3. Method of moments estimators

For each of the following distributions, give the method of moments estimator in terms of the sample averages $\overline{X}_n$ and $\overline{X_n^2}$, assuming we have access to n i.i.d. observations $X_1, \ldots, X_n$. In other words, express the parameters as functions of $\mathbb{E}[X_1]$ and $\mathbb{E}[X_1^2]$ and then apply these functions to $\overline{X}_n$ and $\overline{X_n^2}$.

(a)

1/1 point (graded)

$$X_i \sim \text{Ber}(p), \quad p \in (0,1)$$

(If applicable, write **barX_n** for $\overline{X}_n$.)

Method of moments estimator $\hat{p} =$ barX_n □ **Answer:** barX_n

Solution:

For Bernoulli variables, we have

$$\mathbb{E}_p[X_1] = p,$$

hence

$$\hat{p} = \overline{X}_n.$$

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□ Answers are displayed within the problem

(b)

1/1 point (graded)

$$X_i \sim \text{Poiss}(\lambda), \quad \lambda > 0,$$

which means that each X_1 has density

$$\mathbf{P}_\lambda(X = k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k \in \mathbb{N}.$$

Method of moments estimator $\hat{\lambda} =$ barX_n □ **Answer:** barX_n

Solution:

For a Poisson random variable, we have

$$\mathbb{E}_\lambda [X_1] = \lambda,$$

hence

$$\hat{\lambda} = \overline{X}_n.$$

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Answers are displayed within the problem

(c)

1/1 point (graded)

$$X_i \sim \textsf{Exp}(\lambda), \quad \lambda > 0,$$

which means that each X_1 has density

$$f_\lambda (x) = \lambda e^{-\lambda x}, \quad x > 0.$$

Method of moments estimator $\hat{\lambda} =$ 1/ barX_n **Answer:** 1/barX_n

Solution:

For an Exponential random variable, we have

$$\mathbb{E}_\lambda [X_1] = \frac{1}{\lambda},$$

so

$$\lambda = \frac{1}{\mathbb{E}_\lambda [X_1]}.$$

Hence, the method of moments estimator is

$$\hat{\lambda} = \frac{1}{\overline{X}_n}.$$

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(d)

2.0/2 points (graded)

$$X_i \sim \mathcal{N}\left(\mu, \sigma^2\right), \quad \mu \in \mathbb{R}, \sigma^2 > 0,$$

which means that each X_1 has density

$$f_{\mu,\sigma^2}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

(If applicable, enter **barX_n** for $\overline{X}_n$ and **bar(X_n^2)** for $\overline{X_n^2}$.)

Method of moments estimator $\hat{\mu} =$ barX_n **Answer:** barX_n

Method of moments estimator $\widehat{\sigma^2} =$ bar(X_n^2) - barX_n^2 **Answer:** bar(X_n^2) - barX_n^2

STANDARD NOTATION

Solution:

For a Gaussian distribution, we have

$$\begin{aligned} \mathbb{E}_{\mu,\sigma^2}[X_1] &= \mu \\ \mathbb{E}_{\mu,\sigma^2}[X_1^2] &= \mathsf{Var}_{\mu,\sigma^2}(X_1) + \mathbb{E}[X_1]^2 = \mu^2 + \sigma^2, \end{aligned}$$

which we can invert to obtain

$$\begin{aligned} \mu &= \mathbb{E}_{\mu,\sigma^2}[X_1] \\ \sigma^2 &= \mathbb{E}_{\mu,\sigma^2}[X_1^2] - \mathbb{E}_{\mu,\sigma^2}[X_1]^2. \end{aligned}$$

Plugging in the estimators $\overline{X}_n$ and $\overline{X_n^2}$ then yields

$$\begin{aligned} \hat{\mu} &= \overline{X}_n \\ \widehat{\sigma^2} &= \overline{X_n^2} - \overline{X}_n^2. \end{aligned}$$

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Answers are displayed within the problem

(e)

2.0/2 points (graded)

X_i follows a shifted exponential distribution with parameters $a \in \mathbb{R}$ and $\lambda > 0$. That means each X_i has density

$$f_{a,\lambda}(x) = \lambda e^{-\lambda(x-a)} \mathbf{1}\{x \geq a\}, \quad x \in \mathbb{R}.$$

(If applicable, enter **barX_n** for $\overline{X}_n$ and **bar(X_n^2)** for $\overline{X_n^2}$.)

Method of moments estimator $\hat{a} =$ barX_n - sqrt(bar(X_n^2) **Answer:** barX_n - sqrt(bar(X_n^2) - barX_n^2)

Method of moments estimator $\hat{\lambda} =$ sqrt(1/(bar(X_n^2) - barX_n^2))

Answer: 1/sqrt(bar(X_n^2) - barX_n^2)

STANDARD NOTATION

Solution:

Since X_1 is a shifted exponential random variable,

$$\mathbb{E}_{a,\lambda}\left[X_1\right] = \mathbb{E}_{0,\lambda}\left[a + X_1\right] = a + \frac{1}{\lambda},$$

and

$$\begin{aligned} \mathbb{E}_{a,\lambda}\left[X_1^2\right] = &\ \mathsf{Var}_{0,\lambda}\left(X_1\right) + \mathbb{E}_{a,\lambda}[X_1]^2 \\ = &\ \frac{1}{\lambda^2} + \left(\frac{1}{\lambda} + a\right)^2. \end{aligned}$$

That means

$$a = \mathbb{E}_{a,\lambda}\left[X_1\right] - \frac{1}{\lambda},$$

and plugging this into the equation for the second order moment, we obtain

$$\begin{aligned} \frac{1}{\lambda^2} + \left(\frac{1}{\lambda} + \mathbb{E}_{a,\lambda}\left[X_1\right] - \frac{1}{\lambda}\right)^2 = &\ \mathbb{E}_{a,\lambda}\left[X_1^2\right] \\ \iff \frac{1}{\lambda} = &\ \left(\mathbb{E}_{a,\lambda}\left[X_1^2\right] - \mathbb{E}_{a,\lambda}[X_1]^2\right)^{1/2} \\ \iff \lambda = &\ \left(\mathbb{E}_{a,\lambda}\left[X_1^2\right] - \mathbb{E}_{a,\lambda}[X_1]^2\right)^{-1/2}, \end{aligned}$$

which plugged back into the first equation yields

$$a = \mathbb{E}_{a,\lambda}\left[X_1\right] - \left(\mathbb{E}_{a,\lambda}\left[X_1^2\right] - \mathbb{E}_{a,\lambda}[X_1]^2\right)^{1/2}.$$

Hence, the method of moment estimators are

$$\begin{aligned} \hat{\lambda} = &\ \left(\overline{X_n^2} - \left(\overline{X}_n\right)^2\right)^{-1/2} \\ \hat{a} = &\ \overline{X}_n - \left(\overline{X_n^2} - \left(\overline{X}_n\right)^2\right)^{1/2}. \end{aligned}$$

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Answers are displayed within the problem

讨论

显示讨论

主题： Unit 3 Methods of Estimation:Homework 6 Maximum Likelihood Estimation and Method of Moments / 3. Method of moments estimators

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