Let's now review the main ideas in this unit. We started looking at random processes, processes that evolve in time. And the simplest kind of random processes just involves arrivals over time. In the Bernoulli process, time is divided into slots.

And in each one of the slots, we may or may not have an arrival. And the key assumption that's being made here is that different time slots are independent of each other. We also introduced the Poisson process, which is a continuous time analog of the Bernoulli process. In the Poisson process, time is continuous, and arrivals may occur at any time.

However, the Poisson process can be thought of as a limiting case of the Bernoulli process, in which we take those time slots to be very small, of the order of delta, so that we have a large number of time slots. But during each one of those time slots, there's only a tiny probability of having an arrival. And the analogy is constructed so that the expected number of arrivals in the Bernoulli process, which is n times p, is equal to the expected number of arrivals in the Poisson process, which is lambda times tau.

Of course, lambda here, which is the probability of an arrival divided by the length of the slot, is thought of as an arrival rate. For both of these processes, we looked at various statistical properties. We are interested, for example, in the number of arrivals during a time interval. For the Poisson process, this random variable has a Poisson PMF, whereas for the Bernoulli process, we know that it has a binomial PMF.

Particularly interesting is the time until the first arrival, or more generally, the time between two consecutive arrivals. In the Poisson process, it has an exponential distribution. It's a continuous random variable. Whereas for the Bernoulli process, it has a geometric distribution.

And then the time of the k'th arrival is the sum of k interarrival times. And we discussed that consecutive interarrival times are independent random variables. So the time to the k'th arrival is the sum of k random variables drawn from this distribution. The sum of k exponentials has a so-called Erlang PDF of order k, whereas the sum of k geometrics has a PMF that goes under the name of a Pascal PMF of order k.

In both cases, we derived formulas for the corresponding PDF or PMF, and we did that by exploiting the independence properties, the independence assumption for the Bernoulli or the Poisson process. A few other things that we did in this unit were the following. We considered taking two separate independent arrival streams and merging them into a single stream.

We saw that when we merge Bernoulli processes, we get a new Bernoulli process. When we merge Poisson processes, we get a new Poisson process. And based on the parameters of the original process, we were able to calculate the parameters of the merged process. For the Poisson process, we also looked at the question, given that an arrival occurred, where did it come from?

And we found what the probability of that was. The probability that it comes from a particular stream is proportional to the arrival rate of that particular stream. And we also discussed that the origins of different arrivals are independent random variables. We also discussed splitting of a process-- that is, arrivals come, but we do not accept them all. We only keep some of them.

And for each arrival, we decide whether to keep it or not by flipping a coin. And all those coin flips are independent. They're independent with each other, and also independent of the process that we started with. So we saw that when we split a Bernoulli or Poisson process, what we get is, again, a Bernoulli or Poisson process. And all this discussion, all the properties that we derived, rested in an essential way on the independence assumption that we have been making.

Over and over, we use the fact that whatever happens in disjoint time intervals corresponds to independent random variables. In a loose sense, disjoint time intervals are independent of each other. Finally, our last discussion was about the phenomenon of random incidence. What's happened here is the somewhat counter-intuitive fact that if an observer shows up at a certain time and asks, how long is the interarrival time during which I arrived, what happens is that this interarrival time that gets observed tends to be larger than the usual interarrival times associated with this process.

And we provided an explanation for this phenomenon, and we argued that this is a phenomenon that happens in other contexts as well. It is not just unique to arrival processes like the ones we are discussing here.