8. Review: Power of a Test

Review: Power of a Test for Different Alternative Hypotheses

2/3 points (graded)

Recall that the power π_ψ of a test ψ for the hypotheses

$$H_0 : \theta^* \in \Theta_0$$
$$H_1 : \theta^* \in \Theta_1$$

也就是说：
alpha是under H0，犯错的概率。
power是under H1，能检验出来的概率。

is

$$\pi_\psi \;=\; \inf_{\theta \in \Theta_1} \left(1 - \beta_\psi(\theta)\right)$$

where $\beta_\psi(\theta) = \mathbf{P}_\theta(\psi = 0)$, defined for $\theta \in \Theta_1$, is the **type 2 error rate** of ψ.

Suppose $X_1, \ldots, X_n$ are i.i.d. random variables (in 1 dimension). Assume the theorem of MLE applies so that $\boxed{\hat{\theta}^{\mathrm{MLE}}}$ is asymptotically normal. You use the test

$$\psi \;=\; \mathbf{1}\left(\sqrt{nI}\,\left|\hat{\theta}^{\mathrm{MLE}} - 0\right| > C_\alpha\right),$$

which has level α for some threshold C_α, to test the hypotheses

$$H_0 : \theta^* = 0$$
$$H_1 : \theta^* \neq 0.$$

.

What is the power π_ψ in terms of α?

$\pi_\psi =$ 2*alpha

$2 \cdot \alpha$

Answer: alpha

测试本身就是双侧的！

Now, you use the same test $\psi = \mathbf{1}\left(\sqrt{nI}\,\left|\hat{\theta}^{\mathrm{MLE}} - 0\right| > C_\alpha\right)$ to test a different alternative hypothesis against the same null hypothesis:

$$H_0 : \theta^* = 0$$
$$H_1 : \theta^* = 1.$$

How do the (smallest) level and the power of ψ change with this change of the alternative hypothesis? (Choose one for each column.)

The (smallest) level of ψ ...	the power of ψ ...
○ increases	◉ increases
○ decreases	○ decreases
◉ stays the same	○ stays the same

(In general, how does the level and power of a test vary as Θ_1 shrinks?)

STANDARD NOTATION

power = inf(1 - (测试结果是0 | theta不是0))
= inf(测试结果是1 | theta不是0)
= theta不是0时，大于Ca的概率的最小值

Solution:

The power of ψ with $H_1 : \theta^* \neq 0$ is

$$\begin{aligned}\pi_\psi &= \inf_{\theta \neq 0} \left(1 - \beta_\psi(\theta)\right) \\ &= \inf_{\theta \neq 0} \mathbf{P}_\theta(\psi = 1) = \inf_{\theta \neq 0} \mathbf{P}_\theta\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}} - 0\right| > C_\alpha\right)\end{aligned}$$

Since $\sqrt{nI}\left(\hat{\theta}^{\text{MLE}} - 0\right) \sim \mathcal{N}(0,1)$ (asymptotically), $\mathbf{P}_\theta\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}} - 0\right| > C_\alpha\right)$ decreases as $\theta \to 0$ and approaches $\mathbf{P}_0\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}} - 0\right| > C_\alpha\right) = \alpha$ (sketch the probability as an area to see this). Hence $\pi_\psi = \alpha$ in this case.

If we use the same test ψ for the alternative hypothesis $H_1 : \theta^* = 1,$ then

$$\pi_\psi = \mathbf{P}_{\theta=1}\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}} - 0\right| > C_\alpha\right)$$

which is greater than $\mathbf{P}_{\theta=0}\left(\sqrt{nI}\left|\hat{\theta}^{\text{MLE}} - 0\right| > C_\alpha\right) = \alpha.$ (Again, sketch the probability as an area to see this.)

On the other hand, the alternative hypothesis has no effect on the level of the test once the test has been fixed.

提交 你已经尝试了2次（总共可以尝试2次）

Answers are displayed within the problem

讨论

主题: Unit 4 Hypothesis testing:Lecture 14: Wald's Test, Likelihood Ratio Test, and Implicit Hypothesis Test / 8. Review: Power of a Test

显示讨论