4. Hinge Loss and Objective Function

Hinge Loss and Objective Function

boundary.

How this relates to the hinge loss

that we defined over the training examples.

And how the regularization pushes the margin boundary

as it apart.

All of this together define, then, the objective

function that guides how theta and theta 0 are resolved

as the minimizing values.

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Hinge Loss Exercise 1

3/3 points (graded)

Compute the output of Hinge Loss function (as described in the video) for the following values:

$\text{Loss}_h\,(0) =$ 1 ✔ Answer: 1

$\text{Loss}_h\,(0.2) =$ 0.8 ✔ Answer: 0.8

$\text{Loss}_h\,(-10) =$ 11 ✔ Answer: 11

Solution:

$$\text{Loss}_h\,(z) = \begin{cases} 0 \text{ if } z >= 1 \\ 1 - z \text{ otherwise} \end{cases}$$

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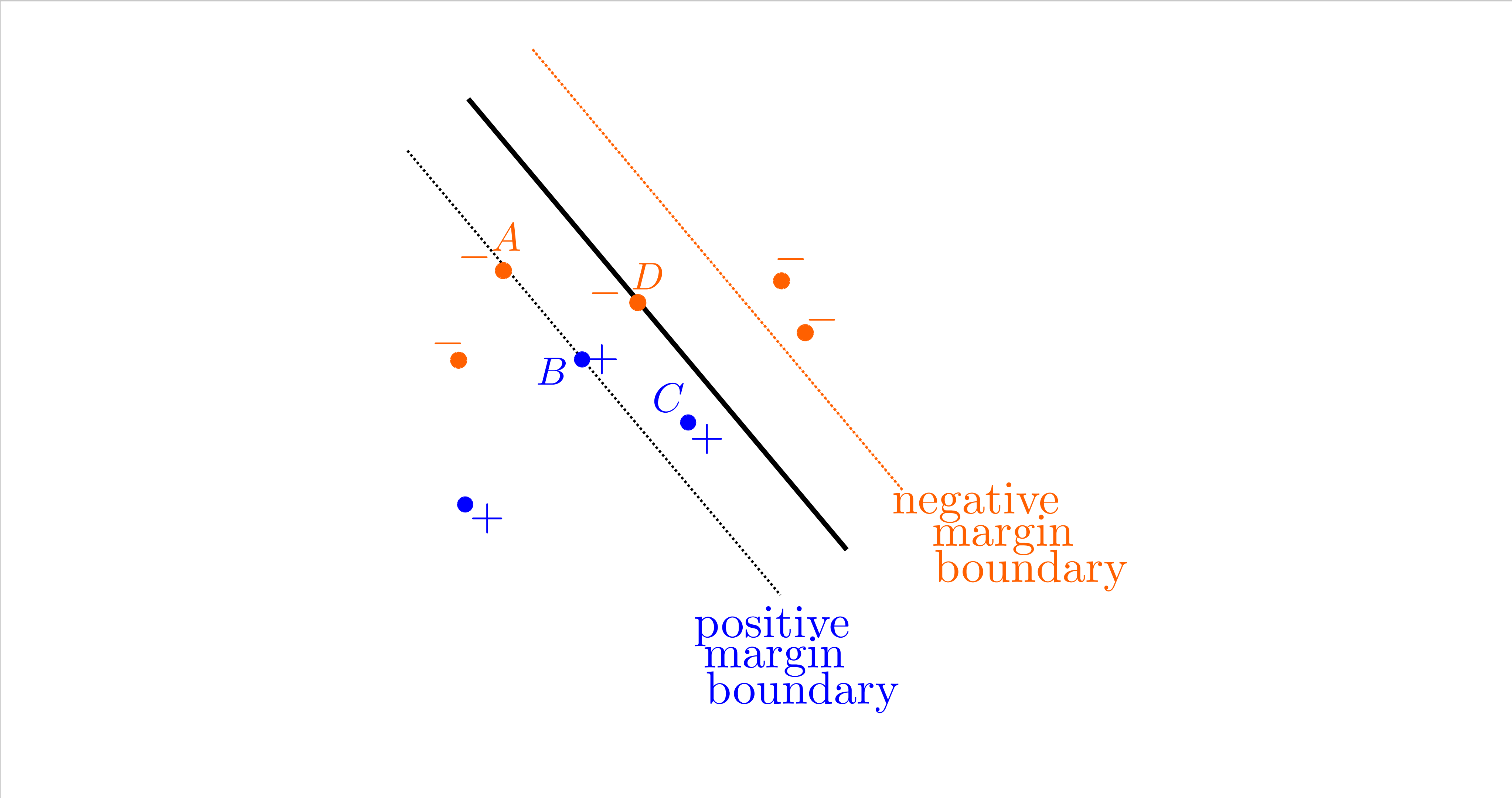
Answers are displayed within the problem

Hinge Loss Exercise 2

4/4 points (graded)

In a 2 dimensional space, there are points A, B, C, D as depicted below. Let $A = (x_a, y_a), B = (x_b, y_b), C = (x_c, y_c), D = (x_d, y_d)$



What is the hinge loss of point A, $\text{Loss}_h\left(y^{(a)}\left(\theta \cdot x^{(a)} + \theta_0\right)\right)$?

