13. Exercise: Convergence in probability

Exercise: Convergence in probability

2/3 points (graded)

a) Suppose that X_n is an exponential random variable with parameter $\lambda = n$. Does the sequence $\{X_n\}$ converge in probability?

Yes ▾ ✔ **Answer:** Yes

b) Suppose that X_n is an exponential random variable with parameter $\lambda = 1/n$. Does the sequence $\{X_n\}$ converge in probability?

No ▾ ✔ **Answer:** No

c) Suppose that the random variables in the sequence $\{X_n\}$ are independent, and that the sequence converges to some number a, in probability. Let $\{Y_n\}$ be another sequence of random variables that are dependent, but where each Y_n has the same distribution (CDF) as X_n. Is it necessarily true that the sequence $\{Y_n\}$ converges to a in probability?

No ▾ ✖ **Answer:** Yes

Solution:

a) In the first case, for any $\epsilon > 0$, we have $\mathbf{P}(X_n \geq \epsilon) = e^{-n\epsilon}$, which converges to zero. Therefore, we have convergence in probability.

b) In the second case, for any $\epsilon > 0$, we have $\mathbf{P}(X_n \geq \epsilon) = e^{-\epsilon/n}$, which converges to one. Therefore, we do not have convergence in probability.

c) Dependence will not make a difference because the definition of convergence in probability involves probabilities of the form $\mathbf{P}\big(|Y_n - a| \geq \epsilon\big)$. These probabilities are completely determined by the marginal distributions of the random variables Y_n, and these marginal distributions are the same as for the sequence X_n.

提交 You have used 1 of 1 attempt

Answers are displayed within the problem

讨论

显示讨论

Topic: Unit 8 / Lec. 18 / 13. Exercise: Convergence in probability