(a) We have

$$\begin{aligned}
\mathbf{P}(Z \le z \mid X = x) &= \mathbf{P}(X + Y \le z \mid X = x) \\
&= \mathbf{P}(x + Y \le z \mid X = x) \\
&= \mathbf{P}(x + Y \le z) \\
&= \mathbf{P}(Y \le z - x),
\end{aligned}$$

where the third equality follows from the independence of X and Y. By differentiating both sides with respect to z, the result follows.

(b) We have, for $0 \le x \le z$,

$$f_{X|Z}(x \mid z) = \frac{f_{Z|X}(z \mid x) f_X(x)}{f_Z(z)} = \frac{f_Y(z - x) f_X(x)}{f_Z(z)} = \frac{\lambda e^{-\lambda(z-x)} \lambda e^{-\lambda x}}{f_Z(z)} = \frac{\lambda^2 e^{-\lambda z}}{f_Z(z)}.$$

Since this is the same for all x, it follows that the conditional distribution of X is uniform on the interval $[0, z]$, with PDF $f_{X|Z}(x \mid z) = 1/z$.