Matrix and National Income Model

Introduction:

Do you know how a Nation earns income? How is the National Income measured? When is an economy said to be in equilibrium? Answer to these questions may not seem to be important for your daily economic activities. But the knowledge of how the economy works will help you understand how different macroeconomic variables influence economic activities and how economic activities affect the macroeconomic variables.

Objectives

The objectives of this module are:

1. *Explain* the concept of National Income
2. *Construct the National Income model*
3. *Solve* the National Income model using matrix algebra

Terminology

1. National Income: Income of a nation that includes, Gross Domestic Product, Government and Private Final Consumption Expenditure, Fixed Capital Formation and other macro-economic aggregates.
2. Great Depression: a worldwide economic downturn where stock markets crash and GDP falls drastically.
3. Gross Domestic Product (GDP): a monetary measure of the market value of all the final goods and services produced in a year
4. Government Final Consumption Expenditure: Final consumption expenditure of administrative departments, expenditure on durable goods, which are used for defence are also treated as part of consumption expenditure of the Government.
5. Private Final Consumption Expenditure: final consumption expenditure of households and non-profit institutions serving households (NPISH) like temples, gurdwaras
6. Capital Formation: additions to fixed assets such as construction and machinery and equipment (including transport equipment and breeding stock, draught animals, dairy cattle and the like; and change in stocks during the accounts period.
7. Induced consumption: part of consumption that varies with disposable income
8. Autonomous consumption: minimum level of consumption, even if a consumer has no disposable income
9. Disposable Income: income in the hands of the consumers after deducting direct taxes

15.1. History of National Income Accounting:

Literature says that William Petty (1623-1687), an English economist, physician, scientist and philosopher introduced the first rigorous assessments of national income and wealth. During his time, he was one to employ the use of quantitative data.

Image 15.1 Sir William Petty

[Source: https://en.wikipedia.org/wiki/William_Petty#/media/File:Sir_William_Petty.jpg]

Petty worked off an estimation that the average personal income was £6 13s 4d (6 pounds 13 shillings and 4 pence) per annum, with a population of six million, which means that national income would be £40m. Petty's theory produced estimates, some more reliable than others, for the various components of national income, including land, ships, personal estates and housing.

After the Great Depression and as a basis for Keynesian macroeconomic stabilization policy and war time economic planning, the first efforts to measure the aggregate economic activity were undertaken by Colin Clark and Simon Kuznets. Collin pioneered the use of Gross National Product (GNP). In 1931, Kuznets took charge of the NBER's work on U.S. national income accounts. Although Kuznets was not the first economist to assess national accounts, his work was comprehensive and meticulous that it set the standard in the field. The first formal national accounts were published by the United States in 1947.

Image 15.2 Collin Clark [2]

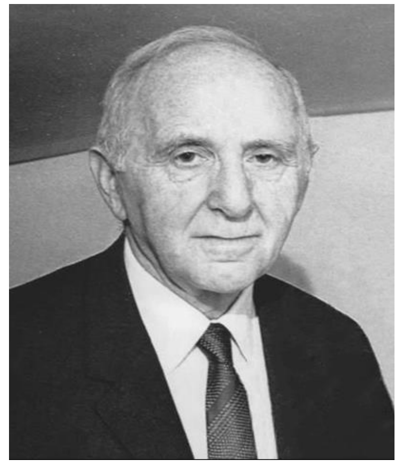

Image 15.3 Simon Kuznets [3]

[2]https://en.wikipedia.org/wiki/Colin_Clark_(economist)#/media/File:Colin_Clark_(economist).png
[3]https://en.wikipedia.org/wiki/Simon_Kuznets#/media/File:Simon_Kuznets_1971b.jpg

Later, Sir John Richard Nicholas Stone, an eminent British economist, received the Nobel Memorial Prize in Economic Sciences in 1984 for developing an accounting model that could be used to track economic activities on a national and later, international level. He is sometimes known as the "Father of National Income Accounting".

Richard Stone mentioned the French economist and physician, Quesnay and the *Tableau économique* (Economic Table) that was explained by Quesnay in 1758. Stone stated that the Tableau *économique* was one of the very first works in economics to examine various sectors on a global level in an analytical way.

Image 15.4 François Quesnay[4]

Image 15.5 Richard Stone[5]

[4] https://en.wikipedia.org/wiki/Fran%C3%A7ois_Quesnay#/media/File:Quesnay_Portrait.jpg
[5] https://en.wikipedia.org/wiki/Richard_Stone#/media/File:Richard_Stone.jpg

15.2. National Income Accounts in India:

National Accounts Division (NAD) of the Central Statistics Office is responsible for preparing the national accounts of India which includes Gross Domestic Product (GDP), Government and Private Final Consumption Expenditure, Fixed Capital Formation and other macro-economic aggregates. The Indian System of National Accounts include regional accounts at the state level and below.

