Mathematical Economics: Input-Output Analysis

17.1 Introduction:

How many industries are there in the Indian Economy? Are the industries producing the goods independently or do they depend on other industries? What happens to the goods that are produced? This module will try to give you a gist of how the different industries and sectors of an economy operate.

Objectives

The objectives of this module are:

1. *Define* the concept of input and output
2. *Explain* the input-output table of an industry
3. *Explore* the input-output table of Indian Economy

Terminology

1. Input-Output Model: It is a mathematical technique that represents the interdependence among the different sectors of an economy
2. Tableau Économique: earliest known economic model and the first model that explains the interdependent systems in economics.
3. Productive sector: also known as primary sector that includes agriculture, forestry, mining and fishing
4. Sterile sector: manufactured goods, servants and foreign produce
5. Input: factors of production needed for production
6. Output: the product produced
7. Leontief Production Function: also known as fixed-proportions production function, is a production function in which inputs are used in fixed proportions
8. Intermediate input: output of one industry that is used as input in another industry
9. Final demand: also called final use that are not used for production purpose. It includes private consumption expenditure, government expenditure, capital formation, stocks, exports and imports.

17.1. Origin of Input-Output Model

One of the early contributions to the Input-Output concept was made by A. A. Bogdanov in 1921. Bogdanov proposed a system of planning with the idea that there is a chain link between different branches of an economy.

The modern Input-Output analysis was developed by the famous economist, Wassily Leontief. Leontief received the Nobel Prize for his contribution to the Input-Output Analysis in 1973. Leontief was the first to represent the national (or regional) economy in matrix form.

Image 17.1 A. A. Bogdanov[1]

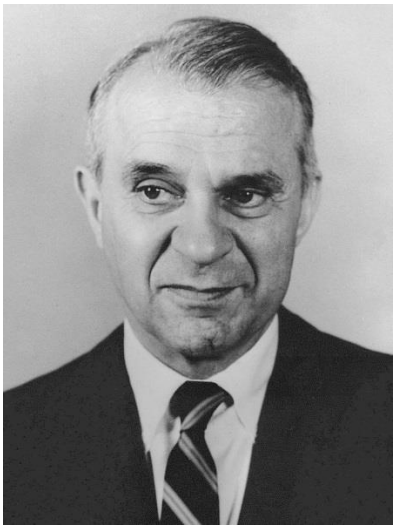

Image 17.2 Wassily Leontief[2]

[1] https://en.wikipedia.org/wiki/Alexander_Bogdanov#/media/File:A_A_Bogdanov.jpg

[2] https://en.wikipedia.org/wiki/Wassily_Leontief#/media/File:Wassily_Leontief_1973.jpg

The Input-Output Model is a mathematical technique that represents the interdependence among the different sectors of an economy or between regional economies. The basic idea of this model is that the output of one industry may be used as an input in some other industry apart from being used for final consumption.

17.2. Quesnay's Tableau Économique

The Tableau Économique or Economic table is the earliest known economic model, first described by Quesnay. This model may be considered as the first model that explains the interdependent systems in economics. This model laid the foundation of the Physiocratic School of Economic Thought.

In the first draft of the Tableau Économique, Quesnay explained the distribution of revenue received on the outputs of the "productive sector" and the sterile sector". Agriculture, forestry, fishing, mining were included in the productive sector and manufactured goods, servants, foreign produce, etc. were included in the "sterile sector". He assumed that both the sectors retain half of the revenue, supplying themselves with half their consumption needs from their own sector and spend the other half on the outputs of other sectors. The circulation process was explained using zigzag lines.

For example, if the total revenue earned was ₹1000; half of it, that is, ₹500 would go to the productive sector and the other half, that is, ₹500 would go to the sterile sector. The productive sector would retain half of ₹500 for its own consumption and spend other half, that is, ₹250 on the other sectors. Further, half of ₹250, that is, ₹125 would be retained by each sector and other half would be spent on the other sector and so.

