# Problem 1: Multiple Choice

**Instructions** :

Be very careful with the multiple choice questions below. Some are choose all that apply and some test your knowledge on the definitions of terms.

As in the rest of this exam, only your last submission will count.

## (a)

1/1 point (graded)
All maximum likelihood estimators are asymptotically normal.

- True
- False ✓

**Solution:**

The maximum likelihood estimator is asymptotically normal only if certain technical conditions are satisfied. One technical condition that is often violated is that the support of the distributions depend on the parameters being estimated; e.g. estimating $\theta$ when $X_1, \ldots, X_n$ come from a Unif$[0, \theta]$.

提交 你已经尝试了1次（总共可以尝试3次）

□ Answers are displayed within the problem

## (b)

1/1 point (graded)
Let $X_1, \ldots, X_n$ be i.i.d. Bernoulli random variables with some unknown parameter $p \in (0, 1)$. Then which of the following is/are valid confidence interval(s) for $p$ with **nonasymptotic** confidence level 95%?
(Choose all that apply.)

- $[\bar{X}_n - 1.96\sqrt{\frac{p(1-p)}{n}}, \bar{X}_n + 1.96\sqrt{\frac{p(1-p)}{n}}]$
- $(0, 1)$ ✓
- $[0, \bar{X}_n + \frac{.83}{n}]$
- None of the Above

✓

**Correction Note:** An earlier version of this problem contained an ambiguity and asked for confidence intervals for $p$ with nonasymptotic level 5%, without stating whether the level refers to a signifance level or confidence level.

**Solution:**

We consider each of the possible choices in turn.

- The first choice is a “confidence interval" for $p$ that depends on $p$ and thus is clearly invalid.

- The second choice is a 100% confidence interval as $p$ is always between $0$ and $1$ and is therefore is also a 95% confidence interval. (Note that a constant interval could also be viewed as a random interval, just as a constant can be viewed as a random variable.)

- Finally, the third choice is invalid. For small $n$ (e.g. $n = 1$), it has (maximal) level $p$, which makes this a 95% nonasymptotic interval if $p > 0.95$ ($p$ should be in the CI with probability 0.95 for all $p$). It is not even a valid 95% nonasymptotic CI, since it has the wrong dependence on $n$, i.e. $\frac{.83}{n}$ rather than $\frac{.83}{\sqrt{n}}$. This makes the interval too narrow to be a valid confidence interval for $p$ with asymptotic or nonasymptotic level 5%.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

## (c)

0/1 point (graded)
Which of the following is/are valid statistical model(s)?
(Choose all that apply.)

- [x] $\left(\mathbb{R}, \{\mathcal{N}(\theta, 1)\}_{\theta > 10}\right)$
- [ ] $\left([\theta, \infty), \{t \mapsto e^{\theta - t}\mathbf{1}(t > \theta)\}_{\theta > 0}\right)$
- [ ] $\left([0, \infty), \{\mathcal{N}(\mu, \sigma^2)\}_{\mu > \sigma^2}\right)$
- [x] $\left([0, \infty), \{x \mapsto e^{\theta x}\mathbf{1}(x > 0)\}_{\theta > 0}\right)$

被坑了，这个东西积分结果不是1。所以不能定义一个概率分布。

**Solution:**

We consider each of the possible choices in turn.

- The first choice is a valid statistical model.

- The second choice is invalid as the sample space of the random variables depends on the parameter of interest $\theta$.

- The third choice is invalid as a normal distribution has support over $\mathbb{R}$ and not the subset $[0, \infty)$. (Do not confuse restricting the parameter space $\Theta$ with incorrectly restricting the support, or sample space, $E$ of the family of distributions.)

- Finally, the fourth choice is invalid as $x \mapsto e^{\theta x}\mathbf{1}(x > 0)\}_{\theta > 0}$ does not define a probability distribution.

提交 你已经尝试了1次（总共可以尝试3次）

Answers are displayed within the problem

## Error and Bug Reports/Technical Issues

显示讨论

**主题：** Midterm Exam 1:Midterm Exam 1 / Problem 1: Multiple Choice