4. GLM: Statistical Model and Notation

We now combine our ingredients together.

We have discussed **canonical exponential famlies** parametrized by θ, with the log-partition function $b(\theta)$ having the property that $b'(\theta) = \mu$. Recall that in GLMs, the point of the link function is to assume $g(\mu(\mathbf{x})) = \mathbf{x}^T\boldsymbol{\beta}$, where μ is the **regression function** : the mean of Y given $\mathbf{X} = \mathbf{x}$, $\mathbb{E}[Y \mid \mathbf{X} = \mathbf{x}]$.

Concept Check: Properties of the Canonical Link Function

1/1 point (graded)

Let $f_\theta = \exp\left(\frac{y\theta - b(\theta)}{\phi} + c(y, \phi)\right)$ for $\phi \neq 0$ describe an exponential family. Which one of the following statements about the function $g(\mu) = \theta$ is **false** ?

- The canonical link function always exists.
- g is identical to $(b')^{-1}$.
- If g strictly increases, then g^{-1} strictly increases.
- Regardless of the value of ϕ, g is strictly increasing. ✔

Solution:

- We can always write down the function $g(\mu) = \theta$.
- Based on the properties of the log-partition function b, we derived previously that $b'(\theta) = \mu$, so we have the identity $g(\mu) = (b')^{-1}(\mu)$.
- It is a general fact that if f is a function that strictly increases, then its inverse is a function that strictly increases. The same holds for strictly decreasing functions.
- g **decreases** if $\phi < 0$. This can be seen from the fact that $\phi \cdot b''(\theta)$ is the variance of a random variable, which means $b'' < 0$. Thus, b' is a decreasing function, which means $(b')^{-1}$ is decreasing. **Ultimately, this demonstrates that there is a "canonical" choice of parametrization.** If $\phi < 0$, all that tells us is that we should re-parametrize by multiplying both ϕ and b by -1. We can always make such a choice, as long as $\phi \neq 0$, so that g is an increasing function. Recall that this is one of the properties we wanted out of link functions of GLMs!

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Answers are displayed within the problem

GLM and Introduction of Beta for Estimation

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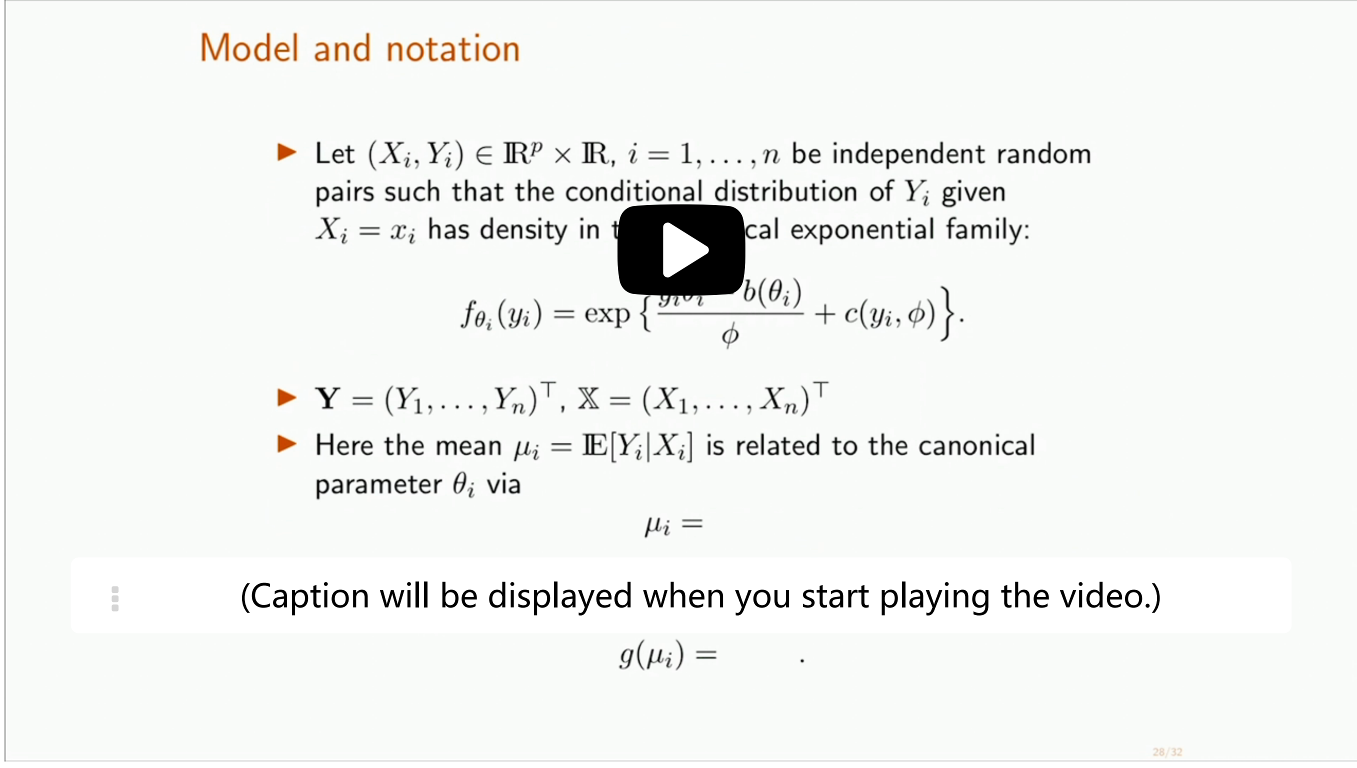

OK, so I've talked about my random component.

And I've talked about how I want to link

the conditional expectation of this random component

to X through this X transpose beta.

So now, if you tell me Y is part of this canonical exponential

family with this very specific choice of b and phi,

and you're telling me a link function,

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Concept Check: Assumptions for GLM

0/1 point (graded)
Choose from the following **the assumptions we make in fitting data using a GLM**.

- [x] We assume a distribution for Y. ✔
- [x] We assume a particular link function $g(\cdot)$. ✔
- [x] We assume a noise model that captures the relationship between $\mathbf{X}$ and Y.

✖

Solution:

In spirit it's the same thing; if you know the whole distribution then you know all of its conditionals, and vice versa.

The only assumptions we make in using a GLM (from the choices we are given in this problem) are **a distribution for** Y and **a link function** $g(\cdot)$. We do not need need to assume a noise model to capture the relationship between Y and $\mathbf{X} = \mathbf{x}$. The assumptions of a distribution for Y and a link function $g(\mu(\mathbf{x}))$ relate Y and $\mathbf{X} = \mathbf{x}$ through the following equation:

$$g(\mu(\mathbf{x}) = \mathbb{E}[Y|\mathbf{X} = \mathbf{x}]) = \mathbf{x}^T \boldsymbol{\beta}.$$

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Discussion

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