We now set out to study the Poisson process, which is a continuous time version of the Bernoulli process. In the Bernoulli process, time is divided into slots. And during each one of the slots, we may either have an arrival or no arrival.

In the Poisson process, time is continuous. And we may get arrivals at any time. We want to define the Poisson process by introducing some assumptions that in some ways parallel the assumptions that we made for the Bernoulli process. What where those assumptions?

The first one we made was the assumption of independence-- namely that what happens in different slots are independent. Similarly, for the Poisson process, we will make the following independence assumption. If we consider two intervals, two time intervals that are disjoint, and look at the random variable that stands for the number of arrivals during this interval and that interval, we will assume that these two random variables are independent.

But even more than that, if we take any collection of disjoint time intervals, and we look at the associated random variables, the associated numbers of arrivals, that collection will also consist of independent random variables.

The second assumption for the Bernoulli processes was one of time homogeneity, namely at each slot, we had the same probability of an arrival. We want to make a similar assumption for the Poisson process, and it's going to be the following. The probability that we have k arrivals during an interval of a certain duration tau is going to be the same no matter where that interval sits.

So if this is an interval that has a certain duration, and this is an interval that has the same duration, the probability of three arrivals in this interval is going to be the same as the probability of three arrivals in that interval. And therefore, since this probability does not depend on where the interval sits, that probability will be fully defined by the number of arrivals that we're interested in and the length of the interval as opposed to the location of the interval.

So we will be using this notation here to indicate this probability. In this notation, we think of tau as a constant. And then, P of k corresponds to probabilities. In particular, if you add over all k's the various probabilities, what you should get would be a value of 1, because this exhausts all the possibilities.

And k here ranges from 0 to infinity, because we allow an arbitrarily large number of arrivals during a given time interval. Now, with this assumption in place, it would still be possible to have arrivals that happen simultaneously, multiple arrivals at the same time point. In order to avoid this situation, we introduce one more assumption which is the following.

It talks about the number of arrivals during a time interval that has a small length delta. During a small time interval, there is negligible probability of having more than one arrival. We will either have one or zero arrivals. And the probability of one arrival is a certain number, lambda times delta. It's proportional to delta. So if the interval becomes smaller and smaller, that probability also goes to 0. But it goes to 0 at a rate proportional to delta.

So you can think of lambda as probability per unit time. These are the units of lambda. Now here, I'm writing an approximate equality. What does that mean? It means that these are not exact expressions.

But they are exact within a second order term. And a second order term is a term that's negligible compared to the first order term when delta is small. More precisely, mathematically, what we mean is that a second order term compared to a first order term goes to 0 as delta goes to 0.

Finally, let me reiterate that lambda should be interpreted as an arrival rate. It is a probability per unit time. The bigger lambda is, the bigger the probability is that we get an arrival during a small time interval.

If we double lambda, then we double the probability that we have an arrival during a small time interval. And so we expect to have about twice as many arrivals, hence the interpretation as an arrival rate. We will see shortly that this is also justified because lambda shows up in expressions for the expected number of arrivals during a time interval.