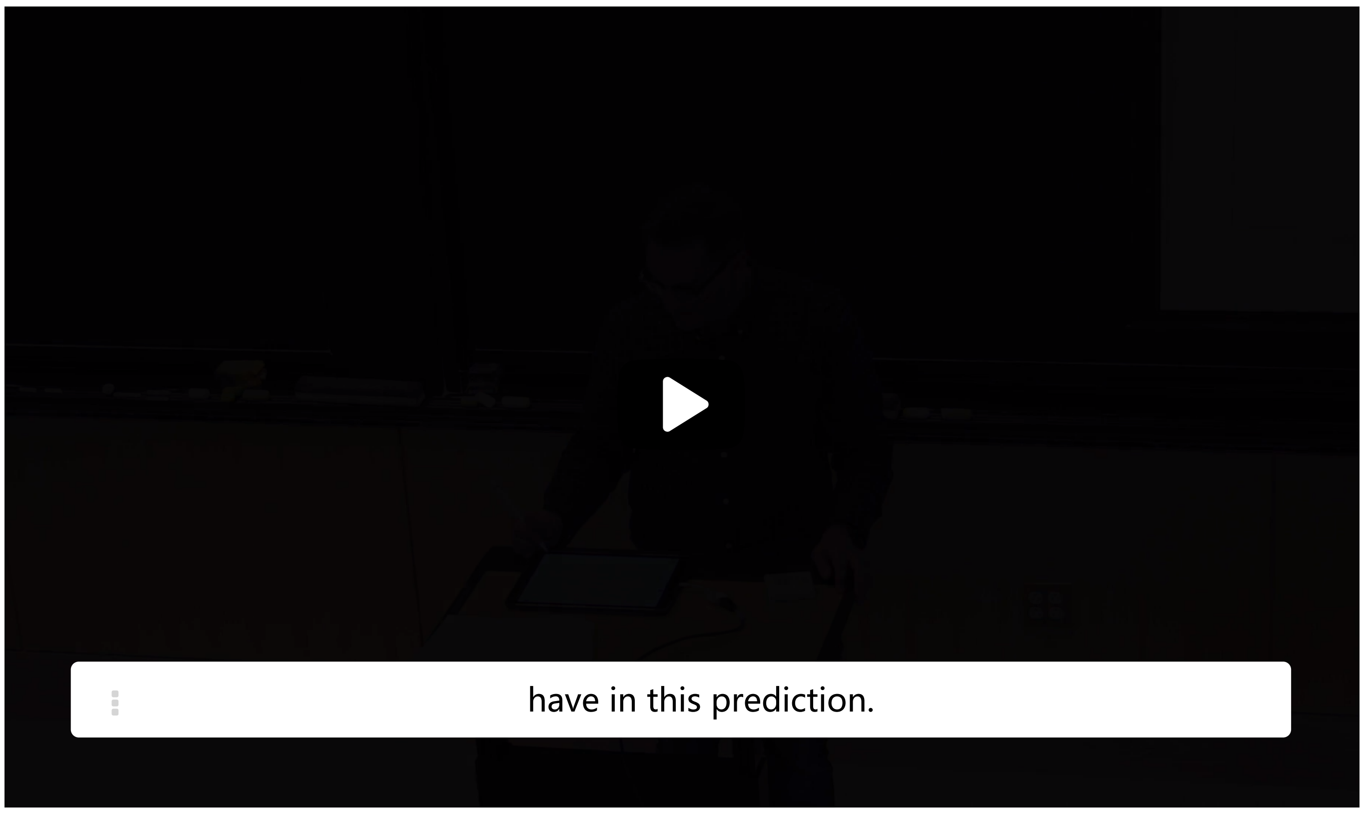
# 5. Partial Modeling, Regression Function, and Conditional Quantiles

## Partial Modeling, Regression Function, and Conditional Quantiles




7:59 / 7:59 1.0x

But you certainly don't want it to stand alone.

You want this number coming with some location number that's

telling you maybe the conditional mean

and the conditional variance or the conditional standard

deviation, OK?

So that you have an idea of how much variability you

**have in this prediction.**

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Given a joint probability distribution $\mathbf{P}$ for the random pair $(X, Y)$, the **regression function** of $Y$ with respect to $X$ is defined as

$$\nu(x) = \mathbb{E}[Y|X=x] = \sum_{\Omega_Y} y \cdot \mathbf{P}(Y = y \mid X = x)$$

which tells us the average value of $Y$ given the knowledge that $X = x$. In the case of continuous distributions where we can compute the conditional density $f(y|x)$, the expression on the right hand side is replaced with an integral:

$$\mathbb{E}[Y|X=x] = \int_{\Omega_Y} y f(y|x)\, dy$$

## A Linear Model

1/1 point (graded)

Assume $(X, Y)$ is a pair such that $Y = 3X + 5 + \varepsilon$ where $\varepsilon \sim \mathcal{N}(0,1)$, independent of $X$. What is $\mathbb{E}[Y|X=x]$?

3*x+5

**Answer:** 3*x+5

$3 \cdot x + 5$

STANDARD NOTATION

**Solution:**

From the definition:

$$\begin{aligned}\nu(x) &= \mathbb{E}[Y|X=x] \\ &= \mathbb{E}[3X+5+\varepsilon \mid X=x] \\ &= \mathbb{E}[3x+5+\varepsilon] \\ &= 3x+5+\mathbb{E}[\varepsilon] \qquad \text{(linearity of expectation)} \\ &= 3x+5.\end{aligned}$$

Linear models of the type $Y = a + bX + \varepsilon$ – hence the name **Linear Regression** – are the main focus of this chapter.

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Answers are displayed within the problem

**Note:** The notions of conditional expectation and conditional variance as random variables, $\mathbb{E}[Y|X]$ and **Var** $(Y \mid X)$, are not to be confused with the expectation and variance of $Y$ given $X = x$. In this lecture, we are interested in these quantites not as random variables, but as constants for each $X = x$.

## Concept Check: Conditional Quantile

1/1 point (graded)

Let $(X, Y)$ be a pair of RVs with joint density $f(x, y) = x + y$, over the sample space $\Omega = [0,1]^2$.

For a given $x$, what is the value $q_\alpha(x)$ such that $P[Y \le q_\alpha(x) | X = x] = 1 - \alpha$? That is, what is the conditional $(1-\alpha)$-quantile function (of $x$) of $Y|X = x$?

sqrt((x+1)^2 - alpha*(2*x

**Answer:** (1/2)*(-2*x + sqrt(4*x^2 + (1-alpha)*8*(x+1/2)))

$$\sqrt{(x+1)^2 - \alpha \cdot (2 \cdot x + 1)} - x$$

STANDARD NOTATION

**Solution:**

We know from a previous problem that for this joint distribution on $(X, Y)$, the conditional pdf $h(y|x), 0 \le x \le 1$ is given as

$$h(y|x) = \frac{x+y}{x+\frac{1}{2}}, \quad 0 \le y \le 1.$$

In order to find the $(1-\alpha)$-quantile value for each $x$, we need to solve for $z$ (hiding the dependency on $x$ for simplicity) in

$$\int_0^z \frac{x+y}{x+\frac{1}{2}} \mathrm{d}y = (1-\alpha),$$

from which we can obtain that $q_\alpha(x) = z = \frac{1}{2}\left(-2x + \sqrt{4x^2 + 8(1-\alpha)(x+0.5)}\right)$.

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Answers are displayed within the problem

## Discussion

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