Lab 8

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Let $x_t = 0.6x_{t-1} - 0.7x_{t-2} + w_t + w_{t-1} + 0.2w_{t-2}$

equivalently

$x_1 - 0.6x_{t-1} + 0.7x_{t-2} = w_t + w_{t-1} + 0.2w_{t-2}$

$x_1(1 - 0.6B + 0.7B^2) = w_t(1 + B + 0.2B^2)$

$x_t\phi(z) = w_t\theta(z)$

with $\phi(z) = 1 - 0.6B + 0.7B^2$ and $\theta(z) = 1 + B + 0.2B^2$

```r
phi_z   = c(1, -0.6, 0.7)
theta_z = c(1,    1, 0.2)
```

```r
roots_of_phi = polyroot(phi_z)
roots_of_phi
```

```
## [1] 0.428571+1.11575i 0.428571-1.11575i
```

```r
roots_of_theta = polyroot(theta_z)
roots_of_theta
```

```
## [1] -1.381966-0i -3.618034+0i
```

$\phi(z)$ has roots at $B = 0.428571 \pm 1.11575i$ $\theta(z)$ has roots at $B = -3.618$ and $B = -1.38197$ $\phi(z)$ and $\theta(z)$ do not share any roots therefore this model of x_t is an ARMA(2,2) model.

1. Determine if it is invertible or stationary/causal.

an ARMA process is invertible only when the roots of $\theta(z)$ lie outside the unit circle.

```r
Mod(roots_of_theta)
```

```
## [1] 1.381966 3.618034
```

This model of x_t is invertible. an ARMA process is stationary/causal only when the roots of $\phi(z)$ lie outside the unit circle

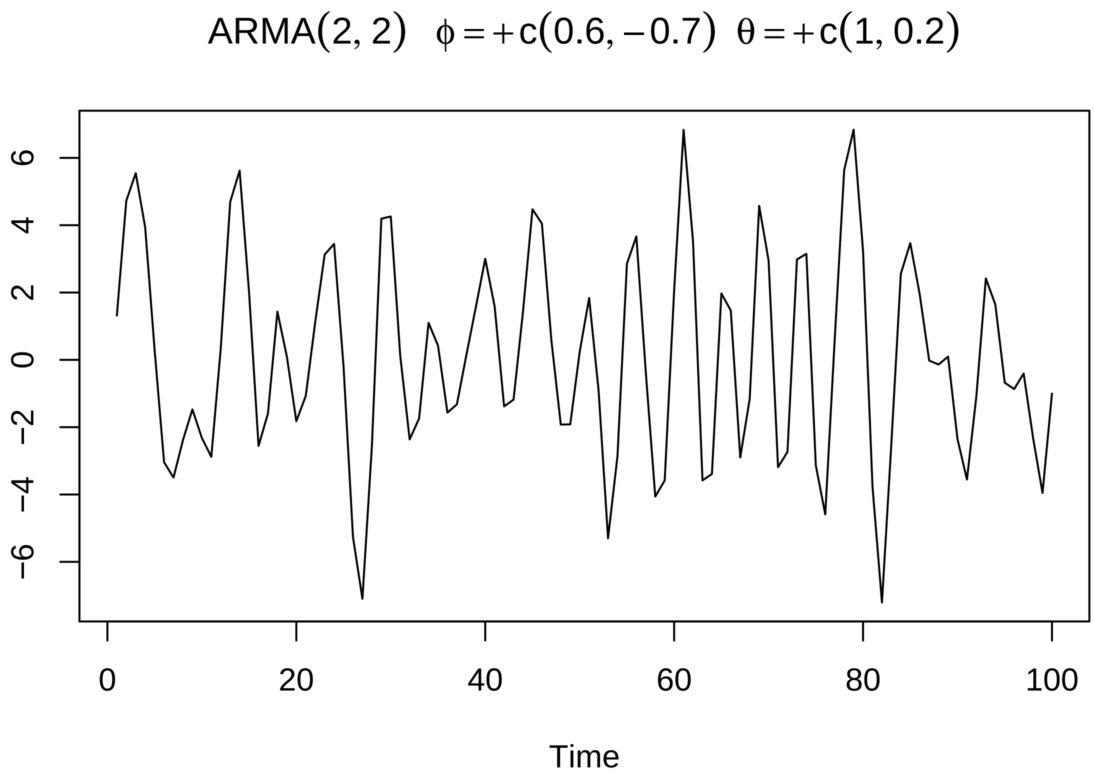
```r
Mod(roots_of_phi)
```

```
## [1] 1.195229 1.195229
```

