

Where to from here?

Hodgkin-Huxley

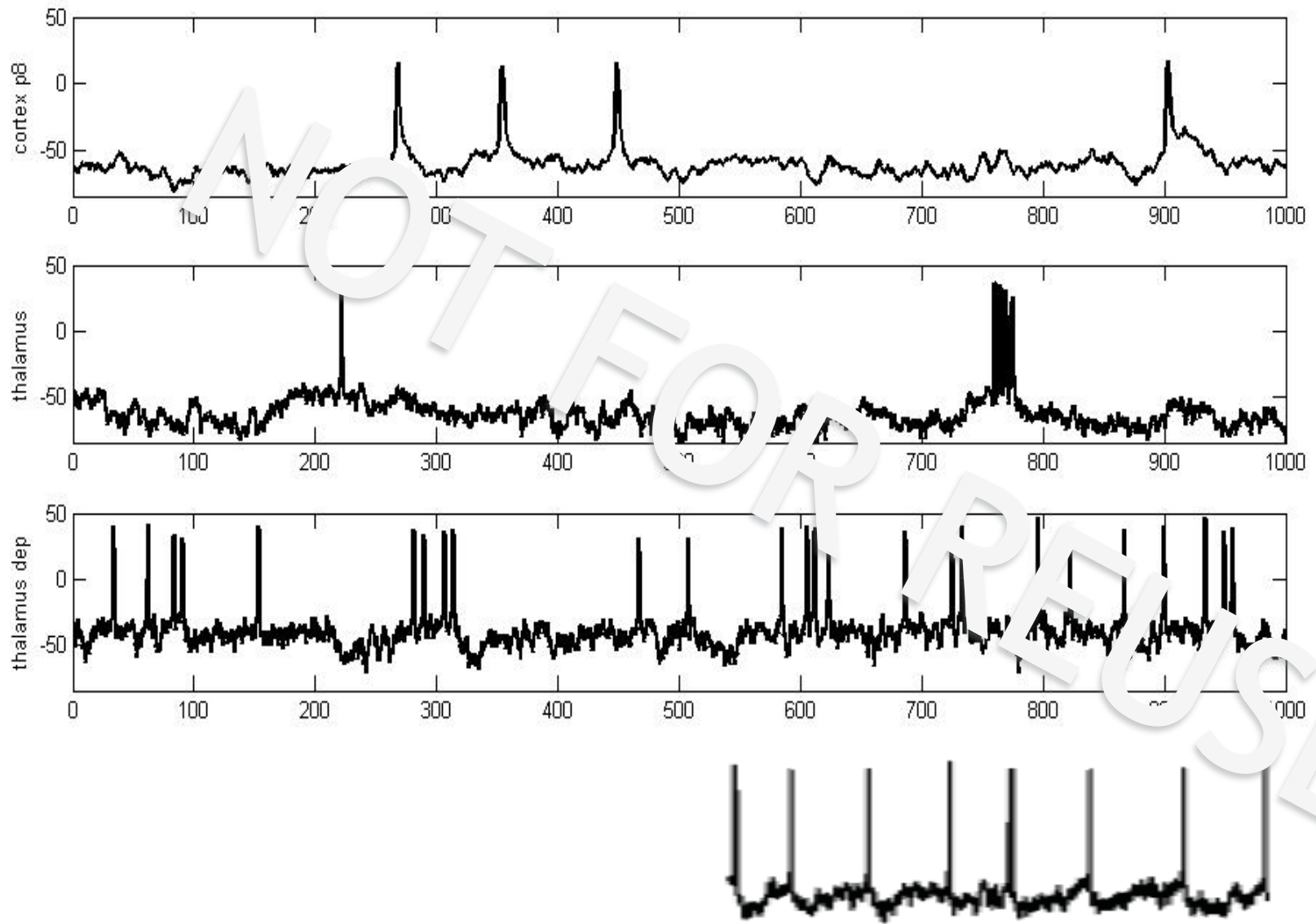
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graph TD; A([Hodgkin-Huxley]) --> B([Biophysical realism<br/>Ion channel physics<br/>Additional channels<br/>Geometry]); A --> C([Simplified models<br/>Fundamental dynamics<br/>Analytical tractability]);
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The diagram illustrates a conceptual split from the Hodgkin-Huxley model. A central red oval labeled 'Hodgkin-Huxley' has two green arrows pointing downwards to two separate red ovals. The left oval, 'Biophysical realism', lists 'Ion channel physics', 'Additional channels', and 'Geometry'. The right oval, 'Simplified models', lists 'Fundamental dynamics' and 'Analytical tractability'.

Biophysical realism
Ion channel physics
Additional channels
Geometry

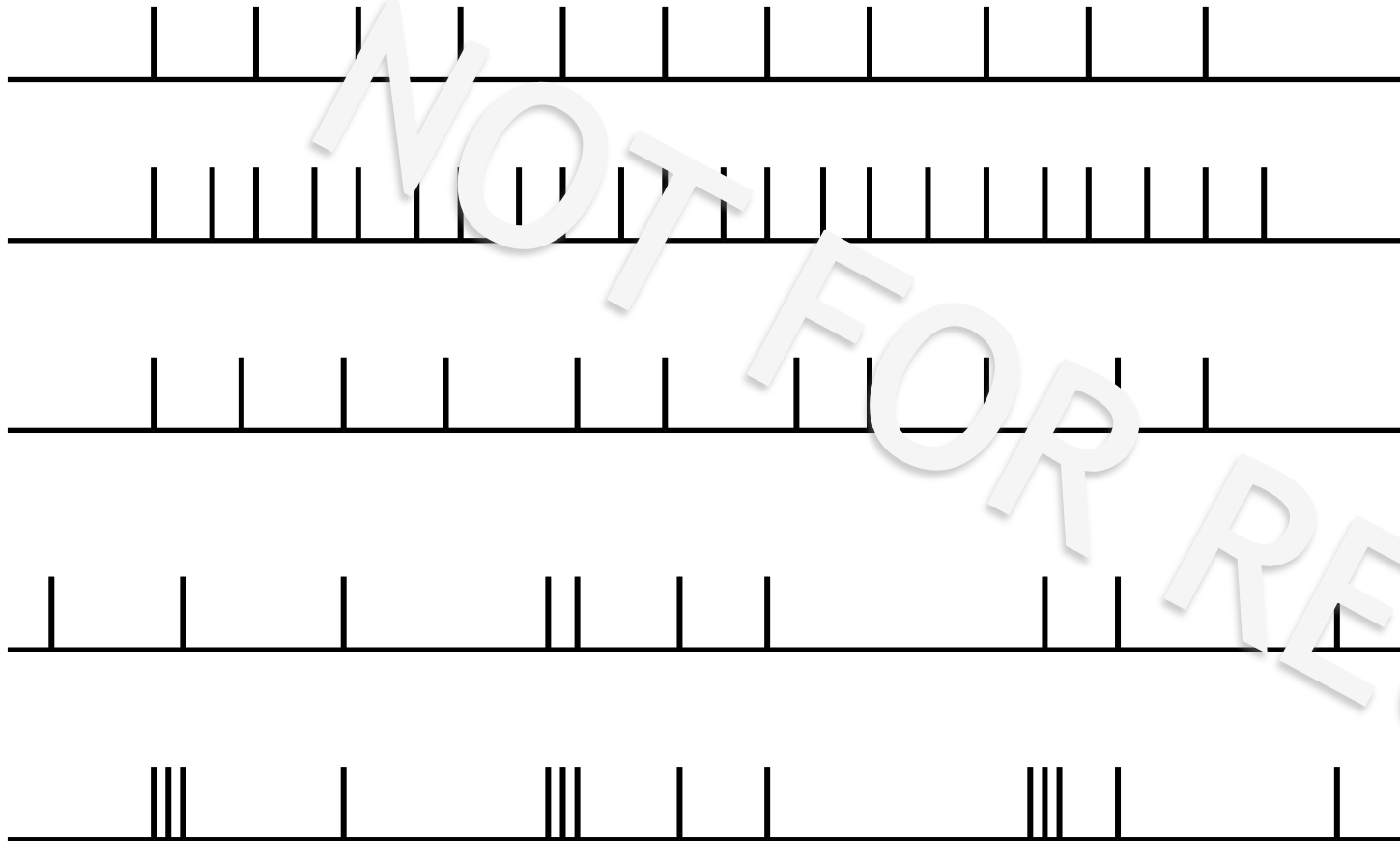
Simplified models
Fundamental dynamics
Analytical tractability

