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Lista de Ejercicios 4

23.- $\frac{dy}{dx} = (x+y+1)^2$

$t = x+y+1; \; t = t(x)$

$\frac{dt}{dx} = 1 + \frac{dy}{dx} \rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$ sustituyendo $\frac{dt}{dx} - 1 = (t)^2$

$\frac{dt}{dx} = t^2 + 1 \rightarrow \frac{dt}{t^2+1} = dx \rightarrow dx - \frac{dt}{t^2+1} = 0$ Integrando $\underbrace{\int dx}_{1} - \underbrace{\int \frac{dt}{t^2+1}}_{1} = \int 0$

$\rightarrow x - \arctan(t) = c$ pero $t = x+y+1$ $\therefore$ $x - \arctan(x+y+1) = c$

24.- $\frac{dy}{dx} = \frac{1-x-y}{x+y} \rightarrow -\frac{dy}{dx} = \frac{x+y-1}{x+y}$ $\quad t = x+y \quad \frac{dt}{dx} = 1 + \frac{dy}{dx} \quad \frac{dy}{dx} = \frac{dt}{dx} - 1$

$-\left(\frac{dt}{dx} - 1\right) = \frac{t-1}{t} \rightarrow -\frac{dt}{dx} + 1 = 1 - \frac{1}{t} \rightarrow \frac{dt}{dx} = \frac{1}{t} \rightarrow t\,dt = dx$

$dx - t\,dt = 0 \rightarrow$ Integrando $\rightarrow \int dx - \int t\,dt = \int 0 \rightarrow x - \frac{t^2}{2} = c$

pero $t = x+y$ $\therefore$ $x - \frac{x+y}{2} = c$

25.- $\frac{dy}{dx} = \tan^2(x+y)$ $\quad t = x+y \quad \frac{dt}{dx} = 1 + \frac{dy}{dx} \rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

$\frac{dt}{dx} - 1 = \tan^2(t) \rightarrow \frac{dt}{dx} = \tan^2(t) + 1 \rightarrow \frac{dt}{\tan^2(t)+1} = dx$

$dx - \frac{dt}{\tan^2(t)+1} = 0$ Integrando $\rightarrow \underbrace{\int dx}_{1} - \underbrace{\int \frac{dt}{\tan^2(t)+1}}_{2} = \int 0$

1 $\int dx = x$ 2 $\int \frac{dt}{\tan^2(t)+1} = \int \frac{dt}{\sec^2(t)} = \int \cos^2(t)$

$= \int \frac{1}{2}dt - \frac{1}{2}\int \cos 2t\,dt = \frac{1}{2}t - \frac{1}{4}\operatorname{sen} t = c$

Sustituyendo

$x - \frac{1}{2}t - \frac{1}{4}\operatorname{sen}(t) = c$ pero $t = x+y$ $\quad x - \frac{x+y}{2} - \frac{1}{4}\operatorname{sen}(x+y) = c$

26= $\frac{dy}{dx} = sen(x+y)$ $t = x+y$ $\frac{dt}{dx} = \left(1+\frac{dy}{dx}\right) \rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

Sustituyendo $\frac{dt}{dx} - 1 = sen(t) \rightarrow \frac{dt}{dx} = sen(t) + 1 \rightarrow \frac{dt}{sen(t)+1} = dx$

$dx - \frac{dt}{sen(t)+1} = 0$ Integrando $\underset{①}{\int dx} - \underset{②}{\int \frac{dt}{sen(t)-1}} = \int 0$

① $\int dx = x$ ② $\int \frac{dt}{sen(t)+1} = \int \frac{dt}{sen(t)-1} \cdot \frac{sen(t)+1}{sen(t)+1} = \int \frac{sen(t)+1}{sen^2(t)-1} dt$

$= \int \frac{sen(t)+1\, dt}{-(1-sen^2(t))} = \int \frac{sen(t)+1\, dt}{-cos^2(t)} = -\left(\int sec^2(t) + \int \frac{sen(t)}{cos^2(t)}\right) = -\left(tan(t) + \int tan\, sec\, dt\right)$

