and baa, but that it doesn't accept the strings b, bb, and babba. Later we use this machine to illustrate the procedure for converting NFAs to DFAs.

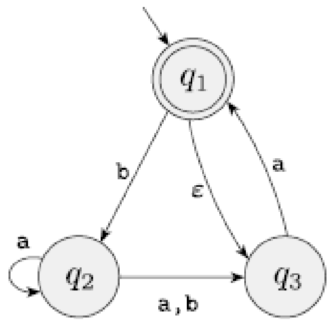


FIGURE 1.36
The NFA N_4

FORMAL DEFINITION OF A NONDETERMINISTIC FINITE AUTOMATON

The formal definition of a nondeterministic finite automaton is similar to that of a deterministic finite automaton. Both have states, an input alphabet, a transition function, a start state, and a collection of accept states. However, they differ in one essential way: in the type of transition function. In a DFA, the transition function takes a state and an input symbol and produces the next state. In an NFA, the transition function takes a state and an input symbol *or the empty string* and produces *the set of possible next states*. In order to write the formal definition, we need to set up some additional notation. For any set Q we write $\mathcal{P}(Q)$ to be the collection of all subsets of Q. Here $\mathcal{P}(Q)$ is called the ***power set*** of Q. For any alphabet Σ we write Σ_ε to be $\Sigma \cup \{\varepsilon\}$. Now we can write the formal description of the type of the transition function in an NFA as $\delta\colon Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$.

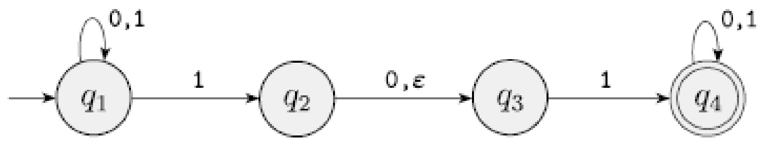
DEFINITION 1.37

A ***nondeterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta\colon Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

EXAMPLE 1.38

Recall the NFA N_1:



The formal description of N_1 is $(Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3, q_4\}$,
2. $\Sigma = \{0,1\}$,
3. δ is given as

	0	1	ε
q_1	$\{q_1\}$	$\{q_1, q_2\}$	$\emptyset$
q_2	$\{q_3\}$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\{q_4\}$	$\emptyset$
q_4	$\{q_4\}$	$\{q_4\}$	$\emptyset$,

4. q_1 is the start state, and
5. $F = \{q_4\}$.

▪

The formal definition of computation for an NFA is similar to that for a DFA. Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA and w a string over the alphabet Σ. Then we say that N ***accepts*** w if we can write w as $w = y_1 y_2 \cdots y_m$, where each y_i is a member of Σ_ε and a sequence of states $r_0, r_1, \ldots, r_m$ exists in Q with three conditions:

1. $r_0 = q_0$,
2. $r_{i+1} \in \delta(r_i, y_{i+1})$, for $i = 0, \ldots, m-1$, and
3. $r_m \in F$.

Condition 1 says that the machine starts out in the start state. Condition 2 says that state r_{i+1} is one of the allowable next states when N is in state r_i and reading y_{i+1}. Observe that $\delta(r_i, y_{i+1})$ is the *set* of allowable next states and so we say that r_{i+1} is a member of that set. Finally, condition 3 says that the machine accepts its input if the last state is an accept state.

EQUIVALENCE OF NFAS AND DFAS

Deterministic and nondeterministic finite automata recognize the same class of languages. Such equivalence is both surprising and useful. It is surprising because NFAs appear to have more power than DFAs, so we might expect that NFAs recognize more languages. It is useful because describing an NFA for a given language sometimes is much easier than describing a DFA for that language.

Say that two machines are ***equivalent*** if they recognize the same language.

THEOREM 1.39

Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

PROOF IDEA If a language is recognized by an NFA, then we must show the existence of a DFA that also recognizes it. The idea is to convert the NFA into an equivalent DFA that simulates the NFA.

Recall the "reader as automaton" strategy for designing finite automata. How would you simulate the NFA if you were pretending to be a DFA? What do you need to keep track of as the input string is processed? In the examples of NFAs, you kept track of the various branches of the computation by placing a finger on each state that could be active at given points in the input. You updated the simulation by moving, adding, and removing fingers according to the way the NFA operates. All you needed to keep track of was the set of states having fingers on them.

If k is the number of states of the NFA, it has 2^k subsets of states. Each subset corresponds to one of the possibilities that the DFA must remember, so the DFA simulating the NFA will have 2^k states. Now we need to figure out which will be the start state and accept states of the DFA, and what will be its transition function. We can discuss this more easily after setting up some formal notation.

Theorem 1.39 states that every NFA can be converted into an equivalent DFA. Thus nondeterministic finite automata give an alternative way of characterizing the regular languages. We state this fact as a corollary of Theorem 1.39.

COROLLARY 1.40

A language is regular if and only if some nondeterministic finite automaton recognizes it.