The electric personalities of neurons



Neuron, what are you trying to tell us?

We see that neurons can have a wide range of firing patterns, which come about partly because of the nature of their dynamics, and partly because of the nature of their inputs.



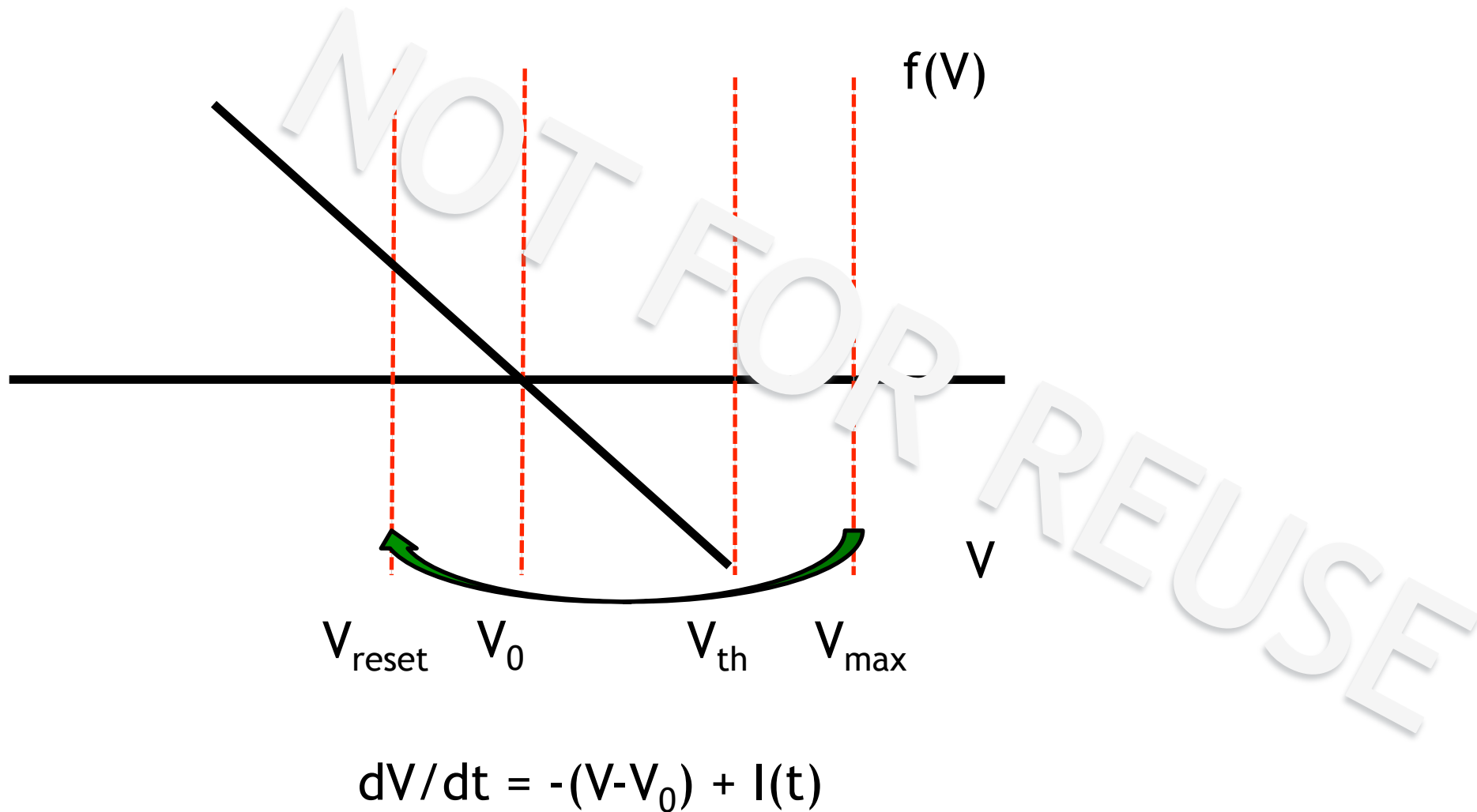
频率不同

频率相同

有一个模式

Capturing the basic dynamics of neurons

Capturing the basic dynamics of neurons



The integrate-and-fire neuron

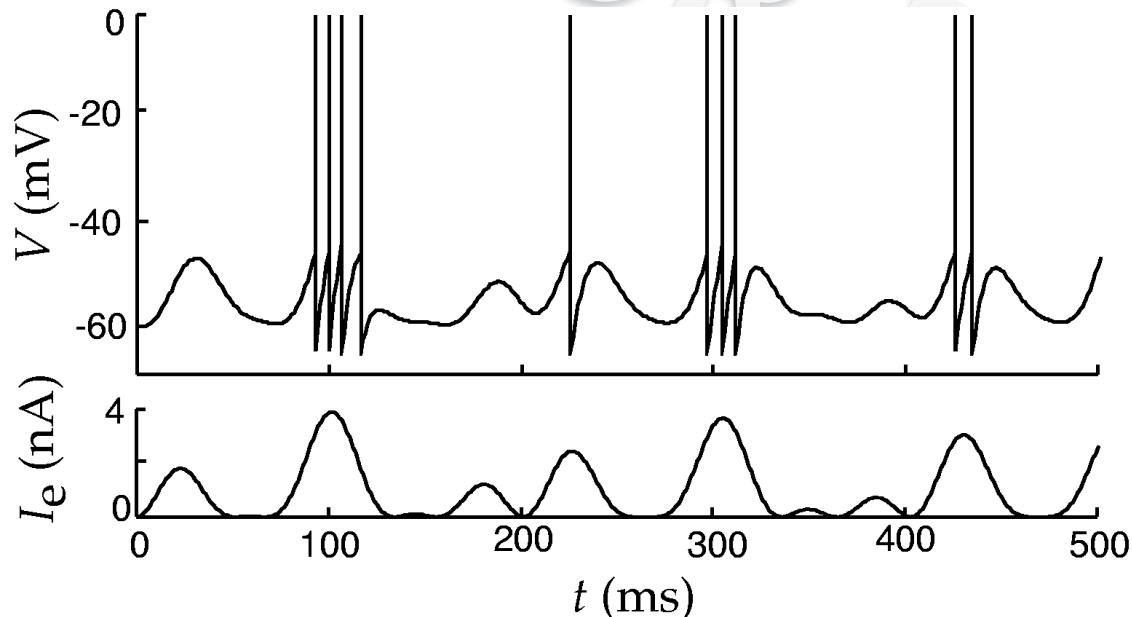
Like a passive membrane:

