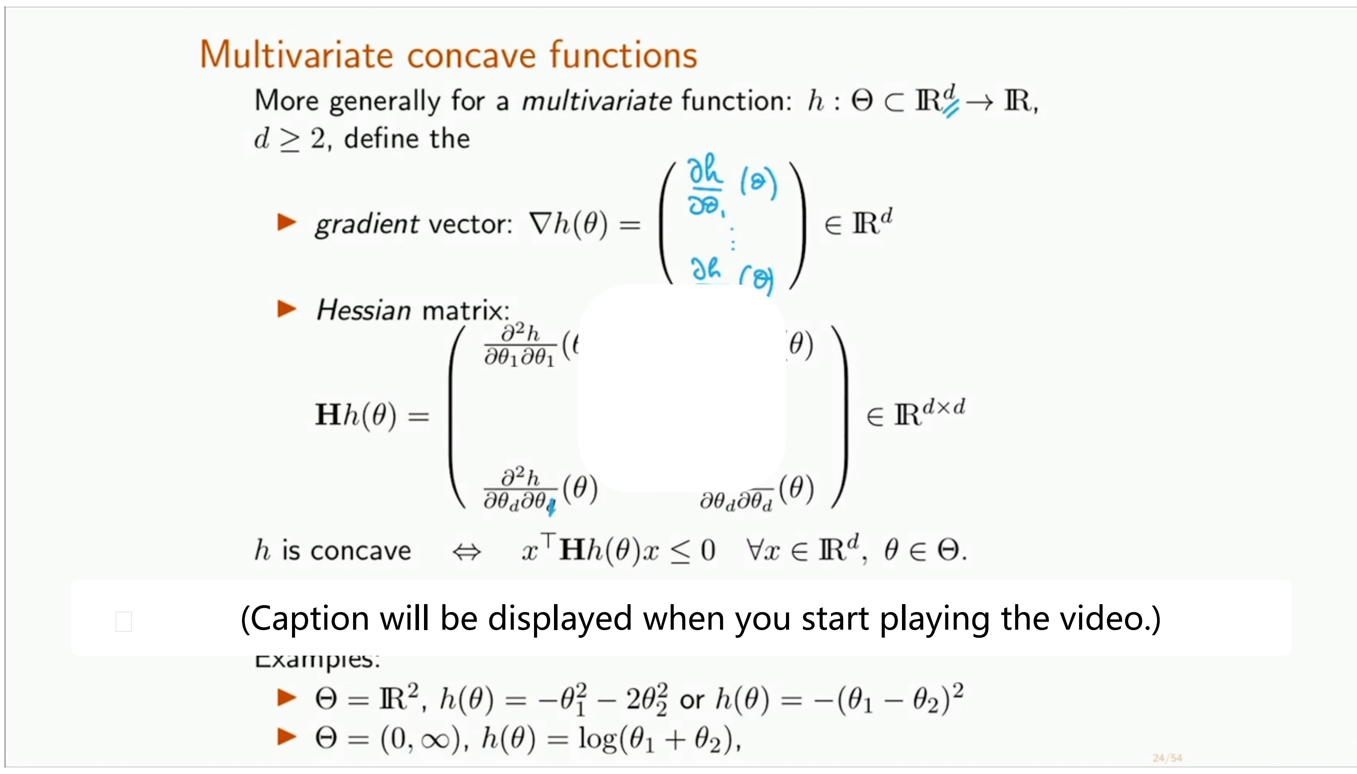
9. Worked Example: Concavity and Composition of Functions

Worked Example: Hessian and Concavity



Start of transcript. Skip to the end.

How about the second one?

Should we do the second one?

So the second one is log of theta 1 plus theta 2.

OK?

So the gradient-- so that's h of theta.

So gradient h of theta is--

well, it's 1 over theta 1 plus theta 2.

The other one is 1 over theta 1 plus theta 2.

视频

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Combination of Convex functions

2/3 points (graded)

Let f_1, f_2 be convex functions on $\mathbb{R}$.

Determine if the following functions are necessarily convex or concave.

Hint: Recall a function $g : I \to \mathbb{R}$ is convex in the interval I is an interval, if for all pairs of real numbers $x_1 < x_2 \in I$

$$g\left(tx_1 + (1-t)\,x_2\right) \leq tg\left(x_1\right) + (1-t)\,g\left(x_2\right) \qquad \text{for all } 0 \leq t \leq 1.$$

- $3f_1 + 2f_2$:

  - (selected) Convex
  - Concave
  - Cannot be determined without more information

- $-10f_1$:

  - Convex

- Concave
- Cannot be determined without more information

- $f_2 f_1$:

- Convex
- Concave
- Cannot be determined without more information

Solution:

Given f_1, f_2 are convex, we have

$$f_1\left(tx_1+(1-t)\,x_2\right)\le t f_1\left(x_1\right)+(1-t)\,f_1\left(x_2\right)\qquad \text{for all } 0\le t\le 1$$

and the same holds for f_2.

- The same inequality holds for $g = 3f_1 + 2f_2$:

$$\begin{aligned} g\left(tx_1+(1-t)\,x_2\right) &= 3f_1\left(tx_1+(1-t)\,x_2\right)+2f_2\left(tx_1+(1-t)\,x_2\right)\\ &\le 3\left(tf_1\left(x_1\right)+(1-t)\,f_1\left(x_2\right)\right)+2\left(tf_2\left(x_1\right)+(1-t)\,f_2\left(x_2\right)\right)\\ &= t\left(g\left(x_1\right)+(1-t)\,g\left(x_2\right)\right).\end{aligned}$$

  Hence $3f_1 + 2f_2$ is also convex.

  Remark: In general, any function $c_1 f_1 + c_2 f_2$ where $c_1, c_2 > 0$ is convex of $f_1,\ f_2$ are.

- $-10f_1$ is concave, because it is negative of a convex function.

- $f_1 f_2$ is not necessary convex For example, is $f_1\left(x\right) = x$, and $f_2 = x^2$, then $\left(f_1 f_2\right)\left(x\right) = x^3$ which is neither convex nor concave. Other examples of f_1 and f_2, e.g. $f_1 = f_2 = x^2$ will lead to $f_1 f_2$ being convex.

提交 你已经尝试了1次（总共可以尝试1次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 3 Methods of Estimation:Lecture 9: Introduction to Maximum Likelihood Estimation / 9. Worked Example: Concavity and Composition of Functions