Neural Networks and Biological Modeling

Professor Wulfram Gerstner
Laboratory of Computational Neuroscience

Question set 12

Exercise 1: Stochastic input

Consider a passive membrane receiving a stochastic input current which is zero for time $t < 0$ and

$$RI(t) = RI_0 + \xi(t), \text{ for } t > 0, \tag{1}$$

where $\xi(t)$ is white noise with

$$\langle \xi(t) \rangle = 0, \tag{2}$$

$$\langle \xi(t)\xi(t') \rangle = \tau_m a^2 \delta(t - t'). \tag{3}$$

The membrane potential obeys the equation

$$\frac{du(t)}{dt} = -(u - u_{rest}) + RI(t), \tag{4}$$

with solution

$$u(t) = u_{rest} + \frac{R}{\tau_m} \int_0^t \exp(-\frac{s}{\tau_m}) I(t - s) ds \tag{5}$$

1.1 Calculate the expected voltage $\langle\, u(t) \,\rangle$, where $\langle\ \rangle$ is the average over multiple repetitions or over a population of neurons having the same dynamics and inputs.

1.2 Calculate the variance of the potential across multiple repetitions: $Var[u](t) = \left\langle\, [u(t) - \langle\, u(t) \,\rangle]^2 \,\right\rangle$.

Exercise 2: Diffusive noise (stochastic spike arrival)

Consider a passive membrane receiving stochastic synaptic input $S(t) = \sum_f \delta(t - t_k^f)$, where the index f runs over the firing times of a presynaptic neuron. The spike train starts only at t = 0, so that $t_k^f > 0$ for all firing times. The membrane potential obeys the equation:

$$\frac{du}{dt} = -\frac{u - u_{\text{rest}}}{\tau} + \frac{q}{C} S(t)\,. \tag{6}$$

where q is the charge brought by each spike and C is the capacitance of the membrane. The solution to this equation writes:

$$u(t) = u_{rest} + \frac{qR}{\tau} \int_0^t \exp(-s/\tau) S(t - s) ds \tag{7}$$

2.1 Calculate the expected voltage $\langle\, u(t) \,\rangle$ as a function of t for a constant presynaptic rate $\langle\, S(t) \,\rangle = \nu$ for $t \geq 0$ ($\nu = 0$ for $t < 0$). Where $\langle\ \rangle$ is the average over multiple repetitions or over a population of neurons having the same dynamics and inputs.

2.2 Calculate $\left\langle u(t)^2 \right\rangle$. Assume that the spike times of the presynaptic neuron are uncorrelated, i.e., $\langle S(t)S(t') \rangle = \nu\delta(t-t') + \nu^2$, and use Eq. 7.

2.3 Calculate the variance of the potential across multiple repetitions: $Var[u](t) = \left\langle [u(t) - \langle u(t) \rangle]^2 \right\rangle$.

Homework:

2.4 Calculate the autocorrelation of the voltage $\langle u(t)u(t') \rangle$ in the steady state regime (replace the upper bound of the integral by ∞ in equation 7).

2.5 Suppose that there are two presynaptic neurons which fire independently with rates $\nu_1 = \langle S_1(t) \rangle$ and $\nu_2 = \langle S_2(t) \rangle$, such that the input to the postynaptic neuron is given by $w_1 S_1(t) + w_2 S_2(t)$ where w_i denote the synaptic weights. Calculate again the mean and autocorrelation of the voltage.

2.6 Redo question 2.5 with correlated spike trains $S_1(t) = S_2(t)$.

Exercise 3: Firing statistics

Consider a stochastic spike generation process in discrete time. The probability of generating a spike in a time Δt is $P_{\Delta t} = \nu \Delta t$. Hence when we take the limit of Δt to 0 the expected value of the quantity $S(t) = \sum_f \delta(t - t_k^f)$ is:

$$\langle S(t) \rangle = \lim_{\Delta t \to 0} \frac{P_{\Delta t}(t)}{\Delta t} = \nu \; ; \; \text{for } t > 0.$$

Consider the probability of having two spikes in different time bins around t and t'. Define $\langle S(t)S(t') \rangle$ in a similar fashion, and show that it is equal to $\nu\delta(t-t') + \nu^2$.

Exercise 4: Renewal process

We consider a neuron with relative refractoriness. Given an output spike at time $\hat{t}$, the probability of firing is given by

$$\rho(t-\hat{t}) = \begin{cases} 0 & \text{for } t - \hat{t} < t_{\text{abs}} \\ [t - \hat{t} - t_{\text{abs}}]\frac{\rho_0}{2} & \text{for } t_{\text{abs}} < t - \hat{t} < t_{\text{abs}} + 2 \\ \rho_0 & \text{otherwise.} \end{cases} \tag{8}$$

Calculate the survivor function and the interval distribution.