

Assignment 1

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1 Task 1

1.1 Task 1.1

We can see, that vectors v_i, w_i form the plane. So this vectors should not lie in the same line to form a plane. If one of the vector of second plane not lie in the first plane, first and second planes would intersect, otherwise they are parallel.

So we can from two matrices $A1 = [v1 \ w1 \ v2]$ and $A2 = [v1 \ w1 \ w2]$. If one of them is invertible (has independent columns), planes would intersect.

The code you can find in Task_1_1.m

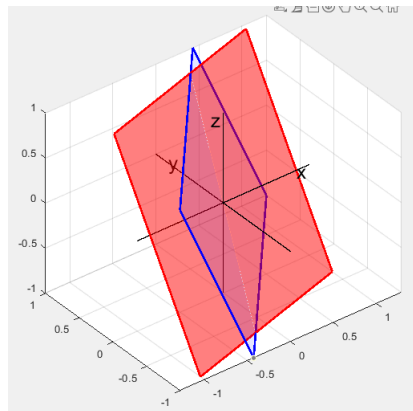


Figure 1: Intersection of 2 planes

1.2 Task 1.2

Find representation in form $n * (r - r_0) = 0$.

n is perpendicular to the plane, so it's perpendicular to vectors v and w
- left null space of matrix $[v \ w]$.

r_0 is equal to any point in that plane. So let it be just p .

You can find a code in Task_1_2.m

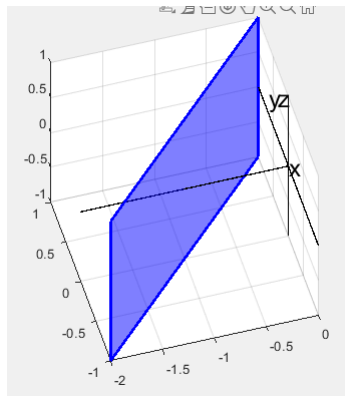


Figure 2: First plane in task 1.2

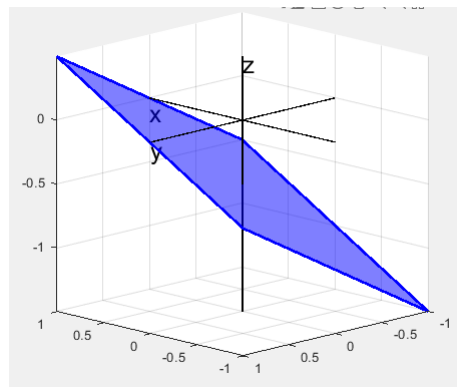


Figure 3: Second plane in task 1.2

1.3 Task 1.3

Line vector is equal to normal of the plane. To find a projection I am using the formula of projection: $g_{proj} = n * (n^T * n)^{-1} * n^T * g$

You can find a code in Task_1_3.m

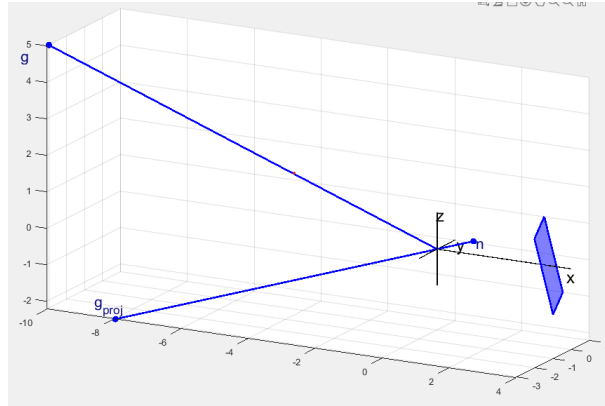


Figure 4: Solution of the task 1.3

1.4 Task 1.4

In this task I decompose vector g on two vectors: projection vector to plane s and perpendicular vector to the plane s equals to distance to that plane (n vector). So symmetrical point will compute as $g^* = g_{proj} - n$

You can find a code in Task_1_4.m

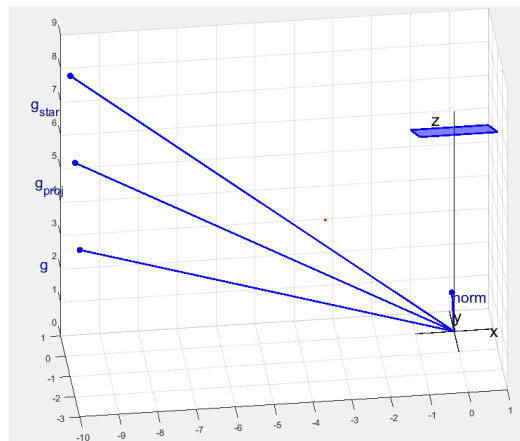


Figure 5: Solution of the task 1.4

2 Task 2

To get a basis of V , we should just compute null space of the matrix. This function is available in matlab.

Finding orthogonal projection onto V and onto the orthogonal complement of V is similar as in task 1.4.

To recover g : $g = g^\perp + g^\parallel$. To prove that g^\perp and g^\parallel are perpendicular we can take scalar multiplication of this vectors, which should be equal to zero.

You can find a code in Task_2.m

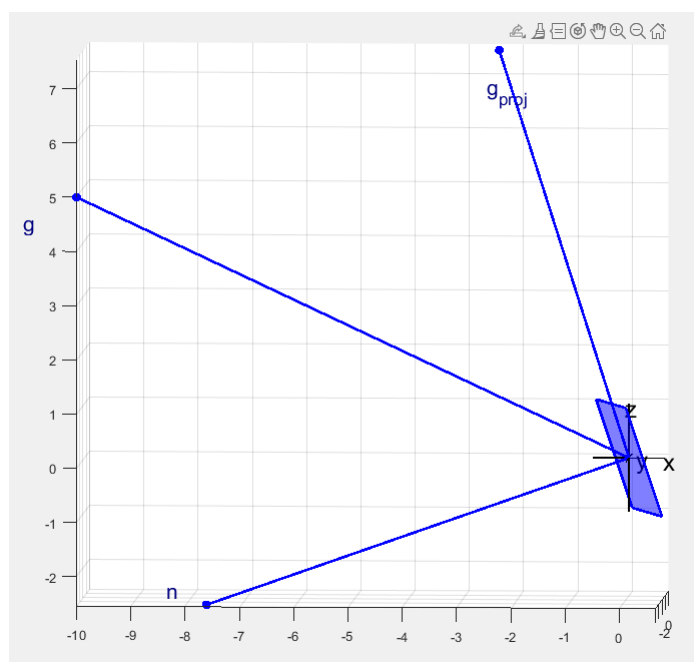


Figure 6: Solution of the task 2

3 Task 3

To rearrange optimization problem in another form we should use the following values:

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix}$$

$$c = [0 \quad -32]$$

$$c0 = 60$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ -1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 6 \\ 8 \\ 0 \\ 0 \\ 9 \end{bmatrix}$$

A solution on CVXPY and visualization you can find in Task_3.ipynb