16. Level of a Statistical test

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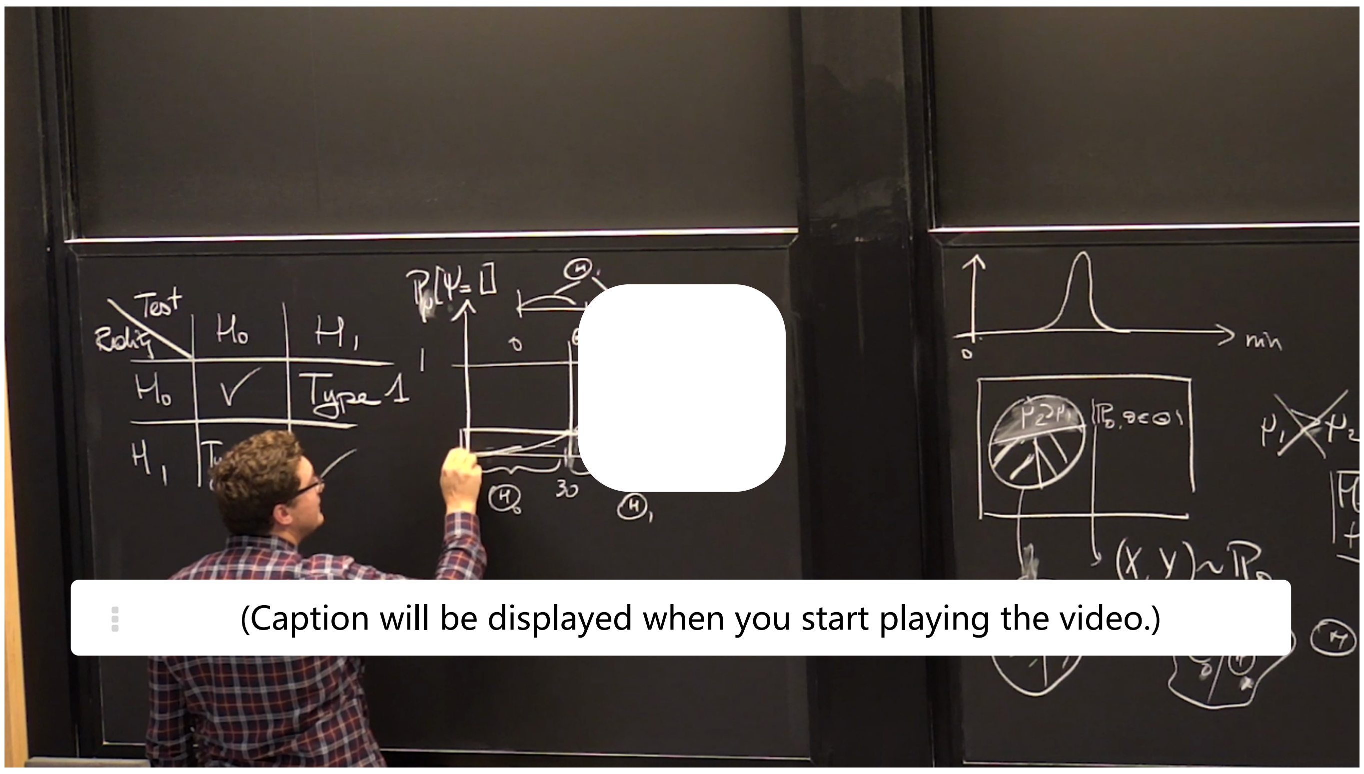

under the constraint that the type I error is at most 5%.

Do your best, under the constraint

that you send to jail at most 5% of innocent people.

That's the one that's important to me.

I want this number to be below 5%.

If that means that you're going to have

to let some guilty people walk free, that's the way it is.

Because I want to put a hard threshold on this 5%.

And then I'm going to ask you to do the best you can.

Now, you could say, oh, great--

that's what you want?

You just won this upper bound of 5%?

Great.

I'm going to be able to call it a day.

I'm going to basically send no one to jail,

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Testing the Support of a Uniform Variable: Level and Threshold

2/2 points (graded)

As in the problems on the previous page, let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathrm{Unif}[0, \theta]$ for an unknown parameter θ and we designed the statistical test

$$\psi_n = \mathbf{1}\left(\max_{1 \le i \le n} X_i > 1/2\right)$$

to decide between the null and alternative hypotheses

$$H_0 : \theta \ \le 1/2$$
$$H_1 : \theta \ > 1/2.$$

Let $\alpha_{\psi_n}(\theta)$ and $\beta_{\psi_n}(\theta)$ be the type 1 and type 2 errors respectively.

