Mathematical Economics: Linear Programming- An Introduction

23.1 Introduction:

Organizing and managing large data is cumbersome. A systematic approach to organize and manage large data saves time, money and energy and at the same time results in efficient and best decisions by maximizing the gains and minimizing the losses.

This module introduces a technique to help decision makers.

Objectives

The objectives of this module are:

1. *Understand the* concept of Programming and linear programming
2. *Explore* the applications of Linear Programming

Terminology

1. Programming: a process of planning, scheduling or performing a program
2. Mathematical Programming: an optimization technique for efficient allocation of resources that uses mathematical models
3. Linear Programming: a mathematical programming that uses linear models
4. Non-linear programming: a mathematical programming that uses non-linear models
5. Algorithm: a process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer.

23.2. Meaning and History

When we come cross the word "programming", we are usually reminded of computer programming. The term programming" means a process of planning, scheduling or performing of a program. A program may be defined as a sequence of activities that are taken to accomplish a certain task or attain a particular goal.

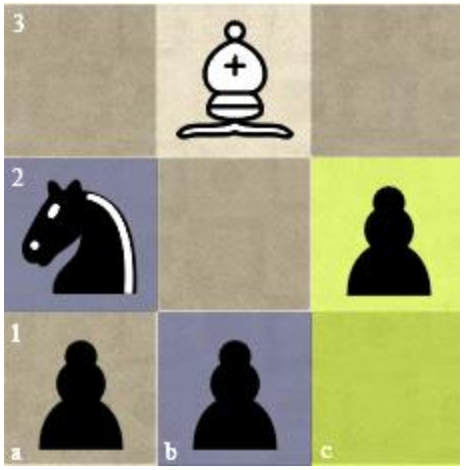

Image 23.1 Lichess: Internet Chess Program

[Source: https://en.wikipedia.org/wiki/Computer_programming]

Lichess is an example of an Internet Chess Program. If the white bishop at b3 moves to a2, capturing the black knight, then the black pawn at b1 is programmed to capture the bishop back. Black has just moved a pawn to c2.

Systematic development of computer programs to solve optimization problems led to the development of mathematical programming. Mathematical programming is also referred to as 'non-classical method of optimization'. The non-classical method generally deals with optimization problems with inequalities.

A. Mathematical Programming:

Mathematical programming is broadly classified into linear programming and non-linear programming. Optimization problems involving linear inequalities are studied in linear programming and problems involving more complex non-linear inequalities are studied non-linear programming. Other branches of mathematical programming are quadratic programming, convex programming, integer programming and stochastic programming.

B. Linear Programming:

The history of linear programming dates back to 1827 when Joseph Fourier initiated a mathematical algorithm for eliminating variables from a system of linear inequalities. Theodore Motzkin, in 1936 is also known to have independently discovered the mathematical algorithm. The Fourier-Motzkin Elimination (FME) Method is named after them.

Image 23.2 Joseph Fourier

[Source: https://en.wikipedia.org/wiki/Joseph_Fourier#/media/File:Fourier2.jpg]

An algorithm is a step-wise process of calculation or a set of rules in solving a problem. Usually the term algorithm is used in computers. A mathematical algorithm describes a step-wise procedure to solve mathematical computations.

Table 23.1: Contributions to development of Linear Programming

Names of mathematicians and economists	**Development in linear programming**
Leonid Kantorovich	Linear programming formulation to plan expenditures and returns in order to reduce costs of the army and to increase losses imposed on the enemy
T. C. Koopmans	Formulated classical economic problems as linear programs
Frank Lauren Hitchcock	Formulated transportation problems
George B. Dantzig	Development of simplex algorithm for solving linear programming problems
John von Neumann	Proposed theory of duality in linear programming
Leonid Khachiyan	Linear programming problem first shown to be solvable in polynomial time
Narendra Karmarkar	Introduced a new interior-point method to solve linear programming problems

Table 23.1 gives a list of some of the mathematicians and economists involved with the development of linear programming. Among them is Narendra Karmarkar, an Indian mathematician who developed the 'Karmakar Algorithm' that is helpful in business and policy decision making.

Tjalling Charles Koopmans (August 28, 1910 – February 26, 1985) was a Dutch American mathematician and economist. He was the joint winner with Leonid Kantorovich of the 1975 Nobel Memorial Prize in Economic Sciences for his work on the theory of the optimum allocation of resources

23.3. Application of Linear Programming

There are varied applications of Linear Programming. Some of the areas where this technique is used are:

a) **Agriculture:** The linear programming tool may be used to solve many practical problems in agriculture such as, optimum allocation of land, farm management problems, least cost ration formulation, optimization of livestock production and optimization structure of plant production.

