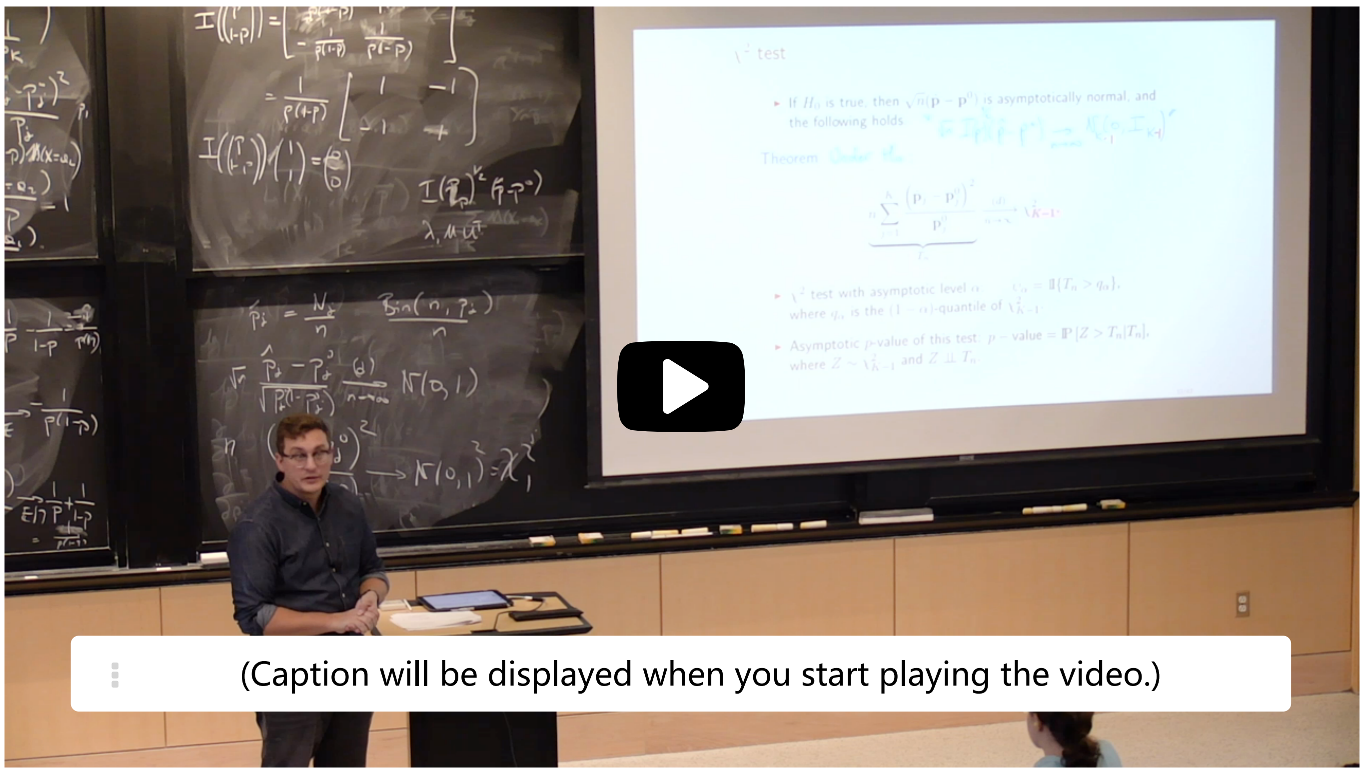
# 10. The Chi-Squared Test - A Few Thoughts

## The Correct Number of Degrees of Freedom Matters in the Chi-Squared Test

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So why is this important?

Because actually if you think about it, having a chi

squared with k degrees of freedom

is going to just give me my critical values associated

to this test.

So those are the critical values of a k squared

k minus 1 degrees of freedom.

They're just going to be larger.

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## The Chi-Squared Test for Two Modalities

1/1 point (graded)

**Note:** This problem is presented in the following video, but we encourage you to try it out (or think about it) before watching the video.

Consider the $\chi^2$ test statistic for $K = 2$:

$$T_n = n \sum_{j=1}^{2} \frac{\left(\hat{p}_j - p_j^0\right)^2}{p_j^0}.$$

We can use this statistic in a chi-squared test with $1$ degree of freedom to determine, with an asymptotic level $\alpha$, whether the observed iid samples follow the distribution $\mathrm{Ber}\left(p_2^0\right)$ under the null hypothesis $H_0$, with the sample space being the two values $a_1 = 0$ and $a_2 = 1$. The chi-squared test with asymptotic level $\alpha$ is

$$\mathbf{1}\left\{T_n > q_\alpha\right\},$$

where $q_\alpha$ is the $(1-\alpha)$-quantile of the chi-squared distribution with 1 degree of freedom.

Is the following statement true or false? "This test is identical (asymptotically) to Wald's test of the Bernoulli statistical model with parameter $p$, null hypothesis $H_0 : p = p_2^0$ and alternative hypothesis $H_1 : p \neq p_2^0$, where $p_2^0$, as defined above, is the probability of $a_2 = 1$ under the null hypothesis."

