

14. Exercise: Natural estimators

Exercise: Natural estimators

3/3 points (graded)

The random variables X_i are i.i.d. and satisfy $\mathbf{E}[X_i^2] = \theta$. Use a natural estimator to calculate an estimate of θ based on the values $X_1 = 1, X_2 = 3, X_3 = -1, X_4 = 2, X_5 = 0$.

✓ Answer: 3

In order to calculate confidence intervals around your estimator, you need information on the variance of your estimator. This variance is determined by $\mathbf{E}[X_i^2]$ and $\mathbf{E}[X_i^a]$ for some other power a . What is the value of a ?

$a =$

✓ Answer: 4

If you do not have any prior knowledge about the value of $\mathbf{E}[X_i^a]$, can you estimate it based on the available data?

Yes



✓ Answer: Yes

Solution:

A natural estimator is

$$\frac{1}{5} \sum_{i=1}^5 X_i^2 = \frac{1}{5} (1 + 9 + 1 + 4 + 0) = 3.$$

To find the variance of the estimator, you need the variance of X_i^2 . Since $\text{Var}(X_i^2) = \mathbf{E}[X_i^4] - (\mathbf{E}[X_i^2])^2$, you need to know $\mathbf{E}[X_i^4]$. This quantity can be estimated using the natural estimator

$$\frac{1}{n} \sum_{i=1}^n X_i^4.$$

提交

You have used 2 of 3 attempts

❗ Answers are displayed within the problem

讨论

显示讨论

Topic: Unit 8 / Lec. 20 / 14. Exercise: Natural estimators