Neural Networks and Biological Modeling

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Question set 13

Exercise 1: From adaptive integrate-and-fire to the SRM

Consider a leaky integrate-and-fire neuron with a spike triggered adaptive current w

$$\tau \frac{du}{dt} = -(u - u_{rest}) - \alpha R w(t) + RI(t), \tag{1}$$

$$\tau_w \frac{dw}{dt} = -w + \tau_w \beta S(t),$$

where the membrane potential u is reset to u_{rest} at the threshold $u = \theta$. Here, we assume the neuron fires at given times $t^f > 0$, which gives the spike train $S(t) = \sum_f \delta(t - t^f)$.

1.1 Set $\alpha = 0$ and assume the neuron is at rest at time $t = t_0$. Integrate Eq. 1, explicitly including a reset of the membrane potential at the spike times. Hint: Use an adequate pulse current injection for the reset.
After this you can set $t_0 \to -\infty$ since the initial time is arbitrary here. Then write the result in the following closed form

$$u(t) = u_{rest} + \int_0^\infty \epsilon(s) I(t-s) ds + \int_0^\infty \eta(s) S(t-s) ds, \tag{2}$$

with two kernels $\epsilon(t)$ and $\eta(t)$. What are the two kernels?

1.2 Now set $\alpha = 1$ and additionally assume that at $t = 0$ the adaptation variable is $w = 0$. Derive a closed form expression similar to the one in the last question, by first integrating w and then u. What are the two kernels now?

Exercise 2: Integrate-and-fire model with linear escape rates

Consider a leaky integrate-and-fire neuron with linear escape rate,

$$\rho_I(t|\hat{t}) = \beta[u(t|\hat{t}) - \theta]_+ = \begin{cases} \beta(u(t|\hat{t}) - \theta) & , \quad if \quad u(t|\hat{t}) > \theta \\ 0 & , \quad otherwise \end{cases}$$

2.1 Start with the non-leaky integrate-and-fire model by considering the limit of $\tau_m \to \infty$. The membrane potential of the model is then

$$u(t|\hat{t}) = u_r + \frac{1}{C} \int_{\hat{t}}^{t} I(t')\, dt'$$

Assume constant input, set $u_r = 0$ and calculate the hazard and the interval distribution.

2.2 Consider the leaky integrate-and-fire model with time constant τ_m and constant input I_0. Determine the membrane potential, the hazard, and the interval distribution.

Exercise 3: Optimization of a free parameter

Consider a very simple model for the membrane potential at time step n as a function of a given input:

$$u_n^{model} = RI_n$$

Further, assume you are given measured data u_n^{data} sampled at the same time steps.

3.1 Optimize the free scalar parameter R by minimizing the sum of squared errors

$$E = \sum_n \left[u_n^{data} - u_n^{model}\right]^2$$

with respect to this parameter (least squares fit).

3.2 Calculate the same for constant input $I_n = I_0$ and interpret the result.

Exercise 4: Likelihood of a spike train

In an in-vitro experiment, a time-dependent current $I(t)$ was injected into a neuron for a time $0 < t < T$ and four spikes were observed at times $0 < t^{(1)} < t^{(2)} < t^{(3)} < t^{(4)} < T$.

4.1 What is the likelihood that this spike train could have been generated by a leaky integrate-and-fire model with linear escape rate defined in exercise 2.

4.2 Rewrite the likelihood in terms of the interval distribution and hazard of time-dependent renewal theory.