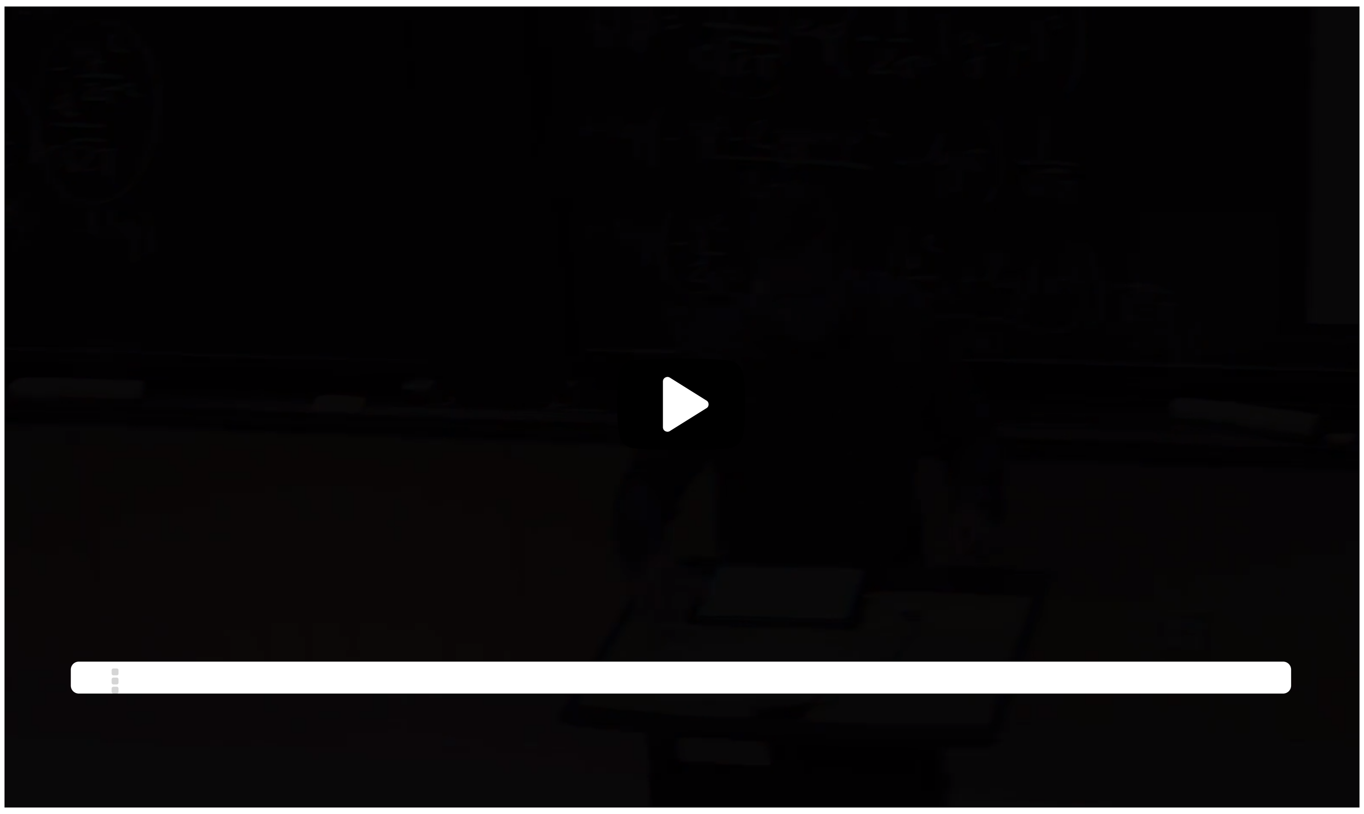
7. Exponential Family: Continuous Examples

Example: Gaussian Distribution



9:01 / 9:01 1.0x

over sigma square

root 2 pi and then the term that I just removed,

which was e to the minus y squared over 2 sigma squared.

So now this thing will be h of y.

This thing is my T1 or T of y.

This is my eta 1 of theta.

And this is my B of theta.

And theta here is really mu.

There's only one unknown parameter.

End of transcript. Skip to the start.

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Exponential Distribution as Exponential Families

4/4 points (graded)

Recall that the exponential distribution with parameter λ is given by the pdf by

$$f_\lambda(y) = \lambda e^{-\lambda y}.$$

Let $\theta = \lambda$. Rewrite $f_\lambda(y)$ in the form

$$f_\theta(y) = h(y)\exp\left(\eta(\theta)\, T(y) - B(\theta)\right),$$

and enter $\eta(\theta)$, $T(y)$, $B(\theta)$ below.

These functions are not unique. To get unique answers, let $h(y) = 1$, and let the coefficient of y in $T(y)$ be $+1$.

$T(y) =$ y ✔ **Answer:** y

y

$\eta(\theta) =$ -theta ✔ **Answer:** -theta

$-\theta$

$B(\theta) =$ -ln(theta) **Answer:** -ln(theta)

$-\ln(\theta)$

If instead of $h(y) = 1,$ we had used $\tilde{h}(y) = C$ for some constant C, then what is $\widetilde{B}(\theta)$ in terms of $B(\theta)$ and C? That is, find $\widetilde{B}(\theta)$ such that the pdf $f_\theta(y)$ of $Y \sim \text{Exp}(\theta)$ is

$$f_\theta(y) = \tilde{h}(y) \exp\left(\eta(\theta) T(y) - \widetilde{B}(\theta)\right).$$

(Enter ***B*** *for $B(\theta)$ and* ***C*** *for C. Your answer should be in terms of only C and $B(\theta)$. Enter "ln" for the natural logarithm.)*

$\widetilde{B}(\theta) =$ B+ln(C) **Answer:** B+ln(C)

STANDARD NOTATION

Solution:

$$f_\theta(y) = \theta e^{-\theta y} = e^{-(\theta)(y)-(-ln(\theta)}$$

Hence $\eta(\theta) = \theta,\ T(y) = y,\ B(\theta) = \ln(\theta)$. If instead $\tilde{h}(y) = C$ is used, then

$$f_\theta(y) = \theta e^{-\theta y} = Ce^{-(\theta)(y)-(-ln(\theta)+\ln(C))}$$

Hence $\widetilde{B}(\theta) = B(\theta) + \ln(C)$.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 7 Generalized Linear Models:Lecture 21: Introduction to Generalized Linear Models; Exponential Families / 7. Exponential Family: Continuous Examples