MATH230: Tutorial Six

Computation and Church Encodings

Key ideas
- Practice β-reduction,
- Encode logic in λ-calculus,
- Encode natural numbers in λ-calculus.
- Encode compound data in λ-calculus.

Relevant notes: Lambda Calculus Slides
Relevant reading: Type Theory and Functional Programming, Simon Thompson

Hand in exercises: 1b, 2, 3b, 4c, 5b
Due Friday @ 5pm to the submission box on Learn.

Discussion Questions

- Compute the normal forms of the following λ-terms:

  $(\lambda x.\ x\ x)(\lambda y.\ \lambda z.\ y\ z\ z)$

AND TRUE TRUE

Tutorial Exercises

1. Compute the normal form for each of the following λ-terms:

   (a) NOT FALSE

   (b) OR TRUE FALSE

   (c) AND FALSE TRUE

   (d) IMPLIES FALSE TRUE

   Only expand those expressions necessary for each step.

2. Write down λ-expressions that represent the propositional binary connectives XOR, NAND, and NOR. Recall that these have the following truth tables.

P	Q	XOR(P,Q)
T	T	F
T	F	T
F	T	T
F	F	F

P	Q	NAND(P,Q)
T	T	F
T	F	T
F	T	T
F	F	T

P	Q	NOR(P,Q)
T	T	F
T	F	F
F	T	F
F	F	T

3. By substituting the explicit λ-expressions (as necessary) and performing β-reduction, determine the normal forms of the following λ-expressions.

   (a) SUCC ONE

   (b) SUM ONE ZERO

   (c) MULT TWO ZERO

   Only expand those expressions necessary for each step.

4. We have defined the following λ-expression to construct pairs of λ-expressions:

$$\mathsf{PAIR} = \lambda x.\ \lambda y.\ \lambda f.\ f\ x\ y$$

   The third input is a built-in place ready to take a selector:

$$\mathsf{FirST} = \lambda x.\ \lambda y.\ x \qquad \mathsf{SecoND} = \lambda x.\ \lambda y.\ y$$

   Reduce these to normal form

   (a) PAIR a b FST

   (b) PAIR a b SND

   (c) PAIR (PAIR a b) (PAIR c d) SND

5. Positive rational numbers are solutions to equations of the form $bx = a$, where $a, b : \mathbb{N}$. Use PAIR to represent positive rational numbers in the λ-calculus and write λ-expressions to compute rational number arithmetic.

   (a) RAT-SUM to calculate the sum of two rational numbers.

   (b) RAT-MULT to calculate the product of two rational numbers.

   (c) RAT-REC(iprocal) to calculate the reciprocal of an integer.