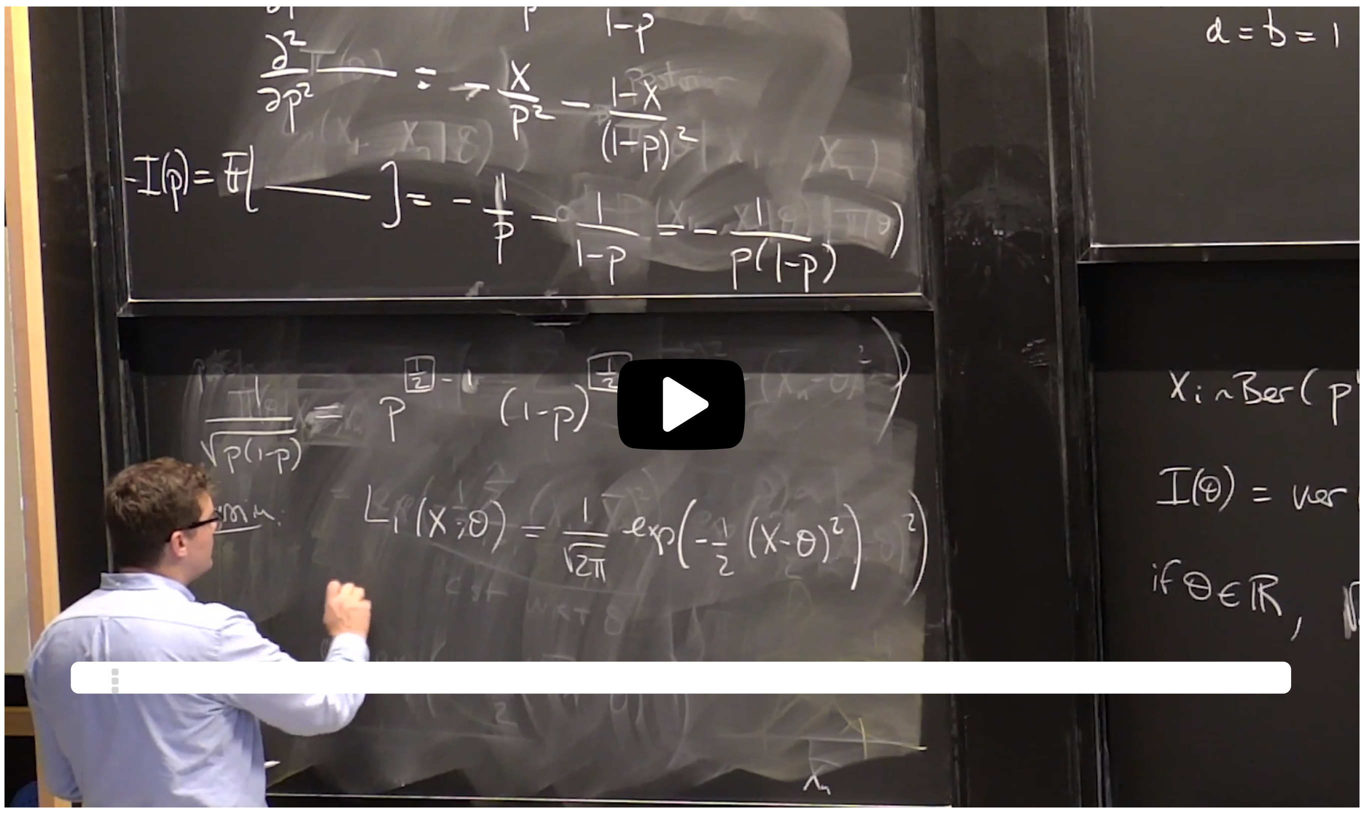
# 7. Jeffreys Prior II: Examples

## Jeffreys Prior II: Examples



4:32 / 6:08 1.0x

is just proportional to 1.

This is the same as the one we had before.

In the Gaussian case, there's no point

that's harder to estimate than any other, right?

This is completely a translation invariant problem.

Nothing is harder than any other point.

And the reason why things can change

is because the variance might depend on your parameter,

but if nothing depends on your parameter,

then it doesn't matter where the mean is.

And so in this case we have this prior.

How did we call such priors?

Improper priors.

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## Computing Jeffreys Prior

2/2 points (graded)

Let $N$ be a Poisson random variable. That is,

$$p_N(n|\lambda) = e^{-\lambda}\frac{\lambda^n}{n!},$$

where $p_N(n|\lambda)$ denotes the conditional pmf of $N$ given the parameter $\lambda$.

