BMMU 7. Hafta

müzik algoritması!
sinyal yön kestirme algoritması

* P izdüşüm ise $R(\bar{P}) \perp N(\bar{P})$

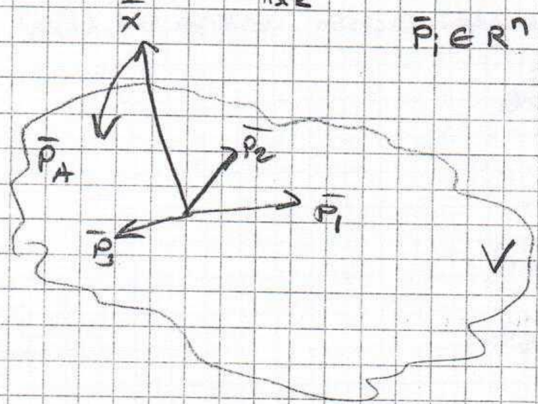
$V \perp W$

izdüşüm matrisi

$V = \text{span}(\bar{P}_1, \bar{P}_2, \ldots, \bar{P}_k)$ k'tane vektörün tarandığı uzay olsun

$\bar{A} = [\bar{P}_1, \bar{P}_2, \ldots \bar{P}_k]_{n \times k}$ $\bar{P}_i \to n \times 1$ $i = 1, 2, \ldots, k$

$\bar{P}_i \in R^n$



Herhangibir $\bar{x} \in R^n$ vektörünü V alt uzayına izdüşümünü oluşturan matris $\bar{P}_A$ olsun

$$\bar{P}_A = \underbrace{\bar{A}_{n \times k} \cdot \underbrace{[\bar{A}^H_{k \times n} \cdot A_{n \times k}]^{-1}}_{k \times k} \cdot A^H_{k \times n}}_{\bar{A}^{\#} \text{ ; } \bar{A}\text{'nın sanal tersi (Pseduoinverse)}}$$

$\bar{P}_A \to n \times n$

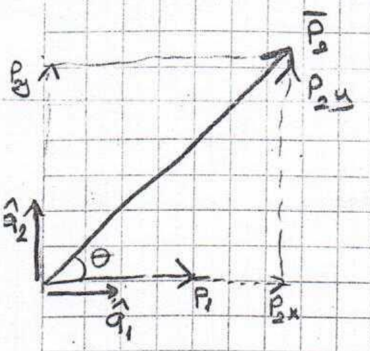
$P_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$P_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\bar{A} = [\bar{P}_1 \; \bar{P}_2] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}_{3 \times 2}$

$$P_A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}_{3 \times 2} \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}_{3 \times 2} \right)^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{2 \times 3} = \text{- - - -}$$

Gram-Schmidt Ortogonalizasyon ~~Prosedürü~~ Algoritması
(Dikleştirme)



$\bar{P}_1 \not\perp \bar{P}_2$

$\theta \neq 90^\circ$

1) $\hat{a}_1 = \frac{\bar{P}_1}{\|\bar{P}_1\|}$

2) $\bar{P}_2$'nin q_1 üzerindeki izdüşüm vektörünü bul ($\bar{P}_2 q_1$)

3) $\bar{P}_2 - \bar{P}_2 q_1 = \bar{P}_2 q_2$

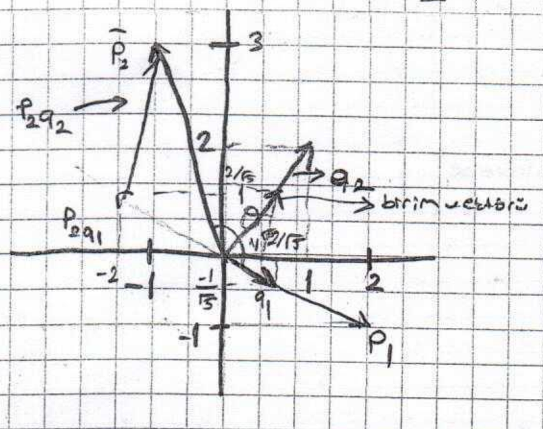
4) $\bar{q}_2 = \frac{\bar{P}_{2y}}{\|\bar{P}_{2y}\|}$

sonuçta $\hat{q}_1 \perp \hat{q}_2$

$\|\hat{q}_1\| = \|\hat{q}_2\| = 1$

Örnek

$\bar{P}_1 = \begin{bmatrix}2\\-1\end{bmatrix}$ $\bar{P}_2 = \begin{bmatrix}-1\\3\end{bmatrix}$



