6. Asymptotic Normality of the ML Estimator - Example Problems

Fisher Information and Asymptotic Normality of the MLE

1/1 point (graded)

Consider the statistical model $(\mathbb{R}, \{\mathbf{P}_\theta\}_{\theta\in\mathbb{R}})$ associated to the statistical experiment $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathbf{P}_{\theta^*}$, where θ^* is the true parameter. Assume that the conditions of the theorem for the convergence of the MLE hold. Which of the following statements about the Fisher information $\mathcal{I}(\theta)$ is true?

- The Fisher information $\mathcal{I}(\theta^*)$ at the true parameter gives a good approximation for θ^*.
- The Fisher information $\mathcal{I}(\theta^*)$ at the true parameter determines the asymptotic mean of the random variable $\hat{\theta}_n^{\text{MLE}}$).
- The Fisher information $\mathcal{I}(\theta^*)$ at the true parameter determines the asymptotic variance of the random variable $\hat{\theta}_n^{\text{MLE}}$.

Solution:

As stated in the theorem,

$$\sqrt{n}\,(\hat{\theta}_n^{\text{MLE}} - \theta^*)$$

converges to a normal random variable $\mathcal{N}\,(0, \mathcal{I}(\theta^*)^{-1})$. Hence, the Fisher information determines the asymptotic variance, and so the third choice is correct.

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

Asymptotic Normality of the MLE

1/1 point (graded)

Consider the statistical model $(\{0,1\}, \{\text{Ber}\,(\theta)\}_{\theta\in(0,1)})$. Let $\ell\,(\theta)$ denote the **log-likelihood of one observation** of this model. You observe samples $X_1, \ldots, X_n \sim \text{Ber}\,(\theta^*)$ and construct the MLE $\hat{\theta}_n^{\text{MLE}}$ for θ^*. By the theorem for the convergence of the MLE (you are allowed to assume that all necessary conditions for this theorem hold), this implies that

$$\sqrt{n}\,(\hat{\theta}_n^{\text{MLE}} - \theta^*) \xrightarrow[n\to\infty]{(d)} \mathcal{N}\,(0, \sigma^2)$$

for some constant σ^2 that depends on θ^*. The quantity σ^2 is referred to as the **asymptotic variance** . Use the theorem for the convergence of the MLE to find the expression for σ^2.

What is σ^2? Express your answer in terms of $T := \theta^*$.

Type **T** for T, using the variable T to stand for θ^*.

$\sigma^2 =$ T*(1-T)   Answer: T*(1- T)

$T \cdot (1 - T)$

STANDARD NOTATION

Solution:

We have that for this model the Fisher information is $\mathcal{I}(\theta) = \frac{1}{\theta} + \frac{1}{(1-\theta)} = \frac{1}{\theta(1-\theta)}$. Applying the theorem for the convergence of the MLE,

$$\sqrt{n}\,(\hat{\theta}_n^{\text{MLE}} - \theta^*) \xrightarrow[n\to\infty]{(d)} \mathcal{N}\left(0, \mathcal{I}(\theta^*)^{-1}\right)$$

Hence,

$$\sigma^2 = \mathcal{I}(\theta^*)^{-1} = \theta^*\,(1 - \theta^*)\,.$$

Remark: Alternatively, the asymptotic variance can be computed directly from the MLE, which is given, in the Bernoulli case, by the sample mean $\overline{X}_n$.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

主题: Unit 3 Methods of Estimation:Lecture 11: Fisher Information, Asymptotic Normality of MLE; Method of Moments / 6. Asymptotic Normality of the ML Estimator - Example Problems