$= -tan(t) - \int tan(t)\, sec(t)\, dt = -tan(t) - sec(t)$

Sustituyendo

$x + tan(t) + sec(t) = C$ pero $t = x+y$ $\therefore$ $\underline{x + tan(x+y) + sec(x+y) = C}$

27= $\frac{dy}{dx} = 2 + \sqrt{y-2x+3}$ $t = y-2x+3$ $\frac{dt}{dx} = \frac{dy}{dx} - 2 + 3 \rightarrow \frac{dt}{dx} = \frac{dy}{dx} + 1 \rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

Sustituyendo

$\frac{dt}{dx} - 1 = 2 + \sqrt{t} \rightarrow \frac{dt}{dx} = 3 + \sqrt{t} \rightarrow \frac{dt}{3+\sqrt{t}} = dx \rightarrow dx - \frac{dt}{3+\sqrt{t}} = 0$

Integrando

$\underset{①}{\int dx} - \underset{②}{\int \frac{dt}{3+\sqrt{t}}} = \int 0$

① $\int dx = x$ ② $\int \frac{dt}{3+\sqrt{t}} = \int \frac{2\sqrt{t}\, du}{u} = 2\int \frac{\sqrt{t}+3-3}{u} du = 2\int \frac{u-3}{u} du$

$u = 3+\sqrt{t}$ $\frac{du}{dt} = \frac{1}{2\sqrt{t}} \rightarrow dt = 2\sqrt{t}\, du$

$= 2\left(\int du - \int \frac{3}{u} du\right) = 2(u - 3\ln|u|) = 2\left((3+\sqrt{t}) - 3\ln|3+\sqrt{t}|\right)$

Sustituyendo

$x - 2\left((3+\sqrt{t}) - 3\ \ln(3+\sqrt{t})\right) = C$ pero $t = x+y$

$x - (6 + 2\sqrt{x+y} - 6\ \ln|3+\sqrt{x+y}|) = C$

$\rightarrow x - 6 - 2\sqrt{x+y} + 6\ \ln|3+\sqrt{x+y}| = C$

28.- $\frac{dy}{dx} = 1 + e^{y-x+5}$

$t = y - x + 5$ $\quad \frac{dt}{dx} = \frac{dy}{dx} - 1 \rightarrow \frac{dy}{dx} = \frac{dt}{dx} + 1$

$\frac{dt}{dx} + 1 = 1 + e^{t} \rightarrow \frac{dt}{dx} = e^{t} \rightarrow \frac{dt}{e^{t}} = dx \rightarrow dx - \frac{dt}{e^{t}} = C \quad \int dx - \int \frac{dt}{e^{t}} = C$

$x + \frac{1}{e^{t}} = C$ pero $t = y - x + 5$ $\quad x + \frac{1}{e^{y-x+5}} = C$

29.- $\frac{dy}{dx} = \cos(x+y)$, $y(0) = \frac{\pi}{4}$

$t = x + y \quad \frac{dt}{dx} = 1 + \frac{dy}{dx} \rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

$\frac{dt}{dx} - 1 = \cos(t) \rightarrow \frac{dt}{dx} = \cos t + 1 \rightarrow \frac{dt}{\cos(t)+1} = dx \rightarrow dx - \frac{dt}{\cos(t)+1} = C$

$\int dx - \int \frac{dt}{\cos t + 1} = \int 0$

$\int dx = x \qquad \int \frac{dt}{\cos t + 1} = \int \frac{dt}{\cos(t)+1} \cdot \frac{\cos t - 1}{\cos(t) - 1} = \int \frac{\cos - 1}{\cos^2(t) - 1} = \int \frac{\cos - 1}{-(1 - \cos^2(t))}$

