8. Exercise: CLT practice

Exercise: CLT practice

6/6 points (graded)

The random variables X_i are i.i.d. with mean 2 and standard deviation equal to 3. Assume that the X_i are nonnegative. Let $S_n = X_1 + \cdots + X_n$.

Use the CLT to find good approximations to the following quantities. You may want to refer to the normal table. In parts (a) and (b), give answers with 4 decimal digits.

Normal Table

Show

a) $\mathbf{P}(S_{100} \le 245) \approx$ 0.9332 ✔ **Answer:** 0.9332

b) We let N (a random variable) be the first value of n for which S_n exceeds 119.

$\mathbf{P}(N > 49) \approx$ 0.8413 ✔ **Answer:** 0.8413

c) What is the largest possible value of n for which we have $\mathbf{P}(S_n \le 128) \approx 0.5$?

$n =$ 64 ✔ **Answer:** 64

Solution:

We will use Z_n to refer to the standardized random variable $(S_n - 2n)/(3\sqrt{n})$.

a) We have

$$\mathbf{P}(S_{100} \le 245) = \mathbf{P}\left(\frac{S_{100} - 2\cdot 100}{3\cdot\sqrt{100}} \le \frac{245 - 2\cdot 100}{3\cdot\sqrt{100}}\right) = \mathbf{P}(Z_n \le 1.5) \approx 0.9332.$$

b) The event $N > 49$ is the same as the event $S_{49} \le 119$. Its probability is

$$\mathbf{P}(S_{49} \le 119) = \mathbf{P}\left(\frac{S_{49} - 2\cdot 49}{3\cdot\sqrt{49}} \le \frac{119 - 2\cdot 49}{3\cdot\sqrt{49}}\right) = \mathbf{P}(Z_n \le 1) \approx 0.8413.$$

c) We want n such that

$$0.5 \approx \mathbf{P}(S_n \le 128) = \mathbf{P}\left(\frac{S_n - 2n}{3\sqrt{n}} \le \frac{128 - 2n}{3\sqrt{n}}\right) = \Phi\left(\frac{128 - 2n}{3\sqrt{n}}\right).$$

But since $0.5 = \Phi(0)$, we must have $(128 - 2n)/(3\sqrt{n}) = 0$, so that $n = 128/2 = 64$.

A faster way to see the answer is to note that since the normal is symmetric around its mean, the relation $\mathbf{P}(S_n \le 128) \approx 0.50$ tells us that 128 should be equal to the mean, $2n$, of S_n.

提交 You have used 2 of 3 attempts