3. Simple Testing

Let $X_1, \ldots, X_n$ be i.i.d. $\mathcal{N}(\theta, 1)$. Consider testing

$$H_0 : \theta = 0 \quad \text{v.s.} \quad H_1 : \theta = 1.$$

(a)

2/2 points (graded)

What would a Type 1 error be in this test?

- (selected) Rejecting H_0 when $\theta = 0$
- Not Rejecting H_0 when $\theta = 0$
- Rejecting H_0 when $\theta = 1$
- Not rejecting H_0 when $\theta = 1$

What would a Type 2 error be in this test?

- Rejecting H_0 when $\theta = 0$
- Not Rejecting H_0 when $\theta = 0$
- Rejecting H_0 when $\theta = 1$
- (selected) Not rejecting H_0 when $\theta = 1$

Solution:

By definition of the type 1 and type 2 errors. The other choices are not errors.

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

(b)

1.0/1 point (graded)

Suppose that the rejection region of a test ψ has the form $R = \{\overline{X}_n : \overline{X}_n > c\}$. Find the smallest c such that ψ has level α.

(If applicable, type **abs(x)** for $|x|$, **Phi(x)** for $\Phi(x) = \mathbf{P}(Z \le x)$ where $Z \sim \mathcal{N}(0,1)$, and **q(alpha)** for q_α, the $1-\alpha$ quantile of a standard normal variable.)

$c \geq$ q(alpha)*(1/sqrt(n)) **Answer:** q(alpha)/sqrt(n)

STANDARD NOTATION

Solution:

Since X_i are Gaussian,

$$\sqrt{n}\overline{X}_n \sim \mathcal{N}(0,1).$$

Given the rejection region $R = \{\overline{X}_n : \overline{X}_n > c\}$, the corresponding test $\psi_{n,\alpha} = \mathbf{1}\left(\overline{X}_n \in R\right)$ has level α for any c such that

$$\mathbf{P}_0\left(\overline{X}_n > c\right) = \mathbf{P}_0\left(\sqrt{n}\overline{X}_n > \sqrt{n}c\right) \leq \alpha.$$

Hence, the smallest such c is $c = \dfrac{q_\alpha}{\sqrt{n}}$.

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Answers are displayed within the problem

(c)

0/2 points (graded)

Suppose that the test ψ has level $\alpha = 0.05$. What is the power of ψ?

(If applicable, type **abs(x)** for $|x|$, **Phi(x)** for $\Phi(x) = \mathbf{P}(Z \leq x)$ where $Z \sim \mathcal{N}(0,1)$, and **q(alpha)** for q_α, the $1-\alpha$ quantile of a standard normal variable, e.g. enter **q(0.01)** for $q_{0.01}$.)

Power of ψ: 1-Phi(1.645/sqrt(n)-1) **Answer:** 1-Phi(q(0.05)-sqrt(n))

What does the power of ψ approach as $n \to \infty$?

$\lim_{n\to\infty} \text{Power} =$ 0.8413 **Answer:** 1

STANDARD NOTATION

Solution:

Since H_1 consists of a single point, the type 2 error of ψ is

$$\mathbf{P}_{\theta=1}(\psi=0) = \mathbf{P}_{\theta=1}\left(\overline{X}_n \leq \frac{q_{0.05}}{\sqrt{n}}\right) = \mathbf{P}_{\theta=1}\left(\sqrt{n}\left(\overline{X}_n - 1\right) \leq q_{0.05} - \sqrt{n}\right)$$

When θ = 1, the test is 0.

$$= \Phi\left(q_{0.05} - \sqrt{n}\right).$$

Hence, the power is $1 - \Phi\left(q_{0.05} - \sqrt{n}\right)$. As $n \to \infty$, this goes to 1.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

主题: Unit 2 Foundation of Inference:Homework 3: Introduction to Hypothesis Testing / 3. Simple Testing