$\int \frac{\cos - 1}{-\operatorname{sen}^2(t)} = \int -\cot(t)\csc(t) + \int \csc^2(t) = \csc(t) - \cot(t)$

Sustituyendo

$x - \csc(t) + \cot t$ pero $t = x + y$

$\therefore x - \csc(x+y) + \cot(x+y) = C$

en $y(0) = \frac{\pi}{4}$

$\frac{\pi}{4} - \csc\left(\frac{\pi}{4} - 0\right) + \cot\left(\frac{\pi}{4} - 0\right) = c$

$\frac{\pi}{4} - \sqrt{2} + 1 = c$ $\quad \underline{x - \csc(x-y) + \cot(x-y) = \frac{\pi}{4} - \sqrt{2} + 1 = c}$

30.- $\frac{dy}{dx} = \frac{3x+2y}{3x+2y+2}$, $y(-1) = -1$

$t = 3x + 2y$ $\quad \frac{dt}{dx} = 3 + 2\frac{dy}{dx} \rightarrow 2\frac{dy}{dx} = \frac{dt}{dx} - 3 \quad \frac{dy}{dx} = \frac{\frac{dt}{dx} - 3}{2}$

$\frac{\frac{dt}{dx} - 3}{2} = \frac{t}{t+2} \rightarrow \frac{dt}{dx} - 3 = \frac{2t}{t+2} \rightarrow \frac{dt}{dx} = \frac{2t}{t+2} + 3$

$\frac{dt}{dx} = \frac{2t + 3(t+2)}{t+2} \rightarrow \frac{dt}{dx} = \frac{5t+6}{t+2} \rightarrow \frac{(t+2)\,dt}{5t+6} = dx \rightarrow dx - \frac{(t+2)\,dt}{5t+6} = 0$

$\underset{①}{\int dx} - \underset{②}{\int \frac{(t+2)\,dt}{5t+6}} = \int 0$

① $\int dx = x$ $\quad$ ② $\int \frac{(t+2)\,dt}{5t+6} = \int \left(\frac{1}{5} + \frac{\frac{4}{5}}{5t+6}\right) dt$

$= \frac{1}{5}\int dt + \frac{4}{5}\int \frac{dt}{5t+6} = \frac{1}{5}t + \frac{4}{5}\cdot\frac{1}{5}\int \frac{dt}{5t+6}$

$= \frac{1}{5}t + \frac{4}{25}\ln|5t+6|$

Sustituyendo

$x - \frac{1}{5}t - \frac{4}{25}\ln|5t+6| = c$ pero $t = (3x+2y)$

$\therefore x - \frac{1}{5}(3x+2y) - \frac{4}{25}\ln|5(3x+2y)+6| = c$

$\underline{x - \frac{3x+2y}{5} - \frac{4}{25}\ln|15x+10y+6| = c}$

$$5x - 3x + 2y - \frac{4}{5}\ln|15x + 10y + 6| = C \cdot 5$$

$$2x + 2y - \frac{4}{5}\ln|15x + 10y + 6| = C$$

$$x + y - \frac{4}{10}\ln|15x + 10y + 6| = C/2$$

$$x + y - \frac{2}{5}\ln|15x + 10y + 6| = C$$

$$\frac{5x}{2} + \frac{5y}{2} - \ln|15x + 10y + 6| = C$$

$$e^{\frac{5x}{2}} + e^{\frac{5y}{2}} - e^{\cancel{\ln}|15x + 10y + 6|} = e^{C}$$

$$= e^{\frac{5}{2}(x+y)} - 15x + 10y + 6 = C$$

en $y(-1) = -1$

$$e^{\frac{5}{2}(-1 + -1)} - 15(-1) + 10(-1) + 6 = C \rightarrow e^{-5} + 15 - 10 + 6 = C$$

$$C = 11 + \frac{1}{e^{5}}$$

$$e^{\frac{5}{2}(x+y)} - 15x + 10y + 6 = 11 + \frac{1}{e^{5}}$$