2. Set operations and probabilities

Problem 2. Set operations and probabilities

3/3 points (graded)
Find the value of $\mathbf{P}\left(A \cup (B^c \cup C^c)^c\right)$ for each of the following cases:

1. The events A, B, C are disjoint events and $\mathbf{P}(A) = 2/5$.

$\mathbf{P}\left(A \cup (B^c \cup C^c)^c\right) =$ 2/5 ✔ **Answer:** 0.4

2. The events A and C are disjoint, and $\mathbf{P}(A) = 1/2$ and $\mathbf{P}(B \cap C) = 1/4$.

$\mathbf{P}\left(A \cup (B^c \cup C^c)^c\right) =$ 3/4 ✔ **Answer:** 0.75

3. $\mathbf{P}\left(A^c \cap (B^c \cup C^c)\right) = 0.7$.

$\mathbf{P}\left(A \cup (B^c \cup C^c)^c\right) =$ 0.3 ✔ **Answer:** 0.3

Solution:

1. Using de Morgan's law, we have $(B^c \cup C^c)^c = B \cap C = \emptyset$ so that

$$\mathbf{P}(A \cup (B^c \cup C^c)^c) = \mathbf{P}(A \cup \emptyset) = \mathbf{P}(A) = \boxed{2/5}.$$

2. Note that A and $B \cap C$ are disjoint. Therefore, using de Morgan's law again, together with the additivity axiom for two disjoint events, we have

$$\mathbf{P}(A \cup (B^c \cup C^c)^c) = \mathbf{P}(A \cup (B \cap C)) = \mathbf{P}(A) + \mathbf{P}(B \cap C) = \boxed{3/4}.$$

3. De Morgan's law implies that $(A^c \cap (B^c \cup C^c))^c = A \cup (B^c \cup C^c)^c$, which is the event of interest. Therefore,

$$\mathbf{P}(A \cup (B^c \cup C^c)^c) = 1 - \mathbf{P}(A^c \cap (B^c \cup C^c)) = \boxed{0.3}.$$

提交 You have used 1 of 3 attempts

Answers are displayed within the problem

讨论

显示讨论

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