3. QQ Plots

Consider an iid sample $X_1, X_2, \ldots, X_n \overset{iid}{\sim} \mathbf{P}$ that has been reordered as $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ where n is very large. In the problems below, we have chosen a different distribution for $\mathbf{P}$ and compared the empirical quantiles to the standard Gaussian quantiles using a QQ plot. Recall that

- the **Laplace distribution** $\mathbf{Lap}(\lambda)$ with parameter $\lambda > 0$ is the continuous probability distribution with density $f_\lambda = \frac{\lambda}{2} e^{-\lambda |x|}$, and
- the **Cauchy distribution** is the continuous probability distribution with density $g(x) = \frac{1}{\pi} \frac{1}{1+x^2}$.

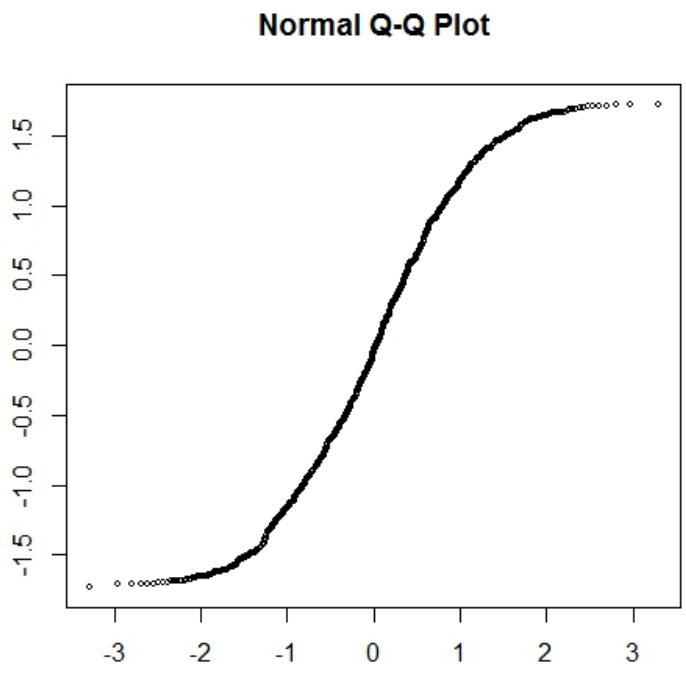
(These were also introduced in Lecture 12.)

For each plot below, match the QQ plot with the correct distribution for $\mathbf{P}$. *Hint:* Each possible distribution will be an answer choice exactly once, so you should use the process of elimination.

Hint: You may use computational tools to graph the pdf of the possible distributions of $\mathbf{P}$.

Matching a Distribution to a QQ Plot I

1/1 point (graded)



- Standard normal: $N(0,1)$
- Cauchy distribution
- Exponential with parameter 1: $\mathbf{Exp}(1)$
- Uniform on the interval $[-\sqrt{3}, \sqrt{3}]$: $\mathbf{Unif}[-\sqrt{3}, \sqrt{3}]$ ✔
- Laplace distribution with parameter $\sqrt{2}$: $\mathbf{Lap}(\sqrt{2})$

Solution:

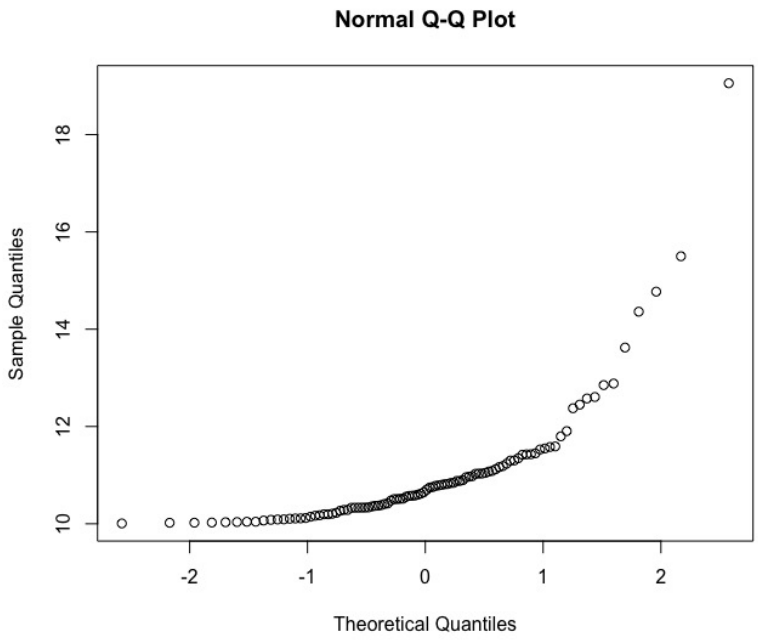
The distribution for this QQ plot is Uniform on the interval $[-\sqrt{3}, \sqrt{3}]$: $\mathbf{Unif}\,[-\sqrt{3}, \sqrt{3}]$. Since the support for this distribution is $[-\sqrt{3}, \sqrt{3}]$, the empirical quantiles $X_{(1)}, X_{(2)}, \ldots, X_{(n)} \in [-\sqrt{3}, \sqrt{3}]$. Since there is nothing plotted outside of the interval $[-\sqrt{3}, \sqrt{3}]$ on the y-axis, we see that the support is restricted to this interval. This implies that the sample was generated from a uniform distribution.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Matching a Distribution to a QQ Plot II

