

3. Hats at a party

Problem 2. Hats at a party

5.0/5.0 points (graded)

Bob goes to a party, wearing a hat that fits him well. At the end of the party, Bob picks up a hat, without looking at it. He knows that the hat he has picked is his own with probability p . As a first check to see if he has picked up his own hat, he tries the hat, and if it fits, he decides that it is his own hat.

However, even if the hat picked by Bob is not his own, it is still possible that it fits with probability q . Given that Bob picked a hat that fits him, what is the probability that he is correct in deciding that the hat is indeed his own?

$p/(p+q-p \cdot q)$

✓ Answer: $p/(p+q-q \cdot p)$

$\frac{p}{p+q-p \cdot q}$

STANDARD NOTATION

Solution:

Let A be the event that the hat is indeed Bob's own hat, and let B be the event that the hat fits. We are asked to find the probability that Bob made a **correct** decision, when he **decided** it was his own hat, namely, we are interested in the probability $\mathbf{P}(A \mid B)$. Using Bayes' rule,

$$\begin{aligned} \mathbf{P}(A \mid B) &= \frac{\mathbf{P}(B \mid A)\mathbf{P}(A)}{\mathbf{P}(B)} \\ &= \frac{1 \cdot p}{\mathbf{P}(B \mid A)\mathbf{P}(A) + \mathbf{P}(B \mid A^c)\mathbf{P}(A^c)} \\ &= \frac{1 \cdot p}{1 \cdot p + q \cdot (1 - p)} \\ &= \frac{p}{p + q - qp}. \end{aligned}$$

提交

你已经尝试了1次（总共可以尝试2次）

❗ Answers are displayed within the problem

Error and Bug Reports/Technical Issues

显示讨论

主题: Final Exam / 3. Hats at a party