In this final lecture, we will first review the various properties of a nice Markov chain, which ensures steady state behavior, and go over some of the notions in more detail with some examples. Providing some insights on how good an approximation we have when we use steady state probabilities to characterize the behavior of a Markov chain, which has run for a long time, but not an infinite amount of time.

We will then consider a classical application of Markov chains, which has to do with the design of a phone system. This is a famous problem, which was posed, analyzed, and solved by a Danish engineer by the name Erlang. It was more than 100 years ago when phones just started to exist, but we will see that this methodology remains relevant to design similar systems in today's world. We will then make use of all what we have learned so far in order to calculate some interesting short term behaviors of Markov chains having more than one recurrent classes.

We will introduce the notion of absorbing states, and we will show how to calculate the probability of ending up in such a state, as well as related quantities such as the expected time it takes to do so. As a classical example, we will look at the gambler continuously playing a simple game of chance, say a lottery, until he either accumulates a given amount of money or loses all his money. Both of these states are absorbing. What are their corresponding probabilities? After this lecture, you will be able to answer such questions.