

### 3. QQ Plots

Consider an iid sample  $X_1, X_2, \dots, X_n \stackrel{iid}{\sim} \mathbf{P}$  that has been reordered as  $X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(n)}$  where  $n$  is very large. In the problems below, we have chosen a different distribution for  $\mathbf{P}$  and compared the empirical quantiles to the standard Gaussian quantiles using a QQ plot. Recall that

- the **Laplace distribution**  $\text{Lap}(\lambda)$  with parameter  $\lambda > 0$  is the continuous probability distribution with density  $f_\lambda = \frac{\lambda}{2} e^{-\lambda|x|}$ , and
- the **Cauchy distribution** is the continuous probability distribution with density  $g(x) = \frac{1}{\pi} \frac{1}{1+x^2}$ .

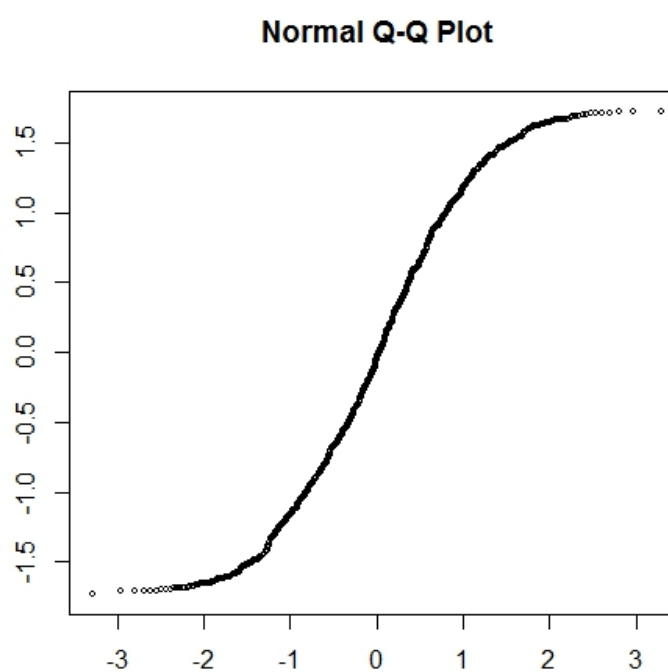
(These were also introduced in Lecture 12. )

For each plot below, match the QQ plot with the correct distribution for  $\mathbf{P}$ . *Hint:* Each possible distribution will be an answer choice exactly once, so you should use the process of elimination.

*Hint:* You may use computational tools to graph the pdf of the possible distributions of  $\mathbf{P}$ .

#### Matching a Distribution to a QQ Plot I

1/1 point (graded)



☐ Standard normal:  $N(0, 1)$

☐ Cauchy distribution

☐ Exponential with parameter 1:  $\text{Exp}(1)$

☒ Uniform on the interval  $[-\sqrt{3}, \sqrt{3}]$ :  $\text{Unif}[-\sqrt{3}, \sqrt{3}]$  ✓

☐ Laplace distribution with parameter  $\sqrt{2}$ :  $\text{Lap}(\sqrt{2})$

**Solution:**

The distribution for this QQ plot is Uniform on the interval  $[-\sqrt{3}, \sqrt{3}]$ : **Unif** $[-\sqrt{3}, \sqrt{3}]$ . Since the support for this distribution is  $[-\sqrt{3}, \sqrt{3}]$ , the empirical quantiles  $X_{(1)}, X_{(2)}, \dots, X_{(n)} \in [-\sqrt{3}, \sqrt{3}]$ . Since there is nothing plotted outside of the interval  $[-\sqrt{3}, \sqrt{3}]$  on the y-axis, we see that the support is restricted to this interval. This implies that the sample was generated from a uniform distribution.

