13. Examples of Maximum Likelihood Estimators: Poisson Model

Note: The following problems will be presented in lecture (video at the bottom of this page), but we encourage you to attempt it first.

Maximum Likelihood Estimator of a Poisson Statistical Model

3/3 points (graded)

Let $X_1, \ldots, X_n \stackrel{iid}{\sim} \text{Poiss}(\lambda^*)$ for some unknown $\lambda^* \in (0, \infty)$. You construct the associated statistical model $(\mathbb{N} \cup \{0\}, \{\text{Poiss}(\lambda)\}_{\lambda \in (0,\infty)})$. Recall that in the sixth question "Likelihood of a Poisson Statistical Model" two sections ago that you derived the formula

$$L_n(x_1, \ldots, x_n, \lambda) = \prod_{i=1}^{n} e^{-\lambda} \frac{\lambda^{x_i}}{x_i!} = e^{-n\lambda} \frac{\lambda^{\sum_{i=1}^n x_i}}{x_1! \cdots x_n!}$$

As in the previous question, we will work with the log-likelihood $\ell(\lambda) := \ln L_n(x_1, \ldots, x_n, \lambda)$.

The derivative of the log-likelihood can be written

$$\frac{\partial}{\partial \lambda} \ln L_n(x_1, \ldots, x_n, \lambda) = -n + \frac{A}{B}$$

where A depends on $\sum_{i=1}^n x_i$ and B depends on λ. Fill in the boxes with the correct expressions for A and B.

(Type **S_n** for $\sum_{k=1}^n x_i$ and **lambda** for λ.)

$A =$ S_n

S_n

Answer: S_n

$B =$ lambda

λ

Answer: lambda

For the Poisson model, given fixed $x_1, \ldots, x_n$, the function $\lambda \mapsto \ln L_n(x_1, \ldots, x_n, \lambda)$ has a unique critical point $\hat{\lambda}$. You are allowed to assume that this critical point gives the expression for the MLE (*i.e.* given observations $x_1, \ldots, x_n$, the global maximum of the log-likelihood is attained at $\hat{\lambda}$). Given this information, suppose you observe the data-set $X_1 = 2, X_2, = 3,$ and $X_3 = 1$. What is $\hat{\lambda}_3^{\text{MLE}}(2, 3, 1)$?

$\hat{\lambda}_3^{\text{MLE}}(2, 3, 1) =$ 2

Answer: 2.0

STANDARD NOTATION

Solution:

Observe that

$$\ln L_n(x_1, \ldots, x_n, \lambda) = \ln\left(e^{-n\lambda} \frac{\lambda^{\sum_{i=1} x_i}}{x_1! \cdots x_n!}\right) = -n\lambda + (\sum_{i=1}^{n} x_i) \ln \lambda - \ln(x_1! \cdots x_n!)$$

Hence,

$$\frac{\partial}{\partial \lambda} \ln L_n(x_1, \ldots, x_n, \lambda) = -n + \frac{\sum_{i=1}^n x_i}{\lambda}.$$

Setting this equal to 0, we recover the critical point

$$\hat{\lambda} = \frac{1}{n} \sum_{i=1}^n x_i.$$

You are encouraged to perform the second derivative test and verify that this critical point is indeed a global maximum. (Don't forget to test the endpoints $\lambda = 0$ and $\lambda = \infty$ as well!)

This verifies that the MLE is

$$\hat{\lambda}_n^{MLE} = \frac{1}{n} \sum_{i=1}^n x_i,$$

which is again the **sample mean**.

Hence, for the observations $X_1 = 2, X_2 = 3, X_3 = 1$, we get the estimate

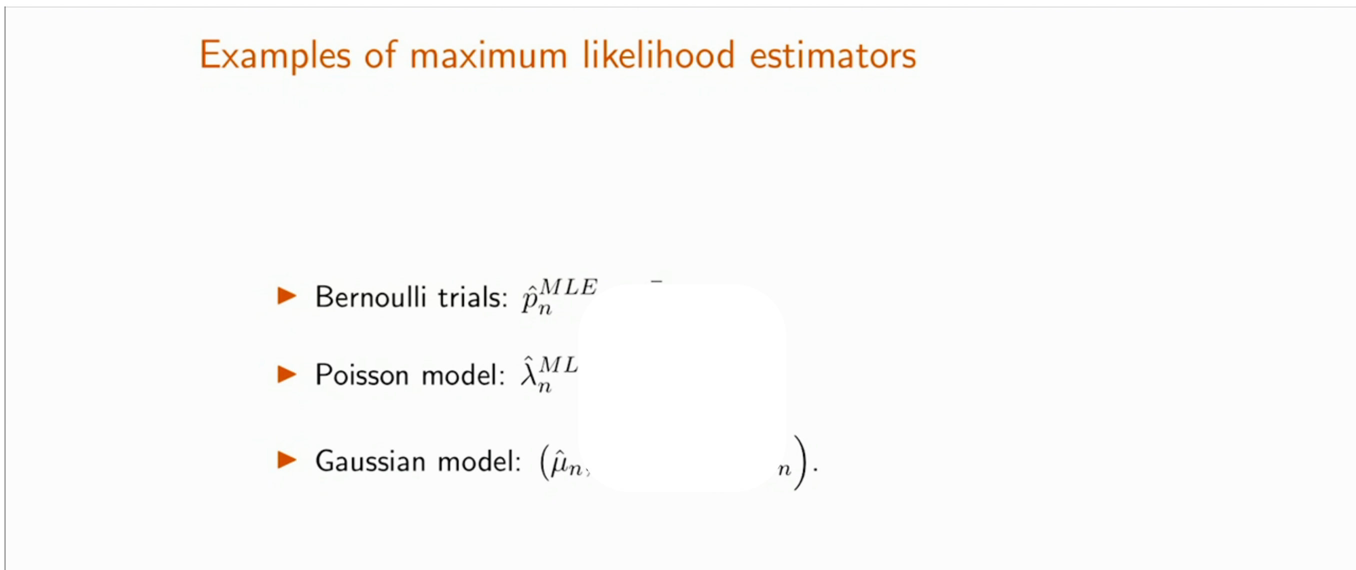
$$\hat{\lambda}_n^{MLE}(2,3,1) = \frac{1}{3}(2+3+1) = 2.$$

Remark: We also see for the Poisson model the conceptually nice fact that the maximum likelihood estimator is the sample mean.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

Maximum Likelihood Estimator of Poisson Statistical Model



(Caption will be displayed when you start playing the video.)

your distribution, et cetera.

Those things don't matter.

I could multiply this thing by the ugliest function

of the xi's it won't matter.

Because when I take the log, it's

going to be plus log of something that

looks like a constant from the perspective of lambda.

So that's it.

So that's h prime of lambda.

h prime prime while it's just minus sum of the xi's

divided by lambda squared, which is clearly non-positive.

My xi's are, after all, Poisson random variables.

They're just non-negative integers.

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讨论

显示讨论

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