

10. Exercise: Conditional variance II

Exercise: Conditional variance II

3/3 points (graded)

The random variable Q is uniform on $[0, 1]$. Conditioned on $Q = q$, the random variable X is Bernoulli with parameter q .

(a) The conditional variance, $\text{Var}(X | Q)$, is equal to:

☐ $1/4$

☐ $q(1 - q)$

☒ $Q(1 - Q)$ ✓

☐ q^2

☐ Q^2

(b) Recall that a uniform random variable on $[0, 1]$ has a variance of $1/12$ and also satisfies $\mathbf{E}[Q^2] = 1/3$. Then:

$$\text{Var}(\mathbf{E}[X | Q]) =$$

1/12

✓ Answer: 0.08333

$$\mathbf{E}[\text{Var}(X | Q)] =$$

1/6

✓ Answer: 0.16667

Solution:

(a) We know that $\text{Var}(X | Q = q) = q(1 - q)$, for all $q \in [0, 1]$, which translates into the abstract statement $\text{Var}(X | Q) = Q(1 - Q)$.

(b) Since $\mathbf{E}[X | Q] = Q$, we have $\text{Var}(\mathbf{E}[X | Q]) = \text{Var}(Q) = 1/12$.

Since $\text{Var}(X | Q) = Q(1 - Q)$, we have

$$\mathbf{E}[\text{Var}(X | Q)] = \mathbf{E}[Q(1 - Q)] = \mathbf{E}[Q] - \mathbf{E}[Q^2] = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

提交

You have used 3 of 3 attempts

i Answers are displayed within the problem

讨论

显示讨论

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