2. Discrete random variables

Normalization constant for the Poisson distribution

1/1 point (graded)
The probability mass function (pmf) of a **Poisson distribution** with parameter λ is given by

$$\mathrm{Poi}\,(\lambda) = \frac{c\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots.$$

Compute the value of c.

$c =$ e^(-lambda) ✔ **Answer:** exp(-lambda)

$e^{-\lambda}$

STANDARD NOTATION

Solution:

In order to obtain a probability distribution, we must have

$$c \sum_{k=1}^{\infty} \frac{\lambda^k}{k!} = 1. \tag{1.1}$$

But

$$\sum_{k=1}^{\infty} \frac{\lambda^k}{k!} = \exp(\lambda) \tag{1.2}$$

by the series definition of the exponential function. Hence,

$$c = \exp(-\lambda).$$

: Probability axioms in lecture 1, *Probability models and axioms*.

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

Moments of Bernoulli variables

3/3 points (graded)
Recall that a **Bernoulli random variable with parameter** p is a random variable that takes the value 1 with probability p, and the value 0 with probability $1 - p$.

Let X be a Bernoulli random variable with parameter 0.7. Compute the **expectation values** of X^k, denoted by $\mathbb{E}[X^k]$, for the following three values of k: $k = 1, 4$, and 3203.

$\mathbb{E}[X] =$ 0.7 ✔ **Answer:** 0.7

$\mathbb{E}[X^4] =$ 0.7 ✔ **Answer:** 0.7

$\mathbb{E}[X^{3203}] =$ 0.7 ✔ **Answer:** 0.7

STANDARD NOTATION

Solution:

Remember, the expectation of a discrete random variable is

$$\mathbb{E}[X] = \sum_{j \in \mathrm{range}(X)} j\,\mathbf{P}(X = j),$$

total expectation theorm

while the higher moments are

$$\mathbb{E}[X^n] = \sum_{j \in \mathrm{range}(X)} j^n\,\mathbf{P}(X = j),$$

0和1的多少次方都没有变化

For a Bernoulli random variable with parameter p, the range is $\{0, 1\}$, and $0^k = 0$, $1^k = 1$ for all $k \geq 1$, so all moments are equal to the first one,

$$\mathbb{E}[X] = 0 \times (1 - p) + 1 \times p = p,$$

and we get the result by plugging in $p = 0.7$.
: Expectation in lecture 5, *Probability mass functions and expectations*.

提交 你已经尝试了3次（总共可以尝试3次）

Answers are displayed within the problem

Variance of Bernoulli variables

3/3 points (graded)
Let X be a Bernoulli random variable with parameter $p \in [0, 1]$. Compute the **variance** of X, which is denoted by $\mathsf{Var}[X]$.

$\mathsf{Var}[X] =$ p*(1-p) ✔ **Answer:** p*(1-p)

$p \cdot (1 - p)$

What value(s) of the parameter p maximize the variance? What values minimize it?

(For each question, enter the values of p as a list of **numbers**, separated by commas. For example, to enter the set $\{0.2, 0.3\}$, type **0.2, 0.3**.)

The values of p for which $\mathsf{Var}[X]$ is minimized: 0,1 ✔ **Answer:** 0, 1

The values of p for which $\mathsf{Var}[X]$ is maximized: 0.5 ✔ **Answer:** 1/2

STANDARD NOTATION

Solution:

Recall from the previous exercise that $\mathbb{E}[X^n] = p$ for all positive integers n. Therefore, the variance is

$$\text{Var}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = p - p^2 = p(1-p).$$

This is a quadratic polynomial with negative leading factor, hence it does not attain a global minimum on $\mathbb{R}$. For the range $p \in [0,1]$ in question, its minima are attained at both boundary points $p = 0$ and $p = 1$. Its maximum can be found by differentiating and setting the derivative equal to zero. It occurs at $p = \frac{1}{2}$.
: Variance in lecture 6, *Variance; conditioning on an event; multiple random variable*.

提交 你已经尝试了2次（总共可以尝试3次）

Answers are displayed within the problem

Sum of Bernoulli variables

1/1 point (graded)
Given n i.i.d. realizations $X_1, \ldots, X_n \sim \text{Ber}(p)$, what is the distribution of $\sum_{i=1}^n X_i$?

- ○ Poisson with parameter pn
- ○ Gamma with parameters n and p
- ◉ Binomial with parameters n and p ✔
- ○ Bernoulli with parameter pn

STANDARD NOTATION

Solution:

We know from probability theory that $\sum_{i=1}^n X_i$ follows a Binomial distribution with parameters n and p.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

Discrete uniform random variables

2/2 points (graded)
Recall that a **uniform random variable** is a random variable that takes values with equal probability,

Let X be a uniform random variable in the finite set $\{1, 2, \ldots, 20\}$.

Compute the following quantities.

The probability that X is an even number:

$\mathbf{P}(X \text{ is an even number}) =$ 1/2 ✔ **Answer:** 1/2

The probability that X is a prime number:

$\mathbf{P}(X \text{ is a prime number}) =$ 2/5 ✔ **Answer:** 2/5

STANDARD NOTATION

Solution:

There are 10 even numbers in $\{1,\ldots,20\}$, therefore

$$\mathbf{P}\left(X \text{ is an even number}\right)=\frac{10}{20}=\frac{1}{2}.$$

There are 8 prime numbers in $\{1,\ldots,20\}$, (namely $\{2,3,5,7,11,13,17,19\}$, so

$$\mathbf{P}\left(X \text{ is a prime number}\right)=\frac{8}{20}=\frac{2}{5}.$$

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提交 你已经尝试了2次（总共可以尝试2次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 0. Course Overview, Syllabus, Guidelines, and Homework on Prerequisites:Homework 0: Probability and Linear algebra Review / 2. Discrete random variables

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