Recall from lecture that a test ψ has **level** α if

$$\alpha \ \ge \ \alpha_{\psi}(\theta) \qquad \text{for all } \theta \in \Theta_0,$$

where $\alpha_{\psi} = \mathbf{P}_{\theta}(\psi = 1)$ is the type 1 error. We will often use the word “level" to mean the “smallest" such level, i.e. the least upper bound of the type 1 error, defined as follows:

$$\alpha \ = \ \sup_{\theta \in \Theta_0} \alpha_{\psi}(\theta)$$

Here, $\sup_{\theta\in\Theta_0}$ stands for the supremum over all values of θ within Θ_0. If Θ_0 is a closed (*resp.* closed half-interval), and if $\alpha_\psi(\theta)$ is continuous (*resp.* continuous and decreasing as it approaches infinity), then its supremum equals the maximum.

Using the graph of the errors on the previous page, what is the smallest level α of the test ψ_n?

$\alpha =$ 0 ✔ Answer: 0

How should the threshold of the test be changed to increase the smallest level α? In other words, consider tests of the form

$$\psi_{n,C} = \mathbf{1}(\max_{1\leq i\leq n} X_i > C)$$

where C is the threshold. In the original test above, $C = 1/2$. What should the value of C be so that the level of $\psi_{n,C}$ is greater than the level of the $\psi_{n,1/2}$?
(Think of how the graph of $\mathbf{P}_\theta(\psi_C)$ changes with the threshold C.)

- ○ $C > 1/2$
- ◉ $C < 1/2$ ✔

Solution:

Since the type 1 error $\alpha_{\psi_n}(\theta)$ is constantly zero over Θ_0, the smallest level of this test ψ is $\alpha = 0$.

To increase the smallest level α from 0, note that $\mathbf{P}_\theta\left(\max_{1\leq i\leq n} X_i > C\right) = 0$ if and only if $\theta \leq C$. This means the constant zero region of graph of $\mathbf{P}_\theta(\psi_C) = 0$ shifts to the right as C increases from $1/2$, and to the left as C decreases from $1/2$. Since the maximum of type 1 error occurs at the boundary $\theta = 1/2$, this means $C < 1/2$ is required for the level to be positive.
Remark: The reason behind increasing the level in this example is to increase the power of the test from 0. In general, one of the first requirements of a test is to have a small-enough level so that the probability of concluding a false positive, (i.e. rejecting the null while the null is true) is controlled.

提交 你已经尝试了2次（总共可以尝试3次）

Answers are displayed within the problem

Testing the Support of a Uniform Variable: Determine the Threshold

0/1 point (graded)
As above, let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathrm{Unif}[0,\theta]$ for an unknown parameter θ and consider tests of the form

$$\psi_{n,C} = \mathbf{1}(\max_{1\leq i\leq n} X_i > C)$$

to decide between the null and alternative hypotheses

$$H_0 : \theta \leq 1/2$$
$$H_1 : \theta > 1/2.$$

Let $\alpha_{\psi_{n,C}}(\theta)$ and $\beta_{\psi_{n,C}}(\theta)$ be the type 1 and type 2 errors respectively.

Determine the smallest threshold C such that the test $\psi_{n,C}$ has level α.

$C =$ alpha/3 ✖ Answer: 1/2*(1-alpha)^(1/n)

$\frac{\alpha}{3}$

Solution:

Following similar computation as in a previous problem where $C = 1/2$, we have $\mathbf{P}_\theta\left(\psi_{n,C}=1\right) = 1-\left(\frac{C}{\theta}\right)^n$. Since the smallest level is

C/theta，是在一次实验中，结果小于C，也就是没有拒绝H0的可能性。
(C/theta)^n，是在n次试验中，结果小于C，也就是没有拒绝H0的可能性。
1-(C/theta)^n，是在n次试验中，存在结果大于C(因此拒绝H0，因为是上确界，所以只要大于C，我们就认为犯了1类错误，其实也可能大于theta)，也就是犯1类错误的概率。

$$\begin{aligned}\alpha &= \max_{\theta\in\Theta_0} p_\theta\left(\psi_{n,C}=1\right)\\ &= p_{1/2}\left(\psi_{n,C}=1\right) = 1-\left(\frac{C}{1/2}\right)^n,\end{aligned}$$

a test with threshold $C=\frac{1}{2}\sqrt[n]{1-\alpha}$ or smaller will have level α.

Remark: Notice the threshold C depends on n, α, as well as the value of θ at the boundary of Θ_0 and Θ_1.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 2 Foundation of Inference:Lecture 6: Introduction to Hypothesis Testing, and Type 1 and Type 2 Errors / 16. Level of a Statistical test

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