4. Estimation of an exponential parameter

(a)

1/1 point (graded)

Let $X_1, \ldots, X_n$ be i.i.d. Exp(λ) random variables, where λ is unknown.

What is the distribution of $\min_i (X_i)$? Enter the pdf $f_{\min}(x)$ of $\min_i (X_i)$ in terms of x.

$f_{\min}(x)$ lambda*n*exp(-1*lambc ✔ **Answer:** n*lambda*e^(-n*lambda*x)

$\lambda \cdot n \cdot \exp(-1 \cdot \lambda \cdot n \cdot x)$

STANDARD NOTATION

Solution:

Recall the cdf of X_i is $F_X(x) = \int_0^x \lambda e^{-\lambda t} dt = 1 - e^{-\lambda x}$.

Compute the cdf of $\min_i (X_i)$:

$$\begin{aligned} \mathbf{P}\left(\min_i (X_i) \leq t\right) = 1 - \mathbf{P}\left(\min_i (X_i) \geq t\right) &= 1 - (\mathbf{P}(X_1 \geq t))(\mathbf{P}(X_2 \geq t))\ldots(\mathbf{P}(X_n \geq t)) \\ &= 1 - (1 - F_X(t))^n = 1 - e^{-n\lambda x}. \end{aligned}$$

Differentiate w.r.t x to get the pdf of $\min_i (X_i)$:

$$f_{\min}(x) = (n\lambda) e^{-(n\lambda)x}.$$

That is, $\min_i (X_i)$ follows an exponential distribution with parameter $n\lambda$. As a sanity check, $\mathbb{E}\left[\min_i (X_i)\right] = 1/(n\lambda) < \mathbb{E}[X_i] = 1/\lambda$ for $n > 1$.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

(b)

1/1 point (graded)

Use the previous question to give an **unbiased** estimator $\hat{\theta}$ for $1/\lambda$.
(Enter min, with no subscripts, for the expression $\min_i (X_i)$.

$\hat{\theta} =$ n*min ✔ **Answer:** n*min

$n \cdot min$

STANDARD NOTATION

Solution:

Since $\mathbb{E}\left[\min_i(X_i)\right] = \frac{1}{n\lambda}$, we have $\mathbb{E}\left[n\min_i(X_i)\right] = \frac{1}{\lambda}$. Therefore $n\min_i(X_i)$ is an unbiased estimator of $\frac{1}{\lambda}$.

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Answers are displayed within the problem

(c)

2/2 points (graded)

What is the variance and quadratic risk of the unbiased estimator $\hat{\theta}$ in the previous part?

Var $\left(\hat{\theta}\right) =$ 1/lambda^2 ✔ **Answer:** 1/lambda^2

$\frac{1}{\lambda^2}$

Quadratic risk of $\hat{\theta}$: 1/lambda^2 ✔ **Answer:** 1/lambda^2

$\frac{1}{\lambda^2}$

STANDARD NOTATION

Solution:

$$\begin{aligned}\text{Var}\left(n\min_i(X_i)\right) &= n^2\text{Var}\left(\min_i(X_i)\right) = \frac{n^2}{n^2\lambda^2} = \frac{1}{\lambda^2}\\ \text{Quadratic risk}\left(n\min_i(X_i)\right) &= \left[\text{bias}\left(n\min_i(X_i)\right)\right]^2 + \text{Var}\left(n\min_i(X_i)\right)\\ &= 0 + \frac{1}{\lambda^2} = \frac{1}{\lambda^2}\end{aligned}$$

Note that the variance and quadratic risk of this estimator stay constant as $n \to \infty$.

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Answers are displayed within the problem

(d)

2/3 points (graded)

Compute $\mathbf{P}\left(\frac{1}{\lambda} \geq \frac{n\min_i X_i}{\ln(5)}\right)$.

$\mathbf{P}\left(\frac{1}{\lambda} \geq \frac{n\min_i X_i}{\ln(5)}\right) =$ 0.8 ✔ **Answer:** 4/5

This computation allows us to compute a confidence interval. The interpretation is as follows:

Let α be a value such that $1-\alpha = \mathbf{P}\left(\frac{1}{\lambda} \leq \frac{n\min_i(X_i)}{\ln(5)}\right)$. (This value depends on the answer you just computed.)

Based on this setup, the corresponding, non-asymptotic, one-sided confidence interval at significance level α for $1/\lambda$ is:

(Type min for $\min(X_i)$.)

(Note the confidence interval is finite.)

[0 ✔ **Answer:** 0 , n*min + 0.84/sqrt(n)*n* ✖ **Answer:** n*min/ln(5)]

0

$n \cdot min + \frac{0.84}{\sqrt{n}} \cdot n \cdot min$

这里直接用上面的答案就好了，我用了plug-in，还plug-in错了哈哈

STANDARD NOTATION

Solution:

$$\frac{1}{\lambda} \geq \frac{n \min_i X_i}{\ln(5)} \iff \min_i X_i \leq \frac{\ln(5)}{n\lambda},$$

Hence,

$$\begin{aligned}\mathbf{P}\left(\frac{1}{\lambda} \geq \frac{n \min_i X_i}{\ln(5)}\right) &= \mathbf{P}\left(\min_i X_i \leq \frac{\ln(5)}{n\lambda}\right) \\ &= 1 - e^{-n\lambda\left(\frac{\ln(5)}{n\lambda}\right)} = \frac{4}{5} = 0.8\end{aligned}$$

Note that when the event $\frac{1}{\lambda} \leq \frac{n \min_i X_i}{\ln(5)}$ occurs, $\frac{1}{\lambda}$ lies in the interval $\left[0, \frac{n \min_i X_i}{\ln(5)}\right]$. Thus, the corresponding confidence interval at significance level 80% is $\left[0, \frac{n \min_i X_i}{\ln(5)}\right]$.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

主题: Unit 2 Foundation of Inference:Homework 1: Estimation, Confidence Interval, Modes of Convergence / 4. Estimation of an exponential parameter

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