AE2111-II Aircraft Report

Calculations on the Fokker F100

Group 51

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AE2111-II: Systems Design

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1 Airfoil analysis

In order to calculate the MAC (mean aerodynamic chord) of the aircraft, the root cord C_r must be first obtained. To calculate this, the wing is assumed to be of a perfectly trapezoidal shape. The taper ratio λ, surface area S and wing span b of the aircraft are known to be 0.235, 93.5 m^2 and 28.08 m, respectively. These values are related to the root chord C_r and tip cord C_t with Equation 1.1.

$$S = b \cdot \left(\frac{C_r + C_t}{2}\right) \qquad \lambda = \frac{C_t}{C_r} \tag{1.1}$$

These equations are combined in Equation 1.2 to get the length of the root cord.

$$S = b \cdot \left(\frac{C_r + (\lambda \cdot C_r)}{2}\right) \rightarrow C_r = 2 \cdot \frac{S}{b} \cdot \frac{1}{1+\lambda} = 2 \cdot \frac{93.5}{28.08} \cdot \frac{1}{1+0.235} = 5.39 \qquad [m] \tag{1.2}$$

With C_r known, it is possible to calculate the C_{MGC}, or the mean geometric cord with Equation 1.3

$$C_{MAC} = \frac{2}{3} \cdot C_r \cdot \left(\frac{1+\lambda+\lambda^2}{1+\lambda}\right) = \frac{2}{3} \cdot 5.39 \cdot \left(\frac{1+0.235+0.235^2}{1+0.235}\right) = 3.75 \qquad [m] \tag{1.3}$$

An additional assumption made is that the $C_{MGC} \approx C_{MAC}$, therefore the value of C_{MAC} is also 3.75 m

To calculate the span-wise location of the MAC, the approximation is made, that the span-wise location of the mean geometric chord is also the span wise location of the MAC. The equation below is used to calculate its position.

$$y_{MGC} = \left(\frac{b}{6}\right) \cdot \left(\frac{1+2\lambda}{1+\lambda}\right) = \left(\frac{28.08}{6}\right) \cdot \left(\frac{1+2 \cdot 0.235}{1+0.235}\right) = 5.57 \qquad [m] \tag{1.4}$$

Therefore the the span-wise location of the MAC is 5.57 m. Using the MAC it is possible to calculate the the mean Reynolds number of the wing at both take-off conditions and cruise conditions. The curse speed of the aircraft is known, while the take off speed is unknown there for it has to be calculated.

To calculate the take off speed depends with the C_l of the air craft during take-off. It is known that the Fokker F100 is a twin engine jet which typically have a take-off C_l in the range of 1.6-2.2, therefor it was assumed the take-off CL of 1.8. By knowing the maximum take-off mass M_{MTOW} as a worst case, the weight can be calculated as follows.

$$W = M_{MTOW} * g = 43090 * 9.80665 = 4.2257 \cdot 10^5 \tag{1.5}$$

It is assumed that the weight of the aircraft is equal to the lift force produced by the wings of the aircraft, therefor $L = 4.2257 \cdot 10^5$. The wing area S is given and the air dencity ρ is assumed to the sea level.

$$L = C_l \cdot \frac{1}{2} \cdot \rho \cdot V^2 \cdot S \rightarrow V = \sqrt{\frac{2 \cdot L}{C_l \cdot \rho \cdot S}} = \sqrt{\frac{2 \cdot 4.2257 \cdot 10^5}{1.8 \cdot 1.225 \cdot 93.5}} = 64 m/s \quad (1.6)$$

Table 1.1: Conditions for take-off and cruise

	Take-off	Cruise
Altitude $[ft]$	0	35000
Velocity $[kt]$	124.5	414

The units in the data need to be converted to metric units. This is done by multiplying the quantities in ft by a factor of 0.3048 to obtain values in m, and multiplying values in kt by a factor of 0.5144 to convert them to ms^{-1}. The results of the conversion is shown in Table 1.2

Table 1.2: Conditions for landing and cruise

	Take-off	Cruise
Altitude $[m]$	0	10668
Velocity $[m \cdot s^{-1}]$	64	213.0

Now with the given altitude it is possible to calculate the density at that altitude assuming a standard atmosphere calculation as the altitude is below 11 km (the upper limit of the troposphere). It is reasonable to assume a constant temperature gradient from sea level to the cruise altitude of -6.5×10^{-3} $°C \cdot m^{-1}$. To calculate the density at 10.668 km Equation 1.7 is used assuming a ground temperate of 15 $°C$.

