3. Matrices

Given a matrix, A, we denote its transpose as A^T and its determinant as det(A). The transpose of a matrix is equivalent to writing its rows as columns, or its columns as rows. Then, ${A^T}_{i,j} = A_{j,i}$. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 1 \end{bmatrix}$

3. (a)

1/1 point (graded)

Compute $det\left(A^T\right)$.

6 ✔ **Answer:** 6

STANDARD NOTATION

Solution:

First compute A^T by writing the first row as the first column. This gives us $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ as the first column. Repeat with rows 2 and 3 to arrive at the solution. Then compute the determinant as follows: $1\left(5-12\right) - 4\left(2-6\right) + 1\left(12-15\right) = 6.$

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Answers are displayed within the problem

3. (b)

1/1 point (graded)

Compute det(A).

6 ✔ **Answer:** 6

STANDARD NOTATION

Solution:

det(A) = $1\left(5-12\right) - 2\left(4-6\right) + 3\left(8-5\right) = 6.$ Note that det(A) = det(A^T). This is not a coincidence. In fact, this useful property holds for all matrices.

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3. (c)

1/1 point (graded)

Let $g = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$. Can we compute gA?

- yes ✔
- no

STANDARD NOTATION

Solution:

The dimension of g is 1×3 and the dimension of A is 3×3. Since the number of columns in g equals the number of rows in A, the product exists.

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3. (d)

1/1 point (graded)
Let g be as above. Can we compute Ag?

- yes
- no ✔

STANDARD NOTATION

Solution:

Unlike part c), the dimension of A is 3×3 and the dimension of g is 1×3. Since the number of columns in A does not equal the number of rows in g, the product does not exist. Note that this example shows that matrix multiplication is not commutative, i.e., $AB \neq BA$.

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3. (e)

1/1 point (graded)
Let $B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 4 \\ 5 & 6 & 4 \end{bmatrix}$. Determine the rank of B. Recall that the rank of a matrix is the number of linearly independent rows or columns.

2 ✔ **Answer:** 2

STANDARD NOTATION

Solution:

Note that the first two rows of B are linearly independent since they are not multiples of each other. Now solve the system $\begin{bmatrix} 2a+b=5c \\ a+4b=6c \\ 4b=4c \end{bmatrix}$. Recall that these three vectors will be linearly independent if the only solution to this set of equations is the zero vector.

Since we find that this system has the solution $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, these vectors are not linearly independent and the rank of the matrix is 2.

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3. (f)

1/1 point (graded)

Let M^{-1} denote the inverse of a matrix M. Let A be as defined above. Compute A^{-1}. What matrix does the product AA^{-1} produce?

- (•) identity matrix ✔
- ( ) zero matrix

STANDARD NOTATION

Solution:

For any matrix A, $AA^{-1} = A^{-1}A = I$, where I is the identity matrix.

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Answers are displayed within the problem

Discussion

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