Week 2 – part 4: Threshold in the Hodgkin-Huxley Model

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 2 – Biophysical modeling:
The Hodgkin-Huxley model

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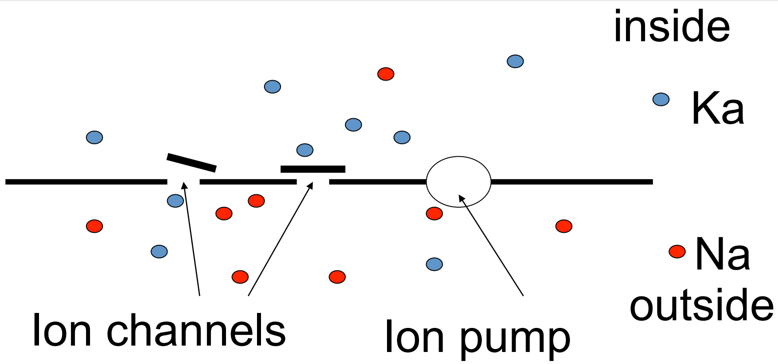
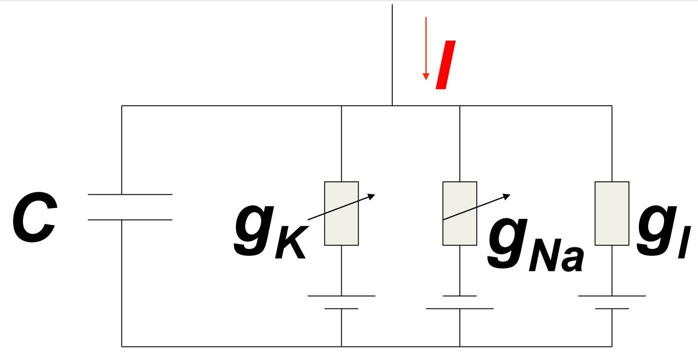
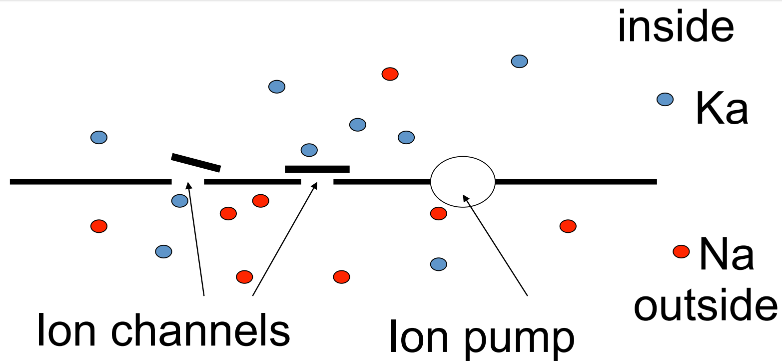
EPFL, Lausanne, Switzerland

Neuronal Dynamics – 2.4. Threshold in HH model



Neuronal Dynamics – 2.4. Threshold in HH model



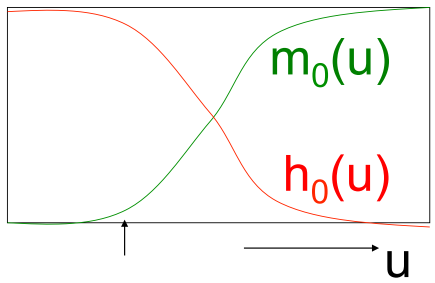


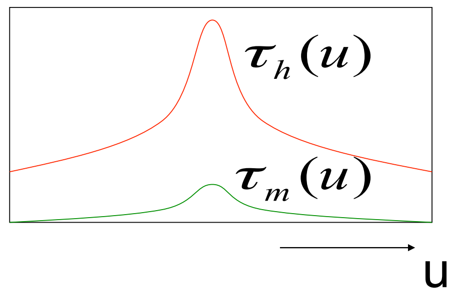
$$C\frac{du}{dt} = \overbrace{-g_{Na}m^3h(u-E_{Na})}^{I_{Na}} - g \ldots \quad \ldots g_l(u-E_l) + I(t)$$

$$\frac{dm}{dt} = \ldots \quad \ldots (u)$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

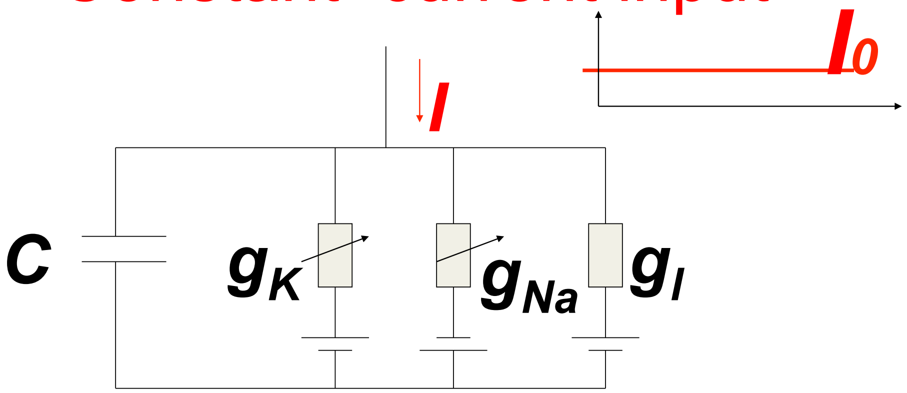
Where is the threshold for firing?

