

17. Exercise: The discrete Bayes rule

Exercise: The discrete Bayes rule

1/1 point (graded)

The bias of a coin (i.e., the probability of Heads) can take three possible values, $1/4$, $1/2$, or $3/4$, and is modeled as a discrete random variable Q with PMF

$$p_Q(q) = \begin{cases} 1/6, & \text{if } q = 1/4, \\ 2/6, & \text{if } q = 2/4, \\ 3/6, & \text{if } q = 3/4, \\ 0, & \text{otherwise.} \end{cases}$$

Let K be the total number of Heads in two independent tosses of the coin. Find $p_{Q|K}(3/4 | 2)$.

✓ Answer: 0.75

Solution:

The Bayes rule for discrete random variables gives

$$p_{Q|K}(3/4 | 2) = \frac{p_Q(3/4)p_{K|Q}(2 | 3/4)}{p_K(2)} = \frac{(3/6) \cdot (3/4)^2}{p_K(2)} = \frac{(3/6) \cdot (3/4)^2}{3/8} = \frac{3}{4}.$$

To find $p_K(2)$, we used the total probability theorem:

$$p_K(2) = \sum_q p_Q(q)p_{K|Q}(2 | q) = (1/6) \cdot (1/4)^2 + (2/6) \cdot (2/4)^2 + (3/6) \cdot (3/4)^2 = 3/8.$$

You have used 1 of 3 attempts