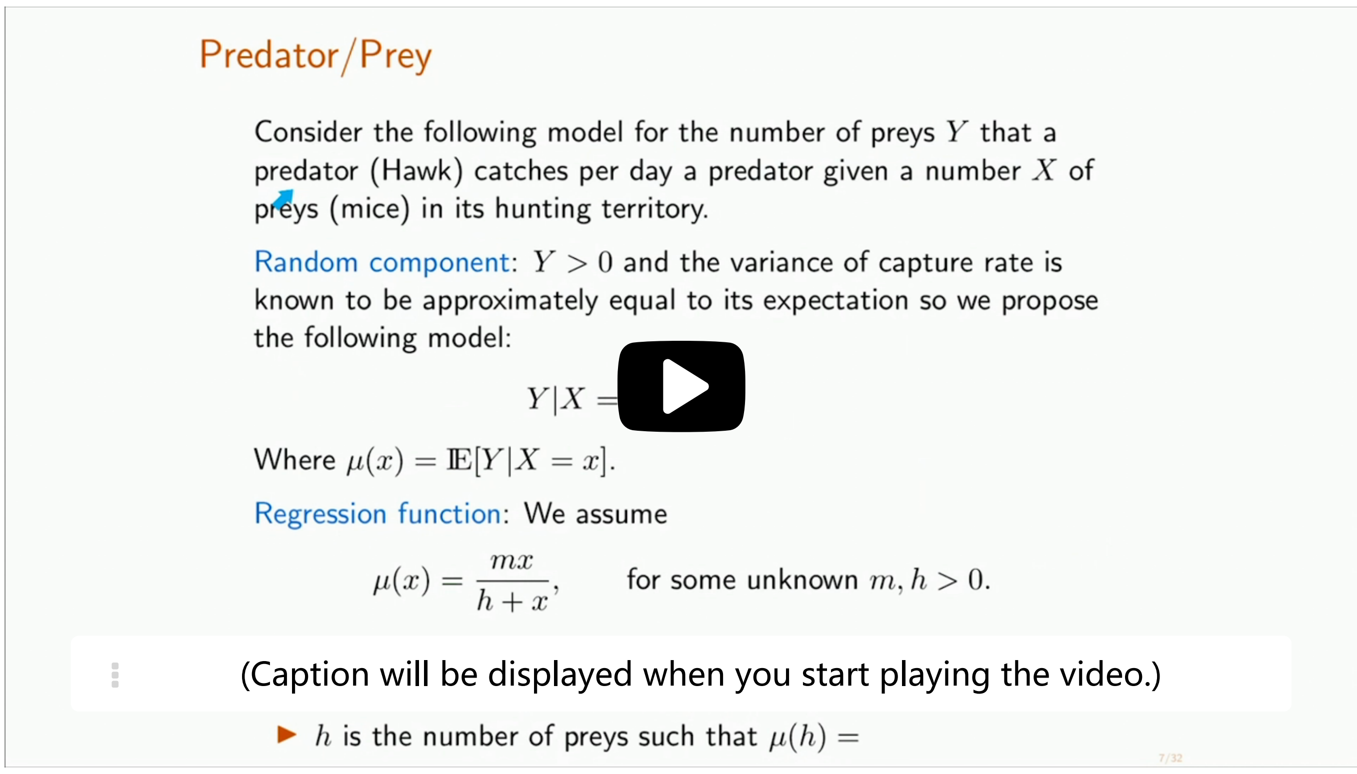
5. Predator-Prey Example: Poisson Link Function

Video note: In the video below, Prof Rigollet made an error when he wrote $\frac{1}{\mu(x)}$ as a linear function of $\frac{1}{x}$. which he corrected near the end of the video. The correct equation is

$$g(\mu(x)) = \frac{1}{\mu(x)} = \frac{1}{m} + \frac{h}{m}\frac{1}{x} = \beta_0 + \beta_1 \frac{1}{x}.$$

Predator-Prey Model: the Random Component and the Link Function

Start of transcript. Skip to the end.



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So here's another example.

This the predator/prey example.

So you have a model for the number of preys--

we'll denote this number Y--

that a predator-- think of a hawk--

catches per day, given a number X of preys,

say mice, in its hunting territory.

So clearly, there is going to be a relationship-- the more

Video

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Link Function Candidates

2/2 points (graded)

Consider random variables $\mathbf{X} = (X_1, X_2)$ and Y. Assume that the regression function $\mu(x_1, x_2) = \mathbb{E}[Y \mid X = (x_1, x_2)]$ for a pair (X, Y) happens to be $\mu(x) = (3x_1 + 2x_2)^3$. Which of the following is an appropriate choice for a link function g? In other words, for which g is it true that $g(\mu(x))$ can be written as a linear function, $x^T\beta$ for some β?

- ○ $g(\mu) = \log(\mu)$
- ○ $g(\mu) = e^{\mu}$
- ○ $g(\mu) = \mu^3$
- ● $g(\mu) = \sqrt[3]{\mu}$ ✔

If instead $\mu(x) = 2^{5x_1}$, which of the following are appropriate choices for the link function g? Choose all that apply.

- [x] $g(\mu) = \log_2(\mu)$ ✔
- [x] $g(\mu) = \ln(\mu)$ ✔
- [ ] $g(\mu) = e^{\mu}$
- [ ] $g(\mu) = \mu^3$
- [ ] $g(\mu) = \sqrt[3]{\mu}$

✔

Solution:

Observe that we always want to compose functions in this order: $g \circ \mu$. For the first problem, observe that the only choice that yields a linear function is the cube root: $g(\mu(\mathbf{x})) = 3x_1 + 2x_2$, so that $\beta = (3, 2)$. For the second problem, we wish to invert an exponential function, so the natural choice is the logarithm, of either base 2 or e: $g(\mu(\mathbf{x})) = \log(2^{5x_1}) = (5\log 2)\, x_1$, so that $\beta = (5\log 2, 0)$. Notice that changing the base, by the change of base formula for logarithms, changes β by a constant factor. This demonstrates an important concept: there can be (infinitely) **many** choices of link functions.

Submit

You have used 2 of 3 attempts

Answers are displayed within the problem

Properties of the link function

4/4 points (graded)

For each one of the proposed link functions below indicate whether they obey the conditions:

1. it is strictly increasing,
2. it is continuously differentiable and
3. its range is all of $\mathbb{R}$.

Choose “Yes" only if the function does satisfy all of the conditions and choose “No" otherwise.

- $g(x) = x^2$.
  - ( ) Yes
  - (•) No ✔

- $g(x) = x^3 - 3$.
  - (•) Yes ✔
  - ( ) No

- $g(x) = 1 - e^{-x}$.

- Yes
- No ✔

- $g(x) = \log x$ for $x > 0$ only.

- Yes ✔
- No

Solution:

- No. Observe that $g(\cdot)$ is not strictly increasing. For instance, even though $-10 < -5$, we have $g(-10) > g(-5)$.
- Yes. This function is a translation of x^3, which does satisfy all the properties.
- No. Note that even though this function is strictly increasing, its range is only $(-\infty, 1)$.
- Yes. This function satisfies all of the properties.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Discussion

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Topic: Unit 7 Generalized Linear Models:Lecture 21: Introduction to Generalized Linear Models; Exponential Families / 5. Predator-Prey Example: Poisson Link Function