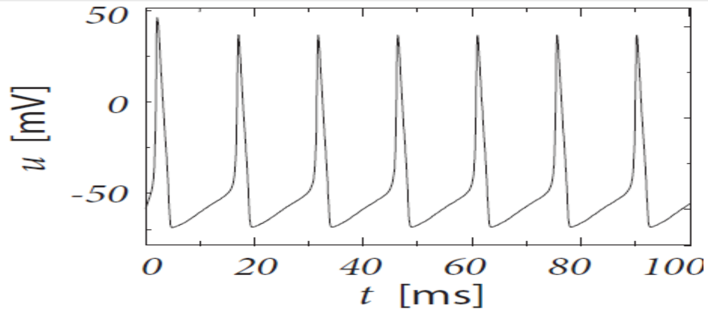


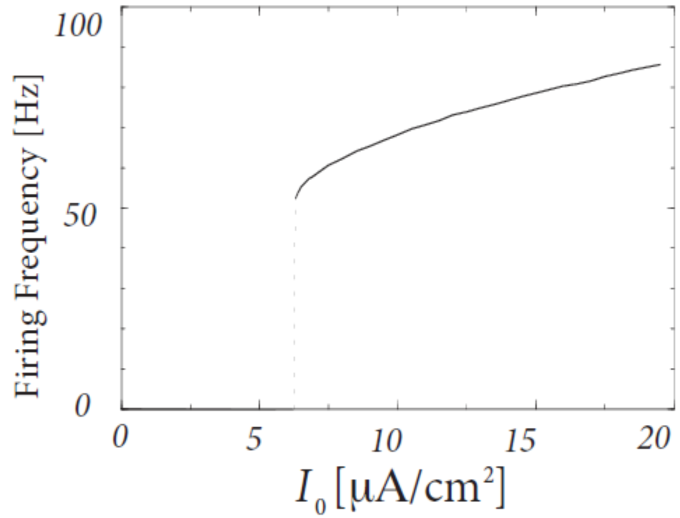


Neuronal Dynamics – 2.4. Threshold in HH model

Constant current input







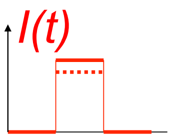
Threshold?

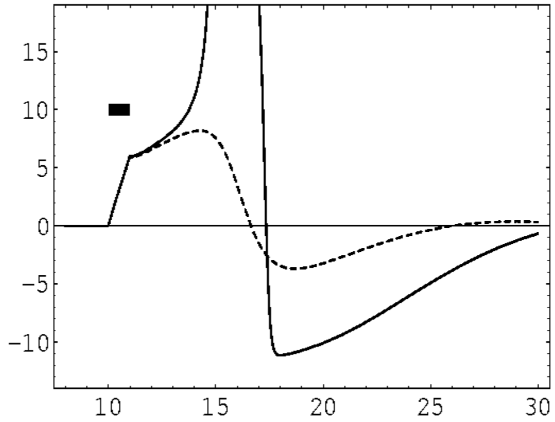
for repetitive firing

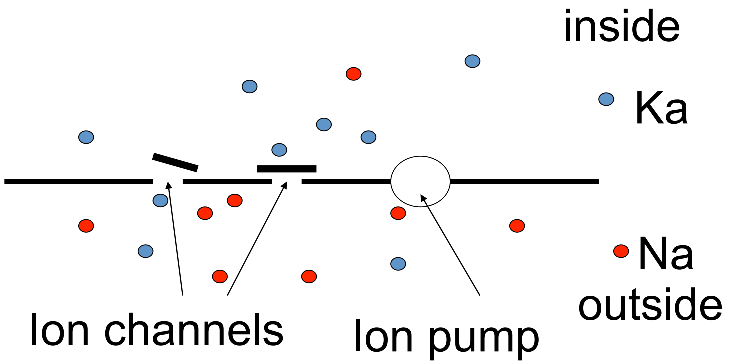
(***current*** threshold)

