9. Properties of the Generalized Method of Moments Estimator

Plus Minus 1 - Method of Moments

3/3 points (graded)
Let X be a random variable that takes on values -1 and $+1$ with probabilities p and $1-p$, respectively. Let $\widehat{m}_1$ be the sample average of n i.i.d. observations of X.

What is the method of moments estimator $\hat{p}_n^{\text{MM}}$?

Use **hatm_1** for $\widehat{m}_1$.

(1 - hatm_1)/2

Answer: (1-hatm_1)/2

Assume that we observe k instances of -1 out of n outcomes. What is the ML estimator $\hat{p}_n^{\text{MLE}}$?

k/n

Answer: k/n

$\frac{k}{n}$

Are the two estimators for the ± 1 random variable equal?

- (•) Yes
- ( ) No

STANDARD NOTATION

Solution:

The expected value of X is $1-2p$.

Therefore, $\hat{p}_n^{\text{MM}} = \frac{1-\widehat{m}_1}{2}$

The ML estimator of p is $\hat{p}_n^{\text{MLE}} = k/n$.

The two estimators are equal because of the following:

$$\begin{aligned}\hat{p}_n^{\text{MM}} &= \frac{1-\widehat{m}_1}{2} \\ &= \frac{1-\frac{(k)\cdot -1+(n-k)\cdot 1}{n}}{2} \\ &= \frac{k}{n}\end{aligned}$$

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Answers are displayed within the problem

Method of Moments - Multiple Estimators

2/2 points (graded)

Let X be a non-zero uniform random variable that we model using the distribution Unif $[0, \theta]$, where $\{\theta \mid \theta > 0\} = \Theta$. Our objective is to estimate θ using a moments estimator constructed out of n i.i.d. samples $X_1, X_2, \ldots, X_n$.

For a random variable $X \sim$ Unif $[0, \theta]$,

$$\begin{aligned} \mathbb{E}[X] &= \frac{\theta}{2}, \\ \mathbb{E}[X^2] &= \frac{\theta^2}{3}. \end{aligned}$$

We have only one parameter to estimate here, and there are two invertible moment functions that we can use to estimate the parameter. Let $\widehat{m}_1$ be the sample average $\frac{\sum_{i=1}^n X_i}{n}$ and let $\widehat{m}_2$ denote $\frac{\sum_{i=1}^n X_i^2}{n}$. By the law of large numbers, $\widehat{m}_1 \to \mathbb{E}[X]$ and $\widehat{m}_2 \to \mathbb{E}[X^2]$ as $n \to \infty$.

To enter your answers to the following, use **hatm_1** for $\widehat{m}_1$, **hatm_2** for $\widehat{m}_2$.

What is the method of moments estimator $\hat{\theta}_{n,1}^{\text{MM}}$ based on $\widehat{m}_1$?

2*hatm_1

Answer: 2*hatm_1 + 0*hatm_2

What is the method of moments estimator $\hat{\theta}_{n,2}^{\text{MM}}$ based on $\widehat{m}_2$?

sqrt(3*hatm_2)

Answer: sqrt(3*hatm_2) + 0*hatm_1

STANDARD NOTATION

Solution:

Note that both $\mathbb{E}[X] = m_1(\theta)$ and $\mathbb{E}[X^2] = m_2(\theta)$ are one-to-one and invertible in Θ. Therefore,

$$\begin{aligned} \widehat{theta}_{n,1}^{\text{MM}} &= 2\widehat{m}_1, \\ \hat{\theta}_{n,2}^{\text{MM}} &= \sqrt{3\widehat{m}_2}. \end{aligned}$$

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Answers are displayed within the problem

Method of Moments Concept Question II

1/1 point (graded)

Let $(E, \{\mathbf{P}_\theta\}_{\theta\in\Theta})$ denote a statistical model associated to a statistical experiment $X_1, \ldots, X_n \overset{iid}{\sim} \mathbf{P}_{\theta^*}$ where $\theta^* \in \Theta$ is the true parameter. Assume that $\Theta \subset \mathbb{R}^d$ for some $d \geq 1$. Let $m_k(\theta) := \mathbb{E}[X^k]$ where $X \sim \mathbf{P}_\theta$. $m_k(\theta)$ is referred to as the **k-th moment of $\mathbf{P}_\theta$**. Also define the moments map:

$$\begin{aligned}\psi : \Theta &\to \mathbb{R}^d \\ \theta &\mapsto (m_1(\theta), m_2(\theta), \ldots, m_d(\theta)).\end{aligned}$$

What conditions on ψ do we have to assume so that the method of moments produces a consistent and asymptotically normal estimator? (Choose all that apply.)

Recall that the method of moments estimator is

$$\hat{\theta}_n^{\mathrm{MM}} := \psi^{-1}\left(\frac{1}{n}\sum_{k=1}^{n} X_i, \frac{1}{n}\sum_{k=1}^{n} X_i^2, \ldots, \frac{1}{n}\sum_{k=1}^{n} X_i^d\right)$$

- [x] The function ψ is one-to-one.
- [x] The function ψ has a differentiable inverse that is continuous.
- [ ] ψ is a polynomial in the entries of θ.
- [ ] ψ^{-1} is a polynomial in d variables.
- [ ] None of the above.

Solution:

We handle the choices in order.

- "The function ψ is one-to-one." and "The function ψ has a differentiable inverse that is continuous." are assumptions included on the theorem regarding the convergence of the method of moments estimator. If ψ is not one-to-one, then we cannot even define ψ^{-1}. Also, the asymptotic covariance matrix is in terms of the inverse of the gradient of ψ, so the second assumption is certainly necessary.

- "ψ is a polynomial in the entries of θ." and "ψ^{-1} is a polynomial in d variables." are incorrect. There are no specific assumptions needed on the form that ψ must take. However, for practical purposes, to be able to perform the method of moments, we need ψ^{-1} to be (efficiently) computable.

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Answers are displayed within the problem

讨论

显示讨论

主题： Unit 3 Methods of Estimation:Lecture 11: Fisher Information, Asymptotic Normality of MLE; Method of Moments / 9. Properties of the Generalized Method of Moments Estimator