4. Maximum likelihood estimators

Instructions:

Let $X_1, \ldots, X_n$ be n i.i.d. random variables with pdf f_θ, where θ is an unknown parameter.

For each of the following questions, compute the likelihood function on paper and then find the maximum likelihood estimator for θ.

To encourage you to do the computations carefully rather than eliminate choices, you will only be given **1 or 2 attempts per question** .

(a)

1/1 point (graded)

Compute the likelihood function and the maximum likelihood estimator for θ for

$$f_\theta(x) = \tau\theta^\tau x^{-(\tau+1)}\mathbf{1}(x \geq \theta), \theta > 0,$$

where $\tau > 0$ is a known constant.

- $\max X_i$
- $\min X_i$
- $\frac{1}{n}\sum X_i$
- $n\tau$
- ∞

Solution:

The likelihood function is

$$L = \tau^n\theta^{n\tau}\prod_i X_i^{-(\tau+1)}\mathbf{1}\{\min_i X_i \geq \theta\}$$

For $\theta \leq \min_i X_i$, the log-likelihood function is

$$l = n\ln\tau + n\tau\ln\theta - (\tau+1)\sum_{i=1} \ln X_i$$

Take the derivative with respect to θ:

$$\frac{\partial l}{\partial \theta} = \frac{n\tau}{\theta} > 0.$$

Thus, L is an increasing function on $(0, \min_i X_i]$, and is 0 for $\theta > \min_i X_i$. Therefore,

$$\hat{\theta} = \min_i X_i$$

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

(b)

3/3 points (graded)

Compute the likelihood function and the maximum likelihood estimator for θ for

$$f_\theta(x) = \sqrt{\theta} x^{\sqrt{\theta}-1} \mathbf{1}(0 \leq x \leq 1), \theta > 0.$$

You will find that the maximum likelihood estimator for θ is of the form

$$\hat{\theta}^{\text{MLE}} = c_1 n^{c_2} \left(\sum_{i=1}^{n} \ln X_i \right)^{c_3}.$$

Enter the numbers c_1, c_2, c_3 below.

$c_1 =$ 1 Answer: 1

$c_2 =$ 2 Answer: 2

$c_3 =$ -2 Answer: -2

STANDARD NOTATION

Solution:

The likelihood function is

$$L = \theta^{n/2} \prod_i X_i^{\sqrt{\theta}-1} \mathbf{1}\{0 \leq X_i \leq 1\}.$$

The log-likelihood function is

$$l = \frac{n}{2} \ln \theta + (\sqrt{\theta} - 1) \sum_i \ln X_i.$$

Take the derivative with respect to θ and set it to 0:

$$\frac{\partial l}{\partial \theta} = \frac{n}{2\theta} + \frac{1}{2\theta^{1/2}} \sum_i \ln X_i = 0.$$

Then we get

$$\hat{\theta} = \frac{n^2}{\left(\sum \ln X_i\right)^2}.$$

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

(c)

4/4 points (graded)

Compute the likelihood function and the maximum likelihood estimator for θ

$$f_\theta(x) = \theta\tau x^{\tau-1}\exp\{-\theta x^\tau\}\mathbf{1}(x \geq 0), \theta > 0,$$

where $\tau > 0$ is a known constant.

You will find that the maximum likelihood estimator for θ is of the form

$$\hat{\theta}^{\text{MLE}} = c_1 n^{c_2}\left(\sum_{i=1}^{n} X_i^{c_3}\right)^{c_4}.$$

Enter the c_1, c_2, c_3, c_4 in terms of τ if applicable.

(Enter **tau** for τ.)

$c_1 =$ 1 **Answer:** 1

1

$c_2 =$ 1 **Answer:** 1

1

$c_3 =$ tau **Answer:** tau

τ

$c_4 =$ -1 **Answer:** -1

-1

STANDARD NOTATION

Solution:

The likelihood function is

$$L = \theta^n \tau^n \prod_i X_i^{\tau-1} \exp\{-\theta \sum_i X_i^\tau\}\mathbf{1}\{X_i \geq 0\}.$$

The log-likelihood function is

$$l = n\ln\theta + n\ln\tau + (\tau - 1)\sum_i \ln X_i - \theta\sum_i X_i^\tau.$$

Take the derivative with respect to θ and set it to 0

$$\frac{\partial l}{\partial \theta} = \frac{n}{\theta} - \sum_i X_i^\tau = 0,$$

we get

$$\hat{\theta} = \frac{n}{\sum_i X_i^\tau}.$$

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

讨论

显示讨论

主题: Unit 3 Methods of Estimation:Homework 4: TV distance, KL-Divergence, and Introduction to MLE / 4. Maximum likelihood estimators

认证证书是什么?