

6. Bob and his coins

Problem 6. Bob and his coins

9.0/10.0 points (graded)

Bob has two coins, A and B, in front of him. The probability of Heads at each toss is p=0.5 for coin A and q=0.9 for coin B.

Bob chooses one of the two coins at random (both choices are equally likely).

He then continues with $\bf 5$ tosses of the chosen coin; these tosses are conditionally independent given the choice of the coin.

Let:

 H_i : the event that Bob's ith coin toss resulted in Heads;

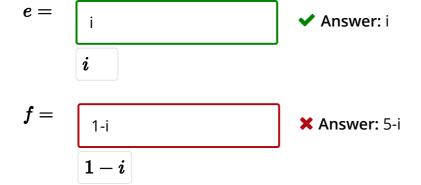
N: the number of Heads in Bob's coin tosses.

1. For $i \in \{0,1,\ldots,5\}$, $p_N(i)$, the pmf of N , is of the form

$$rac{1}{2}inom{5}{a}b^5+cinom{5}{d}q^e(1-q)^f.$$

Find the coefficients a, b, \ldots, f . Your answer can be either a number or an expression involving i.

$$d=igcap i$$
 Answer: i



2. Find $\mathbf{E}[N]$.

$$\mathbf{E}[N] = \boxed{3.5}$$
 \checkmark Answer: 3.5

3. Find the conditional variance of N, in a conditional model where we condition on having chosen coin A and the first two tosses resulting in Heads.

$$Var(N \mid A, H_1, H_2) = 3/4$$
 \checkmark Answer: 0.75

4. Are the events H_1 and $\{N=5\}$ independent?

5. Given that the 3rd toss resulted in Heads, what is the probability that coin \boldsymbol{A} was chosen?

STANDARD NOTATION

Solution:

1. Using the total probability theorem,

$$egin{align} \mathbf{P}(N=i) &= \mathbf{P}(N=i \mid A)\mathbf{P}(A) + \mathbf{P}(N=i \mid B)\mathbf{P}(B) \ &= rac{1}{2}inom{5}{i}p^5 + rac{1}{2}inom{5}{i}q^i(1-q)^{5-i} \ \end{aligned}$$

2. Using the total expectation theorem,

$$\mathbf{E}[N] = rac{1}{2}(\mathbf{E}[N|A] + \mathbf{E}[N|B]) = rac{1}{2}(5p + 5q) = rac{7}{2}.$$

3. Conditioned on H_1, H_2 , we have N=2+M where M is the number of Heads in the last three tosses. Adding a constant does not change the variance. Thus,

$$\mathsf{Var}(N|A, H_1, H_2) = \mathrm{var}(M|A) = 3p(1-p) = 0.75.$$

4. No. It suffices to show $\{N=5\}$ and H_1^c are not independent. By part 1, we have

$${f P}(N=5)=rac{1}{2}p^5+rac{1}{2}q^5
eq 0,$$

whereas

$$\mathbf{P}(N=5|H_1^c)=0.$$

Since $\mathbf{P}(N=5)
eq \mathbf{P}(N=5|H_1^c)$, the events N=5 and H_1^c are not independent.

5. By Bayes' rule we have

$$\mathbf{P}(A|H_3) = rac{\mathbf{P}(H_3|A)\mathbf{P}(A)}{\mathbf{P}(H_3)}.$$

We have $\mathbf{P}(A)=0.5$ and $\mathbf{P}(H_3|A)=0.5$. Also,

$$\mathbf{P}(H_3) = \mathbf{P}(H_3|A)\mathbf{P}(A) + \mathbf{P}(H_3|B)\mathbf{P}(B) = \frac{1}{2}(0.5 + 0.9) = 0.7.$$

Therefore,

$$\mathbf{P}(A|H_3) = rac{0.5*0.5}{0.7} = rac{5}{14} \sim 0.357.$$

提交

You have used 2 of 2 attempts

Answers are displayed within the problem

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