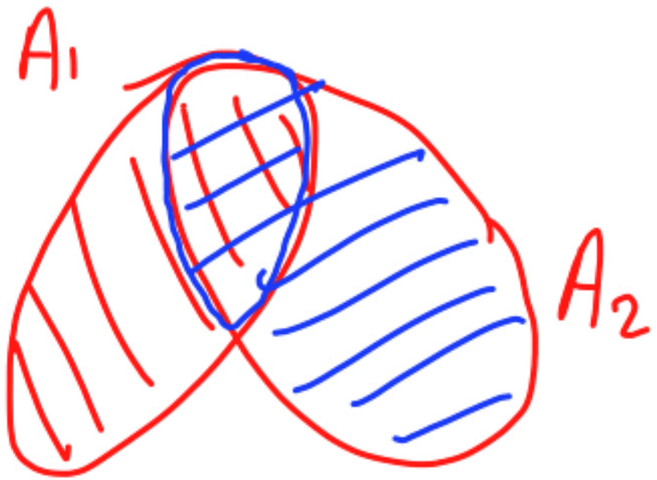
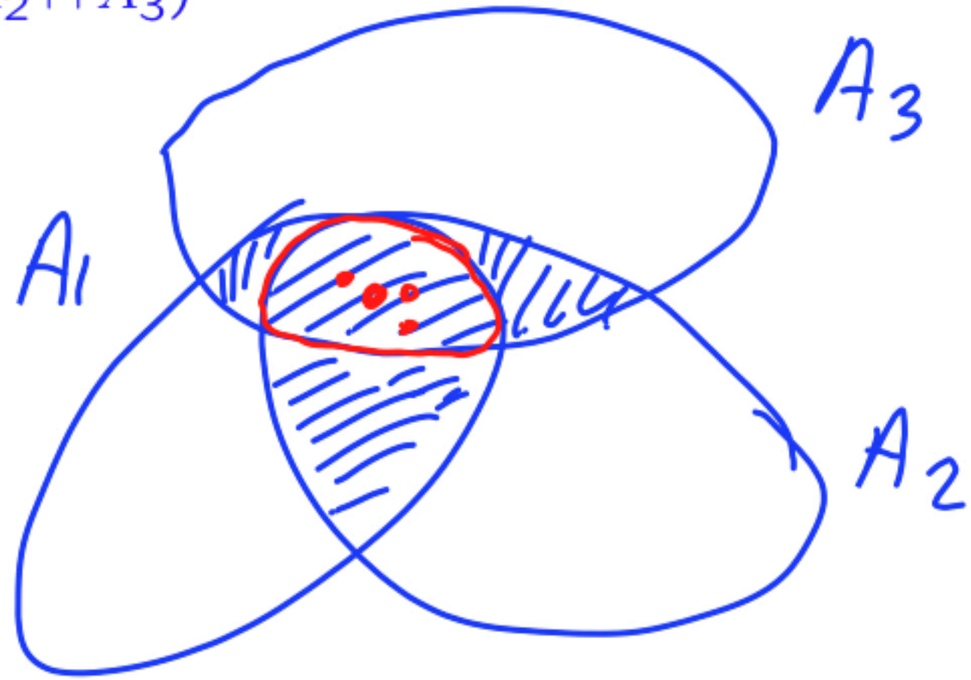
The inclusion-exclusion formula

- $\mathbf{P}(A_1 \cup A_2) = \mathbf{P}(A_1) + \mathbf{P}(A_2) - \mathbf{P}(A_1 \cap A_2)$
- $\mathbf{P}(A_1 \cup A_2 \cup A_3) = \mathbf{P}(A_1) + \mathbf{P}(A_2) + \mathbf{P}(A_3) - \big(\mathbf{P}(A_1 \cap A_2) + \mathbf{P}(A_1 \cap A_3) + \mathbf{P}(A_2 \cap A_3)\big) + \mathbf{P}(A_1 \cap A_2 \cap A_3)$





The inclusion-exclusion formula

- $\mathbf{P}(A_1 \cup A_2 \cup A_3) = \mathbf{P}(A_1) + \mathbf{P}(A_2) + \mathbf{P}(A_3) - \big(\mathbf{P}(A_1 \cap A_2) + \mathbf{P}(A_1 \cap A_3) + \mathbf{P}(A_2 \cap A_3)\big)$
  $+ \mathbf{P}(A_1 \cap A_2 \cap A_3)$

$A_i \leftrightarrow$ indicator X_i

$A_i^c \leftrightarrow 1 - X_i$

$A_i \cap A_j \leftrightarrow X_i X_j$

$A_i^c \cap A_j^c \leftrightarrow (1-X_i)(1-X_j)$

$A_i \cup A_j \leftrightarrow 1 - (1-X_i)(1-X_j)$

$P(A_1 \cup A_2 \cup A_3) = E[\text{indicator of } A_1 \cup A_2 \cup A_3]$

$= E[1 - (1-X_1)(1-X_2)(1-X_3)]$

$= E[1 - 1 + X_1 + X_2 + X_3 - X_1X_2 - X_1X_3 - X_2X_3 + X_1X_2X_3]$

$$\mathbf{P}\Big(\bigcup_{k=1}^{n} A_k\Big) = \sum_i \mathbf{P}(A_i) - \sum_{i_1<i_2} \mathbf{P}(A_{i_1} \cap A_{i_2}) + \sum_{i_1<i_2<i_3} \mathbf{P}(A_{i_1} \cap A_{i_2} \cap A_{i_3}) - \cdots + (-1)^{n-1}\mathbf{P}\Big(\bigcap_{k=1}^{n} A_k\Big)$$