$$C_m \frac{dV}{dt} = -g_L(V - E_i) - I_e$$

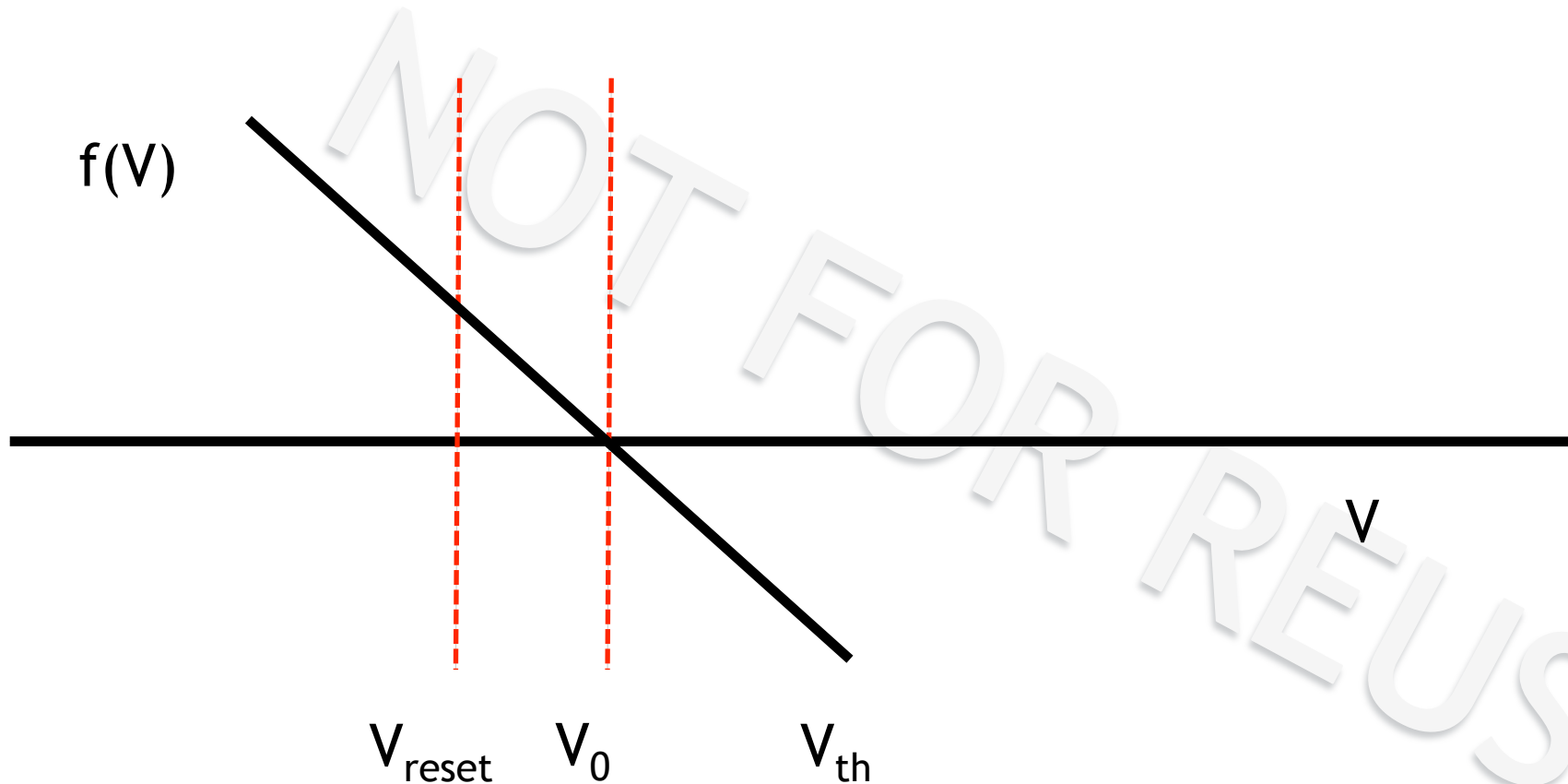
but with the additional rule that

when $V \rightarrow V_T$, a spike is fired
and $V \rightarrow V_{\text{reset}}$.

E_L is the resting potential of the “cell”.



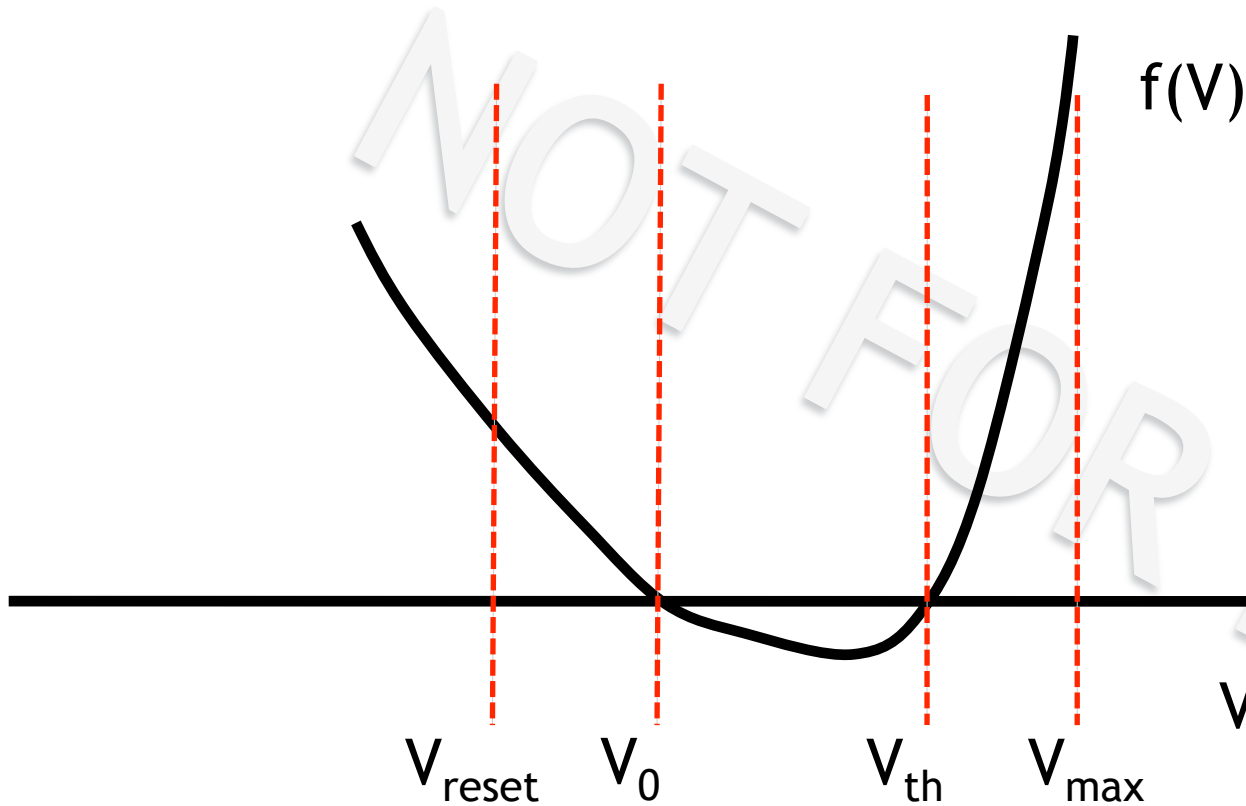
How do we make it excitable?



