

2. Functions of the standard normal

Problem 2. Functions of the standard normal

2/2 points (graded)

The random variable X has a standard normal distribution. Find the PDF of the random variable Y , where:

1. $Y = 5X - 7$.

☐ $f_Y(y) = 5f_X\left(\frac{y-7}{5}\right)$

☒ $f_Y(y) = \frac{1}{5}f_X\left(\frac{y+7}{5}\right)$ ✓

☐ $f_Y(y) = 5f_X\left(\frac{y+7}{5}\right)$

☐ $f_Y(y) = \frac{1}{5}f_X\left(\frac{y-7}{5}\right)$

2. $Y = X^2 - 2X$. For $y \geq -1$,

☐ $\frac{f_X(1 + \sqrt{y+1}) - f_X(1 - \sqrt{y+1})}{2\sqrt{y-1}}$

☐ $\frac{f_X(1 + \sqrt{y+1}) - f_X(1 - \sqrt{y+1})}{2\sqrt{y+1}}$

☒ $\frac{f_X(1 + \sqrt{y+1}) + f_X(1 - \sqrt{y+1})}{2\sqrt{y+1}}$ ✓

$$\frac{f_X(1 + \sqrt{y+1}) + f_X(1 - \sqrt{y+1})}{2\sqrt{y+1} - 2\sqrt{y-1}}$$

Solution:

1. We know that when $Y = aX + b$, with $a \neq 0$, we have

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right).$$

When $Y = 5X - 7$, we have $a = 5$ and $b = -7$. Therefore,

$$f_Y(y) = \frac{1}{5} f_X\left(\frac{y+7}{5}\right),$$

for all y .

2. $Y = X^2 - 2X$. We will find the CDF of Y and differentiate to find the PDF. For $y \geq -1$, we have,

$$\begin{aligned} F_Y(y) &= \mathbb{P}(Y \leq y) \\ &= \mathbb{P}((X-1)^2 \leq y+1) \\ &= \mathbb{P}(-\sqrt{y+1} \leq X-1 \leq \sqrt{y+1}) \\ &= \mathbb{P}(1-\sqrt{y+1} \leq X \leq 1+\sqrt{y+1}) \\ &= F_X(1+\sqrt{y+1}) - F_X(1-\sqrt{y+1}). \end{aligned}$$

Differentiating and using the chain rule, we obtain

$$f_Y(y) = \frac{f_X(1+\sqrt{y+1}) + f_X(1-\sqrt{y+1})}{2\sqrt{y+1}}.$$

提交

You have used 1 of 2 attempts