4. Modeling Assumptions in Regression

Review: Joint, Conditional, and Marginal Distributions

2/2 points (graded)

Let (X, Y) be a pair of random variables with **joint** density $h(x, y) = x + y$ over the space $[0, 1]^2$.

What is the **marginal density** of X? We denote this by writing $h(x)$.

$h(x) =$ x+1/2 ✔ **Answer:** 0.5+x

$x + \frac{1}{2}$

What is the **conditional density** of Y given $X = x$? We denote this by writing $h(y|x)$.

$h(y|x) =$ (x+y)/(x+1/2) ✔ **Answer:** (x+y)/(0.5+x)

$\frac{x+y}{x+\frac{1}{2}}$

STANDARD NOTATION

Solution:

The marginal density $h(x)$ is computed by integrating over y:

$$\begin{aligned} h(x) &= \int_0^1 h(x, y)\, dy \\ &= \left[xy + \frac{y^2}{2}\right]_0^1 \\ &= x + \frac{1}{2} \end{aligned}$$

The conditional density is computed as the ratio:

$$\begin{aligned} h(y|x) &= \frac{h(x, y)}{h(x)} \\ &= \frac{x + y}{x + \frac{1}{2}} \end{aligned}$$

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Answers are displayed within the problem

Conditional Variance Given x

1/1 point (graded)

Consider the joint density setup as in the previous problem. What is the variance of Y given $X = x$?

$\mathrm{Var}(Y|X = x) =$ (4*x+3)/(12*x+6)- ((3*x+: ✔ **Answer:** (x/3+1/4)/(0.5+x) - ((x/2+1/3)/(0.5+x))^2 + y*0

$\frac{4\cdot x+3}{12\cdot x+6} - \left(\frac{3\cdot x+2}{6\cdot x+3}\right)^2$

STANDARD NOTATION

Solution:

The conditional density $h(y|x)$ is

$$h(y|x) = \frac{x+y}{0.5+x}.$$

We need to compute the expectations $\mathbb{E}[Y|X=x]$ and $\mathbb{E}[Y^2|X=x]$ in order to compute the conditional variance of Y given $X=x$.

$$\begin{aligned}
\mathbb{E}[Y|X=x] &= \int_{y=0}^{y=1} \frac{y(x+y)}{0.5+x} \mathrm{d}y \\
&= \frac{1}{0.5+x} \int_0^1 yx + y^2 \mathrm{d}y \\
&= \frac{1}{0.5+x} \left[\frac{x}{2} + \frac{1}{3}\right] \\
&= \frac{\frac{x}{2}+\frac{1}{3}}{0.5+x}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\mathbb{E}[Y^2|X=x] &= \int_0^1 \frac{y^2(x+y)}{0.5+x} \mathrm{d}y \\
&= \frac{1}{0.5+x} \int_0^1 y^2x + y^3 \mathrm{d}y \\
&= \frac{1}{0.5+x} \left[\frac{x}{3} + \frac{1}{4}\right] \\
&= \frac{\frac{x}{3}+\frac{1}{4}}{0.5+x}
\end{aligned}$$

Therefore, the conditional variance given $X=x$ is

$$\mathsf{Var}(Y \mid X=x) = \mathbb{E}[Y^2|X=x] - \mathbb{E}[Y|X=x]^2 = \frac{\frac{x}{3}+\frac{1}{4}}{0.5+x} - \left(\frac{\frac{x}{2}+\frac{1}{3}}{0.5+x}\right)^2.$$

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Answers are displayed within the problem

Review: Joint, Conditional, and Marginal Distributions: Discrete Example

4/4 points (graded)

Let X be a discrete Poisson random variable with parameter λ. Given $X=x$, let Y be the binomial random variable $\mathsf{Binom}(x,p)$, where p is the binomial parameter.

Given $X=x$, what are the values that Y can take?

Lower limit of Y given $X=x$: 0 ✔ **Answer:** 0

Upper limit of Y given $X=x$: x ✔ **Answer:** x

x

Is $\mathbb{E}[Y \mid X=x]$ a linear function of x?

- [x] Yes ✔
- [ ] No

What is $\mathbb{E}[Y]$?

