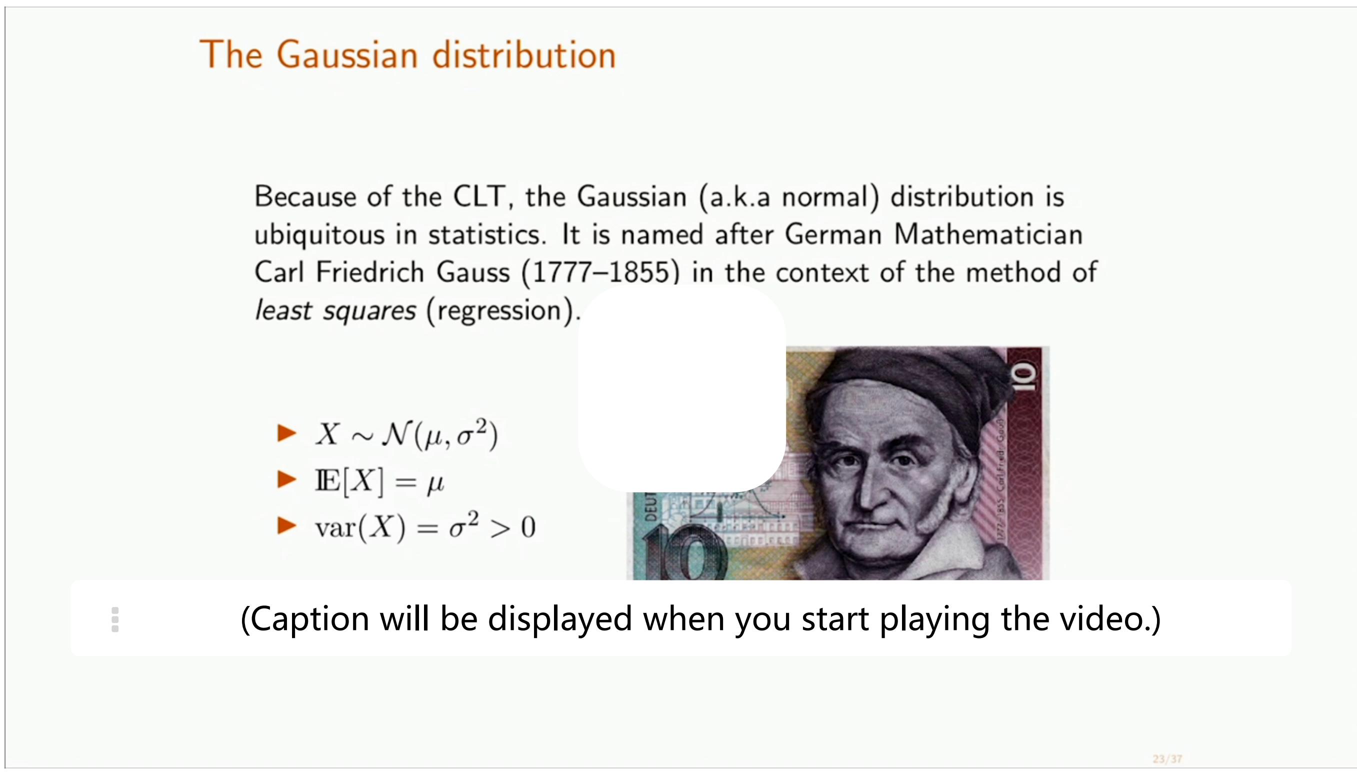
4. Gaussian distribution

Review: Gaussian Distribution



deviation of x
is square root of 6.
Most of the time we'll work with notation.
So you'll see sigma squared, and you
won't have to bother too much.
But when there's numerical examples,
you have to remember which one this is.
And software, for example, also have different conventions.
So some software asks you to enter the standard deviation,
and some software ask you to enter the variance
as the second parameter.
So be careful about that.
OK.
So this is a distribution.
It's on the real line.
It actually has a PDF.
You've probably all seen it before.

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Probability review: PDF of Gaussian distribution

2/2 points (graded)
In practice, it is not often that you will need to work directly with the probability density function (pdf) of Gaussian variables. Nonetheless, we will make sure we know how to manipulate the (pdf) in the next two problems.

If the pdf X is a Gaussian variable is

$$f_X(x) = \frac{n}{3\sqrt{2\pi}} \exp\left(-\frac{n^2(x-2)^2}{18}\right),$$

then what is the mean μ and variance σ^2 of X?
(Enter your answer in terms of n.)

$\mu =$ 2 ✔ Answer: 2

2

$\sigma^2 =$ 9/n^2 ✔ Answer: 9/n^2

$\frac{9}{n^2}$

STANDARD NOTATION

Solution:

Comparing

$$f_X(x) = \frac{n}{3\sqrt{2\pi}} \exp\left(-\frac{n^2(x-2)^2}{18}\right) = \frac{1}{(3/n)\sqrt{2\pi}} \exp\left(-\frac{(x-2)^2}{2(3/n)^2}\right)$$

with

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$$

yields $\mu = 2$ and $\sigma^2 = \frac{9}{n^2}$.

