

1. The number of remaining green fish at time n completely determines all the relevant information of the system's entire history (relevant to predicting the future state.) Therefore it is immediate that the number of green fish is the state of the system and the process has the Markov property:

$$\mathbf{P}(X_{m+1} = j \mid X_m = i, X_{m-1} = i_{m-1}, \dots, X_1 = i_1) = \mathbf{P}(X_{m+1} = j \mid X_m = i).$$

2. For $j > i$, clearly $p_{ij} = 0$, since a blue fish will never be painted green. For $0 \leq i, j \leq k$, we have the following:

$$p_{ij} = \begin{cases} \frac{n-i}{n}, & \text{if } j = i, \\ \frac{i}{n}, & \text{if } j = i - 1, \\ 0, & \text{otherwise.} \end{cases}$$

3. The state 0 is an absorbing state since there is a positive probability that the system will enter it, and once it does, it will remain there forever. Therefore the state with 0 green fish is the only recurrent state, and all other states are then transient.