这里我们需要借助“外力”让他们firing and resting
我们需要找到一个数学公式内在地刻画，不需要外力

$$dV/dt = -a(V - V_0) + I(t)$$

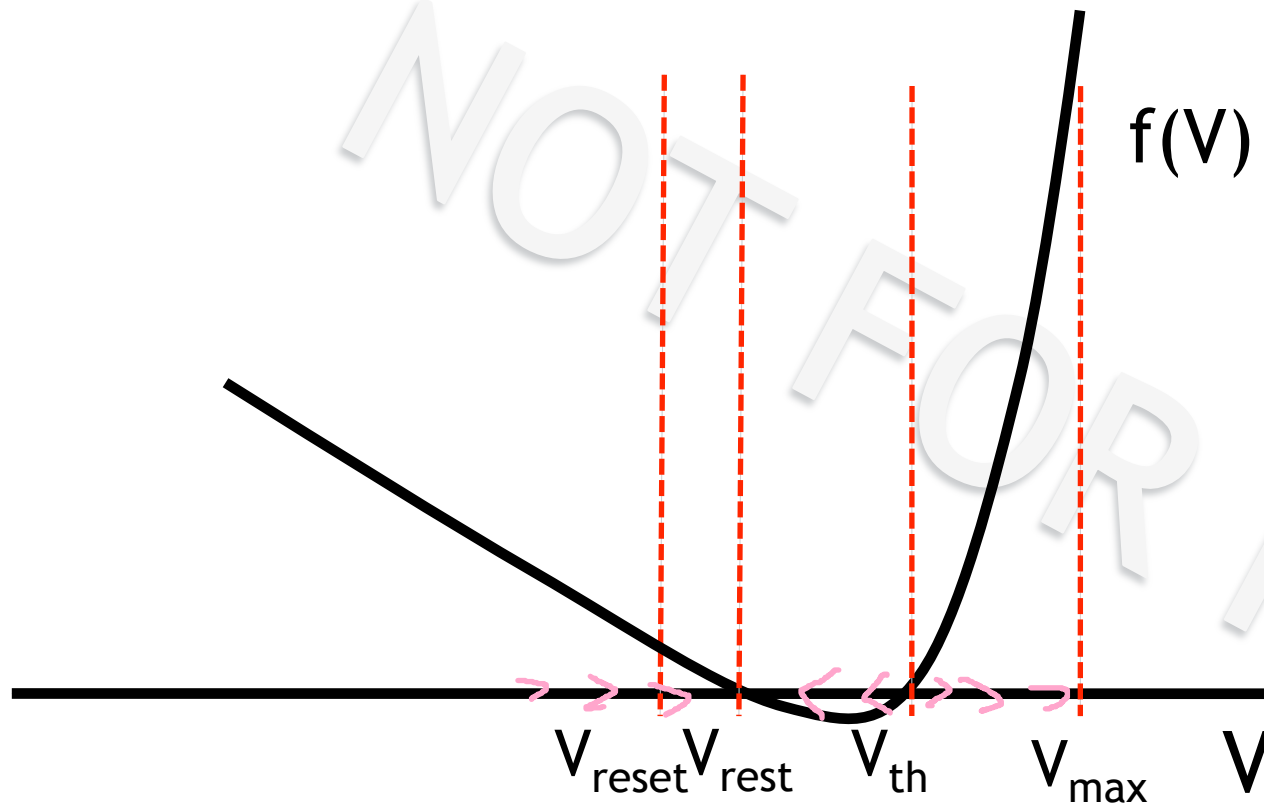
Capturing the basic dynamics of neurons



quasi

$$dV/dt = f(V) + I(t)$$

Exponential integrate-and-fire neuron

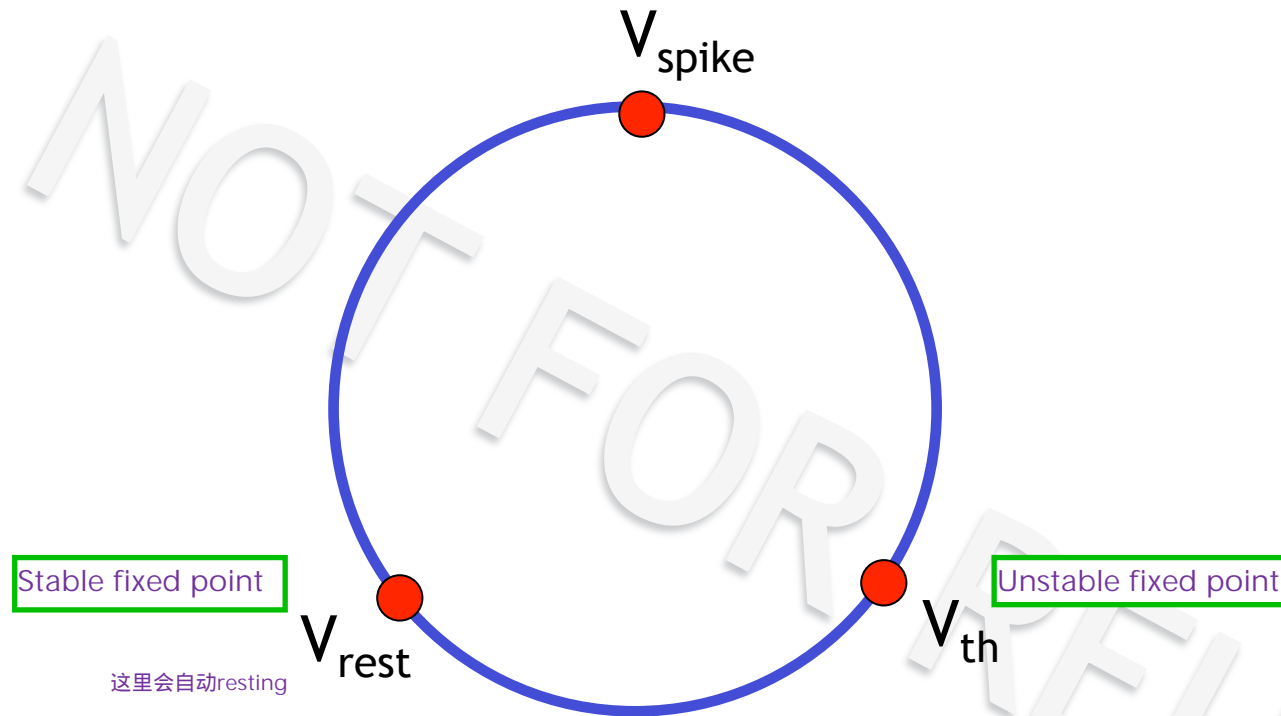


$$f(V) = -a(V-V_0) + \exp([V-V_{th}]/\Delta)$$

这个是负数的时候，这一项很小，因此对前一项的影响很小，所以前面还是linear的

The theta neuron

phase coding

