

- (a) Let X_i be independent Bernoulli random variables that are equal to 1 if the i th flip results in heads. Let N be the number of coin flips, and let H be the number of heads. Using this notation, we have $H = X_1 + X_2 + \dots + X_N$. We also know $\mathbf{E}[X_i] = 1/2$ for all i since the coin is fair, $\text{var}(X_i) = 1/4$ for all i , $\mathbf{E}[N] = 7/2$, and $\text{var}(N) = 35/12$. (The last equality is obtained from the formula for the variance of a discrete uniform random variable). Therefore, using the equations for the expectation and variance of a sum of a random number of independent random variables on p.242 of the text, the expected number of heads is

$$\mathbf{E}[H] = \mathbf{E}[X_i]\mathbf{E}[N] = \frac{1}{2} \cdot \frac{7}{2} = \frac{7}{4},$$

and the variance is

$$\text{var}(H) = \text{var}(X_i)\mathbf{E}[N] + (\mathbf{E}[X_i])^2\text{var}(N) = \frac{1}{4} \cdot \frac{7}{2} + \frac{1}{4} \cdot \frac{35}{12} = \frac{77}{48}.$$

- (b) The experiment in part (b) can be viewed as consisting of two independent repetitions of the experiment in part (a). Thus, both the mean and the variance are doubled and become $7/2$ and $77/24$, respectively.