14. Exercise: The time of the kth arrival

Exercise: The time of the kth arrival

2/2 points (graded)

Let Y_k be the time of the kth arrival in a Poisson process with parameter $\lambda = 1$. In particular, $\mathbf{E}[Y_k] = k$.

Is it true that $\mathbf{P}(Y_k \geq k) = 1/2$ for any finite k?

No ▾ ✔ **Answer:** No

Is it true that $\lim_{k\to\infty} \mathbf{P}(Y_k \geq k) = 1/2$?

Yes ▾ ✔ **Answer:** Yes

Solution:

Consider the special case of $k = 1$. Then, $\mathbf{P}(Y_1 \geq 1) = e^{-1} \neq 1/2.$

When k is large, the central limit theorem applies because Y_k is the sum of k i.i.d. (exponential) random variables. Its (standardized) distribution is approximately normal, hence approximately symmetric around its mean. More formally, using the fact that the variance of an exponential with parameter 1 is 1, we have

$$\lim_{k\to\infty} \mathbf{P}(Y_k \geq k) = \lim_{k\to\infty} \mathbf{P}\left(\frac{Y_k - k}{\sqrt{k}} \geq 0\right) = \Phi(0) = \frac{1}{2},$$

where Φ is is the standard normal CDF.

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

讨论

主题： Unit 9 / Lec. 22 / 14. Exercise: The time of the kth arrival

显示讨论