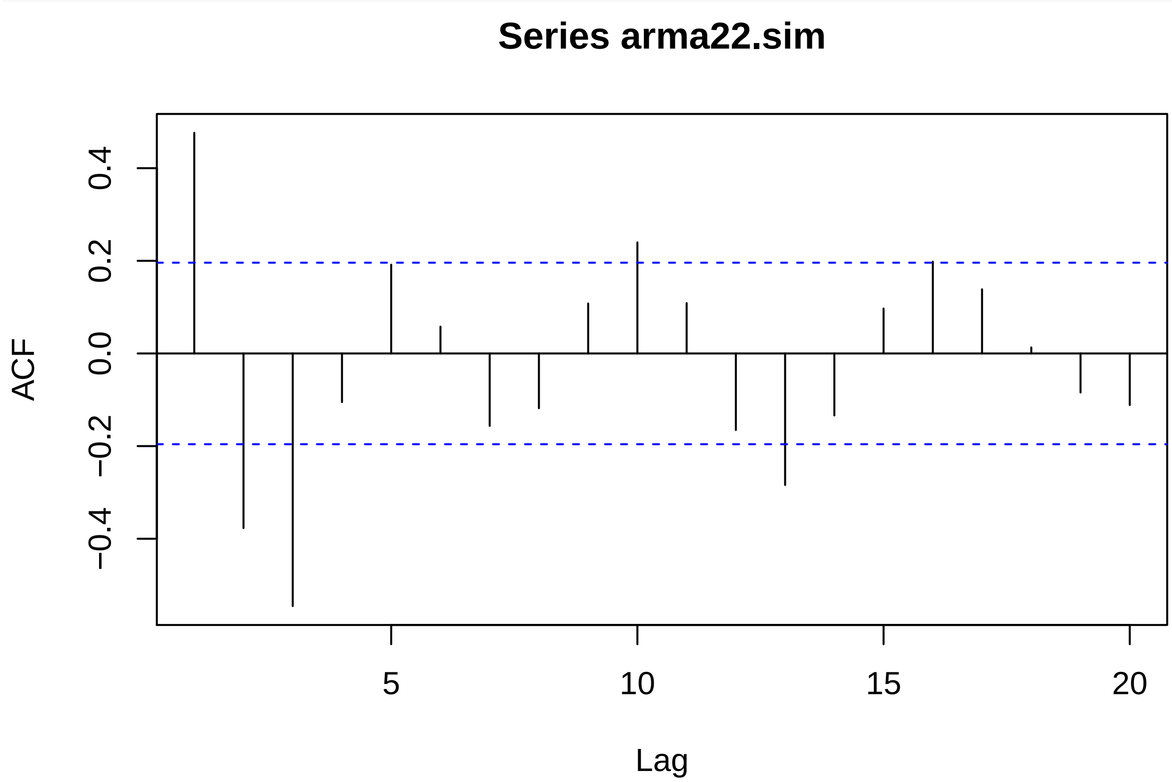
This model of x_t is stationary/causal.

