5. Maximum likelihood estimation

Problem 5. Maximum likelihood estimation

1/1 point (graded)

The random variables $X_1, X_2, \ldots, X_n$ are continuous, independent, and distributed according to the Erlang PDF

$$f_X(x) = \frac{\lambda^3 x^2 e^{-\lambda x}}{2}, \text{ for } x \geq 0,$$

where λ is an **unknown** parameter. Find the maximum likelihood estimate of λ, based on observed values $x_1, x_2, \ldots, x_n$. Express your answer as a function of n and s where $s = x_1 + x_2 + \ldots x_n$.

$\hat{\lambda}_{\text{ML}} =$ 3*n/s ✔ **Answer:** 3*n/s

$\frac{3 \cdot n}{s}$

Solution:

We need to maximize the function,

$$f_X(x; \lambda) = \frac{\lambda^3 x_1^2 e^{-\lambda x_1}}{2} \cdots \frac{\lambda^3 x_n^2 e^{-\lambda x_n}}{2},$$

with respect to λ. Equivalently, we can maximize its logarithm, which is of the form

$$c + 3n \ln \lambda - \lambda \left(\sum_{i=1}^{n} x_i \right),$$

where c is a term that does not involve λ (but can depend on $x_1, x_2, \ldots, x_n$). By taking the derivative with respect to λ and setting it to zero, we obtain,

$$\frac{3n}{\lambda} - \sum_{i=1}^{n} x_i = 0,$$

or equivalently,

$$\lambda = \frac{3n}{\sum_{i=1}^{n} x_i} = \frac{3n}{s}.$$

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem