Problem 4

For simplicity, suppose our rating matrix is a $2x2$ matrix and we are looking for a rank-1 solution UV^T so that user and movie features U and V are both $2x1$ matrices. The observed rating matrix has only a single entry:

$$Y = \begin{bmatrix} ? & 1 \\ ? & ? \end{bmatrix} \tag{6.4}$$

In order to learn user/movie features, we minimize

$$J(U,V) = \left(\frac{1}{2} \sum_{(a,i)\in D} (Y_{ai} - [UV^T]_{ai})^2 \right) + \lambda (U_1^2 + V_1^2) \tag{6.5}$$

where U_1 and V_1 are the first components of the vectors U and V respectively (if $U = [u_1, u_2]$, then $U_1 = u_1$), the set D is just the observed entries of the matrix Y , in this case just $(1,2)$.

Note that the regularization we use applies only to the first coordinate of user/movie features . We will see how things get a bit tricky with this type of partial regularization.

Correction Note (July 30 21:00UTC): An earlier version does not include the clarifcation “where U_1 and V_1 are the first compoments of the vectors U and V respectively."

Correction Note (Aug 4 03:00UTC): Added an example of what U_1 means: if $U = [u_1, u_2]$, then $U_1 = u_1$).

4. (1)

1.0/1 point (graded)

If we initialize $U = [u \quad 1]^T$, for some $u > 0$, what is the solution to the vector $V = [v_1 \quad v_2]^T$ as a function of λ and u?

(Enter V as a vector, enclosed in square brackets, and components separated by commas, e.g. type `[u,lambda+1]` if $V = [u \quad \lambda+1]^T$.)

$V =$ [0, 1/u] ✔ **Answer:** [0,1/u]

STANDARD NOTATION

Solution:

Notice that J only regularizes on the first coordinate. Thus, we only want to minimize $J(v_1, v_2) = \frac{1}{2}(1 - uv_2)^2 + \lambda v_1^2$ given that $V = [v_1, v_2]^T$. We can see that J is minimized when $v_1 = 0, v_2 = \frac{1}{u}$. Therefore,

$$V = \left[0, \tfrac{1}{u}\right]^T. \tag{6.6}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

4. (2)

1/1 point (graded)
What is the resulting value of $J(U,V)$ as a function of λ and u?

(Type `lambda` for λ).

lambda * (u^2)

✔ **Answer:** (lambda*u^2)

$\lambda \cdot (u^2)$

STANDARD NOTATION

Solution:

Notice that J only regularizes on the first coordinate. Therefore,

$$\begin{aligned} J(U,V) &= \frac{1}{2}\sum_{(a,i)\in D}(Y_{ai} - [UV^T]_{ai})^2 + \lambda(U_1^2 + V_1^2) \\ &= \frac{1}{2}(1 - u_1 v_2)^2 + \lambda(u_1^2 + v_1^2) \\ &= \frac{1}{2}\left(1 - u \cdot \frac{1}{u}\right)^2 + \lambda(u^2 + 0^2) \\ &= \lambda u^2 \end{aligned}$$

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Answers are displayed within the problem

4. (3)

1/1 point (graded)
If we continue to iteratively solve for U and V, what would U and V converge to?

- (selected) U goes to $[0,1]$, V goes to $[0,\infty]$ ✔
- U goes to $[0,0]$, V goes to $[0,0]$
- U goes to $[0,1]$, V goes to $[0,0]$
- U goes to $[0,\infty]$, V goes to $[1,0]$

Correction Note (July 29): In an earlier version, V was missing in all choices.

Solution:

The regularization error is minimized when u_1 and v_1 are 0. Over many iterations, u_1 will eventually converge to zero. The squared error term is $\frac{1}{2}(1-u_1v_2)^2$ is minized when $u_1 v_2 = 1$. Since u_1 converges to 0, v_2 diverges to ∞.

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Answers are displayed within the problem

4. (4)

3/3 points (graded)
Not all rating matrices Y can be reproduced by UV^T when we restrict the dimensions of U and V to be 2×1.

For each matrix below, answer “Yes" or “No" according to whether it can be reproduced by such U and V of size 2×1.

$$Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

- ◉ yes ✔
- ○ no

$$Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

感觉full rank就不行，因为这个的假设就是low rank matrix

- ○ yes
- ◉ no ✔

$$Y = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

- ◉ yes ✔
- ○ no

Solution:

In order for matrix Y to be reproduced by UV^T we must have $[u_1, u_2]^T \times [v_1, v_2] = Y$. For the second matrix, this would require $u_1 \times v_1 = 1$, $u_1 \times v_2 = 0$, $u_2 \times v_1 = 0$, $u_2 \times v_2 = 1$, which has no solution.
The first matrix can be represented as $[1 - 1]^T \times [1 - 1]$ and the third can be represented as $[1 - 1]^T \times [11]$.

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Answers are displayed within the problem

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