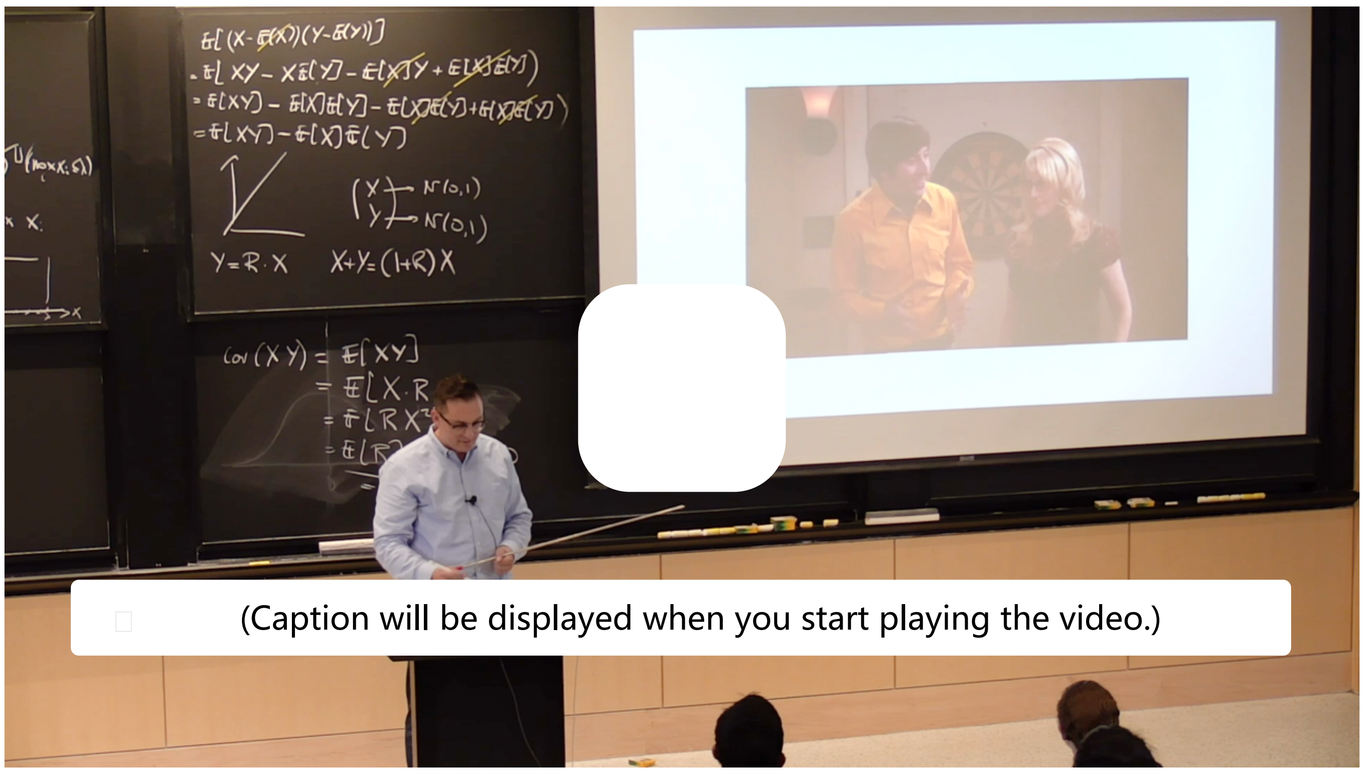
7. Covariance in Real Life

From The Big Bang Theory



Start of transcript. Skip to the end.

So remember this, when I tried to show this?

I finally realized what was the problem that I had.

Hopefully, this will work today.

[VIDEO PLAYBACK]

Hey, Sheldon.

It's me.

I'm going--

视频

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Sample Covariance

4/4 points (graded)

Let $(X_1,Y_1), (X_2,Y_2), \ldots, (X_n,Y_n) \stackrel{iid}{\sim} (X,Y)$ with $\mathbb{E}[X] = \mu_X, \mathbb{E}[Y] = \mu_Y$, and $\mathbb{E}[XY] = \mu_{XY}$. That is, each random variable pair (X_1,Y_1) has the same distribution as the random variable pair (X,Y), and the pairs are independent of one another.

Estimating the covariance between X and Y based on observed sequences is useful because non-zero covariance implies dependence between X and Y. In this problem, we study one way to obtain an unbiased estimator for **Cov** (X,Y).

Consider the following estimator for the covariance:

$$\widetilde{S}_{XY} = \frac{1}{n}\left(\sum_{i=1}^{n}\left(X_i - \overline{X}_n\right)\left(Y_i - \overline{Y}_n\right)\right),$$

where $\overline{X}_n$ and $\overline{Y}_n$ denote the sample mean estimators of μ_X and μ_Y.

What is $\mathbb{E}\left[\frac{\sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{n}\right]$? Provide an expression in terms of n, μ_X, μ_Y, and μ_{XY}.

(Enter **mu_{XY}** for μ_{XY}, **mu_X** for μ_X, and **mu_Y** for μ_Y.)

$\mathbb{E}\left[\frac{\sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{n}\right] =$ mu_{XY} + (n-1)*mu_X*mu_Y

Answer: (1/n)*(n*mu_XY + n*(n-1)*mu_X*mu_Y)

What is $\mathbb{E}[\widetilde{S}_{XY}]$? Provide an expression in terms of n, μ_X, μ_Y, and μ_{XY}.

(Enter **mu_{XY}** for μ_{XY}, **mu_X** for μ_X, and **mu_Y** for μ_Y.)

$\mathbb{E}[\widetilde{S}_{XY}] =$ (n-1)/n*(mu_{XY}-mu_X*mu_Y) **Answer:** ((n-1)/n)*(mu_XY -mu_X*mu_Y)

Is $\widetilde{S}_{XY}$ an unbiased estimator of Cov (X, Y)?

- Yes
- No

If your answer to the above question is "Yes", then type "1" in the following box. Otherwise, find a scaling factor c such that

$$\widehat{S}_{XY} = c \cdot \widetilde{S}_{XY}$$

is an unbiased estimator of Cov (X, Y). Provide your answer in terms of n, μ_X, μ_Y, and μ_{XY}.

(Enter **mu_{XY}** for μ_{XY}, **mu_X** for μ_X, and **mu_Y** for μ_Y.)

$c =$ n/(n-1) **Answer:** (n/(n-1))

STANDARD NOTATION

Solution:

First,

$$\begin{aligned}\mathbb{E}\left[\frac{\sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{n}\right] &= \frac{1}{n}\mathbb{E}\left[\sum_{i=1}^n X_i Y_i + \sum_{i=1}^n \sum_{i \neq j=1}^n X_i Y_j\right] \\ &= \left[\mu_{XY} + (n-1)\,\mu_X \mu_Y\right],\end{aligned}$$

where we have used the property that X_i and Y_j are independent whenever $i \neq j$. Then,

$$\begin{aligned}\mathbb{E}[\widetilde{S}_{XY}] &= \frac{1}{n}\mathbb{E}\left[\sum_{i=1}^n \left(X_i - \overline{X}_n\right)\left(Y_i - \overline{Y}_n\right)\right] \\ &= \frac{1}{n}\mathbb{E}\left[\sum_{i=1}^n X_i Y_i - \frac{\sum_{i=1}^n X_i}{n}\sum_{j=1}^n Y_i - \frac{\sum_{i=1}^n Y_i}{n}\sum_{j=1}^n X_i + \frac{\sum_{i=1}^n X_i \sum_{j=1}^n Y_i}{n}\right] \\ &= \frac{1}{n}\mathbb{E}\left[\sum_{i=1}^n X_i Y_i - \frac{\sum_{i=1}^n X_i \sum_{j=1}^n Y_i}{n}\right].\end{aligned}$$

Using the result in the first part of the problem, we get

$$\begin{aligned}\mathbb{E}[\widetilde{S}_{XY}] &= \frac{1}{n}\left[n\mu_{XY} - \left(\mu_{XY} + (n-1)\,\mu_X \mu_Y\right)\right] \\ &= \frac{n-1}{n}\left[\mu_{XY} - \mu_X \mu_Y\right] \\ &= \frac{n-1}{n}\text{Cov}\,(X, Y).\end{aligned}$$

From the above, we can see that the estimator is biased because $\mathbb{E}[\widetilde{S}_{XY}] \neq \text{Cov}\,(X, Y)$.

However, the bias can be fixed by multiplying $\widetilde{S}_{XY}$ by $\frac{n}{n-1}$ to obtain the following unbiased estimator of Cov (X, Y):

$$\widehat{S}_{XY} = \frac{1}{n-1}\left[\sum_{i=1}^n \left(X_i - \overline{X}_n\right)\left(Y_i - \overline{Y}_n\right)\right].$$

提交 你已经尝试了3次（总共可以尝试4次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 3 Methods of Estimation:Lecture 10: Consistency of MLE, Covariance Matrices, and Multivariate Statistics / 7. Covariance in Real Life