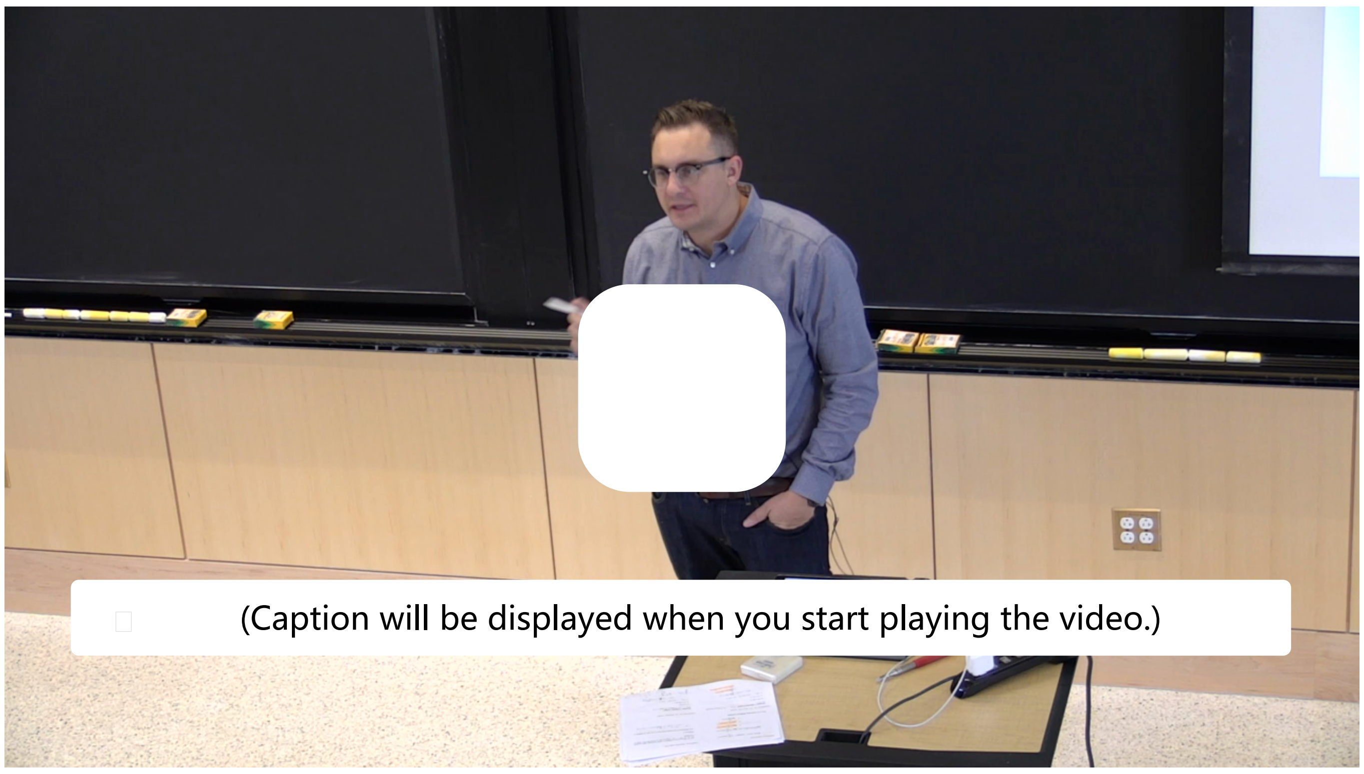
# 2. Review and Likelihood of a Gaussian Distribution

# Concept Check: Likelihoods of a Bernoulli, a Poisson, and a Gaussian Distribution

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Last time we introduced a likelihood,

and the way we motivated it was by saying, OK,

we're looking for a distance between probability

distributions that we can estimate,

and we tried with the total variation, which is perhaps

the most natural distance between probability

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## Is the Likelihood Discrete or Continuous?

2/2 points (graded)

**Setup:**

Consider a **discrete** statistical model $M_1 = (\mathbb{Z}, \{\mathbf{P}_\theta\}_{\theta\in\mathbb{R}})$ and a **continuous** statistical model $M_2 = (\mathbb{R}, \{Q_\theta\}_{\theta\in\mathbb{R}})$. Let $p_\theta$ denote the pmf of $\mathbf{P}_\theta$, and let $q_\theta$ denote the pdf of $Q_\theta$. Assume that $p_\theta$ and $q_\theta$ both vary continuously with the parameter $\theta$.

