Week 1 – part 2: Detour/Linear differential equation

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 1 – neurons and mathematics:
a first simple neuron model

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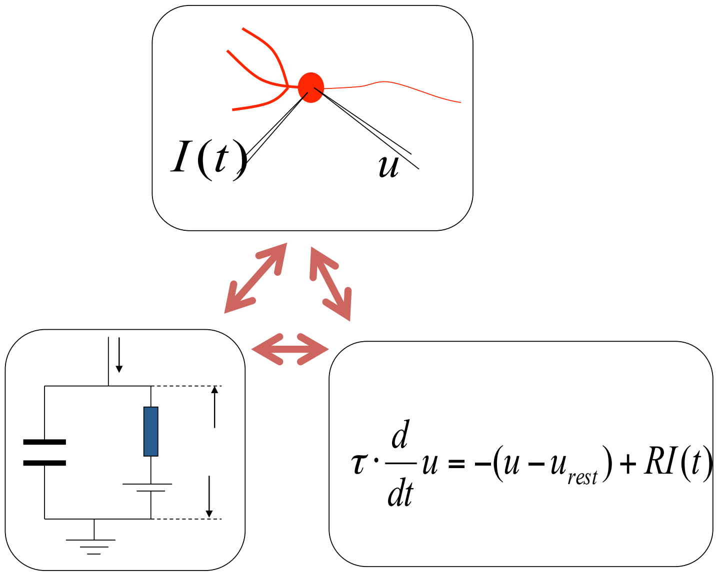
- 1.1 **Neurons and Synapses:**
  Overview
- 1.2 **The Passive Membrane**
  - Linear circuit
  - Dirac delta-function
  - Detour: solution of 1-dim linear differential equation
- 1.3 **Leaky Integrate-and-Fire Model**
- 1.4 **Generalized Integrate-and-Fire Model**
- 1.5. **Quality of Integrate-and-Fire Models**

Neuronal Dynamics – 1.2Detour – Linear Differential Eq.

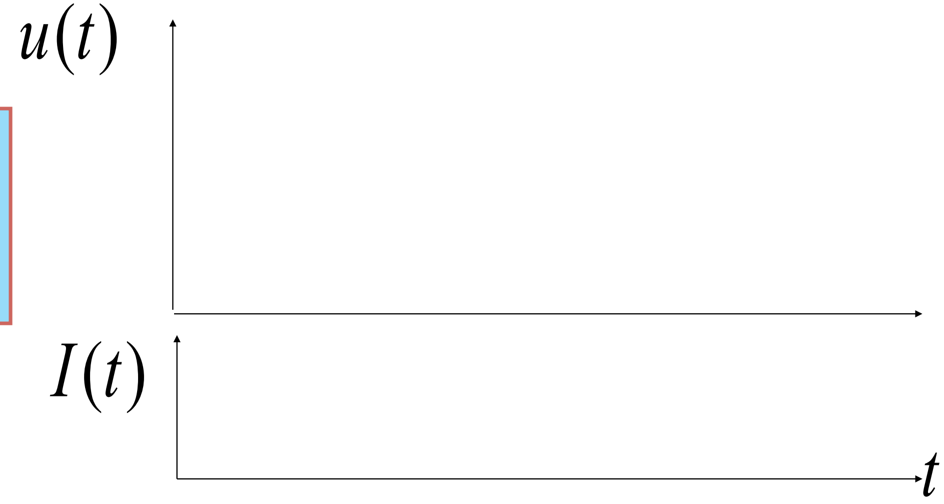
$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

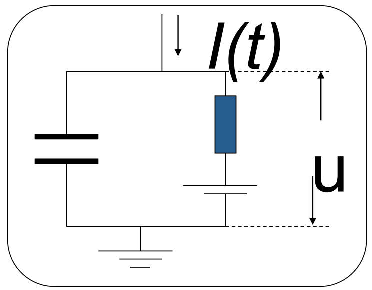


Neuronal Dynamics – 1.2Detour – Linear Differential Eq.

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

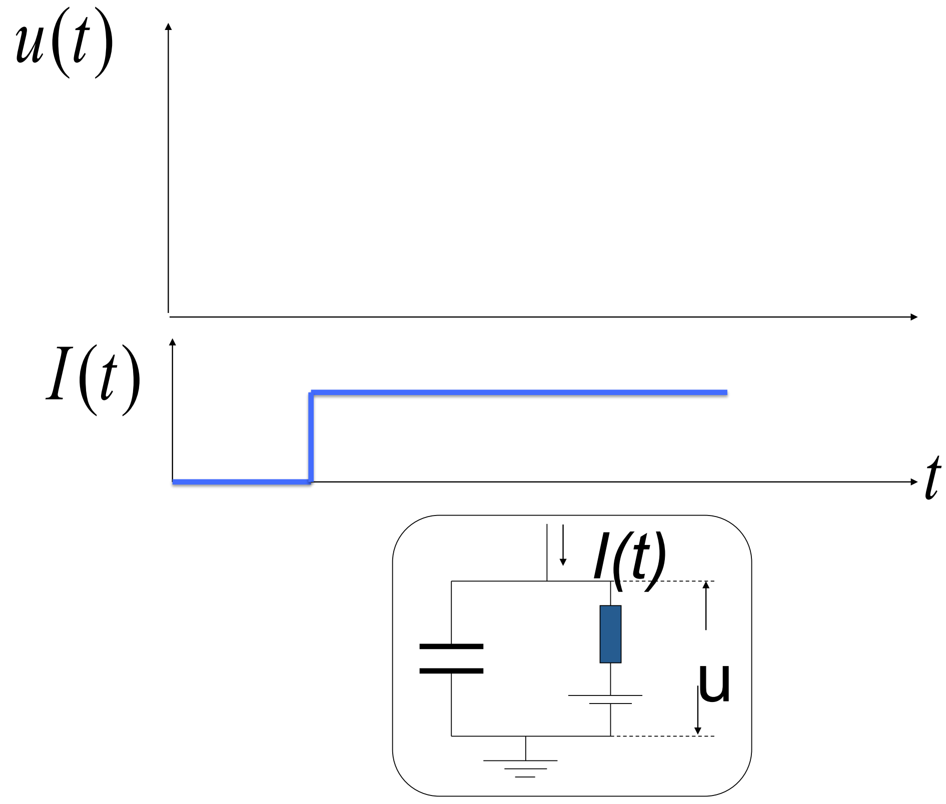
Math development:
Response to step current





Neuronal Dynamics – 1.2Detour – Step current input

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



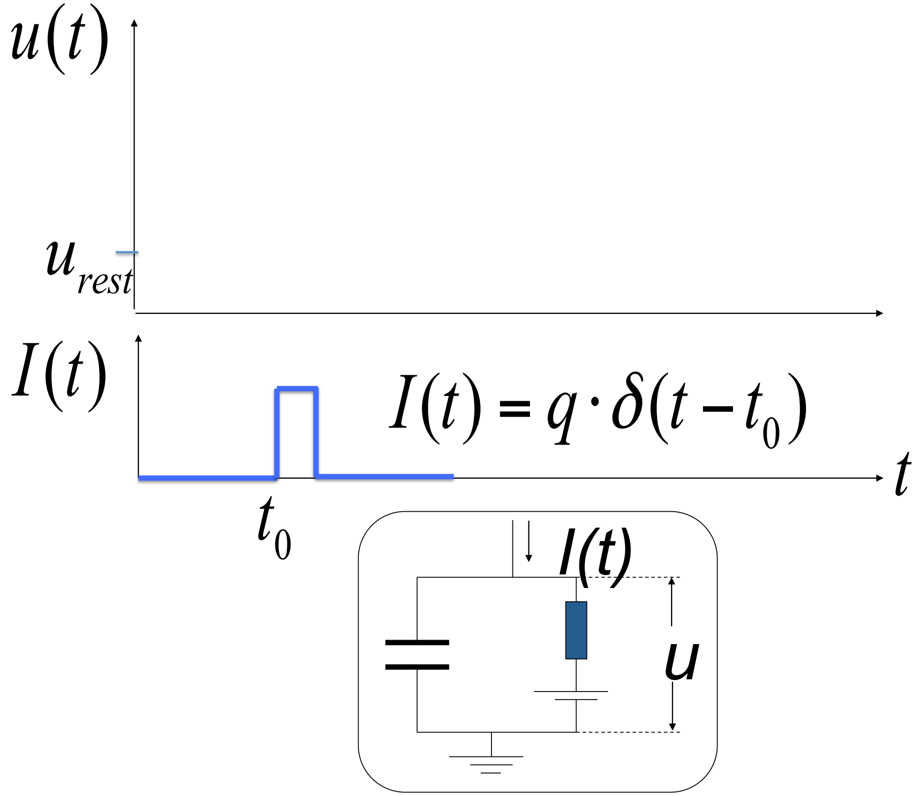
Neuronal Dynamics – 1.2Detour – Short pulse input

$$u(t) = u_{rest} + RI_0 \left[1 - e^{-(t-t_0)/\tau}\right]$$

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

short pulse: $(t - t_0) << \tau$

Math development:
Response to short current pulse

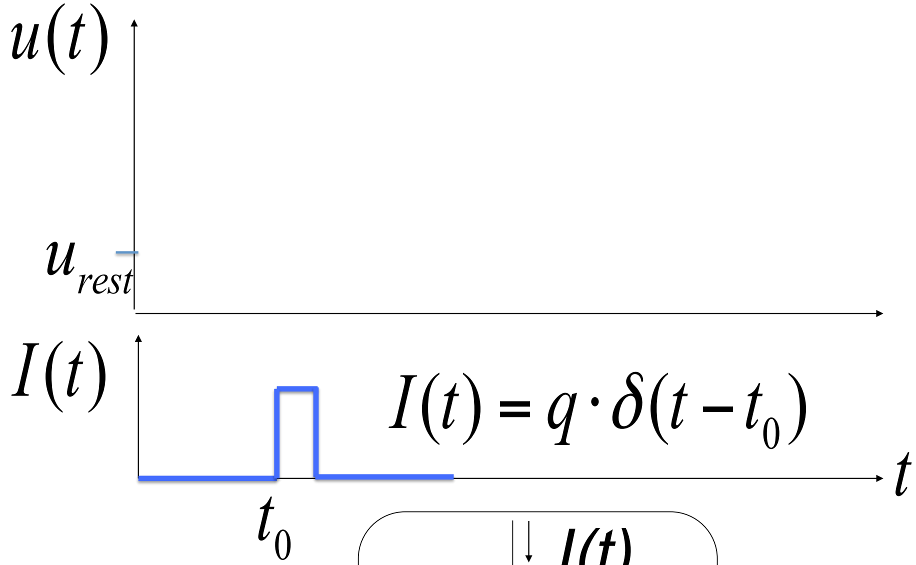


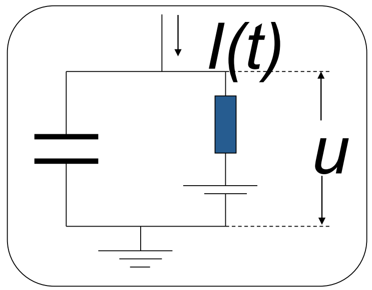
Neuronal Dynamics – 1.2Detour – Short pulse input

