11. Exercise: The effect of a stronger signal

Exercise: The effect of a stronger signal

1/1 point (graded)

For the model $X = \Theta + W$, and under the usual independence and normality assumptions for Θ and W, the mean squared error of the LMS estimator is

$$\frac{1}{(1/\sigma_0^2) + (1/\sigma_1^2)},$$

where σ_0^2 and σ_1^2 are the variances of Θ and W, respectively.

Suppose now that we change the observation model to $Y = 3\Theta + W$. In some sense the “signal" Θ has a stronger presence, relative to the noise term W, and we should expect to obtain a smaller mean squared error. Suppose $\sigma_0^2 = \sigma_1^2 = 1$. The mean squared error of the original model $X = \Theta + W$ is then $1/2$. In contrast, the mean squared error of the new model $Y = 3\Theta + W$ is

1/10

✔ **Answer:** 0.1

Hint: Do not solve the problem from scratch. Think of an alternative observation model in which you observe $Y' = \Theta + (W/3)$.

Solution:

Since Y' is just Y scaled by a factor of $1/3$, Y' carries the same information as Y, so that $\mathbf{E}[\Theta \mid Y] = \mathbf{E}[\Theta \mid Y']$. Thus, the alternative observation model $Y' = \Theta + (W/3)$ will lead to the same estimates and will have the same mean squared error as the unscaled model $Y = 3\Theta + W$. In the equivalent Y' model, we have a noise variance of $1/9$ and therefore the mean squared error is

$$\frac{1}{\frac{1}{1} + \frac{1}{1/9}} = \frac{1}{10}. \quad = \frac{1}{\frac{1}{3^2}+1} \Big/ 3^2$$

提交

You have us

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5 days ago

You can also by-pass the hint by just considering $\Theta' = 3\Theta$, and interpreting $Y = \Theta' + W$. Now the variance of Θ' is simply the Θ's multiplied by 9, and you are good to apply the previous formula.

Edit: Yes, apparently I've made a mistake here. Since we are trying to estimate Θ, as opposed to 3Θ, the value I've suggest should also be divided by 9 (there is an extra step of estimation, from $\Theta' \to \Theta$), admittedly the best way is to do it through the hint.