# Course
# Multipath Propagation and Diversity

**Homework Problems**

**2025–2026**

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## Problem 1

This problem presents a comparison of the performances of wired and wireless communications. In optical fiber communications, the bit error performance can be $p_e = 10^{-12}$. In wireless communication systems over fading channels and using DPSK modulation, the bit error performance is given by

$$P_e = \frac{1}{2(\overline{\gamma}+1)} \tag{1}$$

When $\overline{\gamma} \gg 1$,

$$P_e \approx \frac{1}{2\overline{\gamma}} \tag{2}$$

1. Find the SNR giving the performance of $P_e = 10^{-12}$.
2. Is it easy to have such an SNR in wireless communications?
3. Explain why.

## Problem 2

Consider a radio link over Rayleigh channel with an average SNR of 20 dB. As is known, the PDF of the channel power gain $|h|^2 = R^2 = \Omega$ is given by

$$p_\Omega(\Omega) = \frac{1}{\overline{\Omega}} \exp(-\Omega/\overline{\Omega}) \tag{3}$$

1. Find the PDF of the fading process $R$.
2. What is the joint PDF of R and the channel phase $\vartheta$?
3. Determine the SNR outage probability of the radio link when the SNR threshold is 10 dB below the average SNR.

## Problem 3

In indoor communications the RMS delay spread can be $\sigma_\tau = 50$ ns, while in outdoor communications it can reach $\sigma_\tau = 30$ $\mu$s.

1. Find the data rates for both environments so that the channels can be considered as narrowband channels. The ISI can be ignored when $T_s \geq 10\sigma_\tau$.
2. Compare the rates with those of the available systems.

## Problem 4

1. Find a channel model for an airplane-to-airplane radio link with a line-of-sight (LoS) link and a scattering path via ground reflections. The delay between the two paths is $\tau = 10$ $\mu$s, and the information signal bandwidth is $B_s = 100$ kHz.
2. What would be the model in case of an information signal of $B_s = 10$ kHz?

## Problem 5

1. Find a model for a two-path fading channel with a delay $\tau = 1$ ms. We consider a symbol duration given by $T_s = 0.1$ ms for:
   (a) Rayleigh fading scenario with envelope PDF $p_R(z) = \frac{z}{\sigma_i^2}\exp(-z^2/(2\sigma_i^2))$, where $\sigma_1^2 = 2\sigma_2^2$,
   (b) Rice fading scenario with Rice factor $k = A^2/(2\sigma^2)$.

## Problem 6

Consider a wireless communication system in which 4 users (on the cell edge) are communicating simultaneously with the BS of the cell. The channels are iid Rayleigh fading processes with a common mean SNR given by 10 dB. The SNR threshold is 3 dB below the mean SNR. The system uses DPSK modulation.

1. Find the bit error performance of the radio links.
2. What is the probability to have the 4 radio links in no-outage?

## Problem 7

Consider the path loss model given by

$$L_p(d) = \frac{d^4}{h_{Tx}^2 h_{Rx}^2} \tag{4}$$

1. What is the propagation exponent of this path loss model?
2. Give $L_p(d)$ in dB where $d$ in km and $h_{Tx}$ and $h_{Rx}$ are in m.
3. Compare the present model with the free space path loss model.

## Problem 8

Consider the downlink ( transmission from base station to mobile station) of a mobile communication system operating under free space propagation. The transmit power is assumed to be 10 dBm, and the receiver sensitivity is -90 dBm. The gain of the transmit antenna is 30 dB and that of the receive antenna is 0 dB. Also, the carrier frequency is $F_c = 4$ GHz, the mean power of the receiver noise -110 dBm, and the noise figure of the LNA is $NF = 5$ dB.

