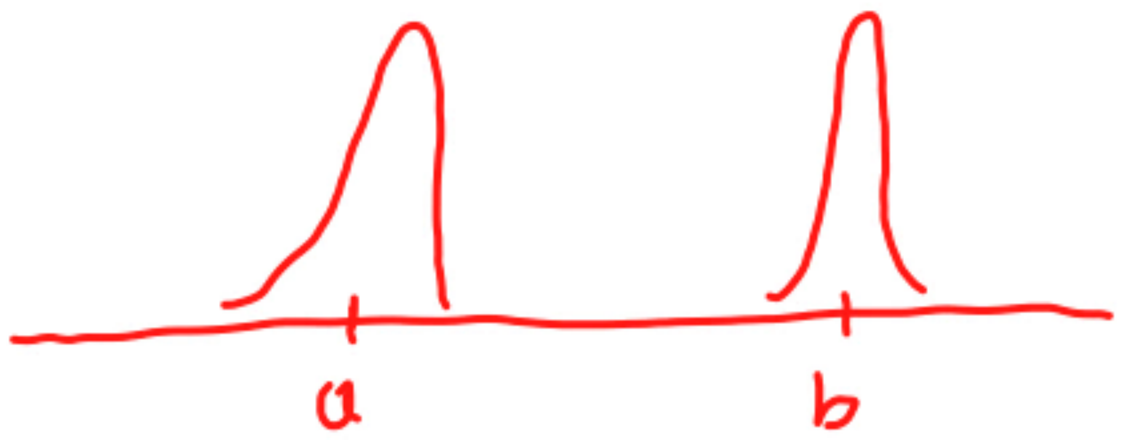
If $X_n \to a$ **and** $Y_n \to b$**, then** $X_n + Y_n \to a + b$ **(in probability)**



If $X_n \to a$ and $Y_n \to b$, then $X_n + Y_n \to a + b$ (in probability)

$\left.\begin{array}{l} a_n \to a \\ b_n \to b \end{array}\right\} \Rightarrow a_n + b_n \to a + b$

$|x+y| \le |x| + |y|$

$a_n \to a$: Fix $\varepsilon > 0$. There exists n_0 such that if $n \geq n_0$, then $|a_n - a| < \varepsilon/2$

$b_n \to b$: There exists some n_0' such that if $n \geq n_0'$, then $|b_n - b| < \varepsilon/2$

if $n \geq \max\{n_0, n_0'\}$ $\quad |\underline{a_n + b_n} - \underline{a - b}| \le |a_n - a| + |b_n - b|$

$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$

If $X_n \to a$ and $Y_n \to b$, then $X_n + Y_n \to a + b$ (in probability)

Fix some $\varepsilon > 0$

$$P(|X_n + Y_n - a - b| \geq \varepsilon) \overset{?}{\longrightarrow} 0$$

$$= P(|(X_n - a) + (Y_n - b)| \geq \varepsilon)$$

$$\leq P(|X_n - a| \geq \varepsilon/2 \text{ or } |Y_n - b| \geq \varepsilon/2)$$

$$\leq \underbrace{P(|X_n - a| \geq \varepsilon/2)}_{\substack{\to 0 \\ n\to\infty}} + \underbrace{P(|Y_n - b| \geq \varepsilon/2)}_{\substack{\to 0 \\ n\to\infty}} \xrightarrow[n\to\infty]{} 0.$$