MITx Video | 6.041x | Spring 2014 | Video transcript | L11-3: A linear function of a normal r.v.

Let us now consider an application of what we have done so far. Let X be a normal random variable with given mean and variance. This means that the PDF of X takes the familiar form.

We consider random variable Y, which is a linear function of X. And to avoid trivialities, we assume that a is different than zero. We will just use the formula that we have already developed.

So we have that the density of Y is equal to 1 over the absolute value of a. And then we have the density of X, but evaluated at x equal to this expression. So this expression will go in the place of x in this formula. And we have y minus b over a minus mu squared divided by 2 sigma squared.

And now we collect these constant terms here. And then in the exponent, we multiply by a squared the numerator and the denominator, which gives us this form here. We recognize that this is again, a normal PDF. It's a function of y. We have a random variable Y. This is the mean of the normal. And this is the variance of that normal.

So the conclusion is that the random variable Y is normal with mean equal to b plus a mu. And with variance a squared, sigma squared. The fact that this is the mean and this is the variance of Y is not surprising. This is how means and variances behave when you form linear functions.

The interesting part is that the random variable Y is actually normal. Intuitively, what happened here is that we started with a normal bell shaped curve. A bell shaped PDF for X. We scale it vertically and horizontally, and then shift it horizontally by b. As we do these operations, the PDF still remains bell shaped. And so the final PDF is again a bell shaped normal PDF.