7. Quiz: Composite Hypotheses for Bernoulli models

(a)

1/1 point (graded)

Let $X_1, \ldots, X_n$ be i.i.d. **Bernoulli** random variables with unknown parameter $p \in (0,1)$.

Find a function $T_{n,p}\left(\overline{X}_n\right)$, which depends on $\overline{X}_n, n,$ and p, such that

$$T_{n,p}\left(\overline{X}_n\right) \xrightarrow[n\to\infty]{\text{(d)}} \mathcal{N}(0,1),$$

by

- using the Central Limit Theorem on $\overline{X}_n$ and
- **substituting any occurrence of p in the variance by a plug-in estimator for p.**

Note: If $T_{n,p} \xrightarrow[n\to\infty]{\text{(d)}} \mathcal{N}(0,1)$, then so does $-T_{n,p}$. For this problem and the next part, use the expression for $T_{n,p}\left(\overline{X}_n\right)$ that is of the form $\left(\overline{X}_n - p\right) f\left(n, \overline{X}_n\right)$ where $f\left(n, \overline{X}_n\right)$ is always **positive**. (Or very loosely speaking, use $\left(\overline{X}_n - p\right)$ and not $\left(p - \overline{X}_n\right)$ where applicable.)

(Enter **barX_n** for $\overline{X}_n$).

$T_{n,p}\left(\overline{X}_n\right) =$ sqrt(n/(barX_n*(1-barX_n)))*(barX_n - p)

☐ **Answer:** sqrt(n) * (barX_n - p)/sqrt(barX_n*(1-barX_n))

STANDARD NOTATION

Solution:

By the Central Limit Theorem and plugging in the variance of a Bernoulli random variable,

$$\frac{\sqrt{n}}{\sqrt{p(1-p)}}\left(\overline{X}_n - p\right) \xrightarrow[n\to\infty]{\text{(D)}} \mathcal{N}(0,1).$$

By the Law of Large Numbers,

$$\overline{X}_n \xrightarrow[n\to\infty]{\mathbf{P}} p,$$

so by Slutsky's Theorem, we can replace $p(1-p)$ by $\overline{X}_n\left(1-\overline{X}_n\right)$ to obtain

$$\frac{\sqrt{n}}{\sqrt{\overline{X}_n\left(1-\overline{X}_n\right)}}\left(\overline{X}_n - p\right) \xrightarrow[n\to\infty]{\text{(D)}} \mathcal{N}(0,1).$$

Hence, the function we are looking for is

$$T_{n,p}\left(\overline{X}_n\right)=\frac{\sqrt{n}}{\sqrt{\overline{X}_n\left(1-\overline{X}_n\right)}}\left(\overline{X}_n-p\right)$$

or

$$T_{n,p}\left(\overline{X}_n\right)=\frac{\sqrt{n}}{\sqrt{\overline{X}_n\left(1-\overline{X}_n\right)}}\left(p-\overline{X}_n\right)$$

提交 你已经尝试了2次（总共可以尝试3次）

☐ Answers are displayed within the problem

(b)

3/3 points (graded)
(This is a quiz, hence only 1 attempt.)

Select a test with asymptotic level α, in terms of the function $T_{n,p}\left(\overline{X}_n\right)$, for each of the following pairs of hypotheses:
(Choose one for each column.)

