The event $\{X < Y < Z\}$ can be expressed as $\{X < \min\{Y, Z\}\} \cap \{Y < Z\}$. Let Y and Z be the 1st arrival times of two independent Poisson processes with rates μ and ν. By merging the two processes, it should be clear that $Y < Z$ if and only if the first arrival of the merged process comes from the original process with rate μ, and thus

$$\mathbf{P}(Y < Z) = \frac{\mu}{\mu + \nu} \ .$$

Let X be the 1st arrival time of a third independent Poisson process with rate λ. Now $\{X < \min\{Y, Z\}\}$ if and only if the first arrival of the Poisson process obtained by merging the two processes with rates λ and $\mu + \nu$ comes from the original process with rate λ, and thus

$$\mathbf{P}(X < \min\{Y, Z\}) = \frac{\lambda}{\lambda + \mu + \nu} \ .$$

Suppose that the event $\{X < \min\{Y, Z\}\}$ has occurred and that X takes on some value t. After time t, the processes with rates μ and ν start fresh. Which one of them will be the first to record an arrival (that is, whether the event $\{Y < Z\}$ will occur) is independent of the past, and therefore independent of the event $\{X < \min\{Y, Z\}\}$. Hence,

$$\begin{aligned} \mathbf{P}(X < Y < Z) &= \mathbf{P}(X < \min\{Y, Z\}) \cdot \mathbf{P}(Y < Z) \\ &= \frac{\lambda \mu}{(\lambda + \mu + \nu)(\mu + \nu)}. \end{aligned}$$