Maximum Entropy Tutorial

Homework

1. In the Taxicab problem described in the videos, suppose your empirical data has mean $\langle x \rangle$ = 7 minutes. Give the maximum entropy distribution $P_{\text{MaxEnt}}(x)$ that constrains this mean.

2. (a) In the Section 4 video (time ~5:30), Simon gives the partial derivative of the entropy S with respect to p_i as

$$\frac{\partial S}{\partial p_i} = -(\log p_i + 1)$$

Show how this expression is derived from the definition of *S*.

(b) In the Section 4 video (time ~ 5:50 to 6:00), Simon gives:

$$\frac{\partial g_1}{\partial p_i} = i \text{ and } \frac{\partial g_2}{\partial p_i} = 1$$

Show how these expressions are derived from the definitions of g_1 and g_2.

3. Referring to the Section 4 video (time ~ 7:25), show how the expression $Z = e^{1+\lambda_2}$ is derived.

4. (a) Referring to the Section 5 video (time ~ 2:25), show how the expression for $\frac{\partial Z}{\partial \lambda_1}$ is derived.

(b) In the same video (time ~ 2:40), Simon writes

$$-\frac{1}{Z}\frac{\partial Z}{\partial \lambda_1} = \frac{e^{-\lambda_1}}{(1 - e^{-\lambda_1})} = 4$$

Show how this expression was derived.

5. In the Section 8 video (time ~ 6:40-7:15) Simon gives the following expressions:

$$\frac{\partial g_1}{\partial p_i} = n \qquad \frac{\partial g_2}{\partial p_i} = n\varepsilon \qquad \frac{\partial g_3}{\partial p_i} = 1$$

Show how these expressions were derived.

6. Show that the maximum entropy distribution that constrains $\langle \log i \rangle$ is the power-law distribution $P(i) \propto i^{\alpha}$ (where α is a constant).

7. Show that the maximum entropy distribution that constrains $\langle \log i \rangle$ and $\langle (\log i)^2 \rangle$ is the log-normal distribution $P(i) \propto e^{-(x-\bar{x})^2/2\sigma^2}$, where $x = \log i$.

Advanced Problem

8. (a) Suppose you roll a six-sided die 100 times, and find that the mean value $\langle x \rangle$ of your rolls is 3. Give the maximum entropy distribution $P_{\text{MaxEnt}}(x)$ that constrains this mean.

(b) Same question, but this time assume the observed mean value $\langle x \rangle$ of your rolls is 2.