The Bayes rule — two continuous random variables

- X, Y independent; $Z = X + Y$

  $Z = x + Y$

  $z = x + z - x$

  – Find $f_{X|Z}(x \,|\, z)$

- First find $\boxed{f_{Z|X}(z \,|\, x) = f_Y(z - x)}$

$$f_{X|Y}(x \,|\, y) = \frac{f_X(x)\, f_{Y|X}(y \,|\, x)}{f_Y(y)}$$

$$\underline{f_{X|Z}(x \,|\, z)} = \frac{f_X(x)\, f_{Z|X}(z \,|\, x)}{f_Z(z)}$$

$$P(Z \le z \,|\, X = x) = P(X + Y \le z \,|\, X = x)$$

$$= P(x + Y \le z \,|\, X = x)$$

$$= P(x + Y \le z) = P(Y \le z - x) = F_Y(z - x)$$

$$d/dy \quad = f_Y(z - x)$$

The Bayes rule — two continuous random variables

- X, Y independent; $Z = X + Y$

- Assume X and Y are exponential with parameter λ

$$f_{X|Z}(x\,|\,z) = \frac{f_X(x)\, f_{Z|X}(z\,|\,x)}{f_Z(z)}$$

$$f_{Z|X}(z\,|\,x) = f_Y(z - x)$$

$f_X(x) = \lambda e^{-\lambda x}, \quad x \geq 0$

$f_Y(y) = \lambda e^{-\lambda y}, \quad y \geq 0$

Fix some $z \geq 0$:

Uniform on [0,z]!

$$f_{X|Z}(x|z) = \frac{1}{f_Z(z)} \lambda e^{-\lambda x} \cdot \lambda e^{-\lambda(z-x)} = \boxed{\frac{1}{f_Z(z)} \lambda^2 e^{-\lambda z}, \quad 0 \leq x \leq z}$$

$x \geq 0$

$z - x \geq 0$

$x \leq z$