

Linear Algebra

HW3 Cosine Transform

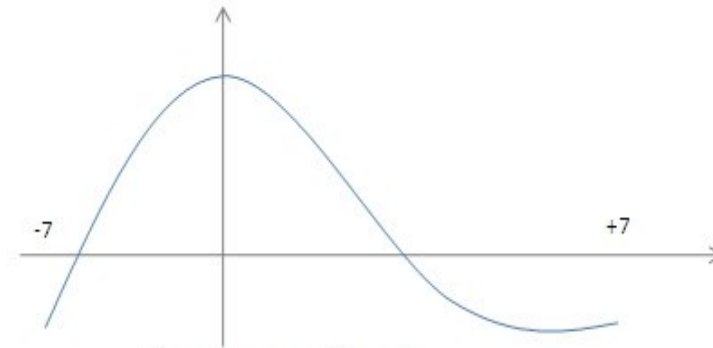
吳思霖

Outline

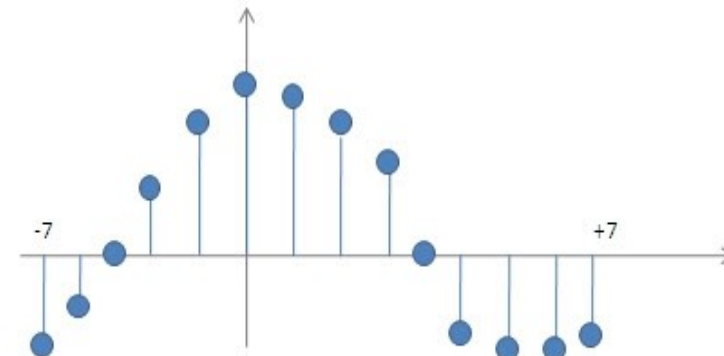
- What is signal
- Fourier Transform
- Cosine Transform
- HW3
 - Input&output
 - Code&util function
 - Rules

What is signal

- 傳遞有關一些現象的行為或屬性的資訊的函數
- Example
 - $f(t)$: 音訊
 - $f(x, y)$: 圖片
- Type
 - Continuous
 - Discrete
- In this homework, we use
 - discrete signal



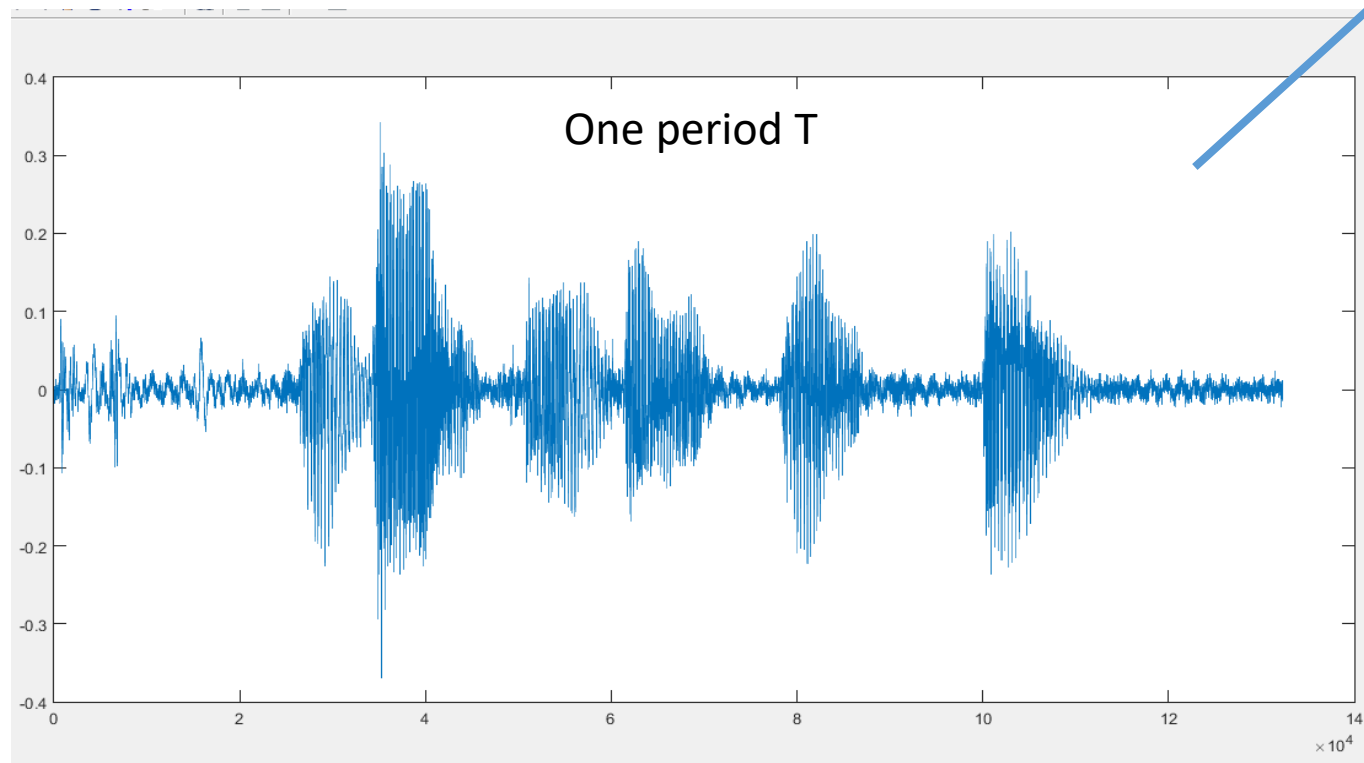
Continuous Signal
(takes values in the set $[-7, 7]$)



