

1. The PDF of the logarithm of X

Problem 1. The PDF of the logarithm of X

7/7 points (graded)

Let \mathbf{X} be a non-negative random variable. Find the PDF of the random variable $\mathbf{Y} = \ln \mathbf{X}$ for each of the following cases:

1. For general f_X , $f_Y(y) =$

☒ $f_X(e^y) e^y$ ✓

☐ $\frac{f_X(e^y)}{e^y}$

☐ $\frac{f_X(\ln y)}{y}$

☐ none of the above

2. When $f_X(x) = \begin{cases} 1/4, & \text{if } 2 < x \leq 6, \\ 0, & \text{otherwise,} \end{cases}$

we have $f_Y(y) = \begin{cases} g(y), & \text{if } a < y \leq b, \\ 0, & \text{otherwise.} \end{cases}$

Give a formula for $g(y)$ and the values of a and b using standard notation.

$g(y) =$ ✓ Answer: $(e^y)/4$

$a =$ ✓ Answer: 0.6931

$b =$ ✓ Answer: 1.791

3.

When $f_X(x) = \begin{cases} 2(x-1), & \text{if } 1 < x \leq 2, \\ 0, & \text{otherwise,} \end{cases}$

we have $f_Y(y) = \begin{cases} g(y), & \text{if } a < y \leq b, \\ 0, & \text{otherwise.} \end{cases}$

Give a formula for $g(y)$, and the values of a and b , using standard notation.

$g(y) =$

$2 \cdot (e^y - 1) \cdot e^y$

✓ Answer: $2 \cdot \exp(2 \cdot y) - 2 \cdot \exp(y)$

$2 \cdot (e^y - 1) \cdot e^y$

$a =$

$\ln(1)$

✓ Answer: 0

$b =$

$\ln(2)$

✓ Answer: 0.6931

STANDARD NOTATION

Solution:

1. $f_Y(y) = f_X(e^y)e^y$. Note that $F_Y(y) = \mathbb{P}(Y \leq y) = \mathbb{P}(\ln X \leq y) = \mathbb{P}(X \leq e^y) = F_X(e^y)$. Differentiating both sides with respect to y and using the chain rule, we obtain

$$f_Y(y) = f_X(e^y)e^y.$$

2. For X between 2 and 6, $Y = \ln(X)$ takes values between $\ln(2)$ and $\ln(6)$. By applying the formula found in the previous part, we obtain

$$f_Y(y) = \begin{cases} \frac{e^y}{4}, & \text{if } \ln 2 < y \leq \ln 6, \\ 0, & \text{otherwise.} \end{cases}$$

3. For X between 1 and 2, $Y = \ln(X)$ takes values between 0 and $\ln(2)$. By applying the formula in part (1), we obtain

$$f_Y(y) = \begin{cases} 2e^{2y} - 2e^y, & \text{if } 0 < y \leq \ln 2, \\ 0, & \text{otherwise.} \end{cases}$$

提交

You have used 2 of 5 attempts