## NEST Topology User Manual

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NEST 2.0 RC2 (r9083 or later)



#### **About the Topology Module**

This user manual gives a short introduction to the use of the Topology Module for the NEST Neural Simulation Tool.

Rüdiger Kupper wrote a first topology library for NEST many years ago entirly in SLI (actually, it pre-dates the NEST kernel). Kittel Austvoll and Hans Ekkehard Plesser designed and wrote a completely new Topology library in 2007/8. That library has been available with the NEST 1.9.x pre-releases since.

For NEST 2.0, Håkon Enger and Hans Ekkehard Plesser re-factored parts of the Topology library code, improved and extended the PyNEST interface for the Topology library, fixed bugs and added tests.

This User Manual describes the NEST 2.0 version of the NEST Topology Library. Please see Chapter sec:changes for a summary of changes.

We plan further improvements to the Topology Module in the future, which may include changes to the API to remove some of the remaining inconsistencies and provide a cleaner user interface.

## **Contents**

1	Intro	oduction	2			
	1.1	Limitations and Disclaimer	2			
2	Layers					
	2.1	Grid-based Layers	4			
		2.1.1 A very simple layer	4			
		2.1.2 Setting the extent	6			
		2.1.3 Setting the center	6			
		2.1.4 Constructing a layer: an example	6			
	2.2	Free layers	7			
	2.3	Periodic boundary conditions	9			
			10			
	2.4		11			
			13			
3	Con	nections	15			
	3.1	Basic principles	15			
		3.1.1 Terminology	15			
			16			
	3.2		17			
	3.3		18			
			18			
			19			
	3.4		21			
	3.5		22			
	3.6		25			
	3.7		26			
	3.8		27			
	3.9		28			
4	Inspecting Layers 30					
			30			
		•	31			
5	Cha	nges from Topology 1.9 to 2.0	33			
Bi	oliog	raphy	34			
Lis	st of 1	Figures	35			

CONTENTS	
List of Tables	36
Index	35

## Chapter 1

## Introduction

The Topology Module provides the NEST simulator<sup>1</sup> (Gewaltig & Diesmann, 2007) with a convenient interface for creating layers of neurons placed in space and connecting neurons in such layers with probabilities and properties depending on the relative placement of neurons. This permits the creation of complex networks with spatial structure.

This user manual provides an introdcution to the functionality provided by the Topology Module. It is based exclusively on the PyNEST, the Python interface to NEST (Eppler et al., 2008). NEST users using the SLI interface should be able to map instructions to corresponding SLI code. This manual is not meant as a comprehensive reference manual. Please consult the online documentation in PyNEST for details; where appropriate, that documentation also points to relevant SLI documentation.

This manual describes the Topology Module included with NEST 2.0, code revision 8995 or later. This version differs from older ones in some important aspects as detailed in Section 5. In our experience, though, most scripts using the Topology Module can be ported to the new version with minimal changes.

In the next chapter of this manual, we introduce Topology layers, which place neurons in space. In Chapter 3 we then describe how to connect layers with each other, before discussing in Chapter 4 how you can inspect and visualize Topology networks.

You will find the Python scripts used in the examples in this manual in the NEST source code directory under topology/doc/user\_manual\_scripts.

#### 1.1 Limitations and Disclaimer

**2D only** The Topology Module provides support for layers in two-dimensional space only. The "3D layers" described in the user manual for earlier versions of the Topology Module were actually layers in two-dimensional space, where each element consisted of several neurons which then were labeled, e.g., as belonging to L2/3, L4, or L6, but were not assigned a physical z-coordinate.

**No extensibility** There is currently no user-friendly way to extend the Topology Module with new kernel functions or masks. We plan to add support for this in the future.

<sup>&</sup>lt;sup>1</sup>NEST is available under an open source license at www.nest-initiative.org.

4

**Undocumented features** The Topology Module provides a number of undocumented features, which you may discover by browsing the code. These features are highly experimental and should *not be used for simulations*, as they have not been validated. A prominent example is the possibility to create layers with neurons freely placed in three-dimensional space.

## **Chapter 2**

## Layers

The Topology Module (just Topology for short in the remainder of this document) organizes neuronal networks in *layers*. We will first illustrate how Topology places elements in simple layers, where each element is a single model neuron. Layers with composite elements are discussed in the following section.

We will illustrate the definition and use of layers using examples.

Topology distinguishes between two classes of layers:

grid-based layers in which each element is placed at a location in a regular grid;

free layers in which elements can be placed arbitrarily in the plane.

Grid-based layers allow for more efficient connection-generation under certain circumstances.

## 2.1 Grid-based Layers

### 2.1.1 A very simple layer

We create a first, grid-based simple layer with the following commands:

The layer is shown in Fig. 2.1. Note the following properties:

- The layer has five *rows* and five *columns*.
- The 'elements' entry of the dictionary passed to **CreateLayer** determines the *elements* of the layer. In this case, the layer contains iaf\_neurons.
- The *center* of the layer is at the origin of the coordinate system, (0,0).
- The *extent* or size of the layer is  $1 \times 1$ . This is the default size for layers. The extent is marked by the thin square in Fig. 2.1.

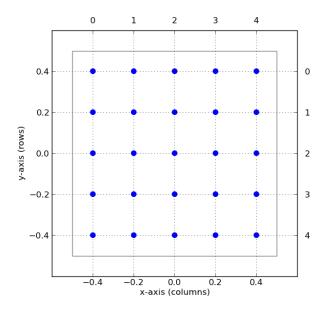


Figure 2.1: Simple grid-based layer centered about the origin. Blue circles mark layer elements, the thin square the extent of the layer. Row and column indices are shown in the right and top margins, respectively.

• The *grid spacing* of the layer is

$$dx = \frac{x\text{-extent}}{\text{number of columns}}$$

$$dy = \frac{y\text{-extent}}{\text{number of rows}}$$
(2.1)

In the layer shown, we have dx = dy = 0.2, but the grid spacing may differ in x-and y-direction.

