8. Warm-up / Review: Proportionality

Distributions with One Parameter

5/6 points (graded)

Match each of the proportionality expressions below to the corresponding well-known distribution, and supply the missing parameters. The variable of interest is θ. In entering the expressions for the parameters, only the variables a, b, or c may be used.

In this problem, the distribution **Geom** (p) is assumed to be over the nonnegative integers. The more explicit specification for the geometric distribution is the number of failure until the first success in a sequence of i.i.d. Bernoulli(p) Trials.

$\pi(\theta) \propto a^{1-\theta}(1-a)^{\theta}$ (for $\theta \in \{0,1\}$, and it is known that $a \in (0,1)$)

- ● Ber (p) ✓
- ○ Exp (λ)
- ○ Poiss (λ)
- ○ Geom (p)

$\text{parameter} =$

1-a ✓ **Answer:** (1-a)+b*0+c*0

$1-a$

$\pi(\theta) \propto c^{a\theta+b}$ (for $\theta \in \mathbb{N} \cup \{0\}$, and it is known that $a \in (0,1)$)

- ○ Ber (p)
- ○ Exp (λ)
- ○ Poiss (λ)
- ● Geom (p) ✓

$\text{parameter} =$

1- c ✗ **Answer:** 1-c^a+b*0

$1-c$

$\pi(\theta) \propto 100e^{a\theta+b}$ (for $\theta \geq 0$, and it is known that $a < 0$)

- ○ Ber (p)
- ● Exp (λ) ✓

○ Poiss (λ)

○ Geom (p)

parameter =

-a

✓ **Answer:** -a+b*0+c*0

$-a$

STANDARD NOTATION

Solution:

- It must be the Bernoulli distribution as this is the only distribution among our choices that has the binary support $\{0,1\}$. The Bernoulli parameter p represents the probability of $\theta = 1$. If we write $f(\theta) = a^{1-\theta}(1-a)^{\theta}$, we get $f(0) = a$ and $f(1) = 1-a$, so the normalization constant is $a + (1-a) = 1$, and we thus have $\pi(0) = a$, $\pi(1) = 1-a$. Hence the parameter is $\boxed{p = 1-a}$.

- It must be the geometric distribution. Our un-normalized PMF $f(\theta) = c^{a\theta+b}$ is characterized by $\boxed{f(0) = c^b}$ and $\boxed{\frac{f(\theta+1)}{f(\theta)} = c^a}$, which define a geometric distribution. The PMF $g(x)$ of the geometric distribution Geom (p) satisfies $\boxed{\frac{g(x+1)}{g(x)} = 1-p}$, thus equating gives $\boxed{c^a = 1-p}$, or that $\boxed{p = 1-c^a}$.

- This is a continuous version of the second item and features a linearly increasing exponent, which implies that it must be the exponential distribution. The PMF $g(x)$ of the exponential distribution Exp (λ) satisfies $\frac{g(x+1)}{g(x)} = e^{-\lambda}$. Computing this quantity for the distribution with un-normalized PMF $100e^{a\theta+b}$ gives e^a, so equating gives $e^{-\lambda} = e^a$, equivalent to $\boxed{\lambda = -a}$.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Distributions with Two Parameters

8/9 points (graded)

Match each of the proportionality expressions below to the corresponding well-known distribution, and then compute the values of the parameter(s) of the distribution in terms of the given a, b, and/or c. The variable of interest is θ. Express the parameters in the order of which they appear in the expression. In entering the expressions for the parameters, only the variables a, b, or c may be used.

In this problem, the distribution N (μ, σ^2) has parameters μ and σ^2.

$\pi(\theta) \propto c$ (for $\theta \in [a,b]$ where $a, b \in \mathbb{R}$, $a < b$)

● Unif $([\alpha, \beta])$ ✓

○ N (μ, σ^2)

○ Binom (n, p)

○ Beta (α, β)

left parameter =

a

✓ **Answer:** a+b*0+c*0

a

$\text{right parameter} =$

b ✓ **Answer:** a*0+b+c*0

b

$\pi(\theta) \propto \theta^a (c - c\theta)^b$ (for $\theta \in [0,1]$ where $a, b > -1$)

○ Unif $([\alpha, \beta])$

○ N (μ, σ^2)

○ Binom (n, p)

● Beta (α, β) ✓

$\text{left parameter} =$

a+1 ✓ **Answer:** a+1+b*0+c*0

$a + 1$

$\text{right parameter} =$

b+1 ✓ **Answer:** b+1+a*0+c*0

$b + 1$

$\pi(\theta) \propto e^{a\theta^2 + b\theta + c}$ (for $\theta \in \mathbb{R}$, and it is known that $a < 0$)

○ Unif $([\alpha, \beta])$

● N (μ, σ^2) ✓

○ Binom (n, p)

○ Beta (α, β)

$\text{left parameter} =$

b/(2*a) ✗ **Answer:** -b/(2*a)+c*0

$\frac{b}{2 \cdot a}$

$\text{right parameter} =$

-1/(2*a) ✓ **Answer:** -1/(2*a)+b*0+c*0

$-\frac{1}{2 \cdot a}$

STANDARD NOTATION

Solution:

- We are given a distribution that is flat over a given finite interval over the real line, which implies that we have a uniform distribution. The parameters of a uniform distribution are the bounds of the interval. Here, they are a and b, so these are also the parameters of the distribution, giving $\textsf{Unif}\left(a, b\right)$.

- Rewriting by dividing the distribution by c^b (which is a constant multiplier) gives $f\left(\theta\right) = \theta^a\left(1-\theta\right)^b$. This resembles the form of a Beta distribution, as discussed in lecture, with parameters $\boxed{\alpha = a+1}$ and $\boxed{\beta = b+1}$.

- We have a support over the real line, so a normal distribution is our only choice here. The standard form of a normal distribution is $\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. Our variable of interest is x, so we may drop the left multiplier, ending up with $e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. Now the exponent is a quadratic in x: $-\frac{x^2}{2\sigma^2} + \frac{\mu}{\sigma^2}x - \frac{\mu^2}{\sigma^2}$. Equating the coefficient with x^2 gives $a = -\frac{1}{2\sigma^2}$, or that $\boxed{\sigma^2 = -\frac{1}{2a}}$. Next, equating the coefficient of x gives $b = \frac{\mu}{\sigma^2} = \frac{\mu}{-\frac{1}{2a}}$. Hence $\boxed{\mu = -\frac{b}{2a}}$.

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Answers are displayed within the problem

Discussion

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