Joint PMF drill #1.

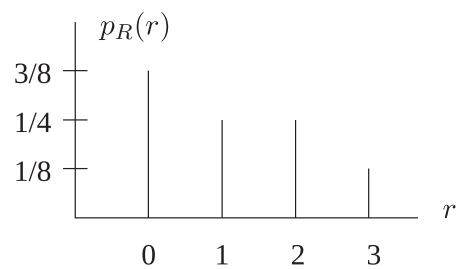
1. $x = 0$ maximizes $\mathbf{E}[Y \mid X = x]$ since

$$\mathbf{E}[Y \mid X = x] = \begin{cases} 2, & \text{if } x = 0, \\ 3/2, & \text{if } x = 2, \\ 3/2, & \text{if } x = 4, \\ \text{undefined}, & \text{otherwise.} \end{cases}$$

2. $y = 3$ maximizes $\mathrm{var}(X \mid Y = y)$ since

$$\mathrm{var}(X \mid Y = y) = \begin{cases} 0, & \text{if } y = 0, \\ 8/3, & \text{if } y = 1, \\ 1, & \text{if } y = 2, \\ 4, & \text{if } y = 3, \\ \text{undefined}, & \text{otherwise.} \end{cases}$$

3.



4. By traversing the points top to bottom and left to right, we obtain

$$\mathbf{E}[XY] = \frac{1}{8}\,(0 \cdot 3 + 4 \cdot 3 + 2 \cdot 2 + 4 \cdot 2 + 0 \cdot 1 + 2 \cdot 1 + 4 \cdot 1 + 4 \cdot 0) = \frac{15}{4}.$$

Conditioning on A removes the point masses at $(0, 1)$ and $(0, 3)$. The conditional probability of each of the remaining point masses is thus $1/6$, and

$$\mathbf{E}[XY \mid A] = \frac{1}{6}\,(4 \cdot 3 + 2 \cdot 2 + 4 \cdot 2 + 2 \cdot 1 + 4 \cdot 1 + 4 \cdot 0) = 5.$$