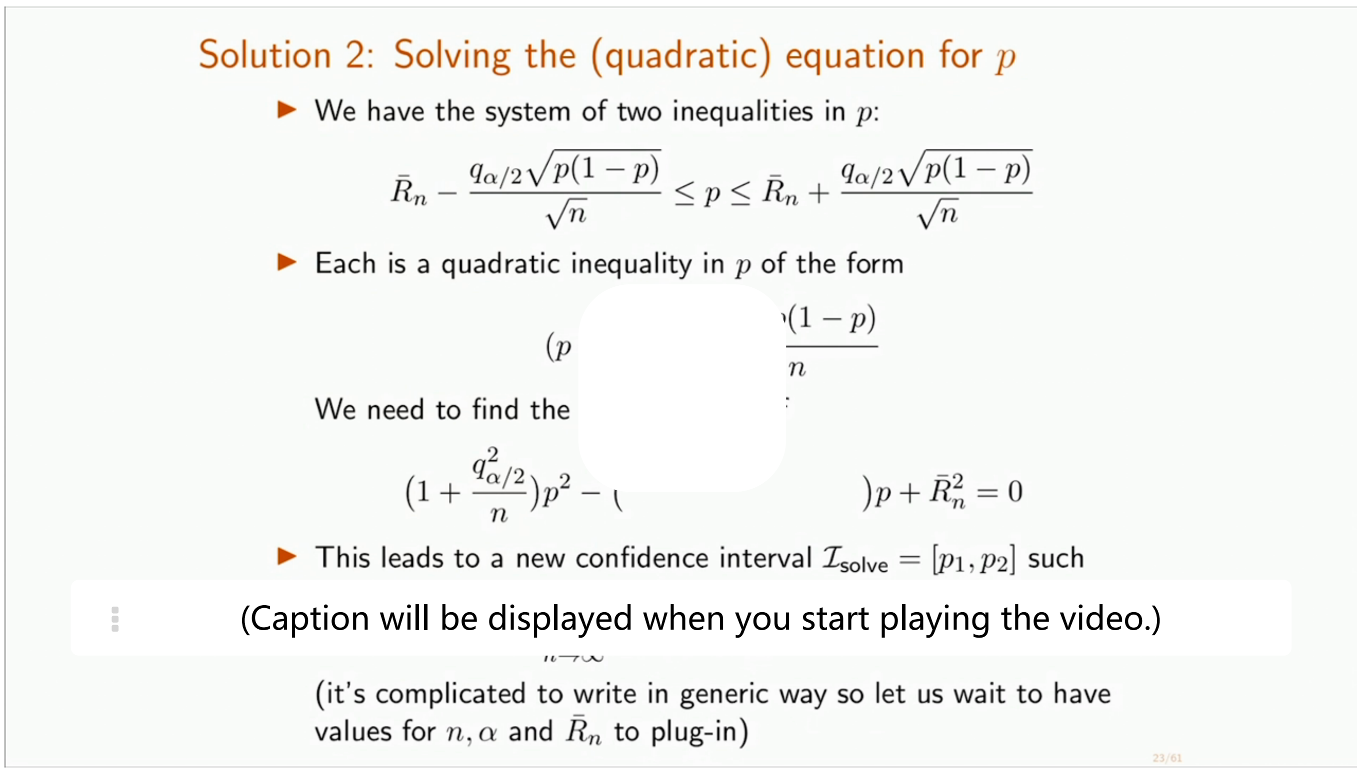
10. Solving for a Confidence Interval

Confidence Interval by Solving for p



Start of transcript. Skip to the end.

Second solution.

This is the slightly--

while I find it's the most fun, but it's also

the most technical one.

And it just so happens that the variance of a Bernoulli

is p1 minus p, all right?

Which is just the quadratic function of p.

So what happens is when I actually

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Solving for a Confidence Interval: Algebra

2/2 points (graded)

In the problems on this page, we will continue building the confidence interval of asymptotical level **95%** by solving for p as in the video.

Recall that $R_1, \ldots, R_n \stackrel{iid}{\sim}$ Ber (p) for some unknown parameter p, and we estimate p using the estimator $\hat{p} = \overline{R}_n = \frac{1}{n}\sum_{i=1}^{n} R_i$.

As in the method using a conservative bound, our starting point is the result of the central limit theorem:

$$\lim_{n\to\infty} \mathbf{P}\left(\left|\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right| < q_{\alpha/2}\right) = 1-\alpha.$$

In this second method, we solve for values of p that satisfy the inequality $\left|\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right| < q_{\alpha/2}$.

