Problem 4

Let $X_1, \ldots, X_n$ be i.i.d. normal variable following the distribution $\mathcal{N}(\mu, \tau)$, where μ is the mean and τ is the variance.

Denote by $\widehat{\mu}$ and $\hat{\tau}$ the maximum likelihood estimators of μ and τ respectively based on the i.i.d. observations $X_1, \ldots, X_n$.

(In our usual notation, $\tau = \sigma^2$. We use τ in this problem to make clear that the parameter being estimated is σ^2 not σ.)

(a)

1/1 point (graded)

Is the estimator $2(\widehat{\mu})^2 + \hat{\tau}$ of $2\mu^2 + \tau$ asymptotically normal?

- (selected) yes
- no
- not enough information to determine

Solution:

Let

只要这个g处处可导，就可以用delta method

$$g(\widehat{\mu}, \hat{\tau}) = 2(\widehat{\mu})^2 + \hat{\tau}$$

Since g is continuously differentiable, the delta method gives

$$\sqrt{n}\left(g(\widehat{\mu}, \hat{\tau}) - g(\mu, \tau)\right) \xrightarrow[n\to\infty]{(d)} \mathcal{N}(0, \nabla g(\mu,\tau)^T (\mathbf{I}(\mu,\tau))^{-1} \nabla g(\mu,\tau)^T.$$

where $\mathbf{I}(\mu, \tau)$ is the Fisher Information matrix of any of the Gaussian variables X_i.

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Answers are displayed within the problem

(b)

1/1 point (graded)

Let

$$g(\mu, \tau) = 2\mu^2 + \tau.$$

and let $\mathbf{I}$ be the Fisher information matrix of $X_i \sim \mathcal{N}(\mu, \tau)$.

The asymptotic variance of $2(\widehat{\mu})^2 + \hat{\tau}$ is...

- $\nabla g(\mu,\tau)^T \mathbf{I}(\mu,\tau) \nabla g(\mu,\tau)$
- (selected) $\nabla g(\mu,\tau)^T (\mathbf{I}(\mu,\tau))^{-1} \nabla g(\mu,\tau)$

- $\nabla g(\mu,\tau)^T \mathbf{I}(\mu,\tau)$
- $\nabla g(\mu,\tau)^T (\mathbf{I}(\mu,\tau))^{-1}$

Solution:

Refer to the solution to the previous problem.

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Answers are displayed within the problem

(c)

1/1 point (graded)

Using the results from above and referring back to homeowork solutions if necessary, compute the asymptotic variance $V\left(2(\widehat{\mu})^2 + \hat{\tau}\right)$ of the estimator $2(\widehat{\mu})^2 + \hat{\tau}$.

Hint: The inverse of a diagonal matrix $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ where $a, b \neq 0$ is the diagonal matrix $\begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}$.

$V\left(2(\widehat{\mu})^2 + \hat{\tau}\right) =$ 16*mu^2*tau+2*tau^2 **Answer:** 16*mu^2*tau+2*tau^2

$16 \cdot \mu^2 \cdot \tau + 2 \cdot \tau^2$

STANDARD NOTATION

Solution:

Recall from homework 6 problem 2 that the Fisher Information of a Gaussian distribution $\mathcal{N}(\mu,\tau)$ where the μ and $\tau = \sigma^2$ are the parameters to be estimated is

$$I(\mu,\tau) = \begin{pmatrix} \frac{1}{\tau} & 0 \\ 0 & \frac{1}{2\tau^2} \end{pmatrix}.$$

Using this and the results from the previous parts, we obtain the asymptotic variance $V\left(2(\widehat{\mu})^2 + \hat{\tau}\right)$ as

$$\begin{aligned} V\left(2(\widehat{\mu})^2 + \hat{\tau}\right) &= \nabla g(\mu,\tau)^T (\mathbf{I}(\mu,\tau))^{-1} \nabla g(\mu,\tau) \\ &= \begin{pmatrix} 4\mu & 1 \end{pmatrix} \begin{pmatrix} \tau & 0 \\ 0 & 2\tau^2 \end{pmatrix} \begin{pmatrix} 4\mu \\ 1 \end{pmatrix} \\ &= 16\mu^2\tau + 2\tau^2. \end{aligned}$$

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Answers are displayed within the problem

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显示讨论

主题: Midterm Exam 1:Midterm Exam 1 / Problem 4