

## 13. Exercise: From joint PDFs to probabilities

### Exercise: From joint PDFs to probabilities

8/8 points (graded)

a) The probability of the event that  $0 \leq Y \leq X \leq 1$  is of the form  $\int_a^b \left( \int_c^d f_{X,Y}(x, y) dx \right) dy$ .

Find the values of  $a, b, c, d$ . Each one of your answers should be one of the following:  $0, x, y$ , or  $1$ .

$a =$	<input type="text" value="0"/>	✓ Answer: 0
$b =$	<input type="text" value="1"/>	✓ Answer: 1
$c =$	<input type="text" value="y"/>	✓ Answer: y
$d =$	<input type="text" value="1"/>	✓ Answer: 1

b) The probability of the event that  $0 \leq Y \leq X \leq 1$  is also of the form

$\int_a^b \left( \int_c^d f_{X,Y}(x, y) dy \right) dx$ . Note the different order of integration as compared to part (a).

Find the values of  $a, b, c, d$ . Each one of your answers should be one of the following:  $0, x, y$ , or  $1$ .

$a =$	<input type="text" value="0"/>	✓ Answer: 0
$b =$	<input type="text" value="1"/>	✓ Answer: 1
$c =$	<input type="text" value="0"/>	✓ Answer: 0
$d =$	<input type="text" value="x"/>	✓ Answer: x

**Solution:**

a) For any given  $y \in [0, 1]$ ,  $x$  ranges from  $y$  to  $1$ , yielding  $\int_0^1 \int_y^1 f_{X,Y}(x, y) dx dy$ .

b) For any given  $x \in [0, 1]$ ,  $y$  ranges from  $0$  to  $x$ , yielding  $\int_0^1 \int_0^x f_{X,Y}(x, y) dy dx$ .