# 3. Consistency

## Quantifying Consistency (optional)

0 points possible (ungraded)

Let $X_1, \ldots, X_n \overset{i.i.d.}{\sim} \mathrm{Ber}(\mathrm{p})$ and let $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ be an estimator $p$.

What is the smallest exponent $c$ such that $n^c\left(\overline{X}_n - p\right)$ does **not** converge to $0$ almost surely as $n \to \infty$?

$c =$ 1/2 ✔ Answer: .5

STANDARD NOTATION

**Solution:**

Let $\sigma = \sqrt{p(1-p)}$ denote the common standard deviation of $X_1, \ldots, X_n$. By the central limit theorem,

$$\frac{\sqrt{n}}{\sigma}\left(\overline{X}_n - p\right) = \frac{\sqrt{n}}{\sigma}\left(\frac{1}{n}\sum_{i=1}^{n} X_i - p\right) \to N(0,1)$$

converge to distribution是最低限度的converge，如果前面的系数大于n^1/2的话，就没法converge了

where the convergence is in distribution. As a result, we see that for $n$ large and $c < 1/2$,

$$n^c\left(\overline{X}_n - p\right) = \frac{\sigma}{\boxed{n^{1/2-c}}}\frac{\sqrt{n}}{\sigma}\left(\overline{X}_n - p\right) \approx \frac{\sigma}{n^{1/2-c}}N(0,1) \to 0$$

必须是一个n的非负次方，不然n越大，整个式子就越大，就没法converge了

almost surely as $n \to \infty$. Hence, $c = 1/2$ is the smallest possible value of $c$ such that

$$n^c\left(\overline{X}_n - p\right) = n^c\left(\frac{1}{n}\sum_{i=1}^{n} X_i - p\right)$$

does *not* converge to $0$ almost surely as $n \to \infty$.

**Remark:** As defined in the third video in this section, this implies that the estimator $\overline{X}_n$ is $\sqrt{n}$-consistent. This means that the estimator $\overline{X}_n$ converges to the true parameter at a relatively fast rate, so this gives us something stronger than just consistency.

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

## 讨论

显示讨论

**主题：** Unit 2 Foundation of Inference:Homework 1: Estimation, Confidence Interval, Modes of Convergence / 3. Consistency

认证证书是什么？

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