15. Likelihood of a Poisson Statistical Model

Review: Statistical Model for a Poisson Distribution

2/2 points (graded)

Let $X_1, \ldots, X_n \stackrel{iid}{\sim} \text{Poiss}(\lambda^*)$ for some unknown $\lambda^* \in (0, \infty)$. Let $(E, \{\text{Poiss}(\lambda)\}_{\lambda \in \Theta})$ denote the corresponding statistical model. What is the smallest possible set that could be E?

- ○ $\mathbb{N} = \{1, 2, 3, \ldots\}$
- ◉ $\mathbb{N} \cup \{0\}$ □
- ○ $\mathbb{Z}$
- ○ $\mathbb{R}$

The parameter space Θ can be written as an interval (a, ∞). What is the smallest value of a so that $\{\text{Poiss}(\lambda)\}_{\lambda \in (a,\infty)}$ represents all possible Poisson distributions?

$a =$ 0 □ **Answer:** 0.0

Solution:

A Poisson random variable takes values on all non-negative integers $\{0, 1, 2, \ldots\}$. Hence, the smallest possible sample space is $\mathbb{N} \cup \{0\}$.

A Poisson random variable is specified by its mean λ, which is allowed to be any positive real number. Hence, $a = 0$ is the correct choice.

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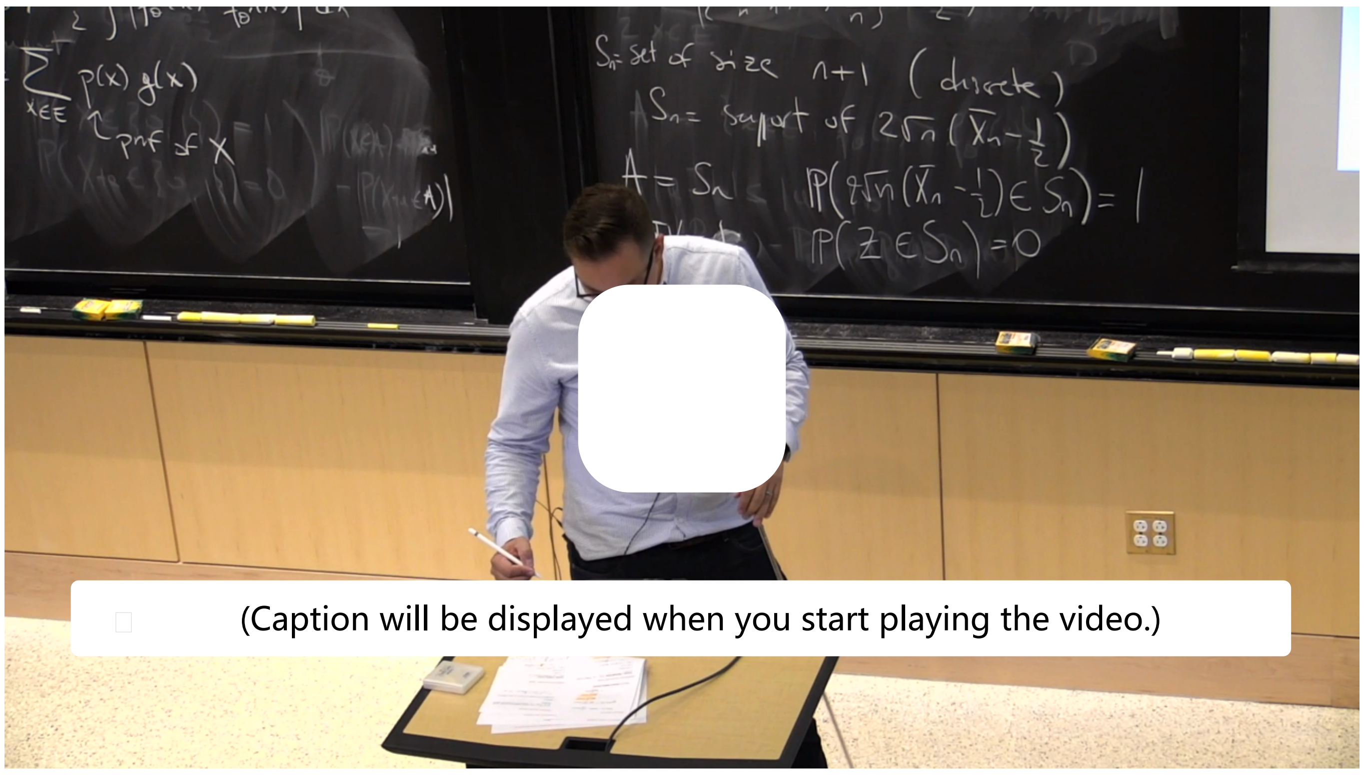
□ Answers are displayed within the problem

