We have developed the convolution formula, which tells us the PDF of the sum of two independent continuous random variables with known PDFs. In this segment, we will see that the convolution formula can be used more generally. It can be exploited to give us the PDF of a more general linear function of two independent continuous random variables.

As an example, we will try and calculate the PDF of the random variable, 2X minus Y. How can we exploit the convolution formula that we have in our hands? Well, the trick is to look at this random variable and think of it as the sum of two random variables-- the random variable 2X and the random variable minus Y. X and Y are independent. Therefore, 2X and minus Y are also independent.

So we're dealing with the sum of two independent random variables, and therefore we can apply the convolution formula to these particular independent random variables and we obtain the following. It's an integral from minus infinity to plus infinity of the density of the first random variable that we're adding, which is the random variable 2X, times the density of the second random variable. And in this case, the second random variable is minus Y.

And all that we need to do now in order to complete the solution of the problem is to figure out these two PDFs, the PDF of 2X and the PDF of minus Y. But this is a problem that we know how to solve. We have seen a formula for the PDF of a linear function of a single random variable, and the special case of this formula takes this form.

So in particular, the density of 2X is equal to 1/2 times the density of X evaluated at the argument divided by 2. And using this formula once more to the random variable minus Y, we obtain-- here, we have a correspondence that a is equal to minus 1. We obtain 1 over the absolute value of minus 1. That's 1. Here we have a minus 1, and so this gives us f of Y at minus y.

Now, what we need here is actually f minus Y evaluated at z minus x. So all we'll need to do is to substitute the right symbols. And when we have z minus x instead of y here, we need to put the negative of the argument that we have on the other side, so this is going to become x minus z.

And now, if we take this expression, substitute it in here, if we take this expression, substitute it in there, we obtain a final answer, which is this formula. Now, this is not an important formula that you should memorize at this point. Instead, you should be comfortable with the way of carrying out these calculations and doing the right substitutions and following the notation and the right set of symbols that have to be used at each step.