Practice: Gamma distribution as Exponential Family

1/1 point (graded)

Recall from the slides that the Gamma distribution can be reparameterized using the two parameters a, the shape parameter, and μ, the mean. The pdf looks like

$$f_{(a,\mu)}(y) \;=\; \frac{1}{\Gamma(a)}\left(\frac{a}{\mu}\right)^{a} y^{a-1}\, e^{-\frac{ay}{\mu}}$$

Let $\boldsymbol{\theta} = \begin{pmatrix} a \\ \mu \end{pmatrix}$ and rewrite this as the pdf of a 2-parameter exponential family. Enter $\boldsymbol{\eta}(\boldsymbol{\theta}) \cdot \mathbf{T}(\mathbf{y})$ below.

$\boldsymbol{\eta}(\boldsymbol{\theta}) \cdot \mathbf{T}(\mathbf{y}) =$ (a-1)*ln(y) - a/mu*y

✔ **Answer:** -a*y/mu+(a-1)*ln(y)

STANDARD NOTATION

Solution:

$$\begin{aligned} f_{(a,\mu)}(y) &= \frac{1}{\Gamma(a)}\left(\frac{a}{\mu}\right)^{a} y^{a-1}\, e^{-\frac{ay}{\mu}} \\ &= \exp\left(\left(-\frac{ay}{\mu} + (a-1)\ln(y)\right) + \left(a\ln\left(\frac{a}{\mu}\right) - \ln(\Gamma(a))\right)\right) \end{aligned}$$

Hence, we have $\boldsymbol{\eta}(\boldsymbol{\theta}) \cdot \mathbf{T}(\mathbf{y}) = \left(-\frac{ay}{\mu} + (a-1)\ln(y)\right)$, where possibly $\boldsymbol{\eta} = \begin{pmatrix} -\frac{a}{\mu} \\ a-1 \end{pmatrix}$ and $\mathbf{T}(\mathbf{y}) = \begin{pmatrix} y \\ \ln(y) \end{pmatrix}$. Here, $\boldsymbol{\eta}$ and $\mathbf{T}$ are not unique since we can multiple $\boldsymbol{\eta}$ by an overall scalar and divide $\mathbf{T}$ by the same.

On the other hand, $B(\boldsymbol{\theta}) = -\left(a\ln\left(\frac{a}{\mu}\right) - \ln(\Gamma(a))\right)$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem