Dr. Igor Gaponov, Simeon Nedelchev, 2021

# [F21] Fundamentals of Robot Control. Assignment 2

**Problem 1 (20 pts)** Consider pendulum with dynamics described by

$$mL^2\ddot{\theta} + mgL\sin\theta + b\dot{\theta} = u$$

a. (10 pts) Find a control law that transforms pendulum dynamics to that of linear spring-mass-damper system

$$mL^2\ddot{\theta} + b_d\dot{\theta} + k_d\theta = 0,$$

with $b_d, k_d$ denoting arbitrary positive values.

b. (10 pts) Write the error dynamics (differential equations for $\tilde{\theta} = \theta_d - \theta$) if the control law is given as the inverse dynamics of the form

$$u = mL^2(\ddot{\theta}_d + k_d\dot{\tilde{\theta}} + k_p\tilde{\theta}) + mgL\sin\theta + \hat{b}\dot{\theta},$$

with $\hat{b}$ denoting our guess about viscous friction $b$, the respective error being $\tilde{b} = \hat{b} - b$, and the terms $k_p = \omega^2, k_d = 2\omega$ denoting positive constant controller gains.

**Problem 2 (20 pts)** Consider a nonlinear system given by the following equations:

$$c^2\left((x_1 + x_2)^2 + 6\right)\ddot{x}_1 + bc(2 - \sin x_2)\ddot{x}_2 + c\frac{x_1\dot{x}_2}{1 + \dot{x}_2^2} + b\dot{x}_2\cos\dot{x}_1 - dx_2 = a(3 + \dot{x}_1^2)u_1 + u_2$$

$$bc(2 - \sin x_2)\ddot{x}_1 + b^2((x_2 - x_1)^2 + 3)\ddot{x}_2 + a(x_1 + x_2)\dot{x}_2^2 + \dot{x}_1\dot{x}_2\sin x_1 = au_1 + (1 + \dot{x}_2^2)u_2$$

a. (15 pts) Write dynamics in the following form (obtain $\boldsymbol{D}, \boldsymbol{\beta}, \boldsymbol{C}$)

$$\boldsymbol{D}(\boldsymbol{x}, \dot{\boldsymbol{x}})\ddot{\boldsymbol{x}} + \boldsymbol{\beta}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \boldsymbol{C}(\boldsymbol{x}, \dot{\boldsymbol{x}})\boldsymbol{u}$$

b. (5 pts) Show that "inertia" matrix $\boldsymbol{D}$ is positive definite.

**Problem 3 (25 pts)** Consider the following class of systems:

$$\dot{\mathbf{x}} = \boldsymbol{f}(\mathbf{x}) + \mathbf{u}$$

That admits a following representation:

$$\dot{\mathbf{x}} + \boldsymbol{\phi}(\mathbf{x})\mathbf{p} = \mathbf{u}$$

where: $\mathbf{x}, \mathbf{u}, \boldsymbol{f} \in \mathbb{R}^n$, $\mathbf{p} \in \mathbb{R}^m$ is vector of constant parameters.

Assuming that exact parameters $\mathbf{p}$ of system are not known but estimates $\hat{\mathbf{p}}$ are given instead, define the robust controller $u$ such that $\tilde{\mathbf{x}} = \mathbf{x}_d - \mathbf{x} \to 0$ provided that parameter uncertainty is bounded by known value $\|\tilde{\mathbf{p}}\| = \|\hat{\mathbf{p}} - \mathbf{p}\| < \rho$

*Tip:* use the following Lyapunov candidate and find controller $\mathbf{u}$ such that its derivative is negative definite:

$$V = \frac{1}{2}\tilde{\mathbf{x}}^T\tilde{\mathbf{x}}$$

**Problem 4 (35 pts)** Consider the flexible joint link governed by following dynamical system:

$$\begin{cases} I\ddot{\theta}_1 + b\dot{\theta}_1 + k(\theta_1 - \theta_2) = u \\ mL^2\ddot{\theta}_2 + mgL\sin\theta_2 = k(\theta_1 - \theta_2) \end{cases}$$

Let the output of interest to be the angle of link $y = \theta_2$, while the state is given by $\mathbf{x} = [\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2]$

a. (15 pts) Find the state transformation $\mathbf{z} = \mathbf{T}(\mathbf{x})$ and controller $u(\mathbf{x})$ that will linearize the system with respect to $\mathbf{z}$ (feedback linearization)

b. (10 pts) Simulate proposed controller in **the colab environement**. Do the regulation from $\theta_{2_0} = 0$ to $\theta_{2_d} = \pi$.

c. (10 pts) Change your simulation in order to track the trajectory $\theta_{2_d} = \pi\sin(4\pi t)$

*Tip:* For convenience you may re-dimensionalize proposed system by setting all coefficients $I = m = l = k = 1$.