Problem 5

(a)

1/1 point (graded)
You have five coins in your pocket. You know a priori that one coin gives heads with probability $0.4,$ and the other four coins give heads with probability $0.7.$

You pull out one of the five coins at random from your pocket (each coin has probability $\frac{1}{5}$ of being pulled out), and you want to find out which of the two types of coin it is. To that end, you flip the coin 6 times and record the results $X_1, \ldots, X_6$ of each coin flip where $X_i = 1$ if "heads" and $X_i = 0$ if "tails".

Let $p = P(X_1 = 1)$. Which of the following is the space of all possible values of the parameter p? In other words, what is the smallest sample space of p?

Note (Added May 6): This question is asking for the sample space (domain) of the prior distribution of p.

- $[0.4, 0.7]$
- $(0, 1)$
- $\{0.2, 0.8\}$
- $\{0.4, 0.7\}$ ✔

Correction Note (added May 4): An earlier version of the statement did not include the second sentence "In other words, what is the smallest sample space of p".

Solution:

Note that by the problem statement that the possible values of p are $.4$ and $.7$ and the result follows.

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Answers are displayed within the problem

(b)

1/1 point (graded)
Find the pmf π that quantifies my prior knowledge of p.

Then, enter the value of the prior evaluated at $p = 0.7$ i.e. $\pi(p = 0.7)$, below.

$\pi(p = 0.7) =$ 4/5 ✔ **Answer:** 0.8

Correction note (added May 5): An earlier version of the problem statement was "Enter $\pi(p = 0.7)$, below." for the second sentence.

Solution:

Since four of the five coins have probability of heads 0.7, we have $\pi(p = 0.7) = 4/5$; the final coin with probability of heads 0.4 gives $\pi(p = 0.4) = 1/5$.

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(c)

1.0/1 point (graded)

Suppose that $X_1+X_2+X_3+X_4+X_5+X_6=3$. Find the maximum likelihood estimate $\hat{p}^{\text{MLE}}$ for p using the given data. (Enter a numerical value accurate to at least 3 decimal places.)

$\hat{p}^{\text{MLE}} =$ 0.4 ✔ **Answer:** 0.5

Grading note: Though not the intention of the question, there is a possibility that you interpreted this question as asking for the constrained MLE,i.e. the value of p within $\{0.4, 0.7\}$ that maximizes the likelihood. Because this was not clearly specified, you wil also receive full credit if you computed the correct constrained MLE.

Solution:

Since $X_i \sim \text{Ber}(p)$, we have $\hat{p}^{\text{MLE}} = \bar{X}_n = 3/6 = 0.5.$ Note that the maximum likelihood estimator does not take into account the prior distribution.

Constrained MLE:

Since the likelihood of obtaining 3 heads and 3 tails of a coin with probability of heads p is $\binom{6}{3}p^3(1-p)^3$, which is $\binom{6}{3}(0.013824)$ for $p=0.4$ and $\binom{6}{3}(0.00926)$ for $p=0.7$, we have $\hat{p}^{\text{MLE}}_{\text{constrained}} = 0.4$. Again, the MLE does not take into account the prior distribution of p.

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Answers are displayed within the problem

(d)

1/1 point (graded)

Find the Bayes estimate $\hat{p}^{\text{Bayes}}$ of p based on $X_1+X_2+X_3+X_4+X_5+X_6=3$.

(Recall the Bayes estimator is the mean of the posterior distribution.)

(Give your answer to 3 decimal places.)

$\hat{p}^{\text{Bayes}} =$ 0.61847 ✔ **Answer:** 0.618

Solution:

Given a coin of probability of heads p, the probability of flipping 3 heads and 3 tails is $\binom{6}{3}p^3(1-p)^3$.

The posterior distribution is

$$\pi\left(p \middle| \sum_{i=1}^{6} X_i = 3\right) \propto L\left(\sum_{i=1}^{6} X_i = 3 \middle| p\right)\pi(p) \propto \begin{cases} 0.2(0.4)^3(0.6)^3 = 0.0027648 & \text{for } p=0.4 \\ 0.8(0.7)^3(0.3)^3 = 0.0074088 & \text{for } p=0.7 \end{cases}.$$

The Bayes estimator is the expectation of p under the posterior distribution:

$$\begin{aligned} p^{\text{Bayes}} &= \frac{0.4\,\pi\left(0.4|\sum_{i=1}^{6} X_i = 3\right) + 0.7\,\pi\left(0.7|\sum_{i=1}^{6} X_i = 3|\sum_{i=1}^{6} X_i = 3\right)}{\pi\left(0.4|\sum_{i=1}^{6} X_i = 3\right) + \pi\left(0.7|\sum_{i=1}^{6} X_i = 3\right)} \\ &= 0.61847. \end{aligned}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

(e)

1/1 point (graded)

Find the maximum a posteriori estimate $\hat{p}^{\text{MAP}}$ of p, i.e. the value of p at which the posterior distribution is maximum, based on $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 3$.

$\hat{p}^{\text{MAP}} =$ 0.7 ✔ **Answer:** 0.7

Solution:

The value of p at which the posterior distribution is maximum is $p = 0.7$. Hence $\hat{p}^{\text{MAP}} = 0.7$.

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Answers are displayed within the problem

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