MITx Video | 6.041x | Spring 2014 | Video transcript | L20-2: The sample mean and some terminology

Let us now discuss a little bit the simplest estimation problem that there is, the problem of estimating the mean of a certain probability distribution, and we will take this occasion to introduce some additional terminology and discuss some desirable properties of estimators.

So the context is as follows. We have n random variables that are independent, and they're identically distributed. They are drawn from some distribution that has a certain mean theta and some variance. We assume that we do not know the value of the mean, and we want to estimate it. The most natural way of estimating the mean is to form the sample mean, that is, we take the n observations and take their average.

Notice, that this quantity, the sample mean or, in this case, it is the estimator that we're using, is a random variable because its value is determined by the values of the random variables X1 up to Xn. Let us discuss some properties of this estimator. The first property is that the expected value of this estimator is equal to the true mean. This is because the expected value of each one of the Xs is theta, and therefore, the expected value of this ratio is theta as well. Now, this is a relation that's true for all possible values of theta.

Let us appreciate the content of this statement. Let us think what this expectation actually is. More generally, suppose that we're dealing with some estimator, which is some function of the data. Then, the expected value of this estimator is using the expected value rule, and assuming that we're dealing with a discrete random variable X, the expected value of theta hat is determined as follows.

And so we see that the expected value for estimator depends, or is affected, by what the true value of theta is. So this is a quantity that generally depends on theta. And what we want in order to have a so-called unbiased estimator is that no matter what theta is, this expectation evaluates to the true unknown value equal to theta.

In general, having this property, having an unbiased estimator, is a desirable one. We do not want our estimates to be systematically high or systematically low, no matter what the true value of theta is. A second property of the sample mean estimator is the following. By the weak law of large numbers, we know that the sample mean converges to the true mean in the sense of convergence in probability.

Once more, this is a property that's true, no matter what the underlying unknown value little theta is. When this is true, this convergence is true, for all values of little theta, then we say that our estimator is consistent or that we have consistency. Having a consistent estimator is definitely a very desirable property. We would like, when we obtain more and more data, that our estimator will give us the correct value.

Finally, we would like to say something about the size of the estimation error. This is measured-- one way of measuring it, but which is the most common, it's measured in terms of the mean squared error. So theta is the unknown value. This is our estimator. This is the error. We square the error, and we take the average. What we've got here for this specific example of the sample mean estimator is the following. Since it is unbiased, we have a random variable minus the mean of that random variable, so this is just the variance of the estimator.

And for the sample mean, we know that its variance is sigma squared over n. So this gives us some very specific knowledge about how the mean squared error behaves as we change n. In this particular example, the mean squared error did not depend on theta. It's the same no matter what the true theta is. But in other situations and with other estimators, you might actually obtain here a function of theta.