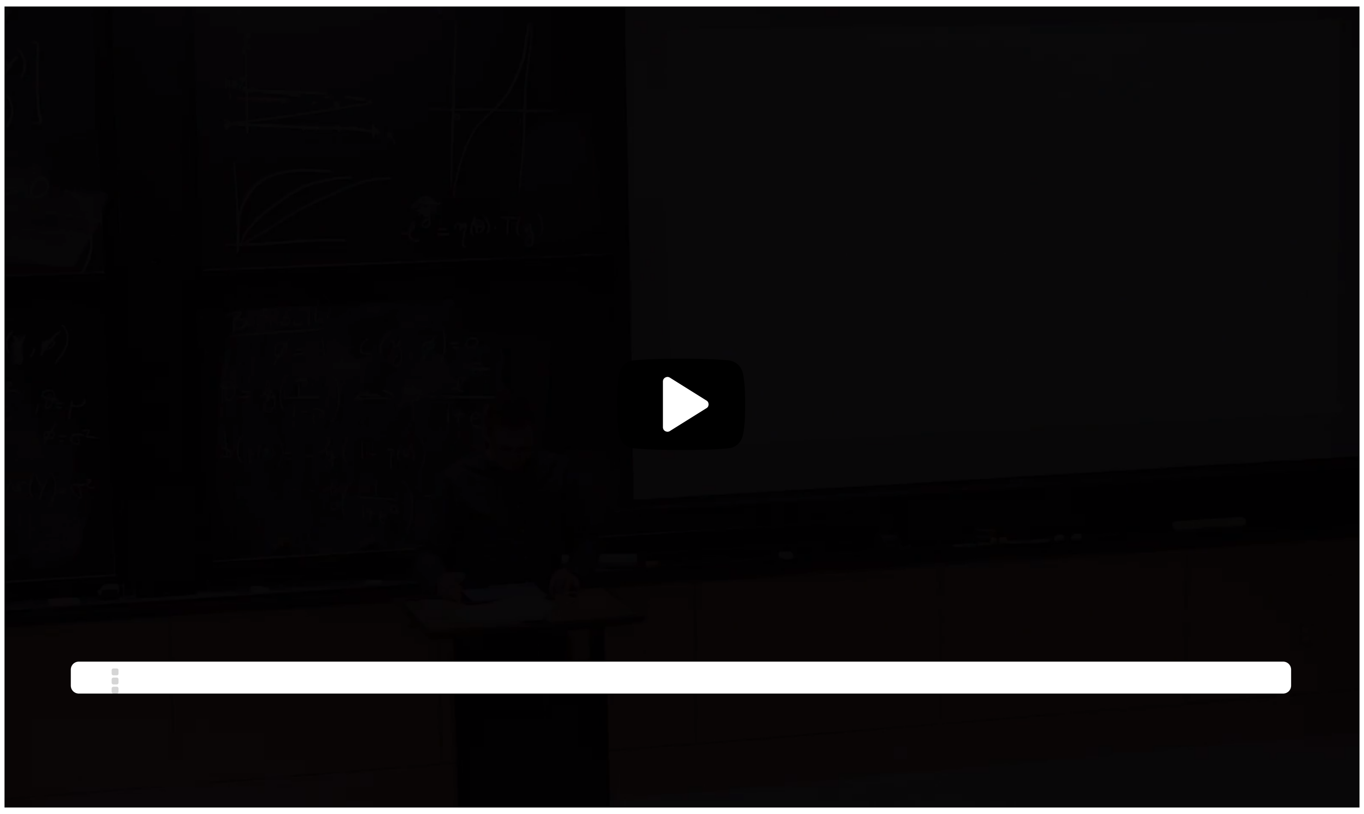
# 13. Variance in terms of the Canonical Parameter

## Variance in terms of the Canonical Parameter



7:43 / 7:43 1.0x

to link functions,

which are just a way to model how mu of x depends on x now

or how, in this case, theta of x depends on x.

Because we could talk about either mu

or we could talk about theta because if there's

only one parameter, it better depend on unknown mean.

So that's it.

