5. Review: Covariance

Note: The following exercises are a review of covariance, and will be discussed in lecture. We encourage that you attempt these exercises before watching the video.

Review: Covariance

2/2 points (graded)

If X and Y are random variables with respective means μ_X and μ_Y, then recall the **covariance** of X and Y (written Cov (X, Y)) is defined to be

$$\text{Cov}\,(X,Y) = \mathbb{E}\left[(X-\mu_X)\,(Y-\mu_Y)\right].$$

Alternatively, one can show that this is equivalent to $\mathbb{E}\left[XY\right] - \mathbb{E}\left[X\right]\mathbb{E}\left[Y\right]$.

For each of the following statements, indicate whether it is true or false.

"Cov $(X, X) =$ Var (X)".

- (•) True
- ( ) False

"Like the variance, the covariance between an arbitrary pair of RVs X and Y is always positive."

- ( ) True
- (•) False

Solution:

- **True.** Cov $(X, X) = \mathbb{E}\left[(X-\mu_X)^2\right] =$ Var (X).
- **False.** Consider (X, Y) which is distributed uniformly over the set $\{(1,-1), (-1,1)\}$. The marginal distributions of both X and Y are uniform over $\{\pm 1\}$, so $\mu_X = \mu_Y = 0$. On the other hand, $\mathbb{E}\left[XY\right] = -1$, so Cov $(X, Y) = -1$.

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

Alternate Formula for Covariance

1/1 point (graded)

Let X and Y are random variables with respective means μ_X and μ_Y. Is it true that $\mathbb{E}\left[(X)\,(Y-\mu_Y)\right] =$ Cov (X, Y)?

- (•) True

False

Solution:

Indeed, $\mathbb{E}\left[(X)\left(Y-\mu_Y\right)\right] = \textsf{Cov}\left(X,Y\right) = \mathbb{E}\left[\left(X-\mu_X\right)(Y)\right]$. That is, it is sufficient to center one random variable around its mean when computing the covariance between two random variables. This can be seen from the following:

$$\begin{aligned}\mathbb{E}\left[(X)\left(Y-\mu_Y\right)\right] &= \mathbb{E}\left[XY\right] - \mathbb{E}\left[X\mu_Y\right] \\ &= \mathbb{E}\left[XY\right] - \mathbb{E}\left[X\right]\mathbb{E}\left[Y\right].\end{aligned}$$

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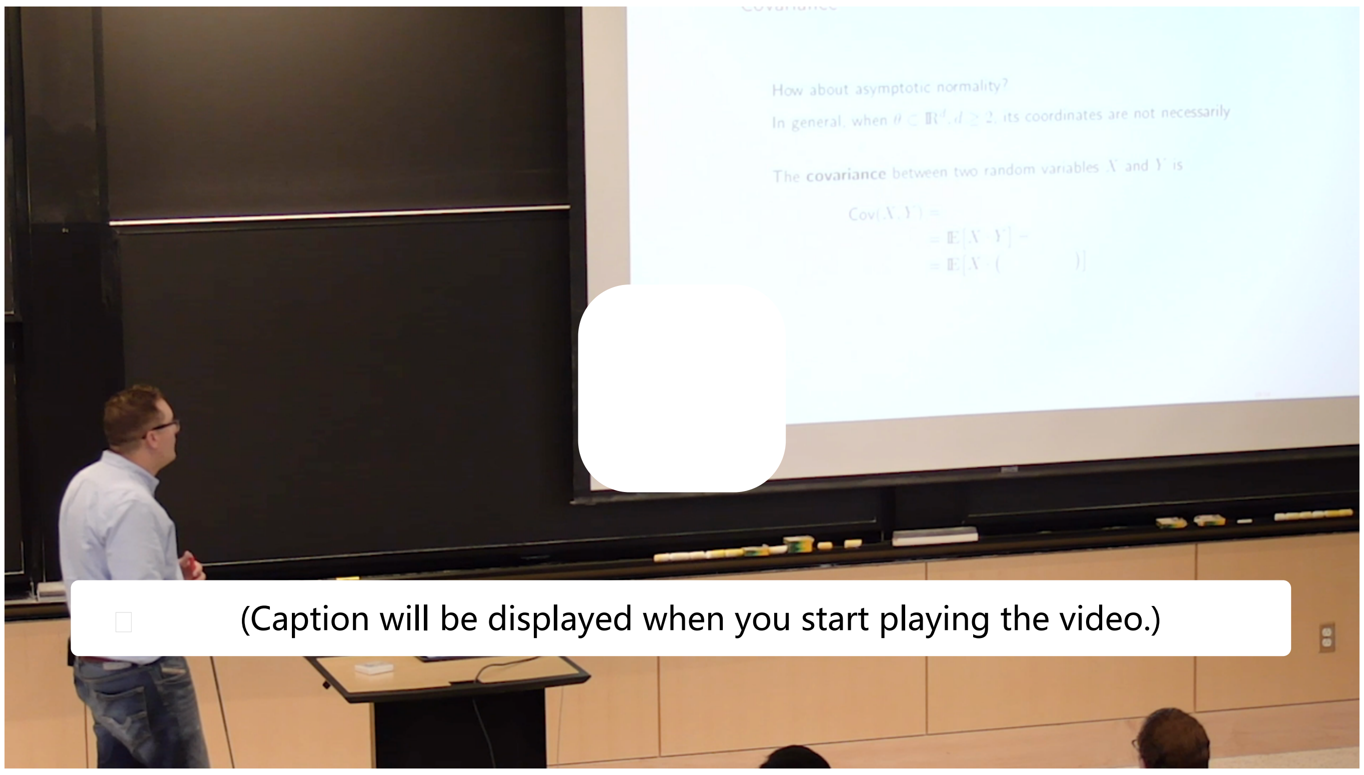
Answers are displayed within the problem

