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论坛

Week 3

Week 3

I don't get this whole matrix operations thing

TL

Thorsten Lemke Week 3 · 2 years ago

I don't get it at all how multiplying an adjacency matrix multiple times with itself can tell us anything about a path being there or not being there. Can someone please try to explain it in a way that a non-mathematician can understand? I read the provided material about paths and matrices and also tried to calculate a simple example with a 3-node graph with only one edge between node 1 and 2. But it doesn't really help.

I also find it extremely confusing that the variable "k" seems to be used alternating for completely different purposes. In one paragraph it is the path length "k" and in the next paragraph it suddenly is the matrix width "k". So what exactly are we talking about if it says A^k[i,j]?

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YK分钟

Yoshimi Kuruma Mentor · 2 years ago · 已编辑

Hi Thorsten. First, we fix the notation.

- A: adjacency matrix of n nodes graph
- $A[i,j]$: the ith row, j th column element of A (index start from 0)
- A^k: A matrix created by multiplying A by k times
- $A^k[i,j]$: the ith row, j th column element of A^k

Next, we define the product of two n-dimentional (n rows and n columns) matrices, A and B, $C = A \cdot B$:

$$C[i,j] = \sum_{l=0}^{n-1}(A[i,l] \cdot B[l,j])$$

for every $0 \leq i \leq n-1$ and $0 \leq j \leq n-1$.

As a special case, the product of the same matrix A and A, $A^2 = A \cdot A$:

$$A^2[i,j] = \sum_{l=0}^{n-1}(A[i,l] \cdot A[l,j])$$

for $0 \leq i \leq n-1$ and $0 \leq j \leq n-1$.

Then, we can investigate the relation between the product of adjacency matrices and pathes with a concrete example. Here is a adjacency matrix corresponding to the ugraph (**but numbering is changed to 0-4 not 1-5**) in this page https://www.coursera.org/learn/algorithmic-thinking-1/resources/8t9fW

```
A=[[0, 1, 0, 0, 0],
   [1, 0, 1, 1, 0],
   [0, 1, 0, 1, 0],
   [0, 1, 1, 0, 0],
   [0, 0, 0, 0, 0]]
```

and the product of A and A, $A \cdot A$ is

```
AA=[[1, 0, 1, 1, 0],
    [0, 3, 1, 1, 0],
    [1, 1, 2, 1, 0],
    [1, 1, 1, 2, 0],
    [0, 0, 0, 0, 0]]
```

How we get this? We can get this from the definition. For example,

$$\begin{aligned} AA[1,1] = A[1,0] \cdot A[0,1] + A[1,1] \cdot A[1,1] + A[1,2] \cdot A[2,1] \\ + A[1,3] \cdot A[3,1] + A[1,4] \cdot A[4,1] = 3 \end{aligned}$$

Remembering $A[i,j] = 1$ if an edge from i to j exists and otherwice $A[i,j] = 0$, we see that $A[i,l] \cdot A[l,j] = 1$ if edges from i to some node l and l to j exist, meaning a path from i to j with length 2 exists.

So, $AA[1,1]$ is the number of path from 1 to 1 with length 2. (1,0,1), (1,2,1), (1,3,1) (or using the original numbering (2,1,2), (2,3,2), (2,4,2)).

We can calculate A^3, A^4, ... along with the definition but we need to consider the meaning of $A^{k-1}[i,l] \cdot A[l,j]$.

http://www.codeskulptor.org/#user41_X7gbOoI9y2qRLJ4.py

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TL Thorsten Lemke · 2 years ago

Thanks a lot for the detailed explanation. I also spend some time on Khan Academy. And now at least I was able to answer the quiz correctly.

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LP

Luoshang (Jeff) Pan · 2 years ago

Gratitudes! I didn't get the reason for Theorem 2 as well. Your explaination is really good.

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Denis Ivanov · 2 years ago

@Yoshimi Kuruma , Hi! How did you get "(1,0,1), (1,2,1), (1,3,1)"?

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YK分钟

Yoshimi Kuruma Mentor · 2 years ago · 已编辑

Hi, Denis. In calculating $AA[1,1]$, we have

- $A[1,0] \cdot A[0,1] = 1$ means (1,0,1) (1 to 0 and 0 to 1)
- $A[1,1] \cdot A[1,1] = 0$ means no path
- $A[1,2] \cdot A[2,1] = 1$ means (1,2,1)
- $A[1,3] \cdot A[3,1] = 1$ means (1,3,1)
- $A[1,4] \cdot A[4,1] = 0$ means no path

3 个赞

Thomas Korrison · 2 years ago

Really interesting property of matrices. Who discovered or proved that matrix multiplication gave rise to this connected phenomena?

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KL

Kenneth LaMantia · 2 years ago

This was also confusing for me, and unfortunately working through examples by hand of graphs with even 5 nodes is pretty time consuming.

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Mohammad Ghaffari · 2 years ago · 已编辑

@Yoshimi Kuruma

I understand the relationship of powers of A and having a path between two nodes if the value at [i][j] > 0.

However I don't understand the questions related to the **number of (shortest) paths** like the followings from the quiz:

- If A is the adjacency matrix for a graph g, what is the number of paths of length k from node i to node j in g?
- If A is the adjacency matrix for g and A^ is the adjacency matrix for the graph g^ created by removing e (but not its endpoints) from g, *what expression* corresponds to the number of shortest paths from node i to node j that include the edge e?

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