14. Exercise: The posterior of a coin's bias

Exercise: The posterior of a coin's bias

2/3 points (graded)

Let Θ be a continuous random variable that represents the unknown bias (i.e., the probability of Heads) of a coin.

a) The prior PDF f_Θ for the bias of a coin is of the form

$$f_\Theta(\theta) = a\theta^9(1-\theta), \quad \text{for } \theta \in [0,1],$$

where a is a normalizing constant. This indicates a prior belief that the bias Θ of the coin is

High ▼ ✔ **Answer:** High

b) We flip the coin 10 times independently and observe 1 Heads and 9 Tails. The posterior PDF of Θ will be of the form $c\theta^m(1-\theta)^n$, where c is a normalizing constant and where

$m =$ 1 ✘ **Answer:** 10

$n =$ 10 ✔ **Answer:** 10

Solution:

a) Because of the high exponent, the term θ^9 is very small when θ is small. This prior, as can also be seen by plotting it, is concentrated on high values of θ and indicates a prior belief in favor of large values.

b) As we saw in the last video, the power to which θ (respectively, $1-\theta$) is raised needs to be incremented by the number of Heads (respectively, Tails) observed, leading to $m = 9+1 = 10$ and $n = 1+9 = 10$. Notice that the resulting posterior is symmetric around 0.5.

This exercise indicates that the strength of the “evidence" incorporated in a prior with $\alpha = 9$ and $\beta = 1$ is exactly counterbalanced by observing 1 Heads and 9 Tails. Differently said, a prior with $\alpha = 9$ and $\beta = 1$ can be thought of as equivalent to prior “evidence" based on 9 Heads and 1 Tails.

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Answers are displayed within the problem

讨论

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How to explain the posterior PDF?

question posted 5 days ago by Belter

According to the result, the posterior PDF is extremely different from the prior PDF. Is it right that the posterior PDF tells us this coin is fair?

此帖对所有人可见。

e_kizildag (Staff)
2 days ago - 2 days ago 前被 Belter 标记为答案

No, the prior initially was heavily biased (towards Heads). However, you get to see many heads, so somehow exponents balance out. However, you still have a probability distribution (aka posterior) for the possible values of bias.

It does **not** tell you directly that it is unbiased. Rather, it compactly expresses (in forms of a distribution) the possibilities of bias. However, with the m, n found in the problem, maximizing $\max_{\theta\in[0,1]} \theta^m(1-\theta)^n$, we see that it happens at $\theta = 1/2$; and thus, MAP estimate would spit out $\widehat{\Theta}_{MAP} = 1/2$, namely would report a fair coin.

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1 other response

-Mayster-
4 days ago

I came up to the same question :)

The prior was highly biased towards high heads, and the observations were instead high tails. So we have essentially two pieces of evidence which are diametrically opposite, and thus average out to even. Essentially more coin tosses will answer the question of whether the coin is fair.

markweitzman (Community TA) 在4 days ago前发表

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