20. Exercise: Discrete unknown, continuous measurement

Exercise: Discrete unknown, continuous measurement

1/1 point (graded)

Let K be a discrete random variable that can take the values 1, 2, and 3, all with equal probability. Suppose that X takes values in $[0, 1]$ and that for x in that interval we have

$$f_{X|K}(x \mid k) = \begin{cases} 1, & \text{if } k = 1, \\ 2x, & \text{if } k = 2, \\ 3x^2, & \text{if } k = 3. \end{cases}$$

Find the probability that $K = 1$, given that $X = 1/2$.

4/11

✔ **Answer:** 0.36364

Solution:

Using the appropriate form of the Bayes rule, we have

$$p_{K|X}(1 \mid 1/2) = \frac{p_K(1)f_{X|K}(1/2 \mid 1)}{f_X(1/2)} = \frac{(1/3) \cdot 1}{f_X(1/2)} = \frac{1/3}{11/12} = 4/11.$$

To find $f_X(1/2)$, we used the total probability theorem:

$$\begin{aligned} f_X(1/2) &= \sum_k p_K(k) f_{X|K}(1/2 \mid k) \\ &= (1/3) \cdot 1 + (1/3) \cdot (2 \cdot (1/2)) + (1/3) \cdot (3 \cdot (1/2)^2) \\ &= 11/12. \end{aligned}$$

提交	You have used 1 of 3 attempts

ℹ Answers are displayed within the problem