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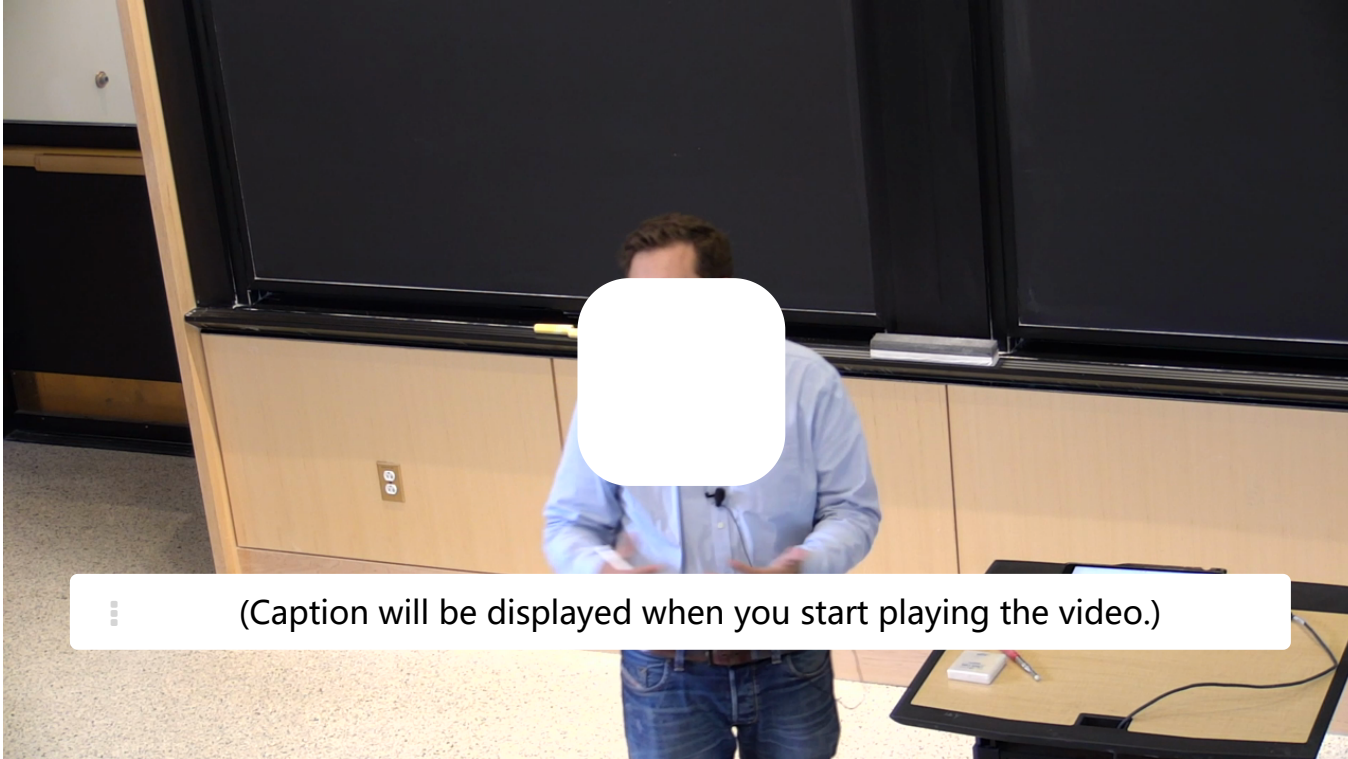
# 3. Confidence Intervals Concept Checks Continued
# Confidence Interval Review

Start of transcript. Skip to the end.

OK.

So let's start with a few exercises

that we left behind us.

And that will be a good way for us

to refresh our memory in terms of what a confidence interval

is and what it does for us.

So now here's the first question.

Hopefully, you've had a minute to think

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## Confidence Interval Concept Check 3

0/1 point (graded)

In a new experiment consisting of 150 couples, 75 couples are observed to turn their heads to the left and the remaining 75 couples turned their heads to the right when kissing. Let $p$ denote the (unknown) parameter which specifies the probability that a couple turns their head to the right.

Which of the following statements are correct regarding this experiment? You are given that **exactly one** but not both of choices 3 and 4 is correct.
(Choose all that apply.)

- ☐ $[0, 0.5]$ is a $50\%$ asymptotic confidence interval for $p$. ✔

- ☐ $[0.5, 1]$ is a $50\%$ asymptotic confidence interval for $p$. ✔

- ☐ $[0.466, 0.533]$ is a $50\%$ asymptotic confidence interval for $p$. ✔

- ☑ $[0.48, 0.52]$ is a $50\%$ asymptotic confidence interval for $p$.

✖

**Solution:**

See next video for presented solution.

The first three answer choices are correct, and the final choice is incorrect.

Let $R_1, R_2, \ldots, R_{150} \overset{iid}{\sim} \mathrm{Ber}\,(p)$ denote the sampled response (without loss of generality, assume that $R_i = 1$ encodes that the $i$-th couple turns their heads to the right, and $R_i = 0$ encodes the couple turns their heads to the left.) Let $P = \mathrm{Ber}\,(p)$ denote the common distribution of $R_1, \ldots R_{150}$.

Consider the sample mean $\overline{R}_n$. By the central limit theorem,

$$\sqrt{n}\left(\frac{\overline{R}_n - p}{\sqrt{p\,(1-p)}}\right) \underset{(d)}{\longrightarrow} N\,(0,1)\,.$$

Now we examine the answer choices in order.

1. Consider the interval $[0, 0.5]$. Since $\overline{R}_n = 0.5$, this interval is a realization of the (random) confidence interval $\mathcal{I} = (0, \overline{R}_n)$. We compute that

$$P\,(\mathcal{I} \ni p) = P\,(p \le \overline{R}_n) = P\,(\overline{R}_n - p \ge 0)\,.$$

   Observe that $\overline{R}_n - p$ is a centered (*i.e.* $\mathbb{E}\,[\overline{R}_n - p] = 0$), symmetric (*i.e.*, $\overline{R}_n - p$ and $-(\overline{R}_n - p)$ have the same distribution) random variable. Therefore $P\,(\mathcal{I} \ni p) = 1/2$. Indeed $[0, 0.5]$ is an asymptotic (in fact, it is even a *non-asymptotic*) confidence interval of level $50\%$.

2. The interval $[0.5, 1]$ is a realization of the (random) confidence interval $\mathcal{I} = (\overline{R}_n, 1)$. We see that

$$P\,(\mathcal{I} \ni p) = P\,(\overline{R}_n \le p \le 1) = P\,(\overline{R}_n - p \le 0)\,.$$

   By the reasoning in the previous part, we must also have that $P\,(\mathcal{I} \ni p) = 1/2$.

3. Given that <mark>either choice 3 or choice 4 is correct but not both</mark>, it must be that the <mark>wider</mark> of the 2 intervals $[0.466, 0.533]$ is a $50\%$ asymptotic confidence interval for $p$. Otherwise, the narrower interval $[0.48, 0.52]$ being a $50\%$ asymptotic confidence interval for $p$ implies the same for the wider interval.

[提交]    你已经尝试了2次（总共可以尝试2次）

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ℹ  Answers are displayed within the problem

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## 讨论

[显示讨论]

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