9. Quantiles of the Pivotal Distribution and P-values

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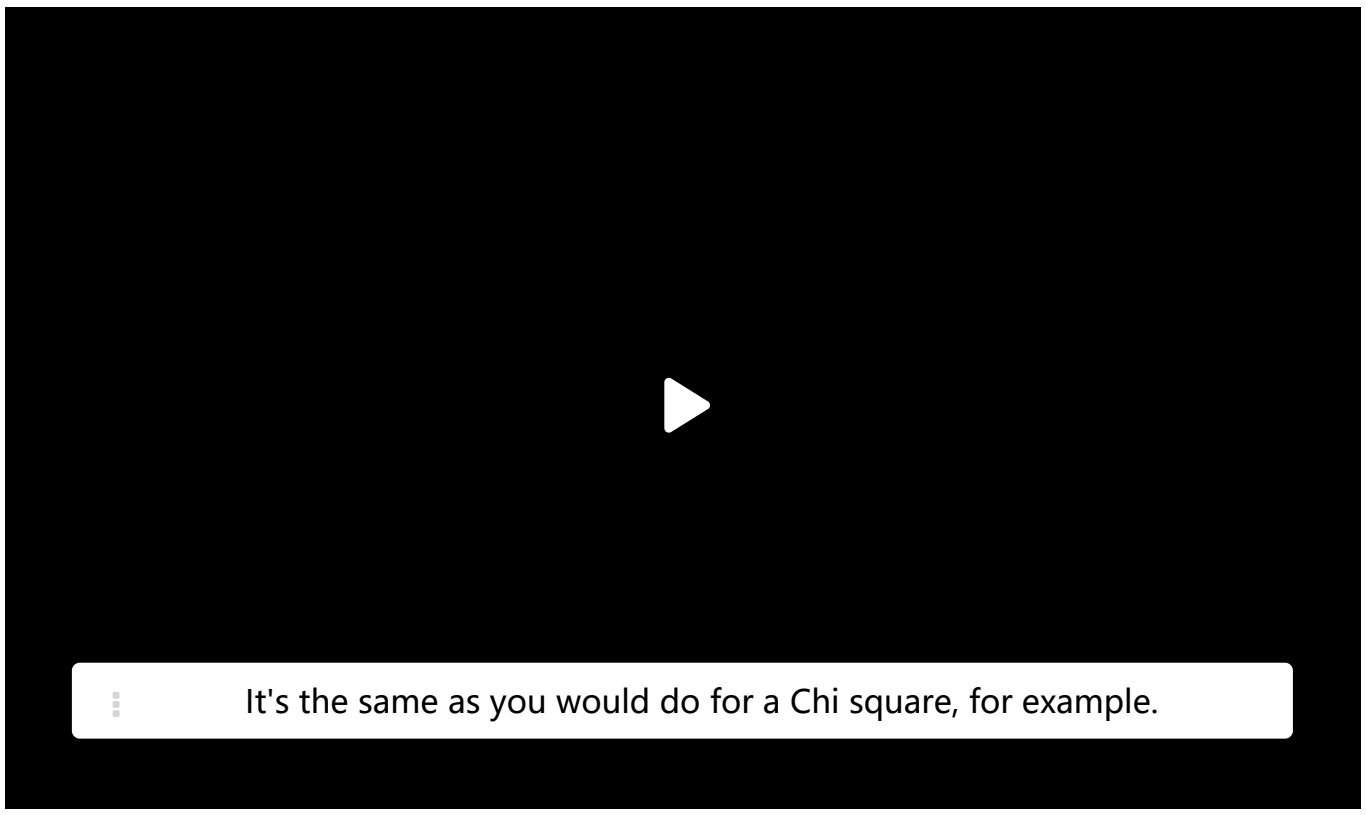

very lower digits.

And therefore, the m that was chosen here

is large enough so that all those digits that you see

have been stabilized.

And so those are the same digits as the true q alpha.

OK?

So everybody knows what you're looking at here, right?

It's the same as you would do for a Chi square, for example.

End of transcript. Skip to the start.

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Kolmogorov-Smirnov Test for a Uniform Distribution

2/2 points (graded)

We use the statistical set-up from a previous problem. Recall that $X_1, \ldots, X_n$ are i.i.d. samples with cdf F, and F^0 denotes the cdf of $\textbf{Unif}([0,1])$. We have the null and alternative hypotheses

$$\begin{aligned} H_0 &: F(t) = F^0 \\ H_1 &: F(t) \neq F^0. \end{aligned}$$

In the last problem, we computed the value of the test-statistic

$$T_n = \sqrt{n} \max_{i=1,\ldots,n} \left\{ \max \left(\left| \frac{i-1}{n} - X_{(i)} \mathbf{1}\left(X_{(i)} \in [0,1]\right) \right|, \left| \frac{i}{n} - X_{(i)} \mathbf{1}\left(X_{(i)} \in [0,1]\right) \right| \right) \right\}.$$

for the data set

$$\mathbf{x} = 0.8, 0.7, 0.4, 0.7, 0.2.$$

You will use the Kolmogorov-Smirnov test

$$\psi_5 = \mathbf{1}\left(T_5 > C\right).$$

What value of C should be chosen so that ψ_5 has (non-asymptotic) level 5%?

Note: Refer to the table in the slide “K-S table". If you are using this table, also note that the quantiles are presented for the statistic $\sup_t |F_n(t) - F(t)|$ and you need to account for the factor $\sqrt{n}$ while entering the quantile value for your answer. That is, the number x in the n-th row of the column labeled by the level α table in the slide “K-S table" is such that

$$P_n^{KS}\left(\frac{T_n}{\sqrt{n}} > x\right) = \alpha.$$

1.25953 ✔ **Answer:** 1.2595

For the Kolmogorov-Smirnov test of level 5%, would you **reject** or **fail to reject** the null hypothesis on the above data set?

- Reject
- Fail to reject ✔

Solution:

Let $Y_1, \ldots, Y_n$ be iid random variables with continuous cdf F. Consider the distribution of the statistic

$$T_n = \sqrt{n} \sup_{t \in \mathbb{R}} |F_n(t) - F(t)|.$$

This statistic is **pivotal**, *i.e.*, for any fixed n, the distribution of T_n does **not** depend on the distribution of Y_i. Let P_n^{KS} denote the distribution of T_n.

The number x in the n-th row of the column labeled by the level α table in the slide “K-S table" is such that

$$P_n^{KS}\left(\frac{T_n}{\sqrt{n}} > x\right) = \alpha.$$

In our example, $n = 5$, and we want our test ψ_5 to have level 5%. The number in the 5'th row and column labeled by 0.05 is 0.56328. Therefore, we need to set $C = \sqrt{5} \cdot 0.56328 \approx 1.2595.$

In the previous question, we computed $T_5 \approx 0.6708$. Therefore, the test ψ_5 will **fail to reject** the null hypothesis that $X_1, \ldots, X_5 \overset{iid}{\sim} U([0,1])$ on the given data set.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Discussion

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