3. Independence: True or false

Problem 2. Independence: True or false

4/4 points (graded)

Determine whether each of the following statements about events A, B, C is always true or not.

1. Suppose that A, B, and C are independent events; then A^c and $B \cup C^c$ are independent.

- (●) True ✔
- ( ) False

2. From now on, we do not assume that A, B, C are independent. Suppose that A is independent of B, given C.

(a) A^c is independent of B, given C.

- (●) True ✔
- ( ) False

(b) $A \cap C$ is independent of $B \cap C$, given C.

- (●) True ✔
- ( ) False

(c) A is independent of B, given C^c.

- ( ) True
- (●) False ✔

Solution:

1. True. This follows from the intuitive meaning of event independence: when A, B, and C are independent, the occurrence or non-occurrence of some of these events does not provide any information on the occurrence or non-occurrence of the remaining ones.

   A formal proof is also possible but is somewhat tedious.

2. (a) True. We know that if A and B are independent, then A^c and B are independent. We now apply this fact to the conditional universe, and obtain the validity of the statement in question.

   (b) True. Within the conditional universe where C is known to have occurred, the events $A \cap C$ and $B \cap C$ coincide with the events A and B, respectively. The truth of the statement follows because we assumed that A and B are independent given C.

   (c) False. Let X and Y be independent binary random variables and let $A = \{X = 1\}$, $B = \{Y = 1\}$, $C = A \cap B = \{X = Y = 1\}$. Given C, the random variables X and Y are deterministic, hence trivially independent. On the other hand, given $C^c = A^c \cup B^c = \{X = 0\} \cup \{Y = 0\}$, the event $X = 0$ allows for both $Y = 1$ or $Y = 0$ to be true, whereas the event $X = 1$ conveys the information that $Y = 0$ must be true. In particular, X and Y are not independent given C. Equivalently, A and B are not independent given C.

提交 You have used 1 of 1 attempt

Answers are displayed within the problem

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Topic: Exam 1 / 3. Independence: True or false

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