Neuronal Dynamics – 2.4. Threshold in HH model

pulse input







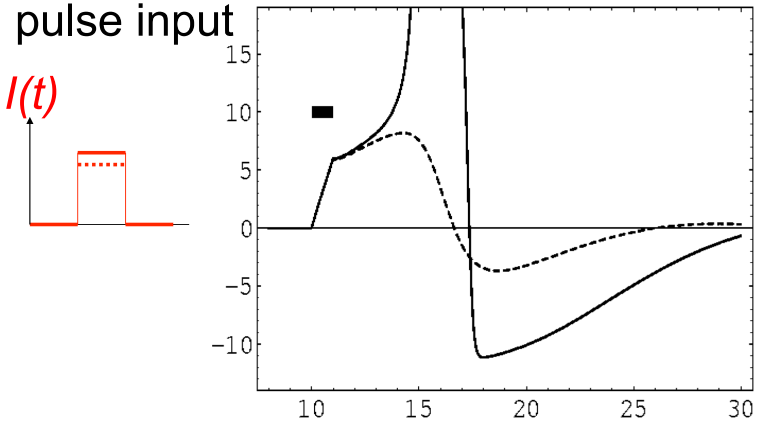
Threshold?

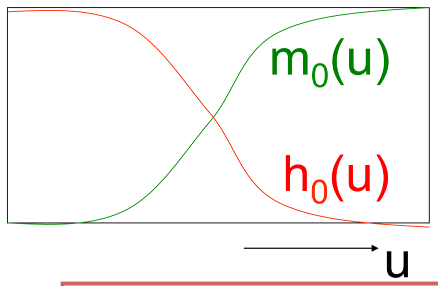
- AP if amplitude 7.0 units
- No AP if amplitude 6.9 units

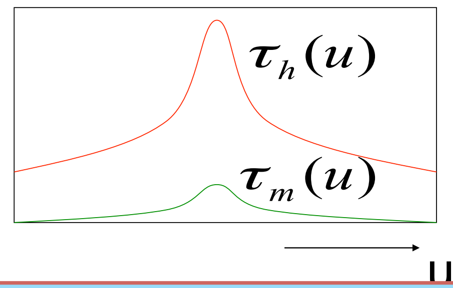
(pulse with 1ms duration)

(and pulse with 0.5 ms duration?)

Neuronal Dynamics – 2.4. Threshold in HH model







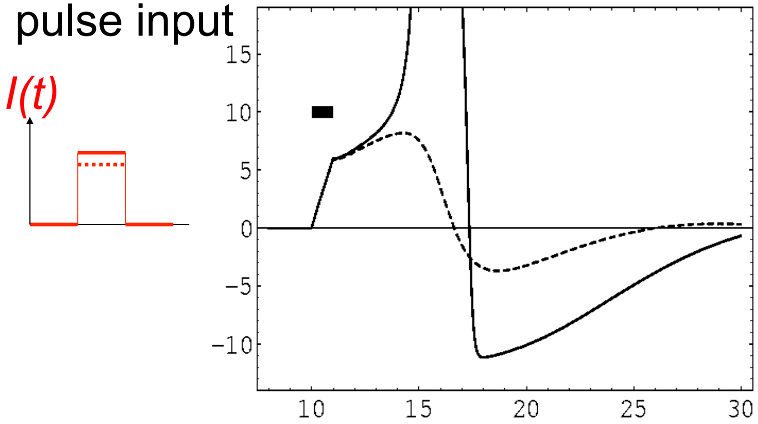
Mathematical explanation

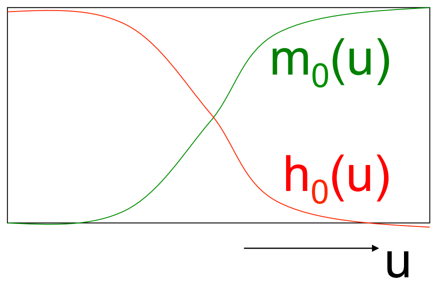
$$C\frac{du}{dt} = \underbrace{-g_{Na}m^3h(u-E_{Na})}_{I_{Na}} \underbrace{-g_K n^4(u-E_K)}_{I_K} \underbrace{-g_l(u-E_l)}_{I_{leak}} + \underbrace{I(t)}_{\text{Stim.}}$$

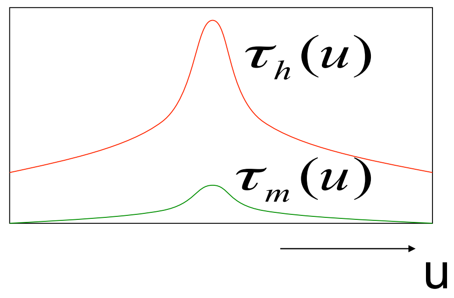
$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

Neuronal Dynamics – 2.4. Threshold in HH model







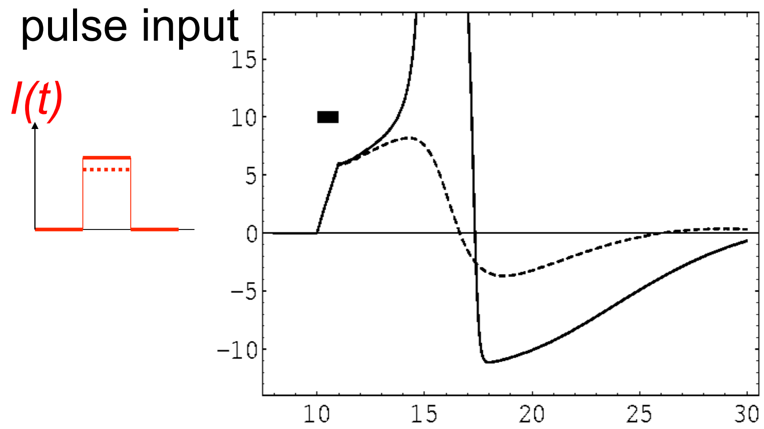
Stim.

