Use same setup as in the previous problems.

**Setup as in the previous problems**

An aerospace manufacturing company would like to assess the performance of its existing planes for its latest design. Based on a sample size of $n = 1000$ flights, each with an identically designed plane, it collects data of the form $(x_1, y_1), \ldots, (x_{1000}, y_{1000})$, where $x$ represents the distance traveled and $y$ represents liters of fuel consumed.

You, as a statistician hired by the company, decide to perform linear regression on the model $y = a + bx$ to predict the efficiency of the design. In the context of linear regression, recall that the mathematical model calls for:

$$\mathbf{Y} = \begin{pmatrix} y_1 \\ \vdots \\ y_{1000} \end{pmatrix} \in \mathbb{R}^{1000}, \quad \boldsymbol{\varepsilon} \in \mathbb{R}^{1000}, \quad \mathbb{X} = \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_{1000} \end{pmatrix} \in \mathbb{R}^{1000\times 2}, \quad \boldsymbol{\beta} = \begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{R}^2.$$

Assume that $\boldsymbol{\varepsilon} \sim \mathcal{N}(0, \sigma^2 I_{1000})$ for some fixed $\sigma^2$, so that $\mathbf{Y} \sim \mathcal{N}(\mathbb{X}\boldsymbol{\beta}, \sigma^2 I_{1000})$.

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Recall the formula for the mean prediction error $\mathbb{E}\|\mathbf{Y} - \mathbb{X}\hat{\boldsymbol{\beta}}\|_2^2$ in terms of $n$, $p$ and $\sigma^2$ (given in the solution of the previous problem). This suggests a formula for an unbiased estimator of $\sigma^2$:

$$\hat{\sigma}^2 = \frac{1}{n-p}\|\mathbf{Y} - \mathbb{X}\hat{\boldsymbol{\beta}}\|_2^2.$$

Assume that based on your findings, the **residual sum** $\|\mathbf{Y} - \mathbb{X}\hat{\boldsymbol{\beta}}\|_2^2$ evaluates to $104.9$. Compute $\hat{\sigma}^2$, to the nearest thousandths ($10^{-3}$) digit.

0.1051 ✔ **Answer:** 0.105

**Solution:**

The formula calls for $n = 1000$, $p = 2$ and $\|\mathbf{Y} - \mathbb{X}\hat{\boldsymbol{\beta}}\|_2^2 = 104.9$. Plugging these numbers in gives $\hat{\sigma^2} = \frac{104.9}{1000-2} \approx 0.105$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem