10. Exercise: Discrete unknown and continuous observation

Exercise: Discrete unknown and continuous observation

2/2 points (graded)

Similar to the last example, suppose that $X = \Theta + W$, where Θ is equally likely to take the values -1 and 1, and where W is standard normal noise, independent of Θ. We use the estimator $\widehat{\Theta}$, with $\widehat{\Theta} = 1$ if $X > 0$ and $\widehat{\Theta} = -1$ otherwise. (This is actually the MAP estimator for this problem.)

a) Let us assume that the true value of Θ is 1. In this case, our estimator makes an error if and only if W has a low (negative) value. The conditional probability of error given the true value of Θ is 1, that is, $\mathbf{P}(\widehat{\Theta} \neq 1 \mid \Theta = 1)$, is equal to

- (selected) $\Phi(-1)$ ✔
- $\Phi(0)$
- $\Phi(1)$

where Φ is the standard normal CDF.

b) For this problem, the overall probability of error is easiest found using the formula

- $\mathbf{P}(\widehat{\Theta} \neq \Theta) = \int \mathbf{P}(\widehat{\Theta} \neq \Theta \mid X = x) f_X(x)\, dx$
- (selected) $\mathbf{P}(\widehat{\Theta} \neq \Theta) = \sum_\theta \mathbf{P}(\widehat{\Theta} \neq \theta \mid \Theta = \theta)\, p_\Theta(\theta)$ ✔

$= P(X \leq 0 \mid \Theta = 1)$

$= P(\Theta + W \leq 0 \mid \Theta = 1) =$

Solution:

a) We have

$$\begin{aligned}\mathbf{P}(\widehat{\Theta} \neq 1 \mid \Theta = 1) &= \cancel{\mathbf{P}(\Theta + W \leq 0 \mid \Theta = 1) = \mathbf{P}(1 + W \leq 0 \mid \Theta = 1)} \\ &= \mathbf{P}(1 + W \leq 0) = \mathbf{P}(W \leq -1) = \Phi(-1).\end{aligned}$$

b) Similar to part (a), $\mathbf{P}(\widehat{\Theta} \neq \theta \mid \Theta = \theta)$ is easy to calculate for either choice of $\theta = -1$ or $\theta = 1$. For this reason, the second formula is easy to implement.

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

讨论

主题: Unit 7 / Lec. 14 / 10. Exercise: Discrete unknown and continuous observation

显示讨论