Multiple Choice Questions (cont.)

(f)

1/1 point (graded)

If $X_1, \ldots, X_n$ are i.i.d. random variables uniformly distributed on $[0, \theta]$ for some unknown $\theta > 0$, Wald's test can be used to test whether $\theta = 1$.

- True
- False ✔

Solution:

Wald's test applies only when the asymptotic normality of the MLE estimate applies. In the case of Unif $[0, \theta]$, the technical conditions for the MLE do not apply as the support this distribution depends on the parameter θ that is being estimated.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

(g)

1/1 point (graded)

Let $X_1, \ldots, X_n \sim X$ be n i.i.d random variables. To test if they follow a Gaussian distribution, (i.e. if their distribution belongs to the Gaussian family), you can use...
(Check all that apply.)

Note: Here, we are not testing whether X follows $\mathcal{N}(\mu, \sigma^2)$ for specific μ and σ^2. We are testing whether there exists μ and σ^2 such that X follows $\mathcal{N}(\mu, \sigma^2)$.

Note (added May 1): The test is not required to be rigorous.

- [x] The χ^2 goodness of fit test ✔
- [ ] The Kolmogorov-Smirnov test
- [x] The Kolmogorov-Lilliefors test ✔
- [x] A normal QQ-test ✔

✖

Grading note: *Since there are a lot of ambiguity in the problem statement and correction notes, you will receive full credit as long as you have selected the Kolmogorov-Lilliefors test as one of your answers. This will take some time.*

Solution:

We consider each of the choices in turn.

- To use the χ^2 test to test if the data follow a Gaussian distribution, first use a consistent estimator to estimate μ and σ^2 (e.g. the MLE), then normalize the data appropriately, bucket various data points, and then perform the test. (For details, see Homework 8.)
  Remarks: The χ^2 test is an asymptotic test, hence n needs to be large.

- The Kolmogorov-Smirnov test is used to test whether the distribution underlying the data is a particular distribution with fixed parameters, e.g. $\mathcal{N}(\mu, \sigma^2)$ where μ and σ^2 are fixed, and not whether or not it belongs to the Gaussian family. **Remark:** The KS test can be used for small n. The KS test statistic is defined using specific parameters. If we instead plug in estimators of μ and σ^2 into the KS test statistic, the KS test may lead us to fail to reject while the more accurate Kolmogorov-Lilliefors test will lead us to reject the null hypothesis that the underlying distribution is Gaussian.

- A Kolmogorov-Lilliefors tests whether the data is Gaussian by testing whether or not the distribution fits the Gaussian of the sample mean and sample variance well and therefore is appropriate.

- Finally, a normal QQ-plot directly checks whether a distribution is Gaussian; if the plot is not nearly linear, then this is an indication that the distribution is not Gaussian. Gaussians with different mean and variances will correspond to linears with different slopes and intercepts.

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Answers are displayed within the problem

(h)

0.67/1 point (graded)
Consider the model $\{\mathbb{R}, \{\mathcal{N}(\theta, 1)\}_{\theta \in \mathbb{R}}$. Then the prior $\pi(\theta) \propto 1$ for all $\theta \in \mathbb{R}$ is called...
(check all that apply).

- [x] Improper ✔
- [ ] A Jeffreys prior ✔
- [ ] None of the above

✱

Grading Note: Partial credit are given.

Solution:

The prior distribution $\pi(\theta)$ is improper since the function $\pi(\theta)$ is not integrable over $\mathbb{R}$, i.e. $\int_{-\infty}^{\infty} \pi(\theta)\, d\theta$ is not finite. Finally note that the likelihood of $L(X; \theta) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(x-\theta)^2}{2}}$ and therefore the log-likelihood is

$$\ell(x; \theta) = \frac{-(x-\theta)^2}{2} - \frac{1}{2}\log(2\pi).$$

Taking the second derivative with respect to θ it follows that

$$\ell''(x; \theta) = -1$$

这个正好是jeffreys prior，我没测。

and therefore the Jeffreys prior is proportional to $\sqrt{I(\theta)} = \sqrt{1} = 1$. Therefore the uniform prior is also the Jeffreys prior.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

(i)

1/1 point (graded)
The maximum likelihood estimator is always unbiased.

- True
- False ✔

Solution:

The maximum likelihood estimator can be biased. For example, consider $X_1, \ldots, X_n \stackrel{i.i.d.}{\sim}$ Unif $[0, \theta]$ .The MLE $\theta^{\backslash\mathrm{MLE}} = \max\left(X_1, \ldots, X_n\right)$ is biased because it always less than or equal to the true θ.

Another example of biased MLE is the MLE $\widehat{\sigma^2}^{\mathrm{MLE}}$ for a Gaussian statistical model.

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Answers are displayed within the problem

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