Remark: The variance of X decreases at the rate of $\frac{1}{n}$. This happens if X is a sample average with sample size n.

提交 你已经尝试了2次（总共可以尝试3次）

Answers are displayed within the problem

Probability review: PDF of Gaussian distribution

1/1 point (graded)

Let $X \sim \mathcal{N}(\mu, \sigma^2)$, i.e. the pdf of X is

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

Let $Y = 2X$. Write down the pdf of the random variable Y. (Your answer should be in terms of y, σ and μ. Type **mu** for μ, **sigma** for σ.)

$f_Y(y) =$ 1/(2*sigma*sqrt(2*pi)) * ✔ **Answer:** 1/(2*sigma*sqrt(2*pi))* exp(-(y-2*mu)^2/(8*sigma^2))

$$\frac{1}{2\cdot\sigma\cdot\sqrt{2\cdot\pi}}\cdot\exp\left(-\frac{(y-2\cdot\mu)^2}{8\cdot\sigma^2}\right)$$

STANDARD NOTATION

Solution:

If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $Y = 2X \sim \mathcal{N}(2\mu, 4\sigma^2)$ by the following general properties of expectations and variance:

$$\begin{aligned} \mathbb{E}[2X] &= 2\mathbb{E}[X] \\ \textsf{Var}[2X] &= 2^2\textsf{Var}[X] = 4\textsf{Var}[X]. \end{aligned}$$

Therefore,

$$f_Y(y) = \frac{1}{2\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-2\mu)^2}{2(4\sigma^2)}\right).$$

Alternate solution: In general, for any continuous random variables X and any continuous monotonous (i.e. always increasing or always decreasing) function g, such that $Y = g(X)$, the pdf of Y is given by:

$$f_Y(y) = \frac{f_X(x)}{g'(x)} \qquad \text{where } x = g^{-1}(y).$$

In this problem, $X \sim \mathcal{N}(\mu, \sigma^2)$, $Y = g(X) = 2X$, and $g'(x) = 2$. Therefore:

$$
\begin{aligned}
f_Y(y) &= \frac{f_X\left(\frac{y}{2}\right)}{g'\left(\frac{y}{2}\right)} \\
&= \frac{1}{g'(y/2)\,\sigma\sqrt{2\pi}} \exp\left(-\frac{(y/2-\mu)^2}{2\sigma^2}\right) \\
&= \frac{1}{2\sigma\sqrt{2\pi}} \exp\left(-\frac{((y-2\mu)/2)^2}{2\sigma^2}\right) \\
&= \frac{1}{2\sigma\sqrt{2\pi}} \exp\left(-\frac{((y-2\mu))^2}{2(4)\sigma^2}\right)
\end{aligned}
$$

and we recover the same answer as above.

