The answers to these questions are found by considering suitable Bernoulli processes and using the formulas of Section 6.1. Depending on the specific question, however, a different Bernoulli process may be appropriate. In some cases, we associate trials with slots. In other cases, it is convenient to associate trials with busy slots.

(a) During each slot, the probability of a task from user 1 is given by $p_1 = p_{1\,|\,B} \cdot p_B = (5/6) \cdot (2/5) = 1/3$. Tasks from user 1 form a Bernoulli process and

$$\mathbf{P}(\text{first user 1 task occurs in slot 4}) = p_1(1-p_1)^3 = \frac{1}{3} \cdot \left(\frac{2}{3}\right)^3.$$

(b) This is the probability that slot 11 was busy and slot 12 was idle, given that 5 out of the 10 first slots were idle. Because of the fresh-start property, the conditioning information is immaterial, and the desired probability is

$$p_B \cdot p_I = \frac{5}{6} \cdot \frac{1}{6}.$$

(c) Each slot contains a task from user 1 with probability $p_1 = 1/3$, independent of other slots. The time of the 5th task from user 1 is a Pascal random variable of order 5, with parameter $p_1 = 1/3$. Its mean is given by

$$\frac{5}{p_1} = \frac{5}{1/3} = 15.$$

(d) Each busy slot contains a task from user 1 with probability $p_{1\,|\,B} = 2/5$, independent of other slots. The random variable of interest is a Pascal random variable of order 5, with parameter $p_{1\,|\,B} = 2/5$. Its mean is

$$\frac{5}{p_{1\,|\,B}} = \frac{5}{2/5} = \frac{25}{2}.$$

(e) The number T of tasks from user 2 until the 5th task from user 1 is the same as the number B of busy slots until the 5th task from user 1, minus 5. The number of busy slots ("trials") until the 5th task from user 1 ("success") is a Pascal random variable of order 5, with parameter $p_{1\,|\,B} = 2/5$. Thus,

$$p_B(t) = \binom{t-1}{4}\left(\frac{2}{5}\right)^5\left(1-\frac{2}{5}\right)^{t-5}, \qquad t = 5, 6, \ldots.$$

Since $T = B - 5$, we have $p_T(t) = p_B(t+5)$, and we obtain

$$p_T(t) = \binom{t+4}{4}\left(\frac{2}{5}\right)^5\left(1-\frac{2}{5}\right)^{t}, \qquad t = 0, 1, \ldots.$$

Using the formulas for the mean and the variance of the Pascal random variable B, we obtain

$$\mathbf{E}[T] = \mathbf{E}[B] - 5 = \frac{25}{2} - 5 = 7.5,$$

and

$$\text{var}(T) = \text{var}(B) = \frac{5\big(1 - (2/5)\big)}{(2/5)^2}.$$