5. Exercise: Continuous convolution

Exercise: Continuous convolution

2/2 points (graded)

When calculating the convolution of two PDFs, one must be careful to use the appropriate limits of integration. Suppose that X and Y are nonnegative random variables. In particular, $f_X(x)$ is equal to some positive function $h_X(x)$ for $x \geq 0$ and is zero for $x < 0$. Similarly, $f_Y(y)$ is equal to some positive function $h_Y(y)$ for $y \geq 0$, and is zero for $y < 0$. Then, the convolution integral $\int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\, dx$ is of the form

$$\int_a^b h_X(x) h_Y(z-x)\, dx,$$

for suitable choices of a and b determined by z. Fix some $z \geq 0$. Find a and b. (Your answer can be an algebraic function of z.)

$a =$ 0 ✔ **Answer:** 0

$b =$ z ✔ **Answer:** z

Solution:

The integrand is equal to $h_X(x) h_Y(z-x)$ only for those choices of x for which the arguments of the functions h_X and h_Y are nonnegative; that is, when $x \geq 0$ and $z - x \geq 0$, which yields $0 \leq x \leq z$. Thus, we should only integrate from 0 to z.

Graphically, the PDF of X extends from 0 to ∞. Also, when we flip the PDF of Y, the resulting PDF extends from $-\infty$ to 0, and when we shift to the right it by z, it will extend from $-\infty$ to z. Thus the two PDFs that we need to multiply in the convolution integral overlap only for values from 0 to z.

提交 You have used 1 of 3 attempts

Answers are displayed within the problem

讨论

显示讨论