We view the Erlang arrival process in the problem statement as part of a Poisson process with rate λ. In particular, the Erlang arrival process registers an arrival once every two arrivals of the Poisson process. For concreteness, let us say that the Erlang process arrivals correspond to even-numbered arrivals in the Poisson process. Let Y_k be the time of the kth arrival in the Poisson process.

Let K be such that $Y_K \leq t < Y_{K+1}$. By the discussion of random incidence in Poisson processes in the text, we have that $Y_{K+1} - Y_K$ is Erlang of order 2. The interarrival interval for the Erlang process considered in this problem is of the form $[Y_K, Y_{K+2}]$ or $[Y_{K-1}, Y_{K+1}]$, depending on whether K is even or odd, respectively. In the first case, the interarrival interval in the Erlang process is of the form $(Y_{K+1} - Y_K) + (Y_{K+2} - Y_{K+1})$. We claim that $Y_{K+2} - Y_{K+1}$ is exponential with parameter λ and independent of $Y_{K+1} - Y_K$. Indeed, an observer who arrives at time t and notices that K is even, must first wait until the time Y_{K+1} of the next Poisson arrival. At that time, the Poisson process starts afresh, and the time $Y_{K+2} - Y_{K+1}$ until the next Poisson arrival is independent of the past (hence, independent of $Y_{K+1} - Y_K$) and has an exponential distribution with parameter λ, as claimed. This establishes that, conditioned on K being even, the interarrival interval length $Y_{K+2} - Y_K$ of the Erlang process is Erlang of order 3 (since it is the sum of an exponential random variable and a random variable which is Erlang of order 2). By a symmetrical argument, if we condition on K being odd, the conditional PDF of the interarrival interval length $Y_{K+1} - Y_{K-1}$ of the Erlang process is again the same. Since the conditional PDF of the length of the interarrival interval that contains t is Erlang of order 3, for every conditioning event, it follows that the unconditional PDF is also Erlang of order 3.