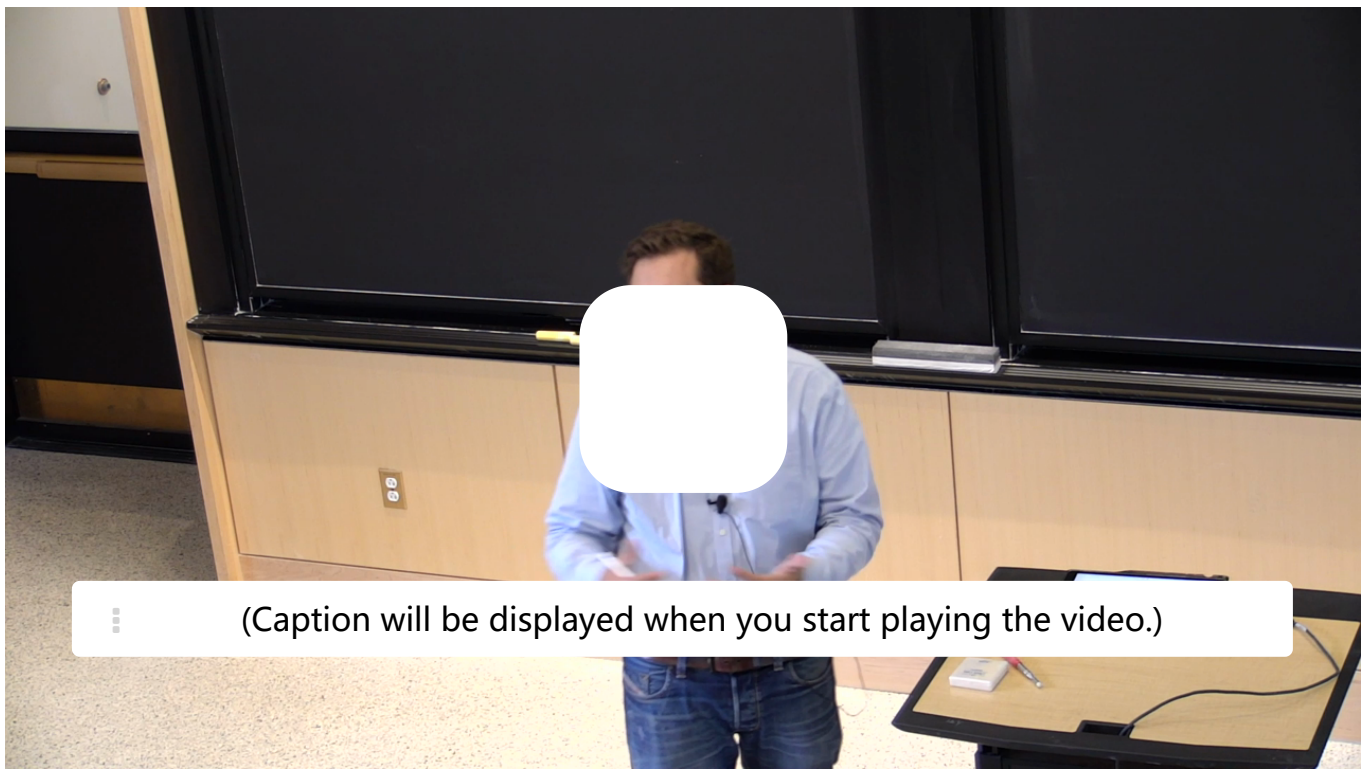
3. Confidence Intervals Concept Checks Continued
Confidence Interval Review



Start of transcript. Skip to the end.

OK.
So let's start with a few exercises
that we left behind us.
And that will be a good way for us
to refresh our memory in terms of what a confidence interval
is and what it does for us.
So now here's the first question.
Hopefully, you've had a minute to think

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Confidence Interval Concept Check 3

0/1 point (graded)
In a new experiment consisting of 150 couples, 75 couples are observed to turn their heads to the left and the remaining 75 couples turned their heads to the right when kissing. Let p denote the (unknown) parameter which specifies the probability that a couple turns their head to the right.

Which of the following statements are correct regarding this experiment? You are given that **exactly one** but not both of choices 3 and 4 is correct.
(Choose all that apply.)

- [ ] $[0, 0.5]$ is a 50% asymptotic confidence interval for p. ✔
- [ ] $[0.5, 1]$ is a 50% asymptotic confidence interval for p. ✔
- [ ] $[0.466, 0.533]$ is a 50% asymptotic confidence interval for p. ✔
- [x] $[0.48, 0.52]$ is a 50% asymptotic confidence interval for p.

✖

Solution:

只要包括了，是不是以平均数为中心不要紧。

See next video for presented solution.

The first three answer choices are correct, and the final choice is incorrect.

Let $R_1, R_2, \ldots, R_{150} \overset{iid}{\sim} \mathbf{Ber}(p)$ denote the sampled response (without loss of generality, assume that $R_i = 1$ encodes that the i-th couple turns their heads to the right, and $R_i = 0$ encodes the couple turns their heads to the left.) Let $P = \mathbf{Ber}(p)$ denote the common distribution of $R_1, \ldots R_{150}$.

Consider the sample mean $\overline{R}_n$. By the central limit theorem,

$$\sqrt{n}\left(\frac{\overline{R}_n - p}{\sqrt{p(1-p)}}\right) \underset{(d)}{\longrightarrow} N(0,1).$$

Now we examine the answer choices in order.

1. Consider the interval $[0, 0.5]$. Since $\overline{R}_n = 0.5$, this interval is a realization of the (random) confidence interval $\mathcal{I} = (0, \overline{R}_n)$. We compute that

$$P(\mathcal{I} \ni p) = P(p \leq \overline{R}_n) = P(\overline{R}_n - p \geq 0).$$

   Observe that $\overline{R}_n - p$ is a centered (*i.e.* $\mathbb{E}[\overline{R}_n - p] = 0$), symmetric (*i.e.*, $\overline{R}_n - p$ and $-(\overline{R}_n - p)$ have the same distribution) random variable. Therefore $P(\mathcal{I} \ni p) = 1/2$. Indeed $[0, 0.5]$ is an asymptotic (in fact, it is even a *non-asymptotic*) confidence interval of level 50%.

2. The interval $[0.5, 1]$ is a realization of the (random) confidence interval $\mathcal{I} = (\overline{R}_n, 1)$. We see that

$$P(\mathcal{I} \ni p) = P(\overline{R}_n \leq p \leq 1) = P(\overline{R}_n - p \leq 0).$$

   By the reasoning in the previous part, we must also have that $P(\mathcal{I} \ni p) = 1/2$.

3. Given that either choice 3 or choice 4 is correct but not both, it must be that the wider of the 2 intervals $[0.466, 0.533]$ is a 50% asymptotic confidence interval for p. Otherwise, the narrower interval $[0.48, 0.52]$ being a 50% asymptotic confidence interval for p implies the same for the wider interval.

提交 你已经尝试了2次（总共可以尝试2次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 2 Foundation of Inference:Lecture 5: Delta Method and Confidence Intervals / 3. Confidence Intervals Concept Checks Continued