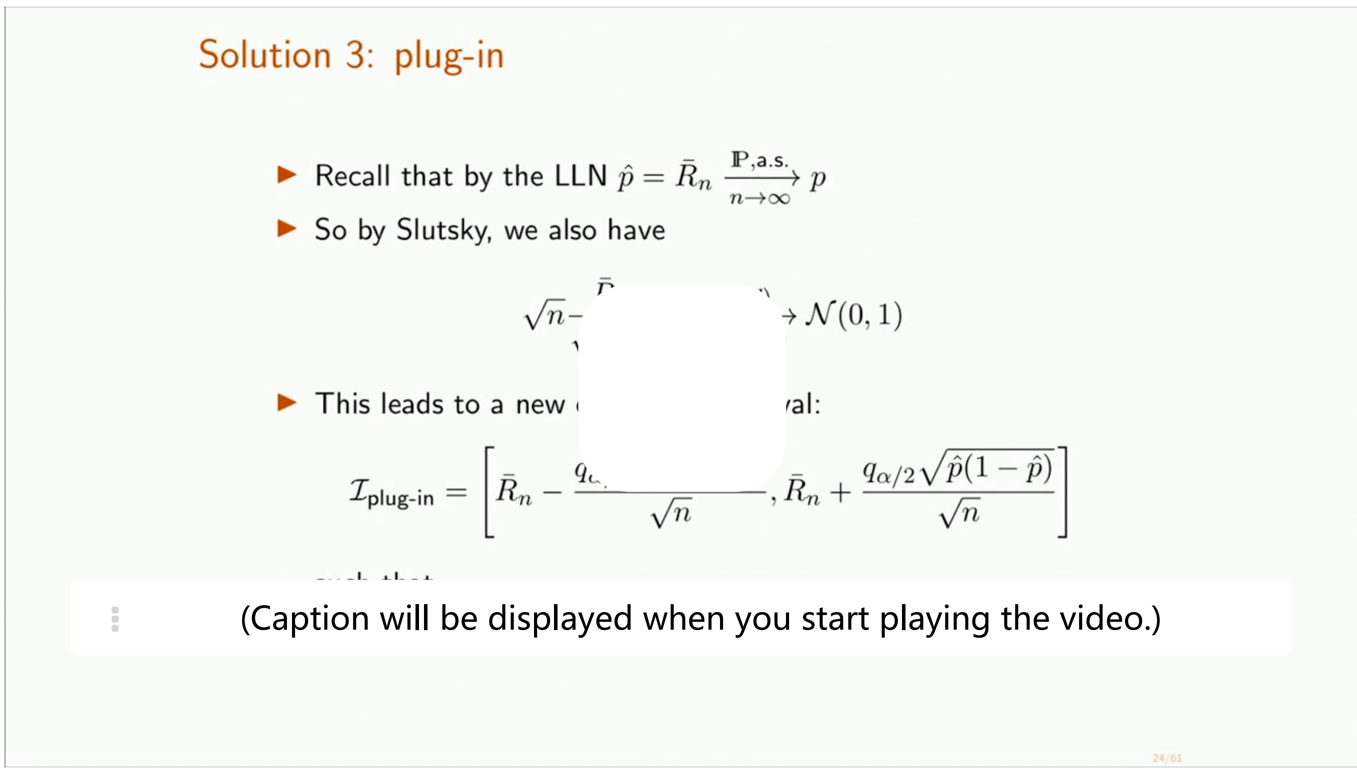
课程 > Unit 2 Foundation of Inference > Lecture 4: Parametric Estimation and Confidence Intervals > 11. Using Slutsky Theorem: Plug-in

11. Using Slutsky Theorem: Plug-in
Confidence Interval by Plug-in



Start of transcript. Skip to the end.

The third one is to say, OK, I don't know p,

but I certainly have an estimator for p.

It's p-hat hat.

And I knew that this p-hat is consistent.

I know that this p-hat should be close to p [? 4 ?]

and large enough.

So if I'm already in the regime where

视频

下载视频文件

字幕

下载 SubRip (.srt) file

下载 Text (.txt) file

Convergences of different quantities

3/3 points (graded)

As in lecture, recall that $R_1, \ldots, R_n \overset{iid}{\sim}$ Ber (p) for some unknown parameter p, and we estimate p using the estimator $\hat{p} = \overline{R}_n = \frac{1}{n}\sum_{i=1}^{n} R_i$.

