We will show that $\mathbf{P}(|X_n + Y_n - a - b| \geq \epsilon)$ converges to zero, for any $\epsilon > 0$. To bound this probability, we note that for $|X_n + Y_n - a - b|$ to be as large as ϵ, we need either $|X_n - a|$ or $|Y_n - b|$ (or both) to be at least $\epsilon/2$. Therefore, in terms of events, we have

$$\big\{|X_n + Y_n - a - b| \geq \epsilon\big\} \subset \big\{|X_n - a| \geq \epsilon/2\big\} \cup \big\{|Y_n - b| \geq \epsilon/2\big\}.$$

This implies, using also the union bound, that

$$\mathbf{P}\big(|X_n + Y_n - a - b| \geq \epsilon\big) \leq \mathbf{P}\big(|X_n - a| \geq \epsilon/2\big) + \mathbf{P}\big(|Y_n - b| \geq \epsilon/2\big),$$

and

$$\lim_{n\to\infty} \mathbf{P}\big(|X_n + Y_n - a - b| \geq \epsilon\big) \leq \lim_{n\to\infty} \mathbf{P}\big(|X_n - a| \geq \epsilon/2\big) + \lim_{n\to\infty} \mathbf{P}\big(|Y_n - b| \geq \epsilon/2\big) = 0,$$

where the last equality follows since X_n and Y_n converge, in probability, to a and b, respectively.