In this unit, we introduced many different concepts, definitions, and formulas, so it may be useful to put in one place a summary of the key concepts and the key formulas that we have developed. We first defined random variables. And then we discussed that random variables are described in terms of a probability mass function that tells you the probabilities of the different values that the random variable can take.

And for the case of multiple random variables, we may use a joint probability mass function. We also defined conditional probability mass functions that refer to the distribution of a random variable X in a universe in which we are told that a certain event has occurred or that a certain random variable takes on a specific value. A key concept that we introduced was the concept of expectation.

We defined the notion of the expected value of a random variable. But if we're given some information, then we are transported to a conditional universe, and we calculate the so-called conditional expectation that takes into account the information that we have available. And this calculation makes use of the corresponding conditional PMF of X, given an event or given the value of another random variable.

The main facts about expectations were the following. We have the expected value rule for calculating the expectation of a function of one or multiple random variables without having to calculate the distribution of this function of random variables. Instead, we can do the calculations directly, using the original PMF of the original random variables. And once more we have conditional versions of the expected value rule that take the same form, except that we need to use conditional PMFs when we carry out the calculations.

The second important fact about expectations is that they're linear. If we have a linear function, let's say, of two random variables, then the expected value of this linear function is the same linear function of the expectations. Another concept that we introduced was the variance of a random variable that measures the dispersion or the spread of the distribution of a random variable.

And if we're talking about a conditional universe where we're given some information, then we have the conditional variance given that an event has occurred or given that a random variable takes a specific value. A useful formula that allows us to calculate in a somewhat easier manner the variance of a random variable is this one. And we had a few opportunities to use it.

Now, an important concept about random variables is the notion of independence. And independence basically means that the joint PMF factors out as a product of marginal PMFs. This is the mathematical definition. The intuitive definition would be that information about one of the random variables gives us no information about the values of the other random variable.

Now, independence has some interesting, nice mathematical consequences. In particular, if X and Y are independent, the expected value of the product is the product of the expectations. And the variance of the sum is equal to the sum of the variances.

Then we extended some of the basic skills that we had introduced earlier in this course-- the multiplication rule and the total probability theorem. Here are two formulas of this kind that are exactly the same as the analogous formula that we had for probabilities, except now that they're written in PMF notation. So the multiplication rule tells us that the probability of several things happening is the product of the probability that one thing happens times the probability that the second thing happens given that the first happened times the probability that the third event happens given that the first two events have happened.

And the total probability theorem allows us to calculate the probability of an event happening by considering different scenarios, different values of Y in this context, looking at the probability of the event of interest under each one of the different scenarios and forming a weighted sum, where the weights are the probabilities of the different scenarios. An extension or variation of the total probability theorem is the so-called total expectation theorem. It is an analogous result. But now, we deal with expectations.

We calculate the expected value of a random variable by considering a number of scenarios, finding the expected value of the random variable under each one of the different scenarios, and then taking a weighted average of these conditional expectations. And finally, in the process of developing all those concepts, we introduced a few special random variables and PMFs and did some calculations with them, for example, calculate their means, variances, or derive certain properties that they had.

And this is the list of the types of random variables that we introduced. In the next unit, we're going to see counterparts of all of these facts and properties but now, in the context of continuous random variables.