Discrete Signal
(takes values at the integers $\{-7, -6, \dots, 0, \dots, 6, 7\}$)

Basis

- How to use basis in signal analysis
- Given a speech signal
 - Can we find basis to describe this signal ?

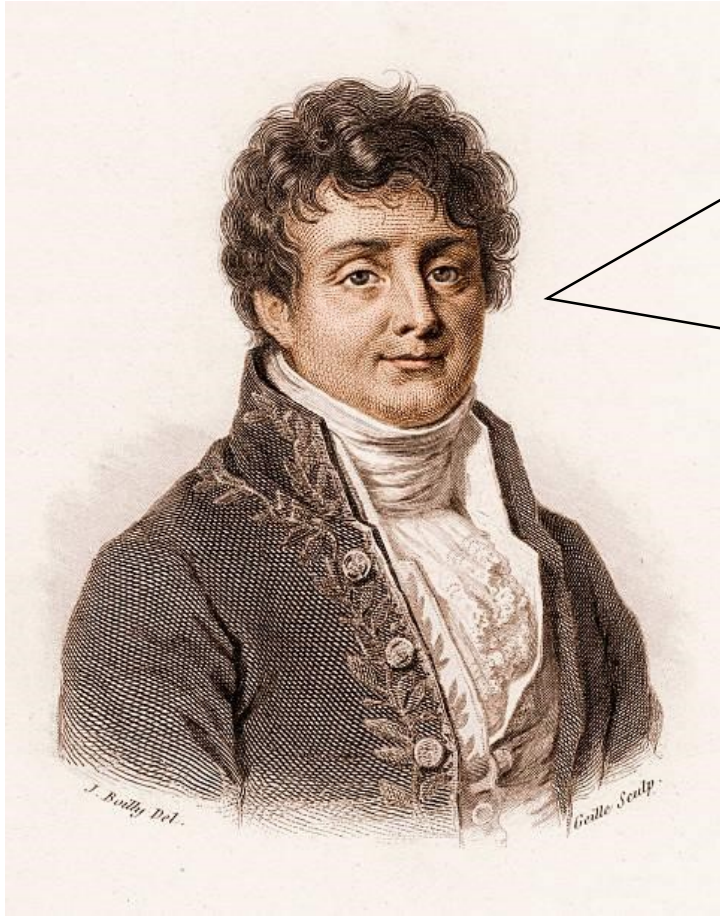


$$\mathbf{x} = [x_0, x_1, \dots, x_{N-1}]$$

$$\mathbf{B} = \{\mathbf{b}_0, \mathbf{b}_1, \dots, \mathbf{b}_{N-1}\}$$
$$[\mathbf{x}]_{\mathbf{B}} = [a_0, a_1, \dots, a_{N-1}] = \mathbf{a}$$
$$\mathbf{x} = \sum_{k=0}^{N-1} a_k \mathbf{b}_k$$

exists **b** that is easy to analysis?
(basis is also a signal)

