The Beta formula

$$f_{\Theta|K}(\theta \mid k) \;=\; \frac{1}{d(n,k)}\,\theta^k(1-\theta)^{n-k}$$

$\Theta \in [0,1]$ $\qquad k \geq 0$

$$d(n,k) = \int_0^1 \theta^k(1-\theta)^{n-k}\,d\theta = \frac{k!\,(n-k)!}{(n+1)!}$$

$$\boxed{\int_0^1 \theta^\alpha(1-\theta)^\beta\,d\theta = \frac{\alpha!\,\beta!}{(\alpha+\beta+1)!}}$$

- Nonnegative integers α, β

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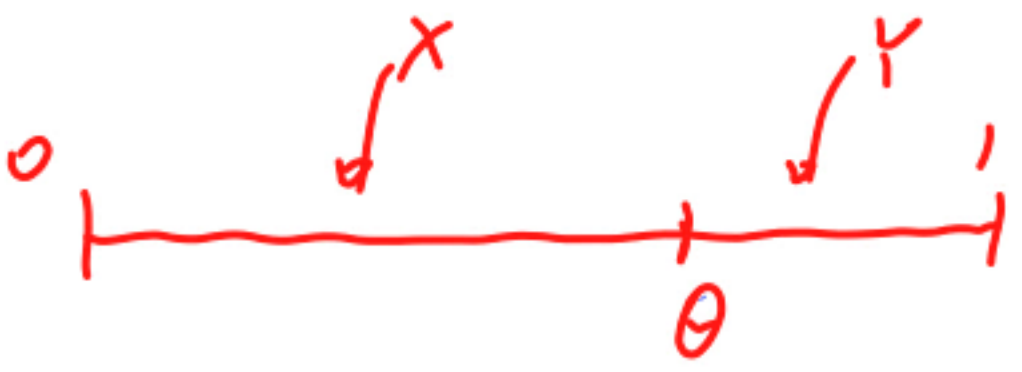
- Let $X_1,\dots,X_\alpha$, Z, $Y_1,\dots,Y_\beta$ independent, uniform[0,1]
- $\mathrm{P}(X_1 < X_2 < \cdots < X_\alpha < Z < Y_1 < Y_2 < \cdots < Y_\beta) = P(A) = \dfrac{1}{(\alpha+\beta+1)!}$

$$P(A) = \int P(A \mid Z=\theta)\, f_Z(\theta)\, d\theta = \int \theta^\alpha (1-\theta)^\beta \frac{1}{\alpha!} \frac{1}{\beta!}\, d\theta$$

$$P(A \mid Z=\theta) = P\left(\begin{matrix} X_1,\dots,X_\alpha < \theta & \text{and} & X_1 < X_2 < \cdots < X_\alpha \\ Y_1,\dots,Y_\beta > \theta & & Y_1 < Y_2 < \cdots < Y_\beta \end{matrix}\right)$$



$$= \theta^\alpha (1-\theta)^\beta \frac{1}{\alpha!} \frac{1}{\beta!}$$