<u>Homework 4: TV distance, KL-</u> <u>Divergence, and Introduction to</u>

课程 □ Unit 3 Methods of Estimation □ MLE

☐ 4. Maximum likelihood estimators

4. Maximum likelihood estimators

Instructions:

Let X_1,\ldots,X_n be n i.i.d. random variables with pdf $f_{ heta}$, where heta is an unknown parameter.

For each of the following questions, compute the likelihood function on paper and then find the maximum likelihood estimator for θ .

To encourage you to do the computations carefully rather than eliminate choices, you will only be given 1 or 2 attempts per question.

(a)

1/1 point (graded)

Compute the likelihood function and the maximum likelihood estimator for $oldsymbol{ heta}$ for

$$f_{ heta}\left(x
ight)= au heta^{ au}x^{-\left(au+1
ight)}\mathbf{1}\left(x\geq heta
ight), heta>0,$$

where au > 0 is a known constant.

max	X_i
	7

lacksquare $\min X_i$ \Box

$$\bigcup \frac{1}{n} \sum X_i$$

 $n\tau$

 \circ ∞

Solution:

The likelihood function is

$$L = au^n heta^{n au} \prod_i X_i^{-(au+1)} \mathbf{1}\{\min_i X_i \geq heta\}$$

For $heta \leq \min_i X_i$, the log-likelihood function is

$$l=n\ln au+n au\ln heta-(au+1)\sum_{i=1}\ln X_i$$

Take the derivative with respect to θ :

$$rac{\partial l}{\partial heta} = rac{n au}{ heta} > 0.$$

Thus, L is an increasing function on $(0,\min_i X_i]$, and is 0 for $heta>\min_i X_i$. Therefore,

$$\hat{ heta} = \min_i X_i$$

提交

你已经尝试了1次(总共可以尝试2次)

☐ Answers are displayed within the problem

(b)

3/3 points (graded)

Compute the likelihood function and the maximum likelihood estimator for $\,m{ heta}\,$ for

$$f_{ heta}\left(x
ight)=\sqrt{ heta}x^{\sqrt{ heta}-1}\mathbf{1}\left(0\leq x\leq1
ight), heta>0.$$

You will find that the maximum likelihood estimator for $oldsymbol{ heta}$ is of the form

$$\hat{ heta}^{ ext{MLE}} \; = \; c_1 n^{c_2} \left(\sum_{i=1}^n \ln X_i
ight)^{c_3}.$$

Enter the numbers c_1 , c_2 , c_3 below.

 $c_1 = \boxed{ }$ 1 \Box Answer: 1

 $c_2 = egin{array}{|c|c|c|c|} 2 & & & \Box & Answer: 2 \\ \hline \end{array}$

STANDARD NOTATION

Solution:

The likelihood function is

$$L= heta^{n/2}\prod_i X_i^{\sqrt{ heta}-1} \mathbf{1}\{0\leq X_i\leq 1\}.$$

The log-likelihood function is

$$l=rac{n}{2} {\ln heta} + (\sqrt{ heta}-1) \sum_i {\ln X_i}.$$

Take the derivative with respect to $oldsymbol{ heta}$ and set it to $oldsymbol{0}$:

$$rac{\partial l}{\partial heta} = rac{n}{2 heta} + rac{1}{2 heta^{1/2}} \sum_i \ln X_i = 0.$$

Then we get

$$\hat{ heta} = rac{n^2}{\left(\sum \ln X_i
ight)^2}.$$

提交

你已经尝试了1次(总共可以尝试2次)

☐ Answers are displayed within the problem

(c)

4/4 points (graded)

Compute the likelihood function and the maximum likelihood estimator for $\,m{ heta}$

$$f_{ heta}\left(x
ight)= heta au x^{ au-1}\exp\{- heta x^{ au}\}\mathbf{1}\left(x\geq0
ight), heta>0,$$

where au>0 is a known constant.

You will find that the maximum likelihood estimator for $oldsymbol{ heta}$ is of the form

$$\hat{ heta}^{ ext{MLE}} \; = \; c_1 n^{c_2} \left(\sum_{i=1}^n X_i^{c_3}
ight)^{c_4}.$$

Enter the c_1, c_2, c_3, c_4 in terms of au if applicable.

(Enter **tau** for au.)

STANDARD NOTATION

Solution:

The likelihood function is

$$L= heta^n au^n\prod_i X_i^{ au-1}\exp\{- heta\sum_i X_i^ au\}\mathbf{1}\{X_i\geq 0\}.$$

The log-likelihood function is

$$l = n \ln heta + n \ln au + (au - 1) \sum_i \ln X_i - heta \sum_i X_i^ au.$$

Take the derivative with respect to $\,oldsymbol{ heta}\,$ and set it to $\,oldsymbol{0}\,$

$$rac{\partial l}{\partial heta} = rac{n}{ heta} - \sum_i X_i^{ au} = 0,$$

we get

$$\hat{ heta} = rac{n}{\sum_i X_i^ au}.$$

提交

你已经尝试了1次 (总共可以尝试2次)

☐ Answers are displayed within the problem

讨论

主题: Unit 3 Methods of Estimation:Homework 4: TV distance, KL-Divergence, and Introduction to MLE / 4. Maximum likelihood estimators

认证证书是什么?

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