Likelihood of a Poisson Statistical Model



I'm going to have to write-- so even let's do it for Xi.

I am going to have to take the product of those guy.

Well, it's lambda to the Xi over Xi factorial, e

to the minus lambda.

That's just my PMF for a Poisson.

Now, if I take the product of those guys--

so my likelihood X1 Xn lambda is just the product.

So it's lambda to the sum of the Xis, the product

of my denominators, Xi factorial, and then

e to the minus, and then I have a product

of those guys n times.

So I get e to the minus n times lambda.

And now if you rearrange the terms,

this is exactly what's written here,

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Practice: Compute Likelihood of a Poisson Statistical Model

3/3 points (graded)

Let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathrm{Poiss}(\lambda^*)$ for some unknown $\lambda^* \in (0, \infty)$. You construct the associated statistical model $(E, \{\mathrm{Poiss}(\lambda)\}_{\lambda \in \Theta})$ where E and Θ are defined as in the answers to the previous question.

Suppose you observe two samples $X_1 = 1, X_2 = 2$. What is $L_2(1, 2, \lambda)$? Express your answer in terms of λ.

$L_2(1, 2, \lambda) =$ exp(-2*lambda)*lambda^(1+2)/(1*2) **Answer:** e^(-2*lambda)*lambda^3/2

$\frac{\exp(-2\cdot\lambda)\cdot\lambda^{1+2}}{1\cdot 2}$

Next, you observe a third sample $X_3 = 3$ that follows $X_1 = 1$ and $X_2 = 2$. What is $L_3(1, 2, 3, \lambda)$?

$L_3(1, 2, 3, \lambda) =$ exp(-3*lambda)*lambda^(1+2+3)/(1*2*3*2) **Answer:** e^(-3*lambda)*lambda^6/12

$\frac{\exp(-3\cdot\lambda)\cdot\lambda^{1+2+3}}{1\cdot 2\cdot 3\cdot 2}$

Suppose your data arrives in a different order: $X_1 = 2, X_2 = 3, X_3 = 1$. What is $L_3(2, 3, 1, \lambda)$?

$L_3(2, 3, 1, \lambda) =$ exp(-3*lambda)*lambda^(1+2+3)/(1*2*3*2) **Answer:** e^(-3*lambda)*lambda^6/12

$\frac{\exp(-3\cdot\lambda)\cdot\lambda^{1+2+3}}{1\cdot 2\cdot 3\cdot 2}$

STANDARD NOTATION

Solution:

The probability mass function of $\mathrm{Poiss}(\lambda)$ is $x \mapsto e^{-\lambda}\frac{\lambda^x}{x!}$ where $x \in \mathbb{N} \cup \{0\}$. Hence by definition

$$L_n(x_1, \ldots, x_n, \lambda) = \prod_{i=1}^{n} e^{-\lambda}\frac{\lambda^{x_i}}{x_i!} = e^{-n\lambda}\frac{\lambda^{\sum_{i=1}^n x_i}}{x_1! \cdots x_n!}.$$

Hence, first we plug in $n = 2$, $x_1 = 1$, and $x_2 = 2$:

$$L_2(1, 2, \lambda) = e^{-2\lambda}\frac{\lambda^{1+2}}{2!1!} = e^{-2\lambda}\frac{\lambda^3}{2}.$$

When the next sample arrives, we can simply evaluate the density of a Poisson at the observation:

$$P(X_3 = 3) = e^{-\lambda}\frac{\lambda^3}{3!}, \quad X \sim \mathrm{Poiss}(\lambda)$$

and multiply this by the previous response:

$$L_3(1, 2, 3, \lambda) = e^{-\lambda}\frac{\lambda^3}{3!}L_2(1, 2, \lambda) = e^{-3\lambda}\frac{\lambda^6}{12}.$$

Remark 1: Observe that we can compute the likelihood sequentially as the data arrives, updating it in the previous fashion after each new observation.

