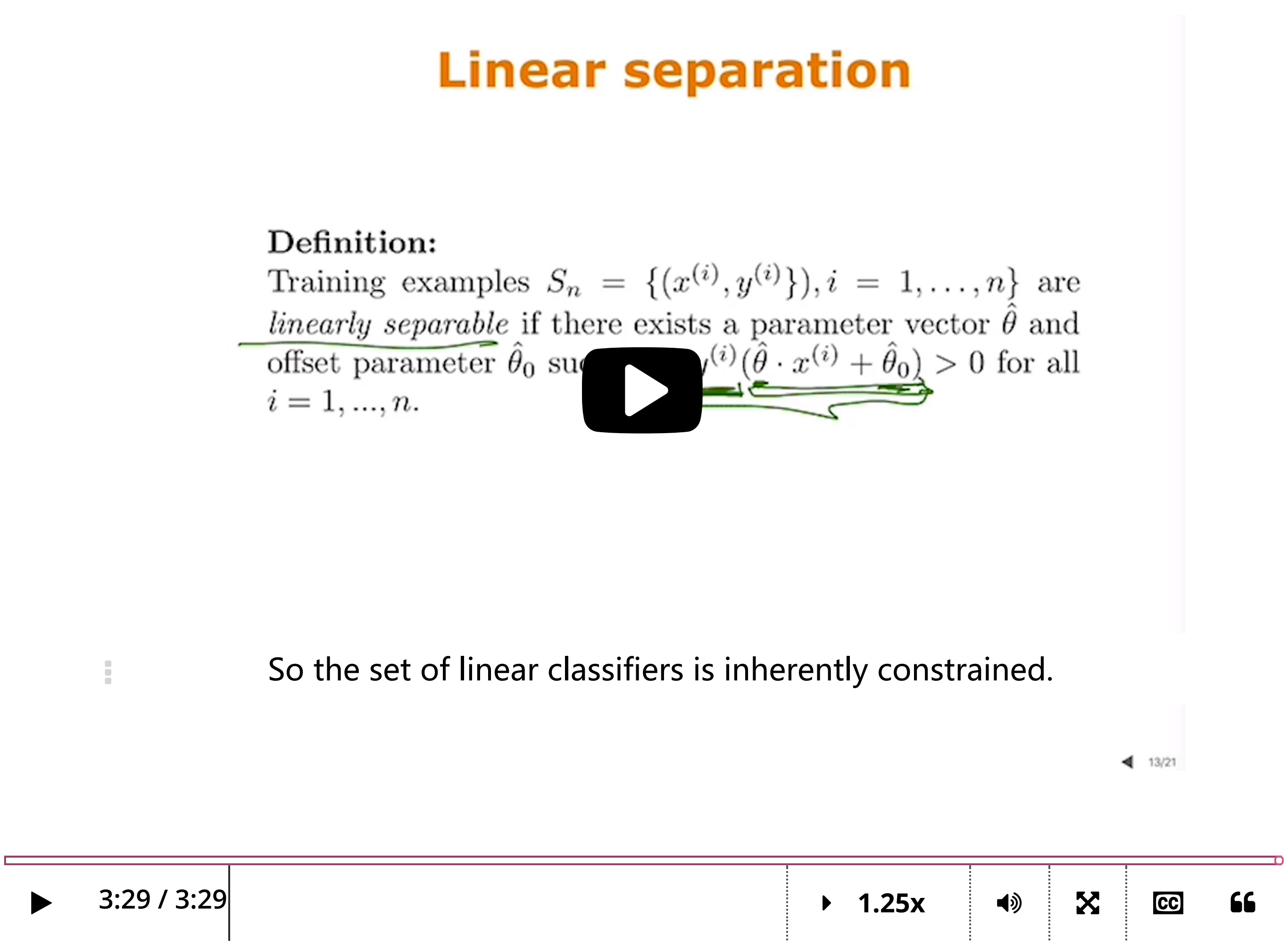
4. Linear Separation

Linear Separation



Linear separation

Definition:
Training examples $S_n = \{(x^{(i)}, y^{(i)}\}), i = 1, \ldots, n\}$ are *linearly separable* if there exists a parameter vector $\hat{\theta}$ and offset parameter $\hat{\theta}_0$ su $^{(i)}(\hat{\theta} \cdot x^{(i)} + \hat{\theta}_0) > 0$ for all $i = 1, \ldots, n$.

