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Problem 7. Charlie joins a reading club

6.66666666667/10.0 points (graded)

Charlie joins a new reading club, from which he receives books to read. Suppose that books arrive as a Poisson process at a rate λ of books per week. For each book, the time it takes for Charlie to read it is exponentially distributed with parameter μ; i.e., on average it takes Charlie $1/\mu$ weeks to finish one book. Assume that the reading times for different books are independent, and also independent from the book arrival process.

The problem with Charlie is that he is easily distracted. If he is reading a book when a new book arrives, he immediately turns to read the new one, and only comes back to the older book when he finishes the new book.

For all of the parts below, give your answers in terms of μ and λ, using standard notation. Enter μ as "mu" and λ as "lambda".

Hint: When Charlie starts reading a book, the total time he spends on reading it can be viewed as the first arrival from a Poisson process of rate μ, and you can then think about merging or splitting of Poisson processes.

1. When Charlie starts a new book, what is the probability that he can finish this book without being interrupted by a new book?

   mu/(mu+lambda) ✔ **Answer:** mu/(mu+lambda)

   $\frac{\mu}{\mu+\lambda}$

2. Given that Charlie receives a new book while reading a book, what is the probability that he can finish both books, the new one and the interrupted one, without further interruption?

   (mu/(mu+lambda))^2 ✔ **Answer:** (mu/(mu+lambda))^2

   $\left(\frac{\mu}{\mu+\lambda}\right)^2$

3. What is the expected reading time of a book given that it is not interrupted? **Hint: The answer is not $1/\mu$.**

   1/lambda ✘ **Answer:** 1/(mu+lambda)

   一个lambda+mu的泊松过程，given我们知道第一个arrival是读完书

   $\frac{1}{\lambda}$

STANDARD NOTATION

Solution:

1. When Charlie starts a book, we can imagine the future as two merged Poisson processes, one with parameter μ and another with λ. We care about the first arrival from this merged process. It will have probability $\mu/(\mu+\lambda)$ to come from the "book finish" process, in which case Charlie will have finished reading without interruption. The answer is then $\mu/(\mu+\lambda)$.

2. Since Charlie's reading is exponentially distributed, after he gets interrupted his old book is a "fresh start". So now in the merged process the first and second arrivals must be from the "book finish" process, the first arrival denoting completion of the second book and the second arrival denoting completion of the first book. These two events are independent, and each one has probability $\mu/(\mu+\lambda)$, so the event of interest happens with probability $(\mu/(\mu+\lambda))^2$.

3. Now we consider the merged process of "book finish" and "book arrival". This is a Poisson process with parameter $\mu+\lambda$. If a book has finished reading before the next book has arrived, then it will be the first arrival in this process. Therefore, we can look at the expected time of the first arrival, which will be $\frac{1}{\mu+\lambda}$.

   A more detailed solution is as follows.

Let T be exponential with parameter μ, which represents the time it would take to read the first book. Let T' be the time until the arrival of a new book. We are interested in $\mathbf{E}[T|T < T']$, which is the same as $\mathbf{E}[\min\{T, T'\}|T < T']$.

We now view T and T' as the first arrival times in two independent Poisson processes with rates μ and λ, respectively, so that $\min\{T, T'\}$ is the first arrival time in the merged process. We also know that the time of the first arrival of a merged process (i.e., $\min\{T, T'\}$) is independent of the origin of the arrival (i.e., the event $\{T < T'\}$). Hence,

$$\mathbf{E}[T|T < T'] = \mathbf{E}[\min\{T, T'\}|T < T'] = \mathbf{E}[\min\{T, T'\}] = \frac{1}{\lambda + \mu}$$

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

SergK (Community TA)
4 days ago

显示讨论

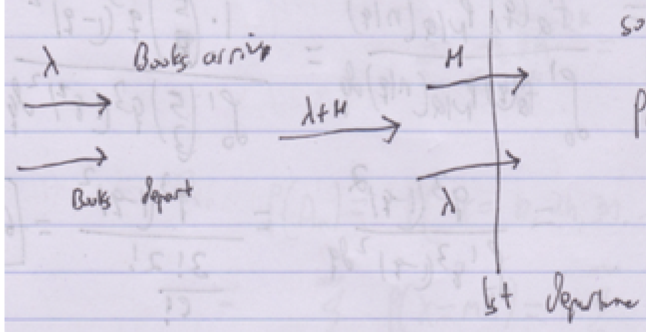
Suppose you have 2 lightbulbs; the burning out times are $T_1 \sim Exp(\lambda)$ for the bulb A and $T_2 \sim Exp(\mu)$ for the bulb B. You are watching the bulbs until one of them burns out; the burning out time is $min(T_A, T_B) \sim Exp(\lambda + \mu)$.

Now suppose the first bulb burned out in small time interval $[t, t + \delta]$. It could happen in 2 ways:

1. Bulb A burned out and bulb B did not, $P_1 = \lambda e^{-\lambda t}\delta \cdot e^{-\mu t}$
2. Bulb B burned out and bulb A did not, $P_2 = \mu e^{-\mu t}\delta \cdot e^{-\lambda t}$

The chances that the burned bulb is bulb A are $\frac{P_1}{P_1+P_2} = \frac{\lambda}{\lambda+\mu}$. The chances do not depend on t, so knowing t does not provide you any information about which bulb burned.

Problem 7 1. Books arrive λ poisson
Books time to read μ exponential
so departures can be modeled as poisson



Prob(finish without interruption)

$= \text{Prob}(\text{1st arrival being } \mu)$

$= \boxed{\frac{\mu}{\mu+\lambda}}$

Also $\int_0^\infty dt\, \mu e^{-\mu t} \cdot e^{-\lambda t} = \frac{\mu}{\mu+\lambda}$

probability of finish at time t in δt — Prob(arrival $> t$)

2. To read 2 books without interruption
read 1st book without interruption

$\left(\frac{\mu}{\mu+\lambda}\right) \cdot$ read remaining book without interruption $= \left(\frac{\mu}{\mu+\lambda}\right)^2$

Also $Y_2 = \mu^2 t e^{-\mu t}$

$$\int_0^\infty \mu^2 t e^{-\mu t} e^{-\lambda t} dt = \mu^2 \int_0^\infty t e^{-(\mu+\lambda)t} dt$$

$$= \mu^2 \left[-\frac{t e^{-(\mu+\lambda)t}}{(\mu+\lambda)} - \frac{e^{-(\mu+\lambda)t}}{(\mu+\lambda)^2} \right]_0^\infty$$

$$= \frac{\mu^2}{(\mu+\lambda)^2}$$

3. $E[\text{Reading} \mid \text{not interrupted}]$

$$= \frac{\int_0^\infty t \mu e^{-\mu t} e^{-\lambda t} dt}{\int_0^\infty \mu e^{-\mu t} e^{-\lambda t} dt} = \frac{\frac{\mu}{(\mu+\lambda)^2}}{\frac{\mu}{\mu+\lambda}} = \boxed{\frac{1}{\mu+\lambda}}$$