

# Aritmetik İşlemlerde Oluşan Hatalar

(ÖRNEK)

$$\Delta(c) = \Delta(a \pm b) = \Delta(a) + \Delta(b)$$

$$\Delta(ab) = |a| \Delta(b) + |b| \Delta(a)$$

$$\Delta\left(\frac{a}{b}\right) = \frac{|b| \Delta(a) + |a| \Delta(b)}{|b|^2}$$

$$\Delta(a^m) = m|a|^{m-1} \Delta(a)$$

$$\delta(c) = \delta(a \pm b) = \frac{|a| \delta(a) + |b| \delta(b)}{|a \pm b|}$$

$$\delta(ab) = \delta(a) + \delta(b)$$

$$\delta\left(\frac{a}{b}\right) = \delta(a) + \delta(b)$$

$$\delta(a^m) = m \cdot \delta(a)$$

## ÖRNEK:

$$X = \frac{a + b^2}{\sqrt{d} - a.c}$$

$$a = 1,4 \pm 0,003$$

$$b = 2,4 \pm 0,01$$

$$c = 0,5 \pm 0,02$$

$$d = 1,6$$

$$\Delta(b^2) = 2|b|\Delta(b) = 2 \times 2,4 \times 0,01 = 0,048$$

$$\Delta(a + b^2) = \Delta(a) + \Delta(b^2) = 0,003 + 0,048 = 0,051$$

$$\Delta(\sqrt{d}) = \frac{1}{2}|d|^{-\frac{1}{2}} \Delta(d) = 0$$

$$\begin{aligned}\Delta(a.c) &= |a|\Delta(c) + |c|\Delta(a) \\ &= 1,4 \times 0,02 + 0,5 \cdot 0,003 \\ &= 0,028 + 0,0015 = 0,0295\end{aligned}$$

$$\Delta(\sqrt{d} - ac) = \Delta(\sqrt{d}) + \Delta(a.c) = 0 + 0,0295 = 0,0295$$

$$\Delta X = \frac{(pay) \times \Delta(payda) + (payda) \Delta(pay)}{|payda|^2}$$

$$= \frac{(a + b^2) \Delta(\sqrt{d} - ac) + (\sqrt{d} - ac) \Delta(a + b^2)}{|\sqrt{d} - ac|^2}$$

$$\Delta X = \frac{(1,4 + 2,4^2) \times 0,0295 + (\sqrt{1,6} - 1,4 \times 0,5) \times 0,051}{|\sqrt{1,6} - 1,4 \times 0,5|^2}$$

$$\Delta X = \frac{(1,4 + 5,76) \times 0,0295 + (1,2649 - 0,7) \times 0,051}{|1,2649 - 0,7|^2}$$

$$\Delta X = \frac{7.16 \times 0,0295 + 0.5649 \times 0,051}{0.5649^2}$$

$$\Delta X = \frac{0,2112 + 0,0288}{0,3191} = \frac{0.24}{0,3191} = 0.7521$$

$$\Delta x = 0,7521$$

$$X = \frac{1,4 + 2,4^2}{\sqrt{1,6} - 1,4 \times 0,5} = \frac{1,4 + 5,76}{1,2649 - 0,7} = 12,6748$$

$$X = 12,6748 \pm 0,7521$$

$$\delta(a) = \frac{0,003}{1,4} = 0,0021,$$

$$\delta(b) = \frac{0,01}{2,4} = 0,0041,$$

$$\delta(c) = \frac{0,02}{0,5} = 0,04,$$

$$\delta(d) = 0$$

$$X = \frac{a + b^2}{\sqrt{d} - a.c}$$

$$\delta(b^2) = 2.0,0041 = 0,0082$$

$$\begin{aligned} \delta(a + b^2) &= \frac{|a| \delta(a) + |b|^2 \delta(b^2)}{|a + b^2|} \\ &= \frac{1,4 \times 0,0021 + 5,76 \times 0,0082}{|1,4 + 5,76|} \\ &= \frac{0,00294 + 0,047232}{7,16} = 0,007 \end{aligned}$$

$$\delta(a) = \frac{0,003}{1,4} = 0,0021,$$

$$\delta(b) = \frac{0,01}{2,4} = 0,0041,$$

$$\delta(c) = \frac{0,02}{0,5} = 0,04,$$

$$\delta(d) = 0$$

$$X = \frac{a + b^2}{\sqrt{d} - a.c}$$

$$\delta(\sqrt{d}) = \delta(d^{\frac{1}{2}}) = \frac{1}{2} \delta(a) \quad \frac{1}{2} \cdot 0 = 0$$

$$\delta(a \times c) = \delta(a) + \delta(c) = 0,0021 + 0,04 = 0,0421$$

$$\delta(\sqrt{d} - ac) = \frac{\sqrt{d} \cdot \delta(\sqrt{d}) + a.c \cdot \delta(a.c)}{\sqrt{d} - ac} = \frac{\sqrt{1,6} \cdot 0 + 1,4 \times 0,5 \times 0,0421}{\sqrt{1,6} - 1,4 \times 0,5}$$

$$= \frac{0 + 0,02947}{0,56491} = 0,0522$$

$$\delta(\sqrt{d} - ac) = 0,0522$$



$$\delta(a) = \frac{0,003}{1,4} = 0,0021,$$

$$\delta(b) = \frac{0,01}{2,4} = 0,0041,$$

$$\delta(c) = \frac{0,02}{0,5} = 0,04,$$

$$\delta(d) = 0$$

$$X = \frac{a + b^2}{\sqrt{d} - a.c}$$

$$\begin{aligned}\delta(X) &= \delta(\textit{pay}) + \delta(\textit{payda}) \\ &= 0,007 + 0,0522 = 0,0592\end{aligned}$$

$$X = 12,6748 \pm 0,7521$$

$$\delta(X) = \frac{0,7519}{12,6748} = 0,0593$$