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1. Points and Vectors

Preamble

You will need to have basic knowledge of probability theory, and a sound foundation of multivariable calculus and linear algebra to follow along in this course. There will be no review of these subjects beyond this unit.

Homework 0 consists of some warmup exercises for the beginning of the course. It does **not** cover all background material you will need. Nonetheless, it can serve as rough guide on how ready you are: if you need to struggle much in these exercises, then you will need to spend substantial amount of time catching up on background material.

The following topics will allow you to understand part of the course material better, although not strictly required:

- Eigenvalues, eigenvectors, and spectral decomposition (linear algebra)
- Lagrange multipliers (multivariable calculus).

Homework 0 is **due Wednesday June 19 23:59 UTC . Please note the UTC time zone and find the corresponding time at your location.** Note that CST on EdX is China Standard Time, not Central Standard Time.

A list of n numbers can be thought of as a point or a vector in n-dimensional space. In this course, we will think of n-dimensional vectors $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$ flexibly as points and as vectors.

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1. (a)

3/3 points (graded)

Note on notation: In this course, we will use regular letters as symbols for numbers, vectors, matrices, planes, hyperplanes, etc. You will need to distinguish what a letter represents from the context.

Recall the dot product of a pair of vectors a and b:

$$a \cdot b \;=\; a_1 b_1 + a_2 b_2 + \cdots + a_n b_n \qquad \text{where } a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \text{ and } b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}.$$

When thinking about a and b as vectors in n-dimensional space, we can also express the dot product as

$$a \cdot b \;=\; \|a\| \|b\| \cos \alpha,$$

where α is the angle formed between the vectors a and b in n-dimensional Euclidean space. Here, $\|a\|$ refers to the length, also known as **norm**, of a:

$$\|a\| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}.$$

What is the length of the vector $\begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}$?

0.5 ✔ **Answer:** 0.5

What is the length of the vector $\begin{bmatrix} -0.15 \\ 0.2 \end{bmatrix}$?

0.25

✔ **Answer:** 0.25

What is the angle (in radians) between $\begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}$ and $\begin{bmatrix} -0.15 \\ 0.2 \end{bmatrix}$? Choose the answer that lies between 0 and π.

(Type **pi** for the constant π.)

pi/2

✔ **Answer:** pi/2

STANDARD NOTATION

Solution:

- Plugging into the equation for norm, we get that the length of $\begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}$ is equal to $\sqrt{0.4^2 + 0.3^2} = 0.5$. Notice that the ratio of x:y is 3:4 so we can use 3:4:5 triangle to speed up our calculation to find the length of the vector.
- We do the same for $\begin{bmatrix} -0.15 \\ -0.2 \end{bmatrix}$.
- Using the second expression for dot product and rearranging, we get $\alpha = \cos^{-1} \frac{x \cdot y}{\|x\|\|y\|}$. Using the first expression for dot product and plugging it in we get that $\alpha = \cos^{-1} \frac{(0.4)(-0.15)+(0.3)(0.2)}{\sqrt{(0.4)^2+(0.3)^2}\sqrt{(-0.15)^2+(0.2)^2}}$

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Answers are displayed within the problem

1. (b)

1/1 point (graded)

Given 3-dimensional vectors $x^{(1)} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $x^{(2)} = \begin{bmatrix} a_1 \\ -a_2 \\ a_3 \end{bmatrix}$, when is $x^{(1)}$ orthogonal to $x^{(2)}$, i.e. the angle between them is $\pi/2$?

- when $2a_1 + 2a_3 = 0$
- when $a_1^2 - a_2^2 + a_3^2 = 0$ ✔
- when $a_1^2 + a_2^2 + a_3^2 = 0$

STANDARD NOTATION

Solution:

Based on the previous equations for the dot product, we find that the angle between $x^{(1)}$ and $x^{(2)}$ is:

$$\alpha = \cos^{-1} \frac{x^{(1)} \cdot x^{(2)}}{\|x^{(1)}\|\|x^{(2)}\|}$$

$$\alpha = \cos^{-1} \frac{a_1^2 - a_2^2 + a_3^2}{a_1^2 + a_2^2 + a_3^2}$$

$x^{(1)}$ is orthogonal to $x^{(2)}$ when $x^{(1)} \cdot x^{(2)} = 0$ or $a_1^2 - a_2^2 + a_3^2 = 0$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

1. (c)

1.0/1 point (graded)
A unit vector is a vector with length, or norm, 1. Given any vector x, what is the unit vector pointing in the same direction as x?

