4. Convolution calculations

Problem 4. Convolution calculations

9/9 points (graded)

1. Let the discrete random variable X be uniform on $\{0,1,2\}$ and let the discrete random variable Y be uniform on $\{3,4\}$. Assume that X and Y are independent. Find the PMF of $X+Y$ using convolution. Determine the values of the constants a, b, c, and d that appear in the following specification of the PMF.

$$p_{X+Y}(z) = \begin{cases} a, & z=3, \\ b, & z=4, \\ c, & z=5, \\ d, & z=6, \\ 0, & \text{otherwise.} \end{cases}$$

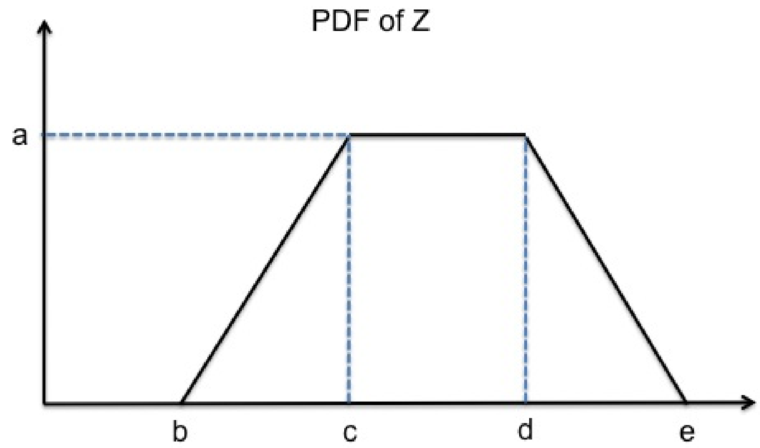
$a =$ 1/6 ✔ **Answer:** 1/6

$b =$ 1/3 ✔ **Answer:** 1/3

$c =$ 1/3 ✔ **Answer:** 1/3

$d =$ 1/6 ✔ **Answer:** 1/6

2. Let the random variable X be uniform on $[0,2]$ and the random variable Y be uniform on $[3,4]$. (Note that in this case, X and Y are continuous random variables.) Assume that X and Y are independent. Let $Z = X+Y$. Find the PDF of Z using convolution. The following figure shows a plot of this PDF. Determine the values of a, b, c, d, and e.



$a =$ 1/2 ✔ **Answer:** 0.5

$b =$ 3 ✔ **Answer:** 3

$c =$ 4 ✔ **Answer:** 4

$d =$ 5 ✔ **Answer:** 5

$e =$ 6 ✔ **Answer:** 6

Solution:

1.
$$p_{X+Y}(z) = \begin{cases} 1/6, & z \in \{3, 6\} \\ 1/3, & z \in \{4, 5\} \\ 0, & \text{otherwise.} \end{cases}$$

2. The answer is easiest to find graphically, by sliding a rectangle of width 1 along a rectangle of width 2, and is:

$$f_{X+Y}(z) = \begin{cases} \frac{z-3}{2}, & 3 \le z < 4, \\ \frac{1}{2}, & 4 \le z < 5, \\ \frac{6-z}{2}, & 5 \le z \le 6, \\ 0, & \text{otherwise.} \end{cases}$$

A more formal approach involves the convolution formula, but requires careful thought in order to identify the appropriate limits of integration. In particular, if $3 \le z \le 6$, we have

$$\begin{aligned} f_{X+Y}(z) &= \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\, dx \\ &= \int_{\max(0,z-4)}^{\min(2,z-3)} \frac{1}{2}\, dx \end{aligned}$$

$$= (\min(2, z-3) - \max(0, z-4))/2$$

which actually agrees with the answer obtained through the graphical method.

提交 You have used 4 of 5 attempts

Answers are displayed within the problem

讨论

显示讨论

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