7. Exercise: Expected value rule and total expectation theorem

Exercise: Expected value rule and total expectation theorem

6/8 points (graded)

Let X, Y, and Z be jointly continuous random variables. Assume that all conditional PDFs and expectations are well defined. E.g., when conditioning on $X=x$, assume that x is such that $f_X(x)>0$. For each one of the following formulas, state whether it is true for all choices of the function g or false (i.e., not true for all choices of g).

1. $\mathbf{E}\big[g(Y)\,|\,X=x\big]=\displaystyle\int g(y)f_{Y|X}(y\,|\,x)\,dy$

   True ▼ ✔ **Answer:** True

2. $\mathbf{E}\big[g(y)\,|\,X=x\big]=\displaystyle\int g(y)f_{Y|X}(y\,|\,x)\,dy$

   False ▼ ✔ **Answer:** False

3. $\mathbf{E}\big[g(Y)\big]=\displaystyle\int \mathbf{E}\big[g(Y)\,|\,Z=z\big]\,f_Z(z)\,dz$

   True ▼ ✔ **Answer:** True

4. $\mathbf{E}\big[g(Y)\,|\,X=x,Z=z\big]=\displaystyle\int g(y)f_{Y|X,Z}(y\,|\,x,z)\,dy$

   True ▼ ✔ **Answer:** True

5. $\mathbf{E}\big[g(Y)\,|\,X=x\big]=\displaystyle\int \mathbf{E}\big[g(Y)\,|\,X=x,Z=z\big]\,f_{Z|X}(z\,|\,x)\,dz$

   False ▼ ✖ **Answer:** True

6. $\mathbf{E}\big[g(X,Y)\,|\,Y=y\big]=\mathbf{E}\big[g(X,y)\,|\,Y=y\big]$

   True ▼ ✔ **Answer:** True

7. $\mathbf{E}\big[g(X,Y)\,|\,Y=y\big]=\mathbf{E}\big[g(X,y)\big]$

False ▼ ✔ **Answer:** False

8. $\mathbf{E}\big[g(X,Z)\,|\,Y=y\big] = \int g(x,z) f_{X,Z|Y}(x,z\,|\,y)\, dy$

True ▼ ✖ **Answer:** False

Solution:

1. True. This is the usual expected value rule, applied to a conditional model where we are given that $X=x$.

2. False. Here the quantity inside the expectation, $g(y)$, is a number (not a random variable). The left-hand side is a function of y, whereas on the right-hand side, y, is a dummy variable that gets integrated away. So, the formula is wrong on a purely syntactical basis (the left-hand side depends on y, while the right-hand side does not).

3. True. This is the total expectation theorem, where we condition on the events $Z=z$.

4. True. This is the usual expected value rule, applied to a conditional model where we are given that $X=x$ and $Z=z$.

5. True. This is the same total expectation theorem as in the third part, except that everything is calculated within a conditional model in which event $X=x$ is known to have occurred.

6. True. When we condition on $Y=y$, we know the value of Y, and we can replace $g(X,Y)$ by $g(X,y)$.

7. False. Given that $Y=y$, we need to somehow take into account the conditional distribution of X, whereas the right-hand side is determined by the unconditional PDF of X.

8. False. The left-hand side is a function of y, whereas the right-hand side (after y is integrated out) is a function of x and z. The correct form (expected value rule, in a conditional model) is:

$$\mathbf{E}\big[g(X,Z)\,|\,Y=y\big] = \int\int g(x,z) f_{X,Z|Y}(x,z\,|\,y)\, dx\, dz.$$

提交 You have used 1 of 1 attempt

Answers are displayed within the problem

讨论

显示讨论

Topic: Unit 5 / Lec. 10 / 7. Exercise: Expected value rule and total expectation theorem