

We now start our discussion of stochastic processes by starting with the simplest stochastic process there is. This is the so-called Bernoulli process, which is nothing but a sequence of independent Bernoulli trials. We let  $X_i$  stand for the random variable that describes the result in the  $i$  trial.

The assumptions that we make are that at each trial there is a certain probability,  $p$ , that the trial results in 1. And in that case, we usually say that there is a success at the  $i$ th trial. And the remaining probability,  $1$  minus  $p$ , is assigned to the possibility that the random variable  $X_i$  takes a value of 0, in which case sometimes we say that there was a failure.

Now, to keep things nontrivial, we will always assume that  $p$  is a number strictly between 0 and 1, because otherwise, in the extreme cases of  $p$  equal to 0 or  $p$  equal to 1, there isn't really any randomness.

This process is something that we have already seen. We have worked plenty of examples involving repeated Bernoulli trials or repeated coin flips. We have solved several problems, we have seen several formulas.

Here we will recapitulate some of them. But we will also start looking at the process from a new point of view. Before continuing, let me emphasize the assumptions that come into the Bernoulli process.

One key assumption is that we have independence. The different trials are independent.

The second assumption that we make is that the model is time homogeneous. What I mean by this is that this probability  $p$  of success at a given trial is the same for all trials. It does not depend on  $i$ .

So this process is as simple as a stochastic process could be. But nevertheless, it can be used as a model in various situations. Sometimes it's clear that we're dealing with a Bernoulli process, but sometimes it also shows up in unexpected ways.

In any case, the first simple example could be the following. Every week you play the lottery, and either you win or you do not to win. And assuming that it is the same kind of lottery that you play each week, the constant  $p$  would be the same, the probability of success. And assuming a lottery that is not rigged in any way, whether you win on one week, should be independent from what happens in other weeks.

Quite often, the Bernoulli process is used as a model of arrivals, in which case, instead of saying probability of success, we would say probability of an arrival.

The idea is that time is slotted, let's say, in seconds, for example. And each second you're sitting at the entrance of a bank and you make a note whether somebody came into the bank, in which case we have an arrival or success, or whether no one came during that time interval.

If one here believes that different slots, different seconds, are independent of each other, then you do have a Bernoulli process. It might not be an exactly accurate model, but it is a good first approximation to start working with a model of this situation.

Similarly, if you have a server, a computer, that takes jobs to process and jobs are coming randomly, you divide time into slots, and during each slot a job might arrive or might not arrive. And as a first approach to a model of this kind, you might as well employ the Bernoulli process.

A final note, why is this process called the Bernoulli process? Well, the name comes from a famous family of Swiss mathematicians, the Bernoulli family.

And one of them, Jacob Bernoulli, did many contributions to many branches of mathematics. But an important one was in the field of probability, where he actually derived quite deep results on a sequence of what we now call Bernoulli trials.