Joseph Fourier

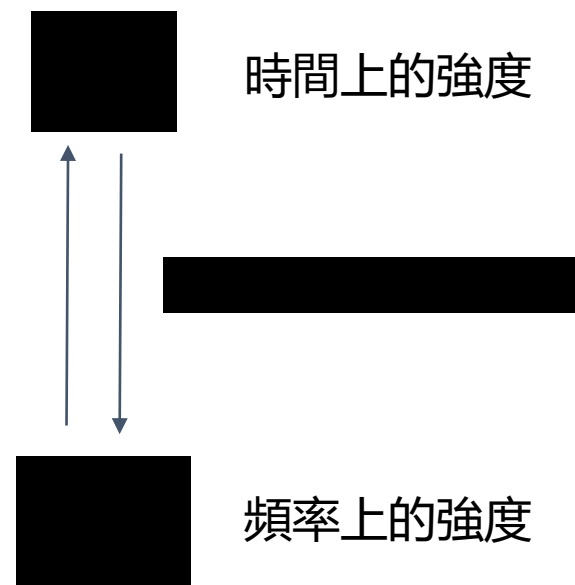
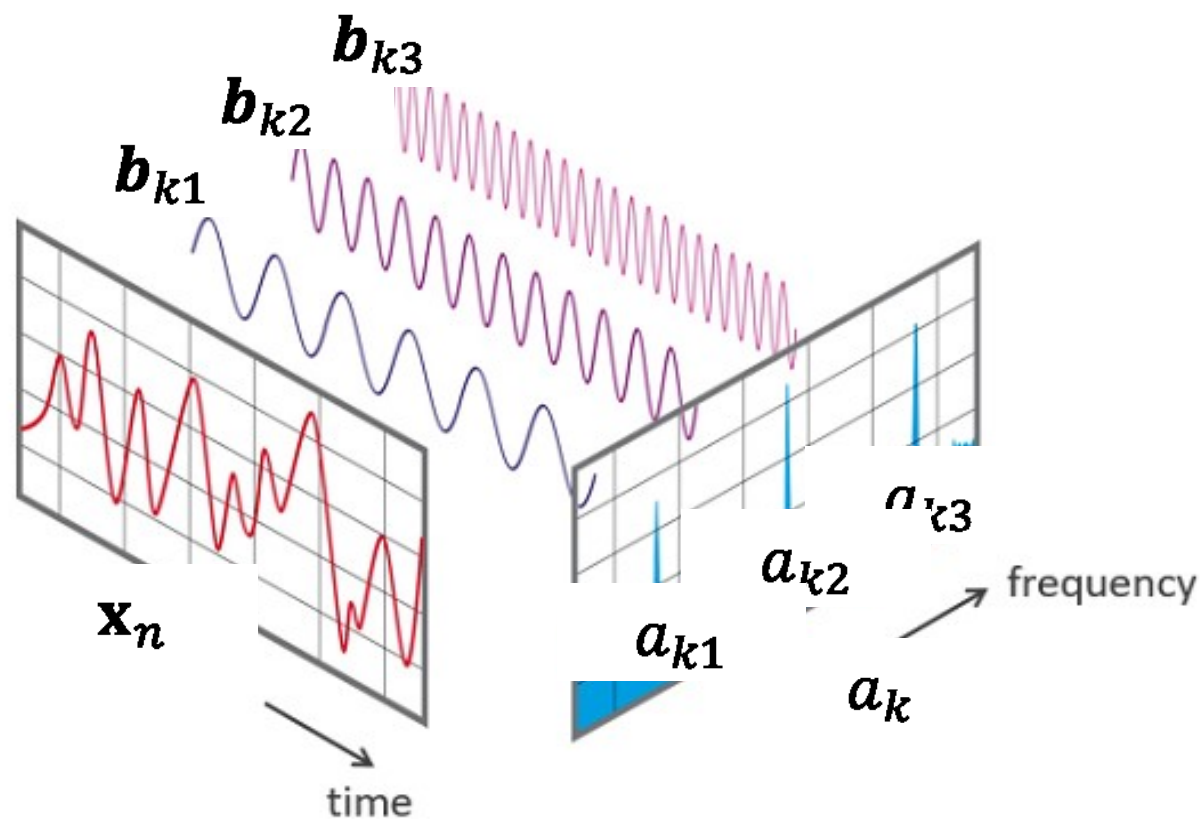


Any **periodic** signal
can be represented
as a sum of
sinusoids.

