10. Identifiability

Preparation: Injectivity

1/1 point (graded)

The notation $f : S \to T$ denotes that f is a function, also called a **map** , defined on all of a set S and whose outputs lie in a set T. A function $f : S \to T$ is **injective** if for all $x, y \in S$, $f(x) = f(y)$ implies that $x = y$.

Alternatively: a function is injective if we can **uniquely** recover some input x based on an output $f(x)$.

Which of the following functions are injective? (Choose all that apply.)

- [x] $f_1 : \mathbb{R} \to \mathbb{R}$, given by $f_1(x) = x$. ✔
- [ ] $f_2 : \mathbb{R} \to \mathbb{R}$, given by $f_2(x) = x^2$.
- [ ] $f_3 : \mathbb{R} \to \mathbb{R}$, given by $f_3(x) = \sin(x)$.
- [x] $f_4 : [0,1] \to \{\text{probability distributions on } \{0,1\}\}$, given by $f_4(p) = \text{Ber}(p)$. ✔

✔

Solution:

The first choice $f_1(x) = x$ is the identity function, so if $f_1(x) = f_1(y)$, then $x = y$ by definition of f_1. So f_1 is injective.

The second choice $f_2(x) = x^2$ is not injective because, for example, both $+1$ and -1 map to the same value, 1, after applying f_2. In general, if $f_2(x) = c$ for some constant $c > 0$, then there are two possible choices for x: either $x = \sqrt{c}$ or $x = -\sqrt{c}$.

The third choice $f_3(x) = \sin(x)$ is not injective. In fact, there are infinitely many points x such that $f_3(x) = 0$. Recall from trigonometry that all values in the set $\{2\pi x : x \in \mathbb{Z}\}$ will map to 0 after applying f_3.

The fourth choice $f_4(p) = \text{Ber}(p)$ is injective: if $p \in [0,1]$, then $f_4(p) = \text{Ber}(p)$, so that p specifies the probability that $X \sim \text{Ber}(p)$ is equal to 1. Since a distribution on $\{0,1\}$ is uniquely determined by $P(X = 1)$, the map f_4 is injective.

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Answers are displayed within the problem

Identifiability

Start of transcript. Skip to the end.

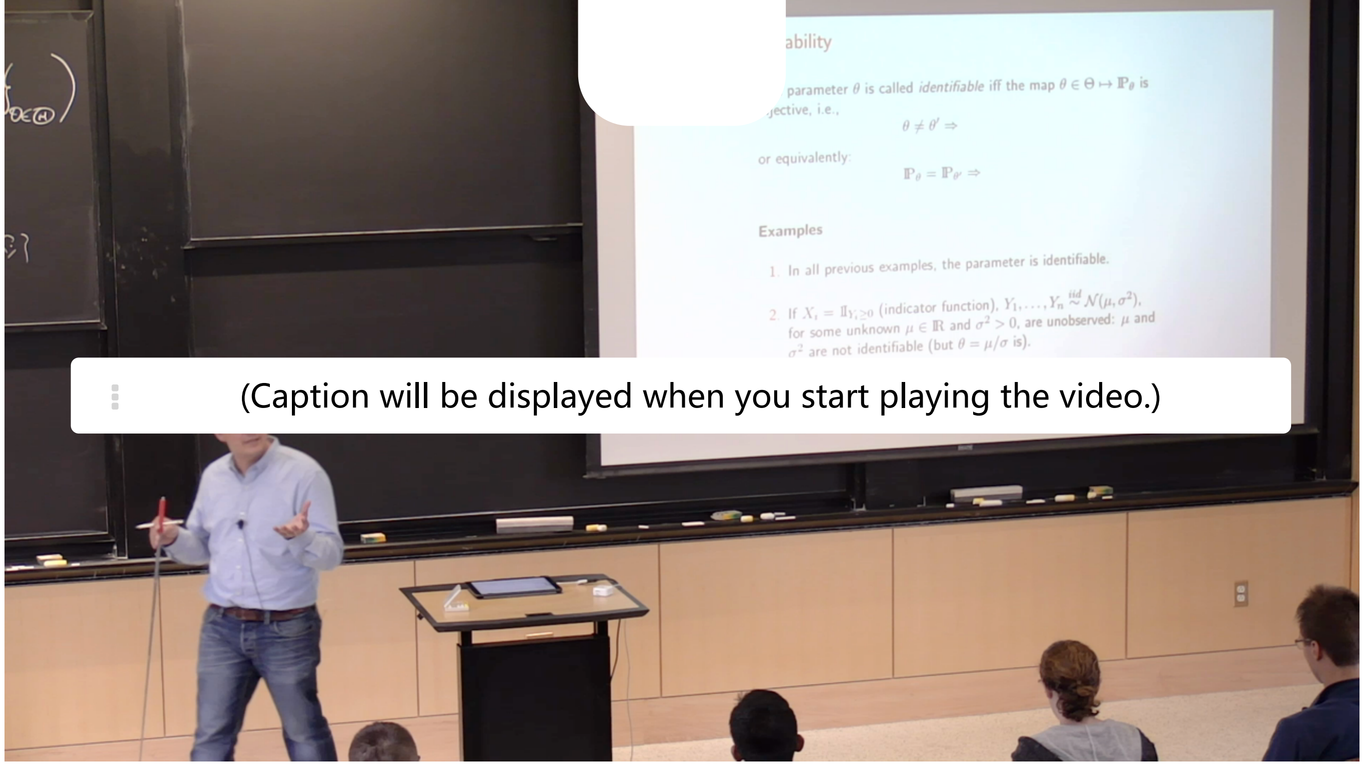
OK, so back from the real word into math.

