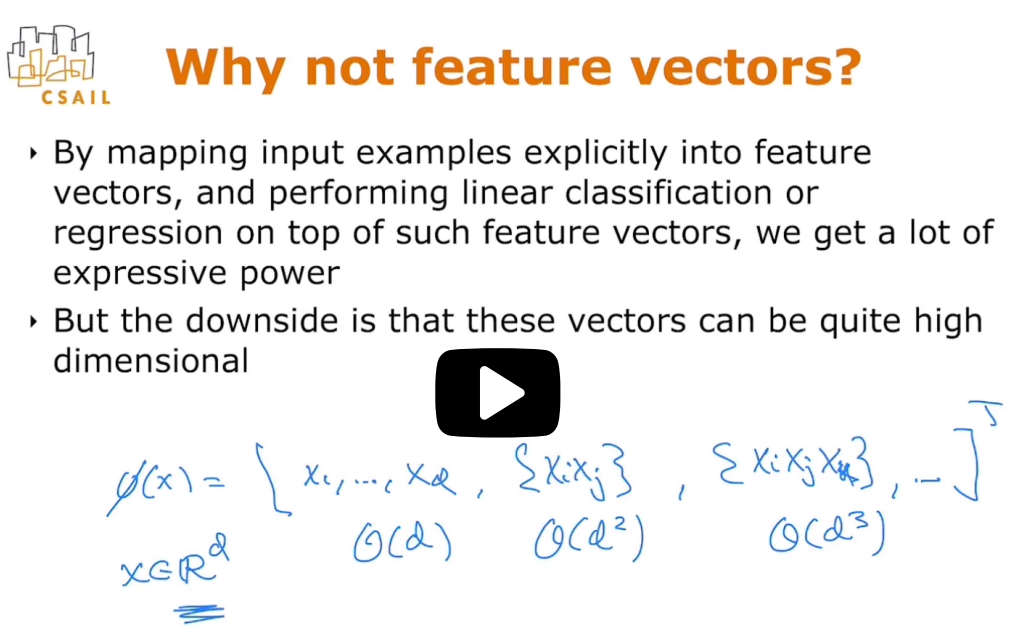
3. Introduction to Non-linear Classification
Introduction to Non-linear Classification



And that is what kernel methods provide us.

dimensional

very quickly if we even started from a moderately dimensional

vector.

OK?

So we would want to have a more efficient way of doing that--

operating with high dimensional feature

vectors without explicitly having to construct them.

And that is what kernel methods provide us.

8:18 / 8:18 1.0x

End of transcript. Skip to the start.

Video
Download video file

Transcripts
Download SubRip (.srt) file
Download Text (.txt) file

Counting Dimensions of Feature Vectors

1/1 point (graded)

Let $x \in \mathbf{R}^{150}$, i.e. $x = [x_1, x_2, \ldots, x_{150}]^T$ where x_i is the i-th component of x. Let $\phi(x)$ be an **order** 3 polynomial feature vector. This means, for example, $\phi(x)$ can be

$$\phi(x) = [\underbrace{x_1, \ldots, x_i, \ldots, x_{150}}_{\deg 1}, \underbrace{x_1^2, x_1 x_2, \ldots, x_i x_j, \ldots {x_{150}}^2}_{\deg 2}, \underbrace{{x_1}^3, {x_1}^2 x_2, \ldots, x_i x_j x_k, \ldots, {x_{150}}^3}_{\deg 3}] \qquad \text{where } 1 \leq i \leq j \leq k \leq 150.$$

Note that the components of $\phi(x)$ forms a basis of the space of all polynomials with zero constant term and of degree at most 3.

What is the dimension of the space that $\phi(x)$ lives in? That is, $\phi(x) \in \mathbb{R}^d$ for what d?

Hint: The number of ways to select a multiset of k non-unique items from n total is $\binom{n+k-1}{k}$. For example, if a ball can be any of 3 colors, then the number of color configurations of 2 balls is $\binom{3+2-1}{2} = \binom{4}{2} = 6$.

$d =$ 585275 ✔ **Answer:** 585275

Solution:

For each of the feature transformations (power 1, power 2, power 3), there are n-multichoose-power combinations. Thus $\binom{150}{1} + \binom{151}{2} + \binom{152}{3} = 585275$. **Remark:** We see that the dimension of the space that the feature vectors live grows quickly as a function of d, the dimension we started with if $x \in \mathbb{R}^d$.

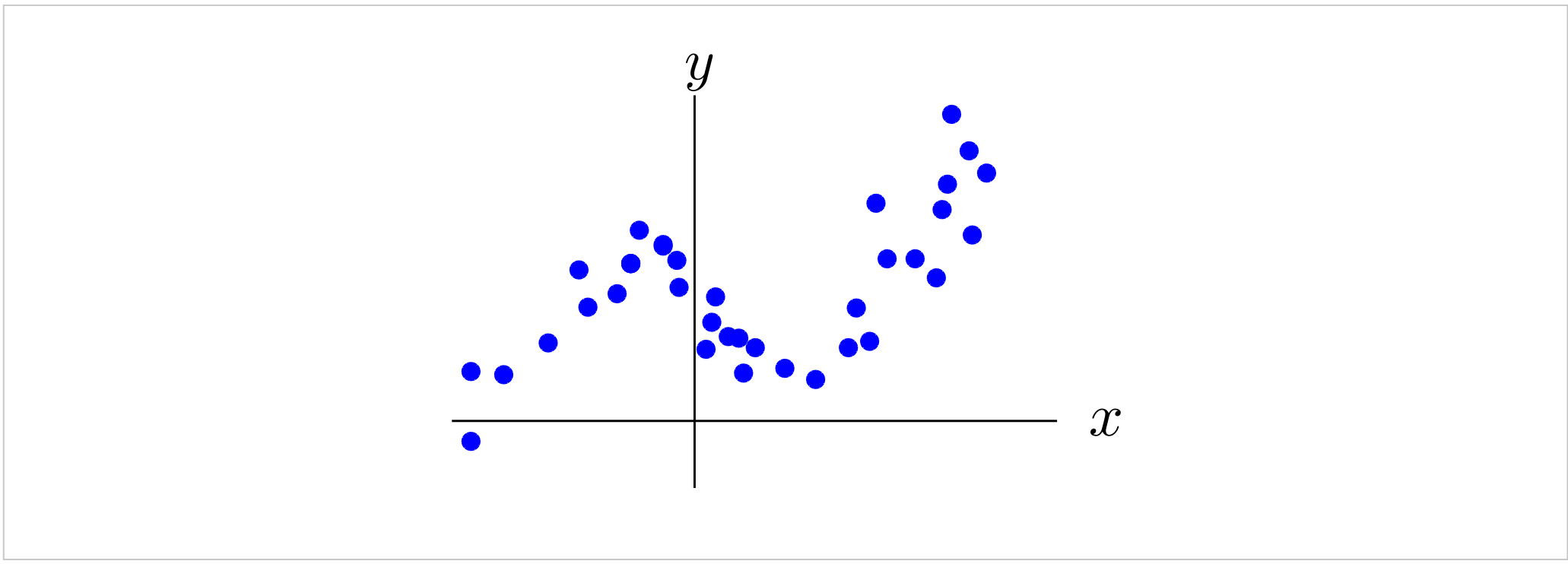
Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Regression using Higher Order Polynomial feature

1/1 point (graded)

Assume we have n data points in the training set $\left\{ \left(x^{(t)}, y^{(t)} \right) \right\}_{t=1,\ldots,n}$ where $\left(x^{(t)}, y^{(t)} \right)$ is the t-th training example:



We want to find a non-linear regression function f that predicts y from x, given by

$$f\left(x; \theta, \theta_0\right) = \theta \cdot \phi\left(x\right) + \theta_0$$

where $\phi\left(x\right)$ is a polynomial feature vector of some order. What (loosely) is the minimum order of $\phi\left(x\right)$?

3 ✔ **Answer:** 3

Solution:

The relationship between y and x can be roughly described by a cubic function, so a feature vector $\phi\left(x\right)$ of minimum order 3 can minimize structural errors.