Fourier Transform

假設 x 主要由三個basis vector組成
→三個頻率的cosine signal組成

$$\mathbf{x} = a_{k1}\mathbf{b}_{k1} + a_{k2}\mathbf{b}_{k2} + a_{k3}\mathbf{b}_{k3} + \dots$$
$$0 \leq k1, k2, k3 \leq N - 1$$



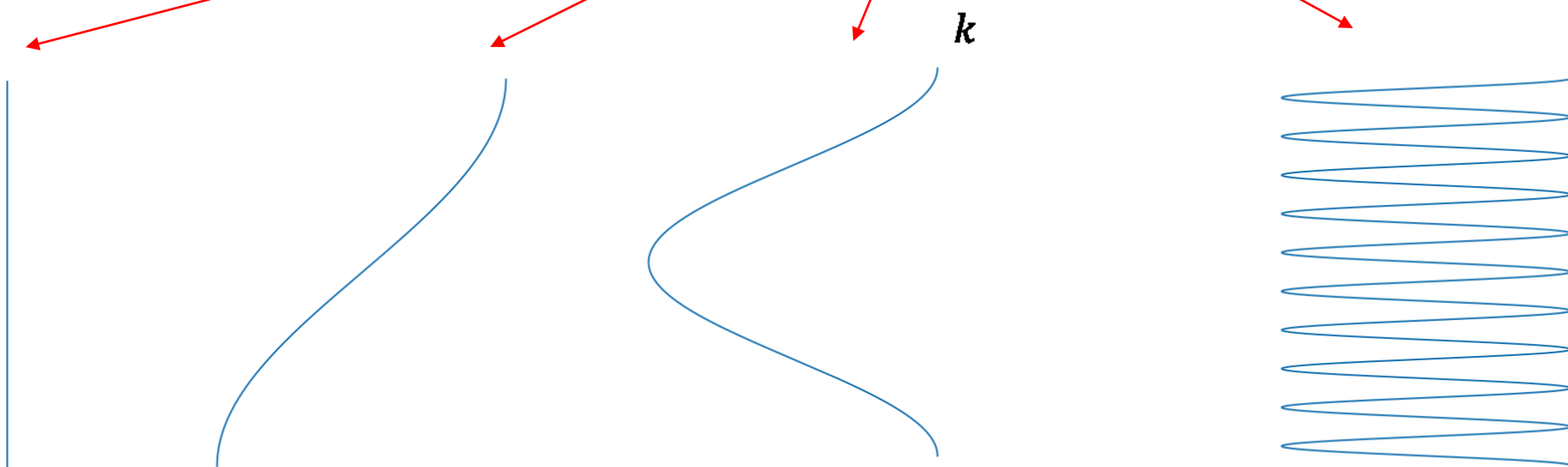
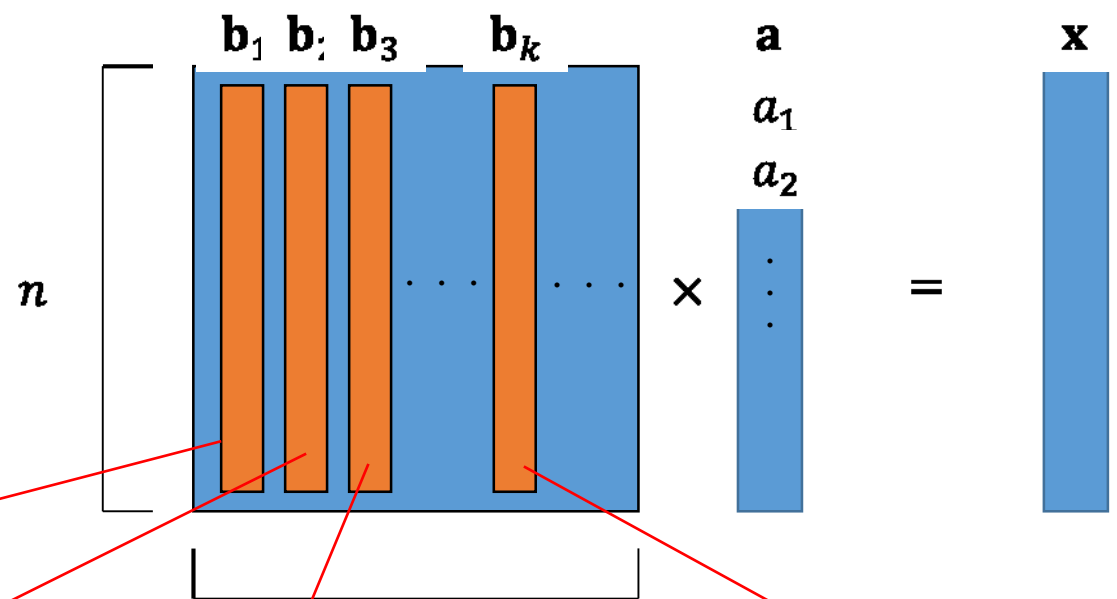
Cosine Transform

