11. Exercise: Independence and CDFs

Exercise: Independence and CDFs

2/2 points (graded)

a) Suppose that X and Y are independent. Is it true that their joint CDF satisfies $F_{X,Y}(x,y) = F_X(x)F_Y(y)$, for all x and y?

Yes ▼ ✔ **Answer:** Yes

b) Suppose that $F_{X,Y}(x,y) = F_X(x)F_Y(y)$, for all x and y. Is it true that X and Y are independent?

Hint: Recall the formula $f_{X,Y}(x,y) = (\partial^2/\partial x \partial y)F_{X,Y}(x,y)$.

Yes ▼ ✔ **Answer:** Yes

Solution:

a) Yes. We have

$$\begin{aligned} F_{X,Y}(x,y) &= \mathbf{P}(X \le x, Y \le y) \\ &= \int_{-\infty}^{y}\int_{-\infty}^{x} f_{X,Y}(x,y)\,dx\,dy \\ &= \int_{-\infty}^{x} f_X(x)\,dx \int_{-\infty}^{y} f_Y(y)\,dy \\ &= F_X(x)F_Y(y). \end{aligned}$$

b) True. Using the formula in the hint, we find that

$$\begin{aligned} f_{X,Y}(x,y) &= \frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y) \\ &= \frac{\partial^2}{\partial x \partial y} F_X(x)F_Y(y) \\ &= \frac{\partial}{\partial x} F_X(x) \frac{\partial}{\partial y} F_Y(y) \\ &= f_X(x)f_Y(y), \end{aligned}$$

and therefore we have independence.