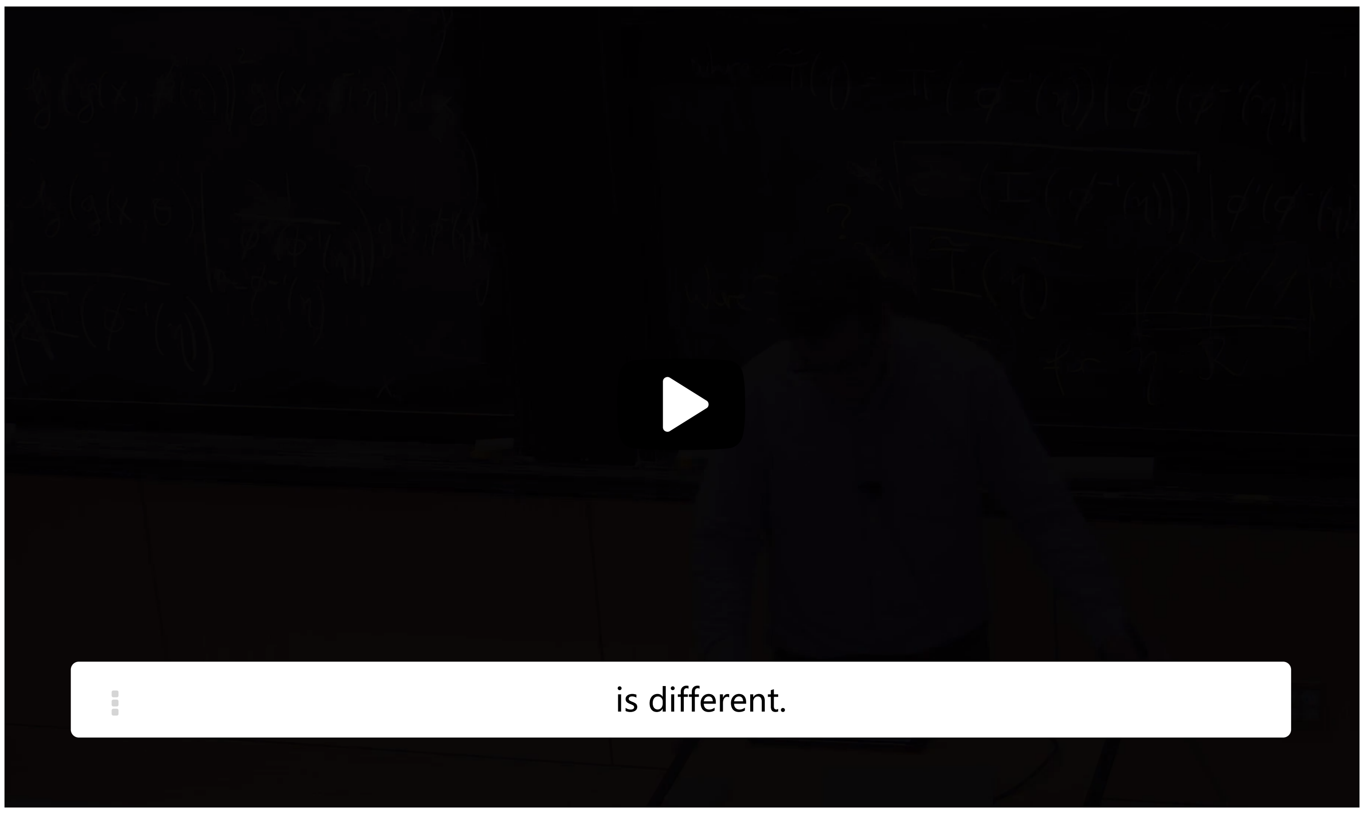
9. Bayesian Statistics for Inference

Bayesian Statistics for Inference



4:50 / 4:50 1.0x

then you just take-- and I'm asking you for a 95%--

I mean level 5% Bayesian confidence region,

you would just put 95% here and spit R as being this region

here.

Is that clear for everyone?

So the same way we've done it before

except that the meaning of the probability distribution

is different.

End of transcript. Skip to the start.

Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

Frequentist Confidence Regions

1/1 point (graded)

Suppose that the parameter space is $\Theta = \{\theta_1, \ldots, \theta_n\}$, and assume that one of these, θ_i, is the true parameter. (Here we are using the frequentist set-up– the parameter is *not* modeled as a random variable.)

