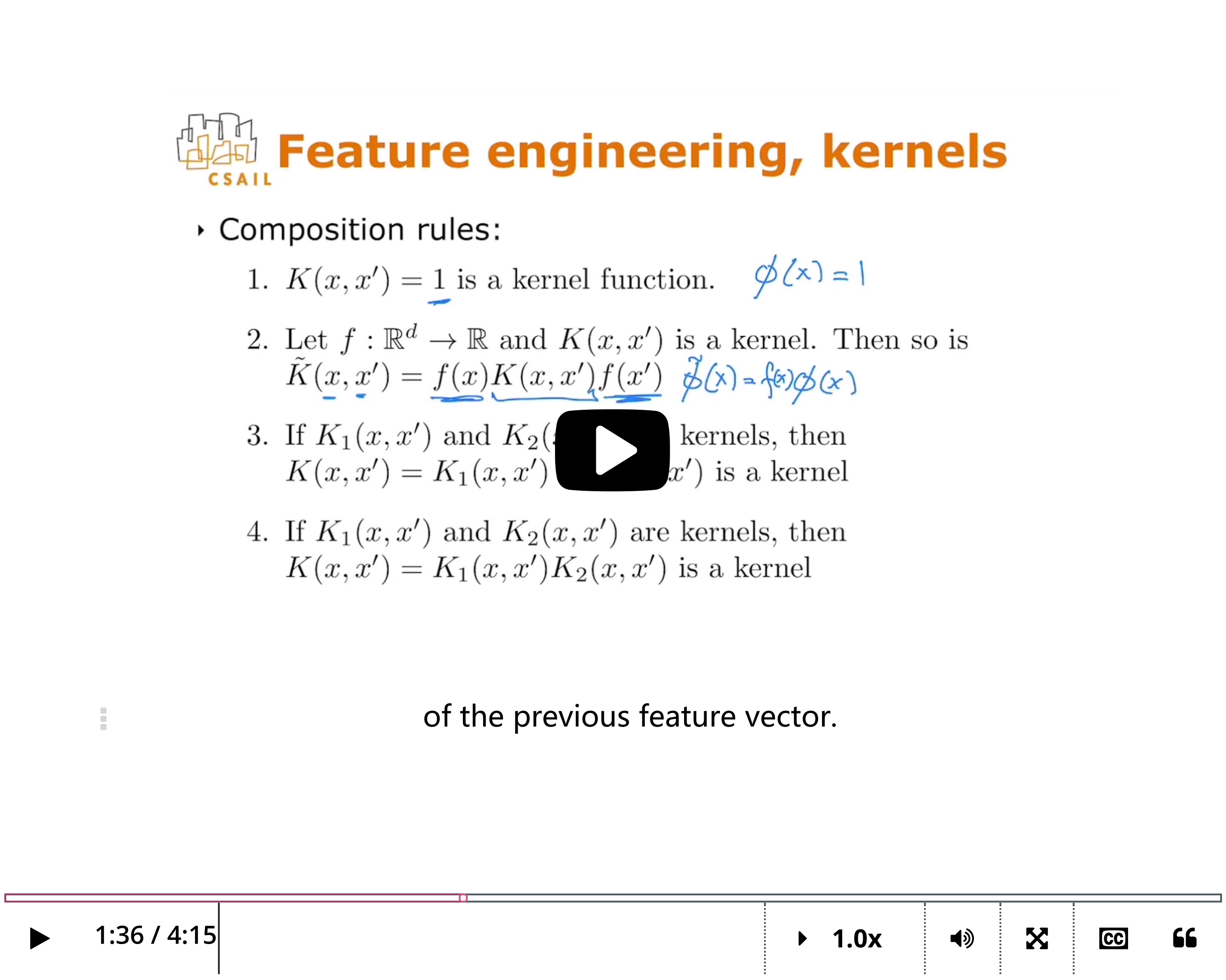
6. Kernel Composition Rules

Kernel Composition Rules



to this kernel, then the new kernel
has a feature vector of phi twiddle, that's
just that scalar function multiplying all the coordinates
of the previous feature vector.
OK?
If I have two kernel functions k 1 and k 2.
So they have their own feature representations.
If I add the two kernels together,
I will get a kernel, a valid kernel function that
now has a feature representation that's just the coordinate
dependent from the two previous kernels.
That means if you take an inner product between these two,
you see that you get two inner products--
one which goes back to the phi 1 representation giving k 1
and added to that as the inner product between the feature

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Kernel Composition Rules 1

1/1 point (graded)

Recall from the video above that if $f : \mathbb{R}^d \to \mathbb{R}$ and $K(x,x')$ is a kernel, so is

$$\widetilde{K}(x,x') = f(x)\,K(x,x')\,f(x').$$

If there exists $\phi(x)$ such that

$$K(x,x') = \phi(x)\cdot\phi(x'),$$

then which of the following φ gives

$$\widetilde{K}(x,x') = \varphi(x)\cdot\varphi(x')?$$

- ○ $\varphi(x) = f(x)\,K(x,x)$
- ○ $\varphi(x) = f(x)\,K(x,x')$

- $\varphi(x) = f(x)$
- $\varphi(x) = f(x)\,\phi(x)$ ✔

Solution:

As $f(x), f(x') \in \mathbb{R}$, we have $(f(x)\,\phi(x)) \cdot (f(x')\,\phi(x')) = \widetilde{K}(x, x')$ Hence $\varphi(x) = f(x)\,\phi(x)$ gives $\widetilde{K}(x, x') = \varphi(x) \cdot \varphi(x')$.

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Answers are displayed within the problem

Kernel Composition Rules 2

1/1 point (graded)
Let x and x' be two vectors of the same dimension. Use the the definition of kernels and the kernel composition rules from the video above to decide which of the following are kernels. (Note that you can use feature vectors $\phi(x)$ that are not polynomial.)
(Choose all those apply.)

- [x] 1 ✔
- [x] $x \cdot x'$ ✔
- [x] $1 + x \cdot x'$ ✔
- [x] $(1 + x \cdot x')^2$ ✔
- [x] $\exp(x + x')$, for x,x' $\in \mathbb{R}$ ✔
- [ ] $\min(x, x')$, for x,x' $\in \mathbb{Z}$

✔

Solution:

We go through the choices in order:

- Yes, for $\phi(x) = 1$.
- Yes, for $\phi(x) = x$.
- Yes, since the sum of kernels are kernels. In this case, we can also easily see $\phi(x) = [1, x]^T$ works.
- Yes, since the product of kernels are kernels. (In this case, factoring the kernel as dot products are more involved, and the composition rule saves this work.)
- Yes, for $\phi(x) = \exp(x)$. $\exp(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$
- No. For example, $\min(-1, -1) = -1 < 0$ and hence cannot be written as a dot product and is not a valid kernel.

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