UNIT 4: Discrete random variables — Summary

- r.v.'s and PMFs: $p_X(x)$, $p_{X,Y}(x,y)$, $p_{X|Y}(x \mid y)$, $p_{X|A}(x)$
- Expectation: $\mathbf{E}[X]$, $\mathbf{E}[X \mid A]$, $\mathbf{E}[X \mid Y = y]$

  Expected value rule: $\mathbf{E}[g(X,Y)]$, $\mathbf{E}[g(X,Y) \mid A]$, $\mathbf{E}[g(X,Y) \mid Z = z]$

  Linearity: $\mathbf{E}[aX + bY] = a\mathbf{E}[Y] + b\mathbf{E}[Y]$
- Variance: $\mathrm{var}(X)$, $\mathrm{var}(X \mid A)$, $\mathrm{var}(X \mid Y = y)$ $\qquad \mathrm{var}(X) = \mathbf{E}[X^2] - \big(\mathbf{E}[X]\big)^2$
- Independence of r.v.'s: $p_{X,Y} = p_X \cdot p_Y$

  $\mathbf{E}[XY] = \mathbf{E}[X] \cdot \mathbf{E}[Y]$ $\qquad \mathrm{var}(X+Y) = \mathrm{var}(X) + \mathrm{var}(Y)$
- Multiplication rule $\qquad p_{X,Y,Z}(x,y,z) = p_Z(z)\, p_{Y \mid Z}(y \mid z)\, p_{X \mid Y,Z}(x \mid y,z)$
- Total probability theorem $\qquad p_X(x) = \sum_y p_Y(y)\, p_{X \mid Y}(x \mid y)$
- Total expectation theorem $\qquad \mathbf{E}[X] = \sum_y p_Y(y)\, \mathbf{E}[X \mid Y = y]$
- Examples: Bernoulli, indicators, uniform, binomial, geometric