4. Exercise: Estimator properties

Exercise: Estimator properties

1/4 points (graded)

We estimate the unknown mean θ of a random variable X (where X has a finite and positive variance) by forming the sample mean $M_n = (X_1 + \cdots + X_n)/n$ of n i.i.d. samples X_i and then forming the estimator

$$\widehat{\Theta} = M_n + \frac{1}{n}.$$

Is this estimator unbiased?

Yes ✖ **Answer:** No

Is this estimator consistent?

Yes ✔ **Answer:** Yes

Consider now a different estimator, $\widehat{\Theta}_n = X_1$, which ignores all but the first measurement.

Is this estimator unbiased?

No ✖ **Answer:** Yes

Is this estimator consistent?

Yes ✖ **Answer:** No

Solution:

We have $\mathbf{E}[\widehat{\Theta}_n] = \theta + (1/n) \neq \theta$, so it is not unbiased. On the other hand, M_n converges (in probability) to θ, and $1/n$ converges to zero. So, their sum, $\widehat{\Theta}_n = M_n + (1/n)$ also converges (in probability) to θ, and the estimator is consistent.

The second estimator is unbiased, because $\mathbf{E}[\widehat{\Theta}_n] = \mathbf{E}[X_1] = \theta$. But it is not consistent. Its value stays the same (equal to X_1) for all n and therefore cannot converge to θ, unless X_1 is guaranteed to be equal to θ. But this is impossible since X has positive variance.

提交 You have used 1 of 1 attempt

Answers are displayed within the problem

讨论

显示讨论

Topic: Unit 8 / Lec. 20 / 4. Exercise: Estimator properties