$$\begin{aligned} \rho_0 \cdot \left(\frac{T_0}{T_0 + \Delta T \cdot (h_1 - h_0)} \right)^{1+\frac{g_0 \cdot M}{R^* \cdot \Delta T}} &= \rho_1 \rightarrow \\ 1.225 \cdot \left(\frac{288.15}{288.15 - 0.0065 \cdot (10668 - 0)} \right)^{1+\frac{9.80665 \cdot 0.0289644}{8.31 \cdot -0.0065}} &= 0.3796 \end{aligned} \quad [kg \cdot m^{-3}] \quad (1.7)$$

The μ at these altitudes was taken from the adsee formula sheet at 10,000 m of altitude.

$$Re = \frac{\rho \cdot V \cdot MAC}{\mu} \quad [-] \quad (1.8)$$

Equation 1.8 gives the Reynolds number in terms of local air density ρ, the wing's air velocity V, the length of the MAC, and dynamic viscosity μ. Therefore, the Reynolds number during cruise is the following:

$$Re = \frac{0.3796 \cdot 213 \cdot 5.57}{0.00001434} = 2.516 \cdot 10^7 \quad [-] \quad (1.9)$$

The Reynolds number during take-off happens at sea level conditions, so $\rho = 1.225 kg \cdot m^{-3}$

$$Re = \frac{1.225 \cdot 64 \cdot 5.57}{0.0000179} = 2.4396 \cdot 10^7 \quad [-] \quad (1.10)$$

1.1. 1a

The program used to simulate the NACA 64-212 was JavaFoil with the setting used to make the geometry of the airfoil summeried in Table 1.3

Table 1.3: Airfoil Geometry

Family	NACA 6-digit
Number of points [-]	61
Thickness (t/c) [%]	12
Thickness Location (xt/c) [%]	40
Design Lift Coefficient C_l [-]	0.2
a [-]	1
A-Modification [-]	0
Modify NACA section to have closed trailing edge	TRUE

1.2. 1b

The characteristic values of the airfoil at the two conditions is summarised in Table 1.4 using Javafoil assuming a smooth surface.

Table 1.4: Airfoil Analysis (Note: Values in ° are rounded to the nearest $0.2°$

Condition	Take-off	Cruise
Re [-]	$2.44 \cdot 10^7$	$2.52 \cdot 10^7$
$C_{dmin}[-]$	0.00417	0.00444
$C_{lmax}[-]$	1.210	1.212
$C_{l\alpha}[-]$	0.115	0.115
$\alpha_0[°]$	-1.8	-1.8
$\alpha_s[°]$	11.6	11.6