- Layer elements are spaced by the grid spacing and are arranged symmetrically about the center.
- The outermost layer elements are placed dx/2 and dy/2 from the borders of the extent.
- Element *positions* in the coordinate system are given by (x, y) pairs. The *coordinate system* follows that standard mathematical convention that the x-axis runs from left to right and the y-axis from bottom to top.
- Each element of a grid-based layer has a *row- and column-index* in addition to its (x,y)-coordinates. Indices are shown in the top and right margin of Fig. 2.1. Note that row-indices follow matrix convention, i.e., run from top to bottom. Following pythonic conventions, indices run from 0.

**Note**: The **definition of the extent has changed** from NEST 1.9 to NEST 2.0. In NEST 1.9, the outermost elements of the layer were placed on the limits of the extent. When working with periodic boundary conditions (see Sec. 2.3), Topology then silently padded the layer with half a grid spacing on all sides, to ensure that nodes at opposite edges did not coincide.

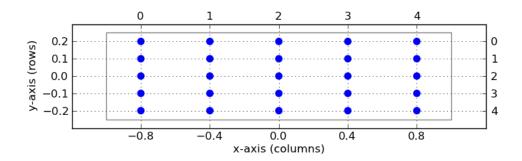


Figure 2.2: Same layer as in Fig. 2.1, but with different extent.

#### 2.1.2 Setting the extent

Layers have a default extent of  $1 \times 1$ . You can specify a different extent of a layer, i.e., its size in x- and y-direction by adding and 'extent' entry to the dictionary passed to **CreateLayer**:

The resulting layer is shown in Fig. 2.2. The extent is always a two-element tuple of floats. In this example, we have grid spacings dx = 0.4 and dy = 0.1. Changing the extent does not affect grid indices.

### 2.1.3 Setting the center

Layers are centered about the origin (0,0) by default. This can be changed through the 'center' entry in the dictionary specifying the layer. The following code creates layers centered about (0,0), (-1,1), and (1.5,0.5), respectively:

The center is given as a two-element tuple of floats. Changing the center does not affect grid indices: For each of the three layers in Fig. 2.3, grid indices run from 0 to 4 through columns and rows, respectively, even though elements in these three layers have different positions in the gobal coordinate system.

### 2.1.4 Constructing a layer: an example

To see how to construct a layer, consider the following example:

- a layer with  $n_r$  rows and  $n_c$  columns;
- spacing between nodes is *d* in *x* and *y*-directions;

2.2 Free layers

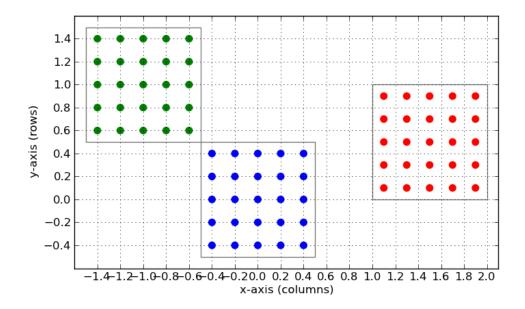


Figure 2.3: Three layers centered, respectively, about (0,0) (blue), (-1,-1) (green), and (1.5,0.5) (red).

- the left edge of the extent shall be at x = 0;
- the extent shall be centered about y = 0.

From Eq. 2.1, we see that the extent of the layer must be  $(n_c d, n_r d)$ . We now need to find the coordinates  $(c_x, c_y)$  of the center of the layer. To place the left edge of the extent at x = 0, we must place the center of the layer at  $c_x = n_c d/2$  along the x-axis, i.e., half the extent width to the right of x = 0. Since the layer is to be centered about y = 0, we have  $c_y = 0$ . Thus, the center coordinates are  $(n_c d/2, 0)$ . The layer is created with the following code and shown in Fig. 2.4:

### 2.2 Free layers

Free layers do not restrict node positions to a grid, but allow free placement within the extent. To this end, the user needs to specify the positions of all nodes explicitly. The following code creates a layer of 50 iaf\_neurons uniformly distributed in a layer with extent  $1 \times 1$ , i.e., spanning the square  $[-0.5, 0.5] \times [-0.5, 0.5]$ :

Note the following points:

2.2 Free layers

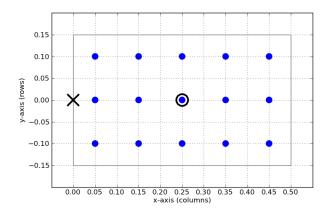


Figure 2.4: Layer with  $n_c = 5$  rows and  $n_r = 3$  columns, spacing d = 0.1 and the left edge of the extent at x = 0, centered about the *y*-axis. The cross marks the point on the extent placed at the origin (0,0), the circle the center of the layer.

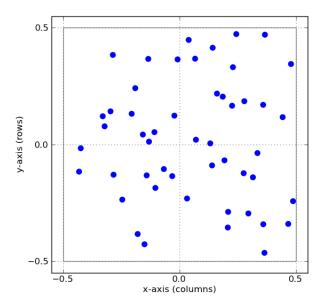


Figure 2.5: A free layer with 50 elements uniformly distributed in an extent of size  $1 \times 1$ .

- For free layers, element *positions* are specified by the 'positions' entry in the dictionary passed to **CreateLayer**. 'positions' is mutually exclusive with 'rows'/'columns' entries in the dictionary.
- The 'positions' entry must be a Python list (or tuple) of element coordinates, i.e., of two-element tuples of floats giving the (x, y)-coordinates of the elements. One layer element is created per element in the 'positions' entry.
- All layer element positions must be *within* the layer's extent. Elements may be place on the perimeter of the extent as long as no periodic boundary conditions are used; see Sec. 2.3.
- Element positions in free layers are *not* shifted when specifying the 'center' of the layer. The user must make sure that the positions given lie within the extent when centered about the given center.

### 2.3 Periodic boundary conditions

Simulations usually model systems much smaller than the biological networks we want to study. One problem this entails is that a significant proportion of neurons in a model network is close to the edges of the network with fewer neighbors than nodes properly inside the network. In the  $5 \times 5$ -layer in Fig. 2.1, e.g., 16 out of 25 nodes form the border of the layer.

