7. LLMS estimation

Problem 6. LLMS estimation

2.5/5.0 points (graded)

Let X and W be independent and uniformly distributed on $[-1, 1]$. We have given the following facts:

$$\begin{aligned} \mathbf{E}[X] = \mathbf{E}[X^3] = \mathbf{E}[X^5] &= 0 \\ \mathbf{E}[X^2] &= 1/3 \\ \mathbf{E}[X^4] &= 1/5 \end{aligned}$$

Suppose that

$$Y = X^3 + W$$

1. Find the LMS estimate of Y, given that $X = x$. (Notice we are trying to estimate Y from X, not the opposite direction.) (Your answer should be a function of x.)

$\hat{Y}_{\text{LMS}}(x) =$ x^3 ✔ Answer: x^3

x^3

2. Find the LLMS estimate for Y, given that $X = x$. (Your answer should be a function of x.)

$\hat{Y}_{\text{LLMS}}(x) =$ x^3 ✘ Answer: 0.6*x

x^3

STANDARD NOTATION

Solution:

1. $\hat{Y}_{\text{LMS}}(x) = \mathbf{E}[Y|X=x] = \mathbf{E}[X^3 + W|X=x] = \mathbf{E}[x^3 + W] = x^3$.

2. Since X, Y are both zero mean, we have $\textbf{Cov}(X,Y) = \mathbf{E}[XY]$, and

$$\begin{aligned} \hat{Y}_{\text{LLMS}}(x) &= \mathbf{E}[Y] + \frac{\mathbf{E}[XY]}{\mathbf{E}[X^2]}(x - \mathbf{E}[X]) \\ &= 0 + \frac{\mathbf{E}[X(X^3 + W)]}{\mathbf{E}[X^2]}x \\ &= \frac{\mathbf{E}[X^4]}{\mathbf{E}[X^2]}x \\ &= \frac{3}{5}x \end{aligned}$$

提交 你已经尝试了2次（总共可以尝试2次）

Answers are displayed within the problem

Error and Bug Reports/Technical Issues

显示讨论

主题: Final Exam / 7. LLMS estimation