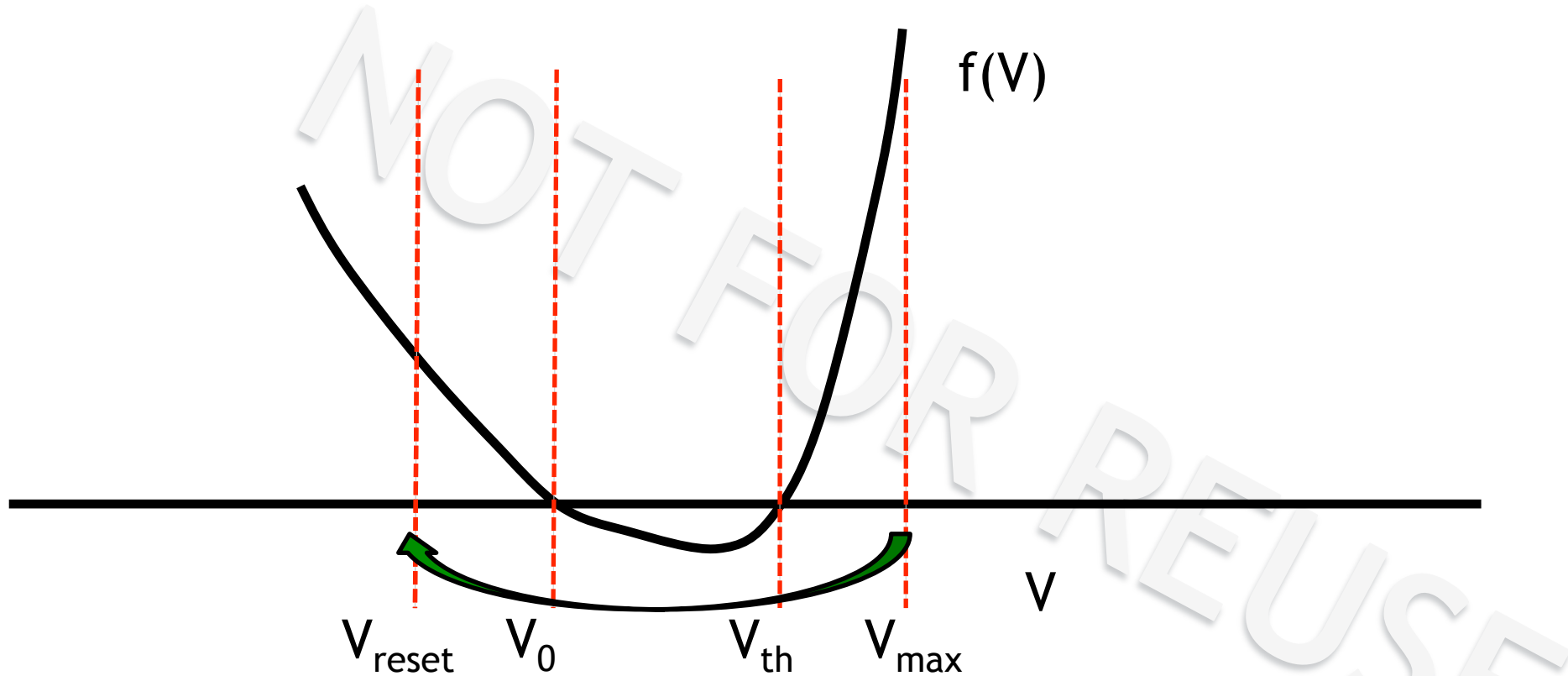


即使没有input，还是能一直firing，它们会一直振荡。因此通常用这样的模型来刻画周期性地firing的神经元。

$$d\theta/dt = 1 - \cos \theta + (1 + \cos \theta) I(t)$$

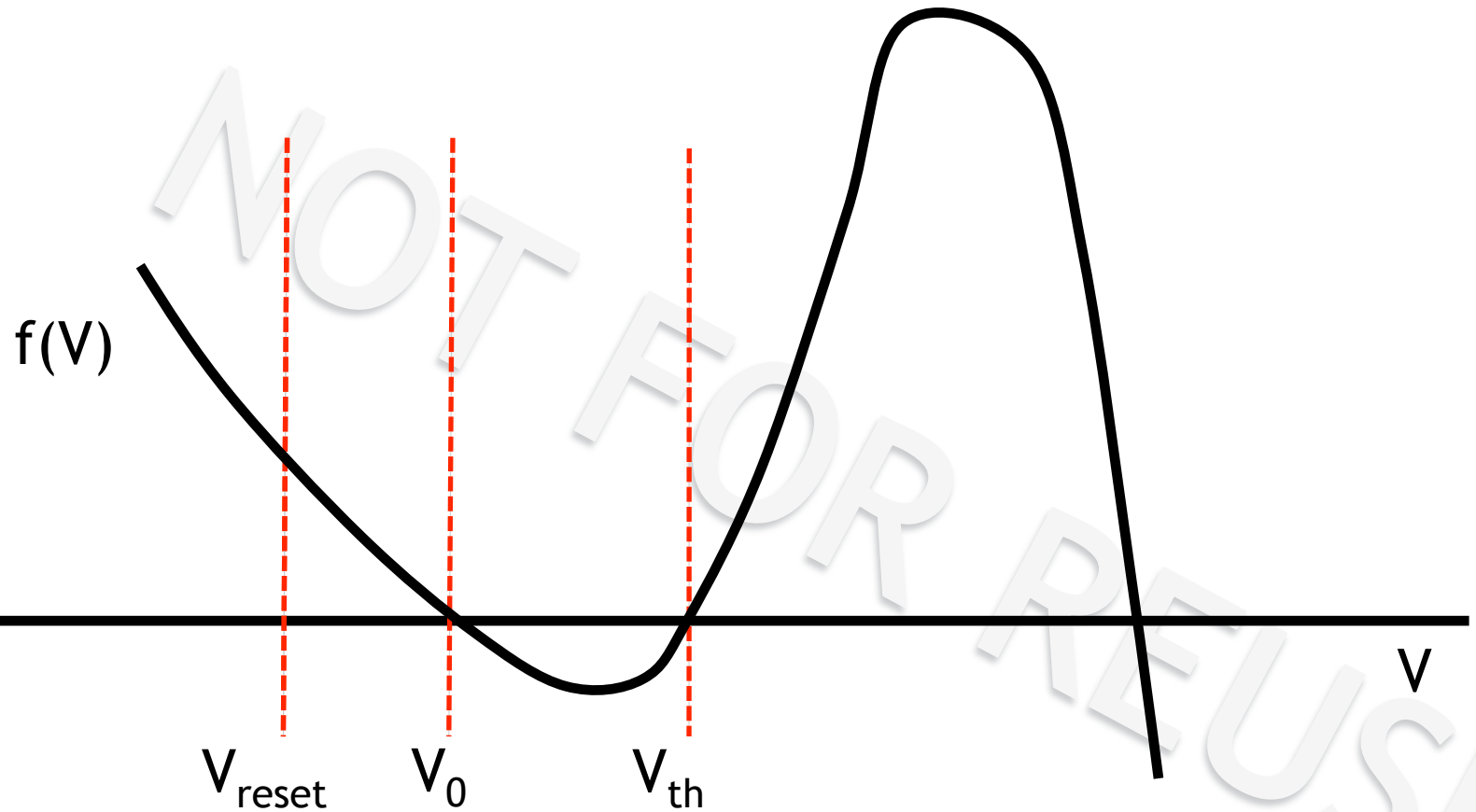
Ermentrout and Kopell

Capturing the basic dynamics of neurons



$$dV/dt = f(V) + I(t)$$

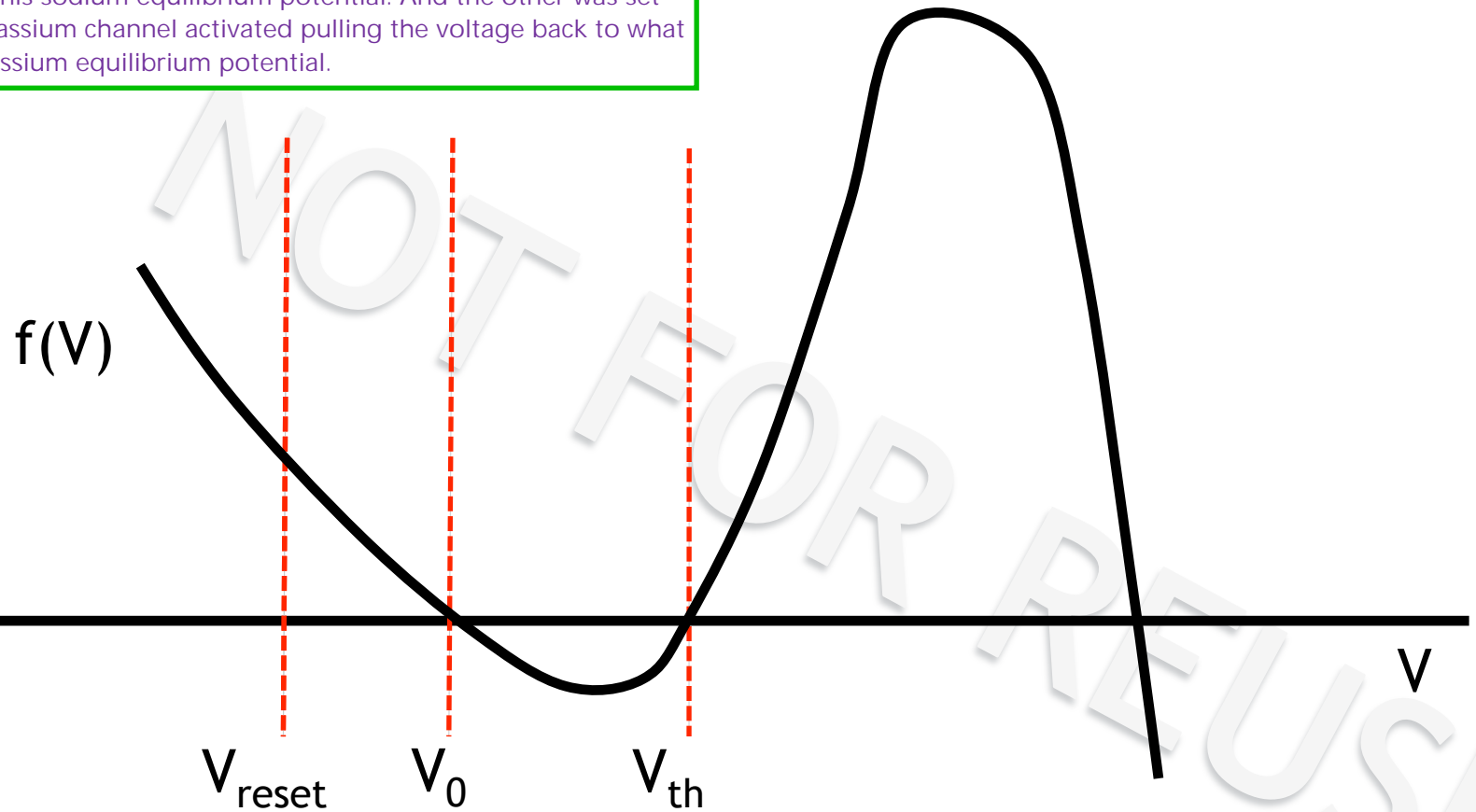
Capturing the basic dynamics of neurons



防止到infinity
但是他回不来，是一个bistable system

