8. Exercise: n-step recursion

Exercise: n-step recursion

3/3 points (ungraded)

Let $r_{ij}(n) = \mathbf{P}(X_n = j \mid X_0 = i)$ be the n-step transition probability of a given homogeneous discrete-time Markov chain with m states. We have shown that $r_{ij}(n)$ satisfies the following recursion for $n \geq 2$: $r_{ij}(n) = \sum_{k=1}^{m} r_{ik}(n-1)p_{kj}$. For each of the following, decide whether it is also a valid recursion formula for $r_{ij}(n)$.

1. $r_{ij}(n) = \sum_{k=1}^{m} p_{ik} r_{kj}(n-1)$ for $n \geq 2$

   Yes ✔ **Answer:** Yes

2. $r_{ij}(n) = \sum_{k=1}^{m} r_{ik}(n-2) r_{kj}(2)$ for $n \geq 3$

   Yes ✔ **Answer:** Yes

3. $r_{ij}(n) = \sum_{k=1}^{m} \sum_{\ell=1}^{m} r_{ik}(n-2) p_{k\ell} p_{\ell j}$ for $n \geq 3$

   Yes ✔ **Answer:** Yes

Solution:

1. Yes. The recursion considers a one-step transition from i to any state k, followed by an $(n-1)$-step transition from k to j.

2. Yes. The recursion considers an $(n-2)$-step transition from i to any state k, followed by a 2-step transition from k to j.

3. Yes. The recursion considers an $(n-2)$-step transition from i to any state k, followed by a one-step transition from k to any state ℓ, followed by a one-step transition from ℓ to j.

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 10 / Lec. 24 / 8. Exercise: n-step recursion