

5. Parametric Hypothesis Testing - Asymptotic Test with Level Alpha

Clinical Trials - Conditions for Slutsky's Lemma for Plug-in

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Asymptotic test

- ▶ Assume that $m = cn$ and $n \rightarrow \infty$
- ▶ Using Slutsky's lemma, we also have

$$\frac{(d)}{n \rightarrow \infty} \rightarrow \mathcal{N}(0, 1)$$

where

$$\hat{\sigma}_d^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X}_n)^2 \quad \hat{\sigma}_c^2 = \frac{1}{m} \sum_{i=1}^m (Y_i - \bar{Y}_m)^2$$

- ▶ We get the the following test at asymptotic level α :

☐ (Caption will be displayed when you start playing the video.)

- ▶ This is a two-sided, two-sample test.

7/47

So let's start with the asymptotic test, OK.
When this did not happen, we were supposed to replace something.
And so what is our trick, what is our [? lemma ?] that allows us to replace sigma squared by sigma hats squared?
Slutsky's right?
OK.

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A Limit

1/1 point (graded)

Let X_1, \dots, X_n be i.i.d. test group samples distributed according to $\mathcal{N}(\Delta_d, \sigma_d^2)$ and let Y_1, \dots, Y_m be i.i.d. control group samples distributed according to $\mathcal{N}(\Delta_c, \sigma_c^2)$. Assume that $X_1, \dots, X_n, Y_1, \dots, Y_m$ are independent.

Let $m = \ln(n)$. Compute the following limit (in probability):

$$\lim_{n \rightarrow \infty} \frac{\sqrt{\frac{\widehat{\sigma}_d^2}{n} + \frac{\widehat{\sigma}_c^2}{m}}}{\sqrt{\frac{\sigma_d^2}{n} + \frac{\sigma_c^2}{m}}}$$

Enter **DNE** for does not exist, **inf** for $+\infty$, if applicable.

1

☐ Answer: 1 + 0*DNE + 0*inf

1

[STANDARD NOTATION](#)

Solution:

$$\lim_{n \rightarrow \infty} \frac{\sqrt{\frac{\widehat{\sigma}_d^2}{n} + \frac{\widehat{\sigma}_c^2}{m}}}{\sqrt{\frac{\sigma_d^2}{n} + \frac{\sigma_c^2}{m}}} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{\widehat{\sigma}_d^2}{n} + \frac{\widehat{\sigma}_c^2}{\ln(n)}}}{\sqrt{\frac{\sigma_d^2}{n} + \frac{\sigma_c^2}{\ln(n)}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt{\widehat{\sigma_d^2 \frac{\ln(n)}{n}} + \widehat{\sigma_c^2}}}{\sqrt{\sigma_d^2 \frac{\ln(n)}{n} + \sigma_c^2}},$$

which is equal to 1 (in probability) by Slutsky and continuous mapping theorem.

提交

你已经尝试了1次（总共可以尝试2次）

☐ Answers are displayed within the problem

Unbiased Estimator for Sample Variance

Recall the notion of sample covariance from [Lecture 10](#). We saw that the scaling by $\frac{1}{n-1}$ leads to an unbiased estimator for the covariance between two random variables. In the following video (and throughout the rest of this lecture), we will use the same scaling factor to refer to an unbiased estimator for the sample variance. That is,

$$\frac{1}{n-1} \sum_{i=1}^n \left(X_i - \bar{X}_n\right)^2$$

is an unbiased estimator for $\text{Var}(X)$, where X_1, \dots, X_n are i.i.d. samples distributed according to the distribution of X .

Clinical Trials - Plug-in Example and P-Value

Asymptotic test

▶ Assume that $m = cn$ and $n \rightarrow \infty$

▶ Using Slutsky's lemma, we also have

$$\frac{\bar{X}_n - \bar{Y}_m - (\Delta_d - \Delta_c)}{\sqrt{\frac{\hat{\sigma}_d^2}{n} + \frac{\hat{\sigma}_c^2}{m}}} \xrightarrow[n \rightarrow \infty]{(d)} \mathcal{N}(0, 1)$$

where

$$\hat{\sigma}_d^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X}_n)^2$$

$$\hat{\sigma}_c^2 = \frac{1}{m} \sum_{i=1}^m (Y_i - \bar{Y}_m)^2$$

▶ We get the the following test at asymptotic level α :

(Caption will be displayed when you start playing the video.)

▶ This is

-sided,

-sample test.

7/47

And q alpha is the quantile--
is the 1 minus alpha quantile of what?
Standard Gaussian.
OK?
Everybody remembers how to do this,
if you don't, please take immediate action.
OK.
So this test, remember, so when we're
talking about terminology
of tests, we had some slightly refined
terminology about a test.
So this is a blah cited blah sample test.
What is it?
Is it a-- one sided.
Why?
Right.
So one sided.

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讨论

主题: Unit 4 Hypothesis testing:Lecture 13: Chi Squared Distribution, T-Test / 5. Parametric Hypothesis Testing - Asymptotic Test with Level Alpha

显示讨论

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