

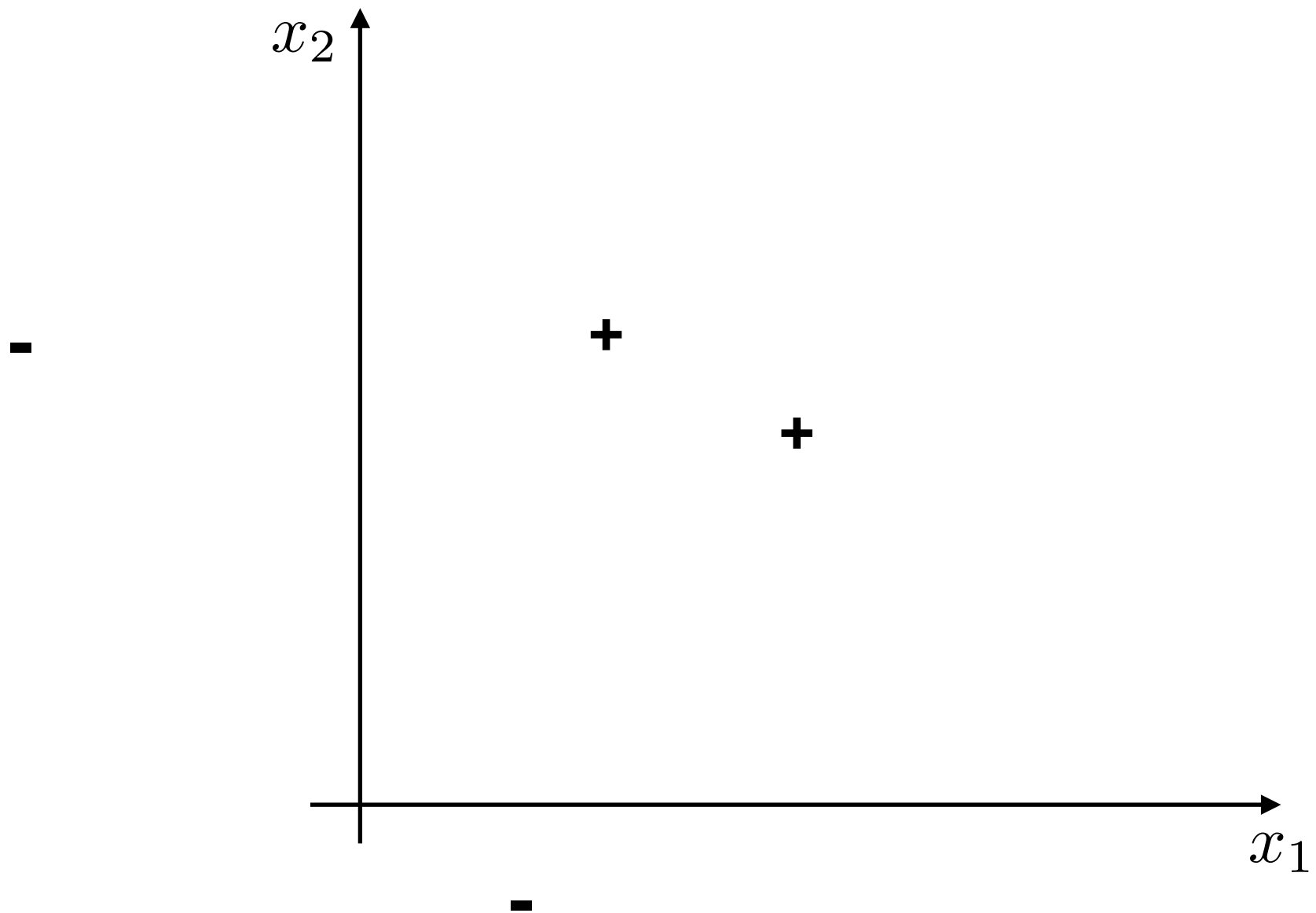
Machine Learning

Lecture 2

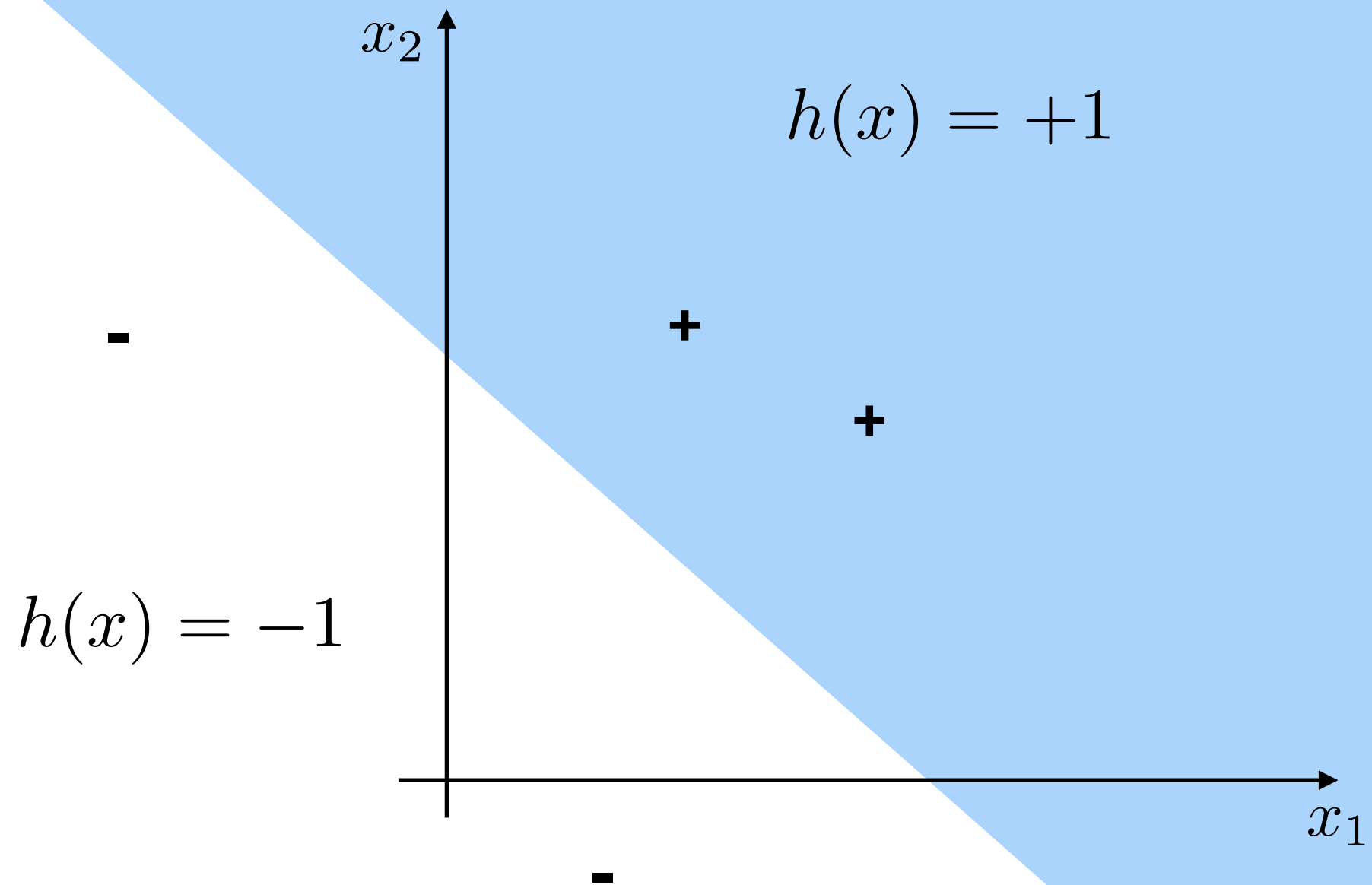
Review of basic concepts

- Feature vectors, labels
- Training set
- Classifier
- Training error
- Test error
- Set of classifiers

