# 15. Type 2 Error and Power of a Statistical Test

## Type 2 Error and Power of a Statistical Test

(Caption will be displayed when you start playing the video.)

particular theta,

which is in theta 1 of what?

Of psi equals 0.

OK, so psi equals 0 corresponds to this column.

And theta [INAUDIBLE] theta 1 corresponds to this row.

OK, and so there is a third notion, which

we won't use as much.

It's just 1 minus the probability of type 2 error.

So that's the probability that you

reject when you really should.

That's a good thing.

So that's a powerful

Thing.

If this number is large, usually you have a powerful test.

That's a good thing.

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## Testing the Support of a Uniform Variable: Type 2 Error of a Test

1/1 point (graded)

As on the previous page, let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathrm{Unif}[0,\theta]$ for an unknown parameter $\theta$ and we designed the statistical test

$$\psi_n = \mathbf{1}(\max_{1\le i\le n} X_i > 1/2)$$

to decide between the null and alternative hypotheses

$$\begin{aligned} H_0 : \theta &\le 1/2 \\ H_1 : \theta &> 1/2. \end{aligned}$$

Recall from lecture that the **type 2 error (rate)** of a test $\psi_n$ is the **function**

$$\begin{aligned} \beta_{\psi_n} : \Theta_1 &\to \mathbb{R} \\ \theta &\mapsto \mathbf{P}_\theta(\psi_n = 0) \end{aligned}$$

where $\mathbf{P}_\theta(\psi_n = 0)$ is the probability of the event $\psi_n = 0$ under the probability distribution $\mathbf{P}_\theta$ when $\theta \in \Theta_1$, i.e. the probability of not rejecting $H_0$ when $H_1$ is true. In this example, the region $\Theta_1$ defining the alternative hypothesis is $(1/2, \infty)$, and $\mathbf{P}_\theta = \mathrm{Unif}[0,\theta]$.

Evaluate $\mathbf{P}_\theta(\psi_n = 0) = \mathbf{P}_\theta\left(\max_{1\le i\le n} X_i \le 1/2\right)$ at $\theta = 1/2$, the boundary between $\Theta_0$ and $\Theta_1$.

$\mathbf{P}_{\theta=1/2}\left(\max_{1\le i\le n} X_i \le 1/2\right) =$ 1 ✔ Answer: 1

**Solution:**

$$\begin{aligned}\beta_{\psi_n}(1/2) &= \mathbf{P}_{1/2}(\max_{1\le i\le n} X_i < 1/2)\\ &= \mathbf{P}_{1/2}(X_1 < 1/2)\ldots\mathbf{P}_{1/2}(X_n < 1/2)\\ &= 1\times 1\ldots\times 1 = 1\end{aligned}$$

where we applied independence of the $X_i$'s in the second line.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

## Testing the Support of a Uniform Variable: Type 2 Error of a Test Continued

3/3 points (graded)

As above, let $X_1,\ldots,X_n \overset{iid}{\sim} \mathrm{Unif}[0,\theta]$ for an unknown parameter $\theta$ and we designed the statistical test

$$\psi_n = \mathbf{1}(\max_{1\le i\le n} X_i > 1/2)$$

to decide between the null and alternative hypotheses

$$\begin{aligned}H_0 : \theta &\le 1/2\\ H_1 : \theta &> 1/2.\end{aligned}$$

Recall from lecture that the **type 2 error** of a test $\psi_n$ is the **function**

$$\begin{aligned}\beta_{\psi_n} : \Theta_1 &\to [0,1]\\ \theta &\mapsto \mathbf{P}_\theta(\psi_n = 0)\end{aligned}$$

where $\mathbf{P}_\theta(\psi_n = 0)$ is the probability of the event $\psi_n = 0$ under the probability distribution $\mathbf{P}_\theta$ when $\theta \in \Theta_1$, i.e. the probability of not rejecting $H_0$ when $H_1$ is true.

In this example, $\Theta_1 = (1/2,\infty)$, and $\mathbf{P}_\theta = \mathrm{Unif}[0,\theta]$.

What is $\beta_{\psi_n}(\theta)$?

$\beta_{\psi_n}(\theta) =$ (1/(2*theta))^n ✔ **Answer:** (1/(2*theta))^n

$\left(\frac{1}{2\cdot\theta}\right)^n$

Find $\lim_{\theta\to 1/2} \beta_{\psi_n}(\theta)$.

$\lim_{\theta\to 1/2} \beta_{\psi_n}(\theta) =$ 1 ✔ **Answer:** 1

$1$

Find $\lim_{\theta\to\infty} \beta_{\psi_n}(\theta)$.