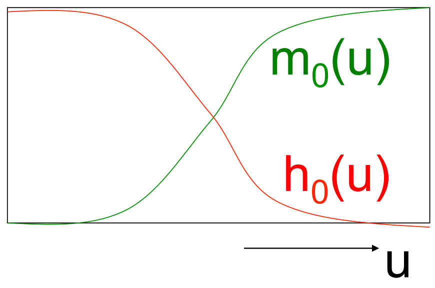
$$C\frac{du}{dt} = \overbrace{-g_{Na}m^3h(u-E_{Na})}^{I_{Na}} - I_K - I_{leak} + I(t)$$

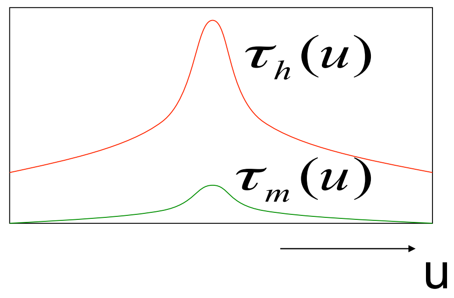
$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

Neuronal Dynamics – 2.4. Threshold in HH model







Why start the explanation with *m* and not *h*?

What about *n*?

Where is the threshold?

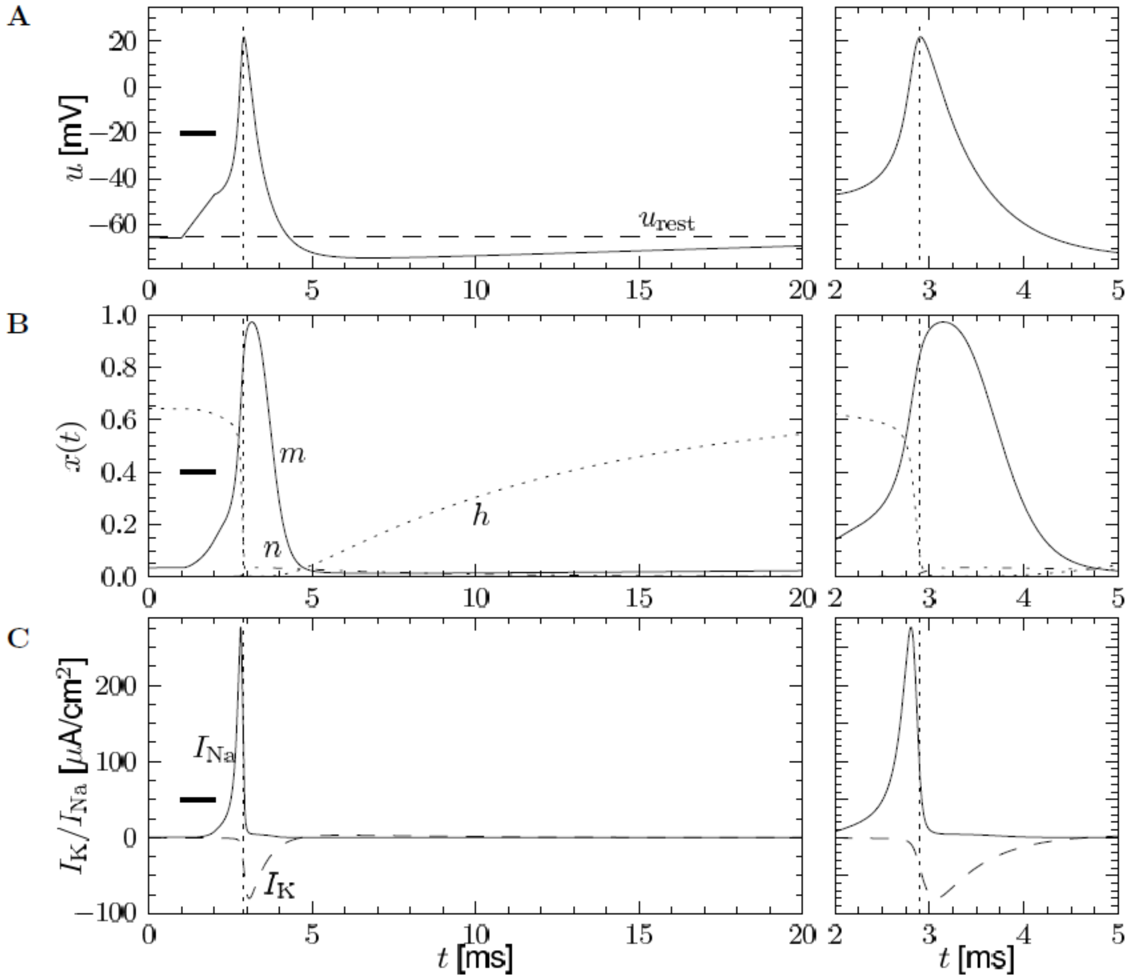
$$C\frac{du}{dt} = \underbrace{-g_{Na}m^3h(u-E_{Na})}_{I_{Na}} \underbrace{-g_K n^4(u-E_K)}_{I_K} \underbrace{-g_l(u-E_l)}_{I_{leak}} + \underbrace{I(t)}_{\text{Stim.}}$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n-n_0(u)}{\tau_n(u)}$$

