lista 6

1- $(2x-1)\partial x + (3y+7)\partial y = 0$

$M(x,y) = 2x-1$
$N(x,y) = 3y+7$

$\frac{\partial M}{\partial y} = 0 \quad \frac{\partial N}{\partial x} = 0$

$\exists U(x,y)$

$\frac{\partial U}{\partial x} = M(x,y) \; ; \; \frac{\partial U}{\partial y} = N(x,y)$

Integrando parcialmente

$\int \frac{\partial U}{\partial x} \partial x = \int (2x-1)\partial x$

$U(x,y) = 2\int x\,\partial x - \int \partial x$

$U(x,y) = x^2 - x + f(y)$

Sustituimos

$\frac{\partial}{\partial x}(x^2 - x + f(y)) = 3y+7$

$f'(y) = 3y+7$

$\int f'(y) = \int (3y+7)\partial y$

$f(y) = \frac{3y^2}{2} + 7y$

$U(x,y) = x^2 - x + \frac{3y^2}{2} + 7y \qquad \therefore \partial U(x,y) = 0$

Integramos

$\int \partial(x^2 - x + \frac{3y^2}{2} + 7y) = \int 0$

$x^2 - x + \frac{3y^2}{2} + 7y = C$

3- $(5x+4y)\partial x + (4x-8y^3)\partial y = 0$

$M(x,y) = 5x+4y$
$N(x,y) = 4x+8y^3$

$\frac{\partial M}{\partial y} = 4 \quad \frac{\partial N}{\partial x} = 4$

$\exists U(x,y)$

$\frac{\partial U}{\partial x} = M(x,y) \; ; \; \frac{\partial U}{\partial y} = N(x,y)$

Integrando parcialmente

$\int \frac{\partial U}{\partial x} \partial x = \int (5x+4y)\partial x$

$U(x,y) = 5\int x\,\partial x + 4y\int \partial x$

$U(x,y) = 5\frac{x^2}{2} + 4xy + f(y)$

Sust

$\frac{\partial}{\partial y}\left(\frac{5x^2}{2} + 4xy + f(y)\right) = 4x + 8y^3$

$4x + f'(y) = 4x + 8y^3$

$f'(y) = 8y^3$

Integrando

$\int f(y) = \int (8y^3)\partial y$

$f(y) = 2y^4$ $\therefore \partial U(x,y)$

Integrando

$\int \partial\left(\frac{5x^2}{2} + 4xy + 2y^4\right) = \int 0$

$\frac{5x^2}{2} + 4xy + 2y^4 = C$

5.- $(2y^2x - 3)\partial x + (2yx^2 + 4)\partial y = 0$

$M(x,y) = 2y^2x - 3$

$N(x,y) = 2yx^2 + 4$

$\frac{\partial M}{\partial y} = 4yx \; ; \; \frac{\partial N}{\partial x} = 4yx$

$\exists U(x,y)$

$\frac{\partial U}{\partial x} = M(x,y) \; ; \; \frac{\partial U}{\partial y} = N(x,y)$

Integrando parcialmente

$\int \frac{\partial U}{\partial x}\partial x = \int (2y^2x - 3)\partial x$

$U(x,y) = 2y^2\int x\,\partial x - 3\int \partial x$

$U(x,y) = y^2x^2 - 3x + f(y)$

Sust

$\frac{\partial}{\partial y}\left(y^2x^2 - 3x + f(y)\right) = 2yx^2 + 4$

$\cancel{2yx^2} - 0 + f'(y) = \cancel{2yx^2} + 4$

$f'(y) = 4$

Integrando

$\int f(y)\partial y = 4\int \partial x$

$f(y) = 4y$ $\therefore \partial U(x,y)$

Integrando

$\int \partial(y^2x^2 - 3x + 4y) = 0 \Rightarrow y^2x^2 - 3x + 4y = C$

7- $(x+y)(x-y)dx + x(x-2y)dy = 0$

$x^2 - xy + xy - y^2 + (x^2 - 2xy)$

$(x^2 - y^2)dx + (x^2 - 2xy)dy = 0$ No es exacta

$2y \qquad 2x - 2y$

9- $(y^3 - y^2 \operatorname{sen} x - x)dx + (3xy^2 + 2y\cos x)dy = 0$

$M(x,y) = y^3 - y^2 \operatorname{sen} x - x$

$N(x,y) = 3xy^2 + 2y\cos x$

$\frac{\partial M}{\partial y} = 3y^2 - 2y\operatorname{sen} x \quad \frac{\partial N}{\partial x} = 3y^2 - 2y\operatorname{sen} x$

