10. Exercise: Exponential PDF

Exercise: Exponential PDF

2/2 points (graded)

Let X be an exponential random variable with parameter $\lambda = 2$. Find the values of the following. Use 'e' for the base of the natural logarithm (e.g., enter e^(-3) for e^{-3}).

a) $\mathbf{E}[(3X+1)^2] =$ 17/2 ✔ **Answer:** 8.5

b) $\mathbf{P}(1 \leq X \leq 2) =$ e^-2 - e^-4 ✔ **Answer:** 0.11702

Solution:

a) By expanding the quadratic, using linearity of expectations, and the facts that $\mathbf{E}[X] = 1/\lambda$ and $\mathbf{E}[X^2] = 2/\lambda^2$, we have

$$\mathbf{E}[(3X+1)^2] = 9\mathbf{E}[X^2] + 6\mathbf{E}[X] + 1 = 9 \cdot \frac{2}{2^2} + 6 \cdot \frac{1}{2} + 1 = \frac{17}{2}.$$

b) We have seen that for $a > 0$, we have $\mathbf{P}(X \geq a) = e^{-\lambda a}$, so that $\mathbf{P}(X \leq a) = 1 - e^{-\lambda a}$. Therefore,

$$\mathbf{P}(1 \leq X \leq 2) = \mathbf{P}(X \leq 2) - \mathbf{P}(X \leq 1) = (1 - e^{-4}) - (1 - e^{-2}) = e^{-2} - e^{-4}.$$

提交 You have used 1 of 3 attempts

Answers are displayed within the problem

讨论

显示讨论

Topic: Unit 5 / Lec. 8 / 10. Exercise: Exponential PDF