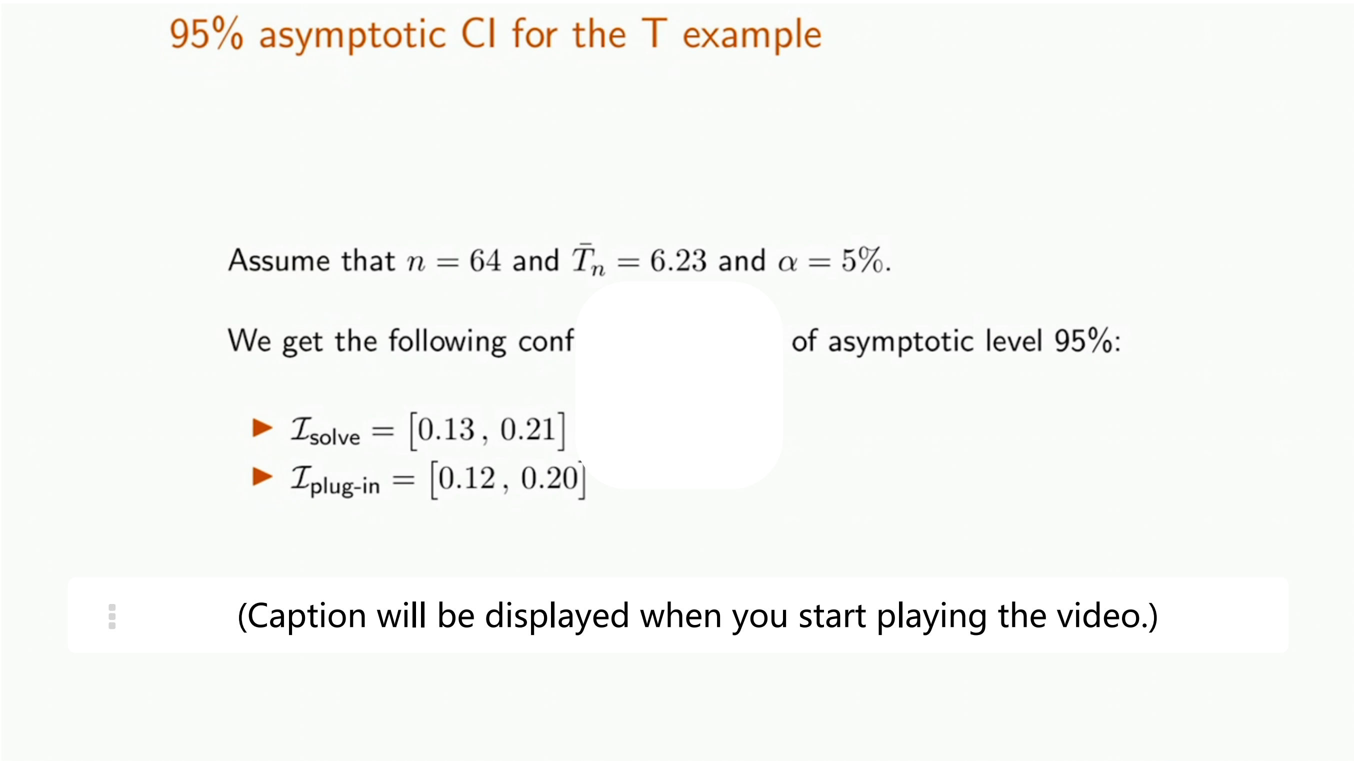
12. Interpretation of the frequentist Confidence Interval

Frequentist Interpretation of a Confidence Interval



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OK, so in the T, if I start plugging in numbers--

now here let's say that I waited for 64 Ts.

And I saw an average waiting time of 6.23 minutes,

and I'm asking you alpha is equal to 5%.

Then, you can get a confidence interval for lambda, all right?

Lambda is the reciprocal of the expected

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Frequentist Interpretation of a Confidence Interval

1/1 point (graded)

In a particular experiment, you gather data in the form of a sample $X_1, \ldots, X_n \overset{iid}{\sim} P_\theta$, and construct a confidence interval $\mathcal{I}$ with level 90% for the true (unknown) parameter θ.

After conducting the experiment, there are two possibilities:

- $\mathcal{I}$ contains θ (We refer to this as a **success**.)
- $\mathcal{I}$ does not contain θ (We refer to this as a **failure**.)

Suppose you repeat the experiment above T total times, and assume that the experiments are jointly independent. Moreover, the value of the unknown parameter θ, is always assumed to be the same. After conducting these T experiments, you will have constructed T confidence intervals $\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_T$.

As T grows very large, what percentage of experiments do you expect to be successes?

90

✔ **Answer:** 90 $\%$

Solution:

By the definition of confidence interval, we know that for the j-th experiment ($1 \leq j \leq T$) that

$$P(\mathcal{I}_j \ni \theta) = 90\%.$$

Consider the indicator random variables $\mathbf{1}(\theta \in \mathcal{I}_1), \mathbf{1}(\theta \in \mathcal{I}_2), \ldots, \mathbf{1}(\theta \in \mathcal{I}_T)$. Since the experiments are jointly independent, this means that $\mathbf{1}(\theta \in \mathcal{I}_1), \mathbf{1}(\theta \in \mathcal{I}_2), \ldots, \mathbf{1}(\theta \in \mathcal{I}_T)$ are independent. Moreover, for all j, the random variable $\mathbf{1}(\theta \in \mathcal{I}_T)$ is Bernoulli because it can only take value 0 or 1. It follows that $\mathbf{1}(\theta \in \mathcal{I}_1), \mathbf{1}(\theta \in \mathcal{I}_2), \ldots, \mathbf{1}(\theta \in \mathcal{I}_T)$ are identically distributed, because for all j,

$$P(\mathbf{1}(\theta \in \mathcal{I}_j) = 1) = P(\mathcal{I}_j \ni \theta) = 90\%.$$

In summary, $\mathbf{1}(\theta \in \mathcal{I}_1), \mathbf{1}(\theta \in \mathcal{I}_2), \ldots, \mathbf{1}(\theta \in \mathcal{I}_T) \stackrel{iid}{\sim} \mathrm{Ber}(0.9)$. By the strong law of large numbers,

$$\lim_{T\to\infty} \frac{\sum_{j=1}^{T} \mathbf{1}(\theta \in \mathcal{I}_j)}{T} = \mathbb{E}[\mathbf{1}(\theta \in \mathcal{I}_j)] = 0.9$$

almost surely. Since

$$\frac{\sum_{j=1}^{T} \mathbf{1}(\theta \in \mathcal{I}_j)}{T} = \frac{\text{Number of successes}}{\text{Total number of experiments}},$$

the correct response is 90%.

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

讨论

主题： Unit 2 Foundation of Inference:Lecture 5: Delta Method and Confidence Intervals / 12. Interpretation of the frequentist Confidence Interval

显示讨论

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