6. Tossing a triple of coins

Problem 6. Tossing a triple of coins

6/8 points (graded)

We have a red coin, for which $\mathbf{P}(\text{Heads}) = 0.4$, a green coin, for which $\mathbf{P}(\text{Heads}) = 0.5$, and a yellow coin, for which $\mathbf{P}(\text{Heads}) = 0.6$. The flips of the same or of different coins are independent. For each of the following situations, determine whether the random variable N can be approximated by a normal.

If yes, enter the mean and variance of N. If not, enter 0 in both of the corresponding answer boxes.

1. Let N be the number of Heads in 300 tosses of the red coin.

   mean: 120 ✔ **Answer:** 120

   Variance: 72 ✔ **Answer:** 72

2. Let N be the number of Heads in 300 tosses. At each toss, one of the three coins is selected at random (either choice is equally likely), and independently from everything else.

   mean: 150 ✔ **Answer:** 150

   variance: 22500 ✖ **Answer:** 75

3. Let N be the number of Heads in 100 tosses of the red coin, followed by 100 tosses of the green coin, followed by 100 tosses of the yellow coin (for a total of 300 tosses).

   mean: 150 ✔ **Answer:** 150

   variance: 7300 ✖ **Answer:** 73

4. We select one of the three coins at random: each coin is equally likely to be selected. We then toss the selected coin 300 times, independently, and let N be the number of Heads.

   mean: 0 ✔ **Answer:** 0

   variance: 0 ✔ **Answer:** 0

Solution:

For each of the following parts let X_i be a random variable that takes value 1 if the ith toss is Heads and takes value 0 otherwise.

1. $N = \sum_{i=1}^{300} X_i$. The CLT applies and N can be approximated by a normal because the X_i are independent and identically distributed Bernoulli random variables with parameter 0.4. Here, $\mathbf{E}[N] = 300 \cdot 0.4 = 120$ and $\text{Var}(N) = 300 \cdot 0.4 \cdot (1 - 0.4) = 72$.

2. $N = \sum_{i=1}^{300} X_i$. The CLT applies and N can be approximated by a normal because the X_i are independent and identically distributed Bernoulli random variables with parameter $\boxed{0.5}$. Here $\mathbf{E}[N] = 300 \cdot 0.5 = 150$ and $\text{Var}(N) = 300 \cdot 0.5 \cdot (1 - 0.5) = 75$. 猜测的解释是：在所有硬币中不断地摸，多次后正面的概率是0.5

3.

Let $Y_1 = \sum_{i=1}^{100} X_i$, $Y_2 = \sum_{i=101}^{200} X_i$, and $Y_3 = \sum_{i=201}^{300} X_i$, such that $N = Y_1 + Y_2 + Y_3$. The CLT applies and Y_1 can be approximated by a normal because the X_i for $i = 1, \ldots, 50$ are independent and identically distributed Bernoulli random variables with parameter 0.4. Using a similar argument, Y_2, and Y_3 can also be approximated by normal random variables. Since Y_1,Y_2, and Y_3 are all independent, we conclude that N can also be approximated by a normal. Here,

$$\mathbf{E}[N] = \mathbf{E}[Y_1] + \mathbf{E}[Y_3] + \mathbf{E}[Y_3] = 100 \cdot 0.4 + 100 \cdot 0.5 + 100 \cdot 0.6 = 150.$$

Similarly,

$$\begin{aligned} \mathsf{Var}(N) &= \mathsf{Var}(Y_1) + \mathsf{Var}(Y_2) + \mathsf{Var}(Y_3) \\ &= 100 \cdot 0.4 \cdot (1 - 0.4) + 100 \cdot 0.5 \cdot (1 - 0.5) + 100 \cdot 0.6 \cdot (1 - 0.6) \\ &= 73. \end{aligned}$$

4. The CLT does not apply in this case as N is approximately a mixture of three normals.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 8 / Problem Set / 6. Tossing a triple of coins