7. A random circle

Problem 6. A random circle

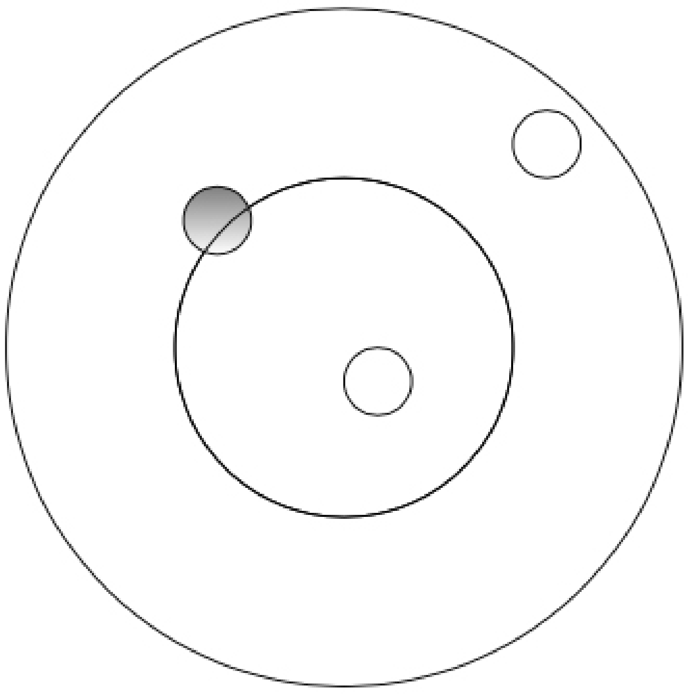
2/2 points (graded)

Terminology:

1. A **circle** of radius r is a curve that consists of all points at distance r from the center of the circle.

2. A **disk** of radius r is the set of all points whose distance from its center is **less than or equal** to r.

Thus, a circle is the boundary of a disk.



Circles of radius 10 and 5. A random circle of radius 1, whose center is inside the larger circle, may or may not intersect the circle of radius 5. In the figure, the circle whose interior is shaded intersects the circle of radius 5. The other two circles of radius 1 do not intersect the circle of radius 5.

(a). We generate a random circle of radius 1, whose center is uniformly distributed inside a disk of radius 10 centered at the origin; see figure above.
Find the probability that the random circle intersects a circle of radius $r=5$, also centered at the origin. (Give a numerical answer.)

0.2 ✔ **Answer:** 0.2

(b). Answer the same question as in Part (a) but for the case where r, instead of being 5, is the realized value of a random variable R that is uniformly distributed between 2 and 5. (Give a numerical answer.)

7/50 ✔ **Answer:** 0.14

Solution:

(a). A circle of radius 1 will intersect a circle of radius 5 if its center is contained within the annulus with inner radius 4 and outer radius 6. Since the probability is uniformly distributed, it is sufficient to compute the area of this annulus and divide by the total area of the disk to find the probability. Letting E denote this event, we have

$$\mathbf{P}(E)=\frac{\pi 6^2-\pi 4^2}{\pi 10^2}=\frac{20}{100}=\frac{1}{5} \qquad (11.1)$$

Note that this is equivalent to working out the total area, 100π, and integrating over the annulus normalized by the total area

$$
\begin{aligned}
\mathbf{P}(E) &= \int_0^{2\pi}\int_4^6 \frac{1}{100\pi}\, r\, dr d\theta \\
&= 2\pi \int_4^6 \frac{1}{100\pi}\, r\, dr \\
&= \frac{1}{50} \times \frac{r^2}{2}\Big|_4^6 \\
&= \frac{20}{100} = \frac{1}{5}
\end{aligned}
$$

(b). Note that conditional on $R = r$, the probability $\mathbf{P}(E|R = r)$ is computed as

$$\mathbf{P}(E|R = r) = \frac{\pi \cdot (r+1)^2 - \pi \cdot (r-1)^2}{\pi \cdot 10^2} = \frac{r}{25}.$$

Hence, using the continuous version of the law of total probability, we have,

$$\mathbf{P}(E) = \int_2^5 \mathbf{P}(E|R = r) f_R(r)\, dr = \int_2^5 \frac{r}{25} \cdot \frac{1}{3}\, dr = 0.14.$$

提交 You have used 2 of 2 attempts

Answers are displayed within the problem

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Topic: Exam 2 / 7. A random circle