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SKEW ENERGY OF DIGRAPHS AND RELATION BETWEEN GRAPH ENERGY AND DISTANCE ENERGY

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ABSTRACT: The energy of a graph is the sum of the absolute values of the eigenvalues of its adjacency matrix. The distance energy of a connected graph is defined as the sum of the absolute values of the D-eigenvalues of its distance matrix. The singular value inequality for the singular value of a matrix sum, including its equality case is used to study how the energy of a graph changes when edges are removed. In this paper we see that the singular value inequality holds good for skew energy of digraphs due to the edge deletion and the also the relation between the graph energy and distance graph energy is discussed for graphs and digraphs.

AMS CLASSIFICATION: 15A45, 05C50.

KEYWORDS: Singular value inequality, Graph energy, Distance graph energy, Skew energy.

1.INTRODUCTION

The energy of a graph has its roots in theoretical chemistry. The energy of a simple graph was first defined by Ivan Gutman [12] in 1978 as the sum of the absolute values of its eigen values of its adjacency matrix. In chemistry, molecular orbital theory (MO) is a method for determining molecular structure in which electrons are not assigned to individual bonds between atoms, but are treated as moving under the influence of the nuclei in the whole molecule. In this theory, each molecule has a set of molecular orbitals in which it is assumed that the molecular orbital wave function Ψ_f may be written as a simple weighted sum of the n constituent atomic orbitals χ_i according to the following equation $\Psi_j = \sum_{i=1}^{n} c_{ij} \chi_i$, where the co-efficients c_{ij} may be determined numerically using Schrodinger equation and application of the variational principle.

The Huckel method or Huckel molecular orbital method (HMO) proposed by Erich Huckel in 1930, is a very simple linear combination of atomic orbitals-molecular orbitals (LCAO-MO) method for the determination of energies of molecular orbitals of pi electrons in conjugated hydrocarbon systems, such as ethane, benzene and butadiene. The extended Huckel method developed by Roald Hoffman is the basis of the Woodward-Hoffmann rules. The energies of extended conjugated molecules such as pyridine, pyrrole and furan that contain atoms other than carbon also known as heteroatoms had also been determined using this method. Huckel expressed the total π-electron of a conjugated hydrocarbon as $E_\pi = n\alpha + 2\beta \sum_{i=1}^{n_2} c_{ij}\,\chi_i$ where α and β are constants and the eigenvalues pertain to a special so called "molecular graph" like ethylene, benzene, butadiene, cyclobutadiene, etc. For the sake of simplicity, the expression for E_π is given, when n is even. The only non-trivial part in the above formula for E_π is $2\sum_{i=1}^{n/2} \lambda_i$. It reduces to graph energy provided $\lambda_{\frac{n}{2}} \geq 0 \geq \lambda_{\frac{n}{2}+1}$ [3].

Huckel's method can be derived from Ritz method in Mathematics with some assumptions. This method predicts the number of energy levels exist for a given molecule in which levels are degenerate and it expresses the molecular orbital energies as the sum of two other energy terms called as α, the energy of an electron in a 2 p-orbital, β, the interaction energy between 2p orbitals which are still unknown but have become independent of the molecule. In addition it enables calculation of charge density for each atom in π frame work, bond order between atoms and overall molecular dipole moment.

Many physical situations require directed graphs. The street map of a city with oneway streets, flow networks with valves in pipes, and electrical networks, for example , are represented by directed graphs. Directed graphs are employed in abstract representations of computer programs, where the vertices stand for the program instructions and the edges specify the execution sequence. The directed graph is an invaluable tool in the study of sequential machines. Directed graphs in the form of signal-flow graphs are used for system analysis in control theory.

Jane Day and Wasin So, [20] in their paper Singular value inequality and graph energy change have shown that the energy of a graph increases, decreases or remains the same when the edges are deleted. In this paper we first discuss about the relation between energy and distance energy of graphs.

2. BASIC DEFINITIONS

Definition 2.1: A linear graph or simply a graph G = (V, E) consists of a set of objects $V = \{v_1, v_2, v_3, ..., v_n\}$ called vertices, and another set $E = \{e_1, e_2, e_3, ..., e_n\}$, whose elements are called edges, such that each edge e_k is identified with an unordered pair (v_i, v_j) of vertices.

