MATH230: Tutorial Two

Propositional Logic: Natural Deductions with Negation

Key ideas

- Write natural deduction proofs involving $\neg$ negation.
- Write natural deductions using Ex Falso i.e. $\perp$ elimination.

Relevant Topic: Propositional Logic
Relevant reading: L∃∀N Chapters 3,4 and Simon section 1.1

Hand in exercises: 1a, 1b, 1c, 1i, 1k
Due Friday @ 5pm to the submission box on Learn.

Discussion Questions

1. Show $A \vdash \neg\neg A$.

2. Show $(A \vee B) \wedge (A \vee C) \ \vdash \ A \vee (B \wedge C)$.

Tutorial Exercises

1. **Minimal Logic.** Provide natural deduction proofs of the following sequents. These deductions require only the use of minimal logic; the introduction and elimination rules for $\wedge$ conjunction, $\vee$ disjunction, $\rightarrow$ implication, and the definition of $\neg$ negation as an implication.

   (a) $\neg A \vdash (C \rightarrow A) \rightarrow \neg C$ [Handin exercise]

   (b) $A \wedge \neg B \vdash \neg(A \rightarrow B)$ [Handin exercise]

   (c) $A \rightarrow C, \quad B \rightarrow D, \quad \neg C \vee \neg D \vdash \neg A \vee \neg B$ [Handin exercise]

   (d) $A, \quad \neg A \vdash \neg B$

   (e) $A \rightarrow B, \ A \rightarrow \neg B \vdash \neg A$

   (f) $A \rightarrow \neg B \vdash B \rightarrow \neg A$

   (g) $\neg(A \wedge B) \vdash A \rightarrow \neg B$

   (h) $A \vdash \neg\neg A$

   (i) $\neg\neg\neg A \vdash \neg A$ [Handin exercise]

   (j) $\neg A \vee \neg B \vdash \neg(A \wedge B)$

   (k) $\neg A \wedge \neg B \vdash \neg(A \vee B)$ [Handin exercise]

   (l) $\neg(A \vee B) \vdash \neg A \wedge \neg B$

   (m) $A \rightarrow \neg B \vdash \neg(A \wedge B)$

   (n) $\vdash \neg\neg(A \vee \neg A)$ [Challenge!]

2. **Intuitionistic derivations.** Provide natural deduction proofs of the following. You do not need to use the *classical* $\perp$ rule for these questions, but may find that the *intuitionistic* $\perp$ rule is necessary.

   (a) $A, \neg A \vdash B$

   (b) $\neg A \vdash A \rightarrow B$

   (c) $\neg A \vee B \vdash A \rightarrow B$

   (d) $A \vee B, \quad \neg A \vdash B$

   (e) $\vdash \neg(B \rightarrow A) \rightarrow (A \rightarrow B)$

   (f) $A \rightarrow B, A \rightarrow \neg B \vdash A \rightarrow C$

   (g) $A \vee B, \neg A, \neg B \vdash C$