1) $\hat{q}_1 = \frac{\bar{P}_1}{\|\bar{P}_1\|} = \begin{bmatrix}2/\sqrt{5}\\-1/\sqrt{5}\end{bmatrix}$ $\|\bar{q}_1\| = 1$

2) $\bar{P}_2$'nin $\bar{q}_1$ üzerindeki bileşeni $\langle \bar{P}_2, \bar{q}_1 \rangle$

$= \|\bar{P}_2\| . \|\bar{q}_1\| . \cos\theta$

$= \bar{P}_2^T . \bar{q}_1$

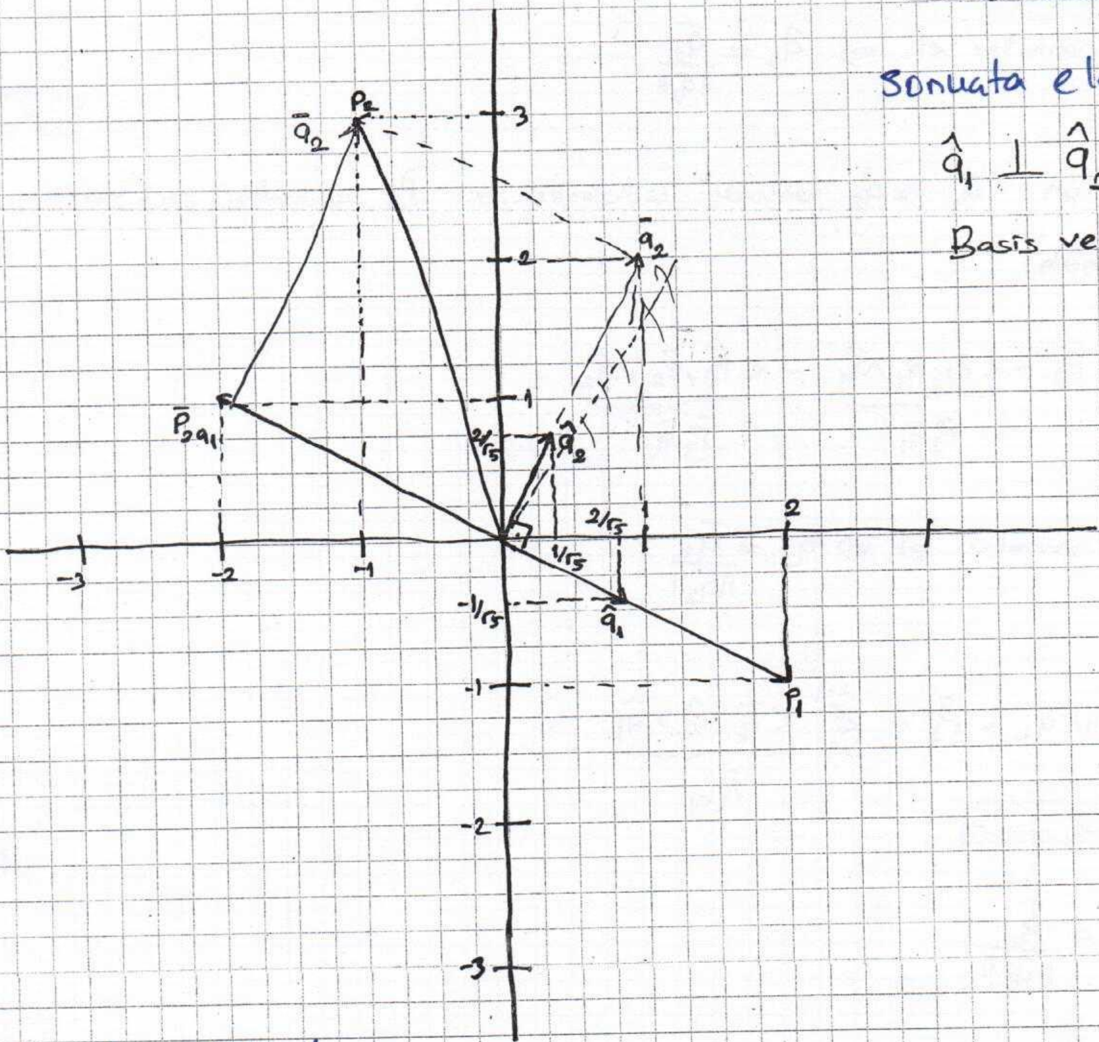
$\begin{bmatrix}-1 & 3\end{bmatrix}\begin{bmatrix}2/\sqrt{5}\\-1/\sqrt{5}\end{bmatrix} = \frac{-2}{\sqrt{5}} - \frac{3}{\sqrt{5}} = \boxed{-\sqrt{5}}$

