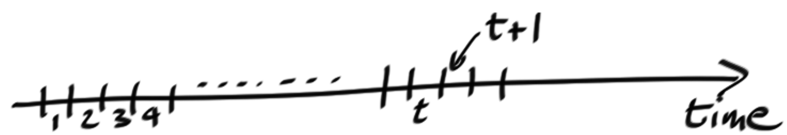
## Markov processes – 1

- checkout counter example
- Markov process definition
- n-step transition probabilities
- classification of states

past future

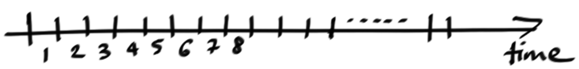
STATE



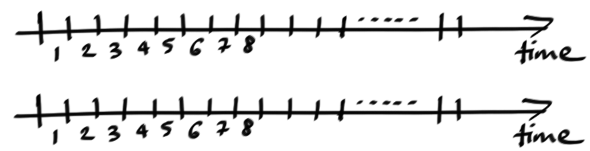

$$\text{state}(t+1) = f(\text{state}(t), \text{noise})$$

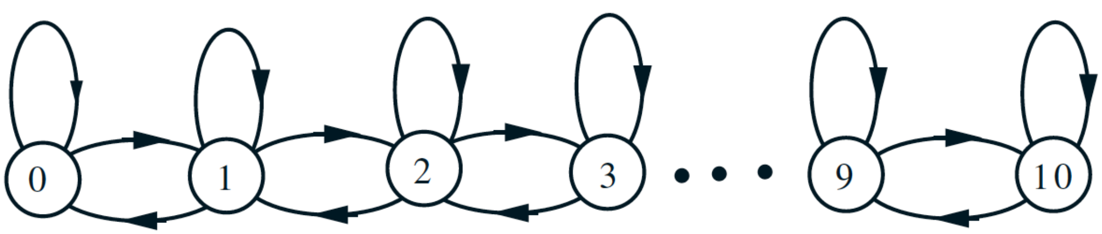
## checkout counter example


- discrete time $n = 0, 1, \ldots$

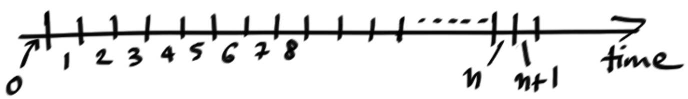



- customer arrivals: Bernouilli $(p)$
- customer service times: geometric $(q)$
- "state" $X_n$ : number of customers at time $n$



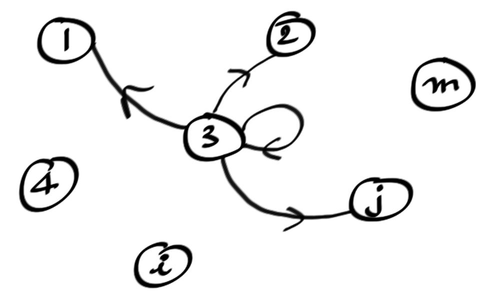



## discrete-time finite state Markov chains




- $X_n$: state after $n$ transitions
  - belongs to a finite set
  - initial state $X_0$ either given or random
  - transition probabilities:

$$\begin{aligned} p_{ij} &= \mathbf{P}(X_1 = j \mid X_0 = i) \\ &= \mathbf{P}(X_{n+1} = j \mid X_n = i) \end{aligned}$$




- Markov property/assumption:
  "given current state, the past doesn't matter"

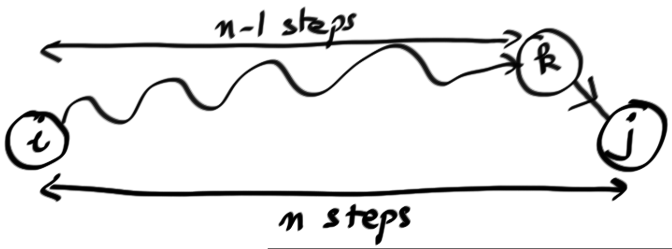
$$\begin{aligned} p_{ij} &= \mathbf{P}(X_{n+1} = j \mid X_n = i) \\ &= \mathbf{P}(X_{n+1} = j \mid X_n = i,\ X_{n-1}, \ldots, X_0) \end{aligned}$$

- model specification: identify states, transitions, and transition probabilities

## n-step transition probabilities

- state probabilities, given initial state i:

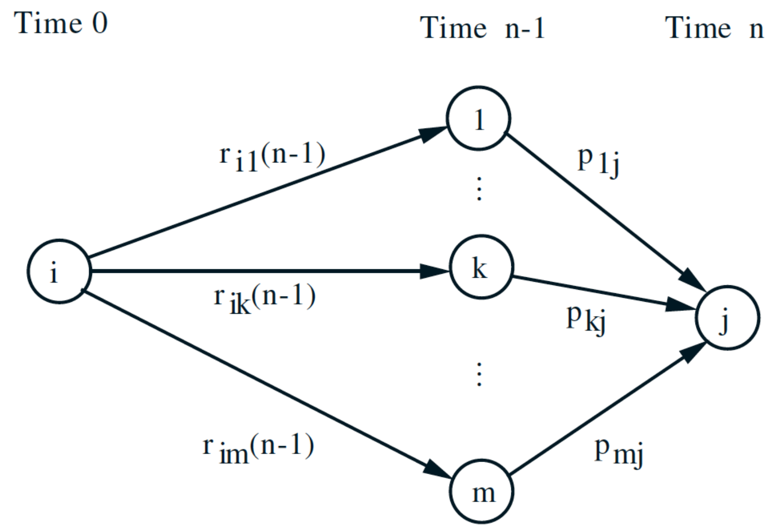
$$
\begin{aligned}
r_{ij}(n) &= \mathbf{P}(X_n = j \mid X_0 = i) \\
&= \mathbf{P}(X_{n+s} = j \mid X_s = i)
\end{aligned}
$$



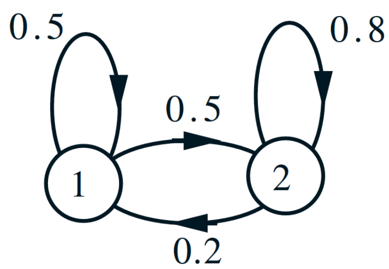

- key recursion: $\boxed{r_{ij}(n) = \sum_{k=1}^{m} r_{ik}(n-1)p_{kj}}$

- random initial state:

$$\mathbf{P}(X_n = j) = \sum_{i=1}^{m} \mathbf{P}(X_0 = i) r_{ij}(n)$$




## example

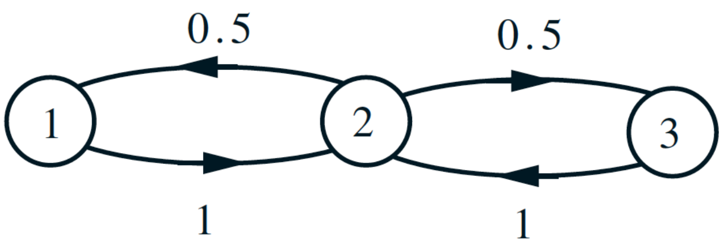



$r_{ij}(n) = \mathbf{P}(X_n = j \mid X_0 = i)$

| | $n=0$ | $n=1$ | $n=2$ | $n=100$ | $n=101$ |
|---|---|---|---|---|---|
| $r_{11}(n)$ | | | | | |
| $r_{12}(n)$ | | | | | |
| $r_{21}(n)$ | | | | | |
| $r_{22}(n)$ | | | | | |

## generic convergence questions

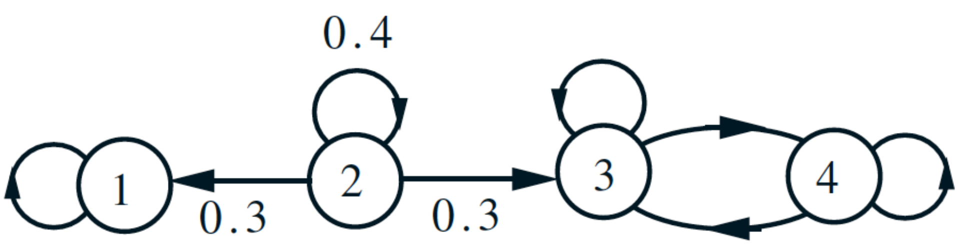
- does $r_{ij}(n)$ converge to something?




$n$ odd: $r_{22}(n) =$

$n$ even: $r_{22}(n) =$

- does the limit depend on initial state?



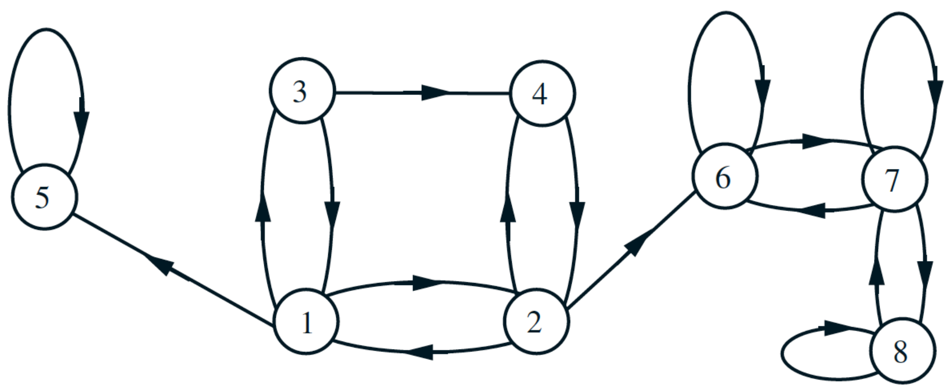

$r_{11}(n) =$

$r_{31}(n) =$

$r_{21}(n) =$

## recurrent and transient states

- state i is recurrent if "starting from i, and from wherever you can go, there is a way of returning to i"
- if not recurrent, called transient




- recurrent class: a collection of recurrent states communicating only between each other