Let $x_1, \ldots, x_n$ be fixed natural numbers and $y_1, \ldots, y_n$ be fixed real numbers. Let $(L_1)_n$ denote the likelihood of the discrete model $M_1$, and let $(L_2)_n$ denote the likelihood of the continuous model $M_2$. Keeping $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ fixed, let's think of $(L_1)_n(x_1, \ldots, x_n, \theta)$ and $(L_2)_n(y_1, \ldots, y_n, \theta)$ as functions of $\theta$.

**Question**

Decide whether the following claims about $(L_1)_n$ and $(L_2)_n$ are true or false.

The map $\theta \mapsto (L_1)_n(x_1, \ldots, x_n, \theta)$ is a continuous function of $\theta$.

- (●) True
- ( ) False

The map $\theta \mapsto (L_2)_n(y_1, \ldots, y_n, \theta)$ is a continuous function of $\theta$.

- (●) True

○ False

**Solution:**

Observe that

$$(L_1)_n(x_1,\ldots,x_n,\theta)=\prod_{i=1}^{n}p_\theta(x_i),\quad (L_2)_n(y_1,\ldots,y_n,\theta)=\prod_{i=1}^{n}q_\theta(y_i).$$

We are given that $p_\theta$ and $q_\theta$ are both continuous function of the parameter $\theta\in\mathbb{R}$. Since products of continuous functions are continuous, this implies that the maps $\theta\mapsto(L_1)_n(x_1,\ldots,x_n,\theta)$ and $\theta\mapsto(L_2)_n(y_1,\ldots,y_n,\theta)$ are continuous functions of the parameter $\theta\in\mathbb{R}$.

**Remark:** It may be confusing that even the likelihood of a discrete statistical model can be continuous. However, considering the likelihood of a Bernoulli (derived in a previous question),

$$L(x_1,\ldots,x_n,p)=\prod_{i=1}^{n}p^{x_i}(1-p)^{1-x_i}=p^{\sum_{i=1}^{n}x_i}(1-p)^{n-\sum_{i=1}^{n}x_i}.$$

we can clearly see that the above varies continuously as a function of the *parameter*. This is also true for a host of other discrete models (for example, the Poisson model).

提交 你已经尝试了1次（总共可以尝试1次）

☐ Answers are displayed within the problem

## Quiz: Likelihood of a Gaussian Statistical Model

3/3 points (graded)

Let $X_1,\ldots,X_n\overset{iid}{\sim}N\left(\mu^*,(\sigma^*)^2\right)$ for some unknown $\mu^*\in\mathbb{R},(\sigma^*)^2>0$. You construct the associated statistical model $\left(\mathbb{R},\{N(\mu,\sigma^2)\}_{(\mu,\sigma^2)\in\mathbb{R}\times(0,\infty)}\right)$.

The likelihood of this model can be written

$$L_n\left(x_1,\ldots,x_n,(\mu,\sigma^2)\right)=\frac{1}{\left(\sigma\sqrt{2\pi}\right)^C}\exp\left(-\frac{1}{A}\sum_{i=1}^{C}B_i\right)$$

where $A$ depends on $\sigma$, $B_i$ depends on $\mu$ and $x_i$. Find $A$,$B_i$ and $C$.

(Choose a $B_i$ that has coefficient $1$ for the highest degree term in $x_i$.)

(Type **sigma** for $\sigma$, **mu** for $\mu$, and **x_i** for $x_i$.)

$A=$ 2*sigma^2

$2\cdot\sigma^2$

☐ **Answer:** 2*sigma^2

$B_i=$ (x_i-mu)^2

$(x_i-\mu)^2$

☐ **Answer:** (x_i - mu)^2

$C =$ n   Answer: n

$n$

STANDARD NOTATION

**Solution:**

The pmf of a Gaussian distribution $N(\mu, \sigma^2)$ is the function $x \mapsto \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$. Hence, the likelihood is

$$L_n(x_1, \ldots, x_n, (\mu, \sigma^2)) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x_i-\mu)^2\right) = \frac{1}{\left(\sigma\sqrt{2\pi}\right)^n} \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right).$$

Hence,

$$A = 2\sigma^2, \quad B_i = (x_i-\mu)^2, \quad C = n.$$

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Answers are displayed within the problem

## 讨论

显示讨论

**主题：** Unit 3 Methods of Estimation:Lecture 9: Introduction to Maximum Likelihood Estimation / 2. Review and Likelihood of a Gaussian Distribution