10. Review of Methods of Estimation

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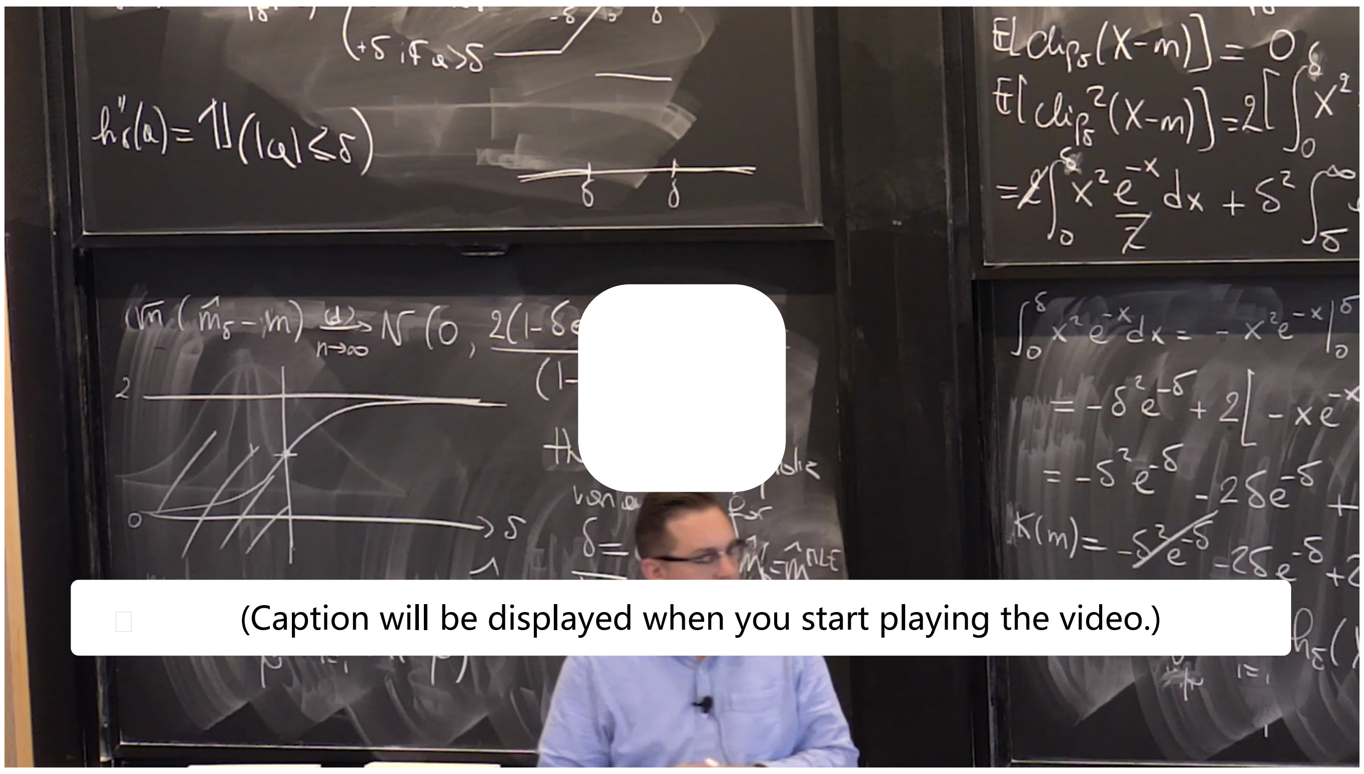

(Caption will be displayed when you start playing the video.)

OK, so let's just wrap up this chapter,

just to make sure that we remind ourselves

everything we've seen.

So we saw essentially three principal methods

for estimation.

And by principal, I mean that we had one before, which was just,

well, if you're parameters an expectation,

just replace it by an average.

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Concept Check: Methods of Estimation I

1/1 point (graded)
Which of the following estimators are defined in terms of an optimization problem? (Choose all that apply.)

- [x] Maximum likelihood estimator.
- [ ] Method of moments estimator.
- [x] M-estimator.

Solution:

The correct responses are "Maximum likelihood estimator." and "M-estimator." The MLE is defined by maximizing the log-likelihood, and an M-estimator is defined by minimizing a loss function. However, the method of moments estimator is constructed by solving a system of equations, so this response is not correct.

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Answers are displayed within the problem

Concept Check: Methods of Estimation II

1/1 point (graded)
All three method of estimation studied in this unit: maximum likelihood estimation, the method of moments, and M-estimation, lead to asymptotically normal estimators if certain technical conditions are satisfied.

In general, an asymptotically normal estimator $\hat{\theta}_n$ can be used to construct a confidence interval for an unknown parameter.

What quantity related to the estimator $\hat{\theta}$ determines the length of an asymptotic confidence interval at level 95%? (Assume that you use the plug-in method and that n is very large.)

- (selected) The asymptotic variance of $\hat{\theta}_n$.
- The rate of convergence of $\hat{\theta}_n$ to the normal distribution $\mathcal{N}(0,1)$.
- The mean of $\hat{\theta}_n$.

Solution:

The correct response is "The asymptotic variance of $\hat{\theta}_n$," as we demonstrate below. Consider an asymptotically normal estimator $\widehat{\theta_n}$, which satisfies

$$\sqrt{n}\left(\widehat{\theta_n}-\theta\right) \xrightarrow[n\to\infty]{(d)} \mathcal{N}\left(0,\sigma^2\right)$$

for some asymptotic variance $\sigma^2 > 0$. Let $q_{\alpha/2}$ denote the $\alpha/2$-quantile of a standard Gaussian. Then we have that

$$P\left(\sqrt{n}\frac{\left|\widehat{\theta_n}-\theta\right|}{\sigma} \geq q_{\alpha/2}\right) \xrightarrow{n\to\infty} \alpha$$

which implies that

$$P\left(\theta \notin \left[\widehat{\theta_n}-q_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \widehat{\theta_n}+q_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]\right) \xrightarrow{n\to\infty} \alpha.$$

Therefore, using the plug-in method, we have that

$$P\left(\theta \notin \left[\widehat{\theta_n}-q_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{n}}, \widehat{\theta_n}+q_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{n}}\right]\right) \xrightarrow{n\to\infty} \alpha,$$

Setting $\alpha = 0.05$, we have that

$$\mathcal{I} := \left[\widehat{\theta_n}-q_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{n}}, \widehat{\theta_n}+q_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{n}}\right]$$

If n is very large, we have that $\widehat{\sigma}_n \approx \sigma$, so the length of $\mathcal{I}$ is approximately $2q_{0.025}\sigma/\sqrt{n}$. That is, the length depends only on the $\alpha/2$ quantile, the sample size, and the asymptotic variance. Therefore, "The rate of convergence of $\hat{\theta}_n$ to the normal distribution $\mathcal{N}(0,1)$." and "The mean of $\hat{\theta}_n$." are incorrect responses.

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Answers are displayed within the problem

讨论

显示讨论

主题： Unit 3 Methods of Estimation:Lecture 12: M-Estimation / 10. Review of Methods of Estimation