Neuronal Dynamics – 2.4. Threshold in HH model



$$C\frac{du}{dt} = -g_{Na}m^3h(u-E_{Na})$$

$$-g_K n^4(u-E_K)$$

$$-g_l(u-E_l)$$

$$+I(t)$$

Neuronal Dynamics – 2.4. Threshold in HH model

There is no strict threshold:

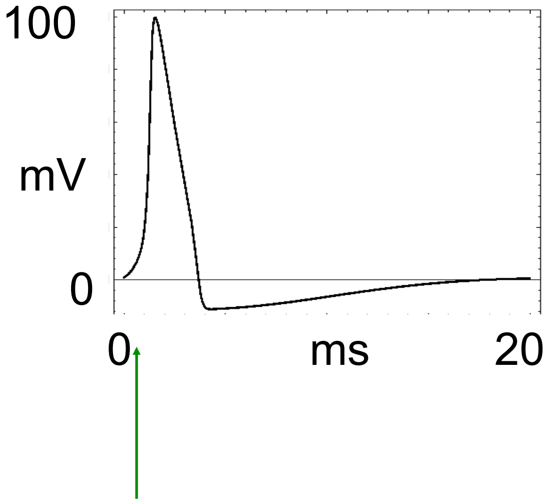
Coupled differential equations

'Effective' threshold
in simulations?

Neuronal Dynamics – 2.4. Refractoriness in HH model

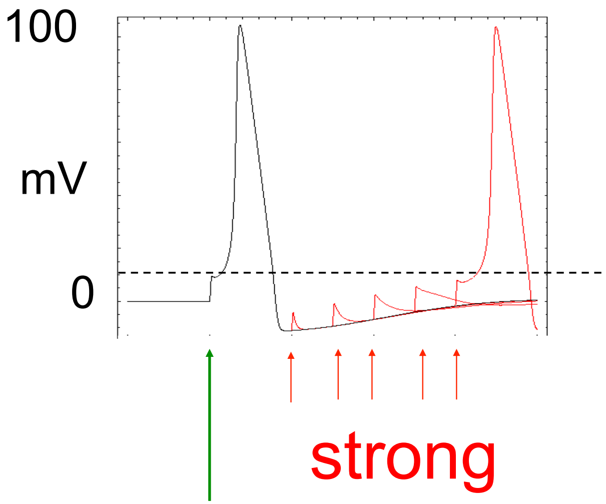
Where is the firing threshold?

Action potential



Strong
stimulus

refractoriness

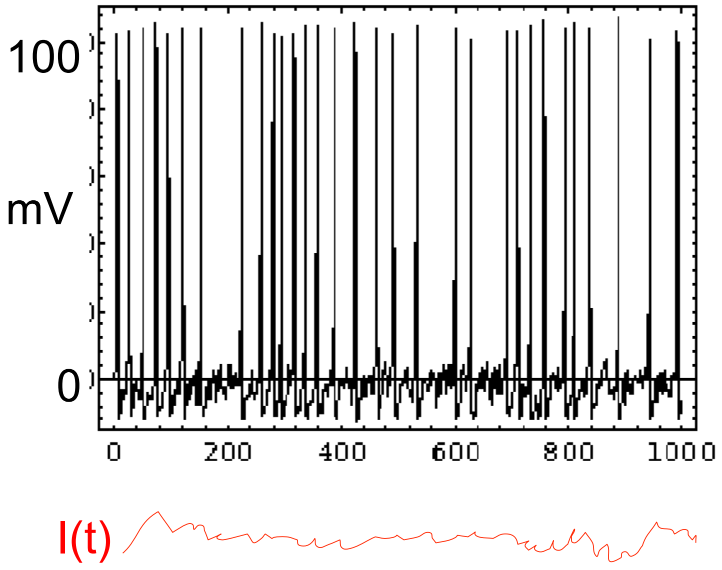


Strong
stimulus

strong
stimuli

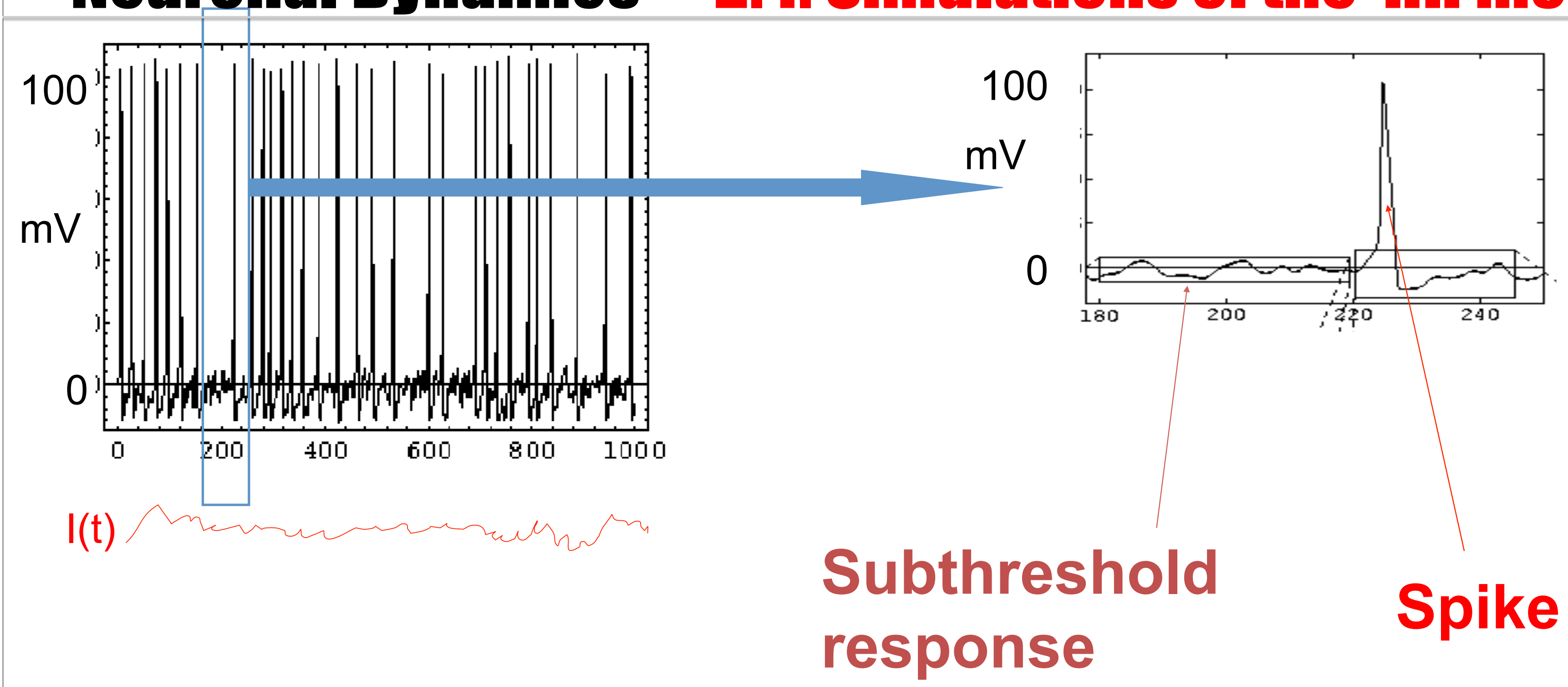
Refractoriness! Harder to elicit a second spike

Neuronal Dynamics – 2.4. Simulations of the HH model



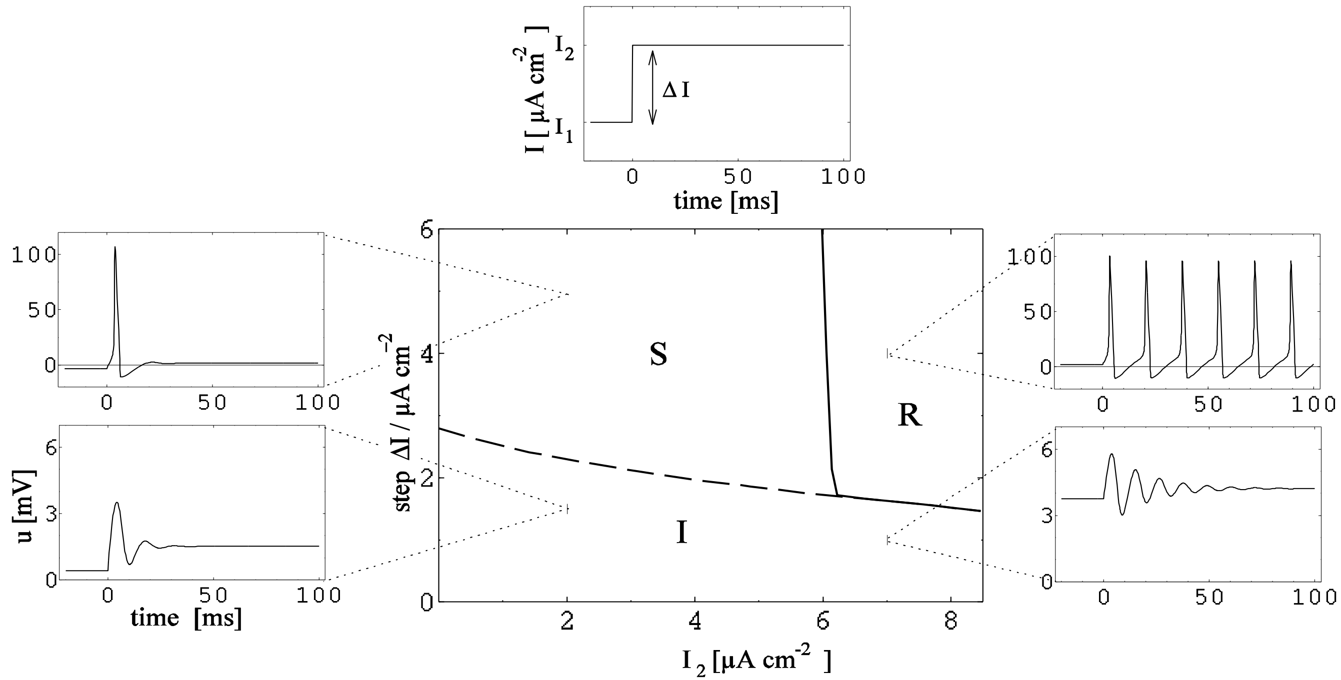
Stimulation with time-dependent input current

Neuronal Dynamics – 2.4. Simulations of the HH model



Neuronal Dynamics – 2.4. Threshold in HH model

Step current input ΔI I_2



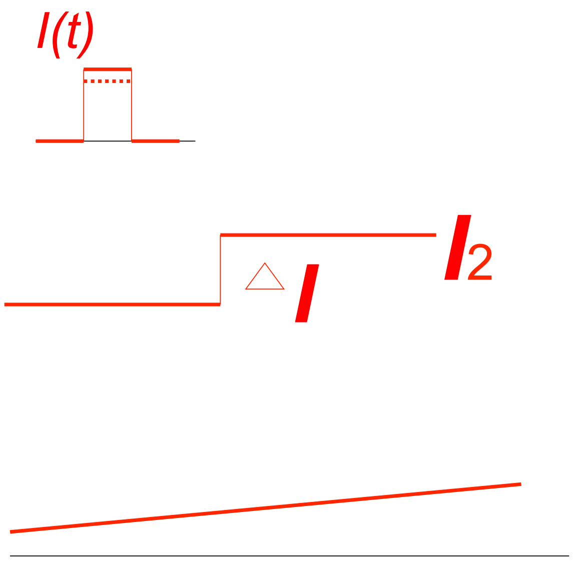
Neuronal Dynamics – 2.4. Threshold in HH model

Where is the firing threshold?

pulse input

step input

ramp input



There is no threshold

- no current threshold
- no voltage threshold

'effective' threshold

- depends on typical input

$$C\frac{du}{dt} = -g_{Na}m^3h(u - E_{Na}) - \ldots$$

Neuronal Dynamics – 2.4. Type I and Type II

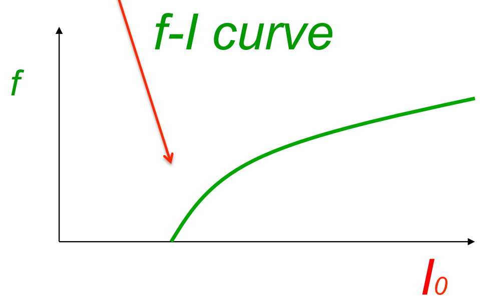
Hodgkin-Huxley model
with other parameters
(e.g. for cortical pyramidal
Neuron)

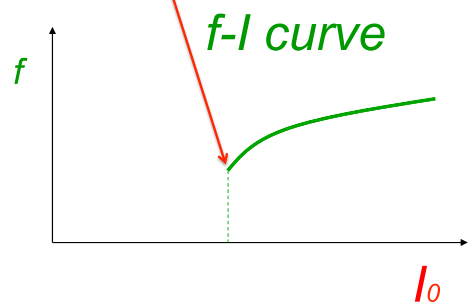
Hodgkin-Huxley model
with standard parameters
(giant axon of squid)

Response at firing threshold?

