MITx Video | 6.041x | Spring 2014 | Video transcript | L23-7: Random incidence in a non-Poisson process

In this segment, we go through another random incidence example, one that does not involve the Poisson process, but a much simpler arrival process. We do this for two reasons. One is because of the simplicity of the example, perhaps the intuition will be a little more transparent. And the second reason is to illustrate that we're dealing with a phenomenon that's not specific to the Poisson process, but is much more general.

The example is as follows. We have an arrival process, in which arrivals happen at random. And the consecutive interarrival times are independent, identically distributed, random variables. However, unlike the Poisson process, these interarrival times are not exponential random variables. But instead, we assume that they are either 5 or 10 minutes with equal probability.

So we have an arrival. The next arrival may happen five minutes later. The next arrival may again happen five minutes later. Then maybe we get an interarrival interval of length 10, then maybe another interarrival interval of length 10, followed by one of five, and so on.

What is the expected value of the kth interarrival time? Well, an interarrival time is with probability 1/2 of length five and with probability 1/2 of length 10. So the average interarrival time is 7.5.

Now, you show up at a random time. And by random we mean a time that's completely uncoordinated with the arrival process. You show up at some point in time, and you look at the interarrival interval in which you fall. And you're interested in the length of that particular interarrival interval. What is the probability that you fall inside a five minute interarrival interval?

Since intervals of length five are as likely as intervals of length 10, in the long run, there's going to be roughly as many five minute intervals as there will be 10 minute intervals. On the other hand, the 10 minute intervals occupy twice as much space on the real line. And for this reason, it is 2 times more likely that you will fall in a 10 minute interval rather than a five minute interval.

In other words, the probability of falling in a five minute interval is going to be 1/3. Whereas the probability of following in a 10 minute interval is going to be 2/3. For this reason, the expected length of the interarrival interval that you get to see when you arrive is equal to, with probability 1/3, you see a five. And with probability 2/3, you see a 10. And this number evaluates approximately to 8.3, which is indeed larger than 7.5.

The conclusion from this example is similar to the one that we had for the Poisson process. Although the average interarrival time is 7.5, when you show up at a random time you are more likely to fall in a larger interval. And for that reason, on the average, you will be seeing longer interarrival intervals.

The calculations that we carried through in that simple example can be generalized for more general arrival processes, called renewal processes. In a renewal process, once more the consecutive interarrival times are independent, identically distributed, random variables. But they have a general distribution.

For this case, there's a formula again that tells us the length or the expected length of the interarrival interval that you get to see. But the main message from this example is that it does make a difference how you sample, how you choose what to watch and what to measure. It makes a difference whether you decide to measure the kth interarrival time and it's average value or to decide to measure an interarrival time that's chosen by showing up at a random time. The two methods of sampling give you different results. And we will see next a few examples that have this particular flavor.