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Homework 6.2: Coding by spikes

First spike in LIF model

1/1 point (graded)

Imagine we are injecting a step current in a neuron that receives no other input. In this exercise we investigate how the time of the first spike T codes for the amplitude of the step current I_0.

You should interpret your results in terms of coding efficiency: How is the time of the first spike coding for the current amplitude in each case? How could you make a precise measurement of the current amplitude with each model?

Determine the timing of the first spike as a function of I_0 for a **leaky integrate-and-fire** model. Assume that the firing threshold is θ and $u\left(t=0\right)=u_{rest}$. The dynamics of the membrane potential for $t\geq 0$ is the following.

$$\tau \frac{d}{dt}u = -\left(u-u_{rest}\right)+RI_0.$$

The time T of the first spike for LIF model is:

- ○ $\frac{\theta-u_{rest}}{RI_0}$
- ○ $\tau \ln\left(1-\frac{RI_0}{\theta-u_{rest}}\right)$
- ◉ $-\tau \ln\left(1-\frac{\theta-u_{rest}}{RI_0}\right)$
- ○ $-\tau \ln\left(\frac{\theta-u_{rest}}{RI_0}\right)$
- ○ $\tau \ln\left(\frac{RI_0}{\theta-u_{rest}}\right)$
- ○ $\tau \frac{\theta-u_{rest}}{RI_0}$

解出来

$$u(t) = u_{rest} + \frac{qR}{\tau}\int_0^t \exp(-s/\tau)S(t-s)ds$$

✔

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✔ Correct (1/1 point)

First spike in poisson neuron

2/2 points (graded)

Now consider a **Poisson neuron** with a firing rate proportional to current: $\rho\left(t\right)=kI\left(t\right)$. Which of the following options are correct? (note that there may be more than one correct answer)

- ☐ The timing of the first spike determines the intensity of the stimulus exactly.
- ☑ One cannot specify the intensity of the stimulus with the timing of the first spike of a poisson neuron because the spikes are stochastic.
- ☑ One can average the number of spikes over some time to approximate the firing rate ν.

✔

- [x] The probability of firing the first spike as a function of time for a poisson neuron is $\nu e^{-\nu t}$.

- [x] The first spike will most probably happen at $t = 0$, regardless of the strength of the stimulus.

因为独立，所以概率一样，但是因为前面可能会有不应期。

- [x] The expected value of the first spike time is ν^{-1}

✔

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✔ Correct (2/2 points)

Leaky integrator neuron with stochastic firing

1/2 points (graded)

Consider a leaky integrator neuron model:

$$\frac{du}{dt} = -\frac{u}{\tau} + \frac{I(t)}{C}$$

where u is the membrane potential above rest. Consider further that firing is stochastic and occurs via escape noise proportional to the potential. Precisely, the instantaneous probability to fire is given by:

$$\rho(t) = \begin{cases} u(t) - a & \text{for } u(t) >= a \\ 0 & \text{for } u(t) < a \end{cases}$$

where a is some threshold potential above which the neuron has a non-zero probability to spike.

For such neuron, one cannot determine the time of the first spike exactly. However we would like to know what the probability distribution of the time of the first spike is.

Imagine a very large time constant τ but a fixed value of C. In this case ($\tau \to \infty$) the potential increases almost linearly and obeys: $u(t) \approx \frac{I_0 t}{C}$. This gives the instantaneous rate as a function of the time of the onset of the stimulus:

$$\rho(t) = \begin{cases} \frac{I_0 t}{C} - a & \text{for } t >= \frac{aC}{I_0} \\ 0 & \text{for } t < \frac{aC}{I_0} \end{cases}$$

Taking the notions of renewal theory, first compute the survivor function for the neuron above. Then compute the probability distribution for the first spike time. It is basically equivalent to computation of the ISI distribution. Around which number the probability distribution is centered?

Hint: Where is the peak of distribution located? To find the peak of distribution, one can find where the first derivative of the distribution is zero.

- ( ) $\frac{C}{I_0}(1 + a)$
- (•) $\frac{C}{I_0} + \sqrt{\frac{aC}{I_0}}$
- ( ) $\frac{C}{I_0} + \sqrt{\frac{C}{aI_0}}$
- ( ) $\sqrt{\frac{C}{I_0}} + \frac{aC}{I_0}$
- ( ) $\frac{C}{I_0}(1 + \frac{1}{a})$
- ( ) $\sqrt{\frac{C}{I_0}}(1 + \sqrt{a})$

✖

You already computed the distribution of the first spike time. What is the **expected value** of this distribution?

Hint: For $a > 0$ we have $\int_0^\infty x^2 e^{-ax^2} dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}$ and $\int_0^\infty x e^{-ax^2} dx = \frac{1}{2a}$.

- $\frac{C}{I_0} + \sqrt{\frac{aC}{I_0}}$
- $\sqrt{2\pi}\sqrt{\frac{C}{I_0}} + \frac{aC}{I_0}$
- $\frac{C}{I_0} + \sqrt{\frac{C}{aI_0}}$
- $\sqrt{\frac{C}{I_0}} + \sqrt{2\pi}\sqrt{\frac{aC}{I_0}}$
- $\frac{C}{I_0}(1 + \frac{\pi}{a})$
- $\sqrt{\frac{\pi}{2}}\sqrt{\frac{C}{I_0}} + \frac{aC}{I_0}$

✔

Submit

You have used 1 of 1 attempt

✱ Partially correct (1/2 points)

Discussion

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