

## Problem 4

### 4. (1)

3/3 points (graded)

Mark the following statements as true or false.

The EM algorithm monotonically increases the likelihood of the data with each iteration. In other words, the likelihood after iteration  $i + 1$  is greater than or equal to the likelihood after iteration  $i$ , for all  $i$ .

☒ True

☐ False



Depending on the initialization, the likelihood of the data the algorithm converges to may be different.

☒ True

☐ False



We are estimating a mixture model with  $K$  components. During random initialization,  $p_1$  was assigned to be zero.  $p_1$  could become non-zero as the algorithm iterates.

☐ True

☒ False



#### Solution:

The M-step chooses the parameter values that maximize the likelihood. Therefore, the previous iteration cannot have parameters of a greater likelihood than the current iteration.

The initial values provide a "starting point" for the algorithm, different initializations lead to different results.

The expected value after the E-step will be zero, therefore  $p_1$  is recomputed in the M-step to be a sum of zeros.

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You have used 1 of 3 attempts

**i** Answers are displayed within the problem

### 4. (2)

1/1 point (graded)

Consider a 1-dim Gaussian mixture model with two components. We set the mixture with  $\mu_1 = 1, \mu_2 = 1, \sigma_1 = 0.5, \sigma_2 = 0.5$ . The mixing proportions are set differently for the two components:  $p_1 = 0.01$  and  $p_2 = 0.99$ . Is it the case that  $\mu_1 = \mu_2$  and  $\sigma_1 = \sigma_2$  after running the EM algorithm regardless of the data?

☒ Yes

☐ No



Will  $p_1 = 0.01$  and  $p_2 = 0.99$  also hold at convergence? (There is no answer box for this question.)


**Solution:** Yes

Initially, it will be 99 times more likely that the points come from cluster 2 as opposed to cluster 1. However, the  $\mu$  and  $\sigma$  that maximize the probability of the clusters generating the given points are the same.

$$p(\text{cluster1}|\text{example}i) = \frac{0.01x}{0.01x+0.99x} \quad p(\text{cluster2}|\text{example}i) = \frac{0.99x}{0.01x+0.99x}$$
 Since all  $p(\text{cluster1}|\text{example}i) * 0.99 = p(\text{clusterz}|\text{example}i)$  for all  $i$ ,  $p_1 * 99 = p_2$ . This holds by induction.

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 Answers are displayed within the problem

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