

6. Goodness of Fit Test of Continuous Distributions: Kolmogorov-Smirnov Test

Kolmogorov-Smirnov Test

Kolmogorov-Smirnov test

- Let $T_n = \sup_{t \in \mathbb{R}} \sqrt{n} [F_n(t) - F^0(t)]$.
- By Donsker's theorem, if H_0 is true, then $T_n \xrightarrow[n \rightarrow \infty]{(d)} Z$, where Z has a known distribution (supremum of a Brownian bridge).
- KS test with asymptotic level α :

$$\delta_\alpha^{KS} = \mathbb{P}(T_n > q_\alpha),$$
 where q_α is the $(1 - \alpha)$ -quantile of Z (obtained in tables).

So we used that.

you have tables, OK?

So this is a asymptotic test, all right?

So this test will have asymptotic level alpha, all right?

We used an asymptotic statement.

What was the asymptotic statement that we used?

Donsker's theorem, OK?

That was the equivalent of--

that's the uniform central limit theorem.

So we used that.



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Concept Check: Goodness of Fit Testing

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Let X_1, \dots, X_n be i.i.d. random variables with unknown cdf F . We will use the tools of goodness of fit testing to test if $X_1, \dots, X_n \stackrel{iid}{\sim} \mathcal{N}(0, 1)$. Let Φ denote the cdf of a standard normal.

Accordingly, you set the null and alternative hypotheses to be, respectively,

$$H_0 : F = \Phi$$

$$H_1 : F \neq \Phi.$$

If the null hypothesis holds and n is very large, you expect the empirical cdf $F_n(t)$ and standard normal cdf $\Phi(t)$ to be...

☒ Similar ✓

☐ Dissimilar

Solution:

We know by the Glivenko-Cantelli theorem that

$$\sup_{t \in \mathbb{R}} |F_n(t) - F(t)| \xrightarrow[n \rightarrow \infty]{a.s.} 0.$$

In particular, for fixed t ,

$$\lim_{n \rightarrow \infty} F_n(t) = F(t).$$

If the null hypothesis holds, then $F(t) = \Phi(t)$, and we have

$$\sup_{t \in \mathbb{R}} |F_n(t) - \Phi(t)| \xrightarrow[n \rightarrow \infty]{a.s.} 0.$$

Therefore, if n is sufficiently large, this uniform convergence guarantees that the empirical cdf $F_n(t)$ and standard normal cdf $\Phi(t)$ are ‘close’ as functions of t . Thus, the correct response for this problem is ‘Similar.’

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You have used 1 of 1 attempt

 Answers are displayed within the problem

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