

Homework 4.4: Piecewise linear FN model

Phase plane analysis 1

1/1 point (graded)

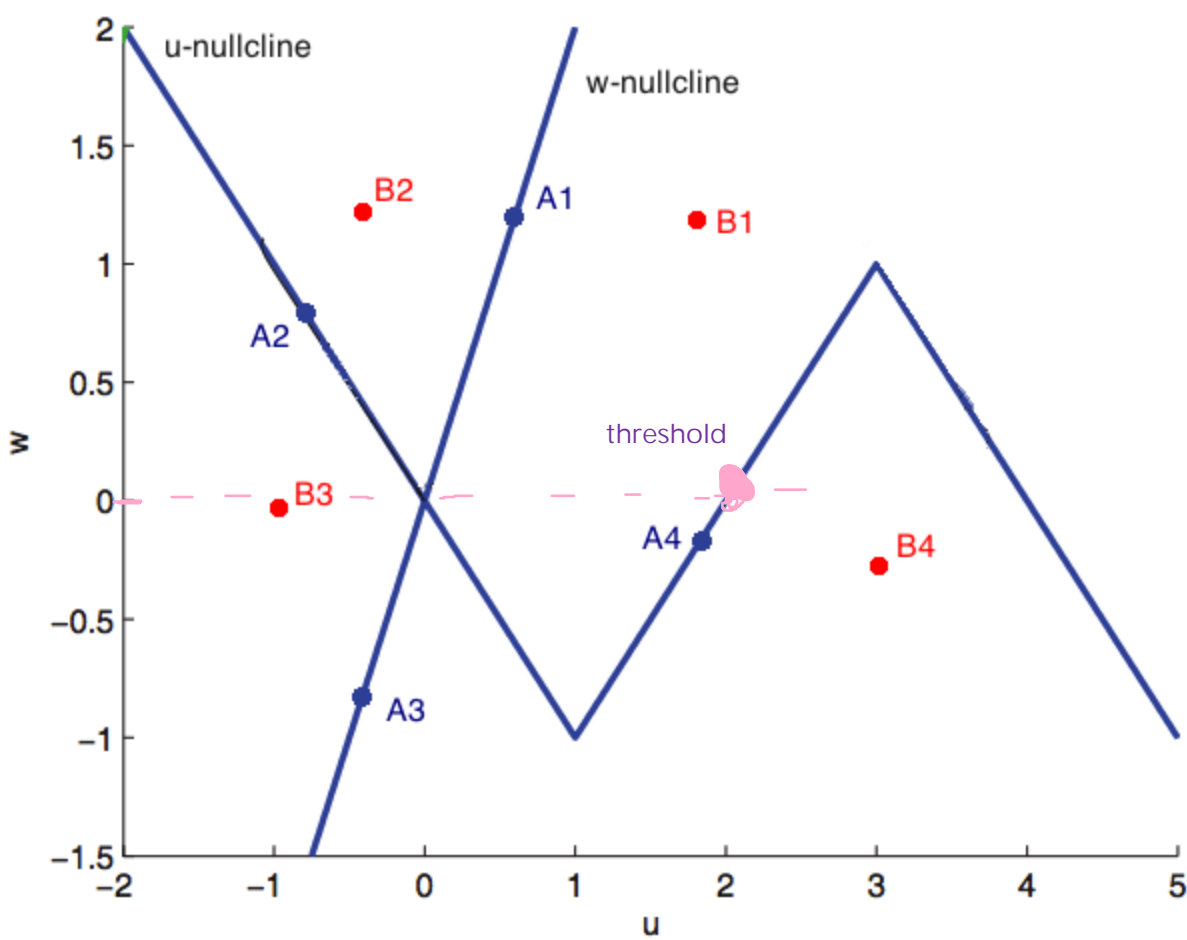
The Fitzhugh-Nagumo model is defined by the equations

$$\begin{cases} \frac{du}{dt} = F(u, w) = f(u) - w + I \\ \frac{dw}{dt} = G(u, w) = \epsilon(bu - \gamma w) \end{cases}$$

Here  $u(t)$  is the membrane potential and  $w(t)$  is a second time-dependent variable.  $I$  stands for the injected current. A simplified model is obtained by considering a piecewise linear  $f(u)$

$$f(u) = \begin{cases} -u & u \leq 1 \\ \frac{u-1}{a} - 1 & 1 < u \leq 1+2a \\ 2(1+a) - u & 1+2a < u \end{cases}$$

In the figure below, you see the nullclines of the system where  $a = 1, b = 9, \gamma = 4$  and there is no external current, i.e.,  $I = 0$ .