Thank you.

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## Practice: the Mean and Variance of Binomial Distribution

2/2 points (graded)

Recall that the pmf of a Binomial distribution $\text{Binom}(n,p)$, with known $n$ can be written as:

$$f_\theta(y) \;=\; \exp\left(\frac{y\theta - b(\theta)}{\phi} + c(y,\phi)\right).$$

Refer to the answer of $b(\theta)$ and $\phi$ to the problem *Practice: Binomial Distribution as a Canonical Exponential Family* 2 pages before this one.

Compute $b'(\theta)$.

$b'(\theta) =$ e^theta*n/(1+e^theta) ✔ **Answer:** n*e^theta/(1+e^theta)

$\frac{e^\theta \cdot n}{1+e^\theta}$

(Is this equal to $\mathbb{E}[y]$?)

Compute $\phi\, b''(\theta)$.

$\phi\, b''(\theta) =$ e^theta*n/(1+e^theta)^2 ✔ **Answer:** n*e^theta/(1+e^theta)^2

$\frac{e^\theta \cdot n}{(1+e^\theta)^2}$

**Note:** Express your answers in terms of the canonical parameter $\theta$.

STANDARD NOTATION

**Solution:**

Recall

$$b(\theta) \;=\; n\ln\left(1+e^{\theta}\right).$$

Taking the derivative gives

$$b'(\theta) \;=\; \frac{db}{d\theta}(\theta) \;=\; \frac{ne^{\theta}}{1+e^{\theta}}.$$

Recall that $\theta = \ln\left(\frac{p}{1-p}\right)$ so $e^{\theta} = \frac{p}{1-p}$. Plugging this in to the equation above gives

$$b'(\theta(p)) \;=\; \frac{ne^{\theta}}{1+e^{\theta}} = np$$

which is, as expected, equal to $\mathbb{E}[Y]$ where $Y \sim$ Binom $(n,p)$.

Take the second derivative of $b(\theta)$:

$$\begin{aligned} b''(\theta) \;&=\; \frac{db}{d\theta}\frac{ne^{\theta}}{1+e^{\theta}} \\ &=\; n\frac{e^{\theta}\left(1+e^{\theta}\right)-\left(e^{\theta}\right)e^{\theta}}{\left(1+e^{\theta}\right)^2} \\ &=\; n\frac{e^{\theta}}{\left(1+e^{\theta}\right)^2} \end{aligned}$$

Recall that $\phi = 1,$ so $\phi b''(\theta) = b''(\theta)$. Rewriting $\phi b''(\theta)$ in terms of $p$ gives $\phi b''(\theta(p)) = np(1-p),$ which is indeed the variance of a binomial variable $Y \sim$ Binom $(n,p)$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

## The log-partition function b

4/4 points (graded)

For each proposed function $b$ shown below, indicate (based only on its second derivative) whether it could potentially be a log-partition function of some exponential family **with dispersion** $\phi = 1$.

- $b(\theta) = \theta^2 - 2\theta + 1$

  - (selected) Valid ✓
  - Invalid

- $b(\theta) = \sqrt{\theta}$

  - Valid
  - (selected) Invalid ✓

- $b(\theta) = \ln \theta$

  ○ Valid

  ◉ Invalid ✔

- $b(\theta) = \theta$

  ◉ Valid ✔

  ○ Invalid

**Solution:**

Recall that in a canonical exponential family, $b''(\theta) \cdot \phi = \text{Var}(Y)$ in lecture, which is always non-negative, i.e. $b(\theta)$ must be convex. Not all of the functions listed satisfy this property.

- Yes. Since the second derivative is positive, it is convex and therefore valid.
- No. Since the second derivative is negative, it is not convex and therefore invalid.
- No. Since the second derivative is negative, it is not convex and therefore invalid.
- Yes. Since the second derivative is non-negative, it is convex and therefore valid.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

## Discussion

Show Discussion

**Topic:** Unit 7 Generalized Linear Models:Lecture 21: Introduction to Generalized Linear Models; Exponential Families / 13. Variance in terms of the Canonical Parameter

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