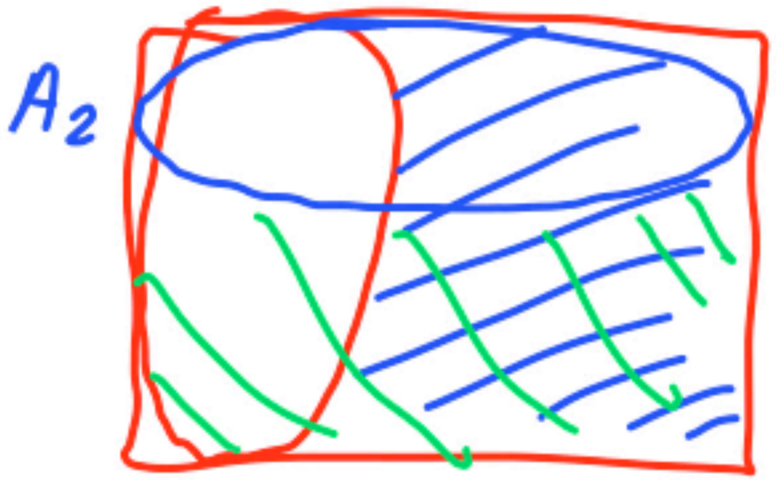
Interpreting the union bound and the Bonferroni inequality

$\mathbf{P}(A_1 \cup A_2) \le \mathbf{P}(A_1) + \mathbf{P}(A_2)$

- Suppose that:
  - very few of the students are smart A_1
  - very few students are beautiful A_2
- Then: very few students are smart or beautiful



- Suppose that:
  - most of the students are smart
  - most students are beautiful
- Then: most students are smart and beautiful

$$\mathbf{P}(A_1 \cap A_2) \ge \underline{\mathbf{P}(A_1)} + \underline{\mathbf{P}(A_2)} - 1$$

The Bonferroni inequality

$\mathbf{P}(A_1 \cap A_2) \geq \mathbf{P}(A_1) + \mathbf{P}(A_2) - 1$

$$\mathbf{P}\left((A_1 \cap A_2)^c\right) = \mathbf{P}(A_1^c \cup A_2^c) \leq \mathbf{P}(A_1^c) + \mathbf{P}(A_2^c)$$

$$\cancel{1} - \mathbf{P}(A_1 \cap A_2) \qquad \leq \cancel{1} - \mathbf{P}(A_1) + 1 - \mathbf{P}(A_2)$$

- $\mathbf{P}(A_1 \cap \cdots \cap A_n) \geq \mathbf{P}(A_1) + \cdots + \mathbf{P}(A_n) - (n-1)$

$$\mathbf{P}\left((A_1 \cap \cdots \cap A_n)^c\right) = \mathbf{P}(A_1^c \cup \cdots \cup A_n^c) \leq \mathbf{P}(A_1^c) + \cdots + \mathbf{P}(A_n^c)$$

$$1 - \mathbf{P}(A_1 \cap \cdots \cap A_n) \qquad \leq \qquad (1 - \mathbf{P}(A_1)) + \cdots + (1 - \mathbf{P}(A_n))$$