4. Exercise: Checkout counter

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2/2 points (ungraded)

Consider our checkout counter example. Assume that there are two types of customers who arrive according to independent Bernoulli processes with rates $p_1 \in (0,1)$ and $p_2 \in (0,1)$, respectively. The overall arrival process of all customers follows a merged Bernoulli process of the two separate Bernoulli processes. All customers who arrive join a single queue, which has a capacity of 10 customers. We are interested in making predictions about the length of the queue at any point in time.

For each of the following parts, choose the correct statement.

1. Assume that service times are not type-dependent and are modelled as independent geometric random variables with parameter $q \in (0,1)$ for all customers in the queue.

   - ○ One can model this queue using the same transition probability graph as in the previous video with $p = (p_1 + p_2)/2$ and q.
   - ◉ One can model this queue using the same transition probability graph as in the previous video with $p = 1 - (1-p_1)(1-p_2)$ and q. ✔
   - ○ One can model this queue using the same transition probability graph as in the previous video with some other appropriate choice of p and q.
   - ○ There are no values of p and q for which one can model the queue using the same transition probability graph as in the previous video.

2. Assume now that service times are type-dependent and are modelled as independent geometric random variables with parameters $q_1 \in (0,1)$ and $q_2 \in (0,1)$, respectively, for the two types of customers.

   - ○ One can model the queue using the same transition probability graph as in the previous video with $p = (p_1 + p_2)/2$ and $q = (q_1 + q_2)/2$.
   - ○ One can model the queue using the same transition probability graph as in the previous video with $p = 1 - (1-p_1)(1-p_2)$ and $q = (p_1 q_1 + p_2 q_2)/(p_1 + p_2)$.
   - ○ One can model this queue using the same transition probability graph as in the previous video with some other appropriate choice of p and q.
   - ◉ There are no values of p and q for which one can model the queue using the same transition probability graph as in the previous video. ✔

Solution:

1. Option 2 is correct. The value of p corresponds to the arrival probability of the merged Bernoulli process.

2. Option 4 is correct. To see this, note that for all of the first three options, the process is a Markov chain. Thus, it suffices to argue that the process with two types of arriving customers is not, in general, a Markov chain. To see this, consider an extreme case where $p_1 = p_2 = 1/2$, $q_1 = 1$, and q_2 is very small. Suppose that the previous state was 0 and the current state is 1. This means that we just had an arrival; by symmetry it is equally likely to have been of either type, and the expected time until the next departure is $(1 + (1/q_2))/2$. If we now observe the next state to be again 1, we are pretty certain that it was an arrival of type 2, and the expected time until the next departure is approximately $1/q_2$. Thus, the statistics of the future of the process are not fully determined by the current state — the past history also plays a role, which violates the Markov property.

   如果离开率是和不同类型的人相关的话，那么假设第一类人来了就不走，第二类人来了马上走。那么如果前一个状态是1，现在的状态还是1，则说明一个来了一个走了，前一个状态是0，则说明各有一定可能。所以这个状态的判定还和前一个状态，而不仅仅是现在的状态相关。