15. Exercise: A function of multiple r.v.'s

Exercise: A function of multiple r.v.'s

2/2 points (graded)

Suppose that X and Y are described by a joint PDF which is uniform inside the unit circle, that is, the set of points that satisfy $x^2 + y^2 \leq 1$. In particular, the joint PDF takes the value of $1/\pi$ on the unit circle. Let $Z = \sqrt{X^2 + Y^2}$, which is the distance of the outcome (X, Y) from the origin. The PDF of Z, for $z \in [0, 1]$, takes the form $f_Z(z) = az^b$. Find a and b.

$a =$ 2 ✔ **Answer:** 2

$b =$ 1 ✔ **Answer:** 1

Solution:

Note that the set of points that satisfy $x^2 + y^2 \leq z^2$ is a circle of radius z, has area πz^2, and probability z^2. Therefore,

$$F_Z(z) = \mathbf{P}(Z \leq z) = \mathbf{P}(X^2 + Y^2 \leq z^2) = z^2,$$

from which it follows that $f_Z(z) = 2z$.

提交 You have used 2 of 3 attempts

Answers are displayed within the problem

讨论

显示讨论

Topic: Unit 6 / Lec. 11 / 15. Exercise: A function of multiple r.v.'s