14. Maximum Likelihood Estimator of Gaussian Statistical Model

Maximum Likelihood Estimator of Gaussian Statistical Model: the mean

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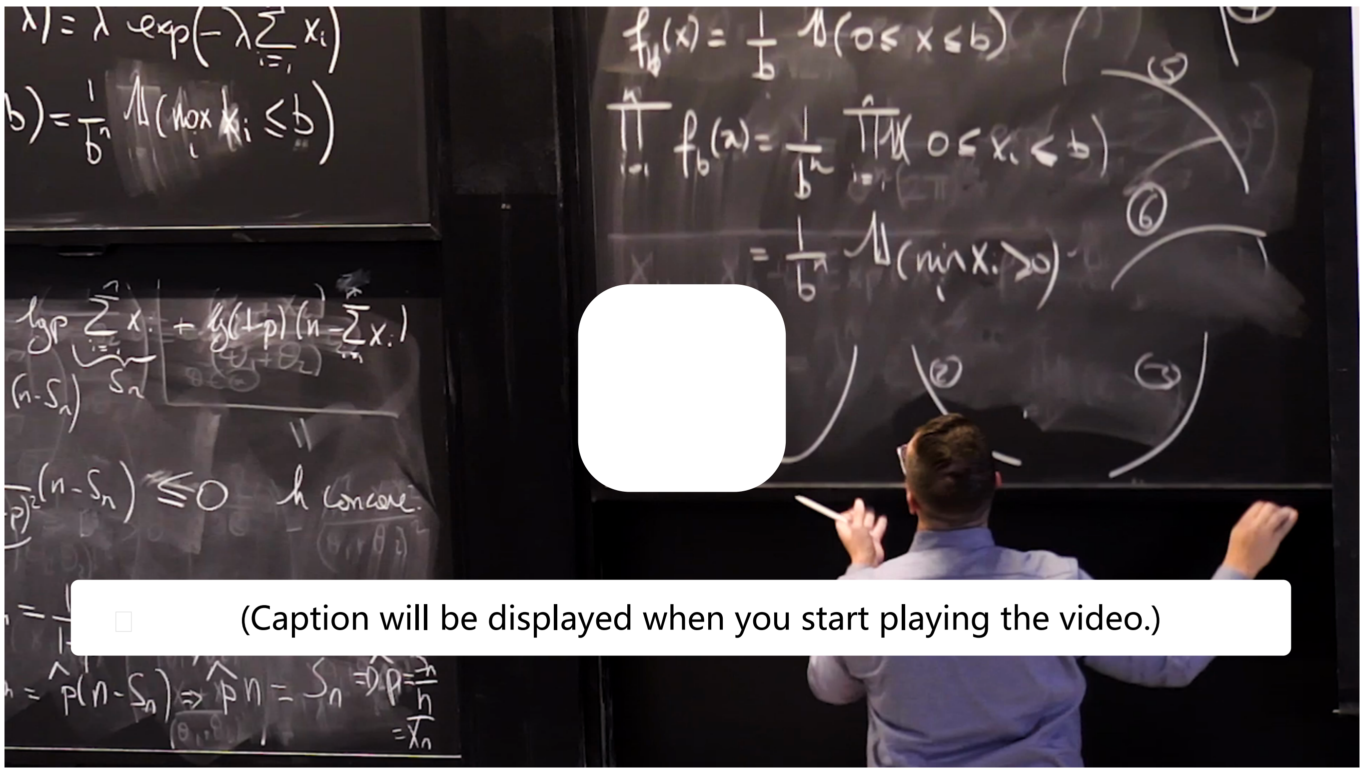

And finally, in the Gaussian model--
so then you write h of mu sigma squared
as being the function of my log likelihood.
So it's going to be--
so remember, that's when I do my flip.
So I really think of it as just a function of the parameters,
and so I do log of l x1, xn, mu sigma squared.
OK. so what do I get here when I do the log

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Note: A variation of the following problem will be presented in lecture (video at the bottom of this page), but we encourage you to attempt it first.

Maximum Likelihood Estimator of the variance a Gaussian Statistical Model with Mean Zero

3/3 points (graded)

Let $X_1, \ldots, X_n \overset{iid}{\sim} N(0, \tau^*)$ for some unknown variance τ^*. You construct the associated statistical model $(\mathbb{R}, \{N(0,\tau)\}_{\tau>0})$. Recall that in the last question from the previous slide, you derived the formula

$$L_n(x_1, \ldots, x_n, (\mu, \sigma^2)) = \frac{1}{(\sigma\sqrt{2\pi})^n} \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^n (x_i - \mu)^2\right).$$

Since we are given $\mu = 0$ and $\tau = \sigma^2$, we may rewrite this

$$L_n(x_1, \ldots, x_n, \tau) = \frac{1}{(\sqrt{2\pi\tau})^n} \exp\left(-\frac{1}{2\tau}\sum_{i=1}^n x_i^2\right).$$

As in the previous two questions, it will be more convenient to work with the log-likelihood $\ell(\tau) := \ln L_n$.

The derivative of the log-likelihood can be written

$$\frac{\partial}{\partial \tau}(\ln L_n(x_1, \ldots, x_n, \tau)) = -\frac{n}{A} + \frac{\sum_{i=1}^n x_i^2}{B}$$

where A and B both depend on τ. Find A and B.

(Type **tau** for τ.)

$A =$ 2*tau

$2 \cdot \tau$

Answer: 2*tau

$B =$ 2*tau^2

$2 \cdot \tau^2$

Answer: 2*tau^2

What is the maximum likelihood estimator for τ^*? (You are allowed to assume that the critical point found by setting $\frac{\partial}{\partial p}\ln L_n(x_1, \ldots, x_n, \tau) = 0$, treating $x_1, \ldots, x_n$ as fixed, gives the global maximum.)

