# 15. Exercise: Sections of a class

## Exercise: Sections of a class

4/4 points (graded)

A class consists of three sections with 10 students each. The mean quiz scores in each section were 40, 50, 60, respectively. We pick a student, uniformly at random. Let $X$ be the score of the selected student, and let $Y$ be the number of his/her section. The quantity $\mathbf{Var}(X \mid Y = y)$ turned out to be equal to $5y$ for each section ($y = 1, 2, 3$).

(a) The random variable $\mathbf{E}[X \mid Y]$ has:

a mean of: 50 ✔ **Answer:** 50

a variance of: 200/3 ✔ **Answer:** 66.66667

(b) $\mathbf{E}\big[\mathbf{Var}(X \mid Y)\big] =$ 10 ✔ **Answer:** 10

(c) $\mathbf{Var}(X) =$ 230/3 ✔ **Answer:** 76.66667

**Solution:**

(a) $\mathbf{E}[X \mid Y = y]$ is the mean of the scores in section $y$. Thus, $\mathbf{E}[X \mid Y]$ is a random variable that takes the values 40, 50, and 60, with equal probability. Its mean is 50 and its variance is

$$\frac{1}{3}\Big((40-50)^2 + (50-50)^2 + (60-50)^2\Big) = \frac{200}{3}.$$

(b) The random variable $\mathbf{Var}(X \mid Y)$ takes the values 5, 10, and 15, with equal probability. Its mean is 10.

(c) From the law of total variance, we just need to add the results from the previous two parts.

提交 You have used 2 of 3 attempts