1.3. 1c

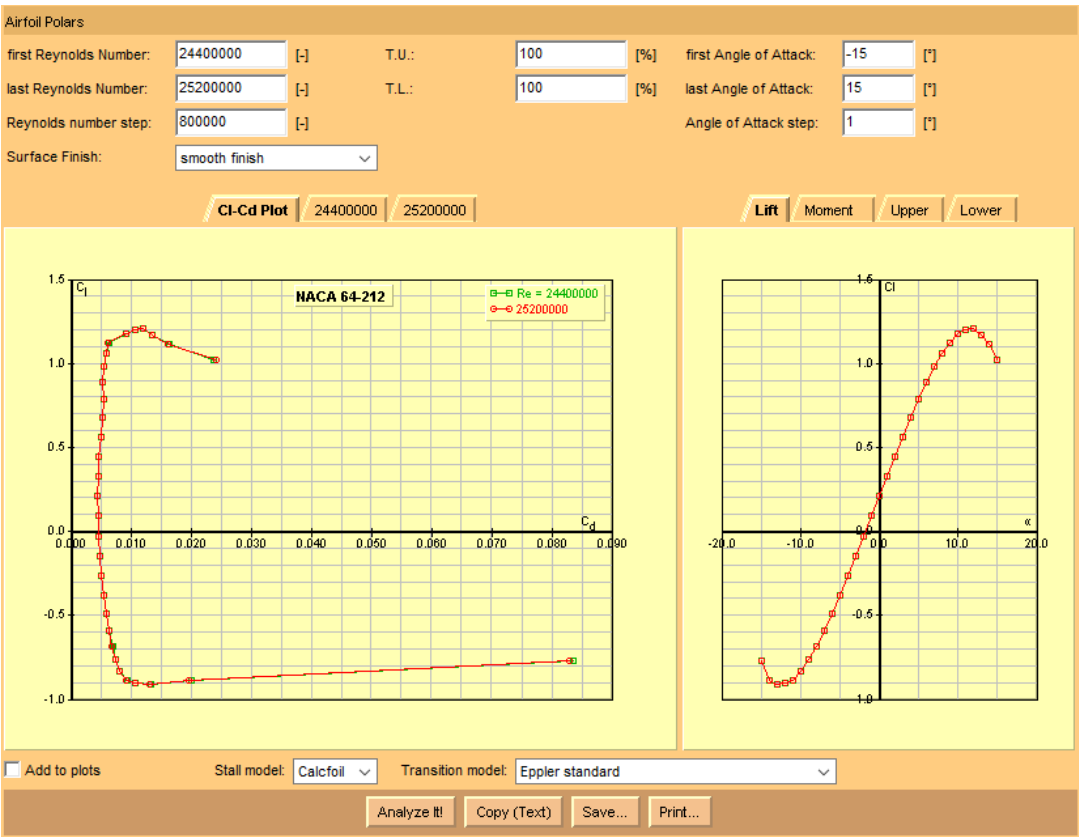

Figure 1.1: NACA 64-212 Lift drag polar (left) and lift curve (right)

1.4. 1d

From the results obtained from the exercise seem quite plausible. By comparing the results to the NACA 64-210 (a slightly thinner airfoil) form the book Introduction to Flight By John D. Anderson Jr (Note: the Re number of the air-foil is $9.0 \cdot 10^6$ at the highest value stated and will be used in comparison). There is a large amount of similarity in the shape of the lift curve, although the NACA 64-210 has a steeper drop-off after stall. In terms of values they are also similar where the NACA 64-210 stall and zero-lift angle occur at approx 13° and $-1°$ respectively. It is therefor possible to say that the results are realistic in terms of lift curves with-in the range of the stall angles.

However there is a significant difference in terms of lift drag polar where the results from Javafoil has approximately straight line running from -0.5 C_l to 1 C_l, the NACA 64-210 has a different shape. The NACA 64-210 airfoil has a strong parabola shape though out the entire range of values of C_l, with a noticeable valley between 0.1 to 0.4 C_l. Yet, comparing C_{dmin} of both yield similar results. It is possible to conclude that the shape and values of the lift-drag polar are not repressive of reality outside of the C_{dmin}.

2
HLD design

2.1. Generating required maximum wing lift coefficient

Considering landing conditions, the aircraft weight must first be calculated. It is given, that $MTOM = 43090kg$, $OEM = 24593kg$, and $M_{fuel} = 8832kg$. To convert those values to weights, they are multiplied by $9.81m/s^2$, which results in $MTOW = 422712.9N$, $OEW = 241257.33N$, and $W_{fuel} = 86641.92N$. Given that data, it is possible to calculate the weight of the aircraft after it has burned 80% of its fuel:

$$W_{Land} = MTOW - 0.8 \cdot W_{fuel} = 422712.9 - 0.8 \cdot 86641.92 = 353399.4 \qquad [N] \quad (2.1)$$

Then, assuming that $1.1W = L$, the following equation can be obtained, where the extra 10% compensates for the negative lift contribution generated by the tail to trim the aircraft:

$$1.1 \cdot W = 0.5 \cdot \rho \cdot V^2 \cdot S \cdot C_L \quad (2.2)$$

This can be rearranged to obtain an equation for the required $C_{L_{max}}$ for landing:

$$C_{L_{max}} = \frac{2 \cdot 1.1 \cdot W_{Land}}{\rho \cdot V^2 \cdot S} = \frac{2 \cdot 1.1 \cdot 353399.4}{1.225 \cdot 65.8^2 \cdot 93.5} = 1.63 \qquad [-] \quad (2.3)$$

From Chapter 1 it is known that $C_{l_{max}} = 1.210$ in landing conditions, which is when flaps are deployed. Converting the airfoil lift coefficient to the wing lift coefficient, so accounting for sweep,

$$C_{L_{max,clean}} = C_{l_{max}} \cdot cos^2(\Lambda) = 1.210 \cdot cos^2(17.45) = 1.10 \qquad [-] \quad (2.4)$$

To achieve this, a $\Delta C_{L_{max}}$ of $1.63 - 1.10 = 0.53$ is needed. The following section describes a method for calculating the maximum $\Delta C_{L_{max}}$.

2.2. Calculating the increase of maximum wing lift coefficient provided by Fowler flaps

The y-positions of ends of the Fowler flaps are constrained by $y = 0.10b/2$ and $y = 0.64b/2$. The x-positions of retracted flaps are also constrained by the trailing edge of the wing and the rear spar, which cannot be penetrated for structural reasons. The rear spar is assumed to be at $x = 0.6c$. Between the leading edge of the retracted flap and the rear spar there must be space for the flap mechanism, so the leading edge of the retracted flaps is positioned at $x = 0.65c$, which means that $\frac{c_f}{c} = 0.35$, where c_f is the chord of the flap.

To obtain the $\Delta C_{l_{max}}$ that the flaps create, first a value of $\frac{c'}{c}$ is needed. It is dependent on flap type and deflection. A graph in ADSEE presentation 3 shows that for Fowler flaps at a deflection of 40 degrees, $\frac{c}{c_f} = 0.6$. Using this, a ratio of chords of landing and clean configurations can be calculated:

$$\frac{c'}{c} = \frac{c+\Delta c}{c} = \frac{c}{c} + \frac{\Delta c}{c_f} \cdot \frac{c_f}{c} = 1 + 0.6 \cdot 0.35 = 1.21 \qquad [-] \quad (2.5)$$

For Fowler flaps:

$$\Delta C_{l_{max}} = 1.3 \cdot \frac{c'}{c} = 1.3 \cdot 1.21 = 1.573. \qquad [-] \quad (2.6)$$

To transform that result to $\Delta C_{L_{max}}$, the additional needed parameters are $\frac{Swf}{S}$, where Swf is the area of the wing that experiences the ΔC_l from the flaps, and $cos(\Lambda_{LE_{flap}})$, which is the cosine of the sweep angle of the leading edge of the flaps. The latter value is calculated as follows:

$$\Lambda_{LE} = tan^{-1}(tan(\Lambda_{c/4}) + \frac{x}{c} \cdot \frac{2 \cdot C_r \cdot (1-\lambda)}{b}) = tan^{-1}(tan(17.45) + \frac{2 \cdot 5.39 \cdot (1-0.235)}{4 \cdot 28.08}) = 21.2[deg] \quad (2.7)$$

Here, C_r was taken from Equation 1.2

$$\Lambda_{LE_{flap}} = tan^{-1}(tan(\Lambda_{LE}) - \frac{x}{c} \cdot \frac{2 \cdot C_r \cdot (1-\lambda)}{b}) = tan^{-1}(tan(21.2) - 0.65 \cdot \frac{2 \cdot 5.39 \cdot (1-0.235)}{28.08}) = 11.1[deg] \quad (2.8)$$

Then, Swf can be calculated for one half of the wing:

$$0.5 \cdot Swf = \int_{0.1 \cdot b/2}^{0.64 \cdot b/2} c(y) \cdot dy \qquad c(y) = C_r - y \cdot \frac{\mathrm{d}c}{\mathrm{d}y} \quad (2.9)$$

To solve this, $c(y)$ must be obtained:

$$c(\frac{b}{2}) = C_r - C_r \cdot \lambda = C_r \cdot (1-\lambda) = \frac{b}{2} \cdot \frac{\mathrm{d}c}{\mathrm{d}y} => \frac{\mathrm{d}c}{\mathrm{d}y} = \frac{2 \cdot C_r \cdot (1-\lambda)}{b} = 0.2938 \quad (2.10)$$

Finally, c in terms of y is in this form:

$$c(y) = 5.39 - 0.2938 \cdot y \quad (2.11)$$

Now, to solve for Swf:

$$Swf = 2 \cdot \int_{0.1 \cdot 28.08/2}^{0.64 \cdot 28.08/2} (5.39 - 0.2938y) \cdot dy = 58.62 \qquad [m^2] \quad (2.12)$$

Now, with all needed components known, $\Delta C_{L_{max}}$ can be calculated:

$$\Delta C_{L_{max}} = 0.9 \cdot \Delta C_{l_{max}} \cdot \frac{Swf}{S} \cdot cos(\Lambda_{LE_{flap}}) = 0.9 \cdot 1.573 \cdot \frac{58.62}{93.5} \cdot cos(11.1) = 0.87 \quad [-] \quad (2.13)$$

Therefore, this $\Delta C_{L_{max}}$ provided by Fowler flaps is larger than required. The requirement of providing a $\Delta C_{L_{max}}$ of at least 0.53 is met, so no modifications to the system are required. After all, a $C_{L_{max}}$ higher than required means lower speeds while landing and taking off, therefore shorter landing and taking off distances and lower wear of tires and undercarriage. Such an aircraft can also be certified to land on shorter runways. However, if a producer would prefer to minimise weight, they can reduce both $\Delta C_{L_{max}}$ and weight of the system by one of the following methods:

- Use simpler flaps, for example slotted flaps.
- Use a combination of flaps and slats, which can be combined with the previous point.
- Decrease the y-direction width of the flap, and therefore require fewer hinges and mechanisms, making the system lighter.

3
Roll characteristics

The F100 is a medium-sized aircraft which puts it into the category 2 for the roll characteristics: it has to be able to roll 45 degrees in 1.4 seconds. From chapter 1 the $c_{l\alpha} = 0.115deg^{-1} = 6.59rad^{-1}$ $S_{ref} = 93.5, b = 28.08$

3.1. Control derivative and roll damping coefficient

Aileron effectiveness τ from the table is 0.47 for the 0.25

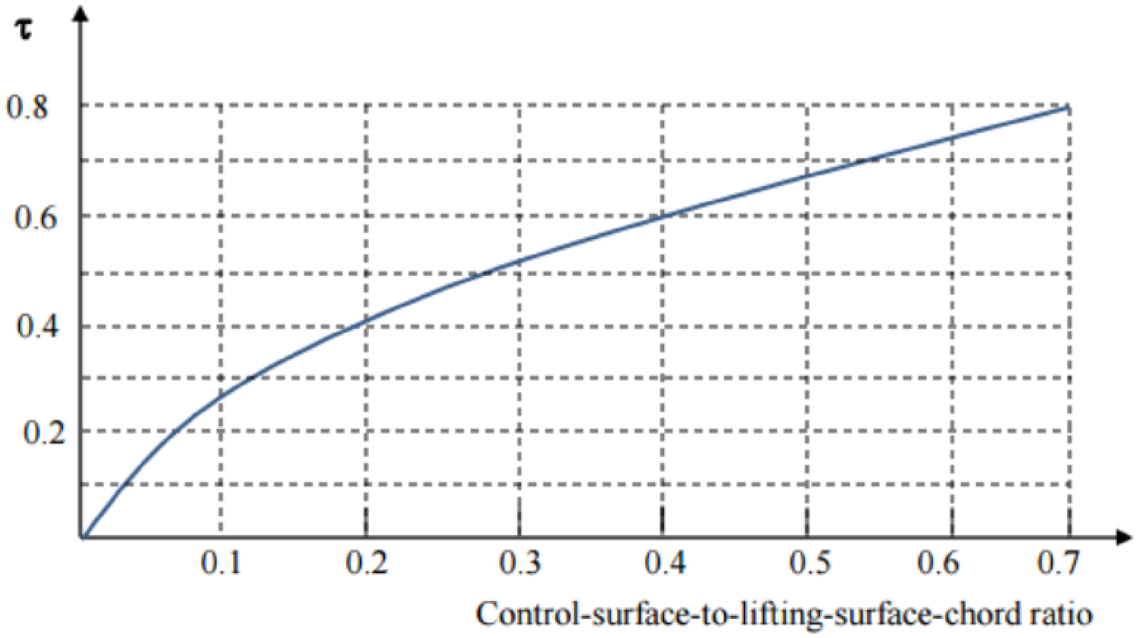

Figure 3.1: Aileron effectiveness related to the aileron chord to chord ratio

From the "Derivation of the C_{l_p} equation" document:

$$c(y) = C_R \cdot \left(1 + \frac{2 \cdot (\lambda - 1) \cdot y}{b}\right) \tag{3.1}$$

This allows to solve the equation for the aileron control derivative with the boundaries of integration being between the beginning and the end of the aileron:

$$C_{l_{\delta\alpha}} = \frac{2 \cdot \tau \cdot c_{l_\alpha}}{S_{ref} \cdot b} \int_{\frac{b \cdot 0.65}{2}}^{\frac{b}{2}} c(y) \cdot y \cdot dy \;= \frac{2 \cdot 0.47 \cdot 0.115}{93.5 \cdot 28.08} \cdot \left[\frac{y^2 \cdot C_R}{2} + \frac{2 \cdot (\lambda - 1) \cdot y^3 \cdot C_R}{b \cdot 3}\right]_{9.126}^{14.04} \tag{3.2}$$

$$C_{l_{\delta\alpha}} = 0.260 \tag{3.3}$$

Roll damping coefficient:
As described in the ”Derivation of the C_{l_P} equation”,

$$C_{l_P} = -\frac{(c_{l_\alpha} + c_{d_0}) \cdot C_R \cdot b \cdot (1 + 3 \cdot \lambda)}{24 \cdot S} \tag{3.4}$$

$$C_{l_P} = -\frac{(6.59 + 0.005) \cdot 5.39 \cdot 28.08 \cdot (1 + 3 \cdot 0.235)}{24 \cdot 93.5} = -0.7584 \tag{3.5}$$

3.2. roll rate at the most demanding condition

$$P = -\frac{C_{l_{\delta\alpha}}}{C_{l_P}} \cdot \delta\alpha \cdot \left(\frac{2 \cdot V}{b}\right); \delta\alpha = 0.5 \cdot (\delta\alpha_{up} + \delta\alpha_{down}) = 0.5 \cdot (25 + 20) = 22.5deg = 0.393rad \tag{3.6}$$

the smallest roll rate is achieved at the approach speed: V = 128 knt = 65.8 m/s

$$P = -\frac{0.260 \cdot 0.393 \cdot 2 \cdot 65.8}{-0.7584 \cdot 28.08} = 0.63rad/s = 36deg/s \tag{3.7}$$

3.3. required modifications

36 deg/sec gives 50.6 degrees in 1.4 seconds, which satisfies the roll rate requirements. No modifications needed

4

Calculation of the wing weight and design iteration

In this chapter, the effect of increase in n_{ult} to the MTOW is considered.

4.1. The initial wing weight

Firstly, initial weight before increasing n_{ult} is calculated. The wing weight is given by Equation 4.1.

$$W_w = (k_w b_s^{0.75}(1 + \sqrt{\frac{b_{ref}}{b_s}})n_{ult}^{0.55}(\frac{b_s/t_r}{ZFW/S})^{0.30})ZFW \tag{4.1}$$

The following constants are given:
$MTOW = 43090\ kg,\ W_{fuel} = 8832\ kg,\ b = 28.08\ m,\ b_{ref} = 1.905\ m,\ S = 93.5\ m^2$
$\Lambda_{0.25} = 17.45°,\ \lambda = 0.235,\ \frac{t}{c} = 0.12,\ k_w = 6.67 \times 10^{-3},\ C_r = 5.39\ m$ (Equation 1.2)

Other values are calculated as following:

$$tan\Lambda_{\frac{x}{c}} = tan\Lambda_{LE} - \frac{x}{c}\frac{2C_r}{b}(1-\lambda) \rightarrow tan\Lambda_{LE} = tan\Lambda_{0.25c} + 0.25\frac{2C_r}{b}(1-\lambda)$$

$$\begin{aligned}\Lambda_{0.5c} &= atan(tan\Lambda_{LE} + 0.5\frac{2C_r}{b}(1-\lambda) \\ &= atan(tan\Lambda_{0.25c} + 0.25\frac{2C_r}{b}(1-\lambda) + 0.5\frac{2C_r}{b}(1-\lambda)) \\ &= atan(tan(17.45°) + 0.75\frac{2 \cdot 5.39}{28.08}(1-0.235) \\ &= 0.419 \qquad [rad]\end{aligned} \tag{4.2}$$

$$b_s = b/cos(\Lambda_{0.5c}) = 28.08/cos(0.498) = 31.96 \qquad [m^2] \tag{4.3}$$

$$n_{max} = 2.1 + (24000/(W_{to} + 10000)) = 2.1 + (24000/(43090 \cdot 0.4536 + 10000)) = 2.912$$
$$n_{ult} = 1.5n_{max} = 1.5 \cdot 2.912 = 4.368 \tag{4.4}$$

$$t_r = C_r \cdot \frac{t}{c} = 5.39 \cdot 0.12 = 0.647 \tag{4.5}$$

$$ZFW = MTOW - M_{fuel} = 43090 - 8832 = 34258 \qquad [kg] \tag{4.6}$$

Finally, inserting given constants and obtained values to Equation 4.1, initial wing weight can be calculated as in Equation 4.7.