Capturing the basic dynamics of neurons

回忆：用两个方程，One was that the sodium, switching of the drive toward this sodium equilibrium potential. And the other was set that potassium channel activated pulling the voltage back to what the potassium equilibrium potential.



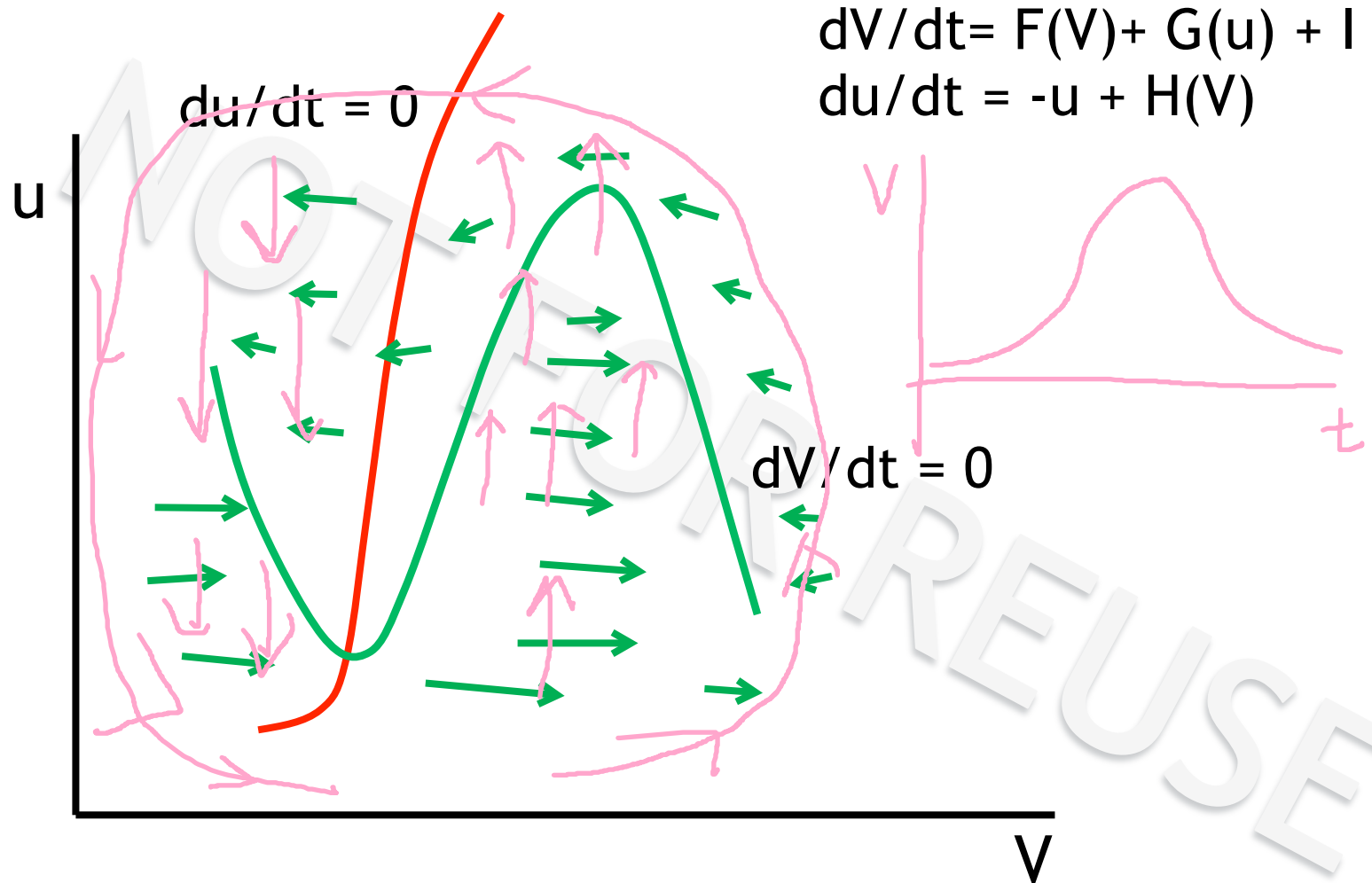
$$dV/dt = f(V) + I(t) \rightarrow$$

$$\begin{aligned} dV/dt &= F(V) + G(u) + I(t) \\ du/dt &= -u + H(V) \end{aligned}$$

