6. Random incidence under Erlang interarrivals
Problem 6. Random incidence under Erlang interarrivals

2/2 points (graded)

A factory produces copper wires. A blue mark is placed on a very long length of copper. The wire is then cut into pieces. The lengths of different pieces are independent, and the length of each piece is distributed according to the same PDF $f_X(x)$. Let R be length of the piece including the blue mark. Determine the expected value of R in each of the following cases.

In each part below, express your answer in terms of μ using standard notation. Enter "mu" for μ.

1. Suppose that $f_X(x) = \begin{cases} \mu e^{-\mu x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

   $\mathbf{E}[R] = $

   | 2/mu |

   ✔ **Answer:** 2/mu

   $\frac{2}{\mu}$

2. Suppose that $f_X(x) = \begin{cases} \frac{\mu^4 x^3 e^{-\mu x}}{6}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

   $\mathbf{E}[R] = $

   | 5/mu |

   ✔ **Answer:** 5/mu

   $\frac{5}{\mu}$

STANDARD NOTATION

Solution:

1. The lengths of the pieces of copper wire are independent and exponentially distributed with parameter μ. As explained in lecture, due to the memorylessness of the exponential, the distribution of the length of the piece of copper wire containing the blue mark is a second order Erlang. Thus, $\mathbf{E}[R] = 2\mathbf{E}[X] = 2/\mu$.

2. Here, X is Erlang of order 4. Think of sections on the copper wire, each exponentially distributed with parameter μ. We then interpret each piece as **four** consecutive sections of exponentially distributed lengths. The piece of the copper wire with the blue mark will have the mark in one of these four sections. By the standard random incidence analysis, the expected length of that section will be $2/\mu$. However, the piece of copper wire containing the blue mark also consists of three other sections, each of which is exponentially distributed with parameter μ, and hence, have an expected length of $1/\mu$. Thus, the total expected length of the piece of copper wire containing the blue mark is $2/\mu + 1/\mu + 1/\mu + 1/\mu = 5/\mu$.

提交   你已经尝试了3次（总共可以尝试3次）

ℹ Answers are displayed within the problem

讨论

显示讨论

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