10. Review of Methods of Estimation

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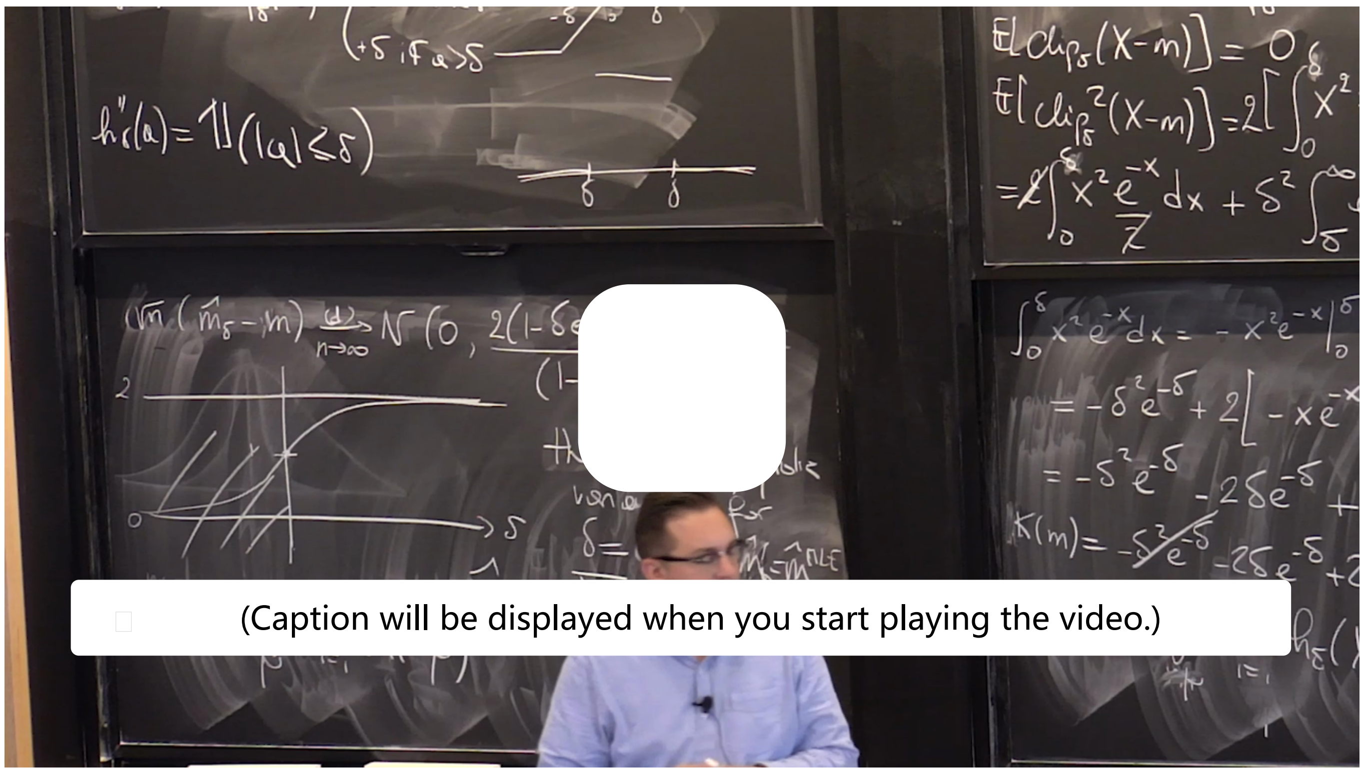

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OK, so let's just wrap up this chapter,

just to make sure that we remind ourselves

everything we've seen.

So we saw essentially three principal methods

for estimation.

And by principal, I mean that we had one before, which was just,

well, if you're parameters an expectation,

just replace it by an average.

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Concept Check: Methods of Estimation I

1/1 point (graded)

Which of the following estimators are defined in terms of an optimization problem? (Choose all that apply.)

- [x] Maximum likelihood estimator.
- [ ] Method of moments estimator.
- [x] M-estimator.

Solution:

The correct responses are "Maximum likelihood estimator." and "M-estimator." The MLE is defined by maximizing the log-likelihood, and an M-estimator is defined by minimizing a loss function. However, the method of moments estimator is constructed by solving a system of equations, so this response is not correct.

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Answers are displayed within the problem

Concept Check: Methods of Estimation II

1/1 point (graded)
All three method of estimation studied in this unit: maximum likelihood estimation, the method of moments, and M-estimation, lead to asymptotically normal estimators if certain technical conditions are satisfied.

In general, an asymptotically normal estimator $\hat{\theta}_n$ can be used to construct a confidence interval for an unknown parameter.

What quantity related to the estimator $\hat{\theta}$ determines the length of an asymptotic confidence interval at level 95%? (Assume that you use the plug-in method and that n is very large.)

- (selected) The asymptotic variance of $\hat{\theta}_n$.
- The rate of convergence of $\hat{\theta}_n$ to the normal distribution $\mathcal{N}(0,1)$.
- The mean of $\hat{\theta}_n$.

Solution:

The correct response is "The asymptotic variance of $\hat{\theta}_n$," as we demonstrate below. Consider an asymptotically normal estimator $\widehat{\theta_n}$, which satisfies

$$\sqrt{n}\,(\widehat{\theta_n}-\theta)\xrightarrow[n\to\infty]{(d)}\mathcal{N}(0,\sigma^2)$$

for some asymptotic variance $\sigma^2>0$. Let $q_{\alpha/2}$ denote the $\alpha/2$-quantile of a standard Gaussian. Then we have that

$$P\left(\sqrt{n}\frac{\left|\widehat{\theta_n}-\theta\right|}{\sigma}\geq q_{\alpha/2}\right)\xrightarrow{n\to\infty}\alpha$$

which implies that

$$P\left(\theta\notin\left[\widehat{\theta_n}-q_{\alpha/2}\frac{\sigma}{\sqrt{n}},\widehat{\theta_n}+q_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]\right)\xrightarrow{n\to\infty}\alpha.$$

Therefore, using the plug-in method, we have that

$$P\left(\theta\notin\left[\widehat{\theta_n}-q_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{n}},\widehat{\theta_n}+q_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{n}}\right]\right)\xrightarrow{n\to\infty}\alpha,$$

Setting $\alpha=0.05$, we have that

$$\mathcal{I}:=\left[\widehat{\theta_n}-q_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{n}},\widehat{\theta_n}+q_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{n}}\right]$$

If n is very large, we have that $\widehat{\sigma}_n\approx\sigma$, so the length of $\mathcal{I}$ is approximately $2q_{0.025}\sigma/\sqrt{n}$. That is, the length depends only on the $\alpha/2$ quantile, the sample size, and the asymptotic variance. Therefore, "The rate of convergence of $\hat{\theta}_n$ to the normal distribution $\mathcal{N}(0,1)$." and "The mean of $\hat{\theta}_n$." are incorrect responses.

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Answers are displayed within the problem

讨论

主题： Unit 3 Methods of Estimation:Lecture 12: M-Estimation / 10. Review of Methods of Estimation

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