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11. Exercise: Conditional variance definition

Exercise: Conditional variance definition

2/5 points (graded)

For each one of the following statements, indicate whether it is true or false.

(a) If X=Y (i.e., the two random variables always take the same values), then $\mathsf{Var}(X\,|\,Y)=0$.

False ▼ **X Answer:** True

(b) If X = Y (the two random variables always take the same values), then $Var(X \mid Y) = Var(X)$.

True ▼ **X** Answer: False

(c) If Y takes on the value y, then the random variable $\operatorname{\sf Var}(X \mid Y)$ takes the value

$$\mathbf{E}[(X - \mathbf{E}[X | Y = y])^2 | Y = y].$$

(d) If Y takes on the value y, then the random variable $\operatorname{\sf Var}(X \mid Y)$ takes the value

$$\mathbf{E}\big[(X-\mathbf{E}[X\,|\,Y])^2\,|\,Y=y.\,\big]$$

False ▼ **X Answer:** True

(e) If Y takes on the value y, then the random variable $\operatorname{\sf Var}(X\,|\,Y)$ takes the value

$$\mathbf{E}[(X - \mathbf{E}[X])^2 \mid Y = y.]$$

Solution:

- (a) Conditioned on Y, X is deterministic, and $\text{Var}(X \mid Y = y) = 0$. This implies that the random variable $\text{Var}(X \mid Y)$ is identically equal to zero. Thus, the statement is true.
- (b) False, because the previous statement is true.
- (c) This statement is just the definition of the numerical value of the conditional variance. We are in a universe where the event Y=y is known to have occurred, and every expectation is replaced by the corresponding conditional expectation.
- (d) The outer expectation places us in a universe where Y=y. Given this information, the value of the random variable $\mathbf{E}[X\,|\,Y]$ becomes a known quantity, equal to $\mathbf{E}[X\,|\,Y=y]$. Thus, this statement is equivalent to the preceding one and is true.
- (e) This is false, because all expectations should be conditional on the universe (Y=y) within which we are working. For a concrete counterexample, suppose that X is zero-mean and that Y=X. Then, as in part (a), $\text{Var}(X \mid Y=y)=0$. On the other hand, since $\mathbf{E}[X]=0$, we have

$$\mathbf{E}[(X - \mathbf{E}[X])^2 \mid Y = y] = \mathbf{E}[X^2 \mid Y = y] = \mathbf{E}[Y^2 \mid Y = y] = y^2.$$

提交

You have used 1 of 1 attempt

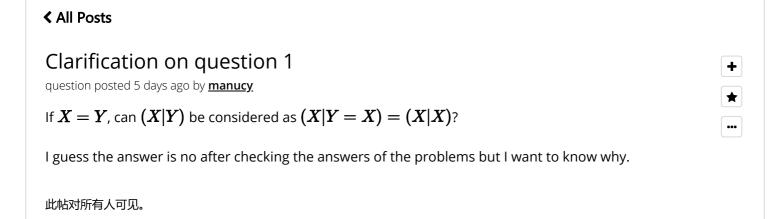
1 Answers are displayed within the problem



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