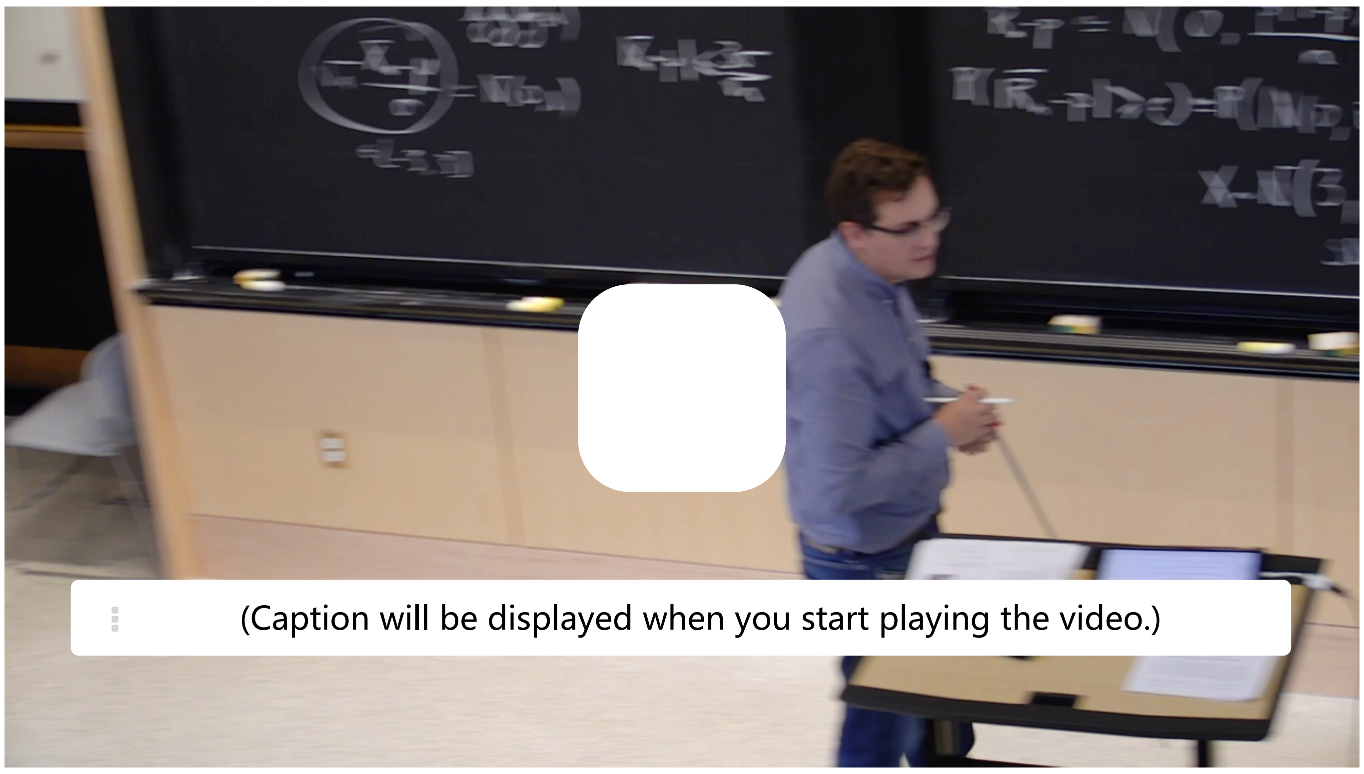
5. Properties of the Gaussian distribution
Affine transformation, standardization, Symmetry

Start of transcript. Skip to the end.



OK.

So what are the useful properties, right?

Now that we've put out of the [INAUDIBLE] thing that we

cannot compute anything, this thing's infinite.

It's kind of slightly awkward.

There's actually some really nice properties.

