

We think of the random variable $2X - Y$ as the sum of the independent random variables $2X$ and $-Y$. Thus, we can apply the convolution formula

$$f_{X+Y}(z) = \int_{-\infty}^{\infty} f_X(x)f_Y(z-x) dz,$$

but with the random variable X replaced by $2X$ and the random variable Y replaced by $-Y$:

$$f_{2X-Y}(z) = \int_{-\infty}^{\infty} f_{2X}(x)f_{-Y}(z-x) dz.$$

Using the formula

$$f_{aX}(y) = \frac{1}{|a|} f_X\left(\frac{y}{a}\right),$$

we have that

$$f_{2X}(x) = \frac{1}{2}f_X(x/2), \quad f_{-Y}(y) = f_Y(-y),$$

which leads us to

$$f_{2X-Y}(z) = \int_{-\infty}^{\infty} \frac{1}{2}f_X(x/2)f_Y(x-z) dz.$$