3. Checking the Markov property
Problem 3. Checking the Markov property

7/7 points (ungraded)
For each one of the following definitions of the state X_k at time k (for $k = 1, 2, \ldots$), determine whether the Markov property is satisfied by the sequence $X_1, X_2, \ldots$.

1. A fair six-sided die (with sides labelled $1, 2, \ldots, 6$) is rolled repeatedly and independently.

(a) Let X_k denote the largest number obtained in the first k rolls. Does the sequence $X_1, X_2, \ldots$ satisfy the Markov property?

Yes ▾ ✔

(b) Let X_k denote the number of 6's obtained in the first k rolls, up to a maximum of ten. (That is, if ten or more 6's are obtained in the first k rolls, then $X_k = 10$.) Does the sequence $X_1, X_2, \ldots$ satisfy the Markov property?

Yes ▾ ✔

(c) Let Y_k denote the result of the kth roll. Let $X_1 = Y_1$, and for $k \geq 2$, let $X_k = Y_k + Y_{k-1}$. Does the sequence $X_1, X_2, \ldots$ satisfy the Markov property?

No ▾ ✔

(d) Let $Y_k = 1$ if the kth roll results in an odd number; and $Y_k = 0$ otherwise. Let $X_1 = Y_1$, and for $k \geq 2$, let $X_k = Y_k \cdot X_{k-1}$. Does the sequence $X_1, X_2, \ldots$ satisfy the Markov property?

Yes ▾ ✔

2. Let Y_k be the state of some Markov chain at time k (i.e., it is known that the sequence $Y_1, Y_2, \ldots$ satisfies the Markov property).

(a) For a fixed integer $r > 0$, let $X_k = Y_{r+k}$. Does the sequence $X_1, X_2, \ldots$ satisfy the Markov property?

Yes ▾ ✔

(b) Let $X_k = Y_{2k}$. Does the sequence $X_1, X_2, \ldots$ satisfy the Markov property?

Yes ▾ ✔

(c) Let $X_k = (Y_k, Y_{k+1})$. Does the sequence $X_1, X_2, \ldots$ satisfy the Markov property?

Yes ▾ ✔

提交   你已经尝试了2次（总共可以尝试3次）

讨论

显示讨论

主题： Unit 10 / Problem Set / 3. Checking the Markov property