2. Generate a sample path using n = 100 observations, and produce the sample ACF and sample PACF

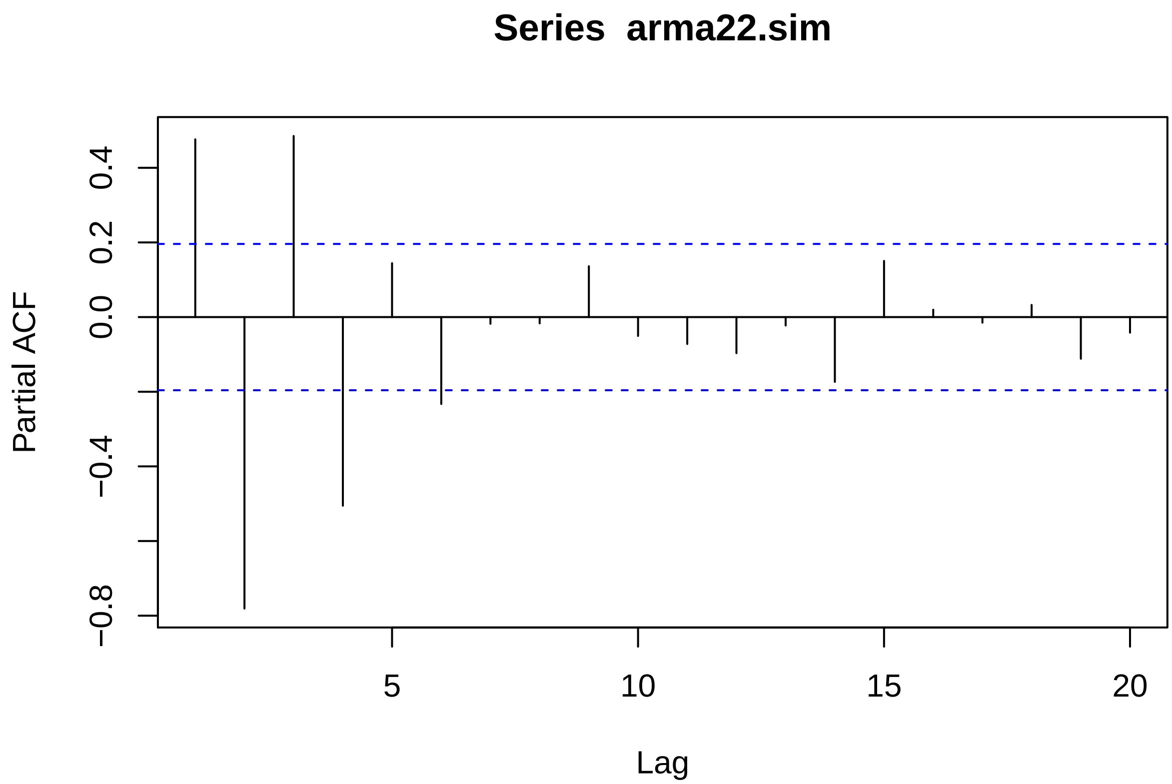
```r
arma22.sim <- arima.sim(list(order = c(2,0,2), ar = c(0.6, -0.7), ma = c(1, 0.2)), n = 100)
ts.plot(arma22.sim,
        ylab="", main=(expression(ARMA(2,2)~~~phi==+c(0.6, -0.7) ~~theta==+c(1, 0.2))))
```




```r
acf(arma22.sim)
```

