

Linear model: $Y|X=x \sim N(\beta^T x, \sigma^2)$
 $= p(x)$ for some known p
GLM:
 $Y|X=x \sim \text{Dist. in exponential family}$
 $E[Y|X=x] = \eta^T \beta$ How to pick η

The $x^T \beta$ is a choice that I made.

 7:05 / 7:05

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1/1 point (graded)

Which one of the following are **valid** link functions? Recall that a link function g is required to be **monotone increasing** and **differentiable**. (Choose all that apply.)

Note: The link function, in general, can be monotone increasing or monotone decreasing. In this class, we have chosen as convention to require it to be monotone increasing.

☒ $g(\mu) = \mu, \mu \in \mathbb{R}$ ✓

☑ $g(\mu) = -\frac{1}{\mu}, \mu > 0$ ✓

□ $g(\mu) = \mu^2, \mu \in \mathbb{R}$

☒ $\ln\left(\frac{\mu^3}{1-\mu^3}\right), 1 > \mu > 0$ ✓

☒ $-\ln\left[-\ln\left(\frac{\mu}{n}\right)\right], 0 < \mu < n \text{ and } n > 0$ known ✓



Solution:

Choices 1, 2, 4, and 5 are valid link functions. One can verify that these functions are differentiable and monotone increasing in their domain.

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You have used 2 of 2 attempts

i Answers are displayed within the problem

Concept Check: Linear Model and Generalized Linear Model

0 points possible (ungraded)

Which one of the following data modeling scenarios require one to **strictly use a generalized linear model over a Gaussian linear model**? (Choose all that apply.)

Note: While it is true that one can use a Gaussian linear model to fit any data (without paying attention to whether it is appropriate or not), in this problem we should use a GLM when it is more appropriate under a given scenario.

☒ We observe data $\mathbf{Y}_i \in \{0, 1, \dots, n_i\}$ as a function of integers $n_i > 0$ and we wish to model the proportions \mathbf{Y}_i/n_i . ✓

☒ We observe $\mathbf{Y}_i \in \mathbb{R}$ that we know are non-linearly related to the explanatory variables \mathbf{X}_i . ✓

☒ The dependent variable $\mathbf{Y} > 0$ has a discrete distribution whose expectation we wish to relate to the explanatory variable \mathbf{X} . ✓



Solution:

All of the scenarios require us to use generalized linear models. We examine the scenarios in order:

- The first choice requires us to model proportions that lie between **0** and **1**. A generalized linear model is clearly a better fit when compared to a linear model.
- The second choice suggests that we should apply a generalized linear model because we know the ground truth that the dependent variable is non-linearly related to the explanatory variables.
- In the third choice, the dependent variable \mathbf{Y} has a discrete distribution and it is stated that $\mathbf{Y} > 0$. If we are to fit the data using a model, a generalized linear model is better than a linear model because of multiple reasons. For one, the restriction $\mathbf{Y} > 0$ can be satisfied if we try to explain \mathbf{Y} for an unobserved data sample \mathbf{X} via the regression function $\mu(\mathbf{X})$ and the link function $g(\cdot)$: $\mu(\mathbf{X}) = g^{-1}(\mathbf{X}^T \boldsymbol{\beta})$. Secondly, a Gaussian linear model assumes that $\mathbf{Y}|\mathbf{X} = \mathbf{x}$ is normally distributed with some mean, which is clearly not the case here because $\mathbf{Y} > 0$ and \mathbf{Y} is discrete.

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