Homework 4.1: Stability analysis

Phase plane stability analysis

1/1 point (graded)
Consider the following linear system

$$\begin{cases} \frac{du}{dt} = \alpha u - w \\ \frac{dw}{dt} = \beta u - w \end{cases}$$

These equations can be written in matrix form $\frac{d}{dt}x = Ax$ where $x = \begin{pmatrix} u \\ w \end{pmatrix}$ and $A = \begin{pmatrix} \alpha & -1 \\ \beta & -1 \end{pmatrix}$.

In the following we want to determine the stability conditions for fixed point of the system, i.e., $(u, w) = (0, 0)$ by studing the eigenvalues of the above matrix A.

What are the eigenvalues λ_+, λ_- of matrix A?

Hint: the eigenvalues are the roots of the equation $\det(A - \lambda I) = 0$ where $\det(.)$ is determinant and I is the identity matrix.

- ✔ $\lambda_\pm = -\frac{1}{2}[(1-\alpha) \pm \sqrt{(1-\alpha)^2 - 4(\beta - \alpha)}]$
- $\lambda_\pm = \frac{1}{2}[(1+\alpha) \pm \sqrt{(1+\alpha)^2 - 4(\beta - \alpha)}]$
- $\lambda_\pm = -\frac{1}{2}[(1-\alpha) \pm \sqrt{(1-\alpha)^2 + 4(\beta - \alpha)}]$
- $\lambda_\pm = \frac{1}{2}[(1+\alpha) \pm \sqrt{(1+\alpha)^2 + 4(\beta - \alpha)}]$

✔

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✔ Correct (1/1 point)

Stability analysis: Case 1

1/1 point (graded)
The eigenvalues calculated above determine the stability of the fixed point $(u, w) = (0, 0)$ in the system. The eigenvalues may be real or complex numbers depending on the parameters. One can show that the fixed points of the system is stable if and only if the real part of both eigenvalues are negative. By taking into account such a hint, analyze the stability of the system in the following situations.

If $\beta < \alpha$, then

- If $\alpha > 1$ the fixed point is stable and if $\alpha < 1$ it is unstable.
- If $\alpha < 1$ the fixed point is stable and if $\alpha > 1$ it is unstable.
- If $\alpha < -1$ the fixed point is stable and if $\alpha > -1$ it is unstable.
- the fixed point is always stable

- (●) the fixed point is always unstable because $\mathbf{sign}\,(\lambda_+.\lambda_-) < 0.$

✔

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✔ Correct (1/1 point)

Stability analysis: Case 2

0/1 point (graded)
If $\beta > \alpha$, then

(Note that there may be more than one correct answer)

- [ ] If $\alpha < 1$ the fixed point is always stable and if $\alpha > 1$ it is always unstable.
- [ ] If $\alpha > 1$ the fixed point is stable and if $\alpha < 1$ it is unstable.
- [x] If $\alpha \leq 1$ the fixed point is always stable.
- [ ] the solution may be cycles (closed curves) if $\alpha = 1$.
- [ ] the fixed point is always unstable because $\mathbf{sign}\,(\lambda_+.\lambda_-) < 0.$
- [ ] the fixed point is always stable.

✖

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✖ Incorrect (0/1 point)

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