So the set of linear classifiers is inherently constrained.

the sign of this, would agree with the corresponding label y.

So if there is a linear classifier

in the set that would correctly classify those training points,

then the training points are said to be linearly separable.

And we've already seen cases where that linear separation

does not succeed.

So the set of linear classifiers is inherently constrained.

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Given θ and θ_0, a **linear classifier** $h : X \to \{-1, 0, +1\}$ is a function that outputs $+1$ if $\theta \cdot x + \theta_0$ is positive, 0 if it is zero, and -1 if it is negative. In other words, $h(x) = \text{sign}(\theta \cdot x + \theta_0)$.

Basics 1

1/1 point (graded)

As described in the lecture above, h is a linear classifier which is defined by the boundary $\theta \cdot x = 0$ (where theta is a vector perpendicular to the plane.) The ith training data is $(x^{(i)}, y^{(i)})$, where $x^{(i)}$ is a vector and $y^{(i)}$ is a scalar quantity. If θ is a vector of the same dimension as $x^{(i)}$, what are $y^{(i)}$ and sign($\theta \cdot x^{(i)}$) respectively?

○ output of the classifier h, label

○ label, dimension of the feature vector

○ label, distance of the point from the linear classifier

● label, output of the classifier h ✓

Solution:

By definition, $y^{(i)}$ is the label of $x^{(i)}$. Also, by the definition of a linear classifier $h\left(x\right)$ =sign($\theta \cdot x^{(i)}$), the output of h is given by sign($\theta \cdot x^{(i)}$).

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Answers are displayed within the problem

Basics 2

1/1 point (graded)

For the ith training data $\left(x^{i}, y^{i}\right)$, what values can $y^{(i)}$ take, **conventionally** (in the context of linear classifiers)? Choose all those apply.

☑ -1 ✓

☑ $+1$ ✓

☐ 0

☐ $+10$

✓

Solution:

By the convention of linear classification, because $y^{(i)}$ is a label, it can take -1 or $+1$. Note that 0 is not a possible value.

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Answers are displayed within the problem

Basics 3

1/1 point (graded)

For the ith training data $\left(x^{i}, y^{i}\right)$, what values can $sign\left(\theta \cdot x^{(i)}\right)$ take? Choose all those apply.

☑ -1 ✓

☑ $+1$ ✓

☑ 0 ✓

☐ $+10$

✓

Solution:

By definition the $sign\left(\theta \cdot x^{(i)}\right)$ function can only take one of $0, -1, +1$ as its value. Remember that a linear classifier outputs one of $-1, 0, 1$.

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Answers are displayed within the problem

When the Product is Positive

1/1 point (graded)

When does $y^{(i)}\left(\theta \cdot x^{(i)}\right) > 0$ happen? Choose all those apply.

- ☑ $y^{(i)} > 0$ and $\theta \cdot x^{(i)} > 0$ ✓
- ☐ $y^{(i)} < 0$ and $\theta \cdot x^{(i)} > 0$
- ☐ $y^{(i)} > 0$ and $\theta \cdot x^{(i)} < 0$
- ☑ $y^{(i)} < 0$ and $\theta \cdot x^{(i)} < 0$ ✓

✓

Solution:

$y^{(i)}\left(\theta \cdot x^{(i)}\right) > 0$ is true if and only if $y^{(i)}$ and $\left(\theta \cdot x^{(i)}\right)$ are both positive both negative. In other words, they have the same sign.

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Answers are displayed within the problem

Intuitive Meanings of Positive Product

1/1 point (graded)
What is the intuitive meaning of $y^{(i)}\left(\theta \cdot x^{(i)}\right) > 0$?

