6. Preparation for the Chi-Squared Test

A Vector Inner Product

1/1得分 (计入成绩)

Let $\mathbf{p}^0$ be the discrete pmf that we wish to test the goodness of fit for an observed sequence of iid samples. Let $\widehat{\mathbf{p}}$ be the MLE upon observing the iid samples.

What is $\sqrt{n}\left(\widehat{\mathbf{p}} - \mathbf{p}^0\right)^T \mathbf{1}$?

Note: This is a vector dot product where $\left(\widehat{\mathbf{p}} - \mathbf{p}^0\right)^T$ is a row vector and $\mathbf{1}$ is the all-ones column vector of appropriate size.

0

Answer: 0

STANDARD NOTATION

Solution:

Both $\widehat{\mathbf{p}}$ and $\mathbf{p}^0$ are pmfs. Let K be the number of modalitites.

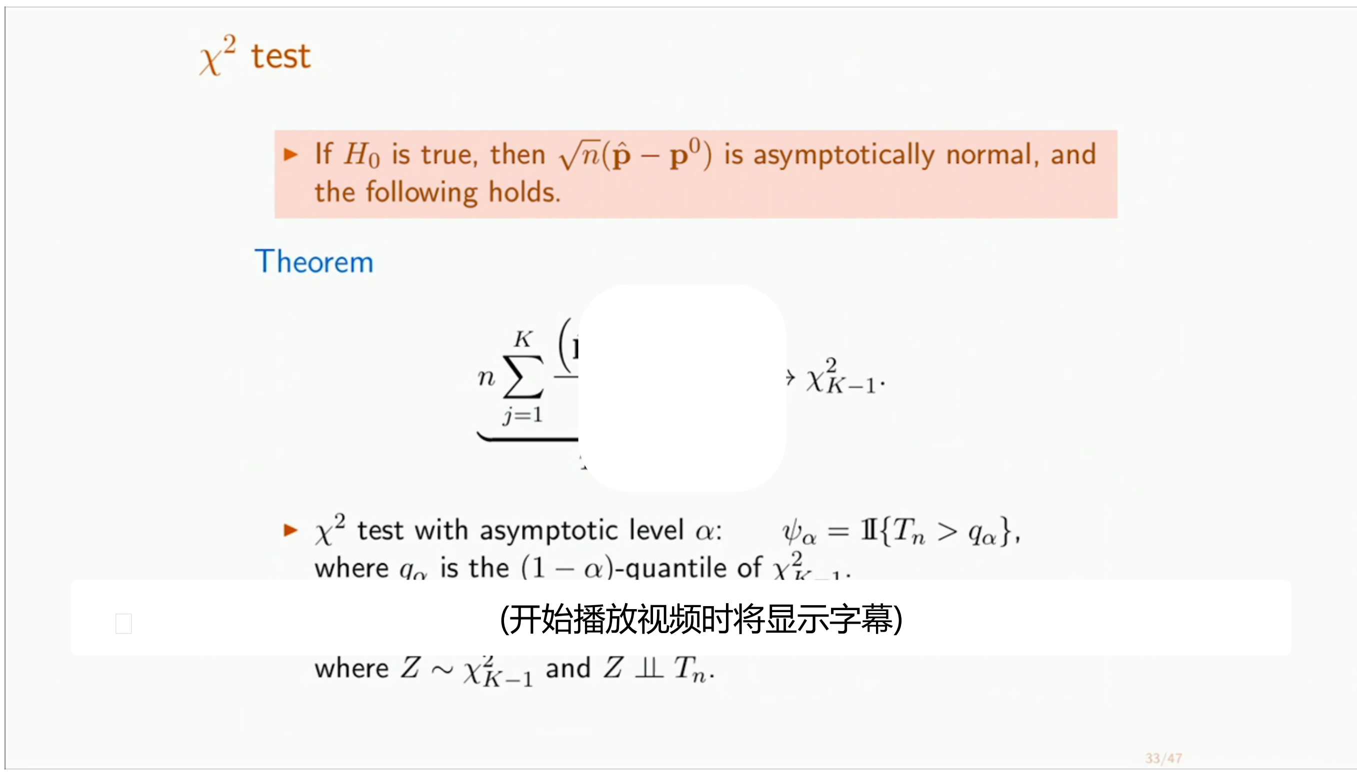
$$\sqrt{n}\left(\widehat{\mathbf{p}} - \mathbf{p}^0\right)^T \mathbf{1} = \sum_{i=1}^{K}\left(\hat{p}_i - p_i^0\right) = \sum_{i=1}^{K}\hat{p}_i - \sum_{i=1}^{K} p_i^0 = 0.$$

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

A Degenerate Gaussian Random Variable

字幕开始。跳转至结尾。



So if H0 is true, then if I think about the vector p

hat minus the vector p0, I multiply it by square root of n

and I claim that this is asymptotically normal.

An asymptotically Gaussian vector.

I was going to do it later.

Let's do it now for one second.

What happens if I take this vector--

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Degrees of Freedom of a Known Test

1/2得分 (计入成绩)

Let us consider a statistical model with parameter $\theta \in \mathbb{R}^d$. Let θ^* be the parameter that generates the n iid samples $\mathbf{X}_1, \ldots, \mathbf{X}_n$. Let $I(\theta)$ be the Fisher information and assume that the MLE $\hat{\theta}_n^{\text{MLE}}$ is asymptotically normal. Assume that $I(\theta^0)$ is a diagonal matrix with positive entries $1/t_1, \ldots, 1/t_d$. We wish to perform a test for the hypotheses $H_0 : \theta^* = \theta^0$ and $H_1 : \theta^* \neq \theta^0$.

Let the test statistic T_n be

$$T_n = n \sum_{i=1}^{d} \frac{\left(\theta_i^0 - \hat{\theta}_i\right)^2}{t_i},$$

where $\left[\hat{\theta}_1 \ \hat{\theta}_2 \ \cdots \ \hat{\theta}_d\right]^T = \hat{\theta}_n^{\text{MLE}}$.

What distribution does the test statistic T_n converge to under H_0 as $n \to \infty$?

Type **chi** for chi-squared distribution, **T** for Student's T distribution, **G** for standard Gaussian distribution.

$T_n \xrightarrow[n\to\infty]{(d)}$ chi

χ

Answer: chi + 0*G + 0*T

What is the number of degrees of freedom of the asymptotic distribution of T_n? If the answer is a standard normal, enter **1**.

d-1

$d - 1$

Answer: d

STANDARD NOTATION

Solution:

The test statistic T_n can be seen to be equivalent to

$$n\left(\hat{\theta}_n^{\text{MLE}} - \theta^0\right)^T I\left(\theta^0\right)\left(\hat{\theta}_n^{\text{MLE}} - \theta^0\right),$$

which is the test statistic for Wald's test. Therefore,

$$T_n \xrightarrow[n\to\infty]{(d)} \chi_d^2.$$

提交 你已经尝试了2次（总共可以尝试2次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 4 Hypothesis testing:Lecture 15: Goodness of Fit Test for Discrete Distributions / 6. Preparation for the Chi-Squared Test