4. Hypothesis Testing and Confidence intervals

(a)

2.0/2 points (graded)

Consider an i.i.d. sample $X_1, \ldots, X_n \sim \textsf{Poiss}(\lambda)$ for $\lambda > 0$.

Starting from the Central Limit Theorem, find a confidence interval $I = [A, B]$ with asymptotic level $1 - \alpha$ that is centered about $\overline{X}_n$ using the plug-in method.

Write **barX_n** for $\overline{X}_n$. If applicable, type **abs(x)** for $|x|$, **Phi(x)** for $\Phi(x) = \mathbf{P}(Z \le x)$ where $Z \sim \mathcal{N}(0,1)$, and **q(alpha)** for q_α, the $1-\alpha$ quantile of a standard normal variable.)

$\mathcal{I} = [A, B]$ for

$A =$ barX_n - q(alpha/2)*sqrt(barX_n/n) **Answer:** barX_n - q(alpha/2)*sqrt(barX_n)/sqrt(n)

$B =$ barX_n + q(alpha/2)*sqrt(barX_n/n) **Answer:** barX_n + q(alpha/2)*sqrt(barX_n)/sqrt(n)

STANDARD NOTATION

Solution:

By the Central Limit Theorem,

$$\sqrt{n}\frac{\overline{X}_n - \lambda}{\sqrt{\lambda}} \xrightarrow[n\to\infty]{(\mathrm{D})} \mathcal{N}(0,1).$$

Since

$$\overline{X}_n \xrightarrow[n\to\infty]{\mathbf{P}} \lambda,$$

by Slutsky's Theorem, we get

$$\sqrt{n}\frac{\overline{X}_n - \lambda}{\sqrt{\overline{X}_n}} \xrightarrow[n\to\infty]{(\mathrm{D})} \mathcal{N}(0,1).$$

That means for $q > 0$ that with

$$I = \left[\overline{X}_n - \frac{q\sqrt{\overline{X}_n}}{\sqrt{n}}, \overline{X}_n + \frac{q\sqrt{\overline{X}_n}}{\sqrt{n}}\right],$$

we have

$$\mathbf{P}_\lambda(\lambda \in I) \xrightarrow[n\to\infty]{} 1 - 2\Phi(q).$$

If we want this quantity to be $1-\alpha$ to guarantee level $1-\alpha$ of the interval, that leads to

$$\Phi(q) = 1 - \frac{\alpha}{2} \iff q = q_{\alpha/2} = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right).$$

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

(b)

2.0/2 points (graded)

Consider the following hypothesis with a fixed number $\lambda_0 > 0$:

$$H_0 : \lambda = \lambda_0 \quad \text{vs} \quad H_1 : \lambda \neq \lambda_0.$$

Define a test for the above hypotheses with asymptotic level α, and rewrite it in the following form:

$$\psi = \mathbf{1}\{\lambda_0 \notin J\},$$

with an interval J.

$\mathcal{J} = [C, D]$ for

$C =$ barX_n - q(alpha/2)*sqrt(barX_n/n) **Answer:** barX_n - q(alpha/2)*sqrt(barX_n)/sqrt(n)

$D =$ barX_n + q(alpha/2)*sqrt(barX_n/n) **Answer:** barX_n + q(alpha/2)*sqrt(barX_n)/sqrt(n)

Solution:

By setting

$$J = I = \left[\overline{X}_n - \frac{q_{\alpha/2}\sqrt{\overline{X}_n}}{\sqrt{n}}, \overline{X}_n + \frac{q_{\alpha/2}\sqrt{\overline{X}_n}}{\sqrt{n}}\right]$$

from part (a), the fact that I is a confidence interval with asymptotic level α means that

$$\mathbf{P}_\lambda(\lambda \in I) \xrightarrow[n\to\infty]{} 1-\alpha \quad \lambda > 0,$$

so

$$\mathbf{P}_\lambda(\lambda \notin I) \xrightarrow[n\to\infty]{} \alpha \quad \lambda > 0.$$

In particular, if we set

$$\psi = \mathbf{1}\{\lambda_0 \notin I\},$$

this means that

$$\mathbf{P}_{\lambda_0}(\psi = 1) = \mathbf{P}_{\lambda_0}(\lambda_0 \notin I) \xrightarrow[n \to \infty]{} \alpha,$$

yielding a hypothesis test with asymptotic level α.

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Answers are displayed within the problem

讨论

显示讨论

主题： Unit 2 Foundation of Inference:Homework 3: Introduction to Hypothesis Testing / 4. Hypothesis Testing and Confidence intervals

认证证书是什么？