

10. Exercise: CLT for the binomial

Exercise: CLT for the binomial

3/3 points (graded)

Let X be binomial with parameters $n = 49$ and $p = 1/10$.

The mean of X is:

✓ Answer: 4.9

The standard deviation of X is:

✓ Answer: 2.1

The CLT, together with the $1/2$ -correction, suggests that

$\mathbf{P}(X = 6) \approx$

✓ Answer: 0.1623

You may want to refer to the [normal table](#).

Normal Table

Show

Note: In this case, the CLT may not provide a great approximation. The range of values that X is likely to take is quite narrow, so that its PMF consists of only a few entries of substantial size. But, regardless, we can still calculate what the CLT suggests.

Solution:

We have $\mathbf{E}[X] = np = 4.9$, and

$$\text{Var}(X) = np(1-p) = 49 \cdot \frac{1}{10} \cdot \frac{9}{10} = \frac{49 \cdot 9}{10^2},$$

so that the standard deviation of X is $21/10 = 2.1$.

The standardized version of X is $(X - 4.9)/2.1$. Thus,

$$\begin{aligned} \mathbf{P}(X = 6) &= \mathbf{P}(5.5 < X < 6.5) = \mathbf{P}\left(\frac{5.5 - 4.9}{2.1} \leq \frac{X - 4.9}{2.1} \leq \frac{6.5 - 4.9}{2.1}\right) \\ &\approx \Phi(0.76) - \Phi(0.29) \approx 0.7764 - 0.6141 = 0.1623. \end{aligned}$$

For comparison, the answer calculated by using the binomial PMF directly is

$$\mathbf{P}(X = 6) = \binom{49}{6} (0.1)^6 (0.9)^{49-6} \approx 0.1507.$$

提交

You have used 3 of 3 attempts

❗ Answers are displayed within the problem

讨论

隐藏讨论