

## 18. Exercise: Correlation properties

### Exercise: Correlation properties

6/6 points (graded)

As in the preceding example, let  $Z$ ,  $V$ , and  $W$  be independent random variables with mean  $0$  and variance  $1$ , and let  $X = Z + V$  and  $Y = Z + W$ . We have found that  $\rho(X, Y) = 1/2$ .

a) It follows that:

$$\rho(X, -Y) = \boxed{-1/2} \quad \checkmark \text{ Answer: } -0.5$$

$$\rho(-X, -Y) = \boxed{1/2} \quad \checkmark \text{ Answer: } 0.5$$

b) Suppose that  $X$  and  $Y$  are measured in dollars. Let  $X'$  and  $Y'$  be the same random variables, but measured in cents, so that  $X' = 100X$  and  $Y' = 100Y$ . Then,

$$\rho(X', Y') = \boxed{1/2} \quad \checkmark \text{ Answer: } 0.5$$

c) Suppose now that  $\tilde{X} = 3Z + 3V + 3$  and  $\tilde{Y} = -2Z - 2W$ . Then

$$\rho(\tilde{X}, \tilde{Y}) = \boxed{-1/2} \quad \checkmark \text{ Answer: } -0.5$$

d) Suppose now that the variance of  $Z$  is replaced by a very large number. Then

$$\rho(X, Y) \text{ is close to } \boxed{1} \quad \checkmark \text{ Answer: } 1$$

e) Alternatively, suppose that the variance of  $Z$  is close to zero. Then

$$\rho(X, Y) \text{ is close to } \boxed{0} \quad \checkmark \text{ Answer: } 0$$

#### Solution:

We saw that a linear transformation  $x \mapsto ax + b$  of a random variable does not change the value of the correlation coefficient, except for a possible sign change if the coefficient  $a$  is negative. Note that in the case of  $\rho(-X, -Y)$ , we have two sign changes, hence no sign change.

For the last two parts, if  $\mathbf{Z}$  has a very large variance, then the terms  $\mathbf{V}$  and  $\mathbf{W}$  become insignificant, and  $\rho(\mathbf{X}, \mathbf{Y}) \approx \rho(\mathbf{Z}, \mathbf{Z}) = 1$ . And if  $\mathbf{Z}$  has very small variance, then  $\mathbf{X}$  and  $\mathbf{Y}$  are approximately independent, so that  $\rho(-\mathbf{X}, -\mathbf{Y}) \approx 0$ . (These conclusions can also be justified by an exact calculation.)

提交

You have used 2 of 3 attempts

**i** Answers are displayed within the problem

讨论

显示讨论

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