- 1. No. Since  $X_i$  for any  $i \ge 1$  is uniformly distributed between -1 and 1, its distribution does not get concentrated on the vicinity of some number.
- 2. Yes, to 0, since for  $\epsilon > 0$ ,

$$\lim_{i \to \infty} \mathbf{P}\left(|Y_i - 0| > \epsilon\right) = \lim_{i \to \infty} \mathbf{P}\left(\left|\frac{X_i}{i} - 0\right| > \epsilon\right)$$
$$= \lim_{i \to \infty} \left[\mathbf{P}\left(X_i > i\epsilon\right) + \mathbf{P}\left(X_i < -i\epsilon\right)\right] = 0.$$

3. Yes, to 0. For  $\epsilon \geq 1$ ,

$$\lim_{i \to \infty} \mathbf{P}(|Z_i - 0| > \epsilon) = \lim_{i \to \infty} \mathbf{P}(|(X_i)^i| > \epsilon) = \lim_{i \to \infty} 0 = 0,$$

since  $X_i$  is uniformly distributed between 1 and -1 and hence  $|(X_i)^i| \le 1$ . For  $0 < \epsilon < 1$ ,

$$\lim_{i \to \infty} \mathbf{P}(|Z_i - 0| > \epsilon) = \lim_{i \to \infty} \mathbf{P}(|(X_i)^i - 0| > \epsilon)$$

$$= \lim_{i \to \infty} \left[ \mathbf{P}\left(X_i > \epsilon^{\frac{1}{i}}\right) + \mathbf{P}\left(X_i < -(\epsilon)^{\frac{1}{i}}\right) \right]$$

$$= \lim_{i \to \infty} \left[ \frac{1}{2} \left(1 - \epsilon^{\frac{1}{i}}\right) + \frac{1}{2} \left(1 - \epsilon^{\frac{1}{i}}\right) \right]$$

$$= \lim_{i \to \infty} \left(1 - \sqrt[i]{\epsilon}\right)$$

$$= 0.$$