Mathematical Economics: Concept of Utility

Introduction

Think about your favourite food. Have you experienced a kind of satisfaction after consuming it? Did you realize that even if it is your favourite food, there is still a limit to your consumption? Do you want to know how much of your favourite food you should consume so that your satisfaction level does not fall?

This module will explore how satisfaction and preferences are measured.

Objectives

The objectives of the module are:

1. Explain the concept of Utility
2. Discover the mathematics behind the theories of consumer behaviour

Terminology

1. Utility: satisfaction derived from consumption
2. Use: consume, employ
3. Cardinal Utility Theory: a theory that says that utility can be measured and added
4. Ordinal Utility Theory: a theory that ranks the preferences of consumers
5. Util: unit of measuring utility
6. Rational: based on reason or logic, not insane
7. Marginal Utility of Money: additional utility derived by spending an extra unit of money
8. Law of diminishing marginal utility: a law that states that utility decreases with more consumption and can also become negative
9. Marginal Utility: additional utility
10. Total Utility: sum of marginal utilities
11. Indifference curve: a curve that shows different combinations of goods that give the same satisfaction
12. Indifference Map: a collection of indifference curves
13. Law of Diminishing marginal rate of substitution: a law that states that the rate at which a good is substituted for another decreases
14. Consistency: stability
15. Transitivity: a mathematical logic; if A is larger than B and B is larger than C, then A is larger than C
16. Level Curve: a curve obtained from a function that has a constant value.

7.1. Utility

Utility is a term used in Economics for satisfaction. Satisfaction as perceived in economics, is the level of pleasure attained when the demands of consumers are fulfilled. The use of this term is generally seen in the 'Consumer Behaviour Theory'. This term was initially used as a measure of satisfaction within the theory of Utilitarianism by Jeremy Bentham and John Stuart Mill.

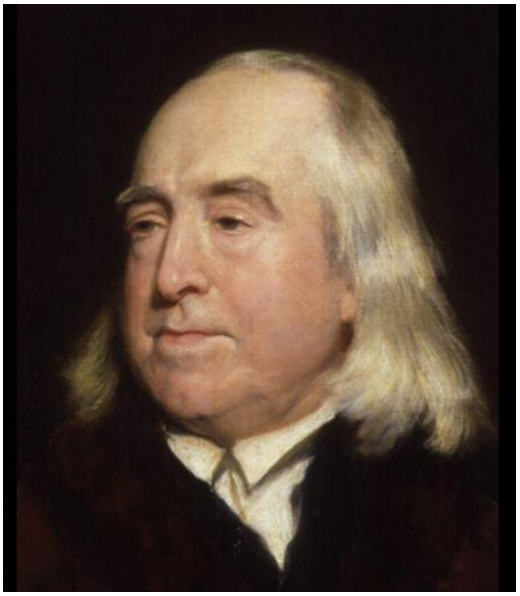


Image 7.1 Jeremy Bentham [1]

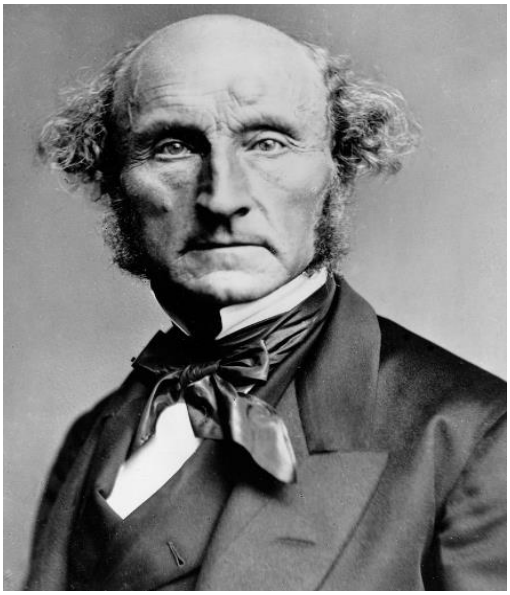


Image 7.2 John Stuart Mill [2]

[1] https://en.wikipedia.org/wiki/Jeremy_Bentham#/media/File:Jeremy_Bentham_by_Henry_William_Pickersgill_detail.jpg
[2] https://en.wikipedia.org/wiki/John_Stuart_Mill#/media/File:John_Stuart_Mill_by_London_Stereoscopic_Company,_c1870.jpg

7.2. Difference between "utility" and "use"

The words "utility" and "use" may look similar to each other, but there is a thin line of distinction between them. The word "use" or "usefulness" is objective, that is, it is good-specific. For example, a refrigerator is considered to be "useful". It may also be said that a refrigerator may be "used" to keep food fresh and cold.

