

Indicator variables: the number of inversions. We will use the method of indicator functions. For all $i < j$, define variables Y_{ij} to be equal to 1 if $X_i > X_j$, and zero otherwise. Then we have

$$N = \sum_{i < j} Y_{ij}.$$

By the linearity of expectation, we have:

$$\begin{aligned} \mathbf{E}[N] &= \mathbf{E} \left[\sum_{i < j} Y_{ij} \right] \\ &= \sum_{i < j} \mathbf{E}[Y_{ij}] \\ &= \sum_{i < j} \mathbf{P}(X_i > X_j) \\ &= \sum_{i < j} \frac{1}{2} \\ &= \sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{1}{2} \\ &= \frac{1}{2} \frac{n(n-1)}{2} = \frac{1}{2} \binom{n}{2}. \end{aligned}$$

Another way to see this is to observe that there are $\binom{n}{2}$ pairs total, and by symmetry, the expected number of pairs in inverted order will be half of the total number of pairs.