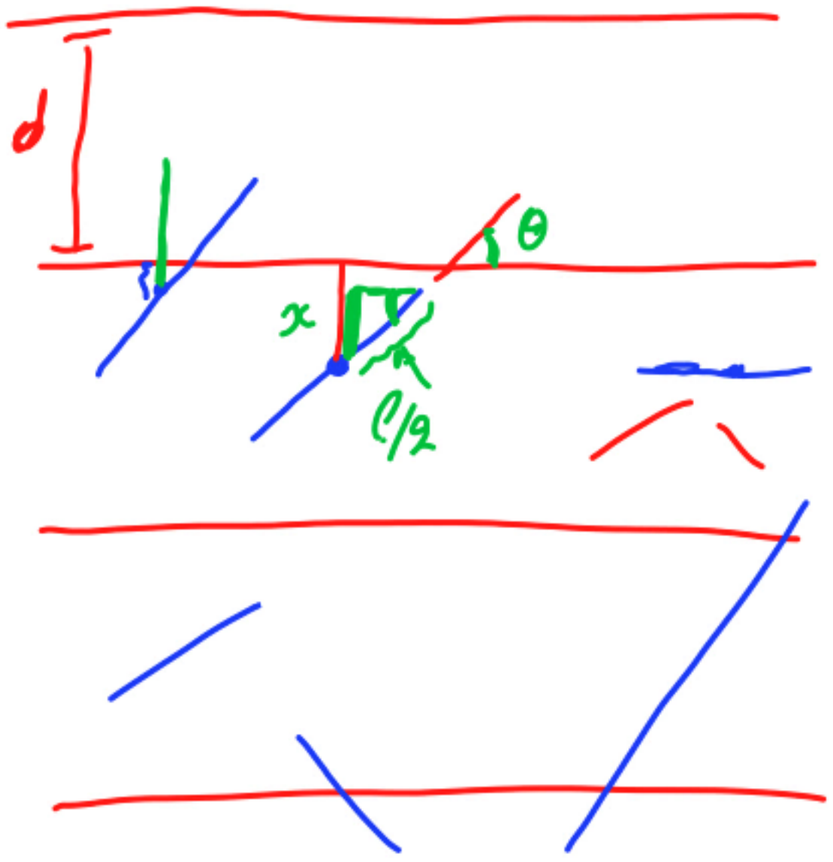
Buffon's needle



- Parallel lines at distance d
  Needle of length ℓ (assume $\ell < d$)
- Find $\mathbf{P}$(needle intersects one of the lines)
- **Model:** (X, Θ) independent

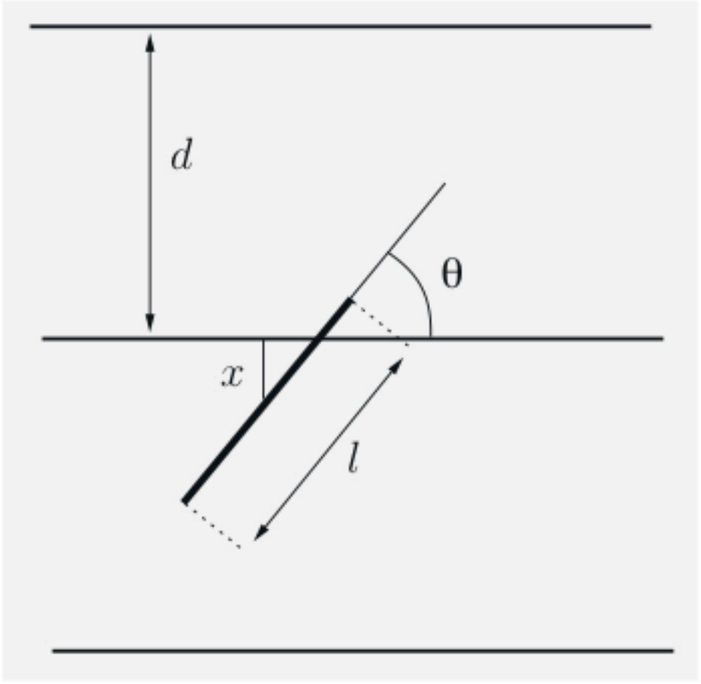
X: $f_X(x) = \frac{2}{d}$, $0 \le x \le \frac{d}{2}$

Θ: $f_\Theta(\theta) = \frac{2}{\pi}$, $0 \le \theta \le \frac{\pi}{2}$

- Intersect if
  - $X \le \frac{\ell}{2} \sin\Theta$

Buffon's needle

$$f_{X,\Theta}(x,\theta) = \frac{4}{\pi d}, \qquad 0 \le x \le d/2, \quad 0 \le \theta \le \pi/2$$



- Intersect if $X \le \frac{\ell}{2}\sin\Theta$

$$\mathbf{P}\left(X \le \frac{\ell}{2}\sin\Theta\right) = \frac{2\ell}{\pi d}$$

$$\mathbb{P}\left(X \le \frac{\ell}{2}\sin\Theta\right)$$

$$= \int_0^{\pi/2}\int_0^{\frac{\ell}{2}\sin\theta} \frac{4}{\pi d}\, dx\, d\theta = \int_0^{\pi/2} \frac{4}{\pi d}\cdot\frac{\ell}{2}\sin\theta \cdot d\theta$$

$$= \frac{2\ell}{\pi d}\,(-\cos\theta)\Big|_0^{\pi/2} = \frac{2\ell}{\pi d}$$

- "Monte Carlo" method for experimental evaluation of π