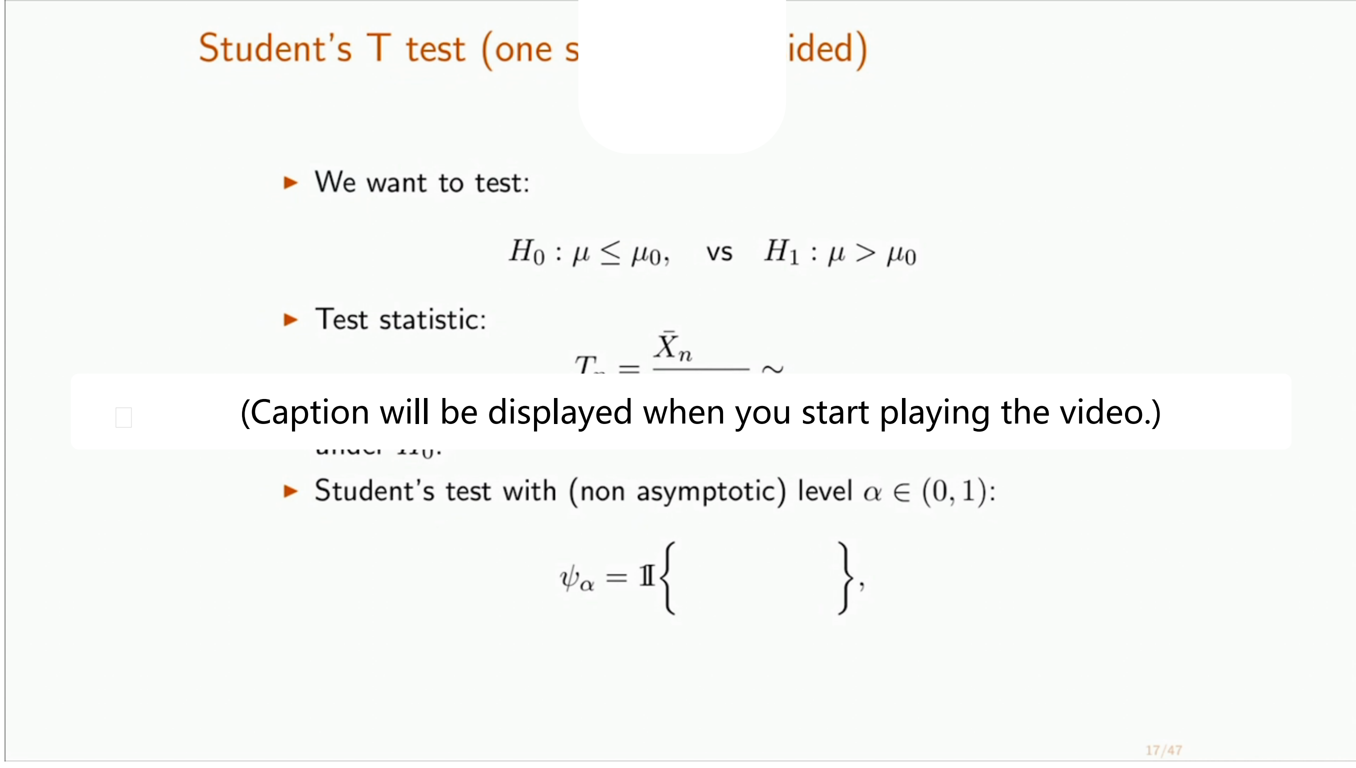

I'm going to say mu, say, less than 0, or less than mu

0, more generally, versus larger than mu 0.

What I'm going to do is, I'm going

to perform the test statistic, which is Xn bar

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Concept Check: Student's T Distribution

3/3 points (graded)

Consider the statistic

$$T_n := \sqrt{n}\left(\frac{\overline{X}_n - \mu}{\sqrt{\frac{1}{n-1}\sum_{i=1}^n \left(X_i - \overline{X}_n\right)^2}}\right).$$

For all $n \geq 2$, the distribution of T_n is a standard Gaussian $\mathcal{N}(0,1)$.

- ○ True
- ◉ False

As $n \to \infty$, what does

$$\frac{1}{n-1}\sum_{i=1}^n \left(X_i - \overline{X}_n\right)^2$$

converge to...