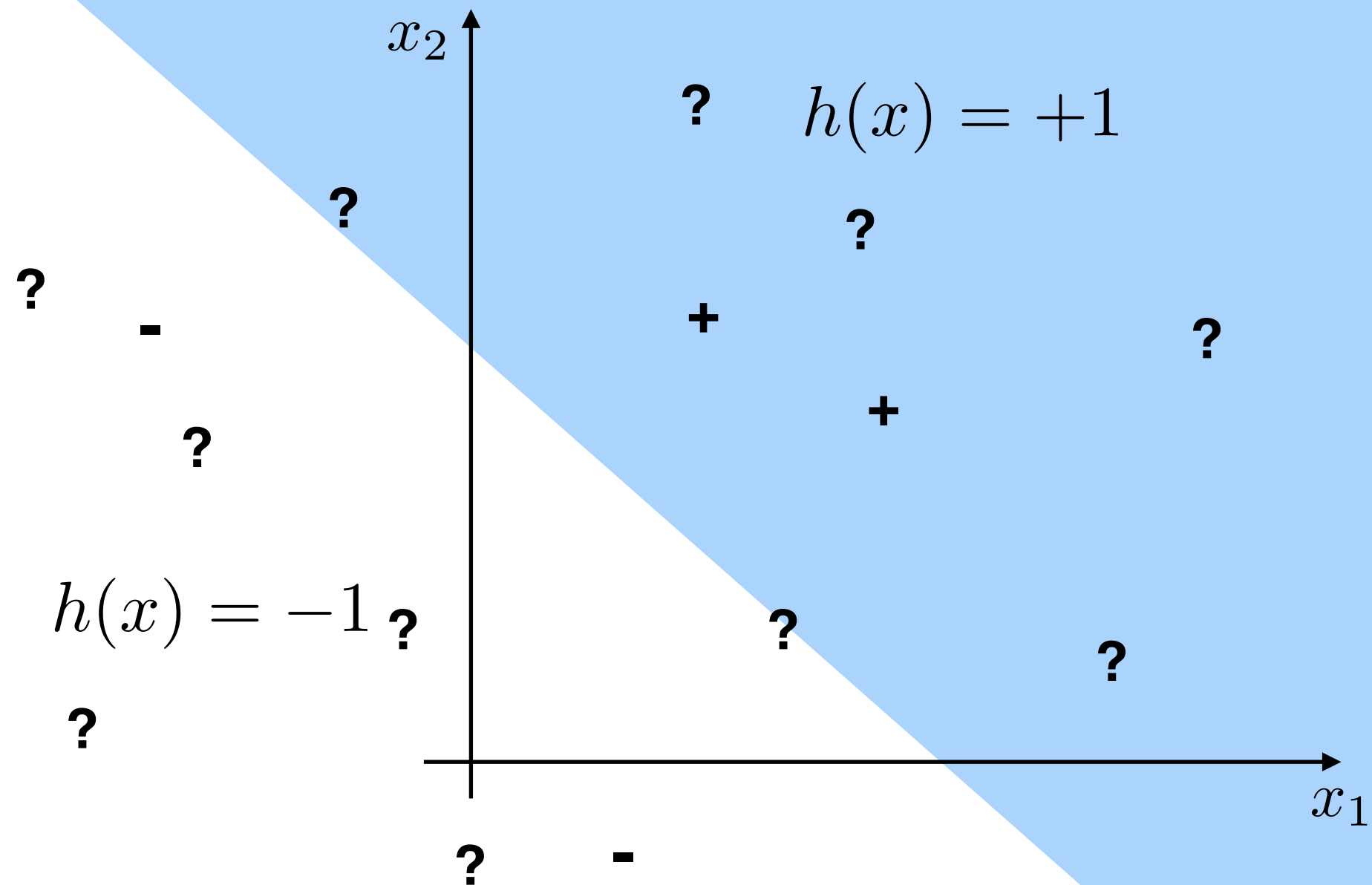
Review: training set



Review: a classifier



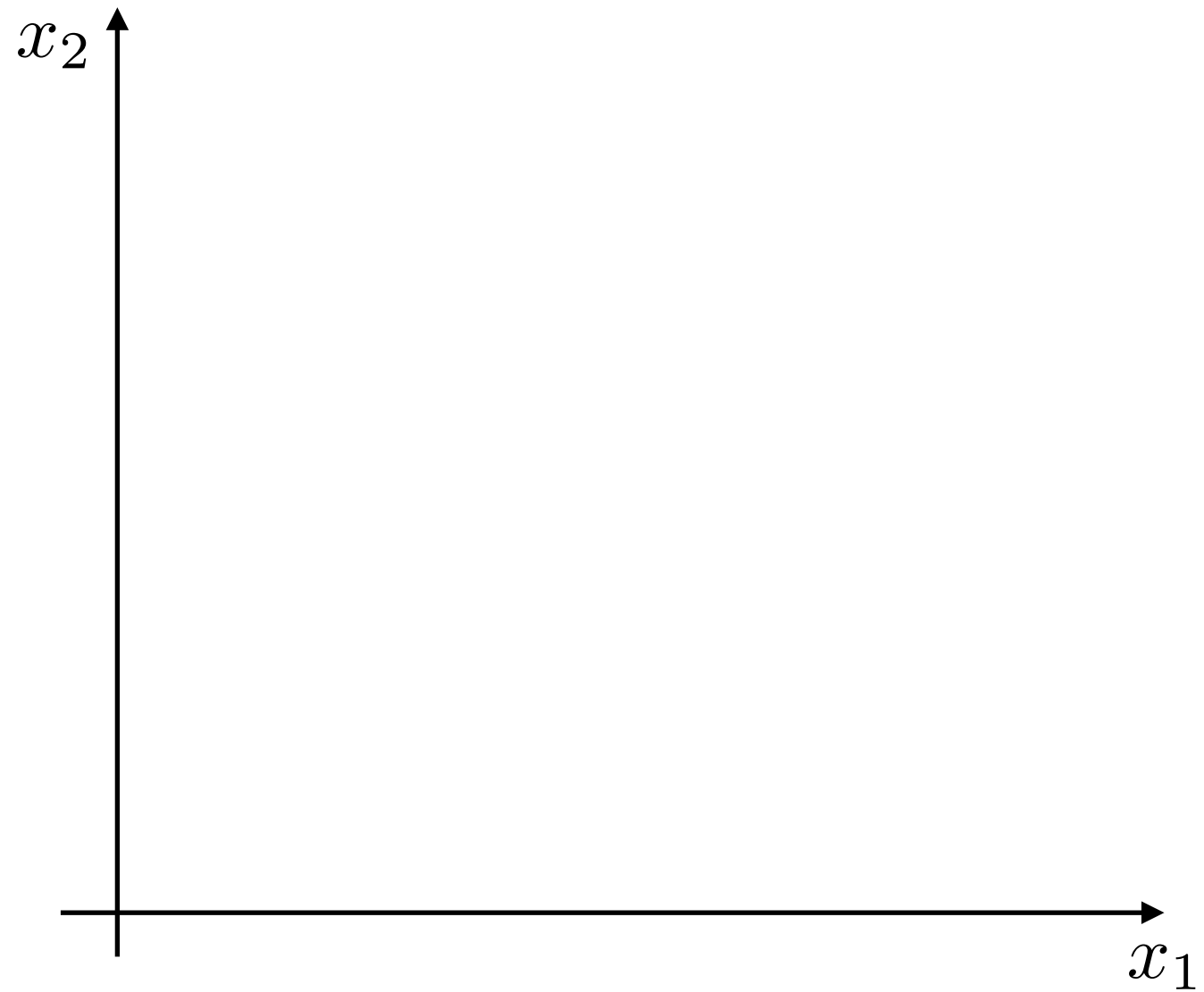
Review: test set



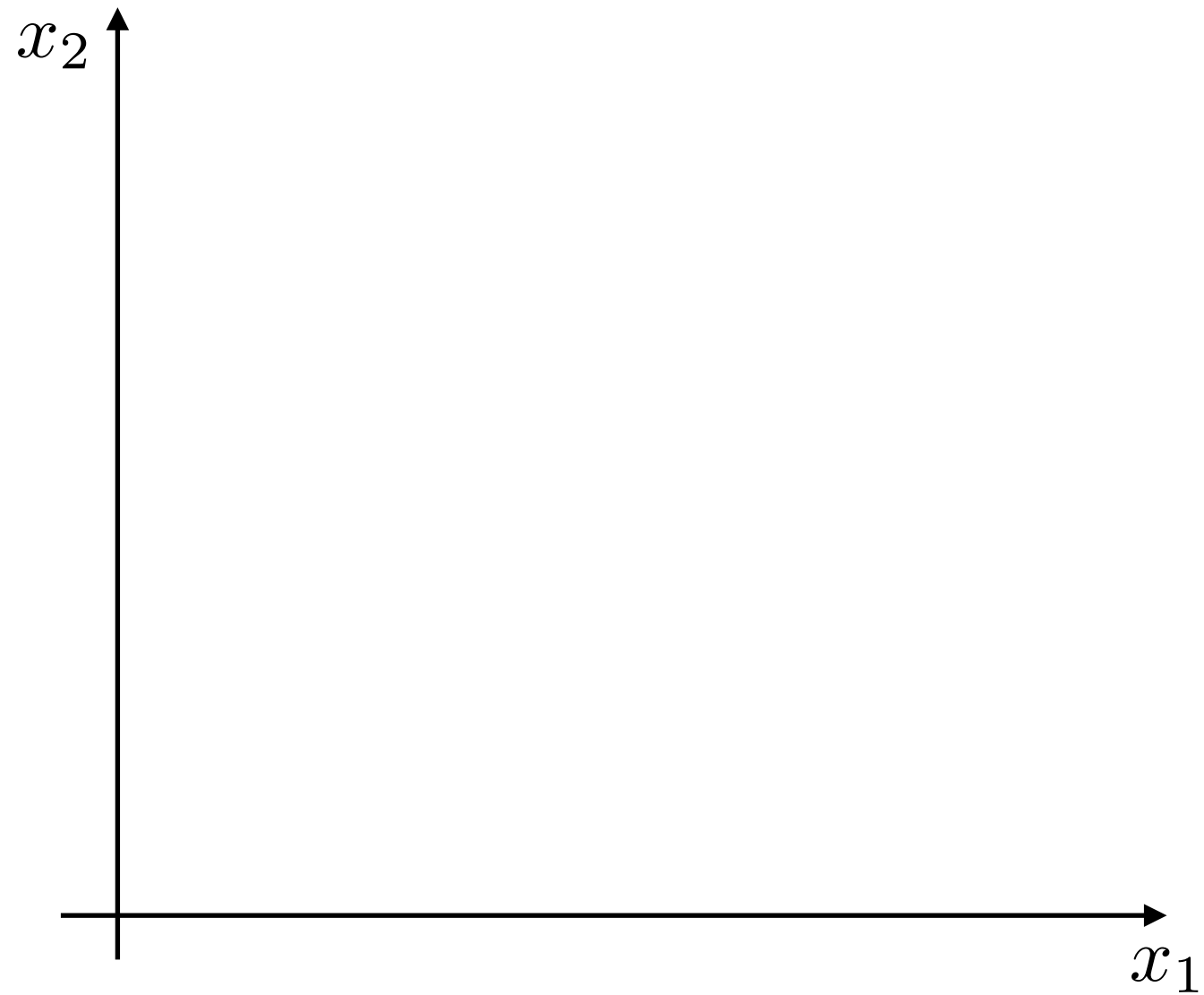
This lecture

