1. Defective Coin
Problem 1. Defective Coin

3/3 points (graded)

A defective coin minting machine produces coins whose probability of Heads is a random variable Q with PDF

$$f_Q(q) = \begin{cases} 5q^4, & \text{if } q \in [0,1], \\ 0, & \text{otherwise.} \end{cases}$$

A coin produced by this machine is tossed repeatedly, with successive tosses assumed to be independent. Let A be the event that the first toss of this coin results in Heads, and let B be the event that the second toss of this coin results in Heads.

1.

   $P(A) =$ | 5/6 | ✔ **Answer:** 5/6

   (Your answer should be a number.)

2. Find the conditional PDF of Q given event A. Express your answer in terms of q using standard notation.

   For $0 \le q \le 1$, $f_{Q|A}(q) =$ | 6*q^5 | ✔ **Answer:** 6*q^5

   $6 \cdot q^5$

3.

   $P(B \mid A) =$ | 5/6 | ✔ **Answer:** 6/7

   (Your answer should be a number.)

STANDARD NOTATION

Solution:

   1. To calculate $P(A)$, we use the continuous version of the total probability theorem:

$$\mathbf{P}(A) = \int_0^1 \mathbf{P}(A \mid Q = q) f_Q(q)\, dq = \int_0^1 q \cdot (5q^4)\, dq = \left[\frac{5}{6} q^6\right]_0^1 = \frac{5}{6}.$$

2. Using Bayes' rule,

$$
\begin{aligned}
f_{Q|A}(q) &= \frac{\mathbf{P}(A \mid Q = q) f_Q(q)}{\mathbf{P}(A)} \\
&= \begin{cases} \dfrac{q \cdot (5q^4)}{5/6}, & \text{if } 0 \le q \le 1, \\ 0, & \text{otherwise}, \end{cases} \\
&= \begin{cases} 6q^5, & \text{if } 0 \le q \le 1, \\ 0, & \text{otherwise}. \end{cases}
\end{aligned}
$$

3. Again, we use the continuous version of the total probability theorem:

$$
\begin{aligned}
\mathbf{P}(B \mid A) &= \int_0^1 \mathbf{P}(B \mid A, Q = q) f_{Q|A}(q)\, dq \\
&= \int_0^1 \mathbf{P}(B \mid Q = q) f_{Q|A}(q)\, dq \\
&= \int_0^1 q(6q^5)\, dq \\
&= 6/7.
\end{aligned}
$$

The second equality holds because for a given value q of Q, the events A and B are (conditionally) independent.

提交   You have used 3 of 3 attempts

ⓘ   Answers are displayed within the problem

讨论

显示讨论

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