Homework 1.1: Passive membrane

Step current

3/3 points (graded)

The voltage across a passive membrane can be described by the equation:

$$\tau \frac{du(t)}{dt} = -\left(u(t) - u_{rest}\right) + RI(t)$$

Consider a step current input $I(t)$ such that $I(t) = 0$ for $t < t_0$ and $I(t) = I_0$ for $t \geq t_0$. We want to calculate the voltage $u(t)$, given that the neuron is at rest at time t_0, i.e., $u(t = t_0) = u_{rest}$.

What is the asymptotic value of $u(t)$ for $t >> t_0$? (Note that $\tau = RC$)

- ○ $u(\infty) = u_{rest}$
- ○ $u(\infty) = u_{rest} - RI_0$
- ○ $u(\infty) = u_{rest} + \frac{C}{\tau} I_0$
- ● $u(\infty) = u_{rest} + RI_0$
- ○ $u(\infty) = RI_0$

✓

What is the functional form of the transition from u_{rest} to the asymptotic value calculated above?

- ○ Quadratic
- ○ Linear
- ● Exponential
- ○ Logarithmic

✓

What is the general form of the solution for the membrane potential $u(t)$ for $t \geq t_0$?

for $t \geq 0, u(t) = ?$

- ○ $u(t) = u_{rest} + RI_0 \left[1 - \frac{\tau}{t - t_0 + \tau}\right]$
- ○ $u(t) = RI_0 + u_{rest} \left[1 - \frac{\tau}{t - t_0 + \tau}\right]$
- ● $u(t) = u_{rest} + RI_0 \left[1 - \exp\left(-\frac{(t - t_0)}{\tau}\right)\right]$
- ○ $u(t) = RI_0 + u_{rest} \left[1 - \exp\left(-\frac{(t - t_0)}{\tau}\right)\right]$

- [ ] $u(t) = u_{rest} + RI_0 \left[1 - \left(\ln\left(\exp(1) + \frac{(t-t_0)}{\tau}\right)\right)^{-1}\right]$
- [ ] $u(t) = RI_0 + U_{rest} \left[1 - \left(\ln\left(\exp(1) + \frac{(t-t_0)}{\tau}\right)\right)^{-1}\right]$

✔

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Pulse current

2/2 points (graded)

Now consider a pulse current input with total electrical charge q, i.e.,

$I(t) = \frac{q}{\Delta}\left[\Theta(t - t_0) - \Theta(t - t_0 - \Delta)\right]$

where $\Theta(.)$ is the Heaviside function and Δ is the pulse duration. For an extreme case $\Delta \to 0$, it can be written as $I(t) = q\delta(t - t_0)$ where $\delta(.)$ is the Dirac delta function. Given that the neuron is at rest at time t_0, what is the correct description of $u(t)$ for $t \geq t_0$? Note that multiple choices may be correct.

- [x] If $I(t)$ is considered as a current pulse of duration Δ and amplitude q/Δ, the voltage starts by increasing exponentially towards the asymptotic value $u_{rest} + Rq/\Delta$ with a time constant τ for the duration of the pulse. The voltage reaches a maximum at the end of the pulse, after which it decreases back to its resting value.
- [ ] It exponentially approaches to the new potential value $u_{rest} + Rq$ and stays fixed there.
- [x] In the extreme case, the membrane potential instantaneously jumps an amount qR/τ and then decays exponentially to its resting value with a time constant τ.
- [ ] The physical behavior of the neuron would be exactly the same as when a step current which is previously injected to a neuron, for sufficiently long time, is removed.

