

1. Normal random variables

Problem 1. Normal random variables

5/5 points (graded)

Let \mathbf{X} and \mathbf{Y} be two normal random variables, with means $\mathbf{0}$ and $\mathbf{3}$, respectively, and variances $\mathbf{1}$ and $\mathbf{16}$, respectively. Find the following, using the standard normal table. Express your answers to an accuracy of **3 decimal places**.

Standard Normal Table

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1.

$$\mathbf{P}(\mathbf{X} > -1) = \boxed{0.8413} \quad \checkmark \text{ Answer: } 0.841$$

2.

$$\mathbf{P}(\mathbf{X} \leq -2) = \boxed{0.0228} \quad \checkmark \text{ Answer: } 0.023$$

3. Let $\mathbf{V} = (4 - \mathbf{Y})/3$. Find the mean and the variance of \mathbf{V} .

$$\mathbf{E}[\mathbf{V}] = \boxed{1/3} \quad \checkmark \text{ Answer: } 1/3$$

$$\mathbf{Var}(\mathbf{V}) = \boxed{16/9} \quad \checkmark \text{ Answer: } 16/9$$

4.

$$\mathbf{P}(-2 < \mathbf{Y} \leq 2) = \boxed{0.2957} \quad \checkmark \text{ Answer: } 0.2957$$

Solution:

1. Since the distribution of \mathbf{X} is symmetric around $\mathbf{0}$, we have,

$$\mathbf{P}(\mathbf{X} > -1) = \mathbf{P}(\mathbf{X} < 1) = \Phi(1) = 0.841.$$

2. Using symmetry again,

$$\mathbf{P}(X \leq -2) = \mathbf{P}(X > 2) = 1 - \mathbf{P}(X < 2) = 1 - \Phi(2) = 0.023.$$

3. We have $\mathbf{E}[V] = 4/3 - \mathbf{E}[Y]/3 = 1/3$, and $\mathbf{Var}(V) = \frac{1}{3^2} \mathbf{Var}(16) = 16/9$.

4. By standardizing Y , and using the normal table, we have

$$\begin{aligned} \mathbf{P}(-2 < Y \leq 2) &= \mathbf{P}\left(\frac{-2-3}{4} \leq \frac{Y-3}{4} \leq \frac{2-3}{4}\right) \\ &= \mathbf{P}(-5/4 \leq Z \leq -1/4) \\ &= \mathbf{P}(1/4 \leq Z \leq 5/4) \\ &= \Phi(5/4) - \Phi(1/4) \\ &\approx 0.8944 - 0.5987 \\ &= 0.2957, \end{aligned}$$

where, Z is a standard normal random variable.

提交

You have used 2 of 3 attempts

i Answers are displayed within the problem

讨论

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