5. Worked Example: a One-Sided Test
Errors, Levels, and Conclusion of a One-Sided Test

(Caption will be displayed when you start playing the video.)

Let's put Q alpha here.
Everybody agrees?
Yeah.
Q1 minus alpha, right?
Because Q alpha is the probability
that exceeds Q alpha goes through this, right?
So this would be Q1 minus alpha.
So if we do a plot of what those guys look like,
I want alpha Q alpha, and I also want 1 minus alpha
to the right.
So that means that if I put alpha here,
I get Q1 minus alpha.
And by symmetry, it turns out that Q1 minus alpha is actually

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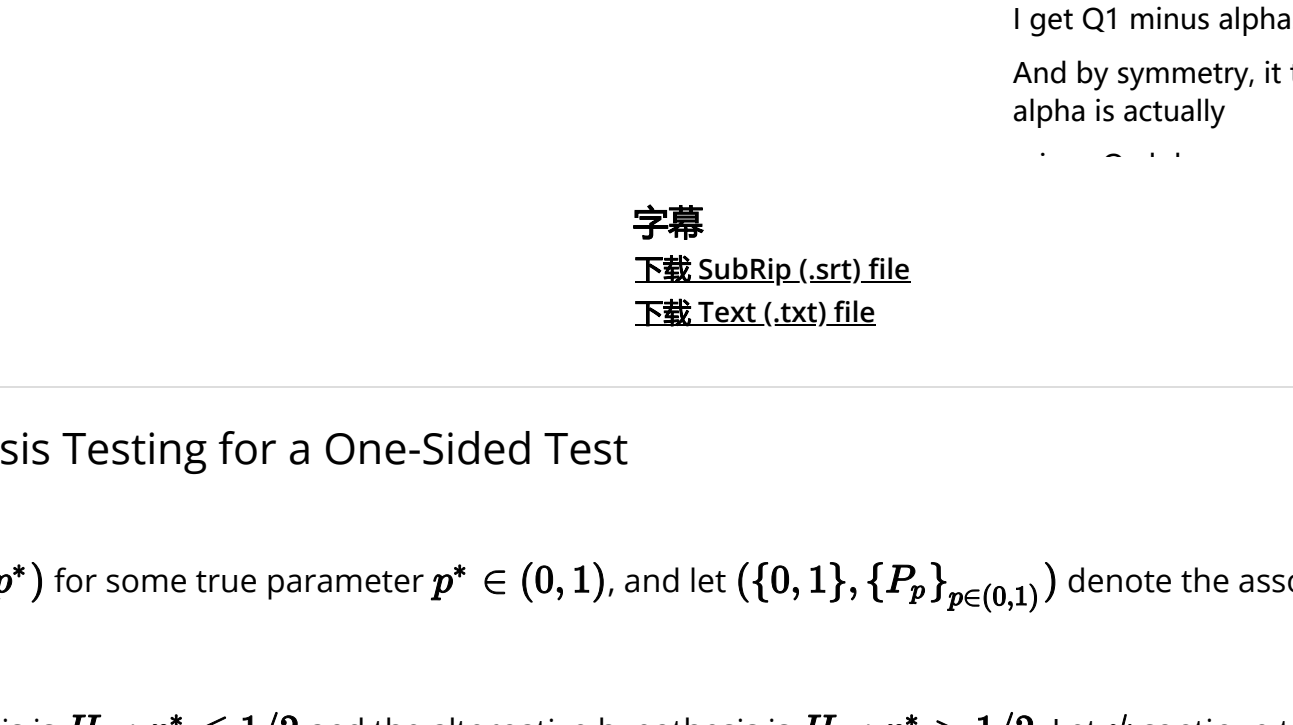
Visualizing Hypothesis Testing for a One-Sided Test

3/3 points (graded)

Let $X_1, \ldots, X_n \overset{iid}{\sim} \mathbf{Ber}(p^*)$ for some true parameter $p^* \in (0,1)$, and let $(\{0,1\}, \{P_p\}_{p\in(0,1)})$ denote the associated statistical model where $P_p = \mathbf{Ber}(p)$.

Suppose the null hypothesis is $H_0 : p^* \leq 1/2$ and the alternative hypothesis is $H_1 : p^* > 1/2$. Let ψ continue to denote the statistical test we will use. (Recall that a test takes value either 0 or 1. Usually it is of the form $\mathbf{1}(T_n > C)$ where C is a threshold to be specified and T_n is known as a **test statistic** . Be careful to not confuse **(tests** with **test statistics**.)

Consider the following graph of this hypothesis testing set-up.



- Continuous curve on the left: type 1 error, α_ψ, graphed as a function of θ.
- Continuous curve on the right: type 2 error, β_ψ, graphed as a function of θ.
- Horizontal axis: the parameter space $\Theta = (0, 1)$.

Which letter indicates Θ_0, the region defined by the null hypothesis?

- A
- B ✔
- C
- D

Which letter indicates Θ_1, the region defined by the alternative hypothesis?

- A
- B
- C ✔
- D

Let $p \in (0, 1)$ denote the point where the power is attained, i.e., the point where

$$\pi_\psi = \inf_{\Theta_1} (1 - \beta_\psi (p)) .$$

Which letter indicates the ordered pair (p, π_ψ)?

- A ✔
- B
- C
- D

Solution:

We consider the questions in order.

For the first question, since we are given that $H_0 : p \leq 1/2$, then the interval $(0, 1/2]$ defines Θ_0. Hence, letter B is the correct response.

For the second question, since we are given that $H_1 : p > 1/2$, then the interval $(1/2, 1)$ defines Θ_1. Hence, letter C is the correct response.

