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6. Upper and lower bounds on the probability of intersection

Problem 6. Upper and lower bounds on the probability of intersection

2/2 points (graded)

Given two events A, B with $\mathbb{P}(A) = 3/4$ and $\mathbb{P}(B) = 1/3$, what is the smallest possible value of $\mathbb{P}(A \cap B)$? The largest? That is, find a and b such that,

$$a \leq \mathbb{P}(A \cap B) \leq b,$$

holds and any value in the closed interval $[a, b]$ is possible.

$a =$

✓ Answer: 1/12

$b =$

✓ Answer: 1/3

Solution:

First, we begin by recalling that, $\mathbb{P}(A \cap B)$ obeys,

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cup B).$$

Now, for the lower bound, note that, $\mathbb{P}(A \cup B) \leq 1$, since the probability of any event can be at most one. Hence,

$$\mathbb{P}(A \cap B) \geq \mathbb{P}(A) + \mathbb{P}(B) - 1 = \frac{1}{12}.$$

Next, for the upper bound, we use the fact that, $A \cap B \subset A$ and $A \cap B \subset B$. Hence,

$$\mathbb{P}(A \cap B) \leq \mathbb{P}(A) \quad \text{and} \quad \mathbb{P}(A \cap B) \leq \mathbb{P}(B).$$

In particular, $\mathbb{P}(A \cap B)$ must be less than or equal to the smallest of $\mathbb{P}(A)$ and $\mathbb{P}(B)$, which means,

$$\mathbb{P}(A \cap B) \leq \frac{1}{3}.$$

提交

You have used 1 of 3 attempts

i Answers are displayed within the problem

讨论

显示讨论

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