

Machine Learning

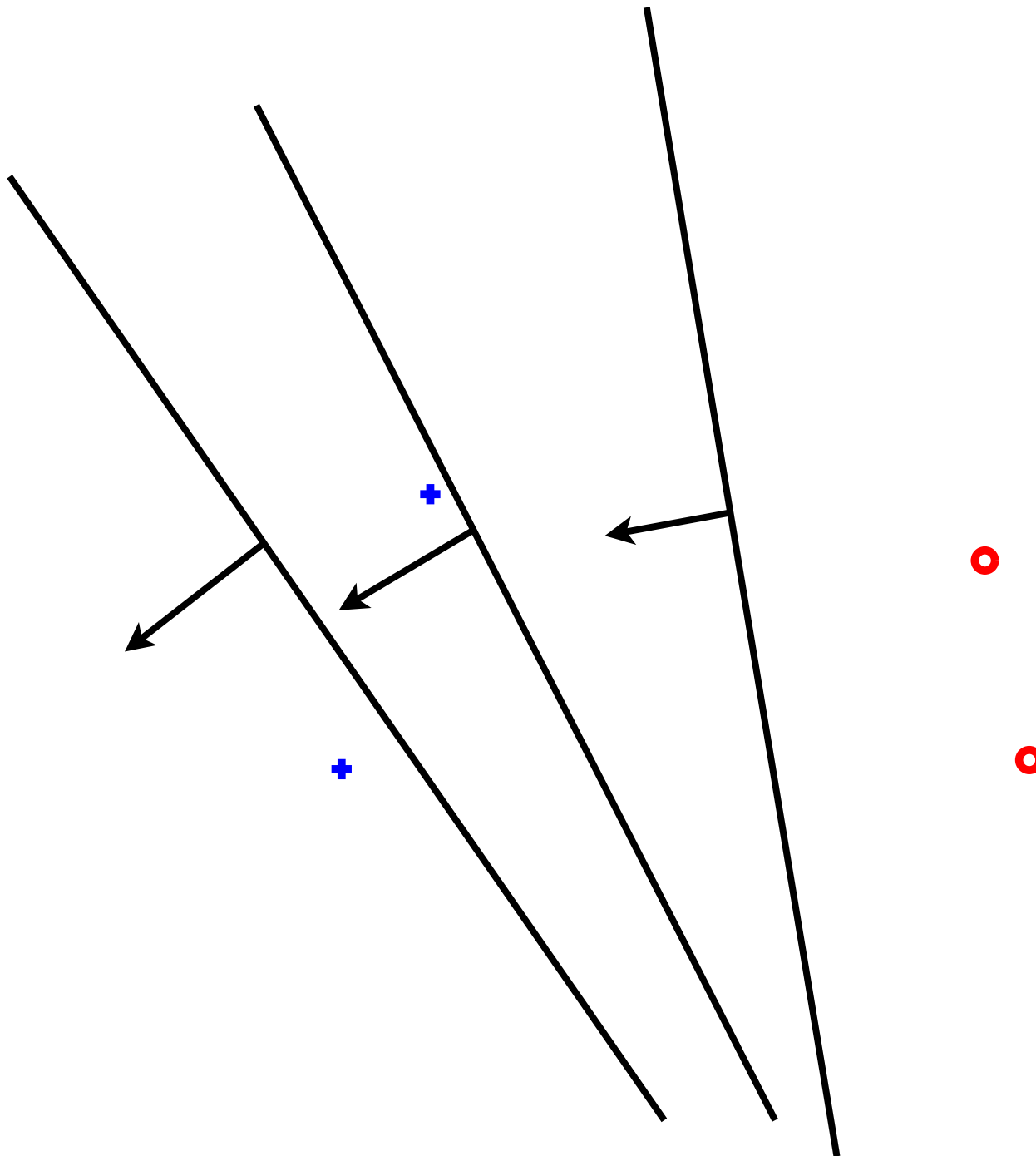
Lecture 3



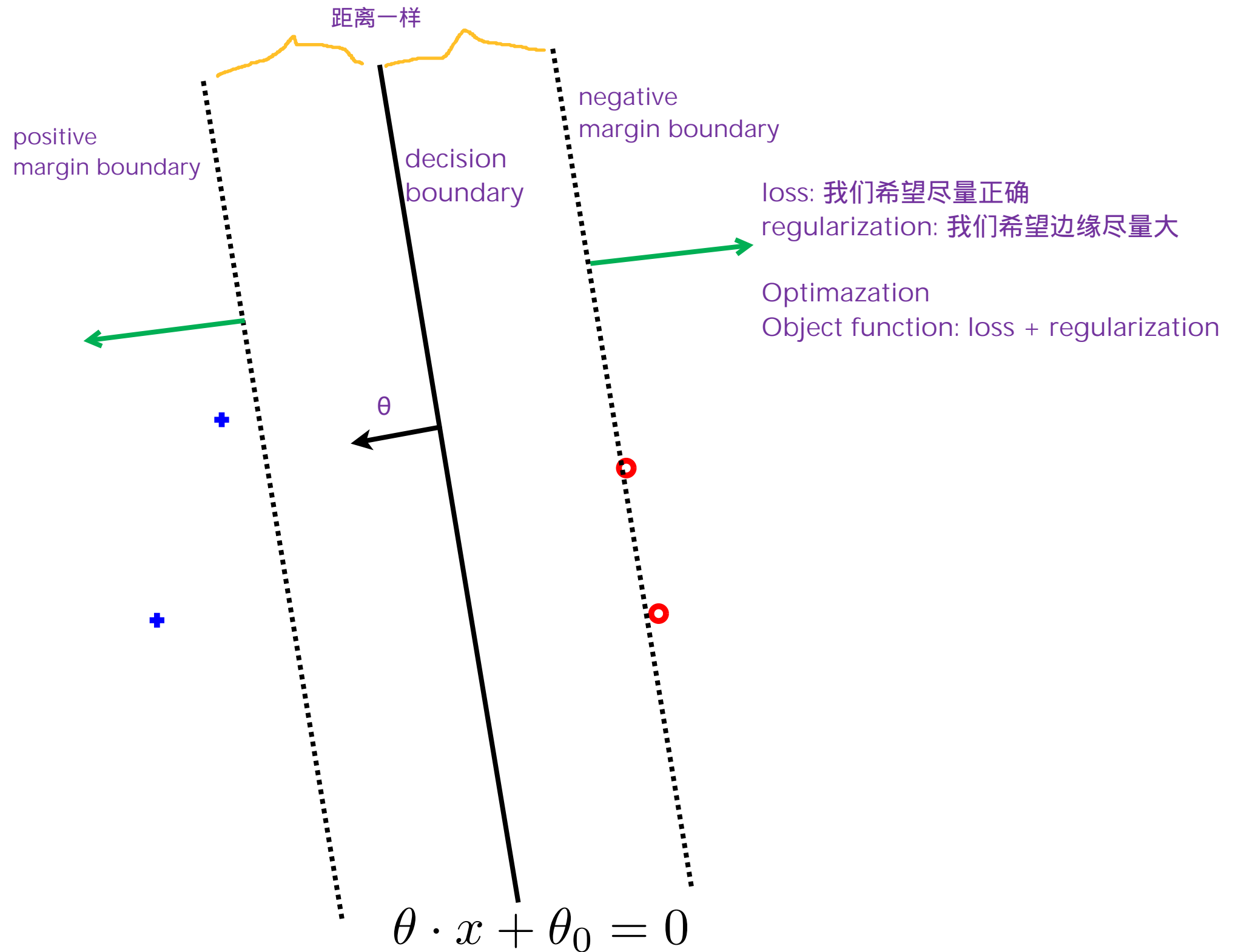
Outline

- Linear, large margin classification
 - margin, hinge loss, regularization
- Learning as an optimization problem

Linear classification



Learning as optimization





Learning as optimization