To do this, we manipulate $\left|\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right| < q_{\alpha/2}$ into an inequality involving a quadratic function $Ap^2 + Bp + C$ where $A > 0$, B, C depend on $n, q_{\alpha/2}$, and $\overline{R}_n$. Which of the following is the correct inequality?

(We will use find the values of A, B, and C in the next problem.)

- (selected) $Ap^2 + Bp + C < 0$ where $A > 0$. ✔

- ○ $Ap^2 + Bp + C > 0$ where $A > 0$.

Let p_1 and p_2 with $0 < p_1 < p_2 < 1$ be the two roots of the quadratic function $Ap^2 + Bp + C$. What values of p satisfy the correct inequality above?

- ○ $(p < p_1) \cup (p > p_2)$
- ◉ $p_1 < p < p_2$ ✔
- ○ $0 < p < p_1$
- ○ $p_2 < p < 1$
- ○ $0 < p < 1$

Solution:

$$\begin{aligned}\left|\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right| < q_{\alpha/2} &\implies \left(\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right)^2 < q_{\alpha/2}^2 \\ &\implies \left(\overline{R}_n - p\right)^2 < \frac{p(1-p)\,q_{\alpha/2}^2}{n} \\ &\implies p^2\left(1 + \frac{q_{\alpha/2}^2}{n}\right) - p\left(2\overline{R}_n + \frac{q_{\alpha/2}^2}{n}\right) + \left(\overline{R}_n\right)^2 < 0\end{aligned}$$

Hence, the inequality is of the form $Ap^2 + Bp + C < 0$ for some $A > 0$.
The quadratic function $Ap^2 + Bp + C < 0$ where $A > 0$ is convex, so the parabola opens up, and the region in which the parabola is below the x-axis is the interval between the two roots. Given $0 < p_1 < p_2 < 1$, the region is $p_1 < p < p_2$.

提交 你已经尝试了1次（总共可以尝试1次）

ℹ Answers are displayed within the problem

Solving for a Confidence Interval: Numerical Descriptions

2/2 points (graded)
Continuing from above, enter numerical values for $A > 0$, B, C such that the inequality in the previous problem is equivalent to $\left|\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right| < q_{\alpha/2}$ for the case when the sample size is $n = 100$, and the observed value of $\overline{R}_n$ is 0.645.

Carry out the computations with the goal of computing a confidence interval of p at asymptotic level 95%. **Note:** Because polynomials differing by only an overall rescaling constant yield the same roots, use $C = \left(\overline{R}_n\right)^2$ here as in the previous problem.

(If necessary, round your answers to the nearest four decimal places (10^{-4}).

$0 < A =$ 1.038416 ✔ **Answer:** 1+(1.96^2)/100

$B =$ -1.328416 ✔ **Answer:** -(2*0.645+1.96^2/100)

$C =$ 0.416025 ✔ **Answer:** 0.645^2

Now, as indicated previously, use the above values (**rounded to the nearest** 10^{-4}) to compute a confidence interval $\mathcal{I}_{\text{solve}}$ of p of asymptotic level 95%.
If necessary, round your endpoints to the nearest two decimal places (10^{-2}).

$\mathbf{P} \in$[0.54744214582419 ✔ **Answer:** 0.5473323 , 0.73182936000777 ✔ **Answer:** 0.7319435]

Solution:

Recall from the previous problem that

$$\left|\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right| < q_{\alpha/2} \implies p^2\left(1+\frac{q_{\alpha/2}^2}{n}\right) - p\left(2\overline{R}_n + \frac{q_{\alpha/2}^2}{n}\right) + \left(\overline{R}_n\right)^2 < 0.$$

Plugging $n = 100$, $\overline{R}_n = 0.645$, and $q_{\alpha/2} = q_{0.025} = 1.96$ into the inequality above gives

$$p^2\left(1+\frac{1.96^2}{100}\right) - p\left(2(0.645) + \frac{1.96^2}{100}\right) + 0.645^2 < 0.$$

The quadratic formula gives the roots $\dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$, which are

$$\begin{aligned} p_1 &= 0.5473323 \\ p_2 &= 0.7319435. \end{aligned}$$

This gives the confidence interval $[p_1, p_2] \approx [0.55, 0.73]$.

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Answers are displayed within the problem

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