6. Correlation coefficients

Problem 6. Correlation coefficients

3/3 points (graded)

Consider random variables X, Y and Z, which are assumed to be pairwise uncorrelated (i.e., X and Y are uncorrelated, X and Z are uncorrelated, and Y and Z are uncorrelated). Suppose that

- $\mathbf{E}[X] = \mathbf{E}[Y] = \mathbf{E}[Z] = 0$,
- $\mathbf{E}[X^2] = \mathbf{E}[Y^2] = \mathbf{E}[Z^2] = 1$,

Find the correlation coefficients $\rho(X-Y, X+Y)$, $\rho(X+Y, Y+Z)$, and $\rho(X, Y+Z)$.

1. $\rho(X-Y, X+Y) =$ 0 ✔ **Answer:** 0

2. $\rho(X+Y, Y+Z) =$ 1/2 ✔ **Answer:** 0.5

3. $\rho(X, Y+Z) =$ 0 ✔ **Answer:** 0

Solution:

1. We have

$$\begin{aligned}
\mathrm{cov}(X-Y, X+Y) &= \mathbf{E}\big[(X-Y)(X+Y)\big] - \mathbf{E}[X-Y]\mathbf{E}[X+Y] \\
&= \mathbf{E}[X^2 - Y^2] - 0 \\
&= \mathbf{E}[X^2] - \mathbf{E}[Y^2] \\
&= 0.
\end{aligned}$$

Hence, $\rho(X-Y, X+Y) = 0$.

2. Since X and Y are uncorrelated, with zero means, we have $\mathbf{E}[XY] = \mathrm{cov}(X, Y) = 0$. Similarly, we have $\mathbf{E}[XZ] = 0$ and $\mathbf{E}[YZ] = 0$.
Hence,

$$\begin{aligned}\mathrm{cov}(X+Y,Y+Z) &= \mathbf{E}[(X+Y)(Y+Z)] - \mathbf{E}[X+Y]\mathbf{E}[Y+Z] \\ &= \mathbf{E}[XY+XZ+Y^2+YZ] \\ &= \mathbf{E}[Y^2] \\ &= 1.\end{aligned}$$

Also,

$$\begin{aligned}\mathsf{Var}(X+Y) &= \mathbf{E}[(X+Y)^2] - (\mathbf{E}[X+Y])^2 \\ &= \mathbf{E}[X^2+2XY+Y^2] - 0 \\ &= 2.\end{aligned}$$

Similarly, $\mathsf{Var}(Y+Z) = 2$.

Therefore, $\rho(X+Y,Y+Z) = \frac{\mathrm{cov}(X+Y,Y+Z)}{\sqrt{\mathsf{Var}(X+Y)\mathsf{Var}(Y+Z)}} = \frac{1}{2}$.

3.

$$\begin{aligned}\mathrm{cov}(X,Y+Z) &= \mathbf{E}[X(Y+Z)] - \mathbf{E}[X]\mathbf{E}[Y+Z] \\ &= \mathbf{E}[XY+YZ] - 0 \\ &= 0.\end{aligned}$$

Hence, $\rho(X,Y+Z) = 0$.

提交 You have used 1 of 3 attempts

Answers are displayed within the problem

讨论

显示讨论

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