The distribution of revenue is shown below:

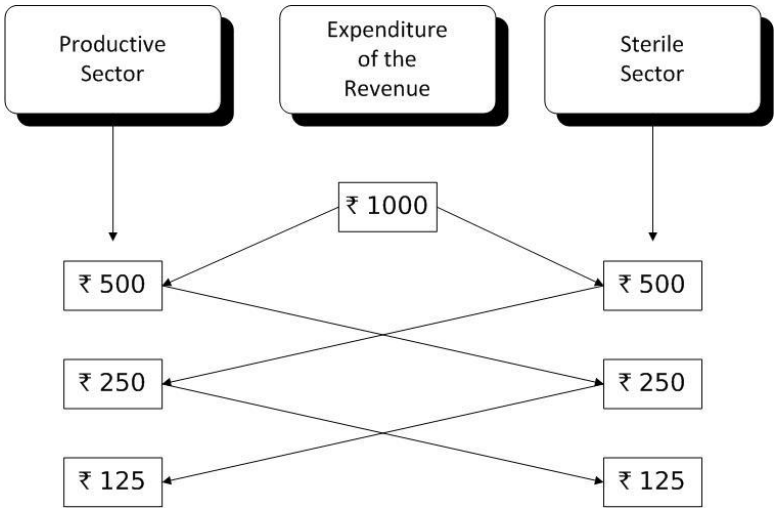

Fig.17.1: Distribution of Revenue

17.3. Input-Output in Mathematics and Economics

In Mathematics, input and output are terms used in a function. We have learned that a function is a relation that expresses the dependence of one variable on one or more other variables. The

determining variable is termed as the independent variable and the value of the variable that depends on the independent variable based on a rule expressed by the function, is termed as the dependent variable.

For example, in a function,

$$Y = a + bX \quad ---------- (17.1)$$

Y is the dependent variable and X is the independent variable.

In equation (1), a rule is established that there is a positive relationship between Y and X

Y may also be termed as the Output variable and X may be termed as the Input variable. Since values of both Y and X changes, they are termed as variables.

In Economics, inputs may be primary input (factors that are used for production) or intermediate input (output of one industry used as input to produce goods in own or other industries). Output is the product that has been processed and may be used for two major purposes:

1. The output of one industry may be used as the input for other industries (intermediate inputs)
2. The output may be used to meet final demand.

For example, a pencil is a final product. Most pencils are made of cedar wood and pencil lead is made from graphite and clay. Wood, graphite and clay are the raw materials. Pencil is produced in the pencil factory and this product may be used in the factory to do paper work. It may be used by other industries or it may go to the households for final consumption.

In the economy at large, there are a large number of products being produced every day and the output of one industry may be used as input for some other industry or it may be used as final consumption. The flow of money between industries that is involved during the process of production is represented in an input-output table.

17.4. Assumptions

Just like any other model, the Input-Output Model in economics also makes certain assumptions. Though these assumptions may look unrealistic, it gives a simple overview of how the Input-Output Analysis may be used to understand the structure of an economy.

Following are the basic assumptions:

1. Each industry produces only one homogenous product. Jointly produced commodities are not allowed.
2. Each industry follows a fixed input ratio. This is known as Leontief Production Function or fixed- proportions production function. In other words, it assumes that there is no substitutability between factors.
3. Constant returns to scale prevail in the economy. That is, if the inputs are changed by a certain proportion, the output is also changed by the same proportion.

17.5. Leontief Production Function and Constant Returns to Scale

The Leontief Production Function is a production function where inputs are used in fixed proportions. If labour and capital are the two inputs used in the fixed proportion of 1:1, that is one unit of labour and one unit of capital. If labour is increased to 2 units, capital will also increase to 2 units and if labour increases to 5 units, capital will also increase to 5 units and so on, such that the input ratio is fixed at 1:1.

Suppose 1 unit of labour and one unit of capital is used to produce 100 units of a particular product, increasing units of labour to 2 units, units of capital remaining the same, will keep the output at 100 units. Similarly, increasing the units of capital to 2 units, units of labour remaining the same, will keep the output at 10 units.

