Orthogonal projection of a point to plane

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A point and a plane is given: point $P(-4,-9,-5)$ and plane defined with three points : $A(0,1,3)$, $B(-3,2,4)$ and $C(4,1,-2)$. So far I've managed to calculate the equation of this plane $5x+11y+4z=23$. How can I calculate the coordinates of the orthogonal projection of this point P to the plane?

linear-algebra vector-spaces

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edited Aug 7 '18 at 10:50
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3,417 2 11 33

asked Aug 7 '18 at 10:49
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27 6

2 Answers

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You asked for another way to do this, so here are a couple. The projection of P is the intersection of the plane defined by the three points and the line through P orthogonal to the plane—parallel to the plane's normal. Since you've already found an equation of the plane, you can use that to compute this point directly in a couple of ways.

- Find the signed distance of P from the plane and move toward it that distance along the normal: The signed distance of a point (x,y,z) from the plane is

  把需要求距离的点的坐标带入hyperplane的方程，得到一个距离

  $$\frac{5x+11y+4z-23}{\sqrt{5^2+11^2+4^2}}=\frac{(5,11,4)\cdot(x,y,z)-23}{9\sqrt2},$$

  得到距离 再用方程的norm进行normalize

  which comes out to $-9\sqrt2$ for P. From the equation that you derived, the corresponding unit normal is

  $$\frac{(5,11,4)}{\sqrt{5^2+11^2+4^2}}=\frac{1}{9\sqrt2}(5,11,4).$$ 复平面被normalize后的vector表示

  We want to move in the opposite direction, so the projection of P onto the plane is

  点的坐标 减去距离乘以复平面的unit normal

  $$(-4,-9,-5)-\frac{-9\sqrt2}{9\sqrt2}(5,11,4)=(-4,-9,-5)+(5,11,4)=(1,2,-1).$$

- Move to homogeneous coordinates and use the Plücker matrix of the line: The line through points $\mathbf p$ and $\mathbf q$ can be represented by the matrix $\mathcal L=\mathbf{pq}^T-\mathbf{qp}^T$. The intersection of this line and a plane π is $\mathcal L\pi=(\mathbf{pq}^T-\mathbf{qp}^T)\pi=(\pi^T\mathbf q)\mathbf p-(\pi^T\mathbf p)\mathbf q$. The quantities in parentheses in the final expression are just dot products of vectors. Applying this to the present problem, we have from the plane equation that you derived $\pi=[5,11,4,-23]^T$. For $\mathbf p$ we can take the point P, i.e., $\mathbf p=[-4,-9,-5,1]^T$, and for $\mathbf q$ the direction vector of the plane normal $[5,11,4,0]^T$. Plugging these values into the above expression, we get

  $$([5,11,4,-23]\cdot[5,11,4,0])\,[-4,-9,-5,1]-([5,11,4,-23]\cdot[-4,-9,-5,1])\,[5,11,4,0]=162[-4,-9,-5,1]+162[5,11,4,0]=[162,324,-162,162].$$

  Dehomogenize this by dividing through by the last coordinate to get the point $(1,2,-1)$. You could of course set up a system of parametric or implicit Cartesian equations and solve them for the intersection of the line and plane, but this method allows you to compute it directly.

answered Aug 7 '18 at 21:34
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If M is the projection of P in that plane so $PM\perp AB$, $PM\perp AC$ and $PM\perp BC$ then you find a system of three equation with three unknowns which are the coordinates of the Point M, you can also use the equation of the plane instead of one of this equation, Can you take it from here?

answered Aug 7 '18 at 11:01
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I've been trying to solve the system but I can't get the number result, I always end up with one unknown expressed with other two. Is there another way? – ckl Aug 7 '18 at 15:15