Type I	type II

ramp input/
constant input





Neuronal Dynamics – 2.4. Hodgkin-Huxley model

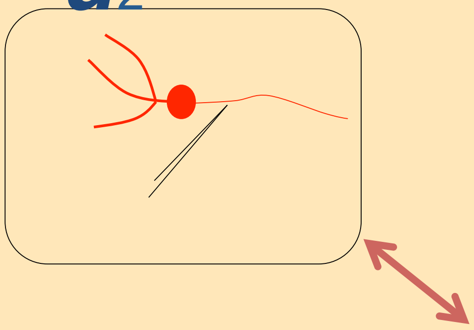
-4 differential equations
-no explicit threshold
-effective threshold depends on stimulus
-BUT: voltage threshold good approximation

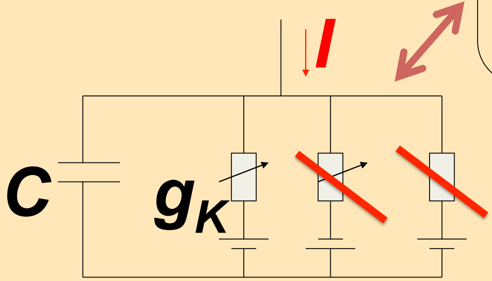
Giant axon of the squid
→ cortical neurons
-Change of parameters
-More ion channels
-Same framework

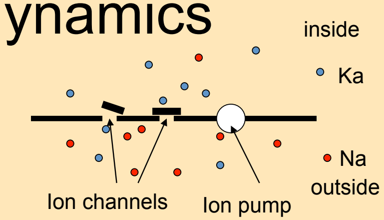
Exercise – 2. 4. Hodgkin-Huxley model – ion channel

voltage step

Determine ion channel dynamics



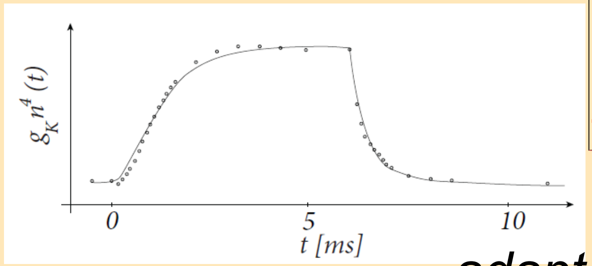


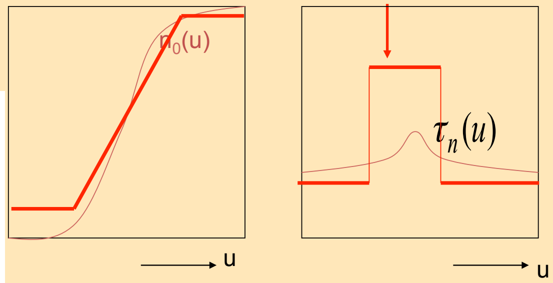


$$C\frac{du}{dt} = -g_K n^4 (u - E_K) + I(t)$$

stimulus

apply voltage step

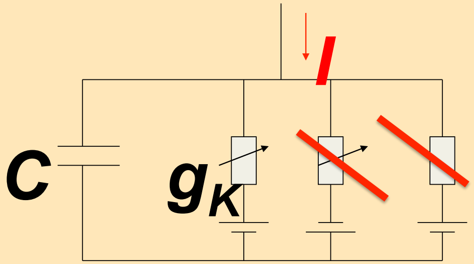


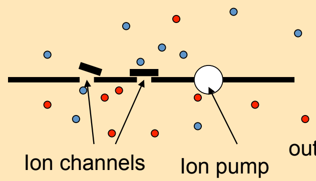


adapted from Hodgkin&Huxley 1952

Exercise – 2. 4. Hodgkin-Huxley model – ion channel

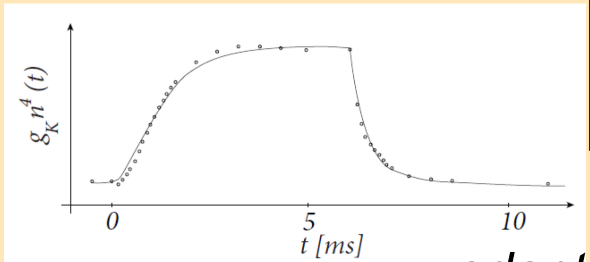
Determine ion channel dynamics

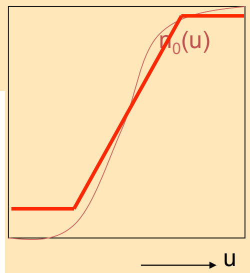


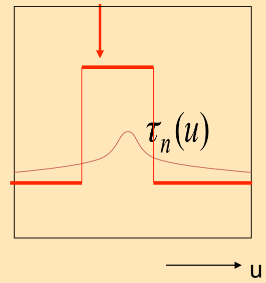


$$C\frac{du}{dt} = -g_K n^4 (u - E_K) + \underbrace{I(t)}_{\text{stimulus}}$$

apply voltage step







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