(a) $\mathbf{E}[X_n] = 0 \cdot \left(1 - \frac{1}{n}\right) + 1 \cdot \frac{1}{n} = \frac{1}{n}$
$\text{var}(X_n) = \left(0 - \frac{1}{n}\right)^2 \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2 \cdot \left(\frac{1}{n}\right) = \frac{n-1}{n^2}$
$\mathbf{E}[Y_n] = 0 \cdot \left(1 - \frac{1}{n}\right) + n \cdot \frac{1}{n} = 1$
$\text{var}(Y_n) = (0-1)^2 \cdot \left(1 - \frac{1}{n}\right) + (n-1)^2 \cdot \left(\frac{1}{n}\right) = n - 1$

(b) Using Chebyshev's inequality, we have
$\lim\limits_{n\to\infty} \mathbf{P}(|X_n - \dfrac{1}{n}| \geq \epsilon) \leq \lim\limits_{n\to\infty} \dfrac{n-1}{n^2\epsilon^2} = 0$
Moreover, $\lim\limits_{n\to\infty} \dfrac{1}{n} = 0$.
It follows that X_n converges to 0 in probability. For Y_n, Chebyshev suggests that,

$$\lim_{n\to\infty} \mathbf{P}(|Y_n - 1| \geq \epsilon) \leq \lim_{n\to\infty} \frac{n-1}{\epsilon^2} = \infty,$$

Thus, we cannot conclude anything about the convergence of Y_n through Chebyshev's inequality.

(c) For every $\epsilon > 0$,

$$\lim_{n\to\infty} \mathbf{P}(|Y_n| \geq \epsilon) \leq \lim_{n\to\infty} \frac{1}{n} = 0,$$

Thus, Y_n converges to zero in probability.

(d) Both the statements are false. A counter example is Y_n. It converges in probability to 0 yet its expected value is 1 for all n.
Since $\text{var}(Y_n) = n - 1$, $\lim\limits_{n\to\infty} \text{var}(Y_n) = \infty$. Therefore the variances don't even converge.

(e) Using the Markov inequality, we have

$$\mathbf{P}\left(|X_n - c| \geq \epsilon\right) = \mathbf{P}\left(|X_n - c|^2 \geq \epsilon^2\right) \leq \frac{\mathbf{E}\left[(X_n - c)^2\right]}{\epsilon^2}.$$

Taking the limit as $n \to \infty$, we obtain

$$\lim_{n\to\infty} \mathbf{P}\left(|X_n - c| \geq \epsilon\right) = 0,$$

which establishes convergence in probability.

(f) A counterexample is Y_n. Y_n converges to 0 in probability, but

$$\mathbf{E}\left[(Y_n - 0)^2\right] = 0 \cdot \left(1 - \frac{1}{n}\right) + (n^2).\frac{1}{n} = n$$

Thus,

$$\lim_{n\to\infty} \mathbf{E}\left[(Y_n - 0)^2\right] = \infty,$$

and Y_n does not converge to 0 in the mean square.