

In this lecture, we introduce Markov chains, a general class of random processes with many applications dealing with the evolution of dynamical systems. As opposed to the Bernoulli and Poisson processes, which are memoryless in a sense that the future does not depend on the past, Markov chains are more elaborate, as they allow some dependencies between different times.

However, these dependencies are of simple and restricted nature, captured by the so-called Markov property. Conditional on the current state of the Markov chain, its future and past evolutions are independent. As mentioned in the unit overview, we will only consider discrete time Markov chains that evolve within finite state spaces. This allows us to concentrate on the main concepts without having to deal with some required technical details needed to study general Markov processes under continuous time and general, possibly uncountable, state spaces.

We will first introduce the basic concepts, using the simple example of a checkout counter at a supermarket, an example of a simple queuing system. We will then abstract from the example and give some general definitions, including the central notions of states, transition probabilities, Markov property, and transition probability graphs.

Afterwards, we will look at various questions, such as predicting what will happen in  $n$ -steps in the future, given the current state of our system. We will define  $n$ -step transition probabilities exactly and show how to calculate them efficiently.

We will also discuss what could happen when we let the Markov chain run for a very long time. We will end this lecture by introducing the notions of recurrent and transient states and their importance in studying Markov chains in the long run.