5. Univariate Gaussians

Univariate Gaussians or normal distributions have a simple representation in that they can be completely described by their mean and variance. These distributions are particularly useful because of the central limit theorem, which posits that when a large number of independent random variables are added, the distribution of their sum is approximated by a normal distribution. In other words, normal distributions can be applied to most problems.

Recall the probability density function of the Univariate Gaussian with mean μ and variance σ^2, $\mathcal{N}(\mu, \sigma^2)$:

$$p_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}$$

5. (a)

1/1 point (graded)

Let $X \sim \mathcal{N}(1, 2)$, i.e., the random variable X is normally distributed with mean 1 and variance 2. What is the probability that $X \in [0.5, 2]$?

(Write a solution with at least 4 decimal places)

0.39841314 ✔ **Answer:** 0.3984

STANDARD NOTATION

```
from scipy.stats import norm
ans = norm.cdf([0.5,2],loc=1,scale = 2**(1/2))
print(ans[1] - ans[0])
```

Solution:

One way to solve this problem is to integrate the PDF, which will give you the answer. Another way is to standardize the normal, giving us the variable $Z = \frac{X-1}{\sqrt{2}}$. We apply Z to the bounds $[0.5, 2]$ and then use a standard normal table to compute the answer.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

5. (b)

1/1 point (graded)

Let $p_X(x; \mu, \sigma^2)$ denote the probability density function of a normally distributed variable X with mean μ and variance σ^2. What value of x maximizes this function?

(Use **mu** to represent the mean and **sigma** to represent the standard deviation.)

mu ✔ **Answer:** mu

μ

STANDARD NOTATION

Solution:

The answer is μ, the mean of the distribution. If you look at the graph of the standardized normal distribution, you see that the maximum is at 0, its mean. Any normal distribution with different mean or variance is simply a shifted (different mean) or stretched (different variance) version of this distribution, so our result holds for any normally distributed variable. Alternatively, you can differentiate the PDF and determine the maximum, which gives you the same result.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

5. (c)

1/1 point (graded)

As above, let $p_X\left(x;\mu,\sigma^2\right)$ denote the probability density function of a normally distributed variable X with mean μ and variance σ^2.

What is the maximum value of $p_X\left(x;\mu,\sigma^2\right)$?

(Use **mu** to represent the mean and **sigma** to represent the standard deviation.)

1/sqrt(2*pi*sigma^2) ✔ **Answer:** 1/sqrt(2*pi*sigma^2)

$\frac{1}{\sqrt{2\cdot\pi\cdot\sigma^2}}$

STANDARD NOTATION

Solution:

From part b), we know that the maximum value occurs when $x=\mu$. Observe the PDF of a normal variable: setting $x=\mu$ forces the exponent of e to 0, leaving us with the answer above.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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