2. Recap: Maximum Likelihood Estimators and Fisher information

Instructions:

For each of the following distributions, compute the maximum likelihood estimator based on n i.i.d. observations $X_1, \ldots, X_n$ and the Fisher information, if defined. If it is not, enter **DNE** in each applicable input box.

(a)

3/3 points (graded)

$$X_i \sim \text{Ber}(p), \quad p \in (0,1)$$

(Enter **barX_n** for the sample average $\overline{X}_n$

$$\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

Maximum likelihood estimator $\hat{p} =$ barX_n □

Hint: Use the definition of Fisher information that leads to the shorter computation.

(If the Fisher information is not defined, enter **DNE**.)

Fisher information $I(p) =$ 1/(p*(1-p)) □

$\frac{1}{p \cdot (1-p)}$

Use Fisher Information to find the asymptotic variance $V(\hat{p})$ of the MLE $\hat{p}$.

$V(\hat{p}) =$ (p*(1-p)) □

$(p \cdot (1-p))$

STANDARD NOTATION

提交 你已经尝试了2次（总共可以尝试3次）

□ 正确 (3/3 分)

(b)

3/3 points (graded)

$$X_i \sim \text{Poiss}(\lambda), \quad \lambda > 0,$$

which means that each X_i has distribution

$$\mathbf{P}_\lambda(X = k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k \in \mathbb{N}.$$

(Enter **barX_n** for the sample average $\overline{X}_n$.)

Maximum likelihood estimator $\hat{\lambda} =$ barX_n

(If the Fisher information is not defined, enter **DNE**.)

Fisher information $I(\lambda) =$ 1/lambda

$\frac{1}{\lambda}$

Use Fisher Information to find the asymptotic variance $V(\hat{\lambda})$ of the MLE $\hat{\lambda}$.

$V(\hat{\lambda}) =$ lambda

λ

STANDARD NOTATION

提交 你已经尝试了1次（总共可以尝试3次）

正确 (3/3 分)

(c)

3/3 points (graded)

$$X_i \sim \mathsf{Exp}(\lambda), \quad \lambda > 0,$$

which means that each X_1 has density

$$f_\lambda(x) = \lambda e^{-\lambda x}, \quad x > 0.$$

(Enter **barX_n** for $\overline{X}_n$ the sample average.)

Maximum likelihood estimator $\hat{\lambda} =$ 1/barX_n (If the Fisher information is not defined, enter **DNE**.)

Fisher information $I(\lambda) =$ 1/lambda^2

$\frac{1}{\lambda^2}$

Use Fisher Information to find the asymptotic variance $V(\hat{\lambda})$ of the MLE $\hat{\lambda}$.

$V(\hat{\lambda}) =$ lambda^2

λ^2

STANDARD NOTATION

提交 你已经尝试了1次（总共可以尝试3次）

正确 (3/3 分)

(d)

4/7 points (graded)

$$X_i \sim \mathcal{N}\left(\mu, \sigma^2\right), \quad \mu \in \mathbb{R},\, \sigma^2 > 0,$$

which means that each X_1 has density

$$f_{\mu,\sigma^2}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

Hint: Keep in mind that we consider σ^2 as the parameter, not σ. You may want to write $\tau = \sigma^2$ in your computation.

(Enter **barX_n** for the sample average $\overline{X}_n$ and **bar(X_n^2)** for the sample average of second moments $\overline{X_n^2}$.)

Maximum likelihood estimator $\hat{\mu} =$ barX_n

(Enter **barX_n** for the sample average $\overline{X}_n$ and **bar(X_n^2)** for the sample average of second moments $\overline{X_n^2}$.)

Maximum likelihood estimator $\widehat{\sigma^2} =$ bar(X_n^2) - barX_n^2

Hint: One of the formulas for Fisher information will lead to a much shorter computation.

(If the Fisher information is not defined, enter **DNE** for all boxes below.)

$[I(\mu,\sigma^2)]_{1,1} =$ 1/(2*sigma^2)

$\frac{1}{2\cdot\sigma^2}$

$, [I(\mu,\sigma^2)]_{1,2} =$ 0

0

$[I(\mu,\sigma^2)]_{2,1} =$ 0

0

$, [I(\mu,\sigma^2)]_{2,2} =$ 1/sigma^4

$\frac{1}{\sigma^4}$

Using the Fisher Information you obtain above, what is the asymptotic variance $V(\widehat{\sigma^2})$ of the MLE $\widehat{\sigma^2}$?Compare this with your result from Homework 5 Problem 3.

$V(\widehat{\sigma^2}) =$ sigma^4

σ^4

STANDARD NOTATION

提交 你已经尝试了1次（总共可以尝试3次）

部分正确 (4/7 分)

(e)

5/6 points (graded)

X_i follows a shifted exponential distribution with parameters $a \in \mathbb{R}$ and $\lambda > 0$. That means each X_i has density

$$f_{a,\lambda}(x) = \lambda e^{-\lambda(x-a)} \mathbf{1}\{x \geq a\}, \quad x \in \mathbb{R}.$$

(Enter **barX_n** for the sample average $\overline{X}_n$, and if applicable, use **min_i(X_i)** for $\min_{1 \leq i \leq n} X_i$).

Maximum likelihood estimator $\hat{a} =$ min_i(X_i)

Maximum likelihood estimator $\hat{\lambda} =$ min_i(X_i)

Hint: Think of the effect of the indicator function on the derivatives. (If the Fisher information is not defined, enter **DNE** in all boxes below.)

$[I(a,\lambda)]_{1,1} =$ DNE

DNE

$, [I(a,\lambda)]_{1,2} =$ DNE

DNE

$[I(a,\lambda)]_{2,1} =$ DNE

DNE

$, [I(a,\lambda)]_{2,2} =$ DNE

DNE

STANDARD NOTATION

提交 你已经尝试了1次（总共可以尝试3次）

部分正确 (5/6 分)

讨论

隐藏讨论

主题： Unit 3 Methods of Estimation:Homework 6 Maximum Likelihood Estimation and Method of Moments / 2. Recap: Maximum Likelihood Estimators and Fisher information

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(d) & (e) : MLE plug in ?	3
Caveat d) input bar(X_n^2)	4
[STAFF] Very minor typo in "2. Recap: Maximum Likelihood Estimators and Fisher information"	2
Doubt about asymptotic variance To compute the asymptotic variance, should we follow the same procedure as in last HW. or is there another approach which I haven't detected yet?	2
Doubts about part e, MLE for lambda	7
a) fisher This question might sound silly to you, but I'm stuck on it. For part a), are we supposed to compute the fisher info based on X_i or based on X_n ?	5
[Staff, Typo] density 社区助教	2
[Staff] Part (e) Should be a and λ instead of μ and σ^2	2
[Staff, Typo] varianace Part (a), last line. 社区助教	2