

15. Exercise: Gambler's ruin

Exercise: Gambler's ruin

5/5 points (ungraded)

Mary loves gambling. She starts out with \$200 and keeps playing rounds of the same game. For each round, she can bet either \$100 or \$200 (assuming she has sufficient funds) and wins with probability p . Assume that whether she wins is independent across rounds and is unaffected by the size of her bet.

If she wins, she receives back double what she bet, and if she loses, she receives back nothing (i.e., she loses the amount she bet). Assume that Mary stops when she either runs out of money or has reached \$400 or more, whichever comes first. What is her optimal betting strategy? Here, "optimal" means the strategy that gives her the greatest probability of reaching \$400 or more, and "strategy" means a rule saying how much she should bet when she has \$100, \$200, and \$300 (the amount she bets need not be the same in these three cases).

1. What is Mary's optimal strategy when she has \$300?

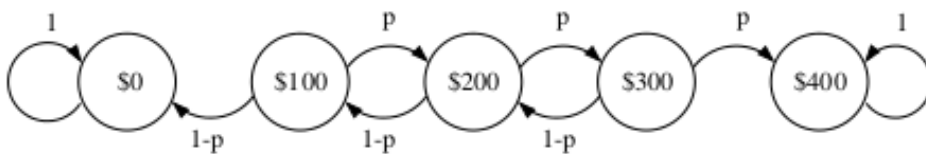
☒ Always bet \$100 ✓

☐ Always bet \$200

☐ It depends on p

2. What is Mary's optimal strategy when she has \$200?

Hint: For this question, the evaluation of the options can be analyzed using the following transition probability graph:



☐ Always bet \$100

☐ Always bet \$200

☒ Bet \$100 only when $p > 1/2$ ✓

☐ Bet \$100 only when $p > 1/3$

☐ None of the above

Solution:

- When Mary has \$300, she should bet \$100. Whether she bets \$100 or \$200, she will meet her goal if she wins the next round. If she bets \$100 and loses, she will then have \$200. If she bets \$200 and loses, she will then have \$100. Everything else being equal, it is more advantageous to have \$200 than to have \$100.
- The more difficult decision is how much to bet when Mary has \$200. We will investigate both possible strategies and decide which is preferable.

First, Mary can bet \$200. This case is easy to analyze. She will either reach her target \$400 or lose all her money on the next round. Therefore, the probability that she will reach her target if she bets \$200 is p .

Second, Mary can bet \$100. In this case, Mary will not immediately reach her target or lose her money, so more analysis is required. As an aid, we use the state transition diagram given in the hint. It is clear that when Mary has \$100, she must bet \$100 because she cannot bet \$200. Hence, the possible transitions from state \$100 are to \$0 and \$200. From part (1), we know that her optimal bet when she has \$300 is \$100. Hence, the possible transitions from state \$300 are to \$200 and \$400. Reaching \$400 and \$0 are the two absorbing states.

We want to find the probability of eventually reaching the \$400 absorbing state, given that she starts with \$200. We will denote the probability that Mary reaches \$400 given that she starts with j hundred dollars by a_j . We have the following system of equations:

$$\begin{aligned}a_1 &= a_2 p \\a_2 &= a_1(1 - p) + a_3 p \\a_3 &= a_2(1 - p) + p.\end{aligned}$$

Solving, we obtain $a_2 = \frac{p^2}{1 - 2p + 2p^2}$.

We need to compare p and a_2 to decide whether to bet \$100 or \$200 when she has \$200. Betting \$200 is better when

$$\begin{aligned}p &> a_2 = \frac{p^2}{1 - 2p + 2p^2} \\1 - 2p + 2p^2 &> p \\(1 - 2p)(1 - p) &> 0,\end{aligned}$$

which implies $1 - 2p > 0 \Rightarrow p < 1/2$. On the other hand, betting \$100 is better when $p > 1/2$. When $p = 1/2$, the two bets give the same probability of reaching \$400.

For an intuitive explanation of the result, consider the case where p is small. If Mary bets \$100, she would need to win at least 2 rounds in order to reach \$400, which is relatively unlikely even accounting for the fact that she can move back and forth between \$100, \$200, and \$300. Hence, it is better just to go “all in” by betting \$200 and count on winning a single round.

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i Answers are displayed within the problem

讨论

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主题： Unit 10 / Lec. 26 / 15. Exercise: Gambler's ruin

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Kelly Criterion and Betting Size to Avoid Gambler's Ruin

discussion posted 3 days ago by [markweitzman](#) (Community TA)

There is a general rule well known to professional gamblers and traders/investors known as the Kelly criterion. For details see:

[KELLY CAPITAL GROWTH INVESTMENT CRITERION, THE: THEORY AND PRACTICE \(World Scientific Handbook in Financial Economics\)](#) by [Leonard C. MacLean](#) (Editor), [Edward O. Thorp](#) (Editor), [William T. Ziemba](#) (Editor)

[Fortune's Formula: The Untold Story of the Scientific Betting System That Beat the Casinos and Wall Street](#) Paperback – September 19, 2006 by [William Poundstone](#) (Author)

Basically the Kelly criterion is simply bet a percentage of your bankroll equal to your edge. So if you are a blackjack card counter and you believe your edge is 1%, and you have **\$10000**, and you wish to seek the maximum probability of doubling your bankroll to **\$20000**, bet 1% of your bankroll on each bet i.e start out with **\$100** bets and increase proportionally to maintain 1% of your bankroll as you win or decrease as you lose. Of course for the game of blackjack this is only an approximation as the total bet is unknown when you first make it as you may double or split etc.

As a well known professional poker player with a mathematical background, many poker players have come up to me asking me about how high they should play relative to their bankroll. My experience has been that most professional poker players (or professional gamblers in general) overplay their bankrolls and often go broke despite having an overall edge. Why is this so? The answer lies in another interesting aspect of the Kelly Criterion. For although the optimum betting amount is equal to your edge, the flip side is (and this can be shown mathematically) if you bet an amount equal to twice your edge, you are certain to go broke. So in the blackjack example above if you start off at **\$200** bets and maintain proportionally, you are certain to be ruined, no matter how far ahead you get.

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So this is the problem most professional gamblers face. It is hard to estimate your edge, you can do computer simulations etc., but it is difficult to take into account all possibilities including mistakes, potential cheating etc. Thus if you think you have a 1% edge and follow the Kelly criterion by betting 1% of your bankroll, but it turns out that your edge is only 0.5%, then you are certain to go broke. So my advice to all professional gamblers is to estimate your edge, and be conservative, and bet half the edge. This way even if your estimate is wrong on the high side, you will avoid ruin.

Finally, if you have a negative edge as most casual gamblers do, the correct betting strategy is to bet the max at every opportunity. The longer you draw out the process, the more likely the casino edge will prevail and get all your money. Of course no one likes to travel thousands of miles to Las Vegas, bet it all and lose and take the next flight home. So you must compromise between maximizing your chance of winning versus the entertainment value of gambling.

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1 response

markweitzman (Community TA)

5 days ago

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As a professional gambler, I will give you general gambling advice, when you have an edge use the Kelly criteria, but when you have a negative expectation, bet as large as you can, the longer you play and the more bets you make, the more the expected value of your loss.

what is "edge" here?

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Muktinath 在about 3 hours ago前发表

Advantage, or % by which game is in your favor.

...

markweitzman (Community TA) 在about an hour ago前发表

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