What exactly the margin boundaries are?
How we can control them?
How far they are from the decision boundary?

$\theta^*x + \theta_0 = 1$
这条线和decision boundary平行

$\theta^*x + \theta_0 = -1$

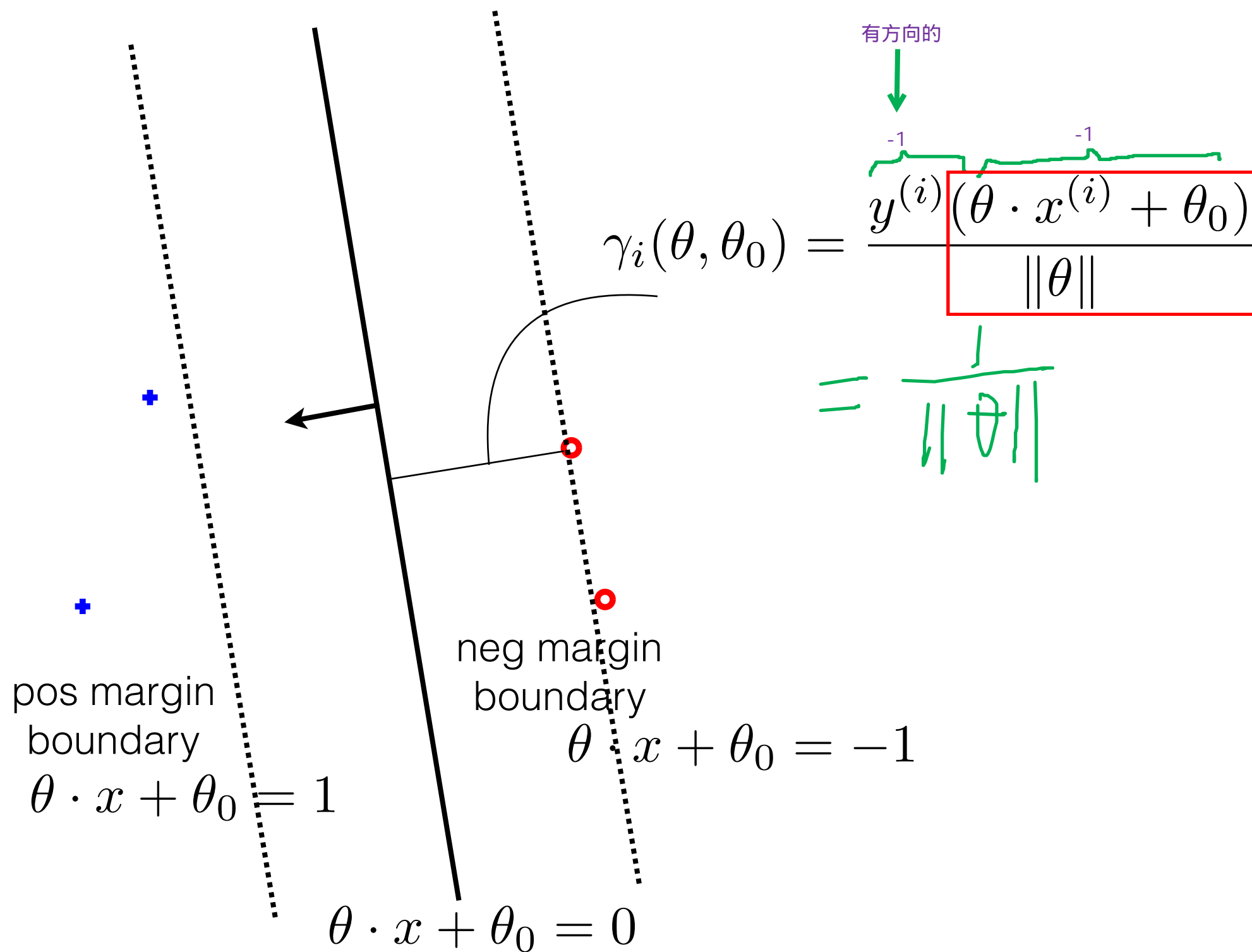
θ 和 θ_0 定义的decision boundary的位置方向
 $\text{norm}(\theta)$ 定义了margin的长度

