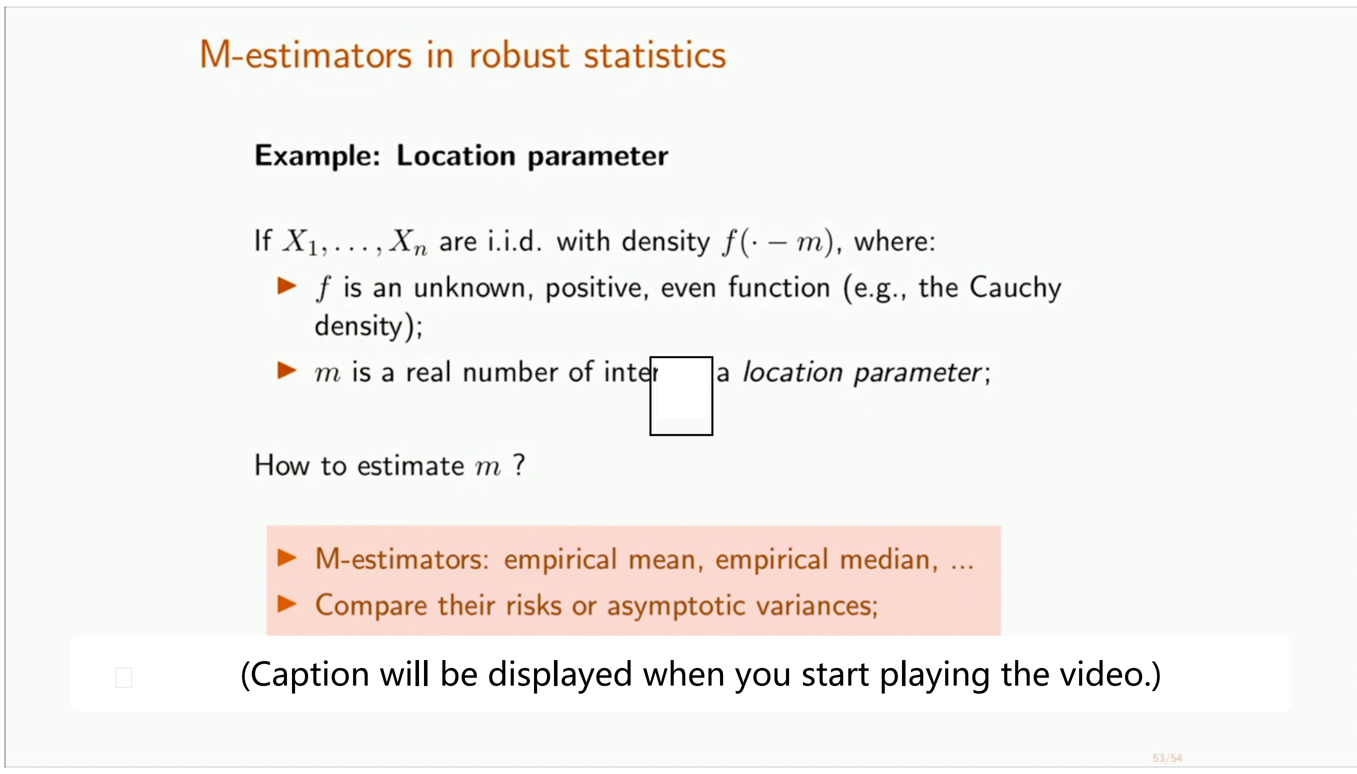
7. Robust Statistics and Huber's Loss

Motivation and Introduction to Huber's Loss

Start of transcript. Skip to the end.



0:00 / 0:00 1.0x

OK, so the question is, how do you estimate this?

Well, you could look at different estimators,

and you could try to compare their asymptotic variance,

right?

That's really nice, and you could actually

conclude that the median is more robust, as we said.

That would be a good estimator.

