ADİ DİFERANSİYEL DENKLEMLER İÇİN CAUCHY PROBLEMİNİN YAKLAŞIK ÇÖZÜMÜ- RUNGE-KUTTA YÖNTEMİ

$y' = f(x, y).$ (1)

$y(x_0) = y_0$ (2)

(1), (2) Cauchy problemini çözümü Runge Kutta yöntemi yardımı ile çözmek için aşağıdaki yaklaşımlar ardışık olarak kullanılır.

Dördüncü dereceden Runge-Kutta yöntemi:

$$y_{i+1} = y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1)$$

$$k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2)$$

$$k_4 = f(x_i + h, y_i + k_3)$$

ÖRNEK:

$$y_{i+1} = y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

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$$k_4 = f(x_i + h, y_i + k_3)$$

$y' = -\frac{y}{1+x}$ $\quad h = 0.2,\ y(0) = 1$ $\quad Runge - Kutta$ *yöntemi ile* çözünüz.

- $x_0 = 0\ , y_0 = 1$

$K_1 = f(x_0, y_0) = -\frac{1}{1+0} = -1$

$K_2 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) = f\left(0.1, 1 + \frac{-1}{2}\right) = f(0.1, 0.5) = \frac{-0.5}{1+0.1} = -0.4545$

$K_3 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right) = f\left(0.1, 1 + \frac{-0.4545}{2}\right) = f(0.1, 0.7728) = -\frac{0.7728}{1+0.1} = -0.7025$

$K_4 = f(x_0 + h, y_0 + K_3) = f(0.2, 1 - 0.7025) = f(0.2, 0.2975) = -\frac{0.2975}{1+0.2} = -0.2479$

$y_1 = 1 + \frac{0.2}{6}(-1 - 2 * 0.4545 - 2 * 0.7025 - 0.2479) = 1 + \frac{0.2}{6}(-3.5619) = \mathbf{0.8813}$

- $x_1 = 0.2 \quad y_1 = 0.8813$

$K_1 = f(x_1, y_1) = -\frac{0.8813}{1+0.2} = -0.7344$

$K_2 = f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right) = f\left(0.3, 0.8813 - \frac{0.7344}{2}\right) = f(0.3, 0.5141) = \frac{-0.5141}{1+0.3} = -0.3955$

$K_3 = f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right) = f\left(0.2 + \frac{0.2}{2}, 0.8813 + \frac{-0.3955}{2}\right) = f(0.3, 0.6836) = -\frac{0.6836}{1+0.3} = -0.5258$

$K_4 = f(x_1 + h, y_1 + K_3) = f(0.2 + 0.2, 0.8813 - 0.5258) = f(0.4, 0.3555) = -\frac{0.3555}{1+0.4} = -0.2539$

$y_2 = y_1 + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$y_2 = 0.8813 + \frac{0.2}{6}(-0.7344 - 2 * 0.3955 - 2 * 0.5538 - 0.2539) = 0.8813 + \frac{0.2}{6}(-2.8309) = \mathbf{0.7869}$

- $x_2 = 0.4 \quad y_2 = 0.7869$