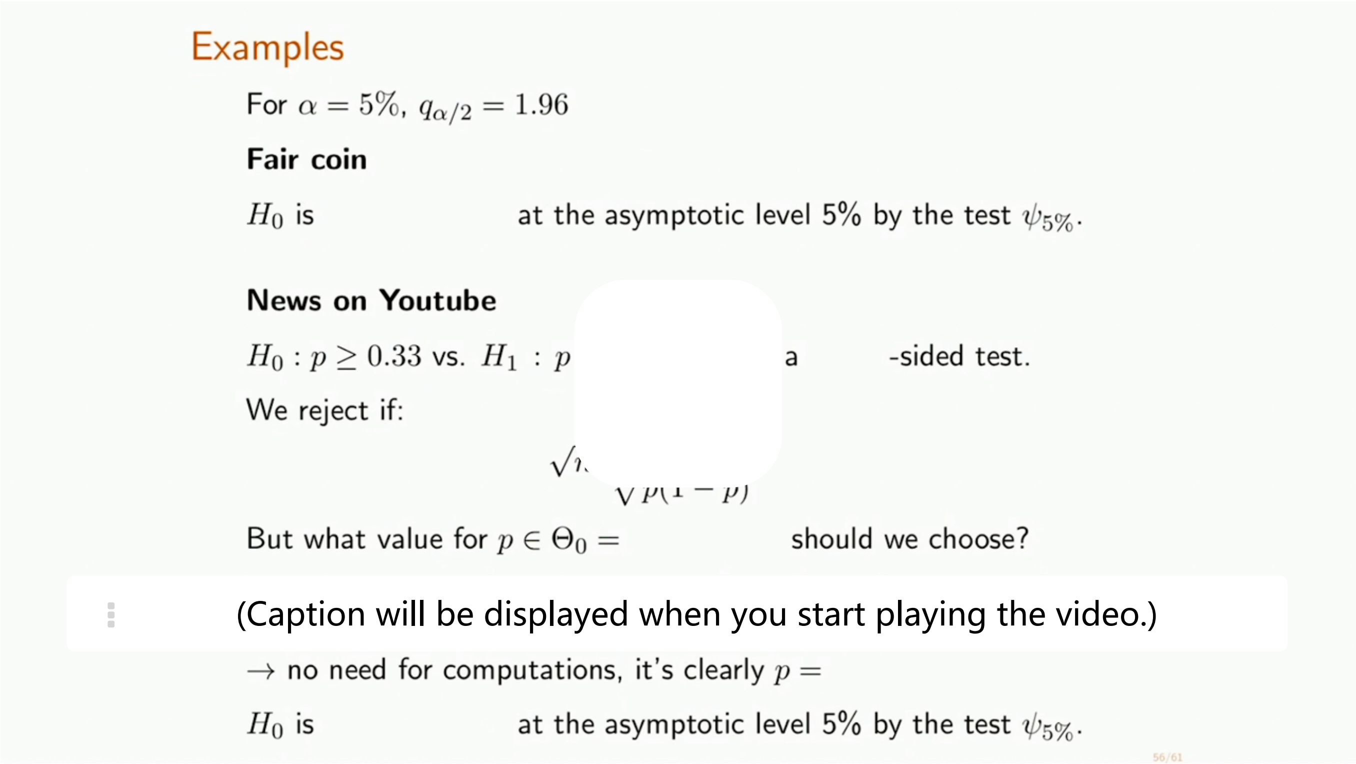
4. Worked Example: Conclusion of a Two-Sided Test

Conclusion and Comments on the Two-Sided Test for a Bernoulli Experiment



Examples

For $\alpha = 5\%$, $q_{\alpha/2} = 1.96$

Fair coin

H_0 is at the asymptotic level 5% by the test $\psi_{5\%}$.

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$H_0 : p \geq 0.33$ vs. $H_1 : p$ a -sided test.

We reject if:

$\sqrt{p(1-p)}$

But what value for $p \in \Theta_0 =$ should we choose?

(Caption will be displayed when you start playing the video.)

→ no need for computations, it's clearly $p =$

H_0 is at the asymptotic level 5% by the test $\psi_{5\%}$.

Start of transcript. Skip to the end.

OK, so let's look at another one.

So when alpha is equal to 5%, we know that q alpha over 2

is equal to 1.96%.

And what I want here is H0.

So for the fair coin, I actually need to remind you the data

that we had.

So for the fair coin, I need to tell you what

Xn bar was actually equal to.

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Rejecting or Failing to Reject the Null Hypothesis I

2/2 points (graded)

In this problem, we will complete the hypothesis testing procedure for testing if a coin is fair.

Setup as before:

You observe $X_1, \ldots, X_n \overset{i.i.d.}{\sim} \text{Ber}(p^*)$ (each X_i models a coin flip) and want to decide if $p^* = 1/2$. The associated statistical model is $\left(\{0,1\}, \{\text{Ber}(p)\}_{p \in (0,1)}\right)$ and the null and alternative hypotheses are

- $H_0 : p^* = 1/2$
- $H_1 : p^* \neq 1/2$.

You design the statistical test:

$$\begin{aligned} \psi_n &= \mathbf{1}\left(T_n > q_{\alpha/2}\right) \\ \text{where} \quad T_n &= \sqrt{n}\frac{\left|\overline{X}_n - 0.5\right|}{\sqrt{0.5\,(1-0.5)}} \end{aligned}$$

where $q_{\alpha/2}$ denotes the $1-\alpha/2$ quantile of a standard Gaussian, and α is determined by the required level of ψ. Note the absolute value in T_n for this two sided test.

Questions:

You flip the coin 200 times and observed 80 Heads. Recall from the problem *Hypothesis Testing: A Sample Data Set of Coin Flips I* in the previous lecture that the value of the test statistics T_n for this data set is $T_{200} = 2.83$.

If the test $\psi = \mathbf{1}\left(T_n > q_{\alpha/2}\right)$ is designed to have asymptotic level 5%, would you **reject** or **fail to reject** the null hypothesis $H_0 : p^* = 1/2$ for this data set?

● Reject ✓

○ Fail to reject

If instead, the test $\psi = \mathbf{1}\left(T_n > q_{\alpha/2}\right)$ is designed to have asymptotic level 10%, would you reject or fail to reject H_0 using the same data set?

● Reject ✓

○ Fail to reject

Solution:

- If ψ is designed to have asymptotic level 5%, this implies that $\alpha = 0.025$, according to the problem on the problem. By using a table or computational tools, we see that $q_{0.025} = 1.96$.

  In the problem "Hypothesis Testing: A Sample Data Set of Coin Flips I", we computed that $T_{200} = |-2.82842| \sim 2.83$. Since $T_{200} = |-2.82842| > 1.96$, we have $\psi = 1$ and we **reject** the null.
- If instead ψ has asymptotic level η where $\eta > \alpha$, then $q_{\eta/2} < q_{\alpha/2}$, i.e. the threshold decreases, leading to $T_{200} > q_{\eta/2}$. Therefore, we again reject H_0.

Remark: A test with a smaller (asymptotic) level is more “stringent" than a test of the same form with a greater (asymptotic) level.

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

讨论

显示讨论

主题: Unit 2 Foundation of Inference:Lecture 7: Hypothesis Testing (Continued): Levels and P-values / 4. Worked Example: Conclusion of a Two-Sided Test

认证证书是什么?