RISHI BIDANI
31883125

1

Q1) $\frac{28.5}{50}$ 57% P

a) $\frac{6.5}{12}$

$$f(x) = \frac{x^2 + 12x + 36}{x^2 - 36}$$

$$f(x) = \frac{x^2 + 6x + 6x + 36}{x^2 - 6^2}$$

$$f(x) = \frac{x(x+6) + 6(x+6)}{(x+6)(x-6)} = \frac{(x+6)(x+6)}{(x+6)(x-6)}$$

3/2

$$f(x) = \frac{x+6}{x-6}$$

$\therefore$ domain = $\{x \in R : x \neq 6, -6\}$

$= (-\infty, -6) \cup (-6, 6) \cup (6, \infty)$

b) $$g(x) = \frac{|x| - 5}{\log_{10}(1-|x|)}$$

$$D_{g(x)} = \{x \in R : (1-|x|) \neq 0\}$$

$$\therefore |x| \neq 1$$

$$D_{g(x)} = R \setminus \{1, -1\}$$

c) $$p(x) = \frac{1}{x^2 - 5} \qquad q(x) = \sqrt{9 - x^2}$$

$$p \circ q(x) = \frac{1}{(\sqrt{9-x^2})^2 - 5}$$

$$p \circ q(x) = \frac{1}{(3-x)(3+x) - 5}$$

$$D_{q(x)} = 9 - x^2 > 0$$

$$= x^2 < 9 \Rightarrow x^2 < 3^2$$

$$\Rightarrow x < 3$$

$$D_{q(x)} = (-\infty, 3]$$

$$D_{p \circ q(x)} = (-\infty, 3] \cap \{9 - x^2 > 5\}$$

$$D_{p \circ q}(x) = (-\infty, 3] \cap \{x^2 < 2^2\}$$

$$D_{p \circ q}(x) = (-\infty, 3] \cap [-\infty, 2]$$

$$\text{Domain}_{p \circ q(x)} = (-\infty, 2]$$

31883125

②

d) $h(x) = \sqrt{6x-3} + 2$

$y = \sqrt{6x-3} + 2$

To find $h^{-1}(x)$, switching ~~the~~ ~~signs~~ x & y

$\Rightarrow x = \sqrt{6y-3} + 2$

$\Rightarrow x-2 = \sqrt{6y-3} \Rightarrow (x-2)^2 = 6y-3$

$\Rightarrow y = \dfrac{(x-2)^2+3}{6}$

$\therefore h^{-1}(x) = \dfrac{(x-2)^2+3}{6}$

Range $h(x)$ = Domain $h^{-1}(x)$

~~$R_{h(x)}$~~ $D_{h(x)} = 6x-3 > 0$

$\Rightarrow x > 1/2$

$= \{x \in R : x > 1/2\}$

$\therefore R_{h(x)} = [2, \infty)$

$\therefore D_{h^{-1}(x)} = [2, \infty)$

Q2) $f(x) = 2 - 7 \times 3^{7x-10}$

3.5

$y = 2 - 7(3)^{7x-10}$

For finding $f^{-1}(x)$, switching x & y

$x = 2 - 7(3)^{7y-10}$

$\Rightarrow \frac{x-2}{-7} = 3^{7y-10}$

$\therefore (7y-10)\ln(3) = \ln\left(\frac{2-x}{7}\right)$

$7y - 10 = \frac{\ln(2-x) - \ln(7)}{\ln(3)}$

$\Rightarrow f^{-1}(x) = y = \frac{1}{7}\left[\frac{\ln(2-x) - \ln(7)}{\ln(3)}\right] + \frac{10}{7}$

$D_{f^{-1}(x)} = R_{f(x)}$

$D_{f(x)} = R$

$(-\infty, 2)$

$\therefore R_{f(x)} = R$

hence $D_{f^{-1}(x)} = R = (-\infty, \infty)$

RISHI BIDANI
31883125

3

Q2)
b)

$$q(t) = 4(1 - e^{-2t})$$

$q(0)$ = charge at $t = 0$

1/2

$$q(0) = 4(1 - e^{-2(0)}) = 4(0) = 0$$

$$q(t)_{t\to\infty} = \lim_{t\to\infty} 4(1 - e^{-2(t)})$$

$$q(t) = \lim_{t\to\infty} 4 - 4e^{-2(t)}$$

RISHI BIDANI
31883125
4

Q3) $2\sqrt{3}\cos(x) - 2\sin(x)$ — ①

in the form $A\sin(x+B)$

7/8

$A\sin(x+B) = A\sin(x)\cos B + A\cos(x)\sin B$ — ②

Comparing coefficients in ① & ②

coeff of $\sin(x)$:

$\Rightarrow A\cos B = -2$ — ③

Coeff of $\cos(x)$: $\Rightarrow A\sin B = 2\sqrt{3}$ — ④

$\frac{④}{③} \Rightarrow \frac{\sin B}{\cos B} = \frac{2\sqrt{3}}{-2} \Rightarrow \tan B = -\sqrt{3}$

$B = \tan^{-1}(-\sqrt{3}) = -\pi/3$

$A\cos\left(\frac{-\pi}{3}\right) = -2 \Rightarrow A = -2(2) = -4$

$A > 0 \therefore A = 4$

4 si

$\therefore 2\sqrt{3}\cos(x) - 2\sin(x) = -4\sin\left(x - \frac{\pi}{3}\right)$

b)

$$2\sqrt{3}\cos(x) + 4 = 2\sin(x)$$

$$2\sqrt{3}\cos(x) - 2\sin(x) = -4$$

$$\Rightarrow \quad \underbrace{-4\sin\left(x - \frac{\pi}{3}\right)}_{\text{[from part (a)]}} = -4$$

$$\Rightarrow \quad \sin\left(x - \frac{\pi}{3}\right) = 1 = \sin\left(\frac{\pi}{2}\right)$$

$$\left[\sin(\pi/2) = 1\right]$$

$$x - \frac{\pi}{3} = n\pi + (-1)^n\left(\frac{\pi}{2}\right)$$

$$x = n\pi + \frac{\pi}{3} + (-1)^n\left(\frac{\pi}{2}\right)$$

$$x = \pi\left(n + 1/3\right) + (-1)^n\left(\pi/2\right)$$

$n = 0 \Rightarrow x = \pi/3 + \pi/2 = 5\pi/6$

$n = 1 \Rightarrow x = 4\pi/3 - \pi/2 = 5\pi/6$

$n = 2 \Rightarrow x = 7\pi/3 + \pi/2 = 17\pi/6$

rejected, because $x \in [-\pi, \pi]$

$n = 3 \Rightarrow x = 10\pi/3 - \pi/2 = 17\pi/6$

rejected

$n = -1 \Rightarrow x = -2\pi/3 - \pi/2 = -7\pi/6$

rejected

$n = -2 \quad x = -5\pi/3 + \pi/2 = 7\pi/6$

$\therefore \quad x = 5\pi/6$

RISHI BIDANI
31883125

5

c) $f(x) = \pi - 2\sin^{-1}(2x-1)$

$D_{f(x)} = \{x \in R : -1 < 2x-1 < 1\}$

$D_{f(x)} = \{x \in R : 0 < 2x < 2\}$

$D_{f(x)} = \{x \in R : 0 < x < 1\}$

$\therefore D_{f(x)} = [0, 1]$

$R_{f(x)} = [f(0), f(1)]$

$R_{f(x)} = [0, 2\pi]$

x-intercept $\Rightarrow y = 0$

$0 = \pi - 2\sin^{-1}(2x+1)$

$2\sin^{-1}(2x-1) = \pi$

$\sin^{-1}(2x-1) = \pi/2$

$2x-1 = \sin(\pi/2)$

$2x-1 = 1$

$\boxed{x = 1}$

y-intercept $\Rightarrow x = 0$

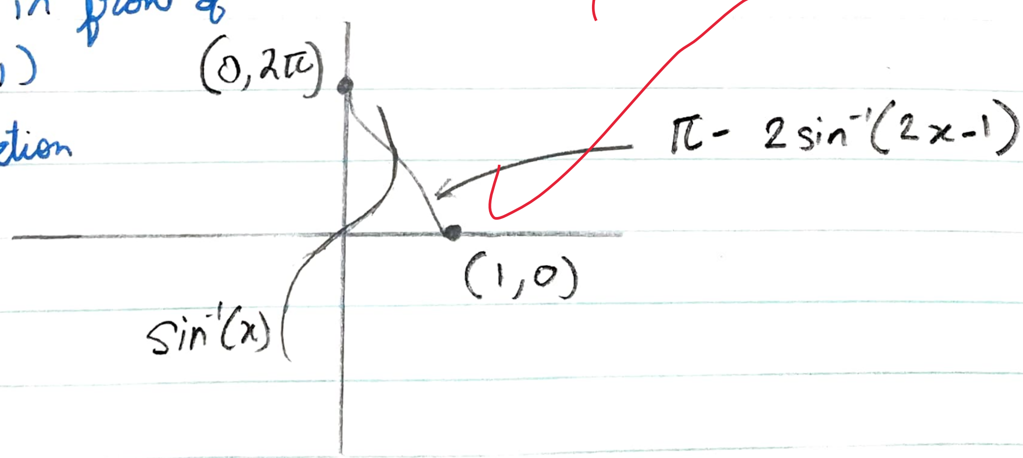
$y = \pi - 2\sin^{-1}(-1)$

$y = \pi - 2\left(-\frac{\pi}{2}\right)$

$y = \pi + \pi$

$\boxed{y = 2\pi}$

-ve sign in front of $\sin^{-1}(2x-1)$ means reflection along y-axis



RISHI BIDANI
31883125 6

A
12

Q4)
a) $$\frac{-1+\sqrt{3}\,i}{-1-i} \times \frac{-1+i}{-1+i}$$

$$\Rightarrow \frac{(-1+\sqrt{3}\,i)(-1+i)}{(-1)^2-(i)^2} = \frac{1-i-\sqrt{3}i+\sqrt{3}i^2}{2}$$

$$\frac{1-\sqrt{3}}{2} - i\left(\frac{1+\sqrt{3}}{2}\right)$$

c) $$\frac{1}{2} z^4 + 1 = \sqrt{3}\, i$$

$$z^4 = 2(\sqrt{3}\, i - 1)$$

$$z^4 = 2\sqrt{3}\, i - 2$$

$$z^4 = 4\left(\frac{\sqrt{3}}{2} i - \frac{1}{2}\right)$$

$$z^4 = -4\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$$

$$z^4 = -4\left(\cos\left(\frac{\pi}{6}\right) - i \sin\left(\frac{\pi}{6}\right)\right) = -4e^{-i\frac{\pi}{6}}$$

$$z^4 = -4\, e^{-i\left(\frac{\pi}{6} + 2k\pi\right)} = -4^{-i\left(\frac{\pi}{6}(1+12k)\right)}$$

$$z = -4^{-i\left(\frac{\pi}{24}(1+12k)\right)}$$

$k = 0 \Rightarrow -4^{-i\left(\frac{\pi}{24}\right)}$

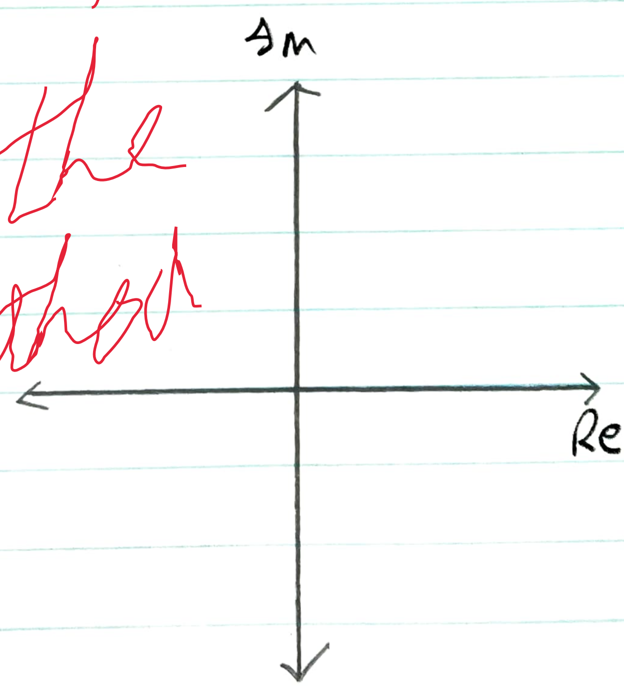
$k = 1 \Rightarrow -4^{-i\left(\frac{13\pi}{24}\right)}$

