11. Exercise: Covariance properties

Exercise: Covariance properties

3/3 points (graded)

a) Is it true that $\textsf{Cov}(X,Y) = \textsf{Cov}(Y,X)$?

True ▼ ✔ Answer: True

b) Find the value of a in the relation $\textsf{Cov}(2X, -3Y+2) = a \cdot \textsf{Cov}(X,Y)$.

$a =$ -6 ✔ Answer: -6

c) Suppose that X, Y, and Z are independent, with a common variance of 5. Then,

$\textsf{Cov}(2X+Y, 3X-4Z) =$ 30 ✔ Answer: 30

Solution:

a) We have $\big(X-\mathbf{E}[X]\big)\big(Y-\mathbf{E}[Y]\big) = \big(Y-\mathbf{E}[Y]\big)\big(X-\mathbf{E}[X]\big)$, and after taking expectations we obtain $\textsf{Cov}(X,Y) = \textsf{Cov}(Y,X)$.

b) We have argued that $\textsf{Cov}(aX+b, Y) = a \cdot \textsf{Cov}(X,Y)$. Note that by symmetry, we also have $\textsf{Cov}(X, aY+b) = a \cdot \textsf{Cov}(X,Y)$. By using these relations,

$$\textsf{Cov}(2X, -3Y+2) = 2 \cdot \textsf{Cov}(X, -3Y+2) = 2 \cdot (-3) \cdot \textsf{Cov}(X,Y) = -6\,\textsf{Cov}(X,Y).$$

c) Using linearity,

$$\begin{aligned}
\textsf{Cov}(2X+Y, 3X-4Z) &= \textsf{Cov}(2X+Y, 3X) + \textsf{Cov}(2X+Y, -4Z) \\
&= \textsf{Cov}(2X, 3X) + \textsf{Cov}(Y, 3X) + \textsf{Cov}(2X, -4Z) + \textsf{Cov}(Y, -4Z) \\
&= 6\,\textsf{Var}(X) + 0 + 0 + 0 = 30,
\end{aligned}$$

where the zeros are obtained because independent random variables have zero covariance.

提交 You have used 1 of 3 attempts