$\exists U(x,y)$

$\frac{\partial U}{\partial x} = M(x,y) \quad \frac{\partial U}{\partial y} = N(x,y)$

Integrando parcialmente

$\int \frac{\partial M}{\partial x} \partial x = \int y^3 - y^2 \operatorname{sen} x - x$

$U(x,y) = y^3 \int \partial x - y^2 \int \operatorname{sen} x - \int x \partial x$

$U(x,y) = xy^3 + y^2 \cos x - \frac{x^2}{2} + f(y)$

sust

$\frac{\partial}{\partial y}(xy^3 + y^2\cos x - \frac{x^2}{2} + f(y)) = 3xy^2 + 2y\cos x$

$3xy^2 + 2y\cos x - 0 + f'(y) = 3xy^2 + 2y\cos x$

$f'(y) = 0$

Integrando

$\int f'(y)\partial x = \int 0$

$f(y) = C$ $\therefore \partial U(x,y)$

forma integrable

$d(3xy^2 + 2y\cos x - \frac{x^2}{2} + C) = 0$

Integran

$\int d(3xy^2 + 2y\cos x - \frac{x^2}{2} + C = \int 0$

$(3xy^2 + 2y\cos x - \frac{x^2}{2} + C_1 = C_2$

$3xy^2 + 2y\cos x - \frac{x^2}{2} = C$

37.500

10,000

¿?

11- $(y\ln y - e^{-xy})dy + (\frac{1}{y} + x\ln y)dy = 0$

$M(x,y) = y\ln y - e^{-xy}$
$N(x,y) = \frac{1}{y} + x\ln y$

$\frac{\partial N}{\partial y} = \ln y + xe^{-xy}$ $\frac{\partial N}{\partial x} = \ln y$ No es exacta

No es exacta

13- $x\frac{dy}{dx} = 2xe^x - y + 6x^2$

$\frac{dy}{dx} = \frac{2xe^x}{x} - \frac{y}{x} + \frac{6x^2}{x} = 2e^x - \frac{y}{x} + 6x$

$dy = (2e^x - \frac{y}{x} + 6x)dx$

$(2e^x - \frac{y}{x} + 6x)dx - dy = 0$

$M(x,y) = (2e^x - \frac{y}{x} + 6x$
$N(x,y) = -1$

$\frac{\partial N}{\partial x} = \frac{1}{x}$ $\frac{\partial N}{\partial x} = 0$

No es exacta !

15- $(1 - \frac{3}{x} + y)dx + (1 - \frac{3}{y} + x)dy = 0$

$M(x,y) = 1 - \frac{3}{x} + y$

$N(x,y) = 1 - \frac{3}{y} + x$

$\frac{\partial N}{\partial y} = 1$ $\frac{\partial N}{\partial x} = 1$ Es exacta

$\exists U(x,y)$
$\frac{\partial U}{\partial x} = M(x,y)$; $\frac{\partial U}{\partial y} = N(x,y)$

Integrando parcialmente

$\int \frac{\partial U}{\partial x}\partial x = \int (1 - \frac{3}{x} + y)\partial x$

$U(x,y) = 1\int \partial x - \int \frac{3}{x}\partial x + y\int \partial x$

$U(x,y) = x - 3\ln|x| + yx + f(y)$

Sust

$\frac{\partial}{\partial x}(x - 3\ln|x| + yx + f(x)) = 1 - \frac{3}{y} + x$

$0 - 0 + \cancel{x} + f'(y) = 1 - \frac{3}{y} + \cancel{x}$

$f'(y) = 1 - \frac{3}{y}$

Integrando

$\int \frac{\partial f(y)}{\partial y} \partial x = \int (1 - \frac{3}{y}) \partial x$

$f(y) = 1 \int \partial x - 3 \int \frac{\partial x}{y}$

$f(y) = y - 3\ln|y|$

forma integrable

$\partial(x - 3\ln|x| + xy + y - 3\ln|y|) = 0$

Integrando

$\int \partial(x - 3\ln|x| + xy + y - 3\ln|y| = \int 0$

$x - 3\ln|x| + xy + y - 3\ln|y| = C$

17- $\left(x^2y^3 - \frac{1}{1+9x^2}\right)\frac{\partial x}{\partial y} + x^3y^2 = 0$

$\left(x^2y^3 - \frac{1}{1+9x^2}\right)\frac{\partial x}{\partial y} = -x^3y^2$

$\left(x^2y^3 - \frac{1}{1+9x^2}\right)\partial x - (x^3y^2)\partial x$

$\left(x^2y^3 - \frac{1}{1+9x^2}\right)\partial x + (x^3y^2)\partial y = 0$

$M(x,y) = x^2y^3 - \frac{1}{1+9x^2}$

$N(x,y) = x^3y^2$

$\frac{\partial M}{\partial y} = 3x^2y^2 \qquad \frac{\partial N}{\partial x} = 3x^2y^2$ son exacta

$\exists\, U(x,y)$

$\frac{\partial U}{\partial x} = M(x,y) \qquad \frac{\partial U}{\partial y} = N(x,y)$

Integrando parcialmente

$\int \frac{\partial U}{\partial x}\partial x = \int\left(x^2y^3 - \frac{1}{1+9x^2}\right)\partial x$

$U(x,y) = y^3\int x^2 \partial x - \int \frac{1}{1+9x^2}\partial x$

$U(x,y) = y^3 \frac{x^3}{3} - \frac{1}{3}\tan^{-1}(3x) + f(y)$

Sust

$\frac{\partial}{\partial x}\left(y^3 \frac{x^3}{3} - \frac{1}{3}\tan^{-1}(3x) + f(x)\right) = x^3 y^2$

$y^2 x^3 \quad - \quad 0 \quad + f'(y) = x^3 y^2$

$f'(y) = 0$

Integrar

$\int \frac{\partial(f(y))}{\partial y} dy = \int 0$

$f(y) = C \qquad \therefore dU(x,y) = 0$

forma integrable

$d\left(y^3 \frac{x^3}{3} - \frac{1}{3}\tan^{-1}(3x) + C\right) = 0$

Integrando

$\int d\left(y^3 \frac{x^3}{3} - \frac{1}{3}\tan^{-1}(3x) + C\right) = \int 0$

$\left(y^3 \frac{x^3}{3} - \frac{1}{3}\tan^{-1}(3x) + C\right) = C$

$y^3 \frac{x^3}{3} - \frac{1}{3}\tan^{-1}(3x) = C$

14- $(\tan x - \operatorname{sen} x \operatorname{sen} y)dx + (\cos x \cos y)dx = 0$

$M(x,y) = \tan x - \operatorname{sen} x \operatorname{sen} y$

$N(x,y) = \cos x \cos y$

$\frac{\partial N}{\partial y} = -\cos y \operatorname{sen} x \qquad \frac{\partial N}{\partial x} = -\operatorname{sen} x \cos y \qquad$ Son exd.

$\exists U(x,y)$

$\frac{\partial U}{\partial x} = M(x,y) \; ; \; \frac{\partial U}{\partial y} = N(x,y)$

Integrando parcialmente

$\int \frac{\partial U}{\partial x} \partial x = \int (\tan x - \operatorname{sen} x \operatorname{sen} y) dx$

$U(x,y) = \int \tan x - \operatorname{sen} y \int \operatorname{sen} x$

$U(x,y) = -\ln|\cos x| + \cos x \operatorname{sen} y + f(y)$

Sust.

$$\frac{\partial}{\partial x}\left(-\ln|\cos x| + \cos x\, \text{sen}\, y + f(y)\right) = \cos x \cos y$$

$$0 + \cos x \cos y + f'(y) = \cos x \cos y$$

$$f'(y) = 0$$

Integrando parcialmente

$$\int f(y) = \int 0$$

$$f(y) = C \quad \therefore \partial U(x,y) = 0$$

forma integrable

$$\partial(-\ln|\cos x| + \cos x\, \text{sen}\, y + C) = 0$$

Integrando

$$\int \partial(-\ln|\cos x| + \cos x\, \text{sen}\, y) = \int 0$$

$$(-\ln|\cos x + \cos x\, \text{sen}\, y = C$$

21.- $(1 - 2x^2 - 2y)\frac{\partial y}{\partial x} = 4x^3 + 4xy$

$$(1-2x^2-2y)\,\partial y = (4x^3 + 4xy)\,\partial x$$

$$(4x^3 + 4xy)\partial x - (1-2x^2-2y)\partial y = 0$$

$$M(x,y) = 4x^3 + 4xy$$

$$N(x,y) = -1 + 2x^2 + 2y$$

$$\frac{\partial M}{\partial y} = 4x \qquad \frac{\partial N}{\partial x} = 4x \quad \text{son exactas}$$

$$\exists\, U(x,y) \quad \therefore \quad \partial U(x,y) = 0$$

$$\frac{\partial U}{\partial x} = M(x,y) \qquad \frac{\partial U}{\partial y} = N(x,y)$$

Integrando

$$\int \frac{\partial U}{\partial x}\partial x = \int (4x^3 + 4xy)\partial x$$

$$U(x,y) = \int 4x^3 \partial x + 4y \int x\,dx$$

$$U(x,y) = x^4 + 2x^2 y + f(y)$$

Sust.

$$\frac{\partial}{\partial y}\left(x^4 + 2x^2y + f(y)\right) = -1 + 2x^2 + 2y$$

$$2x^2 + f'(y) = -1 + 2x^2 + 2y$$

$$f'(y) = -1 + 2y$$

Integrando

$$\int \frac{\partial f(y)}{\partial y}\partial y = \int (-1 + 2y)\partial y$$

$$f(y) = -1\int \partial y + 2\int y\,dy$$

$$f(y) = -y + y^2$$

forma integrable

$$\partial(x^4 + 2x^2 y - y + y^2) = 0$$

Integrando

$$\int \partial(x^4 + 2x^2 y - y + y^2) = \int 0$$

$$x^4 + 2x^2 y - y + y^2 = C$$

23)

$$(4x^3y - 15x^2 - y)\partial x + (x^4 + 3y^2 - x)\partial y = 0$$

$M(x,y) = 4x^3y - 15x^2 - y$
$N(x,y) = x^4 + 3y^2 - x$

$$\frac{\partial M}{\partial y} = 4x^3 - 1 \quad \frac{\partial N}{\partial x} = 4x^3 - 1 \text{ son exacta}$$

$\partial O(x,y) \Rightarrow \partial O(x,y) = 0$

$$\frac{\partial O}{\partial x} = M(x,y) \ ; \ \frac{\partial O}{\partial N} = N(x,y)$$

Integrando parcialmente

$$\int \frac{\partial O}{\partial x}\partial x = \int (4x^3y - 15x^2 - y)\partial x$$

$$O(x,y) = 4y\int x^3 - 15\int x^2 - y\int \partial x$$

$$O(x,y) = x^4y - 5x^2 - xy + f(y)$$

Sust

$$\frac{\partial}{\partial y}(x^4y - 5x^2 - xy + f(y)) = x^4 + 3y^2 - x$$

$$x^4 - 0 - x + f'(y) = x^4 + 3y^2 - x$$

$$f'(y) = 3y^2$$

Integrando

$$\int \frac{\partial(f(y))}{\partial y}dy = 3\int y^2 \partial x$$

$$f(y) = y^3$$

forma integrable

$$\partial(x^4y - 5x^2 + xy + y^3) = 0$$

Integrando

$$\int \partial(x^4y - 5x^2 + xy + y^3) = \int 0$$

$$(x^4y - 5x^2 + xy + y^3 = C$$

25.- $(x+y)^2 \partial x + (2xy + x^2 - 1)\partial y = 0$; $y(1) = 1$

$(x^2 + 2xy + y^2)\partial x + (2xy + x^2 - 1)\partial y = 0$

$M(x,y) = x^2 + 2xy + y^2$

$N(x,y) = 2xy + x^2 - 1$

$\frac{\partial M}{\partial y} = 2x + 2y \quad \frac{\partial N}{\partial x} = 2x + 2x$ Son exacta

$\exists\, U(x,y) : \partial U(x,y) = 0$

$\frac{\partial U}{\partial x} = M(x,y) \; ; \; \frac{\partial U}{\partial y} = N(x,y)$

Integrando parcialmente

$\int \frac{\partial U}{\partial x} \partial x = \int (x^2 + 2xy + y^2)\, \partial x$

$U(x,y) = \int x^2 \partial x + 2y \int x \partial x + y^2 \int \partial x$

$U(x,y) = \frac{x^3}{3} + x^2 y + x y^2 + f(y)$

sust

$\frac{\partial}{\partial y}\left(\frac{x^3}{3} + x^2 y + x y^2 + f(y) \right) = 2xy + x^2 - 1$

$x^2 + 2xy + f'(y) = 2xy + x^2 - 1$

$f'(y) = -1$

Integrando

$\int \frac{\partial f(y)}{\partial y} \partial y = -\int \partial y$

$f(y) = -y$

forma integrable

$\partial\left(\frac{x^3}{3} + x^2 y + x y^2 - y \right) = 0$

Integrando

$\int \partial\left(\frac{x^3}{3} + x^2 y + x y^2 - y \right) = \int 0$

$\frac{x^3}{3} + x^2 y + x y^2 - y = C$

sol general

$\frac{1}{3} + 1 + 1 - 1 = C$

$\frac{4}{3} = C$

sol particular

27.- $(4y+2x-5)dx+(6y+4x-1)dy=0$, $y(-1)=2$

$M(x,y)=4y+2x-5$

$N(x,y)=6y+4x-1$

$\frac{\partial M}{\partial y}=4 \quad \frac{\partial N}{\partial x}=4$ son exacta

$\exists\, U(x,y) : dU(x,y)=0$

$\frac{\partial U}{\partial x}=M(x,y) : \frac{\partial U}{\partial y}=N(x,y)$

Integrando parcialmente

$\int \frac{\partial U}{\partial x}\partial x=\int(4y+2x-5)\partial x$

$U(x,y)=4y\int\partial x+2\int x\partial x-5\int\partial x$

$U(x,y)=4xy+x^2-5x+f(y)$

Sust

$\frac{\partial}{\partial y}(4xy+x^2-5x+f(y))=6y+4x-1$

$4x+0-0+f'(y)=6y+4x-1$

$f'(y)=6y-1$

Integrando

$\int\frac{\partial f}{\partial y}\partial y=6\int y\partial y-\int\partial y$

$f(y)=3y^2-y$

Forma integrable

$d(4xy+x^2-5x+3y^2-y)=0$

Integrando

$\int d(4xy+x^2-5x+3y^2-y)=\int 0$

$4xy+x^2-5x+3y^2-y=C$

sol general

$4(-1)(2)+(-1)^2-5(-1)+3(2)^2-2$

$-8+1+5+12-2=C$

$8=C$

sol particular

29- $(y^2\cos x - 3x^2y - 2x)dx + (2y\,\text{sen}\,x - x^3 + \ln y)dy = 0$; $y(0) = e$

$M(x,y) = y^2\cos x - 3x^2y - 2x$
$N(x,y) = 2y\,\text{sen}\,x - x^3 + \ln y$

$\dfrac{\partial M}{\partial y} = 2y\cos x - 3x^2$ $\quad \dfrac{\partial N}{\partial x} = 2y\cos x - 3x^2$ son exactas

$\exists\, U(x,y) : dU(x,y) = 0$

$\dfrac{\partial U}{\partial x} = M(x,y)$; $\dfrac{\partial U}{\partial y} = N(x,y)$

Integrando parcialmente

$\int \dfrac{\partial U}{\partial x}\partial x = \int (y^2\cos x - 3x^2y - 2x)\partial x$

$U(x,y) = y^2\int \cos x - 3y\int x^2 - 2\int x$

$U(x,y) = y^2\,\text{sen}\,x - x^3y - 2x + f(y)$

sust

$\dfrac{\partial}{\partial y}(y^2\,\text{sen}\,x - x^3y - 2x + f(y)) = 2y\,\text{sen}\,x - x^3 + \ln y$

$2y\,\text{sen}\,x - x^3 + f'(y) = 2y\,\text{sen}\,x - x^3 + \ln y$

$f'(y) = \ln y$

Integrando

$\int \dfrac{\partial(f(y))}{\partial y}\partial y = \int \ln y\,\partial y$

$f(y) = y\ln y - y$

forma integrable

$d(y^2\,\text{sen}\,x - x^3y - 2x + y\ln y - y) = 0$

Integrando

$\int d(y^2\,\text{sen}\,x - x^3y - 2x + y\ln y - y) = \int 0$

$y^2\,\text{sen}\,x - x^3y - 2x + y\ln y - y = C$

sol general

$e^2\,\text{sen}(0) - 0e - 2 0 + e\ln e - e = c$

$C = 0$

sol particular

Determine el valor de K

31- $(y^3 + Kxy^4 - 2x)\partial x + (3xy^2 + 20x^2y^3)\partial y = 0$

$M(x,y) = y^3 + Kxy^4 - 2x$
$N(x,y) = 3xy^2 + 20x^2y^3$

$\frac{\partial M}{\partial y} = 3y^2 + 4Kxy^3 \qquad \frac{\partial N}{\partial x} = 3y^2 + 40xy^3$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

2ost

$3y^2 + 4Kxy^3 = 3y^2 + 40xy^3$
$4Kxy^3 = 40xy^3$
$4K = 40$
$K = \frac{40}{4}$
$K = 10$

33- $(2xy^2 + ye^{11})\partial x + (2x^2y + Ke^x - 1)\partial y = 0$

$M(x,y) = 2xy^2 + ye^{11}$
$N(x,y) = 2x^2y + Ke^x - 1$

$\frac{\partial M}{\partial y} = 4xy + e^{11} \qquad \frac{\partial N}{\partial x} = 4xy + Ke^x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$4xy + e^{11} = 4xy + Ke^x$
$e^{11} = Ke^x$
$K = \frac{e^{11}}{e^x}$
$K = 1$

35- Deduzca una función $M(x, y)$ tal que la siguiente ecuación sea exacta

$$M(x,y)\,\partial x + \left(xe^{xy} + 2xy + \frac{1}{x}\right)\partial y = 0$$

$$N(x,y) = xe^{xy} + 2xy + \frac{1}{x}$$

$$\frac{\partial N}{\partial x} = e^{xy} + xye^{xy} + 2y + \frac{1}{x^2}$$

Si

$$\frac{\partial M}{\partial y} = e^{xy} + xye^{xy} + 2y + \frac{1}{x^2}$$

Integrando ambos con respecto a y

$$\int \frac{\partial M}{\partial y}\partial x = \int \left(e^{xy} + xye^{xy} + 2y + \frac{1}{x^2}\right)\partial y$$

$$M(x,y) = \underbrace{\int e^{xy}\partial x}_{\text{I}} + x\underbrace{\int ye^{xy}}_{\text{II}} + \int 2x + \frac{1}{x^2}\int \partial y$$

$$\text{I} = \int e^{xy}\partial x = \frac{1}{x}\int e^{xy}\partial y = \frac{1}{x}\int e^{u}\partial u = \frac{e^{xy}}{x}$$

$$\text{II} = x\int ye^{xy}\partial y = y\,e^{xy} - \dots\,\partial x = \frac{ye^{xy}}{x} - \frac{e^{xy}}{x^2}$$

$$M(x,y) = \cancel{\frac{e^{xy}}{x}} + x\left(\frac{xe^{xy}}{x} - \cancel{\frac{e^{xy}}{x^2}}\right) + y^2 - \frac{y}{x^2}$$

$$= \cancel{\frac{e^{xy}}{x}} + xe^{xy} - \cancel{\frac{e^{xy}}{x}} + y^2 - \frac{y}{x^2}$$

$$= ye^{xy} + y^2 - \frac{y}{x^2} + f(x)$$

37.- $6xy\,\partial x + (4y + 9x^2)\,\partial y = 0$; $\mu(x,y) = y^2$

$M(x,y) = 6xy$

$N(x,y) = 4y + 9x^2$

$\frac{\partial M}{\partial y} = 6x \quad \frac{\partial N}{\partial x} = 18x$ No exacta

$\mu(x,y) = y^2$

Multiplicando

$y^2(6xy\,\partial x + (4y + 9x^2)\,\partial y)$

$6xy^3\,\partial x + (4y^3 + 9x^2y^2)\,\partial y = 0$

$M_1(x,y) = 6xy^3$

$N_1(x,y) = 4y^3 + 9x^2y^2$

$\frac{\partial M_1}{\partial y} = 18xy^2 \quad \frac{\partial N_1}{\partial x} = 18xy^2$ exacta

$\exists U(x,y) = \partial U(x,y) = 0$

$\frac{\partial U}{\partial x} = M(x,y) \quad \frac{\partial U}{\partial y} = N(x,y)$

Integrando parcialmente

$\int \frac{\partial U}{\partial x}\partial x = \int 6xy^3\,\partial x$

$U(x,y) = 6y^3 \int x\,\partial x$

$U(x,y) = 3x^2y^3 + f(y)$

sust

$\frac{\partial}{\partial y}(3x^2y^3 + f'(y)) = 4y^3 + 9x^2y^2$

$9x^2y^2 + f'(y) = 4y^3 + 9x^2y^2$

$f'(y) = 4y^3$

Integrando

$\int \frac{\partial f(y)}{\partial y}\partial y = 4\int y^3\,\partial x$

$f(y) = y^4$

forma integrable

$\partial(3x^2y^3 + y^4) = 0$

Integrando

$\int \partial(3x^2y^3 + y^4) = \int 0 \Rightarrow 3x^2y^3 + y^4 = C$

Escaneado con CamScanner

39- $(-xy\,\text{sen}\,x + 2y\cos x)dx + 2x\cos x\,dy = 0$; $M(x,y) = xy$

$M(x,y) = -xy\,\text{sen}\,x + 2y\cos x$
$N(x,y) = 2x\cos x$

$\frac{\partial M}{\partial x} = -x\,\text{sen}\,x + 2\cos x$ $\frac{\partial N}{\partial x} = -2x\,\text{sen}\,x + 2\cos x$ No es exacta

Si $M(x,y) = xy$
Multiplicamos

$xy((-xy\,\text{sen}\,x + 2y\cos x)dx + 2x\cos x\,dy)$

$(-x^2y^2\,\text{sen}\,x + 2xy^2\cos x)dx + 2x^2y\cos x\,dy = 0$
$M_1(x,y) = -x^2y^2\,\text{sen}\,x + 2xy^2\cos x$
$N_1(x,y) = 2x^2y\cos dy$

$\frac{\partial M}{\partial x} = -2x^2y\,\text{sen}\,x + 4xy\cos x$ $\frac{\partial N}{\partial x} = 4xy\cos x - 2x^2y\,\text{sen}\,x$ Son exactas

$\exists\, U(x,y) : dU(x,y) = 0$

$\frac{\partial U}{\partial x} = M(x,y)$; $\frac{\partial U}{\partial y} = N(x,y)$

Integramos parcialmente

$\int \frac{\partial U}{\partial x}\partial x = \int(-x^2y^2\,\text{sen}\,x + 2xy^2\cos x)$

$U(x,y) = -y^2\int x^2\,\text{sen}\,x + 2y\int x\cos x$
$U(x,y) = -y^2(-x^2\cos x + 2x\,\text{sen}\,x + 2\cos x) - x\cos x + \text{sen}\,x + f(y)$
$U(x,y) = x^2y^2\cos x + 2xy^2\,\text{sen}\,x + 2y\cos x - x\cos y + \text{sen}\,x + f(y)$

Sol.
$\frac{\partial}{\partial y}(x^2y^2\cos x + 2xy^2\,\text{sen}\,x + 2y\cos x - x\cos x + \text{sen}\,x + f(y)) = 2x^2y\cos x$

$\cancel{2x^2y\cos x} + 4xy\,\text{sen}\,x + 2\cos y - 0 + 0 + f'(y) = 2x^2y\cos x$
$+4xy\,\text{sen}\,x + 2\cos y + f'(y) = 0$
$\therefore f'(y) = -4xy\,\text{sen}\,x + 2\cos y$
Integrando
$\int \frac{\partial f(y)}{\partial y}\partial y = -4x\,\text{sen}\,x\int y + 2\int \cos y$
$f(y) = -2xy\,\text{sen}\,x + 2\,\text{sen}\,y$

forma integrable
$d(x^2y^2\cos x + 2xy^2\,\text{sen}\,x + 2y\cos x - x\cos x + \text{sen}\,y - 2xy\,\text{sen}\,x - 2\,\text{sen}\,y) = 0$
Integrando
$\int d(x^2y^2\cos x + 2xy^2\,\text{sen}\,x + 2y\cos x - x\cos y + \text{sen}\,y - 2xy\,\text{sen}\,x - 2\,\text{sen}\,y) = \int 0$

$x^2y^2\cos x + 2xy^2\,\text{sen}\,x + 2y\cos x - x\cos x + \text{sen}\,y - 2xy\,\text{sen}\,x - 2\,\text{sen}\,y = C$

41.- $(2y^2+3x)\partial x + 2xy\,dy = 0 \quad ; \ \mu(x,y) = x$

$M(x,y) = 2y^2+3x$

$N(x,y) = 2xy$

$\frac{\partial M}{\partial y} = 4y \quad \frac{\partial N}{\partial x} = 2y$ No son exactos

Si $\mu(x,y) = x$

Multiplicamos

$x((2y^2+3x)\partial x + 2xy\,dy)$

$(2xy^2+3x)\partial x + 2x^2y\,dy = 0$

$M_1(x,y) = 2xy^2+3x$

$N_1(x,y) = 2x^2y$

$\frac{\partial M_1}{\partial y} = 4xy \quad \frac{\partial N_1}{\partial x} = 4xy$ ∴ Son exactos

$\exists\, O(x,y) : \partial O(x,y) = 0$

$\frac{\partial O}{\partial x} = M_1(x,y) \quad ; \quad \frac{\partial O}{\partial y} = N_1(x,y)$

Integrando parcialmente

$\int \frac{\partial O}{\partial x}\partial x = \int (2xy^2+3x)\partial x$

$O(x,y) = 2y^2\int x\,\partial x + 3\int x\,\partial x$

$O(x,y) = x^2y^2 + \frac{3x}{2} + f(y)$

Sust

$\frac{\partial}{\partial y}\left(x^2y^2 + \frac{3x}{2} + f(y)\right) = 2x^2y$

$2x^2y + 0 + f'(y) = 2x^2y$

$f'(y) = 0$

Integrando

$f(y) = C$

forma integrable

$d\left(x^2y^2 + \frac{3x}{2} + C\right) = 0$

Integrando

$\int d\left(x^2y^2 + \frac{3x}{2} + C\right) = \int 0 \Rightarrow x^2y^2 + \frac{3x}{2} + C_1 = C_2$

$x^2y^2 + \frac{3x}{2} = C$