## Week 5 – part 4b : Membrane potential fluctuations

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

# Neuronal Dynamics:
## Computational Neuroscience of Single Neurons

## Week 5 – Variability and Noise:
## The question of the neural code

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# Week 5 – part 4b : Membrane potential fluctuations

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**5.1 Variability of spike trains**
- experiments

**5.2 Sources of Variability?**
- Is variability equal to noise?

**5.3 Three definitions of Rate code**
- Poisson Model

**5.4 Stochastic spike arrival**
- Membrane potential fluctuations

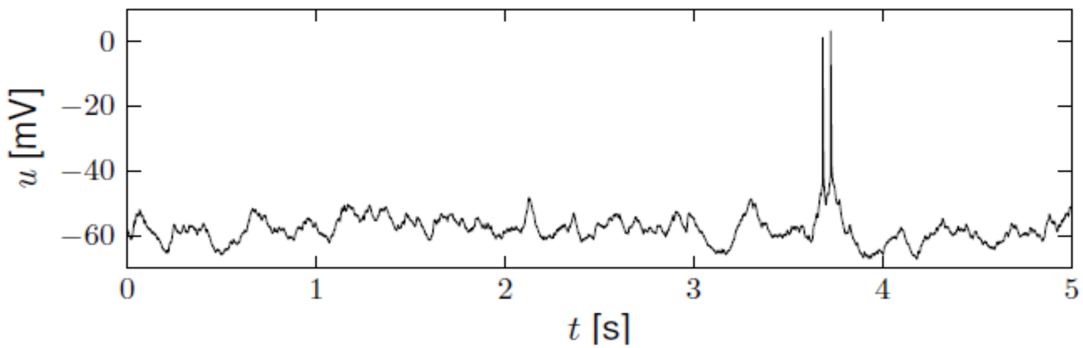
**5.5. Stochastic spike firing**
- subthreshold and superthreshold

# Neuronal Dynamics – 5.4 Variability in vivo

Spontaneous activity *in vivo*

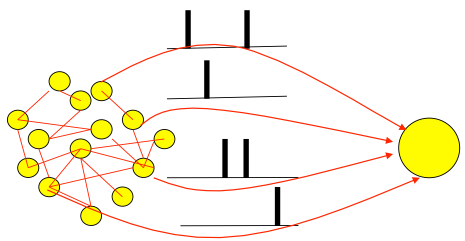
Variability
of membrane potential?

awake mouse, freely whisking,




*Crochet et al., 2011*

# Neuronal Dynamics – 5.4b. Fluctuations of potential



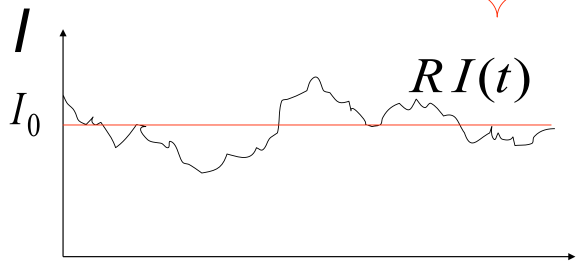
**Synaptic current pulses of shape $\alpha$**

$$RI^{syn}(t) = \sum_k w_k \sum_f \alpha(t - t_k^f)$$

EPSC

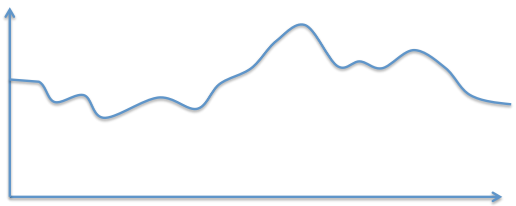
*Passive membrane*

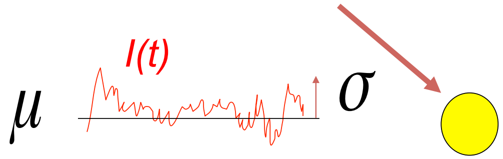
$$\tau \frac{d}{dt}u = -(u - u_{rest}) \quad + RI^{syn}(t)$$




$$I^{syn}(t) = I_0 + I^{fluct}(t)$$

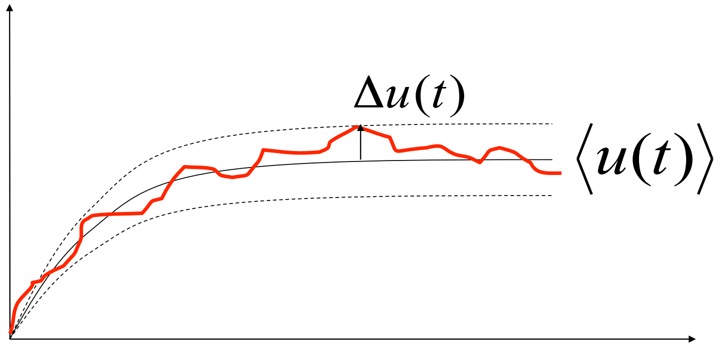
→ Fluctuating potential






*Fluctuating input current*

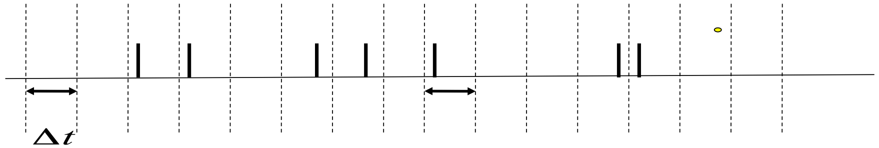
# Neuronal Dynamics – 5.4b. Fluctuations of potential




$$\langle \Delta u(t) \Delta u(t) \rangle = \langle u(t) u(t) \rangle - \langle u(t) \rangle^2 =$$

