UNIT 5: Continuous random variables — Summary

- r.v.'s and PDFs: $f_X(x)$, $f_{X,Y}(x,y)$, $f_{X|Y}(x \mid y)$, $f_{X|A}(x)$
- Expectation: $\mathbf{E}[X]$, $\mathbf{E}[X \mid A]$, $\mathbf{E}[X \mid Y = y]$

  Expected value rule: $\mathbf{E}[g(X,Y)]$, $\mathbf{E}[g(X,Y) \mid A]$, $\mathbf{E}[g(X,Y) \mid Z = z]$

  Linearity: $\mathbf{E}[aX + bY] = a\mathbf{E}[Y] + b\mathbf{E}[Y]$
- Variance: $\text{var}(X)$, $\text{var}(X \mid A)$, $\text{var}(X \mid Y = y)$ $\quad\quad$ $\text{var}(X) = \mathbf{E}[X^2] - \big(\mathbf{E}[X]\big)^2$
- Independence of r.v.'s: $f_{X,Y} = f_X \cdot f_Y$

  $\mathbf{E}[XY] = \mathbf{E}[X] \cdot \mathbf{E}[Y]$ $\quad\quad$ $\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$
- Multiplication rule $\quad$ $f_{X,Y,Z}(x,y,z) = f_Z(z)\, f_{Y|Z}(y \mid z)\, f_{X|Y,Z}(x \mid y,z)$
- Total probability theorem $\quad$ $f_X(x) = \int f_Y(y)\, f_{X|Y}(x \mid y)\, dy$
- Total expectation theorem $\quad$ $\mathbf{E}[X] = \int f_Y(y)\, \mathbf{E}[X \mid Y = y]\, dy$
- Examples: uniform, geometric, exponential, normal

What was new?

- Replace:
  - sums by integrals
  - PMFs by PDFs
- Densities are not probabilities: $\mathbf{P}(x \le X \le x + \delta) \approx f_X(x) \cdot \delta$
- Conditioning on events $\{Y = y\}$ that have zero probability
- CDF: $F_X(x) = \mathbf{P}(X \le x)$
- Bayes' rule variations and mixed (discrete/continuous) models