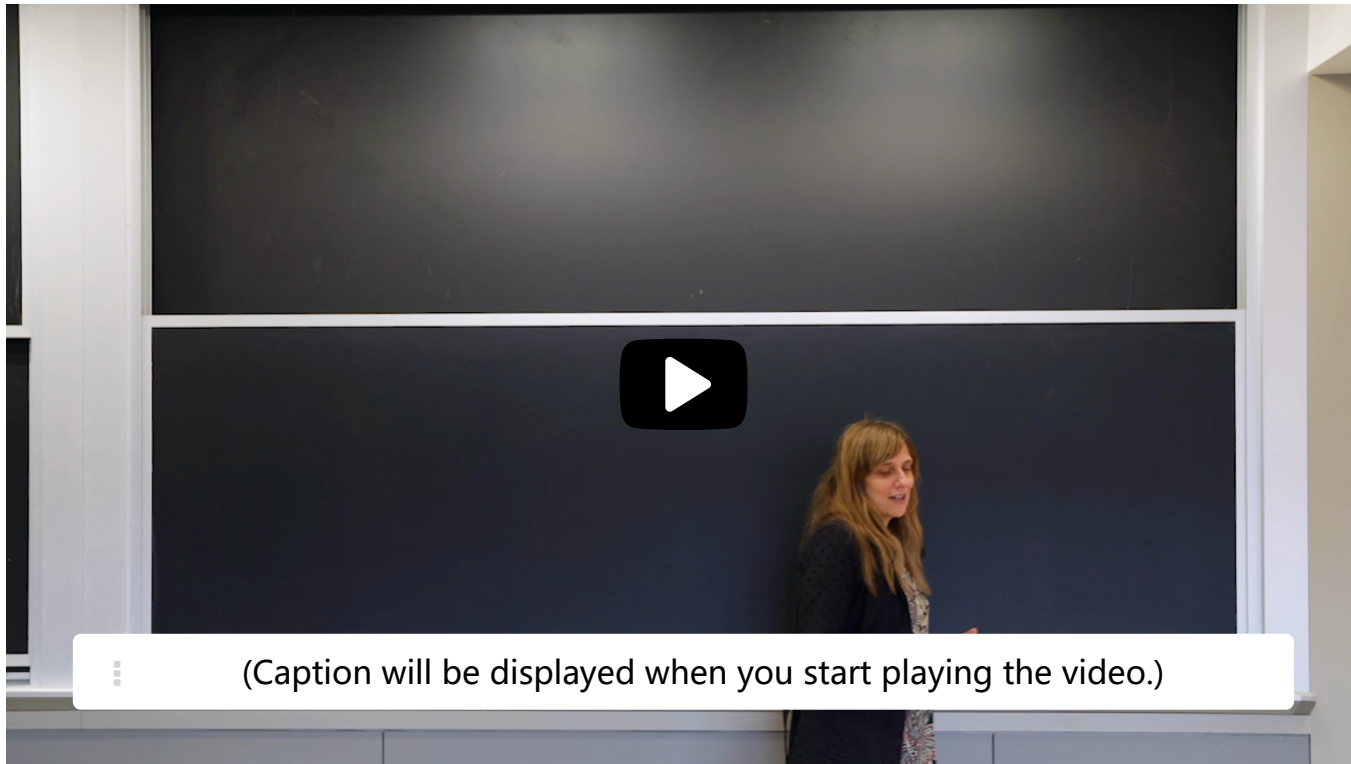


## 2. Recap of Maximum Likelihood Estimation for Multinomial and Gaussian Models

### MLE for Multinomial and Gaussian Models

[Start of transcript. Skip to the end.](#)



Today, we will talk about mixture model and the EM algorithm.

So first, let me start by reminding you the types of distributions that you have already seen in the class.

What we will do today will kind of combine two of them.

So the first distribution-- the first generative model

#### Video

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### MLE under Gaussian Noise I

1/1 point (graded)

Let  $Y_i = \theta + N_i, i = 1, \dots, n$  where  $\theta$  is an unknown parameter and  $N_i$  are iid Gaussian random variables with zero mean. Upon observing  $Y_i$ 's, what is the maximum likelihood estimate of  $\theta$ ?

Choose the correct expression from options below.

☐  $\hat{\theta} = \prod_{i=1}^n Y_i$

☐  $\hat{\theta} = \frac{\prod_{i=1}^n Y_i}{n}$

☐  $\hat{\theta} = \sum_{i=1}^n Y_i$

☒  $\hat{\theta} = \frac{\sum_{i=1}^n Y_i}{n}$  ✓

#### Solution:

$Y_i$ 's are iid Gaussian with mean  $\theta$ . As seen before, the ML estimator of  $\theta$  is  $\sum_{i=1}^n \frac{Y_i}{n}$ .

**i** Answers are displayed within the problem

## MLE under Gaussian Noise II

0/1 point (graded)  
Would the ML estimator **change** if the  $N_i$ 's are **independent** Gaussians with **possibly different variances**  $\sigma_1^2, \dots, \sigma_n^2$  but **same zero mean**?  
看错题了，我以为他问会不会不变，结果问的是会不会变

☐ Yes

☒ No

当然我觉得噪音的方差不一样，estimate不出来

**Solution:**

The log-likelihood (with possibly different variances) is

$$\log P(Y_1, \dots, Y_n | \theta, \sigma_1^2, \dots, \sigma_n^2) = -\frac{1}{2} \sum_{i=1}^n \log(2\pi\sigma_i^2) - \sum_{i=1}^n \frac{(Y_i - \theta)^2}{2\sigma_i^2}.$$

Note here we cannot take  $\sigma_i^2$  out of the summation. Thus, to solve this, we need to take the derivative of the log-likelihood with respect to  $\theta$  and  $\sigma_i^2$  and setting them to zero.  
We will have  $\sigma_i^2 = (Y_i - \theta)^2$  and  $\sum_{i=1}^n \frac{1}{Y_i - \theta} = 0$ . Solving these equations **may** possibly give us different estimator for  $\theta$  than before.

**i** Answers are displayed within the problem

## Discussion

Show Discussion

**Topic:** Unit 4 Unsupervised Learning (2 weeks) :Lecture 16. Mixture Models; EM algorithm / 2. Recap of Maximum Likelihood Estimation for Multinomial and Gaussian Models