One common approach to reducing the effect of boundaries on simulations is to introduce *periodic boundary conditions*, so that the rightmost elements on a grid are considered nearest neighbors to the leftmost elements, and the topmost to the bottommost. The flat layer becomes the surface of a torus. Fig. 2.6 illustrates this for a one-dimensional layer, which turns from a line to a ring upon introduction of periodic boundary conditions.

You specify periodic boundary conditions for a layer using the dictionary entry edge\_wrap:

Note that the longest possible distance between two elements in a layer without periodic boundary conditions is

$$\sqrt{x_{\rm ext}^2 + y_{\rm ext}^2}$$

but only

$$\sqrt{x_{\rm ext}^2 + y_{\rm ext}^2} / 2$$

for a layer with periodic boundary conditions;  $x_{\text{ext}}$  and  $y_{\text{ext}}$  are the components of the extent size.

We will discuss the consequences of periodic boundary conditions more in Chapter 3.

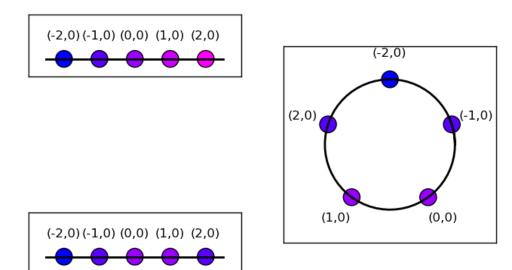


Figure 2.6: Top left: Layer with single row and five columns without periodic boundary conditions. Numbers above elements show element coordinates. Colors shifting from blue to magenta mark increasing distance from the element at (-2,0). Bottom left: Same layer, but with periodic boundary conditions. Note that the element at (2,0) now is a nearest neighbor to the element at (-2,0). Right: Layer with periodic boundary condition arranged on a circle to illustrate neighborhood relationsships.

#### 2.3.1 Topology layer as NEST subnet

From the perspective of NEST, a Topology layer is a special type of *subnet*. From the user perspective, the following points may be of interest:

- Grid-based layers have the NEST model type topology\_layer\_grid, free layers the model type topology\_layer\_free.
- The status dictionary of a layer has a 'topology' entry describing the layer properties (1 is the layer created above):

```
print nest.GetStatus(1)[0]['topology']

{'rows': 5, 'center': [0.0, 0.0], 'edge_wrap': False, 'depth':
    1, 'extent': [1.0, 1.0], 'columns': 5}
```

The 'topology' entry is read-only.

 The NEST kernel sees the elements of the layer in the same way as the elements of any subnet. You will notice this when printing a network with a Topology layer:

```
nest. PrintNetwork (depth=2)
```

```
+-[0] root dim=[1 25]

|

+-[1] topology_layer_grid dim=[25]

|

+-[1]...[25] iaf_neuron
```

The  $5 \times 5$  layer created above appears here as a topology\_layer\_grid subnet of 25 iaf\_neurons. Only Topology connection and visualization functions heed the spatial structure of the layer.

### 2.4 Layers with composite elements

So far, we have considered layers in which each element was a single model neuron. Topology can also create layers with *composite elements*, i.e., layers in which each element is a (possibly nested) collection of model neurons, or, in general NEST network nodes.

Construction of layers with composite elements proceeds exactly as for layers with simple elements, except that the 'elements' entry of the dictionary passed to CreateLayer is a Python list or tuple. The following code creates a  $1 \times 2$  layer (to keep the output from PrintNetwork() compact) in which each element consists of one 'iaf\_cond\_alpha' and one 'poisson\_generator' node

```
+-[0] root dim=[1 2 2]

| +-[1] topology_layer_grid dim=[2 2]

| +-[1] subnet dim=[2]

| | | | | | | +-[1] iaf_cond_alpha

| +-[2] poisson_generator

| +-[2] subnet dim=[2]

| +-[1] iaf_cond_alpha

| +-[2] poisson_generator
```

The network consist of one topology\_layer\_grid with two elements, each of which is a subnet containing one iaf\_cond\_alpha and one poisson\_generator node.

You can create network elements with several nodes of each type by following a model name with the number of nodes to be created:

```
+-[0] root dim=[1 2 13]

+-[1] topology_layer_grid dim=[2 13]

|

+-[1] subnet dim=[13]

|
```

```
| +-[1]...[10] iaf_cond_alpha
| +-[11] poisson_generator
| +-[12]...[13] noise_generator
|--[2] subnet dim=[13]
| +-[1]...[10] iaf_cond_alpha
| +-[11] poisson_generator
| +-[12]...[13] noise_generator
```

In this case, each layer element consists of 10 iaf\_cond\_alpha neurons, one poisson\_generator, and two noise\_generators.

Note the following points:

- Each element of a layer has identical components.
- Each element is implemented as a subnet.
- All nodes within a composite element have identical positions, namely the position of the layer element.
- It is possible to create layers with nested, composite elements, e.g.,

but such nested networks gobble up a lot of memory for subnet constructs and provide no practical advantages. See the next section for design recommendations for more complex layers.

#### 2.4.1 Designing layers

A paper on a neural network model might describe the network as follows<sup>1</sup>:

The network consists of 20x20 microcolumns placed on a regular grid spanning  $0.5^{\circ} \times 0.5^{\circ}$  of visual space. Neurons within each microcolumn are organized into L2/3, L4, and L56 subpopulations. Each subpopulation consists of three pyramidal cells and one interneuron. All pyramidal cells are modeled as NEST iaf\_neurons with default parameter values, while interneurons are iaf\_neurons with threshold voltage  $V_{\rm th} = -52$ mV.

How should you implement such a network using the Topology module? At first sight it appears tempting to reproduce the nested structure suggested by the model description:

There are two shortcomings to this approach:

- The resulting layer will contain many subnets with only four neurons each, resulting in a significant memory overhead.
- It is cumbersome to address the various subpopulations to create selective connections, e.g., from L2/3 pyramidal cells to L4 interneurons.

We thus advise against this nesting approach.

A more efficient approach is to create different models for the neurons in each layer and then define the microcolumn as one "flat" composite element:

```
for lyr in ['L23', 'L4', 'L56']:
   nest.CopyModel('iaf_neuron', lyr+'pyr')
```

<sup>&</sup>lt;sup>1</sup>See Nordlie et al. (2009) for suggestions on how to describe network models.

```
nest.CopyModel('iaf_neuron', lyr+'in', {'V_th': -52.})

1 = tp.CreateLayer({'rows': 20, 'columns': 20, 'extent': [0.5, 0.5], 'elements': ['L23pyr', 3, 'L23in', 'L4pyr', 3, 'L4in', 'L56pyr', 3, 'L56in']})
```

We will discuss in Chapter 3.1 how to connect selectively to different neuron models.

## **Chapter 3**

## **Connections**

The most important feature of the Topology module is the ability to create connections between layers with quite some flexibility. In this chapter, we will illustrate how to specify and create connections. All connections are created using the **ConnectLayers** function.

### 3.1 Basic principles

#### 3.1.1 Terminology

We begin by introducing important terminology:

- **Connection** In the context of connections between the elements of Topology layers, we often call the set of all connections between pairs of network nodes created by a single call to **ConnectLayers** a *connection*.
- **Connection dictionary** A dictionary specifying the properties of a connection between two layers in a call to CreateLayers.
- **Source** The *source* of a single connection is the node sending signals (usually spikes). In a projection, the source layer is the layer from which source node are chosen.
- **Target** The *target* of a single connection is the node receiving signals (usually spikes). In a projection, the target layer is the layer from which source node are chosen.
- **Connection type** The *connection type* determines how nodes are selected when **ConnectLayers** creates connections between layers. It is either 'convergent' or 'divergent'.
- **Convergent connection** When creating a *convergent connection* between layers, Topology visits each node in the target layer in turn and selects sources for it in the source layer. Masks and kernels are applied to the source layer, and periodic boundary conditions are applied in the source layer, provided that the source layer has periodic boundary conditions.
- **Divergent connection** When creating a *divergent connection*, Topology visits each node in the source layer and selects target nodes from the target layer. Masks, kernels, and boundary conditions are applied in the target layer.

**Driver** When connecting two layers, the *driver* layer is the one in which each node is considered in turn.

**Pool** When connecting two layers, the *pool* layer is the one from which nodes are chosen for each node in the driver layer. I.e., we have

Connection type	Driver	Pool
convergent	target layer	source layer
divergent	source layer	target layer

**Displacement** The *displacement* between a driver and a pool node is the shortest vector connecting the driver to the pool node, taking boundary conditions into accout.

**Distance** The *distance* between a driver and a pool node is the length of their displacement.

**Mask** The *mask* defines which pool nodes are at all considered as potential targets for each driver node. See Sec. 3.3 for details.

**Kernel** The *kernel* is a function returning a (possibly distance- or displacment-dependent) probability for creating a connection between a driver and a pool node. The default kernel is 1, i.e., connections are created with certainty. See Sec. 3.4 for details.

**Autapse** An *autapse* is a synapse (connection) from a node onto itself. Autapses are permitted by default, but can be disabled by adding 'allow\_autapses': False to the connection dictionary.

**Multapse** Node A is connected to node B by a *multapse* if there are synapses (connections) from A to B. Multapses are permitted by default, but can be disabled by adding 'allow\_multapses': False to the connection dictionary.

### 3.1.2 A minimal ConnectLayers call

Connections between Topology layers are created by calling **ConnectLayers** with the following arguments<sup>1</sup>:

- 1. The source layer.
- 2. The target layer (can be identical to source layer).
- 3. A connection dictionary that contains at least the following entries:

Either 'convergent' or 'divergent'.

A mask specification as described in Sec. 3.3.

Here is a simple example, cf. 3.1:

<sup>&</sup>lt;sup>1</sup>You can also use standard NEST connection functions to connect nodes in Topology layers.

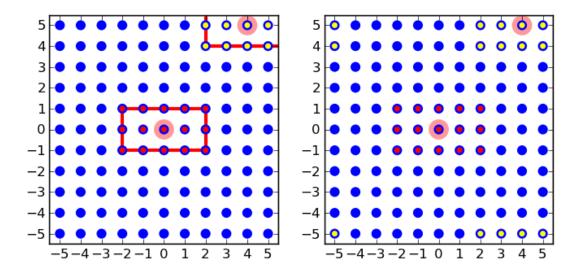


Figure 3.1: Left: Minimal connection example from a layer onto itself using a rectangular mask shown as red line for the node at (0,0) (marked light red). The targets of this node are marked with red dots. The targets for the node at (4,5) (marked light orange) are marked with orange dots. This node has fewer targets since it is at the corner and many potential targets are beyond the layer. Right: The effect of periodic boundary conditions is seen here. Source and target layer and connection dictionary were identical, except that periodic boundary conditions were used. The node at (4,5) now has 15 targets, too, but they are spread across the corners of the layer. If we wrapped the layer to a torus, they would form a  $5 \times 3$  rectangle centered on the node at (4,5).

In this example, layer 1 is both source and target layer. Connection type is divergent, i.e., for each node in the layer we choose targets according to the rectangular mask centered about each source node. Since no connection kernel is specified, we connect to all nodes within the mask. Note the effect of normal and periodic boundary conditions on the connections created for different nodes in the layer, as illustrated in Fig. 3.1.

## 3.2 Mapping source and target layers

The application of masks and other functions depending on the distance or even the displacement between nodes in the source and target layers requires a mapping of coordinate systems between source and target layers. Topology applies the following *coordinate mapping rules*:

1coAhlebtiorsthandskwo-dimensional Euclidean coordinate systems.

2. No scaling or coordinate transformation can be applied between layers.

3.3 Masks 19

3. The displacement d(D, P) from node D in the driver layer to node P in the pool layer is measured by first mapping the position of D in the driver layer to the identical position in the pool layer and then computing the displacement from that position to P. If the pool layer has periodic boundary conditions, they are taken into account. It does not matter for displacement computations whether the driver layer has periodic boundary conditions.

#### 3.3 Masks

A mask describes which area of the pool layer shall be searched for nodes to connect for any given node in the driver layer. We will first describe geometrical masks defined for all layer types and then consider grid-based masks for grid-based layers.