- ○ 0
- ○ between 0 and 1
- ○ 1
- ◉ 2 ✔

What is the hinge loss of point B, $\text{Loss}_h\left(y^{(b)}\left(\theta \cdot x^{(b)} + \theta_0\right)\right)$?

- ◉ 0 ✔
- ○ between 0 and 1
- ○ 1

What is the hinge loss of point C, $\text{Loss}_h\left(y^{(c)}\left(\theta \cdot x^{(c)} + \theta_0\right)\right)$?

- ○ 0
- ◉ between 0 and 1 ✔
- ○ 1

What is the hinge loss of point D, $\text{Loss}_h\left(y^{(d)}\left(\theta \cdot x^{(d)} + \theta_0\right)\right)$?

0

between 0 and 1

1

Solution:

A is on the positive margin boundary but with the label -1, so

$$y^{(a)}\left(\theta \cdot x^{(a)} + \theta_0\right) = -1.$$

Thus its hinge loss is 2. B is on the positive margin boundary and with the label $+1$, so

$$= y^{(b)}\left(\theta \cdot x^{(b)} + \theta_0\right) = 1.$$

Thus its hinge loss is 0. C lies between the decision boundary and the margin boundary. Thus

$$1 > y^{(c)}\left(\theta \cdot x^{(c)} + \theta_0\right) > 0.$$

Thus C's hinge loss is between 0 and 1. Similarly, because D is on the decision boundary,

$$y^{(d)}\left(\theta \cdot x^{(d)} + \theta_0\right) = 0.$$

Thus its hinge loss is 1.**Loss functions tell you in general how bad the prediction is.** The Hinge Loss tells us how undesirable a training example is, with regard to the margin and the correctness of its classification.

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Answers are displayed within the problem

Regularization

1/1 point (graded)

Remember that for points (x, y) on the boundary margin, the distance from the decision boundary to (x, y) is $\frac{1}{||\theta||}$. Thus

$$y^{(i)}\left(\theta \cdot x^{(i)} + \theta_0\right) = 1.$$

And

$$\frac{y^{(i)}\left(\theta \cdot x^{(i)} + \theta_0\right)}{\mid\mid \theta \mid\mid} = \frac{1}{\mid\mid \theta \mid\mid}.$$

Now our goal is to maximize the margin, that is to maximize $\frac{1}{||\theta||}$. Which of the following is **NOT** equivalent to maximizing $\frac{1}{||\theta||}$?

maximizing $\frac{1}{||\theta||^2}$

minimizing $\mid\mid \theta \mid\mid$

maximizing $\sqrt{||\ \theta\ ||}$ ✔

Solution:

Maximizing $\frac{1}{||\theta||}$ is equivalent to maximizing $\frac{1}{||\theta||^2}$. It is also equivalent to minimizing $||\ \theta\ ||$.

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Answers are displayed within the problem

Objective

1/1 point (graded)
Remember that our objective is given as

$$J(\theta, \theta_0) = \frac{1}{n} \sum_{i=1}^{n} \text{Loss}_h\left(y^{(i)}\left(\theta \cdot x^{(i)} + \theta_0\right)\right) + \frac{\lambda}{2} ||\ \theta\ ||^2.$$

Our goal is to minimize this objective J. Now, which of the following is true if we have a large λ?

- ◉ We put more importance on maximizing the margin than minimizing errors ✔
- ○ We put more importance on minimizing the margin than minimizing errors
- ○ We put more importance on maximizing the margin than maximizing errors
- ○ We put more importance on minimizing the margin than maximizing errors

Solution:

Remember that the first term

$$\frac{1}{n} \sum_{i=1}^{n} \text{Loss}_h\left(y^{(i)}\left(\theta \cdot x + \theta_0\right)\right)$$

corresponds to the sum of hinge losses on each training example, and the second term

$$\frac{\lambda}{2} ||\ \theta\ ||^2$$

corresponds to maximizing the margin. If we increase λ, we put more weight on maximizing the margin than minimizing the sum of losses.

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