MATHS MODELLING ASSIGNMENT 1

Akshat Vijayvargia

S3826627

PROBLEM CHOICE

For this assignment, I choose the Diet problem. This is a real-life example of my sister, Dr Disha Vijayvargia, she is a dietician. And this is the diet problem of her patient, Avantika, who wants a diet with minimum cost.

PROBLEM DESCRIPTION

We will investigate on the Diet Problem and will try to make a linear programming out of it. Before going further, let us understand the meaning of linear programming. The Linear Programming (or Linear Optimisation) is technique to accomplish the best outcome, either in profit maximization or minimizing losses, with variables and constraint in mind.

There is a diet chart providing us content of carbohydrates, proteins, and calories in four food items. Here Avantika (our patient) wants to minimise the cost of dieting.

	LENTILS	NAMKEEN	CHAPATI	RICE
CARBOHYDRATES (in grams)	2	7	4	3
PROTIENS(in grams)	5.7	2	0	0
CALORIES	244	500	170	420
COST PER UNIT	$12	$7.8	$10	$5.6

The above chart gives us the nutrients available and cost per unit of Lentils, Namkeen, Chapati and Rice. The diet is planned in such a way, that there are restrictions on intake as at least 63 grams of carbohydrates, 25 grams of protein and 3200 calories.

Now, the question is- “How much should she consume of each food item to minimize her dieting cost?”

MODEL FORMULATION

To formulate the model, one should know the steps behind it. The steps to formulate linear programming model are-

1) Identifying Decision Variables
   Here, our decision variables are the food items.
   X1-> Quantity consumed of Lentils
   X2-> Quantity consumed of Namkeen
   X3-> Quantity consumed of Chapati
   X4-> Quantity consumed of Rice
2) Stating Objective Function
   Now we will write our objective function. For the diet to be optimal we must have minimum cost along with required calories, protein, and carbohydrates.
   MIN: 12X1+ 7.8X2+ 10X3+ 5.6X4
   The Total Cost is calculated by =SUMPRODUCT (B7:E7, B8:E8). The objective function is obtained by multiplying number of units eaten with the cost per unit and aggregating for all the food items.
3) Stating the constraints
   Now, we will look at the constraints, which restrictions or limitation on decision variables. Constraints on the decision variables are as follows-

2X1+ 7X2+ 4X3+ 3X4 >= 63 ---------- CARBOHYDRATES
5.7X1+ 2X2+ 0X3+ 0X4 >= 25---------- PROTIENS
244X1+ 500X2+ 170X3+ 420X4 >= 3200---------- CALORIES

SOFTWARE IMPLEMENTATION

I used Opensolver for Diet Problem. Steps for software implementation is as follows-

Step1-> Identifying decision variables, here, our decision variables are the food items. Avantika consumes 4 unit of Lentils, 8 units of Namkeen, 6 units of Chapati and 3 units of Rice.

DECISION VARIABLE				
	LENTILS	NAMKEEN	CHAPATI	RICE
EATEN	4	8	6	3

Step2-> Formulating objective function. To obtain the optimal solution, we must minimize our cost as well as carbohydrates, proteins, and calories intake.

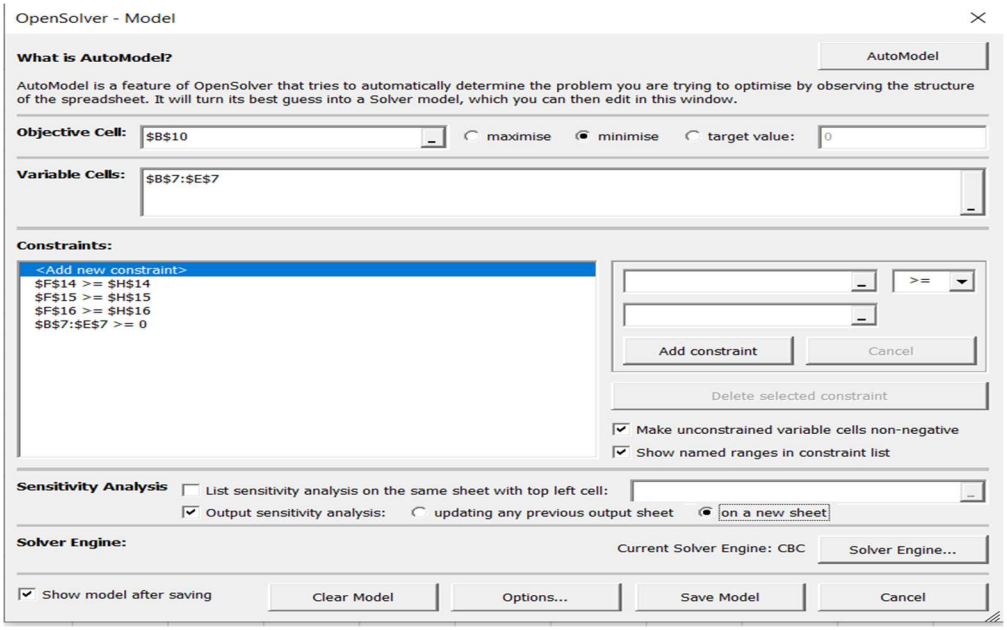
OBJECTIVE FUNCTION				
	LENTILS	NAMKEEN	CHAPATI	RICE
EATEN	0≤ 4	8	6	3
COST PER UNIT	$12	$7.80	$10	$5.60
TOTAL COST	min $187.20			

In the pictorial above, the cells between B7 to E7 represents the unit consumed and cells between B8 to E8 represents the per unit cost of all four food items. The total cost of the diet is aggregated in cell B10 by using sumproduct of number of units consumed and cost per unit of these food items. One can calculate it by =B7*B8+C7*C8+D7*D8+E7*E8 or by using =SUMPRODUCT (B7:E7, B8:E8).