1. What is the radio coverage distance of the system?
2. Find the signal bandwidth.
3. Determine the threshold signal-to-noise ratio for a user located at the edge of the cell.
4. Deduce the capacity (the data rate) of the radio link.
5. The BS is equipped with a dipole array antenna. The gain of a dipole antenna is 1.64 (2.15 dBi). Find the number of dipole elements used (the array gain of the antenna array).
6. Deduce the approximate dimension of the array antenna.
7. Sketch the connection between the power amplifier and the antenna array.

## Problem 9

It is well known that diversity techniques are useful in limiting the effect of multipath fading. We consider the receive diversity system of block diagram shown in Fig. 1. The complex gain of the fading channel is described by $h_i = r_i \exp(j\vartheta)$, where $r_i = \sqrt{-2\sigma^2 \ln(u_i)}$ with $u_i$ being uniformly distributed over $[0, 1]$. The channel phase is uniformly distributed over $[0, 2\pi[$. The fading envelopes $r_1$ and $r_2$ are assumed to be statistically independent with a common mean given by $\mathrm{E}(r_i) = \sigma\sqrt{(\pi/2)}$.

1. Find the PDF of the fading envelope $r_i$.
2. What is the type of fading experienced in the present case of multipath propagation.
3. What is the PDF of $h_I = r_1 \cos\vartheta_1$ and that of $h_Q = r_2 \sin\vartheta_2$.
4. Find the signal available at the combiner output.
5. Find the expression of the signal-to-noise ratio (SNR) available at the output of the diversity combiner.
6. What is the average SNR?
7. Deduce the array gain of the diversity system.

## Problem 10

This problem studies the outage statistics in co-channel interference limited systems, where the effect of additive noise can be neglected. We consider the scenario illustrated in Fig. 2, where the mobile station (MS) receives (over the same frequency) the useful signal from the base station 1 (BS1) and the interfering signal from the base station 2 (BS2). Each of the radio signal undergoes a path loss and a shadow fading. The path loss, at a distance $d$, is given $L_p(d) = 50\mathrm{l}og_{10}(d)$ and the shadowing, in decibels (dB), is statistically described by a Gaussian random variable of zero mean and variance $\sigma^2$ in dB. The transmit power of the BS$_i$ is $P_{T_i}$, $i = 1, 2$.

1. Give, in dB, the power $C$ of the useful signal and the power $I$ of the interfering signal received by the MS. Both the path loss and the shadowing have to taken into account. The shadow fading affecting the useful signal can be denoted by $X_1$ and that affecting the interfering signal is $X_2$.
2. Find the probability density function (PDF) of $C$ and that of $I$.
3. Determine, in dB, the signal-to-interference ratio (SIR). The SIR can be denoted by $\eta$.
4. What is, in dB, the average SIR?
5. Give the PDF of $\eta$.
6. Determine the outage probability caused by the co-channel interference for a threshold $\eta_{th}$.

## Problem 11

The PDF of the Rice fading envelope is given by

$$p_R(z) = \frac{z}{\sigma^2}\exp(-\frac{A^2 + z^2}{2\sigma^2})I_0(\frac{Az}{\sigma^2}) \tag{5}$$

1. Show that
$$p_R(z) = \frac{2z(k+1)}{\overline{P_{Rx}}}\exp(-k - \frac{(k+1)z^2}{\overline{P}_{Rx}})I_0(2z\sqrt{\frac{k(k+1)}{\overline{P}_{Rx}}}) \tag{6}$$
where $\overline{P}_{Rx} = A^2 + 2\sigma^2$ and $k = A^2/(2\sigma^2)$
2. Find the PDF of the channel power gain.
3. Deduce the PDF of the channel power gain of Rayleigh fading.

## Problem 12

In this problem, we show through a simple example that the MGF can used in investigating the BER performance of radio links.

1. Find the MGF $M_\gamma(t)$ of the SNR $\gamma$ in Rayleigh fading channels.
2. Verify that for the DBPSK modulation, the bit error rate is given $P_b(E) = \frac{1}{2}M_\gamma(-1)$.

