6. Gaussian Probability Tables and Quantiles

Review: Gaussian Probability Tables and Quantiles

Start of transcript. Skip to the end.

(Caption will be displayed when you start playing the video.)

So what do those tables look like?

They said that when we're going to want

to compute probabilities, rather than going to a computer that

says, oh, do some numerical integration of my density

between minus infinity and t, or between u and v,

I'm going to actually use tables for that.

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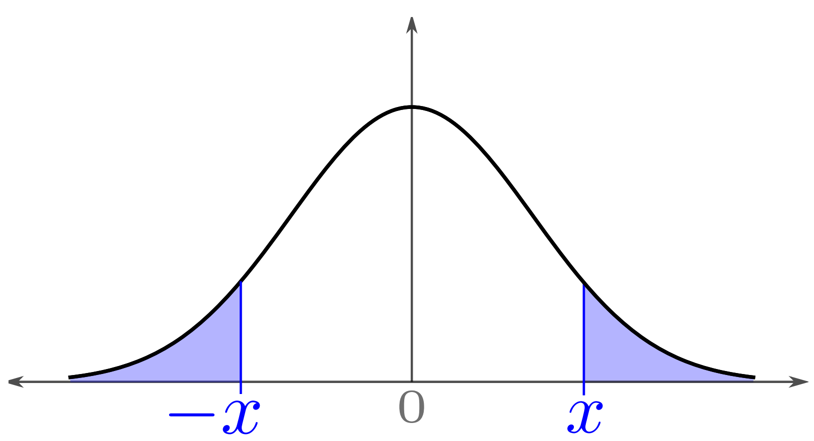
Quantiles

0/1 point (graded)

The **quantile** of order $1-\alpha$ of a variable X, denoted by q_α, is the number such that $\mathbf{P}\left(X \leq q_\alpha\right) = 1-\alpha$.

Graphed below is the pdf of the normal distribution. If the total area of the two shaded regions is 0.03, then what is x?
(Choose all that apply.)



The total area of the two shaded regions is 0.03.

- [ ] $\mathbf{P}\left(|X| \leq 0.03\right)$
- [x] $\mathbf{P}\left(|X| \leq 0.015\right)$
- [ ] 0.07

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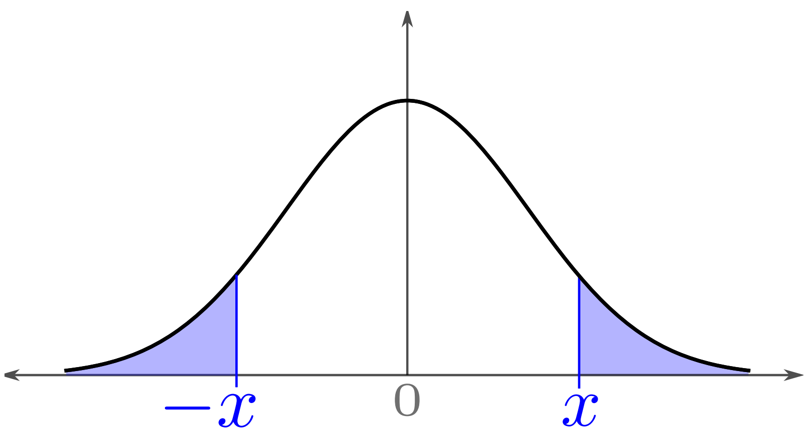
- [ ] 0.085
- [x] $q_{0.03}$
- [ ] $q_{0.015}$ ✔

✖

Solution:



The total area of the two shaded regions equals $\mathbf{P}\left(|X| \geq x\right) = 0.03$. By symmetry, the probability in the positive tail is $\mathbf{P}\left(X \geq x\right) = 0.015$; hence $x = q_{\alpha}$ with $\alpha = 0.015$.

For the wrong choices:

- The first pair of choices mixed up the values of probability with the value of the variable.
- The fourth choice "0.085" can be interpreted as $1-\alpha$ in this example; and the third choice "0.07" as $1-\alpha$ in the case if the area of one of the tails is 0.03.
- The fifth choice would have been correct again if the area of one of the tails is 0.03.

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Answers are displayed within the problem

Quantiles

1/1 point (graded)

Which of the following is the correct ordering of the numbers $q_{0.05}, 2q_{0.5}, q_{0.02}$, which are quantiles of a standard Gaussian variable?

- ( ) $q_{0.02} < 2q_{0.5} < q_{0.05}$
- (•) $2q_{0.5} < q_{0.05} < q_{0.02}$ ✔
- ( ) $q_{0.05} < 2q_{0.5} < q_{0.02}$
- ( ) $q_{0.05} < q_{0.02} < 2q_{0.5}$

Solution:

Recall that q_{α} is the number such that $\mathbf{P}\left(X \geq q_{\alpha}\right) = \alpha$; that is, the probability of the tail to the right of q_{α} is α. Therefore, $q_{0.05} < q_{0.02}$. Since these are quantiles of a standard Gaussian variable, $q_{0.5} = 0$, and $2q_{0.5} = 0$. So the correct ordering is given by $2q_{0.5} < q_{0.05} < q_{0.02}$. **Remark:**. In general, the quantiles of any continous random variable satisfes $q_a > q_b$ if $a < b$.

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Grades in a class

1/1 point (graded)

The score distribution of the final exam in a data science course follows a normal distribution with **mean** 70 and **standard deviation** 10.

Let α in $(0,1)$. As a reminder, the quantile of order $1-\alpha$ of a random variable X is the number q_α such that

$$\mathbf{P}(X \leq q_\alpha) = 1 - \alpha.$$

According to this distribution, what score do you need to get in order to be at the 90th percentile of the class, that is, in order that 90% of all students in the class have a score less than or equal to your score?

Use either the standard normal table below or any online calculator or software.

Normal Table

The entries in this table provide the numerical values of $\Phi(z) = \mathbf{P}(Z \leq z)$, where Z is a standard normal random variable, for z between 0 and $3.49.$ For example, to find $\Phi(1.71)$, we look at the row corresponding to 1.7 and the column corresponding to $0.01,$ so that $\Phi(1.71) = .9564.$ When z is negative, the value of $\Phi(z)$ can be found using the formula $\Phi(z) = 1 - \Phi(-z)$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

*For $z \geq 3.50$, the probability is greater than or equal to $.9998$.

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(Do **Not** round to the integer.)

Required score: 82.8 ✔ **Answer:** 82.82

STANDARD NOTATION

Solution:

Given the final exam grade X follows a normal distribution with $\mu = 70$ and $\sigma = 10$, we can define a variable $Z = \frac{X-70}{10}$ that follows the standard normal distribution $\mathcal{N}(0,1)$.

$$\mathbf{P}(X \leq t) = 0.9 \quad \text{if and only if} \quad \mathbf{P}\left(Z \leq \frac{t-70}{10}\right) = 0.9 \quad \text{if and only if} \quad \frac{t-70}{10} = q_{0.1} = \Phi^{-1}(0.9)\,.$$

where $q_{0.1} = \Phi^{-1}$ is the inverse of the cdf of the standard normal distribution. Since $\Phi^{-1}(0.9) = 1.282$ (e.g. by using **qnorm(0.9)** in R), we have

$$t \;=\; 1.282 * 10 + 70 \;=\; 82.82.$$

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

讨论

显示讨论

主题： Unit 1 Introduction to statistics:Lecture 2: Probability Redux / 6. Gaussian Probability Tables and Quantiles

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