4. Breaking a stick twice

Problem 3. Breaking a stick twice

2/2 points (graded)

Let X be uniformly distributed on $[0,1]$. Given the value x of X, we let Y be uniformly distributed on $[0,x]$.

In lecture, we have seen that the PDF $f_Y(y)$ of Y is,

$$\begin{aligned} f_Y(y) &= \int_0^1 f_{Y|X}(y|x) f_X(x)\,dx \\ &= \int_y^1 \frac{1}{x}\,dx \\ &= -\ln(y), \end{aligned}$$

for $0 < y < 1$.

1. Find the conditional PDF of X, given that $Y = y$. For $0 < y < x < 1$:

   $f_{X|Y}(x \mid y) =$ 1/(-x*ln(y)) ✔ **Answer:** -1/(x*ln(y))

   $\frac{1}{-x \cdot \ln(y)}$

2. The conditional expectation of X given Y, namely $\mathbf{E}[X|Y]$ is of the form $h(Y)$ for some function $h(\cdot)$. Find $h(\cdot)$. For $0 < y < 1$:

   $h(y) =$ (y-1)/ln(y) ✔ **Answer:** (y-1)/ln(y)

   $\frac{y-1}{\ln(y)}$

Solution:

1. Note that conditioned on $X = x$, the PDF of Y is constant, and equal to $1/x$, for $0 < y < x$. Using the Bayes' rule, and for $0 < y < x < 1$,

$$\begin{aligned} f_{X|Y}(x \mid y) &= \frac{f_{Y|X}(y \mid x) f_X(x)}{f_Y(y)} \\ &= \frac{\frac{1}{x} \cdot 1}{-\ln(y)} \\ &= -\frac{1}{x \ln(y)}. \end{aligned}$$

2.

$$\begin{aligned} \mathbf{E}[X|Y=y] &= \int_0^1 x f_{X|Y}(x \mid y)\,dx \\ &= \int_y^1 -x\frac{1}{x\ln(y)}\,dx \\ &= \int_y^1 -\frac{1}{\ln(y)}\,dx \\ &= \frac{y-1}{\ln(y)}. \end{aligned}$$

提交 You have used 1 of 3 attempts

Answers are displayed within the problem

Error and Bug Reports/Technical Issues

Topic: Exam 2 / 4. Breaking a stick twice

显示讨论