7. Examples of Parametric Models

Review: Sample Spaces of Distributions

4/4 points (graded)

Recall that a **sample space** of a random variable X is a set that contains all possible outcomes of X.

Note that the sample space of X is *not unique*. For example, if $X \sim \mathrm{Ber}\,(p)$, then both $\{0,1\}$ and $\mathbb{R}$ can serve as a sample space. However, in general, we associate a random variable with its smallest possible sample space (which would be $\{0,1\}$ if $X \sim \mathrm{Ber}\,(p)$).

Find the **smallest sample space** for each of the following random variables.

$X_1 \sim$ Poiss (λ), a **Poisson** random variable with parameter λ:

○ $\{0,1\}$

● $\{x \in \mathbb{Z} : x \geq 0\}$ ✓

○ $[0,\infty)$

○ $(-\infty,\infty)$

$X_2 \sim \mathcal{N}\,(0,1)$, a **standard Gaussian (or normal)** random variable with mean 0 and variance 1:

○ $\{0,1\}$

○ $\{x \in \mathbb{Z} : x \geq 0\}$

○ $[0,\infty)$

● $(-\infty,\infty)$ ✓

$X_3 \sim \exp\,(\lambda)$, an **exponential** random variable with parameter $\lambda > 0$:

○ $\{0,1\}$

○ $\{x \in \mathbb{Z} : x \geq 0\}$

● $[0,\infty)$ ✓

○ $(-\infty,\infty)$

$X_4 \sim \mathcal{I}\,(Y > 0)$ where Y is standard Gaussian and $\mathcal{I}$ is the **indicator function**.

Recall the definition of the indicator function is:

$$\mathcal{I}(Y>0)=\begin{cases}1 & \text{if } Y>0\\ 0 & \text{if } Y\le 0.\end{cases}$$

● $\{0,1\}$ ✓

○ $\{x\in\mathbb{Z}: x\ge 0\}$

○ $[0,\infty)$

○ $(-\infty,\infty)$

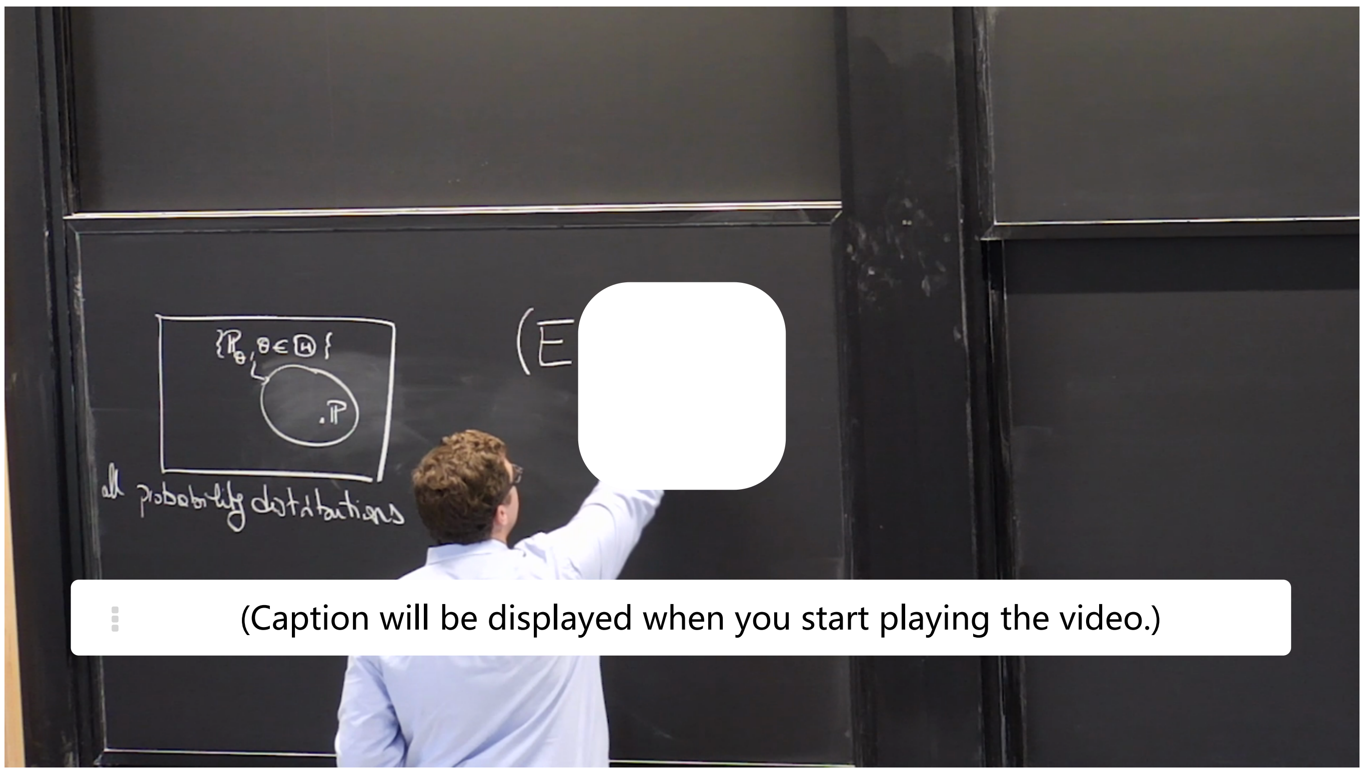
Solution:

- A Poisson random variable is discrete and can take values on all non-negative integers.
- Gaussian random variables can take any real value.
- The Exponential distribution is continuous and is restricted to all non-negative real values.
- The final random variable is an indicator, so it must take values in $\{0,1\}$. Note that X_4 is in fact Bernoulli.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

Examples of parametric and nonparametric models



Start of transcript. Skip to the end.

So let's write some statistical model.

So let's write it here.

A statistical model is a pair E, which is my sample space,

and a family if probability distribution p sub

theta, indexed by theta in some parameter set capital Theta.

So this is a statistical model.

And this is what we're going to be trying

to find for specific examples.

视频
下载视频文件

字幕
下载 SubRip (.srt) file
下载 Text (.txt) file

Statistical Model Definition Concept check

1/1 point (graded)
Which of the following is a statistical model?

○ $\left(\{1\}, (\text{Ber}\,(p))_{p\in(0,1)}\right)$

● $\left(\{0,1\}, (\text{Ber}\,(p))_{p\in(0.2,0.4)}\right)$ ✓

○ Both of the above

○ None of the above

Solution:

Solution in video below.
The set $\{1\}$ is not the sample space of the distribution $\mathrm{Ber}\,(p)$, so the first choice $\left(\{1\}, (\text{Ber}\,(p))_{p\in(0,1)}\right)$ is not a statistical model. On the other hand, $\left(\{0,1\}, (\text{Ber}\,(p))_{p\in(0.2,0.4)}\right)$ is a valid statistical model.
Remark: In the model $\left(\{0,1\}, (\text{Ber}\,(p))_{p\in(0.2,0.4)}\right)$, the parameter p is restricted to be in the interval $(0.2, 0.4)$. Such a restriction is perfectly valid, and can be useful for performing modeling tasks.

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

A Non-Example of a Statistical Model

0 points possible (ungraded)
(This problem is strictly pedagogical and is ungraded.)

Let $\mathcal{U}\,([0,a])$ denote the uniform distribution on the interval $[0,a]$. Let $X_1,\ldots,X_n \overset{iid}{\sim} \mathcal{U}\,([0,a])$ for some unknown $a>0$. Which one of the following is *not* a statistical model associated with this statistical experiment?

● $\left([0,a]\,, (\mathcal{U}\,([0,a]))_{a>0}\right)$ ✓

○ $\left(\mathbb{R}_+, (\mathcal{U}\,([0,a]))_{a>0}\right)$

○ Neither choice above is a statistical model.

Solution:

See video below.
The first choice $\left([0,a]\,, (\mathcal{U}\,([0,a]))_{a>0}\right)$ is not a statistical model because the sample space, as written, depends on an unknown parameter a.
The second choice $\left(\mathbb{R}_+, (\mathcal{U}\,([0,a]))_{a>0}\right)$ is a statistical model because for any value of a, the random variables $X_1,\ldots,X_n$ will have sample space contained in the interval $[0,\infty) = \mathbb{R}_+$.

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

Worked example: Definition of Statistical model

writing exactly the same thing as this guy.

OK?

So make sure that your sample space does not

depend on a parameter.