

8. Example: Assessing the

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8. Example: Assessing the performance of planes Setup:

An aerospace manufacturing company would like to assess the performance of its existing planes for its latest design. Based on a sample size of n=1000 flights, each with an identically designed plane, it collects data of the form $(x_1,y_1),\ldots,(x_{1000},y_{1000})$, where x represents the distance traveled and y represents liters of fuel consumed.

You, as a statistician hired by the company, decide to perform linear regression on the model y = a + bx to predict the efficiency of the design. In the context of linear regression, recall that the mathematical model calls for:

$$\mathbf{Y} = \left(egin{array}{c} y_1 \ dots \ y_{1000} \end{array}
ight) \in \mathbb{R}^{1000}, \quad oldsymbol{arepsilon} \in \mathbb{R}^{1000}, \quad \mathbb{X} = \left(egin{array}{cc} 1 & x_1 \ dots & dots \ 1 & x_{1000} \end{array}
ight) \in \mathbb{R}^{1000 imes 2}, \quad oldsymbol{eta} = \left(egin{array}{c} a \ b \end{array}
ight) \in \mathbb{R}^2.$$

Assume that $m{arepsilon} \sim \mathcal{N}\left(0, \sigma^2 I_{1000}
ight)$ for some fixed $m{\sigma}^2$, so that $\mathbf{Y} \sim \mathcal{N}\left(\mathbb{X}m{eta}, \sigma^2 I_{1000}
ight)$.

Prediction using Regression

1/1 point (graded)

Using the setup as above, you compute the LSE, which comes out to

$$\hat{oldsymbol{eta}} = \left(egin{array}{c} \hat{a} \ \hat{b} \end{array}
ight) = \left(egin{array}{c} 0.8 \ ext{liters} \ 15.0 \ ext{liters} \ / \ ext{km} \end{array}
ight).$$

Just from $\hat{\beta}$, what is a reasonable prediction for the total amount of fuel a plane (in liters) consumes after 200 kilometers?

3000.8

✓ Answer: 3000.8

Solution:

The LSE gives the "best-fitting" model y=0.8+15x. Plugging in x=200 gives y=3000.8.

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You have used 1 of 3 attempts

Answers are displayed within the problem

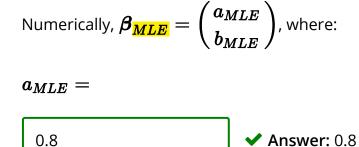
The Maximum Likelihood Estimator

2/2 points (graded)

Using the same setup as the previous problem:

$$\hat{oldsymbol{eta}} = egin{pmatrix} \hat{a} \ \hat{b} \end{pmatrix} = egin{pmatrix} 0.8 ext{ liters} \ 15.0 ext{ liters} / ext{km} \end{pmatrix}.$$

Using n=1000 samples, by thinking of \mathbf{Y} as the vector of observations, we might also consider the Maximum Likelihood Estimator eta_{MLE} . As a reminder, $m{\beta}_{MLE}$ maximizes, over all choices of $m{\beta}$, the likelihood (or the log-likelihood) of $\mathbf{Y} \sim \mathcal{N}\left(\mathbb{X} m{\beta}, \sigma^2 I_{1000}\right)$.



 $b_{MLE} =$

15 **✓ Answer:** 15.0

Solution:

This scenario is the homoscedastic gaussian case, so the MLE coincides with the LSE. Therefore, $a_{MLE}=\hat{a}=0.8$ and $b_{MLE}=\hat{b}=15.0$.

To see why, recall that $\mathcal{N}\left(\mathbb{X}eta,\sigma^{2}I_{1000}
ight)$ follows the density

$$f(\mathbf{Y}) = rac{\exp\left(-rac{1}{2}(\mathbf{Y} - \mathbb{X}eta)^T (\sigma^2 I)^{-1} \left(\mathbf{Y} - \mathbb{X}eta
ight)
ight)}{\sqrt{\left(2\pi
ight)^{1000} \det\left(\sigma^2 I
ight)}}$$

so the log-likelihood function can be written

$$\log f\left(\mathbf{Y}
ight) = -500\log\left(2\pi\sigma^2
ight) - rac{1}{2\sigma^2}(\mathbf{Y} - \mathbb{X}eta)^T\left(\mathbf{Y} - \mathbb{X}eta
ight)$$

The vector $oldsymbol{eta}$ only appears in the second term, so maximizing this expression is the same as minimizing

$$\left(\mathbf{Y} - \mathbb{X}\beta\right)^T \left(\mathbf{Y} - \mathbb{X}\beta\right) = \left\|\mathbf{Y} - \mathbb{X}\beta\right\|^2$$

which, by definition, is attained by the least-squares estimator $\hat{oldsymbol{eta}}.$

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