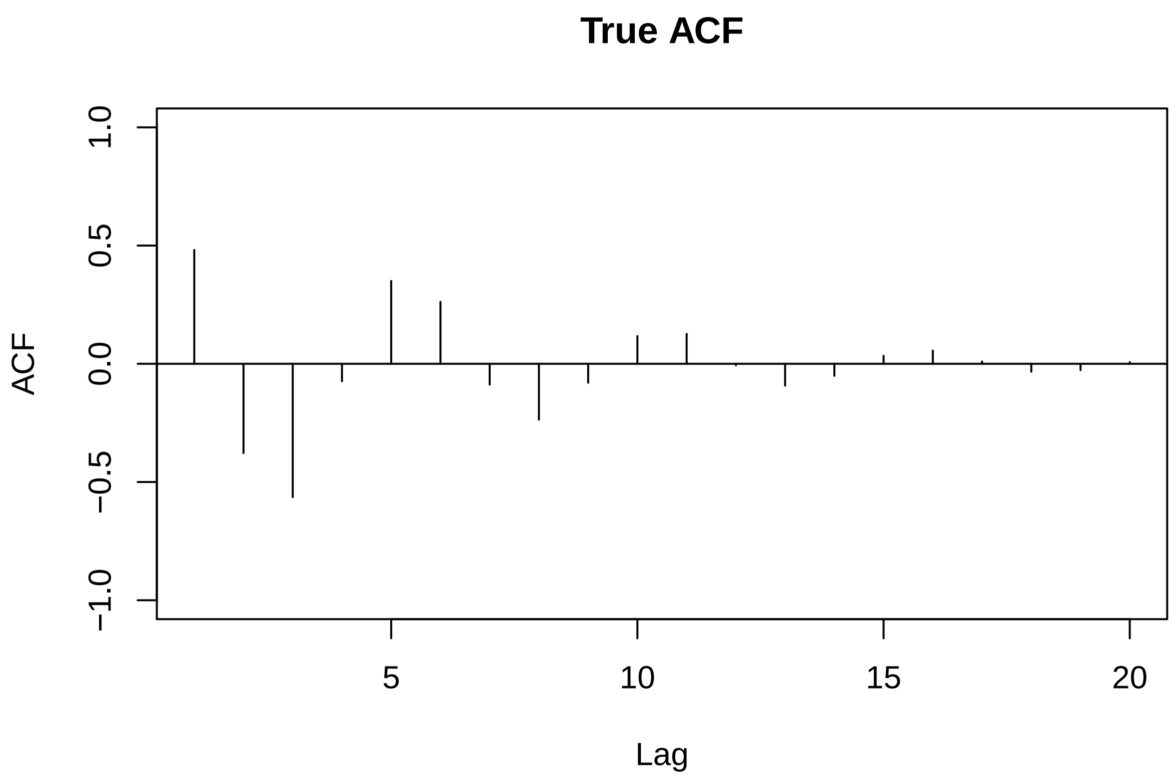


```r
pacf(arma22.sim)
```

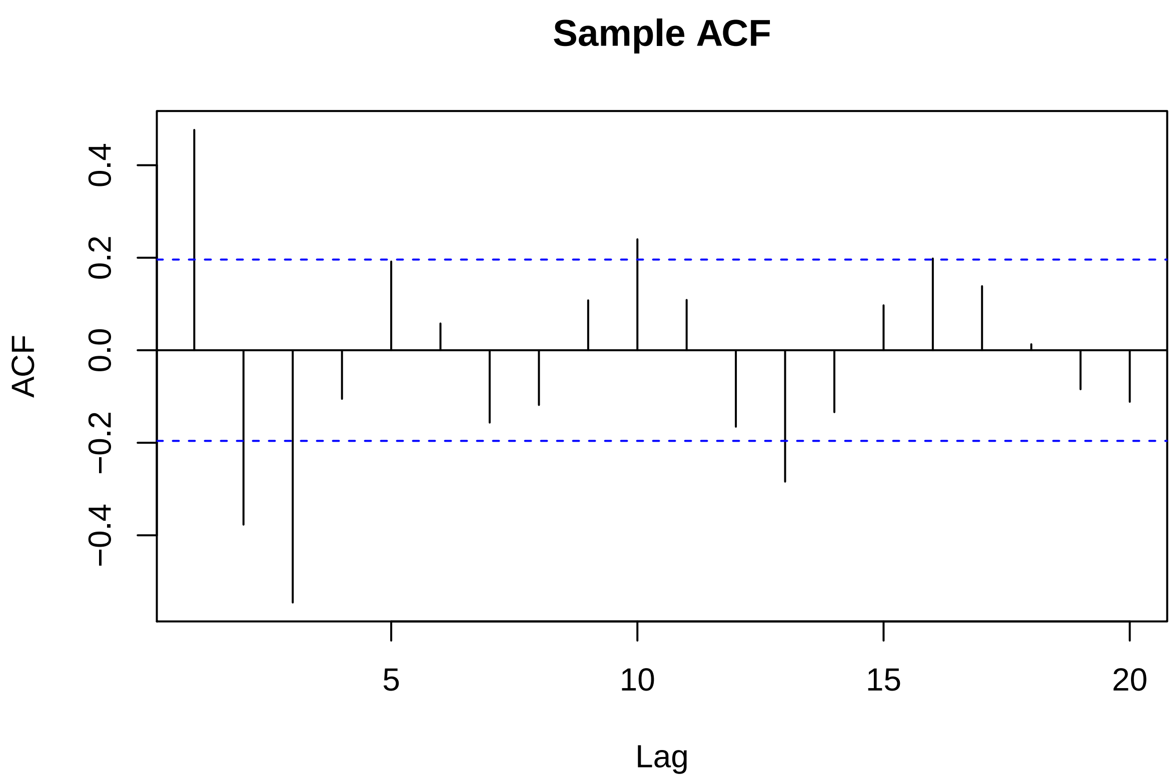


3. Produce the true ACF and the true PACF and compare with the sample ACF and sample PACF, respectively.

```r
