10. Exercise: Independence and expectations II

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2/3 points (graded)

Let X, Y, and Z be independent jointly continuous random variables, and let g, h, r be some functions. For each one of the following formulas, state whether it is true for all choices of the functions g, h, and r, or false (i.e., not true for all choices of these functions). Do not attempt formal derivations; use an intuitive argument.

1. $\mathbf{E}\big[g(X,Y)h(Z)\big] = \mathbf{E}\big[g(X,Y)\big]\cdot\mathbf{E}\big[h(Z)\big]$

   True ▼ ✔ **Answer:** True

2. $\mathbf{E}\big[g(X,Y)h(Y,Z)\big] = \mathbf{E}\big[g(X,Y)\big]\cdot\mathbf{E}\big[h(Y,Z)\big]$

   False ▼ ✔ **Answer:** False

3. $\mathbf{E}\big[g(X)r(Y)h(Z)\big] = \mathbf{E}\big[g(X)\big]\cdot\mathbf{E}\big[r(Y)\big]\cdot\mathbf{E}\big[h(Z)\big]$

   False ▼ ✖ **Answer:** True

Solution:

1. True. Using our intuitive understanding of independence, the pair of random variables (X,Y) does not provide any information on Z. Therefore, (X,Y) and Z are independent. It follows that $g(X,Y)$ and $h(Z)$ are independent, from which the formula follows.

2. False. The random variable Y appears in both functions g and h, so that $g(X,Y)$ and $h(Y,Z)$ will be, in general, dependent. For an example, suppose that $g(X,Y)=h(Y,Z)=Y$, in which case the statement becomes $\mathbf{E}[Y^2]=\big(\mathbf{E}[Y]\big)^2$, which we know to be false in general.

3. True. Using the first part, and then again the independence of X with Y, we have $\mathbf{E}\big[g(X)r(Y)h(Z)\big] = \mathbf{E}\big[g(X)r(Y)\big]\cdot\mathbf{E}\big[h(Z)\big] = \mathbf{E}\big[g(X)\big]\cdot\mathbf{E}\big[r(Y)\big]\cdot\mathbf{E}\big[h(Z)\big]$.

提交 You have used 1 of 1 attempt

Answers are displayed within the problem