4. PMF on a countable set

Problem 3. PMF on a countable set

5.0/5.0 points (graded)

For a fixed real number $k > 1$, define $c_k = \sum_{n=1}^{\infty} n^{-k}$. Let, X and Y be two independent, positive, integer-valued random variables, with

$$\mathbf{P}(X = n) = \mathbf{P}(Y = n) = \frac{1}{c_k} n^{-k}, \quad \text{for } n = 1, 2, \ldots.$$

Note that the constant c_k is defined to ensure that this PMF sums to 1.

1. Find the probability $\mathbf{P}(X = Y)$, in terms of c_k.

○ $\frac{c_{2k}}{c_k}$

○ $\frac{1}{c_k}$

○ $\frac{2c_k}{c_k}$

● $\frac{c_{2k}}{(c_k)^2}$ ✓

○ none of the above

2. Fix a positive integer n, and define the following event:

$$A_n \triangleq \{X \text{ is divisible by } n\}.$$

Find the probability of A_n. Your answer should be entered as a function of n and k.

$\mathbf{P}(A_n) =$ n^(-1*k) ✓ **Answer:** 1/(n^k)

$n^{-1 \cdot k}$

STANDARD NOTATION

Solution:

1. We have:

$$\begin{aligned} \mathbf{P}(X = Y) &= \sum_{n=1}^{\infty} \mathbf{P}(X = Y = n) \\ &= \sum_{n=1}^{\infty} \mathbf{P}(X = n)\mathbf{P}(Y = n) \end{aligned}$$

$$
\begin{aligned}
&= \sum_{n=1}^{\infty} \left(\frac{1}{c_k} n^{-k}\right) \cdot \left(\frac{1}{c_k} n^{-k}\right) \\
&= \frac{1}{(c_k)^2} \sum_{n=1}^{\infty} n^{-2k} \\
&= \frac{c_{2k}}{(c_k)^2}.
\end{aligned}
$$

2. X is divisible by n if and only if X takes a value of the form $n \cdot t$, where t is a positive integer. Thus,

$$
\begin{aligned}
\mathbf{P}(E_n) \ &= \sum_{t=1}^{\infty} \mathbf{P}(X = tn) \\
&= \sum_{t=1}^{\infty} \frac{1}{c_k} (tn)^{-k} \\
&= n^{-k} \sum_{t=1}^{\infty} \frac{1}{c_k} t^{-k} \\
&= n^{-k},
\end{aligned}
$$

where the last line is valid because, from the definition of c_k, we have $\sum_{t=1}^{\infty} \frac{1}{c_k} t^{-k} = 1$.

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

Error and Bug Reports/Technical Issues

显示讨论

主题： Final Exam / 4. PMF on a countable set