3. Forming a committee
Problem 3. Forming a committee

2.0/2.0 points (graded)
Out of five men and five women, we form a committee consisting of four different people. Assuming that each committee of size four is equally likely, find the probabilities of the following events:

1. The committee consists of two men and two women.

    10/21 ✔ **Answer:** 0.476

2. The committee has more women than men.

    11/42 ✔ **Answer:** 0.262

3. The committee has at least one man.

    41/42 ✔ **Answer:** 0.976

    For the remainder of the problem, assume that Alice and Bob are among the ten people being considered.

4. Both Alice and Bob are members of the committee.

    2/15 ✔ **Answer:** 0.133

Solution:

The total number of possible committees is $\binom{10}{4}$, as we are selecting a committee of 4 people out of 10 different people.

1. The number of ways that we can choose 2 out of 5 people is $\binom{5}{2}$. Thus, the number of ways that we can choose two men and two women is

$$\binom{5}{2} \times \binom{5}{2}.$$

Therefore, the desired probability is

$$\frac{\binom{5}{2}^2}{\binom{10}{4}} = 10/21 \approx 0.476.$$

2. The committee can consist of 3 women and 1 man, or all 4 women. The corresponding number of possible committees is $\binom{5}{3}\binom{5}{1}$ and $\binom{5}{4}$, respectively. Therefore, the desired probability is

$$\frac{\binom{5}{3}\binom{5}{1} + \binom{5}{4}}{\binom{10}{4}} = 11/42 \approx 0.262.$$

3. The desired probability is 1 minus the probability of the complementary event. The complementary event, that is, the event that the committee consists entirely of women, happens in 5 different ways (choosing the women not in the committee can be done in $\binom{5}{1} = 5$ different ways), and has probability

$$\frac{\binom{5}{4}}{\binom{10}{4}}.$$

Therefore, the desired probability is

$$1 - \frac{\binom{5}{1}}{\binom{10}{4}} = 41/42 \approx 0.976.$$

4. Having fixed Alice and Bob, in order to form a 4-person committee, there are 2 more members to be selected among the 8 remaining people. The total number of ways of doing so is $\binom{8}{2}$, and therefore, the desired probability is

$$\frac{\binom{8}{2}}{\binom{10}{4}} = 2/15 \approx 0.133.$$

🛈   Answers are displayed within the problem

讨论

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