

6. Exercise: Memorylessness of the exponential

Exercise: Memorylessness of the exponential

1/3 points (graded)

Let X be an exponential random variable with parameter λ .

a) The probability that $X > 5$ is

☐ $\lambda e^{-5\lambda}$

☒ $e^{-5\lambda}$ ✓

☐ none of the above

b) The probability that $X > 5$ given that $X > 2$ is

☐ $\lambda e^{-5\lambda}$

☒ $e^{-5\lambda}$ ✗

☐ $\lambda e^{-3\lambda}$

☐ $e^{-3\lambda}$ ✓

☐ none of the above

c) Given that $X > 2$, and for a small $\delta > 0$, the probability that $4 \leq X \leq 4 + 2\delta$ is approximately

☐ $\lambda\delta$

☒ $2\lambda\delta$ ✗

☐ $\delta e^{-4\lambda}$

☐ $\lambda \delta e^{-4\lambda}$

☐ $\lambda \delta e^{-2\lambda}$

☒ $2\lambda \delta e^{-2\lambda}$ ✓

☐ none of the above

Solution:

a) We have seen in the past that for an exponential random variable with parameter λ , $\mathbf{P}(X > a) = e^{-\lambda a}$, and so $\mathbf{P}(X > 5) = e^{-5\lambda}$.

b) Because of the memorylessness property, given that $X > 2$, the remaining time $X - 2$ is again exponential with the same parameter. Thus,
 $\mathbf{P}(X > 5 \mid X > 2) = \mathbf{P}(X - 2 > 3 \mid X > 2) = \mathbf{P}(X > 3) = e^{-3\lambda}$.

c) By memorylessness, this is the same as the unconditional probability that an exponential takes values in the interval $[2, 2 + 2\delta]$, which is approximately the length, 2δ , of the small interval times the density evaluated at 2, yielding $2\lambda \delta e^{-2\lambda}$.

提交

You have used 2 of 2 attempts

❗ Answers are displayed within the problem

讨论

显示讨论

Topic: Unit 5 / Lec. 9 / 6. Exercise: Memorylessness of the exponential

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