4. Airline overbooking

Problem 4. Airline overbooking

1/1 point (graded)

For any given flight, an airline tries to sell as many tickets as possible. Suppose that on average, 20% of ticket holders fail to show up, all independent of one another. Knowing this, an airline will sell more tickets than there are seats available (i.e., overbook the flight) and hope that there is a sufficient number of ticket holders who do not show up, to compensate for its overbooking. Using the Central Limit Theorem, determine n, the maximum number of tickets an airline can sell on a flight with 400 seats so that it can be approximately 99% confident that all ticket holders who do show up will be able to board the plane. Use the de Moivre-Laplace $1/2$-correction in your calculations. **Hint:** You may have to solve numerically a quadratic equation.

476 ✔ **Answer:** 475

Solution:

The probability that the airline will have to deny passengers from boarding is the probability that more than 400 passengers show up. Let N be the total number of ticketed passengers that show up. Then, N has a binomial distribution with parameters, n and $p = 0.8$. Therefore,

$$\mathbf{E}[N] = 0.8n \quad \text{and} \quad \sigma_N = \sqrt{n(0.2)(0.8)} = \sqrt{0.16n}.$$

Using the de Moivre-Laplace normal approximation to the binomial, we have,

$$\begin{aligned} \mathbf{P}(N > 400) &= \mathbf{P}(N \geq 400 + 0.5) \\ &= \mathbf{P}\left(\frac{N - 0.8n}{\sqrt{0.16n}} \geq \frac{400.5 - 0.8n}{\sqrt{0.16n}}\right) \\ &\approx 1 - \Phi\left(\frac{400.5 - 0.8n}{\sqrt{0.16n}}\right). \end{aligned}$$

In order for the airline to be 99% confident that it will not have to deny boarding any passengers holding tickets, we need

$$\mathbf{P}(N > 400) \approx 0.01, \quad \text{or} \quad \Phi\left(\frac{400.5 - 0.8n}{\sqrt{0.16n}}\right) \approx 0.99.$$

Since, $\Phi(2.33) \approx 0.99$, to find the approximate answer, we solve the following equation:

$$\begin{aligned} \frac{400.5 - 0.8n}{\sqrt{0.16n}} &= 2.33 \\ 400.5 - 0.8n &= 2.33\sqrt{0.16}\sqrt{n} \\ 0.8n + 2.33 \cdot 0.4 \cdot \sqrt{n} - 400.5 &= 0. \end{aligned}$$

We treat this as a quadratic equation in $\sqrt{n}$ and solve to obtain two solutions: $\sqrt{n} \approx 21.7997$ and $\sqrt{n} \approx -22.9647$. Note that in our earlier calculations, $\sqrt{n}$ stands for the positive square root of n. Hence the solution $\sqrt{n} = -22.9647$ can be discarded. Therefore,

$$n \approx 21.7997^2 \approx 475.$$

提交 你已经尝试了1次（总共可以尝试3次）