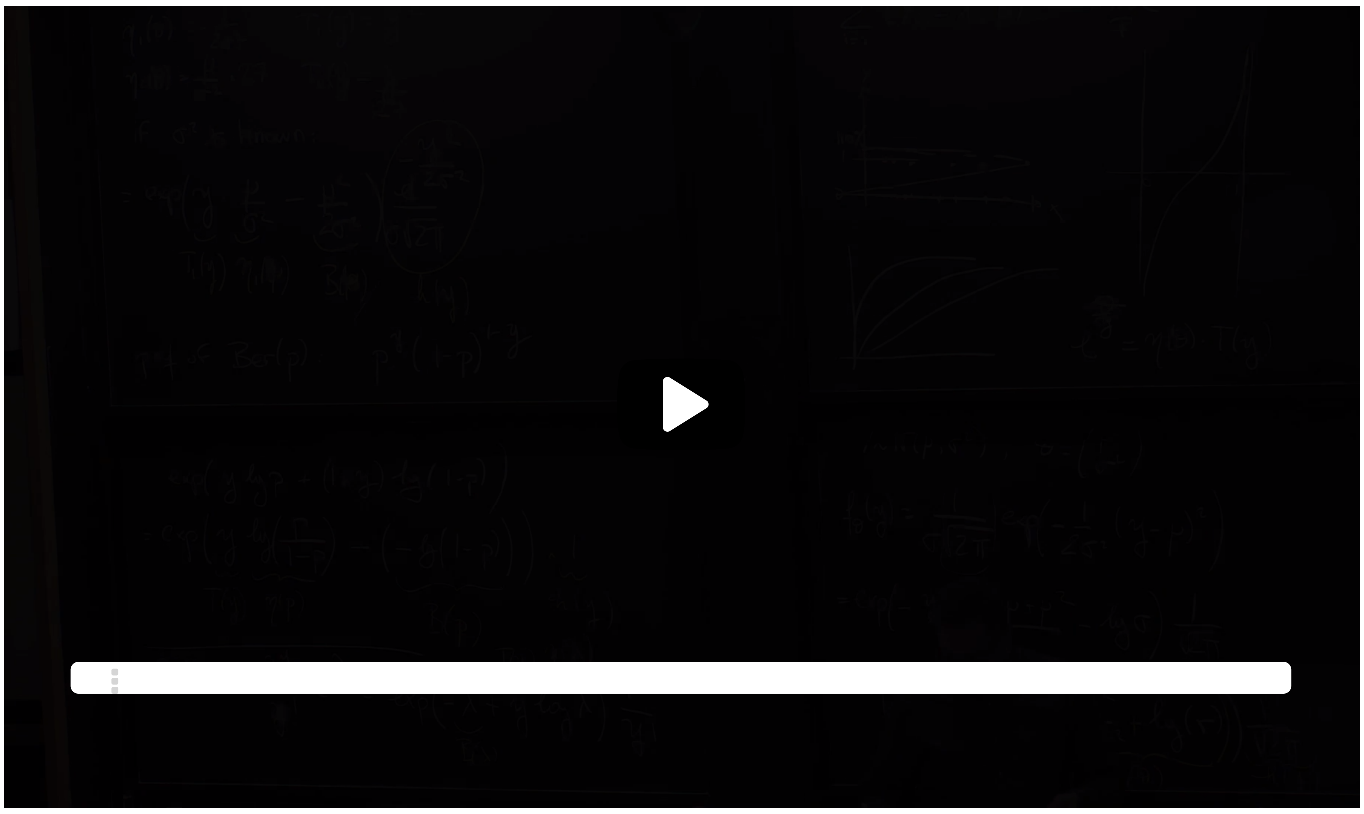
8. Exponential Family: Discrete Examples
Example: Bernoulli and Poisson Distribution



4:52 / 4:52 1.0x

It was just the one parameter.
The y parameter T of y was equal to y.
T of y was equal to y here.
T of y was equal to y here.
And actually in the one-parameter Gaussian, T of y
was also equal to y.
And so this will be a very specific class
which we call canonical exponential family,
and we'll come back to it in a second.

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The Binomial Distribution as an Exponential Family

3/3 points (graded)

Recall that the binomial distribution with parameters n and p is governed by

$$P(Y = y) = \binom{n}{y} p^y (1-p)^{n-y}.$$

Let n be some known number, say $n = 1000$. Then the pmf is

$$f_p(y) = \binom{1000}{y} p^y (1-p)^{1000-y}.$$

Write this as an exponential family of the form

$$f_p(y) = h(y) \exp\left(\eta(p) T(y) - B(p)\right) \qquad \text{where } h(y) = \binom{1000}{y},$$

then enter $\eta(p)$ $T(y)$ and $B(p)$ below. To get unique answers, use 1 as the coefficient of y in $T(y)$.

$\eta(p) =$ ln(p/(1-p)) ✔ **Answer:** ln(p/(1-p))

$\ln\left(\frac{p}{1-p}\right)$

$T(y) =$ y ✔ **Answer:** y

y

$B(p) =$ -1000*ln(1-p) ✔ **Answer:** -1000*ln(1-p)

$-1000 \cdot \ln(1-p)$

STANDARD NOTATION

Solution:

We can write $f_p(y)$ as

$$f_p(y) = \binom{1000}{y} e^{\ln p^y (1-p)^{1000-y}} = \binom{1000}{y} e^{y \ln p + (1000-y)\ln(1-p)} = \binom{1000}{y} e^{y \ln \frac{p}{1-p} - (-1000 \ln(1-p))}.$$

From this, we match up terms to get that $\eta(p) = \ln \frac{p}{1-p}$, $T(y) = y$, and $B(p) = -1000 \times \ln(1-p)$.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 7 Generalized Linear Models:Lecture 21: Introduction to Generalized Linear Models; Exponential Families / 8. Exponential Family: Discrete Examples