Step3-> Now, let us type our constraints on the spreadsheet. In the picture below, Total column is representing the total carbohydrates, proteins and calories and is calculated by SUMPRODUCT () function for every nutrient. Required column shows the required intake quantity of all 3 nutrients and Inequality column shows >= (greater than equal to) as diet should contain at least 63 grams of carbohydrates, 25 grams of protein and 3200 calories.

CONSTRAINTS							
	LENTILS	NAMKEEN	CHAPATI	RICE	TOTAL	INEQUALITY	REQUIRED
CARBOHYDRATES	2	7	4	3	97	>=	≥ 63
PROTIENS	5.7	2	0	0	38.8	>=	≥ 25
CALORIES	244	500	170	420	7256	>=	≥ 3200

Step4-> Now we will open and enter the model in 'Open solver Model' from open-solver from Data tab. After entering the model will then save the model.



Once the model is saved click on the Data tab then click solve. Our spreadsheet will look like the figure below with the optimal cost and new revised value of variables. Here, we have our optimal solution - $83.54 in the cell corresponding to Total Cost.

DECISION VARIABLE							
	LENTILS	NAMKEEN	CHAPATI	RICE			
EATEN	4	8	6	3			
OBJECTIVE FUNCTION							
	LENTILS	NAMKEEN	CHAPATI	RICE			
EATEN	0≤ 1.3649025	8.6100279	0	0			
COST PER UNIT	$12	$7.80	$10	$5.60			
	min						
TOTAL COST	$83.54						
CONSTRAINTS							
	LENTILS	NAMKEEN	CHAPATI	RICE	TOTAL	INEQUALITY	REQUIRED
CARBOHYDRATES	2	7	4	3	63.0000003	>=	≥ 63
PROTIENS	5.7	2	0	0	25.00000005	>=	≥ 25
CALORIES	244	500	170	420	4638.05016	>=	≥ 3200

ANALYSIS AND INTERPRETATION OF RESULTS

After performing linear programming of our Diet Problem, we have generated our sensitivity report and will analyse for more insights. Below is our sensitivity report-

OpenSolver Sensitivity Report - CBC
Worksheet: [Book1.xlsx] Sheet3 Sensitivity
Report Created: 2/09/2020 9:54:40 PM

Decision Variables

Cells	Name	Final Value	Reduced Costs	Objective Value	Allowable Increase	Allowable Decrease
B7	EATEN LENTILS	1.3649025	0	12	10.23000026	9.771428661
C7	EATEN NAMKEEN	8.6100279	0	7.8	8.167251672	3.589473774
D7	EATEN CHAPATI	0	0	10	1E+100	7.720334362
E7	EATEN RICE	0	0	5.6	1E+100	3.890250796

Constraints

Cells	Name	Final Value	Shadow Price	RHS Value	Allowable Increase	Allowable Decrease
F14>=H14	CARBOHYDRATES TOTAL	63	0.56991643	63	24.5	21.85690093
F15>=H15	PROTIENS TOTAL	25	1.9052925	25	154.55	7
F16>=H16	CALORIES TOTAL	4638.0501	0	3200	1438.050139	1E+100
B7>=0	EATEN LENTILS	1.3649025	0	0	1.364902507	1E+100
C7>=0	EATEN NAMKEEN	8.6100279	0	0	8.610027855	1E+100
D7>=0	EATEN CHAPATI	0	7.7203343	0	13.55701754	6.125
E7>=0	EATEN RICE	0	3.8902507	0	18.07602339	6.45970971

It is optimal to consume 1.364 of Lentils and 8.610 of Namkeen. The solution gives us the best numbers for cost minimization. The solution uses the required portion of Carbohydrates and Proteins (i.e. 63 grams of Carbohydrates and 25 grams of Protein respectively) and uses Calories more than required amount, which was 3200.

If we look at the Reduced Costs column, we can say that there is no room for any changes in our objective function before the optimal solution changes. The shadow price shows us that we have some opportunity to reduce our solution just by changing our RHS constraint by one unit.

Allowable increase and decrease help us to maintain our optimal solution and to make sure that shadow price is valid, by changing the coefficients within the range. Like here, if the price of Lentils is raised to 12+10.23=22.23 or reduced to 12-9.77=2.23, the optimal solution will remain unchanged. The allowable increase of Chapati and Rice is 1E+100, which means infinity and that the shadow price is zero (0).

Conclusion of the Diet Problem-

"Avantika (our patient) should not consume any unit of Chapati and

Rice as the corresponding Final values are zeros, but she can intake 1.364 grams of lentils and 8.61 grams of namkeen

to maintain the low-cost diet plan. "