Note that the mask size should not exceed the size of the layer when using periodic boundary conditions, since the mask would "wrap around" in that case and pool nodes would be considered multiple times as targets.

#### 3.3.1 Masks for all layer types

Topology currently provides three types of masks usable for free and grid-based layers. They are illustrated in Fig. 3.2. The masks are

**Rectangular** All nodes within a rectangular area are connected. The area is speficied by its lower left and upper right corners, measured in the same unit as element coordinates. Example:

**Circular** All nodes within a circle are connected. The area is specified by its radius.

**Doughnut** All nodes between an inner and outer circle are connected. Note that nodes *on* the inner circle are not connected. The area is specified by the radii of the inner and outer circles.

By default, the masks are centered about the position of the driver node, mapped into the pool layer. You can change the location of the mask relative to the driver node by specifying an 'anchor' entry in the mask dictionary. The anchor is a 2D vector specifying the location of the mask center relative to the driver node, as in the following examples (cf. Fig. 3.2, bottom row):

3.3 Masks 20

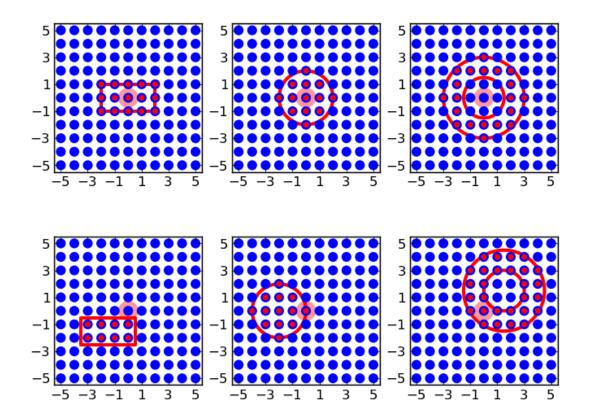


Figure 3.2: Masks for all layer types. For all mask types, the driver node is marked by a wide light-red circle, the selected pool nodes by red dots and the masks by red lines. Top row from left to right: rectangular, circular and doughnut masks centered about the driver node. Bottom row from left to right: the same masks as in the top row, but centered about (-1.5, -1.5), m (-2, 0) and (1.5, 1.5), respectively, using the 'anchor' parameter.

### 3.3.2 Masks for grid-based layers

Grid-based layers can be connected using rectangular *grid masks*. For these, you specify the size of the mask not by lower left and upper right corner coordinates, but give their size in rows and columns, as in this example:

3.3 Masks 21

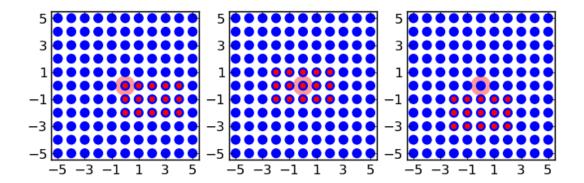


Figure 3.3: Grid masks for connections between grid-based layers. Left:  $5 \times 3$  mask with default alignment at upper left corner. Center: Same mask, but anchored to center node at grid index [1,2]. Right: Same mask, but anchor to the upper left of the mask at grid index [-1,2].

The resulting connections are shown in Fig. 3.3. By default the top-left corner of a grid mask, i.e., the grid mask element with grid index  $[0,0]^2$ , is aligned with the driver node. You can change this alignment by specifying an *anchor* for the mask:

You can even place the anchor outside the mask:

The resulting connection patterns are shown in Fig. 3.3. Connections specified using grid masks are generated more efficiently than connections specified using other mask types.

Note the following:

- Grid-based masks are applied by considering grid indices. The position of nodes in physical coordinates is ignored.
- In consequence, grid-based masks should only be used between layers with identical grid spacings.
- The semantics of the 'anchor' property for grid-based masks differ significantly for general masks described in Sec. 3.3.1. For general masks, the anchor is the center of the mask relative to the driver node. For grid-based nodes, the anchor determines which mask element is aligned with the driver element.

<sup>&</sup>lt;sup>2</sup>See Sec. 2.1.1 for the distinction between layer coordinates and grid indices

3.4 Kernels

Name	Parameters	Function
constant		constant $p \in [0,1]$
uniform	min, max	$p \in [\min, \max)$ uniformly
linear	a, c	
		p(d) = c + ad
exponential	a, c, tau	
1		$p(d) = c + ae^{-\frac{d}{\tau}}$
gaussian	p_center, sigma,	
O	mean, c	$p(d) = c + p_{\text{center}}e^{-\frac{(d-\mu)^2}{2\sigma^2}}$
		p(u) = c + pcenter $c = -c$
gaussian2D	p_center, sigma_x,	
	sigma_y, mean_x, mean_y,rho c	$\frac{(d_x - \mu_x)^2}{(d_x - \mu_x)^2} - \frac{(d_y - \mu_y)^2}{(d_x - \mu_x)(d_y - \mu_y)} + 2\rho \frac{(d_x - \mu_x)(d_y - \mu_y)}{(d_x - \mu_x)(d_y - \mu_y)}$
	inean_y,ino c	$n(d) = c + n_{contex}e^{-\frac{(d_x - \mu_x)^2}{\sigma_x^2} - \frac{(d_y - \mu_y)^2}{\sigma_y^2} + 2\rho \frac{(d_x - \mu_x)(d_y - \mu_y)}{\sigma_x \sigma_y}}{2(1 - \rho^2)}$
		$p(d) = c + p_{\text{center}}e^{-2(1-\rho^2)}$

Table 3.1: Kernel functions currently available in the Topology module. d is the distance and  $(d_x, d_y)$  the displacement.

#### 3.4 Kernels

Many neuronl network models employ probabilistic connection rules. Topology supports probabilistic connections through *kernels*. A kernel is a function mapping the distance (or displacement) between a driver and a pool node to a connection probability. Topology then generates a connections according to this probability.

Probabilistic connections can be generated in two different ways using Topology:

**Free probabilistic connections** are the default. In this case, **ConnectLayers** considers each driver node *D* in turn. For each *D*, it evaluates the kernel for each pool node *P* within the mask and creates a connection according to the resulting probability. This means in particular that *each possible driver-pool pair is inspected exactly once* and that there will be *at most one connection between each driver-pool pair*.

