Since the inequality

$$g(c) + (x - c)g'(c) \leq g(x)$$

is assumed valid for every possible value x of the random variable X, we obtain

$$g(c) + (X - c)g'(c) \leq g(X).$$

For any two random variables, Y and Z, if it is always the case that $Y \leq Z$, then we must have $\mathbf{E}[Y] \leq \mathbf{E}[Z]$. By applying this fact to the inequality above, and choosing $c = \mathbf{E}[X]$, we obtain

$$g\big(\mathbf{E}[X]\big) + \big(\mathbf{E}[X] - \mathbf{E}[X]\big)g'\big(\mathbf{E}[X]\big) \leq \mathbf{E}\big[g(X)\big],$$

or

$$g\big(\mathbf{E}[X]\big) \leq \mathbf{E}\big[g(X)\big].$$