$$\frac{\theta}{\|\theta\|}x + \frac{\theta}{\|\theta\|} = 0$$

相等

$$\theta \cdot x + \theta_0 = 0$$

Linear classification, margin





Large margin as optimization

▸ Hinge loss

$$\text{Loss}_h \left(\overbrace{y^{(i)} (\theta \cdot x^{(i)} + \theta_0)}^{\text{agreement: } z} \right) = \begin{cases} 0 & \text{if } z \geq 1 \\ 1 - z & \text{if } z < 1 \text{ (} z \text{ 越小, penalize 的越厉害)} \end{cases}$$

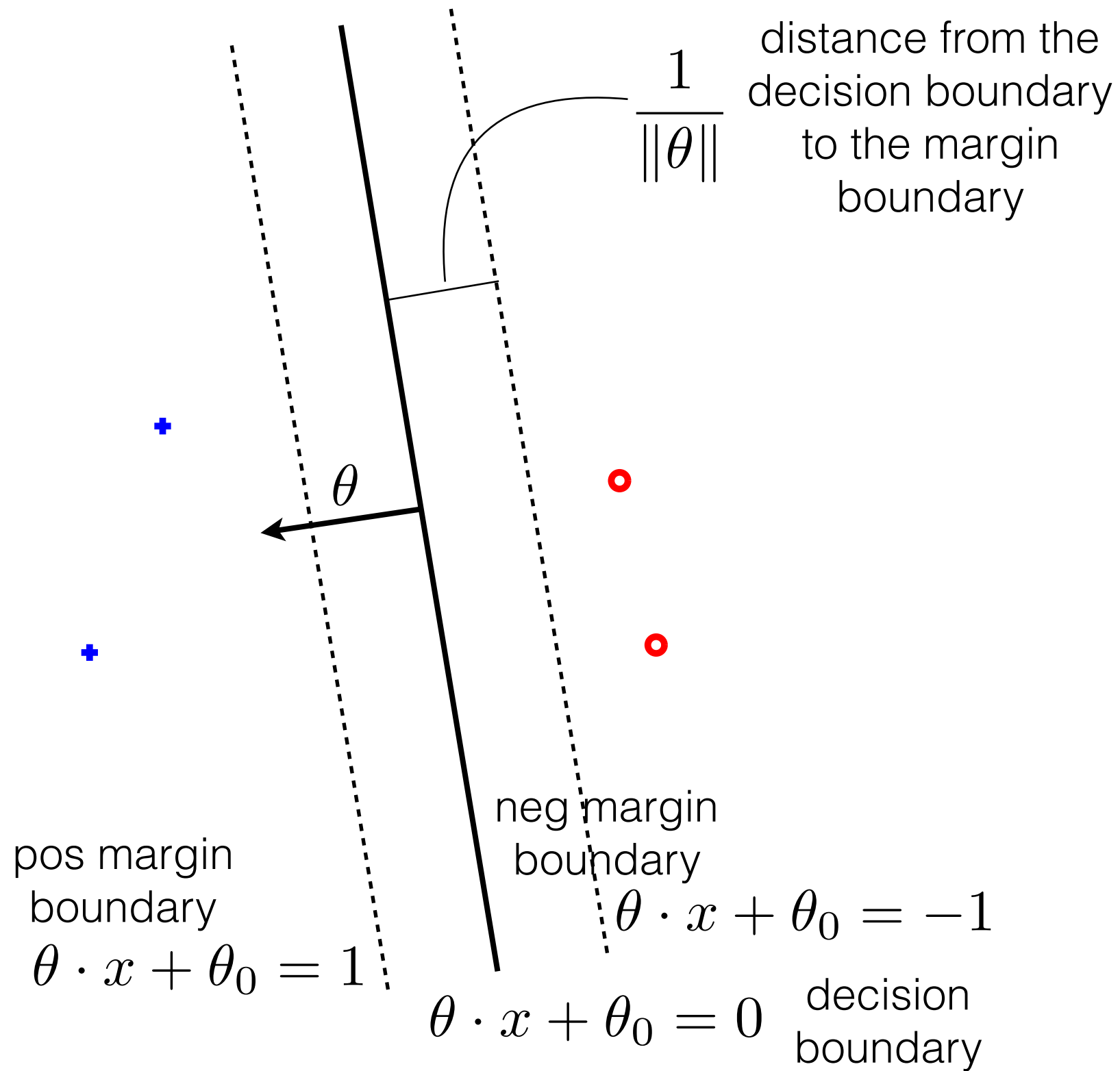
▸ Regularization: towards max margin

$$\max 1/\text{norm}(\theta) \rightarrow \min \text{norm}(\theta) \rightarrow \min 1/2 * \text{norm}(\theta)^2$$

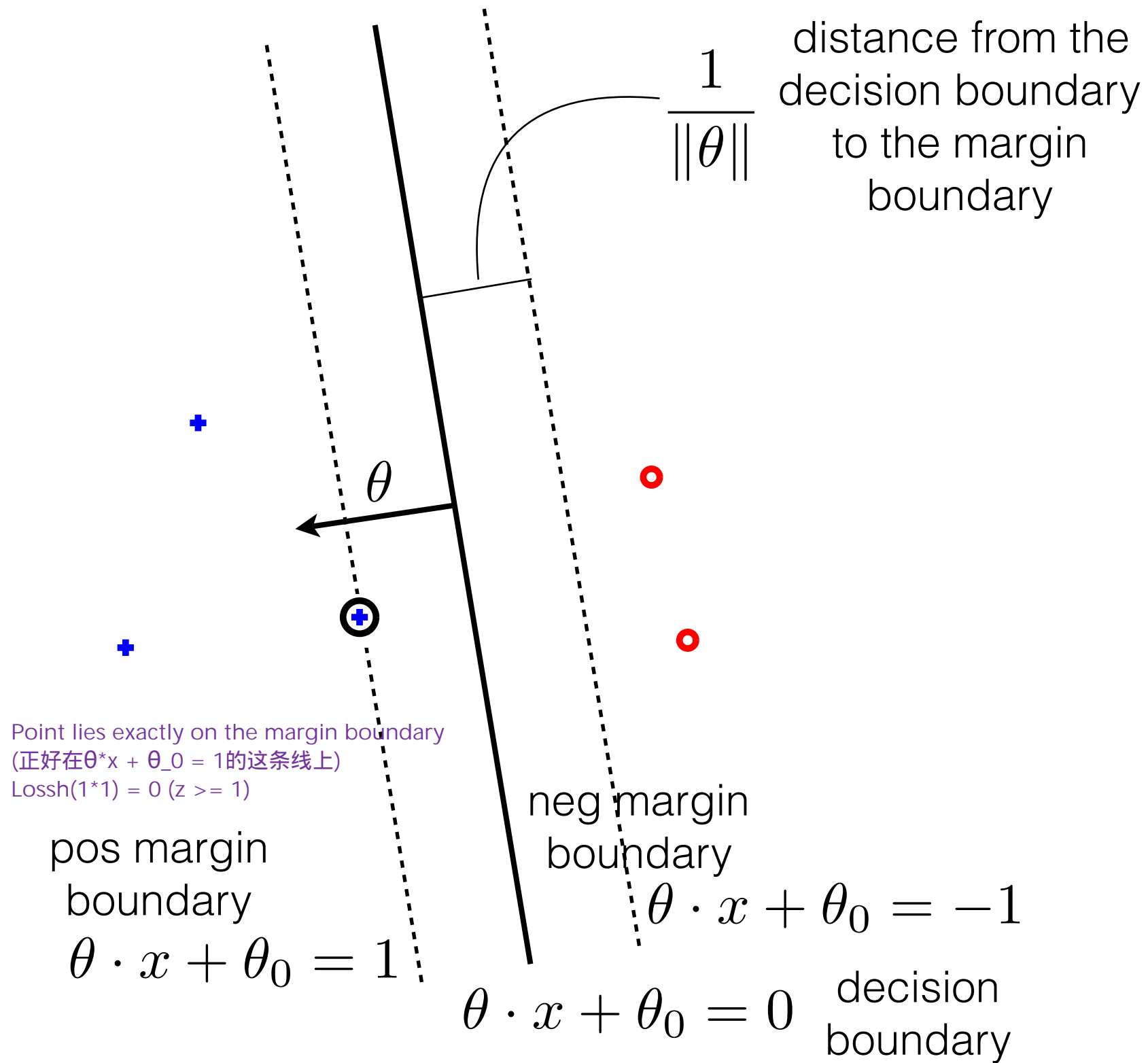
▸ The objective

$$J(\theta, \theta_0) = \frac{1}{n} \sum_{i=1}^n \text{Loss}_h \left(y^{(i)} (\theta \cdot x^{(i)} + \theta_0) \right) + \frac{\lambda}{2} \|\theta\|^2$$

