

Week 1 – part 2: Detour/Linear differential equation



Neuronal Dynamics: Computational Neuroscience of Single Neurons

**Week 1 – neurons and mathematics:
a first simple neuron model**

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✓ 1.1 Neurons and Synapses:

Overview

✓ 1.2 The Passive Membrane

- Linear circuit
- Dirac delta-function
- Detour: solution of 1-dim linear differential equation

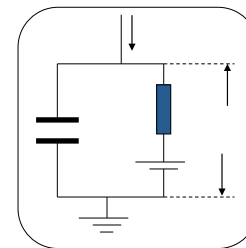
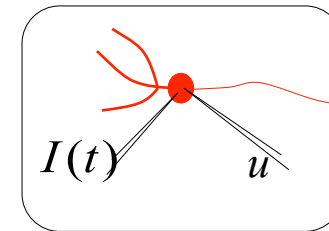
1.3 Leaky Integrate-and-Fire Model

1.4 Generalized Integrate-and-Fire Model

1.5. Quality of Integrate-and-Fire Models

Neuronal Dynamics – 1.2 Detour – Linear Differential Eq.

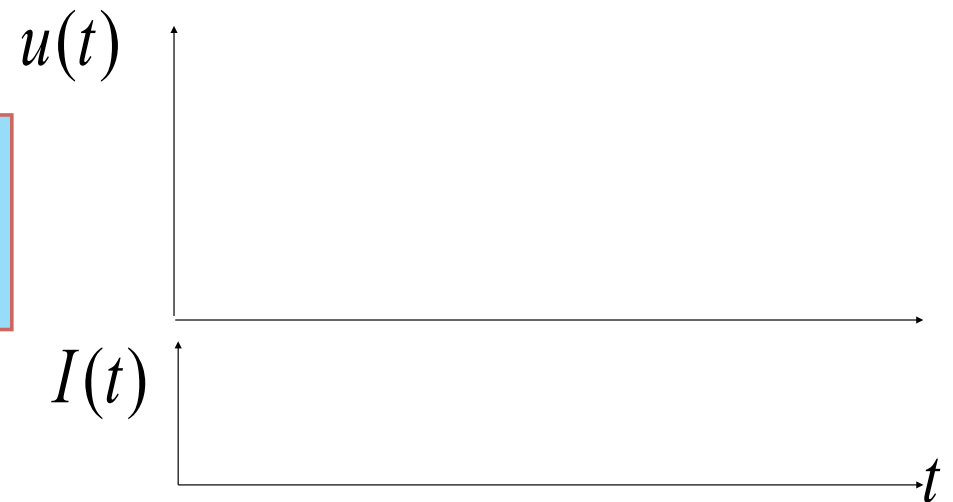
$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



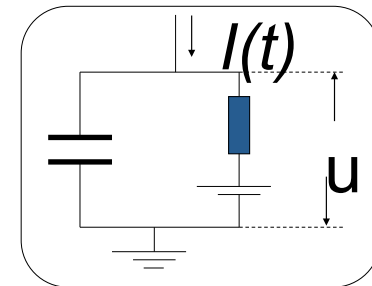
$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

Neuronal Dynamics – 1.2 Detour – Linear Differential Eq.

*Math development:
Response to step current*

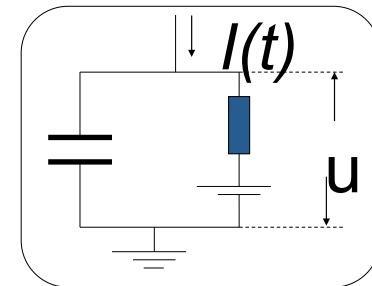
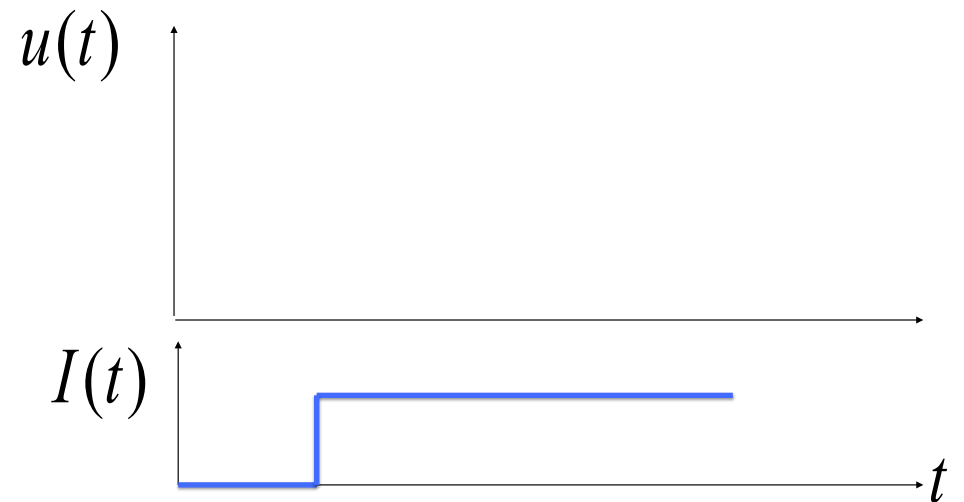
$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$