Definition 2.2: A directed graph G or a digraph G in short consists of a set of vertices $V = \{v_1, v_2, v_3, ..., v_n\}$, a set of edges $E = \{e_1, e_2, e_3, ..., e_n\}$, and a mapping Ψ

that maps every edge onto some ordered pair of vertices (v_i, v_j). A digraph is also referred to as an oriented graph.

Definition 2.3: An edge for which the initial and terminal vertices are the same forms a self-loop. Two directed edges are said to be parallel if they are mapped onto the same ordered pair of vertices. That is, in addition to being parallel in the sense of undirected edges, parallel directed edges must also agree in the direction of their arrows.

Definition 2.4: A digraph that has no self-loop or parallel edges is called a simple digraph. Digraphs in which for every edge (v_i, v_j) that is from vertex v_i to v_j, there is also an edge (v_i, v_j) are called as symmetric digraphs. A digraph that is both simple and symmetric is called a simple symmetric digraph.

Definition 2.5: A complete undirected graph is defined as a simple graph in which every vertex is joined to every other vertex exactly by one edge. For digraphs we have two types of complete graphs. A complete symmetric digraph is a simple digraph in which there is exactly one edge directed from every vertex to every other vertex and a complete asymmetric digraph is an asymmetric graph in which there is exactly one edge between every pair of vertices.

Definition 2.6: A graph is said to be connected if there is at least one path between every pair of vertices. In a digraph, there are two different types of paths. Consequently we have two different types of connectedness. A digraph G is said to be strongly connected if there is at least one directed path from every vertex to every other vertex. A digraph G is said to be weakly connected if its corresponding undirected path is connected, but G is not strongly connected. We can say that a digraph G is connected if its corresponding undirected path is connected that is G can be strongly or weakly connected. A directed path which is not connected is said to be disconnected.

Since there are two types of connectedness in a digraph, we can define two different types of components. Each maximal connected (weakly or strongly) subgraph of a digraph G will be still called as a component of G. But within each components of G, the maximal strongly connected subgraphs will be called as the fragments or strongly connected fragments of G.

3. GRAPH ENERGY

Definition 3.1: Let G be a simple, finite, undirected graph with n vertices and m edges. Let A = (a_{ij}) be the adjacency matrix of graph G. The eigenvalues $\lambda_1, \lambda_2, ..., \lambda_n$ of A (G), assumed in nonincreasing order, are the eigenvalues of the graph G.

The energy of a graph G, denoted by E (G) is defined as E (G) = $\sum_{i=1}^{n}|\lambda_i|$. The set $\{\lambda_1, \lambda_2, ..., \lambda_n\}$ is the spectrum of G and is denoted by Spec G. If the eigenvalues of G are distinct say, $\mu_1 > \mu_2 > ... > \mu_n$ and their multiplicities are $m(\mu_1), m(\mu_2), ..., m(\mu_n)$ then we write

$$\text{Spec } G = \begin{pmatrix} \mu_1 & \mu_2 & \cdots & \cdots & \mu_n \\ m(\mu_1) & m(\mu_2) & \cdots & \cdots & m(\mu_n) \end{pmatrix}.$$

Spec G is independent of labeling of the vertices of G. As A is a real symmetric matrix with zero trace, these eigenvalues are real with sum equal to zero.

Definition 3.2: Let G be a simple connected graph and let its vertices be labelled as $v_1, v_2, ..., v_n$. The distance matrix of a graph G is defined as a square matrix $D(G) = (d_{ij})$ where d_{ij} is the distance between the vertices v_i and v_j in G. The eigenvalues of the distance matrix denoted by $\mu_1, \mu_2, ..., \mu_n$ are said to be the D-eigenvalues of G. Since the distance matrix is symmetric, its eigenvalues are real and can be ordered as $\mu_1 > \mu_2 > ... > \mu_n$. The distance energy of a graph E_D is defined as $E_D = \sum_{i=1}^{n}|\mu_i|$.

The distance energy is defined in analogous to the graph energy as $E = \sum_{i=1}^{n}|\lambda_i|$ where $\lambda_1, \lambda_2, ..., \lambda_n$ are the eigenvalues of the adjacency matrix A (G).

