Let us view T as the first arrival time in a new, independent, Poisson process with parameter μ, and merge this process with the original Poisson process. Each arrival in the merged process comes from the original Poisson process with probability $\lambda/(\lambda+\mu)$, independent of other arrivals. If we view each arrival in the merged process as a trial, and an arrival from the new process as a success, we note that the number K of trials/arrivals until the first success has a geometric PMF, of the form

$$p_K(k) = \left(\frac{\mu}{\lambda+\mu}\right)\left(\frac{\lambda}{\lambda+\mu}\right)^{k-1}, \qquad k = 1, 2, \ldots.$$

Now the number N_T of arrivals from the original Poisson process until the first "success" is equal to $K-1$ and its PMF is

$$p_{N_T}(\ell) = p_K(\ell+1) = \left(\frac{\mu}{\lambda+\mu}\right)\left(\frac{\lambda}{\lambda+\mu}\right)^{\ell}, \qquad \ell = 0, 1, \ldots$$