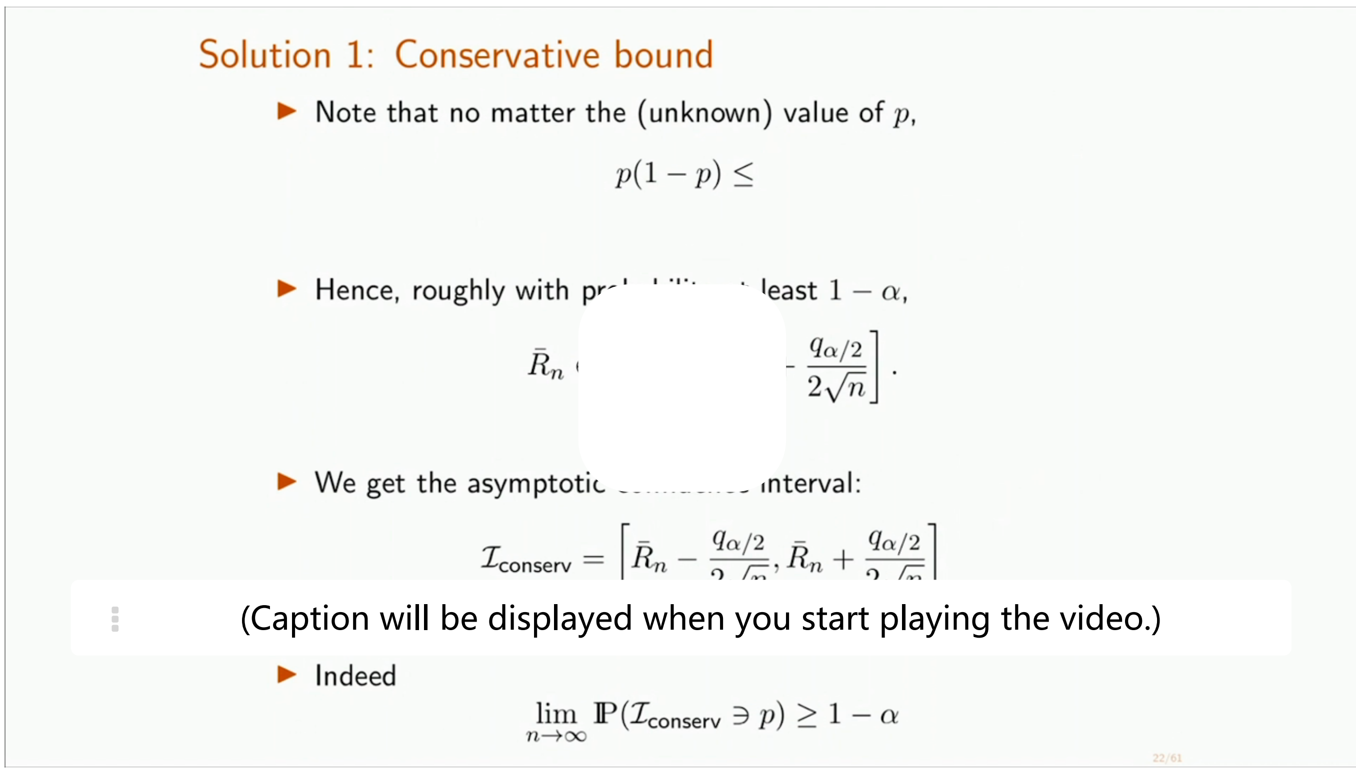
9. Conservative Bound

Confidence Interval using a Conservative Bound



Start of transcript. Skip to the end.

OK, we're going to start from the simplest to the most

complicated one.

The first one is to use the conservative bound.

Remember this picture?

I think I left it on the slide.

So that was p 1 minus p, and that was p.

Now this is the only function that shows up--

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Conservative bound

1/1 point (graded)

As in the video above, let $R_1, \ldots, R_n \overset{iid}{\sim} \mathsf{Ber}(p)$ for some unknown parameter p. We estimate p using the estimator $\hat{p} = \overline{R}_n = \frac{1}{n}\sum_{i=1}^{n} R_i$.

Recall that by the central limit theorem, for any p, $(0 < p < 1)$:

$$\lim_{n\to\infty} \mathbf{P}\left(\left|\sqrt{n}\frac{\overline{R}_n - p}{\sigma_p}\right| < q_{\alpha/2}\right) = \lim_{n\to\infty} \mathbf{P}\left(\overline{R}_n - q_{\alpha/2}\frac{\sigma_p}{\sqrt{n}} < p < \overline{R}_n + q_{\alpha/2}\frac{\sigma_p}{\sqrt{n}}\right) = 1-\alpha$$

where $\sigma_p = \sqrt{p(1-p)}$.

To construct a confidence interval, we need to replace σ_p above by a number c that does not depend on the unknown parameter p.

Which of the following conditions on c will guarantee that for all p in $(0,1)$,

$$\lim_{n\to\infty} \mathbf{P}\left(\left|\sqrt{n}\frac{\overline{R}_n - p}{c}\right| < q_{\alpha/2}\right) \geq 1-\alpha?$$

(Choose all that apply.)

- [x] $c \geq \sigma_p$ for **all** p ✔
- [ ] $c \geq \sigma_p$ for **some** p
- [x] $c = \max_p (\sigma_p)$ ✔
- [ ] $c \leq \sigma_p$ for **all** p
- [ ] $c \leq \sigma_p$ for **some** p
- [ ] $c = \min_p (\sigma_p)$

✔

Solution:

Any number c such that

$$\left(\overline{R}_n - q_{\alpha/2}\frac{c}{\sqrt{n}}, \overline{R}_n - q_{\alpha/2}\frac{c}{\sqrt{n}}\right) \supseteq \left(\overline{R}_n - q_{\alpha/2}\frac{\sigma_p}{\sqrt{n}}, \overline{R}_n - q_{\alpha/2}\frac{\sigma_p}{\sqrt{n}}\right) \qquad \text{for all } p$$

will give the required probability for all p. Hence any $c \geq \max_p (\sigma_p)$ works.

Note: In this example, since $\sigma_p = \sqrt{p(1-p)}$, $\max_p (\sigma_p) = \max_p \left(\sqrt{p(1-p)}\right) = 1/2$.

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 2 Foundation of Inference:Lecture 4: Parametric Estimation and Confidence Intervals / 9. Conservative Bound

认证证书是什么?