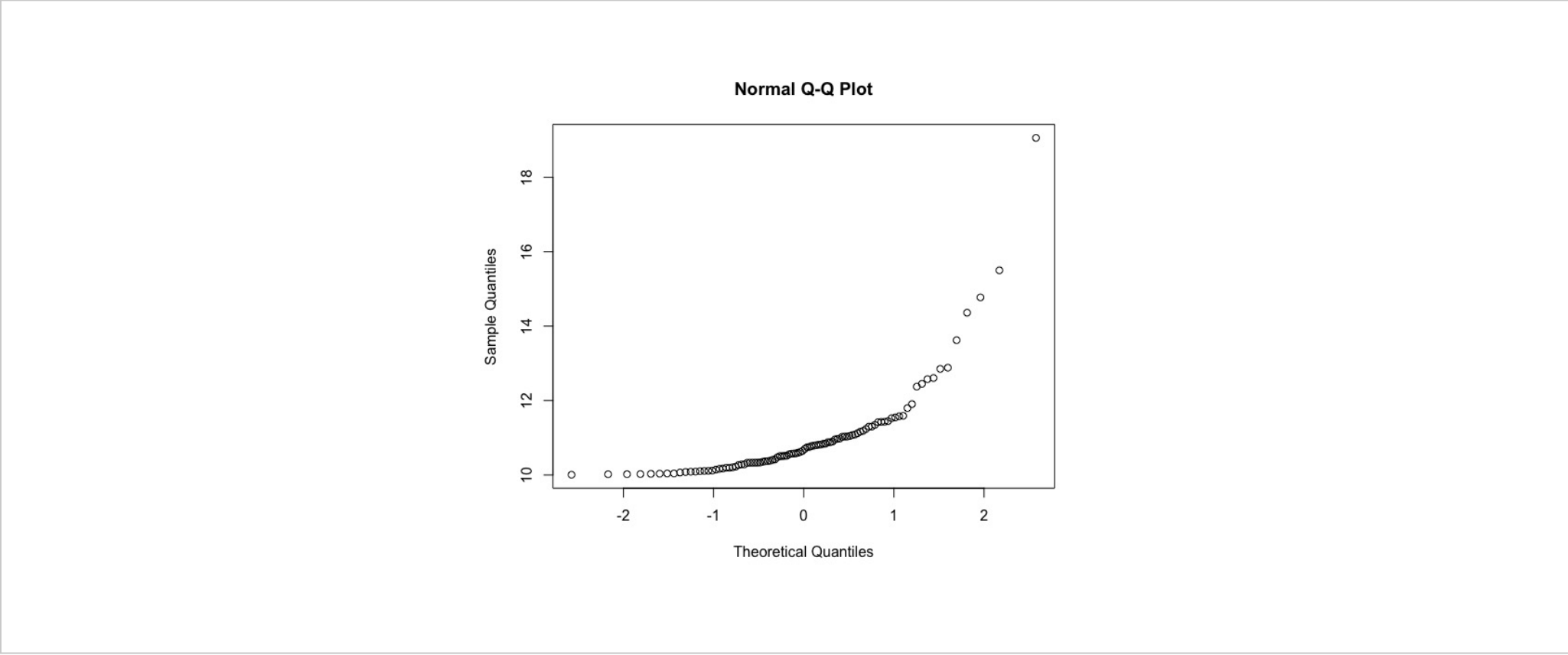
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You have used 1 of 2 attempts

**i** Answers are displayed within the problem

### Matching a Distribution to a QQ Plot II

1/1 point (graded)



- ☐ Standard normal:  $N(0, 1)$
- ☐ Cauchy distribution
- ☒ Shifted exponential with parameter 2.5: **Exp (2.5) + c** for some  $c > 0$  ✓
- ☐ Uniform on the interval  $[-\sqrt{3}, \sqrt{3}]$ : **Unif** $[-\sqrt{3}, \sqrt{3}]$
- ☐ Laplace distribution with parameter  $\sqrt{2}$ : **Lap** $(\sqrt{2})$

#### Solution:

The distribution for this QQ plot is **Shifted exponential with parameter 1 : Exp (2.5) + c**. Note that the exponential distribution (not shifted by any constant) is supported on  $[0, \infty)$ . Hence, the QQ plot will not go below the line  $y = 0$  if it is shifted by a positive constant  $c$ . Moreover, the exponential distribution has **heavier** tails than those of  $N(0, 1)$ , so we expect the QQ plot to be above the line  $y = x$ , which is indeed the case here. Further, the sample quantiles in this example do not start near the value 0 and they rather start near the value 10.

