10. Exercise: Mean squared error

Exercise: Mean squared error

4/4 points (graded)

As in an earlier exercise, we assume that the random variables Θ and X are described by a joint PDF which is uniform on the triangular set defined by the constraints $0 \le x \le 1$, $0 \le \theta \le x$.

a) Find an expression for the conditional mean squared error of the LMS estimator given that $X = x$, valid for $x \in [0, 1]$. Express your answer in terms of x using standard notation.

x^2/12 ✔ **Answer:** x^2/12

b) Find the (unconditional) mean squared error of the LMS estimator.

1/24 ✔ **Answer:** 0.04167

STANDARD NOTATION

Solution:

a) We saw that the conditional PDF of Θ is uniform on the range $[0, x]$. Hence, the conditional variance is $x^2/12$.

b) This is given by the integral of the conditional variance, weighted by the PDF of X. The PDF of X is found using the formula for going from the joint to the marginal, and is $f_X(x) = 2x$, for $x \in [0, 1]$. Thus, the mean squared error is

$$\int_0^1 \frac{x^2}{12} \cdot 2x\, dx = \frac{1}{6} \int_0^1 x^3\, dx = \frac{1}{24}.$$

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{\text{area of } S}, & \text{if } (x,y) \in S, \\ 0, & \text{otherwise.} \end{cases}$$

$$f_X(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y,Z}(x,y,z) \boxed{dy\, dz.}$$

Area of triangular is 1/2, so joint pdf has the value of 2. Then by definition of marginal $f_X(x) = \int_{\theta=0}^{\theta=x} 2 d\theta = 2x$.

提交 You have used 3 of 3 attempts

Answers are displayed within the problem