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True ✔

False

**Solution:**

The answer is true. Wald's test in the above statement is:

$$\mathbf{1}\left\{n\frac{\left(\hat{p}_2-p_2^0\right)^2}{p_2^0\left(1-p_2^0\right)}>q_\alpha\right\},$$

where $q_\alpha$ is the $(1-\alpha)$-quantile of the chi-squared distribution with 1 degree of freedom. The chi-squared test statistic can be re-written as:

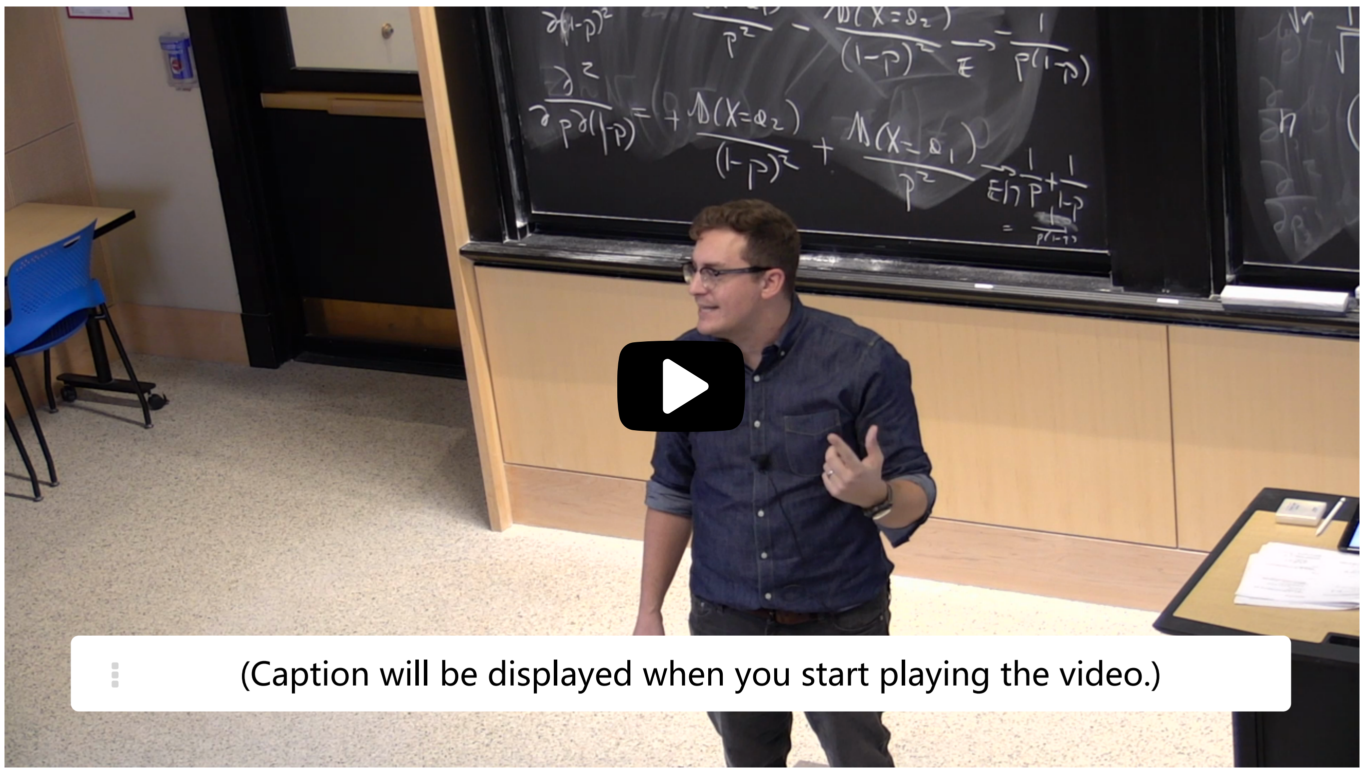
$$\begin{aligned}
T_n &= n\sum_{j=1}^{2}\frac{\left(\hat{p}_j-p_j^0\right)^2}{p_j^0}\\
&= n\frac{\left(\hat{p}_1-p_1^0\right)^2}{p_1^0}+n\frac{\left(\hat{p}_2-p_2^0\right)^2}{p_2^0}\\
&= n\frac{\left((1-\hat{p}_2)-(1-p_2^0)\right)^2}{1-p_2^0}+n\frac{\left(\hat{p}_2-p_2^0\right)^2}{p_2^0}\\
&= n\frac{\left(\hat{p}_2-p_2^0\right)^2\left(p_2^0+1-p_2^0\right)}{p_2^0\left(1-p_2^0\right)}\\
&= n\frac{\left(\hat{p}_2-p_2^0\right)^2}{p_2^0\left(1-p_2^0\right)},
\end{aligned}$$

which is the same as the test statistic for Wald's test.

Submit   You have used 1 of 1 attempt

Answers are displayed within the problem

## Chi-Squared Test for Two Modalities

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Just like Walt's test--

when I had a one dimensional parameter--

was the same as the test that we did with the Gaussian, right?

We had an absolute value that became a square.

And we had a critical value that became

the square of a critical value.

Then we're going to have the same thing

here, if k is equal to 2.