One direction of the "if and only if" condition states that a language is regular if some NFA recognizes it. Theorem 1.39 shows that any NFA can be converted into an equivalent DFA. Consequently, if an NFA recognizes some language, so does some DFA, and hence the language is regular. The other direction of the "if and only if" condition states that a language is regular only if some NFA recognizes it. That is, if a language is regular, some NFA must be recognizing it. Obviously, this condition is true because a regular language has a DFA recognizing it and any DFA is also an NFA.

EXAMPLE 1.41

Let's illustrate the procedure we gave in the proof of Theorem 1.39 for converting an NFA to a DFA by using the machine N_4 that appears in Example 1.35. For clarity, we have relabeled the states of N_4 to be $\{1, 2, 3\}$. Thus in the formal description of $N_4 = (Q, \{\texttt{a},\texttt{b}\}, \delta, 1, \{1\})$, the set of states Q is $\{1, 2, 3\}$ as shown in Figure 1.42.

To construct a DFA D that is equivalent to N_4, we first determine D's states. N_4 has three states, $\{1, 2, 3\}$, so we construct D with eight states, one for each subset of N_4's states. We label each of D's states with the corresponding subset. Thus D's state set is

$$\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}.$$

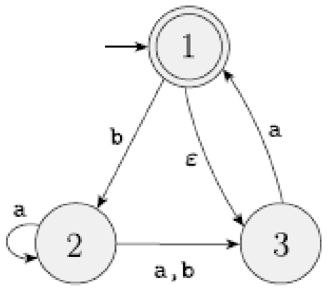


FIGURE 1.42
The NFA N_4

Next, we determine the start and accept states of D. The start state is $E(\{1\})$, the set of states that are reachable from 1 by traveling along ε arrows, plus 1 itself. An ε arrow goes from 1 to 3, so $E(\{1\}) = \{1, 3\}$. The new accept states are those containing N_4's accept state; thus $\{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$.

Finally, we determine D's transition function. Each of D's states goes to one place on input a and one place on input b. We illustrate the process of determining the placement of D's transition arrows with a few examples.

In D, state $\{2\}$ goes to $\{2,3\}$ on input a because in N_4, state 2 goes to both 2 and 3 on input a and we can't go farther from 2 or 3 along ε arrows. State $\{2\}$ goes to state $\{3\}$ on input b because in N_4, state 2 goes only to state 3 on input b and we can't go farther from 3 along ε arrows.

State $\{1\}$ goes to $\emptyset$ on a because no a arrows exit it. It goes to $\{2\}$ on b. Note that the procedure in Theorem 1.39 specifies that we follow the ε arrows *after* each input symbol is read. An alternative procedure based on following the ε arrows before reading each input symbol works equally well, but that method is not illustrated in this example.

State $\{3\}$ goes to $\{1,3\}$ on a because in N_4, state 3 goes to 1 on a and 1 in turn goes to 3 with an ε arrow. State $\{3\}$ on b goes to $\emptyset$.

State $\{1,2\}$ on a goes to $\{2,3\}$ because 1 points at no states with a arrows, 2 points at both 2 and 3 with a arrows, and neither points anywhere with ε arrows. State $\{1,2\}$ on b goes to $\{2,3\}$. Continuing in this way, we obtain the diagram for D in Figure 1.43.

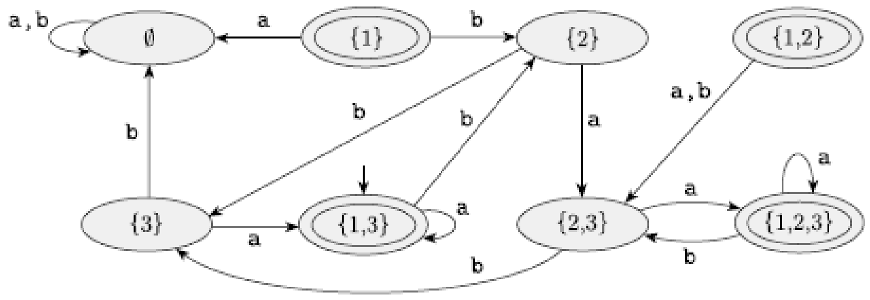


FIGURE 1.43
A DFA D that is equivalent to the NFA N_4

We may simplify this machine by observing that no arrows point at states $\{1\}$ and $\{1,2\}$, so they may be removed without affecting the performance of the machine. Doing so yields the following figure.

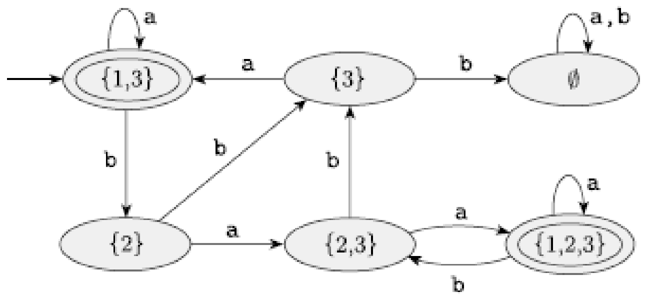


FIGURE 1.44
DFA D after removing unnecessary states

Summary

In this week, we learned about Deterministic Finite Automaton and Non-Deterministic Finite Automaton.