## UNIT 6: Further topics on r.v.'s — Summary

- Derived distributions

- Covariance and correlation

- A deeper view of conditioning

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- Derived distributions:

$Y = g(X)$: find CDF of $Y$; can go directly when $g$ is monotonic

$Y = aX + b$: simple formula

$Z = g(X, Y)$: same method, using CDFs

$Z = X + Y$ $(X, Y$ independent): convolution formula and mechanics

$$p_Z(z) = \sum_x p_X(x)\, p_Y(z-x)$$

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\, dx$$

sum of independent normals is normal

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- Covariance and correlation

$$\text{cov}(X,Y) = \mathbf{E}\Big[(X - \mathbf{E}[X]) \cdot (Y - \mathbf{E}[Y])\Big] \qquad \rho(X,Y) = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y}$$

linearity properties $\qquad |\rho| \leq 1$

used to find $\text{var}(X_1 + \cdots + X_n)$

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- A deeper view of conditioning

  $\mathbf{E}[X \mid Y]$, $\mathrm{var}(X \mid Y)$ as random variables

  Law of iterated expectations: $\mathbf{E}\Big[\mathbf{E}[X \mid Y]\Big] = \mathbf{E}[X]$

  Law of total variance: $\mathrm{var}(X) = \mathbf{E}\Big[\mathrm{var}(X \mid Y)\Big] + \mathrm{var}\Big(\mathbf{E}[X \mid Y]\Big)$

- Sum of a random number of independent r.v.'s: $Y = X_1 + \cdots + X_N$

  $\mathbf{E}[Y] = \mathbf{E}[N] \cdot \mathbf{E}[X]$

  $\mathrm{var}(Y) = \mathbf{E}[N]\,\mathrm{var}(X) + (\mathbf{E}[X])^2\,\mathrm{var}(N)$