The the third question, recall that the power of a test is given by

$$\pi_\psi = \inf_{p \in (0,1)} \left(1 - \beta_\psi\left(p\right)\right).$$

The continuous curve on the right, which graphs β_ψ, attains its maximum at $p = 1/2$, and this maximum is given by $\beta_\psi\left(1/2\right) = 0.8$. Therefore,

$$\pi_\psi = \inf_{p \in (0,1)} \left(1 - \beta_\psi\left(p\right)\right) = 1 - 0.8 = 0.2,$$

which implies that A is the correct response.

提交 你已经尝试了2次（总共可以尝试3次）

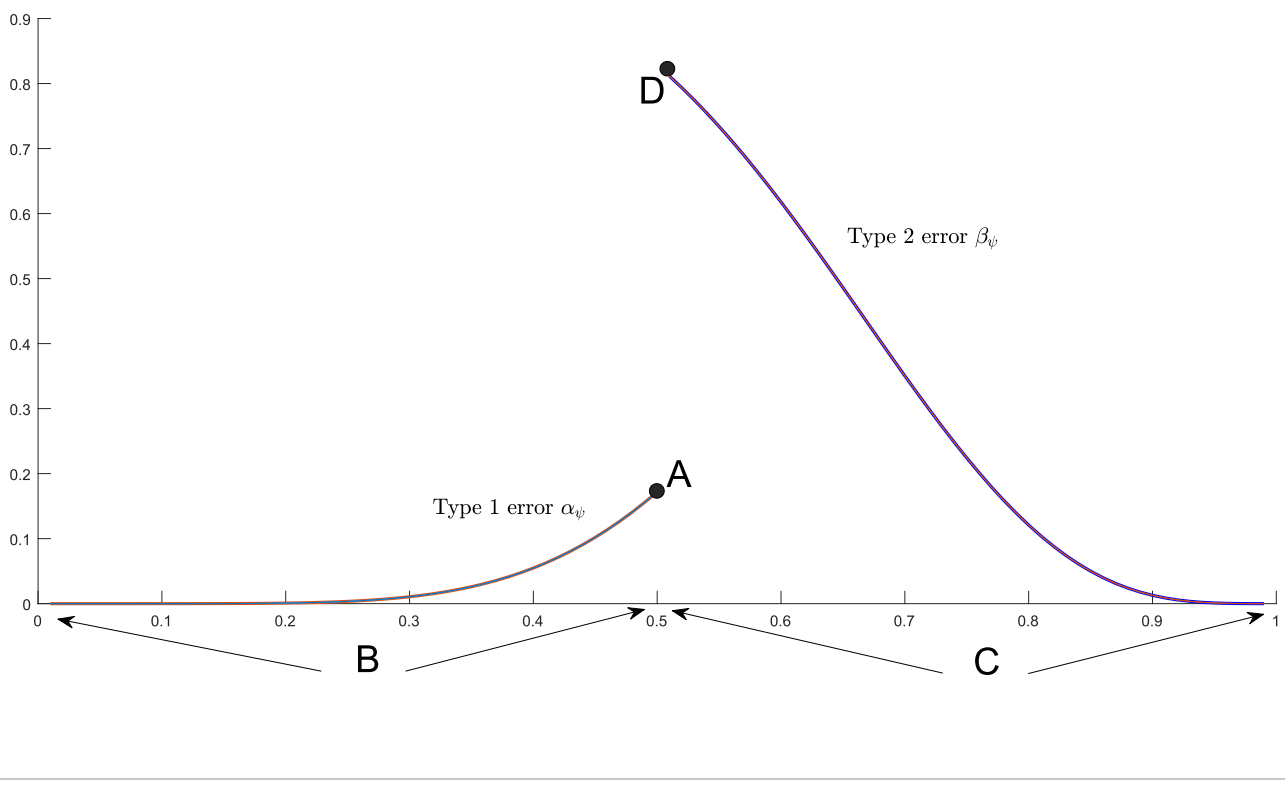
Answers are displayed within the problem

Level of a statistical test

0/1 point (graded)

As in the previous question, let $X_1, \ldots, X_n \overset{iid}{\sim} \mathbf{Ber}\left(p^*\right)$ for some true parameter $p^* \in (0,1)$, and let $\left(\{0,1\}, \{P_p\}_{p \in (0,1)}\right)$ denote the associated statistical model where $P_p = \mathbf{Ber}\left(p\right)$.

Suppose the null hypothesis is $H_0 : p^* \leq 1/2$ and the alternative hypothesis is $H_1 : p^* > 1/2$. Let ψ continue to denote the statistical test we will use. Consider the graphic below from the previous problem.



- Continuous curve on the left: type 1 error, α_ψ, graphed as a function of θ.
- Continuous curve on the right: type 2 error, β_ψ, graphed as a function of θ.
- Horizontal axis: the parameter space $\Theta = (0,1)$.

Which of the following are **levels** of ψ? (Choose all that apply.)

这个就是alpha，答案是犯一类错误的最大值。

- [x] 5 %
- [x] 10 %
- [x] 20 % ✔

Solution:

The level of ψ is given by any real $\alpha \in \mathbb{R}$ such that

$$\alpha_\psi(p) \leq \alpha, \quad \text{for all } p \in \Theta_0 = (0, 1/2]$$

That is, the type 1 error is uniformly bounded above by α. According to the graph, the continuous curve on the left curve stays below 0.2, but not below 0.05 and 0.1. Thus $0.2 = 20\%$ is the correct response.

Remark: In general, we will describe the level of a test by the *smallest* possible level α, but this is not strictly necessary.

提交 你已经尝试了2次（总共可以尝试2次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 2 Foundation of Inference:Lecture 7: Hypothesis Testing (Continued): Levels and P-values / 5. Worked Example: a One-Sided Test

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