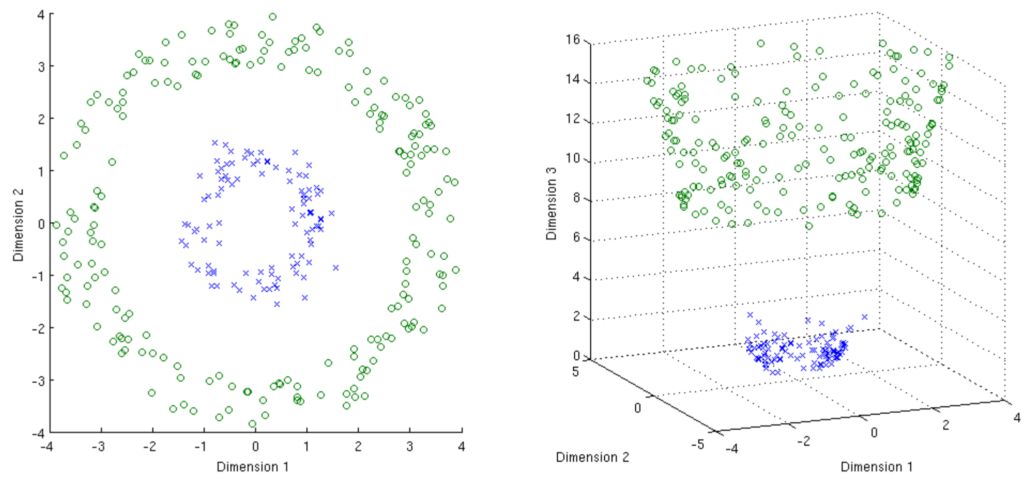
4. Kernels-II

In this question, we will practice some specific kernel methods.

4. (a)

2/2 points (graded)

In the figure below, a set of points in 2-D is shown on the left. On the right, the same points are shown mapped to a 3-D space via some transform $\phi(x)$, where x denotes a point in the 2-D space. Notice that $\phi(x)_1 = x_1$ and $\phi(x)_2 = x_2$, or in other words, the first and second coordinates are unchanged by the transformation.



Which of the following functions could have been used to compute the value of the 3rd coordinate, $\phi(x)_3$ for each point?

- $\phi(x)_3 = x_1 + x_2$
- $\phi(x)_3 = x_1^2 + x_2^2$ ✔
- $\phi(x)_3 = x_1 x_2$
- $\phi(x)_3 = x_1^2 - x_2^2$

Think about how a linear decision boundary in the 3 dimensional space ($\{\phi \in \mathbb{R}^3 : \theta \cdot \phi + \theta_0 = 0\}$) might appear in the original 2 dimensional space.

For example, suppose the decision boundary in the 3 dimensional space is $z = 4$.

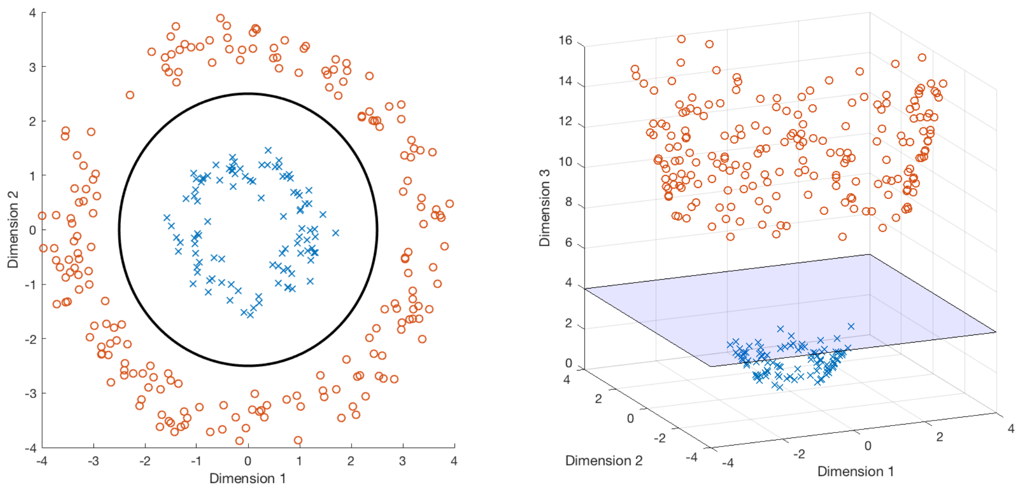
Provide an equation $f(x_1, x_2) = 0$ in the 2 dimensional space such that all the points (x_1, x_2) with $f(x_1, x_2) > 0$ correspond to $z > 4$ in the 3 dimensional space.

