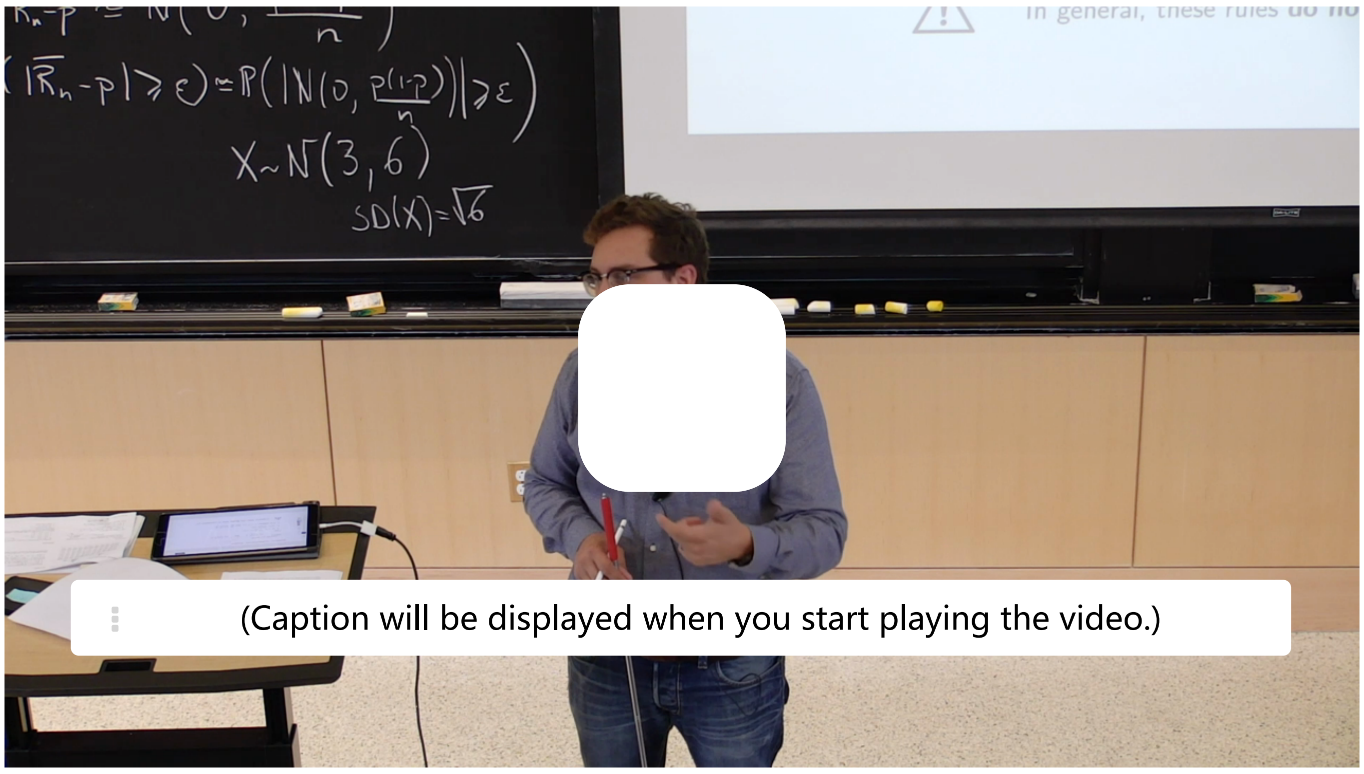
8. Operations on Sequences and Convergence

Addition, multiplicaton, division; Slutsky's Theorem; Continuous Mapping Thoerem



So we're going to replace this with sigma hat,

and then we're going to just have

something that looks like--

maybe we're going to have sigma over sigma hat, something

like this.

And this guy, by the law of large numbers,

is going to converge to one.

The other guy is going to converge to a Gaussian.

How do we combine those things, right?

How do I combine--

there are several limits, multiplying them, adding them?

So when you're actually almost sure and in probability things

go exactly the way you want.

You can add your random variables.

You're going to come to the sum of the limits

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We restate the theorems discussed in lecture below.

