

In this lecture, we will concentrate on the study of Markov chains in the long run, and study under what conditions a Markov chain exhibits steady-state behavior, and under what conditions such steady-state behavior is independent of the initial starting state. More precisely, we will look at long-term state occupancy behavior-- that is, in the n -step transition probabilities when n is large.

So assume that we have a Markov chain which is initially in a given state i , and consider the probability that the chain is in a specific state j after n transitions. Question-- does that probability converge to some constant when n goes to infinity? And if this is the case-- second question-- can this constant be independent of the initial state i ?

We will see that for nice Markov chains, the answers to both questions will be yes. How to characterize nice Markov chains? We will use several new concepts, one dealing with a Markov chain being aperiodic or not, and the other with the notion of recurrent classes.

Without going into details now, let us simply mention that we will show that the existence of convergence will be tied to having an aperiodic Markov chain. And in case we have convergence, the independence from the initial state will be tied to having a single recurrent class.

We will end this lecture by looking in detail at the special and important class of Markov chains usually known as birth-death processes.