14. Review Exercises

Transformations of Random Variables

2/2 points (graded)

Consider a random variable Y with distribution $p_\theta\left(y\right)$ for some θ, coming from a canonical exponential family.

Let $Z = Y + a$, where a is a constant. Denote by $q_\theta\left(z\right)$ the density of $Z,$ which is parametrized by θ.

Is q_θ also a member of some canonical exponential family?

● Yes ✓

○ No

Now instead suppose $Z = \lambda Y$, where $\lambda \neq 0$ is constant. This again determines some density $\tilde{q}_\theta\left(z\right)$ of Z.

Is $\tilde{q}_\theta$ also a member of some canonical exponential family?

● Yes ✓

○ No

Solution:

For the first part: we have $q_\theta\left(z\right) = p_\theta\left(z - a\right)$. In particular,

$$q_\theta\left(z\right) = \exp\left(\frac{\left(z-a\right)\theta - b\left(\theta\right)}{\phi} + c\left(z-a,\phi\right)\right) = \exp\left(\frac{z - \left(b\left(\theta\right) + a\theta\right)}{\phi} + c\left(z-a,\phi\right)\right)$$

Let $\tilde{b}\left(\theta\right) = b\left(\theta\right) + a\theta$ and $\tilde{c}\left(z,\phi\right) = c\left(z-a,\phi\right)$ which demonstrates that this is indeed contained in a canonical exponential family.

A similar argument shows the same answer for the second part, where we instead use $q_\theta\left(z\right) = p_\theta\left(z/\lambda\right)$.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

(Ungraded) Re-parametrization

0 points possible (ungraded)

Ungrading note: The third part of this problem is unclear and need to be reworked. For now, we have ungraded this problem.

Let $\mathbf{x} = \left(X_1, X_2\right)$ where X_1, X_2 are positive random variables, and suppose $\mu\left(x_1, x_2\right) = \mathbb{E}\left[Y|X = \left(x_1, x_2\right)\right]$ is given by

$$\mathbb{E}\left[Y|X = \left(x_1, x_2\right)\right] = 1000\exp\left(x_1^2 - x_2^2\right).$$

Answer the following questions.

- True or False: $\ln \mu(\mathbf{x})$ is linear in $\mathbf{x}$.

  ○ True

  ● False ✓

- True or False: There is an invertible reparametrization $\tilde{\mathbf{x}}$ of $\mathbf{x}$ for which $Y|\tilde{\mathbf{x}}$ is a generalized linear model.

  ● True ✓

  ○ False

- If there *were* a reparametrization $\tilde{\mathbf{x}}$, would Jeffreys prior change? That is, would Jeffreys prior be computed using a different formula?

  ● Yes ✗

  ○ No ✓

Solution:

- No. Note that $\ln \mu(\mathbf{x}) = \ln \delta + \alpha x_1^2 - \beta x_2^2$. In particular, it is quadratic in $\mathbf{x}$.
- Yes. Since x_1, x_2 are positive, so we can equivalently use a reparametrization, $\tilde{\mathbf{x}} = (x_1^2, x_2^2)$. From here, $\ln \mu(\mathbf{x})$ is linear.
- No. This is a consequence of the fact that Jeffreys prior is parametrization-invariant.

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ℹ Answers are displayed within the problem

Discussion

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