视频
下载视频文件

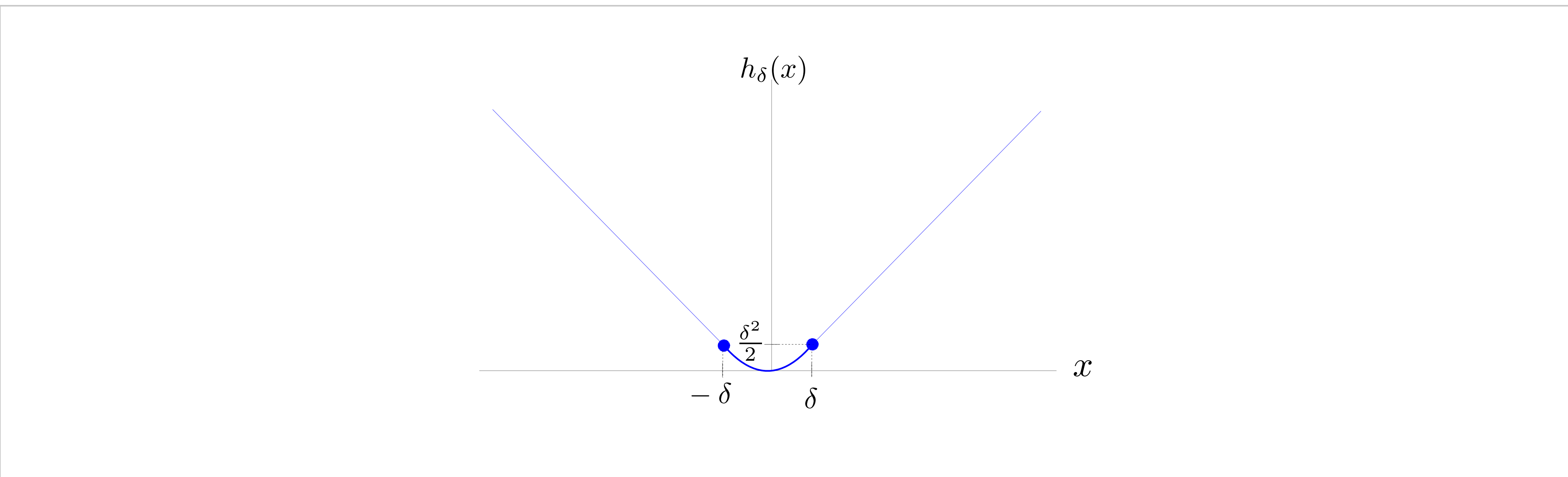
字幕
下载 SubRip (.srt) file
下载 Text (.txt) file

Huber's Loss

3/3 points (graded)

Huber's loss is defined to be

$$h_\delta(x) = \begin{cases} \frac{x^2}{2} & \text{if } |x| < \delta \\ \delta\left(|x| - \delta/2\right) & \text{if } |x| > \delta \end{cases}$$



Let k denote the smallest integer such that the $\frac{d^k}{dx^k} h_\delta(x)$ is **not** a continuous function.

What is k?

2

Answer: 2

The function $\frac{d^k}{dx^k} h_\delta(x)$ is discontinuous at two points $x_1, x_2 \in \mathbb{R}$ where $x_1 < x_2$.

What are x_1 and x_2 in terms of δ?

$x_1 =$ -delta

$-\delta$

Answer: -delta

$x_2 =$ delta

δ

Answer: delta

STANDARD NOTATION

Solution:

Observe that

$$\frac{\partial h_\delta}{\partial x}(x) = \begin{cases} x & \text{if } |x| < \delta \\ \delta & \text{if } x > \delta \\ -\delta & \text{if } x < -\delta, \end{cases}$$

which is a continuous function. However, the next derivative

$$\frac{\partial^2 h_\delta}{\partial^2 x}(x) = \begin{cases} 1 & \text{if } |x| < \delta \\ 0 & \text{if } |x| > \delta \end{cases}$$

has discontinuities at $x = \pm\delta$. In particular, $\frac{\partial^2 h_\delta}{\partial^2 x}(\pm\delta)$ is not defined. Therefore, for the first question, we conclude that $k = 2$. For the second question, $x_1 = -\delta$ and $x_2 = \delta$.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

Comparing Huber's Loss and the absolute value function

1/1 point (graded)

Recall Huber's loss $h_\delta(x)$ as defined in the previous problem. The absolute value function is defined to be $|x|$.

Which of the following statements are true? (Choose all that apply.)

- [ ] Both Huber's loss and the absolute value are differentiable everywhere.
- [x] For $x > 0$ sufficiently large, both Huber's loss and the absolute value are both linear functions.
- [ ] In the intervals where $h_\delta(x)$ is a linear function, both Huber's loss and the absolute value function have the same slope.
- [x] Both Huber's loss and the absolute value function are convex.

Solution:

We examine the choices in order.

- "Both Huber's loss and the absolute value are differentiable everywhere." is incorrect. It is true that Huber's loss is differentiable everywhere. However, $|x|$ is not differentiable at $x = 0$.

- "For $x > 0$ sufficiently large, both Huber's loss and the absolute value are both linear functions." is correct. This is certainly true for the absolute function, as $|x| = x$ if $x > 0$. Moreover, if $x > \delta$, then we have $h_\delta(x) = \delta(x - \delta/2)$ which is also a linear function.

- "In the intervals where $h_\delta(x)$ is a linear function, both Huber's loss and the absolute value function have the same slope." is incorrect. For example, if $x > \delta$, then $|x|$ has slope $+1$. However, $h_\delta(x)$ has slope δ, which is not necessarily equal to 1.

- "Both Huber's loss and the absolute value function are convex." is correct. This is evident from the graphs of $|x|$ and $h_\delta(x)$.

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 3 Methods of Estimation:Lecture 12: M-Estimation / 7. Robust Statistics and Huber's Loss