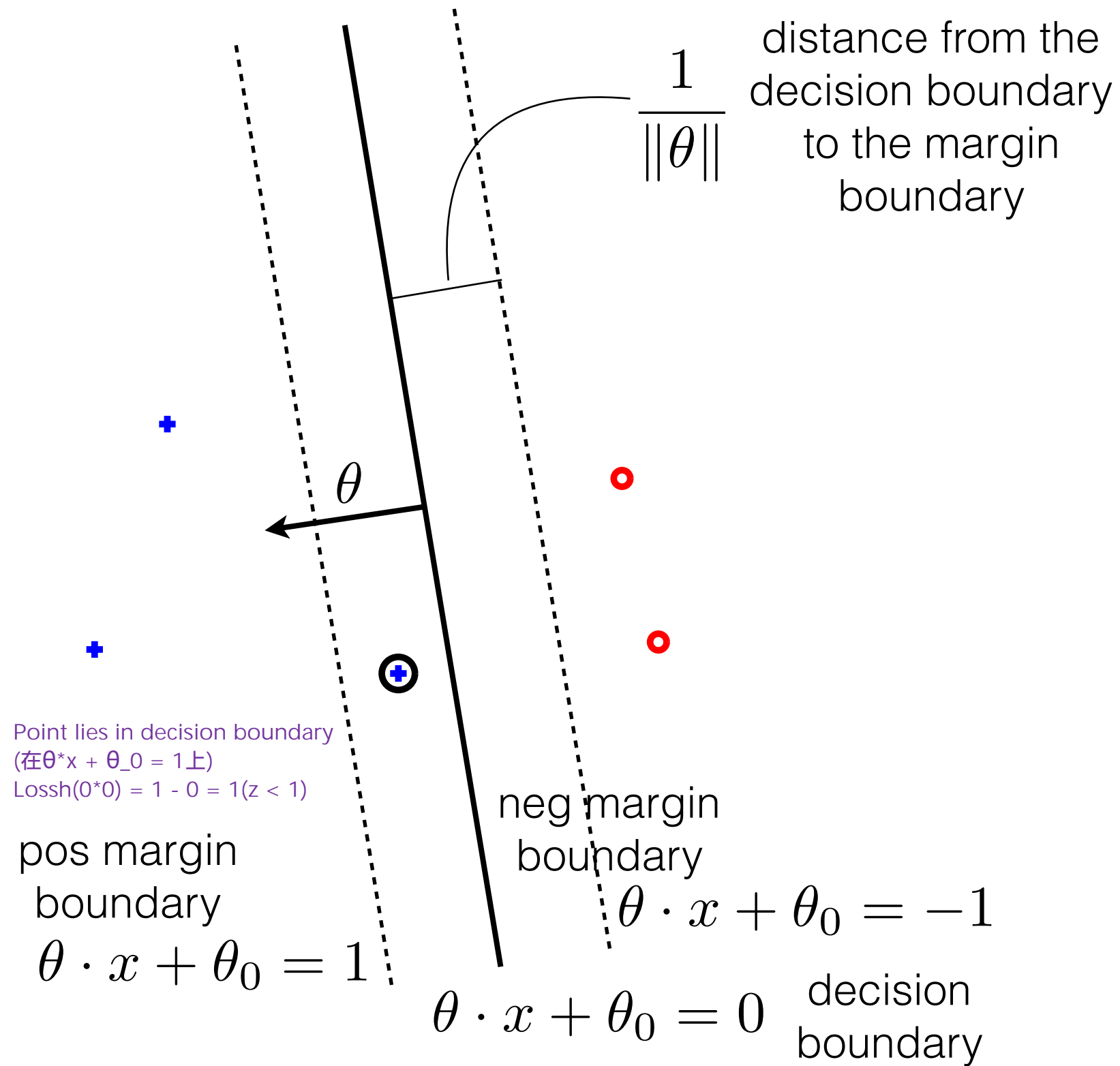
regularization parameter
>0



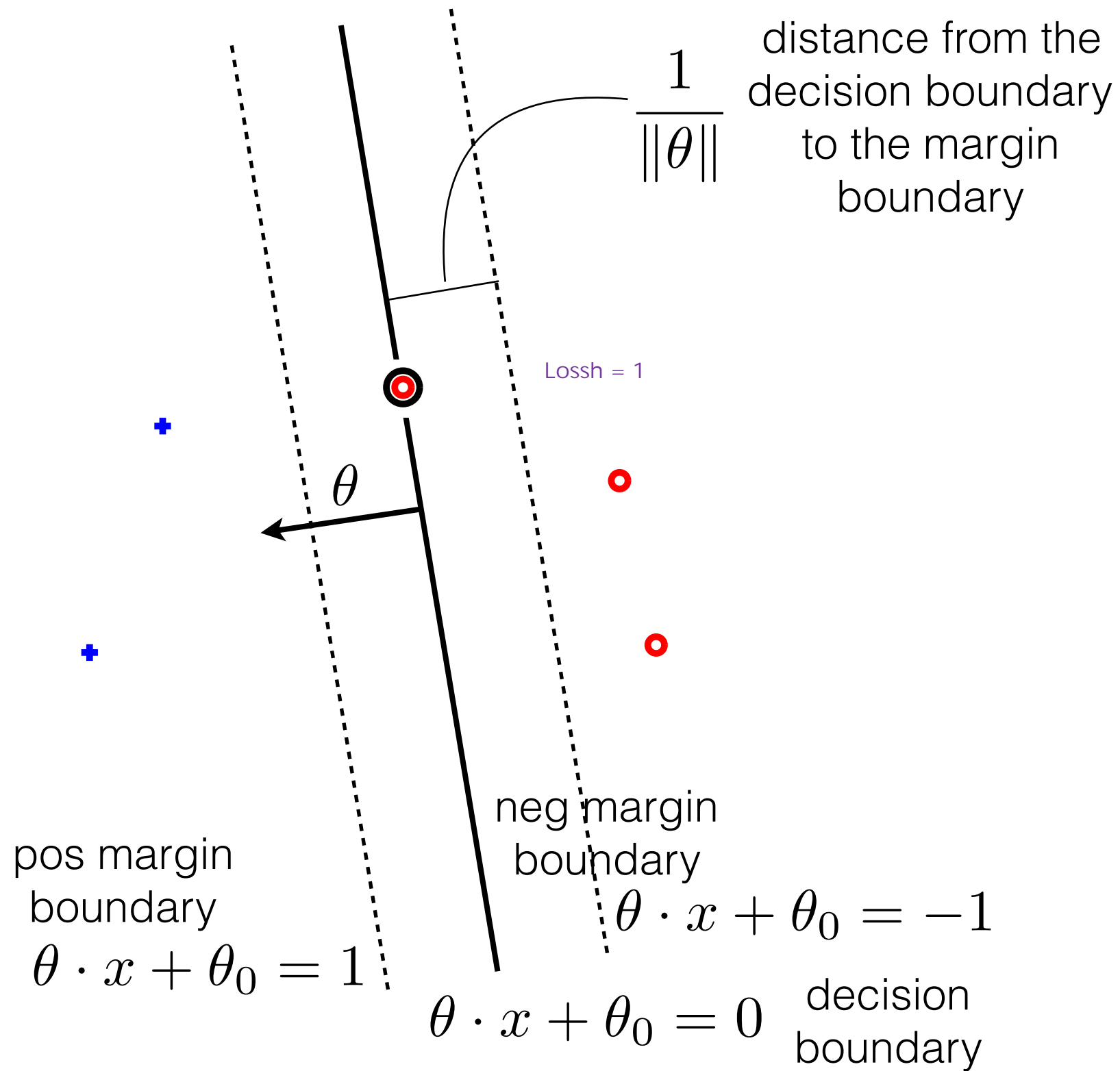
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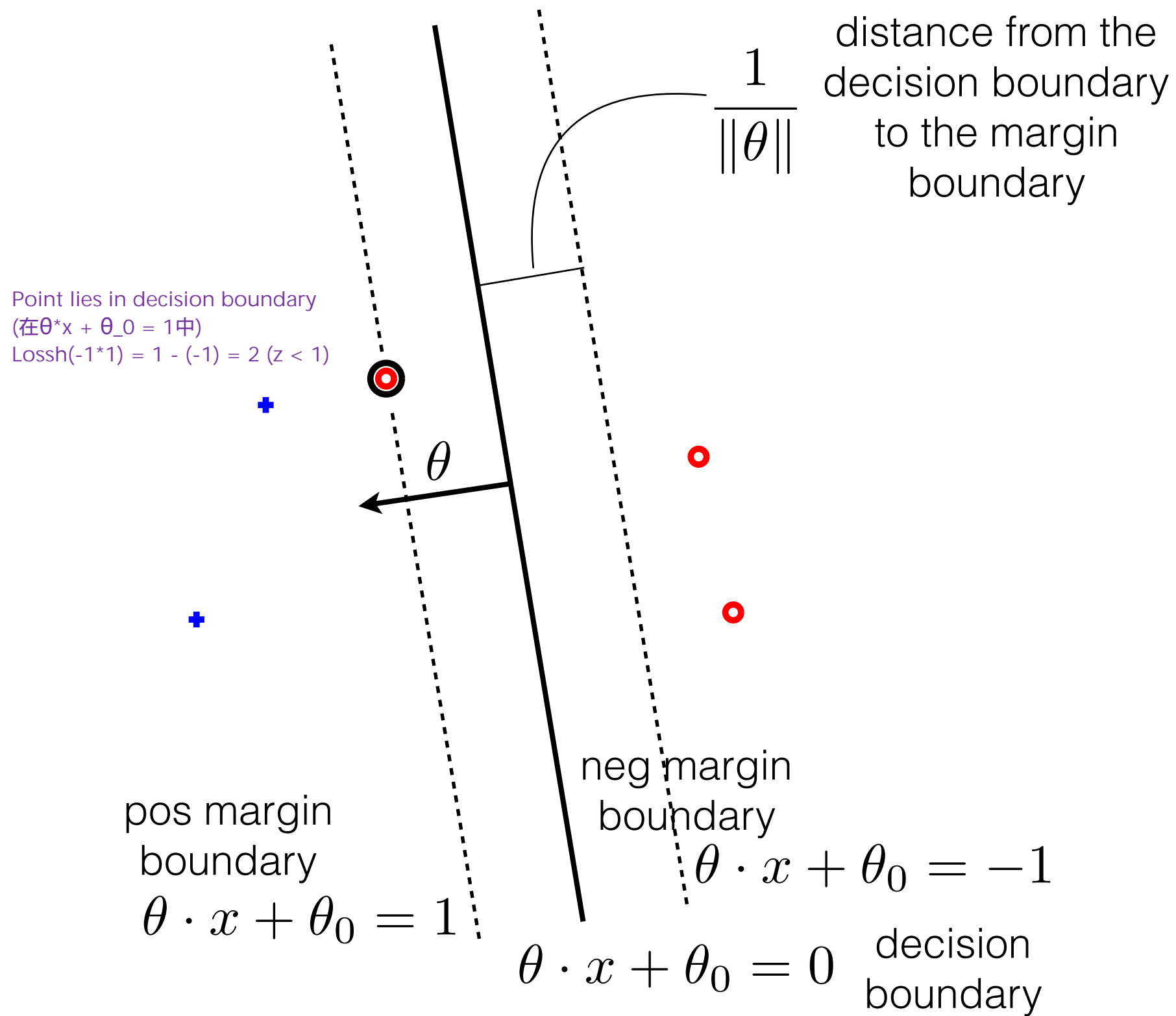
