7. Exercise: Multiple observations

Exercise: Multiple observations

2/2 points (graded)

Consider a model involving multiple observations of the form $X_i = c_i\Theta + W_i$, $i = 1, 2, \ldots, n$, where $\Theta, W_1, \ldots, W_n$ are independent (not necessarily normal) random variables and the c_i's are known nonzero constants. Assume that Θ has positive variance.

a) Are the random variables X_i, $i = 1, 2, \ldots, n$, independent?

No ✔ **Answer:** No

b) Are the random variables X_i, $i = 1, 2, \ldots, n$, conditionally independent given Θ?

Yes ✔ **Answer:** Yes

Solution:

a) The X_i's are dependent because they are all affected by Θ. For a mathematical derivation, you can consider the zero mean case and check that $\mathbf{E}[X_1X_2] = c_1c_2\mathbf{E}[\Theta^2] \neq 0$, whereas $\mathbf{E}[X_1]\mathbf{E}[X_2] = 0$.

b) If we are given that $\Theta = \theta$, then $X_i = c_i\theta + W_i$. In the conditional universe, θ is now a number. Furthermore, the W_i's are independent. Thus, the X_i's (which are equal to W_i plus a number) are also (conditionally) independent.

提交 You have used 1 of 1 attempt

Answers are displayed within the problem

讨论

Topic: Unit 7 / Lec. 15 / 7. Exercise: Multiple observations

显示讨论