Answer by entering the summand below in terms of the variable X_i.

(Type **X_i** for X_i.)

$\hat{\tau}_n^{\text{MLE}} = \frac{1}{n}\sum_{i=1}^n$ X_i^2

X_i^2

Answer: X_i^2

STANDARD NOTATION

Solution:

Observe that

$$\begin{aligned}\ln L_n(x_1, \ldots, x_n, \tau) &:= \ln\left(\frac{1}{(\sqrt{2\pi\tau})^n}\exp\left(-\frac{1}{2\tau}\sum_{i=1}^n X_i^2\right)\right)\\ &= -\frac{n}{2}\ln(2\pi\tau) - \frac{1}{2\tau}\sum_{i=1}^n X_i^2\end{aligned}$$

Taking derivatives, we get

$$\ln L_n(x_1, \ldots, x_n, \tau) = -\frac{n}{2\tau} + \frac{\sum_{i=1}^n X_i^2}{2\tau^2}.$$

Hence $A = 2\tau$ and $B = 2\tau^2$.

Rearranging, this is equal to 0 precisely when

$$\tau = \frac{1}{n}\sum_{i=1}^n X_i^2,$$

which is the “empirical second moment".

You are encouraged to perform the second derivative test to verify that $\tau = \frac{1}{n}\sum_{i=1}^n X_i^2$ is a local maximum. Moreover, it will be a *global* maximum because $\lim_{\tau \to 0} L_n(X_1, \ldots, X_n, \tau) = 0$ and $\lim_{\tau \to \infty} L_n(X_1, \ldots, X_n, \tau) = 0$.

Hence, we derive the formula for the MLE

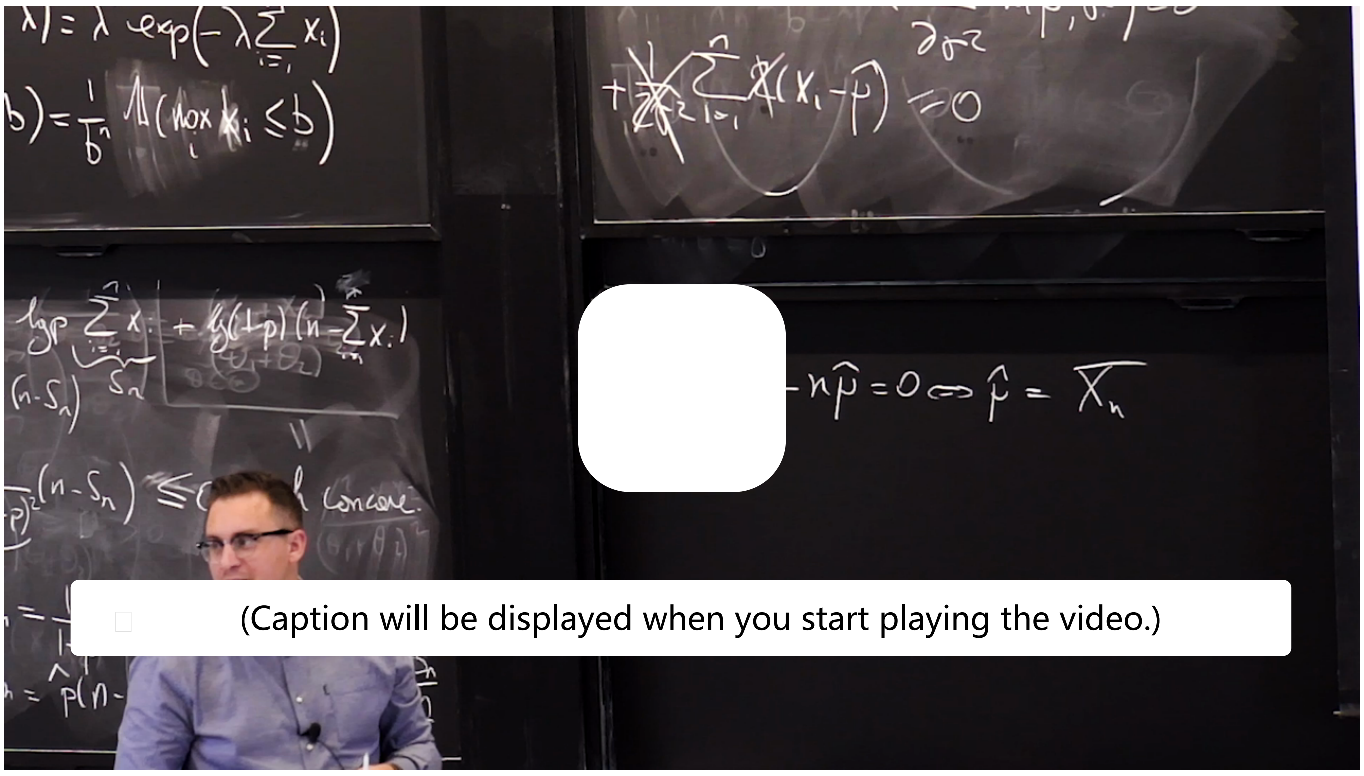
$$\hat{\tau}_n^{MLE} = \frac{1}{n}\sum_{i=1}^{n} X_i^2.$$

Remark: In this example, we want to estimate τ^*, the true variance, and we see the conceptually nice fact that the MLE is the **empirical second moment** $\frac{1}{n}\sum_{i=1}^{n} X_i^2$.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

Maximum Likelihood Estimator of Gaussian Statistical Model: the Variance



We know the maximum--

the asymptotic normality of one random variable,

but what is the asymptotic normality of a random vector?

And we're going to have to talk about covariance matrices

and multivariate Gaussian distribution.

See you then.

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讨论

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