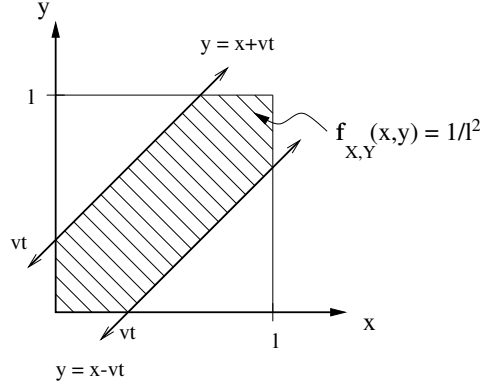


We want to compute the CDF of the ambulance's travel time T , $\mathbf{P}(T \leq t) = \mathbf{P}(|X - Y| \leq vt)$, where X and Y are the locations of the ambulance and accident (uniform over $[0, l]$). Since X and Y are independent, we know:

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{l^2} & , \text{ if } 0 \leq x, y \leq l \\ 0 & , \text{ otherwise} \end{cases}.$$

$$\begin{aligned} \mathbf{P}(T \leq t) &= \mathbf{P}(|X - Y| \leq vt) = \mathbf{P}(-vt \leq Y - X \leq vt) \\ &= \mathbf{P}(X - vt \leq Y \leq X + vt) \end{aligned}$$

We can see that $\mathbf{P}(X - vt \leq Y \leq X + vt)$ corresponds to the integral of the joint density of X and Y over the shaded region in the figure below:



Therefore, because the joint density is uniform over the entire region, we have:

$$F_T(t) = (1/l^2) \times (\text{Shaded area}) = \begin{cases} 0 & , \text{ if } t < 0 \\ \frac{2vt}{l} - \frac{(vt)^2}{l^2} & , \text{ if } 0 \leq t < \frac{l}{v} \\ 1 & , \text{ if } t \geq \frac{l}{v} \end{cases}.$$

By differentiating the CDF, we find the density of T :

$$f_T(t) = \begin{cases} \frac{2v}{l} - \frac{2v^2t}{l^2} & , \text{ if } 0 \leq t \leq \frac{l}{v} \\ 0 & , \text{ otherwise} \end{cases}.$$