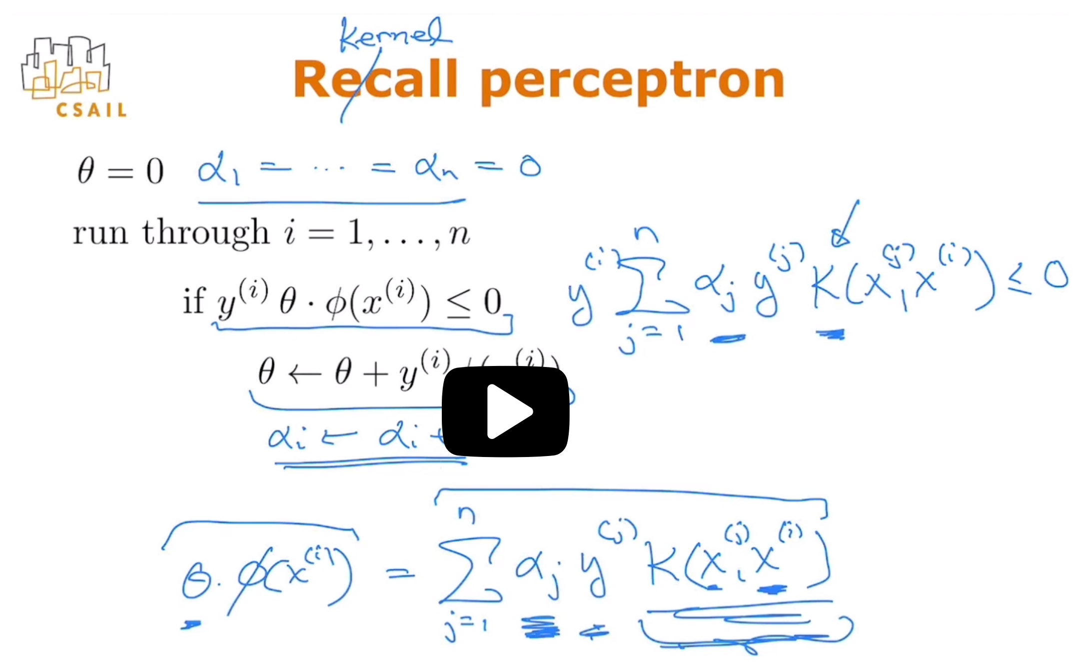
5. The Kernel Perceptron Algorithm

Computational Efficiency



7:34 / 7:34 1.0x

here as a kind of similarity measure.

How similar the j-th example is to the i-th example.

So our predicted value here is now

how important the j-th example is.

Its label times how similar the example

we wish to make a prediction on and the j-th training example.

All right.

That is now the kernel perceptron algorithm.

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How the Kernel Perceptron Algorithm Works: Initalization

1/1 point (graded)
Recall that the original Perceptron Algorithm is given as the following:

Perceptron $\left(\left\{\left(x^{(i)}, y^{(i)}\right), i = 1, \ldots, n\right\}, T\right)$:

initialize $\theta = 0$ (vector);
for $t = 1, \ldots, T$,
for $i = 1, \ldots, n$,
if $y^{(i)}\left(\theta \cdot x^{(i)}\right) \leq 0$,
then update $\theta = \theta + y^{(i)} x^{(i)}$.

In the lecture, it was introduced that we can always express θ as

$$\theta = \sum_{j=1}^{n} \alpha_j y^{(j)} \phi\left(x^{(j)}\right)$$

where values of $\alpha_1, \ldots, \alpha_n$ may vary at each step of the algorithm. In other words, we can reformulate the algorithm so that we somehow initialize and update α_j's, instead of θ.

The reformulated algorithm, or **kernel perceptron** , can be given in the following form:

Kernel Perceptron $\left(\{(x^{(i)}, y^{(i)}), i = 1, \ldots, n, T\}\right)$

Initialize $\alpha_1, \alpha_2, \ldots, \alpha_n$ to some values;
for $t = 1, \ldots, T$
for $i = 1, \ldots, n$
if $(\text{Mistake Condition Expressed in } \alpha_j)$
Update α_j appropriately

Look at the initialization statement of the algorithm. Which of the following is an equivalent way to initialize $\alpha_1, \alpha_2, \ldots, \alpha_n$, if we want the same result as initializing $\theta = 0$?