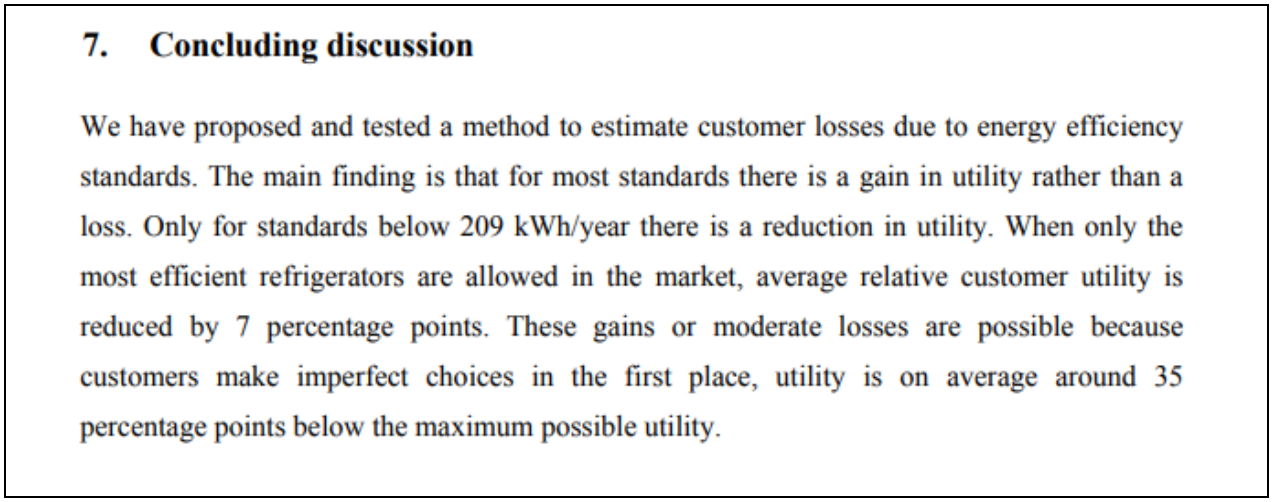
On the other hand, the satisfaction or happiness one derives from the use of the refrigerator is subjective or people-centric. This satisfaction is termed as "utility".

For example, a refrigerator may be useful in many ways, but it does not necessarily give satisfaction to all. A particular consumer may not derive any "utility" from the use of the refrigerator. This may be because of the reason that, that particular consumer sees the refrigerator as a harmful product that kills nutrients of all food and is therefore harmful for health.

Environmentalists may also be of the view that refrigerators are not healthy because it emits harmful gases such as greenhouse gases that pollutes the environment.

In a research paper titled ***"Estimating customer Utility of energy efficiency standards for refrigerators"***, published by Erling Moxnes in *The System Dynamics Group, Department of Information Science, University of Bergen, Norway*; it was concluded that,

"……utility is on average around 35 percentage points below the maximum possible utility."



7. Concluding discussion

We have proposed and tested a method to estimate customer losses due to energy efficiency standards. The main finding is that for most standards there is a gain in utility rather than a loss. Only for standards below 209 kWh/year there is a reduction in utility. When only the most efficient refrigerators are allowed in the market, average relative customer utility is reduced by 7 percentage points. These gains or moderate losses are possible because customers make imperfect choices in the first place, utility is on average around 35 percentage points below the maximum possible utility.

