4. Probability Density Functions

Let X be a **continuous** random variable with probability **density** function (pdf) $p_X(x)$.

Recall the notation that random variables are denoted by capital letters, such as X, while any realization is denoted by small letters, such as x.

Note: In *6.431x Probability–the Science of Uncertainty and Data*), we had used $f_X(x)$ to denote a pdf.

Note: We have replaced the original erroneous paragraph with the above.

4. (a)

0/1 point (graded)
Is the value of $p_X(x)$ always $\in [0,1]$?

- (•) yes ✗ 我傻了！
- ( ) no ✓

STANDARD NOTATION

Solution:

While probabilities are always between 0 and 1, the probability density function (PDF) is not the actual probability of observing a particular outcome. This is an important distinction from probability mass functions, the analog for discrete random variables. So the PDF can be greater than 1, but its integral, which gives the probability must always be $\in [0,1]$.

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ⓘ Answers are displayed within the problem

4. (b)

1/1 point (graded)
When $a < b$, $\int_a^b p_X(x)\,dx \in [0,1]$and represents the probability that the value of X falls between a and b.

- (•) yes ✓
- ( ) no

STANDARD NOTATION

Solution:

Remind yourself that the integral across a range (here, from $-\infty$ to ∞) is the total probability that the value of X lies in that range. Since this range contains all possible values any random variable can take, by definition, not only is the integral finite, but since the total probability must be 1, the integral is always 1.

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4. (c)

1/1 point (graded)

Is the value of $p_X(x)$ always non-negative?

- (•) yes ✔
- ( ) no

STANDARD NOTATION

Solution:

Since $p_X(x)$ denotes relative likelihoods, it must always be ≥ 0.

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4. (d)

1/1 point (graded)

The integral $\int_{-\infty}^{\infty} p_X(x)\,dx$ of $p_X(x)$ from $-\infty$ to ∞ is finite, but the specific value of this integral may vary.

- ( ) yes
- (•) no ✔

STANDARD NOTATION

Solution:

Remind yourself that the integral across a range (here, from $-\infty$ to ∞) is the total probability that X takes values in that range. Since this range contains all possible values any random variable can take, by definition, not only is the integral finite, but since the total probability must be 1, the integral is always 1, i.e. $\int_{-\infty}^{\infty} p_X(x)\,dx = 1$.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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