y = ARMAacf(ar = c(0.6, -0.7),  ma = c(1, 0.2), lag.max = 20)
y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "Lag",
      ylab = "ACF", main = "True ACF")
abline(h = 0)
```

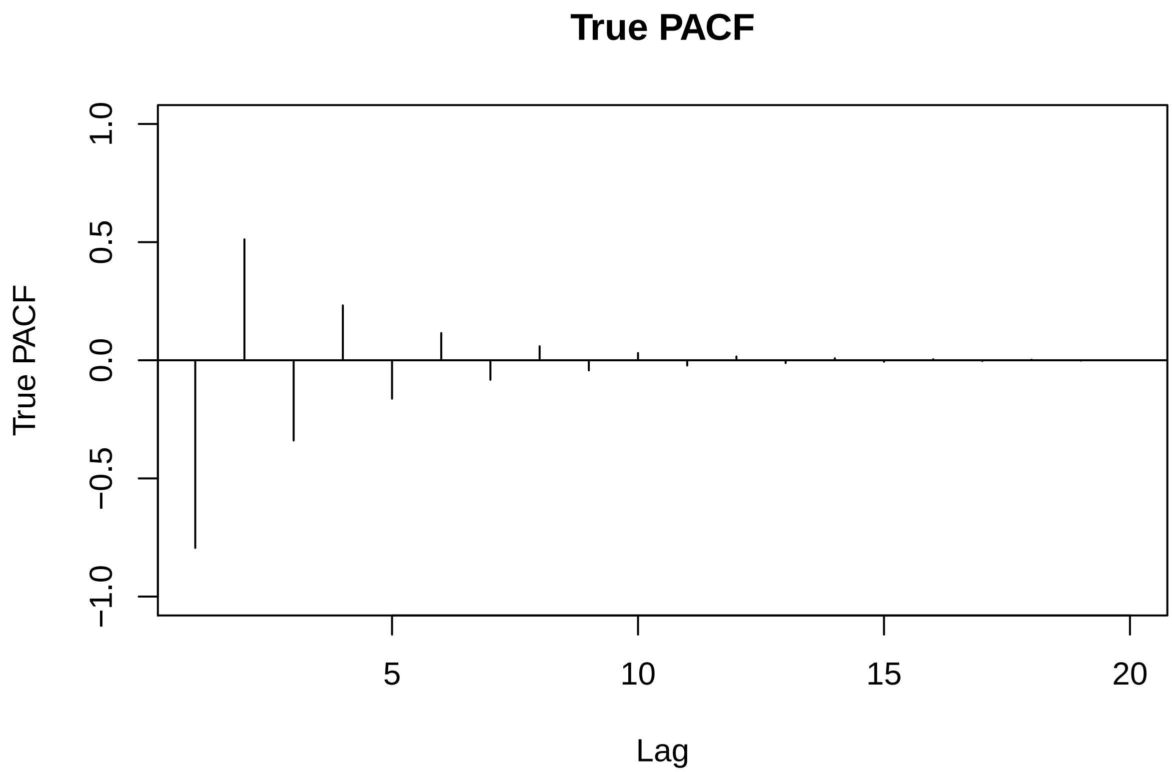



```
acf(arma22.sim, main = "Sample ACF")
```

