MITx Video | 6.041x | Spring 2014 | Video transcript | L22-8: Summary of results

Let us now take stock and summarize what we have done for these two processes, the Bernoulli and the Poisson process, and their relation. The Poisson process runs in continuous time, whereas, for the Bernoulli process, time is discrete and is divided into slots. The Poisson process is defined by a single parameter, lambda, which is the intensity or arrival rate, and tells us the expected number of arrivals per unit time.

For the Bernoulli process, we have, again, one basic parameter, which is the probability of success at any given trial, or the probability of an arrival during each one of the slots. Based on our model, we were interested in three kinds of quantities. And we found the distributions of them.

The first quantity is the number of arrivals during a certain time interval. For the discrete case, the number of arrivals has a binomial distribution, whereas for the one Poisson case, the distribution is that of a Poisson random variable. Then we looked at the time until the first arrival, or the time between consecutive arrivals. For the Bernoulli process, that distribution is geometric. For the Poisson process, that distribution is exponential. Note that in this instance, we're dealing with a discrete random variable, but, here, with a continuous random variable.

And then, as a generalization, we could find the time until a kth arrival, which, in the Poisson case, is given by an Erlang distribution. And for the Bernoulli case, we developed one particular formula. And that formula is actually known under the name of the Pascal distribution.

All of these results, for the Poisson case, were obtained because we used an approximation argument. That is, we had the results for the Bernoulli case. But then we argued that the Poisson process is a limiting case of the Bernoulli process in which we take time, divide it into a large number of slots, during each one of the slots, however, we have a small probability of an arrival. And this is done in a way so that the product of these two numbers stays a constant.

By using a finer and finer discretization, we could approach the Poisson process arbitrarily close by a Bernoulli process. And then we used the Bernoulli formulas in which we took the limit as delta was going to zero. And this gave us the result for the Poisson case.