6. True or False

Problem 6. True or False

7/8 points (graded)

For each of the following statements, state whether it is true (meaning, always true) or false (meaning, not always true):

1. Let X and Y be two binomial random variables.

   (a) If X and Y are independent, then $X+Y$ is also a binomial random variable.

   False ▼ ✔ **Answer:** False

   (b) If X and Y have the same parameters, n and p, then $X+Y$ is a binomial random variable.

   False ▼ ✔ **Answer:** False

   (c) If X and Y have the same parameter p, and are independent, then $X+Y$ is a binomial random variable.

   False ▼ ✖ **Answer:** True

2. Suppose that, $\mathbf{E}[X]=0$. Then, $X=0$.

   False ▼ ✔ **Answer:** False

3. Suppose that, $\mathbf{E}[X^2]=0$. Then, $\mathbf{P}(X=0)=1$.

   True ▼ ✔ **Answer:** True

4. Let X be a random variable. Then, $\mathbf{E}[X^2]\geq\mathbf{E}[X]$.

   False ▼ ✔ **Answer:** False

5. Suppose that, X is a random variable, taking positive integer values, which satisfies $\mathbf{E}[(X-6)^2]=0$. Then, $p_X(4)=p_X(5)$.

   True ▼ ✔ **Answer:** True

6. Suppose that $\mathbf{E}[X] \geq 0$. Then, $X \geq 0$ with probability 1, i.e., $\mathbf{P}(X \geq 0) = 1$.

False ▼ ✔ **Answer:** False

Solution:

1. (a) False. Intuitively, X corresponds to independent coin flips of a coin with a certain bias, and Y corresponds to independent coin flips of another coin, which need not have the same bias as the first coin. Throughout the overall sequence of coin flips, the bias is not kept constant, and so we are in a different situation from the one modeled by binomial random variables.

   For a concrete (and extreme) counter-example, suppose that X and Y are independent Bernoulli random variables, with parameters 0.9 and 0.1, respectively. In particular, they are both binomial with $n = 1$. The sum $X + Y$ takes values in $\{0, 1, 2\}$. So, if it were binomial, it would need to have a parameter n equal to 2. The parameter p of such a binomial would have to satisfy $\mathbf{E}[X + Y] = 2p$. Since $\mathbf{E}[X + Y] = \mathbf{E}[X] + \mathbf{E}[Y] = 0.9 + 0.1 = 1$, we would require $p = 1/2$. This would then imply that $\mathbf{P}(X + Y = 2) = p^2 = 1/4$. However, we can check that

$$\mathbf{P}(X + Y = 2) = \mathbf{P}(X = 1) \cdot \mathbf{P}(Y = 1) = 0.9 \cdot 0.1 \neq 1/4.$$

   The contradiction shows that $X + Y$ is not binomial.

   (b) False. If X and Y have the same parameters, n and p, $X + Y$ is not necessarily a binomial random variable. For example, if the random variables X and Y are dependent and $X = Y$, then the random variable $X + Y$ has zero probability at all odd values of n. Therefore, $X + Y$ is not binomial.

   (c) True. We may interpret $X + Y$ as the number, X, of Heads in some independent tosses of a coin, plus the number, Y, of Heads in some additional independent tosses of the **same** coin. Therefore, $X + Y$ is binomial.

2. False. Consider a random variable with

$$p_X(x) = \begin{cases} 1/2, & \text{if } x = 1, \\ 1/2, & \text{if } x = -1. \end{cases}$$

   We have $\mathbf{E}[X] = 0$, but X takes nonzero values.

3. True. Suppose that X satisfies $\mathbf{E}[X^2] = 0$ but $\mathbf{P}(X = 0) \neq 1$. Then, $\mathbf{P}(X = w) > 0$ for some $w \neq 0$. It would follow that $\mathbf{E}[X^2] \geq w^2 \cdot \mathbf{P}(X = w) > 0$, which would contradict the assumption that $\mathbf{E}[X^2] = 0$.

4. False. Let X be deterministic and equal $1/2$. Then $\mathbf{E}[X^2] = 1/4$, while $\mathbf{E}[X] = 1/2 > \mathbf{E}[X^2]$.

5. True. Since, $\mathbf{E}[(X-6)^2]=0$, and since $(X-6)^2 \geq 0$, we obtain that $(X-6)^2$ must be equal to 0, with probability 1, namely, $p_X(6)=1$. Hence, $p_X(4)=0=p_X(5)$.

6. False. Suppose X is 1 or -1, with equal probability. Then $\mathbf{E}[X]=0$, but $\mathbf{P}(X \geq 0)=1/2 \neq 1$.

提交 You have used 1 of 1 attempt

Answers are displayed within the problem

讨论

显示讨论

Topic: Unit 4 / Problem Set / 6. True or False

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