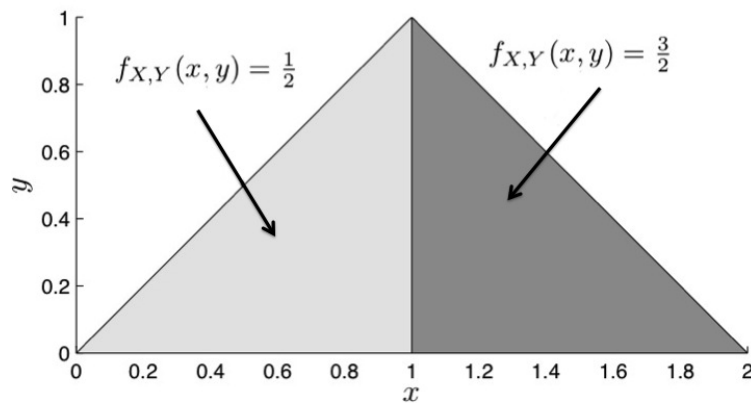# 7. A joint PDF on a triangular region

Problem 7. A joint PDF on a triangular region

8/10 points (graded)

This figure below describes the joint PDF of the random variables X and Y. These random variables take values in $[0, 2]$ and $[0, 1]$, respectively. At $x = 1$, the value of the joint PDF is $1/2$.

1. Are X and Y independent?

○ Yes

◉ No ✔

2. Find $f_X(x)$. Express your answers in terms of x, using the standard notation.

If $0 < x \leq 1$:

$f_X(x) =$

> 1/2*x

✔ Answer: x/2

$$\frac{1}{2} \cdot x$$

If $1 < x < 2$:

$f_X(x) =$

> 3 - 3/2*x

✔ Answer: 3-(3*x)/2

$$3 - \frac{3}{2} \cdot x$$

If $x < 0$ or $x \geq 2$:

$f_X(x) =$

> 0

✔ Answer: 0

$$0$$

3. Find $f_{Y|X}(y \mid 0.5)$.

If $0 < y < 1/2$:

$f_{Y|X}(y \mid 0.5) =$

> 2

✔ Answer: 2

If $y < 0$ or $y > 1/2$:

$f_{Y|X}(y \mid 0.5) =$

> 0

✔ Answer: 0

4. Find $f_{X|Y}(x \mid 0.5)$.

If $1/2 < x < 1$:

$f_{X|Y}(x \mid 0.5) =$

> 1/2

✔ Answer: 1/2

If $1 < x < 3/2$:

$f_{X|Y}(x \mid 0.5) =$

> 2/3

✘ Answer: 3/2

If $x < 1/2$ or $x > 3/2$:

$f_{X|Y}(x \mid 0.5) =$

> 0

✔ Answer: 0

5. Let $R = XY$ and let A be the event that $\{X < 0.5\}$. Find $\mathbf{E}[R|A]$.

$\mathbf{E}[R \mid A] = $ | 0 | ✗ **Answer:** 1/16

STANDARD NOTATION

Solution:

1. In order for X and Y to be independent, the value of X should not give any information about Y. But if X is smaller than say 0.5, then we can infer that $Y < 0.5$.

   In other words, $f_{Y|X}(y \mid 0.5) \neq f_Y(y)$. Therefore, X and Y are not independent.

2. Using the formula $f_X(x) = \int f_{X,Y}(x, y)dy$, we have,

$$f_X(x) = \begin{cases} \int_0^x \frac{1}{2}\, dy, & \text{if } 0 < x \leq 1, \\ \int_0^{2-x} \frac{3}{2}\, dy, & \text{if } 1 < x < 2, \\ 0, & \text{otherwise,} \end{cases}$$

