1. The random variable K can be divided into 4 parts. First, the process will be guaranteed to move from state 1 to state 2 in exactly 1 transition. From state 2, it may self-transition for some random number of trials before eventually moving to state 3. The process the moves from state 3 to state 4 in exactly 1 trial. Finally, from state 4, there will be some random number of trials before the process moves back to state 1. Hence, we can express K as $K = 1 + K_2 + 1 + K_4$, where K_2 and K_4 are independent geometric random variables with parameters 2/3 and 3/5, respectively. Therefore, we have

$$\begin{aligned}
\mathbf{E}[K] &= 2 + 1/(2/3) + 1/(3/5) \\
&= 31/6, \\
\mathrm{var}(K) &= \frac{1-(2/3)}{(2/3)^2} + \frac{1-(3/5)}{(3/5)^2} \\
&= 67/36.
\end{aligned}$$

2. Let us define A to be the event of interest, and let X_n be the state of the process after the nth transition. Since 999 is a large number of transitions, we can approximate using the steady-state probabilities: $\mathbf{P}(X_{999} = i) \approx \pi_i$. By using the law of total probability, we have

$$\begin{aligned}
\mathbf{P}(A) &= \mathbf{P}(X_{999} \neq X_{1000} \neq X_{1001}) \\
&\approx \sum_{i=1}^{4} \mathbf{P}(A \mid X_{999} = i)\pi_i \\
&= 2/3\pi_1 + 2/3\pi_2 + 3/5\pi_3 + 3/5\pi_4 \\
&= 30/93 + 48/155 \approx 0.6323.
\end{aligned}$$