✔

For the extreme case $I(t) = q\delta(t - t_0)$, what is the general solution of $u(t)$ for $t \geq t_0$?

for $t \geq t_0, u(t) = ?$

- [x] $u(t) = u_{rest} + \frac{qR}{\tau}\exp\left(-\frac{t-t_0}{\tau}\right)$
- [ ] $u(t) = u_{rest} + qR\exp\left(-\frac{t-t_0}{\tau}\right)$
- [ ] $u(t) = u_{rest} + \frac{qR}{\tau}\left(\ln\left(\exp(1) + \frac{t-t_0}{\tau}\right)\right)^{-1}$
- [ ] $u(t) = u_{rest} + qR\left(\ln\left(\exp(1) + \frac{t-t_0}{\tau}\right)\right)^{-1}$
- [ ] $u(t) = u_{rest} + \frac{qR}{\tau}\left[\frac{t-t_0+\tau}{\tau}\right]^{-1}$
- [ ] $u(t) = u_{rest} + qR\left[\frac{t-t_0+\tau}{\tau}\right]^{-1}$

✔

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Arbitrary current input

1/1 point (graded)

Assuming that before a given time t_0, the current is null, i.e., $I(t < t_0) = 0$, and the membrane potential is at rest, derive the general solution of the passive membrane equation above, for arbitrary $I(t)$.

$u(t) = ?$

- ( ) $u(t) = u_{rest} + \frac{R}{\tau}\int_{t_0}^{t} \exp\left(-\frac{t}{\tau}\right) I(r)\, dr$
- ( ) $u(t) = u_{rest} + R\int_{t_0}^{t} \exp\left(-\frac{t-r}{\tau}\right) I(r)\, dr$
- (•) $u(t) = u_{rest} + \frac{R}{\tau}\int_{t_0}^{t} \exp\left(-\frac{t-r}{\tau}\right) I(r)\, dr$ t_0之前啥也没有，t时刻的电流
- ( ) $u(t) = u_{rest} + \frac{R}{\tau}\int_{-\infty}^{t} \exp\left(-\frac{t_0-r}{\tau}\right) I(r)\, dr$
- ( ) $u(t) = u_{rest} + \frac{R}{\tau}\int_{t_0}^{t} \frac{\tau}{t+\tau} I(r)\, dr$
- ( ) $u(t) = u_{rest} + R\int_{t_0}^{t} \frac{\tau}{t-r+\tau} I(r)\, dr$
- ( ) $u(t) = u_{rest} + \frac{R}{\tau}\int_{t_0}^{t} \frac{\tau}{t-r+\tau} I(r)\, dr$
- ( ) $u(t) = u_{rest} + \frac{R}{\tau}\int_{-\infty}^{t} \frac{\tau}{t_0-r+\tau} I(r)\, dr$

✔

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With different Initial value

1/1 point (graded)

Now assume that the membrane potential before time t_0 is clamped to u_0 which is not necessarily equal to u_{rest}. The arbitrary current input $I(t)$ is injected at time t_0. How the membrane potential evolves in this situation? Note that $I(t < t_0) = 0$.

It can be shown that $u(t)$ for $t >= t_0$ is again the same as the expression mentioned above except that there is one extra term. In other words,

$u(t) = u_{rest} + g(t) + \int_{t_0}^{t} f(r)\, I(r)\, dr$

where $g(t)$ is added to the previous solution. What would be the expression $g(t)$?

for $t \geq t_0, g(t) = ?$

- ( ) $g(t) = (u_0 - u_{rest})\left(\frac{\tau}{t-t_0+\tau}\right)$
- ( ) $g(t) = (u_0)\left(\frac{\tau}{t-t_0+\tau}\right)$
- ( ) $g(t) = (u_0 - u_{rest})\left(\ln\left(\exp(1) + \frac{t-t_0}{\tau}\right)\right)^{-1}$
- ( ) $g(t) = (u_0)\left(\ln\left(\exp(1) + \frac{t-t_0}{\tau}\right)\right)^{-1}$
- (•) $g(t) = (u_0 - u_{rest})\exp\left(-\frac{t-t_0}{\tau}\right)$ 以exponential的速度从u_0衰减到u_rest

$g(t) = (u_0) \exp\left(-\frac{t-t_0}{\tau}\right)$

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