7. LLMS estimation

Problem 6. LLMS estimation

2.5/5.0 points (graded)

Let X and W be independent and uniformly distributed on $[-1,1]$. We have given the following facts:

$$\begin{aligned} \mathbf{E}[X] = \mathbf{E}[X^3] = \mathbf{E}[X^5] &= 0 \\ \mathbf{E}[X^2] &= 1/3 \\ \mathbf{E}[X^4] &= 1/5 \end{aligned}$$

Suppose that

$$Y = X^3 + W$$

1. Find the LMS estimate of Y, given that $X = x$. (Notice we are trying to estimate Y from X, not the opposite direction.) (Your answer should be a function of x.)

$\hat{Y}_{\text{LMS}}(x) =$ x^3 ✔ Answer: x^3

x^3

2. Find the LLMS estimate for Y, given that $X = x$. (Your answer should be a function of x.)

$\hat{Y}_{\text{LLMS}}(x) =$ x^3 ✖ Answer: 0.6*x

x^3

Because in this problem, our *model* is in the form of $Y = X^3 + W$, but our *observation* is in the form of $X = x$. (We are asked for the LLMS given $X = x$.) Having X^3 in the model doesn't guarantee you can make directly observations in the X^3 form in the real life, even if that would be more convenient.

LLMS estimator varies depending on the representation of your observation, even if both have the same information.

STANDARD NOTATION

Solution:

1. $\hat{Y}_{\text{LMS}}(x) = \mathbf{E}[Y|X=x] = \mathbf{E}[X^3 + W|X=x] = \mathbf{E}[x^3 + W] = x^3$.

2. Since X, Y are both zero mean, we have $\mathbf{Cov}(X,Y) = \mathbf{E}[XY]$, and

$$\begin{aligned} \hat{Y}_{\text{LLMS}}(x) &= \mathbf{E}[Y] + \frac{\mathbf{E}[XY]}{\mathbf{E}[X^2]}(x - \mathbf{E}[X]) \\ &= 0 + \frac{\mathbf{E}[X(X^3+W)]}{\mathbf{E}[X^2]}x \\ &= \frac{\mathbf{E}[X^4]}{\mathbf{E}[X^2]}x \\ &= \frac{3}{5}x \end{aligned}$$

提交 你已经尝试了2次（总共可以尝试2次）

Answers are displayed within the problem

Error and Bug Reports/Technical Issues

显示讨论

主题: Final Exam / 7. LLMS estimation