$$\mathbf{E}\big[g(Y)X \mid Y\big] = g(Y)\mathbf{E}[X \mid Y]$$

assume X, Y discrete

consider outcome where $Y = y$ $P_Y(y) > 0$

$$E\big[g(Y)X \mid Y=y\big] \qquad g(y)E\big[X \mid Y=y\big]$$

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$$g(y)E\big[X \mid Y=y\big]$$

If h is invertible, then $\mathrm{E}[X \mid Y] = \mathrm{E}\left[X \mid h(Y)\right]$

$$Y \leftrightarrow h(Y) \qquad h(Y) = Y^3$$

$$\text{fix outcome for which } Y = y \quad , \quad h(Y) = h(y)$$

$$E[X \mid Y = y]$$

$$E[X \mid h(Y) = h(y)]$$

$$E[X \mid Y = y]$$