6. Maximum Likelihood Estimation for a Multivariate Standard Normal

Let $\mathbf{X}_1, \ldots, \mathbf{X}_n \overset{i.i.d.}{\sim} \mathcal{N}(\mu, \mathbf{1})$, where $\mu \in \mathbb{R}^d$ and $\mathbf{1}$ is the $d \times d$ identity matrix. (The $\mathbf{X}_i$ are random vectors.)

Recall the pdf defining the distribution $\mathcal{N}(\mu, \mathbf{1})$ is

$$f(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\mu)^T \mathbf{1} (\mathbf{x}-\mu)\right)$$

(a)

1 point possible (graded)
What is the likelihood function $L(\mathbf{X}_1, \ldots, \mathbf{X}_n, \mu)$ for μ?

(Enter **(Sigma_i(norm(x_i-mu)^2))** for $\sum_{i=1}^n \|\mathbf{x}_i - \mu\|^2$.)

$L(\mathbf{X}_1, \ldots, \mathbf{X}_n, \mu) =$ [ ]

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(b)

1 point possible (graded)
Compute the maximum likelihood estimator $\hat{\mu}_{MLE}$ for μ.

(Enter **barX_n** for the sample average.)

$\hat{\mu}_{MLE} =$ [ ]

Prove to yourself that the result you obtained above indeed maximizes the likelihood function. Is this step necessary?

STANDARD NOTATION

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(c)

1 point possible (graded)
What is the distribution of $\hat{\mu}_{MLE}$?

- $\hat{\mu}_{MLE} \sim \mathcal{N}(\mu, \frac{1}{n}\mathbf{1})$
- $\hat{\mu}_{MLE} \sim \mathcal{N}(\mu, \mathbf{1})$
- $\hat{\mu}_{MLE} \sim \mathcal{N}(0, \frac{1}{n}\mathbf{1})$

- $\hat{\mu}_{MLE} \sim \mathcal{N}(\mu, \frac{1}{\sqrt{n}}\mathbf{1})$

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(d)

1 point possible (graded)
What is the asymptotic variance of $\mathbf{A}\hat{\mu}_{MLE}$? (here, A is a fixed $m \times d$ matrix)

(If applicable, enter **trans(A)** for the transpose of a matrix A.)

STANDARD NOTATION

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(e)

1 point possible (graded)
What is the asymptotic variance of $\|\hat{\mu}_{MLE}\|^2$?

(If applicable, enter **norm(v)** for the norm $\|\mathbf{v}\|$ of a vector $\mathbf{v}$, and **trans(v)** for the transpose $\mathbf{v}^T$ of a vector $\mathbf{v}$.)

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讨论

隐藏讨论

主题： Unit 3 Methods of Estimation:Homework 6 Maximum Likelihood Estimation and Method of Moments / 6. Maximum Likelihood Estimation for a Multivariate Standard Normal

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d) in doubt For the asymptotic variance of A_mu_MLE, are we supposed to use the delta method or the fisher information?	2
part e I used part d and solved part e. It seemed to be obvious. However answer was wrong. Any hints please?	3
[Staff] Question e Can you please check the answer? Mu cancels out completely and the variance is 1-d numeric which aligns with 1-d [removed].	6
[Staff] Question (a) issue?	8
Part c - Definition of asymptotic distribution	3
(a) I've a question about the notation. Above all it states X_i = (mu,1), where 1 is the d x d identity matrix. Now I wonder, when we compute the likelihood f(), shoul...	2
Pointers for part (e)? Also, likely a bug in part (d).	3
[staff] (d) asymptotic variance instead of variance the question is asking for asymptotic variance instead of variance as the grader doesn't allow the use of "n"	2
[Staff] (e) Possible error: mu not permitted in answer as variable	6

[Staff] No mention of using barX_n in the instruction

Hi there, I guess that the code "barX_n" may be considered standard by now, but up to this point it has always been mentioned explicitly in the instruction, if ex...

2