**Prescribed number of connections** can be obtained by specifying the number of connections to create per driver node. See Sec. 3.7 for details.

Available kernel functions are shown in Table 3.1. There is at present unfortunately no convenient way to add further kernel functions to Topology.

Several examples follow. They are illustrated in Fig. 3.4.

**Constant** The simplest kernel is a fixed connection probability:

**Gaussian** This kernel is distance dependent. In the example, connection probability is 1 for d = 0 and falls off with a "standard deviation" of  $\sigma = 1$ :

**Excentric Gaussian** In this example, both kernel and mask have been moved using anchors:

Note that the anchor for the kernel is specified inside the dictionary containing the parameters for the Gaussian.

**Cut-off Gaussian** In this example, all probabilties less than 0.5 are set to zero:

**2D Gaussian** We conclude with an example using a two-dimensional Gaussian, i.e., a Gaussian with different widths in x- and y- directions. This kernel depends on displacement, not only on distance:

Note that for pool layers with periodic boundary conditions, Topology always uses the shortest possible displacement vector from driver to pool neuron as argument to the kernel function.

## 3.5 Weights and delays

The functions presented in Table 3.1 can also be used to specify distance-dependent or randomized weights and delaysfor the connections created by **ConnectLayers**.

Figure 3.5 illustrates weights and delays generated using these functions with the following code examples. All examples use a "layer" of 51 nodes placed on a line; the line is centered about (25,0), so that the leftmost node has coordinates (0,0). The distance between neighboring elements is 1. The mask is rectangular, spans the entire layer and is centered about the driver node.

#### Linear example

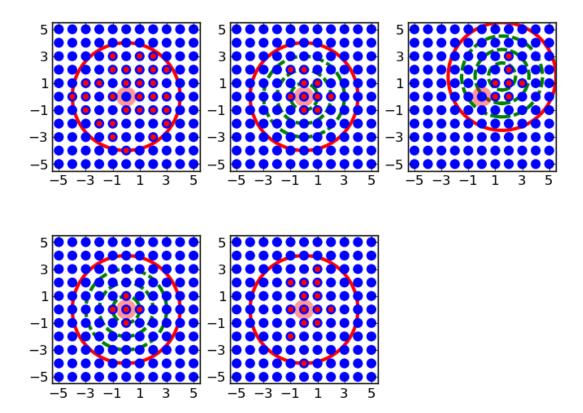


Figure 3.4: Illustration of various kernel functions. Top left: constant kernel, p=0.5. Top center: Gaussian kernel, green dashed lines show  $\sigma$ ,  $2\sigma$ ,  $3\sigma$ . Top right: Same Gaussian kernel anchored at (1.5,1.5). Bottom left: Same Gaussian kernel, but all p<0.5 treated as p=0. Bottom center: 2D-Gaussian.

Results are shown in the top panel of Fig. 3.5. Connection weights and delays are shown for the leftmost neuron as driver. Weights drop linearly from 1. From the node at (20,0) on, the cutoff sets weights to 0. There are no connections to nodes beyond (25,0) since as the mask extends only 25 units to the right of the driver. Delays increase in a stepwise linear fashion, as NEST requires delays to be multiples of the simulation resolution.

#### Linear example with periodic boundary conditions

Results are shown in the middle panel of Fig. 3.5. This example is identical to the previous, except that the (pool) layer has periodic boundary conditions. Therefore, the left half of the mask about the node at (0,0) wraps back to the right half of the layer and that node connects to all nodes in the layer.

#### Various functions

Results are shown in the bottom panel of Fig. 3.5. It shows linear, exponential and Gaussian weight functions for the node at (25,0).

#### Randomized weights and delays

```
cdict = {'connection_type': 'divergent',
```

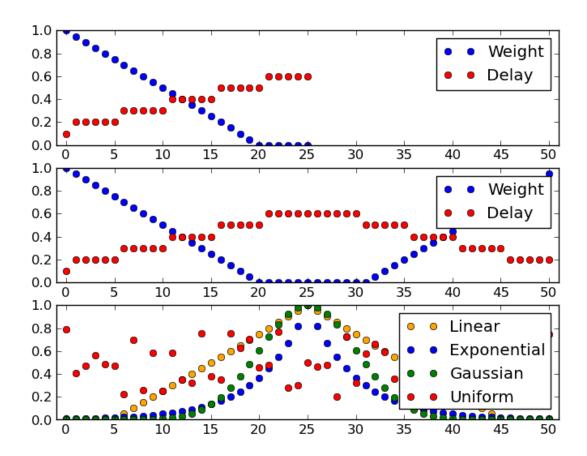


Figure 3.5: Distance-dependent and randomized weights and delays. See text for details.

By using the 'uniform' function for weights or delays, one can obtain randomized values for weights and delays, as shown by the red circles in the bottom panel of Fig. 3.5. Weights and delays can currently only be randomized with uniform distribution.

### 3.6 Periodic boundary conditions

Connections between layers with periodic boundary conditions are based on the following principles:

- Periodic boundary conditions are always applied in the pool layer. It is irrelevant whether the driver layer has periodic boundary conditions or not.
- By default, Topology does not accept masks that are wider than the pool layer when using periodic boundary conditions. Otherwise, one pool node could appear as multiple targets to the same driver node as the masks wraps several times around the layer. For layers with different extents in *x* and *y*-directions this means that the maximum layer size is determined by the smaller extension.

• Kernel, weight and delay functions always consider the shortest distance (displacement) between driver and pool node.

In most physical systems simulated using periodic boundary conditions, interactions between entities are short-range. Periodic boundary conditions are well-defined in such cases. In neuronal network models with long-range interactions, periodic boundary conditions may not make sense. In general, we recommend to use periodic boundary conditions only when connection masks are significantly smaller than the layers they are applied to.