As in the methods before, our starting point is the following result of the central limit theorem:

$$\lim_{n\to\infty} \mathbf{P}\left(\left|\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p\,(1-p)}}\right| < q_{\alpha/2}\right) = 1-\alpha.$$

Choose the correct convergence statement for each quantity below:
(Choose all that apply for each column.)
Note: In the third and fourth choices below, "is approximated by (in distribution)", means that the CDFs are close; i.e. $\lim_{n\to\infty} F_n(x) - G_n(x) \to 0$, where F_n is the CDF of the RV in the question and G_n is the CDF of the normal distribution with mean p and the written variance, e.g. $\mathcal{N}(p, fracp\,(1-p)n)$.

$\overline{R}_n$:

$\sqrt{n}\left(\overline{R}_n - p\right)$:

$\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p\,(1-p)}}$:

☐ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1)$	☐ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1)$	☑ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1)$ ✔
☐ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,p(1-p))$	☑ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,p(1-p))$ ✔	☐ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,p(1-p))$
☑ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{n}\right)$ ✔	☐ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{n}\right)$	☐ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{n}\right)$
☐ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{\sqrt{n}}\right)$	☐ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{\sqrt{n}}\right)$	☐ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{\sqrt{n}}\right)$
☑ $\xrightarrow[n\to\infty]{(\mathbf{P})} p$ ✔	☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} p$	☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} p$
☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} 1$	☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} 1$	☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} 1$
☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} \sqrt{p(1-p)}$	☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} \sqrt{p(1-p)}$	☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} \sqrt{p(1-p)}$
✔	✔	✔

Solution:

1.
   - $\overline{R}_n \xrightarrow[n\to\infty]{(\mathbf{P})} \mathbb{E}\left[\overline{R}_n\right] = p$ by the (weak) law of large number.
   - $\overline{R}_n \xrightarrow[n\to\infty]{(d)} \mathcal{N}\left(\mathbb{E}\left[\overline{R}_n\right], \mathsf{Var}\left(\overline{R}_n\right)\right) = \mathcal{N}\left(p, \frac{p(1-p)}{n}\right)$ by the CLT.
2. $\sqrt{n}\left(\overline{R}_n - p\right) \xrightarrow[n\to\infty]{(d)} \mathcal{N}\left(\mathbb{E}\left[\sqrt{n}\left(\overline{R}_n - p\right)\right], n\mathsf{Var}\left(\overline{R}_n\right)\right) = \mathcal{N}(0, p(1-p))$ by the CLT. Note that with an asymptotic variance that does not depend on n, $\sqrt{n}\left(\overline{R}_n - p\right)$ does not converge in probability to a constant.
3. $\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}} \xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1))$ by the CLT. This is a recaling of the convergence statement immediately above.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

Convergences of different quantities (continued)

3/3 points (graded)
This is a continuation of the previous problem. Choose all that apply for each column below.

$\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)}$:

$\frac{\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)}}{\sqrt{p(1-p)}}$:

$\left(\sqrt{n}\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right)\left(\frac{\sqrt{p(1-p)}}{\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)}}\right)$:

☐ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1)$	☐ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1)$	☑ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1)$ ✔
☐ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,p(1-p))$	☐ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,p(1-p))$	☐ $\xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,p(1-p))$
☐ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{n}\right)$	☐ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{n}\right)$	☐ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{n}\right)$
☐ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{\sqrt{n}}\right)$	☐ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{\sqrt{n}}\right)$	☐ is approximated by (in distribution) $\mathcal{N}\left(p, \frac{p(1-p)}{\sqrt{n}}\right)$
☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} p$	☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} p$	☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} p$
☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} 1$	☑ $\xrightarrow[n\to\infty]{(\mathbf{P})} 1$ ✔	☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} 1$
☑ $\xrightarrow[n\to\infty]{(\mathbf{P})} \sqrt{p(1-p)}$ ✔	☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} \sqrt{p(1-p)}$	☐ $\xrightarrow[n\to\infty]{(\mathbf{P})} \sqrt{p(1-p)}$
✔	✔	✔

Solution:

1. $\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)} \xrightarrow[n\to\infty]{\mathbf{P}} \sqrt{p(1-p)}$ by the continuous mapping theorem.
2. $\dfrac{\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)}}{\sqrt{p(1-p)}} \xrightarrow[n\to\infty]{\mathbf{P}} 1$ since constant multiple of sequences that converge in probability still converge in probability.
3. $\left(\sqrt{n}\dfrac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right)\left(\dfrac{\sqrt{p(1-p)}}{\sqrt{\overline{R}_n\left(1-\overline{R}_n\right)}}\right) \xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1)$ by Slutsky.

提交 你已经尝试了2次（总共可以尝试3次）

Answers are displayed within the problem

讨论

隐藏讨论

主题： Unit 2 Foundation of Inference:Lecture 4: Parametric Estimation and Confidence Intervals / 11. Using Slutsky Theorem: Plug-in

Add a Post

All Posts

Q1 answer

discussion posted 5 days ago by jarkadin

I answered the question 'correctly', and I understand what the point of it was, but it makes no mathematical sense.