Slutsky's Theorem

Let $Phi\left((x+2)/sqrt\left(2\right)\right)$ be two sequences of r.v., such that:

- $T_n \xrightarrow[n\to\infty]{(d)} T$
- $U_n \xrightarrow[n\to\infty]{\mathbf{P}} u$

where T is a r.v. and u is a given real number (deterministic limit: $\mathbf{P}\left(U=u\right)=1$). Then,

- $T_n + U_n \xrightarrow[n\to\infty]{(d)} T+u$,
- $T_nU_n \xrightarrow[n\to\infty]{(d)} Tu$,
- If in addition, $u \neq 0$, then $\frac{T_n}{U_n} \xrightarrow[n\to\infty]{(d)} \frac{T}{u}$.

Continuous Mapping Theorem

If f is a continuous function:

$$T_n \xrightarrow[n\to\infty]{\mathbf{a.s.}/\mathbf{P}/(d)} T \quad \Rightarrow \quad f(T_n) \xrightarrow[n\to\infty]{} f(T).$$

Convergence in distribution

4/4 points (graded)

Let X_n be a sequence of random variables that are converging **in probability** to another random variable X. Let Y_n be a sequence of random variables that are converging **in probability** to another random variable Y.

For each of the statements below, choose true ("This statement is always true") or false ("This statement is sometimes false"). Keep in mind that "convergence in probability" is stronger than "convergence in distribution".

- $X_n + Y_n \longrightarrow X + Y$ in distribution.

  - (•) True ✔
  - ( ) False

- $X_n Y_n \longrightarrow XY$ in distribution.

  - (•) True ✔
  - ( ) False

- $X_n / Y_n \longrightarrow X/Y$ in distribution, provided Y is constant.

  - ( ) True
  - (•) False ✔

- $X_n^2 - 2X_n + 5 \longrightarrow X^2 - 2X + 5$ in distribution.

  - (•) True ✔
  - ( ) False

Solution:

- True. Sums of sequences that converge in probability converge in probability, and convergence in probability implies convergence in distribution.
- True. Since both X_n and Y_n converge in probability to X and Y respectively, $X_n Y_n$ converges in probability, and hence in distribution, to XY.
- False. Even though Y_n converges to a constant, this constant can very well be 0, in which case we do not have the desired convergence.
- True. This is a consequence of continuous mapping theorem, since the function $g(x) = x^2 - 2x + 5$ is continuous, $X_n \longrightarrow X$ in distribution implies $g(X_n) \longrightarrow g(X)$ in distribution.

提交 你已经尝试了2次（总共可以尝试2次）

Answers are displayed within the problem

Applying Slutsky's and the Continuous Mapping theorems

1/1 point (graded)
Given the following:

- $Z_1, Z_2, \ldots, Z_n, \ldots$ is a sequence of random variables that converge in distribution to another random variable Z;
- $Y_1, Y_2, \ldots, Y_n, \ldots$ is a sequence of random variables each of which takes value in the interval $(0,1)$, and which converges in probability to a constant c in $(0,1)$;
- $f(x) = \sqrt{x(1-x)}$.

Does $Z_n \dfrac{f(Y_n)}{f(c)}$ converge in distribution? If yes, enter the limit in terms of Z, Y and c; if no, enter **DNE**.

$Z_n \dfrac{f(Y_n)}{f(c)} \xrightarrow{\text{d}}$ Z ✔ Answer: Z

Z

Solution:

Since f is continuous in $(0,1)$, $f(Y_n)$ converges in probability to $f(c)$ by the continous mapping theorem. Since $f(c)$ is a constant, we have $\frac{f(Y_n)}{f(c)}$ converges in probability to 1. Finally, since Z_n converges in distribution to Z and $Z_n \dfrac{f(Y_n)}{f(c)}$ and $\frac{f(Y_n)}{f(c)}$ converges in probability to a constant, by Slutsky's Theorem, $Z_n \dfrac{f(Y_n)}{f(c)}$ converges in distribution to Z.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

讨论

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主题: Unit 1 Introduction to statistics:Lecture 2: Probability Redux / 8. Operations on Sequences and Convergence

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