$\lim_{\theta\to\infty} \beta_{\psi_n}(\theta) =$ 0 ✔ **Answer:** 0

$0$

STANDARD NOTATION

**Solution:**

For any $\theta \in \Theta_1 = [1/2,\infty)$,

$$\beta_{\psi_n}(\theta) = \mathbf{P}_\theta(\psi_n = 0) = \mathbf{P}_\theta\left(\max_{1\le i\le n} X_i > 1/2\right)$$
$$= \mathbf{P}_\theta(X_1 < 1/2)\dots\mathbf{P}_\theta(X_n < 1/2) = \left(\frac{1/2}{\theta}\right)^n.$$

As $\theta \to 1/2$,

$$\beta_{\psi_n}(\theta) \to \left(\frac{1/2}{1/2}\right)^n = 1.$$

As $\theta \to \infty$,

$$\beta_{\psi_n}(\theta) = \left(\frac{1/2}{\theta}\right)^n \to 0.$$

**Remark**: This test is rather extreme example in that it minimizes type-1 error while maximizing the type-2 error. In general, we want to design tests so that the type-1 and type-2 error are both controlled. These types of trade-offs are crucial to consider in the context of hypothesis testing.

提交 你已经尝试了2次 (总共可以尝试3次)

Answers are displayed within the problem

## Testing the Support of a Uniform Variable: : Power of a Test

1/1 point (graded)

The **power** of the test $\psi_n$ is defined to be

$$\pi_{\psi_n} = \inf_{\theta\in\Theta_1}(1 - \beta_{\psi_n}(\theta)).$$

Continuing from the problem above, what is the power $\pi_{\psi_n}$?

$\pi_{\psi_n} =$ 0 **Answer:** 0

**Solution:**

A priori we have that

$$\pi_{\psi_n} = \inf_{\theta\in[1/2,\infty)}(1 - P_\theta(\psi_n = 0)) = \inf_{\theta\in[1/2,\infty)} P_\theta(\psi_n = 1) \ge 0].$$

Moreover, we computed above that $\beta_{\psi_n}(1/2) = P_{0.5}[\psi_n = 0] = 1$. Thus,

$$\pi_{\psi_n} = 0.$$

**Remark:** The power of a test is the largest lower bound on the probability that if $H_1$ is true, that indeed $H_0$ is rejected in favor of $H_1$. In this example, as $\theta \in \Theta_1$ approaches the boundary $1/2$, the probability of rejecting $H_0$ decreases and approaches $0$.

提交 你已经尝试了1次 (总共可以尝试3次)

Answers are displayed within the problem

## Testing the Support of a Uniform Variable: Graphing the errors

1/1 point (graded)

As above, let $X_1, \ldots, X_n \overset{iid}{\sim} \mathrm{Unif}\,[0, \theta]$ for an unknown parameter $\theta$ and we designed the statistical test

$$\psi_n = \mathbf{1}(\max_{1 \le i \le n} X_i > 1/2)$$

to decide between the null and alternative hypotheses

$$H_0 : \theta \ \le 1/2$$
$$H_1 : \theta \ > 1/2.$$

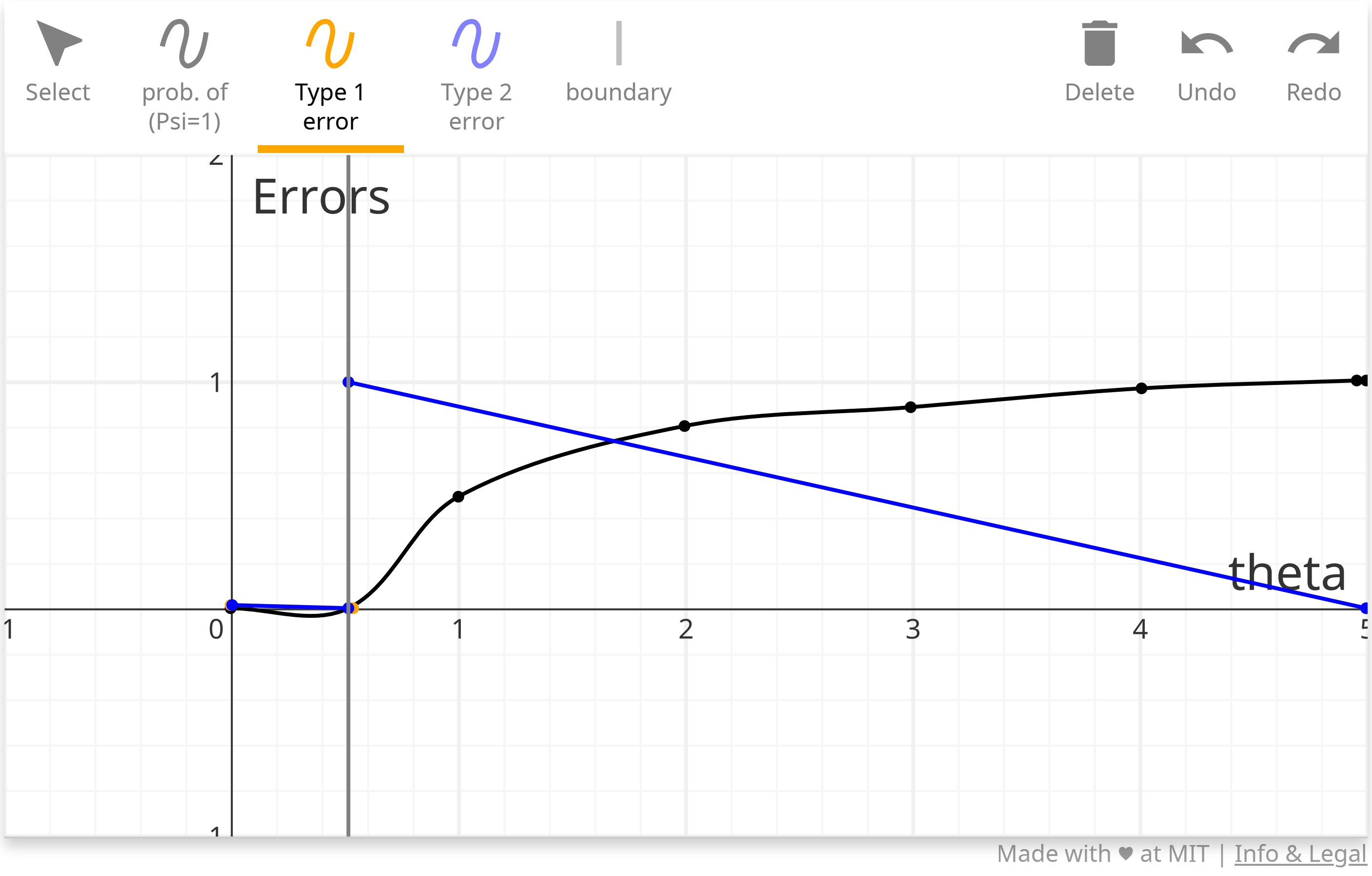
Let $\alpha_{\psi_n}(\theta)$ and $\beta_{\psi_n}(\theta)$ denote the type 1 and type 2 errors respectively.

