

## 17. Exercise: ML estimation

### Exercise: ML estimation

1/1 point (graded)

Let  $K$  be a Poisson random variable with parameter  $\lambda$ : its PMF is

$$p_K(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad \text{for } k = 0, 1, 2, \dots$$

What is the ML estimate of  $\lambda$  based on a single observation  $K = k$ ? (Your answer should be an algebraic function of  $k$  using standard notation.)

✓ Answer: k

Handwritten derivation of the ML estimate for a Poisson distribution. The text shows the maximization of the log-likelihood function:  $\text{Max} [\lambda^k \cdot e^{-\lambda}]$ . The derivative is calculated as  $(\lambda^k \cdot e^{-\lambda})' = k \cdot \lambda^{k-1} \cdot e^{-\lambda} - \lambda^k \cdot e^{-\lambda} = 0$ . This leads to the equation  $\frac{\lambda^k}{\lambda^{k-1}} = \lambda = k$ .

#### Solution:

We maximize the logarithm of the PMF, which is  $k \ln \lambda - \lambda - \ln(k!)$ . Setting the derivative of this expression with respect to  $\lambda$  to 0, we obtain  $(k/\lambda) - 1 = 0$ , so that  $\hat{\lambda}_{ML} = k$ .

You have used 1 of 3 attempts

❗ Answers are displayed within the problem

### 讨论

Topic: Unit 8 / Lec. 20 / 17. Exercise: ML estimation