Submit You have used 1 of 2 attempts

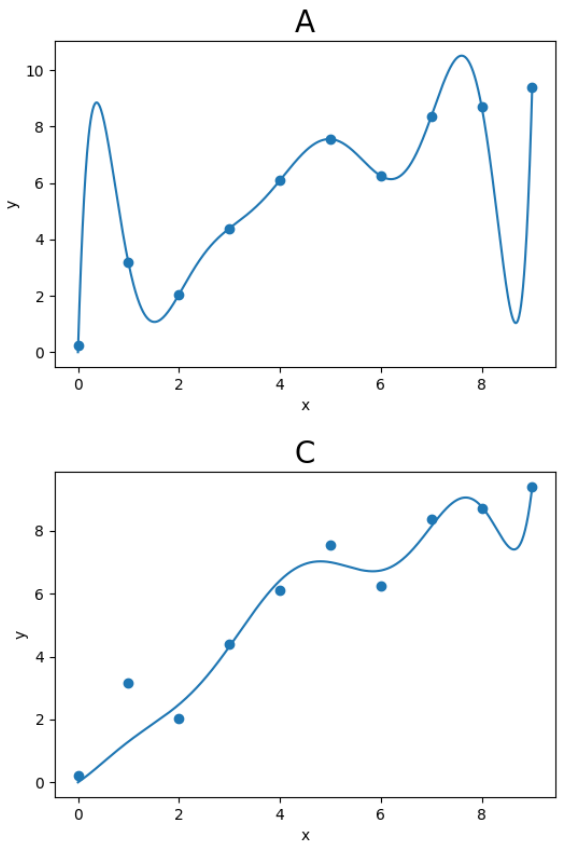
Answers are displayed within the problem

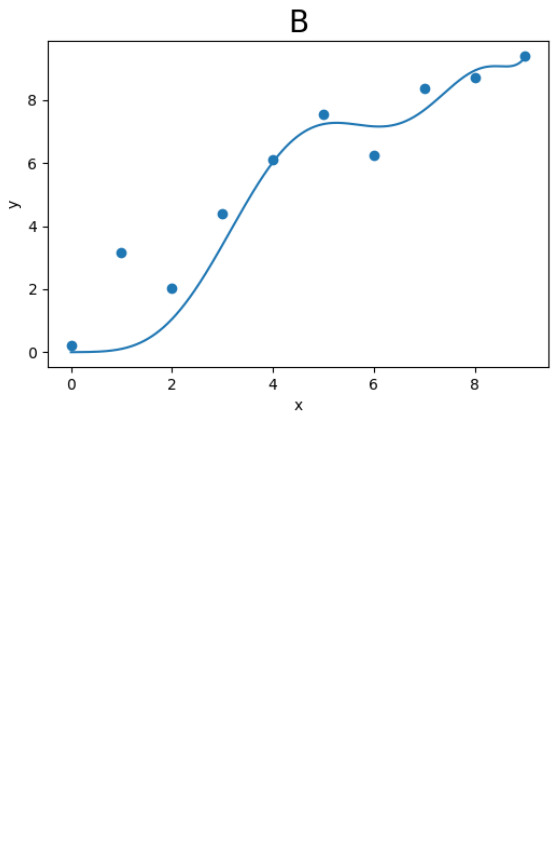
Effect of Regularization on Higher Order Regression

2/2 points (graded)

Let us go back to explore the effect of regularizaion on Higher Order regression.

The three figures below show the fitting result of an 8th order polynomial regression with different regularization parameter λ on the same training data.





Which figure above corresponds to the smallest regularization parameter λ?

- A ✔
- B
- C

Which figure corresponds to the largest regularization parameter λ?

- A
- B ✔
- C

Solution:

The effect of regularization is to restrict the parameters of a model to freely take on large values. This will make the model function smoother, leveling the 'hills' and filling the 'vallyes'. It will also make the model more stable, as a small perturbation on x will not change y significantly with smaller $\|\theta\|$.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

Discussion

Show Discussion

Topic: Unit 2 Nonlinear Classification, Linear regression, Collaborative Filtering (2 weeks):Lecture 6. Nonlinear Classification / 3. Introduction to Non-linear Classification