- Evaluate the Jeffreys prior, $p(\lambda)$ up to a proportionality constant, which only is a function of $\lambda$. Remove outside constants in your answer such that $I(1) = 1$.

1/sqrt(lambda) ✔ **Answer:** 1/sqrt(lambda)

$\frac{1}{\sqrt{\lambda}}$

- Is the Jeffrey's prior proper?

○ Yes

◉ No ✔

STANDARD NOTATION

**Solution:**

- We begin by computing Fisher information, $I(\lambda)$, as follows.

$$\frac{d}{d\lambda}\log p_N(n|\lambda) = -1 + \frac{n}{\lambda}.$$

  Therefore,

$$I(\lambda) = \mathbb{E}\left[\left(\frac{d}{d\lambda}\log p_N(n|\lambda)\right)^2\right] = \mathbb{E}\left[\frac{(N-\lambda)^2}{\lambda^2}\right] = \frac{1}{\lambda^2}\mathbf{Var}(N).$$

  Since $\mathbf{Var}(N) = \lambda$, for a Poisson random variable, we arrive at,

$$I(\lambda) = \frac{1}{\lambda} \implies p(\lambda) \propto \sqrt{I(\lambda)} = \frac{1}{\sqrt{\lambda}}.$$

- Since $\lambda \in (0, \infty)$, we can check that,

$$\int_0^\infty \frac{1}{\sqrt{\lambda}}\, d\lambda = \infty,$$

  and therefore, Jeffreys prior for this problem is improper.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

## Jeffreys Prior for Matrix Case

3/3 points (graded)

In this problem we will consider a model which has a two-dimensional parameter. Then you will calculate Jeffrey's prior using the Fisher information matrix.

Suppose that $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, where $\mu$ and $\sigma^2$ are unknown. In this case, the Fisher information matrix $I(\theta)$ for $\theta = (\mu, \sigma^2)$ will be a $2 \times 2$ matrix, where the off-diagonal entries are 0.

- Find $(I(\theta))_{11}$.

  1/sigma^2 **Answer:** 1/(sigma^2)

  $\frac{1}{\sigma^2}$

- Find $(I(\theta))_{22}$.

  1/(2*sigma^4) **Answer:** 1/(2*sigma^4)

  $\frac{1}{2\cdot\sigma^4}$

- Using your answers to the previous part, determine Jeffreys prior, $\pi(\theta)$, in terms of $\mu$ and $\sigma$. Express your answer in such a form that $\pi((1,1)) = 1$.

sqrt(1/sigma^6)

✔ **Answer:** 1/(sigma^3)

$\sqrt{\frac{1}{\sigma^6}}$

STANDARD NOTATION

**Solution:**

- Clearly, the likelihood model is of form,

$$p_{Y|\mu,\sigma} = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \implies \frac{1}{2}\log p_{Y|\mu,\sigma} = \log\left(\frac{1}{2\pi\sigma^2}\right) - \frac{(x-\mu)^2}{2\sigma^2}.$$

In particular,

$$(I(\theta))_{11} = -\mathbb{E}\left[\frac{\partial^2}{\partial\mu^2}\log p_{Y|\mu,\sigma}\right] = \frac{1}{\sigma^2},$$

using the fact that,

$$\frac{\partial}{\partial\mu}\log p_{Y|\mu,\sigma} = \frac{x-\mu}{\sigma^2} \implies \frac{\partial^2}{\partial\mu^2}\log p_{Y|\mu,\sigma} = -\frac{1}{\sigma^2}.$$

- Using the exact same strategy as above, and the fact that, $\mathbb{E}\left[(X-\mu)^2\right] = \mathbf{Var}(X) = \sigma^2$, we obtain that,

$$(I(\theta))_{22} = \frac{1}{2\sigma^4}.$$

- Since $\pi(\theta) \propto \sqrt{\det I(\theta)}$, we obtain that,

去掉constant

$$\det I(\theta) = \boxed{\frac{1}{2\sigma^6} \implies \pi(\theta) \propto \frac{1}{\sigma^3}.}$$

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

## Discussion

Show Discussion

**Topic:** Unit 5 Bayesian statistics:Lecture 18: Jeffrey's Prior and Bayesian Confidence Interval / 7. Jeffreys Prior II: Examples