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Tareas.

17.- $y'' - 2y' + 5y = e^x \operatorname{Cos} 2x \ldots (1)$

Ec. Homo. ... (1)

$y'' - 2y' + 5y = 0$

(1) Ec. Car.

$m^2 - 2m + 5 = 0$

(2) Raíces

$m_{1,2} = \dfrac{2 \pm \sqrt{4 - 4(1)(5)}}{2} = \dfrac{2 \pm \sqrt{-16}}{2} = 1 \pm 2i \quad \alpha = 1;\ \beta = 2;$

(3) Sol. Li

$y_1 = e^x \operatorname{Cos} 2x$

$y_2 = e^x \operatorname{Sen} 2x$

(4) Sol Gral

$y_c = C_1 e^x \operatorname{Cos} 2x + C_2 e^x \operatorname{Sen} 2x$

$F(x) = \operatorname{Cos} 2x\, e^x \rightarrow y_p = ((A \operatorname{Cos} 2x + B \operatorname{Sen} 2x) e^x) x = Axe^x \operatorname{Cos} 2x + Bxe^x \operatorname{Sen} 2x$

$y_p = Axe^x \operatorname{Cos} 2x + Bxe^x \operatorname{Sen} 2x$

$y_p' = Ae^x \operatorname{Cos} 2x + Axe^x \operatorname{Cos} 2x - 2Axe^x \operatorname{Sen} 2x + Be^x \operatorname{Sen} 2x + Bxe^x \operatorname{Sen} 2x + 2Bxe^x \operatorname{Cos} 2x$

$y_p'' = -3Axe^x \operatorname{Cos} 2x - 4Axe^x \operatorname{sen} 2x + 2Ae^x \operatorname{Cos} 2x - 4Ae^x \operatorname{sen} 2x - 3Bxe^x \operatorname{Sen} 2x + 4Be^x x \operatorname{Cos} 2x$
$+ 2Be^x \operatorname{sen} 2x + 4Be^x \operatorname{Cos} 2x.$

Sustituyendo yp y sus derivadas en (1)

$-3Axe^x \operatorname{Cos} 2x - 4Axe^x \operatorname{sen} 2x + 2Ae^x \operatorname{Cos} 2x - 4Ae^x \operatorname{sen} 2x - 3Bxe^x \operatorname{sen} 2x + 4Bxe^x \operatorname{Cos} 2x + 2Be^x \operatorname{sen} 2x$
$+ 4Be^x \operatorname{Cos} 2x$
$- 2Ae^x \operatorname{Cos} 2x - 2Axe^x \operatorname{Cos} 2x + 4Axe^x \operatorname{Sen} 2x - 2Be^x \operatorname{sen} 2x - 2Bxe^x \operatorname{Sen} 2x - 4Bxe^x \operatorname{Cos} 2x$
$+ 5Axe^x \operatorname{Cos} 2x + 5Bxe^x \operatorname{Sen} 2x = e^x \operatorname{Cos} 2x$

$-4Ae^x \operatorname{sen} 2x + 4Be^x \operatorname{Cos} 2x = e^x \operatorname{Cos} 2x$

$4B = 1 \quad \therefore B = \frac{1}{4}$

$-4A = 0 \quad \therefore A = 0$

$y_p = \dfrac{xe^x \operatorname{sen} 2x}{4}$

$y = y_c + y_p$

$\therefore y = C_1 e^x \operatorname{Cos} 2x + C_2 e^x \operatorname{Sen} 2x + \frac{1}{4} x e^x \operatorname{sen} 2x$

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29. Si $y_1 = x^{-1/2}\cos x$ y $y_2 = x^{-1/2}\operatorname{Sen} x$, forman un conjunto fundamental de soluciones de $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$ en $(0, \infty)$, determine la solución General de.