Input: step + fluctuations

# Neuronal Dynamics – 5.4b. Calculating autocorrelations




## Autocorrelation

$$\langle x(t)x(t')\rangle =$$

$$x(t) = \sum_f \int dt'\, f(t-t')\,\delta(t'-t_k^f)$$

$$= \int dt'\, f(t-t')\, S(t')$$

**Mean:**

$$\langle x(t)\rangle = \int dt'\, f(t-t')\langle S(t')\rangle$$

$$\langle x(t)\rangle = \int ds\, f(s)\ \rho_0$$

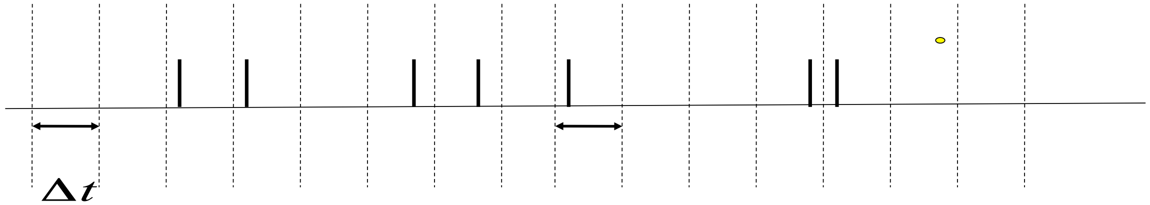
rate of homogeneous Poisson process

$$\langle x(t)x(\hat{t})\rangle = \int dt' \int dt''\, f(t-t')\, f(\hat{t}-t'')\langle S(t')S(t'')\rangle$$

# Neuronal Dynamics – 5.4b. Autocorrelation of Poisson

math detour now!

Probability of spike
in step *n* **AND** step *k*




*spike train*

Probability of spike in time step:

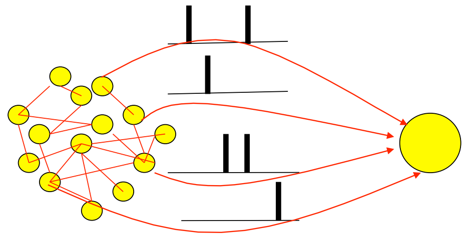
$$P_F = \rho_0 \, \Delta t$$

Autocorrelation (continuous time)

$$\langle S(t)S(t')\rangle = \rho_0 \, \delta(t-t') + [\rho_0]^2$$

# Neuronal Dynamics – 5.4b. Fluctuation of potential

for a passive membrane, we can analytically predict the amplitude of membrane potential fluctuations



Passive membrane

$$u(t) = \sum_k w_k \sum_f \varepsilon(t' - t_k^f)$$

$$= \sum_k w_k \int dt' \varepsilon(t - t') S_k(t')$$

fluctuating potential

$$\langle \Delta u(t) \Delta u(t) \rangle = \langle [u(t)]^2 \rangle - \langle u(t) \rangle^2$$

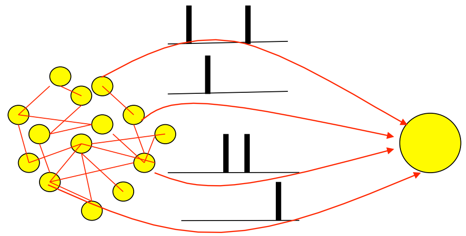
*Leaky integrate-and-fire in subthreshold regime*

# Neuronal Dynamics – 5.4b. Fluctuation of potential

**Stochastic spike arrival:**

for a passive membrane, we can analytically predict the amplitude of membrane potential fluctuations

*Leaky integrate-and-fire in subthreshold regime*



Passive membrane

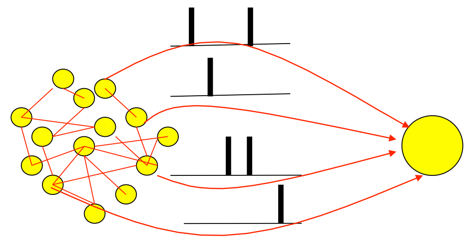
$$u(t) = \sum_k w_k \sum_f \varepsilon(t' - t_k^f)$$

$$= \sum_k w_k \int dt' \varepsilon(t - t') S_k(t')$$

fluctuating potential

$$\langle \Delta u(t) \Delta u(t) \rangle = \langle [u(t)]^2 \rangle - \langle u(t) \rangle^2$$

# Neuronal Dynamics – 5.4b. Fluctuation of potential



Passive membrane

$$u(t) = \sum_k w_k \sum_f \varepsilon(t' - t_k^f)$$

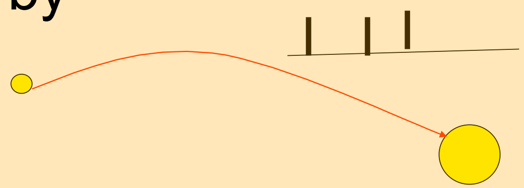
$$= \sum_k w_k \int dt' \, \varepsilon(t - t') \, S_k(t')$$

Fluctuations of potential

$$\langle [\Delta u(t)]^2 \rangle = \langle [u(t)]^2 \rangle - \langle u(t) \rangle^2$$

# Neuronal Dynamics – Quiz 5.4

A linear (=passive) membrane has a potential given by



$$u(t) = \sum_f \int dt' \, f(t-t') \, \delta(t' - t_k^f) + a$$

Suppose the neuronal dynamics are given by

$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + q \sum_f \delta(t - t^f)$$

[ ] the filter *f* is exponential with time constant $\tau$

[ ] the constant *a* is equal to the time constant $\tau$

[ ] the constant *a* is equal to $u_{rest}$

[ ] the amplitue of the filter *f* is *q*

[ ] the amplitue of the filter *f* is $u_{rest}$