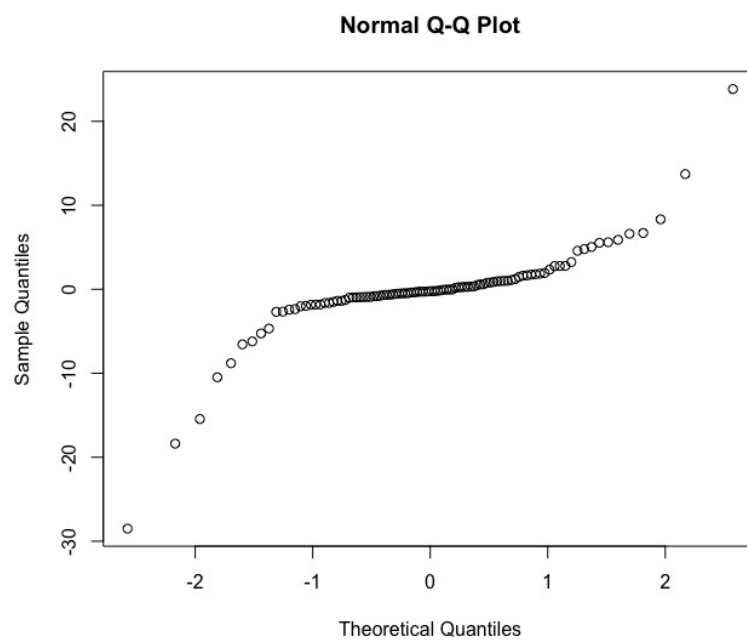
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### Matching a Distribution to a QQ Plot III

1/1 point (graded)



☐ Standard normal:  $N(0, 1)$

☒ Cauchy distribution ✓

☐ Exponential with parameter 1:  $\text{Exp}(1)$

☐ Uniform on the interval  $[-\sqrt{3}, \sqrt{3}]$ :  $\text{Unif}[-\sqrt{3}, \sqrt{3}]$

☐ Laplace distribution with parameter  $\sqrt{2}$ :  $\text{Lap}(\sqrt{2})$

#### Solution:

The distribution for this QQ plot is the **Cauchy distribution**. A Cauchy random variable takes values on all of  $\mathbb{R}$ . Since the pdf  $g(x)$  of the Cauchy distribution decays on the order of  $1/x^2$  as  $x \rightarrow \infty$ , we know that its tails should be much heavier than those of a standard normal, whose tails decay exponentially. On the right, we see that the QQ plot displayed lies very far above the line  $y = x$ . On the left, we see that the QQ plot displayed lies very far below the line  $y = x$ . This indicates that the distribution displayed has much heavier tails than that of a Gaussian, so the Cauchy distribution must be the correct answer.

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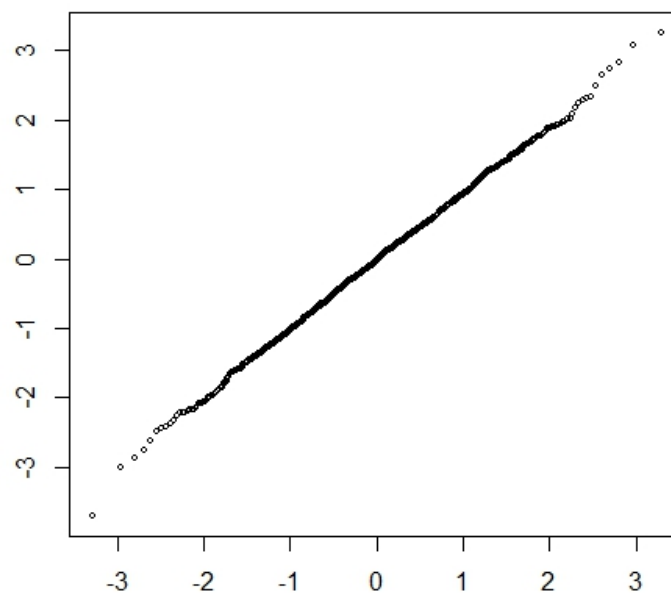
You have used 1 of 2 attempts

