

In this lecture, we discuss the simplest nontrivial stochastic process, the Bernoulli process. Time is divided into slots. At each slot, we have an independent trial such as a coin flip. And each trial results in heads or tails. Or in different language, each trial may or may not result in an arrival or a success.

Pretty much everything we will do will be a simple application of skills that we already have. For example, we already know that the number of arrivals in n time slots is described by the binomial PMF. We will then discuss some consequences of the independence of the different trials.

Basically, the process has no memory. Whatever happens in the future is not affected by whatever happened in the past. By exploiting this property, we will find the distribution of the time of the first arrival and more generally of the time of the k th arrival. We will also find the distribution of the time between consecutive arrivals.

Next, we will take two independent Bernoulli processes, let's say arrivals of men and arrivals of women, and merge them to get an overall arrival process. We will see that the merged process is also a Bernoulli process. We will also look at the reverse operation, namely, splitting a Bernoulli process into two separate processes.

Finally, we will look at a particular asymptotic regime in which we have a large number of time slots but a very small probability of an arrival during each time slot. We will carry out some algebraic manipulations. And we will see that the binomial PMF for the number of arrivals can be well approximated by the so-called Poisson PMF.