5. Poisson Limits

Problem 4. Poisson Limits

0.0/5.0 points (graded)

In this problem, you may find it useful to recall the following fact about Poisson random variables. Let X and Y be two independent Poisson random variables, with means λ_1 and λ_2, respectively. Then, $X+Y$ is a Poisson random variable with mean $\lambda_1+\lambda_2$. Arguing in a similar way, a Poisson random variable X with parameter t, where t is a positive integer, can be thought of as sum of t independent Poisson random variables $X_1, X_2, \ldots, X_t$, each of which has mean 1.

Using the information above, and an appropriate limit theorem, evaluate the following limit:

$$\lim_{n\to\infty} \sum_{k>n+\sqrt{n}}^{\infty} \frac{e^{-n} n^k}{k!}.$$

0

✖ **Answer:** 0.158

Solution:

Let Y be a Poisson random variable with mean n. Then, $\mathbf{P}(Y=k) = \frac{e^{-n}n^k}{k!}$, for $k=1,2,\ldots$. Notice that,

$$\sum_{k>n+\sqrt{n}}^{\infty} \frac{e^{-n}n^k}{k!} = \sum_{k>n+\sqrt{n}}^{\infty} \mathbf{P}(Y=k) = \mathbf{P}(Y>n+\sqrt{n}).$$

Using the hint above, let $Y = Y_1 + Y_2 + \cdots + Y_n$, where $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed Poisson random variables, with mean 1 (thus, $\mathbf{E}[Y_i] = \mathbf{var}(Y_i) = 1$). Then,

$$\mathbf{P}(Y>n+\sqrt{n}) = \mathbf{P}(X_1+\cdots+X_n > n+\sqrt{n}) = \mathbf{P}\left(\frac{X_1+\cdots+X_n-n}{\sqrt{n}} > 1\right).$$

Hence, from the central limit theorem,

$$\lim_{n\to\infty} \mathbf{P}\left(\frac{X_1+\cdots+X_n-n}{\sqrt{n}} > 1\right) = \mathbf{P}(Z>1) \approx 0.158,$$

where Z is a standard normal random variable.

提交 你已经尝试了1次（总共可以尝试2次）

Answers are displayed within the problem

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