11. Identifiability exercises

Identifiability of Statistical Models 2

1/1 point (graded)

Let $X_i = Y_i^2$ where $Y_1, \ldots, Y_n \overset{iid}{\sim} \mathcal{U}([0,a])$ for some unknown parameter a. We observe the i.i.d. samples $X_1, \ldots, X_n$, but not the Y_i's themselves.

Hint: Compute the cdf of X_i.

Is the parameter a identifiable from the common distribution the X_i's?

- (•) Yes ✔
- ( ) No

Solution:

Write $X_i \sim X$ and note that X is supported on the interval $[0, a^2]$. Let us compute the CDF of X in terms of a.

$$\mathbf{P}(X \le t) = \mathbf{P}(Y \le \sqrt{t}) = \min\left(\int_0^{\sqrt{t}} \frac{1}{a}\, dy, 1\right) = \min\left(\frac{\sqrt{t}}{a}, 1\right).$$

For different values of a, the CDF of X are different; hence a is identifiable.

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

Identifiability of Statistical Models 3

1/1 point (graded)

Let $X_i = \mathcal{I}(Y_i \ge a/2)$ where $Y_1, \ldots, Y_n \overset{iid}{\sim} \mathcal{U}([0,a])$ for some unknown parameter a. We observe the independent samples $X_1, \ldots, X_n$ but not the Y_i's themselves.

Is the parameter a identifiable from the common distribution of the X_i's?

- ( ) Yes
- (•) No ✔

Solution:

Note that X is a Bernoulli random variable with parameter $p := P\left(\mathcal{I}\left(Y_i \ge \frac{a}{2}\right) = 1\right) = P\left(Y_i \ge \frac{a}{2}\right).$

For any choice of a, we have by the distribution of Y_i that $p = P(Y_i \ge a/2) = 1/2$. Hence, for any choice of a, the random variable X is distributed as $\mathrm{Ber}(1/2)$. The parameter a is not identifiable.

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Answers are displayed within the problem

Review of terminology

0/1 point (graded)

You have access to samples $X_1, \ldots, X_n \stackrel{iid}{\sim} P_{\theta^*}$ where $\theta^* \in \mathbb{R}$ is a true, unknown parameter specifying the distribution. You construct a statistical model $\left((-\infty, \infty), \{P_\theta\}_{\theta \in \mathbb{R}}\right)$ for this statistical experiment. Your goal is to uncover the true parameter θ^*.

Imagine that somehow you are able to figure out the true distribution P_{θ^*}. Which assumption below implies that it is possible to recover the true parameter θ^*? from the distribution?
(Choose all that apply.)

- [ ] There is another value $\theta' \in \mathbb{R}$ such that $\theta' \neq \theta^*$ but P_{θ^*} and $P_{\theta'}$ are the same distribution.
- [x] The given statistical model $\left((-\infty, \infty), \{P_\theta\}_{\theta \in \mathbb{R}}\right)$ is well-specified.
- [x] The parameter θ is identifiable for the given statistical model. ✔

✖

Solution:

The third choice, "The parameter θ is identified for the given statistical model.", is correct. If θ is identified, then the map $\theta \mapsto P_\theta$ is injective. Hence, given the output P_{θ^*}, which is the true distribution, we can uniquely recover the true parameter θ^*.

The first choice, "There is another value $\theta' \in \mathbb{R}$ such that $\theta' \neq \theta^*$ but P_{θ^*} and $P_{\theta'}$ are the same distribution.", is incorrect because this implies that the parameter θ is *not* identified. This implies that by only knowing the distribution P_{θ^*}, we have no way of saying if θ' or θ^* is the true parameter.

Recall that a statistical model $\left(E, \{P_\theta\}_{\theta \in \Theta}\right)$ associated to a statistical experiment $X_1, \ldots, X_n$ is **well-specified** if there exists θ^* such that $X_1, \ldots, X_n \stackrel{iid}{\sim} P_{\theta^*}$. Note that the problem statement implies that our model is well-specified. However, this assumption is not enough to be able to recover the true parameter θ^* from the distribution P_{θ^*} because the parameter θ may not be identified.

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Answers are displayed within the problem

讨论

显示讨论

主题: Unit 2 Foundation of Inference:Lecture 3: Parametric Statistical Models / 11. Identifiability exercises

认证证书是什么?