13. Exercise: LLMS with multiple observations

Exercise: LLMS with multiple observations

2/3 points (graded)

Suppose that Θ, X_1, and X_2 have zero means. Furthermore,

$$\mathsf{Var}(X_1) = \mathsf{Var}(X_2) = \mathsf{Var}(\Theta) = 4,$$

and

$$\mathsf{Cov}(\Theta, X_1) = \mathsf{Cov}(\Theta, X_2) = \mathsf{Cov}(X_1, X_2) = 1.$$

The LLMS estimator of Θ based on X_1 and X_2 is of the form $\widehat{\Theta} = a_1X_1 + a_2X_2 + b$. Find the coefficients a_1, a_2, and b. *Hint:* To find b, recall the argument we used for the case of a single observation.

$a_1 =$ 1/5 ✔ **Answer:** 0.2

$a_2 =$ 1/5 ✔ **Answer:** 0.2

$b =$ 2/5 ✖ **Answer:** 0

Solution:

By the same argument as in the case of a single observation, we will have $b = \mathbf{E}[\Theta - a_1X_1 - a_2X_2] = 0$. Using the variance and covariance information we are given, the expression we want to minimize is

$$\mathbf{E}\left[(a_1X_1 + a_2X_2 - \Theta)^2\right] = 4a_1^2 + 4a_2^2 + 4 + 2a_1a_2 - 2a_1 - 2a_2.$$

Because of symmetry, we see that the optimal solution will satisfy $a_1 = a_2 = a$, so the expression is of the form $8a^2 + 4 + 2a^2 - 4a$. By setting the derivative to zero, we find that $20a = 4$, or $a = 1/5$
.

提交 You have used 3 of 3 attempts

Answers are displayed within the problem

讨论

显示讨论

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