3. The Canonical Link Function The Canonical Link Function and Bernoulli Example

Example: the Bernoulli distribution

We can check that

$$b(\theta) = \log(1 + e^{\theta})$$

Hence we solve



$$b'(\theta) = \frac{\exp(\theta)}{1 + \exp(\theta)} = \mu \qquad \Leftrightarrow \qquad \theta = \log\left(\frac{P}{1 - P}\right)$$

► The canonical link for the Bernoulli distribution is the

(Caption will be displayed when you start playing the video.)

16:13 / 17:39

▶ 1.0x

X

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Canonical Link function for the Binomial Distribution

1/1 point (graded)

The binomial distribution, with distribution function

$$f_p\left(x
ight) = inom{n}{x} p^x (1-p)^{n-x}$$

can be written as a canonical exponential family, as long as n is a fixed number. For this problem, plug in n=1000.

What is the canonical link function $g\left(\mu\right)$? (With the understanding that $\mu=np$)

ln((mu)/(1000-mu))

✓ Answer: In(mu/(1000-mu))

$$\ln\left(rac{\mu}{1000-\mu}
ight)$$

STANDARD NOTATION

Solution:

For the binomial distribution, $b\left(heta ight)=n\ln\left(e^{ heta}+1 ight)$ if we use the canonical parameter $ heta=\log\left(rac{p}{1-p} ight)$. Therefore, the canonical link	is
$g\left(\mu ight)=\left(b' ight)^{-1}\left(\mu ight)$. A direct computation yields $b'\left(heta ight)=rac{ne^{ heta}}{e^{ heta}+1}$, and so $g\left(\mu ight)=\ln\left(rac{\mu}{n-\mu} ight)$.	

Remark: In some texts, you might see $g(\mu) = \ln\left(\frac{\mu}{1-\mu}\right)$, the logit of μ instead of what we derived. This is due to a re-normalization convention where we think of the likelihood of $\overline{x} = x/n$, so that the mean of \overline{x} is p instead of np. Notice that if you plug in $\mu = np$ into our expression, the n's cancel and we end up with the logit of p, which gives the alternate convention.

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You have used 3 of 3 attempts

• Answers are displayed within the problem

Discussion

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