## Problem 13

Consider a multipath fading channel where the fading envelope is statistically characterized by the following PDF:

$$P_R(z) = 2(\frac{m}{\overline{\Omega}})^m \frac{z^{2m-1}}{\Gamma(m)} \exp(-\frac{m}{\overline{\Omega}} z^2) \tag{7}$$

where $\overline{\Omega} = \mathrm{E}(R^2)$, $m$ is a parameter taking values over $[0.5, \infty)$ and $\Gamma(\cdot)$ is the Gamma function.

1. Check that for $m = 1/2$, $P_R(z)$ corresponds to the one-sided Gaussian PDF ($\Gamma(1/2) = \sqrt{\pi}$).
2. Verify that for $m = 1$, $P_R(z)$ reduces to the Rayleigh PDF ($\Gamma(1) = 1$).
3. Find the PDF of the instantaneous SNR $\gamma$.
4. Deduce the BER of DBPSK modulation.
5. Determine the MGF of $\gamma$.

## Problem 14

The Doppler frequencies are defined by $f = f(\alpha) = f_{max} \cos(\alpha)$, where $\alpha$ are the angles-of-arrival of the waves wrt the direction of motion of the receiver, and $f_{max}$ stands for maximum Doppler frequency given $f_{max} = vF_c/c$. Here, $v$ is the speed of the mobile receiver, $F_c$ is the carrier frequency, and $c$ is the speed of light.

1. Find $f_{max}$ for $F_c = 1$ GHz and $v = 10.8$ km/h. Deduce the coherence time of the corresponding channel.
2. Repeat the above question for $v = 108$ km/h.
3. What conclusion can be drawn from 1) and 2) regarding the time variability of the fading channel?
4. Since the angles-of-arrival are random variables, it follows that the Doppler frequencies are also random variables. Derive the PDF of $f$ for the case where $p_\alpha(\alpha) = 1/2\pi$, with $\alpha \in [0, 2\pi)$.

## Problem 15

The SNR of a received OFDM signal, impacted by the non-linear power amplifier at the transmitter side, can be described by the following quantity,

$$\gamma = K \frac{\Omega}{\Omega \sigma_d^2 + \sigma_w^2} \tag{8}$$

where $\Omega$ is a random variable standing for the power gain of the multipath fading channel, and $K$, $\sigma_2$, and $\sigma_w$ are real constants. It is assumed that $\Omega$ has an exponential distribution with mean $\overline{\Omega}$.

1. What is the channel model assumed in this problem?
2. Give the complex gain of the underlying channel, along with the statistical parameters.
3. Find the PDF of the SNR $\gamma$.

## Problem 16

We consider a propagation environment where the complex gain of the fading channel at symbol $n$ can be approximated be the first-order Gauss-Markov process according to

$$h[n] = \sqrt{1-\kappa}h[n-1] + \sqrt{\kappa}w[n] \tag{9}$$

where $w \sim CN(0,1)$. It can be shown that the auto-correlation function is given by

$$\Gamma_{hh}[m] = E\{\, h[n]h^*[n+m]\} = (1-\kappa)^{m/2} \tag{10}$$

We want to simulate a fading channel using the above model to evaluate the performance of a radio link transmitting at the speed of 0.4 Msps with a correlation, over the time interval of 1 ms, given by 0.5.

1. Find the number of symbols transmitted in the time interval of 1 ms.
2. Deduce the value of $\kappa$ that should be used in the simulator.

## Problem 17

You are working on the design of a cellular system using two-branch receive diversity system operating over a narrow fading channel for which the SNRs $\gamma_1$ and $\gamma_2$ are independent and distributed uniformly over the interval [0, $\gamma_{max}$].

1. Which of the following distortions: Doppler spread, delay spread, multipath fading, can be mitigated by making use of the underlying diversity system?
2. Find the probability density function of the SNR available at the combiner output in case MRC diversity is used.
3. Find the outage probability of the radio link for a threshold $\gamma_{th}$ ($\gamma_{th} < \gamma_{max}$).
4. Compare the result with that corresponding to the reception without diversity.
5. Find the PDF of the real envelope, $R$, of the fading channel.
6. Propose a method for the simulation of $R$.

