MATH230: Tutorial Four

Natural Deductions in First Order Logic

Key ideas

- Read and interpret formulae of first-order logic.
- Write formulae of first-order logic.
- Natural deductions with $\forall$ $\exists$ rules.

Relevant Topic: First-Order Logic.
Relevant reading: Logic and Proof Sections 7,8, and 9.

Hand in exercises: 1a, 1b, 1i, 1j, 1n
Due Friday @ 5pm to the submission box on Learn.

Discussion Questions

1. $\forall x \neg Fx \dashv\vdash \neg \exists x Fx$

Tutorial Exercises

1. Prove the following in the predicate calculus. You will need only the minimal logic rules of inference along with the introduction and elimination rules for the quantifiers.

    (a) $\forall x(Fx \to Gx) \vdash \forall xFx \to \forall xGx$

    (b) $\forall x((Fx \vee Gx) \to Hx), \quad \forall x\neg Hx \vdash \forall x\neg Fx$

    (c) $\forall x(Fx \wedge Gx) \vdash \forall xFx \wedge \forall xGx$

    (d) $\forall xFx \wedge \forall xGx \vdash \forall x(Fx \wedge Gx)$

    (e) $\forall x(P \to Fx) \vdash P \to \forall xFx$

    (f) $P \to \forall xFx \vdash \forall x(P \to Fx)$

    (g) $\exists x(P \to Fx) \vdash P \to \exists xFx$

    (h) $\exists x\neg Fx \vdash \neg\forall xFx$

    (i) $\forall x\neg Fx \vdash \neg\exists xFx$

    (j) $\neg\exists xFx \vdash \forall x\neg Fx$

    (k) $\exists xFx \to P \vdash \forall x(Fx \to P)$

    (l) $\exists x(Fx \to Gx) \vdash \forall xFx \to \exists xGx$

    (m) $\forall xFx \vdash \neg\exists x\ \neg Fx$

    (n) $\exists xFx \vdash \neg\forall x\ \neg Fx$

    (o) $\forall x\ (Fx \to \neg Gx)\ \vdash\ \neg\exists x\ (Fx \wedge Gx)$

    (p) $\vdash\ \exists x\ (Fx \vee Gx) \leftrightarrow \exists x\ Fx\ \vee\ \exists x\ Gx$

2. Prove the following in the predicate calculus. Ex falso or classical modes of reasoning (RAA, LEM, DNE) will be helpful for proving these theorems. Each of these are challenging!

    (a) $\forall x\ (Fx \vee Gx),\ \forall x\ \neg Gx\ \vdash\ \forall x\ Fx$

    (b) $\neg\forall xFx \vdash \exists x\neg Fx$

    (c) $\neg\forall x\ \neg Fx \vdash \exists xFx$

    (d) $\neg\exists x\ \neg Fx \vdash \forall xFx$

Note: The exercises above show that in the presence of classical modes of reasoning it is sufficient to introduce only one of the quantifiers, as the other can be deduced from it. Sequents above show that we could define $\forall = \neg\exists\neg$ and $\exists = \neg\forall\neg$ in classical logic. Following the BHK, however, one needs to define both independently.