视频
下载视频文件

字幕
下载 SubRip (.srt) file
下载 Text (.txt) file

Standardization

1/1 point (graded)

Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with mean μ and variance σ^2. Denote the sample mean by $\overline{X}_n = \frac{\sum_{i=1}^n X_i}{n}$.

Assume that n is large enough that the central limit theorem (clt) holds. Find a random variable Z with approximate distribution $\mathcal{N}(0,1)$, in terms of $\overline{X}_n$, n,μ and σ. (Note that μ and σ^2 refers to the mean and variance of X_i, not $\overline{X}_n$.)

(Type **barX_n** for $\overline{X}_n$, **mu** for μ and **sigma** for σ. Refer to the standard notation button below.)

$Z \sim \mathcal{N}(0,1)$ for $Z =$ sqrt(n)*(barX_n - mu) / s ✔ **Answer:** (barX_n-mu)*sqrt(n)/sigma

STANDARD NOTATION

Solution:

First, compute the mean and variance of $\overline{X}_n$:

$$\mathbb{E}\left[\overline{X}_n\right] = \mathbb{E}\left[X_i\right] = \mu$$

$$\textsf{Var}\left(\overline{X}_n\right) = \frac{\sum_{i=1}^n \textsf{Var}\left[X_i\right]}{n^2} = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}.$$

Then, standardize by defining

Generating Speech Output

$$
\begin{aligned}
Z &= \overline{X}_n - \mathbb{E}\left[\overline{X}_n\right]\sqrt{\text{Var}\left(\overline{X}_n\right)} \\
&= \overline{X}_n - \mu\sqrt{\sigma^2/n} \\
&= \sqrt{n}\overline{X}_n - \mu\sigma.
\end{aligned}
$$

By the clt, $Z \sim \mathcal{N}(0,1)$.

提交 你已经尝试了2次（总共可以尝试3次）

Answers are displayed within the problem

Transformation and Symmetry

1.0/1 point (graded)

Let $X \sim \mathcal{N}(2,2)$, i.e. X is a Gaussian variable with mean $\mu = 2$ and variance $\sigma^2 = 2$. Let $x > 0$.

Write $\mathbf{P}(X \geq -x)$ in terms of the cdf of the **normal** Gaussian variable with a positive argument. In other words, your answer should be in terms of $\Phi(g(x))$ where $g(x)$ is a function of x which takes only **positive** values for $x > 0$.

(For example, if your answer is $1 + \Phi\left(\sqrt{5}x\right)$, type 1+Phi(sqrt(5)*x).)

$\mathbf{P}(X \geq -x) =$ Phi((x+2)/sqrt(2)) ✔ **Answer:** Phi((x+2)/sqrt(2))

Solution:

Standardizing $X \sim \mathcal{N}(2,2)$, we have $\dfrac{X-2}{\sqrt{2}} \sim \mathcal{N}(0,1)$. (The intuition here is that we are translating and re-scaling the density of X so that we end up with a standard Gaussian density.)

$$
\begin{aligned}
\mathbf{P}(X \geq -x) &= \mathbf{P}\left(\frac{X-2}{\sqrt{2}} \geq \frac{-x-2}{\sqrt{2}}\right) \\
&= \mathbf{P}\left(\frac{X-2}{\sqrt{2}} \leq \frac{x+2}{\sqrt{2}}\right) \quad \text{by symmetry} \\
&= \Phi\left(\frac{x+2}{\sqrt{2}}\right).
\end{aligned}
$$

The expression $1 - \Phi\left(\frac{-x-2}{\sqrt{2}}\right)$ gives the same value, but the argument is negative and so is not an accepted answer.

Remark: The symmetry is easiest to see by comparing the areas under the standard normal pdf corresponding to $\mathbf{P}(Z \geq -z)$ and $\mathbf{P}(Z \leq z)$ where $Z \sim \mathcal{N}(0,1)$ and $z > 0$.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 1 Introduction to statistics:Lecture 2: Probability Redux / 5. Properties of the Gaussian distribution

认证证书是什么?

Generating Speech Output