9. Bayes' Formula with the Beta Distribution
Application: Bernoulli Experiment with the Beta Prior

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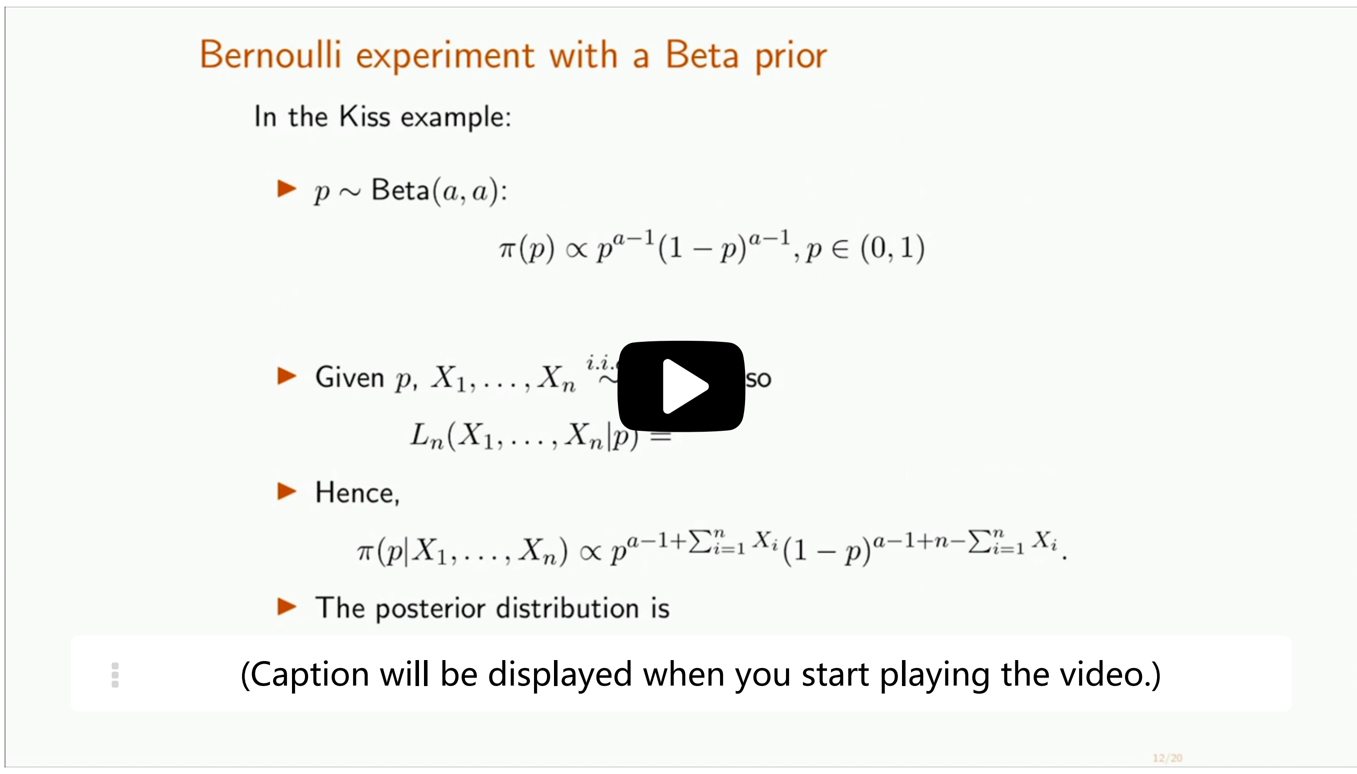

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So in the Kiss example, I'm going to have p.

I'm going to assume that the prior I have on p is this beta

with twice the same parameter.

I don't want to have to play with too many parameters,

so I'm going to take a beta with parameters of a and a.

So I know that the PDF is up to this beta

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Posterior Update : A Concrete Example

2/2 points (graded)

You are playing a computer game, where at each step you either succeed with probability θ or fail with probability $1-\theta$. Assume that the outcomes of the games are independent. Based on previous knowledge, you select a prior $\pi(\theta) \sim \text{Beta}(a, b)$.

Assume that you play this game for 12 rounds, with 7 successes and 5 failures. Find the posterior distribution $\pi(\theta|X_1, \ldots, X_{12})$.

It is known that the posterior is a Beta distribution, so you just need to state the parameters a' and b' below (the primes simply indicate that these parameters are the updated versions of aforementioned a, b).

$a' =$

a+7 ✔ **Answer:** a+7

$a+7$

$b' =$ b+5 ✔ **Answer:** b+5

$b+5$

STANDARD NOTATION

Solution:

- Note that if we interpret $X_i = 1$ for a success and $X_i = 0$ for a failure, the experiment simply is an instance of i.i.d. Bernoulli trials. $L_n(X_1, \ldots, X_n|\theta)$ therefore computes as

$$p_n(X_1, \ldots, X_n|\theta) = \theta^{\sum_{i=1}^n X_i}(1-\theta)^{n-\sum_{i=1}^n X_i}.$$

- Using the update rule for the Beta prior discussed in lecture,

$$\begin{aligned}\pi(\theta|X_1, \ldots, X_n) &\propto p_n(X_1, \ldots, X_n|\theta)\,\pi(\theta)\\ &\propto \theta^{a-1}(1-\theta)^{b-1}\theta^{\sum_{i=1}^n X_i}(1-\theta)^{n-\sum_{i=1}^n X_i}\\ &\propto \theta^{a+\sum_{i=1}^n X_i-1}(1-\theta)^{b+n-\sum_{i=1}^n X_i-1}.\end{aligned}$$

  Therefore, the updated versions are

$$a' = a + \sum_{i=1}^n X_i = a + 7,$$

  and

$$b' = b + n - \sum_{i=1}^n X_i = b + 5.$$

  Thus,

$$\pi(\theta|X_1, \ldots, X_{12}) \sim \mathrm{Beta}(a+7, b+5).$$

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You have used 1 of 3 attempts

Answers are displayed within the problem

Advantages of Bayesian View

0/1 point (graded)

Which one of the following statements below illustrates the advantages of Bayesian view over the frequentist approach?

- ○ The Bayesian approach gives statisticians some freedom to reflect prior belief. ✔
- ◉ The Bayesian approach is computationally more tractable. ✖
- ○ An estimator that takes the maximum of the posterior distribution obtained via Bayes rule is strictly closer to the actual parameter than the maximum likelihood estimator.

Solution:

The first choice is the correct answer.

- The main power of Bayesian approach comes from the fact that, designer can reflect the prior information in terms of a cleverly-engineered prior distribution.
- The second statement is false. Bayesian approach is actually computationally more expensive: normalizing the posterior distribution (via Bayes' rule) involves computing an integral in the denominator which might not have a simple solution.
- The third item is also false. A counterexample is that the maximum a-posteriori and maximum likelihood are actually the same if one has a uniform prior.