8. Exercise: PDF of a general function

Exercise: PDF of a general function

2/2 points (graded)

The random variable X has a PDF of the form

$$f_X(x) = \begin{cases} \frac{1}{x^2}, & \text{for } x \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $Y = X^2$. For $y \geq 1$, the PDF of Y it takes the form $f_Y(y) = \frac{a}{y^b}$. Find the values of a and b.

$a =$ 1/2 ✔ **Answer:** 0.5

$b =$ 3/2 ✔ **Answer:** 1.5

Solution:

For any $y \geq 1$, we have

$$F_Y(y) = \mathbf{P}(Y \leq y) = \mathbf{P}(X^2 \leq y) = \mathbf{P}(X \leq \sqrt{y}) = F_X(\sqrt{y}).$$

By differentiating and using the chain rule, we have

$$f_Y(y) = \frac{1}{2\sqrt{y}} f_X(\sqrt{y}) = \frac{1}{2y^{1.5}}.$$

提交 You have used 1 of 3 attempts

Answers are displayed within the problem