2. Two important probability tools

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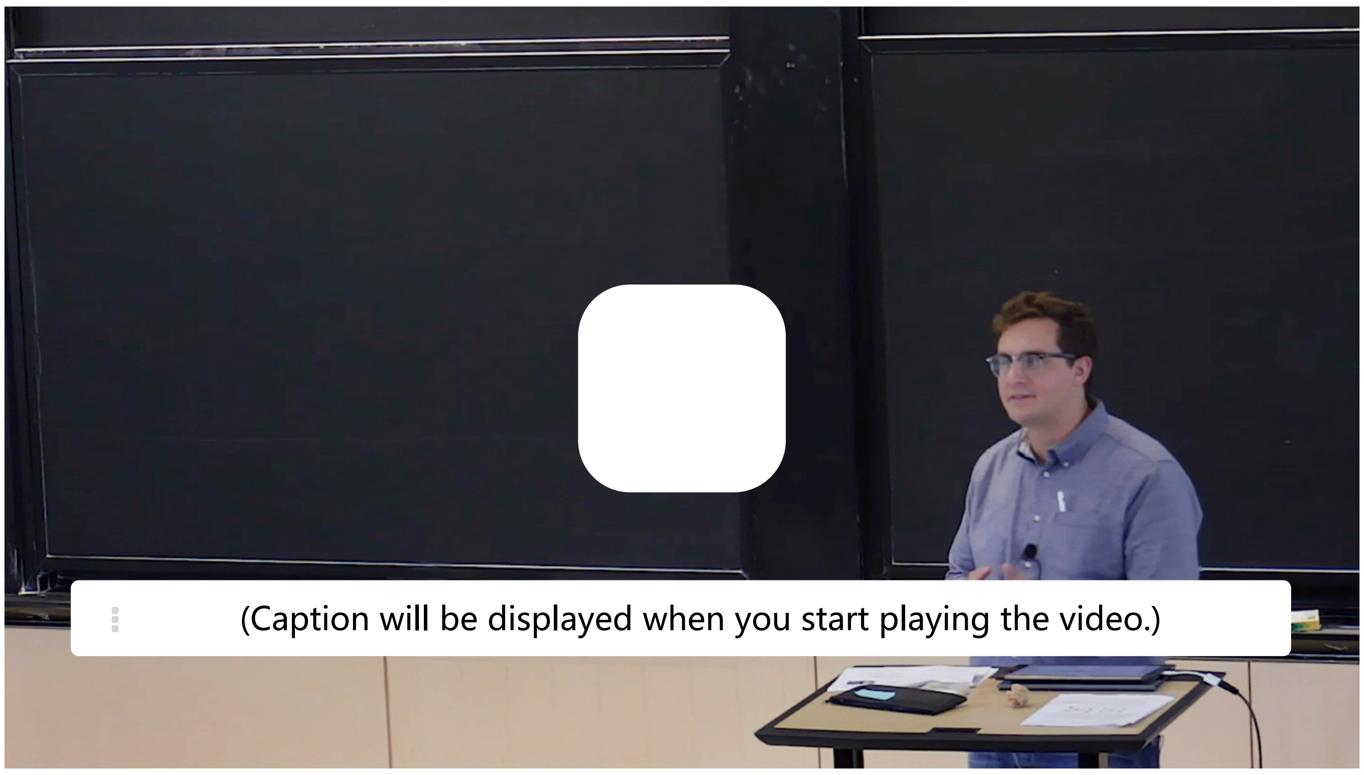

All right.

So welcome back.

So again, this is a statistics class,

and hopefully, I convinced you last time,

probability is an essential part of statistics.

We have, on the one hand, the truth, a stochastic process,

a data generating process, that is generating

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Average of Gaussians

2/3 points (graded)

Let $X_1, X_2, \ldots, X_n$ be i.i.d. **standard normal random variables**. What is the distribution of

$$\overline{X}_n = \frac{X_1 + X_2 + \cdots + X_n}{n}?$$

- (selected) A Gaussian. ✔
- A χ^2-distribution.
- Cannot be determined for finite n, but asymptotically Gaussian.

In terms of n, what are the variance and mean of $\overline{X}_n$?

$\mathrm{Var}\left(\overline{X}_n\right) =$ 1/sqrt(n) ✖ Answer: 1/n

$\frac{1}{\sqrt{n}}$

$\mathbb{E}\left[\overline{X}_n\right] =$ 0 ✔ Answer: 0

0

if independence

$Var(X_1+X_2+ .. X_n) = Var(X_1) + Var(X_2) + \cdots Var(X_n)$

CLT $\Rightarrow$ $X_1+X_2+ .. X_n \sim$ Normal(,

$X_1+X_2+ \ldots X_n \sim N(0, n)$

$= n\bar{X}_n$

$Var(n\bar{X}_n) = n$

$\frac{1}{n^2} \cdot Var(n\bar{X}_n) = \frac{1}{n^2} \cdot n$

$Var(\bar{X}_n) = \frac{1}{n}$.

Solution:

Since the sum of i.i.d. Gaussian random variables is also Gaussian, we deduce first that $X_1 + \cdots + X_n \sim N(0, n)$. Multiplying by $1/n$, we get $\overline{X}_n \sim N(0, 1/n)$, as scaling a random variable with a constant c scales its variance by c^2.

Therefore, $\overline{X}_n$ is a Gaussian random variable with mean 0 and variance $1/n$.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

CLT Concept Check

1/1 point (graded)

Let $X_1, X_2, \ldots, X_n$ be an i.i.d. sequence of random variables with $\mathbb{E}[X] = \mu$, and, $\mathsf{Var}(X) = \sigma^2$. Assuming that n is very large, according to the Central Limit Theorem, what is the best approximate characterization of the distribution of $\overline{X}_n$?

- $N(0, 1)$.
- $N(\mu, \sigma^2/n)$. ✔
- $N(0, \sigma^2/n)$.
- Depends on the distribution of X.

Solution:

The correct choice is the second choice. We know by the Central Limit Theorem that

$$\frac{\sqrt{n}\,(\overline{X}_n - \mu)}{\sigma} \longrightarrow N(0, 1)$$

in distribution. Therefore, we can use approximate normality (as stated in the problem preamble), and get

$$\overline{X}_n \approx N(\mu, \sigma^2/n).$$

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

讨论

主题: Unit 1 Introduction to statistics:Lecture 2: Probability Redux / 2. Two important probability tools

显示讨论

认证证书是什么?