Definition 3.3: Let D be a simple digraph of order n with the vertex set $V(D) = \{v_1, v_2, ..., v_n\}$, and arc set $\Gamma(D) \subseteq V(D) \times V(D)$. We have $(v_i, v_j) \notin \Gamma(D)$ for all i, and $(v_i, v_j) \in (D)$ implies that $(v_i, v_j) \notin (D)$. The skew-adjacency matrix of G is the $n \times n$ matrix $S(D) = (s_{ij})$, where $s_{ij} = 1$, whenever $(v_i, v_j) \in \Gamma(D)$, $s_{ij} = -1$, whenever $(v_i, v_j) \in \Gamma(D)$, and $s_{ij} = 0$ otherwise. S(D) is also the skew-symmetric matrix. The eigenvalues $\{\lambda_1, \lambda_2, ..., \lambda_n\}$ of S (D) are all purely imaginary numbers and the singular values of S(D) coincide with the absolute values $\{|\lambda_1|, |\lambda_2|, ..., |\lambda_n|\}$ of its eigenvalues. Consequently the energy of S (D), which is defined as the sum of its singular values [4], is also the sum of the absolute values of its eigenvalues. This energy is called as the skew energy of the digraph G, denoted by $E_s(D)$. Therefore the skewenergy of a digraph is defined as $E_s(D) = \sum_{i=1}^{n}|\lambda_i|$.

4. SINGULAR VALUE INEQUALITY FOR MATRIX SUM

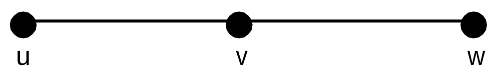
Let A and B be the two $n \times n$ complex matrices. Then the singular value inequality for the matrix sum is given by $\sum_{i=1}^{n} s_i(A+B) \le \sum_{i=1}^{n} s_i(A) + \sum_{i=1}^{n} s_i(B)$.

The equality holds if and only if there exists a unitary matrix P such that PA and PB are both positive semi-definite [20].

5. RELATION BETWEEN ENERGY AND DISTANCE ENERGY DUE TO EDGE DELETION

Let G be a connected simple graph. Let $V(G)$ and $\in (G)$ denote the vertex set and edge set of G respectively. Let $A(G)$ and $D(G)$ be the adjacency matrix and distance matrix of G. The energy of a graph is defined as the sum of the absolute values of the eigenvalues of its adjacency matrix. It is denoted by $E(G) = \sum_{i=1}^{n} |\lambda_i(A(G))|$. The distance energy of a connected simple graph is defined as the sum of the absolute values of the D-eigenvalues of its distance matrix. The distance energy is denoted by $E_D(G) = \sum_{i=1}^{n} |\mu_i(D(G))|$. Since A(G) and D(G) are symmetric matrices, $E(G)$ is defined as the sum of the singular values of $A(G)$ and $E_D(G)$ is defined as the sum of the singular values of $D(G)$. We show that a relation exists between energy and distance energy of graphs by means of an example.

Consider the following graph G



$$A(G) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Characteristic equation of A(G) is $|A - \lambda I| = 0$

The eigenvalues of A(G) are 0, $\pm\sqrt{2}$.

$$E(G) = 2\sqrt{2}$$

Consider the distance matrix of G

$$D(G) = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}.$$

The characteristic equation is $|D - \mu I| = 0$

$$\begin{pmatrix} -\mu & 1 & 2 \\ 1 & -\mu & 1 \\ 2 & 1 & -\mu \end{pmatrix} = 0\,.$$

The D-eigenvalues are $-2,\ 1 \pm \sqrt{3}$.

$$E_D(G) = 2 + 2\sqrt{3}$$

Consider the graph H obtained from G by deleting the edge {v, w}as given below:

The distance matrix of H is

$$D(H) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

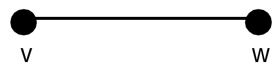
The characteristic equation is

$$|D_H - \mu I| = 0.$$

The D-eigenvalues of H are ± 1.

$$E_D = (H) = 2.$$

Let us consider the graph $G - E(H)$ as given below:



The distance energy of $G - E(H)$ cannot be calculated as it is not connected, but the energy of $G - E(H)$ can be found.

The eigenvalues of $G - E(H)$ are $0, \pm 1$.

$$E(G - E(H)) = 2.$$

It can be easily seen that

$$E(G - E(H)) < E_D(G) + E_D(H) \;\ldots\; \text{I}.$$

Let us assume that $H = \phi$.

Then $E(H) = 0$ and $E_D(H) = 0$.

We get

$$E(G - E(H)) < E_D(G) + E_D(H) \text{ ... II.}$$

Therefore for $H = \phi$ or $H \neq \phi$.

$$E(G - E(H)) < E_D(G) + E_D(H) \text{ ... III.}$$

Similarly we can find

$$E_D(G) - E_D(H) > E(G - E(H)) \text{ ... IV for both } H = \phi \text{ or } H \neq \phi.$$

Combining III and IV , we get

$$E_D(G) - E_D(H) > E(G - E(H)) < E_D(G) - E_D(H) \text{ ... V.}$$

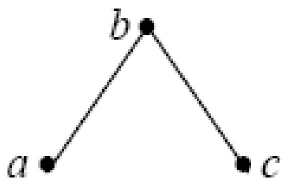
Let us consider a complete graph K_n

$$H = K_n - \{e\}.$$

It can be found that

$$E_D(K_n) = E(K_n) = 2(n-1) \text{ and } E(K_n - \{e\}) = (n-3) + \sqrt{n^2 + 2n - 7}\,[19].$$

For example let $G = K_3$, $H = K_3 - \{e\}$ as given below:



$$E_D(K_3) = 2(3-1) = 4,$$

$$E(H) = (3-3) + \sqrt{3^2 + 2.3 - 7} = \sqrt{8} = 2\sqrt{2}.$$

The distance matrix of $K_3 - \{e\}$ is

$$D(H) = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}.$$

The D-eigenvalues are -2, $1 \pm \sqrt{3}$.

$$E_D(H) = 2 + 2\sqrt{3}$$

It can be also seen that

$$E(G) + E(H) = 1.172$$

$$E(G) + E(H) = 6.828$$

Suppose if $G - E(H) = K_2$

$$E(G - E(H)) = 2(2-1) = 2,$$

$$E_D(G) + E_D(H) = 4 - (2 + 2\sqrt{3}) = 2 - 2\sqrt{3},$$

$$E_D(G) + E_D(H) = 4 + (2 + 2\sqrt{3}) = 6 + 2\sqrt{3}.$$

Therefore for $G = K_n$ and $G \neq H$ we have

$$E_D(G) - E_D(H) < E(G - E(H)) < E_D(G) + E_D(H).$$

And for $G \neq K_n$ and $G \neq H$ we have.

$$E_D(G) - E_D(H) > E(G - E(H)) < E_D(G) + E_D(H).$$

It is clear that for $G \neq K_n$ and $G = H$, we have

$$E_D(G) - E_D(H) < E(G - E(H)) < E_D(G) + E_D(H).$$

It is also seen that for $G = K_n$ and $G = H$, we have

$$E_D(G) - E_D(H) < E(G - E(H)) < E_D(G) + E_D(H).$$

Lemma 5.1: If $A = (a_{ij})$ is a positive semi-definite matrix and $a_{ij} = 0$ for some i, then $a_{ji} = a_{ij} = 0$, for all j. R. Horn and C. Johnson has discussed the above lemma in Matrix Analysis and it is used in the theorem given below.

Theorem 5.2: Let H be a non-empty induced subgraph of a simple connected graph G. Then

$$E_D(G) - E_D(H) < E(G - E(H)) < E_D(G) + E_D(H).$$

The above inequality holds good only for $G = K_n$ with $G \neq H$ and $G = H$ and for also $G \neq K_n$ with $G = H$.