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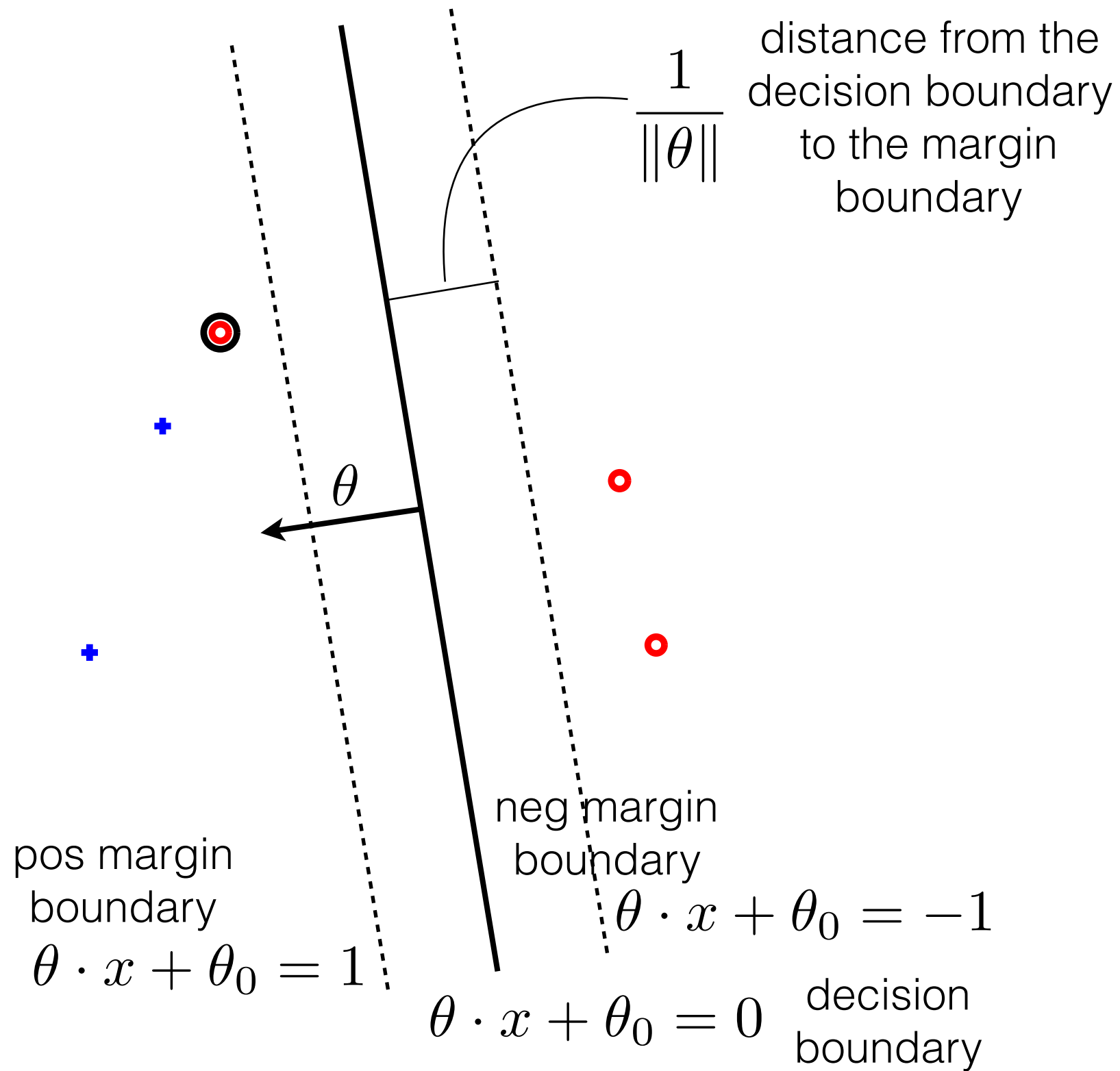
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