- The set of linear classifiers
- Linear separation
- Perceptron algorithm

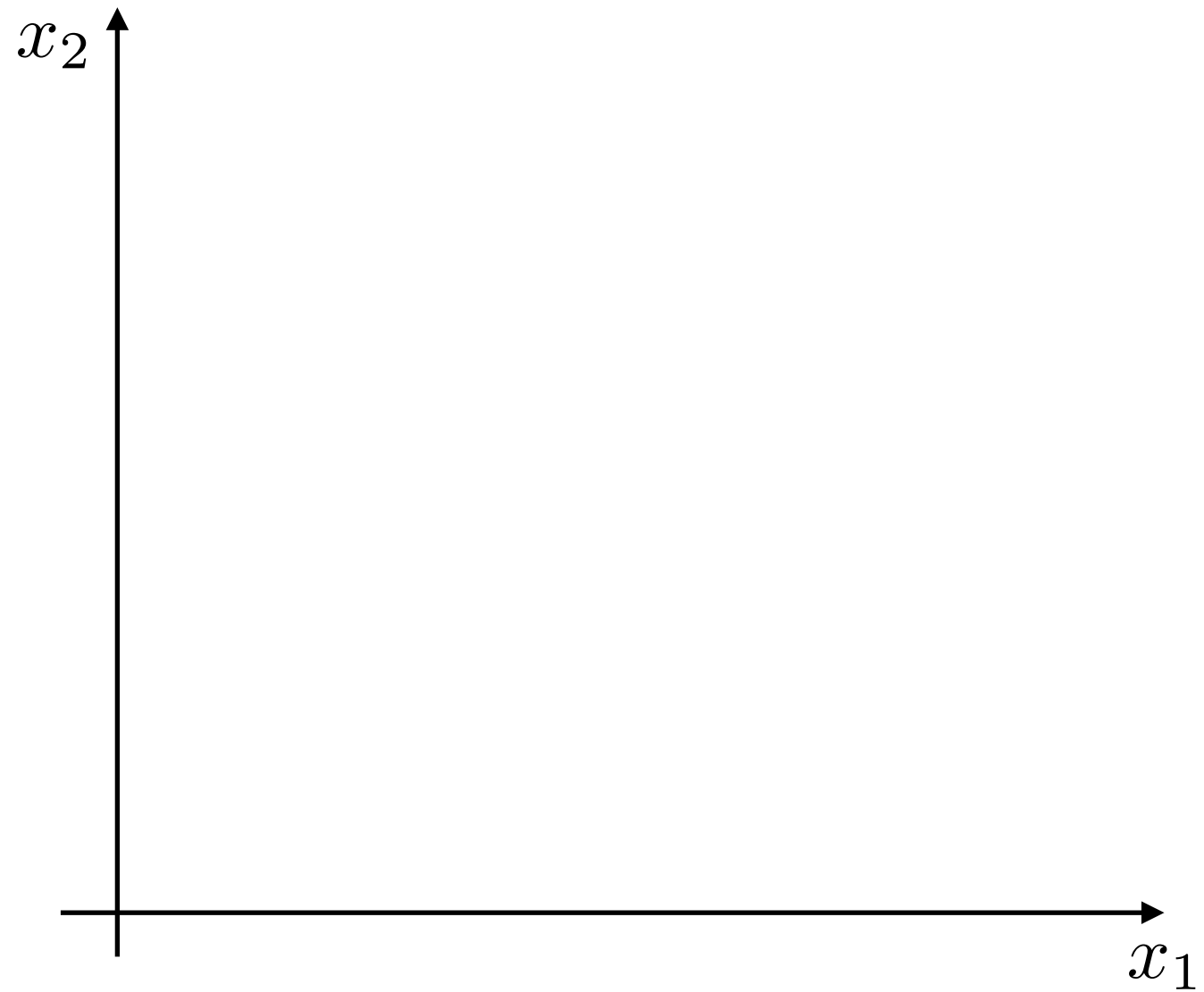
Linear classifiers



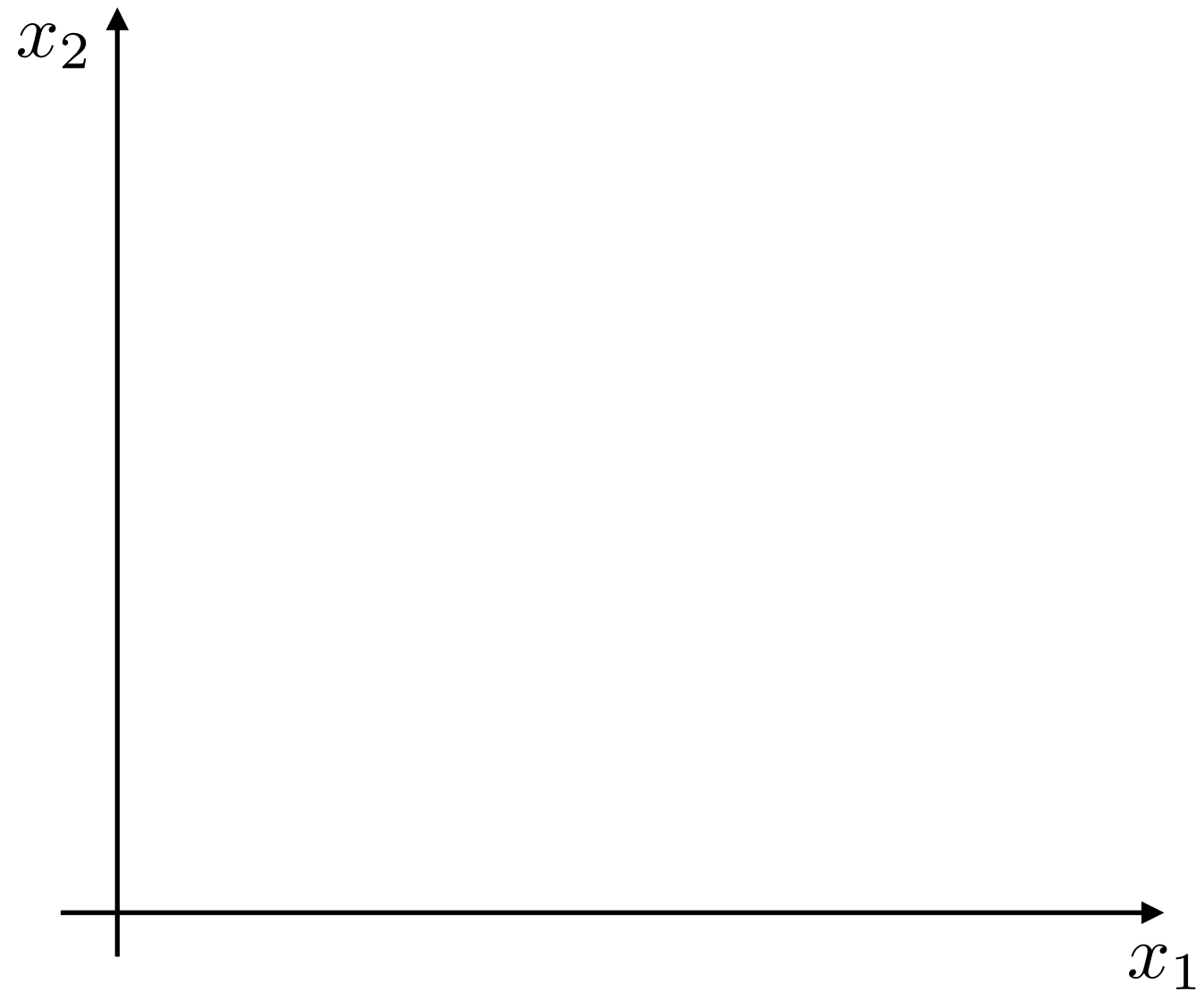
