5. Mixed Bayes rule - discrete unknown and continuous measurements

Problem 4. Mixed Bayes rule - discrete unknown and continuous measurements

3/3 points (graded)

Let S be a discrete random variable that takes the value 1 with probability $p \in (0,1)$, and the value -1 with probability $1-p$. Let X be a continuous random variable whose conditional distribution given S is as follows:

- If $S=1$, then X is exponential with parameter $\alpha > 0$, i.e., $f_{X|S}(x \mid 1) = \alpha e^{-\alpha x}$, for $x \geq 0$.
- If $S=-1$, then $-X$ is exponential with parameter $\beta > 0$, i.e., $f_{X|S}(x \mid -1) = \beta e^{\beta x}$, for $x \leq 0$.

Note that S can be viewed as the “sign" of X. Let $Z = |X|$.

1. Give an expression for $f_X(x)$. (Enter your answer using standard notation; type **alpha** for α, **beta** for β.)

   For $x > 0$:

   $f_X(x) =$

   p*alpha*e^(-alpha*x)

   Answer: p*alpha*exp(-alpha*x)

   $p \cdot \alpha \cdot e^{-\alpha \cdot x}$

   For $x < 0$:

   $f_X(x) =$

   (1-p)*beta*e^(beta*x)

   Answer: (1-p)*beta*exp(beta*x)

   $(1-p) \cdot \beta \cdot e^{\beta \cdot x}$

2. Give an expression for $\mathbf{P}(S=1 \mid Z=z)$, as a function of z. (Enter your answer using standard notation; type **alpha** for α, **beta** for β.)

   $\mathbf{P}(S=1|Z=z) =$

   p*alpha*e^(-alpha*z)/(p*alpha*e^(-alpha*z) + (1-p)*beta*e^(-beta*z))

   Answer: p*alpha*exp(-alpha*z)/(p*alpha*exp(-alpha*z)+(1-p)*beta*exp(-beta*z))

   $\frac{p \cdot \alpha \cdot e^{-\alpha \cdot z}}{p \cdot \alpha \cdot e^{-\alpha \cdot z} + (1-p) \cdot \beta \cdot e^{-\beta \cdot z}}$

STANDARD NOTATION

Solution:

1. The answer is

$$f_X(x) = f_{X|S}(x \mid 1) p_S(1) + f_{X|S}(x \mid -1) p_S(-1),$$

   and the reasoning is as follows. We are dealing with a mixture of two distributions. Hence, when $x > 0$ only the first is nonzero and we obtain $p\alpha e^{-\alpha x}$. When $x < 0$, only the second term is nonzero and we obtain $(1-p)\beta e^{\beta x}$.

2. $Z = |X|$ is always non-negative, and $Z = X$, when $X \geq 0$, and $Z = -X$, when $X \leq 0$. Thus,

$$f_{Z|S}(z \mid s) = \begin{cases} f_{X|S}(z|1) = \alpha e^{-\alpha z}, & \text{if } s = 1 \\ f_{X|S}(-z|s) = \beta e^{-\beta z}, & \text{if } s = -1 \end{cases}$$

Now,

$$\begin{aligned} \mathbf{P}(S = 1 \mid Z = z) &= \frac{f_{Z|S}(z|1)\mathbf{P}(S=1)}{f_{Z|S}(z|1)\mathbf{P}(S=1) + f_{Z|S}(z|-1)\mathbf{P}(S=-1)} \\ &= \frac{p\alpha e^{-\alpha z}}{p\alpha e^{-\alpha z} + (1-p)\beta e^{-\beta z}}. \end{aligned}$$

提交 You have used 2 of 3 attempts

Answers are displayed within the problem

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显示讨论

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