We need to apply the version of Bayes' rule for a continuous random variable conditioned on a discrete random variable:

$$f_{Q|X}(q \mid x) \;=\; \frac{f_Q(q)p_{X|Q}(x \mid q)}{p_X(x)} \;=\; \frac{f_Q(q)p_{X|Q}(x \mid q)}{\int_0^1 f_Q(q)p_{X|Q}(x \mid q)\,dq}.$$

For $x = 0$ and $q \in [0,1]$,

$$\begin{aligned}
f_{Q|X}(q \mid 0) &= \frac{f_Q(q)p_{X|Q}(0 \mid q)}{\int_0^1 f_Q(q)p_{X|Q}(0 \mid q)\,dq} \;=\; \frac{6q(1-q)\cdot(1-q)}{\int_0^1 6q(1-q)(1-q)\,dq} \\
&= \frac{6q(1-q)\cdot(1-q)}{1/2} \;=\; 12q(1-q)^2.
\end{aligned}$$

For $x = 1$ and $q \in [0,1]$,

$$\begin{aligned}
f_{Q|X}(q \mid 1) &= \frac{f_Q(q)p_{X|Q}(1 \mid q)}{\int_0^1 f_Q(q)p_{X|Q}(1 \mid q)\,dq} \;=\; \frac{6q(1-q)\cdot q}{\int_0^1 6q(1-q)q\,dq} \\
&= \frac{6q(1-q)\cdot q}{1/2} \;=\; 12q^2(1-q).
\end{aligned}$$

The distributions $f_Q(q)$, $f_{Q|X}(q \mid 0)$, and $f_{Q|X}(q \mid 1)$ are all in the family of *Beta distributions.*