$k = 2 \Rightarrow -4^{-i\left(\frac{25\pi}{24}\right)}$

$k = -1 \Rightarrow -4^{-\left(\frac{-11\pi}{24}\right)}$

for the method

2



RISHI BIDANI
31883125

Q5) 4.5

$U = 2i - 2j - k$

$V = i + 2j + 2k$

(i) 1

$U + V = 3i + 0j + k$

$4U - V = 7i - 6j - 2k$

(ii)

$U \cdot V = |U||V| \cos\theta$

$\cos\theta = \dfrac{U \cdot V}{|U|\,|V|}$

$\cos\theta = \dfrac{(2i - 2j - k)(i + 2j + 2k)}{\sqrt{4+4+1}\ \sqrt{1+4+4}}$

$\cos\theta = \dfrac{2 - 4 - 2}{3 \cdot 3} = \dfrac{-4}{9}$

$\therefore \theta = \cos^{-1}\left(\dfrac{-4}{9}\right)$

(iii) Scalar resolute of v in the direction of u

$= v \cdot \hat{u}$

$\therefore |u| = \sqrt{1+4+4} = \sqrt{9} = 3$

$\hat{u} = \dfrac{1}{3}(2\hat{i} - 2\hat{j} - \hat{k})$

$= (i + 2j + 2k) \cdot \dfrac{1}{3}(2i - 2j - \hat{k}) = \boxed{\dfrac{2}{3}\hat{i} - \dfrac{4}{3}\hat{j} - \dfrac{2}{3}\hat{k}}$

(iv) $\vec{a}$ = parallel to v = $(v \cdot \hat{u})\hat{u}$

$\vec{b}$ = perpendicular to v = $v - (v \cdot \hat{u})\hat{u}$

~~parallel to v = $[(i + 2j + 2k) \cdot 3]$~~

~~$\vec{a} = (i + 2j + 2k)($~~

$$\vec{a} = \left(\frac{2}{3}\hat{i} - \frac{4}{3}\hat{j} - \frac{2}{3}\hat{k}\right)\frac{1}{3}(2\hat{i} - 2\hat{j} - \hat{k})$$

$$\vec{a} = \frac{4}{9}\hat{i} + \frac{8}{9}\hat{j} + \frac{2}{9}\hat{k}$$

$$\vec{b} = \hat{i} + 2j + 2k - \left(\frac{4}{9}\hat{i} + \frac{8}{9}\hat{j} + \frac{2}{9}\hat{k}\right)$$

$1\frac{1}{2}$ for the method

$$\vec{b} = \frac{5}{9}\hat{i} + \frac{10}{9}\hat{j} + \frac{16}{9}\hat{k}$$

RISHI BIDANI

Q5)

b) $f(x) = |2-x| + 3x - 1$

$f(x) = \lim_{x \to 2} |2-x| + 3x - 1$

$$f(x) = \begin{cases} 2 - x + 3x - 1 & : x > 0 \\ \cancel{x} \cancel{\quad} -2 + 3x - 1 & : x < 0 \end{cases}$$

$$f(x) = \begin{cases} 2 + 2x - 1 & : x > 0 \\ 4x - 3 & : x < 0 \end{cases}$$

What are the limits ?

$$f(x) = \begin{cases} 2 + 4 - 1 & : x > 0 \\ 8 - 3 & : x < 0 \end{cases}$$

$f(x) = 5$

c) $g(x) = \dfrac{16x^2 - 2x + 3}{2x^2 + 10x + 159}$

$$g(x) = \lim_{x \to \infty} \frac{\cancel{x^2}\left(16 - \frac{2}{x} + \frac{3}{x^2}\right)}{\cancel{x^2}\left(2 + \frac{10}{x} + \frac{159}{x^2}\right)}$$

for any constant a

$$\frac{a}{\infty} \approx 0$$

$$\therefore g(x) = \frac{16}{2} = 8$$