This shows that there is no substitutability between the factors, thus giving rise to a right-angled isoquant, as shown in Fig.2

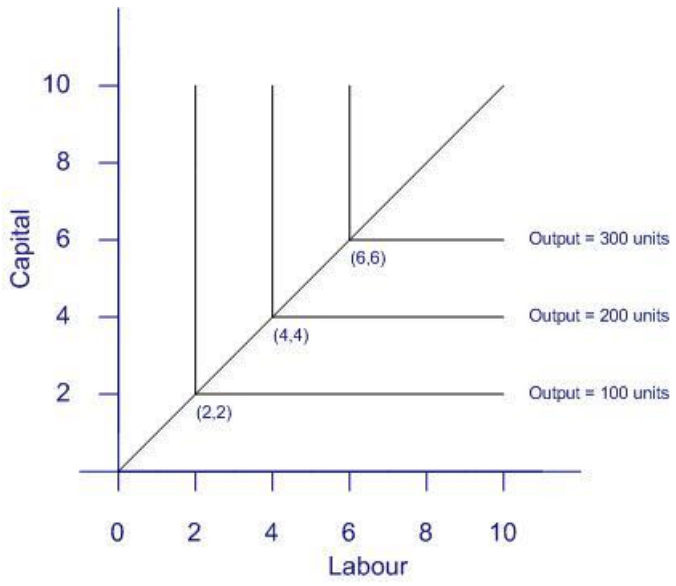

Fig: 17.2 Leontief Production Function and Constant Returns to Scale

Further, another important assumption of the Input output model says that production display constant returns to scale. Thus, if inputs are increased by a certain proportion, the output also increases by the same proportion. In the above figure, when the inputs are doubled, the output also doubles from 100 units to 200 units and when inputs are tripled, output is also tripled to 300 units thus displaying constant returns to scale.

Example:

In order to produce a car, for simplicity, let us assume only two inputs are used– tyre and steering wheel. Tyre and steering wheel are used in the proportion of 1:4, that is, one unit of steering wheel and four units of tyres are required to produce one car. In order to produce 10 cars, the inputs will have to be used in a fixed proportion. Thus, to increase production of cars to ten, steering wheel will have to be increased to 10 and tyres will increase to 40, displaying a fixed proportion of inputs of 1:4. Conversely, if inputs are doubled, the ratio becomes 2:8 and as assumed, the output will double, that is 2 cars. Similarly, if the ratio is tripled, the ratio becomes 3:12, the output will triple, that is 3 cars.

17.6. Input- Output Table

For a layman, the Input-Output table is a tabular representation of the relationship between the Inputs and the Outputs. Generally the number of observations is entered in rows and the variables are entered in columns.

Equation (1) in section 6.1.1 is a single equation with one dependent variable and one independent variable. The rule of the given function is that Y is two times X plus 5. Now, by putting different values for X we get different values for Y.

If numerical values are assumed for the constant ‘a’ and the co-efficient ‘b’ as 5 and 2 respectively, equation (1) may be written as

$$Y = 5 + 2X \quad ---------- (17.2)$$

The relationship between Y and X may be represented in a tabular in table 17.1:

Number of Observation	X (Input)	Y (Output)
1	0	5
2	1	7
3	2	9
4	3	11

Table 17.1: Simple Input-Output Table

In Table 1, there are 4 rows and three columns. The rows show the number of observation and columns show the variables Y and X and the total number of observations. This may be considered as a simple case with one input and one output. Table 1 is a simple Input-Output table. This table may be extended to two or more independent variables.

The Input-Output table of an economy is however far more complex. This is explained in the next section.

17.7. Input-Output Table of an Industry

Consider the Automotive Industry. The Automotive Industry is an industry involved in the manufacturing of motor vehicles, (passenger vehicles and light weight trucks) including engines and bodies of automobiles. For simplicity, suppose that the automotive industry produces light truck and engines and at the same time, trucks and engines are also needed in the manufacturing process for some purpose or the other such as trucks may be needed for transportation and engines may be needed to run machines. It may also be assumed that there is no final demand. That is, whatever is produced in the industry is used as intermediate inputs and not for final demand.