- Fourier Transform includes complex number computation
 - We use cosine transform instead
- Cosine Transform Formula
 - Given a discrete signal $x = [x_0, x_1, \dots, x_n, \dots, x_{N-1}]$ with N length
 - Basis Matrix:
- $\mathbf{B} = \{\mathbf{b}_0, \mathbf{b}_1, \dots, \mathbf{b}_{N-1}\}$

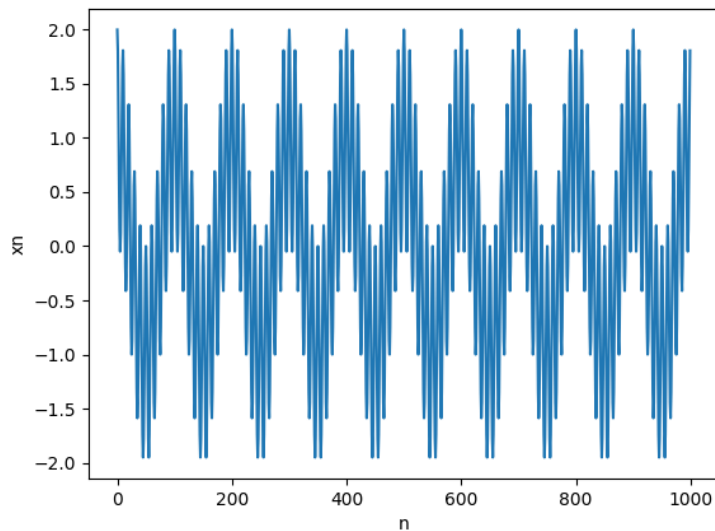
$$\mathbf{b}_{k,n} = \begin{cases} \frac{1}{\sqrt{N}}, & \text{if } k = 0 \\ \frac{\sqrt{2}}{\sqrt{N}} \cos \frac{(n + 0.5)k\pi}{N}, & \text{else} \end{cases} \quad \xrightarrow{\text{How to get a}} \quad \begin{aligned} \mathbf{x} &= \mathbf{B}\mathbf{a} \\ \mathbf{a} &= \mathbf{B}^{-1}\mathbf{x} \end{aligned}$$

n: 時間上的index
k: 頻率上的index

$$\mathbf{b}_{k,n} = \begin{cases} \frac{1}{\sqrt{N}}, & \text{if } k = 0 \\ \frac{\sqrt{2}}{\sqrt{N}} \cos \frac{(n + 0.5)k\pi}{N}, & \text{else} \end{cases}$$

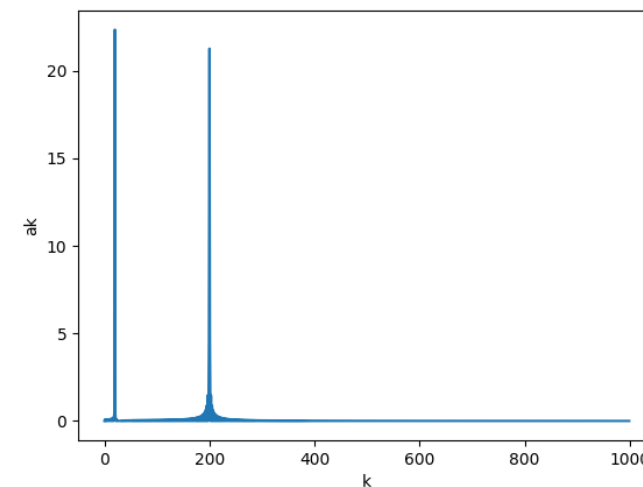


If we want to get the low frequency signal of a mixed signal...



