5. Expectation values

Problem 4. Expectation practice

6.0/6.0 points (graded)

Let X, Y, Z be independent discrete random variables with

$$\mathbf{E}[X] = 2 \qquad \mathbf{E}[Y] = 0 \qquad \mathbf{E}[Z] = 0,$$

$$\mathbf{E}[X^2] = 20 \qquad \mathbf{E}[Y^2] = \mathbf{E}[Z^2] = 16,$$

and

$$\textsf{Var}(X) = \textsf{Var}(Y) = \textsf{Var}(Z) = 16.$$

Let $A = X(Y+Z)$ and $B = XY$.

1. Find $\mathbf{E}[B]$.

   $\mathbf{E}[B] =$ 0 ✔

2. Find $\textsf{Var}(B)$.

   $\textsf{Var}(B) =$ 320 ✔

3. Find $\mathbf{E}[AB]$.

   $\mathbf{E}[AB] =$ 320 ✔

提交 You have used 1 of 2 attempts

Problem 5. Expectation: True or False

3/3 points (graded)

With A, B, X defined as before, determine whether the following statements are true or false:

1. A and B are independent.

- ○ True
- ◉ False ✔

2. A and B are conditionally independent, given $X = 0$.

- ◉ True ✔
- ○ False

3. A and B are conditionally independent, given $X = 1$.

- ○ True
- ◉ False ✔

Solution:

1. Note that

$$\mathbf{E}[AB] = 320 \neq \mathbf{E}[A]\mathbf{E}[B] = 0.$$

Hence, A and B cannot be independent. Intuitively, this is because they are both affected by X.

2. Given $X = 0$ both A and B are identically equal to 0. Hence, they are independent. (Deterministic random variables are always independent; one does not carry any new information on the other.)

3. Given $X = 1$, we have

$$\mathbf{E}[AB \mid X = 1] \;=\; \mathbf{E}[X(Y+Z)XY \mid X = 1] = \mathbf{E}[(Y+Z)Y \mid X = 1]$$

$$= \mathbf{E}[(Y+Z)Y] = \mathbf{E}[Y^2] + \mathbf{E}[Z]\mathbf{E}[Y]$$
$$= \mathbf{E}[Y^2] = 16 \neq 0.$$

On the other hand,

$$\mathbf{E}[A \mid X=1] = \mathbf{E}[X(Y+Z) \mid X=1] = \mathbf{E}[(Y+Z) \mid X=1] = \mathbf{E}[Y+Z] = 0.$$

Thus,

$$\mathbf{E}[AB|X=1] \neq \mathbf{E}[A|X=1]\mathbf{E}[B|X=1],$$

and they are not conditionally independent, given $X=1$. Intuitively, this is because both A and B are affected by Y.

提交 You have used 1 of 1 attempt

Answers are displayed within the problem

Error and Bug Reports/Technical Issues

显示讨论

Topic: Exam 1 / 5. Expectation values

Learn About Verified Certificates