7. Matrices and Vectors

Objectives:

- Recognize the dimensions of the product of two or more matrices.
- (Optional) Understand the concept of rank of a matrix, and how it relates to the invertibility of an $n \times n$ matrix.
- (Optional) Understand the concept of **eigenvalues** and **eigenvectors** of an $n \times n$ matrix.

Matrix Multiplication

6/6 points (graded)

Let $\mathbf{A} = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -4 \end{pmatrix}$ and let $\mathbf{B} = \begin{pmatrix} -1 & 0 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}$. The dimensions of the product $\mathbf{AB}$ are:

2 ✔ **Answer:** 2 rows × 3 ✔ **Answer:** 3 columns.

More generally, let $\mathbf{A}$ be an $m \times n$ matrix and $\mathbf{B}$ be an $n \times k$ matrix. What is the size of $\mathbf{AB}$?

m ✔ **Answer:** m rows × k ✔ **Answer:** k columns.

In addition, if $\mathbf{C}$ is a $k \times j$ matrix, what is the size of $\mathbf{ABC}$?

m ✔ **Answer:** m rows × j ✔ **Answer:** j columns.

Solution:

The size of the output is the number of rows of the left matrix, and the number of columns of the right matrix. The two dimensions on the inside (columns of the left matrix, rows of the right matrix) must match.
In the first part, $\mathbf{AB}$ is 2×3.
For the second and third parts, $\mathbf{AB}$ is $m \times k$ and $\mathbf{ABC}$ is $m \times j$.

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Answers are displayed within the problem

Vector Inner product

1/1 point (graded)

Suppose $\mathbf{u} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. The product $\mathbf{u}^T\mathbf{v}$ evaluates the **inner product** (also called the **dot product**) of $\mathbf{u}$ and $\mathbf{v}$, which evaluates to

$\mathbf{u}^T\mathbf{v} =$ 2 ✔ **Answer:** 2

The inner product of $\mathbf{u}$ and $\mathbf{v}$ is sometimes written as $\langle \mathbf{u}, \mathbf{v} \rangle$.

Solution:

The inner product is always a scalar (a 1×1 matrix). In this case, it evaluates to $1 \cdot -1 + 3 \cdot 1 = 2$. In general, if $\mathbf{u} = (u_1, u_2, \dots, u_n)^T$ and $\mathbf{v} = (v_1, v_2, \dots, v_n)^T$, then $\mathbf{u}^T\mathbf{v} = \sum_{i=1}^{n} u_i v_i$.

$$\begin{pmatrix} u_1 & \cdots & u_n \end{pmatrix} \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = (\cdot)$$

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Answers are displayed within the problem

Vector Outer product

4/4 points (graded)

Suppose $\mathbf{u} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. The product $\mathbf{u}\mathbf{v}^T$ evaluates the **outer product** of $\mathbf{u}$ and $\mathbf{v}$, which is a 2×2 matrix in this case.

What is $(\mathbf{u}\mathbf{v}^T)_{1,1}$?

-1 ✔ Answer: -1

What is $(\mathbf{u}\mathbf{v}^T)_{1,2}$?

1 ✔ Answer: 1

What is $(\mathbf{u}\mathbf{v}^T)_{2,1}$?

-3 ✔ Answer: -3

What is $(\mathbf{u}\mathbf{v}^T)_{2,2}$?

3 ✔ Answer: 3

Solution:

In this case, the outer product evaluates to

$$\mathbf{u}\mathbf{v}^T = \begin{pmatrix} -1 & 1 \\ -3 & 3 \end{pmatrix}.$$

In general, if $\mathbf{u} = \begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$, $\mathbf{u}\mathbf{v}^T$ is an $m \times n$ matrix whose (i,j) entry is $(\mathbf{u}\mathbf{v}^T)_{i,j} = u_i v_j$.

提交 你已经尝试了2次（总共可以尝试3次）

Answers are displayed within the problem

讨论

显示讨论