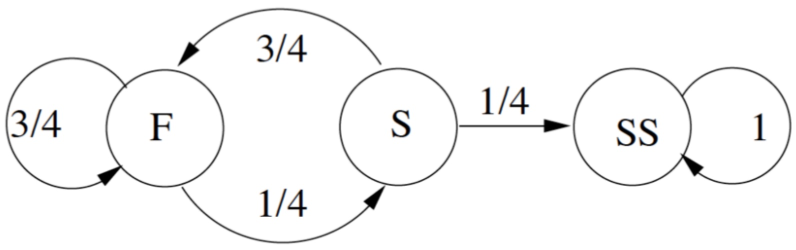
13. Exercise: Time until consecutive successes

Exercise: Time until consecutive successes

3/3 points (ungraded)
Consider a sequence, X_n, of independent Bernoulli random variables with common success probability $p = 1/4$. Let T be the first time at which we have a success immediately following a previous success; that is, $T = \min\{n : X_n = X_{n-1} = \text{success}\}$. We are interested in $\mathbf{E}[T]$. We model this problem using the following Markov chain:



The state S denotes a success, state F denotes a failure, and state SS is an absorbing state denoting the event that we have obtained two successes in a row. Calculate the numerical values of the following quantities.

1.
$\mu_S = \mathbf{E}[T \mid X_0 = S] =$ 16 ✔ **Answer:** 16

2.
$\mu_F = \mathbf{E}[T \mid X_0 = F] =$ 20 ✔ **Answer:** 20

3.
$\mathbf{E}[T] =$ 19 ✔ **Answer:** 19

Solution:

$\mu_S = \mathbf{E}[T \mid X_0 = S]$ and $\mu_F = \mathbf{E}[T \mid X_0 = F]$ are the expected times to absorption starting from states S and F, respectively. We have the following system of equations:

$$\begin{aligned} \mu_S &= 1 + \frac{3}{4}\mu_F \\ \mu_F &= 1 + \frac{3}{4}\mu_F + \frac{1}{4}\mu_S, \end{aligned}$$

and so $\mu_S = 16$ and $\mu_F = 20$. Using the total expectation theorem, we have

$$\begin{aligned} \mathbf{E}[T] &= \mathbf{P}(X_0 = F) \cdot \mathbf{E}[T \mid X_0 = F] + \mathbf{P}(X_0 = S) \cdot \mathbf{E}[T \mid X_0 = S] \\ &= \frac{3}{4} \cdot 20 + \frac{1}{4} \cdot 16 \\ &= 19. \end{aligned}$$

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

主题: Unit 10 / Lec. 26 / 13. Exercise: Time until consecutive successes