5. Arrivals during overlapping time intervals

Problem 5. Arrivals during overlapping time intervals

3/3 points (graded)

Consider a Poisson process with rate λ. Let N be the number of arrivals in $(0,t]$ and M be the number of arrivals in $(0,t+s]$, where $t>0, s\geq 0$.

In each part below, your answers will be algebraic expressions in terms of λ, t, s, m and/or n. Enter "lambda" for λ and use "exp()" for exponentials. Do **not** use "fac()" or "!" for factorials. Follow standard notation.

1. For $0\leq n\leq m$, the conditional PMF $p_{M|N}(m\mid n)$ of M given N is of the form $\frac{a}{b!}$ for suitable algebraic expressions in place of a and b.

   $a=$ (lambda*s)^(m-n)*exp(-1*lambda*s) ✔ **Answer:** lambda^(m-n)*s^(m-n)*exp(-lambda*s)

   $(\lambda\cdot s)^{m-n}\cdot\exp(-1\cdot\lambda\cdot s)$

   $b=$ m-n ✔ **Answer:** m-n

   $m-n$

2. $E[NM]=$ lambda*t*(1+lambda*t+lambda*s) ✔ **Answer:** lambda*t*lambda*s+lambda*t+(lambda*t)^2

   $\lambda\cdot t\cdot(1+\lambda\cdot t+\lambda\cdot s)$

STANDARD NOTATION

Solution:

1. To find $P_{M|N}(m\mid n)$, we assume there are n arrivals in the first t time units, and we are looking for the probability that there are $m-n$ arrivals in the subsequent s time units. This follows a Poisson distribution with parameter λs:

$$p_{M|N}(m\mid n)=\frac{(\lambda s)^{m-n}e^{-\lambda s}}{(m-n)!},\quad \text{for } m\geq n\geq 0.$$

2. We can rewrite the expectation as

$$\begin{aligned}\mathbf{E}[NM] &= \mathbf{E}[N(M-N)+N^2]\\ &= \mathbf{E}[N]\mathbf{E}[M-N]+\mathbf{E}[N^2]\\ &= (\lambda t)(\lambda s)+\left(\text{var}(N)+(\mathbf{E}[N])^2\right)\\ &= (\lambda t)(\lambda s)+\lambda t+(\lambda t)^2,\end{aligned}$$

   where the second equality is obtained because of the independence of the number of arrivals, N and $M-N$, during disjoint time intervals.

提交 你已经尝试了2次（总共可以尝试3次）

Answers are displayed within the problem