Proof: G is a connected simple graph. Let H be an induced subgraph of G, which contains all edges of G joining two vertices of H. Let G – H denote the graph obtained from G deleting all vertices of H and all edges incident with H. Let G – E (H) denote the graph obtained from G by deleting all edges of H, but keeping all vertices of H. If G_1 and G_2 are two graphs without common vertices, let $G_1 \oplus G_2$ denote the graph with vertex set and the edge set $V(G_1) \cup V(G_2)$ $E(G_1) \cup E(G_2)$ respectively. Hence $A(G_1 \oplus G_2) = A(G_1) \oplus A(G_2)$ $D(G_1 \oplus G_2) = D(G_1) \oplus D(G_2)$.

$$D(G)=\begin{pmatrix} D(H) & X^T \\ X & D(G-H) \end{pmatrix}=\begin{pmatrix} D(H) & 0 \\ 0 & 0 \end{pmatrix}+\begin{pmatrix} 0 & X^T \\ X & D(G-H) \end{pmatrix}.$$

X represents edges connecting H and G – H.

$$D(G)=\begin{pmatrix} D(H) & 0 \\ 0 & 0 \end{pmatrix}+\begin{pmatrix} 0 & X^T \\ X & A(G-H) \end{pmatrix}.$$

Also
$$A(G-E(H))=\begin{pmatrix} 0 & X^T \\ X & A(G-H) \end{pmatrix}$$

$$D(G)=\begin{pmatrix} D(H) & 0 \\ 0 & 0 \end{pmatrix}+A(G-E(H)).$$

By singular value inequality theorem

$$E_D(G) \leq E_D(H) + E(G - E(H))$$

$$E_D(G) - E_D(H) \leq E(G - E(H)) \text{ ... i}$$

$$A(G-E(H))=D(G)+\begin{pmatrix} -D(H) & 0 \\ 0 & 0 \end{pmatrix}+\begin{pmatrix} 0 & Y^T \\ Y & 0 \end{pmatrix}.$$

By singular value inequality theorem,

$$E(G - E(H)) \leq E_D(G) + E_D(H) + E_D(Y)$$

$$E(G - E(H)) < E_D(G) + E_D(H) \text{ ... ii}$$

From i and ii, we have

$$E_D(G) - E_D(H) < E(G - E(H)) < E_D(G) + E_D(H).$$

The right inequality holds for all types of connected graphs, but the left inequality holds only for $G = K_n$ and$G \neq H$ and for $G \neq K$ and $G = H$. This means that the singular value inequality holds only for $G = K_n$ with $G = H$ and $G \neq H$ and for also when $G \neq K_n$ and $G = H$.

The above inequality also gives us the relation between energy and distance energy of graphs.

Remark 5.3: Let $G = K_n$, $E_D(G) = 2(n-1)$

$H = K_n - \{e\}$ and $G \neq H$.

For complete graphs, it is known that $E_D(H) > E_D(G)$ [19].

Therefore for $G = K$ and $G \neq H$, we have

$$E_D(G) - E_D(H) < E(G - E(H)) < E_D(G) + E_D(H).$$

For $G = K_n$ and $G = H$, we have

$$E_D(G) - E_D(H) < E(G - E(H)) < E_D(G) + E_D(H).$$

Also for $G \neq K_n$ and $G = H$, we have shown that

$$E_D(G) - E_D(H) < E(G - E(H)) < E_D(G) + E_D(H).$$

Also the left and the right equality holds for complete graphs G with $H = \phi$, where $E(H) = E_D(H) = 0$. That is for $G = K_n$ and $H = \phi$, we have $E_D(G) - E_D(H) = E(G - E(H))$ and $E(G - E(H)) = E_D(G) + E_D(H)$.

Also for $G \neq K_n$ and $G \neq H$ it is seen that the inequality for the relation between energy and distance energy becomes $E_D(G) - E_D(H) > E(G - E(H)) < E_D(G) + E_D(H)$.

Theorem 5.4: Let G be a simple digraph of order n with vertex set $V(G)$ $\{v_1, v_2, v_3, ..., v_n\}$ and arc set $\Gamma(G) \subset V(G) \times V(G)$. That is we have $(v_i, v_j) \notin \Gamma(G)$ for all i, and $(v_i, v_j) \in \Gamma(G)$ implies that $(v_i, v_j) \notin \Gamma(G)$. Let H be an induced subgraph of G, which is also a simple digraph. Then $E_s(G) - E_s(H) < E_s(G - E(H)) < E_s(G) + E_s(H)$.