$$W_w = 6.67 \cdot 10^{-3} \cdot 31.96^{0.75} \cdot (1 + \sqrt{\frac{1.905}{31.96}}) \cdot 4.368^{0.55} \cdot (\frac{31.96/0.647}{34258/93.5})^{0.30}) \cdot 34258 = 4697[kg] \quad (4.7)$$

4.2. Increase n_{ult} by 12%

Here, MTOW is re-calculated under the condition where n_{ult} is increased by 12%. This process is not so simple. When load factor is increased, wing weight is increased as easily seen in Equation 4.1. Increase of wing weight directly increases MTOW. Then, since MTOW is increased, ZFW is increased accordingly. Increase of ZFW affects wing weight, so wing weight changes again. This iterating calculation is carried out below until MTOW is converged. For simplicity, increase of fuel weight due to increase of wing weight is neglected.

First of all, the new n_{ult} becomes 1.12 times bigger than the one calculated in Equation 4.4.

$$n_{ult} = 4.368 \cdot 1.12 = 4.892 \quad (4.8)$$

Using new n_{ult}, the wing weight is calculated again by Equation 4.1.

$$W_w = 6.67 \cdot 10^{-3} \cdot 31.96^{0.75} \cdot (1 + \sqrt{\frac{1.905}{31.96}}) \cdot 4.368^{0.55} \cdot (\frac{31.96/0.647}{34258/93.5})^{0.30}) \cdot 34258 = 4999[kg] \quad (4.9)$$

Since the aircraft weight except the wing part is assumed to be constant, the new MTOW is calculated by adding increase of W_w to original MTOW.

$$MTOW = 43090 + (4999 - 4697) = 43392 \quad [kg] \quad (4.10)$$

Therefore, the new ZFW becomes:

$$ZFW = MTOW - W_fuel = 43392 - 8832 = 34560 \quad [kg] \quad (4.11)$$

Using the new ZFW, W_w is re-calculated again:

$$W_w = 6.67 \cdot 10^{-3} \cdot 31.96^{0.75} \cdot (1 + \sqrt{\frac{1.905}{31.96}}) \cdot 4.368^{0.55} \cdot (\frac{31.96/0.647}{34560/93.5})^{0.30}) \cdot 34560 = 5029[kg] \quad (4.12)$$

The process done in Equation 4.9 to Equation 4.12 is iterated until the difference after one more iteration becomes smaller than $10^{-3}[kg]$. As a conclusion, MTOW converged at 43426 kg. The overview of the evolution is shown in the table Table 4.1 and the graph Figure 4.1.

# iteration	0 (initial)	1	2	3	4	5	6	7
W_w [kg]	4696.5	4998.6	5029.4	5032.5	5032.8	5032.9	5032.9	5032.9
MTOW [kg]	43090	43392	43422	43426	43426	43426	43426	43426
ZFW[kg]	34258	34560	34591	34594	34594	34594	34594	34594

Table 4.1: W_w and MTOW at each iteration

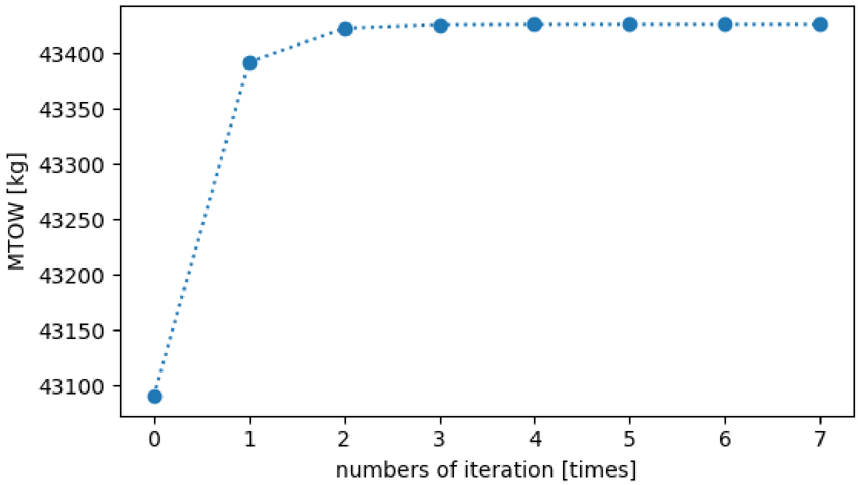

Figure 4.1: MTOW evolution at each iteration