2. Concept Check: Hypothesis Test Using a Single Observation

Let X be a single Gaussian random variable with unknown mean μ and variance 1. Consider the following hypotheses:

$$H_0 : \mu = 0 \quad \text{vs} \quad H_1 : \mu \neq 0.$$

(a)

1.0/1 point (graded)

Define a test $\psi : \mathbb{R} \to \{0,1\}$ with level 5% that is of the form

$$\psi = \mathbf{1}\{f(X) > 0\},$$

for some function $f : \mathbb{R} \to \mathbb{R}$.

We want our test ψ to be symmetric in X and its “acceptance region" to be an interval.
(The **acceptance region** of a test is the region in which the null hypothesis is **not rejected**, i.e. the complement of its rejection region.)

(If applicable, enter **abs(x)** for $|x|$, **Phi(x)** for $\Phi(x) = \mathbf{P}(Z \leq x)$ where $Z \sim \mathcal{N}(0,1)$, and **q(alpha)** for q_α, the $1-\alpha$-quantile of a standard normal distribution, e.g. enter **q(0.01)** for $q_{0.01}$.)

$f(X) =$ abs(X) - q(0.025)

Answer: abs(X) - q(0.025)

STANDARD NOTATION

Solution:

Since our test should be symmetric about zero and its “acceptance region" an interval, it must be of the form

$$\psi = \mathbf{1}\{|X| - q > 0\}.$$

Hence, it remains to determine q such that

$$\begin{aligned} \mathbf{P}_{\mu=0}(|X| > q) &= \quad 0.05 \\ \iff 2(1 - \Phi(q)) &= \quad 0.05 \\ \iff \Phi(q) &= \quad 0.975 \\ \iff q = q_{0.025} &\approx \quad 1.96. \end{aligned}$$

Hence, we can set

$$f(X) = |X| - 1.96.$$

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

(b)

3.0/3 points (graded)

Assume you observe $X = 1.32$.

What is the value of your test?

$\psi(X) =$ 0 ☐ **Answer:** 0

What is the p-value of your test (keeping in mind the symmetry and interval requirements)?
(If applicable, enter **abs(x)** for $|x|$, **Phi(x)** for $\Phi(x) = \mathbf{P}(Z \le x)$ where $Z \sim \mathcal{N}(0,1)$, and **q(alpha)** for q_α, the $1-\alpha$-quantile of a standard normal distribution, e.g. enter **q(0.01)** for $q_{0.01}$.)

p-value $=$ 0.1868 ☐ **Answer:** 2*(1-Phi(1.32))

What is the conclusion of the test?

○ Accept H_0

● Do not reject H_0 ☐

○ Accept H_1

○ Do not reject H_1

STANDARD NOTATION

Solution:

First, since $|1.32| < 1.96$, $\psi(1.32) = 0$.

Next, under the requirements for the test, the p-value is defined as

$$\inf\{\alpha : \psi_\alpha(X) = 1\},$$

where

$$\psi_\alpha(X) = \mathbf{1}\{|X| > q(\alpha)\}.$$

In other words, the p-value is the smallest value so that we could still reject H_0 given the observation, when picking our hypothesis test from a family of hypothesis tests indexed by α. In this case, by the requirement of ψ_α having confidence level α,

$$\begin{aligned} \mathbf{P}_{\mu=0}(\psi_\alpha(X) > q(\alpha)) = {} & 2(1-\Phi(q)) = \alpha \\ \iff \Phi(q) = {} & 1 - \frac{\alpha}{2} \end{aligned}$$

and hence

$$q(\alpha) = q_{\alpha/2},$$

the $1-\alpha/2$ quantile of a Normal variable. Now, by the form of the test ψ_α, we see that we get the infimum of α if $|X| = q_{\alpha/2}$, i.e., if

$$\alpha = 2(1-\Phi(|X|)) = 2 - 2\Phi(1.32) \approx 0.19.$$

We do not reject H_0 because there is not enough evidence for doing so. That does not necessarily mean that we think H_0 true, so we should not “accept" it.

提交 你已经尝试了1次（总共可以尝试3次）

☐ Answers are displayed within the problem

讨论

显示讨论

主题： Unit 2 Foundation of Inference:Homework 3: Introduction to Hypothesis Testing / 2. Concept Check: Hypothesis Test Using a Single Observation

认证证书是什么?