

Let us now look at some numerical examples to get a feel for how the central limit theorem works in practice. Let us look at a discrete random variable that has a uniform distribution in the range here from 1 to 8. If we add two independent random variables, drawn from this PMF, we obtain a random variable whose PMF is the convolution of this PMF with itself. We can even carry out this calculation by hand, and we get a triangular PMF. So this is what we get for the case where n is equal to 2.

Now we can keep doing this. If we add four of these discrete uniforms, of course assumed independent, then we obtain a PMF that starts to have a shape close to that of a normal shape. And if you take as many as 32 of them, then the PMF of the sum of 32 discrete uniforms is almost identical to the shape that you would get if you were to draw a normal PDF.

So with n as small as 32, we have essentially converged. In fact, this convergence is so good that in practice, quite often people use this idea to generate random samples of a normal random variable. What do you do? You draw 32 random samples from a discrete uniform, add them up. And what you get is a sample of essentially a normal random variable.

Now in this example, things worked out well for us because the distribution that we started from was nicely symmetric and didn't have any strange features. Things are not always so favorable. Let us consider starting from a truncated geometric.

If we add eight random variables that are independent and drawn from this distribution, what we obtain is a PMF of this form, which does not really look like a normal shape. The reason is that there's a pronounced asymmetry. So let us add more and more independent X 's.

If we add 16 of them, we start to get something that's a little closer to normal. But the asymmetry is still visible. And if we add 32 of them, we can still see some asymmetry. Namely, this tail here does not look exactly like this tail out there. So in this instance, it's really the asymmetry of the original distribution that makes it difficult to converge. And it will take larger values of n before we can get a very accurate approximation.