10. Exercise: Discrete unknown and continuous observation

Exercise: Discrete unknown and continuous observation

2/2 points (graded)

Similar to the last example, suppose that $X=\Theta+W$, where Θ is equally likely to take the values -1 and 1, and where W is standard normal noise, independent of Θ. We use the estimator $\widehat{\Theta}$, with $\widehat{\Theta}=1$ if $X>0$ and $\widehat{\Theta}=-1$ otherwise. (This is actually the MAP estimator for this problem.)

a) Let us assume that the true value of Θ is 1. In this case, our estimator makes an error if and only if W has a low (negative) value. The conditional probability of error given the true value of Θ is 1, that is, $\mathbf{P}(\widehat{\Theta}\neq 1 \mid \Theta=1)$, is equal to

● $\Phi(-1)$ ✓

○ $\Phi(0)$

○ $\Phi(1)$

where Φ is the standard normal CDF.

b) For this problem, the overall probability of error is easiest found using the formula

○ $\mathbf{P}(\widehat{\Theta}\neq\Theta)=\int\mathbf{P}(\widehat{\Theta}\neq\Theta \mid X=x)\,f_X(x)\,dx$

● $\mathbf{P}(\widehat{\Theta}\neq\Theta)=\sum_\theta\mathbf{P}(\widehat{\Theta}\neq\theta \mid \Theta=\theta)\,p_\Theta(\theta)$ ✓

Solution:

a) We have

$$\mathbf{P}(\widehat{\Theta}\neq 1 \mid \Theta=1) \;=\; \mathbf{P}(\Theta+W\leq 0 \mid \Theta=1)=\mathbf{P}(1+W\leq 0 \mid \Theta=1)$$

$$= \mathbf{P}(1+W \leq 0) = \mathbf{P}(W \leq -1) = \Phi(-1).$$

b) Similar to part (a), $\mathbf{P}(\widehat{\Theta} \neq \theta \mid \Theta = \theta)$ is easy to calculate for either choice of $\theta = -1$ or $\theta = 1$. For this reason, the second formula is easy to implement.

提交 You have used 1 of 1 attempt

ℹ Answers are displayed within the problem

讨论

显示讨论

Topic: Unit 7 / Lec. 14 / 10. Exercise: Discrete unknown and continuous observation