6. Probability tables

Gaussian probabilities

4/4 points (graded)

Let $X \sim N(1, 2.25)$. As a reminder, the 2.25 here represents the value of σ^2. Using the normal probability table below, compute the following probabilities:

Normal probability table

The table lists $P(Z \leq z)$ where $Z \sim N(0, 1)$ for positive values of z.

Second decimal place of z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

*For $Z \geq 3.50$, the probability is greater than or equal to 0.9998.

Hide

$\mathbf{P}(X > 1) =$ 0.5 ✔ **Answer:** 0.5

$\mathbf{P}(|X - 2| \leq 1) =$ 0.4082 ✔ **Answer:** 0.4082

$\mathbf{P}(X^2 > 4) =$ 0.2742 ✔ **Answer:** 0.2752427

$\mathbf{P}(X^2 - 2X - 1 > 0) =$ 0.3472 ✔ **Answer:** 0.3472

STANDARD NOTATION

Solution:

First, note that for $Z \sim \mathcal{N}(0,1)$, $x > 0$, we have

$$\mathbf{P}(Z \leq -x) = \mathbf{P}(Z \geq x) = 1 - \mathbf{P}(Z \leq x),$$

and

$$\mathbf{P}(Z \geq x) = 1 - \mathbf{P}(Z \leq x).$$

Moreover, if $X \sim \mathcal{N}(1, 2.25)$, we can write it as $X = 1.5Z + 1$, where $Z \sim \mathcal{N}(0,1)$. This allows us to reduce all probabilities to the ones that are listed in the table.

In particular,

$$\begin{aligned}
\mathbf{P}(X > 1) = &\ \mathbf{P}(1.5Z + 1 > 1) = \mathbf{P}(1.5Z > 0) \\
= &\ \mathbf{P}(Z \geq 0) = 1 - \mathbf{P}(Z \leq 0) = 1 - 0.5000 = 0.5000,
\end{aligned}$$

$$\begin{aligned}
\mathbf{P}(|X - 2| \leq 1) = &\ \mathbf{P}(-1 \leq (X - 2) \leq 1) = \mathbf{P}(-1 \leq (1.5Z + 1 - 2) \leq 1) \\
= &\ \mathbf{P}(0 \leq 1.5Z \leq 2) \\
\simeq &\ \mathbf{P}(0 \leq Z \leq 1.33) \\
= &\ \mathbf{P}(Z \leq 1.33) - \mathbf{P}(Z \leq 0) \simeq 0.9082 - 0.5000 = 0.4082
\end{aligned}$$

$$\begin{aligned}
\mathbf{P}(X^2 > 4) = &\ \mathbf{P}(|X| > 2) = \mathbf{P}(|1.5Z + 1| > 2) \\
= &\ \mathbf{P}(1.5Z + 1 \leq -2) + \mathbf{P}(1.5Z + 1 \geq 2) \\
= &\ \mathbf{P}(Z \leq -2) + \mathbf{P}\left(Z \geq \frac{2}{3}\right) \\
= &\ 1 - \mathbf{P}(Z \leq 2) + 1 - \mathbf{P}\left(Z \leq \frac{2}{3}\right) \\
\simeq &\ 2 - 0.9772 - 0.7486 = 0.2742
\end{aligned}$$

$$\begin{aligned}
\mathbf{P}(X^2 - 2X - 1 > 0) = &\ \mathbf{P}((X - 1)^2 - 2 > 0) = \mathbf{P}(|X - 1| > \sqrt{2}) \\
= &\ \mathbf{P}(|1.5Z| > \sqrt{2})
\end{aligned}$$

$$
\begin{aligned}
&= \mathbf{P}\left(Z > \frac{\sqrt{2}}{1.5}\right) + \mathbf{P}\left(Z < -\frac{\sqrt{2}}{1.5}\right) \\
&= 2 - 2\mathbf{P}\left(Z < \frac{\sqrt{2}}{1.5}\right) \\
&\simeq 2 - 2\left(0.8264\right) \\
&= 0.3472.
\end{aligned}
$$

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

Approximation of Binomial variables

1/1 point (graded)

Using the normal probability table, evaluate approximately $\mathbf{P}\left(X > 400\right)$, where X is a binomial random variable with parameters 1000 and $.3$.

Normal probability table

The table lists $P\left(Z \leq z\right)$ where $Z \sim N\left(0, 1\right)$ for positive values of z.

	Second decimal place of z									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

*For $Z \geq 3.50$, the probability is greater than or equal to 0.9998.

Hide

$\mathbf{P}(X > 400) \simeq$ 0.0002 ✔ **Answer:** 0.0002

STANDARD NOTATION

Solution:

A binomial distribution with parameters (n, p) has expectation np and variance $np(1-p)$. Hence, by the Central Limit Theorem, we have

$$\frac{1}{\sqrt{np(1-p)}}(X - np) \xrightarrow{(\mathrm{D})} Z \sim \mathcal{N}(0,1).$$

The probability in question can therefore be approximated by

$$\begin{aligned}
\mathbf{P}(X > 400) = & \quad \mathbf{P}\left(\frac{1}{\sqrt{1000 \times 0.3 \times 0.7}}(X - 300) > \frac{100}{\sqrt{1000 \times 0.3 \times 0.7}}\right) \\
\simeq & \quad 1 - \mathbf{P}\left(Z \leq \frac{100}{\sqrt{1000 \times 0.3 \times 0.7}}\right) \\
\simeq & \quad 1 - \mathbf{P}(Z \leq 6.90) \\
\leq & \quad 1 - 0.9998 = 0.0002.
\end{aligned}$$

Note: This is only an estimate, because the probability table ends here. In fact, the probability is approximately 7×10^{-12}.

提交 你已经尝试了2次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 0. Course Overview, Syllabus, Guidelines, and Homework on Prerequisites:Homework 0: Probability and Linear algebra Review / 6. Probability tables

认证证书是什么?