Proof: G is a connected simple digraph. H be an induced subgraph of G, which is also a digraph, which contains all edges of G joining two vertices of H. Let $G - H$ denote the digraph obtained from G deleting all vertices of H and all edges incident with H. Let $G - E(H)$ denote the digraph obtained from G by deleting all edges of H, but keeping all vertices of H. Let S(G) be the skew adjacency matrix of G. If G_1 and G_2 are two digraphs without common vertices, let $G_1 \oplus G_2$ denote the digraph with vertex set and the edge set $V(G_1) \cup V(G_2)$ and $E(G_1) \cup E(G_2)$ respectively.

Hence, $S(G_1 \oplus G_2) = S(G_1) + S(G_2)$

$$S(G) = \begin{pmatrix} S(H) & -X^T \\ X & S(G-H) \end{pmatrix} = \begin{pmatrix} S(H) & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -X^T \\ X & S(G-H) \end{pmatrix}.$$

X represents edges connecting H and $G - H$.

$$S(G) = \begin{pmatrix} S(H) & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -X^T \\ X & S(G-H) \end{pmatrix}.$$

$$S(G-E(H))=\begin{pmatrix}0 & -X^T\\ X & S(G-H)\end{pmatrix}$$

$$S(G)=\begin{pmatrix}S(H) & 0\\ 0 & 0\end{pmatrix}+S(G-E(H)).$$

By singular value inequality theorem

$$E_s(G)\le E_s(H)+E(G-E(H))$$

$$E_s(G)-E_s(H)\le E(G-E(H)) \ldots \text{i}$$

$$S(G-E(H))=S(G)+\begin{pmatrix}-S(H) & 0\\ 0 & 0\end{pmatrix}+\begin{pmatrix}0 & Y^T\\ Y & 0\end{pmatrix}.$$

By singular value inequality theorem,

$$E_s(G-E(H))\le E_s(G)+E_s(H)+E_s(Y)$$

$$E_s(G-E(H))<E_s(G)+E_s(H) \ldots \text{ii}$$

From i and ii, we have

$$E_s(G)-E_s(H)\le E_s(G-E(H))<E_s(G)+E_s(H).$$

This means $E_s(G)-E_s(H)<E_s(G-E(H))<E_s(G)+E_s(H)$.

Also the left equality holds when G = H.

Therefore, $E_s(G-E(H))=E_s(G)-E_s(H)$.

The left and the right equality holds when $H=\phi$, for which $E_s(H)=0$ and therefore we have $E_s(G-E(H))=E_s(G)+E_s(H)$.

We study this example to understand theorem 5.4

Let D be the directed path on four vertices with the arc set {(1, 2), (2, 3), (3, 4)}.

$$S(D)=\begin{pmatrix}0 & 1 & 0 & 0\\ -1 & 0 & 1 & 0\\ 0 & -1 & 0 & 1\\ 0 & 0 & -1 & 0\end{pmatrix}.$$

The characteristic polynomial of $S(D)$ is $\lambda^4+3\lambda^2+1$.

The eigenvalues of $S(D)$ are $\pm i\,\frac{\sqrt{5}+1}{2}$ and $\pm i\,\frac{\sqrt{5}-1}{2}$.

The skew energy of D is $E_s(D) = 2\sqrt{5}$.

Let H be the graph obtained by deleting the edge (3, 4).

The skew energy of H is found to be $2\sqrt{2}$.

The skew energy of D – E (H) is found to be 2.

It can be easily seen that $E_s(G) - E_s(H) < E_s(G - E(H)) < E_s(G) + E_s(H)$.

Let D be the directed cycle on 4 vertices with the arc set {(1, 2), (2, 3), (4, 3), (4, 1)}.

$$S(D) = \begin{pmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

The characteristic polynomial of S(D) is $\lambda^4 + 4\lambda^2 + 4$.

The eigenvalues of S(D) are and $\pm i\sqrt{2}$ and $\pm i\sqrt{2}$.

The skew energy of D is $E_s(D) = 4\sqrt{2}$.

Let H be the graph obtained by deleting the edge (4,1).

The skew energy of H is found to be 4.364.

The skew energy of D – E(H) is found to be 2.

It can be easily seen that $E_s(G) - E_s(H) < E_s(G - E(H)) < E_s(G) + E_s(H)$.

6. CONCLUSION

The singular value inequality can also be analyzed to study the relation between graph energy and distance energy which can be found for different types of graphs. This relation can also be used to find the changes in the energy and distance energy of a graph and also the skew energy of the digraph due to the deletion of the edges.

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