Which of the following options exactly determines the direction of flow arrows on the nullclines while doing phase plane analysis? Note that the flow arrows are given for the points A1(the most left), A2, A3, and A4(the most right), respectively.

☐ ↓ ← ↑ →

☒ ← ↓ → ↑

☐ → ↑ ← ↓

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☐ ← → ↓ ↑

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You have used 1 of 1 attempt

✓ Correct (1/1 point)

### Phase Plane analysis 2

1/1 point (graded)

Which of the following options approximately determines the direction of flow arrows on the regions between the nullclines while doing phase plane analysis? Note that the flow arrows are given for the points B1(the most left), B2, B3, and B4(the most right), respectively.

☐ ↙ ↘ ↗ ↖

☐ ↗ ↖ ↙ ↘

☐ ↘ ↗ ↖ ↙

☒ ↖ ↙ ↘ ↗

☐ ↙ ↗ ↘ ↖

☐ ↖ ↘ ↙ ↗



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### Current injection

2/2 points (graded)

Starting from the condition  $I = 0$ , how does the fixed point move as  $I$  is increased?

☒ It moves to the right along  $f(u)$

☐ It moves to the left along  $f(u)$



and this is because that

☐ the  $u$ -nullcline is shifted downward.

☐ the  $w$ -nullcline is shifted to the left.

☒ the  $u$ -nullcline is shifted upward.

☐ the  $w$ -nullcline is shifted to the right.



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✓ Correct (2/2 points)

Stability of the fixed points

1/1 point (graded)

In the simplified F-N model above,  $f(u)$  is modeled as a piecewise linear function consisting of three linear segments in three regions  $u < 1$ , and  $1 < u < 3$  and  $u > 3$ .

In figure above, the  $w$ -nullcline is crossed with the first linear segment, i.e., in the region  $u < 1$ . By changing the value of injected current  $I$ , one can change the location of fixed point such that the intersection of two nullclines is lied in other linear segments, i.e., in the regions  $1 < u < 3$  or  $u > 3$ , corresponding to the second and third linear segments, respectively. What is the correct answer for stability of the fixed points in these three regions, from left to right respectively?

- ☒ stable - unstable - stable
- ☐ unstable - stable - unstable
- ☐ stable - stable - stable
- ☐ unstable - unstable - unstable
- ☐ stable - unstable - unstable
- ☐ unstable - stable - stable
- ☐ stable - stable - unstable
- ☐ unstable - unstable - stable



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✓ Correct (1/1 point)

Fast dynamic

2/2 points (graded)

Now imagine that  $\epsilon \ll 1$ . Which of the followings is correct?

- ☒ Dynamics of  $u$  is much faster than that of  $w$ .
- ☐ Dynamics of  $w$  is much faster than that of  $u$ .



In such a situation, i.e.,  $\epsilon \ll 1$ , which of the followings is correct?

- ☐ The system moves almost vertically (in the phase plane) until the  $w$ -nullcline is reached.
- ☐ The system moves almost vertically (in the phase plane) until the  $u$ -nullcline is reached.
- ☐ The system moves almost horizontally (in the phase plane) until the  $w$ -nullcline is reached.
- ☒ The system moves almost horizontally (in the phase plane) until the  $u$ -nullcline is reached.



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✓ Correct (2/2 points)

Impulse response

1/1 point (graded)

Still assume that  $\epsilon \ll 1$ . Think about the behaviour of the trajectories of  $u(t)$  in response to a current pulse  $I(t) = q\delta(t)$ .

One can show that the  $u$ -nullcline acts as a threshold when the system is subjected to such current pulses. What is the threshold value for the spike emission according to the figure above? (just enter a number in the box below)

2



2

Sketch these trajectories in the phase plane and in the temporal domain for a few values of  $q$  to see what happens.

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You have used 1 of 1 attempt

✓ Correct (1/1 point)

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