Image 7.3 ***"Estimating customer Utility of energy efficiency standards for refrigerators"***

[Source: https://pdfs.semanticscholar.org/b85f/d988b1ba13a13874dc8b2e5564e927f25ce4.pdf]

Satisfaction or utility therefore, does not necessarily depend on the usefulness of any good. It is subjective and is based on choices made by consumers. Thus, if a consumer likes a good, he/she may derive some utility after consuming that good, even if the good may not be useful.

Thus, something may be considered useful, but may not give satisfaction or utility.

7.3. Theories of Utility

Different economists have proposed different theories of utility based on their observation and understanding of the world around them. Some economists are of the view that utility or satisfaction can be numerically measured, while others are of the view that utility or satisfaction is difficult to measure.

There are two broad Utility theories that are generally studied in Economics:

a) Cardinal Utility Theory
b) Ordinal Utility Theory

Let us try to understand how the basic mathematical tools are used in these Utility Theories.

a) Cardinal Utility Theory

The Cardinal Utility Theory was proposed by neo-classical economists. Alfred Marshall is considered as the father of neo-classical economists. According to the neo-classical economists, the different concepts of demand, supply and consumer behaviour can be studied mathematically by using equations. They used the concept of cardinal numbers to measure human satisfaction. The economic theory on the utility (satisfaction) of human beings stated that the satisfaction derived by consuming goods and services can be objectively measured and added and the unit of measurement was called "util(s)". "Util" is a hypothetical unit measuring utility or satisfaction.

The basic assumptions of the Cardinal Utility Theory are:

1. Rationality: The consumer is assumed to have knowledge about the market and tries to behave in a normal way that is, he/she tries to achieve maximum satisfaction. Thus, more is always better.
2. Constant marginal utility of money: The utility derived by spending an extra unit of money in consuming any good is assumed to be constant, even if the income of consumer increases or decreases.
3. Law of Diminishing marginal Utility: a law that states that the utility derived from successive units of any goods i.e. the marginal utility, initially increases and then gradually decreases and may even become negative.

However, the satisfaction derived from the consumption of one good may differ from person to person because of the difference in the tastes. Thus, consumer A may derive 10 utils of satisfaction by consuming 1 unit of good X, while consumer B may derive 15 utils of satisfaction from the same unit of good X.

Another noticeable point is that, the satisfaction derived from the consumption of different units of the same good or the satisfaction derived from the consumption of different goods may be added because of the independent numerical component. If U_1 = 10 utils is the utility derived from the consumption of 1 unit of good X and U_2 = 20 utils is the utility derived from consuming 1 unit of good Y, then the utility of consumer A, from consuming Good X and Good, say, U, may be added as $U_1 + U_2$

Thus $(U) = U_1 + U_2 = 10\,utils + 20\,utils = 30\,utils$

i) Marginal Utility and Total Utility

The Cardinal Utility Theory identifies two types of utility.

Marginal Utility: In Economics, the concept of marginal utility refers to the additional utility derived from consuming an extra unit of any good.

Total Utility: The sum of marginal utilities gives the total utility.

Table 7.1 Utility from consumption of pastries

No. of Pastries	Marginal utility (in Utils)	Total Utility (in Utils)
1	20	20
2	22	20+22= 42
3	24	20+22+24= 66
4	18	20+22+24+18= 84
5	10	20+22+24+18+10= 94
6	-10	20+22+24+18+10+(-10) = 84

The above example is a hypothetical example of a consumer who loves pastries. The table shows the utility or satisfaction derived by the consumer from each unit of pastry consumed and also the total utility or satisfaction after each extra unit of pastry consumed.

The observations from table 1 may be:

1) Marginal Utility increases from unit 1 to unit 2 to unit 3 and gradually decreases from unit 4, displaying the ***law of diminishing marginal utility.***
2) Marginal Utility becomes negative when the 6^{th} unit is consumed, thus stating that there is a limit of the stomach in consuming food. Negative Utility is generally ignored in Economics. However, this means that the consumer does not derive any satisfaction after while consuming the 6^{th} unit.
3) So far marginal utility is positive, total utility increases.
4) When marginal utility becomes negative, total utility falls.

ii) Relation between Marginal Utility and Total Utility

In mathematical language, the relationship between total utility (TU) and marginal utility (MU) from two units of the same good may be represented as follows:

$$TU = MU_1 + MU_2$$

Or, if Total utility from consuming two units is given as TU_2 and Total utility from consuming one unit is given as TU_1, (or total utility from first unit is the marginal utility, MU_1) then, the MU from consuming the 2^{nd} unit is given as

$$MU_2 = TU_2 - TU_1$$

b) Ordinal Utility Theory

The Ordinal Utility Theory was proposed by modern economists, particularly Hicks. Modern economists were of the view that utility cannot be measured objectively and cannot be added.

