12. Exercise: Continuous unknown and observation

Exercise: Continuous unknown and observation

4/4 points (graded)

Let Θ and X be jointly continuous nonnegative random variables. A particular value x of X is observed and it turns out that $f_{\Theta|X}(\theta \mid x) = 2e^{-2\theta}$, for $\theta \geq 0$.

The following facts may be useful: for an exponential random variable Y with parameter λ, we have $\mathbf{E}[Y] = 1/\lambda$ and $\mathsf{Var}(Y) = 1/\lambda^2$.

a) The LMS estimate (conditional expectation) of Θ is

1/2 ✔ **Answer:** 0.5

b) The conditional mean squared error $\mathbf{E}\big[(\Theta - \widehat{\Theta}_{\text{LMS}})^2 \mid X = x\big]$ is

1/4 ✔ **Answer:** 0.25

c) The MAP estimate of Θ is

0 ✔ **Answer:** 0

d) The conditional mean squared error $\mathbf{E}\big[(\Theta - \widehat{\Theta}_{\text{MAP}})^2 \mid X = x\big]$ is

1/2 ✔ **Answer:** 0.5

Solution:

a) The posterior PDF is exponential with parameter 2. The LMS estimate is the mean of this distribution, which is 1/2.

b) Since $\widehat{\Theta}_{\text{LMS}}$ is the conditional mean, the mean squared error is the conditional variance, that is, the variance of an exponential random variable with parameter 2, and is equal to 1/4.

c) The posterior PDF, which is exponential, is largest at zero.

d) Since $\widehat{\Theta} = 0$, the conditional mean squared error is the second moment of the exponential distribution (that is, of the form $\mathbf{E}[Y^2]$, where Y is exponential with parameter 2). Using the formula $\mathbf{E}[Y^2] = \mathsf{Var}(Y) + \big(\mathbf{E}[Y]\big)^2$, we obtain

$$\mathbf{E}[Y^2] = \frac{1}{4} + \left(\frac{1}{2}\right)^2 = \frac{1}{2}.$$

Note that the LMS estimator results in a smaller mean squared error.

提交 You have used 2 of 3 attempts

Answers are displayed within the problem

讨论

显示讨论

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