The graph shows two vertically aligned plots sharing a common horizontal time axis t . The top plot has a vertical axis labeled $u(t)$ and the bottom plot has a vertical axis labeled $I(t)$. The $I(t)$ plot shows a step function that starts at zero and jumps to a constant positive value at a certain time. The $u(t)$ plot shows a smooth, exponential rise from a baseline level towards a higher steady-state level, starting at the same time as the current step.



Neuronal Dynamics – 1.2 Detour – Step current input

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

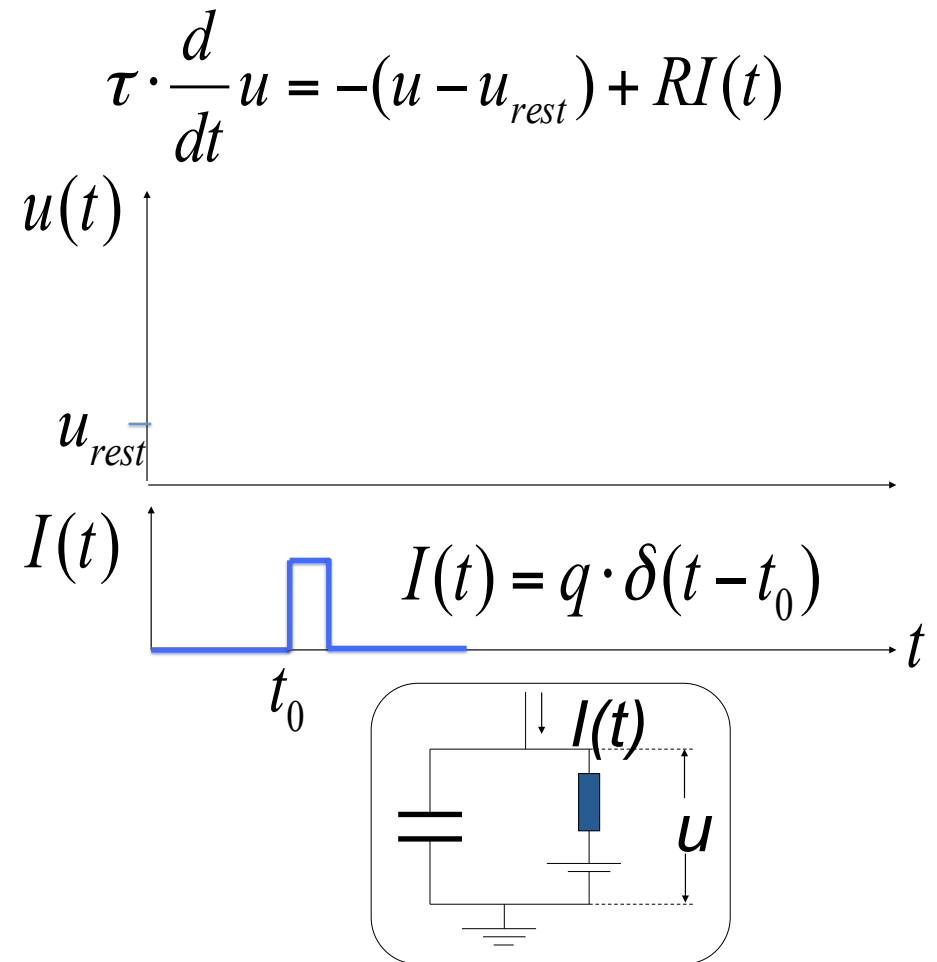


Neuronal Dynamics – 1.2 Detour – Short pulse input

$$u(t) = u_{rest} + RI_0 \left[1 - e^{-(t-t_0)/\tau} \right]$$

short pulse: $(t - t_0) \ll \tau$

*Math development:
Response to short
current pulse*

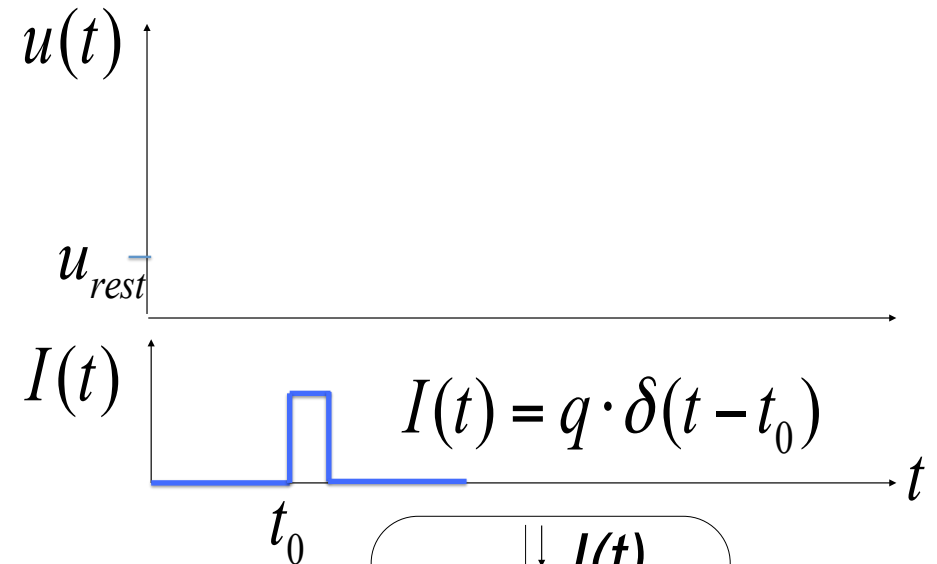


Neuronal Dynamics – 1.2 Detour – Short pulse input

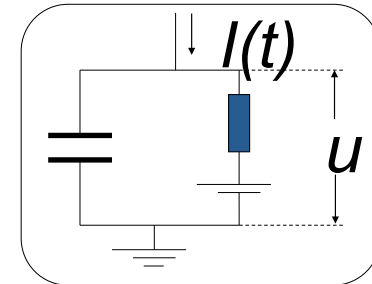
$$u(t) = u_{rest} + RI_0 \left[1 - e^{-(t-t_0)/\tau} \right]$$

short pulse: $(t - t_0) \ll \tau$

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



$$u(t) = u_{rest} + \frac{q}{C} e^{-(t-t_0)/\tau}$$



