

We carry out a probabilistic experiment. For example, we pick a random student out of a given class. At the end of the experiment you may report to me whether the student was female. That is, you may tell me whether a certain event has occurred. But besides telling me which events occurred, you may also wish to report to me some other results of the experiment. For example, report the weight of the selected student. Reporting the weight of a student is not quite the same as reporting an event. Instead you're giving me the numerical value of some quantity associated with the outcome of the experiment. Such a quantity will be called a random variable.

Random variables make the subject of probability much richer and allow us to talk about random numerical quantities and their relations. What we will do in this unit will be to define random variables, talk about ways of describing them, and introduce certain ways of summarizing their properties, namely the expected value, and the variance. There are four main threads in this unit that will be developed in parallel in conjunction with various examples.

One thread is to introduce a fair amount of definitions and notation about the distribution of a random variable. To a large extent, this involves concepts that you're already familiar with but in new notation. A second thread is the definition of the expected value and the variance and a look at some of their properties. A third thread is related to conditioning. We will discuss conditional counterparts of all the concepts that we introduce as well as the concept of independence of random variables. The last thread builds on two basic tools that we developed earlier in this class. The multiplication rule and the total probability theorem.

We will see their counterparts in the context of random variables, as well as an extension, the total expectation theorem. Random variables can be discrete or continuous. Discrete random variables are conceptually and mathematically much easier. For this reason in this unit, we deal exclusively with discrete random variables aiming to develop a solid understanding.

I should caution you. Unless you make sure that you understand very well every single concept and formula in this unit, interpreting the corresponding concepts and formulas will be a real challenge when we move on to continuous random variables. And I would also recommend that you pay special attention to notation. Good notation helps you think clearly.