- ● x^i label and classified result match ✓
- ○ x^i label and classified result do not match
- ○ x^i is on the boundary of the classifier
- ○ training error is positive

Solution:

$y^{(i)}\left(\theta \cdot x^{(i)}\right) > 0$ is true if and only if $y^{(i)}$ and $\left(\theta \cdot x^{(i)}\right)$ are both positive both negative. In other words, they have the same sign.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Intuitive Meanings of Negative Product

1/1 point (graded)
What is the intuitive meaning of $y^{(i)}\left(\theta \cdot x^{(i)}\right) < 0$?

- ○ x^i label and classified result match
- ● x^i label and classified result do not match ✓
- ○ x^i is on the boundary of the classifier
- ○ training error is negative

Solution:

$y^{(i)}\left(\theta \cdot x^{(i)}\right) < 0$ is true if and only if $y^{(i)}$ and $\left(\theta \cdot x^{(i)}\right)$ have different signs.

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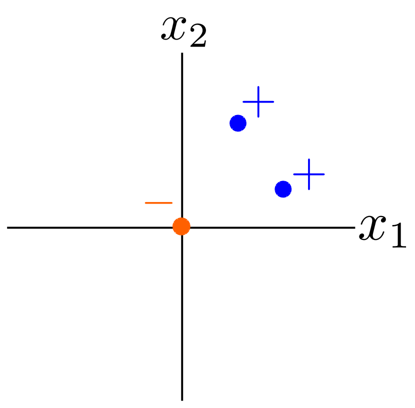
Linear Separation 1

1/1 point (graded)

Of the following, which is linearly separable? Choose all those apply.

☑


✓

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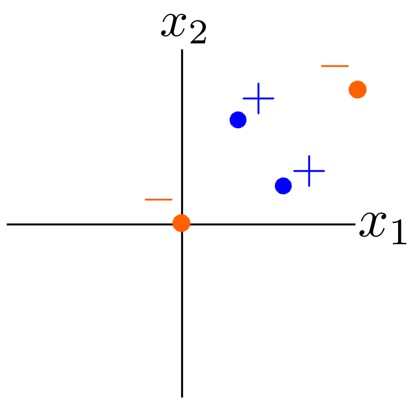

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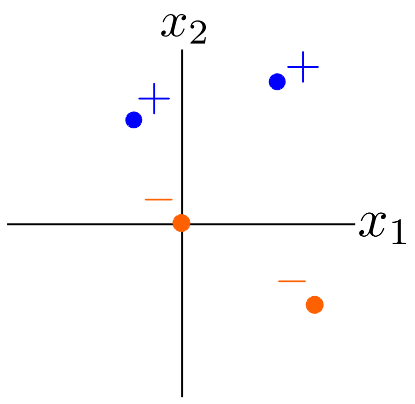

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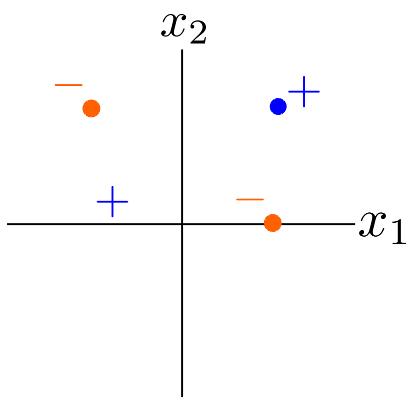

✓

Solution:

Linearly separable data can be separated with $+$ labels on one side of the line and $-$ labels on the other side, by some line on the plane.

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ℹ Answers are displayed within the problem

Linear Separation 2

1/1 point (graded)

A set of Training examples is illustrated in the table below, with the classified result by some linear classifier h and the label y^i. Is it linearly separable?

	$h\left(x^i\right)$	y^i
example 1	-1	-1
example 2	1	1
example 3	1	1
example 4	-1	-1
example 5	-1	-1

● yes ✓

○ no

Solution:

For linearly separable data, a linear classifier can perfectly separate the data. The provided classifier $h\left(x\right)$ classifies all the given points correctly.

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ℹ Answers are displayed within the problem

Discussion

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