The estimates of National income and related aggregates are statistics derived from data available from different primary sources such as such as land records, collection of direct and indirect taxes, civil registration of births and deaths, censuses and sample surveys conducted by official agencies of the Central and State Governments. These estimates are published in "National Accounts Statistics" (NAS), an annual publication by The National Accounts Division of CSO.

15.3 National Income Model: Closed Economy

The National Income Model is an important model in Macroeconomic Analysis. This model may be studied for a closed economy and an open economy. A closed economy is an economy where the values of exports and imports are not taken into consideration. On the other hand, an open economy is an economy where the values of the exports and the imports are taken into consideration.

A) Construction of the model

For a closed economy, the National Income may be defined as the sum of the consumption expenditure, investment expenditure and the government expenditure. This definition may be given a mathematical expression as

$$Y = C + I + G -\!-\!-\!-\!- (15.1)$$

Here Y is National Income
C is the consumption expenditure
I is the Investment expenditure
G is the government expenditure

Equation (1) defines the National Income of an economy and therefore may be called a definitional equation.

In equation (1), let us assume that Investment and Government expenditure are the exogenous variables and Y and C are the endogenous variables. The values of the exogenous variables have been determined from outside the model and may be considered as given, while the value of the endogenous variable is to be determined from within the model.

Now, an economy shows a certain kind of consumption behavior. This may be studied in two parts:

1. **Induced consumption:** It is generally observed that a change in the income induces a change in the consumption expenditure. This type of consumption is known as ***induced consumption.*** More specifically, consumption expenditure depends upon the disposable income. Disposable income is the income left out after deducting the direct tax. People behave in a way that shows a positive relationship between consumption and income. In other words, higher the income, higher will be the consumption and lower the income, lower will be the consumption.
2. **Autonomous consumption:** It may not always be the case that consumption depends on the disposable income. Sometimes, even when the income is zero, there are certain expenses that are made from savings or by borrowing from others. Such consumption expenditure is known as ***autonomous consumption.*** This consumption may be assumed to be a fixed amount or a constant.

Thus, the consumption function may mathematically be written as

$$C = \alpha + \beta\,(Y - T) \;-----\; (15.2)$$

Where $C\ is\ consumption\ expenditure$

$\alpha\ is\ autonomous\ consumption\ (\alpha > 0)$

$Y\ is\ total\ income$

$T\ is\ direct\ tax$

$\beta\ is\ the\ Marginal\ Propensity\ to\ Consume\ or\ MPC\ (0 < \beta < 1)$

$(Y - T)$ is the disposable income or the income left out after deducting the direct tax

from the total income.

MPC is the rate at which the consumption changes when the income changes. In other words, it may also be mathematically represented as $\beta = \frac{\Delta C}{\Delta Y}$

Further, direct tax is a function of total income and when the income is zero, there will be no tax. Thus, there may be a third equation to the National Income Model on the tax component as

$$T = tY \text{ - - - - - } (15.3)$$

Where, t is the rate of change of tax with respect to change in the income. (0<t<1)

Combining equation (15.1), (15.2) and (15.3,) we may therefore construct the National Income Model as

$$Y = C + I + G$$

$$C = \alpha + \beta\,(Y - T) \quad (\alpha > 0)\,(0 < \beta < 1)$$

$$T = tY \qquad (0 < t < 1)$$

The endogenous variables in the model are Y, C and T and the exogenous variables are I and G.

α, β and t are the parameters.

B) Transforming to matrix form

The National Income Model may be solved by using matrix algebra to save time. This is done by following the simple steps of transformation.

Step 1: Re-arrange the variables on the LHS and constants on the RHS

Thus,

$$Y - C = I + G$$

$$C - \beta\,(Y - T) = \alpha$$

$$T - tY = 0$$

Step 2: Re-write all equations in such a way that all variables are included in all the equations and if a variable is absent it is given a co-efficient of 0.

Thus,

$$1 \times Y - 1 \times C + 0 \times T = I + G$$

$$-\beta \times Y + 1 \times C + \beta \times T = \alpha$$

$$- t \times Y + 0 \times C + 1 \times T = 0$$

Step 3: Construct matrices and vectors

Thus,

Matrix of co-efficient may be constructed as $A = \begin{bmatrix} 1 & -1 & 0 \\ -\beta & 1 & \beta \\ -t & 0 & 1 \end{bmatrix}$

Vector of variables may be constructed as $V = \begin{bmatrix} Y \\ C \\ T \end{bmatrix}$

Vector of constants may be written as $C = \begin{bmatrix} I + G \\ \alpha \\ 0 \end{bmatrix}$

Thus the three equations have been transformed into a single equation in matrix form as

AV = C --------------- (15.4)

where A is a 3 x 3 matrix, V is a 3 x 1 vector and C is a 3 x 1 vector.

