4. Neural Network Units

Neural Network Units

whatever
the computation is inside.
It is parameterized by w.
And that computation is affected by w.
So we can learn the parameters of the w, such
that this unit, in the context of the whole network,
will then function appropriately.
And we will get back to that learning
problem in a little bit.

problem in a little bit.

6:51 / 6:51 1.0x

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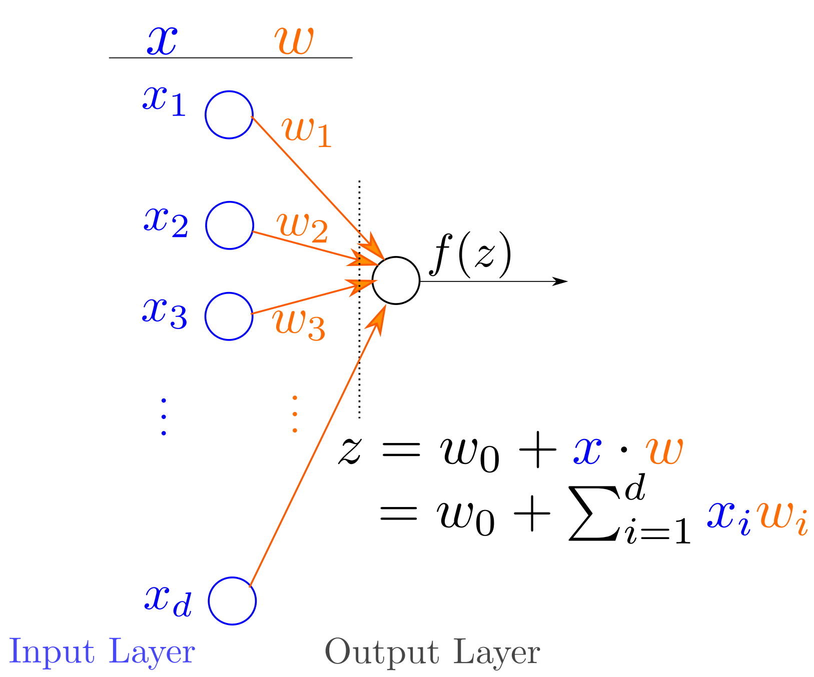
A **neural network unit** is a primitive neural network that consists of only the “input layer", and an output layer with only one output. It is represented pictorially as follows:



A neural network unit computes a non-linear weighted combination of its input:

$$\hat{y} \quad = \quad f(z) \quad \text{where } z = w_0 + \sum_{i=1}^{d} x_i w_i$$

where w_i are numbers called **weights** , z is a number and is the weighted sum of the inputs x_i, and f is generally a non-linear function called the **activation function** .

The above equation in vector form is:

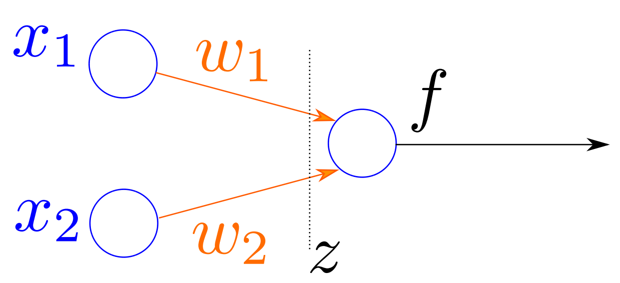
$$\hat{y} = f(z) \quad \text{where } z = w_0 + x \cdot w,$$

where $x = [x_1, \ldots, x_d]^T$ and $w = [w_1, \ldots, w_d]^T$.

Numerical Example - Neural Network Unit

2/2 points (graded)

In this problem, you will compute the output $\hat{y} = f(z)$ in the following neural network unit with 2 inputs x_1 and x_2.



Let

$$\begin{aligned} x &= [1, 0] \\ w_0 &= -3 \\ w &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}$$

First, compute z.

$z =$ -2 ✔ **Answer:** -2

The **rectified linear function (ReLU)** is defined as:

$$f(z) = \max\{0, z\}.$$

Using the ReLU function as the activiation function $f(z)$, compute $\hat{y}$:

$\hat{y} =$ 0 ✔ **Answer:** 0

Solution:

$$\begin{aligned} x &= [1, 0] \\ w_0 &= [-3] \\ w &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ x \cdot w &= [1, 0] \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ x \cdot w &= 1 \\ x \cdot w + w_0 &= 1 - 3 \\ x \cdot w + w_0 &= -2 \\ \text{ReLU}(x \cdot w + w_0) &= \text{ReLU}(-2) \\ \text{ReLU}(x \cdot w + w_0) &= max(0, -2) \\ \text{ReLU}(x \cdot w + w_0) &= 0 \end{aligned}$$

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Answers are displayed within the problem

Hyperbolic Tangent Activation Function

2/2 points (graded)
In this problem, we will recall and refamiliarize ourselves with hyperbolic tangent function, which is commonly used as an activation function in a neural network.

Recall the **hyperbolic tangent function** is defined as

$$\tanh(z) \;=\; \frac{e^z - e^{-z}}{e^z + e^{-z}} = 1 - \frac{2}{e^{2z}+1}.$$

What is the domain of $\tanh(z)$, i.e. for what values of z is $\tanh(z)$ defined?

- ( ) The set of two numbers $\{-1, 1\}$
- ( ) the interval $(-1, 1)$
- (x) All real numbers ✔

Find $\tanh(0)$. (Enter `e` for e.)

$\tanh(0) =$ 0 ✔ **Answer:** 0

Is $\tanh$ odd, even, or neither?

- (x) odd ✔
- ( ) even
- ( ) neither

What is the range of $\tanh$? Answer by giving a greatest lower bound, and a smallest upper bound of the set of all possible values of $\tanh(z)$.

Greatest lower bound: -1 ✔ **Answer:** -1

Lowest upper bound: 1 ✔ **Answer:** 1

Solution:

Observe that $\tanh$ is an odd function since $\tanh(-z) = -\tanh(z)$. Hence $\tanh(0) = 0$. Since $\tanh$ is a strictly increasing function:

$$\frac{d\tanh(z)}{dz} \;=\; \frac{d}{dz}\left(1 - \frac{2}{e^{2z}+1}\right) \;=\; \frac{4e^{2z}}{(e^{2z}+1)^2} > 0,$$

the greatest lower bound (or infimum), and the lower upper bound (or supremum) are given by the limits

$$\lim_{z\to-\infty} \tanh(z) \;=\; 1 - \frac{2}{(\lim_{z\to-\infty} e^{2z}) + 1} \;=\; -1$$

$$\lim_{z\to+\infty} \tanh(z) \;=\; 1 - 0 \;=\; 1$$

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