On the graph below, do the following:

- Place a vertical line at the boundary of $\Theta_0$ and $\Theta_1$ using the **boundary tool**.
- Sketch the graph of $\mathbf{P}_\theta(\psi_n = 1)$ as a function of $\theta$ using the **probabilty of rejecting null** tool.
- Sketch the graph of the type 1 error $\alpha_{\psi_n}(\theta)$ on the **correct domain** using the **type 1 error** tool.
- Sketch the graph of the type 2 error $\beta_{\psi_n}(\theta)$ on the **correct domain** using the **type 2 error** tool.

*Note:* To use the spline tool for sketching the graphs, click on point on the graph, and the tool will connect these points with a smooth curve.

For each curve, you will be graded on its domain, its limiting values, its value on the boundary between $\Theta_0$ and $\Theta_1$, and its shape and continuity.

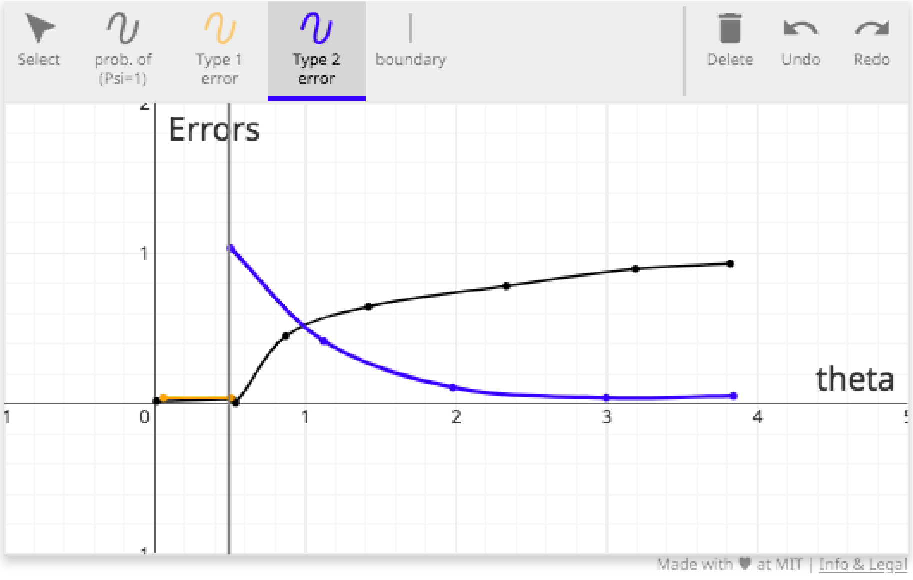



**Answer:** See solution.

Good job on the graph of the probability of rejecting the null! Good job on the graph of the type 1 error! Good job on the graph of the type 2 error!

**Solution:**

theta属于H0才会有type 1 error
theta属于H1才会有type 2 error

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Answers are displayed within the problem

## 讨论

显示讨论

**主题：** Unit 2 Foundation of Inference:Lecture 6: Introduction to Hypothesis Testing, and Type 1 and Type 2 Errors / 15. Type 2 Error and Power of a Statistical Test

认证证书是什么？