Step 4: Solve equation (15.4) by using matrix method

Equation (4) gives AV = C

Thus, $V = A^{-1} C$

Now, $A^{-1} = \frac{Adj\ A}{|A|}$

Proceeding to find A^{-1}, we first find |A|. Thus,

$|A| = \begin{bmatrix} 1 & -1 & 0 \\ -\beta & 1 & \beta \\ -t & 0 & 1 \end{bmatrix}$ = 1 (1 – 0) +1 (-β +βt) + 0 (0 + t) = 1 - β + βt ≠ 0

Since the determinant is not equal to zero, a solution exists.

Next we proceed to find the co-factors of all the elements of matrix A by using the formula

$C_{ij} = (-1)^{i+j} M_{ij}$

Thus,

Co-factor of 1 = $(-1)^{1+1} \begin{vmatrix} 1 & \beta \\ 0 & 1 \end{vmatrix}$ = 1

Cofactor of -1 = $(-1)^{1+2}\begin{vmatrix}-\beta & \beta\\ -t & 1\end{vmatrix}$ = -1(- β +βt) = β - βt

Co-factor of 0 = $(-1)^{1+3}\begin{vmatrix}-\beta & 1\\ -t & 0\end{vmatrix}$ = 1(0 + t) = t

Co-factor of $-\beta$ = $(-1)^{2+1}\begin{vmatrix}-1 & 0\\ 0 & 1\end{vmatrix}$ = -1(-1 + 0) = 1

Co-factor of 1 = $(-1)^{2+2}\begin{vmatrix}1 & 0\\ -t & 1\end{vmatrix}$ = 1

Co-factor of β = $(-1)^{2+3}\begin{vmatrix}1 & -1\\ -t & 0\end{vmatrix}$ = -1 x (0 – t) = t

Co-factor of –t = $(-1)^{3+1}\begin{vmatrix}-1 & 0\\ 1 & \beta\end{vmatrix}$ = 1 (- β + 0) = - β

Co-factor of 0 = $(-1)^{3+2}\begin{vmatrix}1 & 0\\ -\beta & \beta\end{vmatrix}$ = -1(β + 0) = - β

Cofactor of 1 = $(-1)^{3+3}\begin{vmatrix}1 & -1\\ -\beta & 1\end{vmatrix}$ = $1-\beta$

Arranging the co-factors in a matrix gives a 3 x 3 matrix as follows:

$$\begin{bmatrix}1 & (\beta-\beta t) & t\\ 1 & 1 & t\\ -\beta & -\beta & 1-\beta\end{bmatrix}$$

Therefore, Adjoint of A is obtained by transposing the co-factor matrix. This gives

$$Adj\ A = \begin{bmatrix}1 & 1 & -\beta\\ (\beta-\beta t) & 1 & -\beta\\ t & t & 1-\beta\end{bmatrix}$$

Now,

$$A^{-1} = \frac{Adj\ A}{|A|} = \frac{1}{(1-\beta+\beta t)}\begin{bmatrix}1 & 1 & -\beta\\ (\beta-\beta t) & 1 & -\beta\\ t & t & 1-\beta\end{bmatrix}$$

Hence,

$$V = \begin{bmatrix}Y\\ C\\ T\end{bmatrix} = A^{-1}C = \frac{1}{(1-\beta+\beta t)}\begin{bmatrix}1 & 1 & -\beta\\ (\beta-\beta t) & 1 & -\beta\\ t & t & 1-\beta\end{bmatrix}\begin{bmatrix}I+G\\ \alpha\\ 0\end{bmatrix}$$

Or, $\begin{bmatrix} Y \\ C \\ T \end{bmatrix} = \frac{1}{(1-\beta+\beta t)} \begin{bmatrix} I + G + \alpha \\ (\beta - \beta t)(I + G) + \alpha \\ t(I + G) + t\alpha \end{bmatrix}$

Thus,

Equilibrium Income: Y = $\frac{I+G+\alpha}{(1-\beta+\beta t)}$

Equilibrium Concumption: C = $\frac{(\beta-\beta t)(I+G)+\alpha}{(1-\beta+\beta t)}$

Equilibrium Tax: T = $\frac{t(I+G)+t\alpha}{(1-\beta+\beta t)}$