An expression $\lim_{i\to\infty} f(i) = g(i)$ is meaningless: the right hand side cannot depend on i.

此帖对所有人可见。

添加回复

4 responses

markweitzman
5 days ago

This is an asymptotic limit like $n! \to \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$ as $n \to \infty$.

No, that's not mathematically correct. The limit diverges: $n! \to \infty$ as $n \to \infty$, what you have is that $n! \sim (n/e)^n \sqrt{2\pi n}$, which is an approximation, not a limit.

jarkadin 在5 days ago前发表

Yes yes yes, try not to be so pedantic, I said it was an asymptotic.

markweitzman 在5 days ago前发表

添加评论

mrBB
5 days ago

That's actually what I thought as well. I had it in the back of my head to make a post about it when the more pressing issues would have been resolved, but I'm glad the OP saved me the effort. I thought it's probably more correct to say we have convergence to a constant, or even perhaps to a delta function pdf in that particular instance.

Well, I think the statement is more important than that. The convergence in distribution means for large n, I can use a normal distribution with the variance depending on n to calculate probabilities, and confidence intervals for R_nbar, whereas just a statement of convergence to a delta distribution provides no information other than mean of the probability distribution for large n.

markweitzman 在5 days ago前发表

Yes, I realize that and therefore the expression that Serg gives below indeed would be the preferred one.

mrBB 在4 days ago前发表

添加评论

SergK
5 days ago

Good catch. The correct formulation of CLT is

$$\sqrt{n}\left(\bar{X}_n - \mu\right) \xrightarrow{(d)} N\left(0, \sigma^2\right)$$

avoiding n in RHS. The exercise writes it as

$$\left(\bar{X}_n - \mu\right) \xrightarrow{(d)} N\left(0, \sigma^2/n\right)$$

Definitely a better formulation.

markweitzman 在5 days ago前发表

I'm not even convinced that

$$(\bar{X}_n - \mu) \xrightarrow{(d)} N\left(0, \sigma^2/n\right)$$

is meaningful for 'large enough n'.

The standard way of doing this for finite n is to bound the total variation distance between a proposed distribution and the standard normal, but in general you don't get better than a rate of $\frac{1}{\sqrt{n}}$ in tightening of this bound, so the rescaling here seems to break the feasibility of these bounds. And of course SLLN means that if you actually pass the limit, you have a CDF that is a 0 to 1 step function (with a jump at zero).

Derek_edX 在5 days ago前发表

I think you are confusing the scaling, remember the variance scaling as $\frac{1}{n}$ is equivalent to the standard deviation scaling as $\frac{1}{\sqrt{n}}$.

markweitzman 在5 days ago前发表

Not a bad guess... I rarely mess that up since I convert to std deviation first, since std deviation is linear with respect to scaling (though not addition of course).

It seems to be much worse than that -- when I'm tired I sometimes write things upside down! That's what seems to have happened here.

:-|

Derek_edX 在5 days ago前发表

添加评论

younhun (Staff)
3 days ago

All good points. As an alternative to SergK's suggestion, when we write

$$(\overline{X}_n - \mu) \to_d N\left(0, \sigma^2/n\right)$$

what we actually mean is the weak statement of pointwise convergence:

$$\lim_{n\to\infty} F_n(x) - G_n(x) \to 0, \qquad \text{for all } x$$

where F_n and G_n are the CDFs of $\overline{X}_n - \mu$ and $N\left(0, \sigma^2/n\right)$.

Hopefully the intent was clear despite the initial abuse of notation. I'll think about a way to address this.

The condition $\lim_{n\to\infty} F_n(x) - G_n(x) = 0$ is satisfied for a large class of sequences $\{G_n\}$ of cumulative distribution functions. Indeed, if F_n is the CDF of $\overline{R}_n - p$, then the new criterion is that $G_n(x)$ converges to the same step function that is the limit of F_n, namely, the function that jumps from 0 to 1 at $x = 0$ while taking the value $\frac{1}{2}$ at that point. For example, G_n could be the CDF of a uniform $U\left[-\frac{1}{n}, \frac{1}{n}\right]$ or a normal $N\left(0, \frac{\sigma^2}{n^\alpha}\right)$ for any positive exponent α. Any sequence of random variables that converges to 0 in probability and has the minor additional technical property that its CDF's have $G_n(0) \to \frac{1}{2}$ will work.

david301 在about 15 hours ago前发表

添加评论

显示所有的回复

添加一条回复：

Preview

提交

认证证书是什么?