   Example: Farm Management involves organizing and operating a farm for maximum production and profit.

Image 23.3: Harvesting wheat with a combine harvester accompanied by a tractor and trailer

[Source: https://commons.wikimedia.org/wiki/User:Michael_Gäbler]

b) Manufacturing: Linear programming may be used in manufacturing industries to maximize profit, optimization of production scheduling, optimization product-mix problem, optimal use of raw materials, etc.

Example: Material efficiency in construction can be less expensive and energy intensive. Material efficiency include reducing energy demand, reducing harmful emissions and other environmental impacts of industry, and increasing national resource security.

Image 23.4: Building construction can be a materially consumptive endeavor

[Source: https://en.wikipedia.org/wiki/Material_efficiency#/media/File:BuildingSite.jpg]

c) **Transportation**: Transportation problem is based on the supply and demand of commodities or people from different sources to different destinations. Linear programming may be used to reduce the transportation cost.

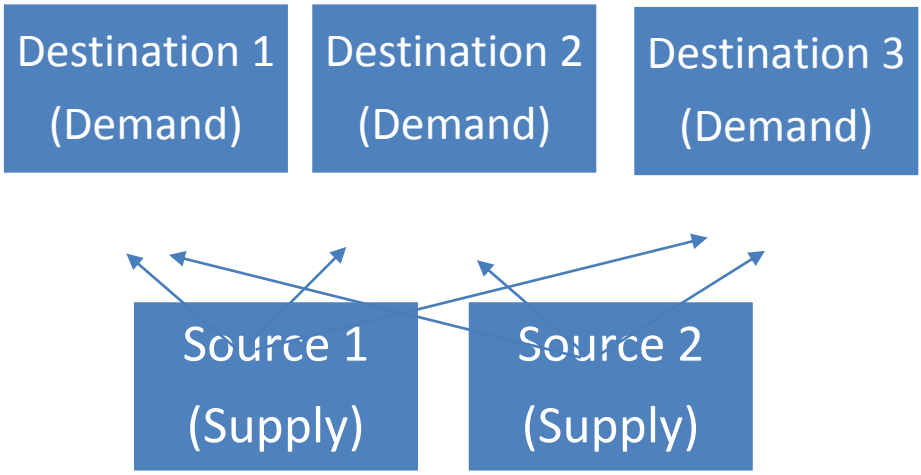

Fig. 23.1: Transportation of goods from 2 sources to 3 destinations

d) **Energy:** Linear programming may be used to maximize energy savings or reduce energy consumption of households.

Example: LED (Light Emitting Diode) bulbs are the most energy efficient lighting option. LED bulbs use 75% less electricity than incandescent light bulbs.

Image 23.5: An assortment of LED lamps commercially available in 2010 as replacements for screw-in bulbs, including floodlight fixtures (left), reading light (center), household lamps (center right and bottom), and low-power accent light (right) applications

[Source: https://en.wikipedia.org/wiki/LED_lamp#/media/File:LED_bulbs.jpg]

e) **Health care**: A balanced diet is a diet that gives the body the minimum nutrients to function properly. Leafy green, cruciferous, and other raw vegetables may contribute to a healthy diet. In the health care field, linear programming may be used to develop balanced diets at lesser costs and adhering to a set of minimum nutritional requirements.

Image 23.6: A Healthy Diet

[Source: https://en.wikipedia.org/wiki/User:Geoffrey.landis]

f) **Military operations:** Linear programming may be used to solve military problems such as army logistics, wartime patient evacuation, cargo distribution and regulate other military operations during a crisis.

Image 23.7: Red Ball Express: Trucks moving through a regulating point
[Source: https://en.wikipedia.org/wiki/Military_logistics#/media/File:Red_Ball_Express_Regulating_Point.jpg]

The technique of maximizing or minimizing an objective is helpful to professionals such as engineers, business managers, financial analysts, computer programmers, research scientists and architects.

g) **Economics:** In Economics in particular, linear programming may be used to solve economic problems such as maximizing profits, revenue and sales, or minimizing the cost, in calculating economic growth, by inserting assumed values for the variables in the equations and solving for the unknowns.

Note:

Simple linear programming problems may be solved manually but computers are widely used in obtaining the solutions.