The concept of sets or ordered pairs is an important characteristic of the Ordinal Utility Theory. The goods consumed by a consumer may be grouped in sets of commodities and consumers can give preferences for the commodities consumed. Satisfaction of the consumers is thus ordinal and can be ranked, as opposed to cardinal theory where commodities are measured in utils.

Let us assume the simple case of an economy producing only two goods, Good X and Good Y. The consumers are opened to infinite options. The different possible combinations of Good X and Good Y may be shown in a commodity space as follows:

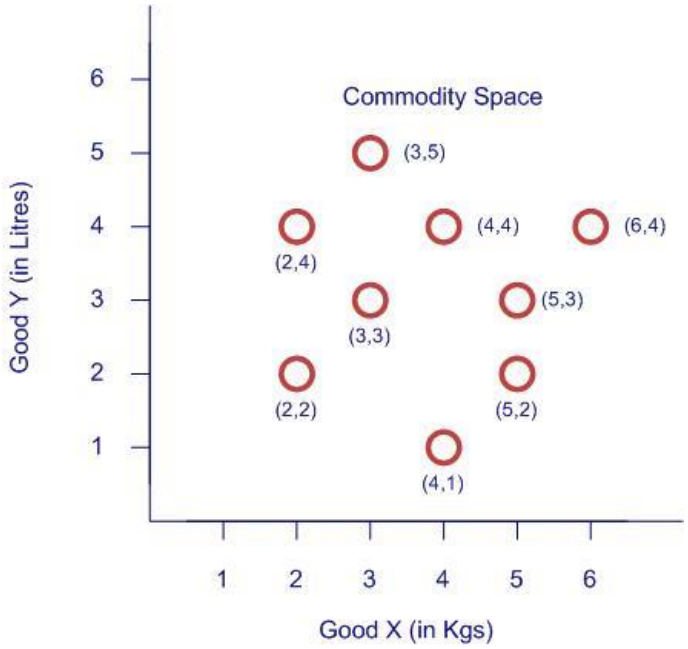


Fig 7.1: Commodity Space

The sets in the commodity space in the diagram above represent different bundles with different units of Good X and Good Y. Thus, (4, 1) represents a bundle with 4 kg of Good X and 1 litre of Good Y.

i) Indifference Curve Theory

The Indifference curve theory is based on ordinal measurement and was propounded by Hicks. This Theory says that a consumer may be indifferent between two or more bundles containing different units of the goods or a consumer may prefer one or more bundles over some other bundles.

Assumptions:

Before proceeding to study the Indifference Curve Theory, it is important to point out the various assumptions, without which the theory would be difficult to understand.

1. Rationality: Just like the Cardinal Utility theory, the consumer is assumed to have knowledge about the market and tries to maximize his/her satisfaction.

2. Ordinal: Different bundles of goods are ranked according to preference of the consumer. Measuring the satisfaction in terms of utility is not necessary.
3. Not additive: Total Utility depends on the quantities of commodities consumed and not the sum of marginal utilities.
4. Law of Diminishing marginal rate of substitution: The rate of substituting one good for the other decreases at a decreasing rate, as the consumption of the other good increases.
5. Consistency and Transitivity: A consumer is consistent in his choice. If he/she prefers Bundle A to Bundle B in one period, he/she cannot prefer Bundle B to Bundle A in another period.
   Further, if a consumer prefers a bundle A to bundle B and prefers bundle B to another bundle C, then the consumer will have to prefer bundle A to bundle C. However, a consumer may be indifferent between any two or more bundles. (i.e. he/she is not sure and cannot say which one to prefer to the other)

Given these assumptions, the ordinal utility theory may be explained with an example.