- ○ The number μ (weakly)
- ◉ The number σ^2 (weakly)
- ○ The distribution $\mathcal{N}(0,1)$
- ○ The distribution χ^2_{n-1}

As $n \to \infty$, the statistic T_n converges in distribution to

- $\mathcal{N}(0,1)$
- $\mathcal{N}(\mu, \sigma^2)$
- χ^2_{n-1}
- χ^2_n

Solution:

The definition of the student's T distribution with $n-1$ degrees of freedom is that it is given by the distribution of $\frac{Z}{\sqrt{V/(n-1)}}$ where $Z \sim \mathcal{N}(0,1)$, $V \sim \chi^2_{n-1}$ and Z and V are independent. Since we are dividing by V, a χ^2 random variable, then T_n will not have the same distribution as $\mathcal{N}(0,1)$ for all $n \geq 2$. 因为没有Slutsky

By the law of large numbers and Slutsky's lemma,

$$\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}_n\right)^2 = \frac{n}{n-1}\left[\left(\frac{1}{n}\sum_{i=1}^{n}X_i^2\right) - \left(\overline{X}_n\right)^2\right] \to \sigma^2$$

in probability.

By the central limit theorem,

$$\sqrt{n}\left(\frac{\overline{X}_n - \mu}{\sigma}\right) \to \mathcal{N}(0,1).$$

Hence, by the law of large numbers and Slutsky's theorem,

$$\sqrt{n}\left(\frac{\overline{X}_n - \mu}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}_n\right)^2}}\right) \xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1).$$

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Answers are displayed within the problem

Quantiles of the T Distribution:

The $(1-\alpha)$-quantile of the t_{n-1} (corresponding to a one-sided test with statistic T_n) can be computed using standard computational tools such as R. One can also find online tables for the quantiles via a simple Google search, which yields results such as this, this, and this.

As a reminder, in this class the $(1-\alpha)$ quantile of the distribution of a random variable T is the number q_α such that

$$P(T \leq q_\alpha) = 1 - \alpha.$$

Concept Check: T Test

1/1 point (graded)

Let $X_1, \dots, X_n \overset{iid}{\sim} \mathcal{N}(\mu^*, \sigma^2)$ for some unknown $\mu^* \in \mathbb{R}$ and $\sigma^2 > 0$. You want to decide between the following null and alternative hypotheses on the mean of $X_1, \dots, X_n$:

$$\begin{aligned} H_0 &: \mu^* = 0 \\ H_1 &: \mu^* \neq 0. \end{aligned}$$

To do so, you define the student's T statistic

$$T_n = \sqrt{n} \frac{\overline{X}_n}{\sqrt{\widetilde{S}_n}}$$

where

$$\widetilde{S}_n = \frac{1}{n-1} \sum_{i=1}^{n} \left(X_i - \overline{X}_n\right)^2$$

is the unbiased sample variance.

The student's T test of level α is specified by

$$\psi_\alpha = \mathbf{1}\left(|T_n| > q_{\alpha/2}\right)$$

where $q_{\alpha/2}$ is the unique number such that $P\left(T_n < q_{\alpha/2}\right) = 1 - \frac{\alpha}{2}$.

Which of the following are true about the student's T test? (Choose all that apply.)

- [ ] The statistic T_n is distributed as a standard Gaussian.
- [x] The test requires the data $X_1, \dots, X_n$ to be Gaussian.
- [x] The distribution of T_n is pivotal, *i.e.*, its quantiles may be found in tables.
- [x] The test is non-asymptotic. That is, for any fixed n, we can compute the level of our test rather than the *asymptotic* level.

Solution:

We examine the choices in order.

- The first choice is incorrect. Due to the fact that T_n has the sample variance $\widehat{S}_n$ in the denominator and not the *true* variance σ^2, the statistic T_n will **not** be standard Gaussian.
- The second choice is correct. It is a key assumption that the data is Gaussian. Otherwise, the test statistic T_n will not necessarily follow the student's T distribution and, hence, may not even be pivotal.
- The third choice is correct. For any fixed n, we may find the quantiles of the student's T distribution in tables. Since the distribution does not depend on the value of the true parameter, the test statistic T_n is indeed pivotal.
- The last choice is also correct. As stated in the previous bullet, for any fixed n, the quantiles of the student's T distribution may be found in tables. Hence, we can find the non-asymptotic level of this test.

Remark: Assuming the data is Gaussian, the student's T test is useful in situations where the sample size is not very large, since the level may be precisely quantified even for small n.

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