The weak law of large numbers tells us that the sample mean-- that is, the average of independent identically distributed random variables, Xi-- converges, in a certain sense, to a number, namely the expected value of the random variables, Xi. But it does not tell us much about the details of the distribution of the sample mean.

The central limit theorem provides us exactly with this kind of detail. It tells us that the sum of many independent identically distributed random variables has approximately a normal distribution. The mean and variance of this normal is easy to find if we know the mean and variance of the original random variables. This enables us to carry out approximate calculations rather quickly by using the normal tables.

We will start with a precise statement of the central limit theorem, and we will emphasize that it is a universal result. It holds no matter what the distribution of the original random variables, and for this reason, it is very useful. We will work through several examples of the typical ways that the central limit theorem is used. We will develop a refinement that can be used when we are dealing with discrete distributions, which provides us with even more accurate approximations.

And finally we will revisit the polling problem, and inquire again about the number of samples that are needed to obtain a certain accuracy with a certain confidence. We will see that the central limit theorem is much more informative, much less conservative, compared to the conclusions that we had gotten before based on the Chebyshev inequality.