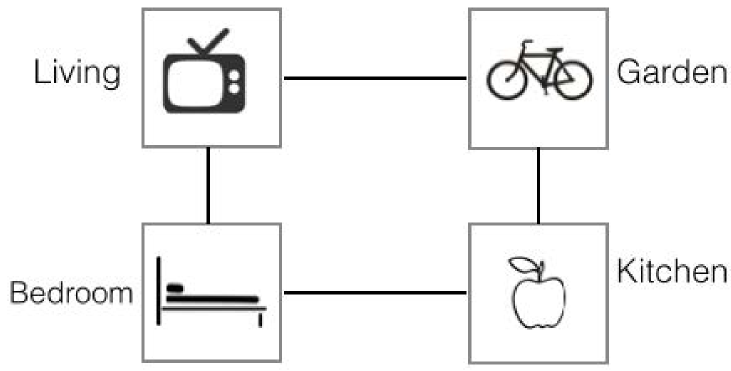
2. Home World Game

Extension Note: Project 5 due date has been extended by 1 **more** day to **September 6 23:59UTC** .

In this project, we will consider a text-based game represented by the tuple $< H, C, P, R, \gamma, \Psi >$. Here H is the set of all possible game states. The actions taken by the player are multi-word natural language **commands** such as **eat apple** or **go east** . In this project we limit ourselves to consider commands consisting of one action (e.g., **eat**) and one argument object (e.g. **apple**).
$C = \{(a,b)\}$ is the set of all commands (action-object pairs).
$P : H \times C \times H \rightarrow [0,1]$ is the transition matrix: $P(h'|h,a,b)$ is the probability of reaching state h' if command $c = (a,b)$ is taken in state h.
$R : H \times C \rightarrow \mathbb{R}$ is the deterministic reward function: $R(h,a,b)$ is the immediate reward the player obtains when taking command (a,b) in state h. We consider discounted accumulated rewards where γ is the discount factor. In particular, the game state h is **hidden** from the player, who only receives a varying textual description. Let S denote the space of all possible text descriptions. The text descriptions s observed by the player are produced by a stochastic function $\Psi : H \rightarrow S$. Assume that each observable state $s \in S$ is associated a **unique** hidden state, denoted by $h(s) \in H$.

You will conduct experiments on a small Home World, which mimic the environment of a typical house. The world consists of four rooms- a living room, a bed room, a kitchen and a garden with connecting pathways (illustrated in figure below). Transitions between the rooms are **deterministic**. Each room contains a representative object that the player can interact with. For instance, the living room has a **TV** that the player can **watch** , and the kitchen has an **apple** that the player can **eat.** Each room has several descriptions, invoked randomly on each visit by the player.

Rooms and objects in the Home world with connecting pathways



Reward Structure

Positive	Negative
Quest goal: $+1$	Negative per step: -0.01
	Invalid command: -0.1

At the beginning of each episode, the player is placed at a random room and provided with a randomly selected quest. An example of a quest given to the player in text is *You are hungry now*. To complete this quest, the player has to navigate through the house to reach the kitchen and eat the apple (i.e., type in command *eat apple*). In this game, the room is *hidden* from the player, who only receives a description of the underlying room. The underlying game state is given by $h = (r,q)$, where r is the index of room and q is the index of quest. At each step, the text description s provided to the player contains two part $s = (s_r, s_q)$, where s_r is the room description (which are varied and randomly provided) and s_q is the quest description. The player receives a positive reward on completing a quest, and negative rewards for invalid command (e.g., *eat TV*). Each non-terminating step incurs a small deterministic negative rewards, which incentives the player to learn policies that solve quests in fewer steps. (see the **Table 1**) An episode ends when the player finishes the quest or has taken more steps than a fixed maximum number of steps.

Each episode produces a full record of interaction $(h_0, s_0, a_0, b_0, r_0, \ldots, h_t, s_t, a_t, b_t, r_t, h_{t+1} \ldots)$where $h_0 = (h_{r,0}, h_{q,0}) \sim \Gamma_0$ (Γ_0 denotes an initial state distribution), $h_t \sim P(\cdot|h_{t-1}, a_{t-1}, b_{t-1})$, $s_t \sim \Psi(h_t)$, $r_t = R(h_t, a_t, b_t)$ and all commands (a_t, b_t) are chosen by the player. The record of interaction observed by the player is $(s_0, a_0, b_0, r_0, \ldots, s_t, a_t, b_t, r_t, \ldots)$ Within each episode, the quest remains unchanged, i.e., $h_{q,t} = h_{q,0}$ (so as the quest description $s_{q,t} = s_{q,0}$). When the player finishes the quest at time K, all rewards after time K are assumed to be zero, i.e., $r_t = 0$ for $t > K$. Over the course of the episode, the total discounted reward obtained by the player is

$$\sum_{t=0}^{\infty} \gamma^t r_t.$$

We emphasize that the hidden state $h_0, \ldots, h_T$ are unobservable to the player.