The flow of trucks and engines in the automotive industry in a year may be represented in a tabular form as shown in Table 2.

	Factory (Industry)		
Commodity	**Trucks**	**Engines**	**Total**
Trucks	90	10	100
Engines	150	50	200

Table 17.2: Input-Output Table of Automotive Industry

Interpretation:

Table 2 shows that, 90 trucks are needed by the truck factory 10 trucks by the engine factory. And 150 engines are needed by the truck factory and 50 engines are needed by the engine factory.

Therefore, the total output of trucks and engines should be the sum of all the intermediate demands.

That is,

Total Output of trucks = 90 +10=100

Total Output of engines = 150+50=200

17.8 Input-Output Table of an Economy

In an economy there are several industries producing a large number of products. Also, a part of the total output is used as intermediate inputs and a part is used as final demand.

For simplicity, the economy may be divided into three sectors- primary, secondary and tertiary. The primary sector includes mining, quarrying, farming, fishing and forestry. The secondary sector includes heavy manufacturing industries, light manufacturing industries, food processing, oil refining and energy production. The tertiary sector provides services to consumers and includes a wide range of businesses such as financial institutions, schools and restaurants.

If an economist wishes to study the flow of resources or money from one sector to the other, they may use the input-output table. This table gives a clear view of the structure of the economy and helps to understand the flow of money in the economy.

Sectors	Primary	Secondary	Tertiary	Final Demand	Gross Output
Primary	30	150	80	340	600
Secondary	100	95	110	395	700
Tertiary	60	100	80	560	800

Table 17.3: Input-Output Table of an Economy

Interpretation:

Table 3 is an arbitrary example of an economy with three sectors. The numbers in the cells shows the flow of money from one sector to another sector. The primary sector produces output worth ₹ 600 crore, secondary sector produces output worth ₹ 700 crore and tertiary sector produces output worth ₹ 800 crore.

The Final Demand may be categorized as Private Final Consumption Expenditure (PFCE), Government Final Consumption Expenditure (GFCE), Gross Fixed Capital Formation (GFCF), Change in Stocks (CIS), Exports of goods and services (EXP) and Imports of goods and services (IMP).

The primary sector spends ₹30 crore for itself, ₹150 crore in the secondary sector, ₹80 crore in the tertiary sector and ₹340 crore in final demand.

Similarly, the secondary sector uses ₹100 crore in the primary sector, ₹95 crore for itself, ₹110 crore in the tertiary sector and ₹395 crore in final demand.

Finally, the tertiary sector uses ₹60 crore in the primary sector, ₹100 crore in the secondary sector, ₹80 crore for itself and ₹560 in final demand.

Generally the output flow is represented in rows, and the columns represent the sectors to which the output goes as an input. The “row sum” gives an idea about the total amount of output of one sector that goes as an input to all the other sectors, as well as to itself and final demand. In other words, “row sum” tells the total supply of each sector to all the sectors. (monetary flow).

17.8. Input-Output Table of Indian Economy

The Central Statistics Office (CSO) prepares the Input-Output Transaction Table of Indian Economy once every five years. The Table includes the input flow matrix and the output matrix. The data for primary and secondary sector along with the final demand vector is compiled from the data used for preparation of national income accounts. The data for registered manufacturing sectors is obtained from Annual Survey of Industries (ASI) and that for unregistered industries is taken from the surveys conducted by National Sample Survey Office (NSSO).

Suppose there are 'k' industries in an economy producing 'n' number of commodities. If Y is the total output of a particular commodity, Z is the total output (products + by-products) of a particular industry, F is the final demand of a commodity, the data available satisfy the following mathematical relations:

Input relations: $Y_j = \sum_k x_{jk} + F_j$ ---------- (17.3); $j = 1,2,3, \ldots . n$

Output relations: $Y_j = \sum_i a_{ij}$ ---------- (17.4); $i = 1,2,3, \ldots . . k$

And $Z_i = \sum_j a_{ij}$ ---------- (17.5)

Here, Y_j is the total output of the j-th commodity.

x_{jk} is the output of j-th commodity used as input in the k-th industry.