So we're going to use word like injectivity.

And so this is actually an important thing,

just like the first thing we said

about a model was whether it was well-specified.



P is in the class of models that I'm considering

or not-- well, specified, which I called mispecified.

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Identifiability of Statistical Models

1/1 point (graded)
Let $\{P_\theta\}_{\theta\in\Theta}$ denote a family of distributions that depends on an unknown parameter $\theta \in \Theta$.

Recall that the parameter θ is **identifiable** if the map $\theta \mapsto P_\theta$ is injective. Here, the notation $\theta \mapsto P_\theta$ denotes a function that takes as input $\theta \in \Theta$ and outputs a probability distribution P_θ. In other words, if $\theta \neq \theta'$ (and both in Θ), then $P_\theta \neq P_{\theta'}$

Which of the following families of distributions has an identifiable parameter? (Choose all that apply.)

- [x] $\{\text{Ber}(p)\}_{p\in[0,1]}$ ✔
- [ ] $\{\text{Ber}(p^2)\}_{p\in[-1,1]}$
- [x] $\{\text{Ber}(\sin(p))\}_{p\in[0,\frac{\pi}{2}]}$ ✔
- [ ] $\{\text{Ber}(\sin(p))\}_{p\in[0,\pi]}$

✔

Solution:

Remark: A family of distributions $\{\text{Ber}(f(p))\}_{p\in S}$ (here $S \subset \mathbb{R}$ is a set where the parameter p lives) has the parameter p identified if and only if the function $f(p)$ is injective.
The function $f(p) = p$ is injective on the interval $[0,1]$, so the first choice $\{\text{Ber}(p)\}_{p\in[0,1]}$ is correct. However, the function $f(p) = p^2$ on the interval $[-1,1]$ is not injective, so the second choice $\{\text{Ber}(p^2)\}_{p\in[-1,1]}$ is incorrect.
Let's look more carefully at the last two choices, $\{\text{Ber}(\sin(p))\}_{p\in[0,\frac{\pi}{2}]}$ and $\{\text{Ber}(\sin(p))\}_{p\in[0,\pi]}$. Observe that the function $f(p) = \sin(p)$ *is* injective on the interval $[0,\frac{\pi}{2}]$ but *is not* injective on the interval $[0,\pi]$. Hence, $\{\text{Ber}(\sin(p))\}_{p\in[0,\frac{\pi}{2}]}$ has an identified parameter, but $\{\text{Ber}(\sin(p))\}_{p\in[0,\pi]}$ does not have an identified parameter.

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Answers are displayed within the problem

讨论

显示讨论

主题： Unit 2 Foundation of Inference:Lecture 3: Parametric Statistical Models / 10. Identifiability