*Lecture*3

rrison

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, Z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \in V$$

1. 封闭性

$$[X, Y] = \begin{bmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - x_2 y_1 \end{bmatrix} \in V$$

2. 双线性

$[aX + bY, Z]$

$$= \begin{bmatrix} ax_2 z_3 + by_2 z_3 - ax_3 z_2 - by_3 z_2 \\ ax_3 z_1 + by_3 z_1 - ax_1 z_3 - by_1 z_3 \\ ax_1 z_2 + by_1 z_2 - ax_2 z_1 - by_2 z_1 \end{bmatrix} = \begin{bmatrix} ax_2 z_3 - ax_3 z_2 \\ ax_3 z_1 - ax_1 z_3 \\ ax_1 z_2 - ax_2 z_1 \end{bmatrix} + \begin{bmatrix} by_2 z_3 - by_3 z_2 \\ by_3 z_1 - by_1 z_3 \\ by_1 z_2 - by_2 z_1 \end{bmatrix}$$

$$= a \begin{bmatrix} x_2 z_3 - x_3 z_2 \\ x_3 z_1 - x_1 z_3 \\ x_1 z_2 - x_2 z_1 \end{bmatrix} + b \begin{bmatrix} y_2 z_3 - y_3 z_2 \\ y_3 z_1 - y_1 z_3 \\ y_1 z_2 - y_2 z_1 \end{bmatrix} = a[X, Z] + b[Y, Z]$$

$[Z, aX + bY]$

$$= \begin{bmatrix} ax_3 z_2 + by_3 z_2 - ax_2 z_3 - by_2 z_3 \\ ax_1 z_3 + bx_1 z_3 - ax_3 z_1 - by_3 z_1 \\ ax_2 z_1 + by_2 z_1 - ax_1 z_2 - by_1 z_2 \end{bmatrix} = \begin{bmatrix} ax_3 z_2 - ax_2 z_3 \\ ax_1 z_3 - ax_3 z_1 \\ ax_2 z_1 - ax_1 z_2 \end{bmatrix} + \begin{bmatrix} by_3 z_2 - by_2 z_3 \\ bx_1 z_3 - by_3 z_1 \\ by_2 z_1 - by_1 z_2 \end{bmatrix}$$

$$= a \begin{bmatrix} x_3 z_2 - x_2 z_3 \\ x_1 z_3 - x_3 z_1 \\ x_2 z_1 - x_1 z_2 \end{bmatrix} + b \begin{bmatrix} y_3 z_2 - y_2 z_3 \\ x_1 z_3 - y_3 z_1 \\ y_2 z_1 - y_1 z_2 \end{bmatrix} = a[Z, X] + b[Z, Y]$$

3. 自反性

$$[X, X] = \begin{bmatrix} x_2 x_3 - x_3 x_2 \\ x_3 x_1 - x_1 x_3 \\ x_1 x_2 - x_2 x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

4. 雅可比等价

$[X, [Y, Z]] + [Z, [Y, X]] + [Y, [Z, X]]$

$$= \begin{bmatrix} x_2(y_1 z_2 - y_2 z_1) - x_3(y_3 z_1 - y_1 z_3) \\ x_3(y_2 z_3 - y_3 z_2) - x_1(y_1 z_2 - y_2 z_1) \\ x_1(y_3 z_1 - y_1 z_3) - x_2(y_2 z_3 - y_3 z_2) \end{bmatrix} + \begin{bmatrix} x_2(y_1 z_2 - y_2 z_1) - x_3(y_3 z_1 - y_1 z_3) \\ x_3(y_2 z_3 - y_3 z_2) - x_1(y_1 z_2 - y_2 z_1) \\ x_1(y_3 z_1 - y_1 z_3) - x_2(y_2 z_3 - y_3 z_2) \end{bmatrix} \begin{bmatrix} x_2(y_1 z_2 - y_2 z_1) - x_3(y_3 z_1 - y_1 z_3) \\ x_3(y_2 z_3 - y_3 z_2) - x_1(y_1 z_2 - y_2 z_1) \\ x_1(y_3 z_1 - y_1 z_3) - x_2(y_2 z_3 - y_3 z_2) \end{bmatrix}$$