3. Statistical Model of a Two Sample Experiment

Preparation: Statistical Model of a Two Sample Experiment

2/2 points (graded)

The observed outcome of a statistical experiment consists of two samples:

$$X_1, X_2, \ldots X_n \overset{\text{i.i.d.}}{\sim} X \sim \text{Ber}(p_1)$$
$$Y_1, Y_2, \ldots Y_m \overset{\text{i.i.d.}}{\sim} Y \sim \text{Ber}(p_2).$$

where in addition, X and Y are independent.

An associated statistical model is $(E, \{P_\theta\}_{\theta\in\Theta})$ where E is the (smallest) sample space of the pair (X, Y), and P_θ, is the joint distribution of (X, Y) with parameter θ. Because X and Y are independent, their joint distribution is the product of their respective distributions.

Identify the sample space E and the parameter space Θ:
(Choose one per column.)

Sample space E:

- $\{0,1\}$
- $\{0,1\} \times \{0,1\} = \{(0,0),(0,1),(1,0),(1,1)\}$ ✔
- $(0,1)$
- $(0,1) \times (0,1) \in \mathbb{R}^2$

Parameter space Θ

- $\{0,1\}$
- $\{0,1\} \times \{0,1\} = \{(0,0),(0,1),(1,0),(1,1)\}$
- $(0,1)$
- $(0,1) \times (0,1) \in \mathbb{R}^2$ ✔

Solution:

Since $X \sim \text{Ber}(p_1)$ and $Y \sim \text{Ber}(p_2)$, the pair (X, Y) takes value in the sample space $E = \{0,1\} \times \{0,1\} = \{(0,0),(0,1),(1,0),(1,1)\}$.

Since X, Y are independent, the joint distribution of (X, Y) is the product $\text{Ber}(p_1) \times \text{Ber}(p_2)$. Hence, the family $\{P_\theta\}_{\theta\in\Theta}$ of joint distributions is parametrized by $\theta = (p_1, p_2)$ and the parameter space is

$$\Theta = \{(p_1, p_2) : p_1 \in (0,1), p_2 \in (0,1)\} = (0,1) \times (0,1) \in \mathbb{R}^2.$$

提交 你已经尝试了2次（总共可以尝试2次）

Answers are displayed within the problem

Preparation: Statistical Model of a Two Sample Experiment II

1/2 points (graded)

Recall the statistical experiment from the lecture: to test whether boarding times by the Window-Middle-Aisle boarding method is shorter than boarding times by the rear-to-front method, we collect a sample of boarding times of each method. We model these boarding times as the following two sets of normal variables:

$$X_1, X_2, \ldots X_n \text{ are } i.i.d. \text{ copies of } X \sim \mathcal{N}(\mu_1, \sigma_1^2) \qquad \text{boarding times of rear-to-front}$$
$$Y_1, Y_2, \ldots Y_m \text{ are } i.i.d. \text{ copies of } Y \sim \mathcal{N}(\mu_2, \sigma_2^2) \qquad \text{boarding times of window-middle-aisle}$$

where X and Y are also independent.

Let $(E, \{P_\theta\}_{\theta\in\Theta})$ be the statistical model associated with this experiment where

- E is the sample space of the pair of random variables (X, Y);
- $\{P_\theta\}_{\theta\in\Theta}$ is the family of joint distributions of (X, Y).

For simplicity, **assume the two variances σ_1 and σ_2 are some known, fixed quantities σ_1^* and σ_2^*.**

Choose a valid candidate for the parametrization θ, which describes the family of joint probability distributions of (X, Y).

- (●) $\mu_1 - \mu_2$ ✖
- ( ) $\left(\mu_1, (\sigma_1)^2, \mu_2, (\sigma_2)^2\right)$ where $(\sigma_1)^2$ and $(\sigma_2)^2$ can each take on more than a single value
- ( ) (μ_1, μ_2) ✔
- ( ) (μ_2, μ_1)

Which of the following are legitimate choice(s) of the parameter space Θ?
(Choose all that apply)

- [ ] $\Theta = \mathbb{R}$
- [ ] $\Theta = [0, \infty)$
- [x] $\Theta = \mathbb{R}^2$ ✔
- [x] $\Theta = [0, \infty) \times [0, \infty)$ ✔

✔

Solution:

Since X, Y are independent, the joint distribution of (X, Y) is the product $\mathcal{N}\left(\mu_1, (\sigma_1)^2\right) \times \mathcal{N}\left(\mu_2, (\sigma_2)^2\right)$

Since the variances σ_1 and σ_2 are fixed and known, the only parameters determining the joint distribution is μ_1 and μ_2. Hence, a choice of the parameter θ is the 2-dimensional vector $(\mu_1 \quad \mu_2)$. (We could also have chosen to construct the statistical model using the pair (Y, X) instead. The family of joint distributions in that case would be parametrized by $(\mu_2 \quad \mu_1)$).

This gives the parameter space

$$\Theta = \{(\mu_1, \mu_2) : \mu_1 \in \mathbb{R}, \mu_2 \in \mathbb{R}\} = \mathbb{R}^2.$$

Because μ_1 and μ_2 model average boarding times, we can further restrict to

$$\Theta = \{(\mu_1, \mu_2) : \mu_1 \in [0, \infty), \mu_2 \in [0, \infty)\} = [0, \infty) \times [0, \infty)\}.$$

提交 你已经尝试了2次（总共可以尝试2次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 2 Foundation of Inference:Lecture 6: Introduction to Hypothesis Testing, and Type 1 and Type 2 Errors / 3. Statistical Model of a Two Sample Experiment

认证证书是什么？