

# 14. Exercise: A variation on merging

## Exercise: A variation on merging

2/2 points (graded)  
We start with two independent Bernoulli processes,  $X_n$  and  $Y_n$ , with parameters  $p$  and  $q$ , respectively. We form a new process  $Z_n$  by recording an arrival in a given time slot if and only if **both** of the original processes record an arrival in that same time slot. Mathematically,  $Z_n = X_n Y_n$ .

The new process  $Z_n$  is also Bernoulli with parameter 

p\*q

 ✓ Answer: p\*q

(Enter an algebraic function of  $p$  and  $q$  using standard notation.)

Suppose that the two Bernoulli processes  $X_n$  and  $Y_n$  are dependent. We still assume, however, that the pairs  $(X_n, Y_n)$  are independent. E.g.,  $(X_1, Y_1)$  is independent from  $(X_2, Y_2)$ , etc. Is the process  $Z_n$  guaranteed to be Bernoulli?

No ▼ ✓ Answer: No

STANDARD NOTATION

### Solution:

The merged process records an arrival if and only if both of the original processes record an arrival, which happens with probability  $pq$ .

In the second case, since the pairs  $(X_n, Y_n)$  are independent, the random variables  $Z_n$  are also independent. However, there is nothing in the statement that would ensure that the  $Z_n$  are identically distributed. Thus,  $Z_n$  is not guaranteed to be a Bernoulli process. For example, consider the special case of  $p = q$  and suppose that  $Y_1 = X_1$  but  $Y_n$  is independent of  $X_n$  for  $n > 1$ . Then  $P(Z_1 = 1) = p$  while  $P(Z_n = 1) = p^2$  for  $n > 1$ , violating the time-homogeneity property of Bernoulli processes.

提交 你已经尝试了1次 (总共可以尝试1次)

我理解的意思是X和Y如果以一个相同的共变系数不独立，那么Z是Bernoulli。但是如果这个共变系数在变，那么Z的p也在变，所以不是一个Bernoulli

Answers are displayed within the problem

## 讨论

主题: Unit 9 / Lec. 21 / 14. Exercise: A variation on merging

显示讨论