4. Introduction to Total Variation Distance

Definition of Total Variation Distance

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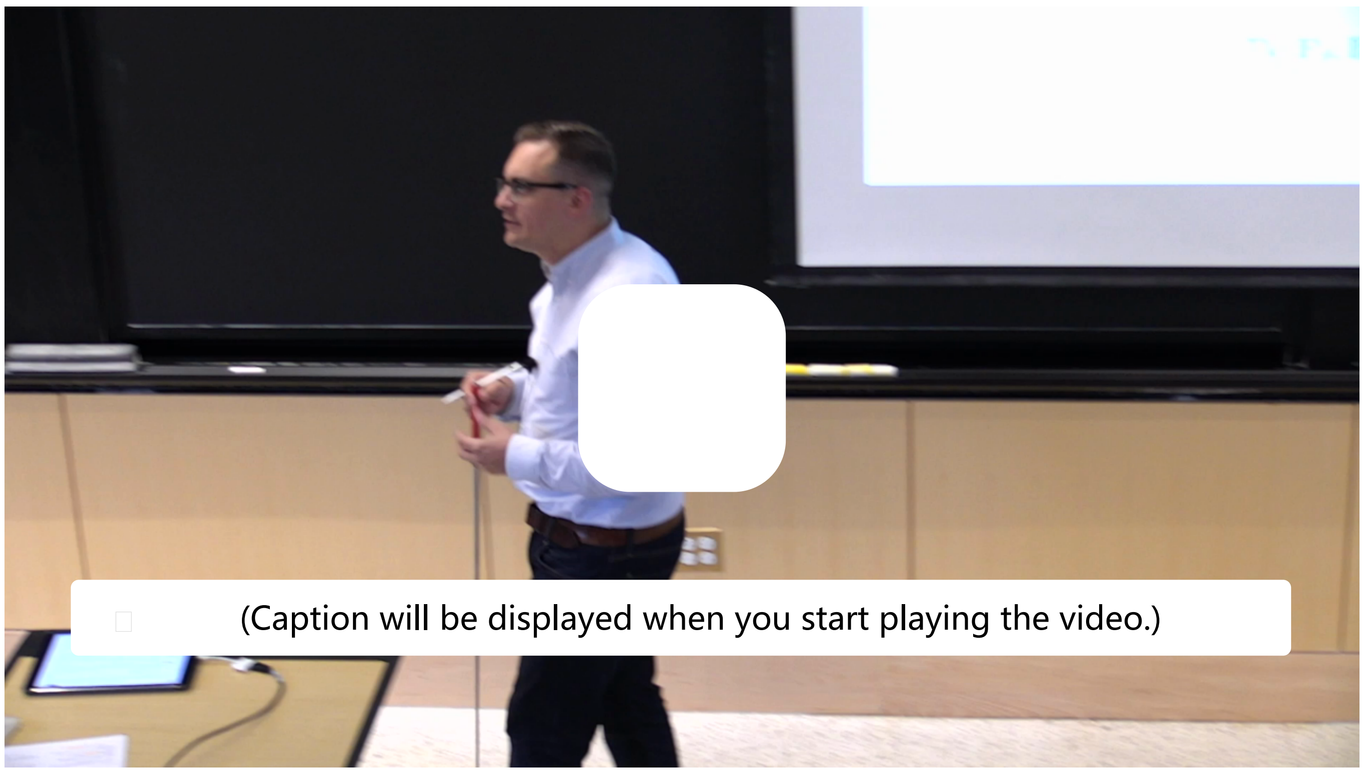

So, arguably, when you're trying to build a statistical method,

you need to understand what your goal is, right?

So if you're trying to understand

what it is to learn a distribution,

that's what we're trying to do.

We're trying to estimate the distribution,

but let's start from the basics and say, what is it we

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Interpreting Total Variation Distance

1/1 point (graded)

Recall from lecture that the **total variation distance** between two probability measures $\mathbf{P}_\theta$ and $\mathbf{P}_{\theta'}$ with sample space E is defined by

$$\mathrm{TV}\left(\mathbf{P}_\theta, \mathbf{P}_{\theta'}\right) = \max_{A \subset E} \left|\mathbf{P}_\theta\left(A\right) - \mathbf{P}_{\theta'}\left(A\right)\right|$$

Let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathbf{P}_{\theta^*}$ where $\theta^* \in \mathbb{R}$ is an unknown parameter. You construct a statistical model $\left(E, \{\mathbf{P}_\theta\}_{\theta \in \mathbb{R}}\right)$ for your data. By analyzing your data, you are able to produce an estimator $\hat{\theta}$ such that the distributions $\mathbf{P}_{\hat{\theta}}$ and $\mathbf{P}_{\theta^*}$ are close in **total variation distance**. More precisely, you know that

$$\mathrm{TV}\left(\mathbf{P}_{\hat{\theta}}, \mathbf{P}_{\theta^*}\right) \leq \epsilon,$$

where ϵ is a very small positive number.

Which of the following can you conclude about the distributions $\mathbf{P}_{\hat{\theta}}$ and $\mathbf{P}_{\theta^*}$? (Choose all that apply.)

- [x] Let A be an event. Then $\left|\mathbf{P}_{\theta^*}\left(A\right) - \mathbf{P}_{\hat{\theta}}\left(A\right)\right| \leq \epsilon$.
- [x] Let $X \sim \mathbf{P}_{\theta^*}$, let $Y \sim \mathbf{P}_{\hat{\theta}}$, and suppose $a, b \in \mathbb{R}$ where $a \leq b$. Then $\left|\mathbf{P}_{\theta^*}\left(a \leq X \leq b\right) - \mathbf{P}_{\hat{\theta}}\left(a \leq Y \leq b\right)\right| \leq \epsilon$.

$|\theta^* - \hat{\theta}| \le \epsilon$.

Solution:

Recall that by definition,

$$\mathrm{TV}\left(\mathbf{P}_{\hat{\theta}}, \mathbf{P}_{\theta^*}\right) = \max_{A \subset E} \left|\mathbf{P}_{\hat{\theta}}(A) - \mathbf{P}_{\theta^*}(A)\right|$$

where the maximum is over all events A. Since we are given that $\mathrm{TV}\left(\mathbf{P}_{\hat{\theta}}, \mathbf{P}_{\theta^*}\right) \le \epsilon$, we conclude that $\left|\mathbf{P}_{\hat{\theta}}(A) - \mathbf{P}_{\theta^*}(A)\right| \le \epsilon$ for every event A. Hence, the first choice is correct.

Let A be the event given by the interval (a, b). Then,

$$\left|\mathbf{P}_{\theta^*}(a \le X \le b) - \mathbf{P}_{\hat{\theta}}(a \le Y \le b)\right| \le \epsilon$$

is the same as saying $\left|\mathbf{P}_{\hat{\theta}}(A) - \mathbf{P}_{\theta^*}(A)\right| \le \epsilon$. Thus, the second choice is true as well.

The third choice, "$|\theta^* - \hat{\theta}| \le \epsilon$.", is incorrect. In general, even if distributions $\mathbf{P}_{\hat{\theta}}$ and $\mathbf{P}_{\theta^*}$ are close, there is no reason to expect the parameters θ^* and $\hat{\theta}$ to be close. To conclude that the estimated parameter is close to the true parameter given their distributions are close, we would need some assumptions on the map $\theta \mapsto \mathbf{P}_\theta$. No such assumption is given here.

提交 你已经尝试了2次（总共可以尝试2次）

Answers are displayed within the problem

讨论

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认证证书是什么?