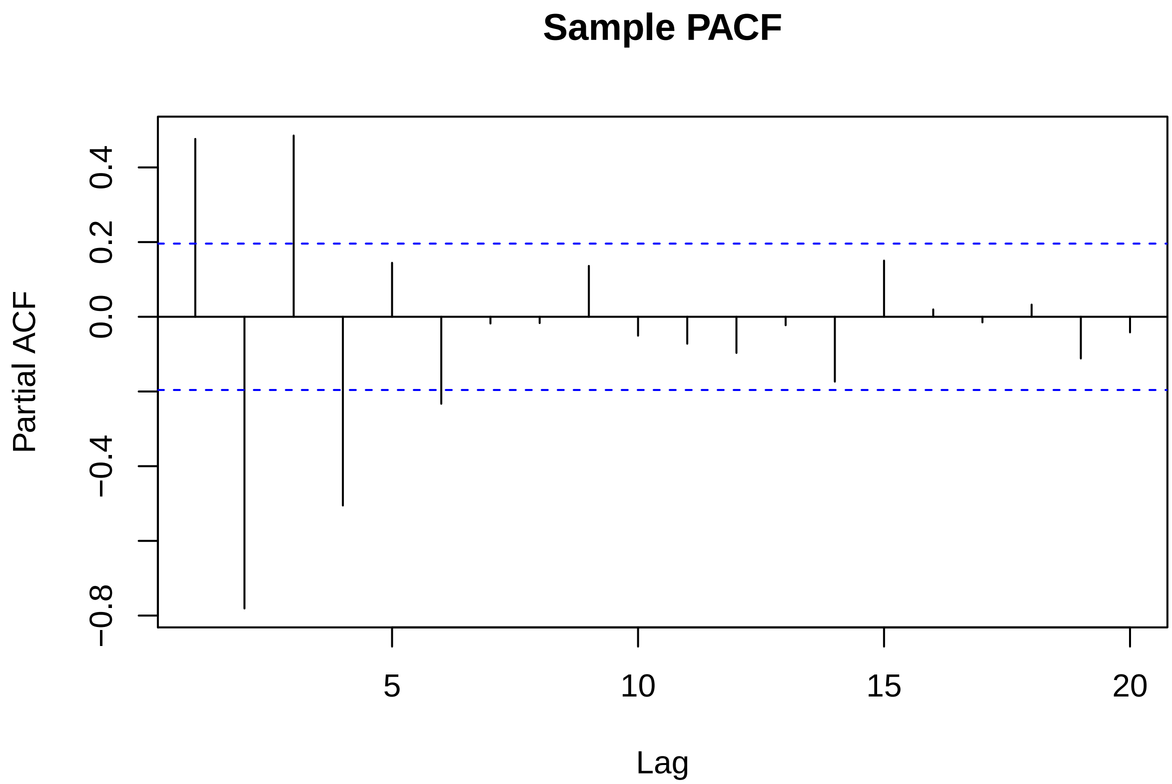



```
y = ARMAacf(ar = c(0.6, -0.7),  ma = c(1, 0.2), lag.max = 20, pacf=TRUE)
y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "Lag",
```

```
      ylab = "True PACF", main = "True PACF")
abline(h = 0)
```




```
pacf(arma22.sim, main = "Sample PACF")
```



The true acf and sample acf both decay and oscillate in pairs The true pacf and sample pacf both decay and strictly alternate sign.