The learning goal of the player is to find a policy that $\pi : S \to C$ that maximizes the expected cumulative discounted reward $\mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R(h_t, a_t, b_t) \mid (a_t, b_t) \sim \pi\right]$,where the expectation accounts for all randomness in the model and the player. Let π^* denote the optimal policy. For each observable state $s \in S$, let $h(s)$ be the associated hidden state. The optimal expected reward achievable is defined as

$$V^* = \mathbb{E}_{h \sim \Gamma_0, s \sim \Psi(h)}\left[V^*(s)\right]$$

where

$$V^*(s) = \max_{\pi} \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R(h_t, a_t, b_t) \,|h_0 = h(s), s_0 = s, (a_t, b_t) \sim \pi\right].$$

We can define the optimal Q-function as

$$Q^*(s, a, b) = \max_{\pi} \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R(h_t, a_t, b_t) \,|h_0 = h(s), s_0 = s, a_0 = a, b_0 = b, (a_t, b_t) \sim \pi \text{ for } t \geq 1\right].$$

Note that given $Q^*(s, a, b)$, we can obtain an optimal policy:

$$\pi^*(s) = \arg\max_{(a,b) \in C} Q^*(s, a, b).$$

The commands set C contain all $(action, object)$ pairs. Note that some commands are invalid. For instance, **(eat,TV)** is invalid for any state, and **(eat, apple)** is valid only when the player is in the kitchen (i.e., h_r corresponds to the index of kitchen). When an invalid command is taken, the system state remains unchanged and a negative reward is incurred. Recall that there are **four** rooms in this game. Assume that there are **four** quests in this game, each of which would be finished only if the player takes a particular **command** in a particular room. For example, the quest "You are sleepy" requires the player navigates through rooms to bedroom (with commands such as **go east/west/south/north**) and then take a nap on the bed there. For each room, there is a corresponding quest that can be finished there.

Note that in this game, the transition between states is deterministic. Since the player is placed at a random room and provided a randomly selected quest at the beginning of each episode, the distribution Γ_0 of the initial state h_0 is uniform over the hidden state space H.

Episodic reward

1.0/1 point (graded)
For an episode with $T+1$ steps (starting from $t=0$), where the agent obtains a reward R_t at time step t. What is the total discounted reward for this episode with a discounted factor $\gamma \in (0,1)$?

Important: If needed, please enter $\sum_{t=0}^{T}(\ldots)$ as a function `sum_t(...)`, including the parentheses.

STANDARD NOTATION

sum_t(gamma^t*R_t)

✔ **Answer:** sum_t(gamma^t*R_t)

Submit You have used 2 of 6 attempts

Answers are displayed within the problem

Relation between value function and Q-function

1/1 point (graded)
Which of the following equation gives the correct relation between Q^* and V^*?

- $Q^*(s,a,b) = \gamma \mathbb{E}\left[V^*(s_0)\,|h_0 = h(s), s_0 = s, a_0 = a, b_0 = b\right]$
- $Q^*(s,a,b) = \gamma \mathbb{E}\left[V^*(s_1)\,|h_0 = h(s), s_0 = s, a_0 = a, b_0 = b\right]$
- $Q^*(s,a,b) = R(s,a,b) + \mathbb{E}\left[V^*(s_0)\,|h_0 = h(s), s_0 = s, a_0 = a, b_0 = b\right]$
- $Q^*(s,a,b) = R(s,a,b) + \mathbb{E}\left[V^*(s_1)\,|h_0 = h(s), s_0 = s, a_0 = a, b_0 = b\right]$
- $Q^*(s,a,b) = R(s,a,b) + \gamma \mathbb{E}\left[V^*(s_0)\,|h_0 = h(s), s_0 = s, a_0 = a, b_0 = b\right]$
- (selected) $Q^*(s,a,b) = R(s,a,b) + \gamma \mathbb{E}\left[V^*(s_1)\,|h_0 = h(s), s_0 = s, a_0 = a, b_0 = b\right]$

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

Optimal episodic reward

1/1 point (graded)
Assume that the reward function $R(s,a,b)$ is given in Table 1. At the beginning of each game episode, the player is placed in a random room and provided with a randomly selected quest. Let $V^*(h_0)$ be the optimal value function for an initial state h_0, i.e.,

$$V^*(h_0) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R(h_t, a_t, b_t)\,|\pi^*\right]$$

Please compute the expected optimal reward for each episode $\mathbb{E}\left[V^*(h_0)\right]$. Note that the initial state h_0 is uniformly distributed in the state space $H = (r,q) : 0 \leq r \leq 3, 0 \leq q \leq 3$.In other words, there are four quests each mapping to a unique room.Assume that the discounted factor is $\gamma = 0.5$

0.55375 **Answer:** 0.55375

Solution:

We can categorize the states $S = \{(s_r, s_q)\}$ into three types:

1. The quest s_q requests a command in the initial room with description s_r. An example of such initial states is **(This room has a fridge, oven, and a sink; you are hungry)** . The optimal policy for such a state is to take the corresponding command to finish the quest and get a reward 1.

2. The quest s_q requests a command in a room next to the initial room with description s_r. An example is **(This area has a bed, desk and a dresser; you are hungry)** . The optimal policy for such a state is first take one step towards the goal room (e.g., **go west,** and get a penalty reward -0.01), and then take the corresponding command to finish the quest (e.g., **eat apple,** and get a positive reward 1). The total discounted reward is: $-0.01 + \gamma \times 1 = 0.49$.

3. The quest s_q requests a command in a room that is not next to the initial room with description s_r, for instance, **(You have arrived at the garden. You can exercise here; you are hungry)** . It is easy to see that the optimal policy would be taking the first steps to arrive at the quested room and then finishing the quest. The total discounted reward would be:

$$-0.01+\gamma \times (-0.01)+\gamma^2 \times 1 = 0.235.$$

Since the room and the quest are selected randomly for the initial state, the probabilities for the above three types of states are $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ respectively. Therefore,

$$\mathbb{E}\left[V^*\left(h_0\right)\right] = \frac{1}{4} \times 1 + \frac{1}{2} \times 0.49 + \frac{1}{4} \times 0.235 = 0.55375.$$

Submit You have used 3 of 6 attempts

Answers are displayed within the problem

Discussion

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