

## 2. A three-sided die

### Problem 2. A three-sided die, part 1

1.0/1.0 point (graded)

The newest invention of the 6.431x staff is a three-sided die. On any roll of this die, the result is 1 with probability  $1/2$ , 2 with probability  $1/4$ , and 3 with probability  $1/4$ .

Consider a sequence of six independent rolls of this die.

Find the probability that exactly two of the rolls result in a 3.

☒  $\binom{6}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4$  ✓

☐  $\binom{6}{2} \left(\frac{1}{4}\right)^2$

☐  $\binom{6}{2} \left(\frac{1}{4}\right)^2 \binom{6}{4} \left(\frac{3}{4}\right)^4$

☐  $\binom{6}{2} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^2$

#### Solution:

Each roll is an independent trial with probability  $1/4$  of resulting in a 3 (a "success"). The probability of exactly 2 successes in 6 trials is given by the binomial probabilities, with  $n = 6$ ,  $k = 2$ , and  $p = 1/4$ :

$$\binom{6}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4$$

**i** Answers are displayed within the problem

## Problem 2. A three-sided die, part 2

3.0/3.0 points (graded)

1. Given that exactly two of the six rolls resulted in a 1, find the probability that the first roll resulted in a 1.

**Note:** Your answer should be a number. Do not enter "!" or combinations in your answer.



2. We are told that exactly three of the rolls resulted in a 1 and exactly three rolls resulted in a 2. Given this information, find the probability that the six rolls resulted in the sequence **(1, 2, 1, 2, 1, 2)**.

**Note:** Your answer should be a number. Do not enter "!" or combinations in your answer.



3. The conditional probability that exactly  $k$  rolls resulted in a 3, given that at least one roll resulted in a 3, is of the form:

$$\frac{1}{1 - (c_1/c_2)^{c_3}} \binom{c_3}{k} \left(\frac{1}{c_2}\right)^k \left(\frac{c_1}{c_2}\right)^{c_3-k}, \quad \text{for } k = 1, 2, \dots, 6.$$

Find the values of the constants  $c_1$ ,  $c_2$ , and  $c_3$ :

$c_1 =$



$c_2 =$



$c_3 =$



