9. Eigenvalues, Eigenvectors and Determinants(Optional)

Eigenvalues and Eigenvectors of a matrix (Optional)

0 points possible (ungraded)

Let $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$\mathbf{Av} = \lambda_1 \mathbf{v}$, where $\lambda_1 =$

3

✔ Answer: 3 .

$\mathbf{Aw} = \lambda_2 \mathbf{w}$, where $\lambda_2 =$

2

✔ Answer: 2 .

Therefore, $\mathbf{v}$ is an eigenvector of $\mathbf{A}$ with eigenvalue λ_1, and $\mathbf{w}$ is an eigenvector of $\mathbf{A}$ with eigenvalue λ_2.

Solution:

$$\mathbf{Av} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} \implies \lambda_1 = 3$$

$$\mathbf{Aw} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \implies \lambda_2 = 2$$

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Answers are displayed within the problem

Geometric Interpretation of Eigenvalues and Eigenvectors (Optional)

0 points possible (ungraded)

Let $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Recall from the previous exercise that $\mathbf{v}$ and $\mathbf{w}$ are eigenvectors of $\mathbf{A}$.

Suppose $\mathbf{x} = \mathbf{v} + 2\mathbf{w} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Then $\mathbf{Ax} = s\mathbf{v} + t\mathbf{w}$, where:

$s =$ 3

✔ Answer: 3

and

$t =$ 4

✔ Answer: 4 .

In particular, s describes the amount that $\mathbf{A}$ stretches $\mathbf{x}$ in the direction of $\mathbf{v}$, and $\frac{t}{2}$ (note the "2" in front of $\mathbf{w}$ in $\mathbf{x}$) describes the amount that $\mathbf{A}$ stretches $\mathbf{x}$ in the direction of $\mathbf{w}$.

Solution:

We have

$$\begin{aligned}\mathbf{Ax} &= \mathbf{A}(\mathbf{v}+2\mathbf{w}) \\ &= \mathbf{Av}+2\mathbf{Aw} \\ &= (3\mathbf{v})+2(2\mathbf{w}) \\ &= 3\mathbf{v}+4\mathbf{w}.\end{aligned}$$

From this, we get $s=3, t=4$.

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Answers are displayed within the problem

Determinant and Eigenvalues (optional)

0 points possible (ungraded)
Recall that the **determinant** of a matrix indicates whether it is singular. For 2×2 matrices, it has the formula

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

but for larger matrices, the formula is more complicated.

What is the determinant of the matrix $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix}$?

6 **Answer:** 6

On the other hand, what is the product of the eigenvalues λ_1, λ_2 of $\mathbf{A}$? (We already computed this in the previous exercises.)

6 **Answer:** 6

Solution:

Plugging into the formula directly gives $3 \cdot 2 - 0 \cdot \frac{1}{2} = 6$. On the other hand, the eigenvalues are $\lambda_1 = 3$, $\lambda_2 = 2$, so the product is 6. This is not a coincidence; for general $n \times n$ matrices, the **product of the eigenvalues is always equal to the determinant**.

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Answers are displayed within the problem

Trace and Eigenvalues

0 points possible (ungraded)
Recall that the **trace** of a matrix is the sum of the diagonal entries.

What is the trace of the matrix $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ \frac{1}{2} & 2 \end{pmatrix}$?

5 **Answer:** 5

On the other hand, what is the sum of the eigenvalues λ_1, λ_2 of $\mathbf{A}$? (We already computed this in the previous exercises.)

5 **Answer:** 5

Solution:

The diagonal sum is $3+2=5$. On the other hand, the eigenvalues are $\lambda_1 = 3$, $\lambda_2 = 2$, so the sum is 5. Just like the determinant, this is also not a coincidence. For general $n \times n$ matrices, the **sum of the eigenvalues is always equal to the trace of the matrix**.

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Answers are displayed within the problem

Nullspace (Optional)

0 points possible (ungraded)

If a (nonzero) vector is in the nullspace of a square matrix $\mathbf{A}$, is it an eigenvector of $\mathbf{A}$?

yes ✔ **Answer:** yes

Which of the following are equivalent to the statement that 0 is an eigenvalue for a given square matrix $\mathbf{A}$? (Choose all that apply.)

- [x] There exists a nonzero solution to $\mathbf{A}\mathbf{v} = \mathbf{0}$. ✔
- [x] $\det(\mathbf{A}) = 0$ ✔
- [ ] $\det(\mathbf{A}) \neq 0$
- [x] $\mathrm{NS}(\mathbf{A}) = \mathbf{0}$
- [ ] $\mathrm{NS}(\mathbf{A}) \neq \mathbf{0}$ ✔

✖

Solution:

- If a vector $\mathbf{v}$ is in the nullspace of $\mathbf{A}$, then $\mathbf{A}\mathbf{v} = \mathbf{0} = (0)\,\mathbf{v}$. So it is an eigenvector of $\mathbf{A}$ associated to the eigenvalue 0.
- If 0 is an eigenvalue for a matrix $\mathbf{A}$, then by definition, there exists a nonzero solution to $\mathbf{A}\mathbf{v} = \mathbf{0}$; that is, $\mathrm{NS}(\mathbf{A}) \neq \mathbf{0}$, and this only happens if and only if $\det(\mathbf{A}) = 0$.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 0. Course Overview, Syllabus, Guidelines, and Homework on Prerequisites:Homework 0: Probability and Linear algebra Review / 9. Eigenvalues, Eigenvectors and Determinants(Optional)