课程 ▫ Unit 3 Methods of Estimation ▫

Homework 6 Maximum Likelihood
Estimation and Method of
Moments

4. Maximum Likelihood Estimation,
Tests, and Confidence Intervals

4. Maximum Likelihood Estimation, Tests, and Confidence Intervals

Setup :

Let $X_1, \ldots, X_n \overset{iid}{\sim} X$ be distributed i.i.d. with probability density function

$$f_\theta(x) = (x/\theta^2) \exp(-x^2/2\theta^2) \, \mathbf{1}(x \geq 0), \theta > 0.$$

(a)

3 points possible (graded)

Let $l(\theta) = \ln L(X_1, \ldots, X_n, \theta)$ denote the log likelihood. Find the critical point of $l(\theta)$. (The critical point is unique because KL divergence is definite.)

(If applicable, enter **barX_n** for $\overline{X}_n$ and **bar(X_n^2)** for $\overline{X_n^2}$.)

Critical point of $l(\theta)$ is at $\theta =$ [   ]

Find the second derivative $l'' = \dfrac{d^2 l}{d\theta^2}$ of $l(\theta)$. Your answer should be a function of θ and the data $X_1, \ldots, X_n$.

(Do **not** evaluate l'' at the critical point at this stage.)

(If applicable, enter **Sigma_i(X_i)** for $\sum\limits_{i=1}^{n} X_i$ and and **Sigma_i(X_i^2)** for $\sum\limits_{i=1}^{n} X_i^2$.)

$l'' = \dfrac{d^2 l}{d\theta^2} =$ [   ]

Using the second derivative test, is the critcal point you obtain above a global maximum, a global minimum, or neither of $l(\theta)$ in the domain $\theta > 0$?

- ○ global maximum

- ○ global minimum

- ○ neither

What can you conclude about the maximum likelihood estimator $\hat{\theta}$ for θ?
(There is no answer box for this question.)

STANDARD NOTATION

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(b)

1 point possible (graded)

What is the Fisher information $I(\theta)$ of the random variables X_i?

$$I(\theta) = $$

[   ]

[   ]

STANDARD NOTATION

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(c)

2 points possible (graded)

Use the theorem for the MLE to write down the asymptotic distribution of the MLE $\hat{\theta}$.

Give an asymptotic 95% confidence interval $\mathcal{I}_{\text{plug-in}}$ for θ using the plug-in method. (You may use I in the answer box below to denote $I(\hat{\theta})$, the Fisher Information, which you found in the previous part, evaluated at $\hat{\theta}$.)

(If applicable, enter **I** for $I(\hat{\theta})$, **hattheta** for $\hat{\theta}$, and **q(alpha)** for q_α for any numerical value α. Recall q_α denotes the value such that $P(Z \geq q_\alpha) = \alpha$ for $Z \sim \mathcal{N}(0,1)$.)

(Do not worry if the parser does not render properly; the graders will work independently. To render properly, add parentheses around **q(alpha)**, i.e. enter **(q(alpha))**.)

$\mathcal{I}_{\text{plug-in}} = [A, B]$ where

$$A = $$

[   ]

$$B = $$

[   ]

STANDARD NOTATION

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(d)

1 point possible (graded)

Use the results from the previous parts to give a test with asymptotic level α for testing

$$H_0 : \theta = 1 \quad \text{v.s.} \quad H_1 : \theta \neq 1.$$

Suppose $n = 100$ and the data gives $\bar{X}_n = 2.5$ and $\bar{X_n^2} = 4.0$. Find the p-value associated to this data for this hypothesis test.

(If applicable, enter **Phi(z)** for the cdf $\Phi(z)$ of a normal variable Z, **q(alpha)** for q_α for any numerical value α.)

p-value: [   ]

STANDARD NOTATION

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讨论

隐藏讨论

主题: Unit 3 Methods of Estimation:Homework 6 Maximum Likelihood Estimation and Method of Moments / 4. Maximum Likelihood Estimation, Tests, and Confidence Intervals

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☐ [(b) : E[X] and Var(X)](#)

3

☐ [[Staff] part C](#)

I am not sure i understand what is wrong with my answer and since it is required for the next question i need help understanding what i am doing wrong ?

1

☐ [Part (a.1) What's the critical point? 1st derivative equals to 0 and solve for theta?](#)

2

☐ [[Staff] part c and d](#)

I found part c very obvious and wrote an answer. Is the grader expecting answer in a different form or a different answer altogether. Perhaps I am wrong. I am ...

2

☐ [[STAFF] Could you please check my second derivative in part a?](#)

I got everything else correct and it may a problem with the grading? or with the entering of the variables? Thanks

3

☐ [MLE doubt](#)

Hi. I'm still not confident about calculating MLE for different scenarios. I know the calculus part, but I lack the procedure of multiplying or elevating the parame...

4

☐ [[Staff] Part C](#)

When I use (q(alpha)) or q(alpha) the grader says ***Invalid Input: alpha not permitted in answer as a variable***. Could you please fix the issue?

3

☐ [[Staff] Question D](#)

3

☐ [[staff] problem a) Invalid Input: X_i not permitted in answer as a variable](#)

problem a) Invalid Input: X_i not permitted in answer as a variable

3