Linear classifiers through origin



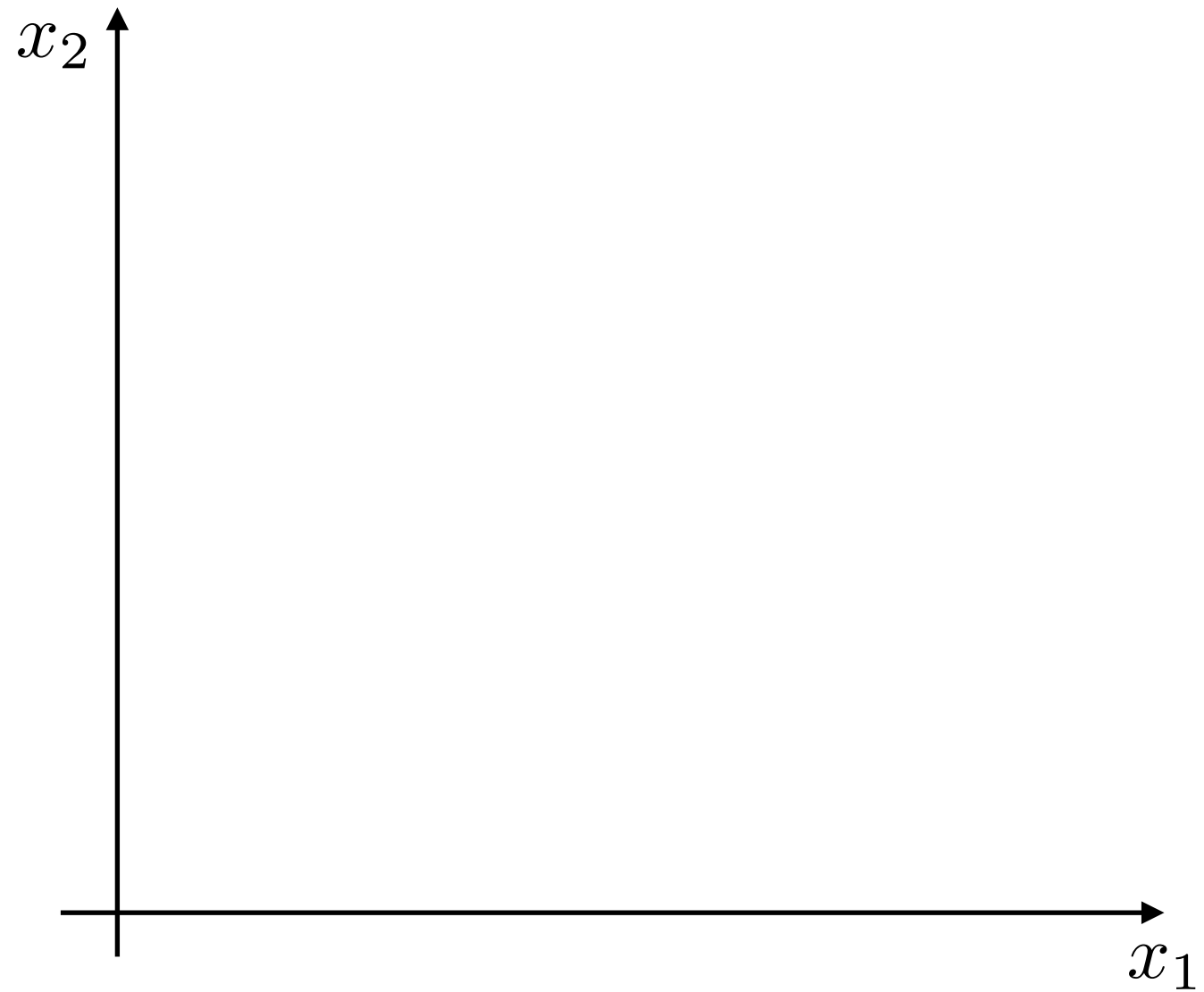
Linear classifiers



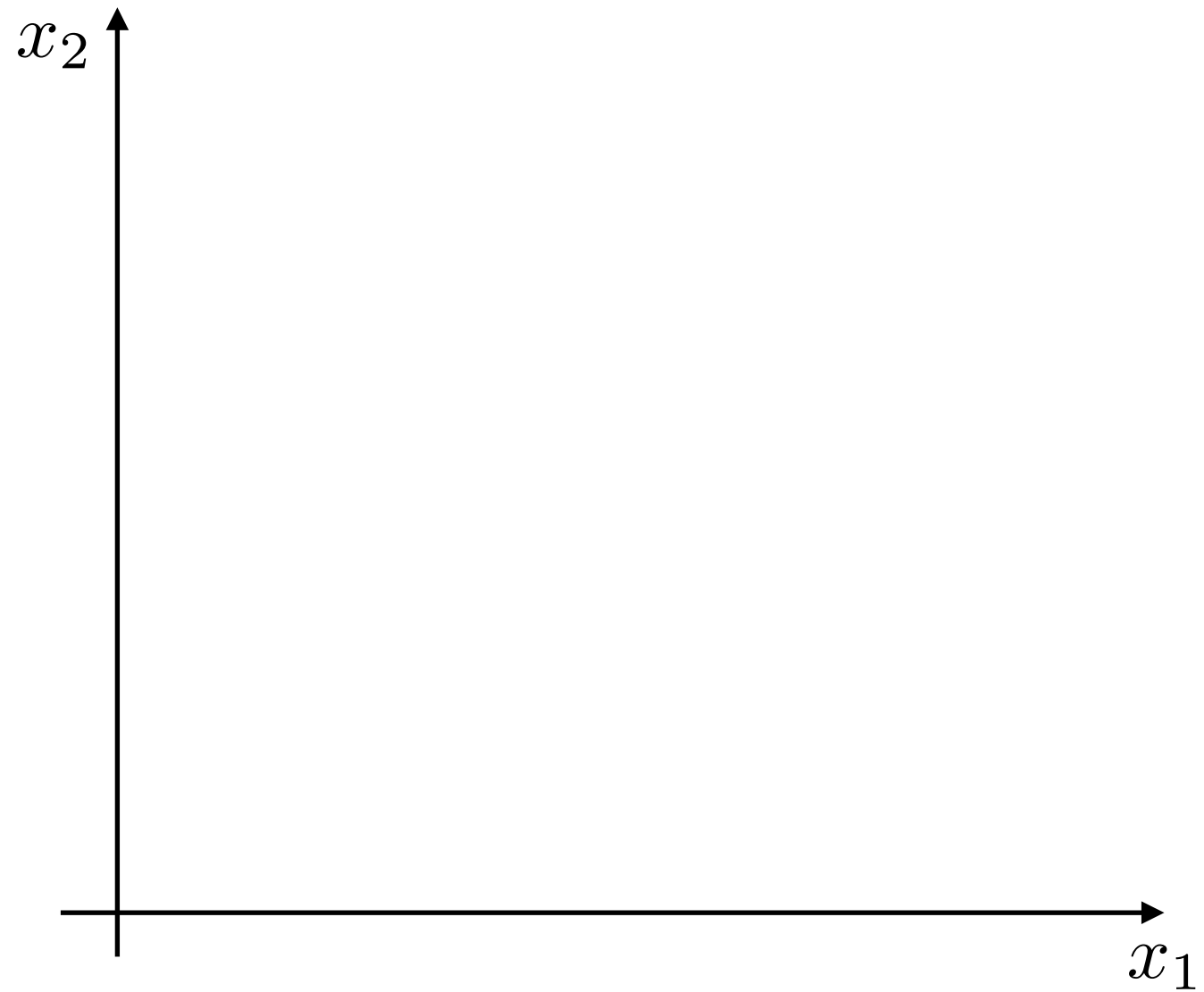
Linear separation: ex



Linear separation: ex



Linear separation: ex



Linear separation

Definition:

Training examples $S_n = \{(x^{(i)}, y^{(i)})\}, i = 1, \dots, n\}$ are *linearly separable* if there exists a parameter vector $\hat{\theta}$ and offset parameter $\hat{\theta}_0$ such that $y^{(i)}(\hat{\theta} \cdot x^{(i)} + \hat{\theta}_0) > 0$ for all $i = 1, \dots, n$.

Learning linear classifiers

- Training error for a linear classifier (through origin)