Nowadays there are a large number of social media sites. Facebook, Twitter, Google Plus, Pinterest are few of them.

If Google Plus is preferred to twitter, the satisfaction derived from Google plus is higher than that derived from Twitter. Therefore, higher the preference, higher will be the Utility.

It can be said that there exists a relationship between Utility of consumers and the goods consumed or used by consumers. This relationship may be represented as a function.

ii) Utility function:

In the Ordinal utility theory, utility may be considered as a function of the quantities of goods consumed. If x_1 and y_1 are the units of two goods X and Y consumed by a consumer, the mathematical function may be written as $U = f(x_1, y_1)$. This is an implicit function that states that the utility of a consumer depends on the consumption of units of good x_1 and good x_2. Here, though U is written on the L.H.S. it is not a dependent variable, but a constant. Depending upon different situations, the function may be a linear function or a non-linear function.

Example 1:

Let U be the Total Utility derived from using Facebook and Twitter. The satisfaction derived from Facebook and Twitter may be measured by the hours spent in Facebook and Twitter in a day. More hours mean higher preference.

Let x_1 be the hours spent in Facebook and y_1 the hours spent in twitter.

Then, mathematically, $U = f(x_1, y_1)$ or Total Utility is a function of the hours spent in Facebook and twitter.

Example 2:

Let the linear function,

$$U = xy$$

be the utility function such that the Utility derived from the product of two goods x and y is a constant. Now, there are infinite possibilities, or infinite number of bundles with different units of x and y that may give the same level of satisfaction, U, to the consumer. Thus, U is a constant and x and y are variables.

A set A = (4, 2) (or bundle of goods x and y with 4 units or x and 2 units of y) and another set B = (2, 4) (or bundle of goods x and y with 2 units of x and 4 units of y) are not equal. They may be referred to as ordered pairs, where the first element of each set is taken from x.

Both the bundles give the same value 8 that is the utility of the consumer from the consumption of the two goods. Likewise, the sets (1, 8), (8, 1) also give the same level of satisfaction 8.

Therefore, all the four bundles with different units of good x and y give the same value for U. Thus U is a constant in the function, U= xy.

Similarly, the sets (1, 4), (4, 1), (2, 2) give the utility as 4, which means that the utility derived from these three bundles is less than the bundles mentioned earlier. Thus, the bundles (2, 4), (4, 2), (1, 8) and (8, 1) are said to be preferred to the bundles (1, 4), (4, 1) or (2, 2)

Graphically, this situation may be represented as follows:

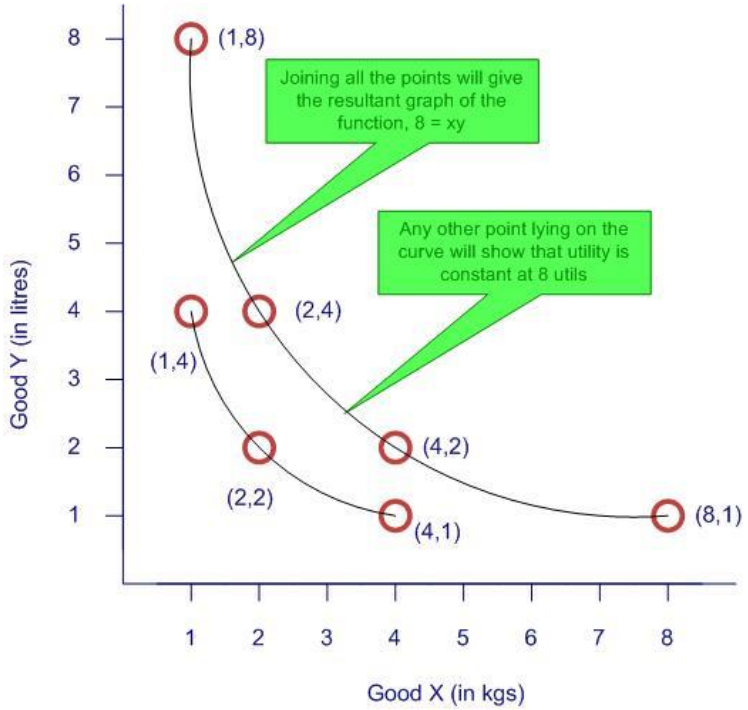


Fig 7.2: Indifference curve

iii) Indifference curves as Level Curves:

A level curve is derived from a function of two variables, say x and y, where the function have a constant value. A utility function, U = f(x, y) may be considered as a level function where the function of x and y is a constant, that is the Utility is a constant. Therefore, indifference curve may be obtained as level curves of a utility function.

