11. Worked Example Part II

Note: The problems in this vertical depend on the final answer from Worked Example Part I. **You must have the answer to the final answerbox in order to answer the questions here.**

We now consider the **Gamma distribution** , which is a probability distribution with parameters $q > 0$ and $\lambda > 0$, has support on $(0, \infty)$, and whose density is given by

$$f(x) = \frac{\lambda^q x^{q-1} e^{-\lambda x}}{\Gamma(q)}.$$

Here, Γ is the Euler Gamma function.

Simplifying the Gamma Distribution

1/1 point (graded)

We will use proportionality notation in order to simplify the Gamma Distribution. But first, we perform a cosmetic change of variables to avoid repetitive notation with our answer in Part I: we write our parameters instead as λ_0 and q_0.

From the expression for the Gamma distribution given above, remove outermost multipliers to simplify it in such a way that our expression for $f(1)$ is $e^{-\lambda_0}$ regardless of the value of q_0.

Use **q_0** for q_0 and **lambda_0** for λ_0.

$f(x) \propto$

x^(q_0-1)*exp(-lambda_(✔ **Answer:** x^(q_0-1)*e^(-lambda_0*x)

$x^{q_0-1} \cdot \exp(-\lambda_0 \cdot x)$

Solution:

Note that we want a function of x, so we are able to pull out factors that do not depend on the variable x. (i.e. are purely constants or a factor whose value only depends on variables other than x). From $f(x) = \frac{\lambda^q x^{q-1} e^{-\lambda x}}{\Gamma(q)}$, we can notice that both λ^q in the numerator and $\Gamma(q)$ in the denominator are independent of x, so removing those reduces our expression to $x^{q-1} e^{\lambda x}$.

Making a slight tweak of variables so that we use λ_0 and q_0 instead, as specified, gives $f(x) \propto \boxed{x^{q_0-1} e^{\lambda x_0}}$, and it can be seen (as an exercise) that this expression for $f(x)$ satisfies $f(1) = e^{-\lambda_0}$.

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Answers are displayed within the problem

Interpreting the Posterior Distribution

3/3 points (graded)

Compare this with the posterior distribution you computed from Part I, which you should see is a Gamma distribution. What is the corresponding variable, and what are its parameters?

Use **SumXi** for $\sum_{i=1}^{n} X_i$.

$x =$

lambda

✔ **Answer:** lambda+a*0+n*0+SumXi*0

λ

$q_0 =$

SumXi+1

✔ **Answer:** SumXi+1+a*0+n*0+lambda*0

$SumXi + 1$

$\lambda_0 =$

a+n

✔ **Answer:** a+n+lambda*0+SumXi*0

$a + n$

STANDARD NOTATION

Solution:

In Part I, we derived the posterior distribution (as a function of x) to be

$$e^{-(a+n)\lambda} \lambda^{\sum_{i=1}^{n} X_i},$$

where our parameter of interest is λ. Here, it is the variable λ that is supposed distributed according to a Gamma distribution, hence we must write $\boxed{x = \lambda}$.

From here, we need to match the remaining variables. The exponent of x (vis. λ) in the general Gamma distribution is $q_0 - 1$ and in our posterior distribution is $\sum_{i=1}^{n} X_i$, so we could write $\boxed{q_0 = (\sum_{i=1}^{n} X_i) + 1}$. Similarly, λ_0 is what multiplies x in the exponent of e, which we see is $a + n$ in our posterior distribution, so $\boxed{\lambda_0 = a + n}$.

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Discussion

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