Let us now summarize what we have done with continuous random variables. This slide is essentially identical to our summary slide for discrete random variables with some differences that are marked in red. Instead of describing random variables by PMFs of all sorts, we started using PDFs.

Symbolically, p's get replaced by f's. But it is important to realize that this is not just a notation change. We are dealing with a similar but different concept.

Other than that, all the concepts and formulas from the discrete setting have continuous analogs, for example, expectation, the expected value rule, the variance, independence, and so on. Our basic tools, the multiplication rule, the total probability theorem, and the total expectation theorem all have continuous analogs in which PMFs get replaced by PDFs and sums by integrals. And we also introduced some examples of useful continuous random variables.

Rather than reviewing all formulas. it may actually be more useful to review what is new or different. I already mentioned the general fact that we replace sums by integrals, and PMFs by PDFs. More important is the fact that PDFs are not probabilities but probability densities. They give the rate at which probability accumulates in the vicinity of a point. If we wish to interpret them as probabilities, then we have to think of them as providing us with the probabilities of small intervals.

The reason for all this is that in a continuous model, individual points have zero probabilities. As a consequence, conditioning on an event of this form is tricky. But we did find a way to get around this difficulty by first defining a conditional PDF algebraically and then defining conditional probabilities by integrating the conditional PDF.

One new concept that we did introduce was the concept of the cumulative distribution function. We did not use it much. But it is a convenient way of describing probability distributions in general. And we will see some uses later in this course.

Finally, we concluded by developing different forms of the Bayes rule. There are four variations, because each one of the two random variables involved can be either discrete or continuous. These variations will be the foundation for our subsequent study of the subject of inference. And so, at this point, we have covered all of the major concepts and formulas that are needed to manipulate probabilities and expectations for all kinds of random variables. The rest of this course will be largely an application of the skills that we have developed.