Hint: Use the tower property of expectation (law of iterated expectation).

lambda*p ✔ **Answer:** p*lambda

$\lambda \cdot p$

STANDARD NOTATION

Solution:

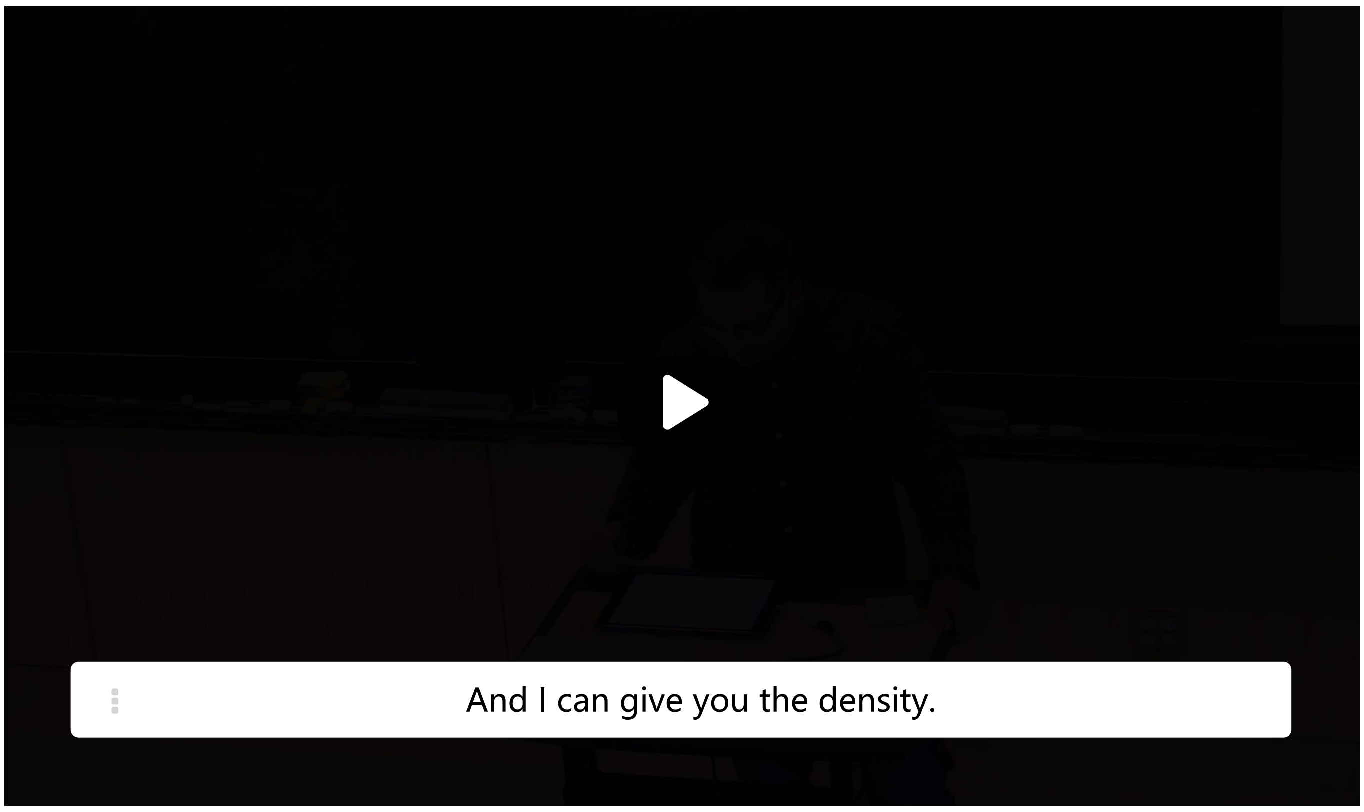
Given $X = x$, it is clear that Y can take values in the set $\{0, 1, \ldots, x\}$. The expectation of Y given $X = x$ is xp as $Y|X = x$ is a binomial random variable with parameters x and p. Therefore, this expectation is a linear function of x. Using the law of iterated expectation,

$$\begin{aligned}\mathbb{E}[Y] &= \mathbb{E}_X[\mathbb{E}[Y|X]] \\ &= \mathbb{E}_X[Xp] = p \cdot \lambda.\end{aligned}$$

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

Modeling Assumptions



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Everybody's with me?

OK, so now if I want to talk about regression,

regression is really about understanding

the conditional distribution of y given x.

And one way to do this is to understand

the conditional density of y given x, because it's telling

me, for a given x, y is a random variable that

will depend on this little x.

And I can give you the density.

End of transcript. Skip to the start.

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