7. Properties of Total Variation Distance

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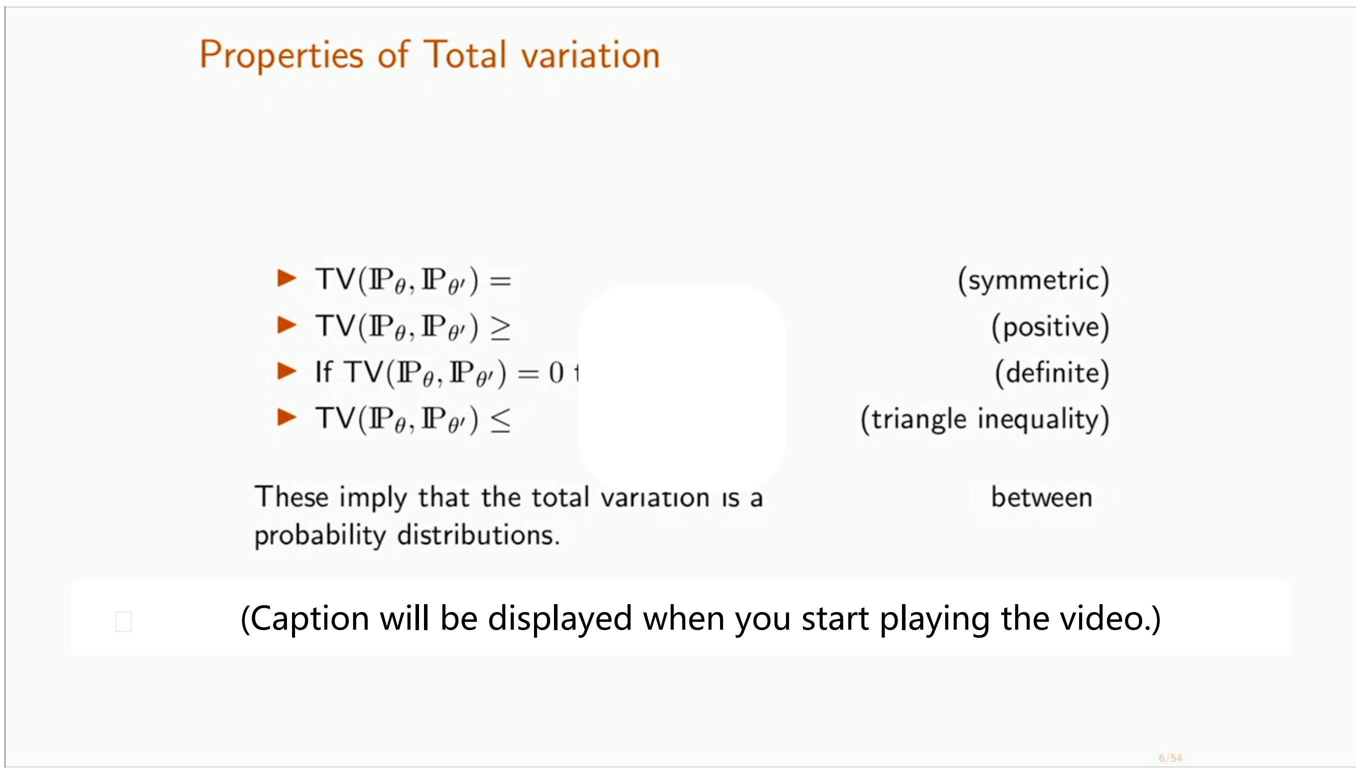

going to get that the absolute value of this difference

is less than the absolute value of f, theta

but f theta is non-negative.

Plus the absolute value of f theta

prime, which is non-negative.

So the absolute values go away.

So I get 1/2 integral of f theta plus 1/2 integral

of f theta prime, but both of them integrate to 1.

So it's 1/2 of 2, which is 1.

OK, so I know that TV is less than 1.

You can certainly check that this is also the case,

here, from those two properties as well,

and you can even check that it's true here.

So it's a little less obvious.

So here, you're not going to use the triangle inequality.

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Let d be a function that takes two probability measures $\mathbf{P}$ and $\mathbf{Q}$ and maps them to a real number $d(\mathbf{P}, \mathbf{Q})$. Then d is a **distance** on probability measures if the following four axioms hold. (Here, $\mathbf{P}, \mathbf{Q}$, and $\mathbf{V}$ are all probability measures.)

- $d(\mathbf{P}, \mathbf{Q}) = d(\mathbf{Q}, \mathbf{P})$ (symmetric)
- $d(\mathbf{P}, \mathbf{Q}) \geq 0$ (nonnegative)
- $d(\mathbf{P}, \mathbf{Q}) = 0 \iff \mathbf{P} = \mathbf{Q}$ (definite)
- $d(\mathbf{P}, \mathbf{V}) \leq d(\mathbf{P}, \mathbf{Q}) + d(\mathbf{Q}, \mathbf{V})$ (triangle inequality)

In the above, $\mathbf{P} = \mathbf{Q}$ means $\mathbf{P}(A) = \mathbf{Q}(A)$ for $A \subset E$, where E is the common sample space of $\mathbf{P}$ and $\mathbf{Q}$.

The total variation distance $(\mathbf{TV})$ is a distance on probability measures.

Symmetry and Definiteness of Total Variation Distance

1/1 point (graded)
Let $\mathbf{P}$ be a probability measure. Which of the following is (are) true?

- [ ] One can find a measure $\mathbf{Q} \neq \mathbf{P}$ such that $\mathbf{TV}(\mathbf{P}, \mathbf{Q}) = 0$.
- [x] $\mathbf{TV}(\mathbf{P}, \mathbf{Q}) = \mathbf{TV}(\mathbf{Q}, \mathbf{P})$.

Solution:

Choice 1 is not true because of the following: By definition, $\mathbf{Q} \neq \mathbf{P}$ means that there is some set A of non-zero measure over which the measures $\mathbf{Q}$ and $\mathbf{P}$ are not the same. Therefore, over this set A, $|\mathbf{P}(A) - \mathbf{Q}(A)| > 0$, which implies that $\mathrm{TV}(\mathbf{P}, \mathbf{Q}) \neq 0$.

Choice 2 (symmetry) is true because for any set A, $|\mathbf{P}(A) - \mathbf{Q}(A)| = |\mathbf{Q}(A) - \mathbf{P}(A)|$.

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Answers are displayed within the problem

Triangle Inequality

1/1 point (graded)
Which of the following quantities is greater than or equal to $\mathrm{TV}(\mathrm{Ber}(.5), \mathrm{Ber}(0.3))$?
(Choose all that apply.)

- [x] $\mathrm{TV}(\mathrm{Ber}(0.5), \mathrm{Ber}(0.1)) + \mathrm{TV}(\mathrm{Ber}(0.1), \mathrm{Ber}(0.3))$
- [x] $\mathrm{TV}(\mathrm{Ber}(0.5), \mathrm{Poiss}(5)) + \mathrm{TV}(\mathrm{Ber}(0.3), \mathrm{Poiss}(5))$
- [x] $\mathrm{TV}(\mathrm{Bin}(7, 0.4), \mathrm{Ber}(0.5)) + \mathrm{TV}(\mathrm{Ber}(0.3), \mathrm{Bin}(7, 0.4))$

Solution:

Recall the triangle inequality states that for distributions $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{V}$:

$$\mathrm{TV}(\mathbf{P}, \mathbf{V}) \leq \mathrm{TV}(\mathbf{P}, \mathbf{Q}) + \mathrm{TV}(\mathbf{Q}, \mathbf{V}).$$

- If we set $\mathbf{P} = \mathrm{Ber}(0.5)$, $\mathbf{V} = \mathrm{Ber}(0.3)$, and $\mathbf{Q} = \mathrm{Ber}(0.1)$, then applying the triangle inequality above gives the first upper bound.
- In the second choice, set $\mathbf{P} = \mathrm{Ber}(0.5)$, $\mathbf{V} = \mathrm{Ber}(0.3)$, and $\mathbf{Q} = \mathrm{Poiss}(5)$ and apply the triangle inequality.
- In the third choice, set $\mathbf{P} = \mathrm{Ber}(0.5)$, $\mathbf{V} = \mathrm{Ber}(0.3)$, and $\mathbf{Q} = \mathrm{Bin}(7, 0.4)$ and apply the triangle inequality.

Remark: Implicitly we are also using the symmetry property of total variation: $\mathrm{TV}(\mathbf{P}, \mathbf{Q}) = \mathrm{TV}(\mathbf{Q}, \mathbf{P})$.

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