Cosine transform

$$\mathbf{a} = \mathbf{B}^{-1}\mathbf{x}$$



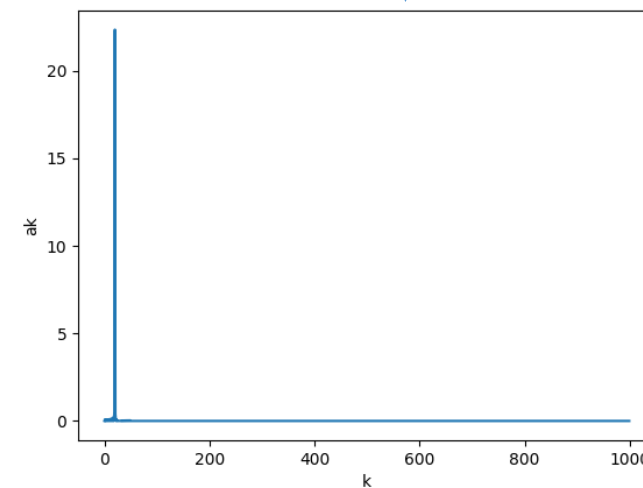
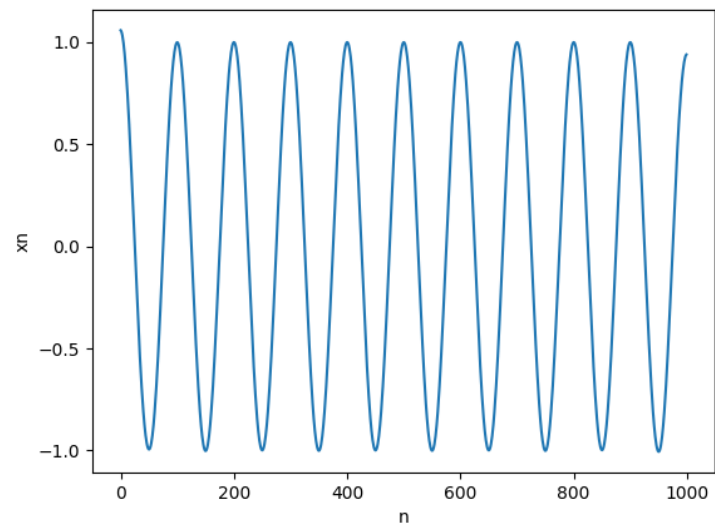
mask [1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
...]

x

↓

Inverse cosine transform

$$\mathbf{x} = \mathbf{B}\mathbf{a}$$



Application

- Filter
 - Human voice
 - Man: 85-180Hz
 - Woman: 165-255Hz
- Remove high frequency noise from speech signal

Input & Output

- $x = \sum_{i=1}^5 \text{Cosine}(2\pi f_i)$
 - $f_1 < f_2 < f_3 < f_4 < f_5$
- Input data
 - Please download the file of <student_id>.txt
 - Total 1000 lines, one value per line.
- **Output f_1 and f_3 signal to following file**
 - f_1 : <student_id>_f1.txt (ex: b01901118_f1.txt)
 - f_3 : <student_id>_f3.txt
 - Use **numpy.savetxt** to output the answer signal
 - Same format as input: 1000 lines
- **Output the picture of a of input signal**
 - <student_id>_freq.png
 - Use util function
- You can use test.txt for testing.
 - Only two cosine signal with different freq are mixed.

```
1 2.0000000000000000e+00
2 1.807043722803219010e+00
3 1.301131695689425438e+00
4 6.732702563537411589e-01
5 1.595661667536837358e-01
6 -4.894348370484646882e-02
7 1.207594915133041180e-01
8 5.958100580910720145e-01
9 1.185323674418810924e+00
10 1.653344919876962527e+00
11 1.809016994374947451e+00
12 1.579530237150736927e+00
13 1.037985621796358338e+00
14 3.755301115537416079e-01
15 -1.715930046262574837e-01
16 -4.122147477075268629e-01
17 -2.731901993959511277e-01
18 1.727366797267690379e-01
19 7.347962859400198887e-01
20 1.177141547059625148e+00
21 1.309016994374947451e+00
22 1.057706881539801413e+00
23 4.963983089606727184e-01
24 -1.836837608106426378e-01
25 -7.462264748456348684e-01
26 -9.99999999999998890e-01
27 -8.718075139042609223e-01
28 -4.343502279392522092e-01
29 1.216356797892221842e-01
30 5.603271072100921568e-01
31 6.909830056250526598e-01
32 4.408924416902699206e-01
33 -1.167622971901245421e-01
34 -7.907706684766655503e-01
```