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## Matching a Distribution to a QQ Plot IV

1/1 point (graded)

Normal Q-Q Plot



☒ Standard normal:  $N(0, 1)$  ✓

☐ Cauchy distribution

☐ Exponential with parameter 1:  $\text{Exp}(1)$

☐ Uniform on the interval  $[-\sqrt{3}, \sqrt{3}]$ :  $\text{Unif}[-\sqrt{3}, \sqrt{3}]$

☐ Laplace distribution with parameter  $\sqrt{2}$ :  $\text{Lap}(\sqrt{2})$

### Solution:

The distribution for this QQ plot is **Standard Gaussian  $N(0, 1)$** . Observe that the QQ plot lies very close to the line  $y = x$ , so this suggests that the data is distributed as  $N(0, 1)$ . By process of elimination, we conclude that the data must have been generated from a standard Gaussian.

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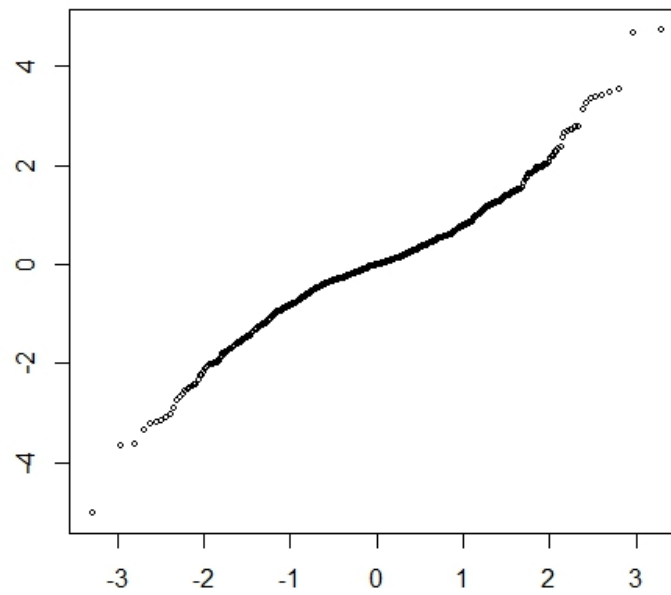
You have used 1 of 2 attempts

**i** Answers are displayed within the problem

## Matching a Distribution to a QQ Plot V

1/1 point (graded)

Normal Q-Q Plot



- ☐ Standard normal:  $N(0, 1)$
- ☐ Cauchy distribution
- ☐ Exponential with parameter 1:  $\text{Exp}(1)$
- ☐ Uniform on the interval  $[-\sqrt{3}, \sqrt{3}]$ :  $\text{Unif}[-\sqrt{3}, \sqrt{3}]$
- ☒ Laplace distribution with parameter  $\sqrt{2}$ :  $\text{Lap}(\sqrt{2})$  ✓

### Solution:

The distribution for this QQ plot is the **Laplace distribution  $\text{Lap}(\sqrt{2})$** . A Laplace random variable takes values on all of  $\mathbb{R}$ . Since the pdf  $f_{\sqrt{2}}$  of the Cauchy distribution decays on the order of  $e^{-|x|}$  as  $x \rightarrow \infty$ , we know that its tails should be heavier than those of a standard normal, whose tails decay at the rate  $e^{-x^2}$ . On the right, we see that the QQ plot displayed lies above the line  $y = x$ . On the left, we see that the QQ plot displayed lies below the line  $y = x$ . This indicates that the distribution displayed **moderately heavier tails than that of a Gaussian**, so by this observation and the process of elimination, the Laplace distribution must be the correct answer.

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### Discussion

Topic: Unit 4 Hypothesis testing:Homework 8 / 3. QQ Plots

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