$$= \begin{cases} x/2, & \text{if } 0 < x \leq 1, \\ -3x/2 + 3, & \text{if } 1 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$
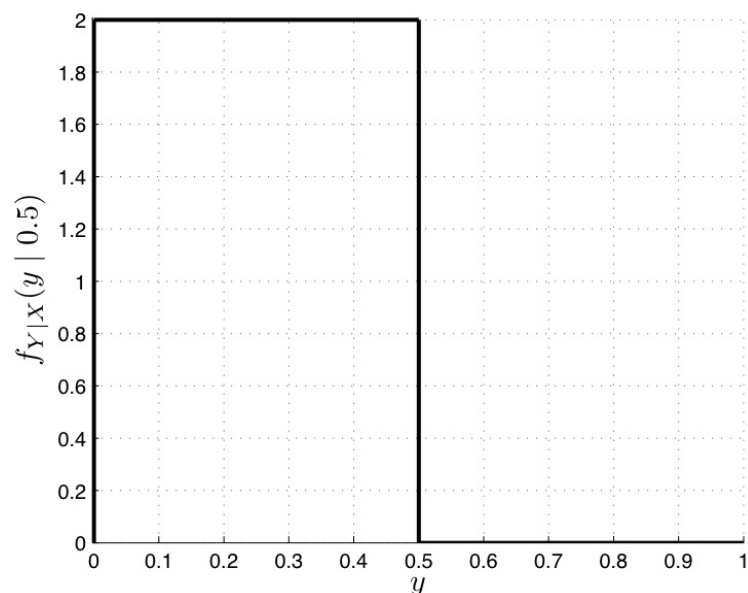
A plot of the PDF is shown below:
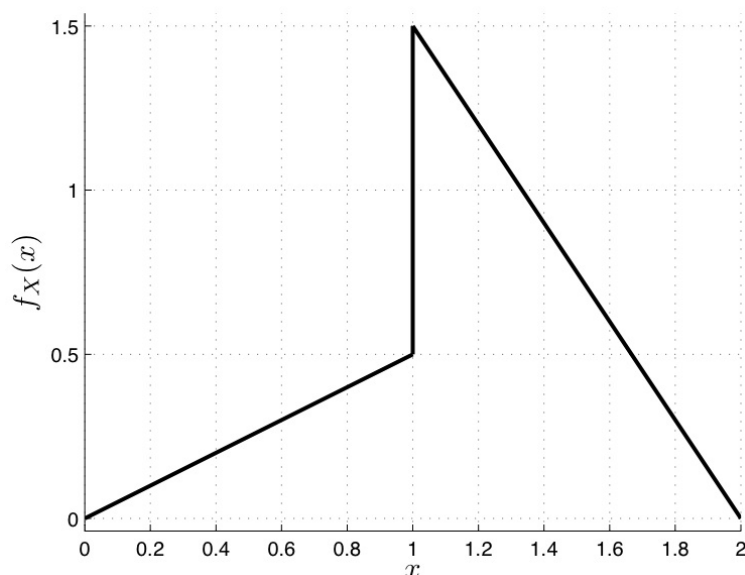
3. Given that $X = 0.5$, Y is uniformly distributed between 0 and $1/2$. Thus,

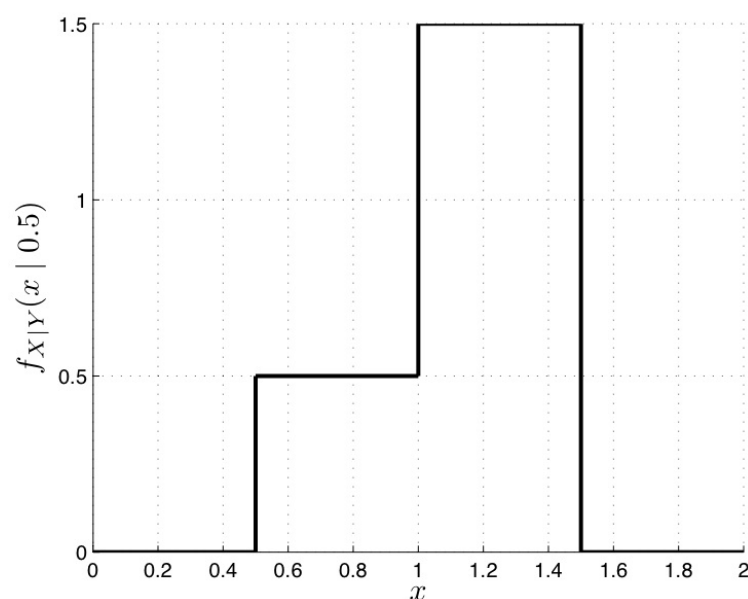$$f_{Y|X}(y \mid 0.5) = \begin{cases} 2, & \text{if } 0 \leq y \leq 1/2, \\ 0, & \text{otherwise.} \end{cases}$$

A plot of the conditional PDF is shown below:

4. Given that $Y = 0.5$, the conditional distribution of X is piecewise constant:

$$f_{X|Y}(x \mid 0.5) = \begin{cases} 1/2, & \text{if } 1/2 \le x \le 1, \\ 3/2, & \text{if } 1 < x \le 3/2, \\ 0, & \text{otherwise.} \end{cases}$$

A plot of the conditional PDF is shown below:

5. Under event A, the pair (X, Y) takes values in a triangular region with sides of length $1/2$, and area $1/8$. The conditional point PDF is uniform, so that $f_{X,Y|A}(x, y) = 8$ on that set. The conditional expectation is

$$\mathbf{E}[R \mid A] = \mathbf{E}[XY \mid A]$$

$$= \int\int xy f_{X,Y|A}(x,y) \ dx \ dy$$

$$= \int_0^{0.5} \int_y^{0.5} 8xy \ dx \ dy$$

$$= 1/16.$$

提交   You have used 5 of 5 attempts

ℹ Answers are displayed within the problem

讨论

Topic: Unit 5 / Problem Set / 7. A joint PDF on a triangular region

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