2. Selecting different days from a year

Problem 1. Selecting different days from a year

6.0/6.0 points (graded)

A set of 60 different days is selected from a given year. Assume that all sets of cardinality 60 are equally likely. Also, for simplicity, assume that the year has only 360 days, divided into twelve 30-day months. Evaluate the probabilities of the following events.

1. Exactly 5 days are selected from each of the 12 months.

$$\frac{\left(\binom{30}{5}\right)^{12}}{\binom{360}{60}}$$ ✔

$$\frac{\binom{30}{5}}{\binom{360}{60}}$$

$$\frac{30^{12}}{360^{60}}$$

$$\frac{360}{\left(\binom{30}{5}\right)^{12}}$$

2. None of the selected days is from January.

$$1 - \frac{\binom{30}{12}}{\binom{360}{60}}$$

$$1 - \frac{\binom{330}{60}}{\binom{360}{60}}$$

$$\frac{\binom{330}{60}}{\binom{360}{60}} \quad \checkmark$$

$$\frac{1}{12}$$

3. There exist **3** different months such that exactly **20** days are selected from each one of these months.

$$\frac{\left(\binom{30}{20}\right)^3}{\binom{360}{60}}$$

$$\frac{\binom{90}{60}}{\binom{360}{60}}$$

$$\frac{\binom{12}{3} \cdot \left(\binom{30}{20}\right)^3}{\binom{360}{60}} \quad \checkmark$$

$$\frac{1}{4}$$

Solution:

1. The number of ways of selecting **60** different days out of **360** days is $\binom{360}{60}$.

The number of ways of selecting 5 days from each month is

$$\left(\binom{30}{5}\right)^{12},$$

since from, say, January, 5 different days can be selected in $\binom{30}{5}$ different ways; the same holds for February, March, etc. Therefore, the answer is

$$\frac{\left(\binom{30}{5}\right)^{12}}{\binom{360}{60}}.$$

2. We have to exclude the 30 days from January and select 60 days among the remaining 330 days. This can be done in $\binom{330}{60}$ different ways. Hence, the answer is

$$\frac{\binom{330}{60}}{\binom{360}{60}},$$

where the denominator is the total number of ways of choosing 60 different days from a given year.

3. We first select 3 different months, out of the 12 months. This can be done in $\binom{12}{3}$ different ways. For these 3 months, we select 20 days from each, which can be done in

$$\left(\binom{30}{20}\right)^{3}$$

different ways. Hence, the answer is

$$\frac{\binom{12}{3} \cdot \left(\binom{30}{20}\right)^3}{\binom{360}{60}}.$$

ℹ   Answers are displayed within the problem

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