$P_2 q_1 = -\sqrt{5} . q_1 = \begin{bmatrix}-2\\1\end{bmatrix}$

3) $\bar{q}_2 = \bar{P}_2 - \bar{P}_2 q_1$

$\bar{q}_2 = \begin{bmatrix}-1\\3\end{bmatrix} - \begin{bmatrix}-2\\1\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}$

4) $\hat{q}_2 = \frac{\bar{q}_2}{\|\bar{q}_2\|}$



Sonuçta elde edilen

$\hat{q}_1 \perp \hat{q}_2 \Rightarrow \langle \hat{q}_1, \hat{q}_2 \rangle = 0$

Basis vektörler'dir.

Algoritma Adımları:

Verilen $T = \{\bar{p}_1, \bar{p}_2, \dots, \bar{p}_n\}$ vektörlerinden birbirine dik olan

$T' = \{\hat{q}_1, \hat{q}_2, \dots, \hat{q}_m\}$ $m \le n$ ve $\hat{q}_i \perp \hat{q}_j$ $\langle \hat{q}_i, \hat{q}_j \rangle = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$

bulmak için

step 1: T kümesinde verilen ilk vektörü normalize et

$$\hat{q}_1 = \frac{\bar{p}_1}{\|\bar{p}_1\|}$$

step 2: a) $\bar{p}_2$ vektörünün $\hat{q}_1$ üzerindeki iz düşümü ile $\bar{p}_2$ arasındaki fark vektörü $\bar{q}_2$'yi hesapla

$$\bar{q}_2 = \bar{p}_2 - \underbrace{(\langle \bar{p}_2, \hat{q}_1 \rangle)}_{\bar{p}_2 \hat{q}_1} \hat{q}_1$$

b) $\bar{q}_2$'yi normalize et $\Rightarrow \hat{q}_2 = \dfrac{\bar{q}_2}{\|\bar{q}_2\|}$

$\boxed{\text{Step 3:}}$ a) P_3' ün $\hat{q}_1$ ve $\hat{q}_2$ üzerindeki izdüşümleri ile $\bar{P}_3$ arasındaki fark vektörü $\bar{q}_3$'ü hesapla

$$\bar{q}_3 = \bar{P}_3 - \underbrace{\langle \bar{P}_3, \hat{q}_1 \rangle}_{\bar{P}_3 \bar{q}_1} \hat{q}_1 - \underbrace{\langle \bar{P}_3, \hat{q}_2 \rangle}_{\bar{P}_3 \bar{q}_2} \hat{q}_2$$

b) $\bar{q}_3$'ü normalize et $\Rightarrow \hat{q}_3 = \dfrac{\bar{q}_3}{\|\bar{q}_3\|}$

$\boxed{\text{Step } k}$ a) $\bar{q}_k = \bar{P}_k - \displaystyle\sum_{i=1}^{k-1} \underbrace{\langle \bar{P}_k, \hat{q}_i \rangle}_{\bar{P}_k q_i} \hat{q}_i$

$\boxed{k = 2, \ldots, n}$

b) $\hat{q}_k = \dfrac{\bar{q}_k}{\|\bar{q}_k\|}$

$\boxed{\text{Tanım}}$ Fonksiyonlar için norm tanımı

$[a,b]$ aralığında tanımlı fonksiyonlar için $(x(t))$

Norm: $\|x(t)\|_2 = \langle x(t), x(t) \rangle^{1/2} = \left[\int_a^b |x(t)|^2 \, dt \right]^{1/2}$

$x(t)$ Periyodik ise $\longrightarrow \sqrt{\frac{1}{T} \int_0^T |x(t)|^2 \, dt}$

root

mean

square

$a = 0$
$b = T$

iç çarpım: $\langle x(t), y(t) \rangle = \int_a^b x(t) \, y^*(t) \, dt$

eşlenik

örnek

$$T = \{1, t, t^2\} \qquad t \in [-1,1]$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ P_1(t) & P_2(t) & P_3(t) \end{array}$$

$P_1 \overset{?}{\perp} P_2$

$P_1 = 1 \quad P_2 = t$

$$\langle P_1, P_2 \rangle = \int_{-1}^{1} 1.t\,dt = \left.\frac{t^2}{2}\right|_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$$

step 1: $$y_1(t) = \frac{P_1(t)}{\|P_1(t)\|} = \frac{P_1(t)}{\underbrace{\left(\int_{-1}^{1} 1.1\,dt\right)^{1/2}}_{\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

step 2: $$e_2(t) = P_2(t) - \langle P_2(t), y_1(t)\rangle . y_1(t) = t$$

$$t - \underbrace{\int_{-1}^{1} t\,\frac{1}{\sqrt{2}}\,dt}_{0}$$

$$y_2(t) = \frac{e_2(t)}{\|e_2(t)\|} =$$

$$\|e_2(t)\| = (\langle e_2(t), e_2(t)\rangle)^{1/2} = \sqrt{\int_{-1}^{1} t.t\,dt} = \sqrt{\left.\frac{t^3}{3}\right|_{-1}^{1}} = \sqrt{\frac{2}{3}}$$

$$\cancel{\frac{t}{\sqrt{2/3}}} \qquad \boxed{\sqrt{\frac{3}{2}}\,t}$$

$$y_2(t) = \sqrt{\frac{3}{2}}\,t$$

step 3: $$e_3(t) = P_3(t) - P_{3y_1}(t) - P_{3y_2}(t)$$

$$P_{3y_1}(t) = \underbrace{\langle P_3(t), y_1(t)\rangle}\, y_1(t) = \frac{\sqrt{2}}{3} \cdot \frac{1}{\sqrt{2}} = \frac{1}{3}$$

$$= \int_{-1}^{1} t^2 . \frac{1}{\sqrt{2}}\,dt$$

$$= \frac{1}{3\sqrt{2}} \left. t^3 \right|_{-1}^{1}$$

$$= \frac{2}{3\sqrt{2}} = \boxed{\frac{\sqrt{2}}{3}}$$

$$P_3 y_2(t) = \underbrace{\langle P_3(t), y_2(t)\rangle}\, y_2(t) =$$

$$\int_{-1}^{1} t^2 \frac{\sqrt{3}}{2} t \, dt$$
$$= a t^4 = 0$$

$$e_3(t) = t^2 - \frac{1}{3}$$

$$y_3(t) = \frac{e_3(t)}{\|e_3(t)\|} = \frac{t^2 - \frac{1}{3}}{\sqrt{\int_{-1}^{1} (t^2-\frac{1}{3})\cdot(t^2-\frac{1}{3})\, dt}} = \text{-----}$$

GRAM-SCHMIDT ALGORİTMASININ GERÇEKLENMESİ:

$$\bar{A} = [\,\bar{P}_1 \; \bar{P}_2 \; \ldots \; \bar{P}_n\,] \qquad \bar{P}_i \in R^{m}$$

$$\bar{Q} = [\,\hat{q}_1 \; \hat{q}_2 \ldots \hat{q}_m\,] \qquad m \leq n$$

R matrisini oluştur.

$$R = \begin{bmatrix} \|\bar{P}_1\| & \langle \bar{P}_2, \hat{q}_1\rangle & \langle \bar{P}_3, \hat{q}_1\rangle & \cdots & \langle \bar{P}_n, \hat{q}_1\rangle \\ & \|\bar{P}_2 q_1\| & \langle \bar{P}_3, \hat{q}_2\rangle & \cdots & \langle \bar{P}_n, \hat{q}_2\rangle \\ & & \|\bar{P}_3 q_2\| & \cdots & \langle \bar{P}_n, \hat{q}_3\rangle \\ & & & \ddots & \langle \bar{P}_n, \hat{q}_{m-1}\rangle \\ & & & & \|\bar{P}_n \hat{q}_m\| \end{bmatrix}$$

k. adımdaki $-\langle \bar{P}_k, \hat{q}_i\rangle \hat{q}_i = R(1:k-1, k)$

$$= Q(:, 1:k-1)^H A(:, k)$$

$$\sum_{i=1}^{k-1} \langle \bar{P}_k, \hat{q}_i\rangle \hat{q}_i = Q(:, 1:k-1)\, R(1:k-1, k)$$