$f(x_1, x_2) = 0 =$ x_1^2+x_2^2 - 4 ✔ **Answer:** x_1^2 + x_2^2 - 4

$x_1^2 + x_2^2 - 4$

Solution:

- With $x = [x_1; x_2]$, one mapping which could satisfy the mapping is $\phi(x)_3 = x_1^2 + x_2^2$. The decision boundary is shown below.
- As a result, the decision boundary at $z = 4$ corresponds to $x_1^2 + x_2^2 = 4$



Submit You have used 1 of 3 attempts

Answers are displayed within the problem

4. (b)

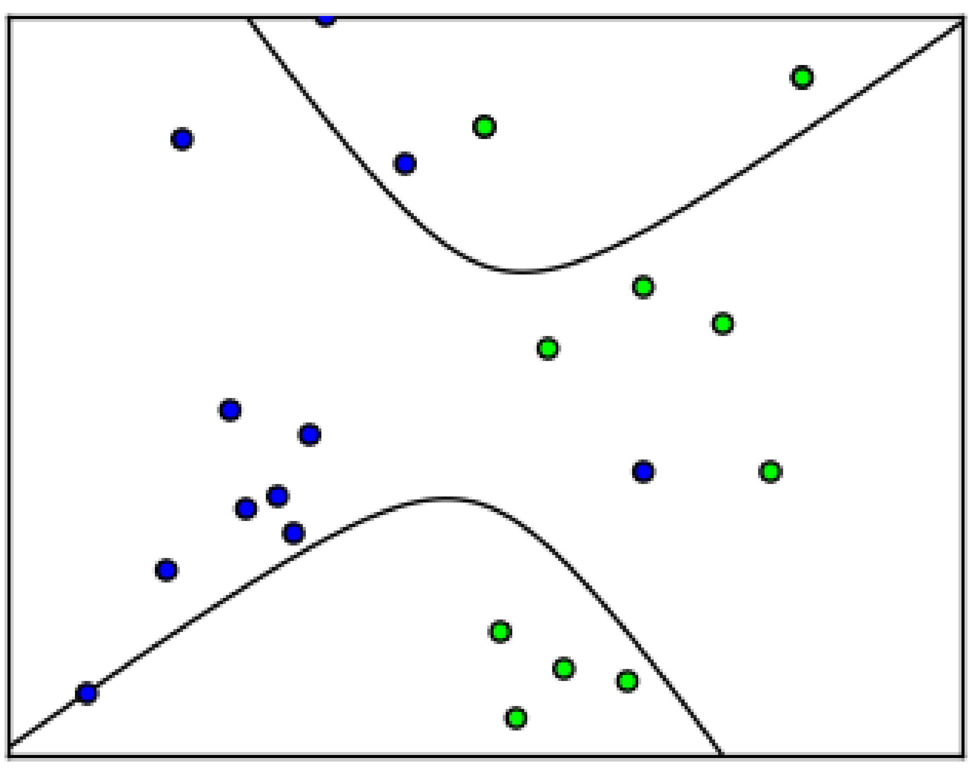
5/5 points (graded)

Consider fitting a kernelized SVM to a dataset $(x^{(i)}, y^{(i)})$ where $x^{(i)} \in \mathbb{R}^2$ and $y^{(i)} \in \{1, -1\}$ for all $i = 1, \ldots, n$. To fit the parameters of this model, one computes θ and θ_0 to minimize the following objective:

$$L(\theta, \theta_0) = \frac{1}{n} \sum_{i=1}^{n} \text{Loss}_h \left(y^{(i)} \left(\theta \cdot \phi \left(x^{(i)} \right) + \theta_0 \right) \right) + \frac{\lambda}{2} \|\theta\|^2$$

where ϕ is the feature vector associated with the kernel function. Note that, in a kernel method, the optimization problem for training would be typically expressed solely in terms of the kernel function $K(x, x')$ (dual) rather than using the associated feature vectors $\phi(x)$ (primal). We use the primal only to highlight the classification problem solved.

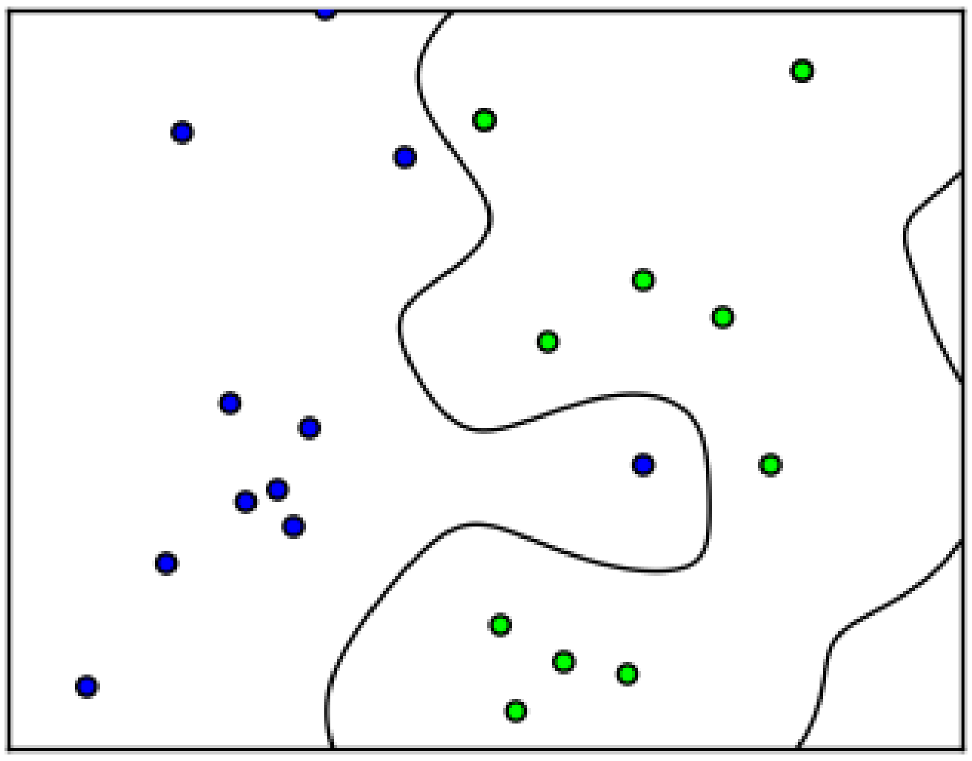
The plots below show 4 different kernelized SVM models estimated from the same 11 data points. We used a different kernel to obtain each plot but got confused about which plot corresponds to which kernel. Help us out by assigning each plot to one of the following models: linear kernel, quadratic kernel, order 3 kernel, and RBF kernel.



Which kernel is used in the above model?

