# 6. Exercise: LMS estimation error
## Exercise: LMS estimation error

3/3 points (graded)

As in the previous exercise, let $\Theta$ be the bias of a coin, i.e., the probability of Heads at each toss. We assume that $\Theta$ is uniformly distributed on $[0, 1]$. Let $K$ be the number of Heads in $9$ independent tosses. We have seen that the LMS estimate of $K$ is $\mathbf{E}[K \mid \Theta = \theta] = n\theta$.

a) Find the conditional mean squared error $\mathbf{E}\big[\big(K - \mathbf{E}[K \mid \Theta = \theta]\big)^2 \mid \Theta = \theta\big]$ if $\theta = 1/3$.

| 2 |
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✔ **Answer:** 2

b) Find the overall mean squared error of this estimation procedure.

| 3/2 |
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✔ **Answer:** 1.5

**Solution:**

a) This is the variance of the conditional distribution of $K$. Since the conditional distribution is binomial with parameters $n = 9$ and $\theta = 1/3$, the conditional variance is $9(1/3)(2/3) = 2$.

b) This is the average of the conditional variance, averaged over all possible values of the observation $\Theta$, which has a uniform distribution:

$$\int_0^1 f_\Theta(\theta)\mathrm{Var}(K \mid \Theta = \theta)\, d\theta = \int_0^1 9\theta(1 - \theta)\, d\theta$$

$$= \left(9\frac{1}{2}\theta^2 - 9\frac{\theta^3}{3}\right)\Big|_0^1$$

$$= 4.5 - 3$$

$$= 1.5.$$

| 提交 | You have used 1 of 3 attempts |
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ℹ Answers are displayed within the problem