Code & Data link

- [Link](#)
- Code
 - hw3.py
- Data
 - <student_id>.txt
 - test.txt

HW3 Code

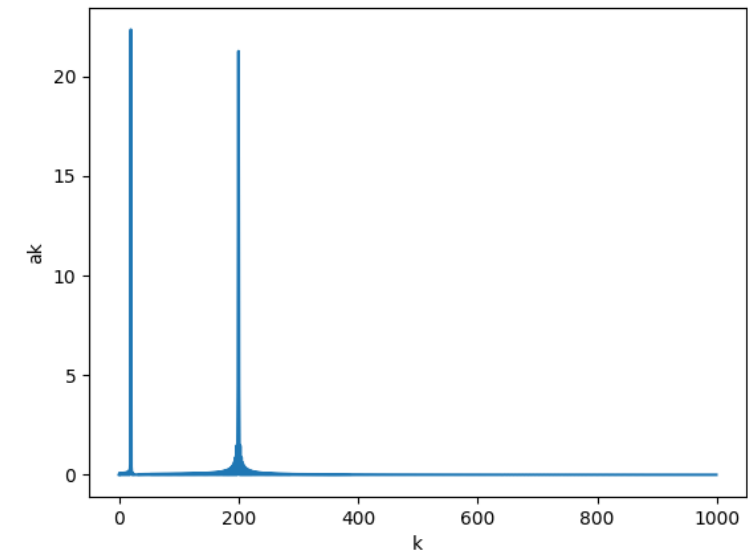
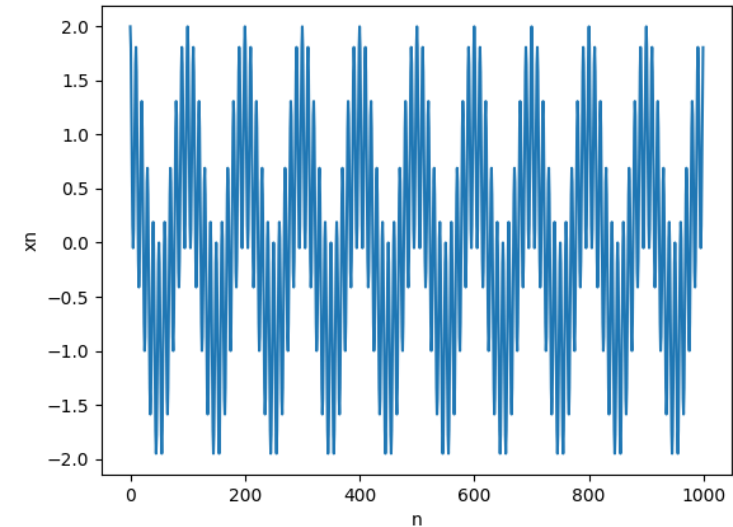
```
5 def CosineTrans(x, B):  
6     # TODO  
7     # implement cosine transform  
8     return  
9  
10 def InvCosineTrans(a, B):  
11     # TODO  
12     # implement inverse cosine transform  
13     return  
14  
15 def gen_basis(N):  
16     # TODO  
17     return  
18  
19 if __name__ == '__main__':  
20  
21     signal_path = sys.argv[1]
```

Run the code

- `python hw3.py <input_signal_txt>`
- Your code should generate 2 txt files and 1 png file.
 - `<student_id>_f1.txt`
 - `<student_id>_f3.txt`
 - `<student_id>_freq.png`
 - These three files should be **in the same folder** with `hw3.py`

Some util function

```
9
10 def plot_wave(x, path = './wave.png'):
11     # util function
12     plt.gcf().clear()
13     plt.plot(x)
14     plt.xlabel('n')
15     plt.ylabel('xn')
16     plt.savefig(path)
17
18 def plot_ak(a, path =  './freq.png' ):
19     # util function
20     plt.gcf().clear()
21
22     # Only plot the mag of a
23     a = np.abs(a)
24     plt.plot(a)
25     plt.xlabel('k')
26     plt.ylabel('ak')
27     plt.savefig(path)
28
```



Scoring

1. Plot the figure of a_k . (2%)
2. Output correct f_1 signal (2%)
3. Output correct f_3 signal (2%)

Submit

- Code you download
|-- hw3.py
- Code you **submit** should be put in a folder and compressed in a **zip** file
r07922072_hw3.zip
|-- ./r07922072_hw3
|-- hw3.py
|-- r07922072_f1.txt
|-- r07922072_f3.txt
|-- r07922072_freq.png

Standard Rules

- 不要抄作業，不要交別人的答案，作弊一律0分計算
- 上傳 zip 檔案到 CEIBA
- 注意繳交的資料夾學號開頭英文用**小寫**
- **DEADLINE: 2018/11/15(四) 23:59 (GMT+8:00)**
- **遲交每過一天：分數 $\times 0.8$ (per day)**
- **格式、檔案、各種奇怪的錯誤讓我無法改作業：分數 $\times 0.8$**

Code Rules

- You **can't**
 - Use cosine transform formula in appendix to generate seperated signal
 - 只准使用inverse matrix的方法產生指定的三個檔案
 - 可以實作，但僅限於檢查inverse matrix的方法是否正確
 - import scipy
 - Or other cosine transform package