Covariance: Definition and Formula



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OK.

So we have consistency in general

about asymptotic normality, right?

That's the next question that we typically ask.

Well, in general, we have to talk about asymptotic normality

of a vector OK.

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Bilinearity of Covariance

1/1 point (graded)
Let X, Y, Z be random variables and a, b be constants. Indicate whether the following statement is true or false.

“Covariance is bilinear, i.e. $\textsf{Cov}\left(aX+bY,Z\right) = a\textsf{Cov}\left(X,Z\right) + b\textsf{Cov}\left(Y,Z\right)$."

Hint: Use the result from the problem immediately above.

True

False

Solution:

True. This can be seen by using the trick of centering only Z when computing covariance. First, note that $\textsf{Cov}\left(aX,Z\right) = \mathbb{E}\left[(aX)\left(Z-\mu_Z\right)\right] = a\mathbb{E}\left[(X)\left(Z-\mu_Z\right)\right] = a\textsf{Cov}\left(X,Z\right)$. Then,

$$\begin{aligned}\text{Cov}\left(aX+bY,Z\right) &= \mathbb{E}\left[\left(aX+bY\right)\left(Z-\mu_Z\right)\right]\\ &= \mathbb{E}\left[\left(aX\right)\left(Z\right)\right]-\mu_Z\mathbb{E}\left[aX\right]+\mathbb{E}\left[\left(bY\right)\left(Z\right)\right]-\mu_Z\mathbb{E}\left[bY\right]\\ &= a\text{Cov}\left(X,Z\right)+b\text{Cov}\left(Y,Z\right)\end{aligned}$$

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

Example of Covariance I

1/1 point (graded)

Let $A = X+Y$ and $B = X-Y$. Let $\mu_X = \mathbb{E}\left[X\right]$, $\mu_Y = \mathbb{E}\left[Y\right]$, $\tau_X = \text{Var}\left(X\right)$, $\tau_Y = \text{Var}\left(Y\right)$ and $c = \text{Cov}\left(X,Y\right)$. In terms of μ_X, μ_Y, τ_X, τ_Y, and c, what is $\text{Cov}\left(A,B\right)$?

(Enter **mu_X** for μ_X, **tau_X** for τ_X.)

tau_X - tau_Y

Answer: tau_X-tau_Y

$\tau_X - \tau_Y$

STANDARD NOTATION

Solution:

Expand out the definition of covariance using bi-linearity (see the solution to the previous question):

$$\begin{aligned}\text{Cov}\left(A,B\right) &= \text{Cov}\left(X+Y,X-Y\right)\\ &= \text{Cov}\left(X+Y,X\right)-\text{Cov}\left(X+Y,Y\right)\\ &= \text{Cov}\left(X,X\right)+\text{Cov}\left(Y,X\right)-\text{Cov}\left(X,Y\right)-\text{Cov}\left(Y,Y\right)\\ &= \text{Var}\left(X\right)-\text{Var}\left(Y\right)\\ &= \tau_X-\tau_Y.\end{aligned}$$

提交 你已经尝试了3次（总共可以尝试4次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 3 Methods of Estimation:Lecture 10: Consistency of MLE, Covariance Matrices, and Multivariate Statistics / 5. Review: Covariance