F_j is the final demand of the j-th commodity

a_{ij} is the output of j-th commodity produced by the i-th industry.

17.8.1. Different matrices that are included in the Input-Output Table of Indian Economy are:

a) Matrix-1: Input Flow (or Absorption) Matrix as the commodity x industry matrix:

This matrix shows the Input and output flows for different sectors in terms of commodities, using the ASICC (Annual Survey of Industries Commodity Codes) for the manufacturing sectors and the IOT (Input Output Table) sector codes for other sectors. The sum of the entries along any row shows the total of the inter-industry and the final use of the commodity. Since the table is commodity x industry transaction presentation, the row totals do not tally with the column totals.

b) Matrix-2: Output (or Make) Matrix as the industry x commodity matrix:

The data for the industry-wise details of output are tabulated to obtain the industry x commodity matrix (Make matrix) by merging the output flows from registered, unregistered manufacturing sectors and the output flows from primary and service sectors.

c) Matrix-3: Input-Output Coefficient Matrix:

This is the matrix of Input-Output co-efficient. This co-efficient expresses the amount of one commodity used as intermediate input to produce one unit of output of another commodity.

If y_{ij} is the total quantity of output of i-th commodity used as input in the j-th commodity

Y_j is the total output of the i-th industry

Then, $b_{ij} = \frac{y_{ij}}{Y_j}$ gives the input-output co-efficient. **(Refer to Module 18, Section 18.2)**

d) Matrix-4: Product Mix Matrix:

In the product mix matrix, column entries show the proportions in which a particular industry produces various commodities, the diagonal elements give the proportion of the main product in the output of the industry while the off-diagonal elements indicate the subsidiary products and by-products.

e) Matrix-5: Market Share Matrix

In the market share matrix, column entries present the proportions in which the various industries produce the total output of a particular commodity, the diagonal elements give the proportions of the output of various commodities produced as main products and the off-diagonal elements show the proportions of the commodities produced as subsidiary products in other industries.

f) Matrix-6: Commodity x Commodity Matrix and Industry x Industry Matrix under the industry technology assumption:

The commodity x commodity input-output table is suitable for multisectoral projections where final demand estimates are obtained on commodity bases. The industry x industry input-output table is useful in detailed planning of industries whose products include by-products also. In a commodity x commodity table both rows and columns represent the

commodity group sectors. In an industry x industry table, on the other hand, both rows and columns represent industry group sectors comprising of a mix of different commodity groups. The row of a sector in this table gives the supply of all products and secondary product (as a mix) produced by the corresponding industry group for different intermediate and final uses.

g) Matrix-7: Leontief Inverse Matrix for commodities

The inverse matrix obtained by using matrix-inversion. **(Refer to Module 18, Section 18.2)**

17.8.2. From Input-Output Table (IOT) to Supply Use Table (SUT)

The Central Statistics Office (CSO) has been publishing Input-Output tables since 1968. The methodology followed for constructing these tables follows to the basic principles articulated in the System of National Accounts. In 2015, CSO brought out a new series of National Accounts Statistics after the recommendations of the System of National Accounts, 2008. There were certain statistical discrepancies as the GDP derived from production side and expenditure side did not match and this could be avoided by compiling the Supply and Use Tables that

"reflects the production of industries in which intermediate products and primary inputs are required, showing where goods and services are produced and where they are used as intermediate consumption, final consumption, gross capital formation and exports." [4]

that offered a detailed analysis of the process of production and the use of goods and services and the income generated in that production.

The CSO published the SUT table for 140 products and 66 industries of Indian Economy. In the Supply and Use Table, there are separate tables for Supply, adjustment for valuation and Use. The compilation of the Use tables is a bit cumbersome.

