(a) Recurrent: 1, 2, 4, 5, 6; Transient: 3; Periodic: 4, 5, 6.

(b) Given that the chain starts in state 3, the only way to remain in state 3 after n transitions is if all n transitions were self-transitions. Therefore, the desired probability is 0.2^n.

(c) The number of trials up to and including the trial on which the process leaves state 3 is a geometric random variable, X, with parameter $p = 0.8$. This is because on each transition, the process leaves state 3 if it transitions to either state 2 or state 4, which happens with a combined probability of $0.5 + 0.3 = 0.8$. Hence, the expected number of trials up to and including the trial on which the process leaves state 3 is $\mathbf{E}[X] = 1/p = 5/4$.

(d) Since the process starts in state 3, it will never enter state 1 if and only if, when it eventually leaves state 3, it transitions to the right to state 4. This probability is

$$\begin{aligned} & 0.3 + (0.2)(0.3) + (0.2)^2(0.3) + (0.2)^3(0.3) + \ldots \\ &= (0.3)(1 + 0.2 + 0.2^2 + 0.2^3 + \ldots) \\ &= (0.3)\left(\frac{1}{1-0.2}\right) \\ &= \frac{3}{8}. \end{aligned}$$

(e) Since states 4, 5, and 6 form a periodic recurrent class with period 3, the process will be in state 4 after 10 trials if and only if it first enters state 4 after 1 trial, or after 4 trials, or after 7 trials, or after 10 trials. These events have a combined probability of

$$0.3 + (0.2)^3(0.3) + (0.2)^6(0.3) + (0.2)^9(0.3) = 0.3024.$$

(f) Let A be the event that the process is in state 4 after 1 trial, and let B be the event that the process is in state 4 after 10 trials. Because of the periodicity, A implies B and hence $A \cap B = A$. Using the definition of conditional probability and our result from part (e), we then have that the desired probability is

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} = \frac{\mathbf{P}(A)}{\mathbf{P}(B)} = \frac{0.3}{0.3024} = 0.992.$$