Conditioning example. Consider repeatedly and independently tossing a coin with probability of heads p. We can interpret $\mathbf{P}(X = i \mid X + Y = n)$ as the probability that we obtained heads for the first time on the ith toss given that we obtained heads for the second time on the nth toss. We can then argue, intuitively, that given that the second heads occurred on the nth toss, the first heads is equally likely to have come up at any toss between 1 and $n-1$. To establish this precisely, note that we have

$$\mathbf{P}(X = i \mid X + Y = n) = \frac{\mathbf{P}(X = i,\ X + Y = n)}{\mathbf{P}(X + Y = n)} = \frac{\mathbf{P}(X = i)\mathbf{P}(Y = n - i)}{\mathbf{P}(X + Y = n)},$$

where the last step follows from the assumption that X and Y are independent. Also

$$\mathbf{P}(X = i) = p(1-p)^{i-1}, \qquad \text{for } i \geq 1,$$

and

$$\mathbf{P}(Y = n - i) = p(1-p)^{n-i-1}, \qquad \text{for } n - i \geq 1.$$

It follows that

$$\mathbf{P}(X = i)\mathbf{P}(Y = n - i) = \begin{cases} p^2(1-p)^{n-2}, & \text{if } i = 1, \ldots, n-1, \\ 0, & \text{otherwise.} \end{cases}$$

Note that for $i \in \{1, \ldots, n-1\}$, this expression does not depend on i. Additionally, $\mathbf{P}(X + Y = n)$ does not depend on i either. Therefore, for any $i \in \{1, \ldots, n-1\}$, $\mathbf{P}(X = i \mid X + Y = n)$ has the same value. Hence,

$$\mathbf{P}(X = i \mid X + Y = n) = \frac{1}{n-1}, \qquad i = 1, \ldots, n-1.$$