5. Constrained maximum likelihood estimator

Instruction:

What can we do when we have prior knowledge about the estimator? Imagine that an expert told you that the parameter θ lies between a and b. Would that additional knowledge change the MLE calculation? We will start by calculating just normal MLE and think about what we can do in part (c).

Let $X_1, \ldots, X_n$ be n i.i.d. random variables with probability density function

$$f_\theta(x) = \theta x^{-\theta-1}, \theta > 0, x \geq 1.$$

To encourage you to do the computations carefully rather than eliminate choices, you will be given only **1-2 attempts per question** .

(a)

1/1 point (graded)

What is the likelihood function for θ?

- (selected) $\theta^n \prod_{i=1}^n x_i^{-\theta-1}$
- $\theta^n \prod_{i=1}^n x_i^{-\theta-1} \mathbf{1}\{\min_i X_i \geq 1\}$
- $\theta^n \prod_{i=1}^n x_i^{-\theta-1} \mathbf{1}\{\min_i X_i < 1\}$
- $\theta^n \prod_{i=1}^n x_i^{-\theta-1} \mathbf{1}\{\max_i X_i \geq 1\}$
- $\theta^n \prod_{i=1}^n x_i^{-\theta-1} \mathbf{1}\{\max_i X_i < 1\}$
- $n \ln \theta - (\theta+1) \sum_{i=1}^n \ln X_i$

Solution:

$$\begin{aligned} L_n &= \prod_{i=1}^n \theta x_i^{-\theta-1} \mathbf{1}\{X_i \geq 1\} \\ &= \theta^n \prod_{i=1}^n x_i^{-\theta-1} \mathbf{1}\{\min_i X_i \geq 1\} \end{aligned}$$

But since we assume our statistical model to be well-specified, $\min_i X_i \geq 1$ will always be satisfied, and so we can drop the corresponding indicator function. Hence, $L_n = \theta^n \prod_{i=1}^n x_i^{-\theta-1}$ is correct under the well-specified assumption.

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

(b)

1/1 point (graded)
What is the maximum likelihood estimator for θ?

- (selected) $\frac{n}{\sum_{i=1}^{n} \ln X_i}$
- $-\frac{n}{\sum_{i=1}^{n} \ln X_i}$
- $\frac{\sum_{i=1}^{n} \ln X_i}{n}$
- $-\frac{\sum_{i=1}^{n} \ln X_i}{n}$
- $\frac{\sum_{i=1}^{n} X_i}{n}$
- $\frac{n}{\sum_{i=1}^{n} X_i}$

Solution:

Take the derivative of the likelihood function with respect to θ.

$$\frac{\partial L_n}{\partial \theta} = \frac{n}{\theta} - \sum_{i=1}^{n} \ln X_i = 0$$

Solving the equation for θ, we get

$$\hat{\theta} = \frac{n}{\sum_{i=1}^{n} \ln X_i}$$

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

(c)

1/1 point (graded)
Suppose we have two numbers $0 < a < b$. We are interested in the value of θ that maximizes the likelihood in the set $[a, b]$.

Let $\hat{\theta}$ denote the maximum likelihood estimator you found in part (b) above, and let $\hat{\theta}_{\text{const}}$ denote the maximum likelihood estimator within the interval $[a, b]$, where $0 < a < b$. Choose all correct answers.

- [ ] If $b \leq \hat{\theta}$, then $\hat{\theta}_{\text{const}} = a$
- [x] If $b \leq \hat{\theta}$, then $\hat{\theta}_{\text{const}} = b$
- [ ] If $b \leq \hat{\theta}$, then $\hat{\theta}_{\text{const}} = \hat{\theta}$
- [ ] If $a < \hat{\theta} < b$, then $\hat{\theta}_{\text{const}} = a$

- [ ] If $a < \hat{\theta} < b$, then $\hat{\theta}_{\text{const}} = b$
- [x] If $a < \hat{\theta} < b$, then $\hat{\theta}_{\text{const}} = \hat{\theta}$
- [x] If $a \geq \hat{\theta}$, then $\hat{\theta}_{\text{const}} = a$
- [ ] If $a \geq \hat{\theta}$, then $\hat{\theta}_{\text{const}} = b$
- [ ] If $a \geq \hat{\theta}$, then $\hat{\theta}_{\text{const}} = \hat{\theta}$

Solution:

Take the second derivative of the likelihood function with respect to θ.

$$\frac{\partial^2 L_n}{\partial \theta^2} = -\frac{1}{\theta^2} < 0$$

Since the second derivative is strictly less then 0, the function is strictly concave with respect to θ. Therefore, depending on the value of $\frac{n}{\sum_{i=1}^{n} \ln X_i}$, which is the maximum, the largest value that likelihood function can take in the set $[a, b]$ changes.

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

讨论

显示讨论

主题: Unit 3 Methods of Estimation:Homework 4: TV distance, KL-Divergence, and Introduction to MLE / 5. Constrained maximum likelihood estimator

认证证书是什么?