Solved Problems: Market Model and National Income Model

This module will use matrix algebra to solve problems related to the market model, its equilibrium and the equilibrium of the National Income Model

16.1. Given a market model

$$D = 50 - 2P$$
$$S = -10 + 3P$$
$$D = S$$

Find the equilibrium Price and Quantity using Matrix algebra

Solution:
Given

$$D = 50 - 2P \text{-----------(1)}$$
$$S = -10 + 3P \text{ ---------(2)}$$
$$D = S \qquad \text{-----------(3)}$$

i) Substituting equation (1) and (2) in equation (3), we get

$$50 - 2P = -10 + 3P$$
$$=> \quad 5P = 60$$
$$=> \quad P = 12$$

Therefore, equilibrium price is 12.

Now, substituting the value P in either of equation (1) or (2) will give the equilibrium quantity.

Thus,

Form equation (1), we get

$$D = 50 - 2(12) = 26.$$

Therefore equilibrium quantity is 26.

ii) An alternative way of finding the equilibrium price and quantity is to put the values of the parameters in the formula for equilibrium price and quantity.

Given the market model

$$D = a - bP\text{-----------}(1)$$
$$S = -c + dP\text{ ---------}(2)$$
$$D = S\qquad\text{-----------}(3)$$

Equilibrium price

$$P = \frac{a + c}{b + d}$$

And

Equilibrium quantity

$$Q = \frac{ad - bc}{b + d}$$

In the above example,

$$a = 50, b = 2, c = 10 \; and \; d = 3$$

Therefore, Equilibrium price

$$\boldsymbol{P = \frac{50 + 10}{2 + 3} = 12}$$

And equilibrium quantity

$$\boldsymbol{Q = \frac{50 \times 3 - 2 \times 10}{2 + 3} = 26}$$

iii) A third alternative method of finding the equilibrium price and quantity is the matrix inverse method.

In this method, we first express the market model in matrix form.

Thus,

$$D = 50 - 2P\text{-----------}(1)$$
$$S = -10 + 3P\text{ ---------}(2)$$
$$D = S$$

May be expressed in matrix form by using the following steps:

a) Re-write all the equations by placing variables on the LHS and constant on the RHS. Thus,

$$D + 2P = 50$$
$$S - 3P = -10$$
$$D - S = 0$$

b) Construct a matrix of the co-efficient of the variables. The variable that is absent in an equation is given a co-efficient 0. The matrix of co-efficient thus constructed is

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 1 & -1 & 0 \end{bmatrix}$$

c) Next we construct a column vector of variables and a column vector of constants. Thus,

$X = \begin{bmatrix} D \\ S \\ P \end{bmatrix}$ is the column vector of variables

$$Z = \begin{bmatrix} 50 \\ -10 \\ 0 \end{bmatrix}$$

d) The matrix form of the given market model therefore may be expressed as $AX = Z$

Where A is a matrix of order 3×3

X is a column vector of order 3×1

Z is a column vector of order 3×1

Hence, $X = A^{-1}Z$

e) To find the values of the elements of matrix X, we find the inverse of A.

By formula, $$A^{-1} = \frac{Adj\ A}{|A|}$$

$$|A| = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 1 & -1 & 0 \end{vmatrix} = 1(0-3) - 0(0+3) + 2(0-1) = -5$$

$$Adj\ A = \begin{bmatrix} -3 & -2 & -2 \\ -3 & -2 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{\begin{bmatrix} -3 & -2 & -2 \\ -3 & -2 & 3 \\ -1 & 1 & 1 \end{bmatrix}}{-5} = \begin{bmatrix} 3/5 & 2/5 & 2/5 \\ 3/5 & 2/5 & -3/5 \\ 1/5 & -1/5 & -1/5 \end{bmatrix}$$

$$\text{Now, } X = \begin{bmatrix} 3/5 & 2/5 & 2/5 \\ 3/5 & 2/5 & -3/5 \\ 1/5 & -1/5 & -1/5 \end{bmatrix}\begin{bmatrix} 50 \\ -10 \\ 0 \end{bmatrix} = \begin{bmatrix} 26 \\ 26 \\ 12 \end{bmatrix}$$

Therefore, Equilibrium Quantity demanded = Equilibrium Quantity supplied =26

And Equilibrium price = 12

iv) There is another method of solving the above problem using matrix algebra and that is the Cramer's Rule.

