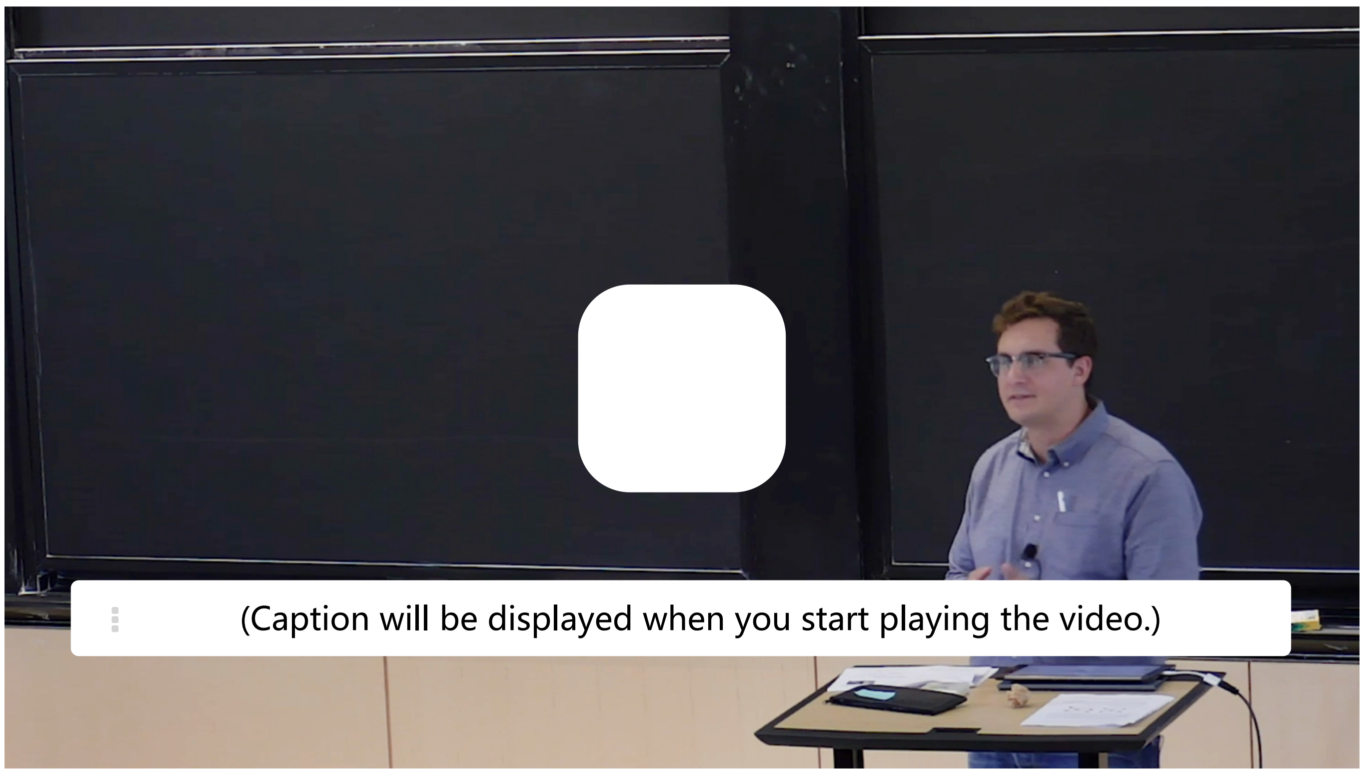
2. Two important probability tools

Two important probability tools



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All right.

So welcome back.

So again, this is a statistics class,

and hopefully, I convinced you last time,

probability is an essential part of statistics.

We have, on the one hand, the truth, a stochastic process,

a data generating process, that is generating

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Average of Gaussians

2/3 points (graded)

Let $X_1, X_2, \dots, X_n$ be i.i.d. **standard normal random variables**. What is the distribution of

$$\overline{X}_n = \frac{X_1 + X_2 + \cdots + X_n}{n}?$$

● A Gaussian. ✓

○ A χ^2-distribution.

○ Cannot be determined for finite n, but asymptotically Gaussian.

In terms of n, what are the variance and mean of $\overline{X}_n$?

$\mathrm{Var}\left(\overline{X}_n\right) =$ 1/sqrt(n) ✗ **Answer:** 1/n

$\frac{1}{\sqrt{n}}$

$\mathbb{E}\left[\overline{X}_n\right] =$ 0 ✓ **Answer:** 0

0

if independence

$Var(X_1+X_2+ .. X_n) = Var(X_1) + Var(X_2) + \cdots Var(X_n)$

CLT => $X_1+X_2+ .. X_n \sim$ Normal (.

$X_1+X_2+ \dots X_n \sim N(0.n)$

$= n\bar{X}_n$

$Var(n\bar{X}_n) = n$

$\frac{1}{n^2} \cdot Var(n\bar{X}_n) = \frac{1}{n^2} \cdot n$

$Var(\bar{X}_n) = \frac{1}{n}$.

Solution:

Since the sum of i.i.d. Gaussian random variables is also Gaussian, we deduce first that $X_1 + \cdots + X_n \sim N(0, n)$. Multiplying by $1/n$, we get $\overline{X}_n \sim N(0, 1/n)$, as scaling a random variable with a constant c scales its variance by c^2.

Therefore, $\overline{X}_n$ is a Gaussian random variable with mean 0 and variance $1/n$.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

CLT Concept Check

1/1 point (graded)

Let $X_1, X_2, \ldots, X_n$ be an i.i.d. sequence of random variables with $\mathbb{E}[X] = \mu$, and, **Var** $(X) = \sigma^2$. Assuming that n is very large, according to the Central Limit Theorem, what is the best approximate characterization of the distribution of $\overline{X}_n$?

- ○ $N(0, 1)$.
- ● $N(\mu, \sigma^2/n)$. ✓
- ○ $N(0, \sigma^2/n)$.
- ○ Depends on the distribution of X.

Solution:

The correct choice is the second choice. We know by the Central Limit Theorem that

$$\frac{\sqrt{n}\left(\overline{X}_n - \mu\right)}{\sigma} \longrightarrow N(0, 1)$$

in distribution. Therefore, we can use approximate normality (as stated in the problem preamble), and get

$$\overline{X}_n \approx N\left(\mu, \sigma^2/n\right).$$

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 1 Introduction to statistics:Lecture 2: Probability Redux / 2. Two important probability tools

认证证书是什么？