Learning linear classifiers

- Training error for a linear classifier

Learning algorithm: perceptron

$$\theta = 0 \text{ (vector)}$$

$$\text{if } y^{(i)} (\theta \cdot x^{(i)}) \leq 0 \text{ then}$$
$$\theta = \theta + y^{(i)} x^{(i)}$$

$$y^{(i)} (\theta_0 + y^{(i)} x^{(i)}) \cdot x^{(i)} = \|x^{(i)}\|^2 > 0$$

Learning algorithm: perceptron

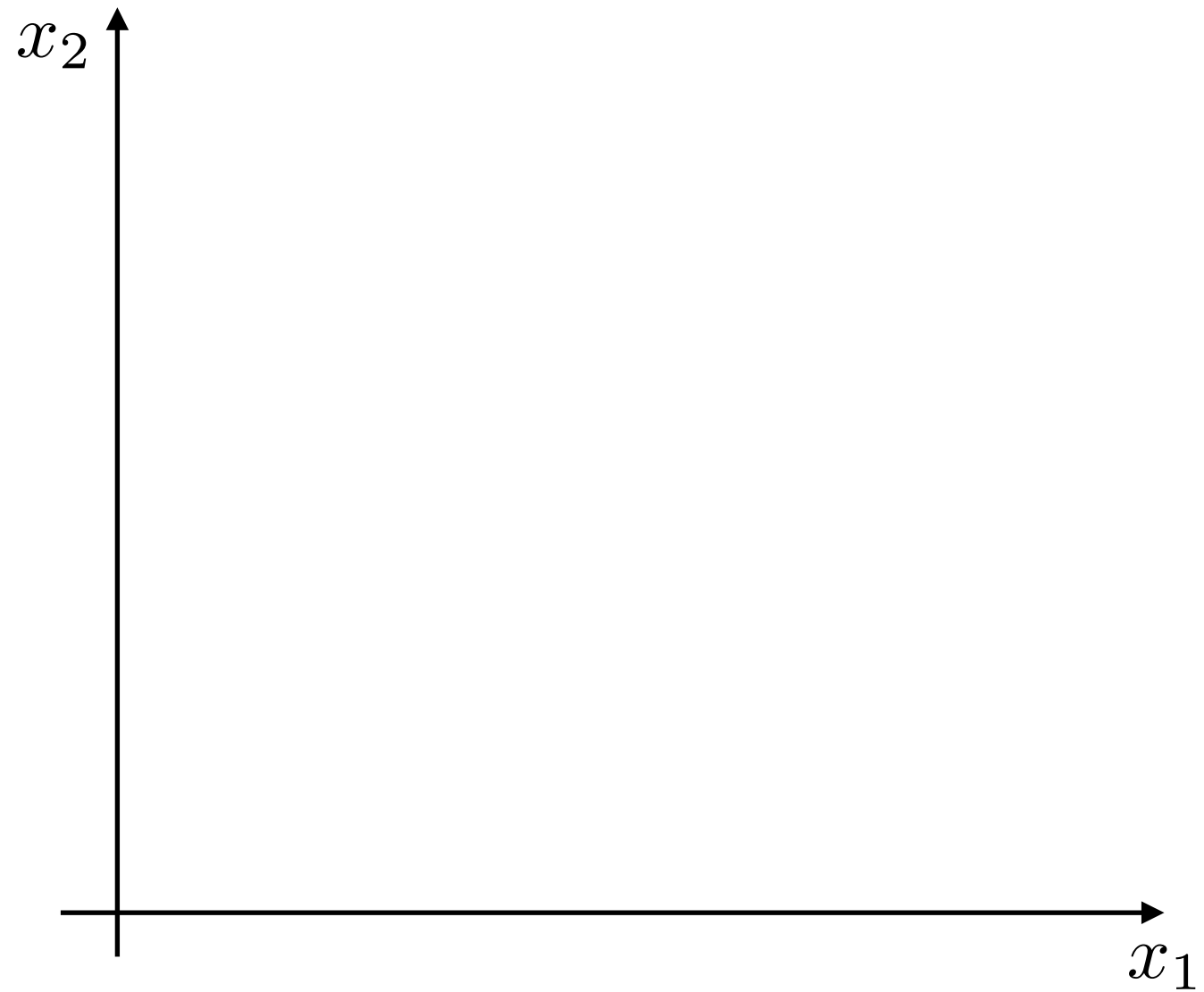
$\theta = 0$ (vector)

```
for  $i = 1, \dots, n$  do  
    if  $y^{(i)}(\theta \cdot x^{(i)}) \leq 0$  then  
         $\theta = \theta + y^{(i)}x^{(i)}$ 
```

Learning algorithm: perceptron

```
procedure PERCEPTRON( $\{(x^{(i)}, y^{(i)}), i = 1, \dots, n\}, T$ )  
   $\theta = 0$  (vector)  
  for  $t = 1, \dots, T$  do  
    for  $i = 1, \dots, n$  do  
      if  $y^{(i)}(\theta \cdot x^{(i)}) \leq 0$  then  
         $\theta = \theta + y^{(i)}x^{(i)}$   
  return  $\theta$ 
```

Perceptron algorithm: ex



Perceptron (with offset)

```
1: procedure PERCEPTRON( $\{(x^{(i)}, y^{(i)}), i = 1, \dots, n\}, T$ )
2:    $\theta = 0$  (vector),  $\theta_0 = 0$  (scalar)
3:   for  $t = 1, \dots, T$  do
4:     for  $i = 1, \dots, n$  do
5:       if  $y^{(i)}(\theta \cdot x^{(i)} + \theta_0) \leq 0$  then
6:          $\theta = \theta + y^{(i)}x^{(i)}$ 
7:          $\theta_0 = \theta_0 + y^{(i)}$ 
8:   return  $\theta, \theta_0$ 
```

Key things to understand

- Parametric families (sets) of classifiers
- The set of linear classifiers
- Linear separation
- Perceptron algorithm