According to Cramer's rule,

$$|A|_1 = \begin{vmatrix} 50 & 0 & 2 \\ -10 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = 50(0-3) - 0(0-0) + 2(10-0) = -130$$

$$|A|_2 = \begin{vmatrix} 1 & 50 & 2 \\ 0 & -10 & -3 \\ 1 & 0 & 0 \end{vmatrix} = 1(0+0) - 50(0+3) + 2(0+10) = -130$$

$$|A|_3 = \begin{vmatrix} 1 & 0 & 50 \\ 0 & 1 & -10 \\ 1 & -1 & 0 \end{vmatrix} = 1(0-10) - 0(0+10) + 50(0-1) = -60$$

Now, Equilibrium Quantity demanded = $\frac{A_1}{|A|} = \frac{-130}{-5} = 26$

Equilibrium Quantity supplied = $\frac{A_2}{|A|} = \frac{-130}{-5} = 26$

Equilibrium Price = $\frac{A_3}{|A|} = \frac{-60}{-5} = 12$

16.2. A company requires three inputs to produce a product. The input requirements for four weeks for the three inputs L, M and N are given as (in numbers of units of each input) by the matrix

$$P = \begin{bmatrix} 2 & 0.5 & 1 & 7 \\ 6 & 3 & 8 & 2.5 \\ 4 & 5 & 2 & 0 \end{bmatrix}$$

The company can buy these inputs from two suppliers, whose prices for the three inputs L, M and N are given (in ₹) by the matrix

$$Q = \begin{bmatrix} 4 & 6 & 2 \\ 5 & 8 & 1 \end{bmatrix}$$

Obtain a matrix that will give the total input bill for the company for four weeks for both the suppliers and interpret the elements of the resultant matrix.

Solution:

The product of matrix Q and P gives:

$$QP = \begin{bmatrix} 4 & 6 & 2 \\ 5 & 8 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0.5 & 1 & 7 \\ 6 & 3 & 8 & 2.5 \\ 4 & 5 & 2 & 0 \end{bmatrix}$$

$$Or,\ QP = \begin{bmatrix} 8+36+8 & 2+18+10 & 4+48+4 & 28+15+0 \\ 10+48+4 & 2.5+24+5 & 5+64+2 & 35+20+0 \end{bmatrix}$$

$$Or, \quad QP = \begin{bmatrix} 52 & 30 & 56 & 43 \\ 62 & 31.5 & 71 & 55 \end{bmatrix}$$

Therefore,

The input bill for Supplier 1 is ₹52, ₹30, ₹56 and ₹43 respectively for Week 1, 2, 3 and 4

The input bill for Supplier 2 is ₹62, ₹31.5, ₹72and ₹55 respectively for Week 1, 2, 3 and 4

16.3. Given the following National Income Model:

$$Y = C + I_0 + G_0$$
$$C = 40 + 0.7(Y - T)$$
$$T = 100 + 0.4Y$$

a) If $I_0 = 1000\ and\ G_0 = 1500$, express the following model in matrix form.
b) Find the equilibrium Income (Y), Consumption (C) and tax (T) using matrix algebra.

Solution:

a) Given the National Income Model

$$Y = C + 1000 + 1500 - - - - - (1)$$
$$C = 40 + 0.7(Y - T) - - - - - - -(2)$$
$$T = 100 + 0.4Y - - - - - - - - -(3)$$

We re-write the equations in such a way that all variables are on the LHS and constants on the RHS. Also, the missing variables are given co-efficient zero. Therefore,

$$Y - C + 0 \times T = 1000 + 1500 - - - - - - -(1)$$
$$-0.7Y + C + 0.7T = 40 \qquad - - - - - - - - (2)$$
$$-0.4Y + 0 \times C + T = 100 \qquad - - - - - - - - (3)$$

In matrix notation,

$$\begin{bmatrix} 1 & -1 & 0 \\ -0.7 & 1 & 0.7 \\ -0.4 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y \\ C \\ T \end{bmatrix} = \begin{bmatrix} 2500 \\ 40 \\ 100 \end{bmatrix}$$

Or, $AX = Z$

Or, $X = A^{-1}Z$

Where X is a column vector of variables of order 3×1
A is a matrix of co-efficient of order 3×3
Z is a column vector of order 3×1

