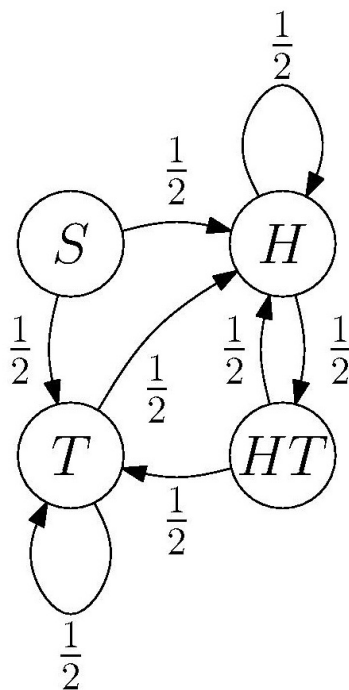


5. Coin tosses revisited

Problem 5. Coin tosses revisited

4/4 points (ungraded)

A fair coin is tossed repeatedly and independently. We want to determine the expected number of tosses until we first observe Tails immediately preceded by Heads. To do so, we define a Markov chain with four states, $\{S, H, T, HT\}$, where S is a starting state, H indicates Heads on the current toss, T indicates Tails on the current toss (without Heads on the previous toss), and HT indicates Heads followed by Tails over the last two tosses. This Markov chain is illustrated below:



Note: State S is in fact unnecessary, and is only included to facilitate understanding. Having the process start at state T , rather than S , makes no difference on what the next state will be; in both cases, the next state is equally likely to be H or T . Therefore S can be dispensed with.

1. What is the expected number of tosses until we first observe Tails immediately preceded by Heads? **Hint:** Solve the corresponding mean first passage time problem for our Markov chain.



2. Assuming that we have just observed Tails immediately preceded by Heads, what is the expected number of additional tosses until we next observe Tails immediately preceded by Heads?



Next, we want to answer similar questions for the event that Tails is immediately preceded by Tails. Set up a new Markov chain from which you can calculate the expected number of tosses until we first observe Tails immediately preceded by Tails.

3. What is the expected number of tosses until we first observe Tails immediately preceded by Tails?



4. Assuming that we have just observed Tails immediately preceded by Tails, what is the expected number of additional tosses until we again observe Tails immediately preceded by Tails?



讨论

主题: Unit 10 / Problem Set / 5. Coin tosses revisited

显示讨论