课程 > Unit 7: Bayesian inf... > Lec. 14: Introductio... > 16. Exercise: Mome...

16. Exercise: Moments of the Beta distribution

Exercise: Moments of the Beta distribution

2/2 points (graded)

Suppose that Θ takes values in [0,1] and its PDF is of the form

$$f_{\Theta}(heta) = a heta(1- heta)^2, \ \ ext{ for } heta \in [0,1],$$

where \boldsymbol{a} is a normalizing constant.

Use the formula

$$\int_0^1 heta^lpha (1- heta)^eta \, d heta = rac{lpha! \, eta!}{(lpha+eta+1)!}$$

to find the following:

a)
$$a = \begin{bmatrix} 12 \\ \checkmark \text{ Answer: } 12 \end{bmatrix}$$

b)
$$\mathbf{E}[\Theta^2] = \begin{bmatrix} 1/5 \end{bmatrix}$$
 \checkmark Answer: 0.2

Solution:

a) Let $I(\alpha, \beta)$ be the integral in the formula given in the problem statement. The normalizing constant must be equal to 1/I(1,2): this is needed for the PDF to integrate to 1. We have I(1,2)=2!/4!=1/12, so that a=12.

b)

$$\mathbf{E}[\Theta^2] = \int_0^1 heta^2 f_\Theta(heta) \, d heta = \int_0^1 a heta^3 (1- heta)^2 \, d heta = a \cdot \mathrm{I}(3,2) = 12 \cdot rac{3! \, 2!}{6!} = rac{1}{5}.$$