Now,

$$A^{-1} = \frac{Adj\ A}{|A|}$$
$$|A| = \begin{vmatrix} 1 & -1 & 0 \\ -0.7 & 1 & 0.7 \\ -0.4 & 0 & 1 \end{vmatrix} = 1(1 - 0) + 1(-0.7 + 0.28) + 0 = 0.58$$

$$Adj\ A = \begin{bmatrix} 1 & 1 & -0.7 \\ 0.42 & 1 & -0.7 \\ 0.4 & 0.4 & 0.3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{Adj\ A}{|A|} = \frac{\begin{bmatrix} 1 & 1 & -0.7 \\ 0.42 & 1 & -0.7 \\ 0.4 & 0.4 & 0.3 \end{bmatrix}}{0.58} = \begin{bmatrix} 1/0.58 & 1/0.58 & -0.7/0.58 \\ 0.42/0.58 & 1/0.58 & -0.7/0.58 \\ 0.4/0.58 & 0.4/0.58 & 0.3/0.58 \end{bmatrix}$$

$$\text{Now, } X = \begin{bmatrix} 1/0.58 & 1/0.58 & -0.7/0.58 \\ 0.42/0.58 & 1/0.58 & -0.7/0.58 \\ 0.4/0.58 & 0.4/0.58 & 0.3/0.58 \end{bmatrix} \begin{bmatrix} 2500 \\ 40 \\ 100 \end{bmatrix} = \begin{bmatrix} 4258.62 \\ 1758.62 \\ 1803.44 \end{bmatrix}$$

Therefore, Equilibrium income (Y) = 4258.62
Equilibrium consumption = 1758.62
Equilibrium tax = 1803.44

Note:

To verify if the values are correct or not, you may replace the variables in the model with the values. If all the equations are satisfied, we may conclude that the values are correct.

Thus, from equation (1), RHS = C +1000+1500 = 1758.62+1000+1500= 4258.62=Y

From equation (2), RHS = 40+0.7(Y-T)= 40 +0.7(4258.62-1803.44)= 1758.62= C

From equation (3), RHS = 100+0.4Y=100+ 0.4(4248.62)= 1803.44=T

Hence the values are verified.

16.4. Given a two-goods market model:

$$D_1 = 25 - 2P_1 + P_2 \qquad D_2 = 20 + 2P_1 - 2P_2$$

$$S_1 = -5 + 4P_1 \qquad S_2 = -10 + 5P_2$$

$$D_1 = S_1 \qquad D_2 = S_2$$

Find the equilibrium prices $P_1\ and\ P_2$ using Cramer's Rule.

Solution:

The demand and supply equations and the equilibrium condition for Market 1 is given as:

$$D_1 = 25 - 2P_1 + P_2 \text{ ---------- (1)}$$

$$S_1 = -5 + 4P_1 \text{ -----------(2)}$$

$$D_1 = S_1 \qquad \text{----------(3)}$$

Substituting equation (1) and (2) in equation (3), we get

$$25 - 2P_1 + P_2 = -5 + 4P_1$$

$$6P_1 - P_2 = 30 \qquad \text{---------(4)}$$

The demand and supply equations and the equilibrium condition for Market 2 is given as:

$$D_2 = 20 + 2P_1 - 2P_2 \quad \text{---------- (5)}$$

$$S_2 = -10 + 5P_2 \qquad \text{-----------(6)}$$

$$D_2 = S_2 \qquad \text{----------(7)}$$

Substituting equation (5) and (6) in equation (7), we get

$$20 + 2P_1 - 2P_2 = -10 + 5P_2$$

$$-2P_1 + 7P_2 = 30 \quad \text{--------------- (8)}$$

Expressing equation (4) and (8) in matrix form, we have

$$\begin{bmatrix} 6 & -1 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 30 \\ 30 \end{bmatrix}$$

Or $AP = D$

Where A is the matrix of co-efficient

P is the matrix of Price variables

D is the matrix of constants

Using Cramer's Rule,

$$P_1 = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} 30 & -1 \\ 30 & 7 \end{vmatrix}}{40} = 6$$

$$P_2 = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} 6 & 30 \\ -2 & 30 \end{vmatrix}}{40} = 6$$

Therefore, the equilibrium prices for the two goods are ₹6 each.

This problem may also be solved by using the Matrix inversion method. (Try it!)