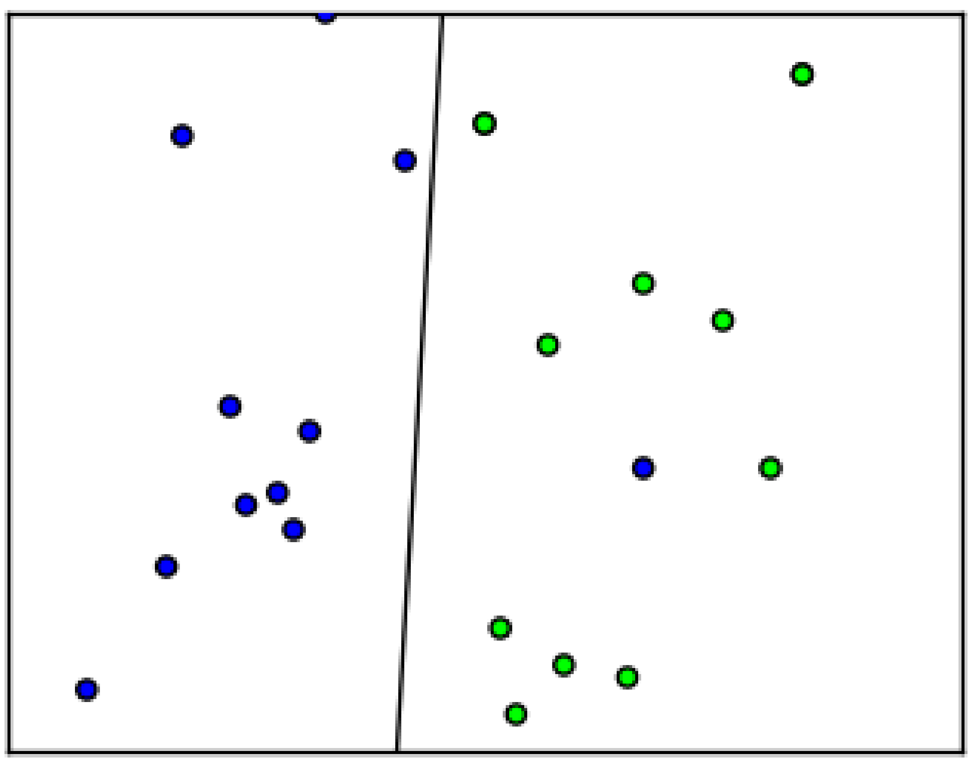
- linear kernel

- ◉ quadratic kernel ✔
- ○ order 3 kernel
- ○ RBF kernel



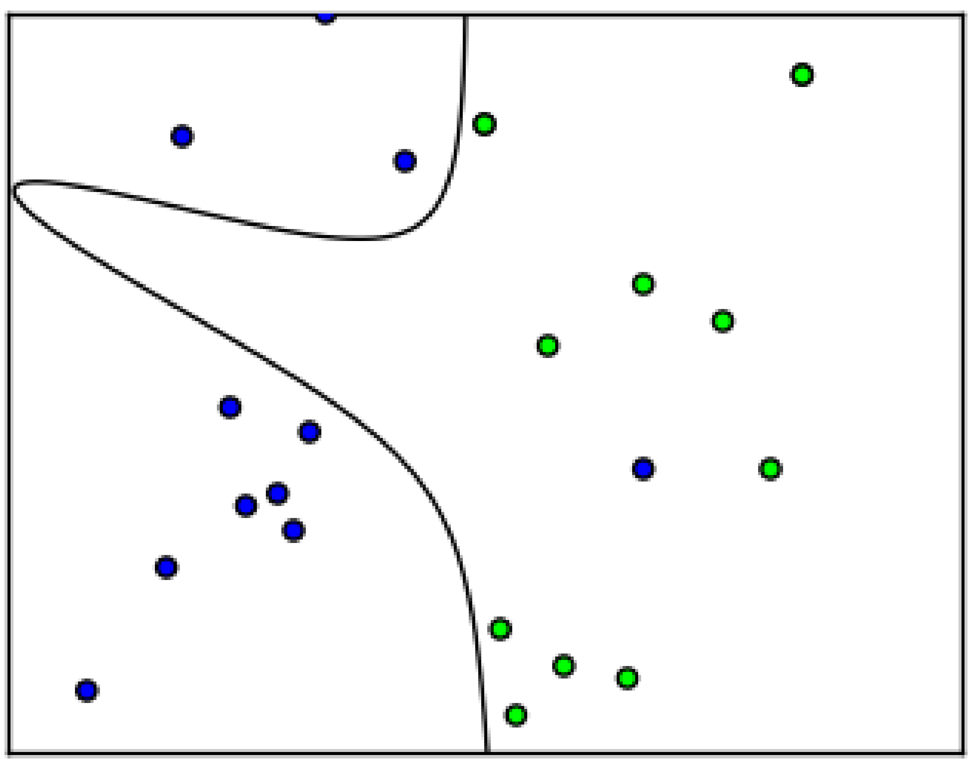
Which kernel is used in the above model?

- ○ linear kernel
- ○ quadratic kernel
- ○ order 3 kernel
- ◉ RBF kernel ✔



Which kernel is used in the above model?

- ◉ linear kernel ✔
- ○ quadratic kernel
- ○ order 3 kernel
- ○ RBF kernel



Which kernel is used in the above model?

- ○ linear kernel
- ○ quadratic kernel
- ◉ order 3 kernel ✔
- ○ RBF kernel

How would you describe qualitatively how the resulting classifiers vary with the value of λ? If the value of λ is increased, the fitting of model would be

- ○ better fit on training data (sharper decision boundary)
- ◉ worse fit on training data (flatter decision boundary) ✔

Solution:

- From examining the number of bends in the decision boundaries:
- 3rd plot corresponds to the linear kernel.
- 1st plot corresponds to the quadratic kernel.

- 4th plot corresponds to the 3rd-order kernel.
- 2nd plot corresponds to the Gaussian RBF kernel.
- Large λ penalty on θ results in flatter/ less “squiggly" lines.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 2 Nonlinear Classification, Linear regression, Collaborative Filtering (2 weeks):Homework 3 / 4. Kernels-II