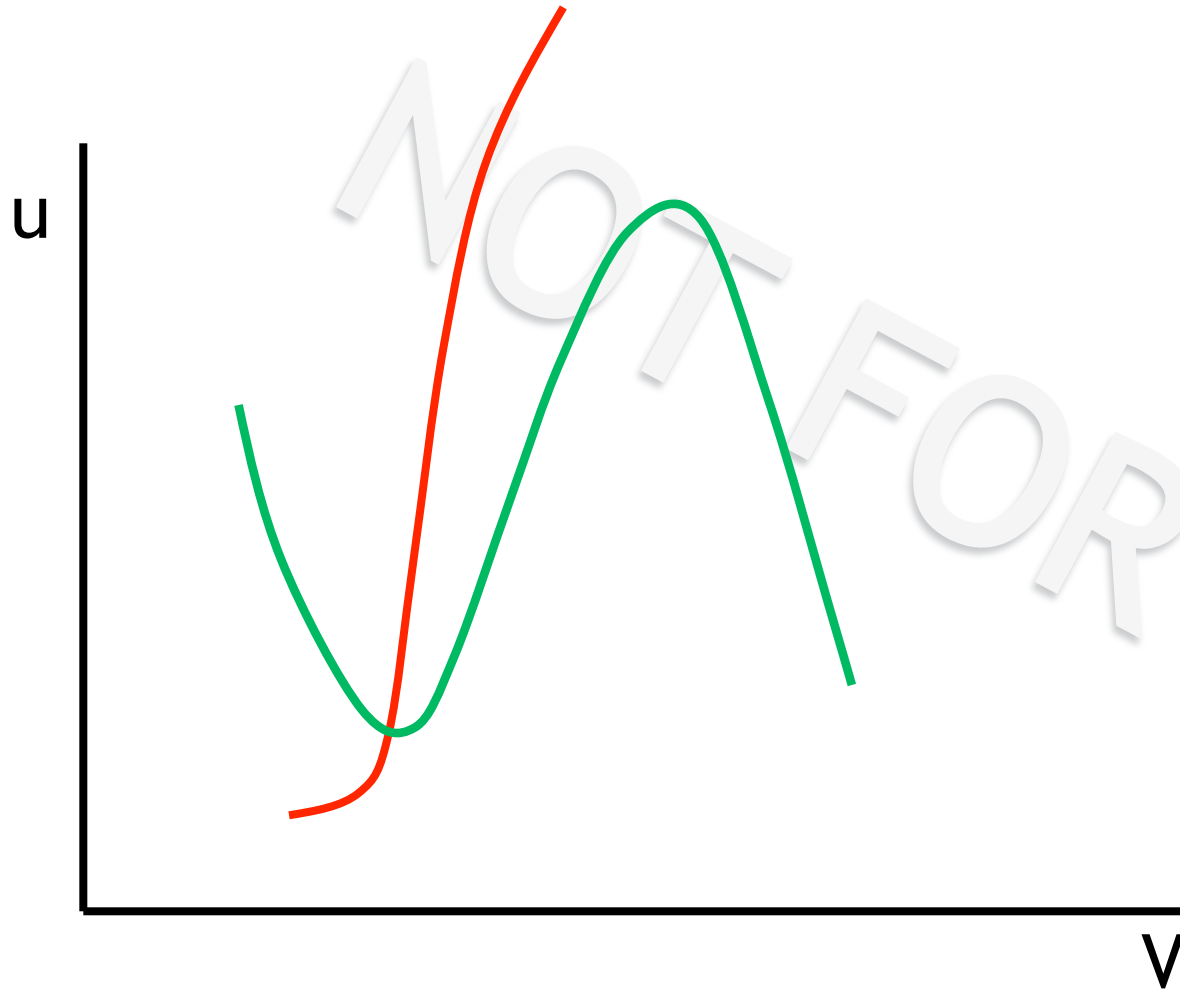
$G(u)$ 是抑制项, u 变大, $G(u)$ 就是一个很大的负数。

V 变大, u 也变大

Two-dimensional models

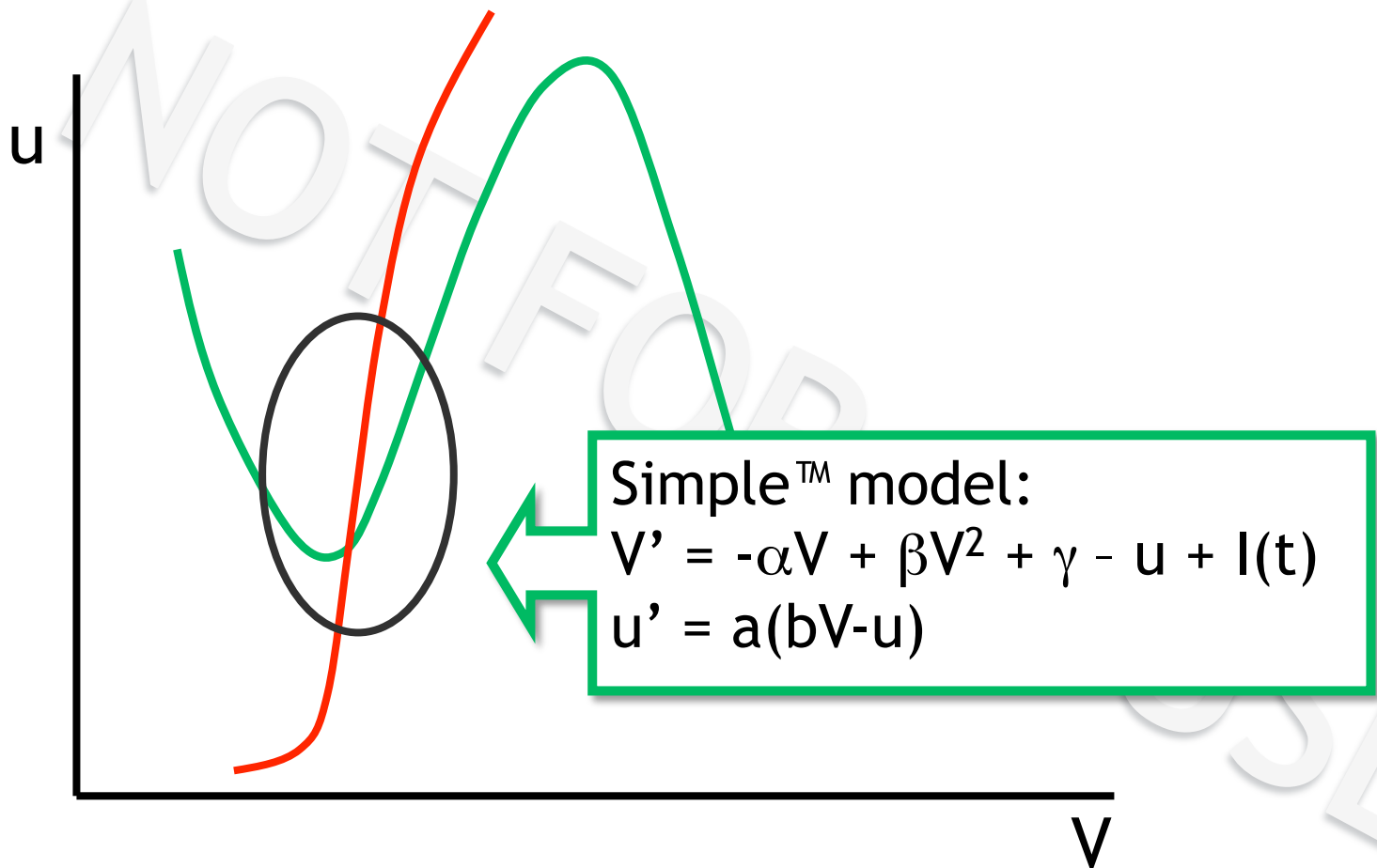


On-line resources about phase plane analysis



- Wulfram Gerstner
- Bard Ermentrout
- Scholarpedia

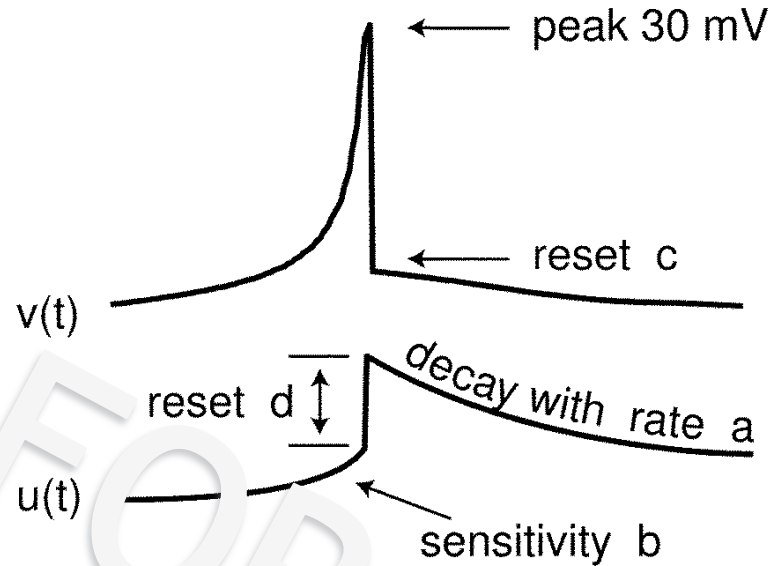
Two-dimensional models



The simple model

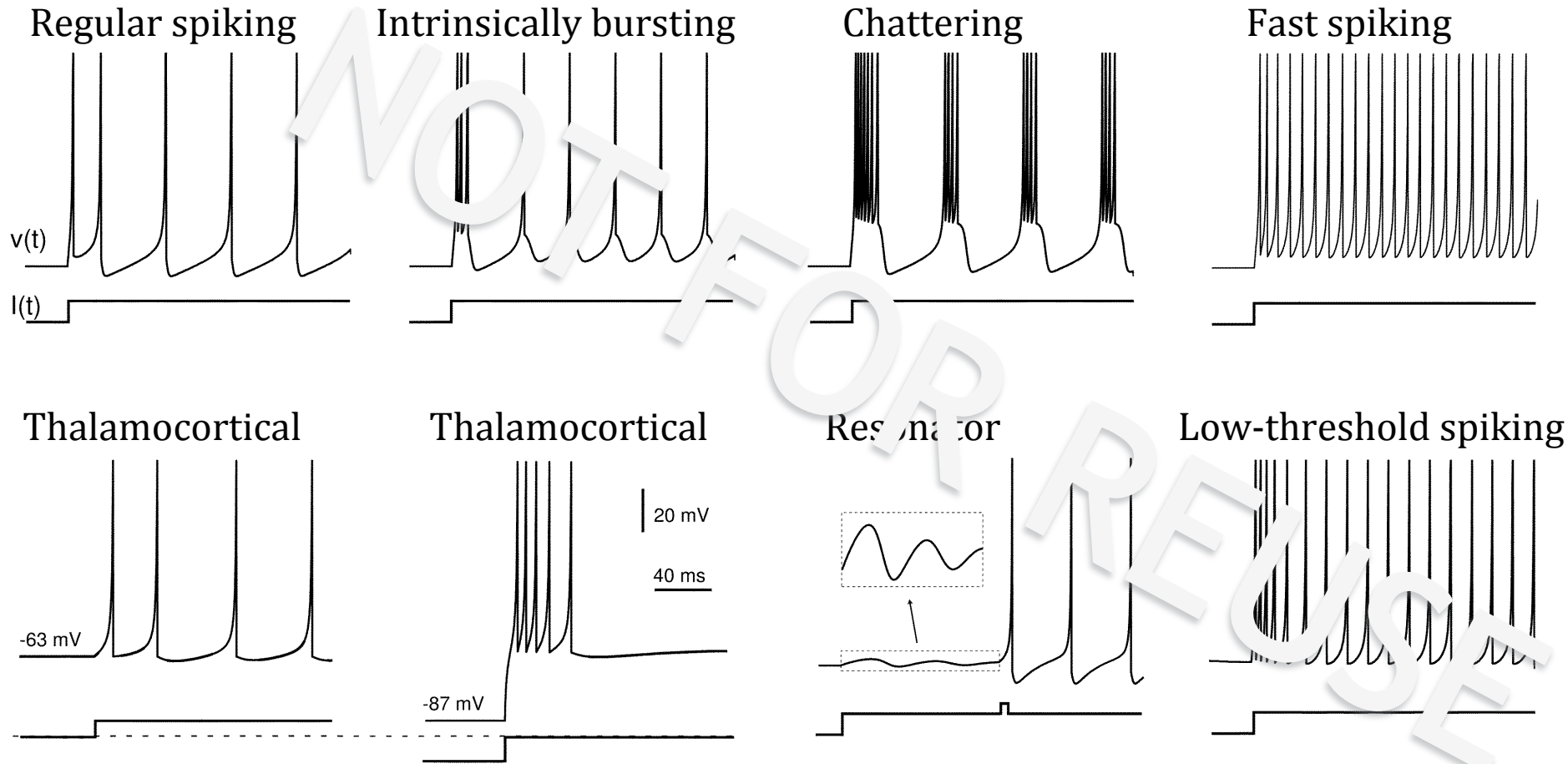
$$v' = 0.04v^2 + 5v + 140 - u + I$$
$$u' = a(bv - u)$$

if $v = 30 \text{ mV}$,
then $\underline{v} \leftarrow c$, $\underline{u} \leftarrow u + d$



The simple model

用4个参数做出来的模型



Where to from here?

The diagram features a central red-outlined oval at the top labeled 'Hodgkin-Huxley'. Two large green arrows point downwards from this central oval to two separate red-outlined ovals below it. The left oval is titled 'Biophysical realism' and lists 'Ion channel physics', 'Additional channels', and 'Geometry'. The right oval is titled 'Simplified models' and lists 'Fundamental dynamics' and 'Analytical tractability'. A faint, large watermark 'CR' is visible in the background.

Hodgkin-Huxley

Biophysical realism
Ion channel physics
Additional channels
Geometry

Simplified models
Fundamental dynamics
Analytical tractability