4. Parking lot problem

Problem 4. Parking lot problem

3.0/3.0 points (graded)

Mary and Tom park their cars in an empty parking lot with $n \geq 2$ consecutive parking spaces (i.e, n spaces in a row, where only one car fits in each space). Mary and Tom pick parking spaces at random; of course, they must each choose a different space. (All pairs of distinct parking spaces are equally likely.) What is the probability that there is at most one empty parking space between them?

Your answer should be a function of n, entered using standard notation (also available through the “STANDARD NOTATION" button just above the “Submit" button.)

(4*n-6)/(n*(n-1))

Answer: (4*n-6)/(n*(n-1))

$$\frac{4 \cdot n - 6}{n \cdot (n-1)}$$

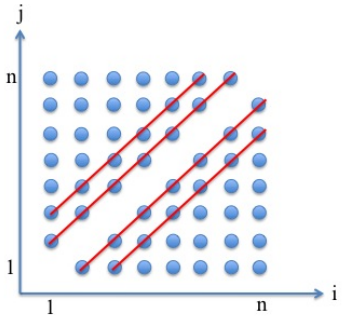
STANDARD NOTATION

Solution:

The sample space is $\Omega = \{(i,j) : i \neq j, 1 \leq i, j \leq n\}$, where outcome (i,j) indicates that Mary and Tom parked in slots i and j, respectively. We apply the discrete uniform probability law to find the required probability. We are interested in the probability of the event

$$A = \{(i,j) \in \Omega : |i-j| \leq 2\}.$$

We first find the cardinality of Ω. There are n^2 pairs (i,j), but since the set Ω excludes outcomes of the form (i,i), the cardinality of Ω is $n^2 - n = n(n-1)$.



If $n \geq 3$, event A consists of the four lines indicated in the figure above and contains $2(n-1)+2(n-2)=4n-6$ elements. If $n=2$, event A contains exactly 2 elements, namely, $(1,2)$ and $(2,1)$, which agrees with the formula $4(2)-6=2$. Therefore,

$$\mathbf{P}(A)=\frac{4n-6}{n(n-1)}.$$

提交 You have used 3 of 7 attempts

Answers are displayed within the problem

讨论

显示讨论

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