3. Jeffreys prior

Note: An extra recitation on Jeffreys prior is now available in the tabs after this homework. The concepts discussed may be helpful to you for these homework exercises.

Instructions:

For each of the following statistical models, compute Jeffreys prior distribution and determine whether it is proper or not.

(a)

2/2 points (graded)

For a family of distribution $\{\text{Ber}(p)\}_{p\in(0,1)}$, Jeffreys prior is proportional to:

$\pi_j(p) \propto$ 1/sqrt(p*(1-p)) ✔ **Answer:** p^(-0.5)*(1-p)^(-0.5)

$\frac{1}{\sqrt{p\cdot(1-p)}}$

Therefore, the Jeffreys prior is:

- (●) Proper ✔
- ( ) Improper

Solution:

Recall that $\pi_j \propto \sqrt{\det(I(\theta))}$.

$$I(p) = \frac{1}{p(1-p)}$$

$$\pi_j \propto \frac{1}{\sqrt{p(1-p)}}$$

Therefore, the prior is proper; $\text{Beta}(0.5, 0.5)$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

(b)

2/2 points (graded)

For a family of distribution $\{\text{Exp}(\lambda)\}_{\lambda>0}$, Jeffreys prior is proportional to:

$\pi_j(\lambda) \propto$ 1/lambda ✔ **Answer:** 1/lambda

$\frac{1}{\lambda}$

Therefore, the Jeffreys prior is:

- ○ Proper
- ◉ Improper ✔

Solution:

Recall that $\pi_j \propto \sqrt{det(I(\lambda))}$.

$$I(\lambda) = \frac{1}{\lambda^2}$$

$$\pi_j \propto \frac{1}{\lambda}$$

Since $\frac{1}{\lambda}$ integrates to infinity, the prior is improper.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

(c)

2/2 points (graded)

For a family of distribution $\{\text{Poiss}(\lambda)\}_{\lambda>0}$, Jeffreys prior is proportional to:

$\pi_j(\lambda) \propto$ lambda^(-1/2) ✔ **Answer:** lambda^(-1/2)

$\lambda^{-\frac{1}{2}}$

Therefore, the Jeffreys prior is:

- ○ Proper
- ◉ Improper ✔

Solution:

Recall that $\pi_j \propto \sqrt{det(I(\lambda))}$.

$$I(\lambda) = \frac{1}{\lambda}$$

$$\pi_j \propto \frac{1}{\sqrt{\lambda}}$$

Since $\frac{1}{\sqrt{\lambda}}$ integrates to infinity, the prior is improper.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

(d) Properties of Jeffreys prior

1/1 point (graded)
For each of the statements below about Jeffreys prior, determine whether it is true or false. Select all the true statements.

- [ ] It allows us to reflect our prior belief about the possible hypotheses. In other words, Jeffreys prior is not obtained from the statistical model alone.
- [ ] Jeffreys prior is always proper.
- [x] For a Bernoulli statistical model, the Jeffreys prior $\pi(\theta_1)$, computed from using $\theta_1 = p^2$ as the parameter (i.e. the model is $\text{Ber}(\theta_1) = \text{Ber}(p^2)$), and the Jeffreys prior $\tilde{\pi}(\theta_2)$, computed from using $\theta_2 = p^3$ as the parameter (i.e. the model is $\text{Ber}(\theta_2) = \text{Ber}(p^3)$), satisfy $\mathbf{P}_{\pi(\theta_1)}(a^2 < \theta_1 < b^2) = \mathbf{P}_{\tilde{\pi}(\theta_2)}(a^3 < \theta_2 < b^3)$ for any $0 < a < b < 1$. That is the probability of θ_1 being between a^2 and b^2 under the distribution $\pi(\theta_1)$ is equal to the probability of θ_2 being between a^3 and b^3 under the distribution $\tilde{\pi}(\theta_2)$, for any pair $a < b$ within $(0, 1)$. ✔

✔

Solution:

- **The first choice is false.** Recall that Jeffreys prior is obtained from the model, there is nothing we reflect about our prior belief. Hence, the first choice is false.
- **The second choice is false.** We have seen examples where it is not necessarily a proper prior as it does not have a finite integral.
- **The third choice is true.** The last choice is true because Jeffreys prior is invariant under reparametrization.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

(e) Review: Reparametrization in the frequentist view

1/1 point (graded)
In the previous units, three of the frequentist methods of estimation we've covered are the maxmium likelihood estimation (MLE), the method of moments, and M-estimation. Let our original parameter is θ, and suppose that our original estimator produces a unique estimate θ^*. We then apply a bijective transformation $f(\theta) = \eta$. For which of the three frequentist methods would the estimator applied to the transformed values η^* be equal to $f(\theta^*)$?

- [x] MLE ✔
- [x] method of moments ✔
- [x] M-estimation ✔

✔

这里想说的我理解的是频率理论认为存在一个true parameter
所以每个estimator都对应一个具体的值。
这个值可以在不同的参数下被转换，而且只要原来的estimator吐出的值一样，那么怎么参数化的结果都一样。

Solution:

The answer is that the estimator applied to the transformed values η^* will always be equal to $f(\theta^*)$. This is because in the frequentist approach, a true parameter is assumed and thus all our estimator functions (of the observation data) will correspond to a particular parameter value. This value can be converted through different parametrizations, and it will correspond to the exact same value as long as the original estimator produces a unique estimate.