6. Optimization and gradients

Gradient ascent/descent methods are typical tools for maximizing/minimizing functions. Consider the function $L(x, \theta)$, where $x = [x_1, x_2]^T$ and $\theta = [\theta_1, \theta_2]^T$. We want to select θ such that we maximize/minimize the value of L.

6. (a)

1/1 point (graded)

The gradient $\nabla_\theta L(x, \theta)$ is a vector with two components:

$$\frac{\partial}{\partial \theta_j} L(x, \theta), j = 1, 2.$$

Let $L(x, \theta) = \log(1 + \exp(-\theta \cdot x))$. Evaluate the gradient. Which of the following is its j^{th} component?

- $\dfrac{\exp(-\theta \cdot x)}{1 + \exp(-\theta \cdot x)}$
- $\dfrac{-x_j \exp(-\theta \cdot x)}{1 + \exp(-\theta \cdot x)}$ ✔
- $\dfrac{-x_j}{1 + \exp(-\theta \cdot x)}$

Note on notation: In this course, we will sometimes abuse notation and use x_j to mean the **vector** whose j^{th} component is x_j (roughly, "x_j for the whole range of j").

STANDARD NOTATION

Solution:

The derivative of $\log(x) = \frac{1}{x}$ and the derivative of $e^{cx} = ce^{cx}$. Applying these rules with the chain rule gives the correct answer.

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Answers are displayed within the problem

6. (b)

0/1 point (graded)

The direction of the derivative of a function gives us the direction of the change in the function with changes in its variables. Under stochastic gradient ascent/descent methods, we make an educated guess about the next values of the variables to try. This corresponds to intelligently choosing values for θ in $L(x, \theta)$. Given $\theta' = \theta + \epsilon \cdot \nabla_\theta L(x, \theta)$, where ϵ is a small positive real number, is the value of $L(x, \theta')$ greater or smaller than the value of $L(x, \theta)$?

- greater ✔
- smaller ✖

STANDARD NOTATION

Solution:

Consider the one-dimensional case. If the gradient is positive, we obtain θ' by moving from θ in the positive direction. This increases $L(x, \theta)$. If the gradient is negative, we move in the negative direction, again increasing $L(x, \theta)$. This analysis extends to higher dimensions. Note that if we used the function above to continue updating θ, we would (in theory) maximize $L(x, \theta)$. Alternatively if our update rule was $\theta' = \theta - \epsilon \cdot \nabla_\theta L(x, \theta)$, we would minimize the function. There are more complications in higher dimensions, but this is the basic idea behind stochastic gradient descent, which forms the backbone of modern machine learning.

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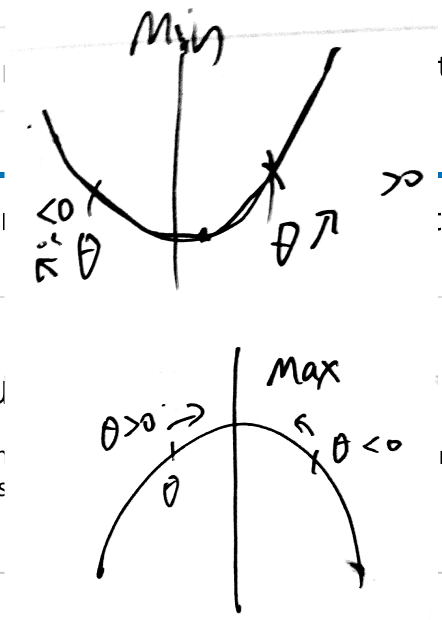

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