Örnek: $f(x) = e^{0,5x}$ fonksiyonunun $x=1$ noktasında türevlerini hesaplayın. $(h=0.1)$

$f_i = f(1)$

$f'(x) = 0,5e^{0,5x}$

$f''(x) = 0,25e^{0,5x}$

f_{i-3}	f_{i-2}	f_{i-1}	f_i	f_{i+1}	f_{i+2}	f_{i+3}
0,7	0,8	0,9	1	1,1	1,2	1,3

$m=1, n=2$ (1. mertebeden türev 3 nokta için)

$f(1) = e^{0,5.1} = e^{0,5} = 1,6487$

$f(1,1) = e^{0,5.1,1} = e^{0,55} = 1,7333$

$f(1,2) = e^{0,5.1,2} = e^{0,6} = 1,8221$

① $f_i' = \frac{1}{2h}(-3f_i + 4f_{i+1} - f_{i+2})$

$= \frac{1}{2.0,1}[-3.f(1) + 4.f(1,1) - f(1,2)]$

$= \frac{1}{0,2}[-3.1,6487 + 4.1,7333 - 1,8221]$

$= 5.0,165 = \boxed{0,825}$

$f(0,9) = e^{0,5.0,9} = e^{0,45} = 1,5683$

$f_i' = \frac{1}{2h}(f_{i+1} - f_{i-1})$

② $= \frac{1}{2.0,1}[f(1,1) - f(0,9)]$

$= \frac{1}{0,2}(1,7333 - 1,5683)$

$= 5.0,165 = \boxed{0,825}$

$f(0,8) = e^{0,5.0,8} = e^{0,4} = 1,4918$

$f_i' = \frac{1}{2h}(f_{i-2} - 4f_{i-1} + 3f_i)$

③ $= \frac{1}{2.0,1}[f(0,8) - 4.f(0,9) + 3.f(1)]$

$= \frac{1}{0,2}(1,4918 - 4.1,5683 + 3.1,6487)$

$= 5.0,1647 = \boxed{0,8235}$

$f'(x) = 0,5e^{0,5x} \Rightarrow f'(1) = 0,5.e^{0,5} = 0,5.1,6487 = \boxed{0,8244}$ → Gerçek değer

①,② $\Rightarrow \delta = \frac{|0,8244 - 0,825|}{0,8244} = \frac{0,0006}{0,8244} = \boxed{0,0007}$ → bağıl hata

③ $\Rightarrow \delta = \frac{|0,8244 - 0,8235|}{0,8244} = \frac{0,0009}{0,8244} = \boxed{0,0011}$

$n=2, n=2$ (2. mertebeden türev 3 nokta için)

$$f_i'' = \frac{1}{h^2}(f_i - 2f_{i+1} + f_{i+2})$$

(1)
$$= \frac{1}{(0,1)^2}[f(1) - 2.f(1,1) + f(1,2)]$$
$$= \frac{1}{0,01}.(1,6487 - 2.1,7333 + 1,8221)$$
$$= 100.0,0042 = \boxed{0,42}$$

$$f_i'' = \frac{1}{h^2}(f_{i-1} - 2f_i + f_{i+1})$$

(2)
$$= \frac{1}{(0,1)^2}[f(0,9) - 2f(1) + f(1,1)]$$
$$= \frac{1}{0,01}(1,5683 - 2.1,6487 + 1,7333)$$
$$= 100.0,0042 = \boxed{0,42}$$

$$f_i'' = \frac{1}{h^2}(f_{i-2} - 2f_{i-1} + f_i)$$
$$= \frac{1}{(0,1)^2}[f(0,8) - 2f(0,9) + f(1)]$$

(3)
$$= \frac{1}{0,01}(1,4918 - 2.1,5683 + 1,6487)$$
$$= 100.0,0039 = \boxed{0,39}$$

$f''(x) = 0,25.e^{0,5x} \Rightarrow f''(1) = 0,25.e^{0,5} = 0,25.1,6487 = \boxed{0,4122}$ ← Gerçek değer

(1),(2) $\Rightarrow \varepsilon = \frac{|0,4122 - 0,42|}{0,4122} = \frac{0,0078}{0,4122} = \boxed{0,0189}$

(3) $\Rightarrow \varepsilon = \frac{|0,4122 - 0,39|}{0,4122} = \frac{0,0222}{0,4122} = \boxed{0,0539}$

Bağıl hata

m=1, n=3 (1 mertebeden türev 4 nokta için)

$$f_i' = \frac{1}{6h}\left[-11f_i + 18f_{i+1} - 9f_{i+2} + 2f_{i+3}\right]$$

$f(1,3) = e^{0,5 \cdot 1,3} = e^{0,65} = 1,9155$ (3)

(1) $= \frac{1}{6 \cdot 0,1}\left[-11 \cdot f(1) + 18 f(1,1) - 9f(1,2) + 2f(1,3)\right]$

