The inclusion-exclusion formula. Let us express the event $B = \cup_{k=1}^n A_k$ in terms of the indicator random variables $X_1, \ldots, X_n$, where $X_k = 1$ if event A_k occurs and $X_k = 0$ if event A_k does not occur. The event B^c occurs when all of the random variables $X_1, \ldots, X_n$ are zero, which happens when the random variable $Y = (1 - X_1)(1 - X_2) \cdots (1 - X_n)$ is equal to 1. Note that Y can only take values in the set $\{0, 1\}$, so that $\mathbf{P}(B^c) = \mathbf{P}(Y = 1) = \mathbf{E}[Y]$. Therefore,

$$\begin{aligned}
\mathbf{P}(B) &= 1 - \mathbf{P}(B^c) \\
&= 1 - \mathbf{E}\big[(1 - X_1)(1 - X_2) \cdots (1 - X_n)\big] \\
&= \mathbf{E}[X_1 + \cdots + X_n] - \mathbf{E}\left[\sum_{i_1 < i_2} X_{i_1} X_{i_2}\right] + \cdots + (-1)^{n-1} \mathbf{E}[X_1 \cdots X_n].
\end{aligned}$$

We note that

$$\begin{aligned}
\mathbf{E}[X_i] &= \mathbf{P}(A_i), \\
\mathbf{E}[X_{i_1} X_{i_2}] &= \mathbf{P}(A_{i_1} \cap A_{i_2}), \\
\mathbf{E}[X_{i_1} X_{i_2} X_{i_3}] &= \mathbf{P}(A_{i_1} \cap A_{i_2} \cap A_{i_3}), \\
\mathbf{E}\big[X_1 X_2 \cdots X_n\big] &= \mathbf{P}(\cap_{k=1}^n A_k),
\end{aligned}$$

etc., from which the desired formula follows.