(Enter **norm(x)** for the norm $\|x\|$ of the vector x. In general, you can enter the norm of a vector using the norm function).

x/norm(x) ✔ **Answer:** x/norm(x)

STANDARD NOTATION

Solution:

We need to scale the vector x so that it is length 1. Right now it is length $\|x\|$ so we need to divide the vector x by $\|x\|$ in order to get the unit vector which points in the same direction.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

1. (d)

3.0/3 points (graded)
Recall from linear algebra the definition of the projection of one vector onto another. As before, we have 3-dimensional vectors $x^{(1)} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $x^{(2)} = \begin{bmatrix} a_1 \\ -a_2 \\ a_3 \end{bmatrix}$.

Which of these vectors is the direction of the projection of $x^{(1)}$ onto $x^{(2)}$?

- ○ $x^{(1)}$
- ◉ $x^{(2)}$ ✔
- ○ $x^{(1)} + x^{(2)}$

What is the signed magnitude c of the projection $p_{x^{(1)}\to x^{(2)}}$ of $x^{(1)}$ onto $x^{(2)}$? More precisely, let u be the unit vector in the direction of the correct choice above, find a number c such that $p_{x^{(1)}\to x^{(2)}} = cu$.

Express your answer in terms of **a_1** for a_1, **a_2** for a_2, and **a_3** for a_3.

$c =$ (a_1^2-a_2^2+a_3^2)/sqrt(a_1^2+a_2^2+a_3^2) ✔ **Answer:** (a_1^2-a_2^2+a_3^2)/sqrt(a_1^2+a_2^2+a_3^2)

$$\frac{a_1^2-a_2^2+a_3^2}{\sqrt{a_1^2+a_2^2+a_3^2}}$$

What is the (vector) projection $p_{x^{(1)}\to x^{(2)}}$ of $x^{(1)}$ onto $x^{(2)}$? Express your answer in terms of the signed magnitude c, $x^{(1)}$, $\|x^{(1)}\|$, $x^{(2)}$, $\|x^{(2)}\|$.

(Enter **x1** for $x^{(1)}$, **norm(x1)** for $\|x^{(1)}\|$, **x2** for $x^{(2)}$, and **norm(x2)** for $\|x^{(2)}\|$. Do not worry if the parser does not display properly, the grader will work independently. For proper display, enclose **x1** and **x2** by brackets, e.g. **(x1)** .)

(*Input update*: In the last question, the grader has been updated to accept answers in terms of c instead of norm(p). Earlier correct answers will remain correct.)

$p_{x^{(1)} \to x^{(2)}} =$ c*x2/norm(x2) **Answer:** c*(x2)/(norm(x2))

STANDARD NOTATION

Solution:

- The definition of the projection of one vector onto another is the part of the first vector which points in the same direction as the second vector. Thus the projection of $x^{(1)}$ onto $x^{(2)}$ points in the direction of $x^{(2)}$
- The vector has magnitude $\|x^{(1)}\| \cos\alpha$. From our previous result $\alpha = \cos^{-1} \frac{x^{(1)} \cdot x^{(2)}}{\|x^{(1)}\| \|x^{(2)}\|}$, the projection thus has magnitude $\frac{x^{(1)} \cdot x^{(2)}}{\|x^{(2)}\|}$. Plugging in our values for $x^{(1)}$ and $x^{(2)}$ we get $\frac{a_1^2 - a_2^2 + a_3^2}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$.
- To find the final vector projection, we scale the unit vector in the direction of the vector projection, which is $\frac{x^{(2)}}{\|x^{(2)}\|}$ by the length, $\|p_{x^{(1)} \to x^{(2)}}\|$. So the answer is $\|p_{x^{(1)} \to x^{(2)}}\| \frac{x^{(2)}}{\|x^{(2)}\|}$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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