$H_0: p=0.5 \quad \text{vs} \quad H_1: p\neq 0.5 \quad :$	$H_0: p\leq 0.5 \quad \text{vs} \quad H_1: p>0.5 \quad :$	$H_0: p\geq 0.5 \quad \text{vs} \quad H_1: p<0.5 \quad :$
● $\mathbf{1}\left(\lvert T_{n,0.5}\left(\overline{X}_n\right)\rvert>q_{\alpha/2}\right)$ ☐	○ $\mathbf{1}\left(\lvert T_{n,0.5}\left(\overline{X}_n\right)\rvert>q_{\alpha/2}\right)$	○ $\mathbf{1}\left(\lvert T_{n,0.5}\left(\overline{X}_n\right)\rvert>q_{\alpha/2}\right)$
○ $\mathbf{1}\left(\lvert T_{n,0.5}\left(\overline{X}_n\right)\rvert>q_{\alpha}\right)$	○ $\mathbf{1}\left(\lvert T_{n,0.5}\left(\overline{X}_n\right)\rvert>q_{\alpha}\right)$	○ $\mathbf{1}\left(\lvert T_{n,0.5}\left(\overline{X}_n\right)\rvert>q_{\alpha}\right)$
○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)>q_{\alpha/2}\right)$	○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)>q_{\alpha/2}\right)$	○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)>q_{\alpha/2}\right)$
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○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<-q_{\alpha/2}\right)$	○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<-q_{\alpha/2}\right)$	○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<-q_{\alpha/2}\right)$
○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<-q_{\alpha}\right)$	○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<-q_{\alpha}\right)$	● $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<-q_{\alpha}\right)$ ☐
○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<q_{\alpha/2}\right)$	○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<q_{\alpha/2}\right)$	○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<q_{\alpha/2}\right)$
○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<q_{\alpha}\right)$	○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<q_{\alpha}\right)$	○ $\mathbf{1}\left(T_{n,0.5}\left(\overline{X}_n\right)<q_{\alpha}\right)$

Solution:

1. By part (a), with

$$T\left(X_1,\ldots,X_n\right)=\frac{\sqrt{n}}{\sqrt{\overline{X}_n\left(1-\overline{X}_n\right)}}\left(\overline{X}_n-0.5\right),$$

we know that

$$\mathbf{P}_{0.5}\left(|T|-q>0\right) \xrightarrow[n\to\infty]{} 2\left(1-\Phi\left(q\right)\right),$$

so to achieve asymptotic level $\alpha = 0.05$, set

$$q = q_{\alpha/2} \approx 1.96,$$

which means

$$\psi = \mathbf{1}\left\{\left|\frac{\sqrt{n}}{\sqrt{\overline{X}_n\left(1-\overline{X}_n\right)}}\left(\overline{X}_n - 0.5\right)\right| - 1.96 > 0\right\}.$$

2. By part (a), with

$$T_{0.5}\left(X_1,\ldots,X_n\right) = \frac{\sqrt{n}}{\sqrt{\overline{X}_n\left(1-\overline{X}_n\right)}}\left(\overline{X}_n - 0.5\right),$$

we know that

$$\mathbf{P}_{0.5}\left(T-q>0\right) \xrightarrow[n\to\infty]{} 1-\Phi\left(q\right),$$

so to guarantee asymptotic confidence level $\alpha = 0.05$, we can set

$$q = q_{\alpha} \approx 1.65.$$

This gives us the required level for $p = 0.5$.

However, for $p < 0.5$, we have that

$$\overline{X}_n \xrightarrow[n\to\infty]{\mathbf{P}_p} p,$$

which entails that

$$\frac{\sqrt{n}}{\sqrt{\overline{X}_n\left(1-\overline{X}_n\right)}}\left(\overline{X}_n - 0.5\right) \xrightarrow[n\to\infty]{\mathbf{P}_p} -\infty,$$

hence in the limit, for $p < 0.5$,

$$\mathbf{P}_p\left(T-q>0\right) \to 0.$$

Overall, we get the desired test by setting

$$\psi = \mathbf{1}\left\{\frac{\sqrt{n}}{\sqrt{\overline{X}_n\left(1-\overline{X}_n\right)}}\left(\overline{X}_n - 0.5\right) - 1.65 > 0\right\}.$$

3. This is exactly analogous to the part above.

提交 你已经尝试了1次（总共可以尝试1次）

☐ Answers are displayed within the problem

讨论

显示讨论

主题： Unit 2 Foundation of Inference:Homework 3: Introduction to Hypothesis Testing / 7. Quiz: Composite Hypotheses for Bernoulli models

认证证书是什么?