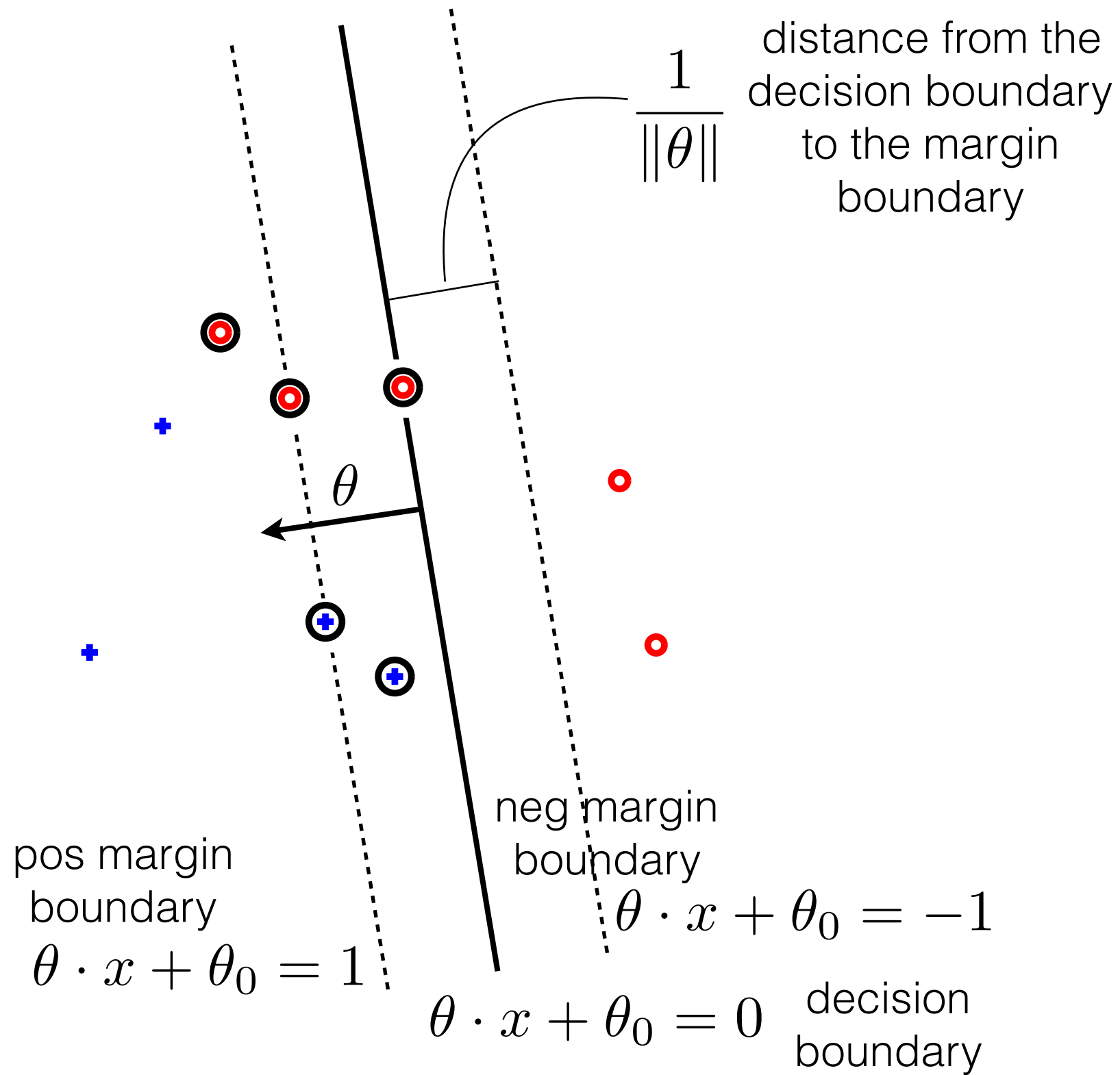
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Things to know

- General optimization formulation of learning

objective function = average loss + regularization

- Large margin linear classification as optimization
 - margin boundaries, hinge loss, regularization

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