2. Three-sided dice

Problem 2. Three-sided dice

9/9 points (graded)

We have two fair three-sided dice, indexed by $i = 1, 2$. Each die has sides labelled 1, 2, and 3. We roll the two dice independently, one roll for each die. For $i = 1, 2$, let the random variable X_i represent the result of the ith die, so that X_i is uniformly distributed over the set $\{1, 2, 3\}$. Define $X = X_2 - X_1$.

1. Calculate the numerical values of following probabilities, as well as the expected value and variance of X:

$\mathbf{P}(X = 0) =$ 1/3 Answer: 1/3

$\mathbf{P}(X = 1) =$ 2/9 Answer: 2/9

$\mathbf{P}(X = -2) =$ 1/9 Answer: 1/9

$\mathbf{P}(X = 3) =$ 0 Answer: 0

$\mathbf{E}[X] =$ 0 Answer: 0

$\mathsf{Var}(X) =$ 4/3 Answer: 4/3

2. Let $Y = X^2$. Calculate the following probabilities:

$\mathbf{P}(Y = 0) =$ 1/3 Answer: 1/3

$\mathbf{P}(Y = 1) =$ 4/9 Answer: 4/9

$\mathbf{P}(Y = 2) =$ 0 Answer: 0

Solution:

1. The sample space for the pair (X_1, X_2) has 9 equally likely outcomes. For each possible value x of X, we count the number of outcomes for which the difference $X_2 - X_1$ equals x, then multiply by $1/9$ to obtain $p_X(x)$.

$$p_X(x) = \begin{cases} 1/9, & x = -2 \text{ or } 2, \\ 2/9, & x = -1 \text{ or } 1, \\ 3/9, & x = 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathbf{E}[X] = \sum_{x=-2}^{2} x p_X(x) = (-2) \cdot \frac{1}{9} + (-1) \cdot \frac{2}{9} + (0) \cdot \frac{3}{9} + (1) \cdot \frac{2}{9} + (2) \cdot \frac{1}{9} = 0$$

We can also see that $\mathbf{E}[X] = 0$ because the PMF is symmetric around 0, or because $\mathbf{E}[X_1] = \mathbf{E}[X_2]$, so that $\mathbf{E}[X] = \mathbf{E}[X_2 - X_1] = \mathbf{E}[X_2] - \mathbf{E}[X_1] = 0$.

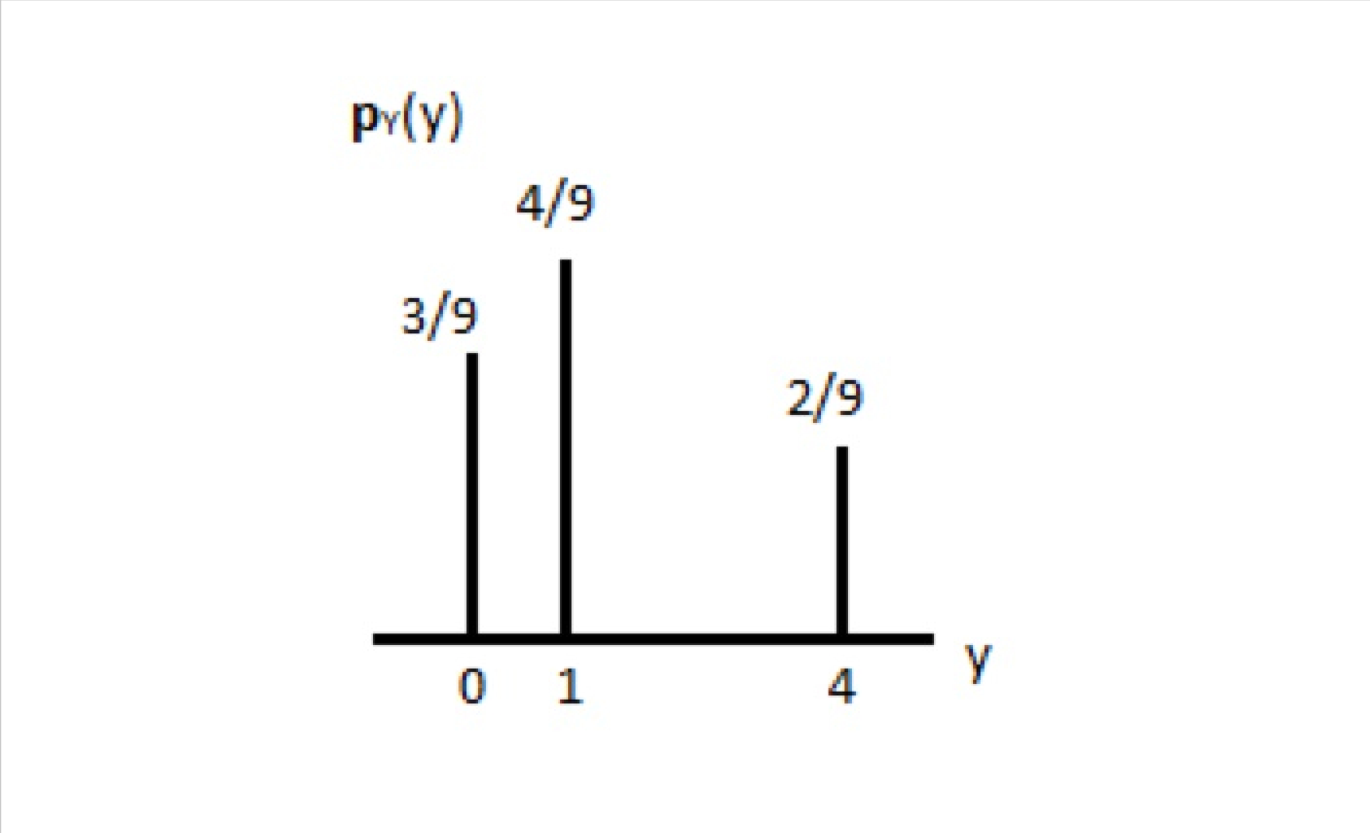
To find the variance of X, we note that $\mathsf{Var}(X) = \mathbf{E}[(X - \mathbf{E}[X])^2] = \mathbf{E}[X^2]$, and so

$$\mathbf{E}[X^2] = \sum_{x=-2}^{2} x^2 p_X(x) = 4 \cdot \frac{1}{9} + 1 \cdot \frac{2}{9} + 0 \cdot \frac{3}{9} + 1 \cdot \frac{2}{9} + 4 \cdot \frac{1}{9} = \frac{4}{3}.$$

2. Let $Y = X^2$. By matching the possible values of X and their probabilities to the possible values of Y, we obtain

$$p_Y(y) = \begin{cases} 2/9, & y = 4, \\ 4/9, & y = 1, \\ 3/9, & y = 0, \\ 0, & \text{otherwise.} \end{cases}$$

A plot of the PMF of Y is shown below:



提交 You have used 2 of 3 attempts

Answers are displayed within the problem

讨论

显示讨论

Topic: Unit 4 / Problem Set / 2. Three-sided dice

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