$= \frac{1}{0,6}\left[-11 \cdot 1,6487 + 18 \cdot 1,7333 - 9 \cdot 1,8221 + 2 \cdot 1,9155\right]$

$= 1,6667 \cdot 0,4958 = \boxed{0,8263}$

$$f_i' = \frac{1}{6h}\left(-2f_{i-1} - 3f_i + 6f_{i+1} - f_{i+2}\right)$$

(2) $= \frac{1}{6 \cdot 0,1}\left[-2f(0,9) - 3f(1) + 6f(1,1) - f(1,2)\right]$

$= \frac{1}{0,6}\left(-2 \cdot 1,5683 - 3 \cdot 1,6487 + 6 \cdot 1,7333 - 1,8221\right)$

$= 1,6667 \cdot 0,495 = \boxed{0,8250}$

$$f_i^{-1} = \frac{1}{6h}\left[f_{i-2} - 6f_{i-1} + 3f_i + 2f_{i+1}\right]$$

(3) $= \frac{1}{6 \cdot 0,1}\left[f(0,8) - 6f(0,9) + 3f(1) + 2f(1,1)\right]$

$= \frac{1}{0,6}\left(1,4918 - 6 \cdot 1,5683 + 3 \cdot 1,6487 + 2 \cdot 1,7333\right)$

$= 1,6667 \cdot 0,4947 = \boxed{0,8245}$

$$f_i' = \frac{1}{6h}\left(-2f_{i-3} + 9f_{i-2} - 18f_{i-1} + 11f_i\right)$$

$f(0,7) = e^{0,5 \cdot 0,7} = e^{0,35} = 1,4191$

(4) $= \frac{1}{6 \cdot 0,1}\left[-2 \cdot f(0,7) + 9f(0,8) - 18f(0,9) + 11f(1)\right]$

$= \frac{1}{0,6}\left(-2 \cdot 1,4191 + 9 \cdot 1,4918 - 18 \cdot 1,5683 + 11 \cdot 1,6487\right)$

$= 1,6667 \cdot 0,4943 = \boxed{0,8238}$

---

(1) $\delta = \frac{|0,8264 - 0,8263|}{0,8264} = \frac{0,0001}{0,8264} = \boxed{0,0003}$ → 3 gül hata

$n=2,\ n=3$ (2. mertebeden türev 4 nokta için)

$$f_i'' = \frac{1}{h^2}(2f_i - 5f_{i+1} + 4f_{i+2} - f_{i+3})$$

$$= \frac{1}{(0,1)^2}\cdot[2f(1) - 5f(1,1) + 4f(1,2) - f(1,3)]$$

(1)

$$= \frac{1}{0,01}\cdot(2.1,6487 - 5.1,7333 + 4.1,8221 - 1,9155)$$

$$= 100 \cdot 0,0038 = \boxed{0,38}$$

$$f_i'' = \frac{1}{h^2}(f_{i-1} - 2f_i + f_{i+1})$$

$$= \frac{1}{(0,1)^2}[f(0,9) - 2f(1) + f(1,1)]$$

(2)

$$= \frac{1}{0,01}(1,5683 - 2.1,6487 + 1,7333)$$

$$= 100 \cdot 0,0042 = \boxed{0,42}$$

$$f_i'' = \frac{1}{h^2}(f_{i-1} - 2f_i + f_{i+1})$$

$$= \frac{1}{(0,1)^2}[f(0,9) - 2f(1) + f(1,1)]$$

(3)

$$= \frac{1}{0,01}(1,5683 - 2.1,6487 + 1,7333)$$

$$= 100 \cdot 0,0042 = \boxed{0,42}$$

$$f_i'' = \frac{1}{h^2}(-f_{i-3} + 4f_{i-2} - 5f_{i-1} + 2f_i)$$

$$= \frac{1}{(0,1)^2}[-f(0,7) + 4f(0,8) - 5f(0,9) + 2f(1)]$$

(4)

$$= \frac{1}{0,01}(-1,4191 + 4.1,4918 - 5.1,5683 + 2.1,6487)$$

$$= 100 \cdot 0,004 = \boxed{0,4}$$

---

(1) $\rightarrow \delta = \dfrac{|0,4122 - 0,391|}{0,4122} = \dfrac{0,0322}{0,4122} = \boxed{0,0781}$ → Bağıl hata

(2)(3) $\rightarrow \delta = \dfrac{|0,4122 - 0,42|}{0,4122} = \dfrac{0,0078}{0,4122} = \boxed{0,0189}$

ÖDEV ! Öğrenci numaranız

ab * * * cde olmak üzere

$f(x) = \sin(e^{0,abx}) + \cos(0,edc \cdot x)$ fonksiyonunun

$x = 1$ noktasında

a) $h = 0,1$ ve b) $h = 0,2$ olmak üzere 1. ve 2. mertebeden türevlerini, ve bağıl hataları hesaplayınız.