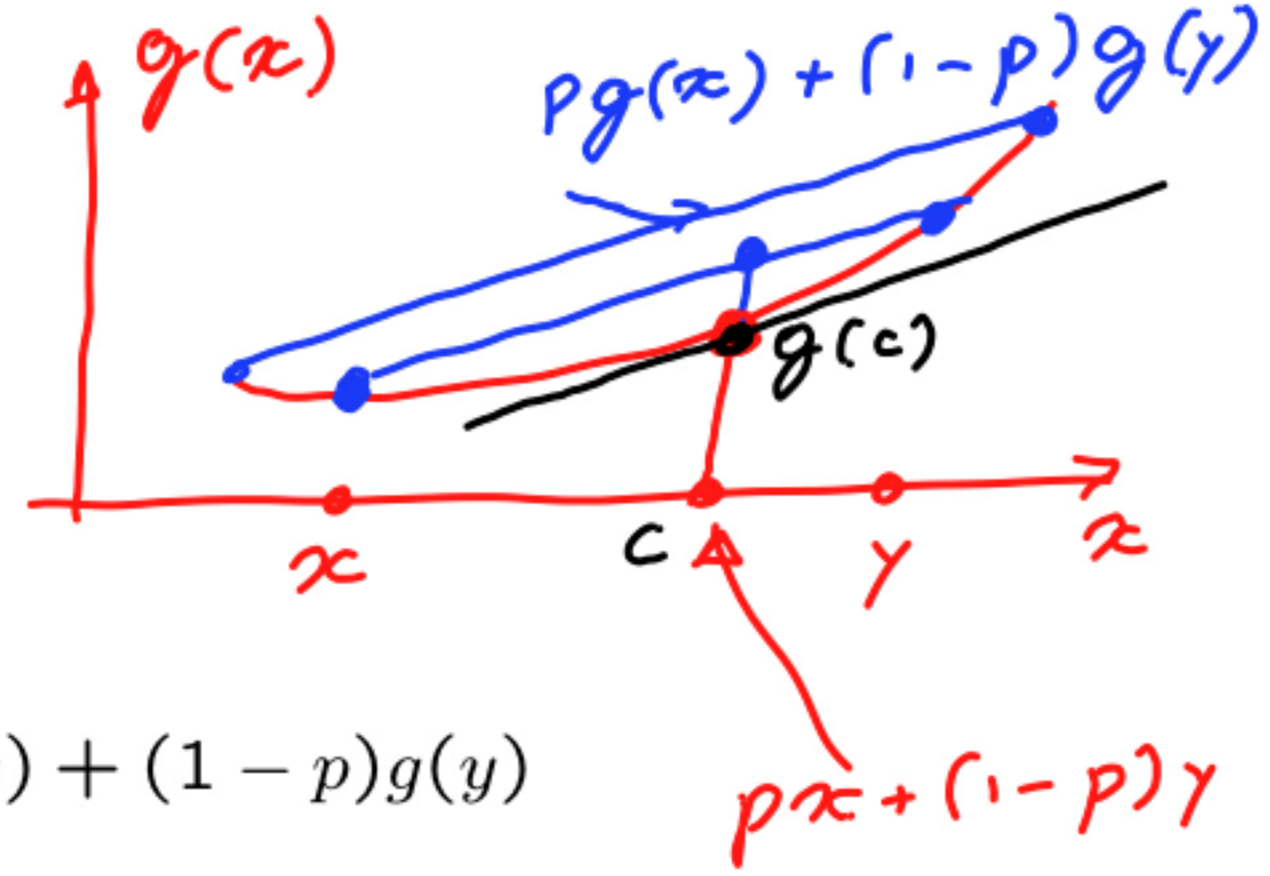
Comparing $\mathbf{E}\big[g(X)\big]$ to $g\big(\mathbf{E}[X]\big)$: Jensen's inequality

- Let g be convex



Then, $g\big(\mathbf{E}[X]\big) \le \mathbf{E}\big[g(X)\big]$.

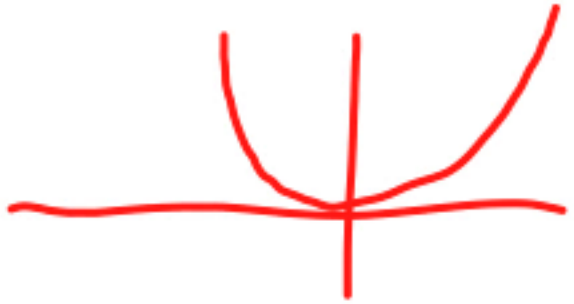
- If $0 \le p \le 1$, then $\underline{g\big(px + (1-p)y\big)} \le pg(x) + (1-p)g(y)$
  - If twice differentiable: $g''(x) \ge 0$
  - for any c, x: $\quad g(x) \ge g(c) + g'(c)(x-c)$

$$g(X) \ge g(E[X]) + \underline{g'(E[X])}\,(X - E[X])$$

$$E[g(X)] \ge g(E[X]) + 0$$

Comparing $\mathbf{E}\big[g(X)\big]$ to $g\big(\mathbf{E}[X]\big)$: Jensen's inequality

- Let g be convex



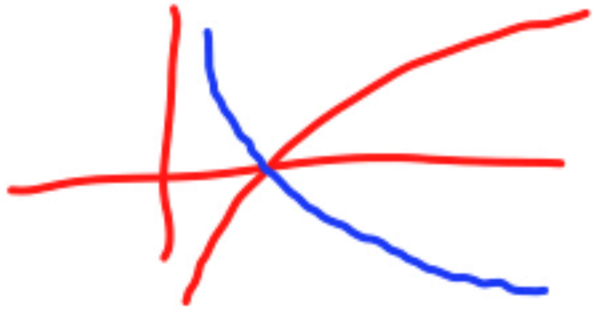
Then, $g\big(\mathbf{E}[X]\big) \leq \mathbf{E}\big[g(X)\big]$.

$g(x) = x^2$ $\quad E[X^2] = Var(X) + (E[X])^2 \geq (E[X])^2$

$\geq g(E[X])$

$E[g(X)]$

$g(x) = x^4$ $\quad (E[X])^4 \leq E[X^4]$



$g(x) = -\log x$ $\quad -\log(E[X]) \leq E[-\log X]$

$\log(E[X]) \geq E[\log X]$