

Lecture 14: Wald's Test, Likelihood Ratio Test, and Implicit Hypothesis

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☐ 7. Wald's Test in 1 Dimension

7. Wald's Test in 1 Dimension

Comparing Quantiles

0/1 point (graded)

Let
$$Z \sim \mathcal{N}\left(0,1
ight)$$
 . Then $Z^2 \sim \chi_1^2$.

The **quantile** $q_{lpha}\left(\chi_{1}^{2}
ight)$ of the $\chi_{1}^{2}-$ distibution is the number such that

$$\mathbf{P}\left(Z^{2}>q_{lpha}\left(\chi_{1}^{2}
ight)
ight)=lpha.$$

Find the quantiles of the χ_1^2 distribution in terms of the quantiles of the normal distribution. That is, write $q_lpha\left(\chi_1^2
ight)$ in terms of $q_{lpha'}\left(\mathcal{N}\left(0,1
ight)
ight)$ where lpha' is a function of lpha.

(Enter **q(alpha)** for the quantile $q_{lpha}\left(\mathcal{N}\left(0,1
ight)
ight)$ of the normal distribution.)

$$q_{\alpha}\left(\chi_{1}^{2}\right)=$$
 (q(2*alpha))^2 \square Answer: (q(alpha/2))^2

STANDARD NOTATION

Solution:

Since $Z^2>q$ for any q>0 if and only if $|Z|>\sqrt{q}$, we have

$$P\left(Z^2>q_{lpha}\left(\chi_1^2
ight)
ight) \,=\, P\left(|Z|>\sqrt{q_{lpha}\left(\chi_1^2
ight)}
ight) \,=\, lpha.$$

Since
$$Z\sim\mathcal{N}\left(0,1
ight),\,P\left(\left|Z
ight|>\sqrt{q_{lpha}\left(\chi_{1}^{2}
ight)}
ight)\,=\,lpha\,$$
 if and only if

inherently two-side
$$\sqrt{q_{lpha}\left(\chi_{1}^{2}
ight)}=q_{lpha/2}\left(\mathcal{N}\left(0,1
ight)
ight)$$

Hence $q_{lpha}\left(\chi_{1}^{2}
ight)=q_{lpha/2}(\mathcal{N}\left(0,1
ight))^{2}.$

For example, for $\alpha=5\%$, using a table (e.g. https://people.richland.edu/james/lecture/m170/tbl-chi.html) or software (e.g. R), we have

$$egin{aligned} q_{0.05}\left(\chi_1^2
ight) &pprox 3.84. \ &\left(q_{0.025}\left(\mathcal{N}\left(0,1
ight)
ight)
ight)^2 &pprox \left(1.96
ight)^2 &pprox 3.84. \end{aligned}$$

提交

你已经尝试了3次(总共可以尝试3次)

Answers are displayed within the problem

Video note: In the video below at 5:27, Prof Rigollet misprinted on the board: the bottom inequality should read:

$$\sqrt{n}rac{\left|\hat{ heta}- heta
ight|}{\sigma}\,>\,q_{lpha/2}\left(\mathcal{N}\left(0,1
ight)
ight).$$

Wald's test

► Hence,

$$\underbrace{n\left(\hat{\theta}_{n}^{MLE}-\theta_{0}\right)^{\top}I(\hat{\theta}^{MLE})\left(\hat{\theta}_{n}^{MLE}-\theta_{0}\right)}_{n\to\infty} \xrightarrow[n\to\infty]{(d)} \underbrace{\chi}_{1}^{(d)}$$

► Wald's test with asyı

=(0,1):

 $_{\alpha}\},$

where q_{α} is the $(1-\alpha)$ -quantile of χ^2_d (see tables).

(Caption will be displayed when you start playing the video.)

So now I'm in good shape.

Because I'm going to want to reject when this thing, which

measures proximity from theta-hat to theta-0.

is large or small?

It's large, right?

This is really measuring how far theta-hat is from theta-star

in the right geometry.

视频

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字幕

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Wald's Test in 1 dimension

In 1 dimension, Wald's Test coincides with the two-sided test based on on the asymptotic normality of the MLE.

Given the hypotheses

$$H_0: heta^{ ext{MLE}}= heta_0$$

$$H_1: heta^{ ext{MLE}}
eq heta_0,$$

a two-sided test of level lpha, based on the asymptotic normality of the MLE, is

$$\psi_{lpha}=\mathbf{1}\left(\sqrt{nI\left(heta_{0}
ight)}\left|\hat{ heta}^{ ext{MLE}}- heta_{0}
ight|>q_{lpha/2}\left(\mathcal{N}\left(0,1
ight)
ight)
ight)$$

where the Fisher information $I\left(heta_{0}
ight)$ is the asymptotic variance of $\hat{ heta}^{ ext{MLE}}$ under the null hypothesis.

On the other hand, a Wald's test of level lpha is

$$egin{aligned} \psi_{lpha}^{ ext{Wald}} &=& \mathbf{1} \left(n I \left(heta_0
ight) \left(\hat{ heta}^{ ext{MLE}} - heta_0
ight)^2 \, > \, q_lpha \left(\chi_1^2
ight)
ight) \ &=& \mathbf{1} \left(\sqrt{n I \left(heta_0
ight)} \left| \hat{ heta}^{ ext{MLE}} - heta_0
ight| > \sqrt{q_lpha \left(\chi_1^2
ight)}
ight). \end{aligned}$$

Using the result from the problem above, we see that the two-sided test of level α is the same as Wald's test at level α .

讨论

显示讨论

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