

Since we are dealing with a discrete uniform model, we will first count the number of favorable configurations, that is, ways in which we can safely place the 8 rooks. We will then divide by the size of the sample space, that is, the total number of possible configurations for 8 rooks on an 8×8 chessboard.

First we count the number of favorable configurations for the rooks. We will place the rooks one by one. For the first rook, there are no constraints, so we have 64 choices. Placing this rook, however, eliminates one row and one column. Thus for our second rook, we can imagine that the illegal column and row have been removed, thus leaving us with a 7×7 chessboard, and thus with 49 choices. Similarly, for the third rook we have 36 choices, for the fourth 25, etc. Thus, the number of favorable configurations is $64 \times 49 \times 36 \times 25 \times 16 \times 9 \times 4$.

To find the size of the sample space, we note that in the absence of any restrictions, other than the requirement that the rooks need to be in distinct squares, there are $64 \times 63 \dots \times 57 = 64!/56!$ total ways we can place the 8 rooks.

Therefore the probability we are after is:

$$\frac{64 \cdot 49 \cdot 36 \cdot 25 \cdot 16 \cdot 9 \cdot 4}{\frac{64!}{56!}}$$

In the above solution, we treated the rooks as distinguishable: there was a 1st rook, a 2nd rook, etc. In an alternative approach, we treat the rooks as being **indistinguishable**. Then, an outcome of the experiment is just an 8-member subset of the chessboard. Since the chessboard has 64 positions, the sample space is of size $\binom{64}{8}$. In how many ways can the event of interest happen? There will be exactly one rook on each row. There are 8 possible positions in the first row; once this is chose, there are 7 positions on the 2nd row, etc., for a total of $8!$ positions. Thus, the desired probability is $\frac{8!}{\binom{64}{8}}$. This expression looks different from that derived earlier. However, it can be checked that the expressions are equal.