### ☐ Composition of Functions

# 9. Worked Example: Concavity and Composition of Functions **Worked Example: Hessian and Concavity**

### Multivariate concave functions

More generally for a *multivariate* function:  $h:\Theta\subset\mathbb{R}^d\to\mathbb{R}$ ,

$$a \ge 2$$
, define the  $b = a \ge 2$ , define the gradient vector:  $∇h(θ) = a \ge 2$ , define the  $b \ge 2$ , define  $b \ge 2$ .

$$\mathbf{H}h(\theta) = \begin{pmatrix} \frac{\partial^2 h}{\partial \theta_1 \partial \theta_1}(\boldsymbol{\ell} & \boldsymbol{\theta}) \\ \frac{\partial^2 h}{\partial \theta_d \partial \theta_d}(\boldsymbol{\theta}) & \frac{\partial^2 h}{\partial \theta_d \partial \theta_d}(\boldsymbol{\theta}) \end{pmatrix} \in \mathbb{R}^{d \times d}$$

$$h \text{ is concave } \Leftrightarrow x^\top \mathbf{H}h(\theta)x \leq 0 \quad \forall x \in \mathbb{R}^d, \ \theta \in \Theta.$$

(Caption will be displayed when you start playing the video.)

$$\blacktriangleright \ \Theta = {\rm I\!R}^2$$
 ,  $h(\theta) = -\theta_1^2 - 2\theta_2^2$  or  $h(\theta) = -(\theta_1 - \theta_2)^2$ 

$$\Theta = (0, \infty), h(\theta) = \log(\theta_1 + \theta_2),$$

Start of transcript. Skip to the end.

How about the second one?

Should we do the second one?

So the second one is log of theta 1 plus theta 2.

OK?

So the gradient-- so that's h of theta.

So gradient h of theta is--

well, it's 1 over theta 1 plus theta 2.

The other one is 1 over theta 1 plus theta 2.

## 视频

下载视频文件

下载 SubRip (.srt) file

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# Combination of Convex functions

2/3 points (graded)

Let  $f_1, f_2$  be convex functions on  $\mathbb{R}$ .

Determine if the following functions are necessarily convex or concave.

*Hint:* Recall a function  $g:I o\mathbb{R}$  is convex in the interval I is an interval, if for all pairs of real numbers  $x_1< x_2\in I$ 

$$g\left(tx_{1}+\left(1-t\right)x_{2}
ight)\leq tg\left(x_{1}
ight)+\left(1-t
ight)g\left(x_{2}
ight) \qquad ext{ for all } 0\leq t\leq 1.$$

•  $3f_1 + 2f_2$ :

Concave

Convex

Cannot be determined without more information

•  $-10f_1$ :

Convex

● Concave □
Cannot be determined without more information
• $f_2f_1$ :
● Convex □
<ul><li>Concave</li></ul>
$lacksquare$ Cannot be determined without more information $\Box$
Solution:
Given $f_1, f_2$ are convex, we have
$f_{1}\left(tx_{1}+\left(1-t ight)x_{2} ight)\leq tf_{1}\left(x_{1} ight)+\left(1-t ight)f_{1}\left(x_{2} ight)\qquad ext{for all }0\leq t\leq1$
and the same holds for $f_2$ .
$ullet$ The same inequality holds for $g=3f_1+2f_2$ :
$egin{array}{lll} g\left(tx_{1}+\left(1-t ight)x_{2} ight) &=& 3f_{1}\left(tx_{1}+\left(1-t ight)x_{2} ight)+2f_{2}\left(tx_{1}+\left(1-t ight)x_{2} ight) \ &\leq& 3\left(tf_{1}\left(x_{1} ight)+\left(1-t ight)f_{1}\left(x_{2} ight) ight)+2\left(tf_{2}\left(x_{1} ight)+\left(1-t ight)f_{2}\left(x_{2} ight) ight) \ &=& t\left(g\left(x_{1} ight)+\left(1-t ight)g\left(x_{2} ight) ight). \end{array}$
Hence $3f_1+2f_2$ is also convex.
<b>Remark:</b> In general, any function $c_1f_1+c_2f_2$ where $c_1,c_2>0$ is convex of $f_1,f_2$ are.
$ullet$ $-10 f_1$ is concave, because it is negative of a convex function.
• $f_1f_2$ is not necessary convex For example, is $f_1(x)=x$ , and $f_2=x^2$ , then $(f_1f_2)(x)=x^3$ which is neither convex nor concave. Other examples of $f_1$ and $f_2$ , e.g. $f_1=f_2=x^2$ will lead to $f_1f_2$ being convex.
提交 你已经尝试了1次 (总共可以尝试1次)
☐ Answers are displayed within the problem

讨论

显示讨论

主题: Unit 3 Methods of Estimation:Lecture 9: Introduction to Maximum Likelihood Estimation / 9. Worked Example: Concavity and Composition of Functions