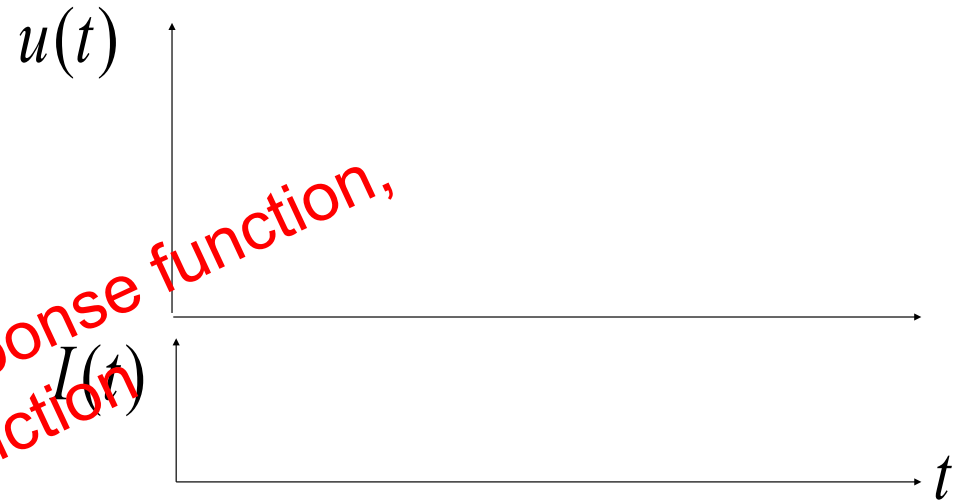
Neuronal Dynamics – 1.2 Detour – arbitrary input

Single pulse

$$u(t) = u_{rest} + \frac{q}{C} e^{-(t-t_0)/\tau}$$

Multiple pulses:

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



$$u(t) = u_{rest} + \int_{-\infty}^t \frac{1}{C} e^{-(t-t')/\tau} I(t') dt'$$

Neuronal Dynamics – 1.2 Detour – Greens function

Single pulse

$$\Delta u(t) = q \frac{1}{C} e^{-(t-t_0)/\tau}$$

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

Multiple pulses:

$$u(t) = u_{rest} + [u(t_0) - u_{rest}] + \int_{t_0}^t \frac{1}{C} e^{-(t-t')/\tau} I(t') dt'$$

Impulse response function,
Green's function

$$u(t) = u_{rest} + \int_{-\infty}^t \frac{1}{C} e^{-(t-t')/\tau} I(t') dt'$$



Neuronal Dynamics – 1.2 Detour – arbitrary input

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

Arbitrary input

$$u(t) = u_{rest} + \int_{-\infty}^t \frac{1}{c} e^{-(t-t')/\tau} I(t') dt'$$

Single pulse

$$\Delta u(t) = \frac{q}{c} e^{-(t-t_0)/\tau}$$

*you need to know the solutions
of linear differential equations!*

Neuronal Dynamics – Exercises 1.2/Quiz 1.2

*If you don't feel at ease yet,
spend **10 minutes** on these
mathematical exercises*