$$u(t) = u_{rest} + RI_0\left[1 - e^{-(t-t_0)/\tau}\right]$$

$$\tau \cdot \frac{d}{dt}u = -(u - u_{rest}) + RI(t)$$

short pulse: $(t - t_0) << \tau$





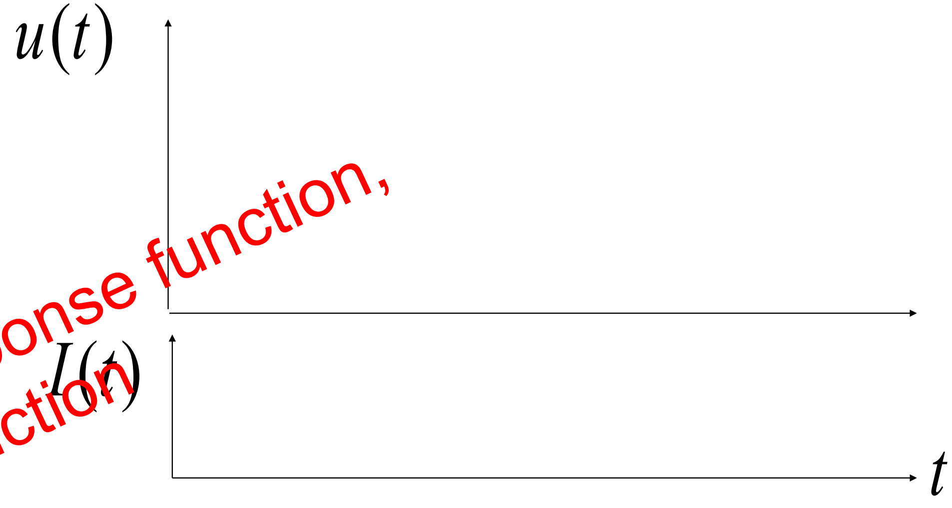
$$u(t) = u_{rest} + \frac{q}{C}e^{-(t-t_0)/\tau}$$

Neuronal Dynamics – 1.2Detour – arbitrary input

Single pulse

$$u(t) = u_{rest} + \frac{q}{C} e^{-(t-t_0)/\tau}$$

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



Multiple pulses:

Impulse response function,
Green's function

$$u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t')dt'$$

Neuronal Dynamics – 1.2Detour – Greens function

Single pulse

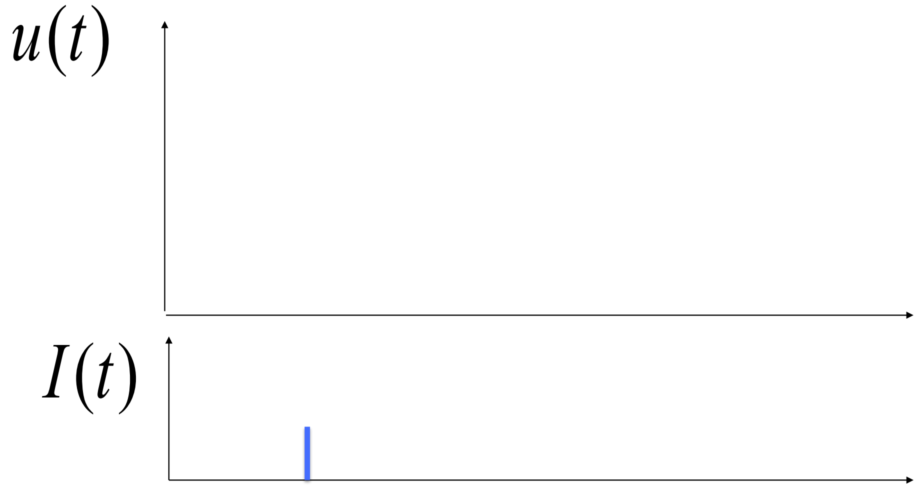
$$\Delta u(t) = q \frac{1}{C} e^{-(t-t_0)/\tau}$$

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

Multiple pulses:

$$u(t) = u_{rest} + [u(t_0) - u_{rest}] + \int_{t_0}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t') dt'$$

Impulse response function,
Green's function



$$u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t') dt'$$

Neuronal Dynamics – 1.2Detour – arbitrary input

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

Arbitrary input

$$u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{c} e^{-(t-t')/\tau} I(t')dt'$$

Single pulse

$$\Delta u(t) = \frac{q}{c} e^{-(t-t_0)/\tau}$$

you need to know the solutions of linear differential equations!

Neuronal Dynamics – Exercises 1.2/Quiz 1.2

*If you don’t feel at ease yet, spend **10 minutes** on these mathematical exercises*