提交 你已经尝试了2次（总共可以尝试3次）

Answers are displayed within the problem

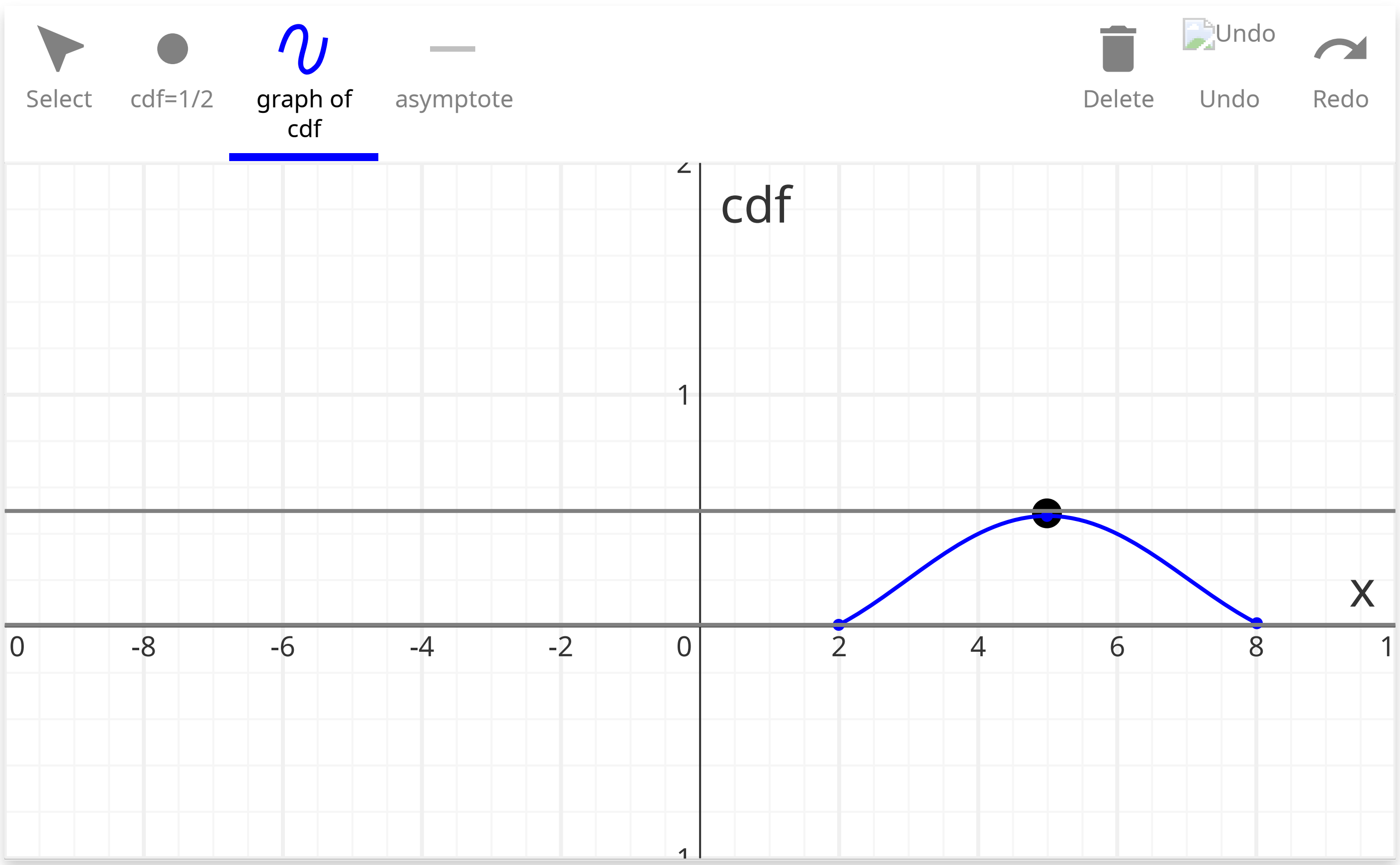
Sketch CDF of a Gaussian distribution

0/1 point (graded)

Let $X \sim \mathcal{N}(5,1)$ and $f_X(x)$ be its pdf. Sketch the cumulative distribution function (cdf) $\Phi(x) = \int_{-\infty}^{x} f_X(t)\,dt$ of X by doing the following:

- Use the **horizontal asymptote** to draw **all** horizontal asymptotes.
- Use the **Curve** tool to sketch the curve of the cdf. Click on points along the curve and the tool will connect these points with a smooth curve.
- Use the **cdf=1/2** point tool to mark the point on the curve of the cdf where the value of the cdf is $1/2$.

Hint: Make sure your curve shows **where in the domain the cdf is very close to its limiting values**. This is to test your understanding of the variance.



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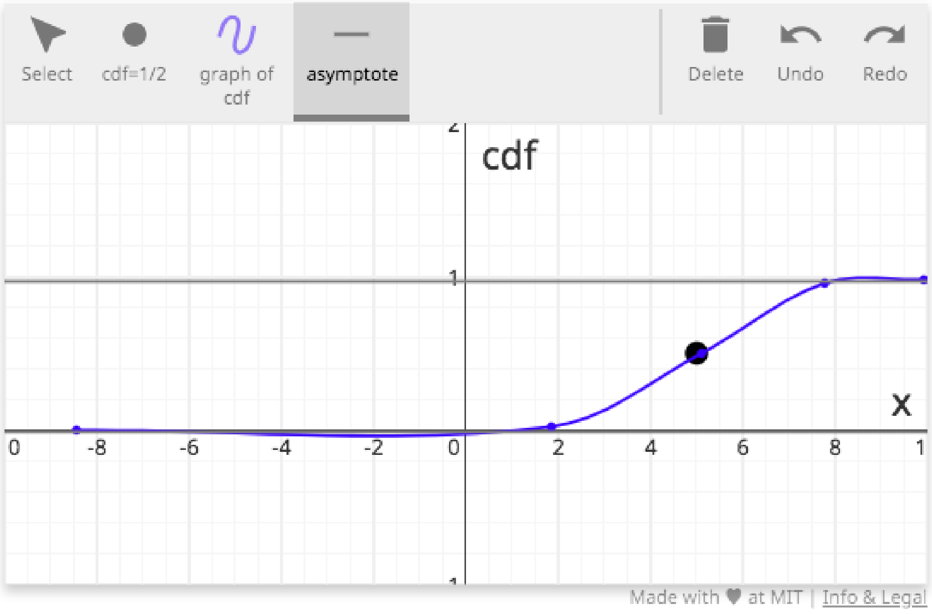
Answer: See solution.

✖

At least one point is not at the correct location.
The shape of the cdf is incorrect. .

Solution:



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Answers are displayed within the problem

讨论

显示讨论

主题： Unit 1 Introduction to statistics:Lecture 2: Probability Redux / 4. Gaussian distribution

认证证书是什么?