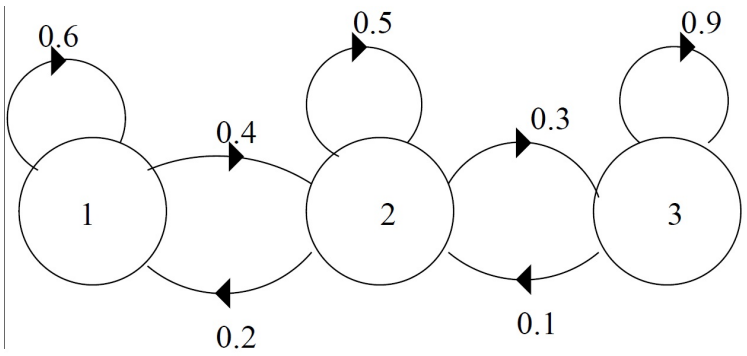
4. A simple Markov chain

Problem 4. A simple Markov chain

10/10 points (ungraded)

Consider a Markov chain $\{X_0, X_1, \ldots\}$, specified by the following transition probability graph.



1. $\mathbf{P}(X_2 = 2 \mid X_0 = 1) =$ 0.44 ✔

2. Find the steady-state probabilities π_1, π_2, and π_3 associated with states 1, 2, and 3, respectively.

   - $\pi_1 =$ 1/9 ✔
   - $\pi_2 =$ 2/9 ✔
   - $\pi_3 =$ 6/9 ✔

3. For $n = 1, 2, \ldots$, let $Y_n = X_n - X_{n-1}$. Thus, $Y_n = 1$ indicates that the nth transition was to the right, $Y_n = 0$ indicates that it was a self-transition, and $Y_n = -1$ indicates that it was a transition to the left.

   $\lim_{n\to\infty} \mathbf{P}(Y_n = 1) =$ 1/9 ✔

4. Is the sequence $Y_1, Y_2, \ldots$ a Markov chain?

   No ✔

5. Given that the nth transition was a transition to the right ($Y_n = 1$), find (approximately) the probability that the state at time $n - 1$ was state 1 (i.e., $X_{n-1} = 1$). Assume that n is large.

   2/5 ✔

6. Suppose that $X_0 = 1$. Let T be the first **positive** time index n at which the state is equal to 1.

   $\mathbf{E}[T] =$ 9 ✔

7. Does the sequence $X_1, X_2, X_3, \ldots$ converge in probability to a constant?

   No ✔

8. Let $Z_n = \max\{X_1, \ldots, X_n\}$. Does the sequence $Z_1, Z_2, Z_3, \ldots$ converge in probability to a constant?

Yes ▼ ✔

提交 你已经尝试了1次（总共可以尝试4次）

讨论

显示讨论

主题： Unit 10 / Problem Set / 4. A simple Markov chain