# 3. LLMS estimation

## Problem 3. LLMS estimation

3/3 points (graded)

Let $X = U + W$ with $\mathbf{E}[U] = m$, $\textsf{Var}(U) = v$, $\mathbf{E}[W] = 0$, and $\textsf{Var}(W) = h$. Assume that $U$ and $W$ are independent.

1. The LLMS estimator of $U$ based on $X$ is of the form $\hat{U} = aX + b$. Find $a$ and $b$. Express your answers in terms of $m$, $v$, and $h$ using standard notation.

   $a =$ v/(v+h) ✔ **Answer:** v/(v+h)

   $\frac{v}{v+h}$

   $b =$ m*h/(v+h) ✔ **Answer:** m*h/(v+h)

   $\frac{m \cdot h}{v+h}$

2. We now further assume that $U$ and $W$ are normal random variables and then construct $\hat{U}_{LMS}$, the LMS estimator of $U$ based on $X$, under this additional assumption. Would $\hat{U}_{LMS}$ be identical to $\hat{U}$, the LLMS estimator developed without the additional normality assumption in Part 1?

   Yes ▼ ✔ **Answer:** Yes

STANDARD NOTATION

**Solution:**

1. In order to write the LLMS estimator we need to find $\mathbf{E}[X]$, $\textsf{Var}(X)$, and $\text{cov}(U, X)$. We have

$$\begin{aligned} \mathbf{E}[X] &= \mathbf{E}[U+W] = \mathbf{E}[U] + \mathbf{E}[W] = \mathbf{E}[U] = m, \\ \textsf{Var}(X) &= \textsf{Var}(U+W) \\ &= \textsf{Var}(U) + \textsf{Var}(W) \qquad \text{since } U \text{ and } W \text{ are independent} \\ &= v + h, \\ \text{cov}(U, X) &= \mathbf{E}[UX] - \mathbf{E}[U]\mathbf{E}[X] \end{aligned}$$

$$
\begin{aligned}
&= \mathbf{E}[U(U+W)] - m^2 \\
&= \mathbf{E}[U^2] + \mathbf{E}[U]\mathbf{E}[W] - m^2 \qquad \text{since } U \text{ and } W \text{ are independent} \\
&= \mathbf{E}[U^2] - m^2 \\
&= \mathbf{E}[U^2] - (\mathbf{E}[U])^2 \\
&= \mathsf{Var}(U) = v.
\end{aligned}
$$

Substituting these results into the formula for the LLMS estimator yields

$$\hat{U} = m + \frac{v}{v+h}(X - m).$$

2. We know that the LMS estimator of $U$ based on $X$, under the normality assumption we have introduced, is linear in $X$. Therefore, it coincides with the LLMS estimator.

提交 You have used 1 of 3 attempts

Answers are displayed within the problem

## 讨论

显示讨论

**Topic:** Unit 7 / Problem Set / 3. LLMS estimation