Appendix - Cosine Transform Formula

- Cosine transform

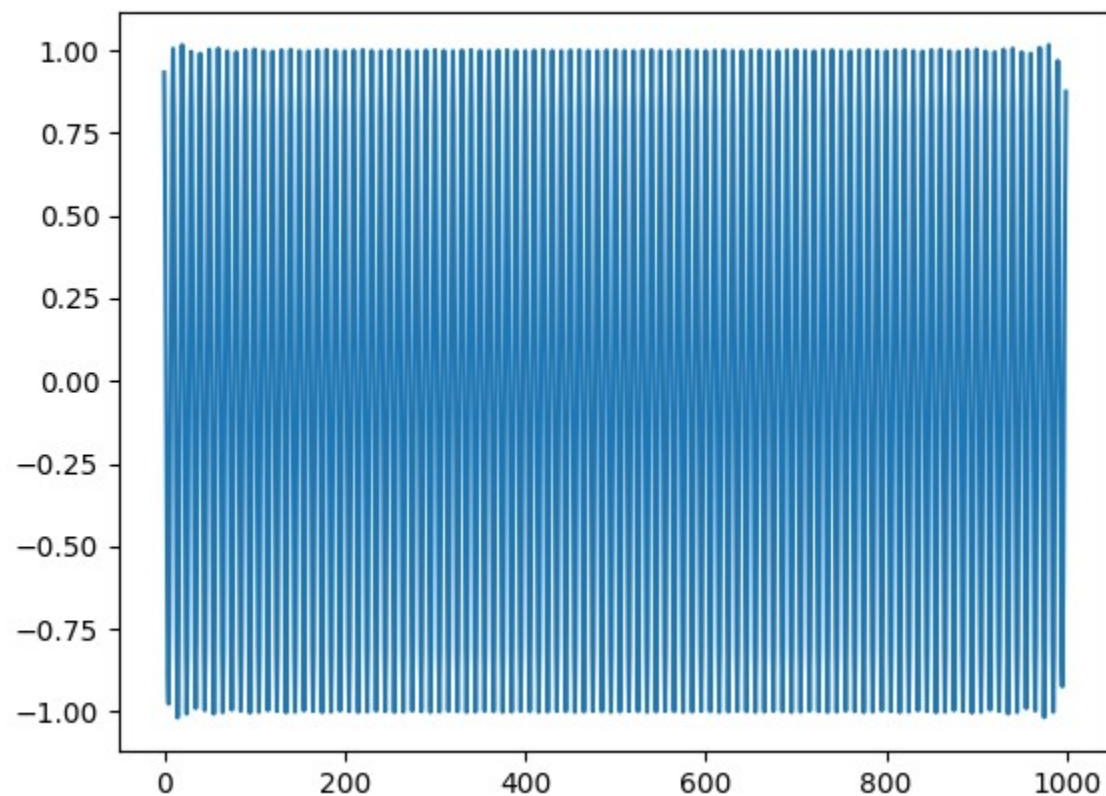
$$a_k = s_k * 2 \sum_{n=0}^{N-1} x_n \cos\left(\frac{\pi}{N} k \left(n + \frac{1}{2}\right)\right)$$

$$\begin{array}{l} \text{if } k = 0, s_0 = \sqrt{\frac{1}{4N}} \\ \text{else, } s_i = \sqrt{\frac{1}{2N}} \end{array}$$

- Inverse Cosine transform

$$x_n = \frac{1}{\sqrt{N}} a_0 + \frac{\sqrt{2}}{\sqrt{N}} \sum_{k=1}^{N-1} a_k \cos\left(\frac{\pi}{N} k \left(n + \frac{1}{2}\right)\right)$$

Update - test.txt的高頻波型



Update - 改作業的流程

假設同學的學號是r07922072，且對應的波檔是r07922072.txt
我會run

```
>>python hw3.py r07922072.txt
```

在程式中，r07922072.txt這個字串是由sys.argv[1]傳進程式裡的

同學的檔案要可以生出r07922072_f1.txt、r07922072_f3.txt、
r07922072_freq.png，且在同一個資料夾下

Update - inverse matrix

同學可以直接用`numpy.linalg.inv`這個函式拿inverse matrix

[Ref](#)

Update - text.txt

- 新增s1.txt/s2.txt, 分別為合成text.txt的兩個波
- 檔案在同個連結裡
 - [Link](#)