

The other thing that we discussed in the last lecture was a classification of the different states of the Markov chain into different types. A Markov chain in general has states that are recurrent, which means that from that recurrent state, you can go somewhere else and then from that somewhere else you can always come back to it.

So if you have a Markov chain of this form and you start in state nine, the options for you is to either go to state three or to state five. But no matter what, if you go to three, you can come back always, and if you go to five, you can always come back as well. So clearly nine here would be a recurrent state, and three for the same reason, and five as well.

Now, if you look at the state six or seven, it is the same thing. Starting from six, the only way that you can go to is to either stay at six or go to seven, and then in that case you always come back. And same thing from seven, you can either go to six and that's it's, actually, and come back.

So both of these are recurrent as well. So in a case the state is not recurrent, we will call it transient. So let's look at for example state one. From state one, if you go from one to two and then go to six, there is no way to come back to one. So the state 1 is transient, and for the same reason the state four will be transient, and the state 2 will be transient.

What about eight? Well, the same reason, the state is transient as well. So what we have seen also is the notion of a recurrent class. A recurrent class is, again, a collection of recurrent states that can communicate between each other.

So here, for this specific example, we have two classes. This is one class, right, so it's a class one. Let's call it recurrent class one. And this is a recurrent class, recurrent class two.

So here again we have two classes instead of one, because if you are in one of these classes, there is no way that you can find a path to go to one state here and vice versa. If you are in one of these states here, there is no path that would lead you to that recurrent class.

In the case where you have two recurrent classes, like here or more, it is pretty clear that in the long run, the steady state behavior of the Markov chain will really depend on where you started. So for example, if your Markov chain started in that recurrent class, there is no probability that in the long run

it will be in that class, and vice versa.

If it started here, the probability of being in that recurrent class in the long run is zero. So the long run behavior of the Markov chain will depend on the initial condition.

In the case where you have only one recurrent class, let's forget about that portion, for example, and you have only that portion here. Then maybe the initial condition will not matter in the long run, but in fact it's not going to be always the case, depending on the recurrent class being periodic or not as we will see in the next clip.