2. Three engines

Problem 2. Three engines

7/7 points (graded)

Suppose that we have three engines, which we turn on at time 0. Each engine will eventually fail, and we model each engine's lifetime as exponentially distributed with parameter λ. The lifetimes of different engines are independent. One of the engines will fail first, followed by the second, and followed by the last. Let T_1 be the time of the first failure, T_2 be the time of the second failure, and T_3 be the time of the third failure. For answers involving algebraic expressions, enter "lambda" for λ and use "exp()" for exponentials. Follow standard notation.

1. Determine the PDF of T_1.
   For $t > 0$,

   $f_{T_1}(t) =$ 3*lambda*exp(-3*lambda*t)

   ✔ **Answer:** 3*lambda*e^(-3*lambda*t)

   $3 \cdot \lambda \cdot \exp(-3 \cdot \lambda \cdot t)$

2. Let $X = T_2 - T_1$. Determine the conditional PDF $f_{X|T_1}(x|t)$.
   For $x, t > 0$,

   $f_{X|T_1}(x \mid t) =$ 2*lambda*exp(-2*lambda*x)

   ✔ **Answer:** 2*lambda*e^(-2*lambda*x)

   $2 \cdot \lambda \cdot \exp(-2 \cdot \lambda \cdot x)$

3. Is X independent of T_1?

   Yes they are independent ▾

   ✔ **Answer:** Yes they are independent

4. Let $Y = T_3 - T_2$. Find the PDF of $f_{Y|T_2}(y \mid T_2)$.
   For $y, t > 0$,

   $f_{Y|T_2}(y \mid t) =$ 1*lambda*exp(-1*lambda*y)

   ✔ **Answer:** lambda*e^(-lambda*y)

   $1 \cdot \lambda \cdot \exp(-1 \cdot \lambda \cdot y)$

5. Is Y independent of T_2?

   Yes they are independent ▾

   ✔ **Answer:** Yes they are independent

6. Find the PDF $f_{T_3}(t)$ for $t \geq 0$.
   For $t \geq 0$,

   $f_{T_3}(t) =$ 3*lambda*exp(-1*lambda*t) - 6*lambda*exp(-2*lambda

   ✔ **Answer:** 3*lambda*e^(-lambda*t)*(1-e^(-lambda*t))^2

   $3 \cdot \lambda \cdot \exp(-1 \cdot \lambda \cdot t) - 6 \cdot \lambda \cdot \exp(-2 \cdot \lambda \cdot t) + 3 \cdot \lambda \cdot \exp(-3 \cdot \lambda \cdot t)$

   Hint: Think of an interpretation of T_3 as a maximum of some exponential random variables.

7. Find $\mathbf{E}[T_3]$.

   $\mathbf{E}[T_3] =$ 11/(6 * lambda)

   ✔

   Answer: 11/(6*lambda)

   $\frac{11}{6 \cdot \lambda}$