3. Independent exponential random variables

Problem 2. Independent exponential random variables

2/2 points (graded)

Let X and Y be two independent, exponentially distributed random variables with parameters λ, and μ, respectively.

For each question below, enter your answers using standard notation; enter **mu** for mu and **lambda** for λ.

1. Find the probability that $X \le Y$.

$\mathbf{P}(X \le Y) =$ lambda/(lambda+mu) ✔ **Answer:** lambda/(mu+lambda)

$\frac{\lambda}{\lambda+\mu}$

2. Let $Z = 1/(1+X)$. For $0 < z < 1$:

$f_Z(z) =$ lambda*e^(-1*lambda*(✔ **Answer:** (lambda*exp(lambda)*exp(-lambda/z))/(z^2)

$\frac{\lambda \cdot e^{-1 \cdot \lambda \cdot \left(\frac{1}{z} - 1\right)}}{z^2}$

STANDARD NOTATION

Solution:

1. Using the law of total probability theorem, and independence of X and Y,

$$\begin{aligned}
\mathbf{P}(X \le Y) &= \int_0^\infty \mathbf{P}(X \le y) f_Y(y)\, dy \\
&= \int_0^\infty \mathbf{P}(X \le y) \mu e^{-\mu y}\, dy = \int_0^\infty (1 - e^{-\lambda y}) \mu e^{-\mu y}\, dy \\
&= \frac{\lambda}{\mu + \lambda}.
\end{aligned}$$

2. We have, for $0 < z < 1$,

$$\begin{aligned}
\mathbf{P}(Z \le z) &= \mathbf{P}\left(\frac{1}{1+X} \le z\right) \\
&= \mathbf{P}\left(1 + X \ge \frac{1}{z}\right) \\
&= \mathbf{P}\left(X \ge \frac{1}{z} - 1\right) \\
&= e^{-\lambda(1/z - 1)} \\
&= e^{-\lambda/z} \cdot e^{\lambda}.
\end{aligned}$$

Differentiating the expression above with respect to z yields,

$$f_Z(z) = \frac{\lambda}{z^2} e^{-\lambda(1/z-1)} = \frac{\lambda e^{\lambda}}{z^2} e^{-\lambda/z}.$$

提交 You have used 2 of 3 attempts

Answers are displayed within the problem

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I used convolution formula to solve Part 1. The answer is correct ,but am I just lucky?

question posted about 3 hours ago by sakimarquis

I am not very familiar with LATEX, please kindly bear with my poor handwriting. Here's my solution.I use $X - Z$ to substitute Y, but when i use $Y + Z$ to substitute X, the result is not defined. So i'm wondering if i was just lucky to coincide with the correct answer.

Suppose, $Z = X - Y$, $Y = X - Z$

$$P(X \le Y) = P(X - Y \le 0) = P(Z \le 0)$$
$$= \int_{-\infty}^{0} f_Z(z)\, dz$$
$$= \int_{-\infty}^{0} \left[\int_{0}^{+\infty} f_X(x) \cdot f_Y(x - z)\, dx \right] dz$$
$$= \int_{-\infty}^{0} \left[\int_{0}^{+\infty} \lambda \cdot e^{-\lambda x} \cdot u \cdot e^{-u(x-z)}\, dx \right] dz$$
$$= \int_{-\infty}^{0} \frac{\lambda u}{\lambda + u} \cdot e^{uz} \cdot dz = \frac{\lambda}{\lambda + u}$$

此帖对所有人可见。

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2 responses

LimYiLe
about 2 hours ago

I used Convolution to solve part 1 too and got the same answer. And your handwriting's nice. :)

Thank you! I did get luck in use convolution formula!

sakimarquis 在5 minutes ago前发表

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alexannan
about an hour ago - 10 minutes ago 前被 **sakimarquis** 标记为答案

Your reasoning looks pretty good to me!

Possibly the reason it didn't work with $y + z$ is that, when you take the outer integral, z takes on negative values. So $y + z$ might be negative, and so you'd need to explicitly take account of the fact that $f_X(x) = 0$ when $x < 0$.

You got lucky with $x - z$ in as much as you accidentally didn't have to take account of the piecewise nature of f_Y, because $x - z$ is always positive when $z < 0$.

Does that make any sense?

I think it's just an issue of the PDFs of X and Y being piecewise, and remembering to take account of that. That's always where I trip up on problems involving integrals. I find it helps to use Iverson brackets to represent piecewise functions. But then that might just be creating more problems!

Thank you for explanation!

sakimarquis 在2 minutes ago前发表

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