19. Exercise: Joint CDFs

Exercise: Joint CDFs

3/3 points (graded)

a) Is it always true that if $x < x'$, then $F_{X,Y}(x,y) \leq F_{X,Y}(x',y)$?

Yes ▼ ✔ **Answer:** Yes

b) Suppose that the random variables X and Y are jointly continuous and take values on the set where $0 \leq x, y \leq 1$. Is $F_{X,Y}(x,y) = (x+2y)^2/9$ a legitimate joint CDF? *Hint:* Consider $F_{X,Y}(0,1)$.

No ▼ ✔ **Answer:** No

c) Suppose that the random variables X and Y are jointly continuous and take values on the unit square, i.e., $0 \leq x \leq 1$ and $0 \leq y \leq 1$. The joint CDF on that set is of the form $xy(x+y)/2$. Find an expression for the joint PDF which is valid for (x,y) in the unit square. Enter an algebraic function of x and y using standard notation.

x + y ✔ **Answer:** x+y

STANDARD NOTATION

Solution:

a) Since $x < x'$, the event $\{X \leq x, Y \leq y\}$ is a subset of the event $\{X \leq x', Y \leq y\}$, and therefore $F_{X,Y}(x,y) = \mathbf{P}(X \leq x, Y \leq y) \leq \mathbf{P}(X \leq x', Y \leq y) = F_{X,Y}(x',y)$.

b) Since the random variables are nonnegative, we have $F_{X,Y}(0,1) = \mathbf{P}(X \leq 0 \text{ and } Y \leq 1) = \mathbf{P}(X = 0 \text{ and } Y \leq 1) \leq \mathbf{P}(X = 0) = 0$, where the last equality holds because X is a continuous random variable. But zero is different from $(0 + 2 \cdot 1)^2/9$. Therefore, we do not have a legitimate joint CDF.

c) The joint CDF is of the form $x^2y/2 + y^2x/2$. The partial derivative with respect to x is $xy + y^2/2$. Taking now the partial derivative with respect to y, we obtain $x + y$.

提交 You have used 2 of 3 attempts