Important changes upon NEST 2.0:

- Prior to NEST 2.0, oversized masks were not prohibited by default and you may
  have created networks with oversized masks by accident. You will now get a
  NEST error message in this case. You can force Topology to accept oversized
  masks by adding 'allow\_oversized\_mask': True to the connection dictionary. Be
  sure to inspect the results, e.g., using DumpLayerConnections.
- Distance- and displacement-dependent kernel, weight and delay functions did not handle periodic boundary conditions correctly prior to NEST 2.0.

#### 3.7 Prescribed number of connections

We have so far described how to connect layers by either connecting to all nodes inside the mask or by considering each pool node in turn and connecting it according to a given probability function. In both cases, the number of connections generated depends on mask and kernel.

Many neuron models in the literature, in contrast, prescribe a certain *fan in* (number of incoming connections) or *fan out* (number of outgoing connections) for each node. You can achieve this in Topology by prescribing the number of connections for each driver node. For convergent connections, where the target layer is the driver layer, you thus achieve a constant fan in, for divergent connections a constant fan out.

Connection generation now proceeds in a different way than before:

- 1. For each driver node, **ConnectLayers** randomly selects a node from the mask region in the pool layer, and creates a connection with the probability prescribed by the kernel. This is repeated until the requested number of connections has been created.
- 2. Thus, if all nodes in the mask shall be connected with equal probability, you should not specify any kernel.
- 3. If you specify a non-uniform kernel (e.g., Gaussian, linear, exponential), the connections will be distributed within the mask with the spatial profile given by the kernel.
- 4. If you prohibit multapses (cf Sec. 3.1.1) and prescribe a number of connections greater than the number of pool nodes in the mask, **ConnectLayers** may get stuck in an infinte loop and NEST will hang. Keep in mind that the number of nodes within the mask may vary considerably for free layers with randomly placed nodes.

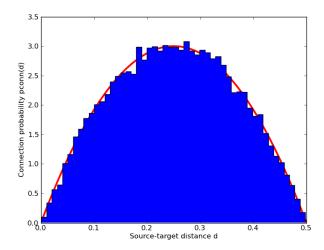


Figure 3.6: Distribution of distances between source and target for a network of 1000 randomly placed nodes, a fixed fan out of 50 connections and a connection probability decaying linerarly from 1 to 0 at d = 0.5. The red line is the expected distribution from Eq. 3.1.

The following code generates a network of 1000 randomly placed nodes and connects them with a fixed fan out of 50 outgoing connections per node distributed with a profile linearly decaying from unit probability to zero probability at distance 0.5. Multiple connections (multapses) between pairs of nodes are allowed, self-connections (autapses) prohibited. The probability of finding a connection at a certain distance is then given by the product of the probabilities for finding nodes at a certain distance with the kernel value for this distance. For the kernel and parameter values below we have

$$p_{\text{conn}}(d) = \frac{12}{\pi} \times 2\pi r \times (1 - 2r) = 24r(1 - 2r) \quad \text{for} \quad 0 \le r < \frac{1}{2}.$$
 (3.1)

The resulting distribution of distances between connected nodes is shown in Fig. 3.6.

Functions determining weight and delay as function of distance/displacement work in just the same way as before when the number of connections is prescribed.

### 3.8 Connecting composite layers

Connections between layers with composite elements are based on the following principles:

- All nodes within a composite element have the same coordinates, the coordinates of the element.
- All nodes within a composite element are treated equally. If, e.g., an element of the pool layer contains three nodes and connection probability is 1, then connections with all three nodes will be created. For probabilistic connection schemes, each of the three nodes will be considered individually.
- If only nodes of a given model within each element shall be considered as sources or targets then this can be achieved by adding a 'sources' or 'targets' entry to the connection dictionary, which speficies the model to connect.

This is exemplified by the following code, which connects pyramidal cells (pyr) to interneurons (in) with a circular mask and uniform probablity and interneurons to pyramidal cells with a rectangular mask unit probability.

```
nest. ResetKernel()
nest.CopyModel('iaf_neuron', 'pyr')
nest.CopyModel('iaf_neuron', 'in')
ldict = {'rows': 10, 'columns': 10, 'elements': ['pyr', 'in']}
cdict_p2i = {'connection_type': 'divergent',
             'mask': {'circular': {'radius': 0.5}},
             'kernel': 0.8,
             'sources': {'model': 'pyr'},
             'targets': {'model': 'in'}}
cdict_i2p = {'connection_type': 'divergent',
             'mask': {'rectangular': {'lower_left': [-0.2, -0.2], 'upper_right': [0]
             'sources': {'model': 'in'},
             'targets': {'model': 'pyr'}}
1 = tp.CreateLayer(ldict)
tp.ConnectLayers(1, 1, cdict_p2i)
tp.ConnectLayers(1, 1, cdict_i2p)
```

### 3.9 Synapse models and properties

By default, ConnectLayers creates connections using the default synapse model in NEST, static\_synapse. You can specify a different model by adding a 'synapse\_model' entry to the connection dictionary, as in this example:

You have to use synapse models if you want to set, e.g., the receptor type of connections or parameters for plastic synapse models. These can not be set in distance-dependent ways at present.

## Chapter 4

## **Inspecting Layers**

We strongly recommend that you inspect the layers created by Topology to be sure that node placement and connectivity indeed turned out as expected. In this chapter, we describe some functions that NEST and Topology provide to query and visualize networks, layers, and connectivity.

### 4.1 Query functions

The following table presents some query functions provided by NEST (nest.) and Topology (tp.). For detailed information about these functions, please see the online Python and SLI documentation.

nest. <b>PrintNetwork</b> ()	Print structure of network or subnet from NEST perspective.
nest. <b>FindConnections</b> ()	Retrieve outgoing connections for a given source; see also http://www.nest-initiative.org/index.php/Connection_Management.
nest. <b>GetNodes</b> ()	Applied to a layer, returns GIDs of the layer elements. For simple layers, these are the actual model neurons, for composite layers the top-level subnets.
nest. GetLeaves()	Applied to a layer, returns GIDs of all actual model neurons, ignoring subnets.
tp. <b>GetPosition</b> ()	Return the spatial locations of nodes.
tp. <b>GetLayer</b> ()	Return the layer to which nodes belong.
tp.GetElement()	Return the node(s) at the location(s) in the given grid-based layer(s).
$tp. {\bf GetTargetNodes}()$	Obtain targets of a list of sources in a given target layer.
$tp. {\bf GetTargetPositions} ()$	Obtain positions of targets of a list of sources in a given target layer.
$tp. \pmb{Find Nearest Element}()$	Return the node(s) closest to the location(s) in the given layer(s).
tp. Find Center Element ()	Return GID(s) of node closest to center of layer(s).

tp.Displacement()	Obtain vector of lateral displacement between nodes, taking periodic boundary conditions into account.
tp.Distance()	Obtain vector of lateral distances between nodes, taking periodic boundary conditions into account.
$tp. \\ Dump Layer Nodes ()$	Write layer element positions to file.
$tp. {\bf Dump Layer Connections} ()$	Write connectivity information to file. This function may be very useful to check that Topology created the correct connection structure.

