2. Maximum Likelihood Estimator of Uniform Statistical Model

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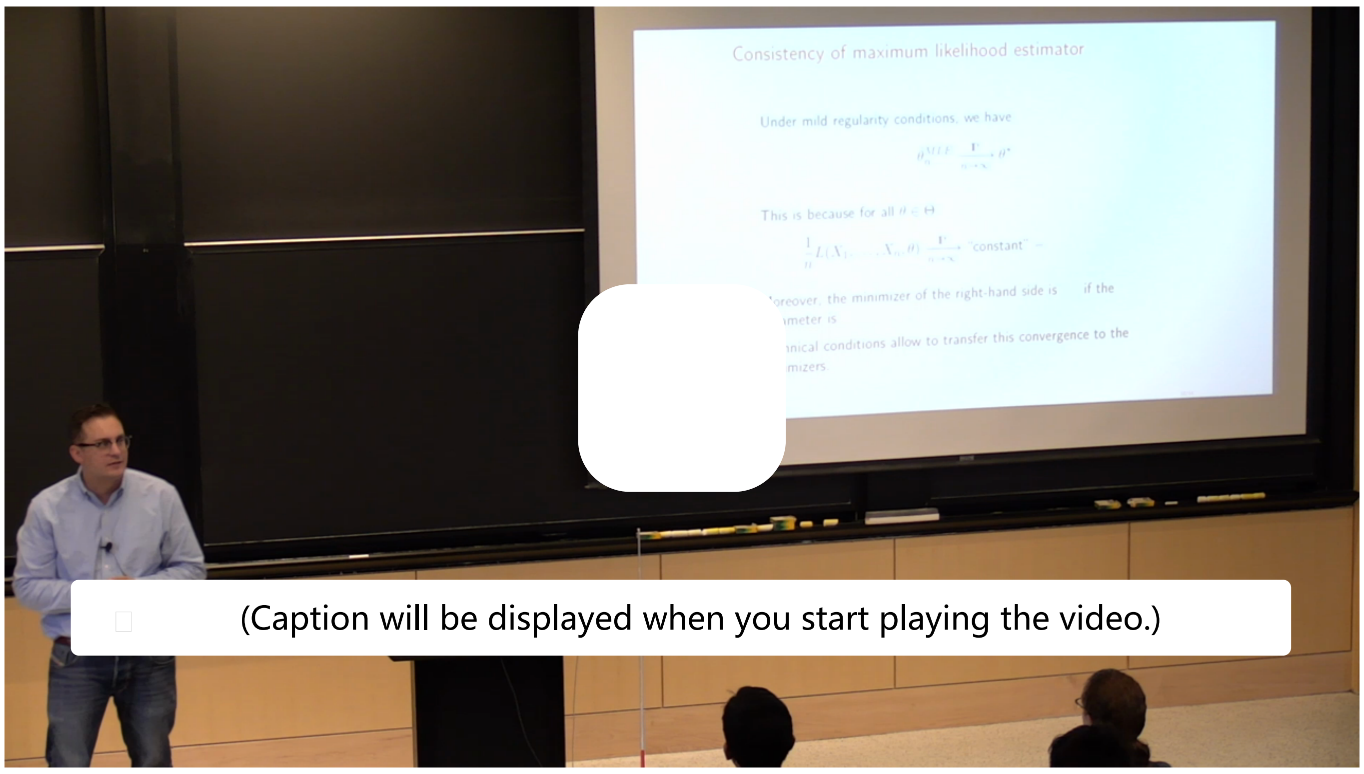

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All right.

So we've computed maximum likelihood estimators

in several examples.

The Bernoulli, the Poisson, and Gaussian we did fairly briefly.

And I wanted to just, before we actually go any further

and talk about some statistical properties

of the maximum likelihood estimator,

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Concept Check: Maximum Likelihood Estimator for a Uniform Statistical Model

1/1 point (graded)

Let $X_1, \ldots, X_n \stackrel{iid}{\sim} \mathrm{Unif}\,[0, \theta^*]$ where θ^* is an unknown parameter. We constructed the associated statistical model $(\mathbb{R}_{\geq 0}, \{\mathrm{Unif}\,[0,\theta]\}_{\theta > 0})$ (where $\mathbb{R}_{\geq 0}$ denotes the nonnegative reals).

For any $\theta > 0$, the density of $\mathrm{Unif}\,[0,\theta]$ is given by $f(x) = \frac{1}{\theta}\mathbf{1}(x \in [0,\theta])$. Recall that

$$\mathbf{1}(x \in [0,\theta]) = \begin{cases} 1 & \text{if } x \in [0,\theta] \\ 0 & \text{otherwise.} \end{cases}$$

Hence we can use the product formula and compute the likelihood to be

$$L_n(x_1, \ldots, x_n, \theta) = \prod_{i=1}^{n} \left(\frac{1}{\theta} \mathbf{1}(x_i \in [0,\theta]) \right) = \frac{1}{\theta^n} \mathbf{1}(x_i \in [0,\theta] \;\; \forall\, 1 \leq i \leq n).$$

For the fixed values $(1, 3, 2, 2.5, 5, 0.1)$ (think of these as observations of random variables $X_1, \ldots, X_6$), what value of θ maximizes $L_6(1, 3, 2, 2.5, 5, 0.1, \theta)$?

5

Answer: 5

Solution:

Observe that

$$L_6\left(1,3,2,2.5,5,0.1,\theta\right)=\frac{1}{\theta^6}\mathbf{1}\left(\{1,3,2,2.5,5,0.1\}\subset[0,\theta]\right).$$

If $\theta<\max\{1,3,2,2.5,5,0.1\}$, then we have $\{1,3,2,2.5,5,0.1\}\not\subset[0,\theta]$. By the definition of the indicator function, this means $L_6\left(1,3,2,2.5,5,0.1,\theta\right)=0$ for $\theta<\max\{1,3,2,2.5,5,0.1\}=5$. Hence, when maximizing $L_6\left(1,3,2,2.5,5,0.1,\theta\right)$, we need to consider $\theta\in[5,\infty)$. Restricted to this interval, we observe that

$$L_6\left(1,3,2,2.5,5,0.1,\theta\right)=\frac{1}{\theta^n}.$$

The above is a decreasing function on $[5,\infty)$, so the maximum is attained when $\theta=\max\{1,3,2,2.5,5,0.1\}=5.$

Remark: In general, the maximum likelihood estimator for θ^* in this uniform statistical model is

$$\widehat{\theta_n}^{MLE}=\max_{1\le i\le n}X_i.$$

提交 你已经尝试了1次（总共可以尝试2次）

☐ Answers are displayed within the problem

讨论

显示讨论

主题: Unit 3 Methods of Estimation:Lecture 10: Consistency of MLE, Covariance Matrices, and Multivariate Statistics / 2. Maximum Likelihood Estimator of Uniform Statistical Model