Let U be a Utility function, such that

$$U = 0.5\ln(x) + 0.5\ln(y)$$

Keeping y fixed at one level and measuring Utility in the y-axis and Commodity x in the x-axis, we obtain the following indifference map. It is seen that for a given level of y, as the value of x increases, the Utility increases but it is to be noted that because of the concept of diminishing marginal utility, the curve increases at a decreasing rate.

For example, if y is fixed at level 1, we have a function as

$$U = 0.5\ln(x) + 0.5\ln(1)$$

Increasing the values of x increases the value of utility and is given. Similarly, for higher levels of y, we obtain indifference curve higher than the previous.

Finally we arrive at an indifference map as shown in the figure below.

Image 7.4 Utility curve when commodity y is constant [4]

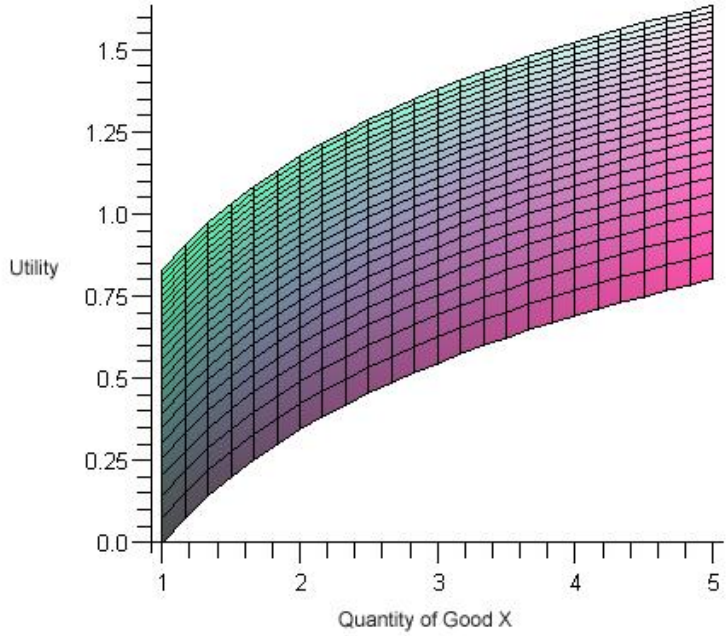


Photo 7.5 Space enclosing the curves in Photo 3.2[5]

[4], [5] https://upload.wikimedia.org/wikipedia/commons/6/6e/Indifference_curve_example.png

Plotting these curves in a three dimensional surface gives a figure as below. Higher points in this surface mean higher levels of Utility that is greater values in the Utility axis.

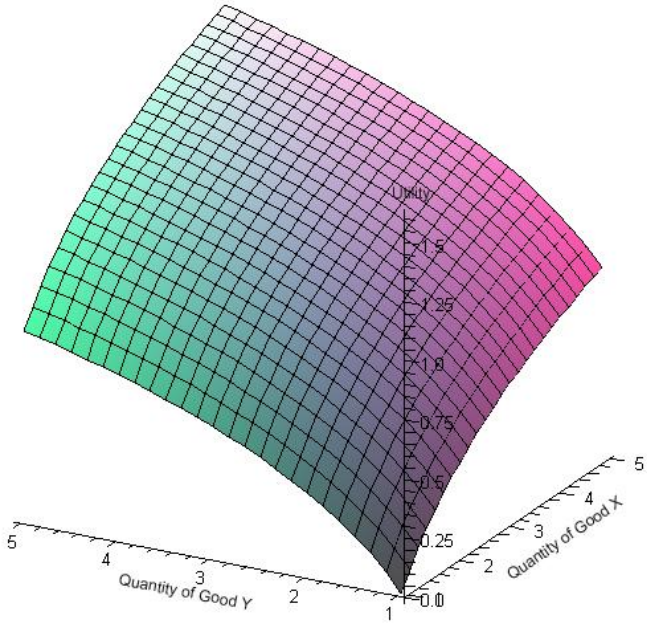


Photo 7.6: 3-D diagram of Utility curves [6]

Turning the above figure to the left we have a figure as shown below.