## Problem 18

In OFDM transmission, a coherence block is defined by the coherence time and coherence bandwidth of the transmission channel. During this time-frequency block, the channel remains approximately constant in the time and frequency domains. The number of symbols transmitted per block is given by

$$N_c = \frac{T_c}{T_{OFDM}} \frac{B_c}{1/T_{OFDM}} = T_c B_c. \tag{11}$$

where $T_c$ is the coherence time and $B_c$ stands for the coherence bandwidth.

1. Find $N_c$ for a mobile receiver with speed $v$=80 km/h and a rms delay spread of $\tau_{max} = 4$ $\mu$s. The carrier frequency is $F_c$=2 GHz.

2. What is the value of $N_c$ if the speed of the receiver is $v$=4 km/h?
3. What is the impact of the speed on the channel estimation period?

## Problem 19

In EGC and MRC receive diversity, the instantaneous SNR can generically be written as

$$\gamma_{EGC/MRC} = \frac{E_s}{N_0\sqrt{L^{1-p+q}}}\left(\sum_{i=1}^{L} R_i^p\right)^q \tag{12}$$

where $L$ is the number of branches, $E_s/N_0$ is the transmitted SNR per symbol, and the values of the parameters $p$ and $q$ depend on the diversity combiner. The correlation coefficient of the envelope $R_i$ is given by

$$\rho(\tau) = \text{sinc}^2(2\pi f_{max}\tau) \tag{13}$$

where $f_{max}$ is the maximum Doppler frequency.

1. Give the values of $p$ and $q$ corresponding to the EGC diversity.
2. Give the values of $p$ and $q$ corresponding to the MRC diversity.
3. What is the minimum separation between the antennas to have uncorrelated diversity branches?

## Problem 20

Consider the wide sense stationary uncorrelated scattering (WSSUS) large band fading channel described by Figure 3. The complex gain corresponding to the path $n$ is given by

$$h_n = \sqrt{\frac{1-\alpha}{1-\alpha^N}}\alpha^{n/2}R_n\text{exp}(j\phi_n) \tag{14}$$

where $0 < \alpha < 1$ and $N$ is the total number of paths. Also, $\phi_n$ is uniformly distributed over $[0, 2\pi[$, and $R_n$ is a random variable having the PDF given by

$$P_R(z) = 2(\frac{m}{\sigma^2})^m\frac{z_n^{2m-1}}{\Gamma(m)}\exp(-\frac{m}{\sigma^2}z_n^2) \tag{15}$$

where $\sigma^2 = \text{E}(R^2)$, $m$ is a parameter taking values over $[0.5, \infty)$ and $\Gamma(\cdot)$ is the Gamma function. The random variables $R_n$ and $\phi_n$ are considered to be statistically independent.

1. What is a WSSUS fading model?
2. Explain the propagation scenario based on the ellipses model.
3. Find the PDF of $R_n$ and that of $\phi_n$.
4. Determine the average power gain per path.
5. Sketch the power delay profile of the channel.
6. Find the overall average power received through the underlying channel.

7. Calculate the RMS delay spread for $N = 2$.
8. Find the PDF of $\Omega_n = |h_n|^2$.
9. Check that $\Gamma(m) = \int_0^{\infty} t^{m-1}\text{exp}(-t)dt$.
10. In the following, we consider the particular case of narrow-band fading channel with $m = 1/2$ ($\Gamma(1/2) = \sqrt{\pi}$).
    (a) Explain the modeling of the propagation scenario based on the ellipses model.
    (b) Find the PDF of the instantaneous SNR per symbol defined by $\gamma = \Omega E_s/N_0$.
    (c) Determine the outage probability for a receiver characterized by a threshold $\gamma_{th}$.
    (d) What is the multipath fading margin?
    (e) How the envelope of the fading channel can be simulated?