1. The number of trains arriving on days 1, 2, and 3 is independent of the number of trains arriving on day 0. Let N denote the total number of trains that arrive on days 1, 2, and 3. Then N is a Poisson random variable with parameter $3\lambda = 9$, and we have

$$\begin{aligned} P(\text{no train on days 1,2,3} \mid \text{one train on day 1}) &= P(\text{no train on days 1,2,3}) \\ &= P(N = 0) \\ &= \frac{e^{-9}9^0}{0!} \\ &= e^{-9}\,. \end{aligned}$$

2. The event that the next arrival is more than three days after the train arrival on day 0 is the same as the event that there are zero arrivals in the three days after the train arrival on day 0. Therefore the required probability is the same as that found in part (a), namely, e^{-9}.

3. The number of trains arriving in the first 2 days is independent of the number of trains arriving on day 4. Therefore, we have

$$\begin{aligned} P(\text{0 trains in first 2 days and 4 trains on day 4}) &= P(\text{0 trains in first 2 days}) \cdot P(\text{4 trains on day 4}) \\ &= e^{-2\lambda}\frac{(2\lambda)^0}{0!} \cdot e^{-\lambda}\frac{\lambda^4}{4!} \\ &= e^{-9}\frac{3^4}{4!}\,. \end{aligned}$$

4. The event that it takes more than 2 days for the 5th arrival is equivalent to the event that there are at most 4 arrivals in the first 2 days. Therefore the required probability is equal to

$$\begin{aligned} \sum_{k=0}^{4} P(\text{exactly } k \text{ arrivals in first 2 days}) &= \sum_{k=0}^{4} e^{-2\lambda}\frac{(2\lambda)^k}{k!} \\ &= e^{-2\lambda}\left(\frac{(2\lambda)^0}{0!} + \frac{(2\lambda)^1}{1!} + \frac{(2\lambda)^2}{2!} + \frac{(2\lambda)^3}{3!} + \frac{(2\lambda)^4}{4!}\right) \\ &= e^{-6}(1 + 6 + 18 + 36 + 54) \\ &= 115e^{-6}\,. \end{aligned}$$