- $\alpha_1 = \ldots = \alpha_n = \theta$
- $\alpha_1 = \ldots = \alpha_n = 1$
- $\alpha_1 = \ldots = \alpha_n = 0$ ✔
- $\alpha_1 = \ldots = \alpha_n = -1$

Solution:

Since $\theta = \sum_{j=1}^{n} \alpha_j y^{(j)} \phi(x^{(j)})$, setting $\alpha_j = 0$ for all j leads to $\theta = 0$.

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Answers are displayed within the problem

How the Kernel Perceptron Algorithm Works: The Update

1/1 point (graded)
As in the previous problem, our goal is to correctly reformulate the original perceptron algorithm. In other words, we want the algorithm to be about updating α_j's instead of θ.

Kernel Perceptron $\left(\{(x^{(i)}, y^{(i)}), i = 1, \ldots, n, T\}\right)$

initialize $\alpha_1, \alpha_2, \ldots, \alpha_n$ to some values;
for $t = 1, \ldots, T$
for $i = 1, \ldots, n$
if $(\text{Mistake Condition Expressed in } \alpha_j)$
Update α_j appropriately

Now look at the line "**Update α_j appropriately**" in the above algorithm. Remember that we express θ as

$$\theta = \sum_{j=1}^{n} \alpha_j y^{(j)} \phi(x^{(j)})$$

Assuming that there was a mistake in classifying the ith data point i.e.

$$y^{(i)}(\theta \cdot x^{(i)}) \leq 0$$

which of the following conditions about $\alpha_1, \ldots, \alpha_n$ is equivalent to

$$\theta = \theta + y^{(i)} \phi(x^{(i)}),$$

the update condition of the original algorithm?

- $\alpha_i = \alpha_i + 1$ ✔
- $\alpha_i = \alpha_i - 1$
- $\alpha_j = \alpha_j + 1$ for all $j \in 1, \ldots, n$

Solution:

Expand θ in the last equation and it turns out only α_i gets updated:

$$\alpha_i y^{(i)} \phi\left(x^{(i)}\right) + y^{(i)} \phi\left(x^{(i)}\right) = \left(\alpha_i + 1\right) y^{(i)} \phi\left(x^{(i)}\right).$$

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How the Kernel Perceptron Algorithm Works: The Mistake Condition

1/1 point (graded)

Kernel Perceptron $\left(\left\{\left(x^{(i)}, y^{(i)}\right), i = 1, \ldots, n, T\right\}\right)$
initialize $\alpha_1, \alpha_2, \ldots, \alpha_n$ to some values;
for $t = 1, \ldots, T$
for $i = 1, \ldots, n$
if $(\text{Mistake Condition Expressed in } \alpha_j)$
Update α_j appropriately

Now look at the line "**Mistake Condition Expressed in** α_j" in the above algorithm. Remember that we express θ as

$$\theta = \sum_{j=1}^{n} \alpha_j y^{(j)} \phi\left(x^{(j)}\right)$$

Which of the following conditions is equivalent to $y^{(i)}\left(\theta \cdot \phi\left(x^{(i)}\right)\right) \leq 0$? Remember from the video lecture above that given feature vectors $\phi(x)$ and $\phi(x')$, we define the Kernel function K as

$$K(x, x') = \phi(x)\, \phi(x').$$

- $y^{(i)} \sum_{j=1}^{n} \alpha_j y^{(j)} K\left(x^j, x^i\right) \leq 0$ ✔
- $y^{(i)} \sum_{j=1}^{n} \alpha_i y^{(j)} K\left(x^j, x^i\right) \leq 0$
- $y^{(i)} \sum_{j=1}^{n} \alpha_j y^{(i)} K\left(x^j, x^i\right) \leq 0$
- $y^{(i)} \sum_{j=1}^{n} \alpha_j y^{(j)} \phi\left(x^{(j)}\right) \leq 0$

Solution:

Substitute θ with $\sum_{j=1}^{n} \alpha_j y^{(j)} \phi\left(x^{(j)}\right)$ in $y^{(i)}\left(\theta \cdot \phi\left(x^{(i)}\right)\right) \leq 0$.

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Answers are displayed within the problem

Discussion

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Topic: Unit 2 Nonlinear Classification, Linear regression, Collaborative Filtering (2 weeks):Lecture 6. Nonlinear Classification / 5. The Kernel Perceptron Algorithm

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