Gaussian Mixture Model: Definitions

1/1 point (graded)

Assume a Gaussian mixture model with K Gaussians such that we know all the means and variances. Assume that we also know the mixture weights $p_1, \ldots, p_K$. Let $\mathbf{x}$ be an observation obtained from the Gaussian mixture model. Let all of the parameters of the Gaussian mixture model be collectively represented as θ.

Which of the following are true?

- [x] We should be able to compute the probability density function (likelihood) $p\left(\mathbf{x}|\theta\right)$ given the information that we know. ✔
- [x] We should be able to compute the probability that $\mathbf{x}$ belongs to each Gaussian component $j = 1, \ldots, K$ given the information that we know. ✔

✔

Solution:

Both the statements are true. The generative Gaussian mixture model means that if we know all of the parameters of the K Gaussians and the mixture weights, the probability density function $p\left(\mathbf{x}|\theta\right)$ can be computed using the law of total probability as

$$p\left(\mathbf{x}|\theta\right) = \sum_{j=1}^{K} p_j \mathcal{N}\left(\mathbf{x}; \mu^{(j)}, \sigma_j^2\right).$$

The posterior probability that $\mathbf{x}$ belongs to a Gaussian component j can then be computed using Bayes rule.

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