5. Quadratic Risk of Estimators

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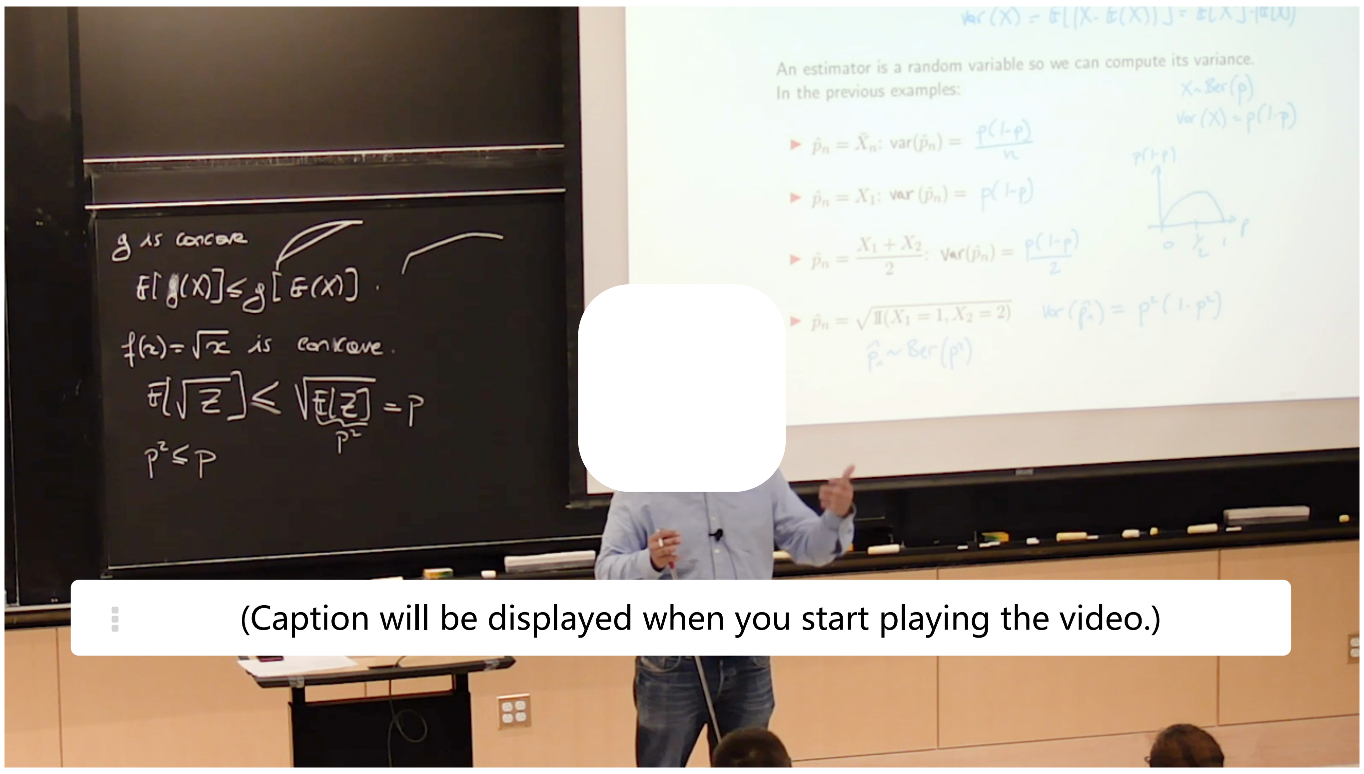

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OK.

So now I have two things.

I have the bias and I have the variance.

And I made things easy for you, because the bias was always

the same.

It was 0000 except for the last one, which is this [INAUDIBLE]..

But I had the bias 0.

So once vou said that the bias is 0.

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Find the Quadratic Risk

1/1 point (graded)

Let $\hat{\theta}_n$ denote an estimator for a true parameter θ. The **quadratic risk** of $\hat{\theta}_n$ is defined to be

$$\mathbb{E}\left[(\hat{\theta}_n - \theta)^2\right].$$

As in the previous problem on variance, let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathcal{U}([a, a+1])$ where a is an unknown parameter. What is the quadratic risk of the estimator $\overline{X}_n - \frac{1}{2}$?

Quadratic risk : 1/(12*n) ✔ **Answer:** 1/(12*n)

$\frac{1}{12\cdot n}$

Solution:

Recall that

$$\text{quadratic risk} = \text{variance} + \text{bias}^2.$$

We showed in a previous question that this estimator is unbiased. Also note that $\mathrm{Var}\left(\overline{X}_n\right) = \mathrm{Var}\left(\overline{X}_n - \frac{1}{2}\right) = \frac{1}{12n}$. Hence, the quadratic risk is also $\frac{1}{12n}$.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

Properties of Estimators

0/1 point (graded)

Let $\hat{\theta}_n$ denote an estimator for a true parameter θ. Here n denotes the sample size. Which of the following properties of $\hat{\theta}_n$ would ensure that $\hat{\theta}_n$ converges in probability to θ as $n \to \infty$? (Choose all that apply.)

- [ ] $\hat{\theta}_n$ is consistent. ✔
- [ ] $\hat{\theta}_n$ is unbiased.
- [x] The quadratic risk of $\hat{\theta}_n$ goes to 0 as $n \to \infty$. ✔
- [ ] The variance of $\hat{\theta}_n$ goes to 0 as $n \to \infty$.

✖

Solution:

The first choice is correct, because by definition, consistency implies that the estimator $\hat{\theta}_n \to \theta$ as $n \to \infty$. The third choice, "The quadratic risk of $\hat{\theta}_n$ goes to 0 as $n \to \infty$.", is correct because if the quadratic risk $\mathbb{E}\left[(\hat{\theta}_n - \theta)^2\right] \to 0$ then $\hat{\theta}_n \to \theta$ in L^2. By the properties of convergence, this implies that $\hat{\theta}_n \to \theta$ in probability.

Recall: Refer to Chapter 1 to review the relationship between the different types of convergence.

The second choice, "$\hat{\theta}_n$ is unbiased.", is incorrect. We give an example that shows that this choice is incorrect. Note that if $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, 1)$, then $\hat{\theta}_n := X_1$ is an unbiased estimator for μ because $\mathbb{E}[X_1] = \mu$. However, it is not consistent: $X_1 - \mu$ does not tend to 0 as $n \to \infty$.

Using this same example, we can also see that the fourth choice "The variance of $\hat{\theta}_n$ goes to 0 as $n \to \infty$." is incorrect. The estimator $\hat{\theta}_n := 0$ has variance 0 for all n, but if $\mu \neq 0$, then $\hat{\theta}_n - \theta = \mu$, which is constant for all n and does not converge to 0.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 2 Foundation of Inference:Lecture 4: Parametric Estimation and Confidence Intervals / 5. Quadratic Risk of Estimators

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