#### 4.2 Visualization functions

Topology provides three functions to visualize networks:

PlotLayer() Plot nodes in a layer.
 PlotTargets() Plot all targets of a node in a given layer.
 PlotKernel() Add indication of mask and kernel to plot of layer. It does *not* wrap masks and kernels with respect to periodic boundary conditions. This function is usually called by PlotTargets.

The following code shows a practical example: A  $21 \times 21$  network which connects to itself with divegent Gaussian connections. The resulting graphics is shown in Fig. 4.1. All elements and the targets of the center neuron are shown, as well as mask and kernel.

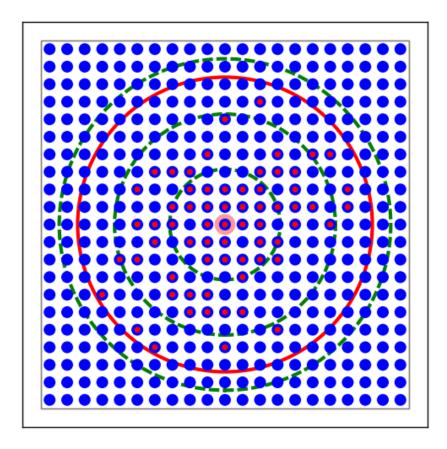


Figure 4.1: 21  $\times$  21 grid with divergent Gaussian projections onto itself. Blue circles mark layer elements, red circles connection targets of the center neuron (marked by large light-red circle). The large red circle is the mask, the dashed green lines mark  $\sigma$ ,  $2\sigma$  and  $3\sigma$  of the Gaussian kernel.

## **Chapter 5**

## **Changes from Topology 1.9 to 2.0**

This is a short summary of the most important changes in the NEST Topology Module from the 1.9-xxxx to the 2.0 version.

- ConnectLayer is now called ConnectLayers
- Several other functions changed names, and there are many new functions. Please see Ch. 4 for an overview.
- All nest.topology functions now require lists of GIDs as input, not "naked" GIDs
- There are a number of new functions in nest.topology, I tried to write good doc strings for them
- For grid based layers (ie those with /rows and /columns), we have changed the definition of "extent": Previously, nodes were placed on the edges of the extent, so if you had an extend of 2 (in x-direction) and 3 nodes, these had x-coordinates -1, 0, 1. The grid constant was extent/(num\_nodes 1).

Now, we define the grid constant as extent/num\_nodes, center the nodes about 0 and thus add a space of half a grid constant between the outermost nodes and the boundary of the extent. If you want three nodes at -1,0,1 you thus have to set the extent to 3, i.e., stretching from -1.5 to 1.5.

The main reason for this change was that topology always added this padding silently when you used periodic boundary conditions (otherwise, neurons are the left and right edge would have been in identical locations, not what one wants).

- The semantics of the anchor entry for kernel functions has changed: the anchor now specifies the center of the probability distribution relative to the driver node. This is consistent with the semantics for free masks, see Sec. 3.3 and 3.4.
- Functions computing connection probabilities, weights and delays as functions of distance between source and target nodes now handle periodic boudary conditions correctly.
- Masks with a diameter larger than the diameter of the layer they are applied to are now prohibited by default. This avoids multiple connections when masks overwrap.

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# **List of Figures**

2.1	Simple grid-based layer
	Layer with non-standard extent
2.3	Layers with different centers
2.4	Layer construction example
2.5	Layer with freely spaced elements
2.6	Periodic boundary conditions
3.1	Minimal connection example
	Masks for all layer types
3.3	Grid masks
3.4	Kernel functions
3.5	Distance-dependent and randomized weights and delays
3.6	Distribution of connection distances
4.1	Example of layer visualization

## **List of Tables**

## Index

grid mask, 19

2D Gaussian kernel, 22	grid spacing, 5 grid-based layer, 4
anchor, 18, 19	8224. 24624. 24.) 22, 2
autapse, 16	hang, 27
center, 4, 6	index, 5
circular, 18	infinite loop, 27
columns, 4	manate 100p/ <b>2</b> /
composite element, 11, 27	kernel, 16, 20
connection, 15	
connection dictionary, 15	layer, 4
	mannin - 17
connection type, 15	mapping, 17
ConnectLayers, 15, 16	mask, 16
constant kernel, 21	model, 28
convergent connection, 15	multapse, 16, 27
coordinate mapping rules, 17	oversized mask, 26
coordinate system, 5, 17	oversized mask, 20
coordinates, 27	periodic boundary conditions, 9, 17, 24, 26
cut-off kernel, 22	pool, 16
dolayo stonyigo 24	position, 5, 9
delays, stepwise, 24	prescribed number of connections, 26
displacement, 16	probabilistic connection rules, 20
distance, 16	production rates, 20
distance-dependent weight, 22	randomized delay, 22
distance-dependent delay, 22	randomized weight, 22
divergent connection, 15	rectangular, 18
doughnut, 18	row, 4
driver, 16	,
egde_wrap, 9	source, 15
element, 4	source layer, 15
•	subnet, 10
excentric kernel, 21	synapse model, 28
extent, 4	15
fan in, 26	target, 15
fan out, 26	target layer, 15
free layer, 7	topology_layer_free, 10
free probabilistic connections, 20	topology_layer_grid, 10
r r	
Gaussian kernel, 21	
' 1 1 10	