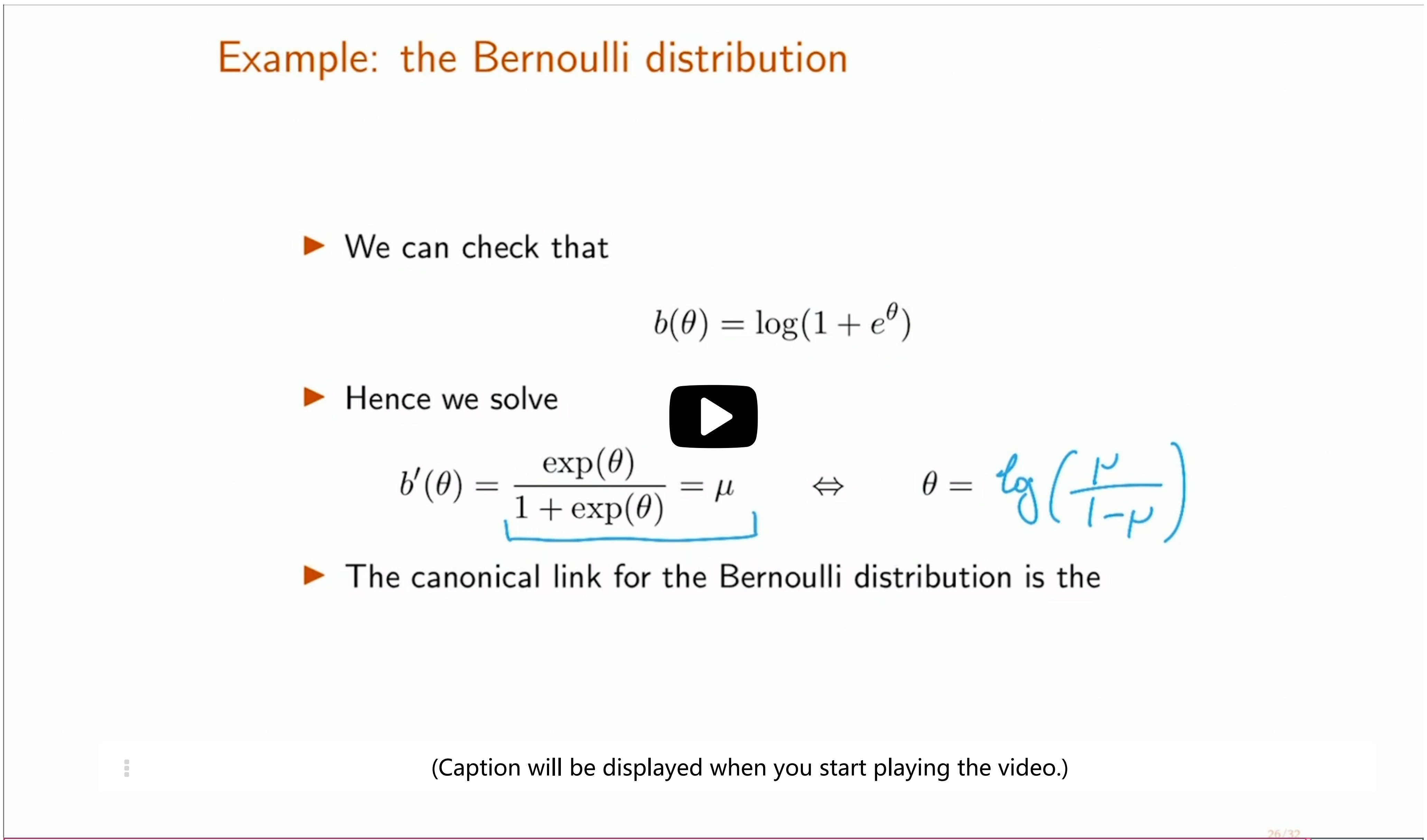
# 3. The Canonical Link Function

## The Canonical Link Function and Bernoulli Example

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Video

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### Canonical Link function for the Binomial Distribution

1/1 point (graded)

The binomial distribution, with distribution function

$$f_p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

can be written as a canonical exponential family, as long as $n$ is a fixed number. **For this problem, plug in $n = 1000$.**

What is the canonical link function $g(\mu)$? (With the understanding that $\mu = np$)

ln((mu)/(1000-mu)) ✔ **Answer:** ln(mu/(1000-mu))

$\ln\left(\frac{\mu}{1000-\mu}\right)$

STANDARD NOTATION

**Solution:**

For the binomial distribution, $b(\theta) = n \ln(e^{\theta} + 1)$ if we use the canonical parameter $\theta = \log\left(\frac{p}{1-p}\right)$. Therefore, the canonical link is $g(\mu) = (b')^{-1}(\mu)$. A direct computation yields $b'(\theta) = \frac{ne^{\theta}}{e^{\theta}+1}$, and so $g(\mu) = \ln\left(\frac{\mu}{n-\mu}\right)$.

*Remark: In some texts, you might see $g(\mu) = \ln\left(\frac{\mu}{1-\mu}\right)$, the logit of $\mu$ instead of what we derived. This is due to a re-normalization convention where we think of the likelihood of $\overline{x} = x/n$, so that the mean of $\overline{x}$ is $p$ instead of $np$. Notice that if you plug in $\mu = np$ into our expression, the $n$'s cancel and we end up with the logit of $p$, which gives the alternate convention.*

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You have used 3 of 3 attempts

Answers are displayed within the problem

## Discussion

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**Topic:** Unit 7 Generalized Linear Models:Lecture 22: GLM: Link Functions and the Canonical Link Function / 3. The Canonical Link Function