(a) We have, from the law of iterated expectations and the fact $\mathbf{E}[X_i \mid Q] = Q$,

$$\mathbf{E}[X_i] = \mathbf{E}\big[\mathbf{E}[X_i \mid Q]\big] = \mathbf{E}[Q] = \mu.$$

Since $X = X_1 + \cdots + X_n$, it follows that

$$\mathbf{E}[X] = \mathbf{E}[X_1] + \cdots + \mathbf{E}[X_n] = n\mu.$$

(b) We have, for $i \neq j$, using the conditional independence assumption,

$$\mathbf{E}[X_i X_j \mid Q] = \mathbf{E}[X_i \mid Q]\,\mathbf{E}[X_j \mid Q] = Q^2,$$

and

$$\mathbf{E}[X_i X_j] = \mathbf{E}\big[\mathbf{E}[X_i X_j \mid Q]\big] = \mathbf{E}[Q^2].$$

Thus,

$$\text{cov}(X_i, X_j) = \mathbf{E}[X_i X_j] - \mathbf{E}[X_i]\,\mathbf{E}[X_j] = \mathbf{E}[Q^2] - \mu^2 = \sigma^2.$$

Since $\text{cov}(X_i, X_j) > 0$, $X_1, \ldots, X_n$ are not independent.

Also, for $i = j$, using the observation that $X_i^2 = X_i$,

$$\begin{aligned}
\text{var}(X_i) &= \mathbf{E}[X_i^2] - \big(\mathbf{E}[X_i]\big)^2 \\
&= \mathbf{E}[X_i] - \big(\mathbf{E}[X_i]\big)^2 \\
&= \mu - \mu^2.
\end{aligned}$$

(c) Using the law of total variance, and the conditional independence of

$X_1, \ldots, X_n$, we have

$$\begin{aligned}
\mathrm{var}(X) &= \mathbf{E}\big[\mathrm{var}(X \mid Q)\big] + \mathrm{var}\big(\mathbf{E}[X \mid Q]\big) \\
&= \mathbf{E}\big[\mathrm{var}(X_1 + \cdots + X_n \mid Q)\big] + \mathrm{var}\big(\mathbf{E}[X_1 + \cdots + X_n \mid Q]\big) \\
&= \mathbf{E}\big[nQ(1-Q)\big] + \mathrm{var}(nQ) \\
&= n\mathbf{E}[Q - Q^2] + n^2\mathrm{var}(Q) \\
&= n(\mu - \mu^2 - \sigma^2) + n^2\sigma^2 \\
&= n(\mu - \mu^2) + n(n-1)\sigma^2.
\end{aligned}$$

To verify the result using the covariance formulas of part (b), we write

$$\begin{aligned}
\mathrm{var}(X) &= \mathrm{var}(X_1 + \cdots + X_n) \\
&= \sum_{i=1}^{n} \mathrm{var}(X_i) + \sum_{\{(i,j) \mid i \neq j\}} \mathrm{cov}(X_i, X_j) \\
&= n\mathrm{var}(X_1) + n(n-1)\mathrm{cov}(X_1, X_2) \\
&= n(\mu - \mu^2) + n(n-1)\sigma^2.
\end{aligned}$$