Because of the definition of g, the random variable Y takes on only non-negative values. Thus $f_Y(y) = 0$ for any negative y. For $y > 0$,

$$\begin{aligned} F_Y(y) &= \mathbf{P}(Y \leq y) \\ &= \mathbf{P}\left(X \in [-y, 0]\right) + \mathbf{P}\left(X \in (0, y^2]\right) \\ &= \mathbf{P}(-y \leq X \leq \sqrt{y^2}) \\ &= F_X(y^2) - F_X(-y). \end{aligned}$$

Taking the derivative of $F_Y(y)$ (and using the chain rule),

$$\begin{aligned} f_Y(y) &= 2y f_X(y^2) + f_X(-y) \\ &= \frac{1}{\sqrt{2\pi}}\left(2ye^{-y^4/2} + e^{-y^2/2}\right). \end{aligned}$$