9. Worked Examples on Total Varation Distance Continued

Note: The following exercises will be presented in lecture, but we encourage you to attempt these yourselves first.

Computing Total Variation IV

1/1 point (graded)

So far, we have defined the total variation distance to be a distance $\mathrm{TV}(\mathbf{P}, \mathbf{Q})$ between **two probability measures** $\mathbf{P}$ and $\mathbf{Q}$. However, we will also refer to the total variation distance between **two random variables** or between **two pdfs** or **two pmfs**, as in the following.

Compute $\mathrm{TV}(X, X+a)$ for any $a \in (0,1)$, where $X \sim$ Ber(0.5).

$\mathrm{TV}(X, X+a) =$

1

Answer: 1

Solution:

Since $a \in (0,1)$, X and $X+a$ have no support points where both pmf's are non-zero. Therefore, the total variation distance is equal to 1.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

Computing Total Variation V

1/1 point (graded)

Compute $\mathrm{TV}\left(2\sqrt{n}\left(\bar{X}_n - 1/2\right), Z\right)$ where $X_i \overset{i.i.d}{\sim}$ Ber(0.5) and $Z \sim \mathcal{N}(0,1)$.

$\mathrm{TV}\left(2\sqrt{n}\left(\bar{X}_n - 1/2\right), Z\right) =$

1

Answer: 1

Solution:

Let $\mathbf{P}$ and $\mathbf{Q}$ denote the probability measures of $2\sqrt{n}\left(\bar{X}_n - 1/2\right)$ and Z, respectively. Recall the total variation distance is defined as

$$\max_{A \subset E} \left|\mathbf{P}(A) - \mathbf{Q}(A)\right|$$

Let $B \triangleq \left\{a_i = 2\sqrt{n}\left(\frac{i}{n} - \frac{1}{2}\right) \mid i = 0, 1, \ldots, n\right\}$ be set of $n+1$ points where the pmf of $2\sqrt{n}\left(\bar{X}_n - 1/2\right)$ is non-zero.

Consider the set $A = \mathbb{R} \setminus B \,(= R \cap B^c)$. For this set, $\mathbf{P}(A) = 0$ and $\mathbf{Q}(A) = 1$. Therefore, $\left|\mathbf{P}(A) - \mathbf{Q}(A)\right| = 1$. We know from a previous problem that the total variation distance is upper bounded by 1 for any two distributions. Since we have produced a set where this bound is met with equality, $\mathrm{TV}\left(2\sqrt{n}\left(\bar{X}_n - 1/2\right), Z\right) = 1$.

提交 你已经尝试了2次（总共可以尝试3次）

Answers are displayed within the problem

Worked Examples on Total Variation Distance Continued