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = x^{3/2}$$

$$G_x = \frac{1}{\sqrt{x}}$$

Solución General: $y_c = C_1 x^{-1/2}\cos x + C_2 x^{-1/2}\operatorname{Sen} x$

Proponemos $y_p = y_1 u_1 + y_2 u_2$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} x^{-1/2}\cos x & x^{-1/2}\operatorname{Sen} x \\ -\operatorname{Sen} x/\sqrt{x} - \cos x/2x^{3/2} & \cos x/\sqrt{x} - \operatorname{Sen} x/2x^{3/2} \end{vmatrix} = \frac{1}{x} \therefore W = \frac{1}{x}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ G(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & x^{-1/2}\operatorname{Sen} x \\ 1/\sqrt{x} & \frac{\cos x}{\sqrt{x}} - \frac{\operatorname{Sen} x}{2x^{3/2}} \end{vmatrix} = -\frac{\operatorname{Sen} x}{x} \therefore W_1 = -\frac{\operatorname{Sen} x}{x}$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & G(x) \end{vmatrix} = \begin{vmatrix} x^{-1/2}\cos x & 0 \\ y_1' & \frac{1}{\sqrt{x}} \end{vmatrix} = \frac{\cos x}{x} \therefore W_2 = \frac{\cos x}{x}$$

$$u_1 = \int \frac{W_1}{W}dx = \int -\frac{\frac{\operatorname{Sen} x}{x}}{\frac{1}{x}} = \int -\operatorname{Sen} x\, dx = \cos x$$

$$u_2 = \int \frac{W_2}{W}dx = \int \cos x\, dx = \operatorname{Sen} x$$

Sustituyendo y_1, y_2, u_1, u_2

$$y_p = \left((x^{-1/2}\cos x)(\cos x) + (x^{-1/2}\operatorname{Sen} x)(\operatorname{Sen} x)\right) = \frac{\cos^2(x)}{\sqrt{x}} + \frac{\operatorname{Sen}^2 x}{\sqrt{x}} = \frac{1}{\sqrt{x}} = x^{-1/2}$$

$y = y_c + y_p$

$$\therefore y = C_1 x^{-1/2}\cos x + C_2 x^{-1/2}\operatorname{Sen} x + x^{-1/2}$$

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30.- Si $y_1 = Cos(In\,x)$ y $y_2 = Sen(In\,x)$ son soluciones conocidas linealmente independientes de $x^2y'' + xy' + y = 0$, en $(0, \infty)$ determine la solución particular de

$$x^2y'' + xy' + y = sec(In\,x)$$

Solución General $y_c = C_1 Cos(In\,x) + C_2 Sen(In\,x)$

Proponemos $y_p = y_1 u_1 + y_2 u_2$ $\qquad G_x = \frac{sec(In\,x)}{x^2}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} Cos(In\,x) & Sen(In\,x) \\ \frac{-Sen(In\,x)}{x} & \frac{Cos(In\,x)}{x} \end{vmatrix} = \frac{1}{x} \quad \therefore \; W = \frac{1}{x}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ G(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & Sen(In\,x) \\ \frac{sec(In\,x)}{x^2} & y_2' \end{vmatrix} = -\frac{Tan(In\,x)}{x^2} \quad \therefore \; W_1 = -\frac{Tan(In\,x)}{x^2}$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & G_x \end{vmatrix} = \begin{vmatrix} Cos(In\,x) & 0 \\ y_1' & \frac{sec(In\,x)}{x^2} \end{vmatrix} = \frac{1}{x^2} \quad \therefore \; W_2 = \frac{1}{x^2}$$

$$u_1 = \int \frac{W_1\,dx}{W} = \int \frac{-\frac{Tan(In\,x)\,dx}{x^2}}{\frac{1}{x}} = -\int \frac{Tan(In\,x)}{x}dx = -\int Tan(u)\,du = -\int \frac{Sen(u)}{Cos(u)}du$$

$u = In\,x$ $\qquad v = Cos\,u$

$du = \frac{1}{x}dx$ $\qquad dv = -Sen\,u\,du$

$$= -\left(-\int \frac{dv}{v}\right) = In|v| = In|Cos\,u| = In|Cos(In\,x)|$$

$$u_2 = \int \frac{W_2}{W}dx = \int \frac{\frac{1}{x^2}dx}{\frac{1}{x}} = \int \frac{1}{x}dx = In|x|$$

Sustituyendo y_1, y_2, u_1, u_2

$$y_p = (Cos(In\,x))(In|Cos(In\,x)|) + Sen(In\,x)(In|x|)$$

$$\underline{y_p = (Cos\,In\,x)(In|Cos\,In\,x|) + (Sen\,In\,x)\,In|x|}$$ Solución Particular.