9. Student's T Distribution
Student's T Distribution: Definition

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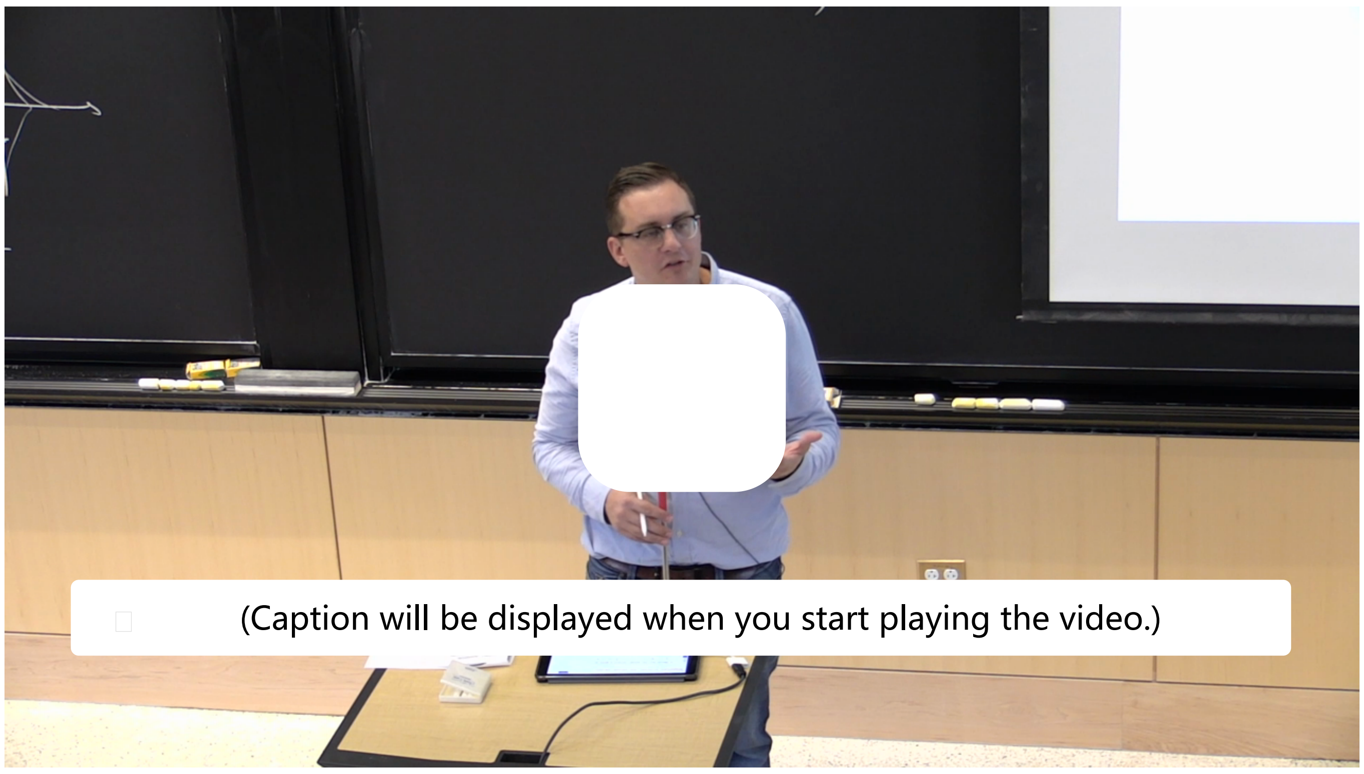

OK, so now we know that the sample variance is this.

We don't have only the sample variance to take into account.

Remember, we had Xn bar minus mu divided by square root

of the sample variance.

Well, Xn bar minus mu divided by sigma is the standard Gaussian.

The sample variance-- sorry, V is not the sample variance

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Comparing Chi-Squared and Student's T Distribution

2/2 points (graded)

Consider the distribution χ^2_n (χ-squared with n degrees of freedom). Let $f_n : \mathbb{R} \to \mathbb{R}$ denote the pdf of χ^2_n, and let A_n denote the maximizer of f_n (*i.e.*, the peak of the pdf of the distribution χ^2_n is located at A_n).

What is $\lim_{n\to\infty} A_n$? (Answer heuristically, based on discussions in the lecture video about how the shape of the chi-squared distribution evolves with n.)

- ○ 0
- ○ 1
- ◉ ∞
- ○ None of the above

Consider the **Student's T Distribution** , which is defined to be the distribution of

$$T_n := \frac{Z}{\sqrt{V/n}}$$

where $Z \sim \mathcal{N}(0,1)$, $V \sim \chi^2_n$, and Z and V are independent. Let g_n denote the pdf of T_n, and let B_n denote the maximizer of g_n (*i.e.*, the peak of the pdf of the distribution T_n is located at B_n).

What is $\lim_{n\to\infty} B_n$? (*Hint:* What is the limit (in probability) of V/n?)

- [x] 0
- [ ] 1
- [ ] ∞
- [ ] None of the above

Solution:

The graph of the pdf of χ^2_n in the slides shows that the peak of the distribution moves to the right as $n \to \infty$. Hence

$$\lim_{n\to\infty} A_n = \infty.$$

This is intuitive since we showed in a previous problem that $\mathbb{E}[X] = n$ if $X \sim \chi^2_n$.

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As $n \to \infty$, the random variable V/n converges to 1 in probability. Hence, as $n \to \infty$,

$$\boxed{T_n \xrightarrow[n\to\infty]{(d)} \mathcal{N}(0,1).}$$

Since the distribution $\mathcal{N}(0,1)$ is peaked at the origin, this implies

$$\lim_{n\to\infty} B_n = 0.$$

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 4 Hypothesis testing:Lecture 13: Chi Squared Distribution, T-Test / 9. Student's T Distribution