(a) Suppose for simplicity that X is integer-valued. The intuitive idea is to partition the unit interval into subintervals, with the kth interval having length $p_X(k)$. Whenever U falls in the kth interval, we assign the value k to the random variable X.

Mathematically, this translates to the following. We assign to X the value k whenever the value u of the random variable U satisfies $F(k-1) < u \leq F(k)$. With this choice,

$$\mathbf{P}(X = k) = \mathbf{P}\Big(F_X(k-1) < U \leq F_X(k)\Big) = F(k) - F(k-1),$$

and $\mathbf{P}(X \leq k) = F(k)$, so that F is indeed the CDF of the random value X we have generated.

(b) In the continuous case, given the value u of U, we assign to X a value x that satisfies $F(x) = u$. (Such a value x exists and is unique because we assumed that F is strictly increasing on the relevant interval.) In terms, of random variables, we have the relation $F(X) = U$. Since F is continuous and monotonic, we have

$$X \leq x \qquad \text{if and only if} \qquad F(X) \leq F(x).$$

Therefore,

$$\mathbf{P}(X \leq x) = \mathbf{P}\big(F(X) \leq F(x)\big) = \mathbf{P}\big(U \leq F(x)\big) = F(x),$$

where the last equality holds because U is uniform. Thus, X has the desired CDF.

(c) The exponential CDF with parameter $\lambda = 1$ takes the form $F(x) = 1 - e^{-x}$, for $x \geq 0$. Thus, to generate values of X, we start with the value u of U and assign to X a value x that satisfies $1 - e^{-x} = u$. Solving for x, we find $x = -\log(1-u)$.