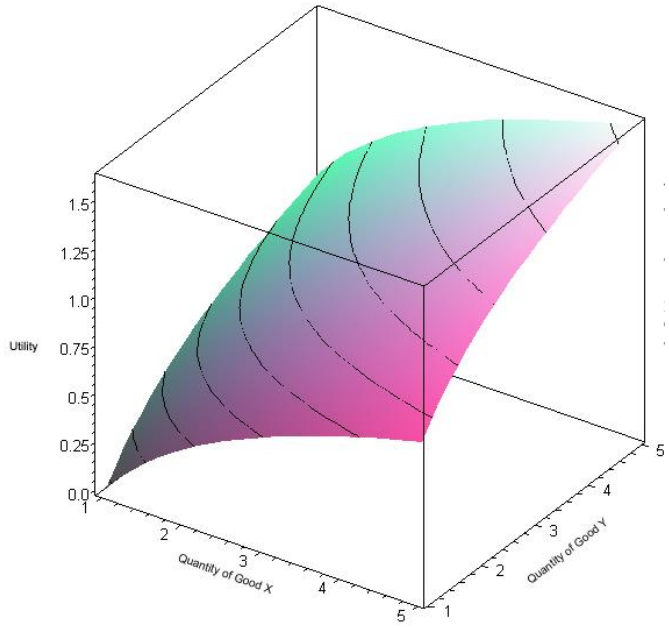


Image 7.7 Level curves [7]

[6], [7] https://upload.wikimedia.org/wikipedia/commons/6/6e/Indifference_curve_example.png

The lines on the surface in Photo 3.5 are level curves. When observed closely, these curves represent points in the surface that are of equal height. Therefore if any two points lie on the same level curve, then the two bundles with some quantity of good x and good y give the same level of utility.

Plotting the level curves in a two dimensional space gives the familiar indifference curves representing combinations of good x and good y associated with same level of utility.

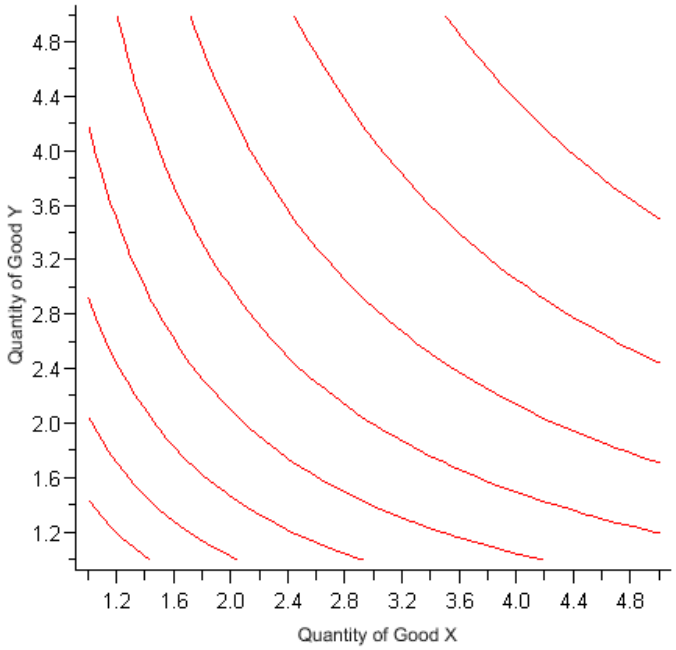


Photo 7.8 Indifference curves [8]

[8] https://upload.wikimedia.org/wikipedia/commons/6/6e/Indifference_curve_example.png

