3. Independent exponential random variables

Problem 2. Independent exponential random variables

2/2 points (graded)

Let X and Y be two independent, exponentially distributed random variables with parameters λ , and μ , respectively.

For each question below, enter your answers using standard notation; enter **mu** for **mu** and **lambda** for λ .

1. Find the probability that $X \leq Y$.

$$\mathbf{P}(X \leq Y) = \boxed{\text{lambda/(lambda+mu)}} \qquad \text{Answer: lambda/(mu+lambda)}$$

$$\frac{\lambda}{\lambda + \mu}$$

2. Let Z = 1/(1+X). For 0 < z < 1:

STANDARD NOTATION

Solution:

1. Using the law of total probability theorem, and independence of $oldsymbol{X}$ and $oldsymbol{Y}$,

$$egin{align} \mathbf{P}(X \leq Y) &= \int_0^\infty \mathbf{P}(X \leq y) f_Y(y) \; dy \ &= \int_0^\infty \mathbf{P}(X \leq y) \mu e^{-\mu y} \; dy \; = \int_0^\infty (1 - e^{-\lambda y}) \mu e^{-\mu y} \; dy \ &= rac{\lambda}{\mu + \lambda}. \end{split}$$

2. We have, for 0 < z < 1,

$$\mathbf{P}(Z \le z) = \mathbf{P}\left(rac{1}{1+X} \le z
ight)$$
 $= \mathbf{P}\left(1+X \ge rac{1}{z}
ight)$
 $= \mathbf{P}\left(X \ge rac{1}{z} - 1
ight)$
 $= e^{-\lambda(1/z-1)}$
 $= e^{-\lambda/z} \cdot e^{\lambda}$.

Differentiating the expression above with respect to z yields,

$$f_Z(z) = rac{\lambda}{z^2} e^{-\lambda(1/z-1)} = rac{\lambda e^{\lambda}}{z^2} e^{-\lambda/z}.$$

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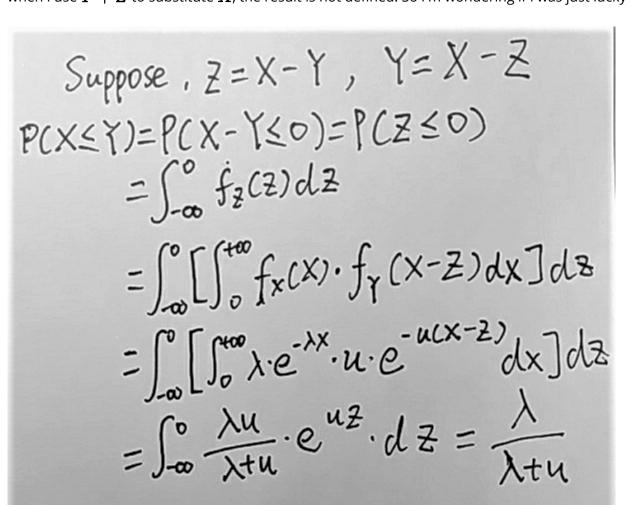
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I used convolution formula to solve Part 1. The answer is correct, but am I just lucky?

question posted about 3 hours ago by sakimarquis

I am not very familiar with LATEX, please kindly bear with my poor handwriting. Here's my solution. I use X-Z to substitute Y, but when i use Y+Z to substitute X, the result is not defined. So i'm wondering if i was just lucky to coincide with the correct answer.



此帖对所有人可见。

添加回复

2 responses

LimYiLe about 2 hours ago I used Convolution to solve part 1 too and got the same answer. And your handwriting's nice. :)	+
Thank you! I did get luck in use convolution formula!	•••
<u>sakimarquis</u> 在5 minutes ago前发表	
添加评论	

<u>alexannan</u>

about an hour ago - 10 minutes ago 前被 <u>sakimarquis</u> 标记为答案



+

Your reasoning looks pretty good to me!

Possibly the reason it didn't work with y+z is that, when you take the outer integral, z takes on negative values. So y+z might be negative, and so you'd need to explicitly take account of the fact that $f_X(x)=0$ when x<0.

positive when $z < 0$.	
Ooes that make any sense?	
think it's just an issue of the PDFs of $m{X}$ and $m{Y}$ being piecewise, and remembering to take account of that. That's always when or oblems involving integrals. I find it helps to use <u>lverson brackets</u> to represent piecewise functions. But then that might just nore problems!	
hank you for explanation!	
akimarquis 在2 minutes ago 前发表	
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