Similarly, we see that

$$L_3\left(2,3,1,\lambda\right)=e^{-3\lambda}\frac{\lambda^6}{12}.$$

Remark 2: Observe that the likelihood of observations $X_1=x_1,\ldots,X_n=x_n$ is independent of the *order* in which these observations arrive.

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Answers are displayed within the problem

Properties of the Likelihood

1/1 point (graded)

Let $\left(E,\{P_\theta\}_{\theta\in\Theta}\right)$ denote a discrete statistical model. Let p_θ denote the pmf of P_θ. Let $X_1,\ldots,X_n\overset{iid}{\sim}P_{\theta^*}$ where the parameter θ^* is unknown. Then the **likelihood** is the function

$$\begin{aligned}L_n:E^n\times\Theta&\to\mathbb{R}\\(x_1,\ldots,x_n,\theta)&\mapsto\prod_{i=1}^n p_\theta\left(x_i\right).\end{aligned}$$

For our purposes, we think of $x_1,\ldots,x_n$ as observations of the random variables $X_1,\ldots,X_n$.

Which of the following are properties of the likelihood L_n? (Choose all that apply.)

Hint: It may be useful to consider your responses from the previous question.

- [ ] The likelihood does not change with the parameter θ.
- [x] The likelihood can be updated sequentially as new samples are observed. For example, $L_3\left(x_1,x_2,x_3,\theta\right)=L_1\left(x_3,\theta\right)L_2\left(x_1,x_2,\theta\right)$.
- [x] The likelihood is symmetric: it doesn't matter the order in which we plug in the observations. For example, $L_4\left(x_1,x_2,x_3,x_4,\theta\right)=L_4\left(x_2,x_3,x_1,x_4,\theta\right)$, and this is true for any rearrangement of x_1,x_2,x_3,x_4.
- [ ] If we eliminate a single observation, then the likelihood remains unchanged. For example, $L_3\left(x_1,x_2,x_3,\theta\right)=L_2\left(x_1,x_2,\theta\right)$.

Solution:

We examine the choices in order.

- "The likelihood does not change with the parameter θ." is incorrect. Rather, it is crucial that we interpret the likelihood L_n as a function of θ. That is, L_n varies as θ ranges over the parameter space Θ. This is evident in the likelihoods for the Bernoulli and Poisson models in the previous problems.

- "The likelihood can be updated sequentially as new samples are observed. For example, $L_3\left(x_1,x_2,x_3,\theta\right)=L_1\left(x_3,\theta\right)L_2\left(x_1,x_2,\theta\right)$." is also correct. In the previous problem, we saw that to compute the likelihood after observing $X_3=3$, we simply took the old likelihood $L_2\left(1,2,\lambda\right)$ and multiplied it by $L_1\left(3,\lambda\right)$. Note that $L_1\left(x_3,\theta\right)=p_\theta\left(x_3\right)$, the density of P_θ evaluated at the new observation. Inspection of the defining formula

$$L_n\left(x_1,\ldots,x_n,\theta\right)=\prod_{i=1}^n p_\theta\left(x_i\right)$$

implies that the likelihood can be updated sequentially in this fashion.

- "The likelihood is symmetric..." is correct. We observed in the previous problem that observing the samples in a different order does not affect the likelihood. This is also evident from the definition of the likelihood: we can take the product

$$\prod_{i=1}^{n} p_\theta\left(x_i\right)$$

  in any order, and the result will still be the same.

- "If we eliminate a single observation, then the likelihood remains unchanged..." is incorrect. In the previous question, we saw that for a Poisson statistical model, $L_2\left(1,2,\lambda\right)$ and $L_3\left(1,2,3,\lambda\right)$ do not have the same formula. Hence, deleting an observation from the sample will change the likelihood.

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Answers are displayed within the problem

讨论

显示讨论

主题： Unit 3 Methods of Estimation:Lecture 8: Distance measures between distributions / 15. Likelihood of a Poisson Statistical Model