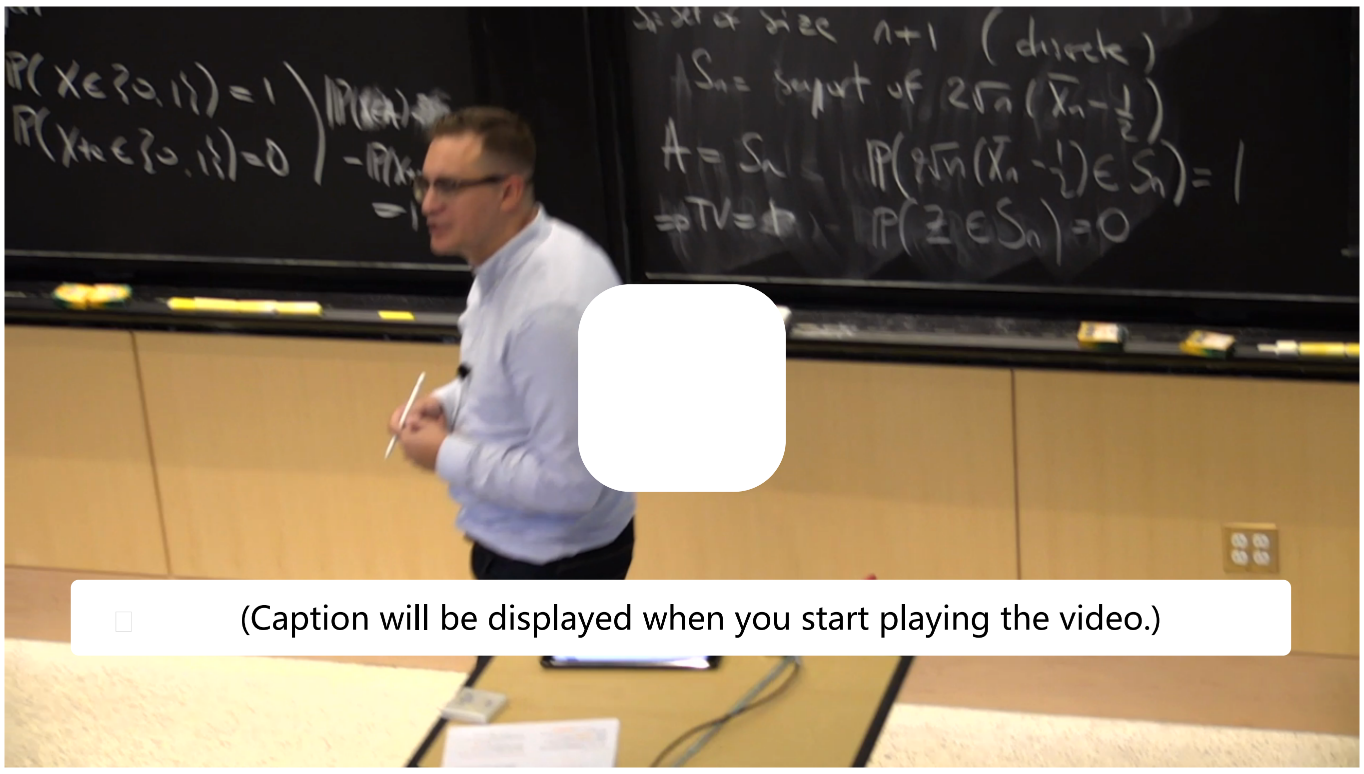
10. Motivation and Introduction to the Kullback-Leibler (KL) Divergence

An Estimation Strategy and Definition of Kullback-Leibler (KL) Divergence



Start of transcript. Skip to the end.

So let's try to find something that does that a little better.

So before that, let's see.

Now I've probably trashed my toleration

distance a little too much.

Maybe you don't want to move on from this.

But let's say it's still something that works.

Let's say we have two continuous distributions

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Definition of Kullback-Leibler (KL) Divergence

Let $\mathbf{P}$ and $\mathbf{Q}$ be **discrete** probability distributions with pmfs p and q respectively. Let's also assume $\mathbf{P}$ and $\mathbf{Q}$ have a common sample space E. Then the **KL divergence** (also known as **relative entropy**) between $\mathbf{P}$ and $\mathbf{Q}$ is defined by

$$\mathrm{KL}\left(\mathbf{P},\mathbf{Q}\right) = \sum_{x\in E} p\left(x\right)\ln\left(\frac{p\left(x\right)}{q\left(x\right)}\right),$$

where the sum is only over the support of $\mathbf{P}$.

Why do we sum only over the support of P?

We use the following limit to justify the definition above. At any point $x \in E$ outside the support of $\mathbf{P}$ but where $q\left(x\right) \neq 0$:

$$\begin{aligned} \lim_{p/q\to 0^+} q\left(\frac{p}{q}\right)\ln\left(\frac{p}{q}\right) &= q \lim_{p/q\to 0^+}\left(\frac{p}{q}\right)\ln\left(\frac{p}{q}\right) \\ &= q\cdot(0) = 0 \qquad (\text{by L'hopital's rule}). \end{aligned}$$

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Analogously, if $\mathbf{P}$ and $\mathbf{Q}$ are **continuous** probability distributions with pdfs p and q on a common sample space E, then

$$\mathrm{KL}(\mathbf{P},\mathbf{Q}) = \int_{x\in E} p(x)\ln\left(\frac{p(x)}{q(x)}\right) dx,$$

where the integral is again only over the support of $\mathbf{P}$.

Computing KL Divergence I

1/1 point (graded)
Let $X \sim \mathbf{P}_X = \mathrm{Ber}(1/2)$ and let $Y \sim \mathbf{P}_Y = \mathrm{Ber}(1/2)$. What is $\mathrm{KL}(\mathbf{P}_X, \mathbf{P}_Y)$?

$\mathrm{KL}(\mathbf{P}_X, \mathbf{P}_Y) =$ 0 **Answer:** 0.0

Solution:

Let p be the pmf of the distribution $\mathrm{Ber}(1/2)$. Note that the sample space is the discrete set $E = \{0, 1\}$. Then

$$\begin{aligned}\mathrm{KL}(\mathbf{P}_X, \mathbf{P}_Y) &= p(1)\ln(p(1)/p(1)) + p(0)\ln(p(0)/p(0)) \\ &= (1/2)\ln(1) + (1/2)\ln(1) = 0.\end{aligned}$$

Remark: Although KL divergence is not a distance on probability distributions (as we defined above), it does satisfy some of the axioms. For example,

- $\mathrm{KL}(\mathbf{P},\mathbf{Q}) \geq 0$ (nonnegative), and
- $\mathrm{KL}(\mathbf{P},\mathbf{Q}) = 0$ only if $\mathbf{P}$ and $\mathbf{Q}$ are the same distribution (definite).

Note that the result of this problem is consistent with the second property.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

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