

In this unit, in a sequence of three lectures, we discuss limits theorems and their uses and conclude with a brief introduction to classical statistics. One reason for our interest in limit theorems is the following. If you're dealing with a very small number of random variables, you can usually calculate quantities of interest either analytically or on a computer. But if the number of random variables is quite a large, let's say 100, calculations can be very difficult. However, the situation often becomes, again, very simple if you take the limit as the number of random variables goes to infinity.

In the first two lectures of this unit, we develop two basic limit theorems. The first one, the weak law of large numbers, tells us that the observed average of a large number of independent identically distributed random variables, which is called the sample mean, converges, in some sense, to the expected value. The second one, the central limit theorem, gives us a lot more information. It provides us with the details of the distribution of the sum of  $n$  independent identically distributed random variables.

We will go through lots of examples to see how these limit theorems are used with a focus on the problem of estimating the unknown mean of a certain distribution and the accuracy of the estimates. Notice that this is a different kind of estimation problem than those that we dealt with in our earlier study of inference. How is it different?

We make no assumptions on the unknown quantity that is being estimated. It is not modeled as a random variable but just as an unknown constant. And we cannot invoke the Bayes rule. We are, therefore, within a very different conceptual setting, the setting of non-Bayesian, or as often called classical statistics.

In the third lecture of this unit we will take a very quick tour through some basic elements of classical statistics. We will look into estimates that rely on the calculation of sample means and also introduce a method of general applicability. Of course, classical statistics is by itself an entire field on which you can take multiple courses. But our objective will be rather limited, to introduce the conceptual framework and maybe whet your appetite to go on to further study.