1/1 point (graded)



- Standard normal: $N\,(0, 1)$
- Cauchy distribution
- Shifted exponential with parameter 2.5: $\mathbf{Exp}\,(2.5) + c$ for some $c > 0$ ✔
- Uniform on the interval $[-\sqrt{3}, \sqrt{3}]$: $\mathbf{Unif}\,[-\sqrt{3}, \sqrt{3}]$
- Laplace distribution with parameter $\sqrt{2}$: $\mathbf{Lap}\,(\sqrt{2})$

Solution:

The distribution for this QQ plot is **Shifted exponential with parameter 1 : $\mathbf{Exp}\,(2.5) + c$**. Note that the exponential distribution (not shifted by any constant) is supported on $[0, \infty)$. Hence, the QQ plot will not go below the line $y = 0$ if it is shifted by a positive constant c. Moreover, the exponential distribution has **heavier** tails than those of $N\,(0, 1)$, so we expect the QQ plot to be above the line $y = x$, which is indeed the case here. Further, the sample quantiles in this example do not start near the value 0 and they rather start near the value 10.

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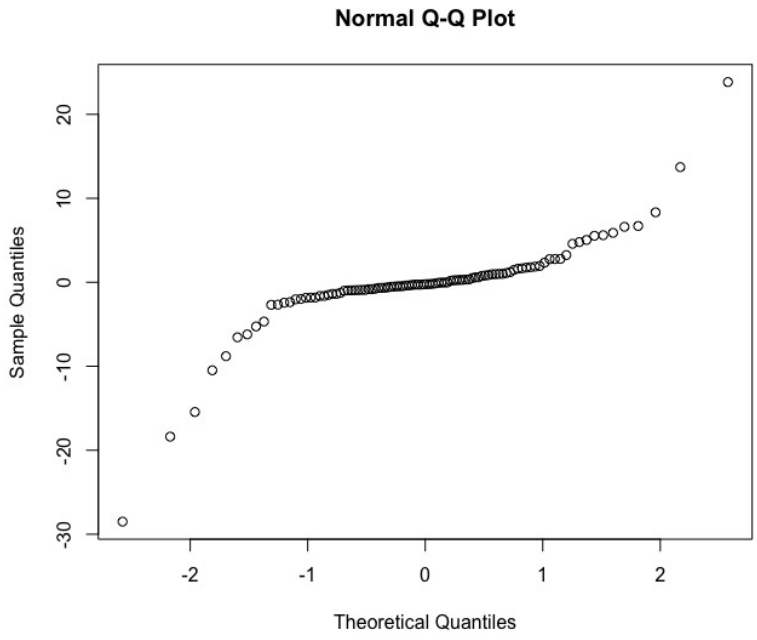
Matching a Distribution to a QQ Plot III

1/1 point (graded)



- Standard normal: $N(0,1)$
- Cauchy distribution ✔
- Exponential with parameter 1: $\mathbf{Exp}(1)$
- Uniform on the interval $[-\sqrt{3},\sqrt{3}]$: $\mathbf{Unif}[-\sqrt{3},\sqrt{3}]$
- Laplace distribution with parameter $\sqrt{2}$: $\mathbf{Lap}(\sqrt{2})$

Solution:

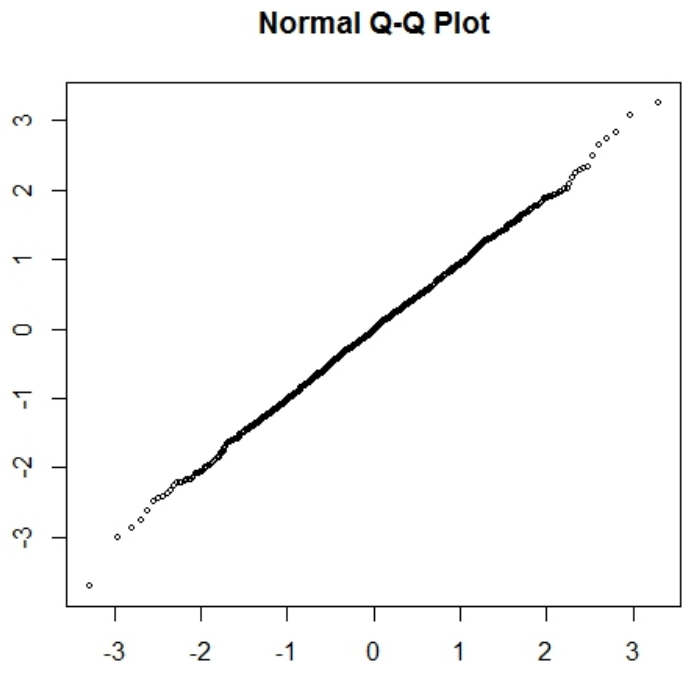
The distribution for this QQ plot is the **Cauchy distribution**. A Cauchy random variable takes values on all of $\mathbb{R}$. Since the pdf $g(x)$ of the Cauchy distribution decays on the order of $1/x^2$ as $x \to \infty$, we know that its tails should be much heavier than those of a standard normal, whose tails decay exponentially. On the right, we see that the QQ plot displayed lies very far above the line $y = x$. On the left, we see that the QQ plot displayed lies very far below the line $y = x$. This indicates that the distribution displayed has much heavier tails than that of a Gaussian, so the Cauchy distribution must be the correct answer.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Matching a Distribution to a QQ Plot IV

1/1 point (graded)



- Standard normal: $N\left(0,1\right)$ ✔
- Cauchy distribution
- Exponential with parameter 1: $\mathbf{Exp}\left(1\right)$
- Uniform on the interval $[-\sqrt{3},\sqrt{3}]$: $\mathbf{Unif}\left[-\sqrt{3},\sqrt{3}\right]$
- Laplace distribution with parameter $\sqrt{2}$: $\mathbf{Lap}\left(\sqrt{2}\right)$

Solution:

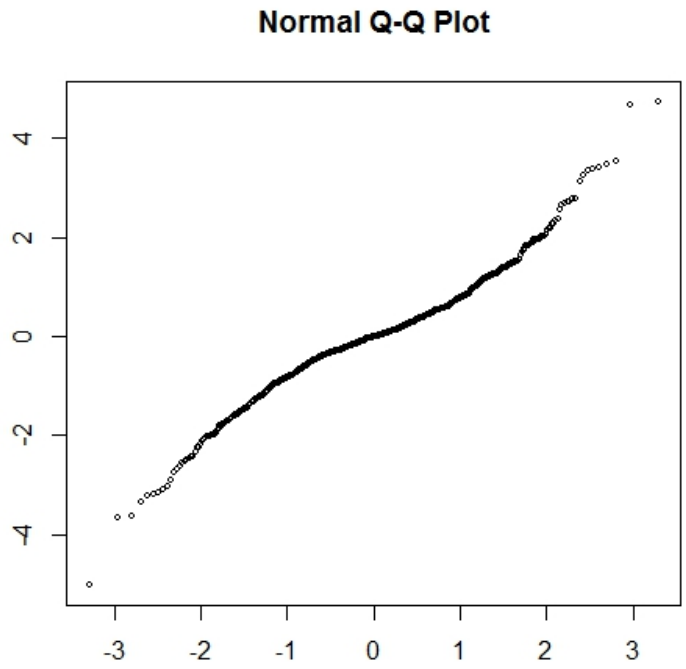
The distribution for this QQ plot is **Standard Gaussian** $\mathbf{N\left(0,1\right)}$. Observe that the QQ plot lies very close to the line $y=x$, so this suggests that the the data is distributed as $N\left(0,1\right)$. By process of elimination, we conclude that the data must have been generated from a standard Gaussian.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Matching a Distribution to a QQ Plot V

1/1 point (graded)



- Standard normal: $N(0,1)$
- Cauchy distribution
- Exponential with parameter 1: $\mathbf{Exp}(1)$
- Uniform on the interval $[-\sqrt{3},\sqrt{3}]$: $\mathbf{Unif}[-\sqrt{3},\sqrt{3}]$
- Laplace distribution with parameter $\sqrt{2}$: $\mathbf{Lap}(\sqrt{2})$ ✔

Solution:

The distribution for this QQ plot is the **Laplace distribution** $\mathbf{Lap}(\sqrt{2})$. A Laplace random variable takes values on all of $\mathbb{R}$. Since the pdf $f_{\sqrt{2}}$ of the Cauchy distribution decays on the order of $e^{-|x|}$ as $x \to \infty$, we know that its tails should be heavier than those of a standard normal, whose tails decay at the rate e^{-x^2}. On the right, we see that the QQ plot displayed lies above the line $y = x$. On the left, we see that the QQ plot displayed lies below the line $y = x$. This indicates that the distribution displayed moderately heavier tails than that of a Gaussian, so by this observation and the process of elimination, the Laplace distribution must be the correct answer.

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Answers are displayed within the problem

Discussion

Topic: Unit 4 Hypothesis testing:Homework 8 / 3. QQ Plots

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