6. Modeling Inter-arrival Times of a Subway System

Review: Exponential Random Variables

3/3 points (graded)

Let $X \sim \exp(\lambda)$ for some $\lambda > 0$. Which of the following is the (smallest possible) sample space for X?

- $\mathbb{N}$
- $\mathbb{Z}$
- $[0, \infty)$ ✔
- $(-\infty, \infty)$

Which of the following is the probability density function (pdf) for X? (Assume that $x > 0$).

- $\lambda e^{-\lambda x}$ ✔
- $\frac{1}{\lambda} e^{-\lambda x}$
- $\lambda e^{\lambda x}$
- $\lambda e^{-\lambda x^2}$

What is $\mathbb{E}[X]$?

(By now, you may simply memorize this and not rederive it everytime.)

$\mathbb{E}[X] =$ 1/lambda ✔ **Answer:** 1/lambda

$\frac{1}{\lambda}$

STANDARD NOTATION

Solution:

- An exponential random variable takes values on all positive real numbers. Therefore, the smallest possible sample space for X is given by $[0, \infty)$.
- By definition, the density of an exponential random variable is given by the function $x \mapsto \lambda e^{-\lambda x}$.
- For completeness, we use the formula for the density to compute the mean of an exponential random variable. By definition and integration by parts,

$$\begin{aligned}\mathbb{E}[X] &= \int_0^\infty x\lambda e^{-\lambda x}\, dx \\ &= -xe^{-\lambda x}\Big|_\infty^0 + \int_0^\infty e^{-\lambda x}\, dx\end{aligned}$$

$$= 0 - \frac{1}{\lambda}e^{-\lambda x}\Big|_{\infty}^{0}$$
$$= \frac{1}{\lambda}.$$

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

Video note: In the video below, the “T" refers to the subway train in the public transportation system in Boston.

Modeling Inter-arrival Times of a Subway System

distributions that
is parameterized by one single real parameter
so that it's easier for me.
Of course, in reality, there are many things
that impact arrival times of the T. For example,
if it rains in Bangkok, the T will be late.
And so you observe-- so the data you collect is very simple.
So rather than just observing arrival times--
and we'll just observe inter-arrival times--
so if I just missed the T, how much
do I need to wait for the next one?
So I observed this time that's in minutes,
and I'm going to call them T1 to Tn.
So this is the number of times--
the times in minutes in between two consecutive T's.
So let's say I have time.

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Memoryless Property of Exponential Random Variables

2/2 points (graded)
Let $X \sim \exp(1)$. What is $\mathbf{P}(X > 3)$?

$\mathbf{P}(X > 3) =$ exp(-3) ✔ **Answer:** exp(-3)

Let $t > 0$. What is $\mathbf{P}(X > t+3 | X > t)$?

$\mathbf{P}(X > t+3 | X > t) =$ exp(-3) ✔ **Answer:** exp(-3)

Solution:

The density of $\exp(1)$ is given by e^{-x}. Therefore,

$$\mathbf{P}(X > 3) = \left.\int_{3}^{\infty} e^{-x} \, dx = - e^{-x} \right|_3^\infty = e^{-3}.$$

Next, by the memoryless property of the exponential distribution, for any $s, t > 0$, it holds that

$$\mathbf{P}(X > s + t | X > t) = \mathbf{P}(X > s).$$

Apply the above equality with $s = 3$ shows that

$$\mathbf{P}(X > 3 + t | X > t) = \mathbf{P}(X > 3) = \exp(-3).$$

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Answers are displayed within the problem

讨论

显示讨论

主题： Unit 2 Foundation of Inference:Lecture 5: Delta Method and Confidence Intervals / 6. Modeling Inter-arrival Times of a Subway System

认证证书是什么?