Let $X_1, X_2, \ldots, X_n$ be observations. You construct an interval

$$I = \left[\overline{X}_n - \sqrt{X_1^2 + \cdots + X_n^2}, \overline{X}_n + \sqrt{X_1^2 + \cdots + X_n^2}\right].$$

Assume that you have observed $X_1, \ldots, X_n$, and you are interested in

$$\mathbb{P}\left(\theta \in I | X_1, \ldots, X_n\right).$$

Suppose that you have access to an all-knowing genie, who says that the probability above is greater than or equal to ϵ, for some $\epsilon > 0$. Using only this information, can you determine

$$\mathbb{P}\left(\theta \in I | X_1, \ldots, X_n\right)?$$

If yes, enter your answer to the input box below. If not, enter -1.

1 ✔ Answer: 1

STANDARD NOTATION

Solution:

As discussed in lecture, conditional on $X_1, X_2, \ldots, X_n$ the confidence interval I is no longer probabilistic, but rather a deterministic one. Since there is no randomness on θ, θ will either be contained in this interval, or not. In particular, the probability above is either 0 or 1. Since we know that it is $\geq \epsilon > 0$, it must therefore be equal to 1.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

The Correct Choice of the Probability Distribution

1/1 point (graded)

Let Θ be the parameter space, let $X_1, X_2, \ldots, X_n$ be random variables, and let $\alpha \in (0,1)$ be a fixed positive real number. Given a candidate Bayesian confidence regions, $\mathcal{R}$, we want to check whether it is indeed a confidence region within α. That is, we want to check if

$$\mathbb{P}(\theta \in \mathcal{R} | X_1, X_2, \ldots X_n) \geq 1 - \alpha,$$

holds. Assuming that Θ is a finite set, the probability above turns out to be

$$\mathbb{P}(\theta \in \mathcal{R} | X_1, X_2, \ldots X_n) = \sum_{\theta \in \mathcal{R}} P_1(\theta),$$

where $P_1(\cdot)$ is some distribution supported on Θ.

Which one of the probability distributions below gives the correct choice of $P_1(\cdot)$?

- $\pi(\theta)$, the prior distribution on θ.
- $L_n(X_1, \ldots, X_n | \theta)$, the likelihood of the model.
- $\pi(\theta | X_1, X_2, \ldots, X_n)$, the posterior distribution of θ, conditional on $X_1, X_2, \ldots, X_n$. ✔
- None of the above.

Solution:

The third choice. As noted in the lecture, we are searching for the probability of θ's being contained in $\mathcal{R}$, having observed $X_1, X_2, \ldots, X_n$, which translates into a conditioning. Therefore, the correct distribution is, $\pi(\theta | X_1, X_2, \ldots, X_n)$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Monotonicity of the Confidence Regions

1/1 point (graded)

Let $\mathcal{R}_1$ be a Bayesian confidence region of level α_1 for a parameter θ given observations $X_1, \ldots, X_n$. Let α_2 be a another confidence level such that $\alpha_2 \geq \alpha_1$. Which one of the statements below is true?

- (selected) $\mathcal{R}_1$ is necessarily a Bayesian confidence region for level α_2. ✔
- $\mathcal{R}_1$ is not necessarily a Bayesian confidence region for level α_2, because it has insufficient probability mass.
- $\mathcal{R}_1$ is not necessarily a Bayesian confidence region for level α_2, because each levels has a unique associated confidence regions.
- None of the above.

Solution:

$\mathcal{R}_1$ is necessarily a Bayesian confidence region for level α_2. To see this, we need to verify that,

$$\mathbb{P}\left(\theta \in \mathcal{R}_1 | X_1, \ldots, X_n\right) \geq 1 - \alpha_2,$$

having known that,

$$\mathbb{P}\left(\theta \in \mathcal{R}_1 | X_1, \ldots, X_n\right) \geq 1 - \alpha_1.$$

But since $\alpha_2 \geq \alpha_1$, $1 - \alpha_1 \geq 1 - \alpha_2$, and the latter implies the former, hence, we are done.
As demonstrated, the region has sufficient amount of probability mass, and furthermore, confidence regions are not necessarily unique (one can simply take a valid one, and any enlargement of that set is also a valid confidence region for the same level, hence uniqueness is not present).

Submit

You have used 1 of 2 attempts

Answers are displayed within the problem

Discussion

Show Discussion

Topic: Unit 5 Bayesian statistics:Lecture 18: Jeffrey's Prior and Bayesian Confidence Interval / 9. Bayesian Statistics for Inference